A new approach to Pairs Trading

Using fundamental data to find optimal portfolios

Author: Erik Jakobsson

Erik Jakobsson
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Abstract

Since its' invention at Morgan Stanley in 1987 pairs trading has grown to be one of the most common and most researched strategies for market neutral returns.

The strategy identifies stocks, or other financial securities, that historically has co-moved and forms a trading pair. If the price relation is broken a short position is entered in the overperforming stock and a long in the underperforming. The positions are closed when the spread returns to the long-term relation. A pairs trading portfolio is formed by combining a number of pairs.

To detect adequate pairs different types of data analysis has been used. The most common way has been to study historical price data with different statistical models such as the distance method. Gatev et al (2006) used this method and provided the most extensive research on the subject and this study will follow the standards set by that article and add new interesting factors.

This is done through an investigation on how the analysis can be improved by using the stocks fundamental data, e.g. P/E, P/B, leverage, industry classification. This data is used to set up restrictions and Lasso models (type of regression) to optimize the trading portfolio and achieve higher returns. All models have been back-tested using S&P 500 stocks between 2001-04-01 and 2015-04-01 with portfolios changed every six months.

The most important finding of the study is that restricting stocks to have close P/E-ratios combined with traditional price series analysis increases returns. The most conservative measure gives annual returns of 3.99% to 4.98% depending on the trading rules for this portfolio. The returns are significantly (5%-level) higher than those obtained by the traditional distance method.

Considerable variations in return levels is shown to be created when capital commitments are changed and trading rules, transaction costs and restrictions on unique portfolio stocks are implemented.

Further research regarding how analysis of P/E-ratios can improve pairs trading is suggested.

The thesis has been written independently without an external client and studied an area that the author found interesting.

Sammanfattning

Sen det uppfanns på Morgan Stanley 1987 har pairs trading (relationshandel) växt till en av de vanligaste och mest omskrivna metoderna till att nå marknadsneutral avkastning.

Strategin identifierar aktier, eller andra finansiella derivat, vars prisserier historiskt haft liknande rörelse. Om förhållandet mellan priserna bryts blankas den övervärderade aktien och den undervärderade köps. Då den långsiktiga relationen återställs stängs de båda positionerna. En portfölj skapas genom att kombinera flera olika par.

Adekvata par hittas genom olika typer av dataanalys. Oftast genomförs denna på historiska prisserier med ”distansmetoden” som den vanligaste. Gatev et al (2006) använder denna modell för att producera den mest heltäckande forskningen på området och skapade då även standarder som använts i senare undersökningar. Denna studie använder dessa standarder och adderar nya intressanta faktorer.

Här introduceras nya typer av fundamentalt data som, P/E, P/B, skuldsättning, industri-klassification och andra variabler. Data används för att filtrera fram par till de statiska modellerna. Även Lasso-modeller (typ av regression) används för att finna optimala portföljer och nå högre avkastning. Samtliga modeller har testats med aktier från S&P 500 under perioden 2001-04-01 till 2015-04-01 med byte av portfölj två gånger per år.

Det viktigaste uppnådda resultatet från studien är att restriktioner på små skillnader i P/E-tal tillsammans med konintegrations-test och distansmodellen ger kraftigt höjd avkastning. Det mest konservativa måttet ger en avkastning på 3.99% till 4.98% beroende på handelsregler för denna portfölj. Dessa är signifikant (5%-nivå) högre än de som uppnås genom distans-modellen utan restriktioner.

Stora förändringar i avkastningen visas även skapas då kapitalbindning, handelsregler, transaktionskostnader och restriktioner på unika aktier i portföljen varieras.

Vidare utforskning i hur framförallt P/E-tal kan höja avkastning föreslås.

Examensarbetet har skrivits självständigt utan extern beställare och undersökt ett område som känts intressant för författaren.

Acknowledgments

This study have been conducted independent without any external clients. This has presented both opportunities and challenges. I have been able to develop the study freely and finding new interesting aspects as the work has moved along.

Support from my surroundings has, at hard times, led me to find new paths and at the end some very interesting results has been reached.

Special thanks goes out to Markus Ådahl who has supported me at the university. Also, even though based in the Czech Republic and the U.S. my close friends Richard Stadig and Alexander Hennig Westöö has given crucial support to the study’s success.

1. Introduction

Stock markets at all-time highs and interest rates at all-time lows has produced an environment where attractive investment opportunities are extremely hard to come by. An increased number of investors has therefore turned towards the hedge fund industry in search for profits. The strategies employed by such hedge funds can often be complex and hard to understand. One of the most common strategies is pairs trading.

1.1. History

The credit of inventing pairs trading has been given to Nunzio Tartaglia. In 1987 Tartaglia, a quantitative analyst, worked at Morgan Stanley and was given the opportunity to form a group that would develop quantitative arbitrage strategies in the stock market. Tartaglia chose bright people from mathematics, physics and programming to help him.

The strategies they developed were solely based on analysis of quantitative data and trades executed automatically. At the time, this was considered somewhat groundbreaking at Wall Street where stock picks were based on almost exclusively fundamental analysis.

During the first year the group produced big profits but after a tougher second and third the project was shut down. However, the ideas and strategies from the group spread with former group members setting up at new firms. Ideas inspired by the work of this group is now the backbone of the ever growing hedge fund industry.

One of the most important trading schemes discovered by the group is what is now known as pairs trading. The simple idea behind it is that when stocks that historically has moved together deviates from each other a short position is entered in the overperforming stock and a long position in the underperforming. When the spread between them returns to normal the positions are closed.

The idea is that this kind of deviation is a form of mispricing of the two securities. The historical co-movement suggests that they should be priced with a fixed ratio and if they deviate it is caused by undervaluation of the first stock, overvaluation of the other or a mixture. The mispricing should be corrected in the future and the stocks priced according to the long-term relation.

Pairs trading is categorized as one of the statistical arbitrage strategies.

1.2. Previous research

Being one of the most widely used hedge fund strategies there exists a wide range of different research on the subject. The most extensive exploration of the strategy was conducted by Gatev, Goetzman and Rouwenhorst (2006) where they studied pairs trading in the US between 1962 and 2002. They found average annual returns over 10%. Gatev et al used the distance method which is a straightforward method screening pairs based on squared differences in historical price series. The study created portfolios of 20 pairs which have since been the standard for research in the subject. Compared to portfolios with only five pairs the risk decreased and profits increased substantially. The researchers also show that returns have dropped since the mid-nineties.

Later research have developed mathematically more advanced models but still almost exclusively rely on analyzing historical price patterns. Vidyamurthy (2004) suggests a co-integration approach and Elliot, van der Hoek and William (2005) examines a model based on the stochastic spread. Together with the distance method these makes up the three main pairs trading strategies. Others, such as the stochastic residual spread (Do and Faff 2006), are generally derived from one of the three main strategies.

All of the above also suggests some form of fundamental analysis before forming pairs. However, how this analysis should be conducted is rarely specified, it is rather presented as something to consider before forming portfolios. Gatev et al (2006) restricts some portfolios to only include stocks from the same industry. Do and Faff (2012) investigates the effects of both industry and sector matching and finds that matching in 48 different sectors improves the profit compared to only major industry matching.

Many researches stresses pairs trading's sensitivity to transaction costs. Do and Faff (2012) finds, after extensive research, the transaction costs to substantially lower profits.

Research, such as Do and Faff (2010), has also shown that pairs trading profitability has decreased, in the U.S., since the mid-nineties. However, the strategy seems to provide high returns during long term market turbulence. They contemplate that the vast amount of trading conducted in pairs trading and other contrary strategies has absorbed a substantial amount of the profits and made the markets more effective. They state instead that the strategy might be profitable with intra-day and high frequency trading (HFT). Further, they speculate that the huge trading volumes from HFT might absorb much of the profits.

Other research has focused on how far back historical prizes should be analyzed, the length of the trading period, how much the stocks should deviate before entering a trade, when a position should be closed, number of pairs in the portfolio, stop-loss implementation and much more.

1.3. Focus of paper

This paper has focused on developing new strategies to perform the fundamental analysis before using the distance method. The Engle-Granger test, from Vidyamurthy 's research, were not used on its' own to choose pairs but provided restrictions before implementation of the distance method. This comes from that Do and Faff (2006) showed the distance method to produce superior results in the U.S stock market.

As mentioned, previous research almost exclusively focus on industry and sector affiliation. In this study the portfolios were formed on basis of how other fundamental data relate between stocks. The variables geographic classification, exchange, price to equity, price to book, dividend yield, leverage and market value were analyzed alongside the traditional metrics.

Pairs with equal, for qualitative, and close, for quantitative, values for the variables were considered by the distance method when implementing the restrictions for the different portfolios.

Also, two Lasso regressions was performed to predict future profitable and co-integrated pairs. This kind of modelling stands in stark contrast to previous fundamental analysis in pairs trading which has, as described above, focused on restricting the quantity of pairs to consider for trading.

Research regarding how pairs trading portfolio is affected by restricting stocks to only be part of one pair of the portfolio has been nonexistent. This relative simple but possibly important restriction were therefore analyzed as well.

The results regarding transaction costs by Do and Faff (2012) adds another important factor that were studied.

To get robust results the models were tested with two separate trading thresholds. The thresholds determines by how much the spread has to deviate before a position is opened.

This will present a new approach to pairs trading and possibly a way to reach higher profits. The portfolio performance will be analyzed on both a monthly and annual basis. Stocks will be chosen to reflect the Standard & Poor's 500 stock index. Return characteristics will therefore be compared with that index. Following the standard set up by Gatev et al (2006) each portfolio will be made up by the top 20 stock pairs based on the corresponding restrictions.

The strategies were tested from the first of April 2001 until March 31 2015. Portfolios were changed every six months.

Data were collected from *Thomson Reuters Datastream* and models developed in *R*.

The significance level when using hypothetical testing will be 5% throughout the paper.

1.4. Key problems

The study can be summarized by some crucial questions it aims to answer.

- Can increased fundamental analysis improve pairs trading returns?
- Can historical data be used to find optimal pairs using Lasso regression?
- How does a restriction on using only unique stocks when forming a portfolio effect its performance?
- How does the returns change when omitting transaction costs?
- Is the result robust to different trading thresholds?

2. Theory

2.1. Co-integration

From the work of R.F. Engle and C.W.J. Granger (1987) two time series, $\boldsymbol{x}$ and $\boldsymbol{y}$, is said to be co-integrated if they can be described by equations ***2.1a*** and ***2.1b*** with a positive error correction.

$$x_t = x_{t-1} + \gamma_x * (y_{t-1} - x_{t-1}) + \varepsilon_{xt} \tag{2.1a}$$

$$y_t = y_{t-1} + \gamma_y * (x_{t-1} - y_{t-1}) + \varepsilon_{yt} \tag{2.1b}$$

$x_t = value\ for\ x\ at\ time\ t, \varepsilon_{xt} = white\ noise\ for\ x\ at\ t, \gamma_x = error\ correction\ for\ x\ at\ t$

$y_t = value\ for\ y\ at\ time\ t, \varepsilon_{yt} = white\ noise\ for\ y\ at\ t, \gamma_y = error\ correction\ for\ y\ at\ t$

The spread between the time series at $t-1$ is described by $(y_{t-1} - x_{t-1})$. The error correction, also called the co-integration coefficient, decreases the spread. A large error correction term would therefore create two time series with a small spread. By reorganizing the terms the increment is found,

$$x_t - x_{t-1} = \gamma_x * (y_{t-1} - x_{t-1}) + \varepsilon_{xt} \tag{2.2a}$$

$$y_t - y_{t-1} = \gamma_y * (x_{t-1} - y_{t-1}) + \varepsilon_{yt} \tag{2.2b}$$

Equations ***2.2a*** and ***2.2b*** clearly shows that the increment in $\boldsymbol{x}\ at\ t$ is expected to be larger if $y_{t-1} > x_{t-1}$. The same naturally applies to the increment in $\boldsymbol{y}$, $y_{t-1} < x_{t-1}$. Simply, the deviation is caused by the white noise and through error correction the spread decreases in future time steps.

The spread between the two price series will when co-integrated be stationary, i.e. have a joint probability distribution that does not change over time. The difference between a stationary and non-stationary stochastic process is shown in ***Figure 2.1*** below.

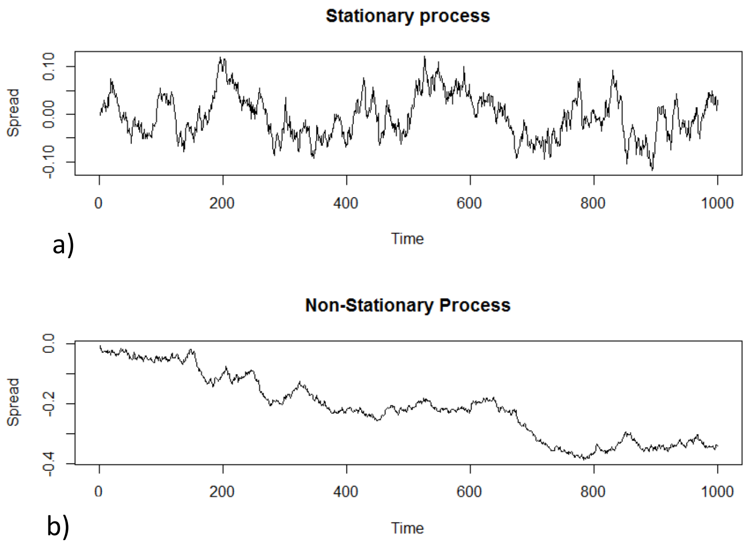

Figure 2.1.
a) Stationary stochastic process
b) Non-Stationary stochastic process.

2.2. Co-integration vs. Correlation

It is important to distinct between a co-integrated time series and a time series with correlated changes. They often get mixed up. This is especially important when analyzing price series and their resulting returns. The simple returns, $\boldsymbol{r}$, for a price series, $\boldsymbol{p}$**,** is defined as,

$$r_t = \frac{p_t - p_{t-1}}{p_{t-1}} \tag{2.3a}$$

With volatility (standard deviation) σ_r,

$$\sigma_r = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(r_t - \bar{r})^2} \tag{2.3b}$$

When referring to correlated stocks it really means stocks with correlated returns. Two co-integrated price series does not need to have correlated returns and vice versa. The correlation, ρ, is estimated by equation ***2.4***.

$$\rho(X,Y) = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 * \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})^2}} \tag{2.4}$$

(Vidyamurthy 2004, p. 12)

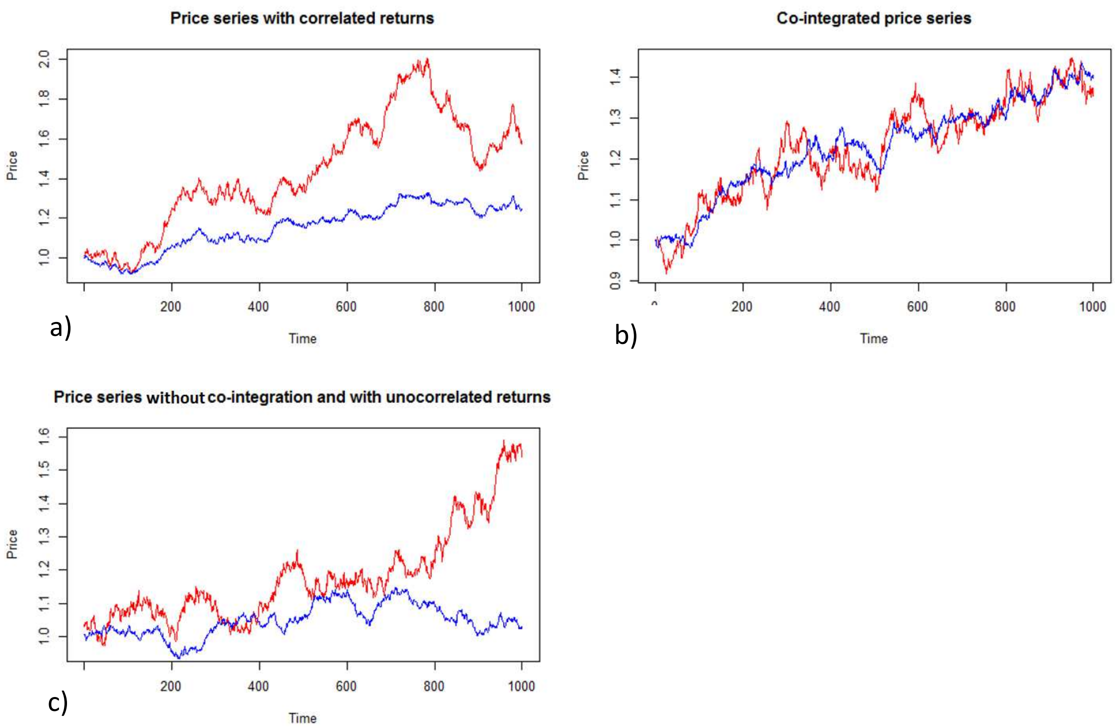

Figure 2.2. Price series comparisons
a) Price series with correlated returns
b) Co-integrated price series
c) Price series without co-integration and with uncorrelated returns

Figure 2.2 clearly illustrates the differences between the price series. As shown by equations ***2.1a*** and ***2.1b*** co-integrated prices is driven back towards each other when the spread is big. This is not the case when the returns is correlated. If the price series deviates from each other nothing says that they will move back towards each other since future prices does not depend on historical. It is evident that co-integrated prices is much more suitable for pairs trading than price series with correlated returns.

2.3. Testing for co-integration

Research has found a wide range of methods for testing the co-integration between two series and through that identifying stock pairs. Gatev et al (2006) uses the distance method when he studies pairs trading between 1962 and 2002. The distance method analyzes historical prices and chooses stock pairs with the smallest squared distances between normalized prices. It is not really a test for co-integration but has produced suitable trading pairs when tested.

Tests for co-integration is also used extensively. Engle and Granger (1987) developed a two-step test that first estimates the linear relationship and then tests for stationarity for the spread. Engle and Granger received *Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel* for the advances they made in this research and ARCH models (Nobel Media AB 2014). It sets up a hypothesis of no co-integration and tries to disprove it.

Many other tests, such as the one-step Johansen test for co-integration, can be used to find stock pairs. The test was developed by Danish statistician Søren Johansen (Johansen 1991). However, only the distance method the Engle-Granger test were used in this study.

2.3.1. Distance method

The distance method is very straight forward. It tries to identify stocks that has moved together through finding stocks with normalized price evolution that has been very similar. When normalizing one or more price series a first value is chosen, like 1 or 100, and every future value is found by combining the simple return, equation ***2.3a***, and the previous value.

The sum of squared differences (SSD) between the normalized price series $\boldsymbol{p_1}\ and\ \boldsymbol{p_2}$ is found by,

$$SSD = \sum_{t=1}^{N} (p_{1t} - p_{2t})^2 \tag{2.5}$$

The distance method simply chooses the stocks with smallest SSD. It does not specifically try to find co-integrated stocks but the strategy has proven to produce large profits and is used extensively (Gatev et al 2006).

2.3.2. Engle-Granger

The Engle-Granger test for co-integration uses two steps. First an OLS is performed to estimate the linear relationship between the price series $\boldsymbol{y}$ and $\boldsymbol{x}$.

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t \tag{2.6}$$

In the next step the residuals, $\boldsymbol{\varepsilon}$**,** produced by the ordinary least squared regression (OLS), equation ***2.6*** is tested for stationarity. OLS is described in more detail in part ***2.5***. The residuals from the OLS is in this case the spread between the two price series after adapting them with the linear relationship. The augmented Dickey Fuller test (ADF) is used for testing of stationarity.

The ADF uses the concept of a unit root. A unit root is said to be present if the autoregressive process ($AR(1)$) of a time series, $\boldsymbol{z}$, the coefficient $|\alpha|$ equals 1 in,

$$z_t = \alpha z_{t-1} + u_t \tag{2.7}$$

where u_t is the error term. If a unit root is present the process is concluded to be non-stationary.

However, it is not right to assume that the residuals follows a $AR(1)$ process. Autocorrelation, correlation between present and previous values, is often present. An $AR(p)$ process looks back p points in time to explain the present value and is an expansion of equation ***2.7***. It is written as,

$$z_t = \alpha z_{t-1} + \beta_1 \Delta z_{t-1} + \cdots + \beta_{p-1} \Delta z_{t-p+1} + u_t \tag{2.8}$$

The autocorrelation is now explained in the model, by equation ***2.8,*** instead of being concealed in the error term. Choosing a good estimation of p is also important. Engle and Granger proposes to fit the model with a grid of different values for p and choose a $\hat{p}$ that yields the smallest information criteria ($AIC(p)$ or $BIC(p)$). An information criteria aims to find an optimal model to fit new data and often used as a substitute to cross-validation.

With L as the maximized likelihood function for the $AR(p)$ AIC, an abbreviation for the Akaike information criteria, is defined by equation ***2.9*** below.

$$AIC(p) = 2p - 2\ln(L) \tag{2.9}$$

(Akaike 1974)

When $\hat{p}$ is found the test can be conducted. Though, for testing simplicity the model is often rewritten by subtracting z_{t-1} from both sides of equation ***2.8***. For modeling the residuals from the OLS, equation ***2.5,*** and setting $\gamma = (\alpha - 1)$ gives,

$$\Delta\varepsilon_t = \gamma\varepsilon_{t-1} + \beta_1 \Delta\varepsilon_{t-1} + \cdots + \beta_{p-1} \Delta\varepsilon_{t-p+1} + u_t \tag{2.10}$$

Now $\gamma = 0$ can be tested instead of $|\alpha| = 1$.

So,

$$H_0: The\ residuals\ are\ not\ stationary, \gamma = 0$$

$$H_1: The\ residuals\ are\ stationary, \gamma > 0$$

The Dickey-Fuller test statistic,

$$DF_\tau = \frac{\hat{\gamma}}{SE(\hat{\gamma})} \tag{2.11}$$

The result from the test is reached after comparing the test statistic with the Dickey-Fuller critical values for a level of significance.

So, if H_0, is rejected the Engle Granger test concludes that the series are co-integrated (Engle and Granger 1987).

Trading pairs can be chosen by finding the pairs with highest probability of co-integration. It can also be combined with the distance method, e.g. only considering pairs that are co-integrated at some chosen level of significance.

2.4. Factor models

To identify risk factors and explain why returns for different stocks looks similar in many ways but different in many others a wide range of factor models has been developed. The first models generally focused on macroeconomic factors but later models has taken both a statistical and fundamental approach. The statistical models focuses on historical security returns and fundamental adds information about the security such as size and earnings (Connor 1995).

2.4.1. Arbitrage pricing theory

Since it was proposed by Stephen A. Ross in 1976 arbitrage pricing theory (APT) has been one of the key tools to explain stock return characteristics. It can be seen as an extension of the simplistic capital asset pricing model (CAPM).

APT explains the return, r_i, with k specific factors exposures $(\beta_{i1}, \beta_{i2}, \dots, \beta_{ik})$, return contributions $(r_1, r_2, \dots, r_k)$ and a firms unique return factor r_{ie} that cannot otherwise be explained by the model.

$$r_i = \beta_{i1}r_1 + \beta_{i2}r_2 + \beta_{ik}r_k + r_{ie} \tag{2.12}$$

All k factors are hard to find and estimate. Chen, Roll and Ross (1986) proposes macroeconomic factors such as inflation, different interest rates, industrial production, consumption, oil prices etc. It is also important to stress that factors in ***2.12*** change over time, e.g. oil prices will probably have a much different impact in 100 years from now.

2.4.2. Fama-French three-factor model

With inspiration from APT Eugene Fama and Kenneth French developed the Fama-French three-factor model. Together with the CAPM variables, risk free rate r_f and market risk premium $(r_m - r_f)$, the Fama-French model incorporates two new variables SMB and HML. SMB stands for "**S**mall (market capitalization) **M**inus **B**ig" and HML for "**H**igh (book-to-market ratio) **M**inus **L**ow. They measure the difference in return between small vs. big cap stocks and value vs. growth stocks.

$$r_i = r_f + \beta_{i1}\left(r_m - r_f\right) + \beta_{i2}SMB + \beta_{i3}HML + \varepsilon_i \tag{2.13}$$

The model has historically explained up to 90% of returns from a diversified portfolio and has been widely used in return analysis (Fama and French 1992). Research regarding which of the factors dominates the other is also extensive. Fama-French uses an error term, ε_i, instead of a firm specific factor in APT but both aims to explain movements not attributed to the models other factors.

2.4.3. BARRA fundamental factors model

Named after the company, Barra Inc., where it was developed the BARRA fundamental factors model is set up much like APT and the Fama-French model but with different factors. It focuses on firm specific fundamental factors. A total of 13 factors, one qualitative and twelve quantitative, is used; industry (55 levels), variability in markets, success (logarithmic return from previous year), size, trade activity, growth, price to earnings, price to book, earnings variability, financial leverage, foreign investment, labor intensity and dividend yield. Connor (1995) describes the model in a similar way as equations ***2.12*** and ***2.13***.

In equation form the BARRA model with 13 factors becomes,

$$r_i = \sum_{j=1}^{13} \beta_{ij} * r_j + \varepsilon_i \tag{2.14}$$

(Connor 1995)

2.5. Lasso

The simplest form of regression is called Ordinary Least Square (OLS). The p regression coefficients $(\beta_0, \beta_1, \dots, \beta_p)$ are found by minimizing the residual sum of squares,

$$\sum_{i=1}^{N} \left(y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j \right)^2 \qquad \textbf{\textit{(2.15)}}$$

$N = number\ of\ observations, y_i = response\ value\ for\ observation\ i, p = number\ of\ variables,$

$x_{ij} = value\ for\ variable\ j\ at\ observation\ i$

A problem with OLS is that it, almost certainly, will give nonzero coefficients to all variables used in the regression model. Even if many of them might have no significant relation to the response variable and the effect shown comes from some random impact. The model will then over fit new data and perform bad estimations when new data is used to predict the response variable (James et al 2013, p. 71-75).

Lasso regression aims to decrease the effect of such variables and create a better model for fitting new data. Lasso is labeled as a shrinkage and selection model since it shrinks the coefficients towards zero. Lasso is an acronym for least absolute shrinkage and selection operator.

The coefficients are found in a very similar way to OLS but also takes the size of the coefficients into account. The coefficients are found by minimizing,

$$\sum_{i=1}^{N} \left(y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j \right)^2 + \lambda \sum_{j=1}^{p} |\beta_j| \qquad \textbf{\textit{(2.16)}}$$

The constant λ controls how much the coefficients will be shrunken and is called the tuning or shrinkage parameter. If $\lambda = 0$ the model will naturally yield the same result as OLS and when $\lambda \to \infty$ all coefficients will be zero. The tuning parameter is found by cross validation.

The data is first divided into K parts. Cross validation then leaves one part of the data out of the model, fits the model with the remaining data and then tests it on the data that was left out. The mean squared error is then calculated. This is repeated for all K parts and the weighted mean of all mean squared errors is lastly calculated. Repeating this for different values of λ and choosing the one with smallest MSE gives a good estimation of the best λ. A common choice of K is 10.

Before estimating a lasso model all variables are required to be standardized. This is important since large coefficients are shrunken faster than small. A drawback with Lasso regression is that the coefficients might be hard to interpret. Lasso is therefore viewed as a prediction model and hard to use for inference (James et al 2013, p. 219-222).

2.6. Return metrics

The simple returns is calculated through equation ***2.3a.*** To analyze how the portfolios performed a range of return metrics were calculated. They compared the portfolios and the S&P 500 index. The study used basis points as a measure for differences in returns and rates. One basis point is equal to 0.01% and denoted as 1 bp.

2.6.1. Annualized returns

The produced returns from the model were monthly. Annualizing them is done by,

$$r_a = (1 + r_m)^{12} - 1 \qquad \textbf{\textit{(2.17)}}$$

The annual standard deviation is approximated by,

$$\sigma_a = \sigma_m * \sqrt{12} \qquad \textbf{\textit{(2.18)}}$$

This type of annualization can easily be reversed or adapted to different time horizons by reorganizing equations ***2.17*** and ***2.18*** (Daníelsson 2011, p. 3).

2.6.2. Sharpe Ratio

The Sharpe Ratio was proposed by William Sharpe (1994) as a risk adjusted measure for investment returns. Instead of solely looking at the magnitude of the return the volatility and the risk free interest rate is also taken into account. With the risk free interest rate denoted as r_f the Sharpe Ratio for asset A with returns, $\boldsymbol{r_a}$,

$$S = \frac{\bar{r}_a - r_f}{\sigma_{r_a}} \qquad \textbf{\textit{(2.19)}}$$

The Sharpe ratio is one of the most used risk metrics and often shown for investment alternatives. It is important to keep in mind that the choice of risk free rate will affect the ratio substantially.

2.6.3. Value at risk

Value at risk is a widely used risk measure. It basically states the magnitude of a loss that will not be exceeded for a level of confidence. The confidence can vary much but is seldom under 95% (Daníelsson 2011, p. 76).

2.6.4. Kurtosis

Kurtosis really measures the "peakedness" of a distribution. However, it is often used as a way to find "fat tails" and is common when analyzing stock returns. A normal distribution has a kurtosis equal to 3. For a return distribution a small kurtosis is desirable since it limits the extreme losses (Daníelsson 2011, p. 16).

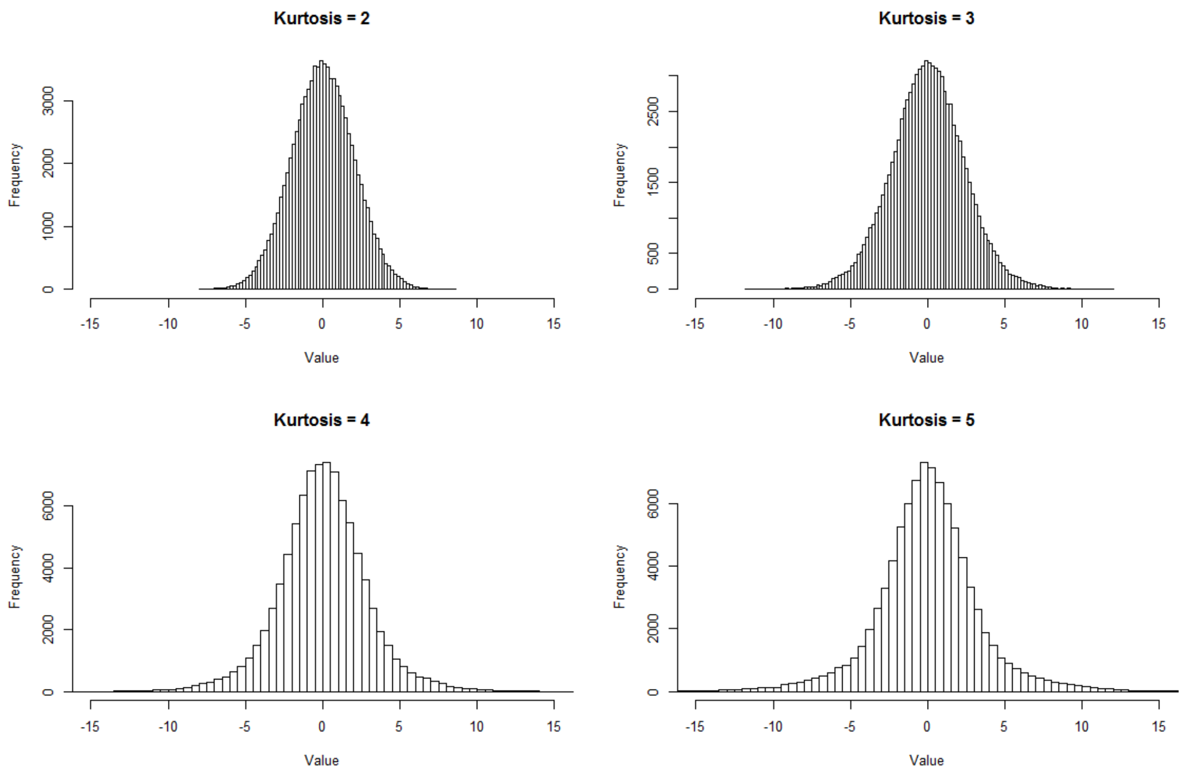

Figure 2.3.
Gaussian distributions with kurtosis of 2, 3, 4 and 5

A distributions level of extreme values, or "fat tails", is clearly dependent on the level of kurtosis. This is distinctly revealed in ***Figure 2.3***.

2.6.5. Skewness

Skewness Is zero for a normal distribution and indicates if a distribution asymmetrical and in which direction the asymmetry is. A distribution is negatively skewed if its mean is less than its mode, most frequent value, and positively skewed if the mean is larger than the mode. (Daníelsson 2011, p. 16).

2.6.6. t-statistic

For testing if the monthly returns, $\hat{r}_m$, are significantly larger than some value, r_{m0}, a one-sided t-test will be performed. From the n monthly returns and their sample standard deviation $s_{\hat{r}_m}$ the standard error is found with equation ***2.20*** and used to get the t-statistic in equation ***2.21***.

$$SE(\hat{r}_m) = \frac{s_{\hat{r}_m}}{\sqrt{n}} \qquad \textbf{(2.20)}$$

So,

$$t_{\hat{r}_m} = \frac{\hat{r}_m - r_{m0}}{SE(\hat{r}_m)} \qquad \textbf{(2.21)}$$

Comparing the t-statistic with critical values for the t-distribution determines the outcome of the test (Marx and Larsen 2006, p. 357).

2.6.7. Correlation with S&P 500

By using equation ***2.4*** monthly return correlation between a pairs trading portfolio and the S&P 500. Developing strategies that have low or negative correlation with the market, i.e. the S&P 500, is a very important field in financial research. The modern portfolio theory developed by Markowitz (1952) shows that this gives investors opportunities to achieve high returns at lower risk levels.

2.7. Short selling

The procedure of taking a short compared with a long position differs much in complexity. Going long in a stock simply means to buy it. Shorting can be described as selling a stock that one not possess. This is done by first borrowing the stock and then selling it to a third party. The stock is bought and returned to the lender to close the short position. The lender can recall the stock but this is very uncommon, especially for large companies (D'Avolio 2002).

2.8. Transaction costs

Transaction costs have historically caused the profitability of pairs trading to decrease considerably.

It is important to remember that the transaction costs are not only made up by commissions. Important factors such as bid-ask spreads and stock liquidity also has to be accounted for. Furthermore, in pairs trading shorting fees must be accounted for as well. Do and Faff (2012) divides the transaction costs for pairs trading into three separate classes. Commissions, market impact (bid-ask-spread, liquidity, price effect of trade) and short selling costs.

The commissions and market impact occurs for every trade while the short selling fee is realized based on the duration of a position.

Gatev et al (2006) estimates a transaction cost for pairs trading to 162 basis points per round trip, opening and closing a position, when only accounting for the market impact (1962-2002). The transaction costs decreased the semiannual profits by 324 basis points. However, Gatev et al notes that this probably is a large estimate of the market impact and also stresses that these costs have decreased over time. However, since commissions and short selling fees are not taken into account the estimate is dubious.

Do and Faff (2012) shows that transaction costs, excluding short selling fees, has dropped from 81 bps (1963-1988) to 33 bps (1989-2009) per one-way trade. This includes, for the latter period, a commission of about 10 bps and a market impact together with bid-ask spread of 23 bps. Since stocks with a large market capitalization tends to be more liquid and also have small bid-ask spreads the S&P 500 offers stocks with relatively small transaction costs.

As this study focuses on an even later period than studied by Do and Faff the transaction costs are estimated to be even lower. Do and Faff examines average transaction costs for all stocks on NASDAQ and NYSE. This study focuses on the largest stocks with generally lesser transaction costs compared to small stocks their estimate will overestimate the true transaction costs.

To short a stock it first has to be borrowed. The borrowing naturally comes with interest and is what makes up the shorting fee. Do and Faff (2012) uses the extensive research by Gene D'Avolio (2002) on short selling costs and constraints to estimate the fee.

D'Avolio starts by criticizing what he calls "arbitrageurs" that assumes that any stocks can be shorted without cost and bases strategies on these unfettered assumptions. D'Avolio concludes that restrictions on short selling is very uncommon among large stocks and that over 99 % of the S&P 500 constituents could be shorted (4/2000-9/2001). Recalling of borrowed S&P 500 stocks is also found to be very uncommon.

D'Avolio finds the borrowing fees associated with short selling to vary much depending on price and market capitalization. For small risky stocks the fee can reach up to 55 % per annum in extreme cases. However, of the 7 879 stocks studied 91 % could be borrowed for an annual interest rate under 1 %. The mean rate was 0.6%, used in Do and Faff (2012), and for S&P 500 stocks it was estimated to 0.17 %.

3. Fundamental data

All price and fundamental data were obtained from Thomson Reuters Datastream using the license of Umeå University.

3.1. Stock screening

The stocks were chosen to reflect the S&P 500 as good as possible. Since accurate historical constituent lists proved hard to obtain the exact collection of stocks from the index could not be used. The S&P 500 constitutes is the, by market capitalization, top 500 stocks listed at NYSE and NASDAQ. The constituents can be replaced at any time due to market cap changes, de-listings, mergers or acquisitions (Standard & Poor Dow Jones Indices 2015).

To find stocks to trade conditions to both satisfy the S&P restrictions and the data requirements for the analysis were arranged.

Apart from being listed at one of the two big American stock exchanges the stocks had to satisfy a few criteria to make the analysis possible. First, to be able to calculate the historical co-integration the stocks must have at least one year of historical price data before the trading period. Second, every stock has to have complete stock fundamental data for the variables that is explained later and can be seen in ***Table 3.1***.

It also has to be possible to short sell the stock. This is often not the case for small stocks with low liquidity but the stocks used are the 500 largest and it is, as explained in ***2.7*** and ***2.8***, reasonable to assume that all of them can be shorted.

Lastly, to mirror the S&P 500, the 500 stocks with highest market capitalization that satisfies the conditions above were chosen as possible parts of a pair. These 500 stocks will not mirror the S&P 500 exactly but give a good estimation of the true historical constituents.

3.2. Fundamental data collection

The fundamental data were collected just before the start of every trading period. The trading periods start at the first of April and October every year. This is chosen to obtain newly updated fundamental data.

In accordance with SEC regulation annual reports, Form 10-K, must be released no later than 90 days after the end of the fiscal year. Quarterly reports, Form 10-Q, must be released no later than 45 days after the end of a fiscal quarter (Securities and Exchange Commission 2015).

So, this reporting periods and trading start dates guaranteed that a relatively new annual or quarterly report would be available when the fundamental data were collected.

3.3. Fundamental data selection

Table 3.1. Fundamental variables			
Abbreviation	***Variable***	***Explanation***	***Type***
MV	Market Value	Total market value for company	Quantitative
PB	Price to Book	Stock price divided by book value per share	
PE	Price to Earnings	Stock price divided by earnings per share	
DY	Dividend Yield	Dividend per share as a percentage of the share price	
DEBT	Debt to Total Capital	(Long Term Debt + Short Term Debt & Current Portion of Long Term Debt) / (Total Capital + Short Term Debt & Current Portion of Long Term Debt) * 100	
EXC	Exchange	NASDAQ or NYSE	Qualitative
Level1	Industry	From FTSE Industrial Classification benchmark. (four levels)	
Level2	Super sector		
Level3	Sector		
Level4	Sub sector		
GEO	Geography Group	Country	

The variables shown in ***Table 3.1*** above were used alongside the measures from the distance and Engle-Granger methods to create the portfolios. In previous studies industry restrictions has been common. Gatev et al (2006) did this with dividing stocks into four different industry groups (Financials, Utilities, Industrial and Transportation). Do and Faff (2012) expanded this to use a total of 48 different sector groups. The FTSE industrial classification benchmark (ICB) offers four different levels of classification with 10 industries, 19 super sectors, 41 sectors and 114 subsectors (FTSE 2012). Using the information ICB provides in pairs trading strategies develops the analysis done by Gatev et al and Do and Faff further.

Researchers have found some price patterns when considering the other variables as well. The studies tries to find factors that explain stock returns. If factors are found to have an effect on returns there is a chance they might be able to explain parts of the co-integration. The more underlying factors that a stock pair have in common the less likely it should be that their prices would diverge substantially. Some factors might work as proxy factors for a real factor underlying factor.

Researchers such as Rolf W. Banz (1981) has shown that small firms historically has performed better than large firms. However, it is not a clear why the size effects exist or if it is a proxy effect for some other variable. With such background, differences in size were deemed to be an interesting factor to consider when pairing stocks.

Price to earnings has for a long time been used to measure if a stock is under or overvalued. A small P/E points to a low valuation. Fama and French (1996) includes the ratio in their multifactor model to explain asset pricing anomalies.

Price to book (P/B), or book to market when reversed, is another way to measure the valuation. Stock portfolios based on P/B has shown similar price momentums as industry portfolios according to Lewellen (2002).

Fama and French (1992) found that leverage, or debt ratio, together with P/E, P/B and company size is one of the factors that have empirical evidence that suggests explanatory power in stock returns.

Dividend yield has also been shown to have some explanatory power to predict future stock returns. Investing in high dividend payout stocks is also a common strategy among investors (Wilkie 1993).

All of the above are included in the 13 BARRA fundamental factors investigated by Connor (1995). It would have been interesting to have included more variables from the BARRA model but Datastream did not provide complete information for the other variables for a large amount of stocks. Including those variables would have forced a substantial amount of stocks to be omitted from the study.

Except for the variables already explained qualitative variables for exchange and country were also included. BARRA assumes that all companies is geographically alike. Investigating this can therefore also be interesting.

NASDAQ and NYSE are very similar. However, technology companies are usually listed on NASDAQ and large industrial companies at the NYSE. These differences should be covered by the industry classifications. Bennett and Wei (2005) identifies some differences in how trades are conducted as well.

Models like BARRA tries to predict return characteristics. Using those factors to predict co-integration between stocks will necessarily not give the same explaining power. Instead, choosing stocks based on these factors, it is more likely that stock pairs with correlated returns are produced.

As seen in ***Figure 2.2*** correlation in returns does not necessarily give a good pair for pairs trading.

3.4. Variable comparison

When creating pairs the stocks' fundamental data were compared. The qualitative variables, i.e. GEO, EXC and the industry levels, were transformed to dummies where the pair gets a 1 if both stocks belong to the same group and 0 otherwise. This is one of the standards set by Gatev et al (2006).

The quantitative variables were compared by taking the absolute difference. Since there is no specific first or second stock in the pair the difference has to be absolute. No pairs trading standards for this comparison has been set since no previous research on qualitative variable comparison has been conducted. However, the choice of comparative measure is here somewhat arbitrary.

The measures from the distance and Engle-Granger methods are numeric and calculated based on one year of historical price data.

3.5. Pairs

500 stocks might not seem as a very large amount of stocks but when they are put together one by one a massive amount of pairs is created. When setting up groups of size, k, from a population of size, n, the total number of groups is explained by the binomial coefficient $\binom{n}{k}$, i.e. *"n choose k".* (Sprugnoli 2006, p 15).

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} \tag{3.1}$$

Using equation ***3.1*** on a population 500 and 2 as group size,

$$\binom{500}{2} = \frac{500!}{2!\,(498)!} = \frac{500 * 499}{2} = 124\,750$$

So, from 500 stocks a total of 124 750 unique pairs can be created.

3.6. Portfolio composition

Table 3.2. Portfolios with single restrictions			
Portfolio	#	Explanation	Restriction type
Baseline	1	No restriction	---
Level1	2	ICB level 1 (Industry)	Group
Level2	3	ICB level 2 (Super sector)	
Level3	4	ICB level 3 (Sector)	
Level4	5	ICB level 4 (Sub sector)	
GEO	6	Geography group	
EXC	7	Exchange	
EG	8	Engle-Granger test (5%)	Test
DY50	9	DY 50%	Absolute difference
DY25	10	DY 25%	
PE50	11	PE 50%	
PE25	12	PE 25%	
MV50	13	MV 50%	
MV25	14	MV 25%	
DEBT50	15	DEBT 50%	
DEBT25	16	DEBT 25%	
PB50	17	PB 50%	
PB25	18	PB 25%	

Table 3.2 shows the portfolios set up by one restriction each. The portfolios were created by first finding all stock pairs that satisfies the restriction. Then, for all pairs that satisfies the restriction the top 20 pairs, based on the distance method, formed the portfolio.

Portfolio 1 has no restriction and its' stock pairs were chosen by the distance method, equation ***2.5***, applied on all stock pairs.

Portfolios 2-7 restricts pairs to only be formed by stocks belonging to the same classification group for each variable before applying the distance method.

The Engle-Granger test were used to find pairs that were co-integrated, during the formation period, at the 5 % significance level**.** Of the co-integrated pairs the top 20 is found by using the distance method. So, the test were not used by itself to find the top 20 pairs but instead as a way to restrict pairs before applying the distance method

The pairs that forms portfolios 9-18 is restricted based on the qualitative variables. The pairs got a value based on the absolute difference in the respective variables. For example, if the P/E for Coca Cola is 26 and for Pepsi 20 the pair will get a value of 6, the absolute difference. Every pair received a value like this and portfolio PE25 choses the 25% pairs with smallest difference as possible constituents of the portfolio. For PE50 50% of the pairs are chosen instead. After this is done the distance method finds the top 20 pairs. This process is shown below in ***Figure 3.1***.

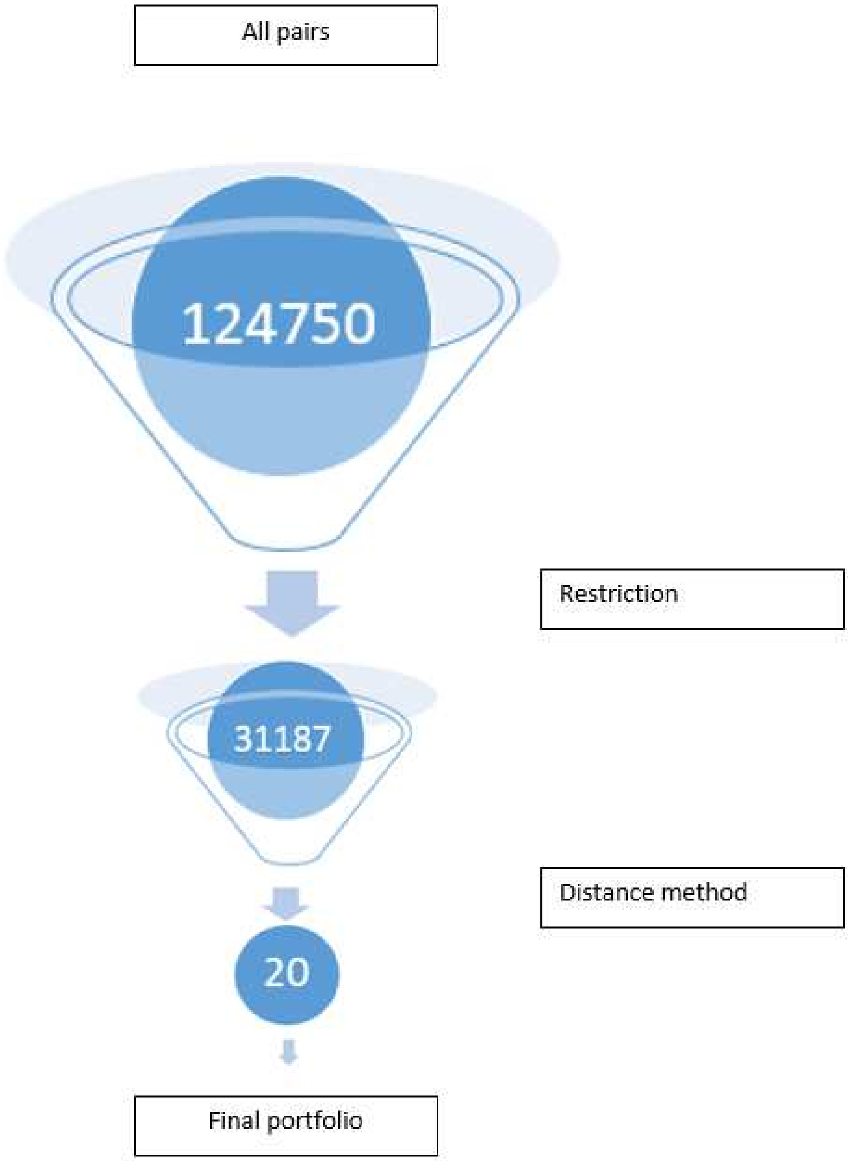

Figure 3.1.
Portfolio formation. Restriction of type 25% is used which chooses the 25% pairs with smallest abs. difference in a variable.
$\mathbf{25\%\ of\ 124750 \approx 31\ 187}$

Restrictions from portfolios 2-18 were also combined pairwise, except within the ICB levels and the 25 and 50% restrictions for the same variable. Only the four best of these will be presented. In terms of the set-up of ***Figure 3.1*** this simply added one more step of restriction before the distance method were applied.

When implementing restrictions the total amount of pairs considered decreases and pairs with higher historical SSD will be traded. When the restrictions gets very tight the risk of choosing pairs without strong signs of co-integration gets obvious.

The Lasso model used all variables from Table 3.1 and the formation period measures for SSD and the significance level for the Engle-Granger test statistic (equation ***2.11***) to fit the model. The observations came from the top 1000 pairs, by the distance method, from the previous four trading periods. This choice is somewhat arbitrary but gives the model a large amount of data that is both reflective and up to date.

So, a total of 4000 observations of both the explaining variables and the response, Profit or SSD for the corresponding trading period, were used to fit the models.

Since four previous trading periods were used to fit the Lasso the period from 1999-04-01 to 2001-04-01 will only be reserved for this. The testing of the strategies therefore begins at the first of April 2001.

The Lasso portfolios were formed based on the magnitude of a pairs predicted profit or the predicted SSD during the trading period. The 20 pairs with highest predicted profit and lowest predicted SSD formed the portfolios. To avoid pairs with low chance of co-integration and, most important, to use similar pairs to those that were used to fit the model only the 1000 pairs with lowest SSD is used. The process is shown in ***Figure 3.2*** below.

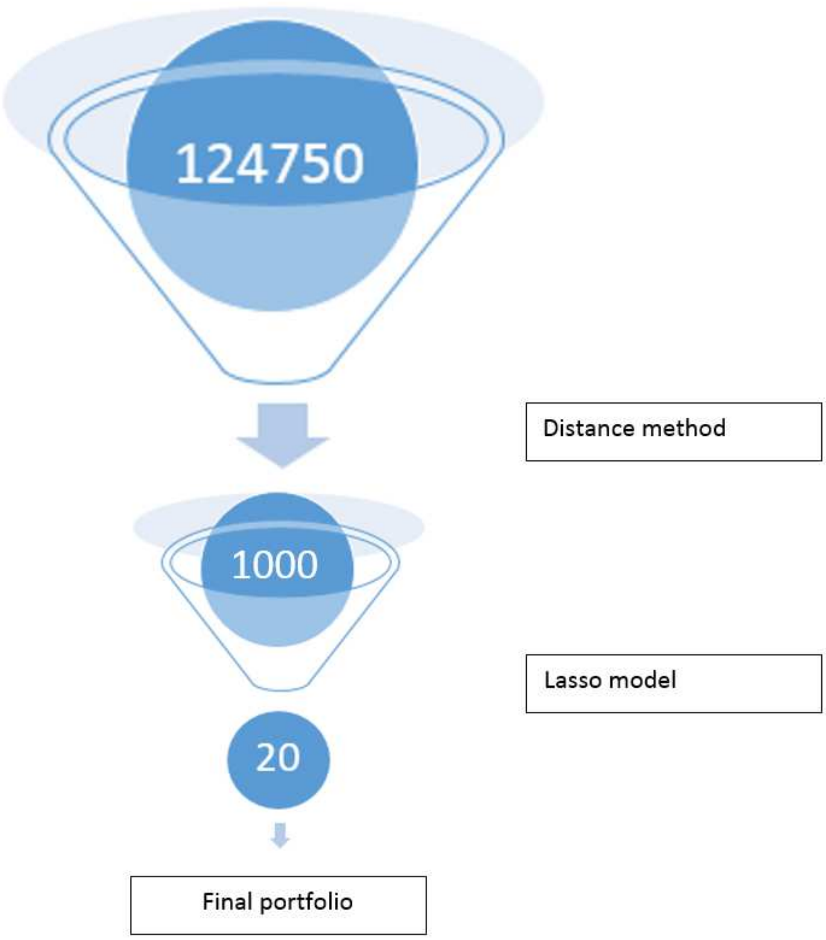

Figure 3.2.
Portfolio formation using Lasso model

Different variables were found to be significant at different periods for the. The relations to the response were at times negative and at others positive. This kind of inconsistency made it hard to draw conclusions about the true relationships between the variables. Prediction could still be powerful however.

4. Methodology

The 500 stocks that satisfies the conditions presented in ***3.1*** creates the total 124 750 possible trading pairs. By applying a set of conditions, ***Table 3.2***, portfolios with the top 20 pairs based on the distance method, equation ***2.5***, were formed.

Trading periods starts at the first of April or October and lasts for six months. The first trading period began 2001-04-01 and the last ended 2015-04-01. This gave a total of 28 trading periods for an overall duration of 14 years.

The strategies were tested using both 1 and 2 historical standard deviations as trading thresholds.

<table>
<tr><th colspan="7">Table 4.1. Strategy time design</th></tr>
<tr><th>Year</th><th colspan="2">Year 1</th><th colspan="2">Year 2</th><th colspan="2">Year 3</th></tr>
<tr><td>Period 1</td><td colspan="2">Formation</td><td>Trading</td><td></td><td></td><td></td></tr>
<tr><td>Period 2</td><td></td><td colspan="2">Formation</td><td>Trading</td><td></td><td></td></tr>
<tr><td>Period 3</td><td></td><td></td><td colspan="2">Formation</td><td>Trading</td><td></td></tr>
<tr><td>Period 4</td><td></td><td></td><td></td><td colspan="2">Formation</td><td>Trading</td></tr>
</table>

Table 4.1 illustrates how formation and trading periods relates to each other. After one year of formation a portfolio is traded for six months. The formation periods overlaps each other with six months. The Lasso models used four formation and trading periods to form the portfolio. Therefore it uses data from three years.

All portfolios were restricted to consist of 40 unique stocks forming 20 pairs. For example, if Microsoft (MSFT) is part of two of the top twenty pairs of the baseline portfolio the pair with higher SSD were omitted and the 21'st best pair chosen instead. This control is made until 40 unique stocks forms the portfolio.

To account for splits, dividends and other unwanted effects on the price series adjusted prices were used.

From the research presented in ***2.8*** 20 bps per trade were chosen as the transaction cost. Since there is a total of four trades for each opening and closing of a position this gives a transaction cost of 80 bps or 0.8% per traded pair. 20 bps might be an underestimation, but not by much. It still highlights the effect of transaction costs on the strategy. Also, in line with the previous research a borrowing cost of 17 bps

So, the total transaction cost estimate for pairs trading is 80 bps per pair trade, consisting of commissions and market impact, together with an annualized short selling fee of 17 bps.

All programming has been performed in R.

4.1. Formation

The formation period begins one year prior to the start of trading and ends on the day before trading. For a stock pair both prices were first normalized to begin at 1 at the start of formation. These series were then analyzed with the distance and Engle-Granger methods. The normalized price series and spread between Commerce Bankshares Inc. (CBSH) and Regions Financial Corp. (RF) is shown below in ***Figure 4.1***.

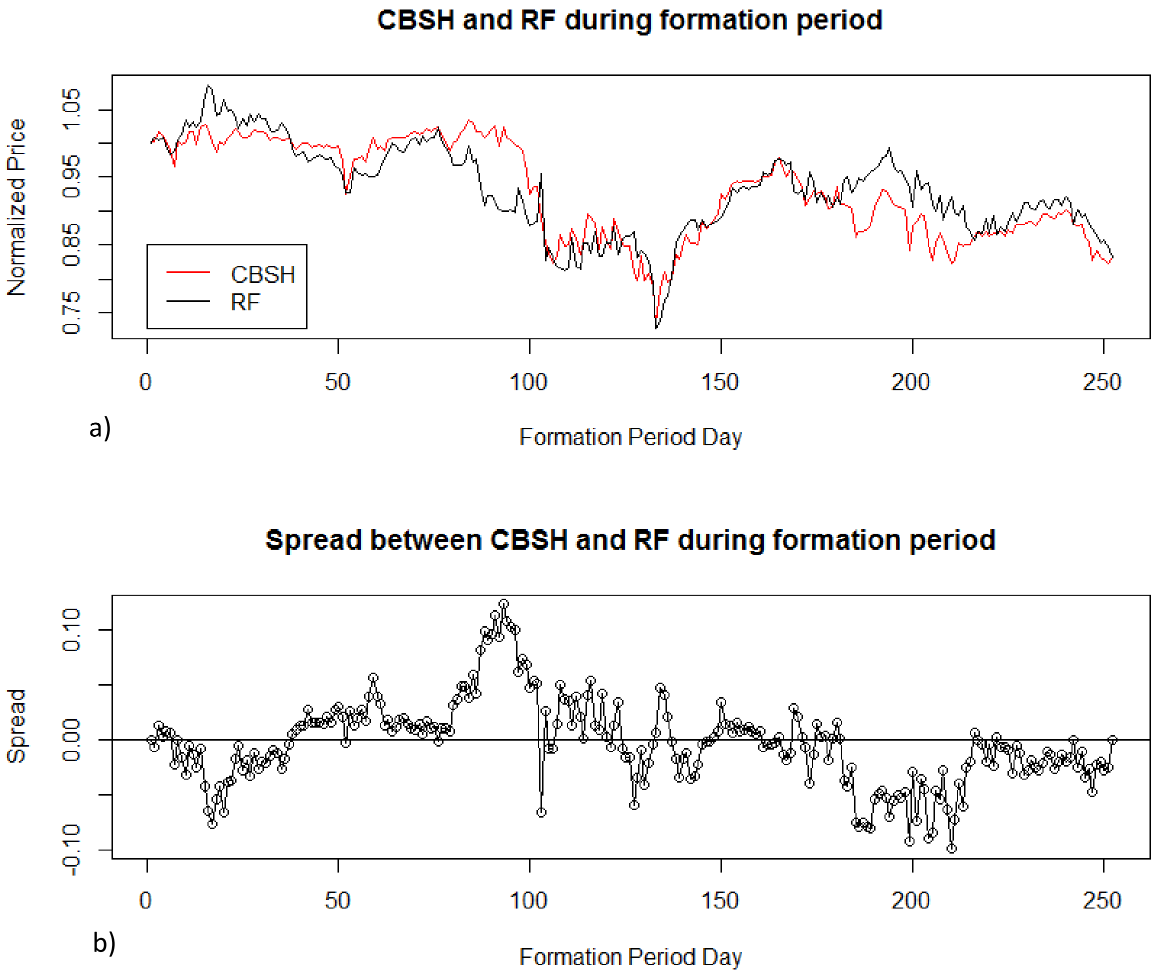

Figure 4.1.
Normalized price spread between Commerce Bankshares Inc. (CBSH) and Regions Financial Corp. (RF).
a) Normalized price series. Red = CBSH, Black = RF
b) Spread between normalized price series
Time. 1998-04-01 to 1999-04-01

As described in ***2.3.1.*** and by equation ***2.5*** the distance method orders stock pairs based on SSD. The normalized prices of CBSH and RF evolves closely with very similar trends. This gives a very small SSD and the distance method would rank the pair high.

To determine if the pair is co-integrated the Engle-Granger method for co-integration were used. ***Figure 4.2*** below shows the residuals from the Engle-Granger test. To reject that the series is not co-integrated the residuals from the OLS performed on the price series has to be stationary.

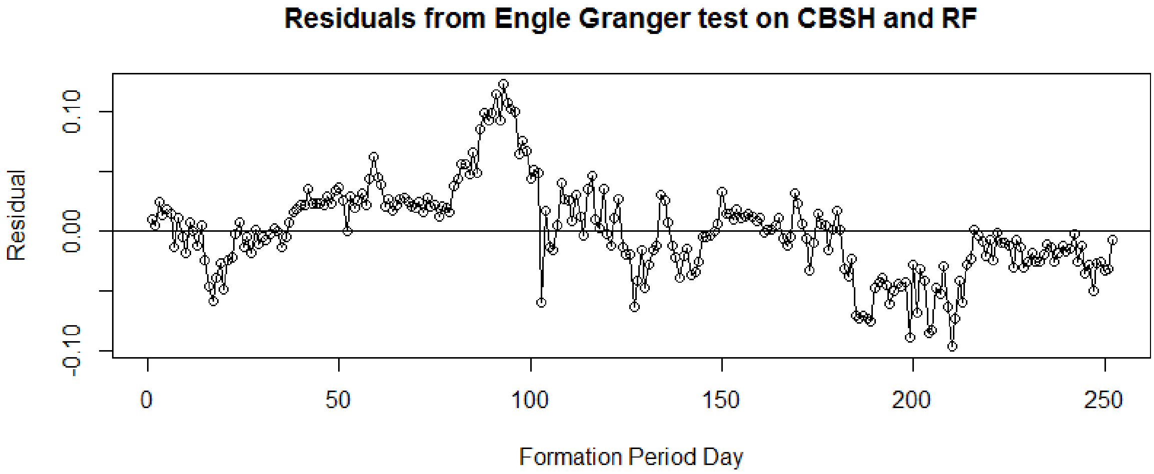

Figure 4.2.
Residuals tested for stationarity in Engle-Granger test on CBSH and RF
Time. 1998-04-01 to 1999-04-01

The residuals in ***Figure 4.2*** shows strong signs of stationarity and the test for this pair concludes that the price series were co-integrated at the 5% significance level.

Besides the price series analytics stock pairs were also analyzed based on their fundamental data relations. ***3.6.*** outlined exactly how the restrictions were created.

The differences between the distance and Engle-Granger method is illustrated in the two previous figures. The spread in ***Figure 4.1b*** is very similar to the residuals in ***Figure 4.2.*** For stock pairs with small SSD this will always be the case.

The distance method finds the stock pairs with smallest squared sum of the spread values and the Engle-Granger test finds the stock pairs with a stationary residuals from the OLS performed on the price series.

4.2. Trading

After identifying a pair the strategy is very straightforward. First, the two trading thresholds were defined. Usually they are a multiple of the historical standard deviation of the spread. In this study one and two standard deviation has been used as thresholds. For simplicity they will be denoted as threshold of 1 and 2. If the spread moves under the lower or over the upper threshold a short position were taken in the overperforming stock and a long in the underperforming stock.

Since the testing is performed on adjusted daily closing prices intraday threshold crossings will not result in a trade if the spread moved back before closing.

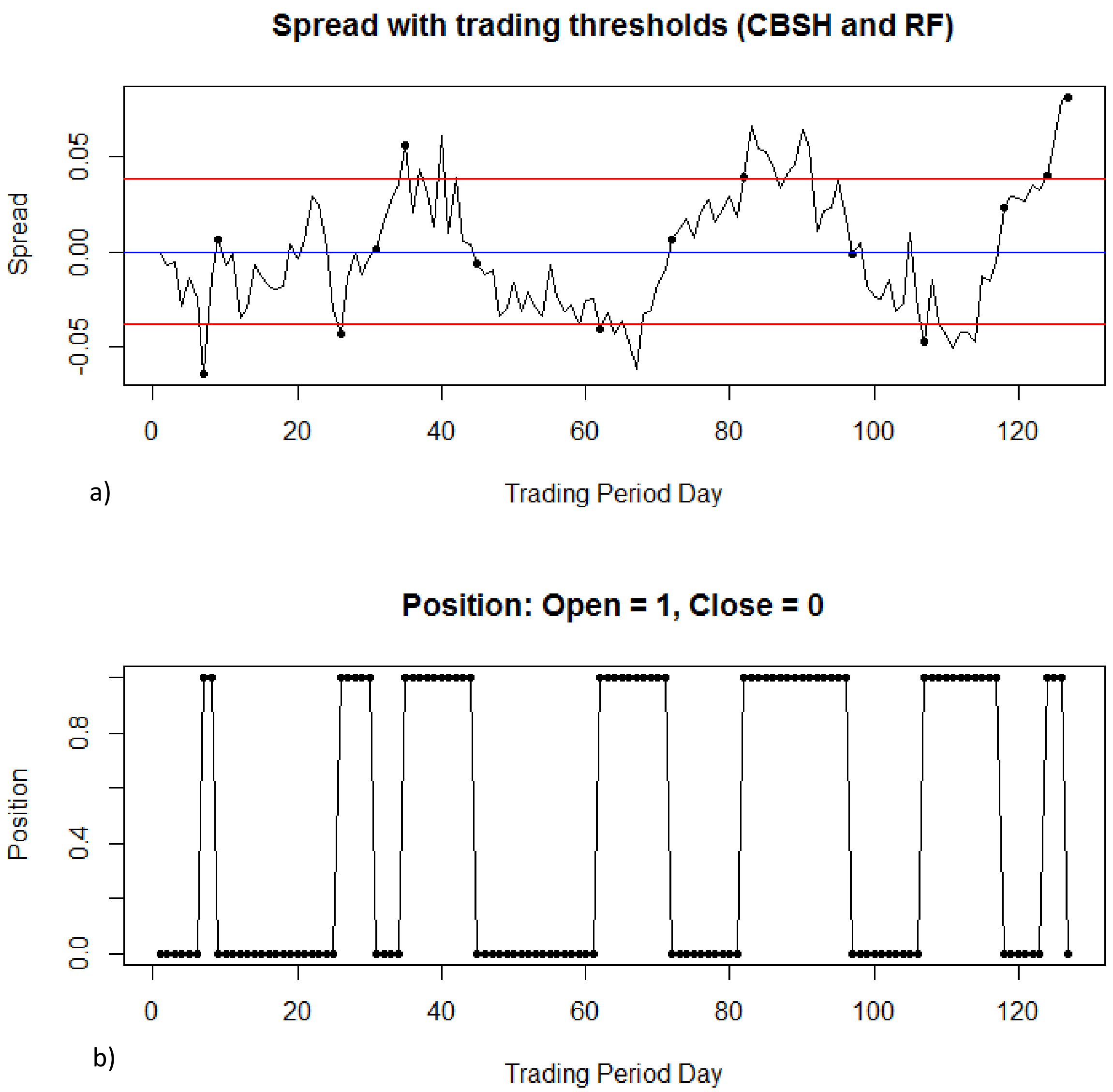

Figure 4.3.
Illustration of trading between Commerce Bankshares Inc. (CBSH) and Regions Financial Corp. (RF).
a) Spread with trading thresholds (+/- one historical standard deviation). Trade at dots.
b) Position, 1=open, 0= closed
Time. 1999-04-01 to 1999-10-01

Part (a) of ***Figure 4.3*** shows the spread, after normalizing prices, between CBSH and RF. The red lines represents trading thresholds for opening a position and the blue line when an open position is closed. The filled dots shows when a trade has been executed. Part (b) of the figure illustrates when the position were open and closed (1=open, 0= closed). The position were opened seven times. The last position were closed at the end of the trading period and gives a negative cash flow. All other trades naturally gave a positive cash flow. To have seven trades during a trading period is considered as very successful.

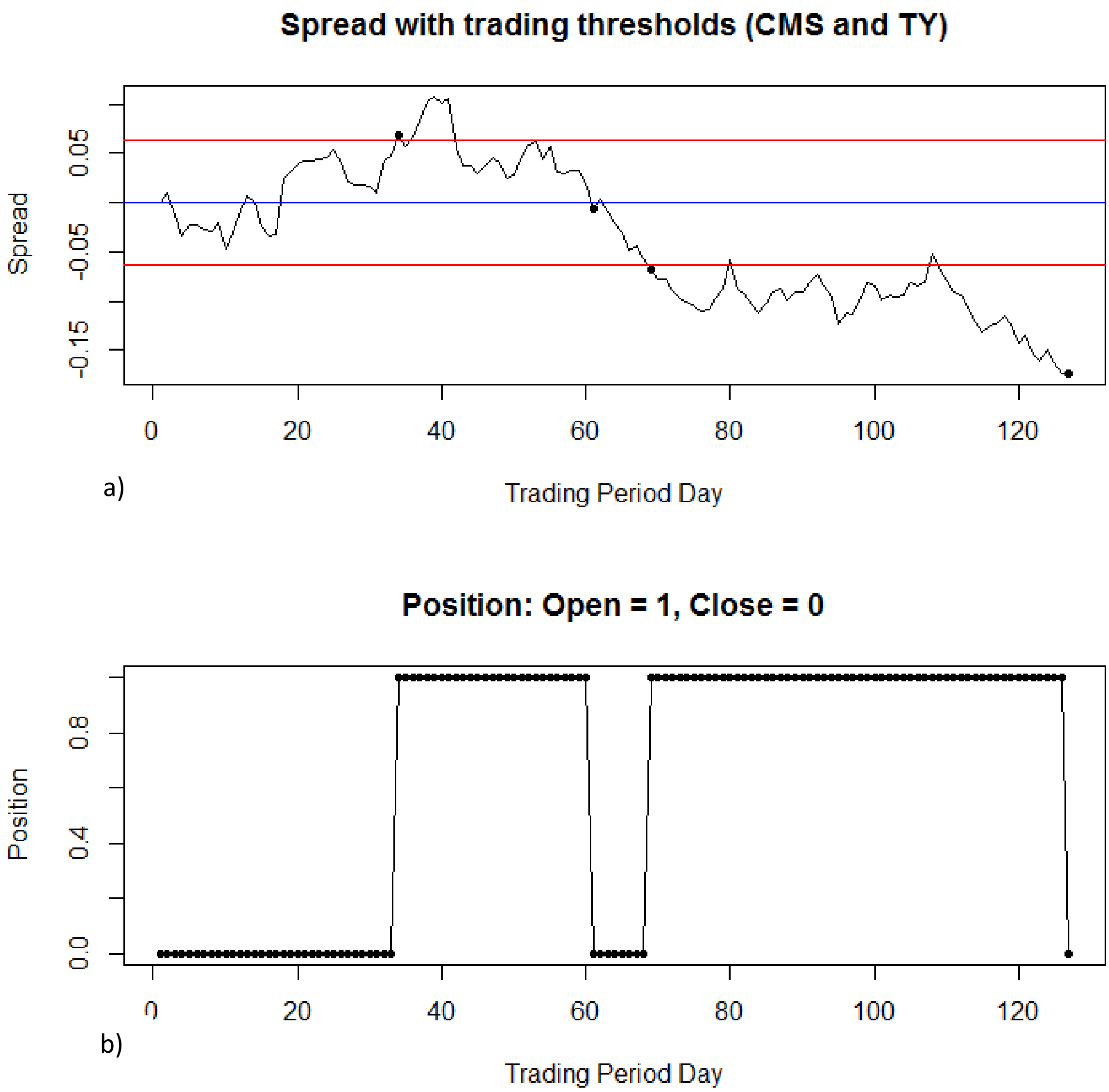

Figure 4.4.
Illustration of trading between CMS Energy Group (CMS) and Trico (TY).
a) Spread with trading thresholds (+/- one historical standard deviation).
b) Position, 1=open, 0= closed
Time. 1999-04-01 to 1999-10-01

Figure 4.4 shows the same thing as ***Figure 4.3*** but for CMS Energy Group (CMS) and Trico (TY). This pair gave a loss since the spread deviated considerably towards the end of the trading period. This also highlights how all pairs certainly won't be profitable and how the number of trades can vary.

4.3. Return computation

Calculating returns for back-testing on pairs trading presents two problems. What capital commitment to use and how to extract the returns from the back-test.

When calculating the Sharpe Ratio (equation ***2.19***) the average (2001-04-01 to 2015-04-01) of the 3-month treasury bill rate were used as risk free rate and estimated to 1.47%.

4.3.1. Capital commitment

Gatev et al (2006) used two different measures for returns. First, return on committed capital calculates the return on the capital that is committed to the pairs at the beginning of the trading period. Second, return on actual employed capital which omits capital employed to pairs that had no trades. The first approach is more conservative but if the threshold isn't very high almost all pairs trades at least once.

Using return on actual employed capital demands that capital can be moved almost instantly between different portfolios. This can actually be the case at a hedge fund where these kind of strategies is most common. Different strategies will probably not need capital at the same time so moving capital between them will be possible at many occasions.

A third, very optimistic, way to calculate returns is proposed in this study. Looking at the number of days a pair has an open position and compounding the returns to monthly afterwards gives a return measure for return on actual employed capital on a daily basis. This assumes that capital can move to and away from the strategy instantaneously without any buffers. Also, since all pairs won't be open at the same time capital can move between pairs.

Moving capital this fast is not possible and this return measure will certainly overestimate the true profits. However, both approaches proposed by Gatev et al (2006) will underestimate the returns for a dynamic hedge fund. The truth lays undoubtedly somewhere in between. For simplicity actually employed capital on a daily basis will be denoted as, simply, employed capital. Another way to calculate returns is to assume that the capital when a pairs position is closed is invested in some very liquid alternative.

Return on committed capital and return on actually employed capital on a daily basis were therefore used as the two methods to calculate return. This will produce the complete range of returns with ***return on committed capital*** as the lower limit and ***return on employed capital*** as the upper limit.

For example, if a pair is open 15 ($t_{employed}$) of 21 (t_{total}) days during one month and gives a trade profit of \$0.05 from the \$1 invested the return on committed capital (cc) is simply given by equation ***2.3a***.

$$r_{cc} = \frac{1.05 - 1}{1} = 5\%$$

To find the return on employed capital (ec) equation ***2.17*** is used with daily and monthly data.

$$r_{ec} = (1 + r_{cc})^{\frac{t_{total}}{t_{employed}}} - 1 = (1 + 0.05)^{\frac{21}{15}} - 1 \approx 7.1\%$$

This illustrates the potentially big differences between the return metrics.

4.3.2. Extracting returns

For simplicity a pair is thought to start with an investment of \$1. After a successful trade with a profit of 5% the pair will have a value of \$1.05. When the spread crosses the threshold the next time all \$1.05 were invested.

Table 4.2. Cash flow from trading with CMS and TY 1999-04-01 to 1999-10-01					
Trading Day	***Trade profit***	***Shorting fee***	***Total Transaction cost***	***Value***	***Position***
1	0	0	0	1	Close
34	0	0	0.0040	0.9960	**Open**
61	0.0754	0.0002	0.0045	1.0669	Close
69	0	0	0.0043	1.0626	**Open**
127	-0.1117	0.0004	0.0042	0.9467	Close

Table 4.2 explains how the profit, transaction cost and value of the portfolio evolves over the trading period for CMS and TY (***Figure 4.4***). The value from the previous time step, after subtracting the transaction cost, is the capital that is invested when a position opens.

So, at day 34 \$0.996 were invested and grows to \$1.0669 at day 61 when the position closed. The shorting fee depends on how long the position is open, since the duration was only 27 days the fee becomes very small. When the second position were opened again at day 69 the profit from the previous trade adds to the capital and \$1.0626 could be invested instead. Since the amount invested has increased the transaction cost rises as well. The duration of the position for the second position was 58 days, leading to a shorting fee of \$0.0004. Instead of reverting back to zero the spread deviates further and the pair closes with a loss.

To obtain monthly returns the value of the portfolio were calculated after each month. If a position is opened its' value is calculated by comparing the spread when the position opened with the up-to-date spread.

5. Results

The result is divided into parts. First the returns, on both committed and employed capital, are presented for portfolios with single restrictions (1-18). This is done for trading threshold on one and two standard deviations.

This way of presenting the result is repeated for portfolios with multiple restriction, based on Lasso models, restricted to unique stocks and without transaction costs.

The results will lastly be compared to the Standard and Poor's 500 index.

All portfolios were tested between 2001-04-01 and 2015-04-01 with a total of 28 six-month trading periods.

5.1. Single restrictions

Returns for threshold of 1 is presented in ***5.1.1*** and 2 in ***5.1.2***.

5.1.1. Threshold = 1

Table 5.1. Annual returns on committed capital for portfolios with single restriction and threshold = 1

Portfolio	#	$\bar{r}_a$	σ_{r_a}	S	$Open$	$Trades$	$Rank(\bar{r}_a)$	$Rank(S)$
Baseline	1	0.0130	0.0459	-0.0371	0.7681	2.8554	9	9
Level1	2	0.0218	0.0508	0.1396	0.7440	2.6786	5	5
Level2	3	0.0165	0.0474	0.0370	0.7352	2.6018	7	8
Level3	4	0.0187	0.0484	0.0823	0.7346	2.5714	6	6
Level4	5	0.0244	0.0501	0.1927	0.7276	2.5339	4	3
GEO	6	0.0128	0.0528	-0.0362	0.7678	2.8321	10	10
EXC	7	0.0045	0.0506	-0.2024	0.7637	2.6000	16	16
EG	8	0.0255	0.0548	0.1964	0.7864	3.0321	3	4
DY50	9	0.0158	0.0495	0.0225	0.7509	2.7446	8	7
DY25	10	0.0088	0.0549	-0.1078	0.7504	2.6589	14	15
PE50	11	0.0302	0.0483	0.3201	0.7586	2.8375	2	2
PE25	12	0.0365	0.0533	0.4096	0.7534	2.7839	1	1
MV50	13	0.0083	0.0615	-0.1037	0.7833	2.6304	15	14
MV25	14	0.0094	0.0649	-0.0817	0.7686	2.6625	12	11
DEBT50	15	0.0041	0.0474	-0.2232	0.7547	2.8036	17	17
DEBT25	16	-0.0018	0.0507	-0.3263	0.7416	2.6857	18	18
PB50	17	0.0098	0.0467	-0.1050	0.7491	2.7964	13	12
PB25	18	0.0103	0.0519	-0.0855	0.7492	2.6839	11	13

$\bar{r}_a$ = annual return (equation 2.17), σ_{r_a}= annual volaility (equation 2.18), S = Sharpe ratio (equation 2.19),
Open = Average fraction of time a pair is open for the portfolio,
Trades = Average number of trades per period per pair.

When trading with a threshold of 1 the baseline portfolio, chosen by the distance method, yielded an average annual return on committed capital of 1.3% as shown in ***Table 5.1***. It was outperformed by the portfolios restricted on P/E and all four different ICB levels. Also, when restricted to choose the 50 % pairs with closest DY a return above the baseline portfolio was realized. Using the Engle-Granger test for co-integration also gave a higher return.

The two portfolios with P/E restrictions lead to, by quite a wide margin, the highest returns. PE50 yielded 3.78% and PE25 3.65%.

The standard deviation did not vary by much which made the ordering of portfolios by the Sharpe Ratio to follow the return relations closely. The baseline portfolio has the lowest standard deviation in returns and MV25 the highest.

Positions were open about 75% of the time and on average a little less than three trades were conducted for each pair during a trading period.

Table 5.2. shows the corresponding monthly returns below.

Table 5.2. Monthly returns on committed capital for portfolios with single restrictions and threshold = 1										
Portfolio	#	$\boldsymbol{r}$	$\boldsymbol{Max}$	$\boldsymbol{Min}$	$\boldsymbol{\sigma_r}$	$\boldsymbol{Skew.}$	$\boldsymbol{Kurt.}$	$\boldsymbol{VaR_{95}}$	$\boldsymbol{r < 0}$	$\boldsymbol{\rho(r, r_{S\&P500})}$
Baseline	1	0.0011	0.0492	-0.0385	0.0133	0.3810	4.2727	-0.0185	0.4762	-0.1504
Level1	2	0.0018	0.0587	-0.0390	0.0147	0.7291	4.9225	-0.0216	0.5119	-0.0711
Level2	3	0.0014	0.0538	-0.0419	0.0137	0.3511	4.1041	-0.0198	0.5060	-0.1054
Level3	4	0.0015	0.0528	-0.0401	0.0140	0.4309	4.0010	-0.0184	0.4821	-0.1661
Level4	5	0.0020	0.0506	-0.0349	0.0145	0.4110	4.1622	-0.0219	0.4702	-0.1854
GEO	6	0.0011	0.0658	-0.0518	0.0152	0.3152	4.8011	-0.0208	0.4821	-0.1644
EXC	7	0.0004	0.0701	-0.0457	0.0146	0.4447	5.9692	-0.0198	0.4940	-0.1002
EG	8	0.0021	0.0611	-0.0431	0.0158	0.5248	4.3852	-0.0228	0.4405	-0.1258
DY50	9	0.0013	0.0485	-0.0423	0.0143	0.2066	4.1651	-0.0197	0.4762	-0.0293
DY25	10	0.0007	0.0879	-0.0354	0.0158	1.1758	8.3750	-0.0259	0.5357	-0.0042
PE50	11	0.0025	0.0463	-0.0517	0.0139	0.1335	4.4049	-0.0175	0.4464	-0.1071
PE25	12	0.0030	0.0506	-0.0495	0.0154	0.0679	3.7847	-0.0207	0.4226	-0.1379
MV50	13	0.0007	0.0538	-0.0547	0.0178	0.3431	4.1208	-0.0249	0.5536	-0.1159
MV25	14	0.0008	0.0608	-0.0419	0.0187	0.2770	3.4798	-0.0305	0.4881	-0.1783
DEBT50	15	0.0003	0.0617	-0.0337	0.0137	0.9135	5.4569	-0.0190	0.5417	-0.1128
DEBT25	16	-0.0002	0.0520	-0.0536	0.0146	0.2278	5.4834	-0.0218	0.5000	-0.0392
PB50	17	0.0008	0.0421	-0.0401	0.0135	0.2937	3.9272	-0.0208	0.5238	-0.0704
PB25	18	0.0009	0.0509	-0.0595	0.0150	-0.2771	4.5664	-0.0224	0.4762	-0.1635

r = monthly return (equation 2.3a), σ_r= monthly volatility (equation 2.3b), Skew. = Skewness (part 2.6.4), Kurt. = Kurtosis (part 2.6.4), VaR_{95} = Value at Risk 95% c.l (part 2.6.3), ($r < 0$) = Fraction of negative returns, $\rho(r, r_{S\&P500})$ = Correlation with S&P 500 (equation 2.4)

All portfolios had a small negative return correlation with the S&P 500. Most portfolios produced about negative returns in 50% of the months. The monthly VaR95 ranged from -1.75% for PE50 to -3.05% for MV25. All portfolios also had kurtosis above 3 which signals “fat tails” in the return distribution. All return distributions except for PB25 had a positive skewness.

The DY25 portfolio had the highest single month return with 8.79 %.

Table 5.3. Annual returns on employed capital for portfolios with single restrictions and threshold = 1								
Portfolio	**#**	$\bar{r}_a$	σ_{r_a}	S	$Open$	$Trades$	$Rank(\bar{r}_a)$	$Rank(S)$
Baseline	1	0.0173	0.0588	0.0439	0.7681	2.8554	9	9
Level1	2	0.0294	0.0660	0.2224	0.7440	2.6786	5	5
Level2	3	0.0220	0.0621	0.1169	0.7352	2.6018	7	8
Level3	4	0.0244	0.0639	0.1526	0.7346	2.5714	6	6
Level4	5	0.0338	0.0673	0.2845	0.7276	2.5339	4	3
GEO	6	0.0178	0.0673	0.0467	0.7678	2.8321	10	10
EXC	7	0.0070	0.0643	-0.1189	0.7637	2.6000	16	16
EG	8	0.0335	0.0691	0.2722	0.7864	3.0321	3	4
DY50	9	0.0233	0.0648	0.1328	0.7509	2.7446	8	7
DY25	10	0.0127	0.0716	-0.0284	0.7504	2.6589	14	15
PE50	11	0.0409	0.0626	0.4179	0.7586	2.8375	2	2
PE25	12	0.0505	0.0701	0.5108	0.7534	2.7839	1	1
MV50	13	0.0135	0.0782	-0.0152	0.7833	2.6304	15	14
MV25	14	0.0177	0.0840	0.0355	0.7686	2.6625	12	11
DEBT50	15	0.0064	0.0617	-0.1341	0.7547	2.8036	17	17
DEBT25	16	0.0003	0.0666	-0.2162	0.7416	2.6857	18	18
PB50	17	0.0147	0.0613	0.0002	0.7491	2.7964	13	12
PB25	18	0.0138	0.0683	-0.0132	0.7492	2.6839	11	13

The annual return on employed capital for the same portfolios as described in ***Table 5.1*** is shown in ***Table 5.3***. The return on employed capital were, compared to the return on committed capital, 43 bps higher for the baseline portfolio. This relation, varying a little in magnitude, naturally holds for all portfolios.

Table 5.4. Monthly returns on employed capital for portfolios with single restrictions and threshold = 1										
Portfolio	**#**	r	Max	Min	σ_r	$Skew.$	$Kurt.$	VaR_{95}	$r < 0$	$\rho(r, r_{S\&P500})$
Baseline	1	0.0014	0.0573	-0.0517	0.0170	0.3000	3.8809	-0.0249	0.4762	-0.1438
Level1	2	0.0024	0.0708	-0.0478	0.0190	0.7046	4.5020	-0.0273	0.5119	-0.0584
Level2	3	0.0018	0.0641	-0.0519	0.0179	0.2712	3.5635	-0.0270	0.5060	-0.0935
Level3	4	0.0020	0.0631	-0.0545	0.0184	0.3148	3.6391	-0.0244	0.4821	-0.1641
Level4	5	0.0028	0.0615	-0.0516	0.0194	0.2515	3.6809	-0.0300	0.4702	-0.1833
GEO	6	0.0015	0.0750	-0.0731	0.0194	0.1424	4.4471	-0.0263	0.4821	-0.1582
EXC	7	0.0006	0.0792	-0.0599	0.0186	0.2858	4.7765	-0.0254	0.4940	-0.0992
EG	8	0.0027	0.0709	-0.0574	0.0199	0.4768	4.2146	-0.0287	0.4405	-0.1185
DY50	9	0.0019	0.0614	-0.0540	0.0187	0.2381	3.9877	-0.0251	0.4762	-0.0221
DY25	10	0.0010	0.1105	-0.0450	0.0207	1.0871	7.5056	-0.0318	0.5357	-0.0015
PE50	11	0.0033	0.0569	-0.0596	0.0181	0.2010	3.8787	-0.0227	0.4464	-0.1007
PE25	12	0.0041	0.0684	-0.0585	0.0202	0.1708	3.5438	-0.0276	0.4226	-0.1314
MV50	13	0.0011	0.0789	-0.0693	0.0226	0.3978	4.2179	-0.0309	0.5536	-0.1075
MV25	14	0.0015	0.0812	-0.0574	0.0242	0.3559	3.5421	-0.0383	0.4881	-0.1705
DEBT50	15	0.0005	0.0719	-0.0410	0.0178	0.8296	4.6767	-0.0254	0.5417	-0.1072
DEBT25	16	0.0000	0.0675	-0.0631	0.0192	0.3135	4.9727	-0.0298	0.5000	-0.0387
PB50	17	0.0012	0.0619	-0.0477	0.0177	0.3516	3.8663	-0.0274	0.5238	-0.0693
PB25	18	0.0011	0.0658	-0.0730	0.0197	-0.1533	4.0561	-0.0307	0.4762	-0.1678

The monthly returns can be seen in ***Table 5.4*** above. For both return measure only the two price to equity restrictions produced monthly returns significantly higher than the baseline portfolio (from t-test).

5.1.2. Threshold = 2

Table 5.5. Annual returns on committed capital for portfolios with single restriction and threshold = 2								
Portfolio	#	$\bar{r}_a$	σ_{r_a}	S	$Open$	$Trades$	$Rank(\bar{r}_a)$	$Rank(S)$
Baseline	1	0.0216	0.0440	0.1573	0.5810	1.5179	4	4
Level1	2	0.0171	0.0434	0.0552	0.5384	1.3554	6	6
Level2	3	0.0156	0.0425	0.0217	0.5224	1.3196	9	9
Level3	4	0.0134	0.0449	-0.0300	0.5146	1.2732	11	11
Level4	5	0.0207	0.0476	0.1268	0.4984	1.2339	5	5
GEO	6	0.0164	0.0505	0.0346	0.5881	1.4982	7	7
EXC	7	0.0127	0.0457	-0.0431	0.5634	1.4143	13	13
EG	8	0.0226	0.0481	0.1638	0.5555	1.4179	3	3
DY50	9	0.0155	0.0428	0.0197	0.5643	1.4196	10	10
DY25	10	0.0162	0.0506	0.0294	0.5605	1.4054	8	8
PE50	11	0.0320	0.0404	0.4277	0.5671	1.4911	1	1
PE25	12	0.0245	0.0435	0.2253	0.5604	1.4375	2	2
MV50	13	0.0121	0.0523	-0.0495	0.5913	1.4661	14	14
MV25	14	0.0051	0.0578	-0.1656	0.5663	1.4339	17	17
DEBT50	15	0.0112	0.0497	-0.0712	0.5645	1.4554	16	16
DEBT25	16	-0.0011	0.0451	-0.3500	0.5447	1.3732	18	18
PB50	17	0.0128	0.0473	-0.0400	0.5537	1.4375	12	12
PB25	18	0.0117	0.0480	-0.0630	0.6009	1.5446	15	15

The performance, in terms of committed capital, of the portfolios with trading threshold of 2, shown in ***Table 5.5***, followed a very similar pattern to those traded with a threshold of 1 (***Table 5.1***). The portfolio with highest returns were in this case PE50 (PE25 with trading threshold of 1). The returns were generally a little lower with the increased threshold. However, the baseline portfolio performed better, 2.16% annual return (1.3% with trading threshold of 1). Only PE50, PE25 and EG outperformed the baseline portfolio.

The amount of trades decreases by more than one per trading period. The position is now only open about 50-60% of the time (approximately 75% with the tighter threshold).

Table 5.6. Monthly returns on committed capital for portfolios with single restrictions and threshold = 2										
Portfolio	#	r	Max	Min	σ_r	$Skew.$	$Kurt.$	VaR_{95}	$r < 0$	$\rho(r, r_{S\&P500})$
Baseline	1	0.0018	0.0497	-0.0376	0.0127	0.5302	5.2651	-0.0173	0.4345	-0.1401
Level1	2	0.0014	0.0561	-0.0346	0.0125	0.9021	5.9523	-0.0161	0.5119	-0.0627
Level2	3	0.0013	0.0482	-0.0409	0.0123	0.6537	5.1495	-0.0168	0.5238	-0.0928
Level3	4	0.0011	0.0416	-0.0355	0.0130	0.2423	3.8598	-0.0213	0.4881	-0.1159
Level4	5	0.0017	0.0484	-0.0337	0.0137	0.4953	4.2868	-0.0228	0.4464	-0.0707
GEO	6	0.0014	0.0584	-0.0394	0.0146	0.5207	4.4936	-0.0215	0.5000	-0.1894
EXC	7	0.0011	0.0616	-0.0408	0.0132	0.6352	5.6466	-0.0190	0.4643	-0.0965
EG	8	0.0019	0.0559	-0.0494	0.0139	-0.0839	5.2829	-0.0180	0.4702	-0.1527
DY50	9	0.0013	0.0396	-0.0359	0.0124	0.3053	4.0922	-0.0170	0.4821	-0.0455
DY25	10	0.0013	0.0840	-0.0367	0.0146	1.0047	8.4970	-0.0235	0.5000	-0.0402
PE50	11	0.0026	0.0365	-0.0473	0.0117	-0.1248	4.9296	-0.0150	0.4226	-0.1046
PE25	12	0.0020	0.0438	-0.0427	0.0126	0.0463	4.3415	-0.0196	0.4286	-0.1782
MV50	13	0.0010	0.0500	-0.0493	0.0151	0.3378	4.4840	-0.0216	0.5000	-0.1001
MV25	14	0.0004	0.0706	-0.0389	0.0167	0.6303	5.1423	-0.0278	0.4524	-0.1901
DEBT50	15	0.0009	0.0540	-0.0508	0.0144	0.4408	5.1916	-0.0179	0.4821	-0.0864
DEBT25	16	-0.0001	0.0631	-0.0441	0.0130	0.6529	6.8045	-0.0203	0.5060	-0.0592
PB50	17	0.0011	0.0452	-0.0436	0.0137	0.4403	4.4296	-0.0192	0.4940	-0.0565
PB25	18	0.0010	0.0631	-0.0364	0.0139	1.0421	6.7380	-0.0184	0.4881	-0.0754

Table 5.6 displays the monthly returns corresponding to ***Table 5.5***. The volatility decreased for all but two portfolios (PB50 and DEBT50) when the trading threshold were increased. The maximum, minimum and value at risk has also generally decreased in absolute terms.

The other metrics did not change much with the thresholds and differences are hard to identify.

Table 5.7. Annual returns on employed capital for portfolios with single restrictions and threshold = 2

Portfolio	#	$\bar{r}_a$	σ_{r_a}	S	$Open$	$Trades$	$Rank(\bar{r}_a)$	$Rank(S)$
Baseline	1	0.0401	0.0743	0.3412	0.5810	1.5179	5	3
Level1	2	0.0341	0.0771	0.2513	0.5384	1.3554	6	6
Level2	3	0.0301	0.0764	0.2012	0.5224	1.3196	11	10
Level3	4	0.0250	0.0836	0.1231	0.5146	1.2732	14	14
Level4	5	0.0419	0.0921	0.2954	0.4984	1.2339	4	5
GEO	6	0.0334	0.0836	0.2238	0.5881	1.4982	8	7
EXC	7	0.0280	0.0779	0.1708	0.5634	1.4143	12	12
EG	8	0.0425	0.0881	0.3160	0.5555	1.4179	3	4
DY50	9	0.0304	0.0751	0.2096	0.5643	1.4196	9	9
DY25	10	0.0336	0.0892	0.2120	0.5605	1.4054	7	8
PE50	11	0.0628	0.0694	0.6935	0.5671	1.4911	1	1
PE25	12	0.0495	0.0768	0.4533	0.5604	1.4375	2	2
MV50	13	0.0252	0.0885	0.1186	0.5913	1.4661	13	15
MV25	14	0.0194	0.1011	0.0467	0.5663	1.4339	17	17
DEBT50	15	0.0240	0.0881	0.1055	0.5645	1.4554	16	16
DEBT25	16	0.0038	0.0796	-0.1369	0.5447	1.3732	18	18
PB50	17	0.0304	0.0833	0.1890	0.5537	1.4375	10	11
PB25	18	0.0246	0.0785	0.1261	0.6009	1.5446	15	13

The increase in returns on employed capital compared to committed capital were greater when the trading threshold is greater. The annual returns on employed capital (***Table 5.7***) were about twice as large as the annual returns on committed capital (***Table 5.5***). As described earlier the capital was employed far less with a threshold of 2 instead of 1 which lead to this larger increase.

Table 5.8. Monthly returns on employed capital for portfolios with single restrictions and threshold = 2

Portfolio	#	r	Max	Min	σ_r	$Skew.$	$Kurt.$	VaR_{95}	$r < 0$	$\rho(r, r_{S\&P500})$
Baseline	1	0.0033	0.0861	-0.0651	0.0214	0.4227	4.8144	-0.0310	0.4345	-0.1273
Level1	2	0.0028	0.0810	-0.0553	0.0223	0.7609	4.4993	-0.0308	0.5119	-0.0305
Level2	3	0.0025	0.0705	-0.0690	0.0221	0.5821	4.1850	-0.0267	0.5238	-0.0749
Level3	4	0.0021	0.0717	-0.0709	0.0241	0.1980	3.4794	-0.0383	0.4881	-0.1218
Level4	5	0.0034	0.0862	-0.0912	0.0266	0.1995	4.0997	-0.0405	0.4464	-0.0927
GEO	6	0.0027	0.0811	-0.0718	0.0241	0.3761	3.9623	-0.0315	0.5000	-0.1817
EXC	7	0.0023	0.0814	-0.0634	0.0225	0.4379	3.9745	-0.0312	0.4643	-0.1049
EG	8	0.0035	0.1103	-0.0859	0.0254	0.1113	5.3247	-0.0357	0.4702	-0.1653
DY50	9	0.0025	0.0752	-0.0658	0.0217	0.2737	4.0981	-0.0313	0.4821	-0.0291
DY25	10	0.0028	0.1458	-0.0711	0.0257	1.0496	8.3838	-0.0365	0.5000	-0.0119
PE50	11	0.0051	0.0545	-0.0626	0.0200	0.1511	3.8142	-0.0226	0.4226	-0.1035
PE25	12	0.0040	0.0740	-0.0618	0.0222	0.1231	3.8257	-0.0314	0.4286	-0.1628
MV50	13	0.0021	0.0819	-0.0964	0.0255	0.1712	4.6956	-0.0333	0.5000	-0.0930
MV25	14	0.0016	0.1138	-0.0653	0.0292	0.5342	4.5715	-0.0479	0.4524	-0.1805
DEBT50	15	0.0020	0.0891	-0.1082	0.0254	0.2321	5.5618	-0.0308	0.4821	-0.0770
DEBT25	16	0.0003	0.0896	-0.0639	0.0230	0.5813	5.0077	-0.0355	0.5060	-0.0621
PB50	17	0.0025	0.0845	-0.0571	0.0240	0.6495	4.1897	-0.0336	0.4940	-0.0745
PB25	18	0.0020	0.1052	-0.0551	0.0227	1.0462	6.2857	-0.0328	0.4881	-0.0664

The monthly returns on employed capital can be seen in ***Table 5.8*** above. None of these portfolios outperformed the baseline with statistical significance.

5.2. Multiple restrictions

The four best portfolios with two restrictions, based on a trading threshold of 1, is presented with both return measures and trading thresholds. The four portfolios are combinations of the restrictions from EG, Level1, PE50 and PE25. ***Table 5.9*** shows the annual returns on committed capital for these.

Table 5.9. Annual returns on committed capital for portfolios with multiple restrictions								
Portfolio	#	$\bar{r}_a$	σ_{r_a}	S	$Open$	$Trades$	$Rank(\bar{r}_a)$	$Rank(S)$
Threshold = 1								
EG and Level1	19	0.0272	0.0604	0.2069	0.7723	2.7554	3	4
EG and PE50	20	0.0429	0.0548	0.5157	0.7765	2.9696	2	2
EG and PE25	21	0.0498	0.0632	0.5555	0.7700	2.8946	1	1
Level1 and PE50	22	0.0293	0.0497	0.2940	0.7411	2.6411	4	3
Threshold = 2								
EG and Level1	19	0.0135	0.0510	-0.0233	0.5626	1.3750	4	4
EG and PE50	20	0.0397	0.0501	0.5003	0.5747	1.4893	2	2
EG and PE25	21	0.0399	0.0464	0.5445	0.5874	1.5625	1	1
Level1 and PE50	22	0.0169	0.0420	0.0525	0.5295	1.3125	3	3

Combining the restrictions increased the returns for these four portfolios compared to when they were implemented one by one. With the tighter threshold portfolio 21 has an annual return on committed capital of 4.98%. Using the corresponding restrictions alone yields 2.55% and 3.65%.

For the wider threshold this only holds for combining the two price to equity restrictions with the Engle-Granger test. The highest performing portfolio were also here 21, however, the return were almost identical to portfolio 20. ***Table 5.10*** below provides monthly return statistics.

Table 5.10. Monthly returns on committed capital for portfolios with multiple restrictions										
Portfolio	#	r	Max	Min	σ_r	$Skew.$	$Kurt.$	VaR_{95}	$r < 0$	$\rho(r, r_{S\&P500})$
Threshold = 1										
EG and Level1	19	0.0022	0.0628	-0.0452	0.0174	0.3216	3.5083	-0.0260	0.4524	0.0344
EG and PE50	20	0.0035	0.0622	-0.0376	0.0158	0.3650	3.5658	-0.0230	0.4405	-0.1226
EG and PE25	21	0.0041	0.0584	-0.0504	0.0183	0.5012	3.6022	-0.0217	0.4762	-0.1629
Level1 and PE50	22	0.0024	0.0630	-0.0388	0.0144	0.5562	4.4020	-0.0186	0.4583	-0.1228
Threshold = 2										
EG and Level1	19	0.0011	0.0478	-0.0435	0.0147	0.3417	4.0185	-0.0199	0.5238	0.0449
EG and PE50	20	0.0033	0.0518	-0.0446	0.0145	0.5460	4.6164	-0.0210	0.4048	-0.1266
EG and PE25	21	0.0033	0.0506	-0.0325	0.0134	0.3413	3.7968	-0.0186	0.4048	-0.0334
Level1 and PE50	22	0.0014	0.0415	-0.0359	0.0121	0.4053	4.0542	-0.0165	0.4643	-0.0803

Besides the increase in return there were no other substantial differences when the multiple restrictions were implemented.

The relations logically holds for return on employed capital on an annual (***Table 5.11***) and on a monthly (***Table 5.12***) basis.

Table 5.11. Annual returns on employed capital for portfolios with multiple restrictions								
Portfolio	**#**	$\bar{r}_a$	σ_{r_a}	S	$Open$	$Trades$	$Rank(\bar{r}_a)$	$Rank(S)$
Threshold = 1								
EG and Level1	19	0.0368	0.0769	0.2879	0.7723	2.7554	3	4
EG and PE50	20	0.0563	0.0702	0.5926	0.7765	2.9696	2	2
EG and PE25	21	0.0672	0.0816	0.6434	0.7700	2.8946	1	1
Level1 and PE50	22	0.0417	0.0655	0.4125	0.7411	2.6411	4	3
Threshold = 2								
EG and Level1	19	0.0324	0.0888	0.1990	0.5626	1.3750	4	4
EG and PE50	20	0.0739	0.0863	0.6853	0.5747	1.4893	2	2
EG and PE25	21	0.0739	0.0781	0.7581	0.5874	1.5625	1	1
Level1 and PE50	22	0.0361	0.0770	0.2784	0.5295	1.3125	3	3

For the best portfolio, combination of PE25 and EG, the annual returns increased from 4.98% to 6.72% with a threshold of 1 and from 3.99% to 7.39% when calculating returns on employed instead of committed capital.

Table 5.12. Monthly returns on employed capital for portfolios with multiple restrictions										
Portfolio	**#**	r	Max	Min	σ_r	$Skew.$	$Kurt.$	VaR_{95}	$r < 0$	$\rho(r, r_{S\&P500})$
Threshold = 1										
EG and Level1	19	0.0030	0.0758	-0.0549	0.0222	0.2923	3.2443	-0.0335	0.4524	0.0373
EG and PE50	20	0.0046	0.0749	-0.0449	0.0203	0.3339	3.3454	-0.0306	0.4405	-0.1158
EG and PE25	21	0.0054	0.0744	-0.0641	0.0235	0.5006	3.5572	-0.0293	0.4762	-0.1575
Level1 and PE50	22	0.0034	0.0764	-0.0455	0.0189	0.5525	3.8360	-0.0235	0.4583	-0.1025
Threshold = 2										
EG and Level1	19	0.0027	0.0918	-0.0660	0.0256	0.5359	3.9025	-0.0388	0.5238	0.0516
EG and PE50	20	0.0060	0.0914	-0.0708	0.0249	0.5301	4.3483	-0.0317	0.4048	-0.1122
EG and PE25	21	0.0060	0.0800	-0.0485	0.0225	0.3681	3.5277	-0.0300	0.4048	-0.0207
Level1 and PE50	22	0.0030	0.0743	-0.0511	0.0222	0.4509	3.4880	-0.0309	0.4643	-0.0505

Portfolio 20 and 21 produced significant higher returns than the baseline portfolio with the narrower threshold but not the wider.

5.3. Lasso models

The Lasso portfolios uses the top 20 pairs based on predicted SSD (for the trading period) and profit to form the portfolios Lasso SSD and Lasso Profit respectively. The annual return statistics on committed capital is presented in ***Table 5.13***.

Table 5.13. Annual returns on committed capital for Lasso portfolios					
Portfolio	$\bar{r}_a$	σ_{r_a}	S	$Open$	$Trades$
Threshold = 1					
Lasso SSD	0.0298	0.0576	0.2613	0.7142	2.2875
Lasso Profit	0.0056	0.0732	-0.1251	0.7521	2.3304
Threshold = 2					
Lasso SSD	0.0153	0.0520	0.0121	0.5189	1.1875
Lasso Profit	-0.0140	0.0736	-0.3892	0.5621	1.3161

Compared with the baseline portfolio the Lasso model predicting forward SSD reached higher returns, however not statistically significant, when traded with the lower threshold. However, when increasing the threshold it got substantially outperformed by the baseline. When predicting profits the returns were very low and actually negative for the wider threshold.

Table 5.14. Monthly returns on committed capital for Lasso portfolios									
Portfolio	r	Max	Min	σ_r	$Skew.$	$Kurt.$	VaR_{95}	$r < 0$	$\rho(r, r_{S\&P500})$
Threshold = 1									
Lasso SSD	0.0024	0.0834	-0.0318	0.0166	1.0826	6.2949	-0.0219	0.4762	-0.0204
Lasso Profit	0.0005	0.0635	-0.0480	0.0211	0.2374	3.4548	-0.0382	0.5000	-0.0077
Threshold = 2									
Lasso SSD	0.0013	0.0541	-0.0367	0.0150	0.7354	4.0569	-0.0207	0.5357	-0.0131
Lasso Profit	-0.0012	0.0699	-0.0788	0.0213	0.0479	5.1449	-0.0324	0.5357	0.0075

The monthly returns on committed capital for the Lasso models in ***Table 5.14*** follows the same patterns as previous results.

Return statistics for employed capital is displayed in ***Table 5.15*** (annual) and ***Table 5.16*** (monthly).

Table 5.15. Monthly returns on employed capital for Lasso portfolios					
Portfolio	$\bar{r}_a$	σ_{r_a}	S	$Open$	$Trades$
Threshold = 1					
Lasso SSD	0.0410	0.0799	0.3297	0.7142	2.2875
Lasso Profit	0.0094	0.0944	-0.056	0.7521	2.3304
Threshold = 2					
Lasso SSD	0.0238	0.1006	0.0909	0.5189	1.1875
Lasso Profit	-0.0205	0.1229	-0.2865	0.5621	1.3161

Table 5.16. Monthly returns on employed capital for Lasso portfolios									
Portfolio	r	Max	Min	σ_r	$Skew.$	$Kurt.$	VaR_{95}	$r < 0$	$\rho(r, r_{S\&P500})$
Threshold = 1									
Lasso SSD	0.0034	0.1203	-0.0517	0.0231	1.0619	6.5267	-0.0305	0.4762	-0.0238
Lasso Profit	0.0008	0.084	-0.0632	0.0273	0.2421	3.3735	-0.0463	0.5000	-0.0095
Threshold = 2									
Lasso SSD	0.0020	0.0888	-0.1178	0.0290	0.2455	4.5775	-0.0387	0.5357	-0.0389
Lasso Profit	-0.0017	0.1167	-0.1102	0.0355	0.1751	4.2915	-0.0716	0.5357	0.0005

5.4. Uniqueness of stocks traded

Stocks are restricted to only be part of one pair in the portfolios. The baseline portfolio (1) and the portfolio combining restriction 8 and 12 (21) were tested for this effect. The returns shown in ***Table 5.17*** and ***5.18*** are for employed capital only.

Table 5.17. Annual returns on employed capital for portfolios with and without unique stocks restriction

Portfolio	#	$Unique$	$\bar{r}_a$	σ_{r_a}	S	$Open$	$Trades$
Threshold = 1							
Baseline	1	30.216	0.0122	0.0685	-0.0361	0.7663	2.8071
		40.000	0.0173	0.0588	0.0439	0.7681	2.8554
EG and PE25	21	30.536	0.0387	0.0802	0.2993	0.7712	2.8857
		40.000	0.0672	0.0816	0.6434	0.7700	2.8946
Threshold = 2							
Baseline	1	30.216	0.0339	0.0815	0.2351	0.5841	1.4946
		40.000	0.0401	0.0743	0.3412	0.5810	1.5179
EG and PE25	21	30.536	0.0711	0.0930	0.6059	0.5792	1.5000
		40.000	0.0739	0.0863	0.6853	0.5747	1.4893

Table 5.17 shows that for both portfolios and with both trading thresholds the restriction increased both returns and Sharpe ratio. The column $Unique$ describes the average amount of unique pairs in the portfolio. With the restriction the amount is naturally 40 and when omitting the restriction this decreases to about 30 for both portfolios.

Table 5.18. Monthly returns on employed capital for portfolios with and without unique stocks restriction

Portfolio	#	$Unique$	r	Max	Min	σ_r	$Skew.$	$Kurt.$	VaR_{95}	$r < 0$	ρ
Threshold = 1											
Baseline	1	30.216	0.0010	0.0793	-0.0482	0.0198	0.5821	4.3961	-0.0285	0.5238	-0.1856
		40.000	0.0011	0.0492	-0.0385	0.0133	0.3810	4.2727	-0.0185	0.4762	-0.1504
EG and PE25	21	30.536	0.0032	0.0833	-0.0480	0.0231	0.5590	3.7582	-0.0339	0.4940	-0.1430
		40.000	0.0054	0.0744	-0.0641	0.0235	0.5006	3.5572	-0.0293	0.4762	-0.1575
Threshold = 2											
Baseline	1	30.216	0.0028	0.0803	-0.0690	0.0235	0.5258	4.2375	-0.0317	0.5357	-0.1676
		40.000	0.0033	0.0861	-0.0651	0.0214	0.4227	4.8144	-0.0310	0.4345	-0.1273
EG and PE25	21	30.536	0.0057	0.1049	-0.0656	0.0269	0.8600	4.7961	-0.0303	0.4762	-0.1588
		40.000	0.0060	0.0800	-0.0485	0.0225	0.3681	3.5277	-0.0300	0.4048	-0.0207

The monthly value at risk decreased for all portfolios shown in ***Table 5.18*** when implementing the restriction. Better figures for kurtosis, volatility and worst case is also achieved by three out of four portfolios.

5.5. Transaction costs

The effects of transaction costs are only presented for annual employed capital.

Table 5.19. Annual returns on employed capital for portfolios with and without transaction costs							
Portfolio	#	$Transaction\ cost$	$\bar{r}_a$	σ_{r_a}	S	$Open$	$Trades$
Threshold = 1							
Baseline	1	No	0.0813	0.0598	1.1134	0.7681	2.8554
		Yes	0.0173	0.0588	0.0439	0.7681	2.8554
EG and PE25	21	No	0.1349	0.0837	1.4354	0.7700	2.8946
		Yes	0.0672	0.0816	0.6434	0.7700	2.8946
Threshold = 2							
Baseline	1	No	0.0861	0.0747	0.9554	0.5810	1.5179
		Yes	0.0401	0.0743	0.3412	0.5810	1.5179
EG and PE25	21	No	0.1208	0.0873	1.2153	0.5747	1.4893
		Yes	0.0739	0.0863	0.6853	0.5747	1.4893

Table 5.19 shows a substantial increase in returns when transaction costs were omitted. The baseline portfolio (EG and PE25 portfolio) increased the annual return by 630 bps (677 bps) with the narrower threshold and by 460 bps (469 bps) with the wider. The increase in return on committed capital is 470 bps (476 bps) with the narrower threshold and 252 bps (248 bps) with the wider.

5.6. Cumulative returns and comparison to S&P 500

Since the stocks were chosen from the S&P 500 it is the natural index to compare the results with.

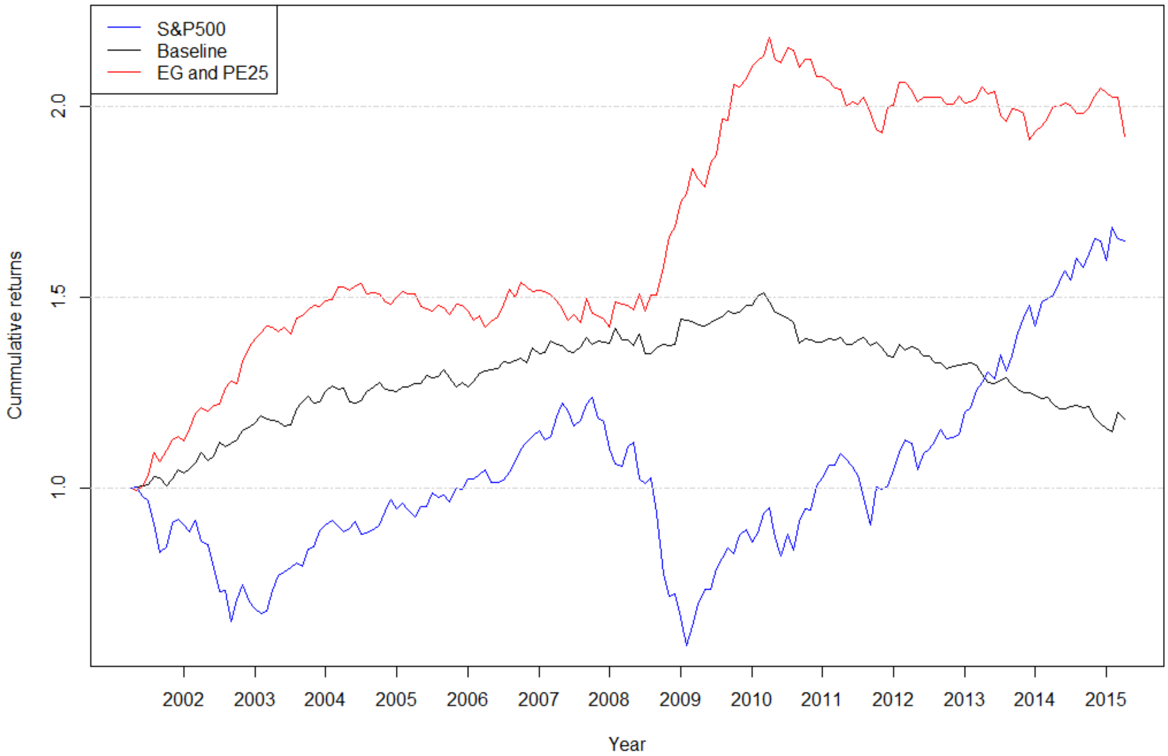

Figure 5.1.
Cumulative returns on committed capital for the baseline, the EG and PE25 portfolio and S&P 500
Portfolios traded with threshold = 1
Time. 2001-04-01 to 2015-04-01

Figure 5.1 shows the cumulative returns from the portfolio 1 and 21 (threshold = 1) compared to the S&P 500. The baseline portfolio has a stable positive development until 2010 when it starts to decline in value. The portfolio combining the EG and PE25 restriction had a rapid increase in the first few years before the curve flattened. From the start of the financial crisis in mid-2008 the portfolio had a very impressive value growth. From 2010 the curve flattened again.

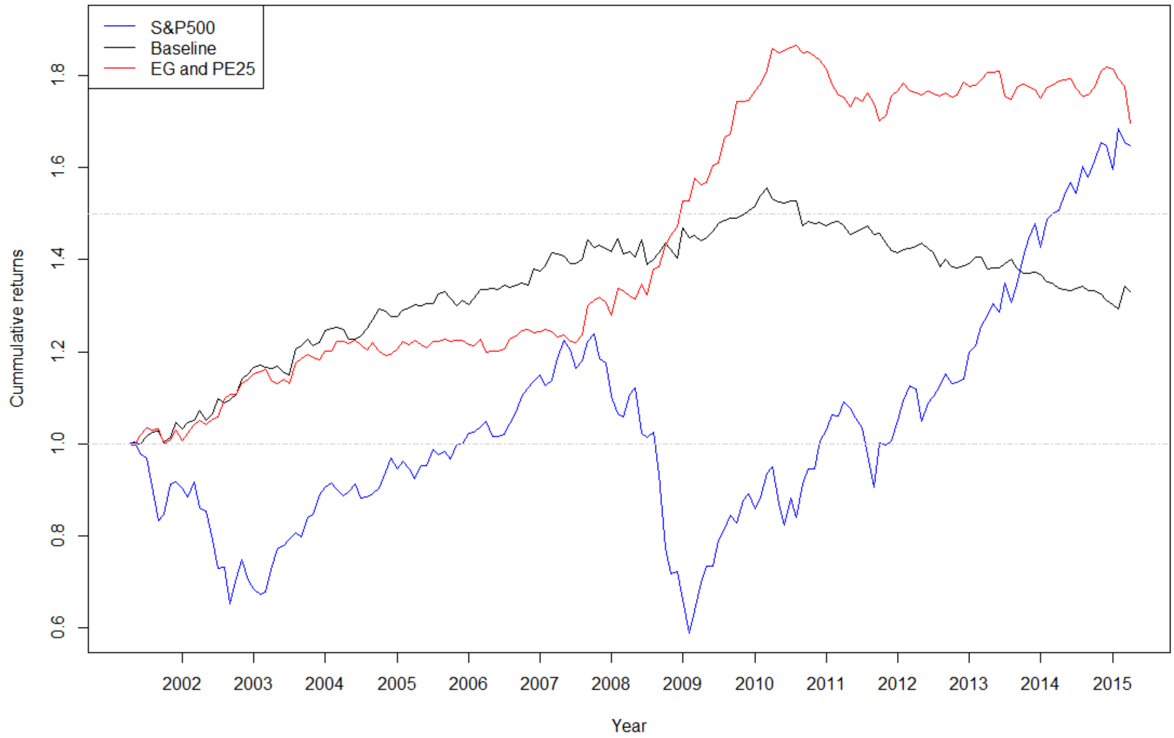

Figure 5.2.
Cumulative returns on committed capital for the baseline, the EG and PE25 portfolio and S&P 500
Portfolios traded with threshold = 2
Time. 2001-04-01 to 2015-04-01

Increasing the threshold to 2 (***Figure 5.2***) does not change the look of the value curves by much.

The cumulative returns on employed capital is presented in the same way in ***Appendix 8.1*** for the tighter threshold and **Appendix 8.2** for the wider.

The returns on committed capital are displayed in ***Appendix 8.3*** for trading threshold of 1, ***Appendix 8.4*** for trading threshold of 2 and ***Appendix 8.5*** for the S&P 500.

From conducting a one-sided t-test the monthly returns are determined to be significantly (5% level) higher for the EG and PE25 portfolio compared with the baseline for both return calculations and both threshold levels.

6. Conclusions

6.1. Discussion of results

Previous research from Do and Faff (2012) finds higher returns when a pairs trading portfolio is restricted by industry classification with two levels and a total of 48 groups. This study increases this to include four levels of classification and the groups to 114 by using the FTSE ICB. The results in ***Table 5.1-5.8*** show that the best portfolio, for both thresholds, is created by restricting to the finest classification, sub sector. However, the magnitude of the outperformance is small. Compared to the loosest restriction, industry (ten groups), the annual returns on committed capital are only 6 bps (threshold = 1) and 36 bps (threshold = 2). The returns does not increase linearly with the level of classification. Restriction on the second, super sector and third, sector, level yields lower returns than the industry restriction. Because of this the result becomes hard to interpret. Does a more specific classification really improve pairs trading? Well, the study does not provide an decisive result to answer this. Classifications such as the FTSE ICB should be considered when implementing pairs trading but more research has to be conducted before a more specific recommendation can be given.

Restrictions on differences in geography classification (GEO), exchange (EXC), debt ratio (DEBT), market value (MV), price to book (PB) and dividend yield does not produce portfolios with improved returns (***Table 5.1-5.8***). However, it is important to stress that using other ways to compare variables, such as ratio or setting up groups, could lead to different results. It is therefore too early to dismiss the variables.

When the Engle-Granger test is used to restrict the portfolio to only use significantly co-integrated pairs the portfolio outperformance the baseline for both trading thresholds. This is together with the fourth level of ICB and the price to equity the only restrictions that outperforms the baseline portfolio. This confirms that Vidyamurthy's (2004) model can improve the distance method.

Of the three restrictions that produced portfolios that consistently outperformed the baseline the two portfolios with price to equity achieved the substantially highest returns. Both Engle-Granger testing and industry restrictions has been examined in previous research. However, this is not the case for price to equity restrictions. It is therefore the most important finding by this study.

Applying Engle-Granger's co-integration restriction increases the returns further (***Table 5.9-5.12***). By adding this new variable to two existing methods described in ***2.3.1*** and ***2.3.2*** the returns increases significantly.

The study shows that transaction costs substantially cut profits (***Table 5.19***). This is completely in line with the results from previous research. Since more trades are conducted and positions are open more frequently with a tighter threshold the trading costs gets higher. When choosing threshold this is very important to keep in mind.

The big differences between returns on committed and employed capital shows the importance to have a dynamic capital structure where capital can be moved from the portfolio when it is not used. For return on committed capital the tighter threshold generally produces higher returns. However, when returns on employed capital is used the wider threshold produces the higher returns instead. An analysis on how dynamic the capital is should therefore be conducted before choosing trading threshold.

When introducing the restriction on uniqueness of stocks the returns increased and volatility generally decreased (***Table 5.17-5.18***). Omitting the restriction lead to about 30 stocks being traded instead of 40. Gatev et al (2006) , as described in ***1.2***, showed that increasing the portfolio size from 5 to 20 pairs decreased the portfolio risk and increased returns. It was concluded that increasing the amount of pairs spread the risk. This study finds that a similar conclusion can be drawn from the restriction implemented here. By increasing the amount of stocks in the portfolio the risk exposures gets more diversified and the portfolio risk decreases. Implementing such a restriction is therefore highly recommended.

The Lasso models produces inconclusive results. Trying to predict future profits proved very hard and when forming portfolios based on the model the annual returns were below one for the tighter threshold and negative for the wider. Including more variables could improve the model but to reach abnormal returns seems unlikely. Predicting future SSD produced higher returns than the baseline portfolio for the narrower threshold but lower for the wider. Since the result proved inconsistent for the two thresholds the study can't recommend nor dismiss the model. Further research using similar models would therefore be interesting.

Figure 5.1-5.2 shows that the portfolios performed best during the financial crisis. This is consistent previous research that suggests that prolonged market turmoil is an ideal environment for the strategy. This is also revealed by the slightly negative market correlation for most portfolios. The strategy produces therefore an extra interesting alternative to traditional investments. Since 2010 the positive returns has disappeared. Other research, as described in ***1.2***, has found this as well. The US market has been relatively stable from then and before dismissing the future for the strategy its' performance during the next market downturn has to be analyzed.

6.2. Suggested research

To use absolute differences when comparing the qualitative variables were somewhat arbitrary and an interesting area for further research would be to try other ways to compare the observations. For example, fractional differences would be an interesting measure.

Conducting similar testing with other variables is also a possible field for new research. To start with the other factors from the BARRA model (***2.4.3***) would provide interesting models. However, without access to a top of the art database the problem with missing data could make this tough.

Applying the models to other markets also provides new research prospects. The decrease in returns from the past few years might be U.S. specific. When investigating new markets the transaction costs and short selling restrictions might be completely different leading to difficulties implementing the strategy and returns decreased by high trading expenses.

Investigating reasons for why the strategy seems to suffer from decreasing returns in recent years is also an interesting research field. The speculations from Do and Faff (part ***1.2***) regarding HFT could be one of the explanations. Can markets with lower level of HFT participation produce higher returns?

6.3. Concluding remarks

Even though the modest returns in recent years the strategy can be interesting for the future. After many years of a rising stock markets a time of turmoil might be closing in. In such an environment much of the data, e.g. ***Figure 5.1***, supports a renaissance for the strategy. Especially when performing a thorough fundamental analysis.

Using restrictions on quantitative variables is one successful example of fundamental analysis that has produced portfolios with abnormal returns. Using these on differences in price to equity ratios and the Engle-Granger test for co-integration has produced the portfolios with highest returns. Other restrictions has also been shown to decrease risk and increase return.

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8. APPENDIX

8.1. Cumulative returns on employed capital with threshold = 1

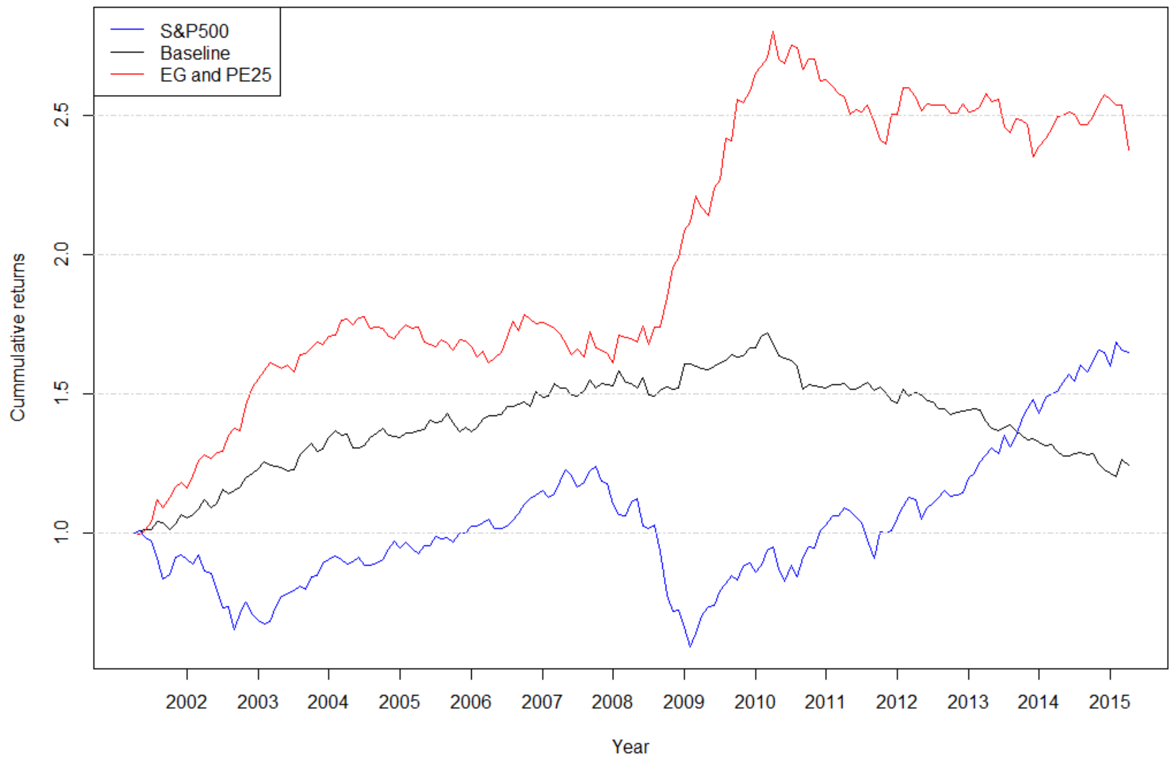

Figure 8.1.
Cumulative returns on employed capital for the baseline, the EG and PE25 portfolio and S&P 500
Portfolios traded with threshold = 1
Time. 2001-04-01 to 2015-04-01

8.2. Cumulative returns on employed capital with threshold = 2

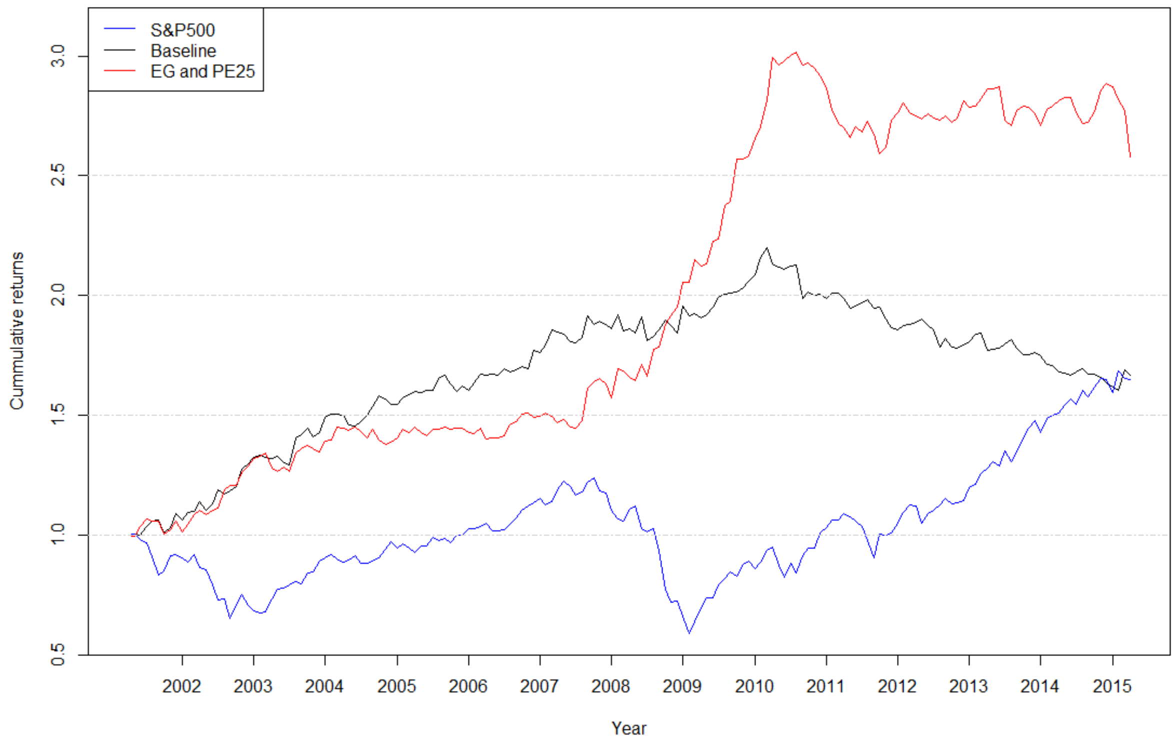

Figure 8.2.
Cumulative returns on employed capital for the baseline, the EG and PE25 portfolio and S&P 500
Portfolios traded with threshold = 2
Time. 2001-04-01 to 2015-04-01

8.3. Monthly returns on committed capital with threshold = 1

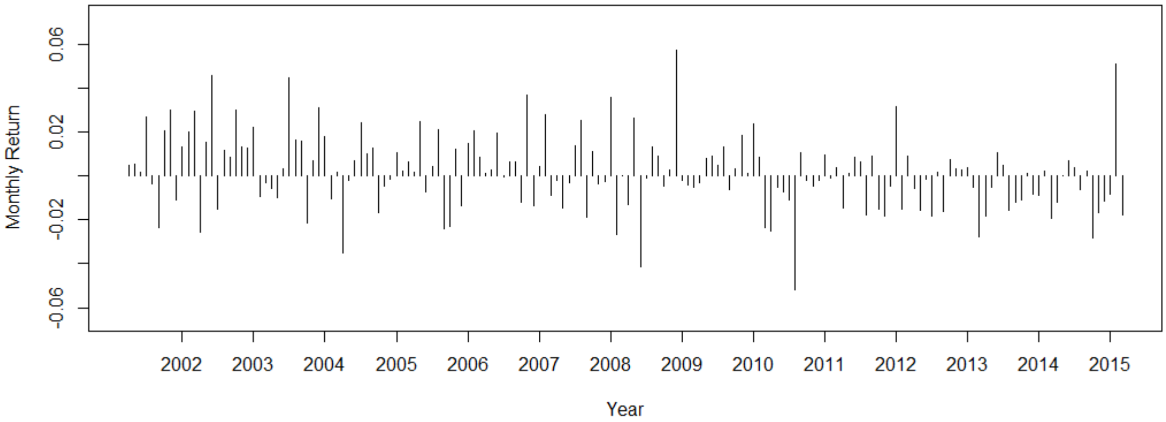

Figure 8.3a.
Monthly returns on committed capital for the baseline portfolio
Portfolios traded with threshold = 1
Time. 2001-04-01 to 2015-04-01

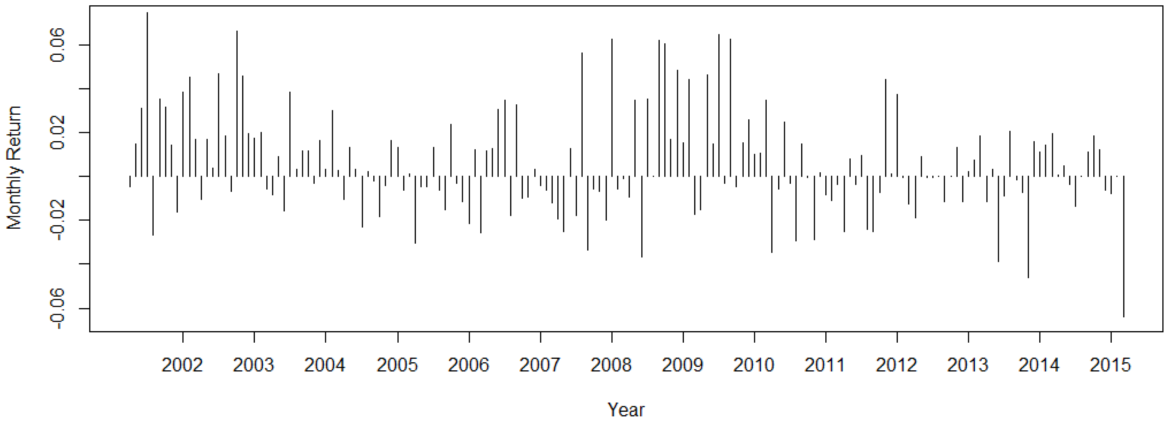

Figure 8.3b.
Monthly returns on committed capital for the EG and PE25 portfolio
Portfolios traded with threshold = 1
Time. 2001-04-01 to 2015-04-01

8.4. Monthly returns on committed capital with threshold = 2

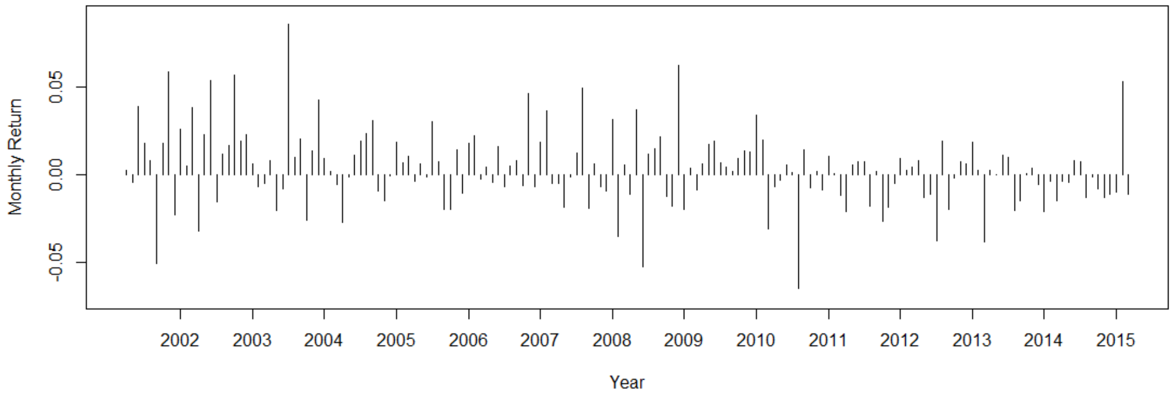

Figure 8.4a.
Monthly returns on committed capital for the baseline portfolio
Portfolios traded with threshold = 2
Time. 2001-04-01 to 2015-04-01

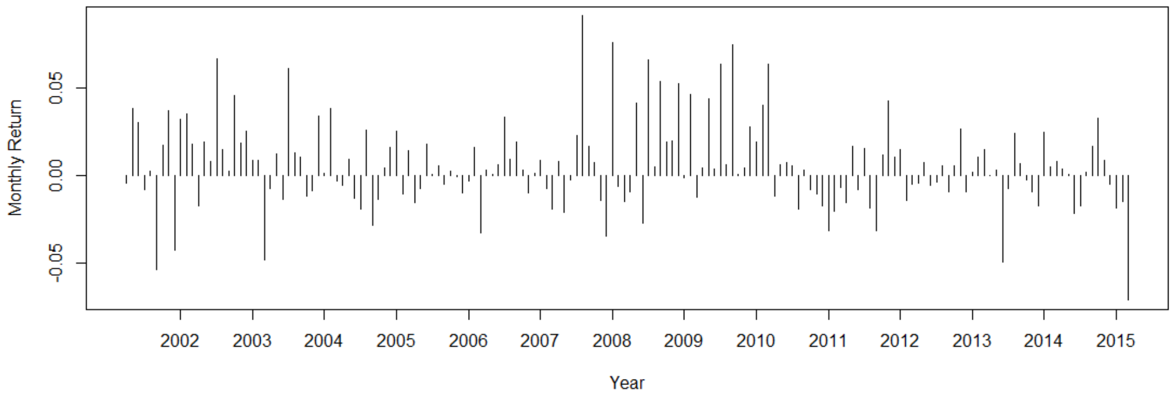

Figure 8.4b.
Monthly returns on committed capital for the EG and PE25 portfolio
Portfolios traded with threshold = 2
Time. 2001-04-01 to 2015-04-01

8.5. Monthly returns for S&P 500

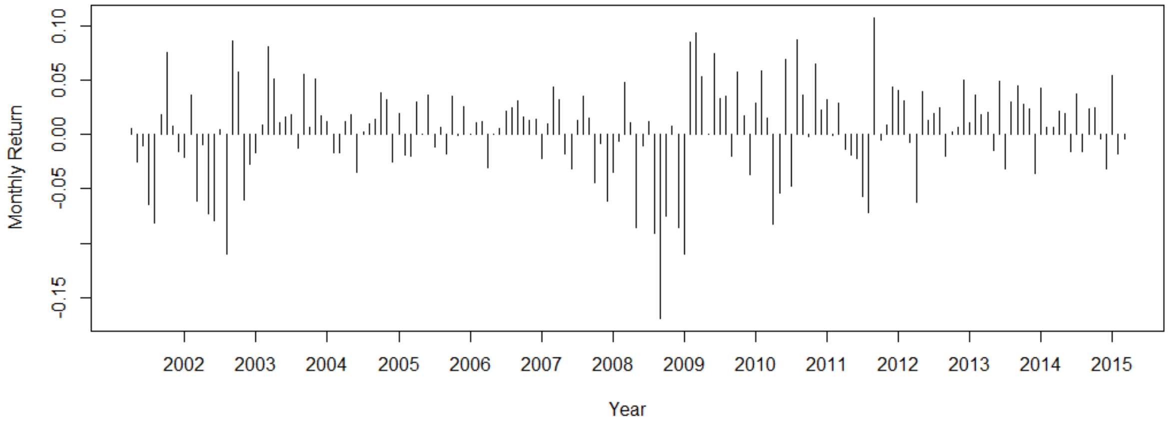

Figure 8.5.
Monthly returns for S&P 500
Time. 2001-04-01 to 2015-04-01