INERTIAL MASS ACTUATORS, UNDERSTANDING AND TUNING

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Abstract

The actuators, the error sensors and the control system are central components in active noise and vibration control systems. The actuators produce the secondary forces acting on the structure while the error sensors measure the quantity to be minimized. Inertial mass actuators are used frequently in a variety of applications, e.g. boat engine mounts and airplane fuselages. To tune the inertial mass actuator to fit a certain application or just to understand the manufactures specification it is important to have an understanding of the construction of the actuator and the equations describing it.

This paper presents a mathematical model of the mechanical and electrical system for the inertial mass actuator. A mobility analogy is introduced in order to determine how the inertial mass and the spring constant of the suspension effect the resonance frequency of the actuator as well as the output force. Does the mobility of the structure, the actuator is mounted on, effect the produced output force in relationship to the rated force of the actuator?

Practical measurement results are presented in order to determine the resonance frequency and output force of an inertial mass actuator manufactured by Metravib in France.

INTRODUCTION

This paper presents some fundamentals about the inertial mass actuator, both from an electrical point of view as well as mechanical. The inertial mass actuator is usually very expensive, if professionally built, why it is important to be able change the characteristics of the actuator fo fit a certain application. The resonance frequency needs to be out of the operating frequency range, and at the same time, the force response should be as flat as possible in the frequency region of interest. The range of applications where inertial mass
The actuator could be used is very large, the authors have used them successfully in active vibration isolation applications in ABB trains, leisure boats [1], Colin Class Submarines etc. The inertial mass actuator used in this paper, for exemplification, is manufactured by Metravib in France.

If electro-dynamic shakers are used, ordinary HiFi stereo amplifiers work well. The use of HiFi amplifiers ensures that the total harmonic distortion, THD, is low, which is important in avoiding spill over of energy into harmonics higher than those targeted by the active system. The advantage of using inertial actuators is that they only need to be attached to the structure at a single point, thus enabling them to be connected in parallel with the passive dampers. In this way the active system does not need to handle the static load. Another advantage with this parallel actuator installation is that if the active system malfunctions, or for some reason needs to be switched off, the passive system still works. The actuator consists of a fixed core to which is attached a cylindrical electrical coil as shown in figure 1. The core is surrounded by a cylindrical mass in the form of a tube, to which permanent magnets are attached on the inside. The mounts should be very flexible in the vertical z-direction, but rigid in the x-y directions. The mass and the flexible mounts will be denoted the mechanical system. The core is usually made of an aluminium alloy to maximize strength and lightness, and is surrounded by the coil, made with copper wire. In order to get a high magnetic field in the air gap between the magnets and the coil, the permanent magnets need to be as strong as possible and the air gap as narrow as possible. Therefore a rare earth metal, for example Soma4, is used to construct the permanent magnets. These magnets have a magnetic-flux density that is almost four times as high as can be achieved using normal ferrite magnets.

**THE ELECTRICAL SYSTEM**

The actuator transforms electrical power into mechanical motion. This is done by driving a current through a coil present in a magnetic field. Assuming that the voltage across the coil is denoted $e(t)$ and the current flowing through it is $i(t)$, the relationship
between voltage and current in the coil can be described by the following equation, [2].

\[ e(t) = L \frac{di}{dt} + R i(t) + C_c \int i(t) dt \]  

(1)

where \( L, R \) and \( C_c \) are the inductance, resistance and capacitance of the coil respectively. For sinusoidal input signals, equation 1 can be transformed into the frequency domain, by using the \( j\omega \)-method as:

\[ E(\omega) = j\omega L I(\omega) + R I(\omega) + \frac{1}{j\omega} C_c I(\omega) \]  

(2)

The electrical impedance is thus given by:

\[ Z_E(\omega) = \frac{E(\omega)}{I(\omega)} = j\omega L + R + \frac{1}{j\omega} C_c \]  

(3)

Once the coil is put into and interacts with the magnetic field, these simple relations are no longer valid. The electrical impedance is now measured only when the mechanical system is blocked, thus \( u(t) = 0 \), where \( u(t) \) is the velocity of the mass with the magnets. Usually \( Z_E \) is called the blocked electrical impedance.

If the actuator as a whole is considered, i.e. when the coil is surrounded by a magnetic field. and a voltage is put across the coil, a current, \( i(t) \), starts to flow. Ampere’s Law,[2] gives rise to a force, \( f(t) \), that will try to move the mass according to:

\[ f(t) = Bli(t) \]  

(4)

Here \( B \) is the magnetic-flux density and since permanent magnets are used, the magnetic-flux density is a constant. The effective length of the electrical conductor that moves across the magnetic field is denoted by \( l \). Conversely, if the mass were to move up and down with a velocity, \( u(t) \), this would induce a voltage across the coil according to:

\[ e(t) = Blu(t) \]  

(5)

The voltage across the coil, with a current \( i(t) \), in a moving magnetic field with a magnetic-flux density, \( B \) is given by:

\[ E(\omega) = Z_E(\omega) I(\omega) + Blu(\omega) \]  

(6)

The electrical impedance for the actuator, \( Z_{E_{tot}}(\omega) \) is now by given by:

\[ Z_{E_{tot}}(\omega) = \frac{E(\omega)}{I(\omega)} = Z_E(\omega) + \frac{BlU(\omega)}{I(\omega)} \]  

(7)

The first term is the normal impedance for the coil, (equation 3) and the second term is added when the coil is put into a moving magnetic field. We can now calculate the total impedance of the actuator from an electrical point of view. The difficulty is that the total impedance will not only depend on the frequency, \( \omega \), but also on the velocity of the moving mass, \( u(\omega) \) and the current, \( i(\omega) \).

The amplifiers have a rated power they can deliver, which usually is valid over \( 2 - 4 \) ohms. For actuators that can deliver hundreds of Newtons of force, or even more, the impedance could easily be as low as \( 0.5 - 1 \) ohms and the amplifier would then exhibit a current limitation.
THE MECHANICAL SYSTEM

The mechanical system can be modelled as a mass that is attached to the coil by a spring and a damper. For the real actuator, the spring and the damper are the flexible mounts on each side of the mass. The equation of motion for a single-degree-of-freedom oscillator of mass $M$, damping constant $C$, stiffness $K$ and displacement $x(t)$ can be written as:

$$Ma(t) + Cu(t) + Kx(t) = f(t)$$  \hspace{1cm} (8)

where $f(t)$ is the driving force on the mass $M$. The derivative of the displacement $x(t)$ is denoted $\dot{x}(t) = u(t)$, which is the velocity of the mass and the second derivative $\ddot{x}(t)$ is the acceleration, which is denoted $a(t)$. For sinusoidal motion the equation can be transformed into the frequency domain:

$$j\omega MU(\omega) + CU(\omega) + \frac{1}{j\omega} KU(\omega) = F(\omega)$$  \hspace{1cm} (9)

where the acceleration is given by $j\omega U(\omega)$ and the displacement is $\frac{1}{j\omega} U(\omega)$. The mechanical impedance, $Z_M(\omega)$, is defined as the ratio between the force $F(\omega)$ and the velocity $U(\omega)$: For a single-degree-of-freedom system, according to equation 9, the mechanical impedance is given by:

$$Z_M(\omega) = \frac{F(\omega)}{U(\omega)} = j\omega M + C + \frac{1}{j\omega} K$$  \hspace{1cm} (10)

In order to obtain the undamped natural frequency, $f_0$, equation 9 is solved with $F(\omega) = 0$ and $C = 0$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K}{M}}$$  \hspace{1cm} (11)

It can also be shown that the resonance frequency is independent of the input force but dependent on the damping $C$, [3]. If the actuator is to be used in an active vibration control application, it is of utmost importance that the resonance frequency is placed below the lower frequency limit where the actuator should work, since the actuator will exhibit a phase shift of 180 degrees at the resonance frequency. In the vicinity of such a sharp phase shift, it is almost impossible for a controller to work properly.

COMBINING THE ELECTRICAL AND MECHANICAL SYSTEMS

A force is produced from the electrical current streaming through the coil in the actuator, but only a portion of this force actually reaches the structure that the actuator is mounted on. By adopting a simple model of a loudspeaker for the system according to figure 2a, the mechanical behavior can be further investigated.

A moving-coil loudspeaker principally consists of a cone with a coil, where the cone is elastically suspended to a supporting frame [4]. To move the cone, an electrical force, $F_E$, is created between the magnet attached to the supporting frame and the voice coil. The mass, $M$, of the total moving system is given by the mass of the cone, voice coil, and
its associated air mass \[4\]. The air in front of the cone has impedance, denoted radiation impedance. The radiation impedance, \(Z_{MR}\), is a load on the loudspeaker, and is associated with the radiation of sound \[4\].

The mechanical situation is a bit different for an inertial actuator, where the electrical force acts between the mass and the structure. Here, the structure also includes the core and the coil since the core is bolted on to the structure. The structure, core and coil are together denoted load. The mechanical model of the actuator is shown in figure 2b.

To be able to determine how the different mechanical components interact, a mobility analogy will be used \[5\]. In this case, the mechanical system is transformed into an electrical circuit where ordinary circuit theory can be applied. With the mobility analogy, a force corresponds to a current and a velocity to a voltage. The mobility is defined as the ratio between the velocity \(U(\omega)\) and the force \(F(\omega)\), i.e. the inverse of the mechanical impedance. The mobility \(Y_M\) of a mass \(M\) is therefore given by:

\[
Y_M = \frac{1}{j\omega M}
\]  

(12)

In mechanical systems, the mass is often a one-terminal quantity. A force acting on the mass, resulting in a velocity of the mass with respect to the earth, the inertial frame, results in the second electrical equivalent terminal being earth. In the same sense, the mobility of a spring with a spring constant \(K\) and a viscous damping \(C\) is:

\[
Y_S = \frac{1}{C + \frac{K}{j\omega}}
\]  

(13)

The flexible mounts of the actuator and the flexible suspension of the loudspeaker consist of a spring and a viscous damper; accordingly the mobility of the mounts and the suspension is denoted \(Y_S\).

The mechanical systems of the loudspeaker and the actuator is depicted in figure 2. Consider figure 2a, representing the loudspeaker. The electrical force \(F_E\) applied to the
mechanical system sets the mass $M$ into motion with the velocity $U_M$ with respect to the supporting frame. The same velocity is also achieved at the flexible suspension and at air in front of the cone. Accordingly, the three mobilities $Y_M$, $Y_S$ and $1/Z_{MR}$ are connected in parallel, resulting in an electrical equivalent of the loudspeaker as shown in figure 3a.

Figure 2b, representing the inertial actuator, may also be transformed according to the mobility analogy. The figure shows that the electrical force acts over the spring and damper. The electrical force sets the two masses, the actuator load (structure, coil and core) and actuator mass $M$, into motion. The mobility of the load $Y_L$ and the mass $Y_M$ are different, resulting in that the load and mass do not have the same velocities with respect to the inertial frame. The velocity of the load and the mass are $U_L$ and $U_M$, respectively, and their directions are opposite. In the electrical equivalent the mobility of the load and mass are connected in series. The velocity difference is balanced by the flexible mount between the load and mass, and in the electrical circuit the mobility of the flexible mount $Y_S$ is parallel with the mobilities $Y_M$ and $Y_L$ connected in series. To demonstrate further that

Figure 3: The electrical equivalent, in a mobility analogy of (a) a moving-coil loudspeaker (b) the inertial actuator

the actuator equivalent, shown in figure 3b properly describes the real physical system, it may be observed that if the spring is extremely stiff, the mobility $Y_S$ becomes almost zero; hence the current(force) $F_S$ will equal the current $F_E$, and the current $F_{ML}$ will be zero, so that the velocity over the both the load and the mass will be zero. This is also what should be expected by the real actuator; if the spring is made very stiff the electrical forces cannot move the mass at all. Also, if the actuator is attached to a very rigid foundation having a mass that is a considerably larger than the mass of the actuator, the mobility of the load, i.e. the foundation $Y_L$ should be very small in comparison to the mobility of the mass, $Y_M$. Therefore, the voltage (velocity) over the load would be very small indicating that the actuator can only move the structure a small amount. The force transmitted into the receiving structure (load) is given by:

$$F_{ML} = F_E \frac{Y_S}{Y_S + Y_M + Y_L}$$  \hspace{1cm} (14)

If the mobility of the spring and damper, $Y_S$, is extremely large, then $F_{ML} \approx F_E$ and all the electrical force would be used to move the mass and the load. In order to make the actuator as efficient as possible, i.e. transmitting as much of the electrical force as possible
to the receiving structure, the mass should be connected free-free, or with as soft springs as possible. In order to measure the maximum force that the actuator can produce, the actuator could be mounted on a very solid foundation, with a mobility $Y_L$ close to zero. The current (force) $F_{ML}$ through the mass would then be at its maximum value. This force can be measured with an accelerometer attached to the actuator mass, yielding the maximum force that the actuator could produce. However, once the actuator is mounted on a structure with a mobility greater than zero, the force produced by the actuator on the mass is reduced.

Measurements were carried out to determine the maximum force that the actuator could produce and its resonance frequency. During these measurements the actuator was mounted on a very large mass of approximately 100 kg, standing on a concrete floor. The current through the coil was measured with a digital clamp ampere meter while the force was measured indirectly with an accelerometer mounted on the moving mass. The rated output force is often given only for a certain frequency, which is usually somewhat misleading, since in most active vibration isolation applications several frequency components need to be addressed. The total force that the actuator needs to produce is the vector sum of all generated forces.

In the measurements, two different mass configurations were tested, one with $M=4\text{kg}$ and the other $M=6\text{kg}$. The maximum force that the actuator can provide is of course

Figure 4: (a) The output force of the actuator vs. frequency, for two different masses $M$ of the actuator. The current $i(t)$ was constant at 5A rms. (b) The electrical impedance, $Z_{E_{tot}}$ at a frequency close to the resonance frequency, In flat range the actuator can produce approximately 80N rms at a coil current of 5A rms. The actuator is rated with a maximum current of 15A rms and 20A rms with cooling. Thus the maximum force that the actuator can produce is 320N rms, which is approximately the value specified by the manufacturer.

In order to correctly dimension the actuator, prior information of the mobility and the velocity of the structure that the active vibration isolation should be applied to needs to be measured. First, the mobility in the mounting positions, $Y_L$ should be measured. During the second measurement, the machinery should be switched on and the velocity measured.
with an accelerometer. The primary forces, \( F_{\text{primary}} \), acting on the load in the mounting positions can be calculated according to:

\[ F_{\text{primary}} = \frac{U(\omega)}{Y_L(\omega)}. \]

The actuator rated force should exceed the forces present in the load by a large margin, or at least be \( F_{\text{ML}} \geq F_{\text{primary}} \).

In more complicated systems, where several actuators are present, and at the same time the structure is moving in different modes, a deeper analysis of the mechanical properties of the structure is required. Several actuators can join forces in order to combat certain modes of the structure. Using several actuators in this way results in less force needing to be produced by each actuator individually.

The electrical impedance of the actuator is of importance in order to dimension the amplifiers driving the actuators correctly, hence deliver the required current.

By rewriting equation 14, the electrical force \( F_E \) is calculated and in the same manner, using equation 4, the electrical coil current \( i(t) \) is obtained. The mechanical impedance, \( Z_M(\omega) \) combined with the frequency domain version of equation 4. The mechanical impedance, \( Z_M(\omega) \) is described by:

\[ Z_M = \frac{BLI(\omega)}{U(\omega)} \quad (15) \]

The total impedance for the actuator in an electrical equivalent is found by substituting equation 15 into equation 7 yielding:

\[ Z_{E\text{tot}}(\omega) = Z_E(\omega) + \frac{(BL)^2}{Z_M(\omega)} \quad (16) \]

The expression for \( Z_{E\text{tot}} \) only consists of the physical properties of the actuator, all of which are measurable. In this way, suitable amplifiers can be sourced. The electrical impedance, shown in figure 4, mainly follows the behaviour of a conventional coil. At the resonance frequency of the actuator, at approximately 16 Hz, there is a peak in the impedance curve. The electrical impedance in the operating frequency range from 20 Hz to 60 Hz spans a coil impedance from 0.7 Ohms to 1.3 Ohms.

**CONCLUSIONS**

This paper presents a detailed analysis of an inertial mass actuator from an active vibration control perspective. The electrical as well as mechanical system is considered and their interaction. The output force and the electrical impedance is formulated. Additionally these quantities where also measured on a real inertial mass actuator.

**REFERENCES**


