Simulation a disposable mass flow meter by an advanced FSI modeling and finite element analysis

Siavash Hooshmand zadeh

Department of Mechanical Engineering
Blekinge Institute of Technology
Karlskrona, Sweden
2015

Supervisors: Lena Klintberg, Ångströmlaboratoriet, Uppsala University
Hanno Ehring, GE-Healthcare, Uppsala
Ansel Berghuvud, BTH
Simulation a Disposable mass flow meter by an advanced FSI Modeling and Finite Element Analysis.

Siavash Hooshmand zadeh

Supervisors:
Dr. Ansel Berghuvud

Department of Mechanical Engineering
Blekinge Institute of Technology
Karlskrona, Sweden

Dr. Lena Klintberg
Microsystem Technology Division
Ångströmlabotariet, Uppsala University
Uppsala, Sweden

Abstract - In this thesis a design of a Coriolis mass flow-meter is chosen by considering all advantages and disadvantages and the project requirements. The chosen geometry is imported into COMSOL, because modelling is implemented by FEM and two different physics should be coupled. To consider both applications of the device include measuring density and flow rate, modelling is divided into two parts: Coriolis density meter and Coriolis mass flow-meter. Both applications are based on Fluid Structure Interaction. The results are compared to existing setup's experimental data.

Keywords: Coriolis mass flowmeter, Fluid Structure Interaction, Modal order, Eigenfrequency, FEM, Acoustic Shell Interaction, Disposable, Polymer.
Acknowledgements

This work was carried out at Microsystem Technology Division, Ångström Laboratorium, Uppsala University, under the supervision of Dr. Lena Klintberg (Uppsala University) and Dr. Ansel Berghuvud (Blekinge Institute of Technology) and Mr. Hanno Ehring (GE-Healthcare). The work is a part of a research project, which is a cooperation between the Microsystem Technology Division, Uppsala University, Department of Mechanical Engineering, Blekinge Institute of Technology and GE-Healthcare. The duration of this project is between, February 2014 to September 2014.

I wish to express my sincere gratitude to Dr. Lena Klintberg, Dr. Ansel Berghuvud and Mr. Hanno Ehring for their guidance and professional engagement throughout the work. Also, I thank Prof. Klas Hjort for the complete guidance and support to accomplish the project.

I would like to thank GE-Healthcare for giving me this wonderful opportunity to work on a truly challenging and multidiscipline thesis and providing me with a great work and living environment.

I would like to thank Dr. Linus Fegerberg (Lightweight Structures) for giving me some insight into Acoustic Shell Interaction and Modal Analysis.

I would like to thank Mikael Fredenberg for helping me in modeling and simulation with COMSOL Multi-physics.

Last but not least I would like to thank all my wife Zarrin, for her kindest supports within the study time.

Uppsala, October 2014

Siavash Hooshmand zadeh.
Table of content

1. Introduction .................................................................................................................. Error! Bookmark not defined.
2. Measuring Principles and previous works ................................................................. 8
   2.1 Available commercial designs. .................................................................................. 10
   2.2 Categorizing different CMF types ........................................................................... 11
      2.2.1 Single Straight-tube ............................................................................................ 11
      2.2.2 Twin Straight-tubes.................................................................................................. 13
   2.3 Curved tube Coriolis mass flow-meters ................................................................... 16
3. Modelling ...................................................................................................................... 19
   3.1 Strategy of the Modelling ......................................................................................... 19
   3.2 Initial Assumption through the Model ..................................................................... 20
   3.3 Design and Geometry ............................................................................................... 22
      3.3.1 Optimized Thickness ........................................................................................... 22
      3.3.2 Analytical calculation ......................................................................................... 23
      3.3.3 Time domain ....................................................................................................... 26
      3.3.4 Geometry ............................................................................................................ 27
   3.4 Implementation in COMSOL ...................................................................................... 28
      3.4.1 Physics ................................................................................................................. 29
      3.4.2 Materials .............................................................................................................. 29
      3.4.3 Mesh ..................................................................................................................... 30
      3.4.4 Eigenfrequency Study ......................................................................................... 31
      3.4.5 Boundary conditions ............................................................................................ 32
   3.5 Order of modes ......................................................................................................... 32
4. Simulation and Results ................................................................................................. 34
   4.2 Condition with flow .................................................................................................. 37
      4.2.1 Solution: ............................................................................................................... 37
      4.2.2 Comparison the results ....................................................................................... 39
   4.3 Simulation by replacing steel tubes to polymers .................................................... 40
      4.3.1 Simulation a disposable flowmeter ..................................................................... 43
5. Conclusion, proposals and future works ..................................................................... 45
   5.1 Comparison .............................................................................................................. 45
   5.2 Proposals and future works ..................................................................................... 45
6. References .................................................................................................................... 48
Abbreviations:

\( a \)  typical radius of curved tube  
\( d \)  typical diameter of curved tube  
\( l \)  straight tube length  
\( b \)  active straight length  
\( r_i \)  flow-meter tube (inner) radius  
\( r_o \)  flow-meter tube (outer) radius  
\( c \)  velocity of sound in fluid  
\( c_1 \)  a coefficient and constant value  
\( f \)  frequency  
\( f_i \)  resonance frequency of transverse fluid vibrations in tube  
\( f_1 \)  frequency of operation of a Coriolis meter  
\( m_f \)  mass of fluid  
\( \delta m \)  small fluid section  
\( m_t \)  mass of tube  
\( n \)  mode number of tube vibration  
\( p \)  pressure; amplitude of pressure fluctuations in fluid during tube vibration  
\( s \)  typical bubble separation distance  
\( t \)  time and wall thickness  
\( u \)  local displacement of tube  
\( V \)  vector velocity of fluid in vibrational motion  
\( U \)  mean fluid's velocity along the tube  
\( l \)  tube's length  
\( d \)  width of U-shaped tube  
\( x \)  displacement of fluid  
\( \rho \)  density; amplitude of density fluctuations in fluid during tube vibration  
\( \Theta \)  twist angle  
\( \Delta \)  internal cross section area of the tube  
\( \omega \)  angular frequency \((=2\pi f)\)  
\( \omega_n \)  fundamental frequency  
\( \omega_{nt} \)  torsional natural frequency/ tortional frequency  
\( \lambda \)  wave-length of sound  
\( \Omega \)  local angular velocity of tube  
\( F_C \)  Coriolis force  
\( a_C \)  Coriolis acceleration  
\( Q_{mA} \)  mass flow rate  
\( \Delta t \)  time lag  
\( \Delta \phi \)  phase shift  
\( K_s \)  shape dependent factor  
\( k \)  temperature dependent stiffness of the tube  
\( K \)  combined stiffness coefficient  
\( I_s \)  the inertia of the U-shaped tube  
\( J \)  the polar inertia of the tube  
\( T_C \)  twisting moment  
\( \text{CMF} \)  Coriolis mass flowmeter in this particular case
The aim of the project is modeling a mass flow meter which is able to investigate how the results are affected by changing material in the sensor. The project requirement consists, 5% accuracy, ability to work in temperature between 0 – 40°C and pressure between 0 – 10 bar and verification the measurement results is desired.

1. Introduction

The Coriolis mass flow meter is a device that measures mass flow rate of a fluid traveling through a tube. It consists of one or several tubes which contains fluid passing through the device. The main characteristic of Coriolis mass flow meters (CMF) is that they measure the mass per unit time (e.g. \( \frac{kg}{s} \)) instead of the volume per unit time (e.g. \( \frac{m^3}{s} \) or \( \frac{Lit}{s} \)). The CMF’s working principle is fundamentally based on fluid-conveying pipes driven at resonance. This structure is vibrating by resonant excitation in symmetric mode when no flow pass the tube. The actual drive frequency (symbolized by \( \omega_n \)) depends on the structure characteristics such as size and geometry of the flow meter as well as material, density and masses of both the CMF’s tube structure and the fluid it contains [2].

When no fluid is flowing, the vibration of the two tubes is symmetrical but in case of a fluid flowing through the pipe, Coriolis forces are developed and cause some twisting of the tubes. The fluid mass \( m_f \) is flowing at a constant velocity \( V \) through a vibration pipe. The fluid mass will experience an angular momentum \( \Omega \) due to the resonance pipe vibration of the overall symmetric mode [2].

According to the Coriolis effect and the “right-hand rule”, Coriolis force \( F_C \) will appear in two opposite directions in case of U-tube shapes and single direction in straight-tube designs. Therefore the arm through which fluid flows away from the axis of rotation must exert a force \( F_C \) on the fluid to increase its angular momentum, so it is lagging behind the overall vibration, on the other hand, the arm through which fluid is pushed back towards the axis of rotation must exert a force on the fluid to decrease the fluid’s angular momentum again, hence that arm leads the overall vibration. Coriolis forces \( F_C \) are:

\[
F_C = -2 \ m_f \ \Omega \times V \quad (1.1)
\]

The bold parameters are vectors, this means the cross product between rotational velocity and linear velocity multiplies only the vector components that are orthogonal. Its direction can be determined by the right hand rule where your index finger (blue) points in the direction of the object velocity (here is fluid flow). Thumb (purple) points in the direction of the axis of rotation (Fig.1.1 (b)) and middle finger (red) will then point in the opposite direction of the resulting Coriolis force. This is indicated by the negative sign in the equation above.
Fig. 1.1. Working principle of a U-shaped Coriolis flow meter, (a) Coriolis effect according to oscillation axis and velocity vector direction \( v \), (b) right hand rule to determine oscillation axis, (c) Fluid mass “\( \delta m \)” which flows at velocity \( V \) through vibrating pipe, (d) gives rise to Coriolis forces \( F_C = -2 \delta m \Omega \times V \) and (highly-exaggerated) twisting motion\[4\].

As is obvious in Fig.1.1(c), the inlet arm and the outlet arms vibrate with the same frequency as the overall vibration, but when there is mass flow the two vibrations are out of sync, it means the inlet arm is behind, the outlet arm is ahead. The two vibrations are shifted in phase with respect to each other, and the degree of phase-shift also called twist angle, \( \theta \) is a measure for the amount of mass that is flowing through the tubes (Fig.1.2).

Please note that the amplitudes of the vibration and twist are extremely small compared to the size of the U-shaped tube. The above graphics are highly exaggerated for illustration purposes. The amplitude of the vibration is too small to be seen (because actuator frequency is very high, typically between 80 to 1000 Hz), but it can be felt by touch.

The motion of the twisted mode is imposed on the pipe’s driven motion. The combined motion does not occur in equilibrium plane simultaneously. The resulting time \( \Delta t \) can be proportional an axial shift in vibration phase \( \Delta \phi = \omega \Delta t \).
The Coriolis force on the small fluid section $\delta m$ is

$$\delta f = \delta m \cdot a_c = - \delta m \cdot 2\hat{\Omega} \times \vec{V}$$

During the down cycle, the tube applies an upward resisting force to the fluid or the fluid pushes the tube down. On the outlet side, the Coriolis force has the opposite direction [7].

To simplify the problem, we assume that the tube has a perfect U shape with a cross section area of $A$. The length and width are $l$ and $d$, respectively and $\rho$ is density of the fluid [7]. The opposite directions of Coriolis forces on inlet and outlet sides result in a twisting moment $T_C$:

$$T_C = F_c \cdot d = m \cdot a_c \cdot d = \rho Al \cdot 2\Omega V \cdot d \quad (1-2)$$

In this particular geometry a $K$-factor can be introduced to compensate for the more generalized U-shape.

$$T_C = K \rho Al \cdot 2\Omega V \cdot d = 2K\Omega Q_m dl \quad (1-3)$$

Where $Q_m = \rho AV$ is the mass flow rate. The governing equation of twisting is

$$I_u \frac{d^2 \theta}{dt^2} + C_u \frac{d \theta}{dt} + K_u \theta = T_C \quad (1-4)$$

Where $I_u$ is the inertia of the U-shaped tube, $C_u$ is the damping coefficient, $K_u$ is the stiffness, $\theta = \Delta \varphi$ is the twist angle, and $t$ is time.

Recall that the Coriolis flow meters are vibrating the U-shaped tube to generate the rotation, the real angular velocity $\Omega$ is function of vibrating frequency $\omega_0$:

$$\Omega = \Omega_0 \cos \omega t$$

Assuming that the damping term $C_u$ is negligible, the equation of twisting becomes

$$I_u \frac{d^2 \theta}{dt^2} + K_u \theta = 2KQ_m dl \Omega_0 \cos \omega t \quad (1-5)$$

The particular solution (steady-state solution) of the twist angle is

$$\theta = \theta_0 \cos \omega t = \frac{2KQ_m dl \Omega_0}{K_u - I_u \omega^2} \cos \omega t \quad (1-6)$$

$$\theta_0 = \frac{2KQ_m dl \Omega_0}{K_u - I_u \omega^2} \quad (1-7)$$
2. Measuring Principles and previous works

Since the early 1980’s the Coriolis mass flow meters have been considerably accepted in many industrial branches. However the first application of the Coriolis effect for mass flow measurement was proposed by Li and Lee in 1953 [2].

The main advantage of the Coriolis mass flow meter (CMF) is that it measures the true mass flow rate directly, unlike some other instruments that measures the volumetric flow rate. Some other advantages and disadvantages are available Table 2.1.

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>No dead volume, no splits, possibility of fully drainable design</td>
<td>Incorrect measurements due to gas bubbles</td>
</tr>
<tr>
<td>Exact measurement of very small to very large flow rates possible by one device</td>
<td>Incorrect measurement when crystallizing and polymerizing media</td>
</tr>
<tr>
<td>High level of accuracy (0.1-0.5%)</td>
<td>High investment costs</td>
</tr>
<tr>
<td>Independent of medium attributes such as conductivity and viscosity</td>
<td></td>
</tr>
<tr>
<td>Suitable for dispersions and suspensions</td>
<td></td>
</tr>
<tr>
<td>Clean-In-Place (CIP) enabled</td>
<td></td>
</tr>
<tr>
<td>Very high product pressures possible</td>
<td></td>
</tr>
</tbody>
</table>

*Table 2.1. CMF’s Advantages and disadvantages [1].*

The bending vibration of straight tubes with flowing fluid was studied in [5] by Ashley and Havilland. The model was a simply supported pipe that was treated like an Euler’s beam. By state of Hamilton’s principle, Chen in [6] derived the out-of-plane equations of motion, of a curved circular tube with flowing fluid. However this thesis is mainly based on mathematical modeling of U-shaped tube CMF that was presented by Sultan and Hemp in [7]. The dynamic response of a straight tube CMF was widely studied by Cheesewright, Clark and others [13].

The basic measurement principle [3] is that a flow tube is caused to vibrate sinusoidal at a resonant frequency (the fundamental natural frequency in most cases [2]) by one or more drivers while two sensors monitor the vibration mentioned above (fig.2.1).
The flow tube geometry and sensor placement are arranged so that the frequency of oscillation can be used to calculate the density of the process fluid, while the phase difference between the two sensor signals provides the mass flow rate. The flowing fluid passing through the vibrating tube produces Coriolis forces acting asymmetrically on the tube. These forces, which are proportional to the mass flow rate, produce the phase difference (Fig.2.2).

Since fluid density is influenced by ambient temperature changes, a temperature-sensor is attached on the structure to measure temperature changes which can cause density changes, and consequently, natural frequency during the measurement. Two pick-up sensors, in same sizes, are mounted symmetrically on structures and measure tube's displacement (Fig.2.1).

In addition, Fig.2.1 shows an actuator on the mid of the structure which oscillates the structure in resonance frequency. Mass of the actuator, pickup and temperature
sensors should be negligible and in some cases if they affect on the structure’s vibration properties then we should recognize them as added masses [1].

2.1 Available commercial designs.

As is mentioned in introduction part, the Coriolis principle is not new to process flow measurement. It is a proven technology that has been employed in a wide variety of markets and applications for more than 20 years. Also, CMFs have been widely used in high flow processes and nowadays in low flow rates as well. Table 2.1 provide some applications in different industries:

<table>
<thead>
<tr>
<th>Industry</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil &amp; Gas</td>
<td>Bulk loading, pipeline conveying, density measurement</td>
</tr>
<tr>
<td>Food &amp; Beverage</td>
<td>Content dosing of juices, CO2 injection, dosing, filling</td>
</tr>
<tr>
<td>Paper &amp; Pulp</td>
<td>Measurement of paper colorants, pulp, additives, bleaches</td>
</tr>
<tr>
<td>Pharmaceutical</td>
<td>Dosing, filling, solvent extraction</td>
</tr>
<tr>
<td>Chemical</td>
<td>Measurement of concentration &amp; density</td>
</tr>
</tbody>
</table>

Table 2.1. Applications of Coriolis Mass flow meters [1].

CMFs are generally made of two parts: a transmitter and a sensor. The transmitter could be mounted on the structure or not. Fig.2.3 shows a micro-CMF and its scale. Obviously, its transmitter cannot be mounted on that. In this thesis we just consider to sensor parts.

As is seen in Fig.2.3, a micro Coriolis mass flow meter with a small size is separated from the transmitter. These kinds of CMFs are made from common silicon or glass substrate. The characters of these micro-CMFs are small size, improved efficiency, and low cost in high volume quantities. Widely used MEMS (Micro Electro-Mechanical
Systems) sensors include pressure sensors, inertial sensors like accelerometers and gyroscopes, ink jet printer heads and optical projector display arrays [4].

2.2. Categorizing different CMF types

Coriolis mass flow meters could be categorized not only by size, ranges, transmitters and materials, but also by their different tubes shapes which mainly can be divided into straight and curved tubes.

2.2.1. Single Straight-tube

Single-straight tube design has gained growing importance since it was first introduced to the market in the late 1980s and early 1990s [12]. The principle of the single-straight tube design is the same as twin-bent tube design. Single-straight tube flow-meters have the advantage of lower pressure drop, and they are also easy to clean and compact. All the attached components e.g. pick up sensors and actuator should be located symmetrically and in the middle of the tube’s length, so, one of the important concerns within designing a single-straight tube meters is to achieve a balancing mechanism. This may add some complexity to the sensor, such as an inner housing cylinder or a mass depending on its torsional motion. The well-known definition of straight tube in this field is, a conveying-pipe which is supported by a vibration absorbing member (or frame) and both ends of the tube are connected to an inlet and outlet of one or more tubes (by manifold). Pickups are provided on the sensor tube.

![Single-tube CMF's schematic plan](image)

Fig.2.4. Single-tube CMF’s schematic plan [3].

Fig.2.5 shows a fluid conveying pipe, from right to left, both ends are fixed and supported perfectly and an actuator mounted at the mid-point along the straight tube. Its angular velocity vector, is perpendicular to the page, two Coriolis forces will appear on both sides and their vector’s directions are perpendicular to the tube, lying on the page with opposite orientation.
If the straight-pipe's length $l$, is divided into 4 sections (each section length $l/4$) then the location of maximum Coriolis force which imposed on the tube body would be expected on $l/4$ and $3l/4$, marked as (1) and (2) in figure 2.6, where is close (but is not exactly same) to the pickups' places. According to Euler cantilever beam theory the reason behind this distance is the slope of the mid-point is not zero ($\frac{d}{dx} \neq 0 , x = \frac{l}{2}$) on the both ends ($\frac{d}{dx} = 0 , x = 0 , l$). Obviously, both pickups should be placed on the points, where the maximum $F_C$ is imposed on the tube's body, otherwise, a dynamic unbalancing would occur through the vibrating straight-tube [1].

![Fig 2.5. The predicted locations of maximum Coriolis effects [3].](image)

According to the Coriolis effect, described clearly in 1.2, all the three quantities, $V$, $\Omega$ and $F$ are orthogonal. As it is mentioned in section 1.2, regarding to the right hand rule Coriolis force $F$ vector is perpendicular to angular velocity vector $\Omega$. This means if overall frequency is sinusoidal (for instance $\Omega \approx \sin(\omega t)$) then the Coriolis force is $F_C = a \sin \left( \omega t + \frac{\pi}{2} \right) = a \cos(\omega t)$. 
2.2.2. Twin Straight-tubes

These types of geometries are basically divided into two different designs: First, the mass flow-meter has a U-shaped tube provided on a support base, which is supported on a vibration absorbing frame, and both ends of the sensor tube are respectively connected to their proportional inlet and outlet pipes separately. Vibrator is provided at an intermediate part of each straight tube part (between the inlet and outlet sides of the sensor tube). Pickups are provided on the upstream and downstream sides of the vibrator so as to detect the displacement of the sensor tube. An external vibration is in an opposite direction between the inlet and outlet sides relative to the vibrating direction caused by Coriolis effect. Since the vibrator mounted between two straight tubes and the displacement on the middle of straight tubes is in opposite direction, then a same flow direction is required on the both tubes inlet and outlet, otherwise the Coriolis forces would be applied in the same orientation on each inlet and outlet sections (Fig.2.6).
In order to solve the problem a longer fluid-convey tube should be used to turn the outlet flow to the same direction with the inlet flow as it is shown in Fig.2.9. The advantages of this complicated design are comprised a dynamically balanced structure, external vibration independent, the design does not require a manifold to divide the inlet flow into two straight pipes. Since this flow meter has two active flow tubes which comprise a dynamically balanced structure that does not require the massive base (such as base of Fig.2.6), the setup could be lighter than that applied for single straight tube [3].

Also, using a large number of connections, longer convey tubes and consequently pressure dropping (may cause different pressure between inlet and outlet) are the disadvantages of the design.

There is another “Twin straight-tubes” design where both tubes convey the fluid in the same direction, here one side of both tubes is inlet and opposite side is outlet, the configuration of splitters can affect deeply on “pressure loss”, the parameter that justified using the straight-tubes. With its twin tube configuration, the measuring tubes are naturally balanced, and, therefore, do not need an extra balancing mechanism. This results in the possibility of a simple and light-weight sensor. Similar to single-straight
tube, the twin-straight tube design still holds the advantages of self-drain ability and compactness.

Fig.2.8. Both tubes convey the same flow direction, the right hand of both, could be inlet and the left end could be outlet and vice versa [12].

However, because of the splitting flow, it may result in a greater pressure drop than a single-straight tube [12]. Also, the dual-tube design requires flow splitters, which are not recommended for applications with fluid that are prone to plugging. Such fluid are often used in the food processing industry where single tube meters are required [8]. Since another reason behind using this geometry that it is easy to clean, designer should consider to choose the convenient splitter (Fig.2.11).

Twin-straight CMFs, also, have a considerable advantage in eliminating the vibration of the center of mass. According to the latest research and development of twin-straight tube Coriolis mass flow-meter [12] that carried out by a vast emphasis on “Minimization Pressure Lose” in twin straight-tubes CMFs, there are three possible splitter configurations, shown in Fig.2.11. Configuration A is simply a direct connection from a larger diameter pipe to two smaller diameter pipes. Fillets with small radii are used to smooth the flow. Configurations B and C use the lofting function in a 3D CAD system, which creates a rather smoother dividing or combining flow [12].

Fig.2.9. Different splitter configurations, proportional to pressure loss and cleaning ability [12].

However, these two configurations (B and C) can cause some problems to access inside the tubes to clean them, on the other hand “Configuration A” lead to the most pressure lose compare to B and C. This means P3>P2 (see Fig.2.9 and Fig.2.10).
Twin-straight tube CMFs have their own advantages compared with other types of flow-meters. Analyses using FEM and CFD at an early stage of the design can accelerate and help to development a flow-meter. A finite element method based on beam theory coupled with “one-dimensional flow” has been further developed for twin-straight tube CFMs [12].

2.3. Curved tube Coriolis mass flow-meters

There is another pipe configuration known as curved-tube which was the first to appear in 1981 [15]. The initial device consisted of a single U-tube. However, the twin-tube are more common in industry because by vibrating in opposition, the Coriolis flow twin-tubes are balanced and isolated from external vibration or movement of the flow-meter.

Generally there are two different designs of curve-tube flow-meters. In the first design, the tubes are bent to form a double loop (Fig.2.11 a). This design behaves similarly to the dual-tube CMF with the difference that the tubes are in series rather than parallel. Such single-tube flow-meters offer the same advantages as dual-tube meters, and they do not have the disadvantage of employing flow splitters (2.2.2).

**Fig.2.10. A comparison between different splitter’s configurations [12].**
The larger Coriolis effect becomes, the larger the time or phase difference between the pickup-sensors becomes, and the easier it is to determine the mass flow. Such magnifying geometrical forms often result in large tube loops that take up much space and have no advantage in zero point stability, because external disturbances are also magnified. Thus, the signal-to-noise ratio remains the same. As electronics have become more and more efficient, the need for such geometrical magnification of the Coriolis effect has disappeared. Therefore, the large loops can be replaced by compact tube design that require little space such as $\Delta$-shaped (Fig2.11b), U-shaped (Fig2.11 e) and even $\Omega$-shaped (Fig2.12) [8].

The second reason for using bent tubes (curved-tubes) in industry is the thermal expansion of the measuring tube. While the fluid temperature may be highly changed even more than 100 degrees Celsius, the temperature of the supporting structure change much less, due to thermal transport, convention and radiation. This can lead to large temperature differences between measuring tube and housing, which increase the axial forces of the tube. For a straight-tube CMF, the axial forces are largest and mainly depend on the expansion coefficient of the tube material. Fig2.11 (c), (f) shows the active part of the straight tube is restricted between two flanges and justifies using curved-tube in industry [8].

By choosing a material with a low expansion coefficient axial forces can be kept below the critical value, even for straight-tube design. Titanium offers higher stress limits than stainless still. Therefore, stainless steel tubes need to have a curved shape to reduce the maximum stress, since the tube can expand into the curve [17].
In this style of micro motion flow-meters that have dual parallel flow tubes, process fluid entering the sensor is split with half of the fluid passing through each flow tube. During operation, a drive coil is energized causing the tubes to oscillate in opposition to one another.

Last but not least, design with dual tubes offer the best performance for the decoupling of the measuring system from the process environment. Similar to a tuning fork, the two tubes vibrate in counter-phase. While the oscillation is maintained, the forces at the fixation points of the two tubes are identical in absolute value but in counter-phase directions. Ideally, this results in zero force acting on the flanges. The
perfect symmetry of the two tubes is unaffected by changes in fluid density, temperature, pressure, viscosity, and so on [8].

So, in next chapter we choose a dual curved shape geometry for our modelling which can be found in industrial designs as an existing CMF.

3. Modelling
The Coriolis mass flow-meter is modelled using the theory of vibrating beams. Tube deformations for the fundamental mode by use of finite element (FE) models to predict the performance of Coriolis mass flow meters is established in this chapter.

3.1. Strategy of the Modelling
As is mentioned in previous chapter Coriolis mass flow-meters have two utilizations; Measuring “fluid density” and “mass flow-rate”. According to these applications the modeling is divided into two parts, Fig3.1 illustrates the basic flow-chart of the strategy:

- Choosing a convenient and existing CMF according to requirements.
- Finding the Eigen-frequency of the corresponded mode shape.
- Induce the frequency on the structure.
3.2 Initial Assumption through the Model

In order to eliminate some factors which affect Coriolis flow-meter accuracy some assumptions are essential within the modeling. As it is mentioned in first chapter (Introduction) there is a wide research [1] has carried out to identify factors affecting Coriolis flow-meter accuracy, precision and robustness. The accuracy of a measurement system is the degree of closeness to its true value. Precision is the degree to which repeated measurements under unchanged conditions give the same results. Robustness is defined to be the ability of a system to cope with (unpredictable) variations in its operating environment with minimal damage, alteration or loss of functionality.

Four factors, suspected to influence Coriolis mass flow-meter accuracy which are investigated by Stephanie Enz [1]:

1. Flow pulsation
2. Asymmetrical actuator and detector positions
3. Imperfect fluid velocity profiles
4. Structural non-uniformities

**Assumption 1:** A pulsation flow caused by gear, piston or peristaltic pump in the pipe system or fast valve opening and closings, can generate severe vibrations in a pipe system. Flow pulsation frequencies, which might cause problems, are identified to be the sum and difference of the drive and Coriolis frequency. The problems are most severe when the frequency is so close or equal to the Coriolis frequency.

So, the first assumption in this model is there is no flow pulsation.

**Assumption 2:** CMF vibration are resonantly driven by an electromagnetic actuator. The actuators should be individually mounted mid-pipe on the flow-meter tube. The
detectors, used to meter the motions of the flow meter pipes, are located near the antinodes of the pipes’ Coriolis mode.

So, in order to find the mid-point easier along the pipe (center point where the actuator should be mounted there) a perfect U-shaped is a convenient geometry. Also, two pickup places are determined mathematically on both sides of the mid-point.

Assumption 3: the internal flow condition, such as imperfect fluid velocity profiles, affect the dynamic behavior of fluid-conveying pipes. It means the velocity profile should be perfectly uniformed along the active section of sensor-tube (Fig.3.3). A perfect fluid velocity profiles into the pipes would be available if and only if the flow to be fully developed. It means a steady state velocity profile would be after entrance length developing flow. These length depends on some parameters such as:

- $D$, is the hydraulic diameter of the pipe.
- $\mu$, is the dynamic viscosity of the fluid [Pa.s]
- $\rho$, is the density of the fluid [Kg/m$^3$] 
- $V$, is the mean velocity of the fluid.

These parameters form Reynolds number, which determines the inactive tube’s length and is considered in designing by improvisation of the straight sections on the geometry.

$$Re = \frac{\rho V D}{\mu} \quad (3-1)$$

So, in order to avoid convening imperfect fluid velocity profile a U-shaped tube with a certain straight section is chosen for modelling.

Assumption 4: Since, this is of relevance for CMF application, where structural properties of flow meter pipes (e.g. damping, stiffness and mass) may change yielding, e.g., altered energy dissipation, inaccurate and imprecise measurements and drifting zero-shift.

If “$r$” is average radios of the pipe and “$t$” is the thickness of the pipe then $r>>t$ to eliminate any changes on the cross-section’s geometry while the U-tube deflected. Obviously along the pipe length both in straight and curved sections the cross section dimensions should be constant. Also just in slender structures on curved pipes the approximation of constant cross-section would be valid [7].
So, the chosen material of the tube must have high-rigidity and uniform cross section to have an un-damped structure as much as possible. Also the conveying fluid assumed in single phase because bubbles can affect dramatically on damping ratio of a liquid conveying pipe [2].

3.3 Design and Geometry

Our object is to give details of a method of modelling the Coriolis mass flow-meter using beam theory. It is assumed that the tube is made of straight and circular lengths joined together. Since, the curved section is a semi-circle then our model implemented on a U-shaped design. Another advantage of this particular geometry is the shape dependent K-factor has been calculated in different straight part's length-semicircle's radios (b/a) ratios where b and a (see fig.3.3) are straight and curve lengths of the tube respectively [7].

3.3.1 Optimized Thickness

With regard to corrosion, erosion and pressure rating the wall thickness of the measuring tubes should be as thick as possible, however, the sensitivity of the instrument to flow induced Coriolis forces decrease with increasing wall thickness. Therefore, tube dimensions have to be optimized for several consideration including the overall pressure loss.
3.3.2. Analytical calculation

As far as it is mentioned in the first chapter the mass flow rate is independent of the amplitude of main vibration and depends only on geometry, stiffness K and the frequencies $\omega$ and $\omega_{nt}$. In this chapter calculation of stiffness constant ($K$) and natural frequencies ($\omega$ and $\omega_{nt}$) is provided as following.

$$K = \left(\frac{Gl}{(l+a)^2}\right) + \left(\frac{3Elx}{(l+a)^3}\right)$$

Where
E is Young's modulus.
G is shear modulus.
I is 2nd moment of area and
$J$ is polar moment of area where the curved radios of the pipe is sufficiently greater than cross sectional radios of pipe[7].
We can determine the polar moment of inertia $J$ about the $z$-axis by the method of composite shapes. This polar moment of inertia is equivalent to the polar moment of inertia of a circle with radius $r_o$ minus the polar moment of inertia of a circle with radius $r_i$, both centered at the origin.

$$J = \frac{\pi((2r_o)^4-(2r_i)^4)}{32} = \frac{\pi((r_o)^4-(r_i)^4)}{2} \quad (3-3)$$

And the same for 2nd moment of area:

$$I_x = \frac{\pi((r_o)^4-(r_i)^4)}{4} \quad (3-4)$$

It means in a pipe structure with homogenous cross-section:

$$J = 2 \ I_x$$

Now, assume that the U-tube is a cantilever beam fixed at one end with total length of $(l + d/2)$. The fundamental frequency, therefore, is:

$$\omega_n = \frac{c_1^2}{(l+d)^2} \left( \frac{EI_x}{(\rho_f+\rho_l)} \right) \frac{1}{2} \quad (3-5)$$

Where $c_1 = 1.8883$ for a cantilever. For un-damped torsional vibration, the Differential equation is:

$$I_z \dot{\theta} + K \theta = 0 \quad (3-6)$$

Where

- $I_z$ is the moment of inertia of the U-tube around $z$-axis
- $K$ is the combined torsional and bending stiffness constant and
- $\theta$ is the twist angle of the U-tube (see Figure 1.2)

From equation (3-6) the $5^{th}$ natural mode frequency (torsional frequency) is:

$$\omega_{nt} = \sqrt{\frac{K}{I_z}} \quad (3-7)$$

Where

$$I_z = \frac{\pi^2}{2} \left( \frac{d}{2} \right)^3 \left( \rho_l r_o^2 - r_i^2 \right) + \rho_f r_i^2 \quad (3-8)$$

Where $r_o$ and $r_i$ are radius of tube cross-section and on the other hand from (1-7):

$$\theta_0 = \frac{2kQ_{mid}l^2\omega_0}{K_u-l_u\omega^2} = \frac{2kQ_{md}l^2\omega_0}{K_u(1-l_u\omega^2)} \quad (3-9)$$
\[ I_z = I_u \] and as is defined above \( K \) is the combined torsional and bending stiffness constant:

\[
K = \frac{K_u}{k} \quad (3-10)
\]

It is essential to select carefully the drive frequency with respect to the natural frequency of twist mode. This is to ensure adequate signal to noise ratio and the accurate metering of fluids of different density without changing calibration \([7]\).

\[
\theta_0 = \frac{2Q_m d \Omega_0}{K (1 - \left(\frac{1}{\omega_{nt}}\right)^2, \omega^2)} = \frac{2Q_m d \Omega_0}{K (1 - \left(\frac{1}{\omega_{nt}}\right)^2)} \quad (3-11)
\]

Consequently the \( \theta_0 \) (=Maximum twist angle) increases as the two frequencies converge. However, \( \theta_0 \) can also be increased by increasing the length of the moment arm and decreasing stiffness of the tube. According to equation above at low driving frequencies, \( \frac{\omega}{\omega_{nt}} \ll 1 \), \( \theta_0 \) varies linearly with \( \Omega_0 \) and \( Q_m \) regardless of density.

When the driving frequency approaches the twisting natural frequency \( \left(\frac{\omega}{\omega_{nt}} = 1\right) \), then a very large increase in twist angle \( \theta_0 \) is obtained. Once the driving frequency becomes much greater than twisting frequency then \( \theta_0 \) reduces inversely with square of frequency. Note that \( I_z \) and therefore by \((3-9)\), \( \omega_{nt} \) depends on fluid density. Thus, if the device is driven at a fixed frequency \( \omega \) (not \( \ll \omega_{nt} \)) and fixed amplitude \( \Omega_0 \) the sensitivity is depends on fluid density.

It should probably be mentioned that the experimental measurement in reality has been our pattern in modeling strategy in some ways. Experimental data can be obtained in following steps:

- First, in laboratory measurements, the CFM was stripped for its protecting casting, and also not mounted in a piping system the recommended way; this implies more sensitively to external disturbances than for a similar CMF under normal operating conditions, and thus variability in measured phase shifts.
- Second, operator with standard impact hummer, applies an instant hit on the sensor’s body and a software records the signals which are detected by pickups.
- Third, after filtering noises by the software, calculated fundamental frequency will be applied on the structure again. Pickups are still detecting the frequency which varies by different fluid properties e.g. density, temperature, etc.
- Finally, when fluid flows along the pipe(s), phase shifts can be investigated by the same setup and components. Resonance frequencies are measuring simultaneously and are continuously recording by the software.

Solution:
Problems associated with fluids of varying density may be eliminated by vibrating the tube at its natural (fundamental) frequency (for example by using a feedback circuit).
The fundamental frequency (1st harmonic) is less than the twisting natural frequency and the $\frac{\omega}{\omega_{nt}}$ ratio is now constant for fluids of different density [7].

3.3.3 Time domain
Since, most of experimental modal analysis can be provided in time domain then we can convert equation (3-11) from twist angle to time variable.

$$Q_m = \frac{\kappa(1-(\frac{\omega}{\omega_{nt}})^2)}{2d \Omega_0} \theta_0$$

Equation above provides the basic relation between mass flow rate and twist. The mass flow is only proportional to the twisting amplitude $\theta_0$ if the angular velocity amplitude $\Omega_0$ of the main vibration is a constant. More usefully it is possible to base the flow measurement on the time difference $\Delta t$ (proportional phase shift $\Delta \varphi$) between signals measured at two arbitrary symmetric points using optical or electromagnetic detectors [1].

On the other hand, the velocity of the turning sides of the U-shaped tube is $\Omega l$ and the displacement difference between these two sides is $\theta \cdot d/2$. Therefore, the time lag $\Delta t$ between these two sides is:

$$\Delta t = \frac{\theta \cdot d}{\Omega l} = \frac{\theta_0 \cdot d}{\Omega_0 l}$$

$$\Delta t = \frac{2Q_m d \Omega_0}{\kappa(1-(\frac{\omega}{\omega_{nt}})^2)} \frac{d}{\Omega_0 l}$$

By measuring the time lag $\Delta t$, the mass flow rate can be obtained

$$Q_m = \frac{\kappa(1-(\frac{\omega}{\omega_{nt}})^2)}{2 \frac{d}{\Delta t}} \Delta t$$

As it is provided in the next chapter, equation (3-14) is applicable in calibration, simulation, set up tests and equation (3-15) is used in real measurement.
3.3.4 Geometry

As is seen in fig. 3.4, \( b \) is the straight part of the U-shaped tube is taken 450mm and \( a \) is the radius of the curved part which is 150mm as well, the diameter of the cross-section is taken \( r \) and thickness of the tube \( t \) are mm and 1.8mm respectively. These dimensions acceptable because also \( a \) and \( b \) are greater than \( r \). The following Fig(3.5) shows the Geometry which is sketched by COMSOL Multi-physics based on the previous investigation [7] as a modeling system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Amplitude</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a=d/2 )</td>
<td>150 [mm]</td>
<td>typical radius of curved tube</td>
</tr>
<tr>
<td>( b=l )</td>
<td>450 [mm]</td>
<td>straight tube length</td>
</tr>
<tr>
<td>( r_o )</td>
<td>2.54 [mm]</td>
<td>flow-meter tube radius</td>
</tr>
<tr>
<td>( r_i )</td>
<td>2.40 [mm]</td>
<td>flow-meter tube inner radius</td>
</tr>
<tr>
<td>( t )</td>
<td>1.8 [mm]</td>
<td>wall thickness</td>
</tr>
</tbody>
</table>

Table 3.1. Dimensions of the chosen geometry [7].

A circle with 50.8mm diameter is sketched on the work plane z-x and extruded through a line on x-y work plane. It means the frequency caused by actuator is in z-direction (Fig.3.6).

**Fig. 3.4.** The geometry is drawn in COMSOL and the cross section is extruded along the straight and curved then uniformed.
Since there is a symmetric geometry (between twin U-tubes), then a tube is taken for modeling and then, in simulation part, half of required flow will be considered. The Coriolis effects induce the force $F_c$ in $z$-direction and on each arms too, but in opposite directions in each period.

### 3.4 Implementation in COMSOL

Since, there are different coupled physics, in Coriolis mass flow meters, including solid mechanics, structural vibration and fluid structure interaction (FSI), COMSOL Multiphysics is able to couple them and provide a converged result. In addition the structural mechanics module is dedicated to the FE-analysis of mechanical structures that are subjected to static or dynamic loads. It is applicable for a wide range of analysis types, including stationary, transient, eigen-mode/modal, parametric and frequency-response [9].
3.4.1 Physics

As is obvious among all the FEA tools, such as COMSOL, there are infinity methods to modeling in different aspects such as element size, number and types (quadratic...) or using different modules such as FSI, or solid mechanics and then coupling them by fluid mechanics, eigenfrequency or frequency domain. In this case, because of the tube’s thickness is slender, then “Shell structure interaction” is a convenient physic [9].

In the case of “Shell structure interaction” physic, the thickness should be defined in structural part as a 3-D FE-Model. Despite in conventional FEM, the elements were used both in depth and on the surface of structure, in “Shell structure interaction” sub-physics we can use a convenient approximation and meshing can be performed on the surface. Note, it is important to realize that in any finite element analysis, the solution of the mathematical model of a physical structure (or more generally physical phenomenon) is numerically approximated using finite element procedures [10].

To describe a shell, user provides its thickness and the elastic material properties. The element used for the shell interface is of Mindlin-Reissner type, which means that transverse shear deformation is accounted for. It can thus also be used for rather thick shells. It has an MITC formulation where MITC means Mixed Interpolation of Tonsorial Components. A general description of this element family can be found in [10].

3.4.2 Materials

Structural steel is defined as the shell material, where “Density” and “Speed of sound” are essential properties as following:

<table>
<thead>
<tr>
<th>Density</th>
<th>Young’s modulus</th>
<th>Poisson’s ratio</th>
<th>Sound’s speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>7850 [Kg/m³]</td>
<td>200 [GPa]</td>
<td>0.33</td>
<td>5850 [m/s]</td>
</tr>
</tbody>
</table>

*Table.3.2 The conventional material properties (Stainless steel)[9].*

The fluid, which the tube contains, is once defined water and then replaced by air and kerosene (which is a well-known low-density liquid) to compare their modal behaviors, but two-phase fluids have never been used in a Coriolis flow meter or at least in our modeling process. When water is enabled as the fluid, then air in Model Builder>Material, should be disabled and vice a versa.

*Fig.3.6. The solid shell is illustrated by blue which covers 3 parts, and the naked part is fluid (gray).*
3.4.3 Mesh

Two types of mesh have been used on our model for the tube surface (shell) and fluid. Mesh for the solid part: As it is mentioned on Acoustic shell interaction description a rectangular mesh has been swept on the surface such as a 2D plate.

The MITC formulation does not take the strain components directly from the basis functions of the element. Instead, meticulously selected interpolation functions are selected for the individual strain components. The values of the interpolated strains are then at selected points in the element tied to the value that would be computed from the basis functions. The interpolation functions and tying points are specific to each element shape and order [10].

Here three basic principles should be considered for mesh sizes:

1. Convergence criteria: it is critical to have a dense enough mesh. If there is no multiple elements per wave length then the solution will not be a good approximation to the “partial differential equation” solution. The problem will still run but the results will be bad [10].

2. Element numbers: We need at least 5 elements per wave length \( \lambda \) (3-14), so as your frequency \( f \) goes up, you will need more elements [9].

\[ \lambda = \frac{c}{f} \quad (3-14) \]
3. If there are places in the model where you expect complex behavior, use a denser mesh in that region [9].

**Fig. 3.8**. A manual mesh is swept on the tube surface (shell) where the wave length is considered. In fluid domains a triangular automatic mesh is used.

For the fluid domains, an automatic rectangular element in “normal” size is used, however, it would be a good idea to do a simple convergence study. After solving make the mesh finer, re-solve and make sure solution did not change much.

### 3.4.4 Eigenfrequency Study

An eigenfrequency analysis finds the damped or un-damped eigenfrequencies and mode shapes of a model. Sometimes referred to as the *free vibration* of a structure. Pre-stress effects can be taken into account [9].

Eigen-frequency can be equal to the natural or even resonance frequency in undamped structures. An “*eigenfrequency study*” solves for the eigenfrequencies and the shape of the eigenmodes. If damping is included in the model, an eigenfrequency solution returns the damped eigenvalues ($\lambda$).

$$f = -\frac{\lambda}{2\pi i} \quad (3-15)$$

In this case, the eigenfrequencies and mode shapes are complex [9]. A complex eigenfrequency can be interpreted so that the real part represents the actual frequency, and the imaginary part represents the damping. Since, an un-damped structure is used for the Coriolis mass flowmeter, then complex mode shapes are not expected as the result.
In order to obtain the eigenfrequency of a particular mode shape we can drive the model at least for first to sixth mode shapes (table.4.1). As is seen, fig.3.9 illustrates the first mode while the tube contains air (right) and water (left)). Obviously, if tube contains a fluid with higher density then eigenfrequency would decrease, for instance if \( \rho_1 = 990 \text{ Kg/m}^3 \) (water) then \( f_1 = 80.417 \text{ Hz} \) and vise versa, for the empty tube which contains air with density \( \rho_2 = 1.2 \text{ Kg/m}^3 \), Eigen-frequency \( f_2 = 110.112 \text{ Hz} \). This comparison is just valid if and only if the vibration is occurred in a same mode (see also 3.5).

### 3.4.5 Boundary conditions

Both tube ends considered as fixed-constrained. Gravity is neglected in the modelling of Coriolis flowmeter. The simplest boundary condition (clamped or pinned) does not precisely represent the true conditions, because deflection, rotation and twist can take place in the brace bars due to their elastic nature. Therefore, if there is any further information about mounting, joints or brace bars, this should be implemented on the boundary conditions [7].

### 3.5 Order of modes

It is typical that a mode shape in its order won’t change within constant geometry and its eigenfrequency just would be changed by changing the density (in this case by changing the fluid’s density). But not always. (Fig3.10)
As is obvious in Fig.3.10, the mode shape that has a deflection just in z-direction is happened in first mode while contains water and as the second mode while containing air. It have not been a well-known phenomenon that the order of mode shape exchanges its order while just densities are changed. It is going to be published after some improvements beside this article.
4. Simulation and Results

In simulation part, both boundary conditions, with and without flow are implemented. Also, a comparison between the model and experimental data is provided.

4.1 The condition without flow:

Fluid (2): Air, Temperature: 23°C, The Sound speed: 332 m/s Density: 1.2 Kg/m³

<table>
<thead>
<tr>
<th></th>
<th>Fluid 1 (Water)</th>
<th>Fluid 2 (Air)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODE1</td>
<td>80.795 Hz</td>
<td>110.056 Hz</td>
</tr>
<tr>
<td>MODE2</td>
<td>119.765 Hz</td>
<td>121.150 Hz</td>
</tr>
<tr>
<td>MODE3</td>
<td>212.817 Hz *</td>
<td>159.516 Hz</td>
</tr>
<tr>
<td>MODE4</td>
<td>438.574 Hz</td>
<td>242.688 Hz</td>
</tr>
<tr>
<td>MODE5</td>
<td>470.055 Hz</td>
<td>287.085 Hz *</td>
</tr>
<tr>
<td>MODE6</td>
<td>487.779 Hz</td>
<td>363.776 Hz</td>
</tr>
</tbody>
</table>

*Table 4.1. The modal-order of torsional natural frequency is marked by (*).

Hence, we can use this model as a density meter, we have changed some densities to find the new eigenfrequencies in their corresponded modes.
Fig. 4.1. The above graph shows the model provides different eigenfrequencies for different densities for the first mode and below line graph for the third mode (torsional mode).
Fig. 4.2. First and third mode are shown on right and left respectively.

By comparing both graph-lines in fig. 4.3, two expected phenomena can be interpreted, first the results are fully linear and second, the ratio each data from graph lines are same exactly (see Table. 4.4). This method could be a suggested to verify the model as a density meter.

<table>
<thead>
<tr>
<th>Density of water [kg/m³]</th>
<th>Natural frequency [Hz]</th>
<th>Temperature [°C]</th>
<th>Speed of sound [m/s]</th>
<th>Natural twisting frequency [Hz]</th>
<th>Frequency ratio</th>
<th>ω / ωnt</th>
</tr>
</thead>
<tbody>
<tr>
<td>960</td>
<td>81.176</td>
<td>97</td>
<td>1543</td>
<td>213.818</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>970</td>
<td>80.986</td>
<td>83</td>
<td>1555</td>
<td>213.317</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>980</td>
<td>80.795</td>
<td>65</td>
<td>1552</td>
<td>212.817</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>990</td>
<td>80.605</td>
<td>45</td>
<td>1526</td>
<td>212.314</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>80.410</td>
<td>1-15</td>
<td>1430</td>
<td>211.800</td>
<td>0.38</td>
<td></td>
</tr>
</tbody>
</table>

Table. 4.2. Different temperatures caused different densities and fundamental frequencies but the ratio of natural frequencies are density independent.

Also, to ensure whether this model can provides the same frequency ratio (ω / ωnt) for other liquids (with different densities) or not, another simulation is carried out as following:

<table>
<thead>
<tr>
<th>Eigenfrequency Kerosene [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
</tr>
<tr>
<td>Mode 2</td>
</tr>
<tr>
<td>Mode 3</td>
</tr>
<tr>
<td>Mode 4</td>
</tr>
<tr>
<td>Mode 5</td>
</tr>
<tr>
<td>Mode 6</td>
</tr>
</tbody>
</table>

Table. 4.3. The Coriolis sensor contains different liquid (Kerosene with ρ=820 Kg/m³) and consequently different eigenfrequencies.
As is seen in table.4.2, despite different density has caused new eigenfrequencies, the eigenfrequency ratio \( \frac{\omega}{\omega_{nt}} = 0.38 \), torsional frequency by fundamental frequency, is obtained in same value with water. It means this ratio is applicable for all liquids with different densities in the equation 3.15.

4.2. Condition with flow

The fluid velocity \( U = 0 - 10 \) [m/s]. Condition of the flow: Turbulent, incompressible fluid, density: 987 [Kg/m^3]. Overall frequency: 80.5 Hz, pressure: 2.5 bar.

The flow is steady state and friction, dynamic drags, pressure loss are neglected.

In fact, a minimum velocity to reach a turbulent flow should be determined, but since we use an analytical equation to solve this condition, an approximation can be implemented otherwise for Transient or Laminar conditions low-flow-rate \( U \ll 1 \) m/s a new solution and corresponded equations would be required.

4.2.1 Solution:

According to analytical expression (3-15):

\[
\Delta t = \frac{2l d^2}{K(1 - \left(\frac{\omega}{\omega_{nt}}\right)^2)} Q_m
\]

Time delay is proportional to mass flow rate and as it is proven in 3.3.2 and shown for different liquids in 4.1, is fluid density independent.

Therefore, the model with the following properties:

<table>
<thead>
<tr>
<th>Density</th>
<th>Young’s modulus</th>
<th>Poisson’s ratio</th>
<th>Sound’s speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>7850 [Kg/m^3]</td>
<td>200 [GPa]</td>
<td>0.33</td>
<td>5850 [m/s]</td>
</tr>
</tbody>
</table>

And the design (with the geometry and dimension table.3.1) provides following values:

<table>
<thead>
<tr>
<th>Fundamentall frequency ( f )</th>
<th>Torsional frequency ( f_{nt} )</th>
<th>Frequency ratio ( \frac{\omega}{\omega_{nt}} = 0.38 )</th>
<th>Combined stiffness coefficient ( K = \frac{K_n}{k} = 46 \times 10^4 )</th>
<th>Curved tube’s diameter ( d = 0.3 ) [m]</th>
<th>Straight tube length ( l = b = 045 ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>80.6 [Hz]</td>
<td>212.6 [Hz]</td>
<td>0.38</td>
<td>( \frac{K_n}{k} ) = 46 \times 10^4</td>
<td>( d = 0.3 ) [m]</td>
<td>( l = b = 045 ) [m]</td>
</tr>
</tbody>
</table>

Table 4.4: Parameter values of stainless still tube contains water

Where \( A = \pi r_i^2 = 18 \times 10^{-4} \) m^2, by substitution the above value in expression and using parametric sweep from 0m/s to 10m/s in \( Q_m = U. A. \rho_f \) the results obtained as below:
Fig. 4.3. Time delay $\Delta t(s)$ versus mean fluid (water) velocity $U (m/s)$.

Fig. 4.4. Time delay $\Delta t(s)$ versus mean fluid (Kerosene) velocity $U (m/s)$.
The time difference is plotted against the mean fluid velocity for two different fluid densities as shown in fig.4.3 and fig.4.4. As is obvious, when there is no flow along the Coriolis sensor, $\Delta t$ is zero consequently, it means, the tube oscillates synchronized and without any phase-shift. One of the significant advantages of time delay measurement rather than other parameters such as $\theta, \theta_0, \Delta \phi, \ldots$ is, as previously mentioned in section 1.4, the Coriolis effect is exceedingly small in relation to the main tube vibration. Thus, very small phase differences are obtained between the output signals of electromagnetic (or optical) detectors. Instead, time delay can be detected and then other parameters can be obtained via corresponding equations.

Values of time difference of frequency $\Delta f$ are provided at various flow rates. The corresponding phase difference $\Delta \phi$ between tube displacements or tube velocities at the positions of the sensing coils is, of course, related to the simple equation $\Delta \phi = 2\pi f \Delta t$.

Overall, the time delay which is appeared by fluid flow is proportional to the fluid densities.

4.2.2. Comparison the results

In order to verify the theoretical model (chapters 3.3 and 4.1) and study the behavior of the meter, a number of experiments were conducted and an account of these is given.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0%</td>
<td>0</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>1</td>
<td>3.7 n/a</td>
<td>2.5</td>
<td>20%</td>
<td>2</td>
<td>5</td>
<td>0%</td>
</tr>
<tr>
<td>2</td>
<td>7.4 7</td>
<td>5%</td>
<td>8%</td>
<td>5</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>9.25 9</td>
<td>2%</td>
<td></td>
<td>6.2 n/a</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>11 n/a</td>
<td>7.5</td>
<td></td>
<td>7</td>
<td>8%</td>
<td></td>
</tr>
<tr>
<td>3.3</td>
<td>12.2 12</td>
<td>1%</td>
<td></td>
<td>8.2 n/a</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>3.8</td>
<td>14 n/a</td>
<td>9.5</td>
<td>-2%</td>
<td>10</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td>15.5 15</td>
<td>4%</td>
<td></td>
<td>10.5 n/a</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>4.7</td>
<td>17.4 16</td>
<td>6%</td>
<td></td>
<td>11.8 12</td>
<td>-1%</td>
<td></td>
</tr>
<tr>
<td>5.8</td>
<td>21.3 20</td>
<td>6%</td>
<td></td>
<td>15 15</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td>24 n/a</td>
<td>16.2</td>
<td>1%</td>
<td>16</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>7.8</td>
<td>28.8 27</td>
<td>6%</td>
<td></td>
<td>19.5 n/a</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>29.5 n/a</td>
<td>20</td>
<td>0%</td>
<td>20</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>32 32 n/a</td>
<td>23</td>
<td></td>
<td>n/a</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>37 n/a</td>
<td>25</td>
<td></td>
<td>n/a</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

*Table 4.6. Comparing available experimental data (conducted by G. Sultan [7]) and the model results (theoretical).*

Table 4.6 provides theoretical and experimental statistics for both liquids water and kerosene. The experiment is conducted by G. Sultan [7] and the available flow rate values (mean velocity of fluid) are based on his experiments, that’s why they are not series or regulated numbers. The scatter of points in the kerosene case is almost certainly related to the tendency of kerosene in the rig to release dissolved gases or to entrain air. Improvement in rig design should overcome this problem [7].
In fig. 4.5 the yellow line represents theoretical results in case of Kerosene and the blue line shows time delay vs mean velocity of water. Obviously a fluid with higher density can cause more time delay in a Coriolis mass flowmeter. There is no experimental data less than 1 m/s as we expected before (see 4.2). At low fluid velocities the frequency variation (due to flow) is extremely small (see Figure 2.14) and as this was the region in which the experiments were performed it was not possible to confirm the effect of fluid velocity on tube frequency [7].

4.3. Simulation by replacing steel tubes to polymers

In this chapter four different polymers are proposed and the model is simulated by each of them. The main approach of this simulation is finding an appropriate material to replace structural steel (conventional material) for a disposable Coriolis mass flowmeter.
The following table provides mechanical properties for Polypropylene, Polyvinylidene fluoride (PVDF), Polyether ether ketone (PEEK):

<table>
<thead>
<tr>
<th>Proposed material</th>
<th>Density ρ [kg/m³]</th>
<th>Speed of sound c [m/s]</th>
<th>Young’s modulus E [GPa]</th>
<th>Poisson’s ratio ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polypropylene</td>
<td>890</td>
<td>2600</td>
<td>0.9</td>
<td>0.42</td>
</tr>
<tr>
<td>PVDF</td>
<td>1780</td>
<td>2250</td>
<td>8.3</td>
<td>0.18</td>
</tr>
<tr>
<td>Peek</td>
<td>1300</td>
<td>2470</td>
<td>6.1</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Table.4.7. The material properties [21]

1. Polypropylene (pp) is a common plastic has already been used in the Äkta system [21]. This thermoplastic polymer is used in a wide variety of applications including, plastic parts and reusable containers of various types, laboratory equipment and automotive components. The significant advantage of using Polypropylene in piping and laboratory equipment’s is rugged and unusually resistant to many chemical solvents, bases and acids.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode2</th>
<th>Mode3</th>
<th>Mode4</th>
<th>Mode5</th>
<th>Mode6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenfrequency [Hz] (air)</td>
<td>7.43</td>
<td>11.33</td>
<td>19.525</td>
<td>40.71</td>
<td>43.84</td>
</tr>
<tr>
<td>Maximum displacement [µm] (air)</td>
<td>9.46</td>
<td>4.61</td>
<td>1.36</td>
<td>0.73</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table.4.8. Eigenfrequencies and Maximum displacements for first six modes.

As is seen in table.4.8, the structure vibrates in a low frequency and this material cannot be appropriate for this structure. The gravity of the tube which contains water can cause a deflection which is not negligible. The eigenfrequency ratio which is fundamental frequency by torsional natural frequency (marked by * in the table 4.8) is \( \omega / \omega_{nt} = 0.38 \) and maximum displacement which is caused by exciter is 9.46 [µm].

2. Polyvinylidene fluoride (PVDF) which has a low density (1780 Kg/m³) compared to the other fluoropolymers, is available as piping products, sheet, tubing, films, plate and as insulator for premium wire. It means it is so convenient to be used in a light pipe structure. Since, using this light material in our model can cause some problem as static deformation when it is filled by high density liquids like water, another application in case of low density flow (gas) is investigated.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode2</th>
<th>Mode3</th>
<th>Mode4</th>
<th>Mode5</th>
<th>Mode6</th>
<th>( \omega / \omega_{nt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenfrequency [Hz] (air)</td>
<td>47.09</td>
<td>67.86</td>
<td>124.07</td>
<td>126.06</td>
<td>248.50</td>
<td>255.11</td>
</tr>
<tr>
<td>Eigenfrequency [Hz] (water)</td>
<td>21.36</td>
<td>32.04</td>
<td>58.18</td>
<td>116.03</td>
<td>125.44</td>
<td>166.89</td>
</tr>
<tr>
<td>Displacement [µm] (water)</td>
<td>1150</td>
<td>58</td>
<td>152</td>
<td>7.8</td>
<td>34</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table.4.9. Eigenfrequencies and Maximum displacements for first six modes in both liquid and gas conditions. Torsional eigenfrequencies are marked by * in both conditions.
Higher frequency in case of gas (more than twice compare to water) can justify this material to be applied in Coriolis gas flowmeter rather than liquid mass flow meter. Because of good ability in resistance to solvents and acidic gases, PVDF can be used in types of flowmeter configurations and applications requiring the highest purity. A disadvantage of using PVDF is its low melting point of around 177 °C and mechanical properties are fully temperature dependent [21]. Hence, the above simulation’s data are restricted in particular temperature (23°C-25°C).

3. Polyether ether ketone (PEEK) is a semi-crystalline thermoplastic with excellent mechanical and chemical resistance properties that are retained to high temperatures. Because of its robustness, PEEK is used to fabricate items used in demanding applications, including bearings, piston parts, pumps, HPLC columns, compressor plate valves, and cable insulation.

<table>
<thead>
<tr>
<th>MODE</th>
<th>Thickness (1.8 mm)</th>
<th>Thickness (2.2 mm)</th>
<th>Thickness (2.6 mm)</th>
<th>Thickness (3.0 mm)</th>
<th>$\omega/2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODE1</td>
<td>18.781 Hz</td>
<td>20.434 Hz</td>
<td>21.866 Hz</td>
<td>23.126 Hz</td>
<td>$\omega/2\pi$</td>
</tr>
<tr>
<td>MODE2</td>
<td>28.474 Hz</td>
<td>31.898 Hz</td>
<td>34.930 Hz</td>
<td>37.617 Hz</td>
<td>$\omega/2\pi$</td>
</tr>
<tr>
<td>MODE3</td>
<td>49.525 Hz*</td>
<td>54.792 Hz*</td>
<td>59.390 Hz*</td>
<td>63.439 Hz*</td>
<td>$\omega_{nt}/2\pi$</td>
</tr>
<tr>
<td>MODE4</td>
<td>102.627 Hz</td>
<td>114.632 Hz</td>
<td>125.036 Hz</td>
<td>134.192 Hz</td>
<td>$\omega_{nt}/2\pi$</td>
</tr>
<tr>
<td>MODE5</td>
<td>110.519 Hz</td>
<td>122.355 Hz</td>
<td>132.377 Hz</td>
<td>141.040 Hz</td>
<td>$\omega_{nt}/2\pi$</td>
</tr>
<tr>
<td>MODE6</td>
<td>159.663 Hz</td>
<td>175.744 Hz</td>
<td>189.280 Hz</td>
<td>201.461 Hz</td>
<td>$\omega_{nt}/2\pi$</td>
</tr>
<tr>
<td>Frequencies ratio $\omega/\omega_{nt}$</td>
<td>0.3792</td>
<td>0.3730</td>
<td>0.3682</td>
<td>0.3645</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4. Different wall thickness and correspond eigenfrequencies.

For two fluid densities, the eigenfrequency ($f = \omega/2\pi$) is plotted against tube wall thickness (for U-shaped tube configuration) in Figure 2.17. It is seen that increasing tube thickness leads to an exponential increase in torsional (=twisting) natural frequency and fundamental frequency. Frequency variation according to tube thickness is indicated in table 4.4. Increase in the tube length leads to considerable increase in at as shown in Figure 2.18.

![Fig. 4.7. different wall thicknesses versus fundamental frequency (blue) and Torsional natural frequency.](image-url)
Since PEEK is a hard, solid material, which can be mechanically shaped into various optical components (like lenses and windows), also it is transparent and measurement operator can observe any interaction, bulbs or fluid phase changing, it could be a convenient CMF’s tube material. So, we need to simulate our model and observe the results.

4.3.1 Simulation a disposable flowmeter
The same dimensions are used and model’s geometry has not been changed. The results of simulation, using PEEK as tubes’ material, is available as following:

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenfrequency [Hz] (air)</th>
<th>Displacement [μm] (air)</th>
<th>(\frac{\omega}{\omega_{nt}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32.84</td>
<td>47.5</td>
<td>0.384</td>
</tr>
<tr>
<td>2</td>
<td>47.65</td>
<td>25.4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>85.41 *</td>
<td>7.07</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>124.07</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>178.60</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>188.68</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.9. Eigenfrequencies in gas conditions maximum displacements is also provided. Torsional eigenfrequencies are marked by *.

As is seen in above table, torsional modes for both cases (water and air) are in third mode, it is why of PEEK is selected among other materials to simulate a disposable Coriolis mass flow meter.

By substituting the obtained values from table 4.9 as well as \(K = \frac{K_u}{K} = 5.53 \times 10^4\) and \(Q_m = U \cdot A \cdot \rho_f\) in the analytical equation \(\Delta t = \frac{2l^2d^2}{K(1-(\frac{\omega}{\omega_{nt}})^2)}Q_m\), then the following results will be obtain. Also it is possible to simulate a Coriolis Gas flowmeter in same way (fig.4.7).
Fig. 4.7: The first graph depicts time delay versus mean air velocity and below one shows time delay changes by mean water velocities from 0 [m/s] to 10 [m/s].
5. Conclusion, proposals and future works

5.1 comparison

PEEK is selected material among other suggested polymers, based on the reasons which explained in previous chapter. The most significant advantages to be an alternative material rather than stainless steel are:

- This material provides a similar modal order in both fluid phases (liquid and gas), unlike stainless steel (see chapter 3.5) and some other suggested polymers.
- Less displacement amplitude and higher resonance frequency are two advantages of using other polymers. High displacement can cause non-linearity both in case of “geometric non-linearity (GNL)” and “Material non-linearity (MNL)”. Low frequency oscillations are prone to be affected by environmental noises.
- Regarding the Young’s modulus of PEEK, the assumption which the gravity can be neglected is still valid, whereas Polypropylene structure which gravity can deform the water containing configuration even in static condition.
- In comprising Stainless steel and PEEK thermal conductivities, PEEK is more convenient material when the fluid temperature should be kept during measuring process.

Disadvantage:

According to this fact that polymers stiffness are temperature dependent (compare to conventional materials like steel), the measurement should be restricted in a limited temperature.

The application of disposable CMF is recommended as Gas mass flow meter. In case of high density liquids (like water), to obtain robust and accurate measurement results, effects of noises should be eliminated (by an accurate signal filtering process). Otherwise, to increase structural resonance frequencies, some changes are needed in geometry (wall thickness) such is implemented in PEEK (please see fig.4.3). Also, to minimize the effect of gravity installation of the disposable CMFs should be vertical (see fig.3.5 that x-y plane is perpendicular to the ground) rather than horizontal.

5.2. Proposals and future works

In this modeling all attempts have been carried out to eliminate errors as much as possible and close it to reality more. Some advantages and disadvantages within this modeling have been found that are already solved or should be considered for future models. Instead of the limitation in one geometry, a general solution could be utilized to model a verity new and former geometries. A research is implemented by Samar&F. Chanon [17] with the title of Modeling of Coriolis mass flow meter of a general plane-shape pipe.
\[
\begin{align*}
D^6 + \left( \frac{2}{a^2} + \frac{m_f U^2}{EI} - \frac{T}{EI} \right) D^4 + 2i\omega \frac{m_f U D^3}{EI} + \left( \frac{1}{a^2} + \frac{m_f U^2}{a^2 Gf} - \frac{m_f + m_t}{a^2 Gf} \omega^2 + \frac{T}{a^2 Gf} \right) D^2 \\
- 2i\omega \frac{m_f U D}{a^2 Gf} + \frac{m_f + m_t}{a^2 Gf} \omega^2 \right] V = 0
\end{align*}
\]

The straight pipe equation will be of the form:

\[
\left[ D^4 - \left( \frac{m_f U^2}{EI} - \frac{T}{EI} \right) D^2 + 2i\omega \frac{m_f U D}{EI} - \frac{m_f + m_t}{EI} \omega^2 \right] V = 0
\]

Because we can assemble all Coriolis mass flow geometries just by this two formulas curved and straight segments.

*Fig.5.1. Delta-shape element and nodes (above). S-shape element and nodes (below)*[17]
Proposal 2:

The model can be utilized for finding the exact points of maximum Coriolis effect as well, for example, in our model the maximum Coriolis force place is not on the straight and curved junctions but also are on the curved part. In future work, if the location of maximum displacement would be desired, then this model is able to provide the exact location on the proportional geometry. The proposal is we can use such model before preparing mentioned components for an experimental setup.

Proposal 3:

This model provides all dynamic and modal characters and amplitudes such as displacement that can be utilized in case of select convenient exciter and pickups (if needed).
6. References
