Roughness induced vibration in a beam-ball system

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Abstract

The dynamic response caused by the rolling contact of rough surfaces is studied experimentally, considering the non-stationary contact-interaction of a steel ball and a steel beam with rough surfaces. This is carried out by building a test rig whereby a steel ball is allowed to roll over a steel beam with a machined surface. The dynamic response is evaluated by measuring the resulting vibrations of the beam. A previously developed theoretical model for the dynamic response caused by the rolling contact of rough surface is applied to this problem, and validated by comparing predicted and measured acceleration levels.

1 Introduction

Ever since surface roughness was recognised as a potential source of dynamic excitation, causing noise and vibration, there has been a strong interest in modelling this phenomenon. In addition to theoretical modelling, considerable effort has been spent on experimental validation. A classic approach is to study the dynamic response of two discs in rolling and/or sliding contact [1–3]. A motivation for this specific type of measurement setup originated from recognising the resonance associated with contact deformation as being an important factor. The advantage of the setup is the possibility of isolating the vibrations of the discs from other sources of vibration in the vicinity of the contact resonance. Fischer [4] approached the problem of surface roughness excitation by studying the dynamic response of a glass plate when a steel ball was rolled over it. By changing the roughness of the glass plate and/or the speed and size of the ball, conclusions regarding the excitation process were able to be drawn. An important application is the noise generated owing to a wheel/rail contact. An early experimental validation of this was made by Remington [5]. This investigation was carried
out by measuring the noise produced by the interaction of the steel wheels of a small personalised rapid transit vehicle with the rails of a test track. Later [6], a similar investigation using a full-scaled transit car on a test track was undertaken. Lately, Thompson et. al. [7] have carried out experimental investigations on full-scale passenger and freight wagon sets, investigating the sound radiation when the train passed by. This work was done as a part of validating the TWINS program\(^1\) where wheel and rail surface roughness spectra provide the excitation. The rail/wheel contact-interaction problem was also the motivation for a test-rig constructed by Feldmann [8]. The experimental setup consisted of a hollow steel cylinder that rolls on two steel beams.

The objective of this paper is to investigate the validity of a previously developed theoretical model and method [9] for predicting the dynamic response caused by the rolling contact of rough surfaces. This is carried out by building a laboratory test rig whereby the dynamic excitation owing to surface roughness excitation is the dominating source. The principle of the rig is simple. A steel ball is allowed to roll down a slope, thereby gaining momentum. At the end of the slope, the ball leaves the slope and continues to roll over a beam that has a machined surface. The dynamic response of the beam is measured using accelerometers attached to the underside of the beam, and the level of response is evaluated by considering the power spectral density of the acceleration registered during the time-interval of the ball being in rolling contact with the beam. The parameters of the contact-interaction model and the resulting dynamic response is computed by applying the previously developed theoretical model, and comparisons between measured and computed acceleration levels are made.

2 Formulation of physical model

The method described in [9] is applied to the case of a steel ball rolling over a beam that has a rough surface, see Figure 1.

2.1 The beam

A clamped-clamped Euler-Bernoulli model [10] is used for the beam, which has a constant cross-section \(A\) with a second moment of inertia \(I\) and the length \(l\). Furthermore, it has the elastic modulus \(E\) and the density \(\rho\). The

\(^{1}\)TWINS "Track-Wheel Interaction Noise Software"
The equation of motion for the beam displacement $w_1(x_1, t)$ is

$$
\rho A \ddot{w}_1 + C \dot{w}_1 + EI \frac{\partial^4 w_1}{\partial x_1^4} = -F_c(t) \delta (x_1 - \xi_1(t)),
$$

where $C$ is a linear homogenous operator with properties as described in [9].

The coupling to the mass is taken into account by the term on the right-hand side, where $\delta(\ldots)$ is the Kronecker delta-function and $\xi_1(t)$ is the position of the moving point of contact along the length of the beam. The velocity of the moving point of contact is thus given by $\dot{\xi}_1(t)$. The velocity also directly affects the contact-interaction force $F_c(t)$ since the force includes the effect of the varying surface topography along the path of rolling contact. Technically, $F_c$ is a function of $\xi_1$, but since $\xi_1 = \xi_1(t)$, the notation $F_c = F_c(t)$ is used. The contact force also depends on the relative motion of the beam and the ball at the point of contact. The complete expression is derived in [9], and is given by

$$
F_c(t) = F_0(t, d_c(t)) + k_0 \cdot d_c(t) + k_0 \cdot [w_1(\xi_1, t) - w_2(t)] + c_0 \cdot [\dot{w}_1(\xi_1, t) - \dot{w}_2(t)],
$$

where the dynamic excitation due to surface roughness is provided by the terms $F_0(t, d_c(t))$ and $k_0 \cdot d_c(t)$. The contact stiffness and damping is provided by $k_0$ and $c$. The physical interpretation of these terms is discussed in [9]. In summary, $F_0(t, d_c(t))$ is the blocked force with respect to $d_c(t)$, and it is the contact force that is brought about when prescribing the displacements of the
contacting bodies such that \( w_2(t) = w_1(\xi_1, t) + d_c(t) \). Equation (4) is a linear expansion of a general non-linear relation with respect to \( w_2(t) - w_1(\xi_1, t) \) at \( d_c(t) \). The expansion is valid if the actual resulting displacements \( w_2(t) \) of the ball is not too far from \( w_1(\xi_1, t) + d_c(t) \).

The dynamic response \( w_1(x_1, t) \) of the beam is expanded in \( N \) modal shape-functions \( u_n \):}

\[
w_1(x_1, t) = \sum_{n=1}^{N} q_n(t)u_n(x_1) = \mathbf{q}^T\mathbf{u}(x_1),
\]

where the elements \( u_n \) of the eigenfunction vector \( \mathbf{u} \) for an undamped clamped-clamped Euler-Bernoulli beam are given by

\[
u_n(x_1) = \begin{bmatrix} \cosh \kappa_n x_1 - \cos \kappa_n x_1 - \left[ \frac{\cos \kappa_n l - \cosh \kappa_n l}{\sin \kappa_n l - \sinh \kappa_n l} \right] \left[ \sinh \kappa_n x_1 - \sin \kappa_n x_1 \right] \end{bmatrix}.
\]

The modal coefficients \( q_n(t) \) are found by introducing the modal expansion (5) in the equation of motion (1), multiplying this equation with \( u_n(x_1) \) and integrating along the length of the beam. Owing to the orthogonality of the modal shape-functions, a state-space system of equations for the modal coefficients is obtained as:

\[
\begin{bmatrix} \ddot{\mathbf{q}} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} -C & -K \\ \mathbf{I} & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}} \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} -u(\xi_1) \frac{F_c(t)}{\rho A} \\ 0 \end{bmatrix},
\]

\[q(0) = \dot{q}(0) = 0,\]

where the matrix \( K \) is diagonal with elements \( \frac{EI}{\rho A} \kappa_n^4 \), \( n = 1 \ldots N \), where \( \kappa_n \) is the \( n \)th bending wave number [10] for a clamped-clamped beam. It is also assumed [9] that \( C \) is a diagonal matrix with elements \( c_n \), \( n = 1 \ldots N \).

### 2.2 The ball

The ball is modelled as a rigid mass \( m_2 \), thus \( m_2\ddot{w}_2 = F_c(t) - m_2g \), where \( g \) is the acceleration of earth gravity, bringing about the state-space system of equations

\[
\begin{bmatrix} \ddot{w}_2 \\ \dot{w}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{w}_2 \\ w_2 \end{bmatrix} + \begin{bmatrix} \frac{F_c(t) - m_2g}{m_2} \\ 0 \end{bmatrix}.
\]

### 2.3 The combined state-space equation system

Equations (7) and (9) are combined to give the \((2N + 2) \times (2N + 2)\) system

\[\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{F},\]

\[\mathbf{z} = \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} -C & -K \\ 0 & 0 \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} -u(\xi_1) \frac{F_c(t)}{\rho A} \\ 0 \end{bmatrix}.
\]
where
\[ z^T = \begin{bmatrix} w_2 & w_2 \end{bmatrix} q^T q^T . \] (11)

The response of the rider is given by the first two components \( \dot{w}_2 \) and \( w_2 \).

The dynamic response of the beam is determined by the components \( \dot{q} \) and \( q \).

The coefficient \( A \) in (10) is given by
\[ A = \begin{bmatrix} 0 & 0 & \cdots & \cdots \\ 1 & 0 \\ \vdots & -C & -K \\ \vdots & I & 0 \end{bmatrix} , \] (12)

and the vector \( F \) by
\[ F^T = \begin{bmatrix} (F_c(t)-m_2g) \\ 0 \\ u^T(\xi_1) \frac{F_c(t)}{\rho A} \\ \cdots \end{bmatrix} , \] (13)
where
\[ F_c(t) = F_0(t,d_c(t)) + k_0 \cdot d_c(t) + k_0 \cdot [w_1(\xi_1,t) - w_2] + c_0 \cdot [\dot{w}_1(\xi_1,t) - \dot{w}_2] = \]
[use expansion (5)]
\[ = F_0(t,d_c(t)) + k_0 \cdot d_c(t) + k_0 \cdot [q^T u(\xi_1) - w_2] + c_0 \cdot \left[ q^T u(\xi_1) - \dot{w}_2 \right] . \] (14)

Omitted elements in (12), as well as the last \( N \) elements in (13), are zero.

The dynamic response of the beam and ball is obtained by solving system (10). This requires an estimation of \( F_0(t,d_c(t)) \), \( d_c(t) \), \( k_0 \) and \( c \) in the contact interaction model (14). Hence, an estimation of the two time-dependent parameters that is valid for the total time of contact interaction is necessary. The time-dependence of these terms is directly attributed to the varying surface topography at the moving point of contact. In other words, an estimation is required that is valid along the complete path of rolling contact. In many cases, this is not possible. Instead, the parameter estimation has to made along a shorter part of the path of rolling contact. From this estimation, a method is required for extrapolating the parameters to the complete path of rolling contact. Such a method is proposed in [9]. From the initial estimation, the corresponding power spectral density is computed. By applying a Monte-Carlo method, a number of representative time-dependent parameters are generated, say \( F_0^j(t,\xi_d(t)) \) and \( d_c^j(t) \), bringing about the corresponding contact interaction force \( F_c^j(t) \). By inserting this in (10) the dynamic response \( z^j \) can be computed. Averaged quantities, such as power spectral densities, are obtained by repeating this procedure a number of times and averaging the response spectra of \( z^j \).
3 Experimental methodology

The experimental apparatus consists of a test-rig in which a steel ball is allowed to roll down a slope, thereby gaining momentum, and then over a steel beam that has a rough surface. The resulting dynamic response of the beam is determined from five accelerometers attached by wax to its underside. The test-rig is schematically shown in Figures 2 and 3. The slope consists of two rails, guiding the ball as it gains momentum. At the end of the slope, the two rails become thinner, gradually lowering the position of the ball until it makes contact with the beam. The small impact upon contact is dealt with by employing a combination of vibration damping and isolation. The first part of the beam is tightly pressed down onto a thin rubber layer, by clamping the beam at the positions indicated in Figure 3, thereby effectively increasing the loss of this part.

The clamping is carried out leaving enough space for the ball to pass through. The vibration caused by the initial impact quickly diminishes in amplitude, owing to the thin rubber-layer under the first part of the beam. Owing to the first clamping, furthermore, most of the vibrations are constrained to stay within this part of the beam. In effect, the accelerometers only register the vibration owing to the ball rolling over the second part of the beam. Two eddy-current proximity probes are positioned over the two clampings. Thus, two pulses are registered as the ball passes by, and the rolling velocity is estimated from the time between these pulses and the distance between the probes. An example of a measured acceleration time-history under the second part of the beam is shown in Figure 4. The ball
Table 1: Instruments used for measurements.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Type</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometer</td>
<td>B&amp;K 4393</td>
<td>5</td>
</tr>
<tr>
<td>Charge amplifier</td>
<td>B&amp;K 2635</td>
<td>5</td>
</tr>
<tr>
<td>Proximity probe</td>
<td>Bentley Nevada 3106</td>
<td>2</td>
</tr>
<tr>
<td>Data acquisition system</td>
<td>HP VXI-system, HP E1421B</td>
<td>1</td>
</tr>
<tr>
<td>Computer</td>
<td>PC 200 MHz, 64MRAM</td>
<td>1</td>
</tr>
</tbody>
</table>

rolls over this part between the times indicated by the dashed vertical lines in Figure 4. The initial response of the impact at $t = 0.05$ s quickly dies out and does not contribute to the dynamic response measured when the ball is in rolling contact with the unconstrained part of the beam.

The instruments used for the measurements are summarised in Table 1.

![Measurement test rig diagram](image-url)
Figure 4: Example of measured acceleration under second part of beam. Ball is in rolling contact with the unconstrained part of the beam between the times indicated by the vertical dashed lines.

A steel beam with a thickness of 6 mm, a width of 50 mm and a length of 700 mm was ground to have an average surface roughness [11] $R_a = 0.3 \mu$m. This was verified by performing surface profile measurements with a Taylor-Hobson stylus instrument [11]. The beam was mounted in the experimental test-rig that applied a torque of 60 Nm at each of the four clampings; tightly pressing down the first 290 mm of the beam against the thin layer of rubber.

The dynamic response of the second part of the beam, between the two pairs of clamping, is registered by the five accelerometers while allowing a steel ball with a diameter of 40 mm to roll over the beam. By changing the starting position of the ball on the slope, the rolling velocity is varied between 1 and 3 m/s. For both of these velocities, the ball is allowed to roll over the beam 25 times. For each rolling velocity, the $5 \times 25$ power spectral densities (PSD) are obtained, and, for both rolling velocities, a single averaged PSD is computed.

4 Response computation

The response computation is carried out by modelling the unconstrained part of the beam as a clamped-clamped Euler-Bernoulli beam. The validity of this approach is investigated experimentally, verifying that this model approximates the dynamic response of the beam fairly well up to 5,000 Hz.
The parameters of the linear contact-interaction model are computed by applying a numerical contact-computation program to the measured three-dimensional topography of the beam. This program is thoroughly discussed in [9].

4.1 Experimental validation of dynamic model

The experimental validation is carried out by performing an Experimental Modal Analysis (EMA) of the complete length of the beam, including the clampings. The geometry used for the EMA is shown in Figure 5. There are 207 measurements points distributed along the beam and eight points positioned directly at the clampings. Frequency response functions (accelerances) are estimated by exciting the beam vertically with an impact hammer at the point indicated in Figure 5, whilst measuring the impact force – with a force transducer integrated in to the impact hammer – and the resulting acceleration using five accelerometers. For each set of response points, the accelerances are estimated, averaging the results of 10 impacts. The accelerometers are then moved to new response positions, and the procedure is repeated until 215 frequency response functions are estimated, providing sufficient input data to the EMA.

Comparisons are made of the accelerance level from the excitation point positioned at the unconstrained part of the beam to response points at the first part of the beam and to response points at the unconstrained part of the beam, respectively, verifying that the level of the latter accelerances are much higher. An example of two such functions is shown in Figure 6.

A single degree-of-freedom peak-picking method is used to obtain the modal parameters. From the resulting mode-shapes, it is verified that the motion of the first part of the beam is negligible compared to the motion of the unconstrained part of the beam. Moreover, the mode shapes of the unconstrained part of the beam agree with anticipated theoretical shapes. A few examples of experimentally estimated mode shapes are shown in Figures 7-9.

The analysis also reveals that the clampings undergo a small rotary motion. This is taken into account by increasing the effective length of the beam, in the model, by 30 mm. An example of measured and theoretical accelerance is shown in Figure 10. The theoretical model approximates the dynamic response of the unconstrained part of the beam quite well, with a tendency to overestimate the dynamic response in off-resonance frequency regions.

It is concluded that the unconstrained part of the beam can be modelled as a clamped-clamped beam.
Figure 5: Modal analysis geometry.

Figure 6: Typical examples of measured accelerances when excited at unconstrained part. *Solid line*: Accelerance response point at unconstrained part of beam. *Dotted line*: Accelerance response point at first part of beam.
Figure 7: Mode shape corresponding to a resonance found at 504 Hz.

Figure 8: Mode shape corresponding to a resonance found at 2,419 Hz.

Figure 9: Mode shape corresponding to a resonance found at 4,465 Hz.
4.2 Numerical estimation of parameters in contact interaction model

An optical non-contacting method [12] is used to measure a small part of the surface topography of the beam, and the digitised three-dimensional topography is used as input to the contact computation program [13] that is described in [9]. This topography corresponds to a rectangular grid having a width of 1.26 mm and a length of 80 mm along the path of rolling contact. The distance between each measurement grid point is 0.02 mm.

The global shape of the measured topography is removed by fitting a second-order polynomial surface to its measured counterpart and then removing the fitted surface from the measured. Also, a three-point median filter is applied to the measured topography. Surface profile measurements established that the average surface roughness $R_a$ of the steel ball is less than 1/10 of the beam roughness. Since this verifies that the steel ball is much smoother than the steel beam, the contact computation is carried out by approximating the spherical surface of the ball to be ideally smooth. A small part of the beam-and-ball topography is shown in Figure 11. The force $F_0(t, d_c(t))$ is obtained by using a constant $d_c(t) = d_0$, where $d_0$ is chosen so that the mean value of the computed $F_0(t, d_0)$ corresponds to the weight of the steel ball. As discussed in [9], the choice of $d_c(t)$ depends on the specific...
Figure 11: Topographies of smooth ball and rough beam.

case that is at hand. Given two relatively smooth surfaces in rolling contact, it is assumed that the relative displacement is of moderate order, justifying this choice of linearisation. From the contact computation, the contact stiffness $k_0$ is estimated to be $6.8 \cdot 10^6$ N/m. It is often quite difficult to choose a suitable value of damping for a contact dynamics analysis. Soom & Chen [14] remarked that one usually chooses the non-dimensional damping value from experience or measurement. For the case studied here, $c_0 = 100$ Ns/m gives reasonable agreement between measurement and computation.

4.3 Numerical solution of state space-equation system

The blocked force $F_0(t,d_0)$, obtained from the contact-computation, is used to generate the power spectral density $S_{FF}(f)$. Applying the Monte-Carlo procedure, five force time-histories $F_j^0(t), j = 1 \ldots 5$, are created, and for each $F_j^0(t,d_0)$ the state-space system (10) is solved numerically for the two rolling velocities 1 and 3 m/s. This is carried out by formulating the state-space system (10) as an initial value problem for a system of first order differential equations. Given $F_j^0(t,d_0)$, the resulting response $z^j(t)$ is obtained by a 4th order Runge-Kutta method\(^2\). The numerical integration is carried out ensuring 0.1 % accuracy in all components of $z(t)$.

The corresponding power spectral density (PSD) at each of the five accelerometer positions is obtained, and, for both rolling velocities, a single

\(^2\)function ode45.m in Matlab 5.3 ©1984-1999 The MathWorks, Inc.
averaged PSD is computed.

In addition to the response computation, a final verification of the numerical procedure and the dynamic model is made by solving for the dynamic response for completely smooth surfaces. If the travel speed of the ball is sufficiently low, the dynamic response of the beam resembles the static case. It is noted [15] that if the ratio of the fundamental period $T_f$ of the beam and the travel time $\tau$ of the moving oscillator system was low enough, $T_f/\tau = 0.1$, there is a close resemblance between the static and dynamic case. For the case studied here, the maximum rolling velocity is 3 m/s. This corresponds to $T_f/\tau = 0.038$. As an assessment of the resemblance, the ratio between the static displacement and the dynamic displacement of the beam at the centre is evaluated. A visual comparison between the dynamic and static displacement at the centre of the beam as the ball moves over the beam is shown in Figure 12. The dynamic and static displacements are indeed indistinguishable.

Figure 12: Ratio between displacement of the beam and maximum static displacement as the oscillator system moves over it with a constant velocity of 3 m/s. The static displacement resulting from a mass moving over the beam is included as reference.
5 Results

The resulting measured averaged spectra, for the two rolling velocities 1 and 3 m/s are shown in Figure 13. Comparisons between measured and computed spectra are shown in figures 14 and 15. The computed acceleration levels compare rather well to the measured response, especially for the lower rolling velocity. The predicted acceleration levels are based upon the computed power spectral density $S_{FF}(f)$ that is displayed in Figure 16. An example of a computed acceleration time-history is shown in Figure 17.

Several interesting features may be observed in Figures 13 and 16. The computation predicts an increasing difference of force level with increasing frequency. A similar trend is evident in the measured spectra. Furthermore, as shown in figures figures 14 and 15, the response falls off rapidly with decreasing frequency below 800 Hz in both the computation and in the measurement.

The computation has a tendency to overestimate the response. This may be due to the fact that the theoretical dynamic model of the unconstrained part of the beam has such a tendency in off-resonance frequency regions (See Figure 10). There is a relatively flat-response-level region around 1 kHz, where the theoretical model overestimates the response for both rolling velocities. This tendency may be due to that reason. It may also be due to inaccuracies in applying the contact-computation procedure. The response is predicted by computing the blocked force $F_0(t, d_0)$ owing to the surface micro-geometry. An alternative method is to directly measure the surface profile deviation $d_c(t)$ that is believed to provide dynamic excitation. This will provide an additional validation, and it may be used for verifying the procedure used here.

Feldmann [8] (1987) presents a theoretical model and performed measurements on a similar system, studying the dynamic response caused by a ring and a beam in rolling contact. Using the Hertzian impact theory, a 10 dB of level increase per a doubling of rolling speed is predicted. As shown in Figure 13, this behaviour is not experimentally confirmed for the case studied here. The level does not increase significantly for low frequencies, whereas it increases by approximately 10 dB at 5,000 Hz. This indicates that the speed behaviour suggested by Feldmann is not valid for all situations.
Figure 13: Measured acceleration PSD level of beam (dB re. 1 m/s²), for rolling velocities 1 m/s (solid line) and 3 m/s (dotted line).

Figure 14: Computed (dotted line) and measured (solid line) acceleration PSD level (dB re. 1 m/s²) for a rolling velocity of 1 m/s.
Figure 15: Computed (dotted line) and measured (solid line) acceleration PSD level (dB re. 1 m/s$^2$) for a rolling velocity of 3 m/s.

Figure 16: Computed force PSD level (dB re. 1 N) for rolling velocities 1 m/s (solid line) and 3 m/s (dotted line).
6 Conclusions

The dynamic response caused by the rolling contact of rough surfaces is studied experimentally. This is carried out by building a test rig, whereby a steel ball rolls over a steel beam with a machined surface. The resulting response is evaluated by measuring the resulting acceleration of the beam caused by the rolling contact with the steel beam. Previously, a theoretical model was developed for predicting the dynamic response caused by the rolling contact of rough surfaces [9]. This model is applied to the specific case of a steel ball rolling over a steel beam with a rough surface. A small part of the beams three-dimensional small-scale surface topography is used as input to a contact computational program, bringing about a blocked-force spectrum that provides dynamic excitation owing to the contact interaction of the ball and the beam. The Monte-Carlo formulation outlined in [9] is applied to the computed force spectrum, providing simulated force time-histories. This approach enables the treatment of non-stationary situations, using measured surface topographies from a short part along the contact interaction path.

The experimental results validate the theory, indicating that the model sufficiently describes the dynamic response in narrow frequency bands over a wide frequency range.
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References


