Wet clutch load modeling for powershift transmission bench tests.

Belastningsmodellering av våta kopplingar i riggprov av powershift-transmission.

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Abstract

In this thesis a model is developed for calculating the loads on the wetclutches in a powershift transmission. This thesis was done at Volvo CE in Eskilstuna and is focused on their 4-speed countershaft transmissions. The goal of the project is to be able to calculate the loads automatically during the transmission tests and thus acquire increased knowledge about what occurs during operation.

The model was developed by first generating a number of concepts and then evaluating them to decide which one should be developed further. The chosen concept was then developed further and implemented into the test equipment as a calculation script. The chosen loads to model were the energy absorbed in each clutch, the coefficient of friction (COF) and the slip distance. The COF was later found to give too unreliable results to be used in any other way than as a benchmark for how well the model was configured and to see any large changes in COF.

The model was validated by calculating the energy absorbed in a HTE-200 series transmission and comparing it to a reference calculation model. It was seen that the results from the new model are very close to the reference result. The energy lost calculated by the new model will be equal to 94 % of the energy lost calculated by the reference model, but only when measuring the inertia phase. The energy lost is equal to 135 % when the torque phase is included. The increasing difference is believed to be because of an error in the reference model that means it does not cover the torque phase.
Sammanfattning

I detta examensarbete har en modell utvecklas för att beräkna belastningarna på de våta kopplingar som finns i en powershift transmission. Arbetet utfördes på Volvo CE i Eskilstuna och fokuserar på deras 4-växlade ”countershaft” transmissioner. Målet med projektet är att kunna beräkna belastningarna automatiskt när riggtesten körs och på så sätt få ökad kunskap om vad som händer under körning.


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Introduction

1.1 Background

Volvo Construction Equipment is one of the world’s leading manufacturers of wheel loaders and articulated haulers, with over 14 thousand employees all over the world and net sales of 64 billion SEK. To maintain this position, constant development of the company and its products is needed [1]. One of the developments that Volvo CE focuses on is the drive-trains in their machines to meet the increasing environmental legislation requirements as well as lowering fuel consumption. This is mainly done at the Volvo Technical Centre in Eskilstuna.

Today some Volvo machines utilize a powershift transmission, this transmission is based on a shifting technique that uses wet clutches to shift gears without losing output torque [2]. This technique requires great understanding of how wet clutches operates to get a smooth gear shift with little wear on the components. During development of a new powershift transmission a lot of tests have to be performed and the current test setup will yield little information on what actually happens to the wet clutches during these tests. This will make it difficult to identify the causes for failure during the test.

1.2 Purpose

The purpose of this project is to create a model that calculates the loads in the clutches during testing and presents it in real time as the test is run. This new data can then be used to determine the cause of failure during certain scenarios and then utilized when designing new more efficient transmissions as well as investigating failure in existing transmissions.
1.3 Goals

The overall goal of this project is to create a model which describes the loads that the clutches are subjected to during test. The project includes:

- Determining appropriate loads to model.
- Creating a model for calculating these loads during testing.
- Validation with existing test data.
- Implementing the model in Volvo test measurement software.
- Investigating how the lifetime and wear can be determined from this model.

1.4 Delimitations

The following delimitations were made on the project:

- The test rig will be unmodified and only the existing hardware will be used.
- The investigation of lifetime and wear will only be a theoretical one and no experiments on this will be performed.
- The model will only be run in a test rig and not tested in a real machine.
- If the model is not producing acceptable results it will not be tested in a test rig.
- The model should be simple and fast enough to be calculated while the test is running.
- The model will be developed for Volvo’s 4-speed countershaft transmissions for wheel loaders.
- Validation of the model will be against a HTE200 transmission.
This chapter will provide a theoretical background to the project and explain basic concepts such as how a powershift works and how it has been modelled previously.

2.1 Powershift

A powershift transmission is a transmission designed to shift gears without interrupting the transmitted torque. This is done by using wet clutches to switch between active gear ratios. [3]

2.1.1 Basic design

A powershift transmission can be designed in many different ways but the common feature is that it will have two or more wet clutches mounted in parallel between which to shift the torque. The transmission can also have multiple wet clutch "gearboxes" mounted in series allowing for additional gear ratios to be achieved. An example transmission can be seen in figure 2.1. This features two gearboxes in series, two directional gears and four speed gears. [4, 3]
2.1. POWERSHIFT

2.1.2 Shifting procedure

The gear shifting procedure in a powershift transmission is a bit more complicated than in a regular transmission. There are four different gear shifting scenarios that can occur but they all share the same shifting phases:

- **Filling phase.** A flow impulse from the actuation system moves the piston to the kissing point, which is the position where the friction surfaces come into contact with each other.

- **Torque phase.** During this phase the torque will be shifted over from the disengaging to the engaging clutch.

- **Inertia phase.** During this phase the speed difference between the plates in the disengaging clutch will increase while the speed difference in the engaging clutch will decrease to zero. This is due to internal inertias that is needed to be overcome.

The order of these phases will depend on the shifting scenario. These can be seen in table 2.1. One common factor is that all scenarios will begin with the filling phase.

---

**Figure 2.1:** Schematic view of a powershift transmission with a separate wet clutch for each gear.
Table 2.1: Phase order in different shifting scenarios.

<table>
<thead>
<tr>
<th></th>
<th>With positive input torque (driving)</th>
<th>With negative input torque (braking)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upshift</td>
<td>Torque phase → Inertia phase</td>
<td>Inertia phase → Torque phase</td>
</tr>
<tr>
<td>Downshift</td>
<td>Inertia phase → Torque phase</td>
<td>Torque phase → Inertia phase</td>
</tr>
</tbody>
</table>

To better explain how a gear shift in a powershift transmission is carried out one can consider a scenario were a person on skis is going uphill between two ropes. The ropes are moving at different speeds and the person is holding on to the slower rope as shown in step 1 in figure 2.2. In order to change speed to the same as the fast rope the person has to perform the following steps:

1. The person is starting out travelling at the same speed as the slow rope.
2. The person then grabs the faster rope with increasing pressure thus decreasing the pull force in the slow rope hand and increasing it in the fast rope hand.
3. At this point the pull force in the hand with the slow rope equal is to zero.
4. The slow rope is now released and the pressure in the fast rope hand is increased to accelerate the person.
5. The person is now travelling at the same speed as the fast rope.

![Figure 2.2: An illustration showing a man going up a hill changing speed in a way equivalent to a powershift.](image)

This scenario is equivalent to an upshift with positive torque. The other shift scenarios will be performed in a similar way but in a different order, as shown in table 2.1. \[2\]

The pressure levels and slipspeeds for an upshift with positive torque can be seen in figure 2.3.
2.2 Wet clutches

Wet clutches are a type of friction clutch that is constantly lubricated to reduce wear and increase the cooling capacity. This makes is possible for the clutch to operate at higher loads than a regular clutch. These types of clutch are used in many applications such as automatic transmissions in cars and also in torque converters as lock-up clutches and different locking devices. A schematic drawing of a wet clutch can be seen in figure 2.4. [5]

These kinds of clutches are usually engaged using a hydraulic piston. To engage the clutch a pressure is applied to the piston. This moves the plates closer together and creates the friction force needed to transfer the torque. A return spring is mounted inside the clutch to push back the piston and disengage the clutch when the piston pressure is decreased. This process of engaging the clutch can be divided into four stages: fully disengaged, filling, engagement and fully engaged. [5]
The friction discs in a wet clutch are often made of a steel core with a paper-based friction material on both sides. This paper-based friction material has grooves in it to give the oil trapped between the plates a way to escape as well as provide a flow of oil for cooling when engaged. Figure 2.5 shows a friction disc with grooves.[7]

The wet clutch exhibit torque loss due to shear forces caused by the oil. This is called drag torque. In Section 2.2.4 the concept of drag torque will be further described.

\[
P_{\text{avg}} = T \times |(\omega_1 - \omega_2)|
\]  

(2.1)

Figure 2.5: Drawing of a friction disc for use in a wet clutch. [7]

2.2.1 Heating and cooling of a wet clutch

Friction heating

When two surfaces slide against each other heat is generated. This happens during the working mode for a wet clutch in normal operation. In some cases "hotspots" will form on the disc due to this kind of heating. This is described in more detail in Section 2.2.2.

The power generated by sliding can be calculated using equation 2.1 were \( P_{\text{avg}} \) is the generated power, \( T \) is the transferred torque and \( \omega_1 \) and \( \omega_2 \) are the angular velocity of the friction disc and mating plate. [8]

\[
P_{\text{avg}} = T \times |(\omega_1 - \omega_2)|
\]

(2.1)

Cooling of the wet clutch

The primary cooling mechanism in a wet clutch during operation is heat transfer to the oil. The effect of the cooling can be varied by for example altering the flow rate or the geometry of the friction disc grooves. [8]
2.2.2 Frictionally-excited thermoelastic instability (TEI)

The formation of hot spots is a common problem in clutches. These are formed due to high localized temperature and pressure zones. These localized temperature zones are created due to non-uniformity in the contact pressure distribution across the surface. Areas that experience higher pressure will also experience higher temperature. The pressure will then increase in the area as the temperature rises due to localized thermal expansion thus increasing the temperature further. This is called thermoelastic instability (TEI), but is normally referred to as "hot spots". The material in these zones will effectively be heat-treated, and for example a martensite structure can be created in the ferrous materials. An example of a steel separator disc with hot spots can be seen in figure 2.6.[9]

![Figure 2.6: Steel separator disc with hot spots.[9]](image)

The types of hotspots can be divided into two main groups: focal hotspots and band hotspots. Focal hotspots are hotspots that are discontinuous in the sliding direction thus forming distributed spots around the disk. Band hotspots are continuous bands which go around the friction surface forming concentric circles. It has been shown that the most common case when dealing with two dissimilar materials is focal hot spots.[9]

2.2.3 Wear and degradation of a wet clutch

The discs in a wet clutch will be subjected to wear during its lifetime and thus some of its properties will change. These properties includes the friction coefficient, real contact area and width of the plate. Tests done on wet clutch wear have shown that the wear rate will be severe in the fist 200 cycles with noticeable change in thickness and then lowered after this run-in period. It has also been seen that the friction plates closest to the piston will suffer the most from this wear. [10]

If the temperature gets too high during operation it may cause the cellulose to decompose leading to degradation of the friction material. This usually happens around 200 to 400 °C [10]. Another failure mode that can occur during operation is glazing of the friction material. This is caused by deposition of fluid degradation products on the
surface creating a darkened smooth or shiny surface. The glazing often causes a decrease in friction and sometimes increases the friction in the boundary regime [10].

2.2.4 Drag torque

The wet clutch suffers from torque loss even when it’s fully disengaged. This is called drag torque. The drag torque is caused by the shear forces from the oil that act on the rotating plates. The drag torque as a function of speed is shown in figure 2.7. It can be seen that the drag torque is proportional to the speed up to a certain point where it drops. This is due to the fact that the amount of oil being pumped out of the clutch is higher than the amount that is pumped into the clutch. [11]

\[ 
\text{Torque} \quad \text{Drag torque peak} \\
\text{Full film} \quad \text{Broken film} \\
\text{rpm} 
\]

\textbf{Figure 2.7: Drag torque as a function of rotational speed.}

In figure 2.8 a schematic view over what happens to the oil can be found. In this figure the broken film due to the previously mentioned conditions is shown in (b).

\[ 
\text{(a)} \quad \text{(b)} 
\]

\textbf{Figure 2.8: Schematic view of the oil film before (a) and after (b) the peak.}

The transition point can be moved to a lower or higher rotation speed by altering the oil flow into the clutch. By for example reducing the oil flow when the clutch is disengaged a low drag torque can be achieved at lower speeds.[12]
2.3 Torque-converter

A torque converter is a device that uses hydrodynamic forces to transfer power in a flexible way. These are often used to replace the clutch between the engine and the transmission in automatic transmission.

A torque converter consists of three major parts: the pump (impeller), turbine and stator. The converter is also filled with a fluid that transfers the power. Figure 2.9 shows a cross-section of a torque converter with the components marked out. [13]

When the pump is rotated by the engine it will cause the fluid to be pumped out from the center and into the turbine. The turbine will then force the fluid to change direction creating a torque on the output shaft. The fluid is now travelling in the opposite direction compared to the pump, This is corrected by the stator that changes back the direction of the fluid. Leading to a higher efficiency since the engine won’t have to provide the torque to change the fluid’s direction back to normal. [13]

2.4 Lifetime estimation

The lifetime is an important parameter to consider when designing a component. If the component is undersized then it will fail too early and lead to a lot of service costs. If the component is oversized then the material and manufacturing cost will be too high. By finding a balance between lifetime and cost a inexpensive and high quality component can be achieved.

This section will cover the two basic tools to approximate the lifetime of a component.

2.4.1 S-N curve

S-N curves are a way to describe the number of load cycles a material can withstand before failure at a certain load. These curves are also called Wöhler curves. An example curve can be seen in figure 2.10.
This curve and the Palmgren-Miner rule provide a powerful tool for calculating the lifetime of a component.

### 2.4.2 Palmgren-Miner rule

One theory to calculate the accumulated damage for an item is to use the Palmgren-Miner rule. This rule will specify how much of the item’s lifetime is spent on each load case. When the summarized damage accompanied with each individual load equals to 1 failure occurs. This is described in the following equation:

\[
\sum_{i=1}^{n} \frac{n_i}{N_i} = 1
\]

were \( n_i \) is the number of cycles of the current load and \( N_i \) is the total number of cycles to failure with the current load. [16]

### 2.5 Powershift simulations

There are many ways to simulate a powershift transmission but one of the more common is to model the transmission with a lumped element model. This model consists of simplified elements that represent different components. This makes it easy to build complex systems that are easy to understand and modify. These models can be built in programs such as MatLab/Simulink. In figure 2.11 a 4- and 15-degrees of freedom (DOF) model can be seen. [17]
2.5. POWERSHIFT SIMULATIONS  

CHAPTER 2. LITERATURE SURVEY

Figure 2.11: Lumped element models of a dual clutch transmission. (a) 15 DOF, (b) 4 DOF. [17]

By reducing the degrees of freedom a faster calculation time can be achieved but since the components are simplified it can cause an error in the final results. The models with a higher number of DOF often include a better representation of the stiffness and inertia for the components. [17]
Method

The following steps were performed to create a working model for calculating the clutch loads: concept generation, concept elimination, further development of the chosen concept, implementation into test equipment and validation of result.

3.1 Concept generation

Three major concepts were generated during the investigation and they are presented in the following sections.

3.1.1 Concept 1

This concept is based on calculating the torque transferred in each clutch by comparing the input and output torque and taking into account the active gear ratios to the different clutches. The losses in the transmission are also needed to get the correct torque in each clutch since some of the torque is lost due to other factors.

3.1.2 Concept 2

This is a simple concept based on calculating the lost energy with a known coefficient of friction. This calculation is made for all the clutches during a shift since the calculations are independent from each other. The power can be calculated by using the following equation:

\[ P_L = \mu \times F \times N \times |\omega_1 - \omega_2| \times \int_{R_I}^{R_O} dr \quad (3.1) \]

where \( \mu \) is the coefficient of friction, \( N \) is the number of friction interfaces and the other constants \( (R_I, R_O, F, \omega_1 \text{ and } \omega_2) \) are described in figure 3.1.
3.1. CONCEPT GENERATION

CHAPTER 3. METHOD

3.1.3 Concept 3

This concept is based on calculating the total power lost in the transmission by comparing the difference in input and output power. The losses in the clutches are then isolated by removing those due to other factors (drag torque etc.) from the total power loss. The transmission will be modelled as a series of clutches and flywheels to calculate the losses more easily and to divide the power. This seen in the figure below.

Figure 3.2: Schematic over the different components in the model and how they are connected.

For each moment in time the power loss in the clutches is divided proportionally to the theoretical amount of power loss in each clutch. This powersplit is illustrated in figure 3.3 were the angle $k$ of the line is determined by the theoretical power losses $P_t$. The measured power losses $P_m$ is determined by finding the point on the line were the sum of the two losses is equal to the total power loss $P_{tot}$ in the clutches.
3.2 Concept elimination

The pros and cons of each concept will be compared to determine which concept will be developed further. The complete comparison table can be found in appendix A. A summary of the most important points for each concept is listed in table 3.1. The expandability and low dependency on COF is the most important property for this application.

<table>
<thead>
<tr>
<th>Concept Name</th>
<th>Pros.</th>
<th>Cons.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept 1</td>
<td>• Independent of COF when slipping occurs in only one clutch.</td>
<td>• Dependent on COF when slipping occurs in both clutches.</td>
</tr>
<tr>
<td>Concept 2</td>
<td>• Won’t require data about which clutch is active.</td>
<td>• Heavily dependent on an exact COF.</td>
</tr>
<tr>
<td>Concept 3</td>
<td>• Easily expanded to compensate for additional losses</td>
<td>• Complex calculation for power split</td>
</tr>
<tr>
<td></td>
<td>• Will not require an exact COF model.</td>
<td>• Only works with a maximum of four clutches (two pairs in series)</td>
</tr>
</tbody>
</table>

From this comparison it was decided that concept three is going to be developed
further mostly due to the fact that it will not be as dependent on how precise the coefficient of friction is and the fact that it can be easily expanded.

3.3 Detailed development

Concept three will be developed and described in more detail. This section will cover how the gear ratios, moment of inertia, power, energy, COF and sliding distance are calculated.

3.3.1 Gear reduction and inertia calculations

The gear ratios and reduced moment of inertia are calculated by using a model described by David Berggren [18]. This model utilizes matrix calculations to describe the transmission and thus becomes very flexible due to the fact that the different designs are calculated in the same way. The constraints of the transmission will be described by the matrix $C$ which is defined so that equation 3.2 is valid. Some examples of this matrix can be found in appendix B.

$$
C \cdot \omega = \begin{bmatrix}
c_{11} & \cdots & c_{1n} \\
\vdots & \ddots & \vdots \\
c_{m1} & \cdots & c_{mn}
\end{bmatrix} \cdot \begin{bmatrix}
\omega_1 \\
\vdots \\
\omega_n
\end{bmatrix} = 0 \quad (3.2)
$$

where $\omega$ is the angular velocity, $m$ the number of constraints and $n$ is the number of shafts in the transmission. Each shaft is indexed with a number from one to $n$. The shafts are then divided into sensor and other shafts. The "sensor" shafts are set to be those with a speed sensor on them and the rest are set to "other". The indexes for the sensor shafts are defined in the vector $N_{S.shaft}$ in the order shown in equation 3.3 and the indexes for the other shafts are defined in the vector $N_{O.shaft}$.

$$
N_{S.shaft} = [n_{turb.} \ n_{inter.} \ n_{outp.}] \quad (3.3)
$$

The gear ratio between the sensor shafts and the other shafts ($R_{O.shaft}$) can be calculated using equation 3.4. An explanation for the $C[::,N]$ notation can be found in appendix B.

$$
R_{O.shaft} = -C[::,N_{O.shaft}]^{-1} \cdot C[::,N_{S.shaft}] \quad (3.4)
$$

The gear ratio between the sensor shafts ($R_{S.shaft}$) is given by an $n \times n$ identity matrix were $n$ is the number of sensor shafts as shown in equation 3.5. This is due to the fact that for simplicity’s sake all the clutches are defined as open and their shafts independent of each other.

$$
R_{S.shaft} = I = \begin{bmatrix}
1 & 0 & \cdots \\
0 & 1 & \cdots \\
\vdots & \vdots & \ddots
\end{bmatrix} \quad (3.5)
$$
The equivalent moment of inertia is calculated by the following steps: first the moment of inertia for each shaft is defined in the vector $J$ as shown in the following equation:

$$ J = \begin{bmatrix} J_1 & J_2 & \cdots & J_n \end{bmatrix}^T $$  \hspace{1cm} (3.6)

These are then divided into two vectors one for other shafts ($J_{O.shaft}$) and one for sensor shafts ($J_{S.shaft}$).

$$ J_{S.shaft} = J[N_{S.shaft}] $$  \hspace{1cm} (3.7)

$$ J_{O.shaft} = J[N_{O.shaft}] $$  \hspace{1cm} (3.8)

By using equation 3.9 the equivalent moment of inertia at each sensor shaft is calculated.

$$ J_{equ} = \text{diag}(J_{S.shaft}) + R_{O.shaft}^T \text{diag}(J_{O.shaft}) R_{O.shaft} $$  \hspace{1cm} (3.9)

The results this equation will have the form:

$$ J_{equ} = \begin{bmatrix} J_{turb.} & 0 & 0 \\ 0 & J_{inter.} & 0 \\ 0 & 0 & J_{outp.} \end{bmatrix} $$  \hspace{1cm} (3.10)

### 3.3.2 Power calculation

The power lost in the clutches will be estimated by first calculating the total power lost in the transmission and then removing the power gained or lost by inertia as well as the power lost due to other effects such as drag torque. This is described by the following equation:

$$ P_{\text{all cl.}} = |P_{\text{in}} - P_{\text{ut}}| - \sum (P_{\text{inertia,i}}) - P_{\text{misc.}} $$  \hspace{1cm} (3.11)

The input and output power can be calculated using the torque and angular velocity as per:

$$ P = T \times \omega $$  \hspace{1cm} (3.12)

were $P$ is the power, $T$ is the torque and $\omega$ is the angular velocity. The following equation can be used to calculate the power lost or gained due to inertia.

$$ P_{\text{inertia,i}} = \frac{d}{dt} \left( \frac{1}{2} J_i \omega_i^2 \right) $$  \hspace{1cm} (3.13)

where $J$ is the moment of inertia and $\omega$ is the angular velocity of the shaft.

The active mode of each clutch needs to be determined in order to decide how to split the power between the clutches. This is done by observing the clamp force ($F_c$) and slip speed ($|\Delta \omega|$). Table 3.2 describes which mode is active at different conditions. The sync mode is when the clutch plates are in contact and no slip occurs, the open mode is when the clutch plates are not in contact. The working mode is when the clutch plates
are in contact and there is a slip between the plates, the clutch will experience frictional heating during this mode.

**Table 3.2:** Table describing which mode is active in different conditions.

<table>
<thead>
<tr>
<th>$F_c \leq 0$</th>
<th>$F_c &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\Delta \omega</td>
</tr>
<tr>
<td>$</td>
<td>\Delta \omega</td>
</tr>
</tbody>
</table>

The total power lost in the clutches can then be divided between the clutch pairs by using the logic described in table 3.3. The power will be split based on which working mode is active in each clutch pair. i.e if one clutch pair does not have any contact slip then no power is assumed to be lost there and thus all the power is set to be lost in the other pair.

**Table 3.3:** Logic table for splitting the power between the two clutch pairs.

<table>
<thead>
<tr>
<th>Working Mode</th>
<th>$C_x/C_y$</th>
<th>$C_f/C_r$</th>
<th>$P_{C_x/C_y}$</th>
<th>$P_{C_f/C_r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W S/O</td>
<td>P_all cl.</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S/O W</td>
<td>0</td>
<td>P_all cl.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W W</td>
<td>Equation 3.14</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If the working mode is active in the two clutch pairs at the same time then equation 3.14 can be used to calculate the theoretical power loss the clutch pairs and then dividing the real power proportional to them.

\[
\begin{align*}
P_{C_x/C_y} &= \frac{P_{\text{all cl.}}}{1 + 1/k_{xy/fr}} \\
P_{C_f/C_r} &= \frac{P_{\text{all cl.}}}{1 + k_{xy/fr}} \\
k_{xy/fr} &= \frac{\mu_x * F_x * N_x * |\Delta \omega_x| * \int_{R_{i,x}}^{R_{o,x}} dr + \mu_y * F_y * N_y * |\Delta \omega_y| * \int_{R_{i,y}}^{R_{o,y}} dr}{\frac{R_{o,f}}{R_{i,f}} * \mu_f * F_f * N_f * |\Delta \omega_f| * \int_{R_{i,f}}^{R_{o,f}} dr + \frac{R_{o,r}}{R_{i,r}} * \mu_r * F_r * N_r * |\Delta \omega_r| * \int_{R_{i,r}}^{R_{o,r}} dr}
\end{align*}
\]

(3.14)

where $\mu$ is the theoretical coefficient of friction, $F$ is the clamp force, $N$ is the number of contact interfaces, $|\Delta \omega|$ is the slip speed and $R_i$ and $R_o$ are the inner and outer radius of the contact interface respectively. The complete calculation of equation 3.14 can be found in appendix C.

The power is then split in a similar way between the clutches in each pair. **Table 3.4** and **equation 3.15** describe how the power is split in the pair of speed clutches.
Table 3.4: Logic table for splitting the power between the two individual speed clutches.

<table>
<thead>
<tr>
<th>Working Mode</th>
<th>$C_x$</th>
<th>$C_y$</th>
<th>$P_x$</th>
<th>$P_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>S/O</td>
<td>$P_{Cx/Cy}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S/O</td>
<td>W</td>
<td>0</td>
<td>$P_{Cx/Cy}$</td>
<td></td>
</tr>
</tbody>
</table>

Equation 3.15

$$
\begin{align*}
P_x &= \frac{P_{Cx/Cy}}{1 + k_{x/y}} \\
P_y &= \frac{P_{Cx/Cy}}{1 + k_{x/y}} \\
k_{x/y} &= \left(\frac{\mu_x F_x N_x \mid \Delta \omega_x}{R_{o,x}} \int R_{i,x} dr\right) R_{o,x} + \left(\frac{\mu_y F_y N_y \mid \Delta \omega_y}{R_{o,y}} \int R_{i,y} dr\right) R_{o,y}
\end{align*}
$$

Equation 3.16 and Table 3.5 describes how the power is split in the pair of directional clutches.

Table 3.5: Logic table for splitting the power between the two individual directional clutches.

<table>
<thead>
<tr>
<th>Working Mode</th>
<th>$C_f$</th>
<th>$C_r$</th>
<th>$P_f$</th>
<th>$P_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>S/O</td>
<td>$P_{C_f/C_r}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>S/O</td>
<td>W</td>
<td>0</td>
<td>$P_{C_f/C_r}$</td>
<td></td>
</tr>
</tbody>
</table>

Equation 3.16

$$
\begin{align*}
P_f &= \frac{P_{C_f/C_r}}{1 + k_{f/r}} \\
P_r &= \frac{P_{C_f/C_r}}{1 + k_{f/r}} \\
k_{f/r} &= \left(\frac{\mu_f F_f N_f \mid \Delta \omega_f}{R_{o,f}} \int R_{i,f} dr\right) R_{o,f} + \left(\frac{\mu_r F_r N_r \mid \Delta \omega_r}{R_{o,r}} \int R_{i,r} dr\right) R_{o,r}
\end{align*}
$$
3.4 Implementation into test equipment

A script has been written to carry out the calculations during tests. The main script is programmed in a calculation program called "imc FAMOS" but some supporting scripts are made in MatLab due to the large amount of matrix operations.

The script can be divided into three parts: pre-processing, data recording and main calculations.
3.4. IMPLEMENTATION INTO TEST . . .

- **The pre-processing** is carried out before the test is started and contains all the calculations that don’t change during the test such as those for inertia and gear ratio.

- **The data recording** part will record sensor data from the test rig and save it so that the main script can process it.

- **The main calculations** will process the recorded data and calculate the energies and COF.

The following sections will describe each part in more detail.

3.4.1 Pre-processing

Some calculations are made before the test is started to increase the calculation speed during the tests. These calculations include: gear ratios, inertia, torque converter look-up table etc. The pre-processing is performed in two different programs. All the calculations that require matrix operations are performed in MatLab and the rest are done in Excel, due to easier accessibility. The parameters specified in each program can be seen in *table 3.6*.

<table>
<thead>
<tr>
<th>Excel</th>
<th>MatLab</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clutch geometry</td>
<td>Gear ratios</td>
</tr>
<tr>
<td>Search paths</td>
<td>Moment of inertia</td>
</tr>
<tr>
<td>Thresholds and tolerances</td>
<td>Torque converter look up table</td>
</tr>
<tr>
<td>Friction model</td>
<td></td>
</tr>
<tr>
<td>Signal names</td>
<td></td>
</tr>
<tr>
<td>Shift phases</td>
<td></td>
</tr>
</tbody>
</table>

3.4.2 Data recording

The sensor data is recorded in the test rig at specified time intervals and each recording contains one gear shift. The data that is needed is the pressures for each clutch, input and output torque and the speeds for the three flywheels. The search path to the recorded data is then queued in the buffer file so that the main script can process it.

3.4.3 Main calculation script

This script will process the recorded data and calculate the energy and COF. A flowchart of the script can be found in *appendix D* and a simplified version is shown in *figure 3.4*. 
Figure 3.4: Simplified flowchart of the main calculations script.

The following sections will describe the different steps carried out by the main calculation script.

Check for new sensor data
The script will check if the buffer file contains any search paths. If it does then it will process the buffered data and remove the search path from the file. If no paths exist then the script will wait for a specified time and then try again.

Load pre-processed data
The script will load all the pre-processed data at the beginning of the calculations to ensure that the most up to date data is loaded in case some minor adjustments are performed during the tests.

Load sensor data
The script will load all the raw sensor data from the file path fetched from the buffer file. The names of the sensor channels loaded are defined in the excel file.

In case the intermediate speed sensor channel is missing then the speeds will be calculated from the other sensors. This creates a new limitation where both clutch pairs cannot be active at the same time. The algorithm for this can be found in appendix D.

Pre-process recorded data
Pre-processing of the recorded data is needed before the energies and COF can be calculated. This includes determining which clutches are active, adjusting the time shift and correcting the rotational directions.

The active clutches are determined by checking if the pressure in each clutch changes from max to min or min to max. If this is the case then the clutch is assumed to be active. If the number of active clutches exceeds four then the calculations will be aborted since the model is not compatible with more than two clutch pairs.

In order to use sensor data from different sources they need to be synchronized. This is achieved by comparing the same sensor signal from the different sources and calculating the time-shift by using built-in functions in FAMOS.
3.5 VALIDATION

The duration of each phase is also measured from the ECU output. The slip distance for each clutch is also calculated, as described in section 3.3.

The final operation is to correct the rotational direction of the measured absolute rotational speed. This is done by a script that looks for points where change in rotational direction should occur and uses these to modify the speed signals. More information on how this is done can be found in appendix D.

Calculate energies and COF

The energies and COF are calculated by using the method described in section 3.3. A more detailed description can be found in appendix D.

Save the results

The results of the calculations are saved in comma separated text files and each time the script is run it will append the new result at the end of the file.

3.5 Validation

The model is validated by comparing the results with those from an existing reference model for calculating the energy. The reference model calculates the energy by utilizing the known gear ratio for a chosen clutch along with the output torque to determine the transferred torque at that clutch. The efficiency of the gears between the clutch and the output shaft is also considered. This can be seen in the following equation:

\[ T_c = \frac{T_{out} \times R}{\eta} \] (3.20)

where \( T_c \) is the clutch torque, \( T_{out} \) is the the output torque, \( \eta \) is the efficiency and \( R \) is the gear ratio between the output and the clutch. The power lost in the clutch can then be calculated with the following equation:

\[ P_c = T_c \times \Delta \omega \] (3.21)

where \( \Delta \omega \) is the slip speed and \( P_c \) is the power lost in the clutch. The total energy is then calculated the same way as in the new model as seen in equation 3.17. A limitation to this reference model is that it’s only valid during the inertia phase and thus the power in the other phases is set to zero.

The analysed gearshift is a shift from third (F3) to fourth (F4) forward gear. Calculations are made on a full shift (torque and inertia phase) and just on the inertia phase since this allows for only one clutch to be experiencing slipping contact. The phases are illustrated in figure 2.3. Several measurements will be analysed to get a mean value for the difference between the two models.
The resulting data will be normalized with mean values for the results from the reference model as seen in the following equation:

\[
\begin{align*}
E_{\text{new, norm}} &= E_{\text{new}} / E_{\text{ref}} \\
E_{\text{ref, norm}} &= E_{\text{ref}} / \bar{E}_{\text{ref}}
\end{align*}
\]  

(3.22)

where \(E_{\text{new}}\) and \(E_{\text{ref}}\) are the results from the new/reference model. \(E_{\text{new, norm}}\) and \(E_{\text{ref, norm}}\) are the normalized results from the new/reference model. \(\bar{E}_{\text{ref}}\) is the mean result from the reference model.
Results

The results from the validation tests will be presented in the following two sections. The first section will cover the results from the test data that only contains an inertia phase and the second section will cover a complete measurement that includes both a torque phase and an inertia phase.

4.1 Inertia phase

The resulting normalized energies from the calculations of the inertia phase measurements can be seen in figure 4.1. Each point represents one shift from F3 to F4.

Figure 4.1: Results from the energy calculation with the new model and the reference model. The calculations are made only on the inertia phase.
4.2. TORQUE AND INERTIA PHASE  

The mean energy calculated by the new model on this series of measurements is about 94% of the mean energy calculated by the reference model.

4.2 Torque and inertia phase

The resulting normalized energies calculated on both the inertia and torque phase can be seen in figure 4.2. Each point represents one shift from F3 to F4.

![Figure 4.2](image)

**Figure 4.2:** Results from the energy calculation with the new model and the reference model. The calculations are made on both torque and inertia phase.

The mean energy calculated by the new model on this series of measurements is about 135% of the mean energy calculated by the reference model.
5

Discussion

5.1 Model assumptions

A number of assumptions are made in the model that need to be discussed. These include wear of the clutches, contact states, spring force, etc.

5.1.1 Friction models

Friction is modelled either as a constant value independent of slip speed or as a linear function of the slip speed. The following models were chosen due to two reasons:

- The constant model was chosen since it would allow the model to be independent of the COF thus allowing the script to run even when the COF is not known. The model becomes independent due to the fact that the COF for each clutch cancel out one another when calculating the split ratio $k$. This is the only occasion were the COF is used.

- The linear model was chosen due to the fact that it would allow for the model to run with a clutch that has a COF which changes a lot when the slip speed changes.

Additional models can be developed if required to get a better representation of how the friction changes with different conditions, for example the temperature of the clutch.

5.1.2 Wear of clutches

In this model the assumption is made that all clutches wear by the same amount at any given time. This is assumed in this way to make it easier to split the power. This assumption is considered valid since wear on the clutches after the run-in period is low and thus the error created will not be very high compared to errors created by other factors.
This assumption can lead to an error when replacing one clutch and not the others since the new clutch will not have gone through the run-in period and will probably experience a different coefficient of friction from the rest of the clutches. This will create an error when dividing the power loss between the clutches since the model will not take the new COF into consideration. This can be corrected by developing a new friction model that takes lifetime into consideration.

This type of problem can also occur when one clutch is subjected to more wear than the rest of the clutches.

5.1.3 Contact states

In the current model it is assumed that there is no travel time between when the clamp force is larger than zero and the kissing point where the plates touch. This assumption is due to the lack of position feedback from the plates and results in a longer sliding working time than in reality. This may cause a significant problem if the travel time is very long thus creating an unwanted power split. This may be filtered away by using feedback from the ECU or by installing a position sensor for each clutch. Another way is to try and identify certain signatures in the pressure signal that indicate contact between the plates thus making it possible to tell if the travel time is over or not.

5.1.4 Spring force

This model contains an assumption that the spring force is constant and independent of the compression. This is due to the lack of positional feedback for the clutch plates. However since the spring force is only used when dividing the power between the clutches one can consider this assumption valid.

If an position sensor would be installed in the clutches then the spring force should be modelled as a function of the compression distance to get a better representation of how big the clamp force actually is and thus dividing the power more accurately.

5.2 Model integration

The current integration with the test equipment utilizes many different applications making it very prone to bugs and crashes since there is very little feedback between the programs and it is very hard to achieve a good error handler. This can be solved by porting the model over to a single program or create a new program that is specific for this application. This program could also include a configuration wizard to increase usability of the program and make it easier to get a correct configuration.

5.3 Signal quality

The data is quite heavily filtered when collected by the sensors and this may cause a couple of problems such as incorrect acceleration and loss of important spikes. If the
acceleration is filtered to an inaccurate value then the power loss/gain in the lumped inertia elements is going to be incorrect and lead to an error in the total energy loss in the clutches. If on the other hand a filter is applied to smooth out the curve one could loose important max and min values. For example the min value where the absolute rotation speed should change direction can be moved upwards thus making it harder for the program to identify it.

5.4 Validation results

Results from the validation test show that the new model gives a result that lies very close to the results from the validation model when only considering the inertia phase. The results differ quite a bit when including the torque phase. This is mainly caused by the reference model not being able to calculate the power loss during the torque phase. This limitation comes from the fact that during the torque phase there will be more than one clutch active at the same time and the reference model has no way to split the torque between the active clutches. This result do on the other hand show that the torque phase does matter to the end result and that it is important to consider this when designing the clutches.

5.5 Accuracy of the COF

The calculated COF value for each clutch will differ quite a lot from reality since it is heavily dependent on how well all the other losses are modelled. If the losses are for example modelled to be lower than in reality then the loss in the clutches will be higher and the COF will increase. It should be noted that the calculated COF can still be used as a measurement on how well all the parameters are tuned in the model. If, for example, a resulting COF of 20 or 100 occurs then one should consider checking if the test parameters are correct. On the other hand if a COF below one is calculated then the model is probably quite well tuned.

The normalized COF can also be used to check for changes over time by comparing it at different run-times. This may be useful for understanding how the clutch wears.

5.6 Lifetime estimation

Approximation of the transmission lifetime could be calculated using the Palmgren-Miner rule combined with an S-N diagram containing the lifetime for a given total energy or shape of the power curve. One problem with this approach is that it will require a large amount of test data to create the S-N diagram. This can possibly be solved by utilizing existing machines and logging the energy values when they are used by the costumers, then collecting it during service of the machine. The problem with this is that one machine is subjected to a lot of different load cases during it’s lifetime so one
should consider utilizing some statistical theory to combine the measurements from all the machines.

One can argue if the shape of the power curve will have a significant impact on the resulting lifetime or if the total energy gives a good enough representation. One thing to consider is that if we have a large power spike then it may lead to problems such as hot spots since the plates are being heated over a very short period of time.

Another approach is to compare the sliding distance for each shift and compare this to a model that describes the wear as a function of this distance, such as the Archard equation.

## 5.7 Further improvements

As mentioned previously there are some points where further improvements have been identified. These include modelling of drag losses, better data handling and filtering and friction models.

One area that needs improvement is the modelling of drag losses in the transmission. Since the model is based on the total amount of losses it is important to try and model all the significant losses correctly. These losses can either be modelled as a constant or as a dynamic function that varies depending on some parameters. The second alternative will probably provide a better result that is closer to reality. If all the losses in the model are modelled correctly it will lead to a lower calculated energy loss in the clutches. This is due to the way that the losses in the clutches are calculated from the total power loss.

Data processing will need some more improvement too, especially the speed data and how the change in rotation direction is handled. This might be done by installing additional hardware in the transmission so that the direction can be measured and not just the absolute speed. Another way may be to create a better system to identify when the change occurs and thus improving the ability to predict the direction in a more reliable way.

The friction models can be improved by implementing a more detailed version that includes the change in friction due to lubrication at different speeds.
6

Conclusions

6.1 Development

The development process resulted in a model that can calculate the energy loss in each clutch as well as estimating the coefficient of friction. The COF is found to be too unreliable and is therefore only used to get a rough estimate of how well the parameters correlate to reality. The model can also calculate the sliding distance for each clutch.

This model was implemented as a FAMOS script to be able to run calculations at the same time as the tests are run.

6.2 Verification

The verification results show that results from the new model lie within 6% of the reference model when calculations are made only on the inertia phase and within 35% when calculations are made for the whole shifting sequence. This is considered to a good result. The increased error when calculating on the whole sequence is believed to be due to flaws in the reference model and another reference model is needed to validate the power split properly.

6.3 Lifetime estimation

The concept of lifetime estimation was discussed and it was found that a large amount of test data was needed to be able to perform a reliable estimation. If this data was available then the estimation could be done by using the Palmgren-Miner’s rule.

6.4 Further development

The following areas were identified to be developed further:
• **Drag loss model.** Accuracy of the results could be increased by adding a better model for other losses in the transmission.

• **Signal Processing.** Signal processing could be developed further to allow for more accurate measurements.

• **Friction models.** Friction could be modelled to include more parameters giving a more accurate representation of the current friction in the disc.

• **Hardware modifications.** The hardware could be modified to include better or new sensors to allow for measurement of, for example, turbine rotation direction or clutch disc travel distance.

• **Wear model.** A model for wear in the clutch could be added to allow for a better power split when the clutches are subjected to different amounts of wear.
7

Bibliography


Appendices
A

Concept Comparison Table
Table A.1: Comparison of the pros and cons for each concept.

<table>
<thead>
<tr>
<th>Concept Name</th>
<th>Pros.</th>
<th>Cons.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept 1</td>
<td>• Independent of COF when slipping occurs in only one clutch.</td>
<td>• Is dependent on COF when slipping occurs in both clutches.</td>
</tr>
<tr>
<td></td>
<td>• Doesn’t require the exact piston pressure when slipping occurs in one clutch.</td>
<td>• Will be very difficult to calculate when two clutches in series are sliding.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Input torque only approximated.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• A lot of gear ratios to keep track of for each case.</td>
</tr>
<tr>
<td>Concept 2</td>
<td>• Quick and easy calculations.</td>
<td>• Heavily dependent on an exact COF.</td>
</tr>
<tr>
<td></td>
<td>• Won’t require data about which clutch is active.</td>
<td>• Will not adapt to changes in COF due to wear, temperature etc.</td>
</tr>
<tr>
<td>Concept 3</td>
<td>• Easily expanded to compensate for additional losses</td>
<td>• Complex calculation for power split</td>
</tr>
<tr>
<td></td>
<td>• Only needs to keep track of a few gear ratios.</td>
<td>• Input power approximated</td>
</tr>
<tr>
<td></td>
<td>• Adapts to changes in COF over time.</td>
<td>• Only works with a maximum of four clutches (two pairs in series)</td>
</tr>
<tr>
<td></td>
<td>• Will not require an exact COF model.</td>
<td></td>
</tr>
</tbody>
</table>
B

Constraint Matrix Examples and Matrix Notations
B.1 Example constraint equations

In the following table a couple of example constraint equations can be seen. These equations are used to build the constraint matrix $C$.

Table B.1: Example constraint equations for different components.

<table>
<thead>
<tr>
<th>Type</th>
<th>Schematic</th>
<th>Constraint eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td><img src="image" alt="Regular Diagram" /></td>
<td>$\omega_1 + \frac{Z_2}{Z_1} \omega_2 = 0$</td>
</tr>
<tr>
<td>Planetery</td>
<td><img src="image" alt="Planetery Diagram" /></td>
<td>$\omega_1 + \frac{Z_2}{Z_1} \omega_2 + (\frac{-Z_2}{Z_1} - 1) \omega_3 = 0$</td>
</tr>
</tbody>
</table>

B.2 Example constraint matrix

An example transmission can be seen in figure B.1. Each shaft is numbered. Speed sensors are located on shafts one and four.
The constraint equations for this transmission can be written as shown in equation B.1 where \( \omega \) is the rotation speed and \( Z \) is the number of teeth for each gear.

\[
\begin{align*}
\omega_1 + \frac{Z_2}{Z_1} \omega_2 &= 0 \\
\omega_2 + \frac{Z_3}{Z_2} \omega_3 &= 0 \\
\omega_4 + \frac{Z_5}{Z_4} \omega_5 &= 0 \\
\omega_5 + \frac{Z_6}{Z_5} \omega_6 &= 0
\end{align*}
\] (B.1)

These can then be written as the constraint matrix \( C \):

\[
C = \begin{bmatrix}
1 & \frac{Z_2}{Z_1} & 0 & 0 & 0 & 0 \\
0 & 1 & \frac{Z_3}{Z_2} & 0 & 0 & 0 \\
0 & 0 & 1 & \frac{Z_5}{Z_4} & 0 \\
0 & 0 & 0 & 1 & \frac{Z_6}{Z_5}
\end{bmatrix}
\] (B.2)
B.3 \(C[:,N]\) notation

The \(C[:,N]\) notation corresponds to a new matrix consisting of the columns given by the vector \(N\). If we have a vector \(C\) as shown:

\[
C = \begin{bmatrix}
11 & 12 & 13 & 14 & 15 & 16 \\
21 & 22 & 23 & 24 & 25 & 26 \\
31 & 32 & 33 & 34 & 35 & 36 \\
41 & 42 & 43 & 44 & 45 & 46
\end{bmatrix}
\]  \hspace{1cm} (B.3)

and the \(N\) vector.

\[
N = \begin{bmatrix}
2 \\
3 \\
6
\end{bmatrix}
\]  \hspace{1cm} (B.4)

then the matrix \(C[:,N]\) would be as follows:

\[
C[:,N] = \begin{bmatrix}
12 & 13 & 16 \\
22 & 23 & 26 \\
32 & 33 & 36 \\
42 & 43 & 46
\end{bmatrix}
\]  \hspace{1cm} (B.5)
Power and Friction Calculations
C.1 Power split

The power is divided between the X/Y and F/R clutch pairs in the following way. First the theoretical power lost can be described by using the equations below:

\[
\begin{align*}
P_{t,Cx/Cy} &= \mu_x \cdot F_x \cdot N_x \cdot |\Delta \omega_x| \cdot \int_{R_{i,x}}^{R_{o,x}} dr + \mu_y \cdot F_y \cdot N_y \cdot |\Delta \omega_y| \cdot \int_{R_{i,y}}^{R_{o,y}} dr \\
P_{t, Cf/Cr} &= \mu_f \cdot F_f \cdot N_f \cdot |\Delta \omega_f| \cdot \int_{R_{i,f}}^{R_{o,f}} dr + \mu_r \cdot F_r \cdot N_r \cdot |\Delta \omega_r| \cdot \int_{R_{i,r}}^{R_{o,r}} dr
\end{align*}
\] (C.1)

An explanation of all the parameters can be seen in the following table:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_t$</td>
<td>Theoretical power loss</td>
</tr>
<tr>
<td>$\mu$</td>
<td>COF</td>
</tr>
<tr>
<td>$F$</td>
<td>Clamp force</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of friction interfaces</td>
</tr>
<tr>
<td>$</td>
<td>\Delta \omega</td>
</tr>
<tr>
<td>$R_o$</td>
<td>Outer radius of friction disc</td>
</tr>
<tr>
<td>$R_i$</td>
<td>Inner radius of friction disc</td>
</tr>
</tbody>
</table>

The sum of the power loss in each pair is equal to the total power loss in all clutches as given in:

\[
P_{\text{all cl.}} = P_{Cx/Cy} + P_{Cf/Cr} \tag{C.2}
\]

The ratio between the two theoretical power losses can be calculated by the following equation:

\[
\frac{P_{t,Cx/Cy}}{P_{t, Cf/Cr}} = \frac{\mu_x \cdot F_x \cdot N_x \cdot |\Delta \omega_x| \cdot \int_{R_{i,x}}^{R_{o,x}} dr + \mu_y \cdot F_y \cdot N_y \cdot |\Delta \omega_y| \cdot \int_{R_{i,y}}^{R_{o,y}} dr}{\mu_f \cdot F_f \cdot N_f \cdot |\Delta \omega_f| \cdot \int_{R_{i,f}}^{R_{o,f}} dr + \mu_r \cdot F_r \cdot N_r \cdot |\Delta \omega_r| \cdot \int_{R_{i,r}}^{R_{o,r}} dr} = \frac{k_{xy/fr}}{k_{fr}}
\] (C.3)

This can then be rewritten in the following form:

\[
P_{Cx/Cy} = k_{xy/fr} \cdot P_{Cf/Cr} \tag{C.4}
\]
Combined with equation C.2 to get the power loss in each clutch pair.

\[ P_{\text{all cl.}} = k_{xy/fr} \cdot P_{Cf/Cr} + P_{Cf/Cr} \]

\[ P_{\text{all cl.}} = P_{Cf/Cr} \cdot (k_{xy/fr} + 1) \]

\[ P_{Cf/Cr} = \frac{P_{\text{all cl.}}}{(k_{xy/fr} + 1)} \quad (C.5) \]

\[ P_{Cf/Cr} = \frac{P_{\text{all cl.}}}{\left(1 + \frac{1}{k_{xy/fr}}\right)} \]

The power is split using the same method for the clutches in each pair. The theoretical power loss in each speed clutch can be calculated with the following equations:

\[
\begin{align*}
P_{t,x} &= \mu_x \cdot F_x \cdot N_x \cdot |\Delta\omega_x| \cdot \int_{R_{i,x}}^{R_{o,x}} dr \\
P_{t,y} &= \mu_y \cdot F_y \cdot N_y \cdot |\Delta\omega_y| \cdot \int_{R_{i,y}}^{R_{o,y}} dr 
\end{align*}
\]

\[ (C.6) \]

The total power loss in the clutch pair can be described with the following equation:

\[ P_{Cf/Cr} = P_x + P_y \quad (C.7) \]

The ratio is then calculated with the following equation:

\[
\begin{align*}
\frac{P_{t,x}}{P_{t,y}} &= \frac{\mu_x \cdot F_x \cdot N_x \cdot |\Delta\omega_x| \cdot \int_{R_{i,x}}^{R_{o,x}} dr}{\mu_y \cdot F_y \cdot N_y \cdot |\Delta\omega_y| \cdot \int_{R_{i,y}}^{R_{o,y}} dr} = \frac{k_{x/y}}{1} \\
&= k_{x/y} 
\end{align*}
\]

\[ (C.8) \]

and can be rewritten in the following form:

\[ P_x = k_{x/y} \cdot P_y \quad (C.9) \]

By combining this with equation C.7 one can calculate the power loss in each clutch:

\[
\begin{align*}
P_{Cf/Cr} &= k_{x/y} \cdot P_y + P_y \\
P_{Cf/Cr} &= P_y (k_{x/y} + 1) \\
P_y &= \frac{P_{Cf/Cr}}{(k_{x/y} + 1)} \\
P_x &= \frac{P_{Cf/Cr}}{(1 + \frac{1}{k_{x/y}})} 
\end{align*}
\]

\[ (C.10) \]
The theoretical power loss in each direction clutch can be calculated with the following equations:

\[
\begin{align*}
P_{t,f} &= \mu_f * F_f * N_f * |\Delta \omega_f| * \int_{R_{i,f}}^{R_{o,f}} dr \\
P_{t,r} &= \mu_r * F_r * N_r * |\Delta \omega_r| * \int_{R_{i,r}}^{R_{o,r}} dr
\end{align*}
\]  
(C.11)

Total power loss in the clutch pair can be described with the following equation:

\[P_{C_f/C_r} = P_f + P_r\]  
(C.12)

The ratio is then calculated with the following equation:

\[
\frac{P_{t,f}}{P_{t,r}} = \frac{\mu_f * F_f * N_f * |\Delta \omega_f| * \int_{R_{i,f}}^{R_{o,f}} dr}{\mu_r * F_r * N_r * |\Delta \omega_r| * \int_{R_{i,r}}^{R_{o,r}} dr} = k_{f/r}
\]  
(C.13)

which can be rewritten in the following form:

\[P_f = k_{f/r} * P_r\]  
(C.14)

By combining this with equation C.12 one can calculate the power loss in each clutch:

\[
\begin{align*}
P_{C_f/C_r} &= k_{f/r} * P_r + P_r \\
P_{C_f/C_r} &= P_r(k_{f/r} + 1) \\
P_r &= \frac{P_{C_f/C_r}}{(k_{f/r} + 1)} \quad \text{(C.15)}
\end{align*}
\]

The COF can be calculated by comparing the theoretical power loss \(P_{t,n}\) and the actual power loss \(P_{model}\) to see what the COF should be so that they are equal.

\[
P_{model} = P_{t,n}
\]

\[
P_{model} = \mu * F * N * |\Delta \omega| * \int_{R_i}^{R_o} dr \]
\[\mu = \frac{P_{model}}{F * N * |\Delta \omega| * \int_{R_i}^{R_o} dr} \]  
(C.16)
D

Calculation Scripts
D.1 Main calculation script

Figure D.1: Flowchart of the main calculations script.
D.2 Intermediate speed calculations.

![Flowchart of the script that calculates the intermediate speed.](image)

**Figure D.2:** Flowchart of the script that calculates the intermediate speed.

D.3 Rotation direction fix

The rotation direction is corrected by identifying the point where a change would occur and then changing the direction between each point pair. A point where a change will occur is identified by the following conditions:

\[
\begin{align*}
\omega_x & \leq \omega_{thres.} \\
\omega_{outp.} & < 0 \\
\frac{d\omega_x}{dt} & = 0
\end{align*}
\]  

(D.1)

where \(\omega_x\) is the rotation speed that is being precessed, \(\omega_{thres.}\) is a threshold value that describes the highest speed where an inversion can occur and \(\omega_{outp.}\) is the output speed from the transmission.
The points are then divided into pairs. If there is an uneven amount of point then an error message will be logged. The rotation speed is then inverted between the points in each pair. It is assumed that rotation will occur in a positive direction at the start of the sequence.

D.4 COF calculations

![Flowchart of the script that calculates the COF of the clutches.](image)

**Figure D.3:** Flowchart of the script that calculates the COF of the clutches.