Camber effect study on combined tire forces

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Abstract

Considering the more and more concerned climate change issues to which the greenhouse gas emission may contribute the most, as well as the diminishing fossil fuel resource, the automotive industry is paying more and more attention to vehicle concepts with full electric or partly electric propulsion systems. Limited by the current battery technology, most electrified vehicles on the roads today are hybrid electric vehicles (HEV). Though fully electrified systems are not common at the moment, the introduction of electric power sources enables more advanced motion control systems, such as active suspension systems and individual wheel steering, due to electrification of vehicle actuators. Various chassis and suspension control strategies can thus be developed so that the vehicles can be fully utilized. Consequently, future vehicles can be more optimized with respect to active safety and performance.

Active camber control is a method that assigns the camber angle of each wheel to generate desired longitudinal and lateral forces and consequently the desired vehicle dynamic behavior. The aim of this study is to explore how the camber angle will affect the tire force generation and how the camber control strategy can be designed so that the safety and performance of a vehicle can be improved.

As the link between the vehicle and the road, the tire ultimately determines the dynamic characteristics of the vehicle. Researchers in the automotive industry have developed various tire models to describe the force and torque generation of a tire. The semi-empirical Magic Formula tire model and the simple physical brush tire model are two common and widely used tire models.

In this study, a quick review of the Magic Formula tire model and the brush tire model is firstly presented. Bearing the advantages and disadvantages of the two models in mind, a new multi-line brush tire model, which places its focus on camber effect on longitudinal and lateral force generation, is developed according to the brush model theory. The newly developed multi-line brush tire model describes longitudinal and lateral force generation at different camber angles accurately and provides some essential information of the effect of camber angle.

However, the multi-line brush tire model consumes huge amount of computational effort thus it is not suitable for real time vehicle level simulations. In order to explore how the camber control strategy can be designed, a simple magic formula model is developed by curve fitting to the multi-line brush tire model. The simple magic formula model takes much less computational effort but represents the force generation at different camber angles quite well as the multi-line brush tire model does. With the help of the simple magic formula model, real time vehicle level simulations are conducted to find the optimal camber control strategies with respect to safety and performance.
Acknowledgement

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<td>$B$</td>
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<td>Longitudinal bristle stiffness</td>
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Resultant force in the XOY plane of each bristle
\( f_{total} \)

Longitudinal, lateral and vertical force on each bristle
\( f_{x,y,z} \)

Strut force in the quarter-car model
\( F_s \)

Longitudinal tire force
\( F_x \)

Lateral tire force
\( F_y \)

Vertical tire load
\( F_z \)

Nominal vertical load of a tire
\( F_{z0} \)

Nominal vertical load of each contact line
\( F'_{z0} \)

Gravitational acceleration
\( g \)

Index of bristles in each contact line
\( i \)

Index of each contact line
\( i_d \)

Index of time step
\( j \)

Spring stiffness
\( k \)

Shape factor of polynomial around cones
\( k_{rs1,2} \)

Rotational spring stiffness in the flexible lateral carcass model
\( l_{s} \)

Load sensitivity scaling factor
\( m_{sprung} \)

Sprung mass in the quarter-car model
\( m_{unsprung} \)

Unsprung mass in the quarter-car model
\( M_{z} \)

Aligning torque
\( M_{rr} \)

Residual torque
\( n \)

Number of bristles
\( R_{r} \)

Radius of the rim
\( R_{w} \)

Tire radius vector for each contact line
\( s_{f_{lateral}} \)

Scaling factor for the lateral forces in the simple magic formula model
\( s_{f_{longitudinal}} \)

Scaling factor for the longitudinal forces in the simple magic formula model
\( S \)

Length of the tire side wall
\( S_{H} \)

Horizontal shift in Magic Formula
\( S_{V} \)

Vertical shift in Magic Formula
\( t \)

Pneumatic trail
\( v_{cx,xy} \)

Components of the velocity of the wheel contact center
\( v_{sxy} \)

Components of the slip velocity \( v_s \) with \( v_{sy} \approx v_{cy} \)
\( V \)

Velocity vector
\( w_{r} \)

Width of the tire rim
\( x \)

Longitudinal motion of the vehicle
\( X,Y \)

Vehicle longitudinal and lateral displacement in road coordinate system
\( y \)

Lateral motion of the vehicle
\( z \)

Vertical position of the sprung mass
\( z \)

Vertical motion of the vehicle
\( z_{t} \)

Vertical position of the tire
1 Introduction

1.1 Background and motivation
The history of the automotive typically begins as early as 1769, with the creation of steam-engined automobiles capable of human transport [1]. Various modes of propulsion, such as internal combustion engine, electric motor as well as the already mentioned steam engine, have been utilized in the early era. With the commercial drilling and production of petroleum began in the mid-1850s, internal combustion engines, which had already been developed before the 19th century, became the predominant mode of propulsion due to the incomparably high energy density of the liquid fossil fuel that internal combustion engines consume.

During the past twenty years, the climate change issues to which the greenhouse gas emission may contribute the most, as well as the diminishing fossil fuel resource, have become more and more concerned. The trend of new vehicle concepts now goes towards full electric or partly electric propulsion systems. Thanks to the introduction of electric power source, more advanced motion control systems, such as active suspension systems and individual wheel steering, can be implemented due to electrification of vehicle actuators. Various chassis and suspension control strategies can thus be developed so that the vehicles can be fully utilized. Consequently, future vehicles can be more optimized with respect to active safety and performance.

The camber angle, $\gamma$, denotes the tilting angle between the wheel center plane and the vertical axle. The sign of the camber angle varies according to different coordinate systems and sign conventions. Cambering a tire create lateral forces even though there is no lateral slip. The effects of camber are particularly important when deriving models for motorcycles since camber produces a large part of the cornering force. For cars and trucks with conventional suspension designs, the achieved camber angles are much smaller and in many applications their effect can be neglected [2]. However, electrification of vehicle actuators enables active control of camber angles instead of passively tilting the wheel according to the suspension geometry.

Active camber control is a method that assigns the camber angle of each wheel to generate desired longitudinal and lateral forces and consequently the desired vehicle dynamic behavior. And one example of suspension system that can realize active camber control is the autonomous corner module (ACM). This is a concept invented by S. Zetterström in 1998 [3]. The name autonomous indicates that the wheel forces and kinematics are individually controlled, supporting a common task [3-5]. More in detail the ACM is a modular based suspension system that includes all features of wheel control, such as control of steering, wheel torques, wheel loads and camber individually.

In order to have the active camber control system work properly, it is absolutely necessary to understand the camber effect on lateral and longitudinal force generation as well as the torque generation correctly. In this context, tire modeling and simulation of the tire behavior becomes rather significant.
As the link between the vehicle and the road, the tire ultimately determines the dynamic characteristics of the vehicle. An extensive amount of work pertaining to tire modeling has been done in the automotive industry. In this thesis, two widely used tire models, the Magic Formula tire model and the brush tire model are reviewed. Based on the brush model theory, a physical multi-line brush tire model focusing on camber effect is developed as the base for a simple magic formula tire model.

It is not sufficient to understand the camber effect only in the tire level. Real time full vehicle level simulations are also conducted in this thesis, trying to optimize the safety and performance of a vehicle by actively control the camber angles.

1.2 Research question
The aim of this master thesis is firstly to explore how the camber angle will affect the tire force generation both in pure slip situations and in combined slip situations. With sufficient understanding of the camber effect in tire level with the help of existing and newly developed tire models, how the camber angle will affect the dynamic behavior of a full vehicle or if it is possible to enhance the safety and performance of a vehicle by means of active camber control is also explored based on real time vehicle level simulations.

The methodology is to combine theory, modeling with simulations.

1.3 Outline of thesis
This thesis is divided into four parts. The first part (Chapter 2) gives a short review of two existing widely used tire models: the Magic Formula tire model and the brush tire model. The second part (Chapter 3 and 4) gives the detailed developing process of a multi-line brush tire model and presents the essential results pertaining to camber effect, then the developing process of a simple magic formula tire model based on the multi-line brush tire model is briefly described. The third part (Chapter 5) presents the process and results of real time vehicle level simulations with vehicle safety and performance orientations. Finally, the work is summarized and discussed.
2 Review of two existing tire models

As the link between the vehicle and the road, the tire ultimately determines the dynamic characteristics of the vehicle. An extensive amount of work pertaining to tire modeling has been done in the automotive industry. The research covers everything from derivation of simplistic expressions describing the tire behavior to advanced finite element methods that predicts the important states at any point of the tire [2]. This chapter gives a short review of two existing widely used tire models: the Magic Formula tire model and the brush tire model. The main part of the contents in this chapter is gathered from previous publications by other authors and rewrite in a slightly new manner.

2.1 The Magic Formula tire model
The first model studied is the semi-empirical Magic Formula tire model. It is a widely used tire model to calculate the steady state tire force and moment characteristics for use in vehicle dynamic studies. The paper [6] presents the ‘Magic Formula’, which quickly became the most predominating tire model. After its first presentation the Magic Formula has been extended to empirically express most of the interesting properties of the tire and also the interaction of many simultaneous actions, such as combined slip. The entire model is presented in [7] and the complete model includes approximately 85 parameters that have to be calibrated from measurement data. In this section only the general form of the model will be discussed.

2.1.1 Model description
The model expresses the longitudinal and lateral forces, as well as the aligning torque, in the form

\[ y(x) = D \sin[C \tan^{-1}((1 - E) x + (E/B) \tan^{-1}(Bx))] \]  

(2-1)

With

\[ Y(X) = y(x) + S_V \]  

(2-2)

\[ x = X + S_H \]  

(2-3)

Where \( Y \) is the output variable and is defined as the longitudinal force \( F_x = y(\kappa) \) or the lateral force \( F_y = y(\alpha) \). \( X \) is defined as the input variable: the longitudinal slip \( \kappa \) as input for longitudinal forces or the lateral slip \( \alpha \) as input for lateral forces. The interpretations of the remaining parameters in Equation (2-1) are given as

- \( B \) Stiffness factor
- \( C \) Shape factor
- \( D \) Peak value
- \( E \) Curvature factor
- \( S_H \) Horizontal shift
- \( S_V \) Vertical shift
The Magic Formula $y(x)$ typically produces a curve that passes through the origin $y = 0$, reaches a maximum and subsequently tends to a horizontal asymptote. For given values of the coefficients $B$, $C$, $D$ and $E$ the curve shows an anti-symmetric shape with respect to the origin. To allow the curve to have an offset with respect to the origin, two shift factors $S_H$ and $S_V$ have been introduced. A new set of coordinates $Y(X)$ arises as shown in Figure 2.1 below. The formula is capable of producing characteristics that closely match measured curves for the lateral force $F_y$ (and if desired also for the aligning torque $M_z$) and for the longitudinal force $F_x$ as functions of their respect slip quantities: the slip angle $\alpha$ and the longitudinal slip $\kappa$ with the effect of load $F_z$ and camber angle $\gamma$ included in the parameters.

Figure 2-1 below illustrates the meaning of some of the factors by means of a typical lateral force characteristic. Obviously, coefficient $D$ represents the peak value (with respect to the central $x$-axis and for $C \geq 1$) and the product $BCD$ corresponds to the slope at the origin ($x = y = 0$). The shape factor $C$ controls the limits of the range of the sine function appearing in Equation (2-1) and thereby determines the shape of the resulting curve. The factor $B$ is the left to determine the slope at the origin and is called the stiffness factor. The factor $E$ is introduced to control the curvature at the peak and at the same time the horizontal position of the peak.

![Figure 2-1 Curve produced by the general form of the Magic Formula][1]

The offset $S_H$ and $S_V$ appear to occur when ply-steer and conicity effects and possibly the rolling resistance cause the $F_y$ and $F_x$ curves not to pass through the origin. Wheel camber may give rise to a considerable offset of the $F_y$ v.s. $\alpha$ curves. Such a shift may be accompanied by a significant deviation.

---

[1]: #Figure 2-1 Curve produced by the general form of the Magic Formula [7]
from the pure anti-symmetric shape of the original curve. To accommodate such an asymmetry, the curvature factor $E$ is made dependent of the sign of the abscissa($x$).

$$E = E_o + \Delta E \cdot sgn(x) \tag{2-4}$$

The aligning torque $M_z$ can be obtained by multiplying the lateral force $F_y$ with the pneumatic trail $t$ and adding the usually small (except with camber) residual torque $M_{zr}$.

$$M_z = -t \cdot F_y + M_{zr} \tag{2-5}$$

The pneumatic trail decays with increasing lateral slip and is described as follows:

$$t(\alpha_t) = D_t \cos[C_t \tan^{-1}(B_t \alpha_t - E_t (B_t \alpha_t - \tan^{-1}(B_t \alpha_t)))] \tag{2-6}$$

Where

$$\alpha_t = \tan \alpha + S_{H_t} \tag{2-7}$$

The residual torque show a similar decay:

$$M_{zr}(\alpha_r) = D_r \cos[\tan^{-1}(B_r \alpha_r)] \tag{2-8}$$

Where

$$\alpha_r = \tan \alpha + S_{H_f} \tag{2-9}$$

Both the aligning and residual torque are modeled using the Magic Formula, but instead of the sine function, the cosine function is applied to produce a hill-shaped curve as shown in Figure 2-2 below. The peaks are shifted sideways. The peak value is defined as $D$, $C$ is the shape factor determining the level $y_a$ of the horizontal asymptote and $B$ influences the curvature at the peak. Factor $E$ modifies the shape at larger values of slip and governs the location $x_o$ of the point where the curve intersects the $x$ axis.
2.1.2 Discussion on the Magic Formula tire model

Generally, the semi-empirical Magic Formula model has high accuracy when describing the steady state tire characteristics and the parameters can be identified quickly due to separate model parameters for describing the force and moment characteristics [8]. However, the parameters included in the model serve for advanced curve fitting instead of having any physical meaning, which makes it difficult to understand the tire behavior with solid physical background. More importantly, it is difficult to extend the model with new operating conditions, such as large camber angles. Combined with the purposes of this thesis, one of which is to understand the camber effect on tire force generation, the original Magic Formula is not selected to be part of the simulations in later chapters. But considering its high accuracy when describing the steady state tire characteristics at moderate camber angles, the original Magic Formula tire model is taken as a reference when developing our own multi-line brush tire model.

2.2 The brush tire model

The brush tire model is a well-known approach to model tire forces, see e.g. [9], [10], or [11]. The model was quite popular in the 1960’s and 1970’s before the empirical approaches became dominating and describes the tire behavior in an educational way. The brush tire model describes the generation of tire forces by dividing the contact patch into adhesion and a sliding region. Forces in the adhesion region are assumed to be caused by elastic deformations in the rubber volume between the tire carcass and the ground. The carcass is assumed to be stiff which means that effects of carcass deformation are neglected. Forces in the sliding region are assumed to be caused by sliding friction. In this section, the basics of the brush tire model will be discussed.
2.2.1 Model description

The brush tire model is obtained by dividing the rubber volume in the contact region into small brush elements. Each element stretches laterally over the entire contact region, but their length is infinitesimal in the longitudinal direction. The elements are regarded as rectangular bristles as shown in Figure 2-3 below. Even though rubber is not linearly elastic in general, this assumption is made in the brush tire model. Positions in the contact region are expressed in a reference system attached to the carcass, with the origin located in the center of the contact region. The length of the contact region is set to be 2a. Each bristle is assumed to deform independently in the longitudinal and lateral directions. In the adhesive region the bristles adhere to the road surface and the deformation force is carried by static friction. In the sliding region the bristles slide on the road surface under influence of sliding friction. Hence, in the sliding region, the resulting force is independent of bristle deformations [2].

For pure lateral slip cases, the brush model moving at a constant slip angle has been depicted in Figure 2-4 below. It shows a contact line which is straight and parallel to the velocity vector \( V \) in the adhesion region and curved in the sliding region where the available frictional force becomes lower than the force which would be required for the tips of the bristles to follow the straight line further.
Consequently, the lateral deformation in the adhesion region reads

\[ v = (a - x) \tan \alpha \]  

(2-10)

Where \( a \) is half the contact region length.

With the lateral stiffness \( c_{py} \) of the bristles per unit length of the assumedly rectangular contact area the following integrals and expressions for the cornering force \( F_y \) and aligning torque \( M_z \) hold:

\[ F_y = c_{py} \int_{-a}^{a} vdx = 2c_{py}a^2\alpha \]  

(2-11)

\[ M_z = c_{py} \int_{-a}^{a} xvdx = -\frac{2}{3} c_{py}a^3\alpha \]  

(2-12)

Consequently, the cornering stiffness and the aligning stiffness become respectively:

\[ C_{Fa} = 2c_{py}a^2 \]  

(2-13)

\[ C_{Ma} = \frac{2}{3} c_{py}a^3 \]  

(2-14)

For pure longitudinal slip cases, linearization for small values of longitudinal slip \( \kappa \) yields a deflection at coordinate \( x \):

\[ u = (a - x)\kappa \]  

(2-15)

And a longitudinal force
\[ F_x = 2c_{px} a^2 \kappa \]  

(2-16)

With \(c_{px}\) the longitudinal bristle stiffness per unit length. This relation contains the longitudinal slip stiffness

\[ C_{FK} = 2c_{px} a^2 \]  

(2-17)

For equal longitudinal and lateral stiffness (\(c_{px} = c_{py}\)) we obtain equal slip stiffness \(C_{FK} = C_{F\alpha}\). In reality, however, appreciable differences between the measured values of \(C_{FK}\) and \(C_{F\alpha}\) may occur which is due to the lateral compliance of the carcass of the actual tire. Still, it is expected that qualitative similarity of both pure slip characteristics remains.

### 2.2.2 Discussion on the brush tire model

In general, the brush tire model describes the tire behavior in a way that is much easier to understand due to its solid physical background. Meanwhile it requires smaller number of model parameters for describing the steady state characteristics and due to its better extrapolation qualities it is more suitable for extending with new operating conditions [8], such as large camber angles. However, the disadvantage of this model is that the model parameters almost influence all force and moment characteristics thus it is more difficult to identify the model parameters. And more importantly, some assumptions, such as the stiff carcass as well as the linear elastic bristles, may affect the accuracy and validity of the model. Therefore, the ideas behind the brush tire model will be kept and based on that a new multi-line brush tire model will be developed in the next chapter.

### 2.3 Conclusion

An extensive amount of research related to tire modeling has been done in the automotive industry and the short reviews given in this chapter do by no means cover the area. The aim is more to show different approaches that have been proposed to describe the tire behavior and their respective advantages and disadvantages. When choosing among the tire models it is always important to consider the purpose of the particular application. As for this thesis, the camber effect on force generation is the main focus. Thus, a new multi-line brush tire model, which takes the basic idea of the brush tire model and then takes the Magic Formula as reference, will be developed in the next chapter.
3 Development of a multi-line brush tire model

In this chapter, a multi-line brush tire model is developed according to the brush tire theory. This multi-line brush tire model comprises three longitudinal contact lines which locate at the left end, the middle, and the right end of the tire. Each of the contact lines contains 50 bristles which have different stiffness in X, Y and Z direction. Deflection of a bristle will result in forces in X, Y, Z direction as well as torques around the X, Y, Z axles. The vector sum of the forces and torques of all the bristles will give resultant forces and torques that act on the tire. Load sensitivity effect, a flexible lateral carcass model and a dynamic tire friction model are implemented in the tire model so that it can have similar behavior compared to the semi-empirical model, Magic Formula, which is a well-known tire model developed by Mr. Hans Pacejka. In order to obtain steady state outputs, a quarter-car model is working with the tire model together. The multi-line brush tire model combined with the quarter-car model takes the velocity of the wheel contact center and the slip velocity, as well as the camber angle, as inputs and gives the longitudinal and lateral forces as outputs. Simulations are conducted in order to obtain some essential dynamic behaviors of the model and the main focus is placed on the effect of the camber angle.

3.1 Specification of the multi-line brush tire model

3.1.1 Coordinate system and sign conventions for force and tire slip

The coordinate system that will be used in the multi-line brush tire model is the ISO coordinate system. Figure 3-1a to 3-1d show the ISO coordinate system and sign conventions for tire slip. And in all these figures, the velocity of the wheel contact center, $v_{cx}$ is positive.

(a) Lateral slip angle, top view   (b) Tire inclination/Camber angle, rear view
The lateral slip angle $\alpha$ is defined as:

$$\tan \alpha = \frac{v_{xy}}{v_{cx}}$$  \hspace{1cm} (3-1)

The longitudinal slip $\kappa$ is defined as:

$$\kappa = -\frac{v_{cx}}{v_{cx}}$$  \hspace{1cm} (3-2)

As can be seen from the figures above, a positive lateral slip angle will result in negative lateral force. Meanwhile, a positive camber angle without any lateral slip will also result in a negative lateral force thus in Figure 3-1d, the dash line is shifted downwards with respect to the zero camber solid line. Furthermore, a positive longitudinal slip will result in positive longitudinal force which means the tire is accelerating.

3.1.2 Fundamental multi-line brush tire model

From the side view, a segment angle is introduced to describe the whole area of interest as shown in Figure 3-2 below.
The bristles of each contact line divide the segment angle equally so that the angle between two neighboring bristles can be calculated as:

$$\theta' = \theta / (n - 1)$$  \hfill (3-3)

Where $\theta = \pi/4$ is the segment angle.

$n = 50$, is the number of bristles in each contact line.

It is noticeable that this segment angle is an arbitrary angle that is basically larger than the angle of the contact patch. This means not necessarily every bristle inside the segment angle is in contact with the road. Whether a bristle is in contact with the road is dependent on the vertical position of the wheel center, which is decided by the vertical load of that wheel and the vertical stiffness of the bristles.

When the tire is standing still, the angle positions of bristles in each contact line can be defined as a vector as:

$$\Phi = [-\frac{1}{2}\cdot \theta; \theta'; \frac{1}{2}\cdot \theta]$$  \hfill (3-4)

However, it is necessary for the tire to roll for some time so that all the bristles can go into the segment angle and finally reach their steady state deflection. Therefore the angle positions of the bristles have to be updated as:
\[
\Phi' = \Phi - \omega \cdot dt
\]  
(3-5)

Where \( \omega = \frac{(v_{cx} - v_{sx})}{R_w} \) is the rolling speed the tire.

\( dt = 0.001 \), is the time step.

It is possible that a bristle may move out of the segment angle, which means:

\[
\Phi' > \frac{1}{2} \cdot \theta \text{ or } \Phi' < -\frac{1}{2} \cdot \theta
\]  
(3-6)

If this happens, then \( \Phi' \) has to be modified as:

\[
\Phi' = \Phi' - \theta \text{ (if } \Phi' > \frac{1}{2} \cdot \theta) 
\]  
(3-7a)

\[
\Phi' = \Phi' + \theta \text{ (if } \Phi' < -\frac{1}{2} \cdot \theta) 
\]  
(3-7b)

Figure 3-3 below shows the rear view of the tire model.

![Figure 3-3 Rear view of the tire](image)

The lateral positions of the contact lines and the respective tire radius can be obtained as:

\[
b = [-w_r/2 + S \cdot \cos \alpha_1; 0; w_r/2 + S \cdot \cos \alpha_2]\]  
(3-8)

\[
R_w = [R_r + S \cdot \sin \alpha_1; R_r + S \cdot \sin \alpha_1 + 0.005; R_r + S \cdot \sin \alpha_2]\]  
(3-9)
where \( w_r = 6 \times 25.4 \times 10^{-3} \) m, is the width of the tire rim.

\[
S = B \times 0.55 \text{ m}, \text{ is the length of the tire side wall.}
\]

\[
R_r = 15/2 \times 25.4 \times 10^{-3} \text{ m}, \text{ is the radius of the tire rim.}
\]

\[
B = 0.195 \text{ m}, \text{ is the width of the tire.}
\]

Introducing the index \( i = 1:50 \) for the bristles in each contact line and the index \( id = 1:3 \) for the 3 contact lines, it is possible to calculate the bristle deflections in X, Y, and Z directions.

\[
Def_z(i, id) = z_t(j) + R_w(2) - (R_w(id) \cdot \cos(\Phi'(i, id)) + b(id) \cdot \sin \gamma) \cdot \cos \gamma
\]

Where \( z_t \) is the Z position of the tire, and the index \( j \) is a time index. \( z_t(1) = -0.0085 \) m is the initial Z position of the tire and \( z_t(j) \) will be updated by the quarter-car model.

Before calculating \( Def_x(i, id) \) and \( Def_y(i, id) \), the sign of \( Def_z(i, id) \) has to be looked into. If \( Def_z(i, id) \) is equal or larger than 0, which means the corresponding bristle is not in contact with the ground and there will be no deflections on that bristle, the corresponding deflections in X, Y, Z directions, \( Def_x(i, id) \), \( Def_y(i, id) \), \( Def_z(i, id) \), should be set to 0. If \( Def_z(i, id) \) is smaller than 0, which means the corresponding bristle is in contact with the ground, calculation of \( Def_x(i, id) \) and \( Def_y(i, id) \) can continue.

As the tire is actually rolling, in order to have all the bristles enter the segment angle and reach steady state deflection, the bristle deflections should be updated. This can be achieved by adding the incremental displacement of each bristle to its deflection in the previous time step.

\[
Def_x(i, id) = Def_x(i, id)_{old} + dx
\]

\[
Def_y(i, id) = Def_y(i, id)_{old} + dy
\]

The initial deflections in X, Y direction are set to 0. The longitudinal incremental displacement \( dx \) and lateral incremental displacement \( dy \) are respectively:

\[
dx(id, i) = \omega \cdot R_w(id) \cdot \cos(\Phi'(i, id)) \cdot dt - v_{cx} \cdot (1 + \psi \cdot b(id)) \cdot dt
\]

\[
dy(id, i) = (-v_{cy} + R_w(id) \cdot \sin(\Phi'(i, id)) \cdot \psi - R_w(id) \cdot \omega \cdot \sin(\Phi'(i, id)) \cdot \sin \gamma) \cdot dt
\]

Where \( \psi = 1/200 \) is the yaw rate.

The vertical force on each bristle is then:

\[
f_z(id, i) = -Def_z(i, id) \cdot \sigma_z - \dot{z}(j) \cdot c_z
\]

Where \( \sigma_z = 555500/(3 \cdot n) \) N/m is the vertical stiffness of each bristle.

\[
c_z = 400/(3 \cdot n) \text{ N/m is the vertical bristle damping per unit length.}
\]
And if $f_z(id, i) < 0$, which means the bristle is not in contact with the ground, $f_z(id, i)$ should also be set to 0.

With the deflections in X, Y directions of each bristle obtained, the forces in X, Y directions of each bristle can be calculated as well given the X, Y stiffness of the bristle.

$$f_x(id, i) = \text{Def}_{f_x}(i, id) \cdot \sigma_x$$  \hspace{1cm} (3-12b)  
$$f_y(id, i) = \text{Def}_{f_y}(i, id) \cdot \sigma_y$$  \hspace{1cm} (3-12c)  

Where $\sigma_x = 900000/(3 \cdot n) \text{ N/m}$ is the longitudinal stiffness of each bristle.

$\sigma_y = 600000/(3 \cdot n) \text{ N/m}$ is the lateral stiffness of each bristle.

Therefore the resultant force in the XOY plane of each bristle is:

$$f_{\text{total}}(id, i) = \sqrt{f_x(id, i)^2 + f_y(id, i)^2}$$  \hspace{1cm} (3-13)  

As is known, the maximum horizontal force is limited by the vertical force and the friction coefficient. If the calculated resultant force in the XOY plane is larger than the available friction force, the bristle will start to slide. The forces in X, Y direction should be re-calculated. The procedure is shown in the following equations.

Firstly, the original calculated lateral and longitudinal force is stored as:

$$f'_x(id, i) = f_x(id, i)$$  \hspace{1cm} (3-14a)  
$$f'_y(id, i) = f_y(id, i)$$  \hspace{1cm} (3-14b)  

If $f_{\text{total}}(id, i) > \mu \cdot f_z(id, i)$, then scale down the forces as:

$$f'_x(id, i) = f_x(id, i) \cdot \mu \cdot f_z(id, i)/f_{\text{total}}(id, i)$$  \hspace{1cm} (3-14c)  
$$f'_y(id, i) = f_y(id, i) \cdot \mu \cdot f_z(id, i)/f_{\text{total}}(id, i)$$  \hspace{1cm} (3-14d)  

Update the deflection of the bristle as:

$$\text{Def}_{f_x}(i, id) = f'_x(id, i)/\sigma_x$$  \hspace{1cm} (3-14e)  
$$\text{Def}_{f_y}(i, id) = f'_y(id, i)/\sigma_y$$  \hspace{1cm} (3-14f)  

Find the new lateral and longitudinal forces as:

$$f_x(id, i) = \text{Def}_{f_x}(i, id) \cdot \sigma_x$$  \hspace{1cm} (3-14g)
\[ f_y (id, i) = D e f_y (i, id) \cdot \sigma_y \]  

(3-14h)

With the forces in X, Y, Z directions of each bristle obtained, it is possible to calculate the total forces acting on the tire as the sum of the forces on each bristle.

\[
F_x (j) = \sum_{id=1}^{3} \sum_{n=1}^{50} f_x (id, i) 
\]  

(3-15a)

\[
F_y (j) = \sum_{id=1}^{3} \sum_{n=1}^{50} f_y (id, i) 
\]  

(3-15b)

\[
F_z (j) = \sum_{id=1}^{3} \sum_{n=1}^{50} f_z (id, i) 
\]  

(3-15c)

3.1.3 Quarter-car model

The multi-line brush tire model can be regarded as a number of springs arranged in parallel. Combined with the fact that the tire in this model is actually rolling, it does take some time for the system to reach a steady state. Therefore, a quarter-car model is introduced to work together with the brush tire model. The quarter car model will take the vertical force from the multi-line brush tire model as an input and output the Z position of the tire \( z_t \) and the vertical velocity of the sprung mass \( \dot{z} \) as new inputs for the brush tire model. The equations are shown as follow:

For the sprung mass:

\[
F_s = k \cdot (z_t (j) - z(j)) + c \cdot (\dot{z}_t (j) - \dot{z}(j)) 
\]  

(3-16a)

\[
\ddot{z} (j + 1) = F_s / m_{sprung} - g 
\]  

(3-16b)

\[
\dot{z} (j + 1) = \dot{z} (j) + \ddot{z} (j + 1) \cdot dt 
\]  

(3-16c)

\[
z (j + 1) = z (j) + \dot{z} (j + 1) \cdot dt 
\]  

(3-16d)

For the un-sprung mass:

\[
\ddot{z}_t (j + 1) = (F_z (j) - F_s) / m_{unsprung} - g 
\]  

(3-17a)

\[
\dot{z}_t (j + 1) = \dot{z}_t (j) + \ddot{z}_t (j + 1) \cdot dt 
\]  

(3-17b)

\[
z_t (j + 1) = z_t (j) + \dot{z}_t (j + 1) \cdot dt 
\]  

(3-17c)

With new \( z_t \) and \( \dot{z} \) obtained, the time index will increase 1.

\[
j = j + 1 
\]  

(3-18)

The multi-line brush tire model will then take the new \( z_t \) and \( \dot{z} \) to calculate the forces in X, Y, Z directions again. This loop will run for 2000 times, equivalently 2 seconds when considering \( dt = 0.001 \), and then the brush tire model will have steady state outputs. As an example, Figure 3-4 below shows the lateral forces v.s. time.
In order to eliminate the numerical fluctuations so that the output can be more accurate, the mean value of the forces in the last 30 time steps will be taken as the ultimate output of the model comprises the multi-line brush tire model and the quarter-car model.

**3.1.4 Load sensitivity effect**

Tire load sensitivity describes the behavior of tires under load. Conventional pneumatic tires do not behave as classical friction theory would suggest. The load sensitivity of most real tires in their typical operating range is such that the coefficient of friction decreases as the vertical load, $F_z$, increases. The maximum force that can be developed does increase as the vertical load increases, but at a diminishing rate.

In the multi-line brush tire model, the load sensitivity effect is implemented for each contact line. When cornering, there will be lateral load transfer so that the load distribution will become uneven: the outer contact line will be over loaded while the inner contact line will be under loaded. This uneven load distribution will decrease the forces that can be developed. For each contact line, the nominal vertical load is:

$$F_{z0}' = \left( m_{sprung} + m_{unsprung} \right) \cdot g / 3 \quad (3-19)$$

The normalized change for each contact line in the vertical load is:

$$df_z(id) = \left( \sum_{i=1}^{50} f_z(id, i) - F_{z0}' \right) / F_{z0}' \quad (3-20)$$

The modified longitudinal and lateral forces will be:

$$f_x(id, i) = f_x(id, i) \cdot (1 - ls \cdot df_z(id)) \quad (3-21a)$$
\[ f_y(id, i) = f_y(id, i) \cdot (1 - ls \cdot df_x(id)) \]  

(3-21b)

Where \( ls = 0.15 \), is a scaling factor.

Figure 3-5a shows the sweep of longitudinal forces of longitudinal slip \( \kappa \) from 0% to 50%, with and without the load sensitivity effect. Figure 3-5b shows the sweep of lateral forces of lateral slip angle \( \alpha \) from -20 degree to 20 degree, with and without the load sensitivity effect. As can be seen in Figure 4a and 4b, implementing the load sensitivity effect will result in reduced lateral forces as well as reduced longitudinal forces. The model behaves as the theory predicted.

![Figure 3-5 Load sensitivity effect on longitudinal and lateral](image)

(a) Longitudinal forces  
(b) Lateral forces

3.1.5 Flexible lateral carcass model

Previous studies about the brush tire model usually considered the carcass as rigid instead of flexible [2]. This multi-line brush tire model will try to implement a flexible carcass model. The lateral force will deform the carcass and result in different load distribution compared with the rigid carcass case. Combined with the load sensitivity effect, the flexible carcass model will result in different lateral and longitudinal force outputs as well.

As can be seen from Figure 3-6 below, from the rear view, this carcass model is rather simple which considers the two tire side walls as two bars that rotate around the upper point of the bar where the tire side wall is connected to the rim. Then two rotational springs are added to each of the rotational bars. To simplify the calculation, the initial inclination of the side walls is neglected and the following equations can be achieved.
For every time step, the angle differences caused by the lateral force are respectively:

\[ \lambda_1 = -0.5 \cdot F_y(j) \cdot S/k_{rs1} \] (3-22a)
\[ \lambda_2 = -0.5 \cdot F_y(j) \cdot S/k_{rs2} \] (3-22b)

Where \( k_{rs1} = k_{rs2} = 500 Nm/\text{rad} \), is the stiffness of the rotational spring.

With the two angle differences obtained, it is possible to calculate the new lateral positions of the three contact lines as well as their respective new tire radius.

\[ b_{ld=1} = -w_r \cdot \cos \gamma/2 - R_r \cdot \sin \gamma + S \cdot \cos(\alpha_1 - \lambda_1) \] (3-23a)
\[ b_{ld=3} = w_r \cdot \cos \gamma/2 - R_r \cdot \sin \gamma + S \cdot \cos(\alpha_2 - \lambda_2) \] (3-23b)
\[ b_{ld=2} = (b_{ld=1} + b_{ld=3})/2 \] (3-23c)
\[ R_{w,ld=1} = R_r \cdot \cos \gamma - w_r \cdot \sin \gamma/2 + S \cdot \sin(\alpha_1 - \lambda_1) \] (3-24a)
\[ R_{w,ld=3} = R_r \cdot \cos \gamma + w_r \cdot \sin \gamma/2 + S \cdot \sin(\alpha_2 - \lambda_2) \] (3-24b)
\[ R_{w,ld=2} = (R_{w,ld=1} + R_{w,ld=3})/2 + 0.005 \] (3-24c)
The newly obtained lateral positions of the three contact lines and their respective new tire radius will be used as inputs for the next time step calculation in the brush tire model. To summarize, this flexible lateral carcass model takes the total lateral force of each time step as an input and output the updated lateral positions of the three contact lines and their respective tire radius as inputs for the next time step calculation in the brush tire model.

Figure 3-7a and 3-7b below show basically how the flexible lateral carcass will deform when there are lateral forces acting on it. In Figure 3-7a, there is a total lateral force of -3000N acting on the carcass and the camber angle is 0 deg. The carcass will deflect to the right side and ‘lift’ the right contact line, resulting in load concentration on the left contact line. In Figure 3-7b, there is a total lateral force of 3000N as well acting on the carcass while there is a 3 deg camber angle of the rim. The carcass will also deflect to the right side and ‘lift’ the right contact line, however, the inclined rim will ‘push down’ the right contact line, resulting in more even load distribution among the contact lines, which is beneficial to the force generation considering the load sensitivity effect.

Figure 3-7 Carcass deflection due to lateral force at different camber angles

Figure 3-8 below shows the differences in lateral forces from -20 degree to 20 degree α angles at 0 degree camber angle. As can be seen in Figure 8, the flexible lateral carcass will result in reduced lateral force when camber angle is 0. This is because the deformed carcass will cause uneven load distribution in the lateral direction which will reduce the maximum available force developed from friction. Or this carcass may be regarded as a camber built in the tire side walls and the lateral force will result in positive camber gain.
3.1.6 Dynamic tire friction model

The friction coefficient is usually considered constant along the contact patch. However, this might not be true in the slide region of the tire. The LuGre friction model, which was developed as a joint cooperation between the Department of Automatic Control at Lund University (Sweden) and Laboratoire d’Automatique de Grenoble (France), describes a dynamic friction phenomenon that arises when friction surface are sliding on each other [12]. An extension of the LuGre tire model is proposed by Deur [13-15]. Here, a much simpler dynamic tire friction model, or more accurately a friction reduction factor, is implemented in the brush tire model in order to obtain similar behavior as the Magic formula tire model. This reduction factor is proportional to the dissipated power. In the adhesion region, the deformation of the bristles will cause no power loss since the energy is stored in the bristles which act as springs. However, in the slide region, the bristles are saturated and there will be power losses. Therefore, in the adhesion region, the reduction factor is set to 1 and will have no effect on the forces. While in the slide region, when \( f_{\text{total}}(id, i) > \mu \cdot f_z(id, i) \), the reduction factor is:

\[
\beta(id, i) = \beta(id, i) - \frac{\mu \cdot f_z(id, i) \cdot f_{pd}}{f_{\text{total}}(id, i)}
\] (3-25)

Where on the right hand side of the equation, \( \beta(id, i) \) is set to 1 in the first time step.

\( f_{pd} \) is a load and velocity dependent scaling factor.

With this factor obtained, Equation (3-14c) (3-14d) will become:

\[
f_x'(id, i) = f_x(id, i) \cdot \mu \cdot f_z(id, i) / f_{\text{total}}(id, i) \cdot \beta(id, i)
\] (3-26a)

\[
f_y'(id, i) = f_y(id, i) \cdot \mu \cdot f_z(id, i) / f_{\text{total}}(id, i) \cdot \beta(id, i)
\] (3-26b)
From Figure 3-9a and 3-9b below it can be seen that when the dynamic friction tire model is implemented, both the longitudinal and lateral forces at larger lateral slip angles will diminish and result in peaks in the curves. This behavior is what can be observed in the Magic Formula model.

![Graph showing the effect of dynamic friction on longitudinal and lateral forces](image)

Figure 3-9 Effect of dynamic friction on longitudinal and lateral forces

Figure 3-10 below shows how the friction reduction factor will behave in the whole contact region. From lateral position 0.12m to -0.035m, the bristles are in the adhesion region and the reduction factor remains 1. While from lateral position -0.035m the bristles start to slide and the reduction factor starts to diminish.

![Graph showing the friction reduction factor along the contact patch](image)

Figure 3-10 Friction reduction factor along the contact patch
3.2 Simulation results from the complete multi-line brush model

3.2.1 Line-wise model visualization

Figure 3-11a and 3-11b show the forces of three directions in each contact line in left front view and side view respectively. The normal load on the tire is $350 \times 9.82N = 3437N$; the lateral slip angle $\alpha = 3^\circ$; the longitudinal slip $\kappa = 0\%$; the camber angle $\gamma = 0^\circ$. Similar to the case Figure 7a, the positive lateral slip angle will result in negative lateral forces and ‘lift’ the right contact line, which is the green line in the figure. Consequently, the vertical force, as well as the lateral force in the right contact line is smaller than that in the left contact line, which is the red line in the figure. Meanwhile, since the tire is turning right due to the negative lateral force, the right contact line becomes an inner contact line and will brake. At the same time, the left contact line becomes an outer contact line and will accelerate. This is why the longitudinal force in the right contact line is negative while in the left contact line it is positive.

(a) Forces in each contact line, left front view                                   (b) Forces in each contact line, side view

Figure 3-11 Multi-line brush tire model line-wise visualization with $\gamma = 0^\circ$

Figure 3-12a and 3-12b show the forces of three directions in each contact line in left front view and side view respectively. The normal load on the tire is $350 \times 9.82N = 3437N$; the lateral slip angle $\alpha = 3^\circ$; the longitudinal slip $\kappa = 0\%$; the camber angle $\gamma = 2^\circ$. Compared with the case in Figure 11a and 11b, the only difference is the camber angle. Similar to the case in Figure 7b, the positive camber angle will diminish the load concentration on the left contact line caused by the flexible lateral carcass deflection, resulting in a more even load distribution. For this specific case, the camber angle even makes the load concentrate on the right contact line which is opposite to the zero camber case. However, the inner braking and outer accelerating effect can still be observed here.
3.2.2 Pure slip

In this section, force generations under pure longitudinal slip and lateral slip are studied. The multi-line brush tire model is firstly taken into comparison with the Magic Formula tire model for several normal load conditions. The camber effect on longitudinal and lateral forces is studied and an attempt to find the optimal camber angle under a certain normal load is made. Finally, the effect of the rim width, as well as the effect of the stiffness of the flexible lateral carcass, is also taken into consideration.
3.2.2.1 Comparison between the multi-line brush tire model and the Magic Formula tire model

Figure 3-13a and 3-13b show the longitudinal force comparison and the lateral force comparison between the multi-line brush tire model and the Magic Formula tire model under different normal loads. The red lines are for the multi-line brush tire model and the blue lines are for the Magic Formula tire model. The cambers angles in both comparisons are set to zero. As can be seen, for both longitudinal and lateral forces, the multi-line brush tire model and the Magic Formula tire model agree in general under different normal loads.

![Figure 3-13 Force comparison between the multi-line brush tire model and the Magic Formula](image)

(a) Longitudinal forces (b) Lateral forces

3.2.2.2 Camber effect on longitudinal and lateral forces

Figure 3-14a shows the camber effect on the longitudinal force. The normal load in this case is $400 \times 9.82 \, N = 3928 \, N$. As can be seen, the cambered tire, both positively and negatively, will result in reduced longitudinal force. Furthermore, the curve for $\gamma = 3 \, ^\circ$ and the curve for $\gamma = -3 \, ^\circ$ overlap each other. These phenomena can be properly explained by the friction circle. Since the available resultant force in the XOY plane is limited by the normal load and the friction coefficient, lateral forces caused by the camber angle will reduce the available longitudinal force, especially when $\kappa$ value is large. And as the model is symmetric, the lateral forces caused by positive and negative camber angles of the same amplitude will be the same, the reduction in available longitudinal forces will be the same and the curves will overlap each other.

Figure 3-14b shows the camber effect on the lateral force. The normal load in this case is $400 \times 9.82 \, N = 3928 \, N$. As can be seen, the camber angle will cause vertical shifts of the curve. In general, compared with the curve with zero camber, a negative camber angle will make the curve shift upwards, while a positive camber angle will make the curve shift downwards. More specifically, at this normal load, the vertical shifts in the vicinity of $\alpha = 0 \, ^\circ$ are relatively small, while the vertical shifts in the vicinity of $\alpha = \pm 8 \, ^\circ$ are much larger. These effects can be very useful when cornering: if the camber angle has the same sign as the alpha angle, the cambered tire will give more available lateral force even when the normal load is unchanged.
3.2.2.3 Search for the optimal camber angles under different normal loads

Figure 3-15a to 3-15c show the attempts to find the optimal camber angles in different load conditions. Since the tire model is symmetric in the lateral direction, the left half of the slip curve is taken into account. The sums of sprung and un-sprung masses are respectively 200kg, 400kg and 600kg, which will result in normal load of 1964N, 3928N and 5892N respectively. Taking the three figures into consideration together, several phenomena can be observed. Firstly, at a certain load, when the amplitude of the camber angle increases, the peak of the curve will laterally move towards $\alpha = 0^\circ$. Secondly, there is a certain camber angle that will maximize the lateral force under a certain normal load and any other camber angles larger or smaller than this ‘optimal’ camber angle will give lower peak lateral forces. Meanwhile, the magnitude of this optimal camber angle will increase when normal load increases, which is reasonable since higher load gives higher lateral forces which will give more carcass deflection and more uneven load distribution among the contact lines, thus a larger camber angle will be required to compensate this uneven load distribution; finally, with a certain camber angle, when the normal load increases, the peak of the curve will laterally move to $\alpha$ of larger amplitude.
Figure 3-15 Search for optimal camber angles under different normal loads

3.2.2.4 Effect of the rim width

Figure 3-16a and 3-16b above show the effect of the rim width on longitudinal force and lateral force respectively. The normal loads in the two cases are \(400 \times 9.82 \text{N} = 3928 \text{N}\). The camber angle is set to 0°. The original rim width is 6 inch and is increased and decreased by 1 inch for comparison. As can be seen, the rim width has almost negligible effect on the longitudinal forces, while has considerable effect on the lateral forces. Figure 16b indicates that the wider the rim is, the higher lateral forces can be generated when there are lateral slips. This is why racing cars, such as F1 cars, usually have wider rim than personal cars do.
3.2.2.5 Effect of the stiffness of the flexible lateral carcass

Figure 3-17a and 3-17b above show the effect of the stiffness of the flexible lateral carcass on longitudinal force and lateral force respectively. The normal loads in the two cases are $400 \times 9.82 \text{N} = 3928 \text{N}$. The camber angle is set to $0^\circ$. The original carcass stiffness is $500 \text{Nm/rad}$ and is increased and decreased by 20% for comparison. As can be seen, the stiffness of the flexible lateral carcass has almost negligible effect on the longitudinal forces, while the has some noticeable effect on the lateral forces. Generally, at $\gamma = 0^\circ$, the stiffer the carcass is, the higher the lateral forces can be generated when there are lateral slips. However, this does not mean that the stiffer carcass is always better. Since here the camber angle is set to $0^\circ$, non-zero camber, which is usually the case when cornering in real situations, will result in uneven load distribution. As discussed before, only when the camber angle has the same sign as the alpha angle, which will make the tire incline to the same direction as the carcass deflect, the cambered tire will give more available lateral force even when the normal load is unchanged.
3.2.3 Combined slip

In this section, force generations in combined slip conditions are studied. Starting with the simplest cases in which the camber angle \( \gamma = 0 ^\circ \), longitudinal force generation combined with different lateral slip angle \( \alpha \), as well as lateral force generation combined with different longitudinal slip \( \kappa \), is studied. Secondly, the lateral slip angle \( \alpha \) is set to constant non-zero values in order to obtain longitudinal force generation with different camber angles; the longitudinal slip \( \kappa \) is set to constant non-zero values in order to obtain lateral force generation with different camber angle. Finally, the camber angles are set to constant non-zero values so that longitudinal and lateral force generations of various combinations of longitudinal and lateral slip can be studied. The normal load in this section is set as \( 400 \times 9.82N = 3928N \).

3.2.3.1 Longitudinal and lateral force generation with \( 0^\circ \) camber angle

Figure 3-18a below shows the longitudinal force generation combined with different lateral slip angles at \( \gamma = 0 ^\circ \). As can be seen, the longitudinal force is reduced due to added lateral slip. Meanwhile, the reductions in longitudinal force are the same if the lateral slip angles have the same magnitude regardless of their directions. This is because the tire model is symmetric laterally and slip angles of the same magnitude will result in lateral forces of the same magnitude. Combined with the friction circle theory, this phenomenon is reasonable and explainable.

Figure 3-18b below on the other hand shows the lateral force generation combined with different longitudinal slips at \( \gamma = 0 ^\circ \). As can be seen, the lateral force is reduced due to added longitudinal slip. However, unlike the case for the longitudinal forces above, same magnitude of longitudinal slips do not result in same reduction of the lateral forces. The reduction in lateral force during braking (when \( \kappa < 0 \)) is smaller than the reduction in lateral force during acceleration (when \( \kappa > 0 \)). This means more lateral force can be obtained during braking than that can be obtained during acceleration, which is beneficial from the safety point of view. This behavior is also obtained from the brush tire model introduced in [7].
3.2.3.2 Camber effect on longitudinal and lateral force generation in combined slip

From Figure 3-19a and 3-19b below, it can be seen that regardless of the direction of the lateral slip, $0^\circ$ camber angle always gives the highest longitudinal forces, which means if we want the maximum accelerating or braking forces when we are cornering, it is better to keep the tire straight up. Meanwhile, it can be seen that if the sign of the lateral slip angle is the same as the camber angle, the longitudinal forces generated will be higher. This can be explained by the load distribution. As discussed before, the camber angle and the lateral slip angle of the same sign will give more even load distribution among the contact lines and thus more available total horizontal force. Even at the same time more lateral force is generated, more longitudinal force can be obtained due to more available total force.

![Figure 3-19 Camber effect on longitudinal force generation in combined slip](image)

Figure 3-20a and 3-20b below show the lateral force generation combined with two constant longitudinal slips at different camber angles. Generally, the curves behave similarly to what they did in the pure slip case. The only difference is that the solid lines, which represent the case at $\gamma = 0^\circ$, have shifted to new positions accordant to the curves in Figure 18b.
3.2.3.3 Longitudinal and lateral force generation with constant non-zero camber angles

As can be seen from Figure 3-21a and 3-21b below, the longitudinal slip curves behave reasonably: the more lateral slip is, the lower longitudinal force can be generated due to the friction circle; if the lateral slip angles have the same magnitude but opposite directions, the one has the same sign as the camber angle will generate a little more force due to more even load distribution.

As can be seen from Figure 3-22a and 3-22b below, the lateral slip curves do not behave so symmetrically as the longitudinal curves do. But still some general information can be extracted: the more longitudinal slip in a certain direction (braking or accelerating) is, the lower lateral force can be generated due to the limitation of friction circle; when the longitudinal slips have the same magnitude
but opposite directions, the one that is braking ($\kappa < 0$) always give a little more available lateral force than the one that is accelerating ($\kappa > 0$) does.

![Graph showing lateral force generation in combined slip with constant non-zero camber angles](image)

(a) Lateral forces at $\gamma = -3^\circ$

(b) Lateral forces at $\gamma = 3^\circ$

Figure 3-22 Lateral force generation in combined slip with constant non-zero camber angles

### 3.2.4 Friction circles

In this section, several friction circles of different camber angles are obtained. The normal load is set to be $400 \times 9.82 \, N = 3928 \, N$ for all cases and the camber angles taken into consideration are respectively:

$$\gamma = 0^\circ, -2^\circ, -4^\circ, 2^\circ, 4^\circ$$

(3-27)

In Figure 3-23 below:

The solid blue lines represent the cases in which $\gamma = 0^\circ$.

The dashed blue lines represent the cases in which $\alpha = 4^\circ$ while the dotted blue lines represent the cases in which $\alpha = -4^\circ$.

The dashed green lines represent the cases in which $\alpha = 8^\circ$ while the dotted green lines represent the cases in which $\alpha = -8^\circ$.

The dashed yellow lines represent the cases in which $\alpha = 12^\circ$ while the dotted yellow lines represent the cases in which $\alpha = -12^\circ$.

The dashed magenta lines represent the cases in which $\alpha = 16^\circ$ while the dotted magenta lines represent the cases in which $\alpha = -16^\circ$.

The dashed red lines represent the cases in which $\alpha = 20^\circ$ while the dotted red lines represent the cases in which $\alpha = -20^\circ$.
Figure 3-23 Friction circles at different camber angles
As can be seen, a negative camber angle will shift the curves to the right side where the lateral slip angles are negative; a positive camber angle will shift the curves to the left where the lateral slip angles are positive. These effects are consistent with what have been observed about the lateral forces in previous sections: if the camber angle has the same sign as the lateral slip angle, in other words if the tire tilts towards the direction of turning, more lateral force can be generated under the same normal load.

Meanwhile, the camber angle will make the friction circles contract in the vertical direction, or in another word, the camber angles will result in reduced longitudinal forces which is consistent with what have been observed about the effect of camber angles on longitudinal forces in Figure 14a. Furthermore, the camber angles will shift the curves a little downwards. This effect is also accordant to what have been observed about the lateral forces in combined slip cases: when the longitudinal forces have the same magnitude but in opposite directions, a little more lateral force can be generated in the braking scenario than that in the accelerating scenario.

### 3.3 Conclusions

In this chapter, the developing process of a multi-line brush tire model was presented. Based on the chosen ISO coordinate system, the fundamental multi-line brush tire model was built up. In order to obtain steady state output from the model, a quarter-car model was developed to cooperate with the tire model. Additionally, the contact line wise load sensitivity effect, a flexible lateral carcass model and a friction reduction factor proportional to the power loss in the contact region were integrated to the model.

Satisfied with the model, several tire level simulations were conducted in order to obtain some preliminary information about the dynamic behavior of the tire model. These simulations mainly focused on the camber effect. Furthermore, the effect of the rim width, as well as the effect of the stiffness of the flexible lateral carcass model, was studied. Generally, the model gave reasonable results not only in the pure slip cases but also in the combined slip cases. Several conclusions can be drawn from the simulation results as follow:

1. For pure slip situations, under a certain normal load, the cambered tire will result in reduced longitudinal forces due to the limitation of the friction circle. Furthermore, the reduction in longitudinal forces is proportional to the magnitude camber angle.
2. For pure slip situations, under a certain normal load, the cambered tire will generate more lateral forces if the tire tilts towards the direction of turning.
3. There is a certain camber angle that will maximize the lateral force under a certain normal load and any other camber angles larger or smaller than this ‘optimal’ camber angle will give lower peak lateral forces. Meanwhile, the magnitude of this optimal camber angle will increase when normal load increases.
4. The rim width has negligible effect on longitudinal force, but a wider rim gives more available lateral force when there is lateral slip.
(5) The stiffness of the flexible lateral carcass has negligible effect on longitudinal force, but in a certain range, the stiffer the lateral carcass is, the more lateral force can be generated when the camber angle is 0°.

(6) If the maximum accelerating or braking forces are wanted when cornering, it is better to maintain a camber angle of 0°.

(7) Similar to the pure slip situations, when the tire is accelerating or braking when cornering, the cambered tire will generate more lateral forces if the tire tilts towards the direction of turning.

(8) When the longitudinal forces have the same magnitude but in opposite directions, a little more lateral force can be generated in the braking scenario than that in the accelerating scenario.

The results pertaining to the camber effect could provide some useful information when formulating the control algorithm in an active camber control suspension system.

At the moment, this multi-line brush tire model is rather complex and it consumes a great amount of computational effort to give steady state outputs, which makes it infeasible to directly implement this tire model into a full vehicle model and do vehicle level real time simulations. Therefore, based on the slip curves and friction circles obtained in this chapter, a much simpler model will be developed for use in vehicle level real time simulations in the next chapter.
4 Development of a simple magic formula tire model

In the previous chapter, a physical multi-line brush tire model has been developed and due to its complexity which consumes a great amount of computational effort, it is decided that a simpler magic formula tire model, which is based on the slip curves and friction circles obtained from the multi-line brush tire model, will be developed in this chapter for use in real time vehicle level simulations.

4.1 Specification of the simple magic formula tire model

Unlike the multi-line brush tire model, which takes the velocity of the wheel contact center and the slip velocity as inputs and gives the lateral and longitudinal forces as outputs, the simple magic formula tire model takes the longitudinal forces and the lateral slip angle $\alpha$ as inputs and gives the lateral forces as outputs. Meanwhile, the load sensitivity effect in the simple magic formula tire model is valid in the tire level instead of in the contact line level as in the multi-line brush tire model. These modifications are made to accommodate with the vehicle model that will be used in the next chapter.

The general form of this simple magic formula model reads

$$F_y = -\sin(\tan^{-1}(C_{f,x} \cdot \alpha + \gamma)) \cdot \sqrt{(\mu \cdot F_z)^2 - (F_x / s_{f_{lontitudinal}})^2} \cdot s_{f_{lateral}} \quad (4-1)$$

Where $C_{f,x}$ is the cornering stiffness for the front or rear wheel.

$\alpha$ is the lateral slip angle.

$\gamma$ is the camber angle.

$\mu$ is the friction coefficient.

$s_{f_{lontitudinal}}$ is a scaling factor for the longitudinal forces.

$s_{f_{lateral}}$ is a scaling factor for the lateral forces.

As concluded in the previous chapter, a cambered tire will result in reduced longitudinal forces while more lateral forces can be generated if the tire tilts towards the direction of turning. Therefore, the longitudinal scaling factor $s_{f_{lontitudinal}}$ is affected by the camber angle while the lateral scaling factor is affected by both the camber angle and the lateral slip angle.

In order to obtain tire behaviors of the simple magic formula tire model as similar as possible to the multi-line brush tire model by means of adjusting the two scaling factors, $s_{f_{lontitudinal}}$ and $s_{f_{lateral}}$, the maximum normalized longitudinal and lateral forces with respect to different normal loads and camber angles are firstly studied based on the multi-line brush tire model.
The results from the multi-line brush tire model are presented in Figure 4-1 below. As can be seen, each point on the surface represents the maximum normalised longitudinal or lateral forces under a certain load at a certain camber angle. For the longitudinal forces, as cambered tire results in reduced longitudinal forces, the peak of the surface is located at $\gamma = 0^\circ$ and when the magnitude of the camber angle increases, the surface shows a descending trend. Since the multi-line brush tire model is symmetric, the reduction in available longitudinal forces is proportional to the magnitude of the camber angle and the surface in Figure 4-1(a) is symmetric. For the lateral forces, as discussed in the previous chapter, when the tire tilts towards the direction of turning or when the camber angle has the same sign with the lateral slip angle, more lateral force can be generated and this is why the peak of the surface locates at negative camber angles (The lateral slip angle is negative in this case). Meanwhile, the peak of the surface is shifting towards camber angle of larger magnitude when the normal load increases, which is reasonable since higher load gives higher lateral forces which will give more carcass deflection and more uneven load distribution among the contact lines, thus a larger camber angle will be required to compensate the uneven load distribution.

Based on the surfaces obtained in Figure 4-1 above, the two scaling factors, $s_{f_{\text{longitudinal}}}$ and $s_{f_{\text{lateral}}}$, can now be identified as:

\[
s_{f_{\text{longitudinal}}} = (1.05 - 0.05 \cdot df_z) - 2 \cdot \gamma \cdot \tanh(20 \cdot \gamma) \cdot (1 - 0.55 \cdot df_z) \tag{4-2}
\]

\[
s_{f_{\text{lateral}}} = (1.05 - 0.05 \cdot df_z) - 2 \cdot \gamma^\ast \cdot \tanh(20 \cdot \gamma^\ast) \cdot (1 - 0.55 \cdot df_z) \tag{4-3}
\]

Where

\[
df_z = (F_z - F_{z0})/F_{z0} \tag{4-4}
\]

\[
\gamma^\ast = \gamma + (0.01 + 0.025 \cdot F_z/2000) \cdot \tanh(-100 \cdot \alpha) \tag{4-5}
\]
The tangent hyperbolic function is used in Equation (4-2), (4-3) and (4-5) in order to avoid sharp points brought by the absolute value function, which the vehicle level optimization program can not deal with. As a result, the scaling factors can be visualized in Figure 4-2 below. As can be seen, though some details on the surfaces are omitted compared with the surface given by the multi-line brush tire model, the general trend of the surfaces given by the two tire models agree quite well. For the longitudinal scaling factor, the surface has a peak at $\gamma = 0^\circ$ and is symmetric with respect to the plane where $\gamma = 0^\circ$. For the lateral scaling factor, the peak of the surface locates at negative camber angles due to negative lateral slip angles and the peak is shifting towards camber angle with larger magnitude when the normal load increases.

![Scale factors for longitudinal](image)

![Scale factors for lateral](image)

(a) Longitudinal scaling factor  (b) Lateral scaling factor (negative lateral slip angle)

Figure 4-2 Longitudinal and lateral scaling factor with respect to different normal loads and camber angles

4.2 Friction circles from the simple magic formula model

In this section, several friction circles of different camber angles from the simple magic formula model are obtained for comparison to the friction circles from the multi-line brush tire model. The normal load is set to be $400 \times 9.82N = 3928N$ for all cases as well and the camber angles taken into consideration are respectively:

$$\gamma = 0^\circ, -2^\circ, -4^\circ, 2^\circ, 4^\circ$$

(4-6)

In Figure 4-3 below:

The solid blue lines represent the cases in which $\alpha = 0^\circ$.

The dashed blue lines represent the cases in which $\alpha = 4^\circ$ while the dotted blue lines represent the cases in which $\alpha = -4^\circ$. 

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The dashed green lines represent the cases in which $\alpha = 8^\circ$ while the dotted green lines represent the cases in which $\alpha = -8^\circ$.

The dashed yellow lines represent the cases in which $\alpha = 12^\circ$ while the dotted yellow lines represent the cases in which $\alpha = -12^\circ$.

The dashed magenta lines represent the cases in which $\alpha = 16^\circ$ while the dotted magenta lines represent the cases in which $\alpha = -16^\circ$.

The dashed red lines represent the cases in which $\alpha = 20^\circ$ while the dotted red lines represent the cases in which $\alpha = -20^\circ$.

Compared with the friction circles produced by the multi-line brush tire model in Figure 3-23, the friction circles produced by the simple magic formula model is much simpler. However, this simple magic formula tire model handles the camber effect quite accurately: the cambered tire will give reduced longitudinal forces which are represented by the contract of the friction circle in the vertical direction when the magnitude of the camber angle increases; if the tire tilts towards the direction of turning or if the camber angle has the same sign as the lateral slip angle, more lateral forces can be generated which are represented by the lateral shift of the friction circle according to the sign of the camber angles.
Figure 4-3 Friction circles at different camber angles from the simple magic formula model
4.3 Conclusions
In this chapter, a simple magic formula tire model, which takes the lateral slip angles, the longitudinal forces as well as the camber angle as inputs and gives the lateral forces as outputs, is developed based on the slip curves and friction circles obtained from the multi-line brush tire model in the previous chapter. The purpose of developing this simple model is to significantly reduce the computational effort that would be required if the physical multi-line brush tire model is used in vehicle level real time simulations. As a result, this simple magic formula tire model can capture the essential effect of camber angles to longitudinal and lateral forces with some less important details of the tire behavior from the multi-line brush tire model omitted. With this simple magic formula model, vehicle level real time simulations will be conducted in the next chapter searching for optimized control strategies with safety and performance orientations.
5 Vehicle level simulations

With sufficient understanding of the camber effect in tire level with the help of existing and newly developed tire models, it is now feasible to investigate how the camber angle will affect the dynamic behavior of a full vehicle. For a vehicle with conventional suspension configurations, the camber angle is predominantly defined by the roll angle of the vehicle. However, with a suspension design that allows active camber control, the camber angle can be adjusted more freely even independent of the roll angle, though commonly the roll angle is still an important input of the control algorithm. For an active camber control system, the essential part is the control strategy or control algorithm which defines the actuators’ outputs based on current motions of the vehicle and the driver’s intentions.

In this chapter, real time vehicle level simulations with vehicle safety and performance orientations are conducted with the help of the simple magic formula tire model developed in the previous chapter, using optimization method which is one identified approach that gives interesting insight when developing a control strategy. Available in the JModelica.org platform [16], there are tools to set up a problem of optimal vehicle control to negotiate path tracking or obstacle avoidance. More vehicle control optimization related studies can be found in [17-20].

5.1 Vehicle models for the simulations
The vehicle model is a 6 degree of freedom (DOF) model developed in MATLAB [21] with the simple magic formula tire model developed in the previous chapter. Longitudinal forces are used directly as input to the tire model. The in-plane motion is described by longitudinal ($x$), lateral ($y$) and yaw ($\psi$) DOFs, and the out-of-plane motion is described by vertical ($z$), pitch ($\theta$) and roll ($\phi$) DOFs. The out of plane dynamics is guided by springs, dampers and anti-roll bars. Figure 5-1 below shows the vehicle model visualized in MATLAB. Parameters of the vehicle are given in Table 5-1 below [22]. For comparison, two models are introduced for the simulations: one is implemented with active camber control as a function of roll angles and the other one is with the conventional suspension design without active camber control functionality. And for the model with active camber control functionality, the maximum magnitudes of the camber angle that can be achieved by the actuators are set to be $|\gamma_0|_{max} = 5°, 10°$ and infinite respectively. Note that this $\gamma_0$ is with respect to the car body while the camber angle $\gamma$ that will be sent to the tire model comprises $\gamma_0$ and the roll angle. The camber angle at the end of the manoeuvre is set to $\gamma = 0°$. 
Table 5-1 Vehicle model parameters [22]

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<td>[m]</td>
</tr>
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<td>[m]</td>
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<td>Wheel track distance</td>
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<td>[m]</td>
</tr>
<tr>
<td>CoG to ground distance</td>
<td>h</td>
<td>0.5</td>
<td>[m]</td>
</tr>
<tr>
<td>Front spring stiffness</td>
<td>c_1, c_2</td>
<td>33000</td>
<td>[N/m]</td>
</tr>
<tr>
<td>Rear spring stiffness</td>
<td>c_3, c_4</td>
<td>56000</td>
<td>[N/m]</td>
</tr>
<tr>
<td>Front stabilizer stiffness</td>
<td>c_{12}</td>
<td>20006</td>
<td>[N/m]</td>
</tr>
<tr>
<td>Rear stabilizer stiffness</td>
<td>c_{34}</td>
<td>16088</td>
<td>[N/m]</td>
</tr>
<tr>
<td>Front damping coefficient</td>
<td>d_1, d_2</td>
<td>4500</td>
<td>[Ns/m]</td>
</tr>
<tr>
<td>Rear damping coefficient</td>
<td>d_3, d_4</td>
<td>3500</td>
<td>[Ns/m]</td>
</tr>
<tr>
<td>Front wheel cornering stiffness</td>
<td>C_f</td>
<td>19.17</td>
<td>[]</td>
</tr>
<tr>
<td>Rear wheel cornering stiffness</td>
<td>C_r</td>
<td>21.3</td>
<td>[]</td>
</tr>
</tbody>
</table>

5.2 Optimization method
The optimization problem is formulated to maximize the entry speed when entering a safety-critical manoeuvre without departure off the road or collide with obstacles. The optimization formulation includes actuator constraints and friction constraints on longitudinal tire force inputs [22]. The vehicle position which is expressed in the inertial system is also constrained so that the vehicle will not leave the
desired tracks or collide with obstacles. A test manoeuvre with cones to mark permitted regions has
been used. Each cone \( n \) capture a forbidden region expressed

\[
Y \geq Y_{\text{cone},n} + \left(k(X - X_{\text{cone},n})\right)^2
\] (5-1)

Where \( X, Y \) are inertial coordinates for the forbidden region associated with the \( n \)th cone positioned at
cone coordinates \((X_{\text{cone},n}, Y_{\text{cone},n})\). \( k (k = 0.3) \) is the shape factor of the polynomial. Since only the
position of center of gravity of the vehicle is considered when formulating the path constraints, the
locations of cones and curve edges are adjusted with the width of the vehicle to compensate it.

The whole optimization problem is described separately from the vehicle model using an extension of
the Modelica language within the JModelica.org platform called Optimica [16, 23], which allows for the
problem to be described in a convenient and intuitive way.

5.3 Simulation setup

The aim of the simulations is to maximize the entry speed of the vehicle in safety-critical situations. The
Consumer union double lane change manoeuvre [24] is chosen as an inspiration of the definition of the
test manoeuvre to evoking over steering. The test is performed by driving the vehicle through a lane
marked by a number of cones, positioned in global coordinates as illustrated in Figure 5-2. This
manoeuvre evaluates the vehicle’s ability to carry out avoidance manoeuvre, typically appearing where
an obstacle suddenly appears in front of the vehicle. The vehicle dynamics problem which sets the limit
is generally over-steering, i.e. the vehicle loses side grip on the rear axle and becomes unstable. The
constraints of the vehicle position throughout the manoeuvre described by Equation 5-1, is shown
around two of the critical cones for this manoeuvre.

The optimization starts when the center of gravity of the vehicle is in the position marked ‘throttle
release’ in Figure 5-2 above. At this starting point the vehicle is placed in the center between the cone
pairs and is directed with zero yaw angle, zero yaw rate and zero lateral velocity. This initial condition
has been selected since it is most likely that a driver in real life situations is not well prepared that an
evasive manoeuvre is to occur.
5.4 Simulation results

5.4.1 Results with limitation on camber actuators to $|\gamma_0|_{\text{max}} = 5^\circ$

Figure 5-3 below shows the vehicle position during the double lane change manoeuvre and Figure 5-4 shows the total longitudinal and lateral forces during the manoeuvre. The solid line represents the vehicle with active camber control while the dash line represents the vehicle without active camber control. From these two figures it can be seen that the vehicle with active camber control functionality tries to utilize the lateral force more than the braking force i.e. the vehicle tries to brake less and utilize the width of the road as much as possible. However, since the maximum achievable camber angle of the camber actuators is set to a relatively low value ($|\gamma_0|_{\text{max}} = 5^\circ$), the difference between the active controlled model and the conventional model is not that much especially in the position plot.

![Figure 5-3 Vehicle position during the manoeuvre with limitation on camber actuators to $|\gamma_0|_{\text{max}} = 5^\circ$](image)
Figure 5-5 below shows the vertical force and camber angle of each wheel during the manoeuvre. As can be seen, the camber actuators try to maximize the lateral forces of the outer wheels by adjusting the camber angles based on the direction of turning and the normal load. For example, at around 10m after the starting point, the normal loads on the outer wheels (wheel 2 and wheel 4) are approaching their maximum values due to load transfer brought by the roll motion. Meanwhile, the camber actuators adjust the camber angle on wheel 2 and wheel 4 to values that will maximize the lateral force based on the direction of turning and the normal load. However, as there is limitation of $|\gamma_0|_{\text{max}} = 5^\circ$ on the camber actuators, the camber actuators are saturated in the vicinity. It also can be seen that the camber angle of wheel 3 at around 10m after the starting point reaches a high magnitude and is opposite to the direction of turning. This might be explained by that the camber thrust caused by the camber angle opposite to the direction of turning will reduce the total lateral force on generated by this wheel so that more longitudinal braking force is available. Since the vehicle is turning left here, more braking force on the left rear wheel (wheel 3) will help the vehicle to turn. Furthermore, as the normal load on wheel 3 is quite small (close to 0) here, the in-plane forces generated by wheel 3 is negligible compared the other three wheels. Similar phenomena can also be seen at around 20m after the starting point where the vehicle is turning to the right.
5.4.2 Results with limitation on camber actuators to $|\gamma_0|_{\text{max}} = 10^\circ$

Figure 5-6 shows the vehicle position during the double lane change manoeuvre and Figure 5-7 shows the total longitudinal and lateral forces during the manoeuvre. The solid line represents the vehicle with active camber control while the dash line represents the vehicle without active camber control. Compared with the results in 5.4.1, it can be seen that if we increase the maximum achievable camber angle to $|\gamma_0|_{\text{max}} = 10^\circ$, the reduction in longitudinal braking forces as well as the increase in lateral forces caused by the active camber control functionality is more conspicuous. As a result, the vehicle with active camber control functionality will move more outwards at around 25m from the starting point compared with the vehicle with conventional suspension design thus more road surface is utilized.
5.4.2 Results with limitation on camber actuators

Figure 5-8 below shows the vertical force and camber angle of each wheel during the manoeuvre. Generally, the camber angles behave similarly compared with the results in 5.4.1, and can be justified comparably. The only difference is that due to the maximum achievable camber angle is set to $|\gamma_0|_{\text{max}} = 10^\circ$ this time, the camber actuators no longer saturate in the vicinity of 10m and 20m from the starting point.

5.4.3 Results without limitation on camber actuators

Figure 5-9 below shows the vehicle position during the double lane change manoeuvre and Figure 5-10 shows the total longitudinal and lateral forces during the manoeuvre. The solid line represents the vehicle with active camber control while the dash line represents the vehicle without active camber control. This time the limitation on the camber actuators is cancelled so that the actuators can generate as much camber angle as desired. Compared with the results in 5.4.2, it can be seen that cancelation of
the limitation on the camber actuators does not bring much gain compared with the setting with limitation of \(|γ_0|_{\text{max}} = 10°\). Meanwhile, from the implementation point of view, actuators without limitations can hardly be achieved. Therefore, this result just presents the theoretical optimal solution of the problem.

Figure 5-9 Vehicle position during the manoeuvre without limitation on camber actuators

Figure 5-10 Total longitudinal and lateral forces during the manoeuvre without limitation on camber actuators

Figure 5-11 below shows the vertical force and camber angle of each wheel during the manoeuvre. Generally the camber angles behave similarly compared with the results in 5.4.1 and can be justified comparably. The only difference is that due to the cancelation of the limitation on the camber actuators, a little more camber angle can be achieved at certain positions on certain wheels.
5.4.4 Results of the control group without active camber control

The vertical forces and camber angles of each wheel from the control group which is without active camber control functionality is also given in Figure 5-12 below. As can be seen, the normal load on each wheel behaves quite similar to previous results from the camber controlled models, which is reasonable since the manoeuvre does not differ conspicuously. The camber angle on each wheel is now accordant with the roll angle so that the curves for each wheel overlap each other.

5.4.5 Maximum entry speed for different settings

The aim of the simulations is to maximize the entry speed of the vehicle in safety-critical situations. Table 5-2 below shows the maximum entry speed of the settings used in the simulations. The model without active camber control functionality is chosen as the control group and as can be seen implementing camber control functionality will result in higher entry speed of the chosen manoeuvre.
The theoretical optimal entry speed is 20.2034m/s (increased by 5.73% compared with the control group), without limitation on camber angle actuators. However, actuators without limitation can hardly be achieved from the implementation point of view and compared with the setting that limit the actuators to $|\gamma_0|_{\text{max}} = 10^\circ$, cancelation of the limitation on the camber actuators does not bring much gain on the entry speed (0.01%). Therefore, the settings with a limitation of $|\gamma_0|_{\text{max}} = 10^\circ$ on the camber actuators would be the practical optimal solution.

<table>
<thead>
<tr>
<th>Model settings</th>
<th>Maximum entry speed [m/s]</th>
<th>Increase [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without active camber control</td>
<td>19.1079</td>
<td>0</td>
</tr>
<tr>
<td>With active camber control, $</td>
<td>\gamma_0</td>
<td>_{\text{max}} = 5^\circ$</td>
</tr>
<tr>
<td>With active camber control, $</td>
<td>\gamma_0</td>
<td>_{\text{max}} = 10^\circ$</td>
</tr>
<tr>
<td>With active camber control, no limitation on $\gamma_0$</td>
<td>20.2034</td>
<td>5.73</td>
</tr>
</tbody>
</table>

5.5 Conclusions
In this chapter, the vehicle level simulations have been conducted using optimization approach. It has been shown that implementing camber control functionality will enhance the safety and performance of the vehicle in safety-critical situations. And around 5% entry speed gain can be expected according to different actuator settings. However, the optimization approach is highly dependent on the simulation model and the 6 DOF vehicle model used in this thesis is sufficient for theoretical analysis while lack the ability of providing accurate quantitative results. As a result, some behaviors of the tires look a bit strange and require further interpretation and validation.
6 Conclusions and recommendations

The aim of this thesis is to investigate the camber effect on vehicle dynamic behaviors. The work starts with a review of two existing tire models: the semi-empirical Magic Formula tire model and the physical brush tire model. Bearing the advantages and disadvantages of these two existing model in mind, also considering the focus of this research which lies on the camber effect, it is decided that a new multi-line brush tire model should be developed. The newly developed multi-line brush tire model takes the basic idea behind the existing brush tire model while is further expanded with load sensitivity effect, a flexible lateral carcass model and a dynamic tire friction model so that this model can have similar behaviors as the empirical Magic Formula model. The multi-line brush tire describes camber effect on tire dynamic behavior in an intuitive way while it consumes a great amount of computational effort to give steady state outputs, which makes it infeasible to directly implement this tire model into a full vehicle model and do vehicle level real time simulations. Thus a much simpler magic formula tire model is developed based on the slip curves and friction circles obtained from the multi-line brush tire model. With this simple magic formula tire model, vehicle level real time simulations with vehicle safety and performance orientations are conducted. Using an optimization approach, the vehicle level simulation results show that implementing the active camber control functionality in the suspension system will enhance the safety and performance of a vehicle in safety-critical situations: higher entry speed is allowed and more road surfaces are utilized.

6.1 Conclusions

The following conclusions can be made concerning the two existing tire models reviewed in this thesis.

(1) The semi-empirical Magic Formula model has high accuracy when describing the steady state tire characteristics and the parameters can be identified quickly due to separate model parameters for describing the force and moment characteristics.

(2) The parameters included in the Magic Formula model serve for advanced curve fitting instead of having any physical meaning, which makes it difficult to understand the tire behavior with solid physical background and it is difficult to extend the model with new operating conditions, such as large camber angles.

(3) The brush tire model requires smaller number of model parameters for describing the steady state characteristics and due to its better extrapolation qualities it is more suitable for extending with new operating conditions.

(4) The disadvantage of the brush tire model is that the model parameters almost influence all force and moment characteristics thus it is more difficult to identify the model parameters. Meanwhile some assumptions, such as the stiff carcass as well as the linear elastic bristles, may affect the accuracy and validity of the model.
The following conclusions concerning the camber effect can be drawn from the simulation results based on the multi-line brush tire model.

(1) For pure slip situations, under a certain normal load, the cambered tire will result in reduced longitudinal forces due to the limitation of the friction circle. Furthermore, the reduction in longitudinal forces is proportional to the magnitude camber angle.

(2) For pure slip situations, under a certain normal load, the cambered tire will generate more lateral forces if the tire tilts towards the direction of turning.

(3) There is a certain camber angle that will maximize the lateral force under a certain normal load and any other camber angles larger or smaller than this ‘optimal’ camber angle will give lower peak lateral forces. Meanwhile, the magnitude of this optimal camber angle will increase when normal load increases.

(4) If the maximum accelerating or braking forces are wanted when cornering, it is better to maintain a camber angle of 0°.

(5) Similar to the pure slip situations, when the tire is accelerating or braking when cornering, the cambered tire will generate more lateral forces if the tire tilts towards the direction of turning.

(6) When the longitudinal forces have the same magnitude but in opposite directions, a little more lateral force can be generated in the braking scenario than that in the accelerating scenario.

The following conclusion can be made according to the vehicle level simulation results.

(1) Implementing camber control functionality will result in higher entry speed of the chosen double-lane change manoeuvre. Around 5% gain can be expected according to different actuator settings.

(2) Though theoretically the entry speed of the manoeuvre can be maximized with a setting that eliminates the limitations on the camber actuators of the maximum achievable camber angle, actuators without limitations can hardly be achieved from the implementation point of view and compared with the settings that has limitations on the actuators, cancelation of the limitation does not bring much gain on the entry speed.

6.2 Recommendations for future work

The newly developed multi-line brush tire model describes the force generation of cambered tires in an intuitive way. However it is far from completion. Several recommendations for future work on the multi-line brush tire model are given as following:

(1) The torque generations are not described by the model at the moment and this area can be explored in the future.
(2) The bristle stiffness can be made independent of the number of bristles and number of contact lines so that the model can be adjusted more conveniently.

(3) With sufficient computational power, it is possible to expand the multi-line brush tire model with even more contact lines and bristles.

(4) If the model were to be eventually validated, field tests on the tires have to be carried out in order to identify the model parameters.

Optimization is a very useful and powerful approach when studying over-actuated systems, however it highly relies on the simulation models. In this thesis, the 6 DOF vehicle model used is sufficient for theoretical analysis while lack the ability of providing accurate quantitative results. Therefore, future works can focus on expanding and validating the vehicle models that would be used in the optimization.
References


Appendix A. Parameter settings of the multi-line brush tire model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 100000 , N/m$</td>
<td>Suspension stiffness</td>
</tr>
<tr>
<td>$c = 4000 , Ns/m$</td>
<td>Suspension damping coefficient</td>
</tr>
<tr>
<td>$z_0 = -0.0419 , m$</td>
<td>Initial vertical position of the suspension</td>
</tr>
<tr>
<td>$\dot{z}_0 = -0.0419 , m/s$</td>
<td>Initial vertical velocity of the suspension</td>
</tr>
<tr>
<td>$z_{t0} = -0.0085 , m$</td>
<td>Initial vertical position of the wheel center</td>
</tr>
<tr>
<td>$\dot{z}_{t0} = -0.0419 , m/s$</td>
<td>Initial vertical velocity of the wheel center</td>
</tr>
</tbody>
</table>
Appendix B. Parameters of the reference Magic Formula tire model

%==========================================================================
% Magic Formula parameters
% Tire: 205/60R15 91V, 2.2bar (Pacejka, 2006)
% Comments:
%==========================================================================

% general parameters
% free unloaded tire radius
R_0        = 0.313
% effective rolling radius (R_e = V_x / Omega_0)
R_e        = 0.305
% radius of the circular tire contour
R_c        = 0.15
% nominal (rated) load
F_z0       = 4000
% reference velocity
V_0        = 16.67
% tire stiffnesses
C_Fx       = 435000 % (taken from ADAMS file)
C_Fy       = 166500 % (taken from ADAMS file)

%==========================================================================

% user scaling factors / default values
% pure slip
lambda_Fz0      = 1.0          % 1.0 nominal (rated) load
lambda_mux      = 1.0          % 1.0 peak friction coefficient (x)
lambda_muy      = 1.0          % 1.0 peak friction coefficient (y)
lambda_muV      = 0.0          % 0.0 with slip speed decaying friction
lambda_KxKap    = 1.0          % 1.0 brake slip stiffness
lambda_KyAlp    = 1.0          % 1.0 cornering stiffness
lambda_Cx       = 1.0          % 1.0 shape factor (x)
lambda_Cy       = 1.0          % 1.0 shape factor (y)
lambda_Ex       = 1.0          % 1.0 curvature factor (x)
lambda_Ey       = 1.0          % 1.0 curvature factor (y)
lambda_Hx       = 0.0          % 1.0 horizontal shift (x)
lambda_Hy       = 0.0          % 1.0 horizontal shift (y)
lambda_Vx       = 0.0          % 1.0 vertical shift (x)
lambda_Vy       = 0.0          % 1.0 vertical shift (y)
lambda_KyGam    = 1.0          % 1.0 camber force stiffness
\[ \lambda_{KzGm} = 1.0 \quad \% \text{1.0 camber torque stiffness} \]
\[ \lambda_t = 1.0 \quad \% \text{1.0 pneumatic trail} \]
\[ \lambda_{Mr} = 1.0 \quad \% \text{1.0 residual torque} \]

\% combined slip
\[ \lambda_{xAlp} = 1.0 \quad \% \text{1.0 alpha influence on } F_x(\kappa) \]
\[ \lambda_{yKap} = 1.0 \quad \% \text{1.0 kappa influence on } F_y(\alpha) \]
\[ \lambda_{VyKap} = 1.0 \quad \% \text{1.0 kappa induces ply-steer } F_y \]
\[ \lambda_s = 1.0 \quad \% \text{1.0 } M_z \text{ moment arm of } F_x \]

\% other
\[ \lambda_{Cz} = 1.0 \]
\[ \lambda_{Mx} = 1.0 \]
\[ \lambda_{My} = 1.0 \]
\[ \lambda_{MPhi} = 1.0 \]

%==========================================================================
%--------------------------------------------------------------------------
% parameters for longitudinal force at pure longitudinal slip
%--------------------------------------------------------------------------
% shape factor
\[ p_{Cx1} = 1.685 \]

% peak value
\[ p_{Dx1} = 1.210 \]
\[ p_{Dx2} = -0.037 \]

% curvature factors
\[ p_{Ex1} = 0.344 \]
\[ p_{Ex2} = 0.095 \]
\[ p_{Ex3} = -0.020 \]
\[ p_{Ex4} = 0 \]

% horizontal shift
\[ p_{Hx1} = -0.002 \]
\[ p_{Hx2} = 0.002 \]

% slip stiffness
\[ p_{Kx1} = 21.51 \]
\[ p_{Kx2} = -0.163 \]
\[ p_{Kx3} = 0.245 \]

% vertical shift
\[ p_{Vx1} = 0 \]
\[ p_{Vx2} = 0 \]
% parameters for overturning couple
q_sx1 = 0
q_sx2 = 0
q_sx3 = 0

% parameters for longitudinal force at combined slip
% stiffness factors
r_Bx1 = 12.35
r_Bx2 = -10.77
r_Bx3 = 0
% shape factor
r_Cx1 = 1.09256
% curvature factors
r_Ex1 = 1.644;
 r_Ex2 = -0.0064359;
% horizontal shift
r_Hx1 = 0.007

% parameters for lateral force at pure side slip
% shape factor
p_Cy1 = 1.193
% peak values
p_Dy1 = -0.990
p_Dy2 = 0.145
p_Dy3 = -11.23
% curvature factors
p_Ey1 = -1.003
p_Ey2 = -0.537
p_Ey3 = -0.083
p_Ey4 = -4.787
p_Ey5 = 0
% slip stiffness
p_Ky1 = -14.95
p_Ky2 = 2.130
\[
\begin{align*}
p_{Ky3} & = -0.028 \\
p_{Ky4} & = 2 \\
p_{Ky5} & = 0 \\
p_{Ky6} & = -0.92 \\
p_{Ky7} & = -0.24 \\
\end{align*}
\]

% horizontal shift
\[
\begin{align*}
p_{Hy1} & = 0.009 \\
p_{Hy2} & = -0.001 \\
p_{Hy3} & = 0 \\
\end{align*}
\]

% vertical shift
\[
\begin{align*}
p_{Vy1} & = 0.045 \\
p_{Vy2} & = -0.024 \\
p_{Vy3} & = -0.532 \\
p_{Vy4} & = 0.039 \\
\end{align*}
\]

% parameters for lateral force at combined slip
%--------------------------------------------------------------------------
% stiffness factors
\[
\begin{align*}
r_{By1} & = 6.461 \\
r_{By2} & = 4.196 \\
r_{By3} & = -0.015 \\
r_{By4} & = 0 \\
\end{align*}
\]

% shape factor
\[
\begin{align*}
r_{Cy1} & = 1.08157 \\
\end{align*}
\]

% curvature factors
\[
\begin{align*}
r_{Ey1} & = 0 \quad \text{% (taken from ADAMS file)} \\
r_{Ey2} & = 0 \quad \text{% (taken from ADAMS file)} \\
\end{align*}
\]

% horizontal shift
\[
\begin{align*}
r_{Hy1} & = 0.009 \\
r_{Hy2} & = 0 \quad \text{% (taken from ADAMS file)} \\
\end{align*}
\]

% vertical shift
\[
\begin{align*}
r_{Vy1} & = 0.053 \\
r_{Vy2} & = -0.073 \\
r_{Vy3} & = 0.517 \\
r_{Vy4} & = 35.44 \\
r_{Vy5} & = 1.9 \\
r_{Vy6} & = -10.71 \\
\end{align*}
\]

% parameters for self-aligning moment at pure side slip
% stiffness factors
q_Bz1 = 8.964
q_Bz2 = -1.106
q_Bz3 = -0.842
q_Bz5 = -0.227
q_Bz6 = 0
q_Bz9 = 18.47
q_Bz10 = 0

% shape factor
q_Cz1 = 1.180

% peak values
q_Dz1 = 0.100
q_Dz2 = -0.001
q_Dz3 = 0.007
q_Dz4 = 13.05
q_Dz6 = -0.008
q_Dz7 = 0.000
q_Dz8 = -0.296
q_Dz9 = -0.009
q_Dz10 = 0
q_Dz11 = 0

% curvature factors
q_Ez1 = -1.609
q_Ez2 = -0.359
q_Ez3 = 0
q_Ez4 = 0.174
q_Ez5 = -0.896

% horizontal shift
q_Hz1 = 0.007
q_Hz2 = -0.002
q_Hz3 = 0.147
q_Hz4 = 0.004

% parameters for self-aligning moment at combined slip
s_sz1 = 0.043
s_sz2 = 0.001
s_sz3 = 0.731
s_sz4 = -0.238
% parameters for normal load
%-----------------------------------------------------------------------------------------------------------
 p_z1 = 15.0

%-----------------------------------------------------------------------------------------------------------
% parameters for turn slip
%-----------------------------------------------------------------------------------------------------------
p_DxPhi1 = 0.4 % (taken from ADAMS file)
p_DxPhi2 = 0 % (taken from ADAMS file)
p_DxPhi3 = 0 % (taken from ADAMS file)
p_DyPhi1 = 0.4 % (taken from ADAMS file)
p_DyPhi2 = 0 % (taken from ADAMS file)
p_DyPhi3 = 0 % (taken from ADAMS file)
p_DyPhi4 = 0 % (taken from ADAMS file)
p_epsGamPhi1 = 0.7 % (taken from ADAMS file)
p_epsGamPhi2 = 0 % (taken from ADAMS file)
p_HyPhi1 = 1.0 % (taken from ADAMS file)
p_HyPhi2 = 0.15 % (taken from ADAMS file)
p_HyPhi3 = 0 % (taken from ADAMS file)
p_HyPhi4 = -4.0 % (taken from ADAMS file)
p_KyPhi1 = 1 % (taken from ADAMS file)
q_BrPhi1 = 0.1 % (taken from ADAMS file)
q_CrPhi1 = 0.2 % (taken from ADAMS file)
q_CrPhi2 = 0.1 % (taken from ADAMS file)
q_DrPhi1 = 1.0 % (taken from ADAMS file)
q_DrPhi2 = -1.5 % (taken from ADAMS file)
q_DtPhi1 = 10.0 % (taken from ADAMS file)

%-----------------------------------------------------------------------------------------------------------
% parameters for rolling resistance moment
%-----------------------------------------------------------------------------------------------------------
q_sy1 = 0.01 % (taken from ADAMS file)
q_sy2 = 0 % (taken from ADAMS file)