Mechanical properties of flax fibers and their composites

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by

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PREFACE

The work presented in this thesis concerns flax fibers as a potential replacement of synthetic fibers in conventional polymer composites. The thesis consists of a general introduction and literature review and two journal papers.

Research nowadays often is a result of team work. Therefore there is a couple of persons that I would like to acknowledge.

First, I thank my supervisors: Prof. Jānis Vārns, Dr. Jānis Andersons, Dr. Roberts Joffē and Prof. Vitauts Tamužs. I would like to thank my co-author Lernart Wallström as well.

Further thanks go to Mr. Vilis Skruls and Mr. Uldis Vilks, research engineers from Institute of Polymer Mechanics, Riga, Latvia. They helped with experimental equipment setup for single fiber tensile tests.

I also thank Dr. Harriëtte L. Bos, who allowed me to use her illustration figure in this thesis.

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SUMMARY

Flax fibers, along with a number of other natural fibers, are being considered as an environmentally friendly alternative of synthetic fibers in fiber-reinforced polymer composites. A common feature of natural fibers is a much higher variability of mechanical properties. This necessitates study of the flax fiber strength distribution and efficient experimental methods for its determination.

Elementary flax fibers of different gauge lengths are tested by single fiber tension in order to obtain the stress-strain response and strength and failure strain distributions. The applicability of single fiber fragmentation test for flax fiber failure strain and strength characterization is considered. It is shown that fiber fragmentation test can be used to determine the fiber length effect on mean fiber strength and limit strain.

Stiffness and strength under uniaxial tension of flax fiber composites with thermoset and thermoplastic polymer matrices are considered. The applicability of rule of mixtures and orientational averaging based models, developed for short fiber composites, to flax reinforced polymers is evaluated.
This thesis comprises the following papers:

**Paper A**  

**Paper B**  

Content of both papers is reported in conferences:


The conference proceedings are not included in the thesis.
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1. Motivation of natural fiber applications in polymer composites

Due to the exponential growth of human population on Earth we face environmental problems more and more. Now, in 21st century, it is clear that we are paying for advanced technology with ecological troubles and even disasters sometimes. And it is in our interests to look for solution. Therefore, from the materials science point of view, there is growing interest in green, environmentally friendly materials. If we consider composites, one of solutions can be use of natural fibers instead of more traditional glass and carbon fibers [1-4]. The possible advantages of such natural fiber composites (NFC) could be

- Lower pollution level during production;
- Energy necessary for fiber production is lower than that of glass;
- CO₂ neutral: amount of CO₂ neutralized during fiber plant growth is comparable with that emitted during processing;
- Lower cost.

However, all these previous statements should be supported by quantitative analysis.

- Natural fibers are renewable resources;
- They are biodegradable; however this can be a drawback during the lifetime of a product;
- Using biodegradable polymers as matrix, we can have totally recyclable materials.

Natural fibers in composites can compete with synthetic fibers by

- Lower density;
- Healthier in use due to their natural origins;
- Less abrasive to the processing equipment.

Low density is the main point why NFC is interesting in automotive sector.

Anyway, beside these environmental advantages, we are about to use natural fibers in composites. The stiffness and strength of fibers are the basis for the reinforcement, but also the interfacial strength (adhesion) is important for efficient reinforcement. Therefore these three parameters are first to be determined and characterized. Let us have a little look at literature dedicated to natural fibers and their composites.
2. State of the art

2.1. Some facts from history

Natural organic fibers have been around for a very long time, from the beginning of the life on Earth. The archeological artifacts suggest that human beings used these materials in fabrics many thousand years ago.

A direct use of the strength of natural fibers is in lines, ropes and other one-dimensional products; miscellaneous applications include early suspension bridges for on-foot passage of rivers and rigging for naval ships in early times and into the nineteenth century. Many kinds of textiles, ropes, canvas and paper produced form natural fibers are in use today.

It may seem surprising, but first natural fiber composites were used more than 100 years ago. In 1896, for example, airplane seats and fuel-tanks were made of natural fibers with a small content of polymeric binders. As early as 1908, the first composite materials were applied for the fabrication of large quantities of sheets, tubes and pipes for electronic purposes (paper or cotton to reinforce sheets, made of phenol- or melamine-formaldehyde resins) [2]. However, these attempts were without recognition of the composite principles and the importance of fibers as the reinforcing part of composites.

The use of natural fibers was suspended due to low cost and growing performance of technical plastics and, moreover, synthetic fibers. A renaissance in the use of natural fibers as reinforcements in technical applications began in 90s of 20th century.

2.2. Overview of cellulose-based natural fibers

Different types and examples of natural fibers classified according to their origin are presented in Figure 1. Asbestos is out of further consideration in this study due to its cancerogenic nature. In addition, asbestos does not possess most of advantages mentioned in Section 1.
Introduction

Natural Fibers

Animal
- Wool
- Silk

Mineral
- Asbestos

Plant or Vegetable

Bast
- Flax
- Hemp
- Jute
- Ramie
- Kenaf

Leaf
- Sisal
- Banana
- Abaka

Seed
- Cotton
- Coir
- Oil palm
- Kapok

Wood
- Soft wood
- Hard wood

Grass stem
- Reed canary grass
- Cereal straw (rye, wheat)

Figure 1. Classification of natural fibers according to origin together with several examples.

Structure and Chemical composition

Generally, plant or vegetable fibers are used to reinforce plastics. The main polymers involved in the composition of plant fibers are cellulose, hemicelluloses, lignin and pectin.

Let us consider very popular flax fibers to understand the intricate structure of plant fibers. The ~1 meter long so-called technical fibers are isolated from the flax plant for the use in textile industry. These technical fibers consist of elementary fibers with lengths generally between 2 and 5 cm, and diameters between 10 and 25 μm. The elementary fibers are glued together by a pectin interface. They are not circular but a polyhedron with 5 to 7 sides to improve the packing in the technical fiber.
Introduction

Figure 2. Composition and built of flax stem. (Illustration by Harriëtte L. Bos. Reproduced by permission)

What are elementary fibers? They are single plant cells. And cellulose \((C_6H_{10}O_5)_n\) is a common material in plant cell walls. It occurs naturally in almost pure form in cotton fiber. Chemical structure of cellulose monomer is represented in Figure 3. Most of the elementary fiber consists of oriented, highly crystalline cellulose fibrils and amorphous hemicellulose. The crystalline cellulose fibrils in the cell wall are oriented at an angle of about ±10 degrees with the fiber axis [3, 6] and give the fiber its high tensile strength.

![Cellulose Structure](image)

Figure 3. Chemical structure of cellulose monomer.
Introduction

Characterization of mechanical properties

The natural organic fibers are basically characterized by the same parameters and properties as all other fibers. However, due to natural origin they show much higher variability of the various parameters than their synthetic counterparts. Chemical composition, crystallinity, surface properties, diameter, cross-sectional shape, length, strength, and stiffness vary from fiber to fiber. Moreover, properties depend on growing (climate), harvesting conditions and processing. This poses two problems: quality characterization of fibers and difficulties in application of traditional composite theories.

Usually the single fiber tensile (SFT) test method is used to measure modulus and tensile strength of natural fibers. It is important to mention that elementary (flax) fibers have considerably higher strength than technical fibers [5]. Elementary fibers of flax [5-8], nettle [6] and wheat straw [9] are tested at single gauge length. It is verified that Weibull distribution is applicable to approximate strength distribution of natural fibers [5, 6, 8]. Nonlinearity of stress-strain response (strain-hardening) is reported [7, 9]. The increase of the Young’s modulus with strain is explained with a reorganization of the cellulose fibrils in the direction of the fiber (loading) axis. From fatigue tests it is established that this effect is irreversible [7].

Another direct method, although less popular, is loop test. Using this method it is possible to determine both tensile and compressive strength of fibers. Analyzing fiber failure qualitatively by ESEM [10] and quantitatively [5] it is obtained that compressive strength of the flax fibers is about 80% of tensile strength.

 Completely different approach is to back-calculate fiber properties from unidirectional model composite tests. Anisotropy of jute is studied using this method [11]. In this way it is calculated that the transverse modulus of fibers is 5-10 times smaller than the longitudinal modulus. Coefficient of thermal expansion is negative along fiber, but in transverse direction – positive and comparable with that of the polymer matrix.

In addition, the effect of heating [9, 12] and moisture [13] uptake of natural fibers has been studied. It is concluded [12] that during production of natural fiber reinforced plastics, only short periods of exposure to high temperatures are allowed. Composite production temperatures higher than 180℃ have to be avoided in order to prevent degradation of fibers.
Unfortunately, in addition to advantages mentioned in Section 1, we have several disadvantages of natural fibers:

- Large scatter of all the parameters;
- Properties depend on growing and processing conditions;
- Degradation of properties (moisture, heat, flame);
- Fibers are short; that means lower performance of their composites;
- Structure is highly inhomogeneous;
- Stress – strain response is nonlinear.

Finally, it is seen from literature that flax fibers have the best potential to substitute glass in polymer composites. A comparison of main parameters for flax, hemp, jute and glass fiber is given in Table 1. Specific properties of natural fibers, especially flax, are promising.

<table>
<thead>
<tr>
<th>Fibers</th>
<th>Modulus (GPa)</th>
<th>Strength (MPa)</th>
<th>Density (g/cm³)</th>
<th>Specific Modulus</th>
<th>Specific Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-glass</td>
<td>72</td>
<td>3530</td>
<td>2.54</td>
<td>28.2</td>
<td>1390</td>
</tr>
<tr>
<td>Flax</td>
<td>50-70</td>
<td>500-900</td>
<td>1.4-1.5</td>
<td>~ 41</td>
<td>~ 480</td>
</tr>
<tr>
<td>Hemp</td>
<td>30-60</td>
<td>300-800</td>
<td>1.48</td>
<td>~ 30</td>
<td>~ 370</td>
</tr>
<tr>
<td>Jute</td>
<td>20-55</td>
<td>200-500</td>
<td>1.3-1.5</td>
<td>~ 27</td>
<td>~ 250</td>
</tr>
</tbody>
</table>

### 2.3. Natural fiber composites

Most of NFC are short fiber composites with non-homogeneous length and orientation distributions. It is known that elastic modulus and the strength of discontinuous fiber composites are moderate compared to continuous fiber-reinforced composites. But there is more flexibility in the selection of their fabrication methods. Many processing methods used for plastics can be applied to the fabrication of discontinuous fiber composites. This leads to the possibility of easier mass production.

### Materials

To process to composites, natural fibers are typically formed into some form of fiber mat. Kenaf, hemp, sisal, coir, jute and some other
fiber composites have been studied. But the flax fibers appear to have received more attention.

Although thermal instability of fibers causes restrictions for matrices, both thermoplastics and thermosets are being used. Polypropylene (PP, melting temperature 160°C) is the most popular thermoplastic matrix, also found to be the best [14] for flax fibers. There is no distinct favorite among thermosets: epoxies, vinylesters, polyesters, and other polymers are used.

There are also attempts to use biopolymers [15-21], such as polyester amide, poly-L-lactic acid and others. The main attention in this area is focused on manufacturing problems. The properties of such biocomposites seem very promising too. However they are not considered in the following review.

**Manufacturing**

Thermoplastic NFC are manufactured mainly by different extrusion methods followed by injection [22] or compression molding [23, 24]. Fibers are chopped during process therefore composites have short fibers (few millimeters at most). Orientation is three-dimensional, but not necessarily isotropic (depends on method: injection process, shape of mould etc.).

Besides, natural fiber mat thermoplastic composite plates with different fiber contents can be manufactured using the film-stacking method [25, 26]. Thermoplastic pultrusion can also be applied for continuous process [27].

Compression molding processes is very typical for thermosets [28]. Resin transfer molding [29, 30] and resin infusion are used as well. These methods ensure relatively longer fibers and more or less inplane orientation in resulting composite. Since the thermal stability of the flax fibers may be increased by chemical treatment, then even autoclave molding technique can be applied [31].

A very important task of NFC manufacturing is to have elementary fiber, not technical fiber, as the reinforcement in the composite [32]. On the other hand, using technical fibers or textile yarns, it is possible make long fiber composites with predefined fiber orientation. However in both cases the basic problem is fiber/matrix adhesion.
Adhesion

All the plant fibers are hydrophilic in nature. That is because of their chemical structure – the hemicelluloses and the pectin are very hydrophilic [1, 14]. In contrary, many of the common matrix polymers in composites are largely hydrophobic in nature. Only thermosets such as phenolformaldehyde and related polymers are less hydrophobic and are therefore less problematic [1]. This discrepancy can lead to the formation of ineffective interfaces between the fiber and matrix. Problem can be solved applying different fiber treatments (both chemical and mechanical) or modifying chemical composition of the matrix. Unfortunately, surface treatments have a negative impact on economical aspect of NFC manufacturing.

There are many publications reporting that properties of PP based composites are improved using maleic anhydride grafted PP (MAPP) as matrix additive [8, 22, 25, 32-37]. The same MAPP [38], acetilation and stearic acid treatment [37] are used for fiber processing. Adhesion increases also after boiling of the flax fibers [35, 39].

For epoxy resin, alkali (NaOH), silane (3-aminopropyltriethoxysilane), isocyanate (phenyl isocyanate) [31], urea [40] and other treatments can be employed. Acrylic acid and vinyl trimethoxy silane of different concentrations are considered for several other thermosets [8].

Single fiber fragmentation (SFF) test is a common method used to measure adhesion quantitatively (via critical fiber length or interfacial shear strength) for synthetic fibers. It is adapted for flax as well [8, 41].

Performance and applications

Usually NFC is compared with glass fiber reinforced polymers of the same type (matrix; manufacturing method; fiber volume fraction, length and configuration). Normally these are fiber mat composites. Composite density must be taken into account, because it is the strong side of NFC. In other words, specific properties must be compared. This subject is studied in many papers. The basic results are that NFC can have very good specific stiffness [24, 42] and reasonable specific tensile strength [24, 42]. Unfortunately, the impact strength is a disaster for NFC [24, 25, 43], however it can be upgraded improving
adhesion [44]. Flexural strength of NFC is also somewhat lower that that of GFRP [32].

As expected, flax fiber composites are the best in properties (jute takes second place). In short, it can be concluded that NFC based on flax fibers can compete with E-glass based GFRP materials in stiffness critical structures, whereas for strength and impact critical applications these materials still need to be optimized further.

Several leading car manufacturers already use flax, banana fibers, for instance as polymer reinforcement in interior parts (car roofs, door panels and other). The use of flax fibers in car disk brakes instead of asbestos fibers, is another example. There is a potential to develop this tendency further [4, 28].

Modeling

Since we are at the beginning of understanding natural fiber composites, the most important parameters, as usual, are stiffness and strength.

Stiffness

A number of theoretical models have been elaborated to model stiffness of short fiber reinforced composites [45, 46]. Although they are very different, they all are based on the same basic assumptions:

- the fibers and the matrix are linearly elastic,
- the matrix is isotropic, and the fibers are either isotropic or transversely isotropic.
- the fibers are axi-symmetric and identical in shape
- the fibers and matrix are well bonded at their interface, and remain that way during deformation.

It is seen that none of them agrees perfectly with nature of natural fibers. Let us keep it in mind.

Historically, shear lag models were the first micromechanics models for short fiber composites. Although classical shear lag models predict the longitudinal modulus only, they are very popular due to their algebraic and physical simplicity. Usually they are implemented by combining the average stress in the fiber with average matrix stress to construct a modified rule of mixtures. It is unusual that the fibers in a short fiber composite are arrayed unidirectionally. Usually some distribution of fiber orientation exists. Therefore we obtain
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\[ E = \eta_{le} \eta_{oe} E_f V_f + E_m \left(1 - V_f \right), \]

where \( \eta_{le} \) is fiber length efficiency factor, and \( \eta_{oe} \) – orientation efficiency factor for Young’s modulus calculation. They both can be calculated having fiber length and orientation distributions.

Another classics are Halpin and Tsai relations:

\[ \frac{P}{P_m} = \frac{1 + \zeta \cdot \eta V_f}{1 - \eta V_f} \quad \text{where} \quad \eta = \frac{(P_f/P_m)-1}{(P_f/P_m)+1} \]

Initially they were developed for continuous fiber composites. However they are efficient for short fiber composites as well. Halpin and Tsai suggested that parameter \( \zeta \) is related to geometry of reinforcement. For longitudinal modulus they found it expressed through aspect ration of fibers \( \zeta = 2 (l/d) \) for instance.

However, there is a need to take both fiber length and orientation distributions into account. It is possible, using laminate analogy approach [47]. In this method composite stiffness is calculated “summing” layers with the same fiber length and orientation according to distribution functions.

Another approach is to use unit cells (matrix bricks with one fiber inside) in combination with finite element method [48]. Unfortunately these models include some parameters that are missing for natural fibers at the moment (fiber Poisson’s ratio and fiber shear modulus, for instance). This is common problem for other models also [49]. Therefore they are not applicable in this study.

Strength

In comparison to the stiffness, strength theory for short fiber composites is still under development. Most of the strength models are also based on the rule of mixtures [50-52] or equivalent laminate [53, 54] approach.

So in simplest case (considering rule of mixtures), misaligned short fiber composite strength

\[ \sigma_{uc} = \eta_{le} \eta_{oe} \sigma_{of} V_f + \left(1 - V_f \right) \sigma_m, \]
where $\sigma_{uf}$ is fiber strength, but $\sigma_m$ – stress in the matrix at the fiber failure strain. $\eta_l$ and $\eta_o$ are fiber length efficiency factor and fiber orientation efficiency factor for strength calculation respectively. For all the models, length efficiency factor $\eta_l$ is related to critical or ineffective fiber length.

Recently, more complicated models have been advanced [48, 55, 56] that employ numerical modeling of deformation and failure processes.

3. Current work

3.1. Objectives

As it follows from Section 2, simple and efficient quality control method for natural fibers is needed. Single fiber tensile test is very time consuming and tiresome. Single fiber fragmentation test has been used as an efficient alternative to SFT test for Weibull distribution parameter determination of synthetic fibers, while for flax fibers the test is mainly applied for the assessment of fiber/matrix adhesion. The principal objective of Paper A is to evaluate the applicability of SFF test for elementary flax fiber strength and failure strain characterization.

The aim of the Paper B is a systematic evaluation of the applicability of elementary mechanical models for flax fiber reinforced polymer matrix composite modulus and strength prediction.

3.2. Paper A

Elementary flax fiber strength and failure strain distributions at several gauge lengths are obtained by SFT tests.

It is found that the modified Weibull distribution [57, 58]

$$P(\sigma) = 1 - \exp\left[\left(\frac{1}{l_0}\right)^\gamma\left(\frac{\sigma}{\beta}\right)^\alpha\right]$$
is preferable for flax fibers considered. The applicability of single fiber fragmentation test for flax fiber failure strain and strength characterization is considered. It is shown that SFF test can be used to determine the fiber length effect on mean fiber strength and limit strain.

### 3.3. Paper B

Stiffness of flax fiber composites was modeled using Cox-Krenchel model. Strength was modeled by two methods: modified Fukuda-Chou and orientational averaging model for strength proposed by van Hattum and Bernardo [54]. To verify obtained results, the following types of flax fiber composites were made.

Flax fiber mats (FFM) produced by FinFlax were used as reinforcement for polymer composites. The resins used were two types of vinylesters, modified acrylic resin, polypropylene and maleic anhydride grafted PP modified polypropylene.

Plates of thermoset natural fiber composites were manufactured by resin transfer molding, while thermoplastic composite specimens were produced from compound obtained by co-extrusion of granulated PP and flax fiber rowing.

Correlation of experimental and theoretical composite characteristics revealed the degree of sensitivity of the models to properties of the constituents and their adhesion.

### 3.4. Conclusions

Strength and failure strain possess the modified Weibull distribution (i.e. independent exponents characterize strength scatter at a fixed gauge length and the dependence of mean strength on fiber length). SFF tests reveal that fiber fragmentation process proceeds in agreement with the two-parameter Weibull distribution of failure strain, but the distribution parameters for individual fibers exhibit a marked scatter. A limited number of SFF tests combined with SFT tests at a fixed gauge length provide the experimental information necessary for characterizing both the scatter of fiber failure strain and strength at a fixed gauge length and the effect of fiber length on the mentioned parameters.
Stiffness and strength under uniaxial tension has been obtained for flax/PP and flax/PPM composites produced from compound obtained by co-extrusion of granulated PP and flax, as well as for FFM/vinylester and FFM/acrylic resin composites manufactured by resin transfer molding. The rule-of-mixtures relations are shown to yield acceptable stiffness prediction of both extruded and FFM-based composites. The sensitivity of a rule-of-mixtures model of strength to the matrix and adhesion properties apparently depends on the relative ineffective fiber length; for relatively long-fiber FFM composites the sensitivity is low and more sophisticated strength models should be applied.

4. Future work

In order to continue the work described in Paper A, correlation between parameters of Weibull strength distribution and structure or morphology of fibers will be sought. This is the subject of a forthcoming paper.

However, the main goal is to have high performance natural fiber composites that can be used in load bearing constructions. The ideal solution is perfectly oriented elementary fibers in the matrix. Unfortunately it is not possible at the moment. But we can improve quality of technical fibers. There are some good examples already. Unidirectional flax fiber composites with remarkable properties are obtained using textile monofilament flax yarn [26]. Could the next step be cross-ply?

One of possible applications of such high performance natural fiber composites could be in building industry. It is a relatively new idea to strengthen and repair concrete or masonry structures with carbon fiber reinforced polymers. In fact, the only important properties of composite in such structures are stiffness and strength. Flax fibers could be better in sound and thermo-insulation. Since possible amounts of used fibers are very large, the economical effect could be very great.

This was just one example. There is a great future of high-performance natural fiber composites in structural parts.
5. References


Strength distribution of elementary flax fibres

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Strength distribution of elementary flax fibres

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Abstract. Flax fibres, along with a number of other natural fibres, are being considered as an environmentally friendly alternative of synthetic fibres in fibre-reinforced polymer composites. A common feature of natural fibres is a much higher variability of mechanical properties. This necessitates study of the flax fibre strength distribution and efficient experimental methods for its determination. Elementary flax fibres of different gauge lengths are tested by single fibre tension in order to obtain the stress-strain response and strength and failure strain distributions. The applicability of single fibre fragmentation test for flax fibre failure strain and strength characterization is considered. It is shown that fibre fragmentation test can be used to determine the fibre length effect on mean fibre strength and limit strain.

1. Introduction

Ecological concerns have resulted in a renewed interest in natural materials, and such issues as recyclability and environmental safety have become increasingly important for the introduction of new materials and products. Structural polymer composites are traditionally utilizing man-made fibres (such as glass or carbon fibres) as reinforcement, but environmental issues have generated a considerable interest in natural fibres. Plant fibres such as flax, hemp, sisal and kenaf are under consideration as environmentally friendly and relatively low-cost alternatives for glass fibres in structural engineering composites [1-3]. However, their use currently is mainly confined to reinforcement in compounded thermoplastic products similar to glass-mat-reinforced thermoplastics. These natural fibre mat thermoplastics are used in the automotive sector, in interior and exterior components [4, 5].

Flax fibres used as reinforcement are located in the bast of the flax plant. The fibres are typically extracted by retting followed by mechanical processing (scutching and hackling). The mechanical properties of the obtained fibres are affected by the natural variability in plant and by the processing stage and damage sustained during
processing [6-8] and thus have considerable scatter. Therefore a simple and efficient method for fibre strength evaluation is needed. The mechanical tests of elementary flax fibres [6, 8-10] revealed that fibre strength is reasonably well approximated by the two-parameter Weibull distribution. Single fibre fragmentation (SFF) test has been used as an efficient alternative to single fibre tensile (SFT) test for Weibull distribution parameter determination of synthetic fibres, while for flax fibres the test is mainly applied for the assessment of fibre/matrix adhesion [10-13]. The principal objective of the present study is to evaluate the applicability of SFF test for elementary flax fibre strength and limit strain characterization.

2. Experimental

2.1. Materials

Flax fibres delivered by FinFlax Oy (Finland) were used. The fibres had been enzyme retted for 22h. The fibres were stored and tested at ambient conditions. The resins used in single fibre composite (SFC) specimens were vinyl ester (VE) and unsaturated polyester (UP).

2.2. Single fibre tensile tests

2.2.1. Specimen preparation

Single fibres were carefully manually separated from the bundles. Fibre ends were glued onto a paper frame according to the preparation procedure described in ASTM D 3379-75 Standard. During mounting the specimens were handled only by the paper frame. Fibre length outside the frame (gauge length) was 5, 10, or 20 mm. Upon clamping of the ends of the paper frame by the grips of the test machine, frame sides were carefully cut in the middle.

2.2.2. Test setup

The tests were carried out on a electromechanical tensile machine equipped with mechanical grips. Load-displacement curve was recorded during the test. Upper grip of the machine was attached through a hinge and thus allowed to self-align. All tests were displacement controlled with the loading rate of 0.5 mm/min.
Since the fibres were not pre-stretched before the test, there was an initial displacement before load was actually applied to the fibre. The amount of this displacement was defined as an interval from the beginning of the test until the point at which load increase is observed. It was discounted later on during data processing.

Although flax fibre cross-section has a polygonal shape (see Fig. 1) and fibre thickness varies somewhat along the fibre [14], we treated each fibre as perfectly round and having a constant diameter in order to simplify analysis. Fibre diameter was evaluated from optical observations under microscope as the average of five apparent diameter measurements taken at different locations along the fibre.

2.3. Fibre fragmentation tests
2.3.1. Specimen preparation

Although single fibre fragmentation test is not strictly standardized, there are certain techniques that are established and widely used on synthetic fibres. The techniques may vary depending on particular experimental setup, but generally they include the following steps: single fibre separation from bundle, fibre pre-stretching (only to straighten the filament), mounting of the fibre on the frame and casting of the resin in the mould with the frame (see e.g. [15]). Our experimental procedure applied for synthetic fibres [16, 17] includes strain measurement by extensometer, therefore certain length of the samples is required. Hence specimen preparation procedure for the short fibres had to be modified in order to obtain specimens with length that would allow use of extensometer. The short flax fibres were modified by attaching fiber extensions on both ends of the
filament, see Fig. 2. Thin fishing line of 90 μm diameter was used as fibre extensions. Once “extended” fibre is obtained it can be handled as any other long fibre. Further specimen preparation is according to the procedure described above (it also includes slight fibre pre-stretching in order to straighten the filament).

![Fig. 2. Modification of flax fibre.](image)

![Fig. 3. SFC specimen.](image)

SFC specimens were prepared by mounting the extended fibres on a 1 mm thick steel frame, using double-sided adhesive tape. The frame was then placed between two flat Teflon-coated aluminum mould plates, separated by spacers of 2 mm thickness and provided with a silicon tube sealing. Mylar film was attached to the mould in order to obtain smooth and transparent surface. After the resin had solidified, the mould was placed in the oven for post-curing. VE was post-cured at 50°C for 2h and 80°C for 5h, UP was post-cured at 50°C for 2h. The plates were cut and polished into specimens with the dimensions...
shown in Fig. 3. Visual inspection revealed that the SFC specimens thus obtained were of uniform quality, with well-aligned fibres and no discernible bubbles or voids.

The effect of fibre extensions on the stress state in a SFC and on SFF test results is discussed in Appendix.

2.3.2. Test setup

The SFF test was performed in a MINIMAT miniature mechanical test machine from Polymer Laboratories Ltd (UK). The test machine was mounted on the \(x\)-\(y\) table of a Zeiss optical microscope. Load was measured by the MINIMAT’s built-in load cell (1000N) and the displacement was registered by the electronic unit of the tensile stage. Fragmentation of the fibres was observed during the loading. Extensometer was used to measure applied strain. In order to avoid pausing the machine to count fibre cracks, the test was carried out at rather low loading rate of 0.1 mm/min. Loading was stopped if the specimen failed, or when the fragmentation saturation level was achieved, namely, no new fibre breaks appeared with strain increase by 0.5%. During the experiment the data were transferred to the PC.

In order to measure the fibre diameter, digital pictures of the fibres were made before the loading. Images were made by the CCD camera attached to the microscope and then transferred to the PC for further processing. As in the case of single fibre tests fibre diameter was evaluated from analysis of digital images as the average of five apparent diameter measurements taken along the fibre.

3. Single fiber tensile test results

3.1. Stress-strain response

Not only limit stress and strain, but also the actual shape of the stress-strain curve was found to vary among fibres, ranging from linear elastic to markedly strain-hardening. (Note that similar variability in mechanical response was also observed for hemp fibres [18]). A typical stress-strain diagram of an elementary flax fibre is shown in Fig. 4. The apparent variation of tangent modulus with strain confined mostly to the initial, small strain part of the diagram (reported for flax fibres in e.g. [14, 19]) is attributed to the orientation
Fig. 4. Typical stress-strain curve of flax fibre.

of the fibrils along the axis of the fibre under load. (This phenomenon is irreversible in that upon unloading, subsequent reloading is linear elastic up to the previously achieved stress level with the modulus equal to the maximum modulus achieved during the previous load cycle [14].) At larger load/strain values, fibre response becomes linear, and we use the linear part of diagram for Young’s modulus calculation. There is a marked scatter of the measured modulus, $E$, values as seen in Fig. 5. The probabilities here and in the following are estimated as median ranks assigned to the measured modulus values at each gauge length using the following approximation

$$P = \frac{i - 0.3}{m + 0.4}$$

where $i$ is the $i$-th number in ascendingly ordered modulus data and $m$ is the sample size (i.e. number of modulus measurements performed at the given gauge length). The average value and standard deviation of the modulus and non-linear strain increment $\varepsilon_n$ are provided in Table 1. In view of the pronounced scatter, these results are in rough agreement with the estimated modulus value of about 80 GPa based on UD composite tests [20, 21] for ArcticFlax fibres produced by FinFlax.
Fig. 5. Young’s modulus distribution.

Fig. 6. Flax fibre modulus as a function of fibre diameter.
Table 1. Flax fibre deformation characteristics

<table>
<thead>
<tr>
<th>Fibre length, mm</th>
<th>Average modulus $\langle E \rangle$, GPa</th>
<th>Average non-linear strain increment $\langle \varepsilon_n \rangle$, %</th>
<th>Standard deviation $s_E$, GPa</th>
<th>Standard deviation $s_{\varepsilon_n}$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>69</td>
<td>20</td>
<td>0.32</td>
<td>0.42</td>
</tr>
<tr>
<td>20</td>
<td>64</td>
<td>21</td>
<td>0.41</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Fibre modulus slightly decreases with diameter increase as noted in [10, 14, 21], but the effect is small and largely overshadowed by the scatter, Fig. 6.

3.2. Strength distribution

The experimental fibre strength distribution yielded by SFT tests is shown in Weibull coordinates in Fig. 7. The Weibull distribution

$$P(\sigma) = 1 - \exp \left[ - \frac{l}{l_0} \left( \frac{\sigma}{\beta_\sigma} \right)^{a_\sigma} \right]$$

parameters from SFT tests were determined by the maximum likelihood method (MLM). In Eq. (1), $a_\sigma$ is the shape parameter, $\beta_\sigma$ – scale parameter of the Weibull distribution, $l$ designates fibre length and $l_0$ – reference length. The parameter values at the gauge lengths tested are summarized in Table 2 (here in Eq. (1) and below the reference length is chosen as $l_0 = 1$ mm). It is seen that the two-parameter Weibull distribution approximates the experimental data at each of the gauge lengths reasonably well. Deviations from Weibull distribution are mostly confined to the weakest fibres and can be tentatively related to the damage done to the fibres in the specimen preparation process.
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\[ l = 5 \text{ mm} \]

\[ l = 10 \text{ mm} \]

\[ \ln(-\ln(1-P)) \]

\[ \sigma, \text{ MPa} \]

\[ 300 \quad 500 \quad 700 \quad 1000 \quad 1500 \quad 2000 \]

\[ -4 \quad -2 \quad 0 \quad 2 \quad 4 \]

\[ 300 \quad 500 \quad 700 \quad 1000 \quad 1500 \]

\[ -4 \quad -2 \quad 0 \quad 2 \]

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Fig. 7. Strength distribution of flax fibres at 5 (a), 10 (b), and 20 mm (c) gauge length. Solid lines – MLM approximation by two-parameter Weibull distribution Eq. (1), dashed lines – by modified Weibull distribution Eq. (3).

Table 2. Weibull distribution parameters of flax fibre strength obtained by SFT tests

<table>
<thead>
<tr>
<th>Data reduction method</th>
<th>Gauge length, mm</th>
<th>Number of specimens</th>
<th>Shape parameter $\alpha_\sigma$, $\beta_\sigma$, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLM</td>
<td>5</td>
<td>90</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>70</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>58</td>
<td>2.5</td>
</tr>
<tr>
<td>Eq. (2)</td>
<td>5…20</td>
<td>-</td>
<td>5.2</td>
</tr>
</tbody>
</table>

The dependence of the average fibre strength on the gauge length is plotted in Fig. 8 in logarithmic scale. The error bars correspond to the average strength standard deviation estimated as $s_\sigma \sqrt{m}$, where $s_\sigma$ and $m$ are standard deviation of the fibre strength and the number of
Fig. 8. Average fibre strength as a function of gauge length obtained by SFT tests. Solid line - approximation by Eq.(2), dashed line - prediction by SFF tests, Eq.(17).

Fig. 9. Flax fibre strength as a function of fibre modulus.
tests at the corresponding gauge length. It follows from Eq. (1) that
the average strength is a power function of gauge length

\[ \langle \sigma \rangle = \beta_\sigma \left( \frac{l}{l_0} \right)^{-\alpha \gamma} \Gamma(1 + 1/\alpha) \]  

(2)

By approximating the data with Eq. (2) as shown in Fig. 8, estimates of the Weibull distribution parameters \( \alpha = 5.2 \) and \( \beta_\sigma = 1430 \text{ MPa} \) are obtained. The Weibull shape parameter derived applying Eq. (2) is considerably higher than that obtained from SFT at a fixed length and amounting to \( \sim 2.9 \) for the gauge length interval considered. This discrepancy in parameter values characterizing the fibre strength scatter at a fixed gauge length and the average strength dependence on the fibre length suggests that the modified Weibull distribution [22, 23]

\[ P(\sigma) = 1 - \exp \left( -\left( \frac{l}{l_0} \right)^{\gamma} \left( \frac{\sigma}{\beta_\sigma} \right)^{\alpha} \right) \]  

(3)

is preferable to (1) for the flax fibres considered. The distribution (3) reconciles the mismatch of the fibre strength scatter at a fixed gauge length (characterized by \( \alpha_\sigma \) ) and the average strength dependence on the fibre length (governed by the exponent, \( \gamma / \alpha_\sigma \) )

\[ \langle \sigma \rangle = \beta_\sigma \left( \frac{l}{l_0} \right)^{-\gamma / \alpha_\sigma} \Gamma(1 + 1/\alpha_\sigma) \]  

(4)

It is suggested in [23, 24, 17] that the distribution (3) reflects the fibre-to-fibre strength parameter variation in a batch of fibres, each of which has the Weibull two-parameter strength distribution (1). The parameters of the distribution (3) determined by MLM from strength data in 5 to 20 mm gauge length interval are as follows: \( \gamma = 0.46 \), \( \alpha_\sigma = 2.8 \), \( \beta_\sigma = 1400 \text{ MPa} \). The slope and location of the corresponding plots in Weibull coordinates, Fig. 7, are in good agreement with the SFT test data. Consequently, the modified Weibull distribution adequately describes both gauge length dependence of strength and the strength distribution at a fixed gauge length for the flax fibres considered.
Both axial stiffness and strength of flax fibres is imparted by the cellulose microfibrils, therefore it appears plausible that these quantities are correlated. Fig. 9 shows that a weak correlation is indeed present.

3.3. Failure strain distribution

Failure strain distributions were determined only for 10 and 20 mm flax fibres. The corresponding Weibull plots are given in Fig. 10. The parameter values of the two-parameter Weibull distribution for failure strain

\[ P(\varepsilon) = 1 - \exp\left[ - \frac{l}{l_0} \left( \frac{\varepsilon}{\beta_{\varepsilon}} \right)^{\alpha_{\varepsilon}} \right] \]  

obtained by MLM are provided in Table 3.

For linear elastic fibres with negligible modulus scatter, the relation between failure strain and strength distributions is trivial – both share the same shape parameter \( \alpha_{\sigma} = \alpha_{\varepsilon} \), and the scale parameter of the strength distribution is given by the product of fibre modulus and the corresponding strain distribution parameter, \( \beta_{\sigma} = E\beta_{\varepsilon} \). Flax fibre stress-strain curve obtained by SFT typically possesses an initial non-linear zone as shown schematically in Fig. 4. Hence, flax fibre strength is related to its modulus, failure strain, and the non-linear strain increment, \( \varepsilon_n \), as follows:

\[ \sigma = E(\varepsilon - \varepsilon_n) \]  

Table 3. Weibull distribution parameters for failure strain of flax fibres

<table>
<thead>
<tr>
<th>Test method</th>
<th>Gauge length, mm</th>
<th>Number of specimens</th>
<th>Shape parameter ( \alpha_{\varepsilon} )</th>
<th>Scale parameter ( \beta_{\varepsilon} ), %</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFT</td>
<td>10</td>
<td>68</td>
<td>2.7</td>
<td>4.15</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>56</td>
<td>2.8</td>
<td>4.79</td>
</tr>
<tr>
<td>SFF</td>
<td>~24</td>
<td>21</td>
<td>6.9</td>
<td>2.42</td>
</tr>
</tbody>
</table>
Fig. 10. Failure strain distribution of flax fibres at 10 (a) and 20 mm (b) gauge length. Solid lines – MLM approximation by two-parameter Weibull distribution Eq. (5).
Note that all the fibre parameters entering Eq. (6) vary considerably between fibres. However, both failure strain and non-linear strain increment are virtually uncorrelated with the fibre modulus. Therefore, Eq. (6) remains valid also for average values of the parameters:

\[ \langle \sigma \rangle = \langle E \rangle \left( \langle \epsilon \rangle - \langle \epsilon_n \rangle \right) \]  

(7)

Thus Eq. (7) relates mean failure strain and fibre strength; accounting for mean stress expression Eq. (4), failure strain as a function of fibre length is given by

\[ \langle \epsilon \rangle = B_\sigma \left( l/l_0 \right)^{-\gamma_{\sigma} / \alpha_{\sigma}} \Gamma \left( 1 + 1/\alpha_{\sigma} \right) / \langle E \rangle + \langle \epsilon_n \rangle \]  

(8)

The prediction by Eq. (8) complies with SFT test data, Fig. 11.

![Graph showing average failure strain as a function of gauge length obtained by SFT tests. Solid line – Eq. (8), dashed line – Eq. (16) based on SFF results.](image-url)
4. SFF tests

4.1. Analysis of fragmentation data

During the initial fragmentation stage when fibre break interaction is negligible the average fragment length, \( \langle l \rangle \), of a fibre with Weibull failure strain distribution (5) is given by

\[
\langle l \rangle = l_0 \left[ \frac{\varepsilon_f}{\beta_{\varepsilon}} \right]^{-\alpha_{\varepsilon}}
\]

(9)

where \( \varepsilon_f \) is the fibre strain, \( \varepsilon_f = \varepsilon_a + \varepsilon_r \), composed of the mechanical strain applied to the SFC, \( \varepsilon_a \), and residual fibre strain, \( \varepsilon_r \). A typical fragmentation diagram of elementary flax fibre is plotted in Fig. 12. The presence of a pronounced initial fragmentation stage with power-law dependence of average fragment length on fibre

![Typical fibre fragmentation diagram showing the dependence of average fragment length on fibre strain and the approximation of the initial part of the diagram employed for Weibull parameter evaluation.](image-url)
Fig. 13. Weibull plot of fibre failure strain distribution, Eq. (5), parameters $\alpha_\varepsilon$ (a) and $\beta_\varepsilon$ (b) obtained by SFF tests.
strain in agreement with Eq. (9) corroborates the applicability of Weibull distribution (5) for failure strain of individual fibres.

We estimated the residual fibre strain using fibre failure strain data obtained by SFT tests at a fixed gauge length so that the average fibre failure strain at the given gauge length derived from SFF tests is equal to the value obtained from SFT. The following iterative procedure was applied. An initial value for $\varepsilon_r$ was assigned, and the Weibull distribution parameters for each SFC specimen were determined approximating the initial linear part of the fragmentation diagram in double logarithmic coordinates by (the logarithm of) Eq. (9) as shown in Fig. 12. Then the average failure strain for each SFF-tested fibre at a length $l^*$, $\varepsilon_i(l^*)$, was estimated using the obtained Weibull parameters as $\langle \varepsilon_i(l^*) \rangle = \beta_i (l^*/l_0)^{1/\alpha_i} \Gamma(1+1/\alpha_i)$. The calculated failure strain values for SFF-tested fibres were averaged producing the fragmentation test estimate for average fibre failure strain at gauge length $l^*$, $\langle \varepsilon(l^*) \rangle = \frac{1}{n} \sum_{i=1}^{n} \langle \varepsilon_i(l^*) \rangle$, where $n$ is the number of SFC tested. The obtained $\langle \varepsilon(l^*) \rangle$ value is compared with average fibre failure strain determined by SFT tests at the given gauge length, and the residual strain estimate adjusted accordingly. Such iterations are repeated until $\varepsilon_r$ value at which $\langle \varepsilon(l^*) \rangle$ is equal to the average failure strain yielded by SFT test is found. Using average fibre failure strain at $l^*=10$ mm gauge length as a benchmark, the residual strain was found to be $\varepsilon_r = -0.41\%$ in the case of VE matrix and $\varepsilon = -0.31\%$ for UP matrix.

Failure strain distribution (5) parameters obtained by SFF tests vary considerably between fibres as the Weibull plots for the parameters reveal, Fig. 13. The average values of the parameters are given in Table 3. The coefficient of variation for shape parameter, $k_\alpha$, amounts to 0.30 and for scale parameter, $k_\beta$, to 0.13. It is interesting to note that a roughly comparable variability of Weibull distribution parameters was also obtained by SFF tests of E-glass fibres [17], namely $k_\alpha = 0.27$ and $k_\beta = 0.10$. 

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4.2. Modified Weibull distribution of fibre failure strain

It has been shown by numerical simulation [24] that the batch strength distribution of fibres, each of which has Weibull strength distribution (1) with the same shape parameter but randomly varying scale parameter (the latter also being Weibull-distributed), closely approximates the modified Weibull distribution (3). Flax SFF tests suggest that failure strain of individual fibres has Weibull distribution (5), and the variability of the scale parameter also agrees with Weibull distribution as revealed by the linearity of the corresponding Weibull plot, Fig. 13b. However, the scatter of the Weibull shape parameter values is not negligible, Fig. 13a. Therefore the approximate relations derived in [24] for parameters of the modified Weibull distribution are not applicable here.

Similar fibre-to-fibre variability of Weibull distribution parameters was also obtained in SFF tests of glass fibres, the strength of which complied with the modified Weibull distribution [17]. We apply the method proposed in [17] to estimate from SFF tests the parameters the modified Weibull distribution of failure strain

\[
P(\varepsilon) = 1 - \exp \left(- \left( \frac{I}{l_0} \right)^\gamma \left[ \frac{E}{\beta} \right]^{\alpha_c} \right)\]  

(10)

Assuming that (i) the difference in Weibull parameter values obtained by SFF tests stems solely from the variation in fibre strength distribution parameters among fibres and (ii) the set of Weibull parameters determined by SFF tests fully describes the interfibre strength variability, we obtain [17]

\[
\langle \varepsilon \rangle = \frac{1}{n} \sum_{i=1}^{n} \beta_i \cdot \left( \frac{l}{l_0} \right)^{-1/\alpha_i} \Gamma \left( 1 + 1/\alpha_i \right) \]  

(11)

\[
s_c = \left\{ \frac{1}{n} \sum_{i=1}^{n} \beta_i^2 \cdot \left( \frac{l}{l_0} \right)^{-2/\alpha_i} \Gamma \left( 1 + 2/\alpha_i \right) - \right. \]  

\[
\left. - \left[ \frac{1}{n} \sum_{i=1}^{n} \beta_i \cdot \left( \frac{l}{l_0} \right)^{-1/\alpha_i} \Gamma \left( 1 + 1/\alpha_i \right) \right]^2 \right\}^{1/2} \]  

(12)
where \( n \) is the number of fibres tested by SFF, and \( \alpha_i, \beta_i \) are the Weibull parameters of the \( i \)-th fibre. Further, equating the ratio of standard deviation and average failure strain provided by Eqs. (12) and (11) at \( l = l_0 \) to the coefficient of variation for limit strain according to Eq. (10), we obtain relation for the shape parameter of distribution (10)

\[
\frac{s_{\varepsilon}}{\langle \varepsilon \rangle_{l=l_0}} = \frac{\Gamma(1+2/\alpha_{\varepsilon})}{\Gamma(1+1/\alpha_{\varepsilon})^{\frac{1}{2}}} - 1
\]

Equating average strength and its derivative with respect to fibre length based on Eq. (11) and on distribution (10) result in

\[
\langle \varepsilon \rangle_{l=l_0} = \beta_{\varepsilon} \Gamma(1+1/\alpha_{\varepsilon}) \\
\frac{d}{dl} \langle \varepsilon \rangle_{l=l_0} = -\frac{\gamma_{\varepsilon}}{l_0 \alpha_{\varepsilon}} \beta_{\varepsilon} \Gamma(1+1/\alpha_{\varepsilon})
\]

Solving Eq. (13) for \( \alpha_{\varepsilon} \) and determining \( \beta_{\varepsilon} \) and \( \gamma_{\varepsilon} \) from Eqs. (14) and (15) yields the numerical values for distribution (10) parameters based on SFF test results as follows: \( \gamma_{\varepsilon} = 0.79, \alpha_{\varepsilon} = 4.97, \beta_{\varepsilon} = 2.5\% \). The average failure strain as a function of fibre length according to Eq. (10)

\[
\langle \varepsilon \rangle = \beta_{\varepsilon} \left( l/l_0 \right)^{-\gamma_{\varepsilon}/\alpha_{\varepsilon}} \Gamma(1+1/\alpha_{\varepsilon})
\]

is plotted in Fig. 11 by dashed line. It is seen that the length dependence for fibre failure strain yielded by Eq. (16) agrees reasonably well with the SFT experimental data, although failure strain variability at fixed fibre length is underestimated. Combining Eqs. (7) and (16), we obtain for average fibre strength

\[
\langle \sigma \rangle = \langle E \rangle \left( \frac{\beta_{\varepsilon} (l/l_0)^{-\gamma_{\varepsilon}/\alpha_{\varepsilon}} \Gamma(1+1/\alpha_{\varepsilon}) - \langle \varepsilon \rangle}{(1+1/\alpha_{\varepsilon}) - \langle \varepsilon \rangle_{n}} \right)
\]
The relation (17) is reproduced in Fig. 8 against SFT experimental data. It is seen that Eq. (17) based on SFF tests can rather accurately predict the length effect of fibre strength. Thus a limited number of SFF tests combined with SFT tests at a fixed gauge length can provide the experimental information necessary for characterizing the scatter of fibre failure strain and strength and the effect of fibre length on the mentioned parameters.

5. Conclusions.

Elementary flax fibre strength and failure strain distributions at several gauge lengths are obtained by SFT tests. Strength and failure strain possess the modified Weibull distribution (i.e. independent exponents characterize strength scatter at a fixed gauge length and the dependence of mean strength on fibre length). SFF tests reveal that fibre fragmentation process proceeds in agreement with the two-parameter Weibull distribution of failure strain, Eq. (5), but the distribution parameters for individual fibres exhibit a marked scatter. A limited number of SFF tests combined with SFT tests at a fixed gauge length provide the experimental information necessary for characterizing both the scatter of fibre failure strain and strength at a fixed gauge length and the effect of fibre length on the mentioned parameters.

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Undergraduate students Henrik Lindberg, Borsini Serena, Isidori Tatiana and graduate student Philippe Lingois were involved in this work.

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Flax fibres for this study were supplied by partner in ECOFINA project, FINFLAX. Fibre surface treatment was performed by partner in ECOFINA project, University of Basque Country.
Appendix. Verification of the extended fibre SFF test method

The intended role of fibre extensions is to hold the fibre in place while the resin solidifies. Once the resin has cured, the perturbation of the fibre stress due to the presence of extensions is expected to extend only by a limited distance, so called stress transfer length, from the fibre ends. This distance, a function of fibre and resin moduli and typically an order of magnitude larger than fibre diameter, is to be excluded from monitoring for fibre cracks during fragmentation test.

![Finite element model of the extended fibre SFC.](image)

In order to illustrate the principal features of stress transfer, we performed linear elastic analysis of the SFC under thermomechanical loading by finite element code ANSYS. In order to simplify analysis, Plane83 2-D 8-node axi-symmetric elements (10000 elements in total) were used to model SFC geometry of coaxial fibre and
extension embedded in a resin cylinder; schematic picture of the model and example of the mesh are shown in Fig. A1. Symmetry conditions are enforced along the top edge of the model and vertical (y) axis, which is the line of axial symmetry. The thermoelastic properties of the constituents were chosen as for glass fibre and VE matrix: fibre modulus $E_f = 73 \text{ GPa}$, Poisson’s ratio $\nu_f = 0.22$, coefficient of thermal expansion $\text{CTE}_f = 5 \cdot 10^{-6} \text{ 1/ºC}$; matrix modulus $E_m = 3 \text{ GPa}$, Poisson’s ratio $\nu_m = 0.35$, coefficient of thermal expansion $\text{CTE}_m = 6 \cdot 10^{-5} \text{ 1/ºC}$. The radius of the extension is taken four times larger than fibre radius, $r_f$, and that of the matrix cylinder is $40r_f$; the length of the extension line is $40r_f$, the fibre length is $160r_f$. The stress state in the SFC was calculated for thermal and for mechanical loading separately. The thermal loading is due to the temperature drop from a stress-free post-cure temperature to room temperature. The mechanical loading is accomplished by applying a uniform y-direction displacement to the bottom (i.e. $y = 0$) line of the model corresponding to the nominal applied strain of 1%. The calculated axial strain at the symmetry axis of the model ($x = 0$) in the extension and the fibre is shown in Fig. A2 as a function of the relative distance form the bottom edge of the model ($y = 0$). It is seen that the perturbation zone of the fibre strain extends for a distance of about $50r_f$ from the fibre/extension joint for the range of extension mechanical characteristics studied. (Note that the slight dependence of the calculated far-field fibre strain under mechanical load on the extension modulus, Fig. A2b, is due to the rather limited cross-section area of the resin in the model as compared to the actual dimensions of the SFC’s tested, Fig. 3.)

In order to verify the applicability of the modified SFF test method, we performed a limited number of extended-fibre SFF tests for an E-glass fibre previously tested by the ordinary SFF method involving long fibres as reported in [17]. Short glass fibres of ca 26 mm length were cut from continuous filaments separated from the glass fibre bundle. The fibre extensions were attached and SFC specimens prepared following the routine described in Section 2.3.1; VE matrix was used. Fragmentation testing and data analysis was carried out in exactly the same way as described in [17] for long-fibre SFCs. The average values of the Weibull shape and scale parameters, based on
Fig. A2. Axial strain distribution in the extension and the fibre under thermal (a) and mechanical (b) loading. Dotted line shows the position of extension/fibre joint.
four SFF tests, amounted to 8.0 (1) and 3150 (120) MPa respectively (here and below the number in brackets is the standard deviation of the corresponding average value). Twelve long-fibre SFF tests yielded the average shape parameter of 8.6 (7) and scale parameter 3090 (90) MPa [17]. It is seen that the fibre strength distribution parameters obtained by the modified SFF test agree within experimental scatter with those obtained by the ordinary SFF procedure.

References


Stiffness and strength of flax fiber/polymer matrix composites

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Stiffness and strength of flax fiber/polymer matrix composites

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Abstract. Flax fiber composites with thermoset and thermoplastic polymer matrices have been manufactured and tested for stiffness and strength under uniaxial tension. Flax/polypropylene and flax/maleic anhydride grafted polypropylene composites are produced from compound obtained by co-extrusion of granulated polypropylene and flax fibers, while flax fiber mat/vinylester and modified acrylic resin composites are manufactured by resin transfer molding. The applicability of rule-of-mixtures and orientational averaging based models, developed for short fiber composites, to flax reinforced polymers is considered.

1. Introduction

Flax fibers are increasingly being studied and used as a reinforcement of traditional thermoplastic (see e.g. [1-15]) and thermoset [16-23], as well as biopolymer [24-25] matrix composites. In order to experimentally determine the most efficient utilization conditions of flax fiber properties, different flax fiber reinforcement types (i.e. rowing, mat) and treatments are considered. Effect of fiber treatment on the stiffness and strength of polypropylene (PP) [1, 10] and epoxy [17, 19, 20] composites has been studied extensively. The reinforcing fiber length [3, 7, 21], fiber content [3, 7, 17, 18, 21, 23], fiber retting degree [13, 20] and decortication stage [4, 20] are also factors affecting the mechanical properties of a composite. It would obviously be helpful to characterize fiber and interface properties by dedicated tests and then apply theoretical models for composite property prediction instead of producing and testing composites to evaluate the overall effect of each particular treatment. However, comparatively few attempts to relate theoretically the mechanical properties of a flax fiber composite to those of the fiber, matrix, and their interface have been made [3, 4, 7, 15, 26]. Variations of rule-of-mixtures relationships were found useful for modulus prediction [26] and longitudinal and transverse strength and modulus correlations [7]
of unidirectionally reinforced composites as well as modulus and strength prediction for random flax/PP composites [3, 4]. Approaches to composite stiffness prediction explicitly accounting for the variability in fiber properties have also been proposed [15, 26].

The aim of the present study is a systematic evaluation of the applicability of elementary mechanical models for flax fiber reinforced polymer matrix composite modulus and strength prediction. Using commercial, well characterized in terms of axial mechanical properties [28] and adhesion [29] enzyme-retted flax fibers, thermoplastic matrix composites were produced by extrusion and thermoset composites by resin transfer molding (RTM). Correlation of experimental and theoretical composite characteristics revealed the degree of sensitivity of the elementary models to properties of the constituents and their adhesion.

2. Experimental

2.1 Materials

Flax fiber mats (FFM) produced by FinFlax were used as reinforcement for polymer composites. The resins used were as follows; thermoset resins: vinylesters XZ92485 and Momentum 411-350 by DOW (denoted as VE₁ and VE₂), modified acrylic resin (AR) Modar 835S by Ashland which can be cured as unsaturated polyester. (Note that Modar 835S was mistakenly referred to as unsaturated polyester (UP) in [28-30]). Thermoplastic: Adstif 770 ADXP Basell polypropylene and maleic anhydride grafted PP (MAPP) modified polypropylene (PPM).

2.2 Composites Manufacturing and Testing

Plates (of 3 mm thickness) of thermoset natural fiber composites (NFC) were manufactured by RTM and postcured for 5h at 80º C after gelation. Rectangular specimens were cut from the plates for tensile tests. Some of the specimens were cut and polished after mechanical tests to evaluate the morphology of the composites. Micrographs in Fig. 1 suggest that the fibers (and fiber bundles) exhibit relatively uniform distribution of orientations within the specimen plane.
Fig. 1. Top (a), edge (b), and cross-section (c) micrographs of a flax mat/thermoset matrix NFC specimen.

Thermoplastic composite specimens were produced from compound obtained by co-extrusion (Krupp Werner & Pfleiderer ZSK 25 WLE twin screw extruder) of granulated PP and flax fiber rowing. Temperature in the extruder at the polymer entrance point was 180°C and it was raised to 190-200°C after the 2/3 of the path along the length of the extruder screw was passed. Afterwards compound was once more heated in the oven to 190-200°C and pressed in the press (pressure of 15-25 tons, temperature of the press \( \approx 40°C \)) under stiff profile that allowed to produce rectangular and dog-bone shaped specimens with constant thickness. Fig. 2 shows representative micrographs of top, edge, and cross-section views of a rectangular specimen. Although a fraction of the fibers appears randomly distributed, there is distinct evidence of a preferential direction in the fiber orientation, namely along the specimen length-wise direction.
Tensile specimens were of rectangular shape, 250 mm long and 25 mm wide. In order to prevent failure of the specimens in the gripping area, very fine metal mesh was applied on the clamped ends (50 mm long) of each sample. Tensile tests were carried out in an Instron 1272 tensile machine with a 25 kN load cell. The experiment was
performed in displacement-control mode at a stroke rate (i.e. cross-head displacement rate) of 2 mm/min. During loading, the longitudinal displacement was measured by an MTS extensometer. All output data (strain, displacement of cross-head, and load) were collected by an acquisition system and transferred to the PC.

3. Elementary models of short fiber composite stiffness and strength

3.1 Stiffness

A number of theoretical models for prediction of the elastic properties of short-fiber reinforced composites (SFC) have been elaborated differing in accuracy and complexity (for recent reviews, see e.g. [31, 32]). The relatively simple Cox-Krenchel model was found to yield good agreement with experimental modulus values for a range of glass fiber lengths and volume fractions [33], and also to perform acceptably for random flax/PP composites [3, 4]. Composite modulus $E$ is related to fiber and matrix moduli, $E_f$ and $E_m$, and fiber volume fraction $\nu_f$ by a rule of mixtures type of relationship:

$$E = \eta_{OE}\eta_{IE}E_f\nu_f + (1 - \nu_f)E_m$$  \hspace{1cm} (1)

where $\eta_{OE}$ is orientation factor and $\eta_{IE}$ is fiber length efficiency factor. For reinforcing fibers of length $l$, the latter is given by

$$\eta_{IE} = 1 - \frac{\tanh(\beta l/2)}{\beta l/2}$$  \hspace{1cm} (2)

where

$$\beta = \frac{1}{r_f} \sqrt{\frac{2G_m}{E_f \ln(R/r_f)}}$$

and $r_f$ stands for fiber radius, $G_m$ is matrix shear modulus, and $R$ relates to the interfiber spacing in the composite. The ratio $R/r_f$ can
be expressed as $R/r_f = \sqrt{K_R/v_f}$, where the numerical factor $K_R$ depends on fiber geometrical packing, being equal to $\pi/4$ for square packing [31].

If the reinforcing fiber length is variable with the distribution density given by $h(l)$,

$$\eta_{OE} = \frac{1}{\langle l \rangle} \int_0^\infty \left(1 - \frac{\tanh(\beta l/2)}{\beta l/2}\right)h(l)dl. \quad (3)$$

Fiber orientation factor $\eta_{OE}$ is determined by the fiber orientation distribution (see e.g. [34, 35]). Neglecting transverse deformations, fiber orientation factor $\eta_{OE}$ value for random in-plane orientation of the fibers (that would approximate fiber arrangement in flax fiber mat) is $3/8$, while for random three-dimensional fiber orientation (approximating fiber arrangement in short-fiber extruded composite) $\eta_{OE} = 1/5$.

Application of more complicated models of SFC stiffness (e.g. [36, 37]) is hampered for flax fibers by the lack of experimental data concerning transverse and shear properties of the fibers. In fact, apparently the only natural fiber that has been exhaustively characterized in terms of thermoelastic anisotropy is jute [38].

### 3.2 Strength

In comparison to the elastic properties, strength theory for SFC is still under development. Most of the SFC strength models are also based on the “rule of mixtures” [39-42] or equivalent laminate [43, 44] approach, i.e. are basically engineering approximations. Recently, more complicated models have been advanced [45-48] that employ numerical modeling of deformation and failure processes.

The strength of a SFC can be expressed as [39-42]

$$\sigma_{uc} = \eta_\sigma \sigma_{of} v_f + \left(1 - v_f\right)\sigma_m \quad (4)$$
where $\sigma_{uf}$ is fiber strength, $\sigma_m$ is stress in the matrix at the fiber failure strain (for linear elastic constituents, $\sigma_m = \frac{\sigma_{uf}E_m}{E_f}$), and $\eta_s$ denotes the fiber efficiency factor. The latter can be decomposed as $\eta_s = \eta_{ls} \cdot \eta_{os}$ [40, 41], where fiber length efficiency factor $\eta_{ls}$ and orientation factor $\eta_{os}$ have similar interpretation as for modulus calculation. (Note that strain concentration factor is also likely to contribute to $\eta_s$ [32, 49]). Fiber length efficiency factor is estimated as [39]:

$$\eta_{ls} = \begin{cases} 1 - l_c / 2l & l \geq l_c \\ l / 2l_c & l < l_c \end{cases}$$

where

$$l_c = \frac{\sigma_{uf}r_f}{\tau}$$

and $\tau$ is interface shear strength (IFSS). Approximating experimental strength data for a range of fiber lengths and volume fractions in glass/PP short fiber composites, a best-fit value of 0.2 for $\eta_{os}$ was found [50]. This fitted parameter value apparently incorporates both fiber orientation and fiber efficiency factors. For flax/PP, the fitted parameter $\eta_{os}$ amounted to 0.075 [3] indicating thus considerably smaller reinforcement efficiency by flax as compared to glass fibers.

Models [40-42] provide means for determining $\eta_s$, but at the cost of introducing other intrinsic material parameters, critical-zone (or process zone) width in Fukuda and Chou [40] model or bridging stress and strength of an oblique fiber in Fu and Lauke [41, 42] model. We consider the simplest critical-zone model by Fukuda and Chou [40], modified according to the suggestions of Jayaraman and Kortschot [35] (see Appendix). The fiber strength as a function of its length is evaluated according to the modified Weibull distribution leading to

$$\sigma_{uf}(l) = \beta_\sigma (l/l_0)^{-\gamma/\alpha} \Gamma(1+1/\alpha)$$

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The Weibull strength distribution parameters entering Eq. (7) are presented in [28]. By combining Eqs. (6) and (7) [44], we obtain for the critical length

$$I_c = \left( \frac{\beta \Gamma(1 + 1/\alpha) \gamma_f}{\tau_0^{\gamma/\alpha}} \right)^{\alpha/(\gamma + \alpha)} \tag{8}$$

The critical zone width \(l_n\) is chosen equal to the critical fiber length \(l_c\). Then for random two-dimensional fiber orientation assumed for fiber mat reinforced composite, it follows from Eqs. (A 4), (A 5) that the strength is given by

$$\sigma_{nc} = \frac{V_f}{4\pi} \int_0^{L_c} \sigma_{nl}(l) \left( 3 \cos^2 \frac{l_0^2}{l_c^2} \right) \left( 3 + 2 \left( 1 - \frac{l_0^2}{l_c^2} \right)^2 \right) \sqrt{1 - \left( 1 - \frac{l_0^2}{l_c^2} \right)^2} \times$$

$$\left( 1 - \frac{l_0^2}{2l} \right) h(l) dl + (1 - V_f) \sigma_m \tag{9}$$

Analogically, for the strength in case of a random three-dimensional fiber orientation assumed for extruded SFC, it follows from Eq. (A 4), (A 6) that

$$\sigma_{nc} = \frac{V_f}{5} \int_0^{L_c} \sigma_{nl}(l) \left[ 1 - \left( \frac{l_0^2}{l_c^2} \right)^5 \right] \left( 1 - \frac{l_0^2}{2l} \right) h(l) dl + (1 - V_f) \sigma_m \tag{10}$$

For application of more complicated models of SFC that provide also strength under combined loading [41, 42, 44] and/or complete stress-strain diagrams accounting for gradual damage accumulation and matrix non-linearity [45-48], more detailed information on fiber and interface properties is needed than is typically available for natural fibers.

4. Results and discussion

Experimental modulus values for flax mat/thermoset matrix NFCs are shown in Fig. 3 compared with polymer matrix modulus. It is
seen that flax fiber volume fraction of 30% provides significant stiffness increase over pure polymer as expected. Theoretical modulus values obtained by Eq. (1) are also presented in Fig. 3. Elementary flax fiber modulus $E_f = 69$ GPa and the average diameter $d_f = 16 \mu$m for FinFlax fibers [28] are taken here and in subsequent calculations. Since the average fiber length in flax mat is evaluated at about 20 mm that exceeds considerably the stress transfer length of the fiber, length efficiency factor in flax mat composites was assumed equal to 1. Due to the relatively uniform planar arrangement of fibers in the mat, Fig. 1, fiber orientation factor for random in-plane orientation, $\eta_{oE} = 3/8$, was applied. It is seen that the Young’s modulus of the composite is slightly but consistently overpredicted. This is likely to be due to the presence of fiber curvature in the mat (as seen in Fig. 1a) that makes the composite somewhat more compliant than the assumed perfectly planar arrangement of straight fibers.

![Graphical representation of experimental and predicted modulus of flax fiber mat/thermoset matrix composites.](image)

Fig. 3. Experimental and predicted modulus of flax fiber mat/thermoset matrix composites.
The dependence of extruded flax/PP modulus on fiber volume fraction is shown in Fig. 4. MAPP modification of the PP matrix has no discernible effect on composite modulus. The distribution of flax fiber length in extruded flax/PP composites is given in Fig. 5 (after [51]). The dependence of modulus on fiber volume fraction estimated according to Eqs. (1), (3) is plotted in Fig. 4 by a solid line. Lacking precise fiber orientation distribution measurements, we employ fiber orientation factor for random three-dimensional fiber orientation, \( \eta_{oe} = 1/5 \), as a rough approximation, notwithstanding the preferential fiber orientation suggested by Fig. 2. The theoretical dependence of Young’s modulus on the fiber volume fraction agrees reasonably well with the test data.

![Graph showing experimental and predicted modulus variation with fiber volume fraction for extruded flax/PP composites.]

Fig. 4. Experimental (markers) and predicted (solid line) modulus variation with fiber volume fraction for extruded flax/PP composites.

For fiber mat reinforced composites, the critical fiber length calculated using the IFSS values obtained in [29] is negligible in comparison to the average fiber length; monodisperse fiber length distribution is assumed. Eq. (9) leads to the following expression for composite strength
\[ \sigma_{wc} = \frac{3}{8} \nu_f \sigma_{uf}(l) + (1 - \nu_f) \sigma_m \] (11)

The flax fiber strength distribution parameters needed for the mean fiber strength at a given length, \( \sigma_{uf}(l) \), evaluation according to Eq. (7) are as follows: \( \alpha = 2.8 \), \( \beta_\alpha = 1400 \text{ MPa} \), \( \gamma = 0.46 \) (and \( l_0 = 1 \text{ mm} \)) [28]. The prediction by Eq. (11) is plotted against FFM/thermoset NFC strength in Fig. 6.

The apparent IFSS of flax fiber and PP matrices are reported in [4, 52] and constitute 8 MPa for PP and 12 MPa for PP/MAPP. The critical fiber length \( l_c \) evaluated by Eq. (8) equals 1.17 mm for flax/PP and 0.82 mm for flax/PPM composites, respectively. The critical length values lie close to the mode of extruded fiber length distribution density, Fig. 5. The theoretical strength values calculated by Eq. (10) are compared to the experimental results for extruded flax/PP and PPM at different fiber volume fractions in Fig. 6. It is seen that the experimental and predicted strength values are...
reasonably well correlated, but the strength is consistently overestimated by Eqs. (10), (11). Therefore we introduce an empirical fiber efficiency factor \( \eta^*_s \) modifying the fiber contribution term in Eq. (A 4) and hence in Eqs. (9) – (11). The value of \( \eta^*_s = 0.63 \) yielded the best fit to extruded flax/PPM strength data. Figs. 5 and 6 show that the same \( \eta^*_s \) value provides reasonably good accuracy of strength prediction also for extruded flax/PP and FFM/thermoset NFCs.

![Graph showing experimental vs. theoretical strength](image)

Fig. 6. Experimental strength vs. theoretical prediction for extruded flax/PP and PPM at \( v_f = 0.13 (\circ) \), 0.2 (\( \Delta \)), 0.29 (\( \circ \)) and for flax mat/thermoset matrices, \( v_f = 0.3 (\circ) \). The slope of the dashed line is 1 that would correspond to equal experimental and predicted strength.

The adhesion of flax and PP is apparently insufficient to provide enhancement of the NFC strength above that of the polymer, while addition of MAPP leads to a notable strength increase, Fig. 7. The higher IFSS due to MAPP, reflected in the decrease of the critical fiber length, is relatively well translated into NFC strength increment by the modified Fukuda and Chou model. By contrast, the model predicts virtually the same strength for FFM/AR and FFM/VE₂.
NFCs, Fig. 8, although their IFSS differ by a factor of two [29]. This is apparently due to the fact that IFSS enters the strength model only via the critical fiber length. Therefore the average fiber length in the flax mat, being large in comparison with $l_c$, makes the effect of variations in $l_c$ insignificant. Hence, in order to better reflect the role of adhesion on the strength of relatively long fiber NFC, a more sophisticated model is needed.

![Experimental (markers) and predicted (solid lines) strength variation with fiber volume fraction for extruded flax/PP composites.](image)

Fig. 7. Experimental (markers) and predicted (solid lines) strength variation with fiber volume fraction for extruded flax/PP composites.

Although the detailed information on the anisotropic elastic and strength properties of the fibers required by most of the strength models is largely missing for flax, relatively rough estimates can still be made. Consider an orientational averaging model for strength proposed by van Hattum and Bernardo [44]. The model represents misaligned SFC as an assembly of unidirectionally reinforced (UD) plies. Tsai-Wu strength tensor [53, 54] of the SFC is obtained by first averaging limit strains of the mentioned UD plies over all directions weighted by fiber orientation distribution function, and then transforming the limit strain tensor to strength tensor by means of
SFC compliance tensor (obtained by orientational averaging of UD stiffness tensors). UD composite stiffness is derived using Halpin-Tsai relations [55], longitudinal strength – by the rule of mixtures, while transverse and shear strength is approximated by that of the matrix in the case of perfect adhesion [44].

Fig. 8. Experimental and predicted strength of flax fiber mat/thermoset matrix composites.

Regarding transverse and shear stiffness of flax fiber, we follow the assumption made in [7] that for flax and jute, both being lignocellulosic fibers, the mechanical behavior is qualitatively similar and the degree of anisotropy of stiffness properties is approximately equal. Therefore the ratios of transverse and shear moduli to the axial fiber modulus for flax fiber are taken equal to the corresponding ratios for jute fiber at room temperature [38] (i.e. although the moduli of flax and jute fibers differ considerably, their ratios are assumed equal). Longitudinal tensile strength of a UD layer is estimated by the modified Fukuda and Chou model, while transverse and in-plane
shear strengths are evaluated by the micromechanical relations of [55] assuming that the former is governed by the matrix strength while the latter – by fiber and matrix adhesion characterized by IFSS. The predicted FFM/AR and FFM/VE\textsubscript{2} NFC strengths, based on the van Hattum and Bernardo model with the assumptions discussed above and IFSS values taken from [29], are presented in Fig. 8. It is seen that the orientational averaging model, despite the approximate nature of the material properties used as input, captures the difference between FFM/AR and FFM/VE\textsubscript{2} strength considerably better than the rule of mixtures approach. Such an enhanced sensitivity to matrix and adhesion characteristics apparently stems from the implicit accounting for the SFC failure modes due to matrix fracture and fiber debonding inherent in the orientational averaging model [44].

5. Conclusions

Stiffness and strength under uniaxial tension has been obtained for flax/PP and flax/PPM composites produced from compound obtained by co-extrusion of granulated PP and flax, as well as for FFM/vinylester and FFM/acrylic resin composites manufactured by resin transfer molding. The rule-of-mixtures relations are shown to yield acceptable stiffness prediction of both extruded and FFM-based composites. The sensitivity of a rule-of-mixtures model of strength to the matrix and adhesion properties apparently depends on the relative ineffective fiber length; for relatively long-fiber FFM composites the sensitivity is low and more sophisticated strength models should be applied.
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Appendix. Modification of Fukuda-Chou strength theory.

Jayaraman and Kortschot [35] have noted an inconsistency in the way the load born by the short fibers is calculated for SFC modulus [34] and strength [40] estimation, and also provided the corrected model for modulus [35]. Based on the statistical derivations presented in [35], it is straightforward to correct also the strength model [40]. A basic part of the analysis is calculation of the force sustained by the fibers crossing a scan line (arbitrary line normal to the applied load in a thin rectangular specimen) [35]. This involves finding the number of fibers of length $l$ and orientation $\theta$ that cross the scan line, determining the load-direction component of the force such fibers carry at failure, and finally integrating over fiber length and orientation to find the total force sustained by the fibers. The number of fibers of length between $l$ and $l + dl$ and orientation between $\theta$ and $\theta + d\theta$ that cross the scan line is [35]

$$N(l, \theta) = V_f \frac{A_c l}{A_f \langle l \rangle} \cos \theta h(l) g(\theta) dl d\theta$$  \hspace{1cm} (A 1)

where $h(l)$ and $g(\theta)$ are fiber length and orientation distribution density correspondingly, $\langle l \rangle$ denotes the average fiber length, $A_f$ stands for fiber cross-section area, and $A_c$ – composite specimen cross-section normal to the applied load. The force carried by a fiber in the load direction (i.e. normal to the scan line) is [40]
\[ F(l, \theta) = \begin{cases} 
A_f \sigma_0(l) \cos^3 \theta & \cos \theta \geq l_n / l \\
0 & \cos \theta < l_n / l 
\end{cases} \]  \quad (A 2)

where \( l_n \) is the critical zone width and \( \sigma_0(l) \) is related to fiber strength \( \sigma_{sf} \) and critical fiber length \( l_c \) as follows:

\[ \sigma_0(l) = \begin{cases} 
(1 - l_c / 2l) \sigma_{sf} & l \geq l_c \\
l / 2l_c \sigma_{sf} & l < l_c 
\end{cases} \]  \quad (A 3)

The total force is given by \[ F_T = \int_0^{\omega \pi / 2} \int_0^\infty N(l, \theta) F(l, \theta) \, d\theta \, dl . \]

Therefore the corrected expression for tensile strength of SFC by the rule of mixture [40] is as follows:

\[ \sigma_{wc} = \nu_f \int_{l_n}^{\infty} \int_0^{\theta_0} \sigma_0(l) \cos^4 \theta - \frac{h(l)}{l} g(\theta) d\theta dl + \left(1 - \nu_f\right) \sigma_m \]  \quad (A 4)

where \( \theta_0 = \cos^{-1} \frac{l_n}{l} \).

Let us consider SFC with fibers of random two-dimensional orientation. Then \( g(\theta) = 2 / \pi \) and the integral in (A 4) is given by:

\[ \int_{l_n}^{\infty} \int_0^{\theta_0} \sigma_0(l) \cos^4 \theta - \frac{h(l)}{l} g(\theta) d\theta dl + \left(1 - \nu_f\right) \sigma_m \]

\[ \frac{1}{4\pi} \int_{l_n}^{\infty} \left[ 3 \cos^{-1} \frac{l_n}{l} + \frac{l_n}{l} \left( 3 + 2 \left( \frac{l_n}{l} \right)^2 \right) \left( 1 - \left( \frac{l_n}{l} \right)^2 \right) \right] h(l) dl \]  \quad (A 5)

For random three-dimensional orientation, \( g(\theta) = \sin \theta \) [40] and the integral in (A 4) reduces to:

\[ \frac{1}{5} \int_{l_n}^{\infty} \left[ 1 - \left( \frac{l_n}{l} \right)^5 \right] h(l) dl \]  \quad (A 6)
References
