Performance Analysis of DF Relay Systems with Keyhole and Correlation Effects

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Abstract

In recent years, the innovation of multiple-input multiple-output (MIMO) has completely changed the face of wireless communication system. Especially, the concept of MIMO is evolved to fulfill the demand of multimedia services. However, challenges have risen to the system provider for maintaining the quality of service (QoS) and reliability of MIMO system. Additionally, very recently, cooperative communication has been proposed in order to face and deal with the shortcomings of MIMO system. In cooperative communication, the addition of relay between source and destination enhances the QoS and reliability of wireless system.

The main purpose of this paper is to derive the closed form analytical expressions of symbol error rate (SER) of M-ary phase shift keying (MPSK) for MIMO decode-and-forward (DF) relay system. Besides, SER derivation is based on moment generating function (MGF) of the overall system. In our case, we only deal with a downlink MIMO DF relay system. Here, we consider an $n_t$-antenna at the source $S$, an $n_r$-antenna at the relay $R$ and an $n_d$-antenna at the destination $D$, where the propagated signal from the source passes through keyhole and correlated channel. Further, the same orthogonal space time block codes (OSTBC) is applied to source-destination, source-relay and relay-destination link to exploit the maximal diversity gain.

Furthermore, we analyze the effects of MPSK modulation on MIMO DF system under different values of correlation coefficient by combining the different antenna pairs at different nodes. Beside this, we compare keyhole and no keyhole MIMO relay systems with fix value of correlation coefficient and antenna pairs. With the use of cooperative scheme, effects of keyhole on MIMO DF architecture can be minimized to certain level. Eventually, our simulation analysis excellently matches with analytical results, which is sufficient to validate our work.

**Keywords:** Correlated channel, decode-and-forward, keyhole effects, moment generating function, orthogonal space-time block codes, symbol error rate.
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Sagar Raj Mahat
Contents

Abstract i
Acknowledgments i
Contents ii
List of Figures iv

1 Introduction 1
  1.1 Introduction ................................................. 1
  1.1.1 Literature Review ........................................ 3
  1.1.2 Problem Statement ....................................... 5
  1.1.3 Thesis Outlines ....................................... 6

2 Background 7
  2.1 MIMO System .............................................. 7
  2.2 Cooperative Communication ................................ 8
  2.3 Relay Selection .......................................... 9
  2.4 Maximum Likelihood Detection ........................... 10
  2.5 Keyhole Effects ........................................... 11

3 Our Contribution 13
  3.1 System and Channel Model .................................. 13
  3.2 Symbol Error Rate Analysis of Correlated Keyhole MIMO DF Systems ... 15
    3.2.1 Symbol Error Rate Analysis during the First Hop Transmission ... 16
    3.2.2 Symbol Error Rate Analysis during the Dual Hop Transmission ... 18
    3.2.3 Average Symbol Error Rate of Decode and Forward Protocol ... 20
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>Simulation Results and Analysis</td>
<td>21</td>
</tr>
<tr>
<td>3.3.1</td>
<td>SER of DF protocol</td>
<td>22</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Simulations on SER for Different Values of Correlation Coefficients</td>
<td>23</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Simulation on SER for Different Modulation Techniques</td>
<td>27</td>
</tr>
<tr>
<td>3.3.4</td>
<td>Simulation on SER for Arbitrary Number of Antennas</td>
<td>30</td>
</tr>
<tr>
<td>3.3.5</td>
<td>Simulation on SER with Comparison of Keyhole and No Keyhole System</td>
<td>31</td>
</tr>
<tr>
<td>4</td>
<td>Conclusion and Future Work</td>
<td>32</td>
</tr>
<tr>
<td>4.1</td>
<td>Conclusion</td>
<td>32</td>
</tr>
<tr>
<td>4.2</td>
<td>Future Work</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td><strong>Bibliography</strong></td>
<td>34</td>
</tr>
</tbody>
</table>
List of Figures

2.1 Cooperative based system. .................................................. 9
2.2 Keyhole effect on Rayleigh channel. ................................. 12

3.1 Block diagram of MIMO DF system. ................................. 13
3.2 SER versus SNR comparison of cooperative and noncooperative system for (2, 2, 2) MIMO DF system for 8PSK modulation. ....................... 22
3.3 SER of (2, 2, 2) 8-PSK modulation over MIMO DF relay keyhole channel versus SNR for several values of correlation coefficients. ................... 23
3.4 SER of (2, 4, 2) 8-PSK modulation over MIMO DF relay keyhole channel versus SNR for several values of correlation coefficients. ................... 24
3.5 SER of (4, 2, 2) 8-PSK modulation over MIMO DF relay keyhole channel versus SNR for several values of correlation coefficients. ................... 24
3.6 SER of (4, 2, 4) 8-PSK modulation over MIMO DF relay keyhole channel versus SNR for several values of correlation coefficients. ................... 25
3.7 SER of (4, 4, 2) 8-PSK modulation over MIMO DF relay keyhole channel versus SNR for several values of correlation coefficients. ................... 26
3.8 SER of (2, 2, 2)-OSTBC MIMO DF relay systems with keyhole and correlation effects versus SNR for M=2,4,8. ................................. 27
3.9 SER of (2, 4, 2)-OSTBC MIMO DF relay systems with keyhole and correlation effects versus SNR for M=2,4,8. ................................. 28
3.10 SER of (4, 2, 2)-OSTBC MIMO DF relay systems with keyhole and correlation effects versus SNR for M=2,4,8. ................................. 28
3.11 SER of (4, 2, 4)-OSTBC MIMO DF relay systems with keyhole and correlation effects versus SNR for M=2,4,8. ................................. 29
3.12 SER of (4, 4, 2)-OSTBC MIMO DF relay systems with keyhole and correlation effects versus SNR for M=2,4,8. ................................. 29
3.13 SER of MIMO DF relay systems with keyhole for $8-PSK$ modulation and $r = 0.5$ versus SNR for different antenna pairs. 30

3.14 SER of $(4, 2, 2)$-OSTBC MIMO DF relay systems with keyhole and $(4, 2, 2)$ Rayleigh channel for $8-PSK$ modulation versus SNR with correlation effects. 31
1 Introduction

1.1 Introduction

In a wireless communication networks, there is a probability of significant degradation in the transmitted signals due to the effect of fading channels. One of the approach to improve the performance of communication system over a wireless medium is to use “diversity”. Diversity scheme can be achieved through various ways, such as time, frequency, space, polarization, spatial diversity, channel coding and MIMO antenna systems. Among these diversity methods, spatial diversity with MIMO signal processing techniques have been widely accepted. MIMO communication through space-time coding (STC) has become an interesting topic because it increases QoS in point-to-point communication systems in terms of combating fading effects. Furthermore, MIMO also helps to enhance the error performance, system throughput and link reliability with the use of multi co-located antennas at the transmitting and the receiving side. But, due to its cost, size and hardware complexity, MIMO techniques with multiple co-located antennas are often regarded as impracticable in many applications such as sensor or ad-hoc networks [1].

Cooperative communication is a diversity technique where the virtual MIMO channel is generated without increasing antenna terminals. In cooperative communication, relays are introduced in between source and destination. Where the relay channel between the source and destination is regarded as an auxiliary channel to direct channel. Relay nodes are placed several wavelengths far from the source. Hence, relay channel introduces a full-rank MIMO channel between the source and the destination and also because it fades independently of the direct channel [2]. These relays can be used to generate independent paths between transmitter and receiver. Relays handle signals received from transmitter and retransmit the signals to receiver end.
1 Introduction

Cooperative transmission uses various relaying strategies. Generally, two main strategies have been used, i.e., amplify and forward (AF) and decode and forward (DF) [3]. Perhaps, two more relaying schemes, compress and forward (CF) and estimate and forward (EF) are also mentioned in many research.

In AF relaying scheme, AF relay terminal simply amplifies the received signal and retransmits the scaled version of signal into the destination. Whereas, in case of DF, DF relay terminal decodes the received signal and retransmits to the receiver. Therefore hardware complexity is more in DF relaying scheme. Hence, spatial diversity can be attained using this technique. However, DF strategy is preferred mainly because it reduces the effects of additive white Gaussian noise (AWGN) as compared to AF relay. While, in DF system there is the high probability of forwarding correctly detected signals to the receiver than incorrectly decoded signal depending upon the threshold value.

In cooperative communication, relay nodes system is helpful to exploit the wireless medium and is pioneer in both macro and micro diversities. The importance of using relays is to get multiple terminals equipped within a single antenna. With this relay system a virtual MIMO array with the realization of spatial diversity is achieved.

It is a well known fact that, the use of MIMO system with space time block coding is beneficial to wireless systems. Generally, the capacity of MIMO channel is bigger than single input single output (SISO) channel when transmitters and receivers set statistically independent transmission paths [4]. The capacity of MIMO channel is determined by the degree of spatial correlation. Perhaps, maximum channel capacity is possible during the absence of spatial correlation. A higher data transmission is achievable whenever STC is introduced in a MIMO channel, which exploits the spatial diversity in a MIMO channel. Besides, spatial correlation, an effect known as keyhole effect also jeopardizes the performance of MIMO channel [5, 6]. Because of the key-hole effect, the rank of MIMO channel matrix is reduced to one, even in the absence of antenna correlation. Although, keyhole channel is considered as low rank MIMO channel, it may have sufficient scattering around source and destination to receive low correlated or uncorrelated signals. Channel has low rank matrix due to propagation effects like diffraction or wave guiding. Because of this reason, the performance of MIMO channel is highly affected by the keyhole effects. In a key-hole affected MIMO channel, signals have to pass through keyhole channel before
heading toward receivers. The unique characteristic of MIMO keyhole channel is that, it has a single nonzero eigenmode. Most often this keyhole channel is regarded as the worst scenario for MIMO propagation [7, 8].

1.1.1 Literature Review

To cope with high speed data transmission, many works have been done to design a system with maximum channel capacity, less spatial correlation and better power allocation. Nowadays, wireless communication system with relays on key-hole MIMO channel have been widely researched. In paper [9], exact SER of OSTBC over spatially correlated double Rayleigh fading keyhole, MIMO channel is derived and evaluated. The most interesting thing about this research work is the use of a pre-coder at the transmitter which holds channel side information (CSI). Due to the keyhole effects, SER performance degradation and loss of diversity is seen. The SER derived in this paper is applied to MIMO key-hole channels at both the source and destination antennas with spatial correlations. Even in uncorrelated channel there is the effect of keyhole. Analyzing the simulation results achieved from SER expressions, performance improvement is possible when pre-coding is done at the transmitter side.

Furthermore, the authors extended their work to derive exact expression of SER of an key-hole affected OSTBC MIMO system with spatially correlated antennas using MGF approach. Consequently, exact SER expression is minimized by using constrained gradient decent minimization algorithm, to design minimum SER (MSER) pre-coder. Experimental analysis shows that there is an improvement of system performances with the use of pre-coding at MIMO channel input. MIMO key-hole channel consists of complex and real correlation matrix. Since, the transmitter has CSI about channel correlation matrix, pre-coding is done at transmitter, which is importantly helpful to enhance the performance of OSTBC MIMO architecture [10].

Recently, another study has been done to derive outage probability of an OSTBC MIMO relay system with spatially uncorrelated Rayleigh flat-fading channels. They have derived the expression of closed form for the outage probability of the signal to noise ratio (SNR) of DF relay. Finally, they have validated that cooperative scheme surpass non-cooperative scheme in terms of diversity and system performance [11].
Likewise, it has been shown in [12] that the more number of DF relay nodes, the better SER performance of OSTBC signal will be. But it does not mean we can increase the maximum number of relays node since relays require extra power allocation during signal transmission. In general, DF relay has the ability to decode the source message of the whole OSTBC code word as such DF relay is selected for analysis.

The diversity order of signal relay system is lesser than the diversity order of multi-relay system in OSTBC MIMO relay channel. Moreover, SER analysis of AF relay is done in the presence of CSI at receiver rather than transmitter [13]. Most recently, SER analysis of DF MIMO relay channels with OSTBC transmission is successfully done. MGF for overall SNR is derived and analyzed. Hence, asymptotic SER analysis for direct link as well as without direct link is presented by adopting OSTBC in DF MIMO relay channels and maximal ratio combining (MRC) receptions. That is how, full diversity order is achieved for direct link and without direct link. Monte Carlo simulation method is used for the validation for analytical and theoretical expressions [14].

Universal keyhole mitigation through relay deployment technique is proposed in [15]. DF relay is deployed over keyhole MIMO channel. The channel between source to destination and relay to destination is assumed to have keyhole effect. However, channel between source to relay is considered as keyhole free. Simulation results shows that, above cut-off, relay transmit power, the impact of keyhole effect, can be minimized to some extent. Moreover, cutoff value is considered as a combination function of source transmit power and channel gain matrices.

The authors in [16] have also extended their research work on the same topic but they have considered the cutoff value as independent of source transmit power and channel gain matrix, in the absence of CSI at transmitters. Perhaps, power required by relay during signal transmission is lesser than power required by the source.

More recently, a study is conducted on a MIMO AF relay with OSTBC transmission over spatially correlated keyhole channel. Expression for symbol error probability (SEP) and outage probability is derived for downlink AF relay network based on MGF approach.
of instantaneous SNR. It is shown that even if the system is affected by keyhole effects and spatial correlation, a cooperative diversity can be obtained with the use of AF relay channel under minimum number of transmit and relay antennas [17].

So far, performance analysis of AF/DF relays over MIMO channel with OSTBC transmissions is studied [11, 12, 13, 14]. Lots of studies have been done in cooperative communication for the analysis of SER. Moreover, in various systems, AF and DF strategies have been introduced in wireless communication. We have only considered DF in our work, as a work of similar type has been done in [17] using AF relay. However, to the best of our knowledge, SER analysis for DF relay over OSTBC aided MIMO with spatially correlated keyhole channels has never been accomplished till date.

1.1.2 Problem Statement

Motivation

It has been known that the use of multiple antennas at transmitter and receiver increases speed of data transmission between communication terminals. However, MIMO channel capacity is degraded by correlation and keyhole effects. But to get high data rate communication, system should have full rank MIMO channel. Moreover, different schemes have been analyzed till now to overcome these issues. Furthermore, researchers have also implemented the concept of cooperative scheme to cope with these limitations. In [10], pre-coder is introduced to minimize the effects of keyhole and correlation.

In contrast to past research, we focus on the implementation of DF relay on correlated keyhole MIMO channel. In addition, we derive the analytical expression of SER for overall system with M-PSK modulation and maximum likelihood (ML) detection. In this paper, simulation and analysis are carried out in Matlab and Mathematica, respectively. We put forward numerical results that yield the comparisons of analytical and simulated results, to verify our work.

Motivated by paper [10], we derive SER expression during first hop transmission and similarly we calculated SER during dual hop transmission based on MGF approach. Later, average SER expression of DF protocol is derived by considering the probability at which the input signal is decoded properly. The experimental result verifies that with the use
of DF relay, there is significant improvement in the system performance which has been affected by keyhole and correlation effects.

Main Issues

- We propose half duplex DF relaying scheme that take SNR in account.
- Performance analysis for different SNR regime.

Assumptions

- The MIMO Rayleigh channel is supposed to have correlation and keyhole effects.
- Each channel coefficients is treated as product of two random Gaussian variables.
- We generate exponential correlation matrix for all the available antennas.
- OSTBC is applied between S-D link and the cooperative link
- CSI at relay and destination is known.

1.1.3 Thesis Outlines

The remaining part of this paper is managed as follows

- Chapter 2: We briefly describe the background of MIMO and cooperative system related to our research.
- Chapter 3: In this chapter, we present the system and channel model and later we derive expressions for SER for overall system. Finally, we present our analytical and simulation results.
- Chapter 4: This chapter concludes our research.
2 Background

2.1 MIMO System

It has not been such a long time that the thought of point to point or point to multi-point communication has been replaced by newly developed wireless network technology. The new wireless technology allows end user to receive and process information to other nodes involved in the network, so that the performance gain of the signal is improved. Most recently, in communication field, a wireless technology with multi antennas at transmitter and receiver, which is referred to as MIMO system, is widely accepted. The main reason behind using such system is that it increases data rates, improves signal capacity, and also shapes and combines signals from multiple transmit antennas to receive antennas with the help of a 3 dimensional digital signal processing.

Multi-path fading is one of the major reasons for degrading signal quality. Multi-path fading occurs when transmitted signal from source arrives at the receiver sides with different frequencies or angles at various time instances. Hence, almost exact copy of input signal is not received at the destination point. Due to these fading effects, power variation occurs in time, space and frequency domain in the receiving section. To cope up with this problem caused by multi-path fading, power limitation and frequency scarcity concept of MIMO system was developed.

As the receiver of MIMO system consists coherent combiner, therefore combination of signals result in significant increment of SNR at receiver. Moreover, array gain leads system to withstand the effect of noise and also improves long distance transmission. Array gain increases signal to noise to interference ratio by increasing noise and interference resisting power. The main advantage of MIMO system is, same signal is transmitted from different antenna to receiver. Hence probability of getting unfaded signal at receiver is
2 Background

higher. If a MIMO system has \( n_t \) and \( n_d \) transmitter and receiver antenna respectively then the spatial diversity order becomes \( n_t \times n_d \). The variation in signal level is minimized by spatial diversity gain.

With MIMO configuration, multiple channels are created between transmitter and receiver. If these multiple channels are low correlated or independent of one another, then there is a lesser probability of channel link failure. Reliability of trans-receiver detection can be achieved if the number of antenna pairs is increased. But to get low correlated and independent channel we have to separate the antennas with some standard distances. Perhaps, antennas separation is dependent on scattering at the antennas neighborhoods and signal carrier frequencies. Although, power enhancement is possible through MIMO techniques, there arises hardware and software complexities with the use of multiple antenna terminals and 3 dimensional digital signal processing [2].

2.2 Cooperative Communication

Recently, high data rate communication system like long term evolution (LTE), ultra mobile broadband (UMB) and Wimax has gained popularity. In coming years, a wireless system called mobile broadband wireless access (MBWA) can promise up to 260 Mbps in downlink and 60 Mbps in uplink. But this high data rate communication is possible if the wireless system exhibits full rank MIMO channel. To achieve full rank MIMO system not only multiple antennas are required at transmitter and receiver, but also independent channel fading between the antenna terminals are required. However, full rank MIMO channel is not possible if the channel is highly correlated. Taking consideration of this problem, researchers need to think about new techniques that can cope up with high data rate communication requirements.

In recent years, researchers have developed a new paradigm called cooperative communication in order to get rid of the limitations of MIMO techniques. Cooperative communication is a diversity technique where the virtual MIMO channel is generated without increasing the number of antennas. Higher capacity of information rates can be achieved for relay channels, compared to direct transmission [18].
2 Background

In cooperative communication, a cooperating node acting as a relay is introduced between transmitter and receiver. The relay channel, which is just a few wavelengths away from the source, independently fade from the direct channel and hence the result is full rank MIMO channel between source and destination. Finally, at receiver, the relayed information can be combined using various techniques. Hence, spatial diversity is achieved. With this cooperative architecture, different receiver shares the antennas of MIMO configuration and therefore transmit diversity is possible through the application of virtual MIMO channel. The end to end transmission via cooperative relay is possible in two stages:

- Broadcast phase: Information is transmitted to receiver and relay via same channel.
- Relaying phase: Information received from source is retransmitted to receiver to enhance the capacity of direct communication. All relay nodes may not use the same channel for transmission thereby reducing the possibility of co-channel interference.

A general cooperative scheme is shown in Fig. 2.1

![Cooperative based system.](image)

Figure 2.1: Cooperative based system.

2.3 Relay Selection

Generally, two types of relays, i.e., AF and DF are widely used in research. Along with these two relaying strategies, CF and EF are also used, though these are not so popular [19]. AF relay amplifies the source signal and retransmits it to receiver, whereas DF relay re-encodes the received signal and retransmits it to destination. If the relayed information is not decoded properly then DF relay does not retransmit the information to the receiver. Only source to receiver transmission is possible.
2 Background

If the source transmits $s$ signals to the receiver and relays then the signal received at relay is denoted by $\hat{s}$. But the signal transmitted by relay to the destination is written as $\sqrt{P_2}\hat{s}$, where $P_2$ is transmitted power at relay. However, if incorrect signal is transmitted by relay, there is no use of decoding signal at receiver, it is ineffective. Perhaps, the diversity order of such system is only one. There is a limitation in the performance of a system in case of worst link and the worst link can be source-relay or source-destination or both in unison. If erroneous signal is transmitted to the receiver, erroneous propagation occurs, hence affects the performance of system.

Hence, fixed DF mode relay is selected because it only retransmits the properly decoded information.

2.4 Maximum Likelihood Detection

Maximum Likelihood (ML) detector is also known as an optimal detection technique, which is used at the receiver end. If a system uses ML detection at the receiver, the diversity order of the system is dependent on the receiver antennas rather than on transmit antennas. Hence, it is said that the diversity order is the same as the number of received antennas. As the number of receiver antennas increases, the signal to noise ratio also grows significantly. Therefore, ML detection can be used even if the number of transmit antennas is higher than that of receive antennas. The main property of ML detection is to minimize the error probability.

There are various ways for decoding the received signal correctly, but in our case we have considered ML detector.

For symbol detection, prior knowledge of ML metric is required and it is decoupled into the sum of $M$ terms. These $M$ terms are dependent closely on one complex symbol $s_n$, where $n = (1, \ldots, M)$. However, the detection of $s_n$ is decomposed from detection of $s_p$ where $n \neq p$ [20]. When $Z$ satisfies the equation [21, (7.4.1)], the ML metric decomposing is demonstrated as [21, (7.4.2)]

$$\| YHZ \|^2 = \| H \|^2 \cdot \sum_{n=1}^{M} s_n - \frac{ReTr\{Y^HHA_n\} - ImTr\{Y^HHB_n\}}{\| H \|^2} |^2 + constant \quad (2.1)$$
2 Background

2.5 Keyhole Effects

Hypothetically, keyhole is an effect which occurs in the wireless system, where the MIMO system efficiency degrades even for uncorrelated signals. From our study, we have found that “keyhole” effect is possible in all open air propagation. The main effect is that it degrades the rank of matrix of the MIMO channel to one, even though there is no channel correlation [7, 16]. Initially MIMO system have multiplexing gain of \( \min(n_r, n_d) \) and diversity gain \( n_r \times n_d \) but after keyhole effects system is equivalent to SISO system hence diversity gain is \( \min(n_r, n_d) \). During communication in keyhole MIMO channel, the original signal is constrained by environmental factors before reaching its destination. Generally, there are three types of keyhole in open air propagation, they are

- **Spatial keyhole**
  Radio waves propagates through single hole of metal obstacles.

- **Modal keyhole**
  It occurs when there is huge separation between transmit and receive antenna arrays e.g. hallway and subway tunnel. Hallway is regarded as a single mode waveguide in which little scattering cannot be able to boost up channel capacity.

- **Diffraction edge keyhole**
  When there is diffraction in roof edge near mobile then vertical antenna array suffers keyhole effects.
Typical keyhole effect on Rayleigh channel can be observed on Fig. 2.2., where $\nu_1, \ldots, \nu_M$ represent channel gains between transmitter and keyhole and $\Omega_1, \ldots, \Omega_M$ represent channel gains between receiver and keyhole.

**Figure 2.2:** Keyhole effect on Rayleigh channel.
3 Our Contribution

In this chapter, we present our main contribution, i.e., performance analysis of the MIMO relay system under correlation and keyhole effects. Average SER analysis of DF protocol is chosen for discussion. Finally, analytical expression of SER is verified by Monte-Carlo simulation.

3.1 System and Channel Model

We consider a half duplex DF relay based system operating over a keyhole and correlated MIMO channel as shown in Fig. 3.1. A single DF relay is introduced between source and destination. The communication between these terminals occurs in two phases. During the first phase source transmits symbol to destination and relay. Whereas in the second phase relay forwards correctly decoded signal to destination. The source, relay and destination are equipped with $n_t$, $n_r$ and $n_d$ antennas respectively.

As shown in Fig. 3.1, $H_0$, $H_1$ and $H_2$ are assumed to have poor scattering environment. Due to this reason, all these three channels are supposed to be low ranked. We also assume correlation effects caused due to less spatial separation between antennas of $n_t$, $n_r$ and

![Figure 3.1: Block diagram of MIMO DF system.](image-url)
3 Our Contribution

Here we denote $R_t$, $R_r$ and $R_d$ as the correlation matrix of transmitter, relay and receiver, respectively.

Now we briefly explain OSTBC transmission during broadcasting and relaying phases. The input signal $x = (x_1, ..., x_k)$ is encoded via OSTBC, where $x_k \in A$. $A$ is modulating signal set that has complex values. In this paper, MPSK modulation technique is considered. Consequently, the output of OSTBC encoder is assumed to be $C$, which has a $B \times N$ matrix where $B$ is the space dimension and $N$ is the time dimension. However, the elements of OSTBC codeword $C$ is formed when $x_1, ..., x_k$ and their complex conjugates combines linearly. Hence, the transmission rate is $R_1 = K/N$ at transmitter but the transmission rate at relay is $R_2$, where $K$ is the information bits. Then after, the codeword $C$ is transmitted through correlated keyhole MIMO channel.

As we suppose signal transmission is possible in two time slots, during the first time slot the received signals at the destination and the relay is given by, respectively

$$Y_0 = H_0 x_k + Z_0$$  \hspace{1cm} (3.1)

$$Y_1 = H_1 x_k + Z_1$$  \hspace{1cm} (3.2)

where $Z_0$ and $Z_1$ complex iid Gaussian random variable with variance $N_0/2$. Due to the keyhole effects each channels are outer product of complex Gaussian vectors, i.e., $H_0 = h_{t0} h_{r0}$, and $H_1 = h_{t1} h_{r1}$ where $h_{t0} = h_{tw0} \sqrt{R_t}$, $h_{r0} = h_{rw0} \sqrt{R_d}$, $h_{t1} = h_{tw1} \sqrt{R_t}$ and $h_{r1} = h_{rw1} \sqrt{R_r}$ in which $h_{tw0} \sim \mathcal{CN}(0, n_t I_{n_t})$, $h_{tw1} \sim \mathcal{CN}(0, n_t I_{n_t})$, $h_{rw0} \sim \mathcal{CN}(0, n_d I_{n_d})$, and $h_{rw1} \sim \mathcal{CN}(0, n_r I_{n_r})$ are iid complex Gaussian vectors of sizes $1 \times n_t, 1 \times n_t, n_d \times 1$ and $n_r \times 1$, respectively.

During the second time slot, the relay forwards correctly decoded signal to the destination, the received signal $Y_2$ is given by

$$Y_2 = H_2 x_k + Z_2$$  \hspace{1cm} (3.3)

where $Z_2$ is complex iid Gaussian variables with variance $N_0/2$. Since $H_2$ is a keyhole channel, $H_2 = h_{t2} h_{r2}$ where $h_{t2} = h_{tw2} \sqrt{R_r}$ and $h_{r2} = h_{rw2} \sqrt{R_d}$ in which $h_{tw2} \sim \mathcal{CN}(0, n_r I_{n_r})$ and $h_{rw2} \sim \mathcal{CN}(0, n_d I_{n_d})$ are iid complex Gaussian vectors of sizes $1 \times n_r$ and $n_d \times 1$, respectively.
Finally, the OSTBC signal at the relay and destination is decoded by ML detector and both the relay and receiver have CSI.

During the first hop transmission, source transmit symbols to the destination and the relay. Hence, SNR at relay and destination is

$$\gamma_r = \eta_1 \| H_1 \|^2$$

$$\gamma_0 = \eta_0 \| H_0 \|^2$$

During the second hop transmission, when message is correctly decoded, maximum ratio combining combines the signal that arrives from the source and the relay. Hence, total instantaneous SNR at destination is expressed as in [11, (1)]

$$\gamma = \eta_0 \| H_0 \|^2 + B_r \eta_2 \| H_2 \|^2$$

where $B_r$ is state of relay. Here, the threshold value of SNR is $\gamma_{th}$

$$B_r = \begin{cases} 
0 & \text{if } \gamma_r < \gamma_{th} \\
1 & \text{if } \gamma_r \geq \gamma_{th}
\end{cases}$$

where $\eta_0 = \bar{\gamma}/(n_t \times R_1)$ and $\eta_2 = \bar{\gamma}/(n_r \times R_2)$. $\bar{\gamma} = E_s/N_0$ is average SNR.

### 3.2 Symbol Error Rate Analysis of Correlated Keyhole MIMO DF Systems

The main objective of this paper is to derive average SER of DF protocol depending upon the propagation of correctly decoded or incorrectly decoded signal, i.e., $B_r = 1$ and $B_r = 0$. For this we need the average SER during the first hop and the second hop transmission. At last SER of DF protocol is calculated depending upon the probability of decoding the signal.


### 3 Our Contribution

#### 3.2.1 Symbol Error Rate Analysis during the First Hop Transmission

In this section, we use MGF based approach [22] to derive SER expressions of MPSK modulation technique for DF relaying protocol. Since the channel suffers from keyhole effects, there exist two MGF \( \phi_{y_t}(s) \) and \( \phi_{y_r}(s) \) between transmitter and keyhole and keyhole and receiver, respectively. To find out the MGF of right side of keyhole and left side of keyhole, the signal \( y_0 \) at keyhole and signal \( y_{r0} \) at receiver is required. The signal \( y_0 \) can be shown as [10, (7)]

\[
y_0 \triangleq \| h_{t0} \|^2 = \sum_{i=1}^{n_t} \lambda_{ti} \| \{ \hat{h}_{tw0} \}_i \|^2
\]  

(3.8)

where \( \lambda_{ti}, i = 1, \ldots, n_t \) are eigen values of transmitter correlation matrix \( R_t \) and \( \hat{h}_{tw0} \) be the iid Gaussian vector of size \( 1 \times n_t \).

Similarly, we obtain \( y_{r0} \) as equation (3.8)

\[
y_{r0} \triangleq \| h_{r0} \|^2 = \sum_{j=1}^{n_d} \lambda_{rj} \| \{ \hat{h}_{rw0} \}_j \|^2
\]  

(3.9)

where \( \lambda_{rj}, j = 1, \ldots, n_d \) be the eigenvalues of \( R_d \) and \( \hat{h}_{rw0} \) be the iid Gaussian vector of size \( n_d \times 1 \). We suppose \( \{ \hat{h}_{tw0} \}_i \) and \( \{ \hat{h}_{rw0} \}_j \) are ith and jth elements of \( \hat{h}_{tw0} \) and \( \hat{h}_{rw0} \), respectively.

The MGF of \( y_0 \) and \( y_{r0} \) can be calculated as [23, (14)]

\[
\phi_{y_0}(s) = \prod_{i=1}^{n_t} \frac{1}{1 + \lambda_{ti}s}
\]  

(3.10)

In similar way \( \phi_{y_{r0}}(s) \) can be calculated as equation (3.10).

Expanding equation (3.10), we get the following partial fraction.[10, (10)]

\[
\phi_{y_0}(s) = \sum_{i=1}^{n_t} \frac{\alpha_i}{1 + \lambda_{ti}s}
\]  

(3.11)

in which \( \alpha_i \) is set coefficients of \( \phi_{y_0}(s) \).

By taking inverse Laplace transform of \( \phi_{y_{r0}}(s) \), pdf of \( y_{r0} \) is expressed as

\[
f_{y_{r0}}(y_{r0}) = \sum_{j=1}^{n_d} \frac{\beta_j}{\lambda_{rj}} \exp \left( \frac{-y_{r0}}{\lambda_{rj}} \right)
\]  

(3.12)
3 Our Contribution

where \( \beta_j \) is set of expansion coefficients of \( \phi_{y_0}(s) \).

MGF of output SNR, when \( B_r = 0 \) is given by

\[
\phi_{SD}(s) = \int_{0}^{\infty} \phi_{y_0}(sy_0)f_{y_0}(y_0)dy_0
\]

(3.13)

where \( Y_0 = y_0y_r \). \( y_0 \) and \( y_r \) are independent to each other [24].

Hence, substituting values of \( \phi_{y_0}(s) \) and \( f_{y_0} \) from equation (3.11) and equation (3.12) in equation (3.13), expression for MGF of source to destination is given by [10, (15)]

\[
\phi_{SD}(s) = \sum_{i=1}^{n_t} \sum_{j=1}^{n_d} \alpha_i \beta_j \lambda_{ij} \int_{0}^{\infty} \frac{\exp \left( \frac{-y_0}{\lambda_{ij}s} \right)}{1 + \lambda_{ti}sy_0} dy_0
\]

(3.14)

\[
\phi_{SD}(s) = -\sum_{i=1}^{n_t} \sum_{j=1}^{n_d} \frac{\alpha_i \beta_j}{\lambda_{ij}s} \exp \left( \frac{1}{\lambda_{ij}s} \right) \Xi_i \left( \frac{1}{-\lambda_{ij}s} \right)
\]

(3.15)

where \( \lambda_{ij} = \lambda_{ti}\lambda_{rj} \).

The exponential integral \( \Xi_i(t) \) in equation (3.15) is expressed as [25, (8.211)]

where

\[
\Xi_i(t) = -\int_{-t}^{\infty} \frac{\exp(-x)}{x} dx, \quad (t < 0)
\]

(3.16)

By using MGF approach average symbol error rate of MPSK modulation with instantaneous SNR \( \eta_0 \) is written as in [22, (8.23)]. Then, the conditional SER is

\[
P_{MPSK1} = 1 \pi \left( \frac{M-1}{M} \right) \int_{0}^{\pi} \exp \left( -\gamma_0 \frac{g_{MPSK}}{\sin^2 \theta} \right) d\theta
\]

(3.17)

Now, the average \( SER_1 \) of MPSK signalling is denoted as

\[
P_{MPSK1} = 1 \int_{0}^{\pi} \Phi_{YSD} \left( \eta_0 \frac{g_{MPSK}}{\sin^2 \theta} \right) d\theta
\]

(3.18)
3 Our Contribution

After putting value of $s = \eta_0 g_{\text{MPSK}} \sin^2 \theta$ in equation (3.15), we obtain

$$P_{\text{MPSK}1} = -\frac{1}{\pi} \sum_{i=1}^{n_t} \sum_{j=1}^{n_d} \frac{\alpha_i \beta_j}{\lambda_{ij} \eta_0 g_{\text{MPSK}}} \left(\frac{(M-1)\pi}{M}\right) \int_0^{\sin^2 \theta} \exp\left(\frac{\sin^2 \theta}{\lambda_{ij} \eta_0 g_{\text{MPSK}}}\right) \Xi_i \left(\frac{-\sin^2 \theta}{\lambda_{ij} \eta_0 g_{\text{MPSK}}}\right)$$

(3.19)

where $\zeta_1 = \eta_0 g_{\text{MPSK}}$, $P_{ij}(\theta_1, \theta_2, \zeta_1)$ can be written as in paper [10, (19)].

For simplicity, we suppose

$$P_{ij} (\theta_1, \theta_2, \zeta_1) \triangleq -\int_{\theta_1}^{\theta_2} \sin^2 \theta \exp\left(\frac{\sin^2 \theta}{\lambda_{ij} \zeta_1}\right) \Xi_i \left(\frac{-\sin^2 \theta}{\lambda_{ij} \zeta_1}\right)$$

(3.21)

3.2.2 Symbol Error Rate Analysis during the Dual Hop Transmission

In this section, we derive average SER during the dual hop transmission. To derive SER, we need MGF of output SNR $\gamma$, when $B_r = 1$. During the second hop transmission, when relay transmits message to the destination then instantaneous SNR at the destination is

$$\gamma_2 = \eta_2 \|H_2\|^2$$

(3.22)

The MGF of R-D link is derived as MGF of S-D link because both link have correlation and keyhole effects.

When relay forwards correctly decoded signal $Y_2$ to the destination, the MGF of the received signal $Y_2$ is derived as [10, (16)]

$$\phi_{RD}(s) = -\sum_{m=1}^{n_r} \sum_{n=1}^{n_d} \alpha_m \beta_n \exp\left(\frac{1}{\lambda_{mn} s}\right) \Xi_i \left(\frac{-1}{\lambda_{mn} s}\right)$$

(3.23)

where $\lambda_{mn} = \lambda_{tm} \lambda_{rn}$, $m = 1, ..., n_r$ and $\lambda_{rn}$, $n = 1, ..., n_d$ are eigenvalues of $R_r$ and $R_d$, respectively. The values of $\alpha_m$ and $\beta_n$ are derived as in [10, (11)].

Hence, the total MGF is the product of MGF of S-D link and MGF of R-D link, when $B_r = 1$ is expressed as

$$\phi(s) = \phi_{SD}(s) \phi_{RD}(s)$$

(3.24)
After substituting expressions of $\phi_{SD}(s)$ and $\phi_{RD}(s)$ from equation (3.15) and (3.23), we obtain

$$\phi(s) = \sum_{i=1}^{n_t} \sum_{j=1}^{n_d} \sum_{m=1}^{n_r} \sum_{n=1}^{n_d} \alpha_i \beta_j \alpha_m \beta_n \frac{1}{\lambda_{ij} \lambda_{mn} s^2} \exp \left( \frac{1}{\lambda_{ij} s} + \frac{1}{\lambda_{mn} s} \right) \Xi_i \left( -\frac{1}{\lambda_{ij} s} \right) \Xi_i \left( -\frac{1}{\lambda_{mn} s} \right)$$  

(3.25)

The average SER of MPSK signalling after input message is correctly decoded at relay [19, page 33 (14)]

$$P_{MPSK2} = \frac{1}{\pi} \int_0^{(\frac{M-1}{2})\pi} \Phi_{SD} \left( \frac{\eta_0 g_{MPSK}}{\sin^2 \theta} \right) \Phi_{RD} \left( \frac{\eta_2 g_{MPSK}}{\sin^2 \theta} \right) d\theta$$  

(3.26)

After substituting value of $s = \eta_0 g_{MPSK}$ and $s = \eta_2 g_{MPSK}$ for $\phi_{SD}(s)$ and $\phi_{RD}(s)$, respectively in equation (3.25), we get

$$\phi(s) = \sum_{i=1}^{n_t} \sum_{j=1}^{n_d} \sum_{m=1}^{n_r} \sum_{n=1}^{n_d} \alpha_i \beta_j \alpha_m \beta_n \frac{1}{\lambda_{ij} \lambda_{mn} \eta_0 \eta_2 g_{MPSK}^2} \sin^4 \theta \exp \left( \frac{\sin^2 \theta}{\lambda_{ij} \eta_0 g_{MPSK}} + \frac{\sin^2 \theta}{\lambda_{mn} \eta_2 g_{MPSK}} \right) \Xi_i \left( -\sin^2 \theta \right) \Xi_i \left( -\sin^2 \theta \right)$$  

(3.27)

Then,

$$P_{MPSK2} = \frac{1}{\pi} \sum_{i=1}^{n_t} \sum_{j=1}^{n_d} \sum_{m=1}^{n_r} \sum_{n=1}^{n_d} \alpha_i \beta_j \alpha_m \beta_n \frac{1}{\lambda_{ij} \lambda_{mn} \eta_1 \eta_2 g_{MPSK}^2} \int_0^{(\frac{M-1}{2})\pi} \sin^4 \theta \exp \left( \frac{\sin^2 \theta}{\lambda_{ij} \eta_1} + \frac{\sin^2 \theta}{\lambda_{mn} \eta_2} \right) \Xi_i \left( -\sin^2 \theta \right) \Xi_i \left( -\sin^2 \theta \right)$$  

(3.28)

where $\zeta_2 = \eta_2 g_{MPSK}$.

For simplicity, we suppose

$$U_{ijmn} (\theta_1, \theta_2, \zeta_1, \zeta_2) \triangleq \int_{\theta_1}^{\theta_2} \frac{\sin^4 \theta}{\lambda_{ij} \lambda_{mn} \zeta_1 \zeta_2} \exp \left( \frac{\sin^2 \theta}{\lambda_{ij} \zeta_1} + \frac{\sin^2 \theta}{\lambda_{mn} \zeta_2} \right) \Xi_i \left( -\sin^2 \theta \right) \Xi_i \left( -\sin^2 \theta \right)$$  

(3.29)
3 Our Contribution

Hence,

\[ P_{MPSK2} = \frac{1}{\pi} \sum_{i=1}^{n_t} \sum_{j=1}^{n_d} \sum_{m=1}^{n_r} \sum_{n=1}^{n_d} \alpha_i \beta_j \alpha_m \beta_n U_{ijmn} \left( 0, \left( \frac{M-1}{M} \right) \pi, \zeta_1, \zeta_2 \right) \]  

(3.30)

3.2.3 Average Symbol Error Rate of Decode and Forward Protocol

Under this section, we derive average SER of DF protocol. For this we need the value of \( P_c \). Here \( P_c \) is the probability at which relay decodes the input signal correctly. During the first time slot the received signal at input of relay is denoted as \( Y_1 = y_{t1} y_{r1} \), where \( y_{t1} \) and \( y_{r1} \) are independent variables. Now the probability of decoding signal by relay can be expressed as

\[ P_c = P_r \left\{ y_1 < \frac{\gamma_{\text{SNR}} R}{\pi} \right\} \]  

(3.31)

where \( SNR = \frac{\pi}{\pi} y_1 \), \( SNR < \gamma_{\text{th}} \) and \( P_r \{ \} \) probability operator.

To get the value of \( P_c \), we further derive expression for cumulative density function (CDF) of source to relay link. Then, CDF of \( Y_1 \) is

\[ F_{Y_1} (y_1) = E_{y_1} \left\{ F_{y_{t1}} < \frac{y_1}{y_{r1}} \right\} \]  

(3.32)

where \( E_{y_1} \{ \} \) is expected value.

The cdf \( F_{y_{t1}} \), i.e., cdf of source to keyhole is given by

\[ F_{y_{t1}} (y_{t1}) = \alpha_k \left( 1 - \exp \left( \frac{-y_{t1}}{y_{t1} \lambda_{tk}} \right) \right) \]  

(3.33)

The pdf \( f_{y_{r1}} \), i.e., pdf of keyhole to relay is calculated as [10, (12)]

\[ f_{y_{r1}} (y_{r1}) = \sum_{l=1}^{n_r} \beta_l \frac{1}{\lambda_{rl}} \exp \left( \frac{-y_{r1}}{\lambda_{rl}} \right) \]  

(3.34)

Now, from (3.33) and (3.34), it follows that the CDF of \( Y_1 \) is

\[ F_{Y_1} (y_1) = \int_{0}^{\infty} F_{y_{t1}} (y_{t1}) f_{y_{r1}} (y_{r1}) dy_{r1} \]  

(3.35)

\[ F_{Y_1} (y_1) = \sum_{k=1}^{n_t} \alpha_k - \sum_{k=1}^{n_t} \alpha_k \int_{0}^{\infty} \exp \left( \frac{-y_{1}}{y_{r1} \lambda_{tk}} \right) \sum_{l=1}^{n_r} \beta_l \frac{1}{\lambda_{rl}} \exp \left( \frac{-y_{r1}}{\lambda_{rl}} \right) dy_{r1} \]  

(3.36)
3 Our Contribution

After substituting value of \( y_1 = \frac{n_t n_t R}{\lambda_t} \) from equation (3.31) in equation (3.36), we get

\[
F_{Y_1} \left( \frac{n_t n_t R}{\lambda_t} \right) = 1 - \sum_{k=1}^{n_t} \sum_{l=1}^{n_r} 2 \alpha_k \beta_l \sqrt{\frac{n_t n_t R}{\lambda_t \lambda_t \lambda_t \lambda_t}} K_1 \left( 2 \sqrt{\frac{n_t n_t R}{\lambda_t \lambda_t \lambda_t \lambda_t}} \right)
\] (3.37)

where \( K_v \{ \} \) is modified Bessel function of second kind of order first [25, page 368 (9)].

The value of \( P_c \) is calculated in software package called Mathematica.

Finally, the average SER of DF protocol which is associated with \( P_{MPSK1} \{B_r = 0\} \) and \( P_{MPSK2} \{B_r = 1\} \) is given by

\[
P_{MPSK} = P_c P_{MPSK1} + (1 - P_c) P_{MPSK2}
\] (3.38)

3.3 Simulation Results and Analysis

Under this section, numerical results are presented in order to validate our analysis of SER expression derived in the previous section. Since we are considering correlated channel, exponential correlation model is selected [26]. For analysis and simulation software packages called Mathematica and Matlab is used, respectively. The correlation matrix \( R_t, R_r \) and \( R_d \) is calculated as \( \{ R_t \} = r_t^{u,v} \), \( \{ R_r \} = r_r^{u,v} \) and \( \{ R_d \} = r_d^{u,v} \) where \( u, v = 1, \ldots, n_d \).

The value of correlation coefficients are assumed to be \( r = 0.3, 0.6, 0.9 \) for Fig. 3.3, Fig. 3.4, Fig. 3.5, Fig. 3.6, Fig. 3.7 but \( r = 0.5 \) is chosen for rest of all figures. Depending on low correlated or highly correlated case, correlation coefficient can be varied from 0 to 1. Since, we have used MPSK modulation techniques, value of \( M = 4, 8, 16 \) is selected for Fig. 3.8, Fig. 3.9, Fig. 3.10, Fig. 3.11 and Fig. 3.12. Whereas, value of \( M \) is 8 for remaining figures.

In this paper, transmit, relay and destination antennas are also varied to see the impact of antenna variation on SER under keyhole and correlation effects. For \( 2 \times 2 \times 2 \) and \( n_t \times 2 \times 2 \) Alamouti code is used [27]. Whereas, for \( 4 \times n_r \times 2, 4 \times 2 \times n_d \) and \( n_t \times 4 \times 2 \) OSTBC MIMO system \( C_4 \) of [28] is chosen. We have also generated SER curves for sys-
3 Our Contribution

tems with $4 \times n_r \times 4$ and $4 \times 4 \times n_d$ as $g_4$, in paper [29].

The threshold value of $SNR = 3dB$ is selected. If maximum threshold value is chosen then there is lesser probability of error propagation to the destination. It has been shown that if $SNR$ of relay is greater than threshold value the relay decodes correctly, otherwise transmission from the relay is not possible.

3.3.1 SER of DF protocol

![Graph of SER versus SNR comparison of cooperative and noncooperative system for (2,2,2) MIMO DF system for 8PSK modulation.](image)

**Figure 3.2:** SER versus SNR comparison of cooperative and noncooperative system for (2,2,2) MIMO DF system for 8PSK modulation.

Fig. 3.2 shows SER curve of a MIMO system and MIMO relay system. As can be seen from the plot, in high SNR regime cooperative scheme improves SER performance compared to noncooperative scheme. Hence co-operative scheme outperforms noncooperative scheme under correlation and keyhole effects.
3.3.2 Simulations on SER for Different Values of Correlation Coefficients

The effect of correlation coefficients variation on different antenna combination is demonstrated by Fig. 3.3, Fig. 3.4, Fig. 3.5. In these figures SER curves of MIMO DF relay channel with keyhole effects are considered. The SER of DF system when \( r = 0.3 \) is less compared to \( r = 0.6 \) and \( r = 0.9 \). It is clear from these figures that as the value of correlation coefficients increases, performance of system degrades. However, the effects of correlation is minimized to some extent by increasing number of transmit antennas. As clearly seen from figures, there is an exact match between our analytical and simulation results. This clearly validates our simulation results with analytical one.
3 Our Contribution

**Figure 3.4:** SER of $(2, 4, 2)$ $8 - PSK$ modulation over MIMO DF relay keyhole channel versus SNR for several values of correlation coefficients.

**Figure 3.5:** SER of $(4, 2, 2)$ $8 - PSK$ modulation over MIMO DF relay keyhole channel versus SNR for several values of correlation coefficients.
3 Our Contribution

Figure 3.6: SER of (4, 2, 4) 8-PSK modulation over MIMO DF relay keyhole channel versus SNR for several values of correlation coefficients.

It is evident from Fig. 3.6 that as the number of transmit and receive antennas increases, the effect of correlation and keyhole is lesser than other antenna pairs. At almost SER 10^{-7} there is about 8 dB power loss, when value of r varies from 0.3 to 0.9. Furthermore, Fig. 3.6 also shows improvement of SER curve decreases as the value of r decreases. This can be justified from the gap of SER curve, where SER improvement from r : 0.9 to 0.6 is more than r : 0.6 to 0.3.
Our Contribution

Figure 3.7: SER of (4, 4, 2) 8-PSK modulation over MIMO DF relay keyhole channel versus SNR for several values of correlation coefficients.

Fig. 3.7 shows SER versus SNR plot of (4, 4, 2) system under 8-PSK modulation and keyhole effects. This figure is plotted to see the impact on DF system performance under different values of correlation coefficients. It can be observed from the figure, at SER $10^{-6}$ approximately $5dB$ gain is obtained, when $r = 0.3$ compared to $r = 0.9$. 
3 Our Contribution

3.3.3 Simulation on SER for Different Modulation Techniques

Figure 3.8: SER of (2, 2, 2)-OSTBC MIMO DF relay systems with keyhole and correlation effects versus SNR for M=2,4,8.

Fig. 3.8, Fig. 3.9, Fig. 3.10, Fig. 3.11 and Fig. 3.12 illustrate the SER performance of OSTBC MIMO DF relay system with keyhole and spatial correlation effects. As mentioned earlier different antenna combinations are used but correlation coefficient is fixed, i.e., $r = 0.5$. All figures shows that when value of $M$ is increased SNR is also increased. Hence, system capacity decreases as value of $M$ increases. However, as value of $M$ decreases bandwidth requirement becomes higher but power requirement becomes lesser. Moreover, system capacity is pronounced more in Fig. 3.11 as SER curve is lowered when the number of transmit antennas increases.
Figure 3.9: SER of (2, 4, 2)-OSTBC MIMO DF relay systems with keyhole and correlation effects versus SNR for M=2,4,8.

Figure 3.10: SER of (4, 2, 2)-OSTBC MIMO DF relay systems with keyhole and correlation effects versus SNR for M=2,4,8.
3 Our Contribution

![Graph](image)

Figure 3.11: SER of (4, 2, 4)-OSTBC MIMO DF relay systems with keyhole and correlation effects versus SNR for M=2,4,8.

![Graph](image)

Figure 3.12: SER of (4, 4, 2)-OSTBC MIMO DF relay systems with keyhole and correlation effects versus SNR for M=2,4,8.
3 Our Contribution

3.3.4 Simulation on SER for Arbitrary Number of Antennas

In Fig. 3.13, we plotted SER versus SNR curve under 8–PSK modulation and keyhole effects. To see the effects on system performance, we varied antennas at transmitter, relay and receiver with correlation coefficient $r = 0.5$. It can be seen from the figure, $4 \times 4 \times 4$ MIMO DF system has better SER performance than $3 \times 3 \times 3$ and $2 \times 2 \times 2$ MIMO DF system. As can be seen by SER plots, degradation on performance of system caused by correlation effects can be reduced by increasing $n_t$, $n_r$ and $n_d$ antennas. Although there arises a question about the maximum number of antennas that can be deployed, but there is always tradeoff between number of antennas and performance of system.
3 Our Contribution

3.3.5 Simulation on SER with Comparison of Keyhole and No Keyhole System

![Graph showing SER comparison]

**Figure 3.14:** SER of (4,2,2)-OSTBC MIMO DF relay systems with keyhole and (4,2,2) Rayleigh channel for 8-PSK modulation versus SNR with correlation effects.

As a benchmark, we have also demonstrated SER versus SNR comparison of keyhole and no keyhole MIMO relay system in Fig. 3.14, under correlation coefficients $r = 0.5$. As clearly seen from figure at SER $10^{-7}$ approximately $11dB$ gain is obtained for no keyhole system compared to keyhole system. At high SNR regime $4 \times 2 \times 2$ antenna combination, no keyhole system has better performance in terms of SER.
4 Conclusion and Future Work

4.1 Conclusion

In this paper, we have analyzed the performance of DF relay with keyhole and correlation effects in MIMO relay channel. During signal transmission we selected the $3dB$ threshold level for decoding the signal. Here, we determined the value of $P_c$, which is the probability at which the relay decodes the signal correctly. Moreover, based on MGF approach we determine the closed form expression of average SER of the MIMO DF system under key-hole and correlation effects. We also illustrated the performance analysis of OSTBC for arbitrary number of antennas with MPSK modulation technique to verify our analytical expression.

Based on our analysis, it is evident that cooperative scheme with keyhole channel out-performs the keyhole affected non cooperative scheme. Even MIMO relay system with Rayleigh channel shows better SER performance than MIMO relay system with double Rayleigh channel. Though the keyhole effect cannot be avoided completely, but with this cooperative scheme, effect of it on system performance is minimized to a certain level.

From the observation of simulation results, we noticed that effect of correlation on system performance is minimized to some extent by increasing the number of antennas at transmitter, relay and receiver. Based on our analysis, we also concluded that 4-PSK system has better SER performance than 8-PSK and 16-PSK systems for different SNR regime. As clearly seen from plotted figures, there is an exact match between our analytical and simulation results. So without any ambiguity this validates our simulation results with analytical one.
4 Conclusion and Future Work

4.2 Future Work

In this paper performance analysis has been done only for single DF relay. In future, we can carry out this work in using multiple relays.

We have done performance analysis using only one performance parameter. Concurrently, performance analysis can also be done using another performance parameter, i.e., outage probability.

It is a well known fact that the use of cooperative system enhances the system performance. However, in paper [10] the use of pre-coder at transmitter reduces the SER significantly. It will be interesting to observe the implementation of “pre-coder at transmitter” and “with relay over correlated keyhole channel” in future.
Bibliography


35
Bibliography


