Textbooks in mathematics education
a study of textbooks
as the potentially implemented curriculum

Monica Johansson
Textbooks in mathematics education

A study of textbooks
as the potentially implemented curriculum

by

Monica Johansson

Department of Mathematics
Luleå University of Technology
SE-971 87 Luleå, SWEDEN

November 2003
To my family
Abstracts

Textbooks are a most important feature of teaching mathematics in the classroom, in Sweden as well as in many other countries. For teachers and students, the textbooks often determine what is school mathematics and also what is mathematics. Previous studies on textbooks and the use of textbooks in teaching and learning mathematics raise important questions about textbooks as representations of the curriculum. One important question concerns their role as a link between curriculum and activities in classrooms. In this thesis, some international investigations in this connection are reviewed and analyzed. Moreover, in order to illustrate the role of textbooks as the potentially implemented curriculum, a content analysis of a textbook series, is conducted. The development of the textbook series, a commonly used schoolbook in Sweden, is portrayed in the light of the curriculum development. Some findings from the analysis of the textbooks show that the objective of mathematics, formulated in the national curriculum, are only partially realized.
Acknowledgments

First of all, I want to express my deepest gratitude to my three supervisors for their unconditional support and encouragement during this first half period of the graduate school program. Their wisdom and experience have contributed to this work as well as my personal development.

I want to thank my main supervisor, Professor Lars-Erik Persson, for many things. As an experienced supervisor he makes me feel safe and secure. From the very beginning, he had faith in me, and, for that I am most grateful. I also thank my co-supervisors Professor Barbro Grevholm and Professor Rudolf Sträßer. Their encouragement and hard work made it possible to finish this work. Not only have I learned about how to do research in didactics of mathematics, the numerous meetings with them have, hopefully, made me a better researcher as well as a better writer.

Riksbankens Jubileumsfond gives financial support to The Graduate School for Mathematics and Teaching Methods and I have the privilege to attend this national graduate school. I am most grateful for that and for the support from the management team and the other students of the graduate school. Thanks for joint courses and inspiring meetings.

Furthermore, I thank everyone at the Department of Mathematics at Luleå University of Technology for the inspiring atmosphere. In particular, I want to thank my colleagues Christina and Anna for many laughs and good company and for their support.

Finally, I want to thank my family. Many thanks to my husband Henry, my true love, he deserves special credit for his unconditional support, and our children, Andreas, Jenny, and Linnea for their love and for being who they are.
Contents

1. Introduction 1

2. Curriculum 4
   The concept curriculum 4
   Curriculum development 9
      Pressuring factors 10
      Obstacles 10
      Strategies to be employed 11
   The stages 13
   TIMSS Curriculum Study 14
   Concluding comments 19

3. Research on textbooks 20
   What is a mathematics textbook? 20
   Subject matter in research on textbooks 22
   Content and structure of textbooks 24
   The use of textbooks 28
   Educational authorities’ view of textbooks 31
   Some effects of reform-based curriculum material 33
   Concluding comments 38

4. On curriculum development in Sweden 40
   Background 40
   The status of textbooks 41
   Overview of the curriculum reforms 44
      From 1962 to 1969 44
      From 1969 to 1980 48
      From 1980 to 1994 50
      From 1994 and onwards? 52
5. **Curriculum reforms and textbooks 56**

   Research design 56
   Findings 65
     - Findings in the first step 65
     - Findings in the second step 68
   Summary and conclusion 73

6. **Summary, conclusions and recommendations 75**

   Suggestions for further studies 78

7. **References 80**

8. **Appendix 87**
List of figures

Figure 1: Textbooks and the tripartite model 6
Figure 2: The conceptual framework for TIMSS 16
Figure 3: The three aspects of the TIMSS frameworks 17
Figure 4: The three aspects and major categories of the mathematics framework 18
Figure 5: A grouping of the mathematics and science textbooks in the TIMSS 26
Figure 6: The triangle of educational philosophy 53
Figure 7: A comparison between the different types of curricula 55
Figure 8: The textbooks in the study 57
Figure 9: Example of blocks 58
Figure 10: The classical problem about the tower of Hanoi 60
Figure 11: Instructional narrative – with perspective code 61
Figure 12: Aspects of real life tasks 62
Figure 13: The topics of word problems 64
Figure 14: Physical features of the textbooks 65
Figure 15: Performance expectations in the textbooks 66
Figure 16: Block types in the textbooks 67
Figure 17: Block types in the textbooks 67
Figure 18: Word problems in the chapter on percentages 69
Figure 19: The realism of the textbooks tasks 70
Figure 20: The number of word problems in the chapter on percentages, grouped into area of interest 70
Figure 21: Interest areas in word problems on percentages 71
Figure 22: The topics in blocks with a perspective code 72
Figure 23: The number of blocks associated to the perspectives 73
1. Introduction

The general interest behind the present work is to contribute to understanding the role of textbooks in teaching and learning of mathematics.

Textbooks are a most important feature of teaching mathematics, in the classrooms in Sweden as well as in many other countries. This is natural considering that textbooks are designed for the purpose to help teachers to organize their teaching. In textbooks they can find the topics to be covered during the school year. The topics are perhaps ordered in a suitable way. What is more important, textbooks provide exercises and suggest activities to the students. In view of the fact that textbooks often determine what is school mathematics they also come to define what is mathematics, for teachers and students.

In a reform of the mathematics curriculum, textbooks can have a prominent position and role. Developing textbooks and curriculum material can be seen as a quick and easy way to change teaching. However, textbooks can also be regarded as an obstacle to the development. In fact, teachers’ reliance on textbooks has been a concern from time to time. Unlike teachers, textbooks authors and publishers do not necessarily follow the national curriculum, at least if there is no evaluation of textbooks done by a central authority. Considering that in most countries publishing is business, the purpose of design and production of textbooks is more likely to be intended to get a bigger share of the market than to follow guidelines from a curriculum.

Internationally, there is research on mathematics textbooks and the use of textbooks. However, it is still a rather unexplored field, especially in Sweden and especially regarding textbooks for compulsory school. So, even if textbooks cannot be blamed for all kind of problems associated with teaching of mathematics, we need to learn more about their role and influence. If it is true that ‘mathematics is for teachers and students simply what is written in the textbooks’, then it is important to know what is written in the textbook.

---

1 See Skolverket, 2003, p. 28.
The conceptual framework for this work is based on the view of the textbooks as mediators between general intentions and classroom instruction – the potentially implemented curriculum. The global questions are: In what way can textbooks be regarded as a tool to accomplish educational goals? Can they be a link between objectives and reality? To what extent are textbooks the potentially implemented curriculum?

Chapter 2 offers a characterization and describes some levels of the concept curriculum. It also reports on influential aspects in curriculum development. Furthermore, a major part of this chapter is devoted to the Third International Mathematic and Science Study, TIMSS, and the related curriculum analysis.

Chapter 3 starts with a discussion about textbooks and offers a definition. What is a mathematics textbook? Thereafter, some results of previous research on textbooks are described. Topics that are reported on are the content of textbooks and the use of them in the teaching of mathematics. In addition, it entails a discussion about the view of textbooks and the role of textbooks in curriculum reforms.

The study of the development of textbooks should be portrayed in the light of the curriculum development. Therefore, there is a brief description of the curriculum reforms in Sweden in Chapter 4. It begins with the reform in 1962 when the first step toward a coherent compulsory school was taken and ends with a short description of the current school system.

Chapter 5 describes the development of a specific mathematics textbook series in relation to the development of the curriculum. One purpose of the case study is to find connections between two levels of the curriculum, the intended and the potentially implemented. Documents related to the intended curriculum of Sweden, namely the läroplan and the kursplan, and a representative of the potentially implemented curriculum, namely a mathematics textbook series, are examined.

The case study explores how one specific textbook series changed when parts of the intended curriculum in Sweden were revised. Different editions of the textbook series were compared illustrating some changes in school mathematics in Sweden during the last twenty-five years. The objective was to find evidence of relations between the intended curriculum in mathematics, especially in relation to the revisions of the läroplan in 1969,
1980 and 1994, and the textbook series. Another purpose of the case study is to look into the evolution of a textbook.

In the concluding chapter, Chapter 6, the study is discussed in general terms and some conclusions are offered. It also contains some recommendations for further research.
2. Curriculum

Curriculum can be considered as a multifaceted concept with each aspect linked to a context or level of educational activity (e.g., Robitaille et al., 1993). The aim of this chapter is to present a characterization and some levels of the concept curriculum and to describe factors that influence the development of curricula. Furthermore, a major part of this chapter is devoted to the TIMSS Curriculum Study. The relation between the case study described in chapter four and the TIMSS Curriculum Study will be identified.

The concept curriculum

To define the term curriculum has been a concern in several studies of educationalists, nevertheless, a unanimous definition is difficult to offer. A narrow view of the curriculum development as merely the production of new syllabuses and texts is though unhelpful for the development. Neither content nor method can be viewed in isolation – the assessment of the course and the students must also be considered (Howson, Keitel, & Kilpatrick, 1981).

Curriculum, therefore, must mean more than syllabus – it must encompass aims, content, methods and assessment procedures. One can not truly talk, then, of a ‘national curriculum’ for it depends upon individual teachers, their methods and understanding, and their interpretation of aims, guidelines, texts, etc. (Howson et al., 1981, p. 2).

If I ought to provide a definition of the curriculum I would use Kilpatrick’s’:

The curriculum can be seen as an amalgam of goals, content, instruction, assessment and materials (Kilpatrick, 1996, p. 7).

In the Third International Mathematics and Science Study, TIMSS², as well as in other studies of the International Association for the Evaluation of

---

² The TIMSS was a cross national study that took place in about 50 countries in 1994/5.
Educational Achievement, IEA, curriculum is considered in a broad meaning. The different levels of the curriculum are, in the IEA model, labeled as the implemented, intended and attained curriculum (Robitaille et al., 1993). A fourth level of the concept curriculum, the potentially implemented curriculum, is later added to the model (Schmidt et al., 2001; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002).

**Intended curriculum:**
The intended curriculum is at the educational system level. It is seen in national policies and official documents reflecting societal visions, educational planning, and official or political sanctioning for educational objectives (Robitaille et al., 1993; Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). In the curriculum study of TIMSS, written documents were collected and analyzed in order to describe the intended curriculum in the participating countries “… the intended curriculum is embodied in textbooks, in curriculum guides, in the content of examinations, and in policies, regulations, and other official statements generated to direct the educational system” (Robitaille et al., 1993, p. 27).

**Implemented curriculum:**
The implemented curriculum is at the classroom level. Thus, intention and objectives at the level of teacher and classroom activity are considered as the implemented curriculum (Schmidt et al., 1997). The implemented curriculum is influenced by, but not identical to, the intended curriculum. It is the mathematics content as it is interpreted by teachers and made available to students. The implemented curriculum concerns institutional arrangements such as teaching practice, aspects of classroom management, use of resources, teacher attitudes and teacher background (Robitaille et al., 1993).

**Attained curriculum:**
The attained curriculum is on the student level. The result of what takes place in classrooms at the level of student outcomes is therefore considered as the attained curriculum. Part of these student attainments can be documented through academic achievement and student belief measures (Schmidt et al., 1997). The attained curriculum also concerns institutional arrangements students make for their own learning. At the individual level it is about the amount of homework the student does, the effort the student expends, the student’s classroom behavior patterns, and so on. These fac-
tors are greatly influenced by both the system- and classroom-level arrangements but differ in that the individual student has some control over these arrangements (Robitaille et al., 1993). Influential factors are for instance “…the attitudes about education that students bring to school, their aspirations, their perceptions of their own ability to succeed, parental expectations for their success, and the economic well-being of their families” (Robitaille et al., 1993, p. 29).

**Potentially implemented curriculum:**
The official status, design and use of textbooks vary greatly among countries participating in the TIMSS. However, textbooks are an instantly recognizable resource in classrooms and they and other organized resource materials are therefore regarded as the *potentially implemented* curriculum in the TIMSS. “Textbooks served as intermediaries in turning intentions into implementations. They helped make possible one or more potential implementations of mathematics curricular intentions” (Schmidt et al., 1997, p. 178).

![Figure 1: Textbooks and the tripartite model](image)

An alternative to the partitioning of the concept curriculum offered by TIMSS is introduced by Porter and Smithson (2001). They work on the de-

---

velopment and expansion of curriculum indicators, with the objective to measure instructional content and its relationship to assessment and standards. They distinguish between the intended, enacted, assessed, and learned curriculum. For them, as well as in the IEA studies, the intended curriculum is seen in policy tools as curriculum standards, frameworks, or guidelines that outline the curriculum teachers are expected to deliver. They maintain that there are two important types of information to consider when examining the intended curriculum: the description of the curriculum in the policy documents and the characteristic of the policy documents themselves.

**Enacted curriculum:**
The enacted curriculum is linked to the actual curricula that students in the classroom engage in, in other words the classroom practice (Porter & Smithson, 2001). The enacted curriculum is the context where most learning occurs and it is therefore seen as an important feature of curriculum indicator systems. However, Porter and Smithson comment that the “Description of the enacted curriculum still lie at the heart of our work” (p. 2) and that they look at “… the intended, assessed, and learned curricula in combination with the enacted curriculum in order to describe the context within which instruction occurs” (ibid.).

**Assessed curriculum:**
The assessed curriculum is represented by high-stakes tests. Porter and Smithson emphasized that a curriculum indicator system should be able to expose the tension between the assessed curriculum, represented by high-stakes tests, and the intended curriculum, represented by curriculum standards, frameworks, or guidelines.

**Learned curriculum:**
The learned curriculum is defined as all knowledge that students acquire as a result of their schooling experience. The result on high-stakes test may provide a moderate measure of student learning but as an isolated factor they tell us little about the learned curriculum (Porter & Smithson, 2001). “… indicator measures of the learned curriculum need to describe the content that has been learned as well as the level of proficiency offered by test scores” (p. 3).
The aspects of the curriculum, described above, are only some of the dimensions of the concept. Further facets are, for instance, the hidden curriculum⁴.

Hidden curriculum:
The hidden curriculum is a widely known expression – a metaphor to describe the unintended curriculum, that is what students learn from the culture and climate of schools. The reward system of the school is linked to success in both the ‘official’ and the hidden curriculum. The ‘official’ curriculum can for example emphasize independent thinking, and problem-solving ability. Where as, from the student viewpoint, assessment and teaching procedures suggest memorizing facts and theories to achieve success.

Testmakers describe a person as ‘test-wise’ when he has caught on to the tricks of the test construction sufficiently well to answer questions correctly even though he does not know the material on which he is being examined (Jackson, 1990, p. 35)

Examples of the different levels of the curriculum – the case of Sweden
There is no single corresponding term for the curriculum, in its extensive meaning, in the Swedish language (Svingby, 1978). Representatives for the intended curriculum in Sweden are the official documents, offered by the National Agency of Schools, Skolverket. These are three documents, the Education Act, Skollagen, which include the timetable, the curriculum, läroplanen, and the syllabus, kursplanen (Skolverket, 2001b)⁵. However, these are not the only representatives of the intended curriculum. The national tests are an additional example of how the intended curriculum is made recognizable. The results on these tests can be seen as representatives of the assessed curriculum and serve as contributors to the evaluation of the attained and the learned curriculum. Textbooks can, to some extent and in several classrooms, be regarded as the implemented curriculum and associate to the enacted curriculum, at least for students in year 4-5 onward⁶. Beliefs about the nature of mathematics are somewhat implicit in

---

⁵ These are the official translation from Swedish to English, retrieved from www.skolverket.se.
⁶ This will be discussed in chapter four and five.
textbooks. Thus, if textbooks are the implemented curriculum then textbooks analysis can tell us something about the hidden curriculum.

**Curriculum development**

Initiative for a development of a curriculum can be local in nature – a group of schools in a region, a single school or an individual teacher. But it can also be a large-scale movement (Howson et al., 1981). In the 1960s and early 1970s there was a considerable interest in research on curriculum, especially curriculum as a social construction. This was, however, a period of extensive change in the Western world and in the world of schooling. Much of the research was normative due to the desire of influential scholars to reform education. At the same time, a wide range of curriculum reform movements were actively seeking ‘to revolutionize school curricula’ (Goodson, 1994, p. 17). “Therefore it was unlikely that scholars or curriculum reformers would wish to focus upon, let alone concede, the areas of stability, of unchallengeable ‘high ground’ that may have existed within the school curriculum” (ibid.). In a rather poetical way, Goodson expressed his view of curriculum reforms of that time:

One might characterize curriculum reform in the 1960s as a sort of ‘tidal wave’. Everywhere the waves created turbulence and activity but actually they only engulfed a few small islands; more substantial land masses were hardly affected at all, and on dry land the mountains, the high ground, remained completely untouched. As the tide now rapidly recedes the high ground can be seen in stark silhouette. If nothing else, our scrutiny of the curriculum reform should allow recognition that there is not only high ground but common ground in the world of curriculum (Goodson, 1994, p. 17).

The characteristics of curriculum development in mathematics are described by Howson, Keitel and Kilpatrick (1981). They argue that curriculum developers should not ignore the social, political and educational systems in which the school curriculum is embedded. To introduce a ready-made successful curriculum that has worked in another country and in some other context is rarely beneficial, such imports seldom survive the move. Curricula are developed in response to specific pressures, dependent on time and place. How well they might work in another context depends in part on the similarity between the two contexts, the one of the originators
and the one of the receivers (Howson et al., 1981). The following four subsections consist of a summary of the analysis of Howson, Keitel and Kilpatrick (1981) concerning: (a) pressuring factors that initiate curriculum development; (b) obstacles of curriculum development; (c) strategies to be employed when implementing new curricula; and (d) stages of curriculum development.

**Pressuring factors**

Pressure that initiates curriculum development can arise from: the society; through the mathematical development; within the educational system; and from individual or groups of innovators. Most of all, significant pressure may be a consequence of the economical and technological development of a society. Progress in these areas has not only meant that society can provide more education, but that it must do so.

It is a result of societal pressures that secondary education is obtainable for a larger group of people and no longer exclusive for a selected minority. The technological and economical development during the 20th century has caused changes in mathematics as well. For example, mathematics has found a wide variety of new applications and the improvement of computers has revolutionized numerical mathematics. University courses have been revised – and changes at one level of education put pressure on lower levels.

When new technology is available, for instance computers and hand-held calculators, pressure for change can arise within the educational system. It can also arise as a result of research, new educational theories, or the pioneering developmental work of individuals.

On the micro-level, individual educators’ desire to change and act as innovators could be a starting point for a development of the curriculum, since many educators welcome the opportunity to deviate from practice that is becoming routine. On the macro-level, countries fear that they will be left behind is yet another supporting factor in the initial phase.

**Obstacles**

Different types of barriers hinder the development of a curriculum. Howson et al. (1981) categorized them as: value barriers; power barriers; practical barriers; and psychological barriers.
[...] radical changes in the curriculum occur somewhat infrequently and many attempted changes fail to disturb the equilibrium that would appear to exist in the classroom. One is almost tempted to believe that there is a curricular analogue to Newton’s Third Law ‘To every action there is an equal and opposite reaction’ (Howson et al., 1981, p. 6).

Value barriers, such as reactions to mixed-ability teaching, can impede changes. So also the attitudes toward mathematics as a school subject. For instance, to what extent it is seen as a service subject with the aim of supporting science teachers and prospective employers. Moreover, curriculum development can shift the balance of power, sometimes away from the teacher, sometimes away from central authority. A desire to exercise more power can be expressed by universities, parents, students and employers.

Economical concerns are considerable practical barriers toward reform of curricula. Suppose a new topic in mathematics, for instance category theory, will be put on the agenda of school mathematics. It is extremely expensive and perhaps logistically impossible to mount an in-service teacher education program to really prepare teachers for that. Additional money and personnel would be needed to produce the necessary textbooks and new revised examinations. Some traditional topic would have to be omitted to make room for the new one. Practical problems, such as lack of educated teachers, could also arise when the mathematics courses changes from optional to obligatory in the educational programs.

Furthermore, there will often be psychological barriers to overcome before an innovation is accepted. Innovation implies risk-taking and the known provides some degree of security. A powerful barrier to curriculum development concerns incentives, or the lack of incentives. The innovator can most likely identify spurs and motives for a change but what about the ordinary teacher asked to adopt the innovation?

**Strategies to be employed**

Howson et al. (1981) distinguished between three strategies to be employed: the *power-coercive*, the *rational-empirical* and the *re-educative*.
**The power-coercive strategy:**
The power-coercive strategy is in use when educational authority (local or national), a head teacher or a head of department uses organizational authority to impose an innovation on subordinates.

**The rational-empirical strategy:**
The rational-empirical strategy involves emotional methods with arguments such as ‘if you cared for pupils,…if you were up-to-date, you would be…’. But also concrete methods, like having a higher pass rate for the ‘modern’ syllabus in the centrally controlled public examinations and paying higher salaries to those who teach the new courses.

**The re-educative strategy:**
The re-educative strategy attempts to change the attitudes, skills and values of those within the educational system in the belief that this will activate them to become innovators.

The choice of strategy are somehow related to the educational system.

Generally speaking one finds that innovations are disseminated throughout centralized national or local education systems by means of the two coercive strategies, whilst ‘large scale’ projects working in non-centralized systems make greater use of ‘rational-empirical’ arguments. ‘Local’ projects will tend to use the re-educative strategy in which in-service education and curriculum development are closely coordinated (Howson et al., 1981, p.11).

Howson et al. (1981) believed that the role and status of the teachers within the educational system could be deduced from the strategy employed. Moreover, teachers’ initial acceptance of innovation and commitment to it when difficulties arise will be crucially dependent on how the innovations are disseminated: “If the teacher visualizes himself as the servant of the system, to be told what to teach (and, preferably, when), then he is unlikely to use successfully materials devised by innovators who see the teacher as an autonomous producer/director of what happens in the classroom” (p.12).
The stages
Howson et al. (1981) describes the different stages of curriculum development, here presented in the order they occur.

The identification stage:
In the first phase, a need or a possibility is identified, usually in response to a particular pressure.

The formulation stage:
Then next step will be to formulate a course of action.

The acceptance stage:
After the strategy has been formulated there is usually a need to convince other interest groups, such as schools, teachers and financiers, to accept and support the suggested innovation. “There is a need for negotiation, a need which is likely constantly to recur” (Howson et al., 1981, p. 14).

The development stage:
A key stage in most curriculum development projects is the preparation of new materials and other physical resources. Sometimes the work is based on existing research findings, in other cases research work has to be commissioned.

The dissemination stage:
If an innovation is to be adopted throughout an educational system then many teachers will have to be introduced to the new material and methods.

The implementation stage:
Once innovation has been accepted there is a need to help sustain it over a period of time. In the implementation stage, as in earlier stages, there will be a need to consider and overcome potential barriers to change, and to adopt a suitable strategy for propagation.

The evaluation stage:
In addition to the formative evaluation that occurs at the developmental stage, summative and comparative evaluation will also take place. Actually, such evaluation can lead to the identification of new problems and consequently help to initiate another round of innovation.
TIMSS Curriculum Study

Results from cross national comparisons of curricula and students’ achievement can also be a starting point for curriculum development, especially for countries at the lower half of the ranking list. However, the main purpose with the Third International Mathematics and Science Study, TIMSS, is not to describe the achievement of the students in the participating countries in terms of high or low. The aim of the study is rather to explain resulting differences in terms of variations in curriculum, instructional practices, or some other variables. Thus, countries are provided with means to evaluate their own curricula in the light of the outcomes achieved by their students and, if required, instruments to guide suitable educational changes (Robitaille et al., 1993).

The concept Opportunity To Learn, OTL, could be understood as “… the configuration of social, political and pedagogical conditions to provide pupils chances to acquire knowledge, to develop skills and to form attitudes concerning school subjects” (Valverde et al., 2002, p. 6). This concept has, in former studies of the IEA, and others, been measured by having teachers examine the test beforehand and report whether the students have been taught the skills necessary to carry out the items on the test sufficiently well (Bianchi, Houang, Babcock, & Schmidt, 1998; Robitaille et al., 1993). This was the case in the First International Mathematics Study, FIMS, which took place in twelve countries in 1964, as well as in the Second International Mathematics Study, SIMS, which took place in twenty countries in 1980-1982 (Travers & Westbury, 1989).

Freudenthal (1975) was a tough critic of the way IEA applied the concept OTL and how they organized FIMS. He claimed that the influence of mathematicians and mathematics educationalists was negligible in the preparation phase of the test. He questioned why the organizers did not avail themselves with experts of mathematics education, for instance The International Commission on Mathematical Instruction, ICMI. In addition, Freudenthal hesitated about the validity and the objectivity of the test:

Though it may be correct that pupils in The Netherlands learned as little as 60-70 % of the mathematics represented in the test instruments, it is no less true that they learned quite a lot more in mathematics – I estimate twice or thrice that represented in the test instruments – and similar statements can be made with re-
spect to pupils of other countries. I am sorry – the variable “opportunity to learn” is suggesting – this does not count. … Who decides that the test instruments of IEA coincide with the 100 % opportunity to learn? Commissions that include one outstanding didactician of mathematics? An irresponsible team of test editors? Or the country that protests the most forcefully? (Freudenthal, 1975, p. 139).

TIMSS curriculum analysis is conducted as an attempt to use a more informative concept of educational opportunity (Bianchi et al., 1998). The starting point of the model of educational opportunities is a tripartite model of curriculum with an analytic distinction between curriculum as system goals, curriculum as instruction and curriculum as student achievement. In that order, the three dimensions correspond to the concepts the intended curriculum, the implemented curriculum and the attained curriculum (Robitaille et al., 1993; Valverde et al., 2002).

Work on the mathematics curriculum framework for TIMSS began in 1989 and was mainly concentrated on the development of a methodology for conducting an analysis of textbooks. A common framework was needed in order to make a valid analysis of curricula and meaningful comparisons of countries with widely different educational environments. The TIMSS curriculum framework had two main purposes. On the one hand, to provide a common basis for the analysis of curricular intentions and documents that educational systems use. On the other hand, to provide guidance for the design of test components intended for assessing the performance of students (Robitaille et al., 1993).

In TIMSS, as well as in previous IEA studies, the curriculum was considered as a complex construct with several facets, each linked to a context or level of educational activity: the implemented, intended and attained curriculum described in the beginning of this chapter. In the conceptual model of TIMSS (Figure 2), the variables influencing education are seen as situated in a sequence of contexts – starting from the most global (society) and moving to the most personal. The narrow contexts are influenced, but not only subsets, of the broader contexts in which they are embedded (Robitaille et al., 1993).
In the TIMSS Curriculum Study, textbooks, syllabi, and curriculum guides are analyzed as representatives of the intended curriculum. A considerable amount of these kinds of written documents were collected from the about 50 participating countries. The analysis of the implemented curricula relies primarily on responses to questionnaires by teachers and other school personnel. The attained curriculum, the outcomes of the students, was mainly evaluated through multiple-choice and free-response items, but a small number of so-called ‘performance tasks’ were also included (Robitaille et al., 1993).

Each section of every textbook, syllabus and curriculum guide in the curriculum analysis is characterized in terms of three parameters: subject matter content; performance expectations; and perspectives or context. These are the three dimensions or aspects of the TIMSS framework (Figure 3). Each test item or block of content is coded relatively to one or more of the

---

8 The resulting sample included for instance 241 curriculum guides and 318 mathematics textbooks (Schmidt et al., 1997).
three aspects and to one or more categories within each aspect (Robitaille et al., 1993).

![Diagram of TIMSS frameworks]

**Figure 3: The three aspects of the TIMSS frameworks**

The *content* aspect represents the content of school mathematics, partitioned into ten major categories (Figure 4), each with two to twenty subcategories.

The *performance expectations* aspect describes the kind of performance that students will be expected to demonstrate while engaged with the content. There are five main categories: knowing, using routine procedures, investigating and problem solving, mathematical reasoning, and communicating.

The *perspectives* aspect has particular relevance for the analysis of documents such as textbooks. It is intended to illustrate curricular goals that focus on the development of students’ attitudes, interests, and motivations in mathematics teaching. This aspect makes it possible to describe learning outcomes or curriculum materials that are intended to promote positive attitudes and mathematical modes of thought or habits of mind, as well as goals that are encouraging toward careers in mathematics, science, or technology. Intended learning experiences that promote participation of

---

9 Robitaille et al., 1993, p. 43.
groups currently under-represented in these fields (for example, girls) are also included.

The TIMSS framework included an analytic distinction between the intended, implemented and attained curriculum. The important role of textbooks as mediators between general intentions and classroom instruction lead to the introduction of a fourth dimension, the potentially implemented curriculum, an additional important component, added to the model (see figure 1), with a feature that involved textbooks and other organized resource materials. Within this conceptual framework, textbooks and other organized resource materials play a key role as mediators between the intended and the implemented curriculum (Schmidt et al., 1997; Valverde et al., 2002). The case study, described in chapter five, gives an example of the relation between a specific textbook series and the intended curriculum in Sweden. By looking at the influence of the intended curriculum on textbooks, a more illustrative picture of the role of textbooks in the model and in mathematics education can be given.

---

10 Robitaille et al., 1993, p. 46.
Concluding comments

The initiative for a development of the curriculum is usually emerging from forces within society. International comparative studies of students’ achievement, like the TIMSS, can be a starting point for a reform of the curriculum. The fear to be ‘left behind’ when other countries improve their education is especially noticeable when results from such studies are released.

The participating countries of the TIMSS reacted in a variety of ways to the accomplishment of their students. Macnab (2000) conducted a survey based on a questionnaire and information from TIMSS country reports, official documents, and related sources. He found that the reaction to the TIMSS was especially extensive and intense by the United States. From the test results it appeared that the United States average scores in mathematics were below the international average score for two of the three populations in the TIMSS. The considerable interest in the outcome of TIMSS in the US has resulted in a national curriculum development program, *Attaining Excellence*, which has been prepared with the aim of improving achievement in mathematics and science. The US results of the TIMSS have also been analyzed, discussed and documented in books.\(^{11}\)

However, the reaction from Argentina was at the other extreme – the results were not reported in official TIMSS reports and the governmental interest in the outcomes was low. A number of countries, for instance Sweden, reported on ongoing curricular change in mathematics education but not directly because of TIMSS (Macnab, 2000). Reports with results from the TIMSS have also been published by the National Agency of Schools in Sweden.\(^{12}\)

\(^{11}\) See for instance *A Splintered Vision* and *Facing the Consequences* by Schmidt et al. (1997, 1998).

3. Research on textbooks

What is a mathematics textbook?

To begin with, we need a definition of textbooks. What is a mathematics textbook? In a broad sense it can be a complex package of materials. It can include books, booklets, work cards and worksheets. But it can also have associated material like teachers’ guides or computer software. However, textbooks in form of fairly large and printed objects, which intend to guide students’ work throughout the year, are probably the most common form of textbooks. For this study, the concept textbook refers to the latter type of material. In addition, I will use the definition offered by Stray. For him, a textbook is:

[... ] a book designed to provide an authoritative pedagogic version of an area of knowledge (Stray, 1994, p. 2).

Textbooks play an important role in mathematics education because of their close relation to classroom instruction. They identify the topics and order them in a way students should explore them. They also attempt to specify how classroom lessons can be structured with suitable exercises and activities. In some sense they provide an interpretation of mathematics to teachers, students and their parents. Moreover, textbooks have a prominent position in curriculum reforms and are considered as the most important tool for the implementation of a new curriculum in many countries (Valverde et al., 2002).

In Sweden, an evaluation of schools in 40 municipalities (out of 290) was done in the years 2001-2002. An observation that is reported from the inspectors concerns textbooks. They noticed that from year 4-5 and onwards, the teaching of mathematics is in principle based on the use of textbooks and states that “Mathematics is, for both students and teachers, simply what is written in the textbook” (Skolverket, 2003, p. 39, my translation). Hence, the textbook seems to be the definition of ‘school mathematics’ as well as the ‘learning path’ for the majority of students, at least in lower and upper secondary school.

---

13 A detailed discussion about pedagogical texts can be found in Selander (1988), p. 17-18.
14 See for instance the analysis of textbooks from 48 different countries, Valverde et al. (2002).
A strong reliance on textbooks in the teaching of mathematics is not a phenomenon exclusive of Sweden. Mathematics textbooks are widely used in countries all over the world.

There is a good deal of evidence that many teachers like the security and freedom from responsibility that a text series provides. [...] when using a text series, teachers need not involve themselves in ordering the topics, in ensuring that notation is consistent, nor in concerning themselves whether a student will have met the necessary pre-requisites for a new topic (Love & Pimm, 1996, p. 384).

An ordinary organization of text in a textbook is the ‘exposition-examples-exercises’ model. It is appropriate for classroom practices and facilitates a flexible use by teachers. In the exposition part of the textbooks, the authors may see their task as support students’ concept formation. Perhaps they present material or situations to be worked on which are intended to elicit the formation of concepts, sequences of tasks which can be described as ‘guided discovery’ (Love & Pimm, 1996). However, the text is usually moving to a particular destination and “... students are often impatient with the exposition and skip to the ‘essential’ results (Love & Pimm, 1996, p. 387).”

The exposition part in a textbook is usually followed by examples. For the students, these examples offer a prototype to copy for the next part, the section with sets of exercises. The exercise sets are often ‘graded’ and progresses from easier to more difficult. They can also be offered in parallel sets with different levels of difficulty (Love & Pimm, 1996).

Such gradings have in-built assumptions concerning the ability of the student to make further generalisations and what is meant by a question ‘being similar’ to an example. More importantly, they reflect beliefs that learning is best achieved by the student progressing in small, well-defined steps (Love & Pimm, 1996, p. 388).
Exercises are quantitatively dominant in many modern textbooks\textsuperscript{15}. But in an ‘old’ study of German textbooks intended for secondary school, Sträßer (1978) found that texts were most predominant in German textbooks. The distribution of text and task was 60,2\% and 39,8\% respectively.

An exception from the ‘exposition-examples-exercises’ model is the ‘activities-cours-exercises’ model, which can be found in textbooks from France. The objective with the ‘activities’ is to introduce a notion to the students through small investigations. The ‘cours’ describes what needs to be taught, in words and in worked examples (Pepin & Haggarty, 2001).

Textbooks can be used for different kinds of activities. Teachers use textbooks for teaching in order to lay down rules and conditions. They can also use them to explain processes through worked examples. But first and foremost, many teachers use textbooks to provide students with exercises\textsuperscript{16}.

Moreover, the teachers can use textbooks in different ways. Some teachers follow the book page by page through the entire book whilst others select from, reorganize and add to the text in the textbook, experienced teachers often do so (Love & Pimm, 1996; Sosniak & Stodolsky, 1993). When teachers select from the textbooks they can be guided by the objective to only teach lessons directly related to basic mathematical concepts and skills. But they can also be influenced and guided by recommendations from for example educational authorities\textsuperscript{17}.

Subject matter in research on textbooks

The interest in textbooks and their role in mathematics education is not new. The most comprehensive review of the relevant literature in the area, in my opinion, is made by Pepin and Haggarty (2001). They give a thorough review of the research area and suggest that existing studies on content and structure of textbooks can be divided into four main areas:

(a) The mathematical intentions of the textbooks — the mathematics represented in textbooks, beliefs about the nature of the mathematics in textbooks and presentation of the mathematical knowledge;

\textsuperscript{15} See for instance the study of mathematics textbooks and their use in France, Germany and England conducted by Pepin & Haggarty, 2001.
\textsuperscript{17} See for instance Freeman & Porter, 1989.
(b) The pedagogical intentions of the textbooks — ways in which the learner is helped (or not) through the content, method and rhetorical voice of the text;

(c) The sociological contexts of textbooks;

(d) The cultural traditions represented in textbooks.

Studies with respect to the use of textbooks by teachers and pupils can be organized into six main themes:

(e) Whether textbooks are used or not for teaching and learning;

(f) The authority of textbooks;

(g) Who uses the textbooks (pupil/teacher) and who makes the decision on who uses the textbooks;

(h) How textbooks are used, who makes the decisions on how they are used and what teachers value about textbooks;

(i) The teacher as the mediator of the text;

(j) Educational traditions as a potential influence of what happens in classrooms.

For the attempt to better understand and describe textbooks as the potentially implemented curriculum I have to gain more knowledge about textbooks and the use of them. To learn more about textbooks, it was crucial to examine the content and structure of textbooks. Therefore, the first part of this chapter reports on studies based on content analysis of textbooks. It includes mainly results from the TIMSS Curriculum Study and other cross-national comparisons. Textbooks are widely used in classrooms all over the world, and accordingly, the second part of the chapter reviews some studies of the use of textbooks.

Besides these two general issues, I am interested in the status of textbooks and if they are helpful in the endeavor to change mathematics education. Are textbooks ‘good tools’ for the accomplishment of new ideas of teaching and learning mathematics? What can we expect when we introduce new teaching materials? The third part of the chapter contains a dis-
cussion on the view of textbooks in different countries. The fourth part reports on studies of the influence of reform-based curriculum materials.

The reviewed literature in these four parts of the chapter is, in a wide sense, related to the research questions. They serve as a conceptual framework and motivation for the study. The fifth and final part of the chapter is a summary that includes some comments and remarks related to the review and the study.

Previous studies of textbooks and the use of textbooks are, for this study, organized into four main themes:

I. Content and structure of mathematics textbooks.

II. The use of textbooks in mathematics education.

III. The status of textbooks.

IV. Textbooks in curriculum reforms.

Content and structure of textbooks

Content analysis can give you an idea about the characteristics of textbooks. The studies discussed in this subsection are only examples of the range of variation of both focus and results. Some researchers make an attempt to distinguish textbooks from different countries (e.g., Pepin & Haggarty, 2001; Valverde et al., 2002). Content analysis can also concern restricted areas of mathematics (e.g., Harries & Sutherland, 1999; Li, 2000) or how adherent the textbooks are toward the official goals and objectives of mathematics education (e.g., Chandler & Brosnan, 1995). Generally speaking, which method to use for the analysis depends on the question to be answered.

As one part of the body of work for the TIMSS, Valverde et al. (2002) examine a set of characteristics, physical as well as pedagogical, of textbooks in mathematics and science from across the world. They noticed that students are challenged with pedagogical tools that are dissimilar concerning the number of pages, the total surface area of pages and the amount of graphics.

The United States is an exceptional case regarding the number of pages in mathematics textbooks. Only one of the textbooks intended for nine-
year-olds have fewer than 500 pages, and it had 448 pages. The average number of pages in mathematics textbooks for fourth graders is 125. Furthermore, textbooks also differ in depth and in the types of behavior they attempt to elicit from students (Valverde et al., 2002). Through a holistic view of textbooks, Valverde et al. find that mathematics and science textbooks can be categorized and grouped into five types of textbooks:

**Group 1:**
These textbooks have a small number of strands and emphasize exercises and problem solving to a high degree. No science textbook belongs to this group and there are a slightly higher percentage of books intended for nine-year-olds in this group.

**Group 2:**
These textbooks are small and focus on a small number of topics. They have lessons presented mostly by narrative. A little more than one fifth of the textbooks in this group are mathematics textbooks.

**Group 3:**
These textbooks are large books with many pages. They cover a moderate array of topics. The books are fragmented and mostly made up of exercises. Almost all of them are mathematics textbooks.

**Group 4:**
These textbooks emphasize narrative and have many thematic shifts. They have moderate breadth and depth as well as a moderate number of pages. Almost all of them are science textbooks.

**Group 5:**
These textbooks are very large in terms of the number of pages. They are thematically very cumbersome and have at least twice as many occasions where they shift themes as books in the other groups. This group consist of science textbooks entirely, thus there are no mathematics textbook in this group.
The distribution of textbooks is showed in Figure 5.

Essentially all mathematics textbooks in the United States belong to Group 3 whereas all mathematics textbooks in Cyprus belong to Group 1. Nevertheless, such characteristic types of textbooks in a country are relatively uncommon across the TIMSS sample. Most countries have textbooks with different structures (Valverde et al., 2002).

Valverde et al. (2002) maintain that textbooks are a critical factor, with respect to educational opportunities, as a mediator between systems goals and the realities of classrooms.

However, it is the primary professional responsibility of teachers to be concerned with their implementation. Understanding teachers’ instructional behaviors is necessary to characterize educational opportunities (Valverde et al., 2002, p. 167).

Another comparative study of textbooks was conducted by Pepin and Haggarty (2001). They examined textbooks and the use of textbooks, at the lower secondary level, in English, French and German classrooms. The preliminary result of the study, based on an analysis of textbooks, classroom observations and interviews with teachers, shows that mathematics textbooks occupy an important position in English, French and German classrooms and education. An analysis of one of the best-selling textbook series in each of the three countries illustrates differences concerning the structures of the textbooks. For example, a part that distinguishes French
from English and German textbooks is the first section of each chapter, *activités*. It contains small investigations, practical or cognitive activities which are intended to introduce pupils to a notion (Pepin & Haggarty, 2001).

An underlying assumption in a study made by Li (2000) is that “The textbook provides a blueprint for content coverage and instructional sequences” (p. 236). Li developed a three dimensional framework for a content analysis of textbook problems. Mathematics features, contextual features and performance requirements in American and Chinese middle school mathematics textbooks were investigated in order to identify the cross-national similarities and differences. In particular, problems presented immediately after the introduction of the content on addition and subtraction of integers were analyzed. The problem-response types are classified as:

a) A numerical answer only (A);

b) A numerical expression only (E);

c) Explanation or solution required (ES).

Li found that American textbook problems are more various and emphasizes conceptual understanding more than Chinese textbook problems. However, the problems are similar regarding complexity in computation requirement and contextual features. He also noted that Chinese textbooks include more advanced problems and that the subject ‘addition and subtraction with rational numbers’ is introduced at an earlier stage in Chinese textbooks. Li recommends extended studies of textbooks across or within different educational systems.

Such studies are feasible and important for furthering our knowledge of the potential effects of textbooks on classroom instruction and students’ mathematics achievement (Li, 2000, p. 240).

Harries and Sutherland (1999) did a comparative study of mathematics textbooks from England, France, Hungary, Singapore and the USA. They examined how the textbooks handle the introduction of multiplication and division for pupils at the age from 6 to 8. According to Harries and Suther-
The seven most frequently used textbook series in sixteen school districts of Ohio were investigated concerning correspondence to a proficiency test, the *Ohio Ninth Grade Proficiency Test* (Chandler & Brosnan, 1995). Percentages of mathematics textbook content were compared with percentages of mathematics content on the test. Chandler and Brosnan noticed that the content in the mathematics textbooks is disproportionate to the content of the proficiency test. The areas of greatest mismatch are arithmetic, measurement and algebra. Chandler and Brosnan argue that “According to the NCTM Standards, not only should curriculum and assessment content be aligned, but also the goals, objectives, and instructional approaches should be aligned with the assessment tasks” (p. 120). Since the Ohio test is constructed with the aim to encounter the goals and objectives of the Standards, they recommend teachers, school administrators, and parents to be aware of the mismatch (Chandler & Brosnan, 1995).

**The use of textbooks**

Pepin and Haggarty (2001) made a cross national comparison of textbooks and the use of textbooks in English, French and German classrooms. They investigated similarities and differences of mathematics textbooks and the way teachers use the textbooks at the lower secondary level. Teachers in all three countries emphasize the use of textbooks for exercises. There are, however, differences in how they use them with respect to the theoretical part and the worked examples. Mostly, French teachers introduce the core of the lesson in a different way than it is done in the textbooks and German teachers use worked examples from other sources than textbooks. The English pupils had a limited access to the textbooks. Therefore, they have to rely on the teachers, which in turn rely on the textbooks, as the only source of mathematics (Pepin & Haggarty, 2001).

To what extent textbooks are used has been studied in different countries. Röj-Lindberg (1999) made a survey of curriculum material and mathematics education in Swedish-speaking middle schools in Finland. She noticed that the teaching of mathematics is mostly guided by textbooks. Eighty-six percent of the teachers state that they frequently use textbooks in their
teaching of mathematics (Röj-Lindberg, 1999). In Sweden\textsuperscript{18}, the impact of mathematics textbooks in education has been a subject of discussion in more than seventy years\textsuperscript{19}.

The indication of the mathematics teachers’ frequent use of textbooks is not a local story. The result of Röj-Lindberg is pertinent to an earlier study by Barr (1988) conducted in the Chicago area in the United States. Nine fourth-grade mathematics teachers were interviewed and observed during a school year. Barr noticed that seven out of nine teachers used their textbooks by moving lesson by lesson through the book in a non-flexible way. In addition, the teachers spend a lot of their time on review. Barr suggested that teachers should base review work on the need of the individual student and not on the sequence and nature of the textbook. A comparison of mathematics and reading textbooks shows that mathematics textbooks overemphasize review and Barr suggests that the design of textbooks should be examined.

Moreover, teachers using the same textbooks differed in the mathematics lessons presented and the problem assigned. An influential factor on instruction was the characteristics of classes of students. The teachers of less able and more diverse classes are more focused on what they considered as the essential core and less focused on difficult areas like division and fractions (Barr, 1988).

The use of textbooks by Singaporean mathematics teachers at lower secondary level was investigated through a questionnaire survey (Fan & Zhu, 2002). Twenty-eight teachers using the most commonly used textbooks were asked about their background (gender, education and experience of teaching) and teachers’ general use of textbooks. How they use different categories of problems that are provided in the textbooks was examined through multiple-choice questions. Fan and Zhu maintain that three general conclusions could be drawn from the study:

a) Textbooks are still the most important resource for mathematics teachers in their teaching practice, especially when they assign homework;

\textsuperscript{18} A more detailed description of the view of textbooks in Sweden will follow in chapter four.
\textsuperscript{19} One of the first evaluation of mathematics textbooks was done in 1869 (Marklund, 1987).
b) Textbooks do not play a role as important as many people would think in Singaporean classrooms, parts of the textbooks were excluded by the teachers;


c) The qualities of schools/students, teachers’ gender and teaching experiences do not significantly affect the ways in which teachers use their textbooks.

The interaction between the use of textbooks and the characteristics of the book was examined by Stodolsky (1989). She suggests that the use and influence of textbooks should be analyzed with respect to topics, content of the book and the teacher guides that correspond to student texts. During two weeks, she observed how six fifth-grade teachers used the student and teacher editions of their math textbooks. A wide range of styles of use was observed, from fidelity to autonomy toward the content in the textbooks. Stodolsky noticed that there was high conformity between textbook topics and instructional topics and less agreement between suggested activities in the teacher edition and classroom practices. Furthermore, the teachers made no use of the manipulative activities and suggestions for enrichment in the teacher editions of the textbooks, at least not during the two weeks of observations (Stodolsky, 1989).

Another study in this area was made by Freeman and Porter (1989). They described the way teachers use textbooks and examined the overlap between content taught and textbooks content. During one school year, four fourth-grade teachers kept daily logs of their instructional activities in mathematics. Time devoted to mathematics instruction, content covered and examples of problems in use were recorded. Three styles of textbooks’ use are identified among the teachers in this study and categorized as:

a) *Textbooks bound*, in which the teacher followed the book page by page through the entire book;

b) *Focus on the basics*, in which the teacher only taught lessons directly related to basic mathematical concepts and skills;

c) *Focus on district objectives*, in which the teacher relied on recommendations on the topics to be taught provided by the current district.

Freeman and Porter came to the conclusion that “… textbooks are not the content policy instrument they are billed to be” (Freeman & Porter, 1989,
Moreover, they question the long held conviction that students are better served with teachers who go beyond their textbooks than by those who follow it closely. Freeman and Porter noticed in their study that teachers, who follow the book closely, were the teachers who placed the most emphasis on applications and conceptual understanding.

Sosniak and Stodolsky claimed that “… to understand textbook use, it is necessary to consider teachers’ thought and action and their relationships, teachers’ work within and across subjects, and the fuller context of teachers’ conditions of work” (Sosniak & Stodolsky, 1993, p. 252). They looked at four fourth-grade teachers to see how consistent individual teachers are regarding the use of textbooks across the subjects they teach (language arts, mathematics and social studies). Sosniak and Stodolsky report a considerable variability across teachers and, for a single teacher, from one subject to another. They noticed that none of the teachers in the study used textbooks in the same way when teaching different subjects. Also, the teachers viewed textbooks as tools and properties rather than blueprints. According to their findings, textbooks cannot be blamed for “… the variety of problems associated with elementary education” (p. 272). Furthermore, and in contrast to Barr (1988), they state that textbooks “… do not control the elementary curriculum to the extent ordinarily assumed, and textbook content does not necessarily directly influence what students learn” (Sosniak & Stodolsky, 1993, p. 272).

In a summary of the study of the TIMSS, Foxman (1999) discusses evidence for relationships between the degree of teacher use of textbooks and students’ performance. He reports that teachers in primary schools in three Pacific Rim countries and in the Netherlands place greater reliance on textbooks than their counterparts in England and Scotland. Nevertheless, students from these countries performed better than their counterparts in England and Scotland in the TIMSS study. “However, such relationships in TIMSS are correlational and not necessarily causal, and are not universal” (Foxman, 1999, p. 28).

**Educational authorities’ view of textbooks**

One way to recognize the position textbooks have in the education policy of a state is by looking at documents and reports published by the educational authorities in that state.
A national curriculum reflects the intended curriculum in many countries. New syllabus and/or material are usually one of the outcomes of a reform. In the absence of a national curriculum, as in the case of the United States, changes will be proposed and materials produced in the hope that they will be widely used (Howson et al., 1981). From 1989 and onwards, the National Council of Teachers of Mathematics, NCTM, published several documents with the intention to shape content, instruction, and assessment in school mathematics in the United States—these documents are known as the Standards. One of the outcomes was that middle school mathematics curricula, with the ambition to reflect the Standards, were produced (Chávez-Lópes, 2003).

Whether textbooks and other teaching materials are deliberately involved or not in a reform of the curriculum depend to some extent on the curriculum developers. Van den Heuvel-Panhuizen (1998) maintain that a negative attitude towards textbooks, and the use of them, are common in reforms of mathematics education and many reform movements are aimed at getting rid of textbooks. In contrast we have the case of the Netherlands where the improvement of mathematics education depends largely on textbooks (van den Heuvel-Panhuizen, 1998).

The roots of the reform movement of mathematics education in the Netherlands go back to the beginning of the seventies when the first ideas for the realistic approach to mathematics education were conceptualized. To some extent, it was initiated as a reaction to the ‘New Math’ movement. The use of problems in context is very significant in this approach. It emphasizes on making something real in students’ mind. The context of the problems presented can be a real-world context, but not by necessity (van den Heuvel-Panhuizen, 1998). “The fantasy world of fairy tales and even the formal world of mathematics can be very suitable contexts for a problem, as long as they are real in the student’s mind”.

The reform work in mathematics, in the Netherlands, the Realistic Mathematics Education, is developed at the the Freudenthal Institute, and is still an ongoing process (van den Heuvel-Panhuizen, 1998, p. 12)\textsuperscript{20}.

In the Netherlands, the textbook series are developed by commercial publishers. However, their work is supported by ideas for teaching activi-

\textsuperscript{20} Realistic Mathematics Education, RME, is developed by Hans Freudenthal, 1905-1990, and his colleagues at the former IOWO.
ties from the developmental research done at the Freudenthal Institute, and its predecessors. Currently, about eighty percent of the primary schools use a mathematics textbook series based on the RME (van den Heuvel-Panhuizen, 1998).

Generally speaking, textbooks can either be perceived as a resource or a hinder for the development of the curriculum in mathematics and there are at least three ways to look upon textbooks in relation to curriculum development.

- Guidelines can be formulated and producers of textbooks can choose to follow the principles in order to stay on the market.
- Research institutes can develop and evaluate teaching activities and provide textbooks authors with ideas.
- The education authorities can strive to put pressure on the teachers and make them feel poor if they depend upon the textbook.

Some effects of reform-based curriculum material

The NCTM Standards called for changes in both contents and teaching practice. As a result, several producers of textbooks designed new textbooks guided by the Standards. Consequently, attention to the effect of reform-based curriculum materials on the achievement of both students and teachers emerged as an important area of study.

The effect of a new curriculum on students learning was examined in a study of proportional reasoning among 7th grade students (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998). Ben-Chaim et al. compared two groups of students’ achievement on the subject of proportional reasoning. One group of students, 187 students from eight seventh-grade classes, was involved in the Connected Mathematics Project, CMP, during a complete one-year course\(^\text{21}\). The control sample consisted of 128 students from six seventh-grade classes and was taught in a traditional way. The two groups of students were tested with a written exam. Ben-Chaim et al. (1998) find that the CMP students performed better or significantly better than the control students on all items. In addition, an analysis of the written answers

\(^{21}\) The curriculum of the Connected Mathematics Project was developed under the influence of the NCTM Standards documents (Ben-Chaim et al., 1998).
and support work shows that the CMP students are generally more successful in selecting effective strategies to the given tasks. Ben-Chaim et al. assume that “… the superior performance of the CMP students might be attributed to the problem solving approach with its inherent discourse presented by the CMP curriculum” (Ben-Chaim et al., 1998, p. 270). However, at least two factors were disregarded by the researchers in this study, time allocation for each topic and the backgrounds of the teachers.

That content and structure of curriculum material do make a difference is the conclusion of a study of students’ achievement in a local test, Missouri Assessment Program, MAP (Reys, Reys, Lapan, Holliday, & Wasman, 2003). The student sample consisted of eight-grade students from six school districts. Three districts used Standards-based curriculum material, MATH Thematic, MT, or Connected Mathematics Project, CMP. The other districts were selected to serve as comparison districts. Prior student achievement and socioeconomic level were taken into account in selecting these. Reys et al. (2003) noticed that the use of Standard-based curriculum material influenced students’ achievement on the MAP. The students who used MT or CMP performed equal or better than the students from the comparison districts (Reys et al., 2003).

The way teachers adapt to new textbooks and other curriculum material has been studied from different points of view. Wilson and Goldenberg (1998) explore the struggles of a veteran middle school mathematics teacher as he tries to implement reform ideas and change his practice. The experiences of the teacher are observed during a two-year period. For the analysis of the teacher’s beliefs about mathematics and mathematics teaching, a model of intellectual development, a reconstructed version of Perry’s scheme, is used. Wilson and Goldenberg (1998) noticed that the teacher made minor progress regarding the implementation of the reform ideas.

But we believe Mr. Burt’s dualistic view of teaching in general, and of mathematics and mathematics teaching in particular, was the primary factor preventing him from shifting away from an environment dominated by teacher-judged correct ways of operating, toward a student-centered environment in which exploration and more experimentalist notions of right and wrong (e.g., an idea is right because it works and makes sense) were valued and emphasized (Wilson & Goldenberg, 1998, p. 287).
The teacher in the study of Wilson and Goldenberg is very experienced, he has approximately 21 years of teaching behind him. Another study related to the way experienced teachers adapt to new textbooks is made by Remillard (2000). Remillard investigates the potential and limits of reform-based curriculum material. She examines if and how the use of a reform-oriented textbook can contribute to teachers’ learning and change their way of teaching mathematics. Two fourth-grade teachers, who use a newly adopted mathematics textbooks, are observed during a school year. Remillard noticed that the teachers “… reading was selective and interpretive. They read different parts of the text and drew on their own perspectives to make meaning of what they read” (Remillard, 2000, p. 336).

The teachers use the textbook in two ways, as a source of tasks or as a source of mathematical and representational ideas. Remillard (2000) identifies two kinds of situations when the textbooks contribute to the teachers’ learning: when the teachers read the text in order to make curricular decisions and in the process of enacting tasks in the classroom. Missed opportunities for learning were also recognized. Additionally, Remillard suggests that “… if curriculum materials are to support teachers’ efforts to learn to teach differently, they must be written for teachers as readers” (p. 347). She criticizes the teachers’ guide for being designed to provide teachers with a collection of tasks to give students. “It communicated by speaking through teachers, by guiding their actions. It did not speak to them about these tasks or the ideas underlying them” (ibid.).

For the improvement of teachers’ guides, Remillard suggests that they can provide images of classroom interaction through sample dialogues. She gives two concrete ideas:

(a) Sample dialogues could provide the reader access to the teachers’ thinking by the use of annotations;

(b) Sample dialogues could also include examples in which students or teachers have come to an impasse, along with commentary on how these struggles might be assessed.

Finally, Remillard states that “… in order to promote productive use of curriculum material, professional development opportunities need to foster teachers’ reading and decision making, deepen and broaden their mathematical knowledge, and develop their knowledge of the workings of the curriculum development industry” (Remillard, 2000, p. 347). According to
Remillard, development in these three areas could, “… enhance teachers’ abilities to assess students, mathematical tasks, and goals and to critically examine the offerings in their textbooks in order to adapt and transform them” (ibid.).

Similar to the studies of Remillard and Wilson & Goldenberg, Lloyd (1999) conducted an empirical study in which she focused on how teachers view and understand a reform-oriented curriculum material. She examined the conceptions of two veteran high school mathematics teachers as they adapt to a new curriculum material. The Core-Plus material attempts to support teachers in enacting many recommendations of the NCTM Standards. Teachers using the material are directed to organize the classroom for exploratory activities and cooperation (Lloyd, 1999).

Lloyd found that the teachers interpret the recommended activities in the Core-Plus material differently. But she is puzzled by the fact that “… the teachers did not change the Core-Plus problems and activities to better suit their personal goals” (Lloyd, 1999, p. 245). One of the teachers in the study highlighted an interesting notion:

When a reform-minded teacher uses traditional materials in the classroom, he or she may be afforded more room for personalization because the goals of the materials are so different from his or her own goals (p. 246).

Manouchehri and Goodman (1998) investigate the process of evaluation and implementation of four different Standards-based curriculum programs. Sixty-six middle school mathematics teachers from 12 school districts in Missouri are interviewed and observed during a period of two years. At the beginning, all the participating teachers are enthusiastic and have a positive attitude toward the curriculum material. Nearly all the teachers report that they observed an increased interest of learning mathematics and a better involvement in class activities from the students. However, only 20 of the 66 teachers report that they were regularly using the programs in their classes after the first five months. Some of the problems that teachers face when implementing a new curriculum are of practical nature. Parents’ expectations about using traditional material and approaches are also reported as obstacles. According to Manouchehri and Goodman, the way curriculum material are perceived, evaluated, and used by the teachers are strongly related to:
a) The amount and quality of teachers’ experiences;

b) Teachers’ professional knowledge base about curriculum and instruction;

c) The contexts within which teachers’ worked;

d) Teachers’ own personal theories on effective teaching and learning practice.

In a follow up study, Manouchehri and Goodman (2000) look more closely at the teaching and learning processes taking place in the classroom. Two seventh-grade teachers are observed during a period of two years. Manouchehri and Goodman maintain that the teachers’ mathematical knowledge is the greatest influence on how they evaluate and implement the textbook.

In order for teachers to help students grow mathematically, as it was envisioned by the new textbook, they needed to have an understanding of the central ideas of mathematics, see relationships among concepts, be able to reason mathematically, and have an elaborated knowledge base that included multiple representation of concepts (Manouchehri & Goodman, 2000, p.27).

A report from another study in this area was recently presented in a doctoral dissertation of Chávez-Lópes (2003). He conducted a study of a group of 53 teachers participating in the Middle School Mathematics Study, and case studies with three of the 53 teachers. Teachers from six of the eleven schools used Standards-based curricula, funded by the National Science Foundation (NSF). The rest of the schools were using a variety of textbooks. The majority of the teachers in this study report using their textbooks very frequently.

Chávez-Lópes (2003) noticed that textbooks are used in two primary ways, as a source of tasks and as a guide for preparation of lessons. Teachers using NSF-funded curricula seem to adhere more closely to the textbooks. They are also more likely to read and use the teacher guide. Furthermore, Chávez-Lópes noticed that the enacted curriculum is shaped, to
the same extent, by the textbook and by the teachers’ beliefs about mathematics and mathematics teaching.

Teachers’ view of the curriculum and the match, or lack of it, between their own views about mathematics and mathematics teaching and the philosophy of the textbook – whether it is explicit or not – were the primary factors that determined how the textbook was used. However, the primary factor that determined what tasks were presented to students was the textbook (p. 157).

Findings from the case studies of the three teachers support the belief that the choice of textbook matters. “It is clear that in these cases, the nature of the mathematical tasks presented to students, their richness and quality, does depend on the textbook used by the teacher” (Chávez-López, 2003, p. 159).

**Concluding comments**

There are certainly differences among textbooks from different countries concerning structure, content and performance expectation of students. However, there are many similarities too. The study by Valverde et al (2002) shows that most of mathematics textbooks seem to belong to group 1 or 3 (Figure 5). At least this is true for the TIMSS sample. Thus, either they contain a small number of strands with topics that are commonly intended across the world and give emphasis to problem solving or they have many pages and many shifts of topics. Both types of books emphasize exercises heavily.

A long held conviction is that mathematics teachers follow their textbooks very closely. To some extent, some of the studies mentioned above confirm these claims (see Barr, 1988; Röj-Lindberg, 1999). However, the data were mostly from surveys and not from classroom observations. Research on teachers’ actual use of textbooks in classrooms challenges that view (see Sosniak & Stodolsky, 1993).

Curriculum material have frequently been used for the implementation of new curricula. On the one hand, a common standpoint among researchers is that such a strategy has limitations. The examples above show that there are several reasons for unsuccessful adoptions of new teaching material (see Remillard, 2000; Wilson & Goldenberg, 1998). Teachers’ mathematical
knowledge, interpretation of the material, and attitudes towards textbooks are some of the obstacles for the use of reform-oriented material. On the other hand, textbooks and other curriculum material can make a difference concerning the learning of mathematics by students. The studies by Ben-Chaim et al. (1998) and Reys et al. (2003) show that students who were taught through reformed-based (or –guided) curricula performed better in tests.

In summary, the role of textbooks in mathematics education cannot be illustrated in a simple picture. Nevertheless, when reading the literature some statements can be maintained:

- Textbooks are widely used by teachers and students all over the world.
- The view of textbooks – as a resource or as a hinder in reforms of mathematics education – differ among the educational authorities.

However, teachers make decisions that influence the role textbooks and other curriculum material will have in mathematics classrooms, hence:

- The fact that teachers use the same textbook does not imply that they teach in the same way.
- A reform of the textbook that is used by a teacher will not necessarily change the teacher’s practice.

Finally, results from research in the US schools directed me to an additional conclusion. In studies of the way reform-based curriculum material effect students learning there is a common notion that:

- Curriculum materials do make a difference.
4. On curriculum development in Sweden

Background

In curriculum studies it is important to distinguish between the organizational part and the pedagogical part of the curriculum. The former describes at least the accessible school topics and time allocation. The latter describes the goals for the education and gives instruction on how to accomplish them (Svingby, 1981). Both parts are represented in the following description of curriculum reforms in the Swedish educational system. However, the main focus is the pedagogical part.

The concept läroplan is for this chapter defined as the political document, published by the educational authorities, which describes the overall contents and goals for compulsory school. The contents and goals for the school topics, for instance the course in mathematics, are called kursplan.

Läroplanen and kursplanen are both parts of the intended curriculum, according to the definition presented in the second chapter. However, these are not the only representatives of the intended curriculum. For example, The Education Act, Skollagen, and the national tests are also examples of how the intended curriculum is made recognizable by the National Agency of Education.

The characteristics of läroplaner have impact on the running of schools, and vice versa. The national goals of education are in a ‘rule regulated system’ interpreted by the center and in a ‘management by objectives system’ interpreted by the periphery. The view of knowledge differs between these two systems. In a rule-regulated system, people are expected to know what to do – to apply the rules and directives in a correct way. But in a management by objectives system, people need knowledge to interpret by themselves. Another difference between these two systems is that evaluation and assessment are more complicated in a management by objectives system.

---

22 The later is not applicable for the new, goal directed, curriculum from 1994.
23 The translation by the National Agency of Education in Sweden is curriculum for läroplan and syllabus for kursplan. In order to avoid misunderstanding, I will remain faithful to the definition of the concept curriculum from previous chapters.
24 The concept ‘management by objectives’ occurred as a technic to govern companies and not public service.
systems than in a system where evaluation only controls if rules and directions have been followed (Blidberg, 2001).

The content as well as the characteristic of läroplaner and kursplaner have changed during the last thirty years. The fact that the number of pages is almost five hundred in the first national läroplan for compulsory school and about twenty in the current, indicates a shift toward a less detailed läroplan as regards interpretation. The kursplan shifted from being a rather detailed description of the course to a goal-oriented document – leaving a broad opportunity, and responsibility, to teachers and textbooks’ authors to describe the course themselves.

Currently, the National Agency of Education in Sweden uses evaluation of schools as one instrument to control the educational system. A quality inspection of schools in 40 municipalities (out of 290) was done in the years 2001-2002 and one of the observations that are reported from the inspectors is about textbooks. They find that the teaching of mathematics, more than any other school subject, relies on the use of textbooks.

The evaluation shows the surprisingly dominant role of the textbooks in teaching ... especially from year 4-5 and onwards … Content as well as arrangement of teaching are to a high degree directed by the textbook. Mathematics is, for both students and teachers, simply what is written in the textbook (Skolverket, 2003, p. 39, my translation). [Granskningen visar att det är frapperande vilken dominerande roll läroboken har i undervisningen … framförallt från år 4-5 och uppåt … Såväl innehåll, uppläggning som undervisningens organisering styrs av boken i påfallande hög grad. Matematik är för både elever och lärare kort och gott det som står i läroboken.]

The status of textbooks

The concern of government authorities about the textbook and its influence on education is indeed a long story. In a report from 1931 there is a discussion, based upon the view of the education committee from 1812, about textbooks. In that time, the committee thought that textbooks should not be too detailed because that could change the focus away from the responsibil-

25 SOU 1931:2 (reference in Marklund, 1987)
ity of teachers and students’ interest. Furthermore, the school committee, *Skolkommissionen*, from 1946 claimed that the education was dependent on the textbook to an unacceptably high degree. They meant that the reliance on textbooks was an obstacle to the required development of a model, for education and work, in accordance with the demand from a democratic school.

The educational authorities suggested the preparation of an ‘organized plan for teaching’, a so-called *studieplan*, so that teachers dependence on textbooks should decrease. *Studieplanerna* were small booklets with examples of exercises and working methods. However, the efforts were unsuccessful. At a time of shortage of teachers, there was little interest for the *studieplan*. Many teachers did not have the time or ability for the thorough preparation of the mathematics lessons related to an involvement of the *studieplan*. Nevertheless, some authors said that they used the *studieplan* when writing textbooks so it was not a total failure (Marklund, 1987).

Whether teaching material should be put under scrutiny to be approved or not has also been a question for the educational authorities in Sweden for a long time. The first complete inspection of curriculum material for the elementary school was done in 1934\textsuperscript{26} and resulted in an exclusion of some textbooks. Inspections were, after that, done on a regular basis and the most recent committee in charge for that was *Statens Institut för Läromedelsinformation*, SIL (Marklund, 1987). SIL was a government authority, active in 1974–92, and responsible for inspection, approving course material, *basläromedel*, and information to schools.

An evaluation of mathematical textbooks, designed for grade seven to nine, is reported in *Matematikgranskning* (Areskoug & Grevholm, 1987) and indicates clear opportunities for improvement in several aspects. The committee states that textbooks are monotonous, characterless and uninteresting and claims that an improvement of textbooks is the most important effort for the development of mathematics education. For the survey, three aspects are taken into account: the content, the working forms and the methodical design. The content is analysed from different perspectives such as readability, comprehensibility and choice of pictures. Other issues concern the tasks and their relevance for the pupils, for example: Do pupils from different social environments recognise themselves? Are questions like

\textsuperscript{26} SOU 1935:4 SOU 1935:45 (reference in Marklund, 1987)
solidarity and equity dealt with? The working forms are investigated according to two questions: Are the tasks suited for laboratory work; explorative work; thematic work; group work or co-operation with other school subjects? Are the tasks stimulating joy for exploration and creativity? The textbook series chosen for the case study described in the next chapter is evaluated in this survey too. In the report, the committee expresses rather harsh critic for this textbook series (Areskoug & Grevholm, 1987). The result from this evaluation that became most discussed afterwards was the gender imbalance in pictures and text\textsuperscript{27}.

\textit{Läromedelsöversynen} made another survey of curriculum material in the late 80’s. The work is obtainable in a three-part report, \textit{Skolböcker}. The focus is on textbooks quality and students’ access to them and the importance of quality and of students’ right to have their own books, so-called gift books, is emphasized (Grevholm, Nilsson, & Bratt, 1988; Marklund, 1987; Rönström, 1988).

At present, there is no governmental control or evaluation of textbooks in Sweden.

In comparison with the current läroplan, it is quite clear that textbooks gained more attention in the 80s’. There is even a special part concerning textbooks in the läroplan from 1980, Lgr 80, with a recommendation for the use of printed material produced for teaching.

For schoolwork, there exist special teaching material, produced for teaching. Printed material that covers essential parts of a subject, a group of related subjects or sections of a course, should play an important role, in order to bring firmness and connections to the studies (Skolöverstyrelsen, 1980, p. 50, my translation). [För skolarbetet finns också speciella läromedel, producerade för undervisning. Tryckt material, som täcker väsentliga delar av ämne, ämnesgrupp eller kursmoment, måste spela en viktig roll, bl a för att ge fasthet och sammanhang i studierna.]

\textsuperscript{27} Personal reflection by Barbro Grevholm, November 2003.
Overview of the curriculum reforms

From 1962 to 1969
The extensive reform of the compulsory school in Sweden, 1962, was established due to both economical and political-ideological causes and implemented by means of a power-coercive strategy\textsuperscript{28}. A shift from a segregated and selective system to an open and elective system in grade 7-9 was the most radical change. According to that, students could choose between different courses, one theoretical alternative, one less demanding theoretical alternative and eight alternatives with a more practical direction. But the system with choice of course program was complicated and difficult to grasp and when more than sixty percent of the students preferred the theoretical alternative it caused organizational problems (Marklund, 1987; Svingby, 1978).

The pedagogical problem with heterogeneous classes was conceived as manageable through teaching methods based on individualization. The development of a special material for this purpose was one of the objectives of the IMU-project\textsuperscript{29} which was initiated in the middle of the 60s’ (Kilborn, Lundberg, Selander, & Öhlund, 1977; Lundgren, 1983). Initially, the purpose of the IMU-project was to solve problems related to teaching in mixed ability classes at a school in Älghult. Correspondence courses were used as special resources to the more gifted students and the results were positive. The IMU-project “… was taken under the wing of a regional educational authority” (Howson et al., 1981, p. 45) and the experimental work was disseminated and developed. At a later stage it involved continuation courses for teachers, elaboration of teaching material and research studies.

A third version of the IMU material was tried out in a nationwide field experiment between 1968 and 1971. The rationale of the material involved students working by themselves, using tests to guide their progress. About 12 000 students were using the material by 1971 (Howson et al., 1981; Wallby, Carlsson, & Nyström, 2001).

In 1968, the Swedish National Board of Education, Skolöverstyrelsen, recommended a nationwide regular introduction of the IMU system. By doing that they maintained the “…Swedish myth […] that the government

\textsuperscript{28} Strategies to be employed in curriculum development are described in chapter 2.

\textsuperscript{29} Individualized Mathematics Education (Individualiserad Matematik Undervisning).
should set the framework for curriculum development” (Howson et al., 1981, p. 45). The IMU researchers were not consulted and reacted against the proposal. They asserted that more research was needed prior to a compulsory introduction of the system into the schools. The reaction from the researchers led to a policy where the IMU system only was introduced if the local school and its teachers requested it. Subsequently, the IMU author group developed a similar, but simplified, instructional system, *Hej matematik*\(^{30}\), which was in the form of a textbook series available on the market (Howson et al., 1981).

The läroplan from 1962, Lgr 62, can be regarded as the first national läroplan for the compulsory school, and the need for a continuing revision of the läroplan, to keep pace with changes in society, was discussed from the very beginning (Svingby, 1978). The decision for a continuous reform had substantial consequences for the development of the läroplan. After this decision, the work was done by a small group of administrators and specialists. Hence, the control and administration of the school were in principle moved from being a political concern to an issue of “… a handful of people” (Lundgren, 1983, p. 118, my translation).

The call for a revision of Lgr 62, initiated a new läroplan, Lgr 69, only seven years after the previous one was implemented. The external organizational structure with different programs in grade 7-9 changed to an internal organizational structure where students were integrated into classes and differentiated within the class. Alternative courses were offered in Mathematics, English, French, and German. They can be regarded as a compromise and a concession to teachers and other critics of a complete comprehensive compulsory school (Marklund, 1987; Svingby, 1981; Wallby et al., 2001).

In addition to the adjustment of the organizational structures came the reform of mathematics as a school topic. The ‘New Math’ reform movements was a global request for improvement and renewal of mathematics and were a matter of concern in Sweden as well as in many other countries during the second part of the twentieth century. Another influential factor for the initiation of a reform of school mathematics was the results from an international study of 13-years old students’ achievement in mathematics\(^{31}\)

---

\(^{30}\) The authors were Matts Håstad, Curt Öreberg and Leif Svensson.

\(^{31}\) The study was the International Study of Achievement, IEA.
which was published in 1967 (Kristiansson, 1979). As regards the achievement-ranking, Israel and Japan was at the top and USA and Sweden at the bottom (Steiner, 1980).

It was not only mathematicians who took part in the discussion concerning the ‘New Math’. Many other professionals like educationalists and teachers agreed to the demand that content as well as methods must be improved. In Sweden, the ‘New Math’ involved for instance set theory and laboratory work. The introduction of set theory was considered as an instrument to describe mathematics and make connections between the different subjects within mathematics. Laboratory material was considered as important resources for students in order to discover, perceive and understand the structure of mathematics (Kristiansson, 1979).

At the same time as the ‘New Math’ movement, discussion concerning terminology, notations and the overall structure (uppställningar) initiated an evaluation of the use in the classrooms. Later, a working group was appointed to elaborate instructions for schools. The result of the working group was published in 1966 in the report *Matematikterminologi i skolan*. The revision of terminology and new intentions in the läroplan caused a renewal of textbooks. Hence, the reform of teaching in mathematics is somehow an outcome of the evaluation of the terminology as well as Lgr 69 (Kristiansson, 1979).

The reform of mathematics in Lgr 69, which included an introduction of set theory, was a problem for many teachers. Hence, about 40 000 teachers were offered competence development courses through a special project called *Delta* in order to prepare for the new läroplan and reinforce the mathematical knowledge (Emanuelsson, 2001).

The responsible group for the preparation of the course description in mathematics, the supplementary part in the Lgr 69, encountered a fair amount of difficulties and criticism. The disapproval in the initial phase was mostly about the introduction of set theory and the deficiency of basic arithmetic. The final result is strongly criticized in a report from the pedagogical institute in Göteborg, published eight years after Lgr 69 (Kilborn et al., 1977). Kilborn et al. claim that work for the supplementary part of the läroplan was done with insufficient resources, both concerning personnel and economy, and that preparation was fragmentary. They argue that an evaluation of the previous läroplan, Lgr 62, should have been done before the work on the new one. However, the decision to omit an assessment of
the current läroplan was justified through the principle that new methods can not be assessed through old (Kilborn et al., 1977).

Furthermore, Kilborn et al. state that ‘New Math’ together with the so-called ‘educational techniques’ had a significant influence on school mathematics in Sweden in the beginning of the 70s’. These two factors had a major impact on both textbooks and teaching. The ‘educational techniques’ came from USA and were partly implemented in Sweden via the IMU-project. According to Kilborn et al., the vision of the project comprised a radical change of the role of the teacher (Kilborn, 1982; Kilborn et al., 1977).

A study of the effect of the new läroplan was conducted by Kristiansson (1979). She compared two groups of students that have worked according to Lgr 69 or Lgr 62. The achievement on three different tests shows a range of variation which could not solely be explained by the revision of the läroplan (Kristiansson, 1979).

**Lgr 69**
The läroplan for the nine-year compulsory school from 1969, Lgr 69, consists of two parts, a general and a supplementary part. The general part of the läroplan consists of more than 200 pages and describes goals and guiding principles, timetable and kursplan. Instructions and guidance for school activities such as recommendations concerning teaching methods and curriculum materials are included.

[...] more suitable to let students work completely at their own speed, with individual tasks according to individual instructions. This is motivated by the wish to handle different prerequisites of students, which mainly concern time needed to learn certain topics. Such ‘individualization by speed’ can be used in for instance mathematics with the help of material associated with self-instructional programs (Skolöverstyrelsen, 1969a, p. 63, my translation). [...] vara lämpligare att eleverna arbetar helt i egen takt med individuella uppgifter enligt individuella instruktioner. Detta är motiverat bl a av att elevernas olika förutsättningar där tar sig uttryck huvudsakligen på det sättet, att de behöver olika lång tid för att lära sig ett visst moment. En sådan s k hastighets-individualisering tillämpas t ex i matematik med hjälp av självinstruerande programmerat material.]
The supplementary part for mathematics, associated with the Lgr 69, contains suggestions for the plan of study. It is a rather detailed and instructive text. For example, one proposal is: “Through measuring exercises, the students can prove the probability of some basic theorems regarding angles such as sum of the angles of a triangle, sum of the supplementary angles, size of a vertical angle, and angles at parallel lines” (Skolöverstyrelsen, 1969b, p. 14, my translation). As well as in the general part of the läroplan, there are recommendations concerning differentiation. Topics that are central in the general course and the special course are described. For instance, multiplication and division of negative numbers and fractions are topics that, according to the supplementary part of the läroplan, can be considered at a later stage of compulsory school or excluded from the general course (Skolöverstyrelsen, 1969b).

The kursplan as well as the supplementary part of Lgr 69 emphasizes mathematical accuracy, especially with respect to terminology:

Formation of concepts should be supported through a clear and concise language in teaching, and when a mathematical terminology is introduced it must be correct (Skolöverstyrelsen, 1969a, p. 138; 1969b, p. 4, my translation). [Begreppsbildningen bör understödjas genom att ett klart och koncist språk används vid undervisningen, och när en matematisk terminologi införs, måste denna vara korrekt.]

The importance of set theory is subdued in the general part of Lgr 69. The subject is even not mentioned in the list of main topics. However, the use of set theory is pointed out under the headline Lärostoff in the kursplan of mathematics. One small part of the subchapter describes how set theory can be used to show connections between mathematical concepts. In contrast to the general part of Lgr 69, the supplementary part makes extensive use of set theory in the suggestions for the mathematical topics.

From 1969 to 1980
The constructors of a läroplan are dealing with a critical problem when deciding how detailed the document should be. An exhaustive läroplan can have a restraining influence on improvements and make some people feel controlled whilst others feel safe and secure (Marklund, 1987). The kursplan for mathematics from 1969 is a rather descriptive text, both concern-
ing the topics and the decisions when they should be introduced. This is in accordance with the centralized way to govern school and other public services at that time. A first deviation from the detailed management of compulsory school is taken when the läroplan from 1980, Lgr 80, is implemented. The schools were then requested to formulate their own local school plan and were free to decide on the time distribution within the frame of the three levels of compulsory school (Utbildningsdepartementet, 2001).

In the late 1970s, there was a period when mathematics seemed to play an utilitarian role. The most important goal was at that time to be able to use concepts and skills for solving ‘real-world’ problems (Wyndhamn, 1993). The emphasis on problem solving was a global trend as well as restrictions and a setback of set theory. In the läroplan, Lgr 80, the emphasis on problem solving is evident.

The alternative courses are not mentioned in the kursplan of mathematics in the läroplan from 1980. Their abolition was however discussed in the preparatory work and the National Board of Education, Skolöverstyrelsen, suggested a transitional period of five years long. However, the alternative courses in Mathematics and English were still a reality in many schools till the beginning of the 90’s (Wallby et al., 2001).

Even if Lgr 80 entails new ways to teach mathematics, for instance the emphasis on problem solving, competence development courses for teachers were not offered in relation to the implementation phase (Emanuelsson, 2001).

**Lgr 80**

The läroplan for compulsory school from 1980 consists of two parts, a general part and a booklet with comments, Kommentarmaterial, which was published two years after the läroplan. The latter includes a description of the difference between Lgr 80 and the previous läroplan, Lgr 69.

With respect to the goals in previous kursplaner, a considerable difference is noticed in the goals for the kursplan in mathematics in Lgr 80. Whereas the main focus formerly was to build up

---

32 For further readings on problem solving, see for instance: Schoenfeld, A. (1988)
33 Guidelines for the alternative courses were offered in the previous läroplan, see p. 45.
mathematics for further studies it is now clearly stated that the objective for mathematics is to provide each and everyone with useful knowledge so one can avail oneself of ones rights and fulfill ones duties as a member of society (Skolöverstyrelsen, 1982, p. 9, my translation). [I förhållande till tidigare kursplaners mål märks en väsentlig skillnad i målet för kursplanen i matematik i Lgr 80. Medan man tidigare främst siktat på att bygga upp en matematik för vidare studier anges nu klart, att matematiken främst syftar till att ge var och en brukbara kunskaper så att man skall kunna ta vara på sina rättigheter och fullgöra sina skyldigheter som samhällsmedlem.]  

The emphasis on problem solving and realistic mathematics is clearly stated in the Lgr 80 in the sense that students shall deal with tasks related to everyday life. In the description of the goal of mathematics one can read:

The teaching in mathematics shall start from the experience and the need of the students and prepare them for the role as adult citizens. The students shall therefore first and foremost obtain good ability to solve such mathematical problems that commonly occur in everyday life. [Undervisningen i matematik skall utgå från elevernas erfarenheter och behov och förbereda dem för rollen som vuxna medborgare. Eleverna skall därför i första hand skaffa sig god förmåga att lösa sådana matematiska problem som vanligen förekommer i vardagslivet (Skolöverstyrelsen, 1980, p. 98, my translation).]

Subjects like subtraction of negative numbers and multiplication by two negative numbers are especially pointed out as tasks that 'barely exist in practical situations' – a rather clear message, to teachers and textbooks authors, to exclude these subjects from the mathematics course.

From 1980 to 1994
At the end of the 80s’, the Swedish Government had problems concerning the economy. This excluded the continuation of the expansion of public service. At the same time, there was a pressure from the citizens concerning better service and improvement of quality. The democracy issue was pushed in the background by the demands for effectiveness and service. In order to meet the demands, the ongoing development aimed at goals and
results. An increasing autonomy for those who were involved in the public services, which includes the schools, was an important feature. The hope was that by giving responsibility to the local institutions and persons the actual economical problems could be handled better than by central control (Blidberg, 2001).

A logical consequence of the decentralization of schools was that the municipalities took over the responsibility of teachers and other employees in schools. This happened in 1991. Two years later, the municipalities obtain a general Government grant instead of the previous economical support especially directed to schools (Blidberg, 2001).

The decision process in the school system is the following: The government and Riksdag (Parliament) stipulate the national objectives for the school in the läroplan. The municipality formulates a school plan from their own local prerequisites and their main concerns, ideas and visions of how the school can work toward the national objectives. In their part, each school elaborates a work plan. That is a specification of the school plan and also a plan for the forthcoming work in the school (Blidberg, 2001).

In a goal directed school system, evaluation is equally important as the objectives. The governmental control through inspections is therefore essential. However, goals and rules are present in the new system as well as in the old one. But the difference between the two systems is that the center interprets the national objective in the former. In the latter, it is the periphery (Blidberg, 2001).

**Lpo 94**
The differences between Lpo 94 and its predecessors are that the national educational objectives are elucidated and that directions and instructions concerning teaching methods are absent. The kursplan has been revised and the new one is utilized since year 2000. Lpo 94, as a whole, is utilized since 1995/96 and the new kursplan from 1996/97 (Blidberg, 2001).

The läroplan as well as the kursplan in Lpo 94 is particularly focused on the goal for education and written in even more general terms than Lgr 80. At present, the schools are regulated by this läroplan in conjunction with the Education Act, *Skollagen*, and each employed person at Swedish schools is bound to follow them (Skolverket, 1994).
The kursplan for mathematics describes:

a) The aim of mathematics and its role in education;

b) Goals to aim for;

c) The structure and nature of mathematics;

d) Goals to attain, for students at the end of the fifth year and the ninth year;

e) Assessment criteria.

An example of ‘goals to attain’ for students at the end of the ninth year is that they “…should have developed their understanding of numbers to cover whole and rational numbers in fraction and decimal form … should be able to reproduce and describe important properties of some common geometrical objects” (Skolverket, 2001a, p. 26). However, there is no explanation of what is meant by ‘developed their understanding’, ‘important properties’ or ‘common geometrical objects’.

The vision that students should know about the importance of mathematics is evident in the description of the goals for mathematics as well as in the assessment criteria.

Mathematics is an important part of our culture and the education should give pupils an insight into the subject’s historical development, its importance and role in our society (Skolverket, 2001a, p. 23)

In the assessment criteria, it is clearly stated that in order to have the grade ‘passed with distinction’ (VG) “The student gives examples on how mathematics has developed and been used through the history and what importance it has in our times in some different areas” (Skolverket, 2000).

From 1994 and onwards?
A summary of the development of the kursplaner, from Lgr 62 to Lpo 94, is made by Englund (1995). He claims that the läroplan from 1962, Lgr 62,
is influenced by the *realskolan*\(^{34}\) from the old school system, at least for the last years of compulsory school. Lgr 62 involves a structure where school topics are segregated and the main objective is to prepare for further studies and working life. Lgr 69 has a different focus, it stresses that the starting point shall be students’ understanding. On the other hand, Lgr 69 recommends the ‘educational techniques’, which makes the kursplan rather contradictory. To train students in order to make them full members of society is emphasized in Lgr 80. This goal is also recognized in the kursplaner. At the level of the kursplan, Englund maintains that Lpo 94 entails a retrogression where the school subjects are yet again in focus. As an indication of that, he points out the emphasis on Mathematics and Swedish (Englund, 1995).

![Figure 6: The triangle of educational philosophy\(^{35}\)](image)

---

\(^{34}\) Realskolan was the junior secondary school that existed before 1962 as one possible stream parallel to *folkskolan* (elementary school).

\(^{35}\) Englund, 1995, p. 151. The picture is slightly simplified.
Teveborg (1999) offers a different way to look upon the development of the läroplaner. He suggests that one can put the following epithet on them:

Lgr 62: *The student in focus*

Lgr 69: *A democratic school*

Lgr 80: *A school for all*

Lpo 94: *A school with liberty of choice*

Despite extensive efforts to decentralize the control of education, the kursplaner for mathematics and other school subjects are nevertheless formulated by the center. In a report from a working party, appointed by the Minister of Education in 2000 with the purpose to evaluate the role of läroplaner as a tool for organization, there is a proposal for locally formulated kursplaner. However, the working group comments that a reassignment of responsibility from the center to the periphery might not lead to increased variety and local variations. Even when schools are free to choose content there is an immediate risk for central control through textbooks, national exams and tests. Experience from other countries shows that these factors can have strong impact on the content (Utbildningsdepartementet, 2001).

One can say that different types of läroplaner illustrate different types of solutions of political and organizational problems. In a comparison of the läroplaner in the Nordic countries, Sivesind et al. (2003) classified the läroplan in Sweden as *Politically standardized* (type III, see figure 7). The focus in this type of läroplan is principles and normative values that should permeate the schoolwork, how to concretize is locally determined. The läroplan in Finland is also of this type. The Norwegian läroplan is however *Formally regulated* (type I). This type of läroplan consists of rules for the content as well as description of the obligations and rights of the subordinates of the school system. Additionally, the läroplan in Norway includes guiding rules concerning methodology based on didactical principles. It can therefore be regarded as a *Description of content* (type II) type of läroplan as well. The Danish läroplan is comparable with the Norwegian.

If the proposals for forthcoming läroplaner (see Utbildningsdepartementet, 2001, p. 103-126) in Sweden is realized, then it can be seen as a move
toward a *Standards giving* (type IV) läroplan. In this type of läroplan there is no description of the content of a course. Content and method will be decided on at local level and the result will be evaluated by the center. The quality of outcomes is in this type of läroplan superior to the professional work (Sivesind et al., 2003).

<table>
<thead>
<tr>
<th>Types of curricula</th>
<th>Demands for structural quality</th>
<th>Demands for quality of process</th>
<th>Individual demands on process</th>
<th>Demands for quality of outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formally regulated (I)</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Descriptions of content (II)</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Politically standardized (III)</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Standards giving (IV)</td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Figure 7: A comparison between the different types of curricula

---

36 Sivesind et al., 2003, p. 12.
5. Curriculum reforms and textbooks

This chapter describes a case study of some relations within the curriculum in mathematics in Sweden. Documents related to the intended curriculum of Sweden, namely läroplanen and kursplanen, and a representative of the potentially implemented curriculum, namely a mathematics textbook series, were examined. The objective was to find evidence of relations between the intended curriculum in mathematics, especially related to the revisions in 1969, 1980 and 1994, and the textbook series. Another purpose of the case study was to look into the evolution of a textbook.

The following questions were investigated:

- To what extent does a reform of the curriculum influence the development of mathematics textbooks?
- In what respect does a textbook differ from the late 70s’ to a modern book?

Research design

Läroplaner and kursplaner from 1969, 1980 and 1994, and a commonly used textbook for seventh graders were chosen for the analysis. Before the alternative courses in mathematics ceased to exist in practice, which was in the beginning of the 90s' (Wallby et al., 2001), the textbook was divided into two different books, one for the general course and one for the more special course (Undvall, Olofsson, & Forsberg, 1979a, 1979b, 1985a, 1985b). In the new edition of the textbook there is still an external division for grade eight and nine but not for grade seven. Instead, there is an internal partition where students can choose exercises on different levels: A, B or C, where A is the most elementary and recommended as a first stage for the majority of the students (Undvall, Olofsson, & Forsberg, 2001). The two older editions of the textbook series investigated is from 1979 and 1985, the newer unified version is from 2001.

There are two reasons why I chose this particular textbook: 1) Even though twenty-seven years passed between the first\textsuperscript{37} and latest edition, the

\textsuperscript{37} The first edition of the textbook is from 1974.
group of authors is the same all over time. 2) This was one of the two textbook series selected for the TIMSS curriculum study\textsuperscript{38}.

The analysis of the textbooks is made in two steps:

**Step 1**
In order to acquaint myself and learn more about the selected textbooks I did a general survey by using a technique for coding inspired by the TIMSS Curriculum Study. Five books were chosen for this part of the study, the two books in the textbooks series from 1979 and 1985 and the single book from 2001, *Matematikboken X*. One of the books from 1979 and one of the books from 1985 is intended for the general course, *Matematikboken ak 7* and *Matematikboken 7A* respectively. The two other ‘old’ books are intended for the more advanced special course, *Matematikboken sk 7* from 1979 and *Matematikboken 7S* from 1985. All five textbooks are published at least five years after a revision of the corresponding läroplan.

Occasionally I will use the following abbreviations:

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Intended for</th>
<th>Year</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matematikboken ak 7</td>
<td>General course 7</td>
<td>1979</td>
<td>Ak79</td>
</tr>
<tr>
<td>Matematikboken sk 7</td>
<td>Special course 7</td>
<td>1979</td>
<td>Sk79</td>
</tr>
<tr>
<td>Matematikboken 7A</td>
<td>General course 7</td>
<td>1985</td>
<td>A85</td>
</tr>
<tr>
<td>Matematikboken 7S</td>
<td>Special course 7</td>
<td>1985</td>
<td>S85</td>
</tr>
<tr>
<td>Matematikboken X</td>
<td>Year 7</td>
<td>2001</td>
<td>X2001</td>
</tr>
</tbody>
</table>

Figure 8: The textbooks in the study

I also tried to ‘read’ the books in order to discover characteristics that are undetectable through the coding method. I focused especially on the structure, language, number of pages and exercises.

**Details on step 1**
For the survey, the books are first divided into smaller units labeled as ‘blocks’. A block is for instance a set of exercises, worked examples or in-
I used a total of ten block types in characterizing the three textbooks:

- Instructional narrative
- Related picture
- Worked example
- Unrelated picture
- Exercise set
- Summary
- Word problem
- Repetition
- Suggested activity
- Other, e.g. homework

Each block is studied from different perspectives. The three aspects are content, performance expectations and perspectives.

---

39 From Undvall et al., 1979a and 1979b, p. 120 and p. 75 respectively.
Content:
The content of a block is examined from two directions:

1. Number  A block can consider natural numbers, negative numbers, fractions, decimals or a combination of them.
2. Operation For example, the topic can be geometry, multiplication or division.

Performance expectations:
The task for the student, when working with each block, can be:

a) Reading and understanding;
b) Using a routine procedure;
c) Problem solving;
d) Reasoning;
e) Communicating and work in groups;
f) and others – for instance homework.

Perspectives:
This aspect refers to any overarching orientation to the subject matter, for instance, how and when one can use the specific mathematical knowledge in the everyday world or something about the history of mathematics.

Each block of content is coded relatively to one or more of the three aspects and to one or more categories within each aspect. The partition of the content aspect into number and operation makes it possible to distinguish between, for instance, multiplication of natural numbers and negative numbers.

In order to categorize the blocks regarding the performance expectations I tried to ‘read’ the textbooks as a student. If a task is similar to a worked example or if the task is located in a clearly defined subchapter, for instance ‘Multiplication of fractions’, which guides me to the right choice of
method to solve the task – then I considered the performance expectations as to be *using a routine procedure*.

If a block has the performance expectation *reasoning*, then it involves a demand to reflect upon a certain issue. These kinds of blocks can contain exercises like:

Your friend believes that ‘zero point thirteen’ is a bigger number than ‘zero point three’. Is she right or wrong? Explain why (Undvall et al., 2001, p. 43, my translation).

Problem solving is not an easy area so I decided to be very restrictive in categorizing an exercise set as problem solving. It only happened for non-routine tasks that the authors explicitly stated as problems. This can for instance be a classical problem like ‘The tower of Hanoi’.

![Figure 10: The classical problem about the tower of Hanoi](image)

The perspective code is only applied when a block describes something from the history of mathematics or clearly states something about applications for the mathematics in question.

---

40 Undvall et al., 1985a, p.197.
The main objective for the examination of the textbooks in this first step was to find out the differences and similarities. What characteristics distinguish the books intended for the general course with the books intended for the special course? What type of changes is recognizable in the new edition?

**Step 2**
In the second part of the analysis, some characteristics of the läroplaner and kursplaner from 1980 and 2001 are observed, features that distinguish them from their predecessors. Special attention is directed towards the goals of mathematics.

**Details on step 2**
In the kursplan for mathematics in Lgr 80 I find that the words *realistic* and *every-day-life* can illustrate the main vision also the main dissimilarity to the previous läroplan, Lgr 69.

In order to examine if the 1985 edition of the textbook corresponds to the idea of realistic tasks and problems connected to every-day-life, as declared in the Lgr 80, I looked at word problems in one specific chapter, namely the chapter on the topic percentages. Part of the framework, developed by Palm (2002) is used in order to classify the word problems. The framework includes eighteen aspects of real life task situations, the three most important of them are used in this study. The decision to exclude the majority

---

42 Percentage is a subject in mathematics which is frequently used for solving problem in every-day life.
43 For a complete list of the 18 aspects, see appendix.
of aspects is mainly based on the judgment of their relative importance for the study. In addition, several aspects are difficult to apply to textbooks problems. The aspects *Realism* and *Purpose* are however relevant for the study and perhaps even possible to apply to textbooks problems. But difficulties to find facts, for instance the correct prize of gasoline in 1979 or the life of young people in the 1985, hindered me from using these aspects.

The word problems in the chapter dealing with percentages\(^{44}\) in all five books are categorized using three aspects. The aspects are briefly described in figure 12 and explained through some example of tasks from the textbooks, which I have translated\(^ {45}\) into English.

1. **Event**  
The match in the event in the school task and the event in a corresponding simulated real life task situation.

2. **Question**  
The match in the question posed in the school task and the question posed in a simulated real life task situation.

3. **Existence**  
The match in the existence of the important information/data that is available in the school task and the important information/data available in the simulated situation.

![Figure 12: Aspects of real life tasks\(^ {46}\)](image)

**Example 1:**  
(Undvall et al., 2001, p. 256)

*There are red, green, and white marbles in a box. 25 % of the marbles are red and 15 % are green. What is the percentage of white marbles?*

**Example 2:**  
(Undvall et al., 1985a, p. 144)

*Stefan got an exiting book as a Christmas gift. He read \(\frac{3}{4}\) of the book on Christmas Eve. What percentage was left to read?*

---

\(^{44}\) The section intended for repetition and homework is excluded from all books.  
\(^{45}\) Free translation.  
\(^{46}\) This is a section of the table in (Palm, 2002, p. 8 in paper II), for the complete list see appendix.
Example 3:
(Undvall et al., 1985a, p. 152)

Åreskutan is 1,420 m high. The highest mountain in Sweden, Kebnekajse, is 48.6% higher. What is the height of Kebnekajse?

Example 4:
(Undvall et al., 1985a, p. 151)

The price of a pair of skis during the winter season was 950 SKr. The price went down 40% at the end of the season. What was the new price?

Example 5:
(Undvall et al., 1985b, p. 130)

Eric was paid 35 SEK per hour. The salary went up 8% due to a new agreement. How much was he paid after that?

Event

Example 1 is not corresponding to a real event. One can however argue that there are marbles in a box somewhere and therefore regard the example as a real event. But this can be said about almost any task in a textbook. Example 4 is on the other hand a real event and even a familiar situation for students.

Question

Example 2 is not regarded as a real question. A relevant question in this task would for instance be: How many pages are left to read? Example 5 is however a real question.

Existence

The information in Example 3 is not available in a real situation when someone asks about the height of a mountain. If the question is posed as in Example 3, then the height of Kebnekajse is known. This type of word problems is sometimes called ‘pseudo-problems’. The phenomenon is discussed in 1943 in relation to a survey by Fritz Wigforss.\(^{47}\) It was yet under discussion in a report published by Grevholm et al. in 1988. In these type
of tasks, the reality is turned upside-down – the facts that are given are those which would be calculated in an authentic problem (Grevholm et al., 1988, p. 272).

The information in Example 5 is in general available in a real situation so the aspect ‘existence’ is fulfilled in that task.

In addition to the categorization of word problems described above, I also looked at the topics. Problems can for example involve private economy or issues related to other school subjects. I arranged the problems in groups in an inductive way. The word problems are organized into four groups (Figure 13).

<table>
<thead>
<tr>
<th>Economy</th>
<th>Tasks concerning private economy: wages; taxes; prices on products or services; or rental costs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>School</td>
<td>Tasks concerning school, for instance the distribution of female and male students.</td>
</tr>
<tr>
<td>Other school subjects</td>
<td>Tasks related to social studies (for instance demography, politics), geography, biology, chemistry, physics or other school subjects.</td>
</tr>
<tr>
<td>Other</td>
<td>Tasks related to sports, games or other ‘odd’ subjects.</td>
</tr>
</tbody>
</table>

Figure 13: The topics of word problems

The role of mathematics in society and history is never mentioned in the Lgr 80. However, in the läroplan from 1994, Lpo 94, this is clearly emphasized. The vision that students should know about the importance of mathematics is evident in the description of the goals for mathematics as well as in the assessment criteria.

In order to examine if the new edition of the textbook pays more attention to the historical development, the importance and role of mathematics in society than the editions from 1985 and 1979, I investigated the narrative blocks with a perspective code. In addition, I used the classification of the word problems, described above, to examine how mathematics tasks are used in order to show the important role of mathematics in society.
Findings

Findings in the first step
The first step of this study was to examine the textbooks to get an overall picture of them. Looking at the physical features, like structure and number of pages or exercises, I found that the number of pages has increased. The newest textbook, X2001, has 367 pages\(^{48}\), which is more than twice as much as the number of pages in the editions of 1979. But the number of exercises\(^ {49}\) is approximately the same (Figure 14).

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Abbreviation</th>
<th>Number of pages</th>
<th>Number of exercises</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matematikboken ak 7</td>
<td>Ak79</td>
<td>180</td>
<td>1 022</td>
</tr>
<tr>
<td>Matematikboken sk 7</td>
<td>Sk79</td>
<td>179</td>
<td>1 112</td>
</tr>
<tr>
<td>Matematikboken 7A</td>
<td>A85</td>
<td>228</td>
<td>1 120</td>
</tr>
<tr>
<td>Matematikboken 7S</td>
<td>S85</td>
<td>238</td>
<td>1 240</td>
</tr>
<tr>
<td>Matematikboken X</td>
<td>X2001</td>
<td>367</td>
<td>1 107</td>
</tr>
</tbody>
</table>

Figure 14: Physical features of the textbooks

There is no considerable difference between the books if we look at the structure. For every new subject there is first an introduction, followed by some worked examples and a set of exercises. The topics are clearly identifiable and arranged into chapters and subsections with headings like *Addition and subtraction of fractions* and *The area of a triangle*.

Nevertheless, one detectable difference between X2001 and the other books is that X2001 has special subsections, at the end of each chapter, devoted to reasoning and suggestion for group work. Hence, four percent of the blocks in X2001 are coded as *Suggested activity*. The performance expectations are for these blocks *Reasoning* or *Communicate*.

Two percent of the blocks in X2001 and six percent of the blocks in A85 and S85 are viewed as *Problem solving* in relation to the performance ex-

---

\(^{48}\) The average numbers of pages are 225 for lower secondary school (eight grade) mathematics textbooks in TIMSS Curriculum Study (Valverde et al., 2002).

\(^{49}\) The sections with repetition and homework are excluded.
Expectations. At first, it can be perceived as a higher degree of emphasis on problem solving in A85 and S85 than in X2001. However, it must be taken into account that each of them only contains one task and that each block of problems in the X2001 consists of twelve exercises. None of the blocks in Ak79 and Sk79 are considered as problems (Figure 15).

![Figure 15: Performance expectations in the textbooks](image)

The emphasis on *Instructional narratives* is decreasing. In the editions from 1979, eighteen percent of the blocks consist of narratives. In the S85 and A85 the proportions are sixteen percent and in X2001, only thirteen percent of the blocks are instructional narratives.

The four ‘old’ books are fairly comparable regarding blocks with *Exercise sets*. However, the percentages of *Instructional narratives* and *Word problems* are lesser in the editions from 1985 in favor of *Worked examples*. The share of *Word problems* is decreasing if we compare Sk79 with S85 and Ak79 with S85. On the contrary it is slightly higher in the X2001 than in all the other books. There are more blocks of *Word problems* in comparison to *Exercise sets, Instructional narrative, Worked example* and *Suggested activity* in X2001. It is the reversed in the other textbooks, i.e. fewer blocks with *Word problems*. Blocks with suggestions on activities, *Suggested activity*, are only present in X2001. These are placed at the end of each chapter in the book (Figure 16).

66
There are more pictures, both related and unrelated, in the X2001 than in the older books. Related pictures are more frequent in the books for the general course\textsuperscript{50}.

\textsuperscript{50} A detailed discussion about pictures in textbooks can be found in Grevholm et al., 1988, p. 247-249.
When comparing the books intended for the general course, Ak79 and A85, with the books intended for the special course, Sk79 and S85, I find that the difference between these two books is primarily about including or excluding specific concepts. Topics like multiplication and division of negative numbers and fractions were, according to Lgr 69, not necessary for the general course and these topics only appear in the Sk79. However, in geometry there is a difference between the books irrespective of the suggestions in the syllabus. Notions like supplementary angle and vertical angle are excluded from the Ak79 and geometrical objects like rhombus, parallelogram and diagonal are also absent. The only polygons introduced are triangles, squares and rectangles. Another observable difference is the size of the font, Sk79 has smaller letters than Ak79. This can indicate a belief that for many students mathematical problems are based on reading difficulties.

The alternative courses are not mentioned in Lgr 80 but the textbooks still appear in two versions in the editions from 1985. Moreover, suggestions regarding differentiation are absent in Lgr 80. Nevertheless, the textbooks S85 and A85 differ in the same way as described above regarding the editions from 1979.

Findings in the second step
There is a strong emphasis on real mathematics in Lgr 80, stating that students shall deal with tasks to attain knowledge that is useful in practical situations. In contrast to that, a comparison of word problems in the chapter on percentages shows that the number of word problems is decreasing (Figure 18).
The analysis of the word problems regarding their degree of realism shows that most problems are formulated as real events. A hundred percent of the word problems in Ak79, Sk79 and A85 describes real events. Corresponding figures for S85 and X2001 are 97% and 96% respectively.

The question is also relevant in most cases. However, the emphasis on authentic questions are somewhat higher in the books intended for the general course where 98% in both Ak79 and A85 are questions that can be posed in a real situation. Corresponding figures are 95% for Sk79 and S85 and 88% for X2001.

Several word problems are formulated so that the facts that are given are what you will calculate if it was a problem of the real world. The books intended for the special course emphasis problems with realistic information to a higher degree than the other books, the figures for Sk79 and S85 are 71% and 74%, respectively. A little more than half of the word problems in the other books are giving realistic information. The proportions are 55% in Ak79, 58% in A85 and 54% in X2001.
The word problems in the chapter on percentages involve different interest areas. Most tasks concern private economy, for example: prices, wages, rents and taxes. The second most common problem in the editions from 1979 and 1985 is related to other school subjects such as Social studies, Geography and Chemistry. The results on the last general election, the distribution of employees in the industry and the population of China are examples of tasks that students have to deal with. Sk79 and S85 emphasizes to a high degree subjects related to other school subjects. The number of word problems of this type is 31 (of total 87) and 25 (of total 74) (Figure 20). However, if we consider the distribution, then the proportion of tasks related to other school subjects is approximately the same in the editions from 1979 and 1985, but there is a considerable difference between X2001 and the other books (Figure 20). The second most common problem in X2001 is school issues, for instance the proportion of girls/boys in a class.
There is a strong emphasis on the role of mathematics in modern society and in the history of mathematics in the kursplan from 1994. If the textbooks were a concrete form of the läroplan, then it would be natural to find stories about the history of mathematics or role of mathematics in modern society in the new edition. However, only six narrative blocks in X2001 are coded with a perspective code. Five of them are about when and why to use a specific mathematical knowledge, the sixth block contains a five-sentence short story about the history of measuring time. There are some narrative blocks with a perspective code in the textbooks from 1979 and 1985. However, when and why the mathematical knowledge is useful is described in more general terms than in the X2001.
Table

<table>
<thead>
<tr>
<th>Topic</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1 Rough estimate</td>
<td>When you are buying things in a store – rough estimate is helpful if you want to find out how much the costs it.</td>
</tr>
<tr>
<td>P2 Rounding</td>
<td>Stores utilize rounding. If the total sum is 14,47 you must pay 14,50 because there are only whole and half crowns.</td>
</tr>
<tr>
<td>P3 Time</td>
<td>If you want to know how long time a trip will take – then you must know how to compute a difference in time.</td>
</tr>
<tr>
<td>P4 Diagrams</td>
<td>The newspapers and the TV often use diagrams to illustrate facts and connections. Diagrams can also be used to illustrate a trip.</td>
</tr>
<tr>
<td>P5 Statistics</td>
<td>Collected data can be more understandable if you compute the mean and the median.</td>
</tr>
<tr>
<td>P6 History</td>
<td>A story about the historical development concerning mathematics.</td>
</tr>
<tr>
<td>P7 Hand-held calculators</td>
<td>Hand-held calculators are used for solving practical problems in every-day life.</td>
</tr>
<tr>
<td>P8 Equations:</td>
<td>Solving equations is relevant mainly in physics and chemistry.</td>
</tr>
</tbody>
</table>

Figure 22: The topics in blocks with a perspective code

The narrative block in X2001 with a historical connection contains a story, which in a free translation would be:

In the twelfth century before Christ, the Egyptians divided the day and the night into twelve hour each. This implied that the length of an hour varied at different times of the year. The system was abandoned in the fourteenth century after Christ. A couple of hundred years before Christ was born, Greek astronomers introduced the partitioning into 60 minutes and 60 seconds. The number 60 came from the Babylonian numerical system (Undvall et al., 2001, p. 236, my free translation).
The distribution of the narrative blocks with perspective codes are summarized in the following table:

<table>
<thead>
<tr>
<th>Perspectives</th>
<th>Ak79</th>
<th>Sk79</th>
<th>A85</th>
<th>S85</th>
<th>X2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1. Rough estimate</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P2. Rounding</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>P3. Time</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>P4. Diagrams</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>P5. Statistics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>P6. History</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P7. Hand-held calculators</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>P8. Equations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 23: The number of blocks associated to the perspectives

**Summary and conclusion**

The first part of the analysis of the textbooks indicate that, apart from some special units, the books are comparable. Special units, with for instance problem solving and thematic work, are added to X2001 so the number of pages is higher, but the number of exercises are, if we exclude these units, almost the same. This can imply that students are not working through the whole book and it has to be decided which part of the book they should leave out. This decision can be made by: (a) the teacher; (b) the individual student; (c) the student together with the teacher; or (d) the teachers of a school as a collective group. So even if the new edition of the textbook series investigated in this study is more varied with respect to suggestions for students’ activities, it is easy to ignore the parts of the book dedicated to problem solving and other enrichments. Teachers could use the new book and teach in the same way as with the old one. Students can basically work with the same type of exercises as the students did in the beginning of the 80’s. So, even if we assume that there is a strong dependence on textbooks in mathematics education we are not able to draw too general conclusions about the consequences of that from this study. Further studies on the interaction between textbooks and the use of textbooks in mathematics educa-
tion are therefore needed to determine on the influence of textbooks on the teaching of mathematics.

The second part of the analysis shows that the goals of mathematics, explicitly stated in läroplaner and kursplaner, have not been realized through the textbooks analysed. The editions of the textbooks from 1985 did not respond to the emphasis on problem solving and authentic tasks, which are stressed in Lgr 80.

Moreover, even if the kursplan in Lpo94 is written in general terms, it is explicitly stated that the education shall provide students with knowledge about the history of mathematics and the importance and role of mathematics in modern society. Such proposal could be handle, by textbooks authors, in at least two ways.

a) They could add short stories in order to explain the role of mathematics in our society or short stories about the history of mathematics.

b) They could increase the number of word problems with a subject matter that deal with questions related to the use of mathematics in our society. Such word problems could for example concern issues related to social studies, chemistry or other school subjects.

I claim that the textbook authors have not succeeded to sufficiently well incorporate the intention in the actual kursplan and läroplan in the newest edition of the textbook series, X2001. In the entire book there are only six narrative blocks related to the history of mathematics or applications. Additionally, the number of tasks in the chapter on percentages that are related to other school subjects are only 7 (out of 48). Actually, most of the problems concern private economy or questions related to the school. In comparison with the other editions, X2001 is not improved in this field. In fact, there are fewer word problems related to other school subjects in X2001 than in the other editions. Hence, the findings suggest that textbooks are not a corresponding image to the vision of the educational authorities.
6. Summary, conclusions and recommendations

In part, textbooks provide reflections of the intended curriculum. Therefore, the study of textbooks was central in the research design of the *Third International Mathematics and Science Study*, TIMSS. In the curriculum model, textbooks are regarded as the potentially implemented curriculum, the link between aims and actuality (Schmidt et al., 1997; Valverde et al., 2002). Does this view coincide with the role of the textbooks in Sweden?

The situation of Mathematics in schools in Sweden is assessed and described in a report from the National Agency of Schools (Skolverket, 2003). One part of the report entails a discussion concerning the influence of textbooks in teaching and learning of mathematics. In particular, the school inspectors react to the dominant role of the textbooks. If the report gives a true picture of the mathematical activities, then textbooks are the implemented curriculum, at least for the majority of students in the last years of compulsory school. The textbook series in this study is, and has been, commonly used in Sweden. This implies that the chosen textbook can be regarded as a representative for the implemented curriculum.

The running of compulsory school in Sweden has changed since the first national läroplan, Lgr 62, was implemented. Earlier, the educational system was administered by the center. The läroplan was a detailed document describing the objectives as well as content and teaching methods. Today, the schools are administered by the periphery. However, the norms and objective are still expressed by the center, the National Agency of Schools, Skolverket, in the läroplan and kursplan (Skolverket, 1998).

Do textbooks reflect the läroplan? The case study, described in chapter five, shows that the specific textbook series, which I have analyzed, is not a complete image of the läroplan. The objectives of mathematics, stated in the kursplan, are only partially realized through the investigated series of textbooks. However, it is not the responsibility of textbooks authors to care for fulfillment of goals. In fact, it is the school that is responsible for the completion (Skolverket, 1998). In a personal communication with one of the authors of the textbook series, Lennart Undvall says that the present kursplan of mathematics is written in too general terms. For the latest ver-

---

sion of the textbook\textsuperscript{52}, the authors used the national tests as a concrete form and an interpretation of the kursplan.

In Sweden, the educational authorities have, from time to time, expressed concerns about a too strong reliance on textbooks. But for what reason are they concerned? What do we know about the use of textbooks in Swedish classrooms? Not much, I must say. In the third chapter of this text, some studies from other parts of the world regarding the use of textbooks in classrooms are described. These examples show that the role of textbooks in the teaching of mathematics is not easily determined. So, general information that textbooks are commonly used in Sweden, as well as in other countries all over the world, does not help to better understand the consequences.

If we think about the development of the curriculum in Sweden, we find that power-coercive strategies\textsuperscript{53} are used. An example is the IMU-project. It started as a local project but was later on absorbed by the authorities with the objective to disseminate the idea to all schools. The IMU-project entailed material developed to facilitate an individualized approach to teaching in mathematics. The objective was a self-instructional and self-controllable material that was graded by degree of difficulty. When analyzing the textbooks I consider them to be constructed in this manner. Therefore, I cannot resist from asking the question: Is it possible that the beliefs of the ‘educational techniques’ still have influence on the teaching of mathematics in Sweden? The question goes beyond the aim of this study so I will leave it open here. However, according to Wallby et al. (2001), there was a renewal of textbooks in connection to the IMU-project.

So, which strategies can be employed for a reform of teaching of mathematics in Sweden? Under the assumption that ‘mathematics is for teacher and students simply what is written in the textbook’, changing the textbooks is a necessary condition. However, textbooks will possibly not change through a revision of the läroplan. From the historical development of a commonly used textbook we learn that some things are hard to change – especially vision and aims concerning teaching in mathematics. Can restrictions of the use of textbooks be a solution? Well, for a teacher with little or no training in mathematics this may be a bad solution.

\textsuperscript{52} Matematikboken from 2001.
\textsuperscript{53} See chapter 2, page 12.
Perhaps we can learn from other countries? The United States has the Standards, which are elaborated by educationalists and can be regarded as a national curriculum. In order to cope with the demand from the Standards, so called reform based teaching material and textbooks are published. However, bearing in mind the experience from several studies related to the influence of textbooks in curriculum reforms\textsuperscript{54}, changing the textbooks is not a sufficient condition. According to some researchers (see for instance Manouchehri & Goodman, 2000), teachers’ competence in mathematics is the most important factor in a reform.

The role of curriculum material in teacher learning and instructional reform is discussed by Ball and Cohen (1996). They remark that textbooks and other curriculum material play an uneven role in practice, despite their central role in the school system. They offer three explanations for that: 1) curriculum developers’ failure to embrace teachers and their need to learn how to use the new material; 2) teachers’ necessity to select material that suits their own students; and 3)

[...] that educators often disparage textbooks, and many reform-oriented teachers repudiate them, announcing disdainfully that they do not use texts. This idealization of professional autonomy leads to the view that good teachers do not follow textbooks, but instead make their own curriculum (Ball & Cohen, 1996, p. 6).

The fact that many authors of textbooks are teachers themselves, at least in Sweden, can have an effect on the development of school mathematics – both in terms of good and bad influence. Acquaintance with classroom life makes it possible to construct textbooks that are easy to work with. However, this can also be a factor that causes stagnation. If teachers have no or little time to prepare group work or thematic work and if students often ignore narrative blocks and if authors are aware of that – then there is no reason to construct a textbook which puts an emphasis on other activities than solving routine tasks.

The design of ‘teaching units’\textsuperscript{55} in mathematics has almost exclusively been a task for teachers and textbooks’ authors. However, Wittmann (1995) claims that researchers should pay more attention to ‘teaching units’. He

\textsuperscript{54} See chapter 3 about the role of textbooks in curriculum reforms.

\textsuperscript{55} Textbooks can be regarded as a collection of teaching units.

77
suggests that the design of them is “… a difficult task that must be carried out by the experts in the field” (Wittmann, 1995, p. 365).

By no means can it be left to teachers, though teachers can certainly make important contributions within the framework of design provided by experts, particularly when they are members of or in close connection with a research team (ibid.).

Finally, I agree with Ball and Cohen who believe that “… if we want the intended curriculum best to contribute to the enacted one, we must find ways to design the first with the second clearly in view” (Ball & Cohen, 1996, p. 8).

**Suggestions for further studies**

I claim that we know too little about mathematics textbooks in Sweden. Only a few studies has been done in this area (for example, Bremler, 2003; Brändström, 2003; Lithner, 2000). The case study, described in Chapter 5, is evidently a limited analysis of the selected textbook series. Instead of just looking at three editions one can extend the study in different respects: One could analyze all six editions of the books. This would be a more comprehensive analysis, but not as focussed on the relations between textbooks and curriculum change. Another enlargement of the study would be the inclusion of the books intended for year eight and nine of ‘grundskolan’ (what would have definitely been too large an analysis for this text).

What if the specific textbook series, which was analyzed is a special case, not representing a standard, ‘normal’ Swedish textbook series? Through an analysis of the most widely used, if not *all* textbook series on the market one can find a way around that problem.

What about other interesting aspects of the textbooks that could be analyzed in detail? One could for instance look into the way textbooks introduce a certain mathematical topic, how the topics are organized and how they are connected in the books. Another interesting area concerns teachers’ guides and computer softwares. Several textbook series have already associated material to the series. How is this additional material connected

---

56 Compulsory school.
to the textbook and to the teaching? Does the material offer additional, different ways to teach a certain topic?

These additional pieces of research would simply enlarge the detailed study of the relation between the intended and the potentially implemented curriculum. What is more important, we need more information about the use of mathematics textbooks in classrooms. Not only *how much* textbooks are used in relation to other activities should be analyzed but also *how* and *why* textbooks are used. A study of the use of textbooks in classrooms would offer further knowledge about the role of textbooks in teaching and learning of mathematics in schools. This can for example be done through observations in classrooms studying the actual use of textbooks in classrooms. Another research would be trying to know more about the role textbooks play in the everyday practice of teachers, for instance by asking them to write down a logbook of their work for a detailed study of the role textbooks play in the professional life of teachers.
7. References


8. Appendix

Aspects of real task situations proposed to be of importance in simulations (Palm, 2002)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table 1</strong></td>
<td>Aspects of real life task situations proposed to be of importance in simulations</td>
<td></td>
</tr>
<tr>
<td>A. Event</td>
<td>The match in the event in the school task and the event in a corresponding simulated real life task situation</td>
<td></td>
</tr>
<tr>
<td>B. Question</td>
<td>The match in the question posed in the school task and the question posed in a simulated real life task situation</td>
<td></td>
</tr>
<tr>
<td>C. Purpose</td>
<td>The clarity of the purpose of the school task</td>
<td></td>
</tr>
<tr>
<td>C1. Purpose in the social context</td>
<td>The match in the school situation and the simulated situation regarding the clarity of the purpose of solving the task in the simulated situation</td>
<td></td>
</tr>
<tr>
<td>C2. Purpose in the figurative context</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D. Information /data</td>
<td>The match in the existence of the important information/data that is available in the school task and the important information/data available in the simulated situation</td>
<td></td>
</tr>
<tr>
<td>D1. Existence</td>
<td>The realism of the information/data available in the school task</td>
<td></td>
</tr>
<tr>
<td>D2. Realism</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D3. Specificity</td>
<td>The match in the specificity of the information/data available in the school task and the specificity of the information/data available in the simulated situation</td>
<td></td>
</tr>
<tr>
<td>E. Language</td>
<td>The match in the form (oral, written, graphic etc.) of the question and the information in the school task and the form of the question and the information in the simulated situation</td>
<td></td>
</tr>
<tr>
<td>E1. Form</td>
<td>The match in the semantics (terminology, sentence structure, text amount) used in the school task and in the simulated situation</td>
<td></td>
</tr>
<tr>
<td>E2. Semantics</td>
<td>The match in the possibilities of asking about, and discussing the meaning of, assignment and information in the school task and such possibilities in the simulated situation</td>
<td></td>
</tr>
<tr>
<td>E3. Discussion opportunities</td>
<td>The match in the available student strategies and plausible strategies for solving the simulated task situation</td>
<td></td>
</tr>
<tr>
<td>F. Solutions strategies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F1. Availability</td>
<td>(table continues on next page)</td>
<td></td>
</tr>
</tbody>
</table>

87
<table>
<thead>
<tr>
<th>Table 1 (continued)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F2. Experienced plausibility</strong></td>
<td>The match in the strategies experienced as plausible for solving the school task and the strategies experienced as plausible for solving the task in the simulated situation</td>
</tr>
<tr>
<td><strong>G. Circumstances</strong></td>
<td></td>
</tr>
<tr>
<td><strong>G1. Availability of external tools</strong></td>
<td>The match in the availability of external tools (e.g. calculators, computers and maps) in the school task situation and in the simulated situation</td>
</tr>
<tr>
<td><strong>G2. Guidance</strong></td>
<td>The match in guidance (explicit or implicit hints, e.g. on solution methods and types of answers required) in the school task and in the simulated situation</td>
</tr>
<tr>
<td><strong>G3. Consultation and collaboration</strong></td>
<td>The match in the possibilities of interaction (consultation and collaboration) with e.g. literature (paper and IT-based) or other persons in the school situation and in the simulated situation</td>
</tr>
<tr>
<td><strong>G4. Time</strong></td>
<td>The match in time constraints on solving the school task and time constraints on solving the task in the simulated situation</td>
</tr>
<tr>
<td><strong>G5. Consequences</strong></td>
<td>The match in the consequences of success or failure in solving the school task and the consequences of success or failure in solving the task in the simulated situation</td>
</tr>
<tr>
<td><strong>H. Solution requirements</strong></td>
<td>The match in explicit or implicit requirements on the solution of the school task and the requirements on the solution of the task in the simulated situation. Requirements can for example be expressed in the task text or in the judgement on the solution.</td>
</tr>
</tbody>
</table>