Optical Frequency Modulated Continuous Wave (FMCW) Range and Velocity Measurements

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To all my friends and relatives...
Today a number of different optical techniques capable of measuring range, velocity or both exist. With recent advancements in laser diode manufacturing, small tunable sources have become available. By modulating current supplied to the laser the optical output frequency can be modulated. If the outgoing modulated light is divided into two parts, a transmitted and a local oscillator part, and a photodiode is used as detector, the frequency difference between the two parts can be obtained directly from the photodiode current. This frequency difference is often referred to as the beat frequency. If triangular frequency modulation is used, both range and radial velocity can be determined from the measured beat frequency. The common name for this technique, which was first implemented using regular radar, is frequency modulated continuous wave (FMCW).

The thesis consists of an introduction and five papers. Paper one is a theoretical investigation where contributions from different noise sources are discussed. We have investigated how the signal to noise ratio for the photodiode is affected by how the optical power is divided between the transmitted part and the local oscillator part.

Paper number two introduces a new modulation scheme that avoids ambiguity problems resulting from a Doppler shift larger than the frequency shift associated with the range. As a result of this new modulation scheme other benefits are also gained. The modulation scheme was tested and verified in our lab system built with a tunable laser diode and a fiber optic coupler.

Paper number three presents a single stage OP-amp solution suited for an FMCW system. Our circuit combines a high gain in the desired frequency region with minimal gain at dc, without using any inductors. The risk of saturation or clipping due to the local oscillator can thereby greatly be reduced. Inserting a high pass filter between the photodiode and the first amplifier stage, to remove the dc level, is often not practical when using a regular current to voltage converter. A cascade of two stages, with a high pass filter between the two, is therefore commonly used. Our solution has similar performance as the cascade solution, but since it uses only one OP-amp, it is less sensitive to external disturbances.

Paper number four and five deal with a common problem in an FMCW systems where the optical frequency is used as the carrier wave, and the modulation is obtained by modulating a current to the laser. For optimum performance, the frequency sweep should be as linear as possible, but due to thermal effects, a linear current ramping seldom results in a linear frequency ramping. At lower modulation frequencies, below 1 MHz, the temperature behaviour of the laser has a large influence on the frequency behaviour. To model the frequency behaviour we hence started by investigating the thermal behaviour. This work is presented in paper four. The goal is to obtain a model for the frequency...
behaviour with respect to changes in the laser drive current. In paper five, this model has been tweaked a bit and is used to obtain modulation currents that gives a linear frequency ramping.
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To this day I have spent close to ten years in Luleå. First as a regular student at the Electrical Engineering Program, and now as a Ph.D. student. At the end of my master studies I was sure of two things. I wanted to stay in Luleå and I wanted to work and develop myself within the field of electronics. When the opportunity to do my master thesis at the university, and with a possibility to continue as a Ph.D. student presented itself, I jumped at it. There are many people worth mentioning here, but I will keep it short. Only a few of you are mentioned by name, but the rest of you have not been forgotten. First I would like to thank my supervisor Kalevi Hyyppä for his guidance. Secondly, I would like to thank Johan Carlson for the thesis template and also send a greeting to all the rest of you at EISLAB and the Department of Electrical Engineering and Computer Science. And last but not least, a special thank you to my girl friend Ann-Catrin, mom, dad, the rest of my family, relatives and friends.
Part I
Chapter 1

Thesis Introduction

1.1 Background

Most people with a drivers licence, and especially previous owners of one, are probably aware of that the police uses laser light to measure the speed of cars. Many boat owners and fishermen are likely familiar with instruments that can detect large quantities of fish and measure the distance to the sea bottom. In war movies, we can often see warships or air planes use sophisticated equipment to detect and track enemy vessels. During the Second World War, most battle ships were equipped with active sonar. The active sonar transmits a very wide sound pulse. If the sound pulse makes contact with a submarine, the sound will echo back to the sender. The direction of the submarine can be determined from the direction of the pulse echo, and the distance to the submarine can be calculated from the delay time of the echo.

As we strive to make more things automatic, the need for cheaper, less power consuming, more accurate and more reliable distance and velocity measuring equipment increases. Instead of relying on a human to monitor and control the fluid level in a tank, a range sensor and a computer can do the job. For an unmanned vehicle to be able to navigate in an unknown environment, it has to be able to sense that environment by measuring the distance to nearby objects. Range measurements can also be used to improve safety. One typical example of this is collision avoidance between cars. By measuring the distance to the car in front of you, your car could automatically give a warning, or even slow down, if the distance decreases too fast.

Range and velocity measurements are often performed by transmitting some form of energy towards the target and monitor the returned signal. Both sound waves [1] and electromagnetic waves can be used. Electromagnetic waves travels with the speed of light, which is approximately equal to $3 \cdot 10^8$ m/s in free space. The speed is effected by the propagation mediums refractive index. The speed of sound depends on both medium and temperature, and is around 340 m/s in air at room temperature. Sound with a frequency above 20 kHz is referred to as ultrasound.

An electromagnetic wave with a wavelength between 1 nm and 1 mm is normally referred to as an optical wave, while wavelengths between 1 mm and 0.1 m are referred to as microwaves. A ranging system utilizing microwaves is often referred to as a Radar...
[2], where Radar stand for "radio detection and ranging". A ranging system using optical waves is often called Lidar or Ladar, where Lidar stands for "light detection and ranging" and Ladar for "laser detection and ranging".

The same measuring principles can often be used in both radar and lidar systems. For a recent review of different laser ranging techniques see [3]. One common ranging method is the so-called time of flight method, where a pulse is transmitted towards a target and the return time of the pulse echo is measured. Due to the high speed of electromagnetic waves, accurate distance measurement requires high performance electronics with high time resolution.

A second ranging method is the so-called frequency modulated continuous wave (FMCW) method [4]. In FMCW, the outgoing wave is frequency modulated and the frequency difference between the reflected wave and a local oscillator wave is measured. From the measured frequency difference, often referred to as beat frequency, it is possible to determine the distance to the target. One advantage with the FMCW method is that short distance measurements can be performed with relatively low detector bandwidth, since the beat frequency for a given distance depends on the modulation parameters and the modulation parameters can be selected to fit a certain application.

As the speed of sound is much lower than the speed of light, a time of flight system using sound requires less time resolution compared to an optical or microwave system. However, since ultrasound has much longer wavelength than light, it is harder to focus an ultrasound wave than an optical wave. The far-field half-angle beam divergence $\theta_{FF}$ is given by [5]

$$\theta_{FF} = \frac{\lambda}{\pi w_0},$$

where $\lambda$ is the wavelength and $w_0$ the radius of the beam waist. If we have an ultrasound transmitter with a wavelength of 5 mm, and a laser source with a wavelength of 1500 nm, to obtain the same $\theta_{FF}$ as the laser beam, the ultrasound systems requires a beam waist 3333 times larger than the optical system. Using a narrow and focused beam is an advantage in scanning applications, as it makes it easier to resolve finer surface details and achieve high angular resolution. The wavelength of ultrasound can often be large compared to the irregularities of the reflecting surfaces, and hence the surface reflections can often be specular. If the reflection is specular the first back reflection might miss the detector. Due to the short wavelength of light, even a smooth wall surface will give a diffuse reflection instead of a specular one. One approach to solve these problems with ultrasound is to use an array of transmitters and receivers as reported in [6]. Another drawback with sound is the temperature dependence of the speed of sound.

Non-metallic surfaces, like wood and rock, generally produce very weak reflections at radar frequencies, and some materials may even produce no detectable reflection at all. Depending on the application, this can be either an advantage or disadvantage. For indoor applications, a wavelength around 1 mm works reasonably well, as can be seen in [7]. But an optical system still has the advantage that it is easier to obtain a narrow and focused beam, and an optical system also has fewer problems with specular reflections. For these reasons, optical ranging systems are well suited for indoor navigation applications.
1.2 Motivation

Eye safety is an important aspect when dealing with optical systems. The human eye is sensitive to wavelengths from approximately 380 nm to 770 nm, with peak sensitivity in the middle of that spectrum. The wavelength, spot size, power of the outgoing light and exposure time determines if a system is eye safe. Lasers are commonly classified as either class 1, 2, 3A, 3B or 4. A class 1 laser is totally safe, but the diffuse reflection from a class 4 laser damages the eye and the direct beam burns skin and ignites flammable material.

In recent years, the use of fiber optics has become very common in long-range communication system. By simultaneously transmitting data at different optical wavelengths, the data throughput of these fibers can be increased. For this reason there is an interest in tunable laser sources with narrow spectral width. The lasers used with fiber optics typically operate around 1520-1570 nm, since optical fibers have minimal attenuation losses at these wavelengths. This wavelength range is also attractive for eye safety reasons.

1.2 Motivation

With recent advancements in optical telecommunication hardware, both laser sources and fiber optical components, new opportunities have become available for measuring applications. By using a tunable laser diode and fiber components, small, compact, eye safe range and radial velocity measuring systems can be realized utilizing a frequency modulating continuous wave (FMCW) technique.

Our main objective is to investigate and develop the FMCW technique. The intended application is indoor robotic navigation, with a maximum distance around 15-20 m, where the robot is also connected to Internet through wireless communication. When this research project started, one of the main questions we wanted to investigate was if it was possible to construct a relatively cheap FMCW system suited for this type of application.

1.3 Thesis Content

Chapter 2 gives an introduction to the main components used in an FMCW system. The chapter starts with some basic laser diode theory and gives an overview of different available tunable laser sources. Detector options, detector electronics and noise are also discussed.

Chapter 3 gives a short introduction to different optical range measuring techniques. The FMCW method is explained in more detail and important performance issues and references related to optical FCMW are discussed. Chapter 4 contains a short summary of each paper in the thesis and chapter 5 contains the conclusions.

The work presented in the papers deals with solving four different problems related to FMCW. First, we investigated how to optimally use the output power of the laser, with respect to the signal to noise ratio at the photodiode. Secondly, we have attempted to solve the ambiguities occurring when the Doppler shift becomes larger than the frequency shift associated with the range. Thirdly, we have looked at the photodiode amplifier and
how we can combine a high gain in the desired frequency region without risking saturation or clipping due to the dc current generated in the photodiode. The challenge was to use only one OP-amp and no inductors. And finally, we have investigated the wavelength behavior of the laser with respect to changes in the laser drive current. The goal is to obtain a linear ramping of the optical frequency, as a non-linear frequency ramping decreased the performance.
2.1 Laser diodes

The word laser was originally a short for "light amplification by stimulated emission of radiation". The first laser diode was constructed in 1962, two years after the first laser effect was achieved. The basic physics behind a light emitting diode (LED) are similar to that of a laser diode. LEDs are constructed from a regular p-n junction. A "p" material is a positively doped semiconductor material and an "n" material is negatively doped one. When a "p" and "n" material are brought together a junction is formed. The free electrons on the "n" side will propagate into the "p" side, and holes from the "p" side propagates to the "n" side. Even though the materials are doped they are still charge neutral before being brought together. After the carrier transport at the junction has taken place, the "n" side close to the junction will be slightly positively charged and the "p" side slightly negatively, giving rise to a built in electrical field. The junction formed between the "p" and "n" side is often refereed to as the depletion region, since it is depleted of free carriers. In a LED, some of the recombination of electron-hole-pairs in and close to the junction gives rise to the generation of photons by so called spontaneous emission. The energy levels of the materials used to form the p-n junction determine if it is suited for photon generation and which wavelengths that are transmitted. The electron flow through the junction, and the excitation of new electrons, are achieved by applying a forward voltage to the diode.

A LED is relatively cheap but not very efficient in converting electrical power to optical power. The optical wavelength spectrum transmitted from a LED is also much wider compared to even less sophisticated diode lasers. By improving upon the basic LED structure, the laser effect can be achieved.

- **Carrier confinement.** To lower losses, as much as possible of the current supplied to the laser should be guided through the active region, where the photons are generated.
- **Light confinement.** Guiding as much as possible of the generated photons in the right directions increases the efficiency. Figure 2.1 displays the cross section of a laser
Components
diode where both carrier and light confinement are achieved by surrounding the active region with an isolator that has suitable reflective properties. The current propagating from the contacts will hence be guided through the active region, and reflections that guides the light back into the active region will occur at the isolator junction.

![Figure 2.1](image)

_Figure 2.1: Displays the cross section of a laser diode where carrier and light confinement are achieved by surrounding the active region with a reflective isolator. The light direction is outwards from the figure._

- **Light Feedback.** By coupling back some of the generated photons to the active region, more photons of the same wavelength will be generated due to stimulated emission. Stimulated emission is when a photon of a certain wavelength stimulates an exited electron to recombine with a hole and release a photon of the same wavelength and phase as the photon passing by. In a laser diode, the reflections due to the refractive index difference between the active section in the semiconductor and the surrounding usually creates sufficient light feedback. When needed, more feedback can be obtained by adding mirrors.

- **Population inversion.** For the electron hole pair recombination and generation of photon process in a diode laser to sustain, and for sufficient stimulated emission to take place, a large number of free electrons and holes must be available in the same region. This state, where most of the electrons occupy higher energy levels, is refereed to as population inversion. It is not a state that can be self-sustained, since electrons and holes also recombine spontaneously. To obtain population inversion in a p-n junction, an external forward bias is applied that shifts the energy levels so that the high concentration of free electrons on the n-side resides above the free holes on the p-side, as displayed in Figure 2.2. This is also refereed to as electrical pumping of the laser diode.

### 2.1.1 Homojunction and Heterojunction laser diodes

A homojunction laser diode, displayed in Figure 2.3 (a), suffers from low carrier and light confinement. In a homojunction laser diode the depletion region acts as the active region. In a single heterojunction laser diode, displayed in Figure 2.3 (b), the p-GaAs layer is made very thin and a "p" doped GaAlAs layer is added next to the p-GaAs layer. In this case the p-GaAs layer will act as the active region. The single heterojunction structure
Figure 2.2: Population inversion in the p-n junction is obtained by applying a forward bias that shifts the energy levels so that the free electrons on the n-side reside above the free holes on the p-side.

has improved carrier and light confinement compared to the homojunction case, but even further improvements can be gained by using a so called double heterojunction structure.

Figure 2.3: Displays a homojunction laser diode structure (a), a single heterojunction diode structure (b), and a double heterojunction diode structure (c).

In a double heterojunction laser diode an "n" doped GaAlAs layer is added next to the active p-GaAs region, as displayed in Figure 2.3 (c). This gives two main benefits. First, the refraction index difference between the n-GaAlAs regions and the active p-GaAs region is larger then the difference between n-GaAs and p-GaAs, which improves light confinement. Secondly, n-GaAlAs has a larger band gap compared to n-GaAs, as seen in Figure 2.4, which increases the carrier confinement to the active p-GaAs region. In Figure 2.4, the dashed lines represents the Fermi level of the materials. An energy state at the Fermi level has a probability of 0.5 of being occupied by an electron. At a temperature of zero Kelvin, every energy state up to the Fermi level is filled with electrons and the energy states above the Fermi level are empty. As the temperature is increased, more and more electrons will occupy energy levels above the Fermi level. When two different materials are brought together, the Fermi levels of the two will align at the junction. By adding a voltage bias to the junction, it is possible to move the Fermi levels and create a population inversion region.
Components

2.1.2 Vertical Cavity Surface Emitting Lasers

The first types of diode lasers built were so called in-plane cavity lasers. Starting around 1990 development of Vertical Cavity Surface Emitting Lasers (VCSEL) gained large interest. In a VCSEL the light is transmitted in the same direction as the materials are grown during manufacturing. Figure 2.5 displays the difference between in-plane and VCSEL. Some advantages of the VCSEL types of lasers are small achievable cavity dimensions with good coupling characteristics to fibers and the ability to perform device tests directly on the wafer. One disadvantage is that including additional mirrors around the active area is often needed after the VCSEL has been cleaved from the wafer.

2.1.3 Bragg structures

In a regular laser cavity, the cavity length and the materials used determines the number of different wavelengths that can oscillate. Each lasing wavelength must fill the cavity
2.1. Laser diodes

length with an integer number of half wavelengths, but the energy gaps in the material generally only support a few closely spaced wavelengths. Figure 2.6 displays the oscillating wavelengths determined by the cavity length, usually referred to as modes, and the gain curve determined by the material. The frequency separation between two modes, $\delta f$, can be expressed as $\delta f = c/2Ln$, where $L$ is the cavity length, $c$ the light velocity in vacuum and $n$ the refractive index of the cavity.

$$\delta f = \frac{c}{2Ln}$$

![Figure 2.6: Displays the intensity, $I$, of the different laser modes and the gain curve of the laser material.](image)

**Figure 2.6:** Displays the intensity, $I$, of the different laser modes and the gain curve of the laser material.

![Figure 2.7: A Bragg structure consisting of repeated discontinuities.](image)

**Figure 2.7:** A Bragg structure consisting of repeated discontinuities.

The Bragg grating structure is a repeating structure. A simple sketch of a rectangular Bragg structure can be viewed in Figure 2.7. Reflections are built up at each discontinuity and the reflections that are in phase will add up together. In a Bragg structure, the distance between two discontinuities, rather than the entire cavity length, determines which wavelengths that will oscillate. Since that distance is much smaller compared to the entire cavity length, the separation between two oscillating frequencies, $\delta f$, will increase a lot. If the separation between two oscillating wavelengths is large enough, only one wavelength will be supported by the gain curve of the material. By utilizing Bragg structures it is hence possible to build diode lasers that are monochromatic and have fairly long coherence lengths (typically around 1 meter) for diode lasers also referred to as single-mode lasers.

Two examples of laser diodes utilizing Bragg structures are "distributed feedback" (DFB) lasers and "distributed Bragg reflector" (DBR) lasers. In a DFB laser, the Bragg
structure stretches through the entire cavity. The Bragg section in a DFB laser can either be active itself, or be spaced very closely to the active region. Besides having one contact to supply the bias, the DFB laser can be divided into more than one section and have additional contacts used for fine tuning of the optical output frequency. DBR lasers generally have three sections, one active section, one Bragg section and one phase compensation section. The currents to the Bragg and phase sections are used to fine-tune the optical output frequency of the laser. Figure 2.8 shows a simple sketch of a three section DBR laser.

![Diagram of a three section DBR laser](image)

*Figure 2.8: A three section in-plane distributed Bragg reflector laser (DBR).*

### 2.1.4 External Cavity Lasers

If anti reflective coating is applied to at least one of the laser cavity ends, an external component can be used to set which wavelengths that are reflected back into the laser cavity. Hence the external component, rather than the cavity dimension of the laser, will determine the lasing wavelengths.

If the external component is a fibre bragg grating, the wavelength can be tuned by applying a force to the grating, which changes the grating dimensions and hence the back reflected wavelength.

Another common external cavity configuration is the so called Littman-Metcalf cavity arrangement, displayed in Figure 2.9. The Littman-Metcalf arrangement uses a diffraction grating and a retro reflecting mirror to select the back reflected wavelength. Wavelength tuning of this arrangement can be achieved by rotating the retro reflecting mirror, which will change the grating wavelength.

External cavity lasers often have very linear tuning characteristics, but the tuning rate is often much lower compared to current modulated laser diodes.

### 2.1.5 Current modulation

By modulating currents to the laser diode the output frequency can be controlled. Some lasers, like the three sections DBR laser, have special connections for this purpose. In other laser, for instance a one-section DFB laser, the frequency is effected by changes in the drive current to the active section. Modulating a current to the laser effects the
optical frequency through two different mechanism. First, as the current increases the
cavity temperature rises which affects both the relative cavity length and the gain curve
of the material. Secondly, increasing the current increases the number of free electrons
in the material, which influence the refractive index and hence also changes the optical
path length. The later is refereed to as the plasma effect, and the time constant for the
plasma effect has been reported to be in nanoseconds regime [8]. At lower modulation
frequencies temperature effects tend to dominate the frequency behaviour, but at higher
frequencies, the plasma effect becomes the dominant one.

2.2 Detection and amplification

A large number of different optical detectors are available today. Photodiodes are small,
compact and do not require any complicated supporting circuits to operate. The pho-
todiode consist of a single p-n junction, or a p-n junction with a lightly or not doped
section inserted between the "p" and "n" materials. The later is refereed to as p-i-n (PIN)
structure, where the "i" stands for intrinsic, and the "i" material is inserted to increase
the size of the depletion region. The depletion region responds to incoming photons by
absorbing the photon energy and generating an electron-hole pair. The electron and
the hole are then set in motion by the electrical field in the depletion region, creating a
current proportional to the incoming light. The width of the depletion region, influenced
by the materials and the doping gradients, determines the width of the responsive region
and the shunt capacitance of the diode. By applying a reverse bias voltage the width of
the depletion region can be increased, which increases the responsivity and decreases the
capacitance. By monitoring either the output current, refereed to as photoconductive
mode, or changes in the voltage over the diode, refereed to as photovoltaic mode, the
incoming light can be monitored.

The photodiode can be combined with a number of different amplifier circuits suited
for different applications. One common solution is to use one or more FET input stage
OP-amps. The so-called "current to voltage converter" or transimpedance circuit, dis-
Figure 2.10: A simple current to voltage converter, also referred to as a transimpedance circuit, consisting of a photodiode, an OP-amp and a feedback resistor $R_f$.

played in Figure 2.10, is a simple solution with many benefits. The virtual ground at the OP-amp (-) input keeps a constant low voltage over the photodiode, which isolates the diode capacitance from the output that benefits both linearity and bandwidth. Most of the current generated in the photodiode will propagate through the feedback resistor $R_f$, generating an output voltage $v_{out} = i_d \times R_f$. Reverse bias can be added to the photodiode by inserting a voltage source between the photodiode and ground. Common drawbacks associated with the reverse bias are increased diode leakage current and noise contributions from the bias source. For low noise applications, the input capacitance of the amplifier and the shunt capacitance of the photodiode should be kept as low as possible [9].

2.2.1 Noise in electrical circuits

Noise is often defined as any unwanted disturbance that obscures or interferes with the useful signal. In electronics, one often uses the word noise to refer to the noise being generated by the components themselves, while the word disturbance is used when the source is external to the component. Some examples of disturbances are radiation from electrical equipment, such as power lines, radio transmitter, other components in the same circuit and vibrations giving rise to micro phonics. To reduce the influence of these disturbances one often applies shielding, filtering and layout optimization.

The noise signal is a purely random signal and hence it is impossible to predict its value at any given time. When describing a noise source mathematically, it is common to refer to the noise spectral density. The spectral density, which is defined as the Fourier transform of the auto correlation function for the noise, gives the average noise power as a function of frequency. If the noise spectral density is constant, the noise is often called white noise. In this case the total noise power can be obtained by multiplying the spectral density with the noise bandwidth. The definition of the noise bandwidth is not the same as the -3 dB bandwidth. The noise bandwidth is defined as the frequency span of a rectangularly shaped power gain curve equal in area to the actual power gain vs frequency curve.

Components generates noise due to the physics of the device itself. The three fundamental noise mechanisms are thermal noise, shot noise and so called 1/f noise or flicker
noise.

A random motion of charge carriers due to temperature effects is the cause for thermal
noise. The higher the temperature, the larger this carrier motion will be and the thermal
noise hence increases with temperature. Imagine a resistor where the temperature gives
rise to random carrier motions and hence a noise current. The voltage noise generated by
that resistor will then be proportional to the noise current multiplied by the resistance.
For thermal noise the voltage noise spectral density is given by

\[ E_n^2 = 4kTR, \]

where \( k \) is Boltzmann’s constant, \( T \) the temperature and \( R \) the resistance. The unit for \( E_n^2 \) is V\(^2\)/Hz.

Shoot noise exists in all components containing pn-junctions, such as transistors and
diodes, but also in tubes. When the carriers cross the pn-junction their motion will not
be smooth and continuous, instead they will have more of a pulsating motion. This
pulsation carrier motion, and the fact that the current trough the pn-junction consist of
relatively few carriers, is the cause of the shoot noise and the shoot noise increases when
the current trough the pn-junction is increased. The current spectral density of the shoot
noise is given by

\[ I_n^2 = 2qI_D, \]

where \( q \) is the electron charge and \( I_D \) the dc current trough the pn-junction.

The physical cause for 1/f noise, also called flicker noise, is often said to be due
to surface properties and contacts. As the name implies, this noise increases as the
frequency decreases. Flicker noise only exist where there is a direct current flow. The
current spectral density of the flicker noise is given by

\[ I_n^2(f) = K_1I_a^a/f^b, \]

where \( a \) is a constant in the range 0.5-2 and \( b \) is a constant in the range 0.8-1.2. \( K_1 \) is a
constant that depends on the component.

A noise analysis of a complex circuit with many different components can be quite
cumbersome since the noise contribution from each component has to be considered
separately. A complex circuit, such as an operational amplifier, is therefor normally
represented with a noise model that consists of a noiseless component with equivalent
input voltage sources \( E_n \), and equivalent input current noise sources, \( I_n \), as shown in
Figure 2.11. An equivalent input noise source refers all the noise sources to the signal
input.

### 2.2.2 Avalanche Photo Diodes

In an Avalanche Photo Diode (APD), a very high electrical field accelerates electrons to
high enough kinetic energy to release other static electrons in the structure by collision,
also referred to as impact ionization. The electrical field will accelerate the released
electrons so that the released electrons in turn can collide and release even more electrons.
Figure 2.11: An operational amplifier noise model with an equivalent input voltage source, $E_n$, and current input noise sources $I_n$.

This avalanche effect gives rise to a built in gain. APDs are generally built up by an n-p-i-p+ structure, where the (+) indicates a more heavily doped region. The depletion region formed by the n-p structure are where most of the photons are absorbed, while most of the impact ionization and multiplication of electrons takes place in the high field "i" region. One drawback with avalanche diodes is the large reverse bias voltage required to achieve the avalanche effect. APD and PIN diodes are built in both Silicon and InGaAs. Silicon components are sensitive to wavelengths between 300-1000 nm while InGaAs can cover the 0.9-2.6 μm region. The choice between using a PIN or an APD depends on many things, for instance the noise levels in the amplifier electronics. Generally speaking, APD diodes have higher sensitivity but also more noise. If the amplifier noise is very low, a PIN diode may result in a lower detection limit. But if the amplifier noise is larger than the APD noise, an APD might be preferred.

More theory, and information about different optoelectrical components, fibers and tunable laser diodes can be found in [10-13].
3.1 Measuring techniques

A number of different optical distance and velocity measuring techniques exists today. This section contains a short summary of the more common ones. For a recent review of different laser ranging techniques see [3].

3.1.1 Time of flight

By sending out a light pulse and measure the time delay of the reflection, the distance to a target can be determined. Overall system performance is determined by a number of factors, for instance the peak output power of the laser pulse, variations in the shape of the light pulse, the time resolution and dynamic range of the receiver electronics. Measuring short distances with high resolution is hence a challenging task. The resolution can be improved by sending out multiple pulses and average the delays. For more detailed information see for example [14-16].

3.1.2 Frequency Modulated Continuous Wave (FMCW)

This technique was first implemented using regular radar, but with recent advancements in laser sources and optical components the interest for optical systems have increased. The FMCW principle can be applied in an optical system in two different ways. First, the frequency modulation can be added to the optical frequency directly [17-43], as described in section 3.1.2.1, secondly, it can be added to the outgoing light intensity, as described in section 3.1.2.2. The focus in this thesis is on the first method.

Optical frequency domain reflectometry (OFDR) [44-47] is closely related to FMCW. Both use the same principals, but in the case of OFDR the technique is used to study optical fiber networks.
3.1.2.1 Optical frequency modulation

The optical output frequency of the laser is modulated and the outgoing light wave is divided into two parts. One part is transmitted towards the target and the second part is used as a local oscillator. If a regular photodiode is used as a detector, the frequency difference between the local oscillator wave and the reflected wave can be obtained directly by mixing (=summing) the two parts on a photodiode and monitor the photodiode current. This is due to the fact that the output current of the photodiode is proportional to the squared sum of the two waves, and that high frequencies are filtered out by the photodiode. The frequency difference between the local oscillator and the reflected signal is usually referred to as the beat frequency. Due to the square law mixing process the amplitude of the beat frequency component is proportional to the product of the amplitudes of the local oscillator and the reflected signal. If sawtooth frequency modulation is used, only the range can be determined and a Doppler shift will result in an error. By using triangular modulation, as displayed in Figure 3.1, both range $R_a$ and radial velocity $v_t$ can be extracted from the measured beat frequencies $f_1$ and $f_2$. If the frequency shift associated with the range, $f_R$, is larger than the Doppler shift, $f_D$, the range can be obtained from

$$f_R = \frac{f_1 + f_2}{2} = \frac{4R_a \cdot \Delta f}{c \cdot T_{mod}},$$

and the velocity from

$$f_D = \frac{f_2 - f_1}{2} = \frac{2 \cdot v_t}{\lambda},$$

where $\lambda$ corresponds to the wavelength and the other symbols can be found in Figure 3.1. A positive $v_t$, and hence $f_D$, corresponds to a decreasing distance between source and target.

One advantage with the FMCW approach is the possibility to optimize the system for a certain application. To improve the resolution of a time of flight system, the time resolution has to be improved. With the FMCW approach, for a given value of $\Delta f$ one can choose a value of $T_{mod}$ that suits the application, as long as the laser is not the limiting factor and $T_{mod}$ is larger than the optical round trip time. For instance, if the detector has large bandwidth, but less frequency resolution, a smaller value of $T_{mod}$ will improve the measurement resolution at shorter distances.

When the optical frequency is used as the carrier, the FMCW method demands a single mode laser with fairly long coherence length, as the coherence length influence the stability of the local oscillator compared to the reflected wave. At measuring distances much shorter then the coherence length, as long as the frequency sweep is linear the beat frequency spectrum peak will be narrow and sharp. But when the measuring distance is increased, the beat frequency spectrum becomes wider and the height of the beat frequency peak decreases. This is referred to as phase noise, and it is discussed in more detailed in [17],[20],[36],[40] and [42].

Since this method demands a very coherent laser source, it is sensitive to speckle that occurs in diffuse coherent reflections. Different speckle spots in the reflected light can
be out of phase when compared to each other. When the speckle spots in the returned pattern are summed on the detector they can hence cancel each other out.

The theoretical noiseless spatial resolution of a system depends on the size of the modulation frequency span $\Delta f$ and the linearity of the frequency sweep, placing additional requirements on the laser source. In practice the frequency resolution of the electronics can also limit the spatial resolution. Besides using DFB and DBR diode lasers, external cavity lasers, Nd:YAG and CO$_2$ lasers, which offer higher output power but are less compact, provides other alternatives. When Nd:YAG lasers are used the frequency can be modulated by applying a voltage to a piezoelectric transducer (PZT) mounted on the laser cavity, thereby controlling the cavity length [48].

When the frequency modulation is obtained through current modulation, a linear current ramp seldom results in a linear frequency ramping. A non-linear frequency ramping will cause the beat frequency from a stationary target to vary during the current ramp. This smearing of the beat frequency will make it harder to detect and decrease the measurement resolution. One elegant solution to this problem is presented in [30]. Here a part of the laser output is used in a reference arm with a fixed optical path length difference to generate a beat frequency. This beat frequency, and an external reference signal with a frequency close to that of the beat frequency, are used as inputs to a lock-in amplifier. The error signal from the lock-in amplifier is then added to the lasers current ramp and the result is a linearization of the frequency ramp. A second approach to
this problem, presented in [40], is to use an optical reference path and divide the beat frequency from the reference path with the one obtained from the target.

To my knowledge, the highest resolution obtained so far at shorter distances is in the sub micrometer regime [37], using a two section DFB laser and a mirror target. The longest reported measuring range is 18 km [45]. These measurements were done in optical fibers using a so-called frequency shifted feedback (FSF) laser source, which had very fast tuning rate.

3.1.2.2 FMCW with sub-carrier

This is similar to the optical frequency modulation method, but instead of using the optical frequency as the carrier wave, modulation is applied to the outgoing light intensity to create a sub-carrier [49-52]. This method is less sensitive to the selection of laser diode, as the coherence length is not as critical, and if the light is less coherent it is not so much influenced by speckle. Obtaining the desired frequency difference is not as straightforward as when the optical frequency is modulated, since no filtering is done directly in the photodiode, but it is possible to combine electrical and optical mixing as in [52]. Obtaining large frequency sweeps, and hence good spatial resolution, also becomes more complicated compared to when the optical frequency is modulated.

3.1.3 Phase measurement method

The outgoing light intensity is sinusoidally modulated with a constant frequency, and the phase difference between the local wave and the received wave is measured. The phase difference can be obtained by measuring the time differences between the zero crossings of the reference wave and the reflected wave. In both cases the maximum distance that can be measured, while avoiding ambiguities, is determined by the selected modulation frequency. Two extensions of this technique, to improve the accuracy and avoid ambiguities when determining the distance, involve the use of more than one modulation frequency and/or adding a 90 degree phase shift to the local reference waveform [53-54].

3.1.4 Triangulation

Triangular ranging utilize trigonometry to calculate the side of a triangle that correspond to the distance to a spot of interest. If the length of one side in the triangle is known, for instance the distance between two cameras, the distance to the target can be calculated from two measured angels. The basic principle is displayed in Figure 3.2, where the distance B can be calculated from

\[ B = A \frac{\sin \theta}{\sin(\theta + \phi)}. \]

Triangular ranging systems can be either passive or active. A passive system uses only the background light while an active system emits a beam to illuminate a certain spot. In the active system the beam emitter is one corner of the triangle and the camera
3.1. Measuring techniques

Corresponds to the second corner. For the passive system the type and amount of background light has a large impact on the performance, and in some circumstances artificial light has to be provided. In a passive two camera system, it can also be problematic to match points viewed in one camera to the same points viewed in the other camera. For both passive and active systems it becomes increasingly difficult to determine longer distances with good accuracy. For a given angle measurement error, the error in the distance measurement increases with range.

Figure 3.2: By measuring the two angles $\theta$ and $\phi$ the range $B$ can be determined.
4.1 Summary of contributions

This chapter gives a brief summary of the published and submitted papers.

4.1.1 Paper A - Dividing Optical Power for Maximum Signal-to-Noise Ratio in a Self Mixing FMCW System

Authors: Daniel Nordin and Kalevi Hyyppä

The outgoing light from the laser is divided into two parts, the transmitted part and the local oscillator part. In this paper we have carried out a theoretical investigation on how to divide the optical power between the two, for maximum Signal-to-Noise Ratio (SNR) at the receiver photodiode. Other noise sources in the systems are also mentioned. A formula for the optimal amount of transmitted power as a function of photodiode parameters and target reflection ratio is derived. Our investigation shows that in a system where most of the transmitted power is lost, as is common in open-air measurements, maximum SNR with respect to the photodiode noise is obtained by transmitting most of the available power. Increasing the local oscillator power only increases SNR to a certain point.

Theoretical work and most of the writing was done by Daniel Nordin. The main idea for the paper came from Kalevi Hyyppä.

4.1.2 Paper B - Advantages of a new modulation scheme in an optical self-mixing FMCW system

Authors: Daniel Nordin and Kalevi Hyyppä
In this paper we have presented a new modulation scheme that avoids ambiguities resulting from a Doppler shift larger than the frequency shift associated with the range. Besides avoiding this ambiguity, the new scheme also increases the freedom when choosing the modulation parameters of the laser. In systems with relatively low frequency modulation span, $\Delta f$, and/or low modulation frequency $1/T_{\text{mod}}$, which is common with today’s tunable laser diodes, this problem occurs even for relatively low velocities. By inserting a constant frequency region in the triangular modulation scheme both the range and velocity can be determined without ambiguities. The new modulation scheme is verified and tested in our lab system built around a one section DFB laser and a 50/50 fiber coupler.

The main idea for the paper came from Daniel Nordin. Theory, measurements and most of the writing was done by Daniel Nordin.

### 4.1.3 Paper C - Single stage photodiode OP-amp solution suited for a self mixing FMCW system

**Authors:** Daniel Nordin and Kalevi Hyyppä  

Paper number three presents a single stage OP-amp solution suited for an FMCW system. The current delivered by the photodiode in a FMCW system contains both the useful harmonic signal and a dc current. As a high gain is required, the direct current can easily saturate the receiver electronics or result in clipping of the useful signal. Removal of the dc current before the first current to voltage converter stage is usually not practical, and a cascade of stages, with a filter in between them, is therefore commonly used. Our solution uses only one OP-amp, no inductors, and can combine a high gain in the desired frequency region with minimal gain at dc.

The main idea for the paper came from Daniel Nordin. Theory, simulations, measurements and most of the writing was done by Daniel Nordin.

### 4.1.4 Paper D - Finding the thermal time constants of a DFB laser module used in an FMCW ranging system

**Authors:** Daniel Nordin and Kalevi Hyyppä  
**Reproduced from:** Proceedings of the ODIMAP IV Topical Meeting on Distance/Displacement Measurements and Applications, Oulu, Finland, June, 2004.

Paper number four presents a thermal investigation of our laser module, with the purpose of creating an electrical equivalent model of the lasers thermal and hence frequency behavior. The idea is to later be able to simulate the frequency behavior of the laser, with respect to changes in its drive current, in an electrical circuit simulator. The paper presents some theoretical work, measurements and simulations.

The idea to model the lasers thermal behaviour came from Kalevi Hyyppä. Daniel
Nordin contributed with theory, simulations, and measurements. Most of the writing was done by Daniel Nordin.

**4.1.5 Paper E - Using a discrete temperature model to obtain a linear frequency ramping in an FMCW system**

*Authors: Daniel Nordin and Kalevi Hyypää*
*Submitted to: Journal of Optical Engineering*

In this paper we have continued on the work started in paper D. The thermal model derived in paper D is modified and used to obtain current modulation wave forms that resulted in a linearisation of the optical frequency sweep. The approach was tested with good results for modulation frequencies between 125-2000 Hz.

The idea to use the thermal model to obtain a linear frequency ramping came from Daniel Nordin. Theory, simulations, measurements, and most of the writing was done by Daniel Nordin.
Conclusions and Future Work

5.1 General conclusions

The FMCW approach where the optical frequency is used as the carrier wave has a promising future. By utilizing lasers and fiber components developed for telecom purposes, relatively compact, eye safe and high performance systems can be made. The performance of the laser is critical for the final performance of the system. Both the output power and the coherence length influence how far one can measure. A wide, fast and linear frequency tuning is also required to obtain high spatial resolution. A DFB laser diode filling these requirements is currently quite expensive, and costs approximately 100 times that of a regular heterojunction laser diode. The price of a fiber optical circulator is around 3000 USD today, which is also relatively expensive. By comparison, the price for detector and the electronics are much lower, but the final price of the system will be relatively high. Since this method requires a very coherent laser, speckles are also especially problematic. For these reasons, until a lower priced high performance source becomes available, this method seems less suited for a low cost indoor measuring system.

The laser diode we have used is a single section DFB with a lasing wavelength of 1510 nm. Unfortunately, most fiber optical circulators are built for the so-called c-band and only work for wavelengths between 1525-1570 nm. Hence we had to make do with a fiber coupler. The specified output power from our laser chip was 8 mW, but a large fraction of this power is lost when coupling the light to the fibre. The amount of power aimed towards the target was approximately 1 mW in our system. Our laser operates in single mode, but the coherence length, which is around 0.3 m, is pretty short for a DFB laser. These three unfortunate characteristics, low power, short coherence length and a wavelength outside the bandwidth of available fiber optical circulator, limited our measuring range to approximately 1 meter on diffuse targets.

5.2 Paper conclusions

The modulation scheme suggested and tested in paper B makes it possible to use a laser diode with a lower frequency modulation span $\Delta f$, and still avoid ambiguities resulting
from a Doppler shift larger than the frequency associated with the range. If the suggested modulation scheme is used, it is hence possible to use a somewhat cheaper laser diode, as long as it still operates in single mode, in applications where high spatial resolution can be sacrificed.

Our current system is built around a 50/50 fiber coupler. The theoretical investigation carried out in paper A indicates that the local oscillator power obtained due to the reflection caused by the refractive index difference between the fiber core and air, at the output of such a system, is more than sufficient. In our setup a large amount of the optical power from the laser will be wasted, since 50 percent of both the outgoing and incoming power is lost due to the coupler. The coupler setup also exhibits problems caused by internal reflections and interference fluctuations of the local oscillator power, as reported in paper A and B. Replacing the coupler with a fiber optical circulator should remove the power losses associated with the coupler and eliminate the problems caused by internal reflections and the fluctuations in the local oscillator power. Unfortunately our current laser operates at 1.51 µm, which is outside the wavelength range of most of today’s available fibre optical circulators.

Even if the local oscillator power is relatively low, the resulting dc current can easily saturate the receiver electronics if a regular transimpedance circuit with a high gain is used. The amplifier solution presented in paper C can combine a high gain in the desired frequency region with minimal gain at dc, and hence avoids this problem. Compared to a cascade solution our circuit lacks the added noise associated with the second operational amplifier, and using only one operational amplifier also decreases the cost, power consumption and the size of the circuit.

One thing worth mentioning from paper D is that according to our study, there are at least two measurable thermal time constants associated with the laser diode cavity. One was in the same order of magnitude as the calculated time constant of the whole laser cavity, while the second was in the same order of magnitude as the simulated time constant of the Bragg region. In reference [8], the authors have performed thermal time constants measurements on two different lasers diodes that were similar to our laser diode. The two time constants that they measured for each laser were of the same magnitude as ours, but they assumed that the shorter time constant corresponded to the laser diode cavity, while the longer time constant corresponded to the heat sink.

When testing a new laser diode source in an FMCW system, the method outlined in paper E can be used to obtain a fairly linear frequency ramping, if the required laser diode parameters are know. However, to obtain a very good frequency linearization either further tweaking of model parameters, or the use of the linearization method described in [30], are required.

### 5.3 Future work

To further investigate the performance possibilities of optical FMCW, a new laser source and a matching fiber optic circulator are needed. A direct performance comparison to the intensity modulated FMCW method would also be interesting.

Another possibility for future work would be to further investigate the dynamic fre-
5.3. Future work

quency behaviour of the laser. The model outlined in paper E and D could be im-
proved by measuring the dynamic frequency behaviour directly using a high performance
monochromator, with the ability to time resolve the optical spectra from the laser.
Conclusions and Future Work
References


Conclusions and Future Work


Part II
Dividing Optical Power for Maximum Signal-to-Noise Ratio in a Self Mixing FMCW System

Authors:
Daniel Nordin and Kalevi Hyypä

Reformatted version of paper originally published in:

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An examination on how to divide the optical power between the local oscillator radiation and radiation sent towards the target for maximized signal-to-noise ratio, has been carried out. From the self-mixing equations, by substituting electrical fields with optical power and including relevant noise sources, an expression for the signal-to-noise ratio (SNR) for the photodiode has been obtained. From this equation the conditions for maximum SNR has been derived.

1 Introduction

Frequency modulated continuous wave (FMCW) is a well-known technique from the field of radar applications. The reflected part of an outgoing frequency modulated wave (w1) is mixed with a local oscillator wave (w2), and as a result the frequency difference between the two signals can be obtained. If the frequency modulation is chosen as a triangular wave, both velocity and range of a target can be determined from the frequency difference, usually referred to as beat frequency. By using a tunable laser as the source, with the light frequency as the carrier frequency and a photodiode as detector, the beat frequency can be extracted directly from the diode current [1-4]. This phenomena, usually referred to as the ”self-mixing-effect”, is due to the fact that the photodiode delivers a current proportional to the square of the sum of the two electrical fields w1 and w2. The most common type of lasers used are diode lasers of DFB or DBR type, CO2 or Nd:YAG, and systems can be built either as classical open Michelson interferometers or with optical fibers.

2 System overview

Fiber based systems are attractive because of their simplicity. If constructed as in Figure 1 a minimum of alignment is required since the same lens is used both for collimating the output beam and for collecting the reflected light. A local oscillator can be achieved either by a separate reflector at the free fiber end or by utilizing the refractive index difference between fiber core and air at one of the two fiber ends. If we allow two reflections, one at each fiber end, to build up the local oscillator, the oscillator will vary in intensity as a result of phase differences imposed by temperature changes that stretches the fibers.
Since both transmitted and received light is divided in the coupler a loss of transmitted power will occur in one fiber end and a fraction of the received light will be lost at the isolator end. The isolator is required to keep reflected light from disturbing the laser.

![Figure 1: Simple fiber based system using a tunable laser diode.](image)

### 3 Theory

#### 3.1 Modulation

If triangular modulation as in Figure 2 is used the range $R_a$ can be calculated from the frequency shift $f_R$ corresponding to the distance according to

$$f_R = \frac{f_1 + f_2}{2} = \frac{4R_a \cdot \Delta f}{c \cdot T_{mod}}, \quad (1)$$

where $c$ is the speed of light. The other symbols are defined in Figure 2. The target velocity $v_t$ can be calculated from the doppler shift $f_D$ according to [5]

$$f_D = \frac{f_2 - f_1}{2} = \frac{2 \cdot v_t}{\lambda}, \quad (2)$$

where a positive $v_t$, and hence $f_D$, corresponds to a decreasing distance between source and target. The time $T_e = 2R_a/c$ in Figure 2 represents an error interval, corresponding to the flight time to target and back, where no useful measurements can be made. To achieve linear modulation can be problematic, especially for larger values of $\Delta f$, since the relation between current and frequency can be nonlinear. Nonlinear modulation leads to a smeared out beat frequency but solutions to this problem have been suggested by other authors as in [6].

The theoretical spatial resolution $\Delta R_a$ for a noiseless FMCW system is given by [7]

$$\Delta R_a = \frac{c}{2\Delta f}. \quad (3)$$

One advantage of the FMCW method, compared to time-of-flight, is the possibility to increase the spatial resolution without increasing the bandwidth and hence the noise of the receiver. If $\Delta f$ and $T_{mod}$ is increased by the same amount, so that the slope
of the modulation line and hence the beat frequency remains constant, more periods of the beat frequency will fit under the longer time $T_{mod}$ and as a result the spatial resolution is increased without increasing the bandwidth. In a time-of-flight system the spatial resolution depends on the time measurement resolution and hence the receiver bandwidth. Increasing the receiver bandwidth deteriorates the achieved spatial resolution due to a higher receiver noise floor.

![Beat Frequency Diagram](image)

**Figure 2:** The upper part shows the frequencies of the outgoing and incoming waves. The lower part shows the corresponding beat frequencies.

### 3.2 Self mixing effect

If the received electrical field is denoted as $E_r \cos(\omega_r t + \theta_r)$, and the local oscillator electrical field is denoted $E_{lo} \cos(\omega_{lo} t + \theta_{lo})$, the optical power received by the photodiode, ignoring reflections at the photodiode, will equal

$$
\Phi(t) = \frac{A}{Z} (E_r \cos(\omega_r t + \theta_r) + E_{lo} \cos(\omega_{lo} t + \theta_{lo}))^2,
$$

where $A$ is the area and $Z$ the characteristic impedance of the medium in front of the detector. By expanding equation (4) and removing terms with frequencies to high for the photo diode to respond to, the current from the diode can be expressed as

$$
i(t) = \frac{RA}{Z} \left( \frac{E_{lo}^2}{2} + \frac{E_r^2}{2} + E_{lo} E_r \cos(\omega_{b} t + \theta) \right) + i_b,\n$$

where $\omega_{b}$ and $\omega_{r}$ are the beat frequency and the original frequency of the incoming wave respectively.
where $\omega_b/2\pi$ is the beat frequency and $R$ the responsivity of the detector. The phase $\theta$ can be calculated from the received signal phase $\theta_r$ and the local oscillator phase $\theta_{lo}$ according to $\theta = \theta_r - \theta_{lo}$. By expressing $E_r$ as a function of $E_o$, the amplitude of the outgoing wave, we obtain

$$i(t) = \frac{RA}{Z} \left( \frac{E_o^2}{2} + \Re E_o^2 + \sqrt{\Re E_{lo} E_o \cos(\omega_b t + \theta)} \right) + i_b,$$

(6)

where $\Re$ is the fraction of the optical power $\Phi_o$, sent towards the target, that returns back to the receiver. Substituting the electrical fields amplitudes, $E_{lo}$ and $E_o$, with the corresponding optical powers, according to $\Phi_{lo} = A E_{lo}^2 / 2Z$ and $\Phi_o = A E_o^2 / 2Z$, gives

$$i(t) = R \left( \Phi_{lo} + \Re \Phi_o + 2\sqrt{\Re \Phi_o (\Phi_{tot} - \Phi_o) \cos(\omega_b t + \theta)} \right) + i_b.$$  

(7)

Replacing $\Phi_{lo}$ with $\Phi_{tot} - \Phi_o$, where $\Phi_{tot}$ is the total optical power that is divided between $\Phi_o$ and $\Phi_{lo}$, results in

$$i(t) = R \left( (\Phi_{tot} - \Phi_o) + \Re \Phi_o + 2\sqrt{\Re \Phi_o (\Phi_{tot} - \Phi_o) \cos(\omega_b t + \theta)} \right) + i_b.$$  

(8)

A much simplified approximation of $\Re$, assuming a Lambertian reflector and that the receiver optics and the spot on the target are aligned, is $\Re = d^2/4R^2a$, where $d$ is the diameter of the receiving aperture. This approximation greatly overestimates $\Re$ since it neglects speckle and assumes a reflection coefficient of one at the reflecting target surface. Another loss not accounted for in this investigation is polarization mismatch. The parameter $i_b$ can include both dark current and current generated by background light propagating into the system. The amount of current generated by background light of course also depends on $R$, $A$ and $Z$.

### 3.3 Signal-to-noise ratio

An equation for the signal-to-noise ratio for a regular photodiode, neglecting $1/f$-noise, can be obtained from [8]

$$SNR = \frac{I_s^2}{B(I_D^2 + I_s^2)}.$$  

(9)

The spectral density of the shot noise is $I_D^2 = 2eI$, where $e = 1.602 \times 10^{-19}$ C is the electron charge and $I = R ((\Phi_{tot} - \Phi_o) + \Re \Phi_o) + i_b$ obtained from equation (8) is the dc-current through the pn-junction. The spectral density of the thermal noise $I_s^2$, often referred to as Nyquist or Johnson noise, can be derived from $I_s^2 = 4kT/r_d$, where $k$ is boltzmanns constant, $T$ the temperature in Kelvin and $r_d$ the shunt resistance of the photodiode. The term $B$ corresponds to the limiting bandwidth, determined by design choices in the measurement system. The useful signal power $I_s^2$ in our case equals

$$I_s^2 = \left\langle (2R\sqrt{\Re \Phi_o (\Phi_{tot} - \Phi_o) \cos(\omega_b t + \theta)})^2 \right\rangle = 2R^2\Re \Phi_o (\Phi_{tot} - \Phi_o),$$  

(10)
where $\langle \rangle$ denotes time average. By inserting the expressions for $I^2_D$, $I^2_r$ and $I^2_s$ into equation (9) we obtain

$$SNR = \frac{2R^2\Re\Phi_o (\Phi_{tot} - \Phi_o)}{B \left( (2e (R\Re\Phi_o + (\Phi_{tot} - \Phi_o)) + i_b) + \frac{4kT}{r_d} \right)}.$$ \hfill (11)

In a photodiode $r_d$ is dominated by the leakage resistance which usually is larger than $10^8 \Omega$. Shot noise hence dominate, unless $i_b$ or the local oscillator power is unrealistically low. Neglecting thermal noise in equation (11) results in

$$SNR = \frac{R^2\Re\Phi_o (\Phi_{tot} - \Phi_o)}{cB (R\Re\Phi_o + (\Phi_{tot} - \Phi_o)) + i_b)}.$$ \hfill (12)

Figure 3 displays two plots of equation (12) obtained using the values in Table 1. Plot 1 displays $SNR$ against $\Phi_o/\Phi_{tot}$ and $\Re$, where $\Re = 0 \rightarrow 1$, and plot 2 shows the interval $\Re = 0.00001 \rightarrow 0.0001$. It should be pointed out that equation (12) can not be used to evaluate the expected performance of a system since a number of error sources, for instance laser RIN noise, detector amplifier noise, and phase noise [9-12], have been neglected, but the equations are adequate for our purpose of finding the optimum relation between $\Phi_{tot}$ and $\Phi_o$ with respect to the signal-to-noise ratio for the photodiode.

**Table 1: Parameters used for the plots in Figure 3**

<table>
<thead>
<tr>
<th>$\Phi_{tot}$</th>
<th>1 mW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>0.8 A/W</td>
</tr>
<tr>
<td>$i_b$</td>
<td>100 nA</td>
</tr>
<tr>
<td>$B$</td>
<td>10 MHz</td>
</tr>
</tbody>
</table>

### 3.3.1 The optimum

The exact expression for the optimum relation between $\Phi_o$ and $\Phi_{tot}$ for $\Re < 1$, obtained by differentiating (12), equals

$$\Phi_{oopt} = \frac{1}{2 (-R\Re + R)} \left( 2R\Phi_{tot} + 2i_b - 2\sqrt{(R\Phi_{tot}i_b + i_b^2 + R^2\Re\Phi_{tot}^2 + R\Re\Phi_{tot}i_b) } \right).$$ \hfill (13)

For the parameters in Table 1, and with $\Re = 0.0001$, the optimum is found at $\Phi_{oopt} \approx 0.9852\Phi_{tot}$. If $\Re \rightarrow 1$ and $i_b \rightarrow 0$ equation (13) approaches $\Phi_{oopt} = 0.5\Phi_{tot}$ as expected. If thermal noise $I^2_r$ is not neglected the optimum can be obtained from (11), which results in
Figure 3: Two plots from equation (11). The parameters from Table 1 are used in both plots but the \( \Re \) intervals are different.

\[
\Phi_{\text{opt}} = \frac{1}{4(-eR\Re + eR)} \left( 4eR\Phi_{\text{tot}} + 4eI + 2I^2 - 2\sqrt{(4e^2R\Phi_{\text{tot}}I + 2eR\Phi_{\text{tot}}I^2)} \right.
\]

\[
\left. + 4e^2I_b^2 + 4eI_bI + I^4 + 4e^2R^2\Re\Phi_{\text{tot}}^2 + 4e^2R^2\Re\Phi_{\text{tot}}Ib + 2eR\Re\Phi_{\text{tot}}I^2 \right).
\]

In a fiber based system the amount of \( \Phi_{\text{tot}} \) sent towards the target can be adjusted by changing the refractive index at the fiber-air junction and thereby changing the reflected amount.

4 Conclusions

As the quality of the laser source improves, as mentioned in section 3.3, and the noise generated by the laser decreases, the noise generated in the photodiode will be of more importance. Examining equation (11) or (12) reveals that increasing the local oscillator power while the received power remains constant and without saturating the receiver electronics will only increase SNR to a certain point, since \( \Phi_{\text{tot}} - \Phi_0 \) is included both in the dominator and the nominator. According to our calculations the condition for maximum SNR, equation (13) and (14), depends on the expected amount of received light \( \Re\Phi_0 \), and hence the range to the target. In a system where a large percentage of the transmitted power will be lost, maximum SNR is usually obtained by transmitting most of the available power.

Acknowledgments

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References


PAPER B

Advantages of a new modulation scheme in an optical self mixing FMCW system

Authors:
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Advantages of a new modulation scheme in an optical self mixing FMCW system

Daniel Nordin and Kalevi Hyypää

Abstract

A new frequency-modulated continuous wave modulation scheme, which gives correct result even when the Doppler shift is larger than the frequency difference associated with the range, is presented and tested with a tunable laser diode and fiber based system. By inserting a constant frequency region in the modulation scheme, both the magnitude and the sign of all beat frequencies can be determined. When they are known, the correct frequency difference as a result of the range can be calculated. This new scheme gives more freedom when choosing the modulation parameters of the laser, because increasing the modulation frequency, and/or the frequency sweep, to avoid ambiguities resulting from a large Doppler shift no longer becomes necessary. This is especially useful when using a somewhat cheaper laser diode source, since the maximum obtainable modulation frequency and frequency sweep can be somewhat limited. The suggested modulation scheme makes it possible to use some laser diodes in an application they otherwise would not be suited for.

1 Introduction

Frequency-modulated continuous wave (FMCW) is a range and radial velocity measuring technique well known from the field of radar applications. The reflected part of an outgoing frequency modulated wave (w1) is mixed with a local oscillator wave (w2), and as a result the frequency difference between the two signals can be obtained. If the frequency modulation is chosen as a triangular wave, both radial velocity and range of the target can be determined from the frequency difference, usually referred to as beat frequency. By using a tunable laser as the source, with the light frequency as the carrier frequency and a photodiode as detector, the beat frequency can be extracted directly from the photodiode current. This phenomena, usually referred to as the "self-mixing-effect", is due to the fact that the photodiode delivers a current proportional to the squared sum of the two electrical fields w1 and w2. The most common type of lasers used are diode lasers of DFB or DBR type, CO₂ or Nd:YAG [1]-[9]. Systems can be built either as classical open Michelson interferometers or with optical fibers. Our intended application is indoor range measurements of distances up to 10 m against a diffuse reflecting surface.
2 General FMCW theory

2.1 Self mixing effect

If the received electrical field is denoted as $E_r \cos(\omega_r t + \theta_r)$, and the local oscillator electrical field is denoted $E_{lo} \cos(\omega_{lo} t + \theta_{lo})$, the optical power received by the photodiode, ignoring reflections at the photodiode, will equal

$$\Phi(t) = \frac{A}{Z} (E_r \cos(\omega_r t + \theta_r) + E_{lo} \cos(\omega_{lo} t + \theta_{lo}))^{2},$$

where $A$ is the active area and $Z$ the characteristic impedance of the medium in front of the detector. By expanding (1) and removing terms with frequencies too high for the photodiode to respond to, the current from the diode can be expressed as

$$i(t) = \Re \Phi(t) + i_b = \Re A Z \left( \frac{E_{lo}^2}{2} + \frac{E_r^2}{2} + E_{lo} E_r \cos(\omega_b t + \theta_r - \theta_{lo}) \right) + i_b,$$

where $f = 2\pi \omega_b$ is the beat frequency, $\Re$ the responsivity of the detector, and $i_b$ the direct current in the detector, which can include both dark current and current generated by background light. It should also be pointed out that (2) is only valid when the two waves have the exact same polarization states, and that polarization mismatch degrades the performance.

2.2 Modulation

The simplest modulation scheme consists of a sawtooth wave that only allows range measurements to a stationary target. The more sophisticated FMCW scheme used today is displayed in Fig. 1.

It has the advantage that both range $R$ and radial velocity can be calculated from two measured beat frequencies $f_1$ and $f_2$. The time $T_e = 2R/c$ in Fig. 1 represents an error interval, corresponding to the flight time to target and back, where no useful measurements can be made. If we assume that the range and velocity of the target are constant during one modulation period, $f_1$ and $f_2$ can be expressed as

$$f_1 = |f_R - f_D|,$$

$$f_2 = |f_R + f_D| = |f_R - f_D|,$$

where $f_R$ is the magnitude of the frequency difference due to the range and $f_D$ is the Doppler shift. If $|f_D| < f_R$, the frequency $f_R$ can be expressed as

$$f_R = \frac{f_1 + f_2}{2},$$

and $f_D$ as

$$f_D = \frac{f_2 - f_1}{2}.$$
If $|f_D| > f_R$, the frequencies would instead have to be calculated from

$$f_R = \left| \frac{f_2 - f_1}{2} \right|,$$

and

$$f_D = \begin{cases} 
  \frac{f_2 + f_1}{2}, & f_2 > f_1 \\
  -\left( \frac{f_2 + f_1}{2} \right), & f_2 < f_1. 
\end{cases}$$

If we have determined the correct values of $f_R$ and $f_D$, the range and radial velocity can be calculated. Since the modulation parameters $T_{mod}$ and $\Delta f$ are known, $f_R$ will depend on the range according to

$$f_R = \frac{4R\Delta f}{cT_{mod}}.$$  

The range $R$ can then be calculated from

$$R = \frac{cRfT_{mod}}{4\Delta f},$$

and the radial velocity $v_t$ of the target is given by [10]

$$v_t = \frac{f_D\lambda}{2},$$
where $\lambda$ is the optical wavelength. If the distance between source and target decreases, the sign of $f_D$, and hence $v_t$, will be positive.

3 Problem description and background

When using triangular modulation, it is commonly assumed that $|f_D| < f_R$ and the maximum allowed Doppler shift is hence upper bounded by $f_R$, since (5) and (6) give incorrect results if $|f_D| > f_R$. To deal with larger Doppler shifts, it hence becomes necessary to increase $\Delta f$ and/or $f_{\text{mod}} = 1/T_{\text{mod}}$, thereby increasing the value of $f_R$ for a given range. However, depending on the laser source used, this may not be a practical solution. Both parameters are commonly upper bounded, and the performance will suffer if they are increased too much. One example of this is the one section DFB laser used in the experiments described in Sec. 6. In a one section laser diode, the current modulation must be added to the bias current. If the laser is to operate in single mode, the current may not drop below a certain value, and increasing the current modulation too much also increase the non-linearity of the optical sweep. If the optical frequency modulation is mostly due to thermal effects, as appears to be the case with our laser, increasing the modulation frequency $f_{\text{mod}}$ too much also decreases $\Delta f$. Others have also reported this effect, as well as the problem caused by larger Doppler shifts [11]. Some laser diodes also have a built in low pass filter to protect the diode from spikes in the drive current. This, of course, also limits $f_{\text{mod}}$.

4 The new modulation scheme

To overcome the explained problem, we have to know if $|f_D| > f_R$. Our suggested solution is to include a constant frequency region in the modulation scheme, as shown in Fig. 2.

It would also be possible to insert the constant region at the peak of the modulation triangle. By inserting this region, $|f_D|$ can be measured separately. If $f_2 > f_1$, the sign of $f_D$ must be positive, and negative if $f_2 < f_1$. The measured $f_D$ can be compared to the calculated one by inserting the measured values of $f_1$ and $f_2$ in (2). If the difference between the measured and calculated $f_D$ is large, we can assume that $|f_D| > f_R$, and $f_R$ can then be calculated from (7).

5 Choice of parameters

If the suggested modulation scheme is used, the maximum allowed $f_D$ is no longer upper bounded by $f_R$, and as a result we have more freedom when choosing the $\Delta f$ and $T_1$ parameters. Note that $T_1$ in Fig. 2 corresponds to $T_{\text{mod}}$ in Fig. 1. Reducing $\Delta f$ and/or increasing $T_1$ decreases the frequency $f_R$ for a given range $R$, as seen in (9). Decreasing $f_R$ decreases the larger one of the two measured frequencies $f_1$ and $f_2$, and the required bandwidth of the detector can be reduced. By reducing $\Delta f$, it is easier to achieve linear modulation and avoid mode hopping by using the laser where the relation between current
and frequency is as linear as possible. However there is one major drawback associated with lowering $\Delta f$, since the theoretical spatial resolution $\Delta R$ is given by \[12\]

\[
\Delta R = \frac{c}{2\Delta f},
\]

i.e., decreasing $\Delta f$ increases $\Delta R$.

The maximum achievable range will depend on $T_1$. If $T_2 \geq T_1/2$,

\[
R_{\text{max}} < \frac{cT_1}{4}.
\] \[13\]

This indicates that if maximum possible range for a selected value of $T_1$ is desired, measurements can only be taken near the turning points of the frequency modulation waveform, as shown in Fig. 3.

$\textit{Figure 2: The new modulation scheme with a constant frequency region.}$

$\textit{Figure 3: Measurement intervals at maximum possible range.}$
However, commonly the largest realistic flight time is much smaller than the time required to establish linear modulation when the modulation changes from a negative to a positive derivative, or vice versa. Hence that time, rather then the flight time, determines the start of the measurement interval. When the frequency difference between $f_D$ and $f_R$ is small, and either $f_1$ or $f_2$ approaches zero, the measurement interval also determines the lowest measurable beat frequency, since at least one period of the frequency must fit under the measurement interval. The time $T_2$ must be set large enough for the output laser frequency to stabilize and for a sufficient amount of periods of the Doppler shift to fit under the remaining time. Detecting smaller Doppler shifts requires a longer $T_2$. Depending on application and modulation parameters, it can also be advantageous not to include $T_2$ in each period. In a rapidly scanning system, the introduction of $T_2$ can lead to less accurate distance measurements, for instance to static walls inside a room.

6 Measurement setup

Fiber-based systems are attractive because of their simplicity. Several other authors, for instance in [11], have built and reported on systems using a fiber optical coupler in a similar way, as we have done in our experiments. A second attractive alternative is to use a fiber optical circulator instead of a coupler [13]. By using a circulator, the loss of optical power associated with the coupler, as well as other problems described later, can be avoided.

![Measurement Setup Diagram](image)

*Figure 4: Our measurement setup using a simple fiber-based system and a tunable laser diode.*

The test system we used is displayed in Fig. 4. One advantage with this type of systems is that a minimum of alignment is required. A local oscillator can be achieved either by a separate reflector at the free fiber end or by utilizing the refractive index difference between fiber core and air at one of the two fiber ends. If we allow two reflections, one at each fiber end, to build up the local oscillator, the interference between the two beams can cause the local oscillator to vary in intensity. This kind of interferometer is very sensitive and responds to small changes in the fiber lengths, caused by, for instance, temperature and pressure changes. In this application, this effect is a disturbance. In-
cluding an isolator in the free fiber end would reduce it. Due to the coupler, we will also lose a part of both the outgoing and reflected light. The isolator in front of the laser is required to keep reflected light from disturbing the laser.

Our system is built around a 10 mW, 1.51 \( \mu \)m one-section DFB laser, where 10 mW is the output power of the laser cavity, which leaves around 1 mW that propagates out of the collimated end in our system. We used none polarization maintaining single mode fibers and a 50/50 coupler. By modulating the drive current to the laser, variations in the current influence the cavity optical length, which results in a modulation of the optical output frequency. The cavity optical length is effected by temperature changes and changes in refractive index, both caused by the varying current. In our experiments the current was swept using our suggested modulation scheme. As no special measure are taken to insure that the resulting optical modulation is linear this will result in a slightly nonlinear modulation of the optical frequency. At our measuring distance of 0.5-1.0 m, and with our choice of modulation parameters, the nonlinear optical sweep will cause the beat frequency to increase slightly during the modulation ramp. If the modulation amplitude is increased, the nonlinearity increases, as expected. Our system also lacks any form of temperature regulation.

The target was coated with retro reflective tape and moved back and forth between 576-976 mm distance measured to the fiber end. The movement was controlled using a linear displacement unit, and the laser was driven using voltage waveforms generated in a PC. The voltage was converted to current by our laser drive and protection circuit. The current was ramped from 91.54 mA to 97.51 mA, and we picked \( T_1 = 8.5 \) ms and \( T_2 = 1.5 \) ms. This current modulation was chosen as it gave a fairly linear ramping of the optical output frequency. The diode laser used in our setup has a built-in low-pass filter to protect the diode against spikes in the drive current. This low-pass filter and the fact that \( T_1 \) also influences \( \Delta f \), which is common for our type of laser diodes since the optical frequency modulation is mainly caused by temperature effects, lead to our selected value of \( T_1 \). A decrease in \( T_1 \) otherwise resulted in a decrease of \( \Delta f \). In our case, \( T_2 \) was chosen from the required time for the laser frequency to stabilize, and the Doppler shift was measured at the end of \( T_2 \). With a target speed of 0.05 m/s, the target will move approximately 0.5 mm during one measurement period \( T_1 + T_2 \).

### 6.1 Measurement results

The measurements were performed to verify the suggested modulation waveform and to investigate how the constant frequency region would effect the modulation. Our system is under development and the presented data should not be used to critically evaluate range and velocity measurement accuracy. The measurements also serve the purpose of illustrating how a Doppler shift would cause problems, even at relatively low velocities, if regular triangular modulation is used.

As we have a relatively low value of \( \Delta f \), and a long \( T_1 \), the measured beat frequencies to a stationary target, \( f_1 \) and \( f_2 \) listed in Table 1, will be fairly low. The \( f_1 \), \( f_2 \) and \( f_D \) values in Table 1 were obtained by measuring the mean value of the last 10 periods for each modulation ramp N number of times. The clear difference between \( f_1 \) and \( f_2 \) appears to be caused by the insertion of the \( T_2 \) region, since the laser cavity temperature
decreases during $T_2$. A slight difference between $f_1$ and $f_2$ is always present due to the triangular modulation form. This difference also increases with the amplitude of the current modulation. But with our modulation parameters and with $T_2 = 0$, the difference between $f_1$ and $f_2$ was negligible. From all ten listed values of $\Delta f$, we obtained an overall mean $\Delta f$ of 4.885 GHz. From this $\Delta f$ and the measured mean values of $f_1$ and $f_2$, we calculated the approximate distance using (10). In this case both the difference in $\Delta f$ on the up and down ramps and the nonlinear optical sweep are assumed to be the main contributors to the overall error. Normally one would assume a constant distance during one period and use both $f_1$ and $f_2$ to calculate the range, which brings the calculated values closer to the real values for our measurements.

In the experiments lined out before, we performed measurements to a fixed target at known distances. This was done in order to calibrate the system and to obtain an approximate value of $\Delta f$. In the lab, $\Delta f$ appears to be pretty stable, since the temperature in the room is approximately constant. In our setup we also have a fixed beat frequency resulting from internal reflections in the system. Part of the local oscillator wave is reflected at the fiber-photodiode junction. This reflection will travel back through the coupler and a second reflection will occur at the fiber air-junction. When that reflection reaches the photodiode, it will be mixed with the local oscillator wave. The obtained beat frequency can then be used to calculate $\Delta f$, since the fiber length involved is known. By monitoring that beat frequency, it should be possible to continuously calibrate the system. In a system lacking this type of reflection, for instance a system built around a fiber optical circulator, a fixed reference beat frequency can be obtained by inserting a coupler before the circulator and have that signal travel down a reference path of known length. That signal can then be mixed with the local oscillator signal using a second coupler.

Some simultaneous measurements of $f_1$, $f_2$ and $f_D$ against a moving target were also performed. For the case in Fig. 5, we obtained $f_1 = 59.88$ kHz, $f_2 = 73.26$ kHz and $f_D = 65.79$ kHz. Since $f_2 > f_1$ the sign of $f_D$ must be positive. The edgy and fat appearance of channel 2 is a result of the oscilloscope display. Inserting the measured values of $f_1$ and $f_2$ in (6) gives $f_D = 6.69$ kHz. Since the measured $f_D$ is significantly different compared to the calculated one, we can assume that $|f_D| > f_R$ and (7) gives $f_R = 6.69$ kHz. From our calculated mean $\Delta f$, this would correspond to a distance of 873.32 mm. In this measurement the target was moved between 776 mm and 876 mm. The measurement verify the modulation scheme and show how it can avoid errors that can be problematic at short distances even for relatively low velocities. If regular triangular modulation were used, a laser source similar to the one in our test system would not be suitable for many types of applications. For instance, if the intended application is robotic navigation or just plane range scanning in an environment containing moving objects, even a relatively low velocity can result in $|f_D| > f_R$. 

Figure 5: Displays the simultaneous measurements of all three beat frequencies during one specific period.

Table 1: Measurement results from our laboratory system.

<table>
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<tr>
<th>Measurement</th>
<th>N</th>
<th>Mean $f_1$ Hz</th>
<th>Std. Dev. $f_1$ Hz</th>
<th>Std. Err. $f_1$ Hz</th>
<th>Mean $\Delta f$ GHz</th>
<th>Calc. $R_{mm}$</th>
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<tr>
<td>$f_1$ at 576 mm</td>
<td>10</td>
<td>4476</td>
<td>22.29</td>
<td>7.05</td>
<td>4.954</td>
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<td>18.08</td>
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<td>700.69</td>
</tr>
<tr>
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<td>10</td>
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<td>22.4</td>
<td>7.095</td>
<td>4.828</td>
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<td>64.43</td>
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<td>$f_D$ at 0.05 m/s</td>
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<td>66352</td>
<td>1081</td>
<td>241.8</td>
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</table>

7 Conclusion

We have presented a new FMCW modulation scheme that avoids errors resulting from Doppler shifts larger than the frequency associated with the range. The modulation scheme was tested and verified using a tunable laser diode and a fiber-based system. One advantage to using this modulation scheme is that it no longer becomes necessary to increase $\Delta f$ and/or $f_{mod}$ to deal with larger Doppler shifts. This is especially useful when using a somewhat cheaper laser diode, since the maximum obtainable $\Delta f$ and $f_{mod}$ can
be limited. The gained freedom when choosing the system parameters can also simplify both the detection task, by lowering the required bandwidth, and the task of achieving linear modulation by sacrificing spatial resolution. One disadvantage observed in our systems is an increased difference between the measured $f_1$ and $f_2$ as $T_2$ is inserted. In some systems, the drawback of this effect can be reduced by not including $T_2$ in each modulation period.

Our experimental system used to verify the modulation waveform is primitive and in an early stage of development. By replacing the coupler with a fiber optical circulator, improving the linearity of the optical sweep and tuning both receiver optics and electronics, we aim to improve the performance of our system up to a point where measurements to diffuse targets at ranges up to 10 m can be performed. Continuous calibration can then be achieved by adding a reference path of known length to the fiber optical circulator-based system and mix the reference signal with the local oscillator signal.

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References


Single stage photodiode OP-amp solution suited for a self mixing FMCW system

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Single stage photodiode OP-amp solution suited for a self mixing FMCW system

Daniel Nordin and Kalevi Hyypää

Abstract

The current delivered by the photodiode in a self mixing FMCW or OFDR system consist of a DC-current resulting from the local oscillator, the reflected signal, dark current in the photodiode, and current generated from background light. The current also contains the useful harmonic signal with a beat frequency corresponding to the range and radial velocity of a target. To avoid saturation and clipping due to the DC current generated in the photodiode, it is desirable to minimize the gain at DC while maintaining a high gain in the beat frequency region. We have investigated some different solutions and present a modified current-to-voltage converter using bootstraping and added voltage gain, which addresses this problem using only one OP-amp and no DC shorting inductors.

1 Introduction

Frequency modulated continuous wave (FMCW) is a range and radial velocity measuring technique well-known from the field of radar applications. The reflected part of an outgoing frequency modulated wave (w1) is mixed with a local oscillator wave (w2), and as a result the frequency difference between the two signals can be obtained. If the frequency modulation is chosen as a triangular wave, both radial velocity and range of the target can be determined from the frequency difference, usually referred to as beat frequency. By using a tunable laser as the source, with the light frequency as the carrier frequency and a photodiode as detector, the beat frequency can be extracted directly from the photodiode current. This phenomena, usually referred to as the ”self-mixing-effect”, is due to the fact that the photodiode delivers a current proportional to the squared sum of the two electrical fields w1 and w2. The most common type of lasers used are diode lasers of DFB or DBR type, CO2 or Nd:YAG [1]-[14]. Systems can be built either as classical open Michelson interferometers or with optical fibers. Optical frequency domain reflectometry (OFDR) is closely related to FMCW. Both utilize the same principals but in the case of OFDR the technique is used to study optical fiber networks.

Besides the beat frequency, a dc current will also be generated in the photodiode. This paper deals with different ways, while focusing on one of them, to avoid saturation due to this dc current and still maintain a high gain in the beat frequency region. Before going into the problem description and the solution we start with a short introduction to the theory behind FMCW.
2 FMCW Theory

2.1 Frequency Modulation

If triangular modulation of the optical frequency is used, as displayed in Fig. 1, the range \( R_a \) can be calculated from the frequency shift \( f_R \) corresponding to the distance according to

\[
f_R = \frac{f_1 + f_2}{2} = \frac{4R_a \cdot \Delta f}{c \cdot T_{mod}},
\]

where \( c \) is the speed of light, \( f_1 \) and \( f_2 \) the output beat frequencies as shown in Fig. 1, and \( T_{mod} \) the modulation cycle.

\[\text{Figure 1: Triangular modulation scheme where the outgoing frequency corresponds to the local oscillator wave and the incoming frequency corresponds to the reflected wave. The lower part shows the corresponding beat frequencies.}\]

The target velocity \( v_t \) can be calculated from the Doppler shift \( f_D \) according to [15]

\[
f_D = \frac{f_2 - f_1}{2} = \frac{2 \cdot v_t}{\lambda},
\]

where a positive \( v_t \), and hence \( f_D \), corresponds to a decreasing distance between source and target. The above calculations are derived under the assumptions that the range and speed of the target are constant during one modulation period and that \( |f_D| < f_R \).

2.2 Self mixing effect

If the received electrical field is denoted as \( E_r \cos(\omega_r t + \theta_r) \), and the local oscillator electrical field is denoted \( E_{lo} \cos(\omega_{lo} t + \theta_{lo}) \), the optical power received by the photodiode,
ignoring reflections at the photodiode, will equal

$$\Phi(t) = \frac{A}{Z} (E_r \cos(\omega_r t + \theta_r) + E_{lo} \cos(\omega_{lo} t + \theta_{lo}))^2,$$

(3)

where $A$ is the active area of the detector and $Z$ the characteristic impedance of the medium in front of the detector. By expanding (3) and removing terms with frequencies too high for the photodiode to respond to, the current from the diode can be expressed as

$$i(t) = R \Phi(t) + I_B = \frac{RA}{Z} \left( \frac{E_{lo}^2}{2} + \frac{E_r^2}{2} + E_{lo}E_r \cos(\omega_{lo} t - \theta_r) \right) + I_B,$$

(4)

where $f = 2\pi\omega_b$ is the beat frequency, $R$ the responsivity of the detector and $I_B$ the DC current in the detector, which can include both dark current and current generated by background light. The total dc current $I_D$ generated by the photodiode can hence be expressed as

$$I_D = \frac{RA}{Z} \left( \frac{E_{lo}^2}{2} + \frac{E_r^2}{2} \right) + I_B.$$

(5)

It should be pointed out that equation (4) is only valid when the two waves have the exact same polarization states and that polarization mismatch degrades the performance.

### 3 Problem description

In a system where most of the transmitted power will be lost it can be shown theoretically that maximum signal to noise ratio for the photodiode is obtained by transmitting most of the available power [16]. But even for such a design the dc current, resulting mainly from the local oscillator, can cause saturation or clipping in the amplifier if no special care is taken to avoid it.

When combining an OP-amp with a photodiode for current to voltage conversion it is often advantageous to keep the voltage over the photodiode constant. Commonly a constant reverse bias or zero voltage is desired. The latter case can be accomplished by placing the photodiode between the OP-amps (+) and (-) inputs and utilize feedback. One possible solution to avoid saturation due to the dc current is to place a high pass filter between the photodiode and the first amplifying stage. A drawback with this approach is that it forces you to use a reverse biasing source and the resulting reverse bias over the diode will vary due to variations in the dc current. A second drawback is that the filter introduces noise.

Another possibility is to use a cascade of two OP-amps. The gain in the first stage must be kept low, to avoid saturation and clipping, and a high pass filter that removes the DC level is inserted between the stages. The disadvantages of this approach are the extra OP required and the noise that it will contribute directly in the signal path. Yet another possibility is to utilize inductances as a dc short circuit. However, when a wide passband, starting from a few kHz, is desired the resulting inductance value tend to be of the order of mH or larger, and hence unpractically large.
4 Solution

Our circuit is based on a current to voltage converter with added voltage gain using bootstrapping [17], as displayed in Fig. 2. The current is first converted to voltage by $R_1$.

![Diagram of a current to voltage converter using bootstrapping and added voltage gain.](image)

Figure 2: A current to voltage converter using bootstrapping and added voltage gain.

The voltage obtained over $R_1$ is then amplified by the voltage gain $(1 + R_3/R_2)$. The voltage over the photodiode is kept constant by placing it between the OP-amp inputs, referred to as bootstrapping. By including $C_2$, as seen in Fig. 3, the gain at lower frequencies and up to the passband can be expressed as

$$
\frac{V_{out}}{I_d} = R_3 + R_1 \left(1 + \frac{j\omega R_3 C_2}{j\omega R_2 C_2 + 1}\right). \tag{6}
$$

When deriving (6) we have assumed infinite open loop OP-amp gain. From now on, this assumption is always made when deriving transfer functions.

Adding only $C_2$ to the circuit in Fig. 2 reduces the gain at dc by reducing the voltage gain $1 + R_3/R_2$ to unity. The total gain at dc then equals $R_1 + R_3$, since the DC current generated in the photodiode will propagate through both of these resistances. In Fig. 3 the photodiode has also been replaced with an equivalent model consisting of a current source, shunt capacitor $C_D$ and a leakage resistance $R_D$. Note that a complete equivalent model should also include an ideal diode in parallel with the current source. $R_D$ is usually larger than $10^8 \Omega$ and can often be approximated as infinite. If the dc gain must be reduced even further, compared to (6), we can insert $C_4$ and $R_4$. The gain for the circuit at lower frequencies and up to the passband then equals

$$
\frac{V_{out}}{I_d} = R_3 + \frac{j\omega R_1 R_4 C_4}{j\omega C_4 (R_1 + R_4) + 1} \left(1 + \frac{j\omega R_3 C_2}{j\omega R_2 C_2 + 1}\right). \tag{7}
$$

Inserting $C_4$ grounds the OP-amp (+) input at dc, creating a virtual ground at the (-) input that reduces the dc gain for the entire circuit to simply $R_3$. However, this introduces
The complete transfer function with all capacitors included can be obtained from (7) and is expressed in (8), where $C_d = C_0 + C_{icm}$, as the common mode input capacitance of the OP appears in parallel with $C_0$. Fig. 4 displays the simulated transfer function and the ones obtained from (7) and (8). As (8) assumes infinite open loop gain, a slight difference between the calculated and the simulated gain can be observed at higher frequencies. OP-amp OPA655 and the component values in Table 1 were used in the simulation. The listed values of $C_{icm}$ and the differential input capacitance of the OP $C_{idm}$, used in the calculation, were obtained from the data sheet for the OPA655. The value of $R_3$ was selected based on tests done in our FMCW system.

With $C_4$ and $R_4$ inserted reverse bias should be added to avoid forward biasing of the photodiode. Reverse biasing can be accomplished by inserting a voltage source between $R_1$ and ground. One drawback associated with the bootstrapping placement is that the voltage obtained over $R_4$, and hence over both of the OP-amp inputs, is a common mode signal that will be amplified by the non-zero common mode gain of the OP-amp resulting in an unwanted contribution to $v_{out}$. It is also possible to remove the voltage gain by...
Table 1: Parameters used in the SPICE simulations and calculations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>22 kΩ</td>
</tr>
<tr>
<td>$R_2$</td>
<td>56 Ω</td>
</tr>
<tr>
<td>$R_3$</td>
<td>2.2 kΩ</td>
</tr>
<tr>
<td>$R_4$</td>
<td>10 MΩ</td>
</tr>
<tr>
<td>$R_D$</td>
<td>1 GΩ</td>
</tr>
<tr>
<td>$C_1$</td>
<td>10 pF</td>
</tr>
<tr>
<td>$C_2$</td>
<td>2.2 µF</td>
</tr>
<tr>
<td>$C_3$</td>
<td>10 pF</td>
</tr>
<tr>
<td>$C_4$</td>
<td>2.2 µF</td>
</tr>
<tr>
<td>$C_0$</td>
<td>10 pF</td>
</tr>
<tr>
<td>$C_{id}$</td>
<td>1.2 pF</td>
</tr>
<tr>
<td>$C_{icm}$</td>
<td>1.0 pF</td>
</tr>
</tbody>
</table>

Figure 4: The SPICE simulated transfer function and the calculated ones from (7) and (8).

shorting the output to the (-) input. Doing so, in conjunction with inserting $C_4$ and $R_4$, completely removes the gain at dc but also decreases the overall gain.

4.1 Noise performance

A complete noise analysis including all noise sources, as seen in Fig. 5, and transfer functions in the circuit is tedious due to the large number of components and the circuit layout. We have therefore chosen to focus on the main contributors separately.

The photodiode generates a noise current consisting of both thermal and shot noise. Usually the shot noise resulting from the dc current through the diode junction will be the dominant one. The noise current generated in the photodiode will be added to the useful signal current and both will be amplified by the circuits transfer function. It is hence not interesting from a circuit analysis point of view.

The gain for the root spectral density of the OP-amp input noise voltage $V_n$, neglecting $C_4$, assuming $R_1 >> R_2$ and infinite $R_D$, can be expressed as

$$
\frac{V_{no}}{V_n} = \left| 1 + \frac{j \omega R_3 C_2}{(j \omega C_2 R_2 + 1) (j \omega C_3 R_3 + 1)} \right| \left| \frac{1 + j \omega R_x (C_D + C_{id} + C_0 + C_1 + C_{icm})}{1 + j \omega R_x (C_0 + C_1 + C_{icm})} \right|,
$$

where $R_x = R_1 // R_4$. A SPICE simulation of the input voltage noise gain and the calculated one from (9) are displayed in Fig. 6.

The input voltage noise gain gives rise to noise gain peaking at higher frequencies. The voltage gain $(1+Z_3/Z_2)$ itself gives rise to one peak and the second factor in (9) gives
rise to a second peak. As the second factor will be multiplied by one as $Z_3/Z_2 \to 0$, the second peak will not be completely cut off until limited by the decreasing gain of the OP-amp at higher frequencies. The difference between the simulated and calculated curves are assumed to result from our assumption of infinite OP-amp gain at all frequencies. We also used a slightly larger $C_D$ than commonly found in photodiodes for these types of applications, to visualize the second peak around 1 MHz.

The transfer function of the root spectral density of the input current noise $I_{n+}$ to the output can be expressed as

$$\frac{V_{no}}{I_{n+}} = \left| \frac{R_x}{j\omega R_x C_x + 1} \right| \cdot \frac{j\omega C_2 R_3}{(j\omega R_2 C_2 + 1)(j\omega R_3 C_3 + 1)}; \quad (10)$$

where $C_x = C_1 + C_0 + C_{icm}$. The simulated and calculated current noise gains are displayed in Fig. 7. As $I_{n-}$ only propagates through $R_3/C_3$ without being amplified by the voltage gain its contribution will be significantly lower than that of $I_{n+}$. 

Figure 5: Noise model of our circuit without the noise sources associated with the photodiode.

Figure 6: The SPICE simulated and from (9) calculated noise voltage gains.
Thermal noise contributions in the circuit can be kept low by using relatively small resistance values. However, this may not always result in the largest signal to noise ration. The parallel combination of $R_4$ and $R_1$ will usually be the largest thermal noise contributer as the noise current $I_{nR_x} = \sqrt{4kT/R_x}$ appears in parallel with $I_{n+}$ and will hence be amplified with the same gain function.

### 4.2 Choice of components

Minimal gain at dc, without including any inductors, is achieved by minimizing $R_3$, and $R_1$ if $C_4$ is not used. By keeping both $R_1$, $R_3$ and $R_2$ relatively low, but $R_3 \gg R_2$, the dc gain can be kept low while the voltage gain still make a significant contribution to the overall gain. $R_2$ and $R_3$, $C_2$ would also have to be chosen to obtain the desired low frequency break points. The break point due to the zero equals $(2\pi R_3C_2)^{-1}$ and the one due to the pole $(2\pi R_2C_2)^{-1}$. If $R_1 \ll R_4$, $R_x \rightarrow R_1$ will determine the overall gain in the passband. $R_x$ also effects both the signal gain, as seen in (7), and the noise. Increasing $R_x$ increases the current noise gain as seen in (10), with the same amount as the useful signal gain. The thermal noise contribution from $R_x$ at the output increases with $\sqrt{R_x}$, as $R_x$ increases. Increasing $R_x$ also shifts the beginning of the noise gain peaking, due to the input voltage noise, to lower frequencies. If the input voltage noise peaking effect dominates the total noise spectra a larger value of $R_x$ and $R_3$ can improve the overall signal to noise ratio. Break points due to the capacitors $C_0$, $C_1$ and $C_3$ can be approximated as

$$f_{C_1} = \frac{1}{2\pi R_2C_1}, \quad f_{C_0} = \frac{1}{2\pi R_x(C_0 + C_{icm})},$$

and

$$f_{C_3} = \frac{1}{2\pi C_3R_3}.$$
5 Comparisons

The output noise of a regular current to voltage converter, also referred to as a transimpedance amplifier, consist of contributions from three different noise sources. First, the input voltage noise of the OP-amp will result in a noise gain peaking at higher frequencies according to

\[
\frac{V_{no}}{V_n} = \left| \frac{1 + j\omega R_f(C_D + C_{idm} + C_{isc} + C_f)}{1 + j\omega R_f C_f} \right|, \tag{11}
\]

where \(R_f\) and \(C_f\) are the feedback resistance and capacitance. Second, the input current noise of the OP-amp will propagate through \(R_f // C_f\) generating a noise voltage at the output. Third, \(R_f\) itself will also contribute thermal noise. Increasing \(R_f\) increases the gain but also amplifies the input current noise by the same amount and increases the self generated thermal noise at the output by \(\sqrt{R_f}\). The beginning of the noise gain peak due to the input voltage noise is also shifted to lower frequencies as \(R_f\) increases. If the noise contribution from the input voltage noise dominates the noise spectra, which is common, increasing \(R_f\) can improve the overall signal-to-noise ratio. However, the high gain at dc excludes a transimpedance circuit with very large gain as a practical solution in our case, unless one introduces a high pass filter after the photodiode, but before the op-amp. Then it is still possible to use this circuit and the drawbacks with this solution were discussed in section III.

If a cascade of two amplifier stages are used the dc level can be completely removed before the second stage by inserting a simple high pass link after the first stage. The first stage in the cascade usually consists of a current to voltage converter and the second stage commonly consisting of a non-inverting voltage amplifier. The gain in the first stage can be kept as high as possible without risking saturation and cutting of the useful signal. The noise generated by all three noise sources in the first stage, as mentioned above, will be amplified by the voltage gain of the second stage along with the input voltage noise of the second amplifier. In this case, the input voltage noise of the second amplifier does not generate any noise gain peaking at the output and the other noise sources generated by the second stage can often be neglected in comparison.

Compared to our circuit, the cascade has similar or slightly higher noise floor mainly due to the input voltage noise of the second OP-amp. The noise differences between the two solutions with the same bandwidth and gain can however often be neglected compared to the other noise sources in the system, as the laser and photodiode shot noise, laser phase noise and laser output noise [18]-[21]. If the same OP-amps are used in both the single and cascade case, the cascade solution has a higher theoretical maximum bandwidth for a given gain. Using only one amplifier will lower the power consumption and reduce the influence of external noise.

6 Realization

The circuit has been assembled and tested in our fiber based system built around a one-section 10 mW DFB laser at 1510 nm. We used the same components values as in Table
but a photodiode with $C_d = 1.5 \text{ pF}$. Our local oscillator signal was achieved by utilizing the refractive index difference between the fiber core and air. Measuring the noise spectra was somewhat complicated since external noise effects dominated the spectra. To verify the transfer function we amplitude modulated our laser and compared the gain in the passband to the gain at lower and higher frequencies. The measured and simulated results showed decent agreement with deviations up to about 15 percent. However, because of restrictions in the instruments we were not able to verify the lowest pole around 0.01 Hz, as seen in Fig. 4. In our setup, and with our suggested amplifier circuit, with a transimpedance gain of just below 1 MΩ in the passband, the output voltage offset due to the dc current $I_D$ was just below 200 mV. As the maximum supply voltage of our OP-amp is 5.5 V, a regular current to voltage converter with a transimpedance gain of only 56 kΩ saturated when tested in our lab system.

7 Conclusions

We have presented a single OP-amp solution that can combine minimal gain at dc with a high gain in the desired frequency region without the use of inductors. This makes it suitable for use in an optical self-mixing FMCW system. The circuit has been analyzed theoretically, simulated using SPICE and tested in our system. Unfortunately the simulated and theoretical noise results were difficult to verify practically. One drawback associated with adding $R_4$ and $C_4$ is that reverse biasing in most cases becomes necessary and the resulting reverse bias will depend on the generated dc current. Compared to a cascade of two OP-amps, the theoretical noise performance of our solution is similar or slightly better. Using only one amplifier can also reduce the power consumption, the size of the circuit and the influence of external noise. The cascade solution does, however, have a slightly higher theoretical bandwidth for a given gain.
References


Finding the thermal time constants of a DFB laser module used in an FMCW ranging system

Authors:
Daniel Nordin and Kalevi Hyypää

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Finding the thermal time constants of a DFB laser module used in an FMCW ranging system

Daniel Nordin and Kalevi Hyyppä

Abstract

In a frequency modulated continuous wave (FMCW) range and velocity-measuring system, knowledge of the thermal time constants of the laser module source is useful to obtain linear modulation. For each part of our DFB laser module a thermal resistance and capacitance were calculated, but measurements showed that our model was missing one significant time constant. FEMLAB simulations revealed that the missing time constant is of the same order of magnitude as the time it takes to heat up the area above the active region.

1 Introduction

Frequency modulated continuous wave (FMCW) is a range and radial velocity measuring technique well known from the field of radar applications. The reflected part of an outgoing frequency modulated wave (w1) is mixed with a local oscillator wave (w2), and as a result the frequency difference between the two signals can be obtained. If the frequency modulation is chosen as a triangular wave, both radial velocity and range of the target can be determined from the frequency difference, usually referred to as the beat frequency[1]. By using a tunable laser as the source, with the light frequency as the carrier frequency and a photodiode as detector, the beat frequency can be extracted directly from the photodiode current. The photodiode delivers a current proportional to the squared sum of the two electrical fields w1 and w2. One suitable type of light source for FMCW is a distributed feedback (DFB) laser diode. By modulating the current to the DFB laser the optical frequency can be modulated. Changes in the current affects the temperature of the laser, which in turn affects the wavelength. Knowledge of the thermal time constants involved is therefore valuable and for this purpose a simple SPICE equivalent temperature model of our DFB laser module was derived. Measurements were also made to verify the model but the shortest and most significant time constant was missing in the model. To investigate the origin of the missing time constant a simple FEMLAB thermal simulation of the laser body was performed. The simulation showed that the missing time constant is of the same order of magnitude as the time it takes to heat up the area between the active region and the top of the laser body.
# 2 The Laser Module

Our laser module consists of a 600 $\mu$m long single section DFB, with a lasing wavelength of 1.51 $\mu$m, placed in a Radians Inova KAP-10 module. The laser chip is mounted on a silicon carrier plate. The silicon plate is connected to a copper plate and the copper plate is mounted on a Peltier device connected to a much larger aluminium block. A thermistor is also connected to the copper plate. The large aluminium block is considered to be at ambient temperature.

## 2.1 The Spice temperature model

For each part of the laser module, including the laser chip and the active region, thermal capacitances and resistances have been calculated. The resulting equivalent thermal model is displayed in Fig. 1.

![Figure 1: The SPICE equivalent thermal model](image)

In the model voltage corresponds to temperature and current corresponds to power. Injecting a current, the only current not corresponding to power in the model, into the peltier device results in a current/energy flow into the cold side. $P_{th}$ can be calculated from $P_{th} = I_D V_D - \Phi$, where $I_D$ is the laser drive current, $V_D$ the laser voltage, $\Phi$ the optical output power and $T_{Amb}$ represents the external temperature. A resistor represents the thermal resistance of the material and determines a materials ability to transfer heat. The thermal resistance can be calculated from

$$R_{th} \approx \frac{1}{k} \left( \frac{\text{Thickness}}{\text{Area}} \right)$$

where $k$ is the thermal conductivity. The thermal conductivity depends on the material, temperature, geometry and other environmental factors, but it can often be treated as a constant, which we have done in our calculations. A capacitor represents the thermal capacitance for a certain thermal body. The thermal capacitance determines the materials ability to store energy in the form of heat. The thermal capacitance can be calculated from

$$C_{th} \approx C_s \cdot m = C_s \cdot \text{Volume} \cdot \text{Density}$$

where $C_s$ is the specific heat and $m$ is the mass of the body. $C_s$ depends somewhat on the initial temperature and the temperature interval, but can be assumed to be constant.
in most cases. The only things we know about our laser are the dimension of the laser body and the active area. The time constant for the active region was calculated to 1.7710-11 s, which is small enough to be neglected. To calculate the thermal resistance of the laser body we assumed that the whole body could be modelled as an Indium Phosphide (InP) cylinder with radius \( r_2 \). The active channel was modelled as a cylinder in the middle of the laser body with radius \( r_1 \). A cylinder has a lateral surface area equal to \( 2\pi r l \), where \( l \) in this case corresponds to the length of our laser chip and the radius \( r \) corresponds to the distance from the active channel to the point in the laser that we are considering. By integrating over the cylinder we obtained the following equation for the thermal resistance from the active channel to the outside of the laser body

\[
R_{\text{InP-laser}} \approx \int_{r_2}^{r_1} \frac{1}{2\pi k \cdot r \cdot l} dr = \frac{1}{2\pi k \cdot l} \ln \left( \frac{r_2}{r_1} \right)
\]

In our case \( r_2=50 \mu m \) and \( r_1=0.7 \mu m \). The material properties, dimensions and the calculated thermal components for the model displayed in Fig. 1 are listed in Table 1. Here \( r \) corresponds to the density, \( l \) to the length, \( w \) to the width and \( h \) to the height of the material. The dimensions for the active region are \( w=1.5 \mu m, h=90 \text{ nm}, l=469 \mu m \).

The thermal properties for the thermistor, \( R_{\text{Therm}}=125 \text{ K/W} \) and \( C_{\text{Therm}}=0.008 \text{ J/K} \), were obtained from the thermistor datasheet.

<table>
<thead>
<tr>
<th>Material</th>
<th>( k ) W/mK</th>
<th>( C ) J/K°C</th>
<th>( \rho ) Kg/m³</th>
<th>( l ) mm</th>
<th>( h ) mm</th>
<th>( w ) mm</th>
<th>( R ) K/W</th>
<th>( C ) J/K°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>InP-Body</td>
<td>68</td>
<td>310</td>
<td>4810</td>
<td>0.6</td>
<td>0.2</td>
<td>0.1</td>
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</tr>
<tr>
<td>Si</td>
<td>141.2</td>
<td>700</td>
<td>2330</td>
<td>11</td>
<td>10</td>
<td>5</td>
<td>0.032</td>
<td>0.09</td>
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<tr>
<td>Cu</td>
<td>386</td>
<td>385</td>
<td>8960</td>
<td>20</td>
<td>18</td>
<td>2</td>
<td>0.014</td>
<td>2.484</td>
</tr>
</tbody>
</table>

2.2 Measurements

Changes in the drive current affect the lasing wavelength mainly by means of two different mechanisms, the plasma effect and thermal effects[2]. The plasma effect refers to how the refractive index is affected by changes in the current density. Changes in the temperature affect both the cavity dimensions and the energy levels of the lasing material. The time constant resulting from the plasma effect is in the nanosecond region[3] and hence to short to be measurable with our measurement set up displayed in Fig. 2.

A current step of 7 mA was applied to the laser diode drive current and the obtained beat frequency \( f_b \), resulting from the optical path length difference \( T \) between the target and the air-fibre junction, as a function of time was recorded. The optical lasing frequency shift as a result of the step modulation can be written as

\[
\Delta f(t) \approx \Delta I \left( \frac{\partial f_0}{\partial I} + \frac{\partial f_1}{\partial I} \left( 1 - e^{-\frac{t}{\tau_1}} \right) + \frac{\partial f_2}{\partial I} \left( 1 - e^{-\frac{t}{\tau_2}} \right) \right)
\]
where $\partial f_1/\partial I$ and $\partial f_2/\partial I$ are the frequency deviations resulting from the two thermal time constants $\tau_1$ and $\tau_2$. $\partial f_0/\partial I$ corresponds to the frequency shift as a result from the plasma effect and the remaining thermal effects. If the time delay $T$ is much smaller than the time variations in frequency, the beat frequency can be expressed as $f_b \approx T \cdot df/dt$ [2]. By taking the time derivative of (4) an equation for the time dependency of the beat frequency can be obtained

$$f_b(t) \approx \Delta IT \left( \frac{1}{\tau_1} \cdot \frac{\partial f_1}{\partial I} e^{-\frac{t}{\tau_1}} + \frac{1}{\tau_2} \cdot \frac{\partial f_2}{\partial I} e^{-\frac{t}{\tau_2}} \right) \quad (5)$$

The measured beat frequency as a function of time was fitted to (5) using non-linear regression. Fig. 3 displays the result from a curve fit graphically and Table 2 contains the calculated parameters.

**Table 2: The parameters obtained from the non-linear regression curve fit.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\tau_1$ (µs)</th>
<th>$\tau_2$ (µs)</th>
<th>$\partial f_1/\partial I$ / (GHz/mA)</th>
<th>$\partial f_2/\partial I$ / (GHz/mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>12.4</td>
<td>319</td>
<td>0.495</td>
<td>0.718</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.57</td>
<td>68</td>
<td>0.001</td>
<td>0.096</td>
</tr>
</tbody>
</table>

$\tau_2$ is in the same order of magnitude as $R_{InP-Body}C_{InP-Body}$, but $\tau_1$ is missing in our model.

### 3 FEMLAB Simulation

To find the missing time constant $\tau_1$ we decided to simulate the laser body in FEMLAB. Since we were unable to obtain any detailed information about the laser chips internal structure, we had to use a microscope to detect the placement of the active region. Two photos from the microscope view of the output surface of the laser chip are displayed in Fig. 4. From these photos it is apparent that the active region is close to the top of the laser body and that the distance to the top is approximately 5 µm.
Figure 3: Measured time variation of the beat frequency as a function of time for 2 different measurement runs with $T = 3.36$ ns.

![Figure 3: Measured time variation of the beat frequency as a function of time for 2 different measurement runs with $T = 3.36$ ns.](image)

Figure 4: Visual inspection revealed the placement of the active region.

![Figure 4: Visual inspection revealed the placement of the active region.](image)

This information, along with the thermal properties of InP and the dimensions of the laser body and the active region, were used in the FEMLAB simulation. The initial temperature of the laser body and its surroundings was set to 290 K and the heat conductivity from the laser body to air was set to 8 mW/K. At $t(0)$ the temperature of the active region was increased with 1 K and the temperature profile of the laser body as a function of time was investigated. Fig. 5 displays the temperature as a function of time 2.5 $\mu$m above the active region. From Fig. 5 we can see that the temperature rise time in this region is of the same order of magnitude as our missing time constant $\tau_1$. 
In a so-called index-coupled DFB laser the Bragg region is located either below or above the active region. Since the Bragg region directly affects the wavelength of the laser it is reasonable to assume that the thermal time constant associated with the Bragg region will also affect the lasing wavelength. The simulated rise time for the temperature of the whole laser body is also in the same order of magnitude as the measured time constant $\tau_2$ and $R_{InP-Body}C_{InP-Body}$.

4 Conclusions

To find the thermal time constants of our laser module we derived a simple SPICE equivalent thermal model. The lab measurements showed that one important time constant was missing in our model. A FEMLAB simulation revealed that the missing time constant is of the same order of magnitude as the time it takes for the heat from the active area to raise the temperature in the region between the active area and the top of the laser body. Since this is where the Bragg section most likely will be found, it is reasonable to assume that this time is linked to the missing time constant.
References


Using a discrete thermal model to obtain a linear frequency ramping in an FMCW system

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Using a discrete thermal model to obtain a linear frequency ramping in an FMCW system

Daniel Nordin and Kalevi Hyyppä

Abstract

The lasing wavelength of a single section distributed feedback (DFB) laser diode can be modulated by modulating the drive current. This makes it possible to utilize the DFB laser diode in a frequency modulated continuous wave (FMCW) range and velocity measuring system. In FMCW, the frequency of the laser is ramped, and the frequency difference between the reflected wave and a local oscillator wave is monitored. For maximum performance the frequency ramping should be linear. Due to thermal phenomena, a linear ramping of the current seldom results in a linear ramping of the optical frequency. We have derived a discrete thermal model, using resistors and capacitors, of our laser module. The thermal model was then used as a starting point to model the frequency behavior of the laser and to derive modulation currents that resulted in a linear frequency ramping at some different modulation frequencies.

1 Introduction

Frequency modulated continuous wave (FMCW) is a range and radial velocity measuring technique well known from the field of radar applications. The reflected part of an outgoing frequency modulated wave (w1) is mixed with a local oscillator wave (w2), and as a result the frequency difference between the two signals can be obtained. If the frequency modulation wave form is triangular, both radial velocity and range of the target can be determined from the frequency difference between w1 and w2, usually referred to as the beat frequency.

By using a tunable laser as the source, with the light frequency as the carrier frequency and a photodiode as detector, the beat frequency can be extracted directly from the photodiode current. One suitable type of light source is the distributed feedback (DFB) laser diode. By modulating the current to the DFB laser the optical frequency can be modulated. Unfortunately, a linear ramping of the current seldom results in a linear ramping of the optical frequency. One solution to this problem is to use an optical reference path and divide the beat frequency from the reference path with the one obtained from the target [1]. A second approach is to use an electrical feedback path to control the current ramp [2]. The non-linear frequency response of a multi section DFB was also investigated and compensated for in [3].

Our approach presented in this paper, is to derive a model of the frequency behavior and use that model to find a suitable modulation current. This approach can be combined with the ones in [1] and [2].
2 Modulation and Detection

The simplest modulation scheme consists of a saw-tooth wave form that only allows range measurements to a stationary target. A more sophisticated FMCW scheme is displayed in Figure 1.

\[ f_1 = |f_R - f_D| \]
\[ f_2 = |-f_R - f_D| = |f_R + f_D|, \]

Figure 1: Triangular modulation scheme where the solid line corresponds to the local oscillator wave and the dashed line to the reflected wave. The lower part shows the corresponding beat frequencies.

It has the advantage that both range \( R \) and radial velocity can be calculated from two measured beat frequencies \( f_1 \) and \( f_2 \). If we assumes that the range and velocity of the target are constant during one modulation period, \( f_1 \) and \( f_2 \) can be expressed as [4]
where \( f_R \) is the magnitude of the frequency difference due to the range and \( f_D \) is the Doppler shift. If \( |f_D| < f_R \) the frequency \( f_R \) can be expressed as

\[
f_R = \frac{f_1 + f_2}{2} = \frac{4R\Delta f}{cT_{\text{mod}}},
\]

and \( f_D \) as

\[
f_D = \frac{f_2 - f_1}{2} = \frac{2v_t}{\lambda},
\]

where \( \lambda \) is the optical wavelength. The range \( R \) can be calculated from \( f_R \) and the radial velocity \( v_t \) can be calculated from \( f_D \). If the distance between source and target decreases, the sign of \( f_D \), and hence \( v_t \) will be positive [5]. For maximum performance, the frequency ramping should be as linear as possible. If the frequency ramping is non-linear, the beat frequency will vary during the ramping and this smearing will make the beat frequency harder to detect. See Section 6 for a more detailed explanation on how a non-linear frequency ramping affects the beat frequency.

When the reflected wave (w1) and the local oscillator wave (w2) are mixed on a photodiode, the photodiode current will be proportional to the squared sum of the electrical fields of w1 and w2. Since the photodiode has limited bandwidth, the optical frequency components will be filtered out and only the beat frequency component, corresponding to the frequency difference between w1 and w2, will be observable in the photo diode current.

### 3 Laser Sources, System and Performance

The most common type of lasers used in FMCW systems today are diode lasers of DFB (Distributed Feed Back) or DBR (Distributed Bragg Reflector) type, CO\(_2\) or Nd:YAG. The ideal source should operate in single mode, have long coherence length and high output power. Sub micrometer resolution over shorter distances has been reported by some authors [6], while others have achieved a maximum range of more then 18 km [7]. The modulation of the optical frequency can often be realized by modulating a current to the modulation section of the laser. Some lasers have special connections for this purpose, but other lasers lacking the modulation section can often be modulated by adding a modulation to the regular drive current. Our laser is a single section DFB and hence current modulation is added to the drive current.

Changes in the drive current affects the lasing wavelength by two different mechanisms, the plasma effect and thermal effects. The plasma effect refers to how changes in the current density affect the refractive index of the material, which in turn affects the wavelength. Changes in temperature affects both the optical gain profile of the material and the dimension of the material, both of which affects the lasing wavelength. The time constant associated with the plasma effect has been reported to be in the nanosecond region [8].
4 Modelling the frequency behavior

At modulation periods much longer then the time constant associated with the plasma effects, the thermal effects will dominate the dynamics of the frequency behavior. For this reason the thermal behavior of our laser module was investigated in [9]. The idea is to obtain a simple model of the temperature dynamics, and hence the frequency behavior of the laser module with respect to changes in the drive current.

![Figure 2: A simple cross section sketch of the laser modules different thermal parts. The parts are not drawn to scale.](image)

A sketch of our laser modules different thermal parts is shown in Figure 2. The DFB laser chip is mounted on a silicon plate and the silicon plate is mounted on a copper plate. The copper plate is mounted on a Peltier element and the other side of the Peltier element is connected to a large aluminium block. As a starting point for our model we calculated a thermal capacitance and resistance for each thermal part. The thermal resistance of a plate can be calculated approximately from

\[
R_{th} = \left( \frac{1}{k} \right) \left( \frac{\text{Thickness}}{\text{Area}} \right),
\]

where \( k \) is the thermal conductivity. The thermal conductivity depends on the material, temperature, geometry and other environmental factors, but in our calculations we have treated \( k \) as a constant. The thermal capacitance determines the material's ability to store energy in the form of heat and can approximately be calculated from

\[
C_{th} = C_s \cdot m,
\]

where \( C_s \) is the specific heat and \( m \) the mass of the body.

To calculate the thermal resistance of the laser body from the active channel to the surface, we assumed that the whole body could be modelled as an Indium Phosphide (InP) cylinder with radius \( r_2 \). The active channel was modelled as a cylinder in the
Table 1: Displays the data for the different thermal parts and the resulting component values.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\kappa$ ($\text{W m}^{-1}\text{K}^{-1}$)</th>
<th>$C_f$ ($\text{J K}^{-1}$)</th>
<th>$\rho$ ($\text{kg m}^{-3}$)</th>
<th>$l$ (mm)</th>
<th>$h$ (mm)</th>
<th>$w$ (mm)</th>
<th>$R$ (K)</th>
<th>$C$ ($\text{J K}^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inp-Body</td>
<td>68</td>
<td>310</td>
<td>4810</td>
<td>0.6</td>
<td>0.2</td>
<td>0.1</td>
<td>16.65</td>
<td>17.8 $\mu$</td>
</tr>
<tr>
<td>Si</td>
<td>141.2</td>
<td>700</td>
<td>2330</td>
<td>11</td>
<td>10</td>
<td>0.5</td>
<td>0.032</td>
<td>0.09</td>
</tr>
<tr>
<td>Cu</td>
<td>386</td>
<td>385</td>
<td>8960</td>
<td>20</td>
<td>18</td>
<td>2</td>
<td>0.014</td>
<td>2.484</td>
</tr>
</tbody>
</table>

middle of the laser body with radius $r_1$. A cylinder with radius $r$ has a lateral surface area equal to $2\pi rl$, where $l$ in this case corresponds to the length of our laser chip. By integrating over the cylinder we obtained the following equation for the thermal resistance from the active channel to the outside of the laser body

$$R_{\text{Laser}} = \frac{1}{k} \int_{r_1}^{r_2} \frac{1}{2\pi l r} \partial r = \frac{1}{2\pi kl} \ln \left( \frac{r_2}{r_1} \right).$$

The measurements reported in [9] showed that two thermal time constants had the largest influence on the frequency behavior. The shortest one was around 12.5 $\mu$s, and the second one around 320 $\mu$s. The time constant for the active region was calculated to 18 ps with our model. This is too small to be measurable with our equipment. The FEMLAB simulation, also reported in [9], of the laser body indicated that 12.5 $\mu$s was of the same order of magnitude as the time it takes to heat the Bragg region of the laser, while 320 $\mu$s is of the same order of magnitude as the calculated time constant of the whole laser chip. The calculated thermal components and the measured time constants became the starting point of our model as displayed in Figure 3. In the model, voltage corresponds to temperature and current corresponds to power. To account for both the 12.5 $\mu$s and the 320 $\mu$s measured time constants, we used four components, $R_1$, $R_2$, $C_1$ and $C_2$, to model the laser body. The silicon plate is modelled by $R_{\text{Si}}$ and $C_{\text{Si}}$, and the copper plate by $R_{\text{Cu}}$ and $C_{\text{Cu}}$. Since the model is a very crude approximation with few components for each thermal blocks, it is not obvious if $R_x$, where $x$ ∈ (1, 2, Cu, Si), should be placed to the left or the right of $C_x$. Hence we tried both placements, using the measurement set up described in Section 6, and kept the one in the figure, as it showed a much better agreement between the measured and simulated results.

![Figure 3: Model used to simulate the thermal behavior of the laser. The $R_1$, $R_2$, $C_1$ and $C_2$ components models the laser body. The silicon and copper plate are modelled by $R_{\text{Si}}$, $C_{\text{Si}}$, $R_{\text{Cu}}$ and $C_{\text{Cu}}$. In the model, voltage corresponds to temperature and current corresponds to power.](image)

The input power, represented by current source $P_{th}$, can be calculated from $P_{th} = I_D V_D - \Phi$, where $I_D$ is the laser drive current, $V_D$ the laser voltage, $\Phi$ the optical output.
power. The dc voltage source $T_{\text{Amb}}$ corresponds to the external temperature. The idea is that if the laser temperature is directly proportional to the lasing wavelength, the voltage at node at $T_{\text{Body}}$ in Figure 3 will be directly proportional to the lasers lasing wavelength. With an accurate model, it should be possible to replace the current source $P_{\text{th}}$ with a voltage source $T_{\text{th}}$, as in Figure 4, and linearly ramping voltage $T_{\text{th}}$ to simulate a linear ramping of the temperature/wavelength. But what we really want, is a linear ramping of the frequency. If the wavelength ramping is linear, it can be written as

$$\lambda(t) = \lambda_0 + \lambda_{\Delta} \cdot t,$$  \hspace{1cm} (1)

where $\lambda_0$ and $\lambda_{\Delta}$ are constants. For a linear frequency ramping $\delta f/\delta t$ should be constant. From $f = c/\lambda$ and (1) we can obtain

$$\frac{\delta f}{\delta t} = \frac{-c \cdot \lambda_{\Delta}}{\lambda_0^2 + \lambda_{\Delta}^2 \cdot t^2 + 2 \lambda_0 \cdot \lambda_{\Delta} \cdot t},$$  \hspace{1cm} (2)

which is approximately constant since $\lambda_0 \gg \lambda_{\Delta} \cdot t$. In our case $\lambda_0 \approx 1500000 \cdot \lambda_{\Delta} \cdot t$. From now on, we will hence make the assumption that a linear wavelength ramping results in a linear frequency ramping.

$\text{Figure 4: Approach used to obtain a current waveform that results in a linear ramping of the temperature and hence frequency.}$

The high and low values of $T_{\text{th}}$, which corresponds to the start and end voltages of the ramp, used when simulating the circuit in Figure 4, were obtained by simulating the circuit in Figure 3 with the selected values of $I_D$ and $V_D$ and record the high and low voltages at node $T_{\text{Body}}$. From the current supplied by $T_{\text{th}}$, $P_{\text{thc}}$ in Figure 4, the compensated drive current to the laser can be derived using

$$I_{\text{Dc}} = \frac{P_{\text{thc}} + \Phi}{V_D}.$$  \hspace{1cm} (3)

When deriving the compensated drive current $I_{\text{Dc}}$, the voltage-current and current-output-power relationships of the laser have to be accounted for. These relationships can often be modelled in the electrical circuit simulator used to simulate the thermal/wavelength model. With the current values used in our experiments, both the voltage-current and current-output-power relationships were approximately linear with $V_D = 1.45$ V at $I_D = 59.2$ mA, $V_D = 1.55$ V at $I_D = 72.4$ mA, $\Phi \approx 7$ mW at $I_D = 59.2$ mA and $\Phi \approx 7.4$ mW at $I_D = 72.4$ mA. Since the relationships were approximately
linear, we used a triangular voltage and current source to account for the effect of $V_D$ and $\Phi$, where the voltage source representing $V_D$ was ramped from 1.45 V to 1.55 V, and the current source representing $\Phi$ from 7.0 mA to 7.4 mA. In our case the drive circuit for the laser, which converts an input voltage to a drive current, also exhibited some non-linearities that had to be compensated for.

The calculated component values of the whole laser body, $R_{\text{InP-Body}}$ and $C_{\text{InP-Body}}$, were divided between $R_1$, $R_2$, $C_1$ and $C_2$ according to $R_2 = R_{\text{InP-Body}} - 6 \, \Omega$, $C_2 = C_{\text{InP-Body}} - 2 \, \mu F$, $R_1 = 6 \, \Omega$ and $C_1 = 2 \, \mu F$. These values, and the calculated component values for the Si and Cu plate in Table 1, were used as the initial values in our model.

## 5 Measurements

Figure 5 displays the measurement system. An arbitrary waveform/function generator was used to generate the voltage waveforms. The laser drive circuit then converts this voltage into a current. The beat frequency $f_b$, resulting from the optical round trip delay $T$, as a function of time was monitored with an oscilloscope. In the experiments reported in this section $T$ was equal to 2.7 ns. We also tested with $T = 7.1$ ns and $T = 1.1$ ns, and the linearization improvements were similar for all three values of $T$.

![Figure 5: The measurement system used to monitor the beat frequency behavior resulting from the optical round trip delay T.](image)

Figure 6 shows the oscilloscope trace obtained with a linear current ramp from 59.2 mA to 72.4 mA and with $T_{\text{mod}} = 2$ ms. As can be seen in the figure, $f_b$ is not constant during the ramp and there is always a delay at the beginning of the ramp before the first solid beat frequency period can be measured. In this case, the linear current ramping resulted in a non-linear frequency ramping, which caused $f_b$ to increase during the ramp. Figure 7 shows a crude example of the current, frequency and beat frequency behaviour for a case like this. When the current is increased, we expect the lasing wavelength to increase and the frequency to decrease. If $df/dt$ is approximately constant during $T$, then $f_b/T \approx df/dt$ and $f_b \approx T \cdot df/dt$. If the lasing frequency $f(t) \approx f_0 + A \cdot e^{t/\tau}$, where $f_0$ is the frequency at the turning point of the ramp and $A$ is either positive or negative, depending on if the frequency increases or decreases, then $f_0(t) \approx T \cdot f'(t) = T A \cdot e^{t/\tau}$. Since $f'(t) = \frac{T A}{\tau} e^{t/\tau}$ has the same sign as $f_0(t)$, we expect $|f_b|$, and hence the measured $f_b$, to increase on both the up and down slope of the current ramp.
Figure 6: The lower oscilloscope trace shows the beat frequency resulting from a linear current ramping. The upper trace shows the input voltage to the drive circuit.

Figure 7: Crude example of a linear current ramping that results in a non-linear frequency ramping, which causes $f_b$ to increase during both the up and down ramping of the current. If $df/dt$ is approximately constant during $T$, then $f_b/T \approx df/dt$.

The compensated modulation current obtained with our initial values for $R_1$, $C_1$, $R_2$ and $C_2$ resulted in a noticeable improvement compared to a linear ramping, but further optimization of the model was still needed. After some testing we ended up with $R_2 = 14 \Omega$, $C_2 = 45 \mu F$, $R_1 = 2 \Omega$ and $C_1 = 10 \mu F$. The resulting modulation current at $T_{mod} = 2$ ms is displayed in Figure 8, and Figure 9 displays the oscilloscope view.
As a comparison we have plotted $f_b$ vs time for both the regular current ramp and the compensated one in Figure 10. When measuring a beat frequency period, the uncertainty on each period measurement was around 1.5 $\mu$s. This uncertainty, or noise, remained constant for all the different values of $T_{\text{mod}}$ that we tried, and as a result, the measured $f_b$ values became more noisy at shorter beat frequency periods.

The approach was tested for $T_{\text{mod}} = 8$ ms, $T_{\text{mod}} = 4$ ms and $T_{\text{mod}} = 1$ ms. In each of these cases we obtained a very good linearization of the frequency modulation, similar to what we did for $T_{\text{mod}} = 2$ ms. At $T_{\text{mod}} = 0.5$ ms the result was somewhat worse compared to lower modulation frequencies, but still much improved compared to linear current ramping as can be seen in Figure 11.

We also tested with two different current amplitudes, and in both cases we obtained similar improvements as the one shown in Figure 10.
Figure 10: Displays the behavior of the beat frequency during the ramping of the current for $T_{\text{mod}} = 2$ ms. The dashed line is the result of a linear current ramping, while the solid line is the result obtained with the compensated modulation current.

6 Discussion

The model is not perfect and there are rooms for improvement. More components could be added to account for more time constants and even more time can be spent trying to find better component values. In all of our experiments there is always a delay at the start of the ramp before the first period can be observed. This is expected, because at the turning point of the ramp $\frac{\delta I}{\delta t} \to \infty$ and $f_b \to 0$. In this region the short time constants that are not included in our model, as the one associated with the plasma effect and the heating of the active region of the laser, will also have a larger influence on the beat frequency.

The $R*C$ time constants that worked well in our model, are not exactly the same as the 12.5 $\mu$s and 320 $\mu$s measured in [9], but they are in the same order of magnitude. In [9], the 12.5 $\mu$s and 320 $\mu$s time constants where obtained by applying a current step to the laser, assume that $f_b \approx T \cdot df/dt$, and monitoring the beat frequency using the same measurement setup as in Figure 5. It’s also worth noting that $C_1 + C_2$ is larger then the calculated capacitance for the whole laser body.
Figure 11: Displays the behavior of the beat frequency during the ramping of the current for $T_{mod} = 0.5 \text{ ms}$. The dashed line is the result of a linear current ramping, while the solid line is the result obtained with the compensated modulation current.

7 Conclusions

Starting with a thermal analysis of our laser module, a simple discrete module of the temperature behavior was derived. By assuming that the lasing frequency is directly proportional to the temperature, we used the model to obtain modulation currents that resulted in a linear ramping of the optical frequency. The approach was tested with some different modulation frequencies and current amplitudes with good result. Even if the result was not perfect at higher modulation frequencies, the linearity of the frequency sweep was noticeably improved compared to a triangular current ramping.
References


