Abstract

The interest for developing miniaturized satellites is increasing, which facilitates the need for propulsion systems that can deliver small impulse bits for more precise maneuvers and thrust levels. This is best achieved with electric propulsion systems and one of the most widely used techniques in that category is pulsed plasma thrusters (PPT). One such system to be installed on-board the miniaturized satellite system VELOX, is currently under development by the Undergraduate Satellite Research Program at Nanyang Technological University. In order to make this system as efficient as possible and extend the lifetime of the satellite, the design parameters must be optimized.

In the present work, the complex discharge process of the PPT is reviewed in order to investigate how it is influenced by the design parameters and how modifications in the design can result in enhanced thruster efficiency. An analytical electromechanical model describing the plasma dynamics of the PPT operation has been established to facilitate the study of how various performance parameters are affected by the geometrical design. As a tool to assist in this, a graphical user interface has been implemented in Matlab and designed to determine the dependencies of specific impulse, impulse bit, exhaust velocity, inductance gradient, thrust level and thruster efficiency on the electrode shape, dimensions and flare angle as well as on the propellant dimensions and propellant feeding mechanism. In addition, a new type of propellant feeding mechanism, combining both side-fed and breech-fed design, is investigated. The aim of the study is to develop well-motivated design suggestions for optimized thruster performance. The analytical model and evaluation tool are meant to serve as design guidelines for further experimental optimization of the miniaturized PPT prototype developed for use on-board VELOX.

The investigation using the evaluation tool based on the established analytical model has enabled identification of certain trade-off factors between the design parameters that needs to be considered in order to enable an efficient thruster function, which constitutes the base for successful operation of the VELOX satellite system. The conducted study also provides a theoretical foundation to reinforce previous empirical results and confirms data from experimental studies on micro-PPTs, showing that the implementation of tongue-shaped electrodes at optimal flare angle likely exceeds performance improvements caused by other modifications of the thruster geometry, such as increasing the aspect ratio.
Till mamma och pappa
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Nomenclature

\begin{align*}
A & \quad \text{Propellant area exposed to discharge} \\
B & \quad \text{Self-induced magnetic field} \\
B_\parallel & \quad \text{Magnetic field in direction of length elements} \\
C & \quad \text{Total capacitance of capacitor bank} \\
E & \quad \text{Electric field between electrodes} \\
E_0 & \quad \text{Discharge energy} \\
E_{\text{kin}} & \quad \text{Kinetic energy} \\
F & \quad \text{Force} \\
F_L & \quad \text{Lorentz force} \\
I_{\text{bit}} & \quad \text{Impulse bit} \\
I_{\text{sp}} & \quad \text{Specific impulse} \\
L & \quad \text{Total system inductance} \\
L' & \quad \text{Inductance change per unit distance of plasma sheet displacement} \\
L_0 & \quad \text{Initial system inductance} \\
L_M & \quad \text{Mutual inductance} \\
P & \quad \text{Power} \\
R & \quad \text{System resistance} \\
T & \quad \text{Thrust force} \\
T_{\text{EM}} & \quad \text{Electromagnetic thrust component} \\
T_{\text{ET}} & \quad \text{Electrothermal thrust component}
\end{align*}
$T_{\text{avg}}$ Average thrust force during one pulse

$T_{\text{inst}}$ Instantaneous thrust force

$V_0$ Initial capacitor voltage

$V_{\text{bar}}$ Volume of one propellant bar

$V_p$ Volume of plasma sheet

$V_{\text{tot}}$ Total propellant volume

$\Delta L$ Change in circuit inductance achieved during one pulse

$\Delta l$ Length element in direction of magnetic field

$\Delta v$ Net velocity change of spacecraft

$\Delta \bar{L}$ Average inductance change

$\Phi$ Magnetic flux

$\alpha$ Electrode flare angle

$\ddot{x}_p$ Acceleration of plasma sheet

$\delta$ Plasma sheet thickness

$\dot{x}_p$ Velocity of plasma sheet

$\eta$ Thruster efficiency

$\frac{E_0}{A}$ Ratio of discharge energy to propellant area exposed to discharge

$\mu$ Permeability constant

$\mu_0 \approx 1.2566370614 \times 10^{-6} \text{ Hm}^{-1}$ Vacuum permeability constant

$\rho$ Volumetric mass density

$\tau$ Expected satellite lifetime

$\theta$ Angle between magnetic field and y-axis

$a$ Interelectrode aspect ratio

$f$ Pulse frequency

$g \approx 9.8 \text{ ms}^{-2}$ Gravitational acceleration constant

$h$ Interelectrode spacing

$h_0$ Base interelectrode spacing for complex electrode geometries
\( h_{\text{prop}} \) Propellant bar height
\( i \) Discharge current
\( j \) Current density
\( l \) Electrode length
\( l_{\text{prop}} \) Propellant bar length
\( m \) Total propellant mass
\( m_{\text{bit}} \) Mass ablated per pulse
\( m_f \) Final total mass of spacecraft
\( m_i \) Initial total mass of spacecraft
\( m_q \) Charged particle mass
\( q \) Electric charge
\( r_g \) Larmor gyroradius
\( t' \) Pulse duration
\( v \) Charge velocity
\( v_e \) Velocity of particle exhaust stream
\( v_\perp \) Velocity component perpendicular to magnetic field
\( w \) Electrode width
\( w_0 \) Base electrode width for complex electrode geometries
\( w_{\text{prop}} \) Propellant bar width
\( x_p \) Displacement of plasma sheet from propellant surface

ESU Energy storage unit
GUI Graphical user interface
LEO Low Earth orbit
PPT Pulsed plasma thruster
PPU Power processing unit
Chapter 1

Introduction

1.1 Background

The interest for development of smaller satellites has increased over recent years, mainly due to the possibility to decrease manufacturing, testing and launching costs and times, as compared to larger satellites. In addition, for many applications building a cluster of small satellites could also be more beneficial than constructing a single larger satellite, in terms of cost, robustness and versatility.

Larger spacecraft commonly use chemical thrusters, due to their high torque and high impulse capability, while missions carried out by smaller satellites require more precise maneuvers and thrust levels for stabilization, attitude control and drag compensation. For this reason, the thruster must be able to provide very small and accurate force, which is something that can be better achieved with electric propulsion systems. Current research within the area of miniaturized satellites focus on so-called microthrusters, i.e. thrusters which deliver impulse bits on the \( \mu \text{Ns} \) scale. \[30\]

1.1.1 Satellite research at NTU

Since 2009 the Satellite Research Center at Nanyang Technological University hosts an Undergraduate Satellite Program, which offers students to get acquainted with satellite design through the development of Singapore’s first nanosatellite operating in low Earth orbit (LEO). The base platform nanosatellite is accompanied by a picosatellite intended to carry the scientific payload. These two miniature satellites are separated in order to be used for intersatellite communication experiments and together they constitute the VELOX satellite system. Part of the scientific payload of the VELOX system is also to acquire high-resolution images of Earth from LEO and transmit them back to the ground station. \[29\]

While a typical satellite weighs more than 1000 kg, a picosatellite refers to a satellite of a mass below 1 kg and dimension of 1U (10cm x 10cm x 10cm).
Launched on November 21th 2013, the picosatellite VELOX-PII was the second satellite produced in Singapore and the first one constructed by students. The slightly larger nanosatellite VELOX-I has a mass of 4.28 kg and a dimension of 3U.

The satellite structure, attitude determination and control system, power supply, thermal management system and vision payload are developed at NTU, while commercial hardware is used for the on-board data handling and communication boards. A ground station for mission control is set up at the NTU campus, operating in the very high (144-148 MHz) and ultra high (430-440 MHz) armature radio frequency bands. [29]

1.2 Fundamentals of spacecraft propulsion

The mechanism behind spacecraft propulsion relies on basic physical principles and is based on Newtons third law of motion, i.e. for every action there is an equal, but oppositely directed, reaction. A spacecraft is propelled forward by acceleration of propellant in the antiparallel direction of motion and the reaction force that is caused by this mass acceleration is called thrust. Neglecting gravitation and drag, the equation of motion of a spacecraft follows from the conservation of the total momentum of the vehicle and the propellant stream, and the thrust generated by the system is simply

\[ T = m \frac{dv}{dt} = \frac{dm}{dt} v_e \]  

(1.1)
1.2. Fundamentals of spacecraft propulsion

where \( m \) is the mass of the spacecraft, \( \frac{dv}{dt} \) is the vehicle acceleration, \( \frac{dm}{dt} \) is the propellant mass flow rate, i.e. rate of change of the spacecraft mass due to the propellant expulsion, and \( v_e \) is the velocity of the exhaust stream measured relative to the rocket.

Propulsion techniques are used to change the velocity of a spacecraft and are required to launch artificial satellites out of the Earth’s atmosphere and into orbit. In addition, propulsion is needed for attitude control to compensate for disturbing torques and to maintain the right orientation, as well as for orbital maintenance for position adjustment to keep the satellite in the desired mission orbit for longer periods of time, as they are subject to gravitational forces and drag from the atmosphere.

The difficulty to perform a velocity change is influenced by the mass of the spacecraft and therefore the quantity normally used to evaluate a propulsion system is momentum, or more specifically change in momentum. The purpose of a propulsion system is to create an impulse, which is the cumulative effect of the applied force over the time for which it is applied. The law of conservation of momentum implies that when a propulsion method changes the momentum of a spacecraft, it must also change the momentum of something else, which is usually some mass to be accelerated away in order to push the spacecraft forward. This mass is called reaction mass and since in the vacuum of space the amount of matter that a propulsion system can accelerate is negligible, it must be included in the system and is consumed when used. For this reason, it is often used to determine the efficiency of a propulsion system.

The amount of impulse that can be obtained per unit of reaction mass is called the specific impulse. It is used to evaluate the performance of a propulsion system and is defined as the ratio of thrust to the rate of expulsion of propellant per unit weight on Earth and it is measured in seconds\(^1\)

\[
I_{sp} = \frac{T}{\frac{dm}{dt}} = \frac{v_e}{g}
\]  

(1.2)

where \( g \) is the standard gravitational acceleration constant.

A propulsion system can achieve a given impulse either by producing small accelerations during a long time or produce larger accelerations during shorter periods. An impulse bit is the smallest change in momentum that can be delivered by the thruster and is required to allow for e.g. fine attitude and orbit control. For a thrust force \( T \) applied during a single pulse of duration \( t' \), the impulse bit is defined to be

\[
I_{bit} = \int_{0}^{t'} T \, dt
\]

(1.3)

\(^1\)For vehicles in space, the weight on Earth is insignificant and therefore specific impulse can also be measured in terms of impulse per unit mass, which has the unit of ms\(^{-1}\). It is then equal to the exhaust velocity.
If all usable propellant of a spacecraft were to be exhausted through the engines in a straight line in free space, a net velocity change $\Delta v$ would be produced. The maximum change of speed for an ideal rocket with no external forces acting on it can be described through the Tsiolkovsky rocket equation

$$\Delta v = v_e \ln \frac{m_i}{m_f}$$  \hspace{1cm} (1.4)

where $m_i$ is the initial total mass at the beginning of the thrust period, including the propellant, and $m_f$ is the final total mass at the end of the thrust period. By rearranging the above equation we get the proportion of the spacecraft’s initial mass that can be delivered to the vehicle’s destination.

$$\frac{m_f}{m_i} = e^{-\frac{\Delta v}{v_e}}$$  \hspace{1cm} (1.5)

As can be seen from this relation, a high exhaust velocity is required for the spacecraft to deliver a large portion of its initial mass to the destination. In order to accelerate the mass, the kinetic energy of the exhaust must be increased. Consequently, the system needs to be able to add energy into the flow and convert it to kinetic energy. There are different ways to add energy into an exhaust flow, and here we focus on the electric technique described in the next section. \cite{1,33}

### 1.2.1 Electric propulsion

Being an active area of research, there are several propulsion techniques, each with different benefits and disadvantages. The idea of electric motors to gain mechanical energy in spacecraft dates back to the beginning of the 20th century and the technique has been tested both on ground and in space since the 1960’s. In recent years it has gained popularity, mainly due to the fact that it offers a more mass-efficient alternative compared to conventional chemical propulsion systems.

Nevertheless, most spacecraft use chemical propulsion, where the energy required to heat the fuel and expand it thermodynamically is stored in the chemical bonds of the propellant and the reaction products are flowing out the back through a nozzle, providing the reaction mass. In contrast, in electric propulsion devices the thermodynamic expansion can instead be replaced by direct application of body forces to particles in the propellant stream. In such systems, electric energy is used to create the kinetic energy in the expelled reaction mass, which is constituted by electrically charged propellant particles that are accelerated by electric and/or magnetic fields to generate thrust.

While using a very small amount of fuel, electric propulsion systems require huge amounts of energy because of their ability to accelerate particles to high exhaust velocities, which means that large impulses can be achieved with less reaction mass. These high exhaust speeds combined with low propellant utilization favors electric propulsion systems over chemical alternatives. However, since the energy required to produce the impulse is proportional to the exhaust velocity, thrusters with a
1.2. Fundamentals of spacecraft propulsion

high exhaust velocity are typically less energy efficient. As a result, due to practical power source constraints, electric propulsion systems typically provide lower thrust than chemical engines by several orders of magnitude.

There are various types of electric propulsion systems and common for all such techniques is that they produce a plasma which is accelerated to exhaust velocity. The electric propulsion techniques are usually categorized based on the type of force that is used to accelerate the plasma particles and the common categories are listed below:

- **Electrostatic**
  Application of a high voltage static electric field causes particles to be accelerated by the Coulomb force.

- **Electrothermal**
  The propellant is heated thermally to increase the exhaust velocity of the particles.

- **Electromagnetic**
  The particles are accelerated either by the Lorentz force or by the effect of an electromagnetic field.

1.2.2 Pulsed plasma thrusters

The propulsion system under development by the satellite research centre (SaRC) in NTU for its future VELOX program is Pulsed Plasma Thrusters (PPT); an electromagnetic propulsion technique in which the particles of the plasma are accelerated by the force created when the discharge current interacts with a self-induced magnetic field. While most thrusters are set out to optimize the ratio \( \frac{m_i}{m_f} \) in equation (1.4), the principle of PPTs is to increase \( v_e \) in order to achieve high particle velocities with low propellant consumption. The term 'pulsed' simply refers to the unsteady operation of the device [16].

In 1964, PPT was the first type of electric thruster to be deployed in space, when it was used for attitude control from parking at Earth orbit to March on the Soviet probe Zond 2 [31][32]. The development of the PPT was first inspired by phenomenological observation and the basic process can be demonstrated by simple familiar devices. Consider a pair of parallel rectangular electrodes and a large capacitor with each term connected to one of the electrodes. If the capacitor is charged to high voltage and switched across the electrodes an arc will initiate between them at the shortest possible distance from the capacitor circuit and then propagate down the electrodes [15]. The sequence of physical events will be the base of the PPT mechanism and is schematically illustrated in Figure 1.2 below.
Figure 1.2: Schematic overview of the idea behind the PPT mechanism.

The main components of the PPT are depicted in Figure 1.3 below. The power source can be any source of electrical power but, because the thruster operates at low power levels, solar cells are most commonly used. The power source is connected to the power processing unit (PPU), which includes energy storage power supply and ignition power supply. The main functions of the PPU is to provide the charging voltage for the energy storage unit (ESU) and to provide the command and telemetry functions required to operate the PPT.

Figure 1.3: PPT functional schematic.

The thruster unit consists of the discharge chamber, electrical parts and ESU, which is based on one or several high-current capacitors connected to the electrodes. The two electrodes form the accelerator unit of the thruster, which also includes a mechanism for discharge initiation constituted by a spark plug igniter. The thruster unit also contains one or several fuel bars which are fed into the thruster channel by a mechanism that varies between different PPT models. Most commonly a spring is used at the breech of the propellant to push it forward as it is consumed and the
bar is held in position by a small incision in the lower electrode that works as a fuel retaining shoulder.

1.2.3 PPT operation

The creation of the plasma occurs by an arc discharge from the capacitor bank placed in parallel with a pair of electrodes between which a source of insulating solid propellant, usually Teflon\(^2\), is positioned. The energy stored in the capacitor is provided by the thruster’s power supply and a PPU converts the provided voltage to charge the capacitor, which provides high-current pulses through the thruster to perform work, so that the energy is transformed into kinetic energy of the propellant.

Because of the insulating solid propellant, no current can flow under the given conditions and since the vacuum of space does not provide a conducting path, an electric discharge must be triggered by an igniter mounted close to the propellant. By a high voltage this igniter produces a spark that releases a large supply of free electrons, ions and neutral particles and will impact the surface of the propellant with high velocities and form a conducting path between the electrodes. Once enough electrons are liberated from the fuel surface, the discharge rapidly forms an arc across it and the potential difference between the electrodes quickly decreases. The capacitor will be discharged and the heat transfer from the discharge will cause the surface of the Teflon between the electrodes to evaporate by causing it to undergo a phase transition from solid to plasma. The vaporized Teflon will create a closed circuit, allowing electric current to flow between the anode and the cathode and as it does, more electrons will be ripped from the atoms causing the propellant to be ionized. The particles will keep colliding with the propellant so that more plasma is created. As the plasma composes both electrons and heavy particles of various charged states it is affected by electromagnetic fields, and due to its conductive properties the capacitor is further discharged through the plasma.

In the initial acceleration stage, where the particles have not yet entered into the electric field, thrust is produced by the thermal energy generated from the gas expansion caused by the discharge. At the beginning of the discharge, such electrothermal acceleration is primarily near the propellant surface. As the formation of the plasma makes the electrodes become conductive and the electromagnetic discharge occurs, more particles will be produced and the temperature and pressure near the propellant surface will increase. The particles will then be accelerated by this high pressure.

In the subsequent electromagnetic acceleration stage the current will induce a magnetic field within the plasma so that the particles will be accelerated out of the thruster by the Lorentz force resulting from the interaction between the discharge current and this self-induced magnetic field. Regardless of the sign of the current

\(^2\)Polytetrafluoroethylene, C\(_2\)F\(_4\), is chosen for its attributes of high specific impulse, impulse bit and zero surface charging; meanwhile, the search for alternative propellant materials with improved characteristics remains [3].
giving rise to this field, the force will always act to expand the area of the circuit loop thus driving the plasma away from the external conductors.

As the propellant evaporates with each discharge, the remainder is forced forward axially by a simple spring mechanism so that a constant fuel source is provided. Once the capacitor bank is fully discharged, no further acceleration of the plasma sheet can take place and the pulse is shot down. The capacitor bank is then ready to recharge from the power supply and the pulse cycle is repeated. The pulse frequency is normally around 1 Hz, meaning that the capacitor is charged and discharged once every second. This technique produces a quick and repeatable burst of impulse. [3][31]

Figure 1.4: Operation principle of PPT.

The cyclic operation of the PPT is illustrated schematically in Figure 1.4 above. As seen in the figure, the following axis conventions are adapted in the present work: the x-axis points in the direction of the acceleration, the y-axis in the direction
transverse to the width, and the z-axis in the antiparallel direction to the current flow. The measure of the point of origin will differ between different feedings, and should be below the spark plug where the acceleration starts, but for simplicity we set it as in Figure 1.7.

1.2.4 PPT characteristics: benefits and efficiency issues

There are many advantages of PPTs compared to other alternative propulsion systems and of the electric propulsion systems under consideration for microsatellites, PPTs have the benefit of being especially amenable to miniaturization due to several reasons. For instance, the technique delivers a high specific impulse, while allowing more precise control because of the ability to generate low impulse bits, due to the pulsed nature of the thrust produced. Being a self-field thruster it also requires uniquely low electric power and fuel consumption. Another advantage is the use of solid propellant, which makes the propellant bar the only moving part and the fact that it is passively fed by a spring adds to the structural simplicity by eliminating the need for active mechanical controls and subcomponents like tankage and pumps, which are difficult to downscale. This results in a simple, compact, robust and light construction of reduced volume with significantly lowered risk of propellant leakage, which will not only keep the development time and cost low but also promise to have a rather constant size to mass ratio when miniaturized. Additionally, the complexity of the electrical circuit is low, compared to other electric propulsion systems, which allows for easy adjustment of the ESU and modifications to the electrode geometry and thereby propellant surface area. Due to this simple mechanical and electrical design, PPTs are highly reliable and scalable to performance requirements. Also, PPTs can provide both impulse and continued force as well as discrete impulse bits compatible with digital logic, which also adds to the flexibility to meet different kinds of mission requirements. [18]

However, despite these beneficial characteristics, the technique also suffers from efficiency issues compared to other propulsion systems and typically has a thruster efficiency below 10%. Listed below are some of the main issues causing decreased efficiency in the PPT thrust production process.

- **Inefficient energy transfer into propellant acceleration.** The basic qualification for improved thruster efficiency is maintained efficiency of the conversion process of electric energy into kinetic energy of the exhaust stream. The acceleration process is affected by design parameters such as electrode dimensions and discharge energy so in order to minimize losses and to transform the highest possible fraction of the initial energy provided by the power source into kinetic energy of the accelerated propellant, these parameters can be optimized.

- **Non-uniform conditions.** Ideally, the arc that forms during the discharge will cover the entire exposed propellant surface area, providing uniform conditions for heat transfer to the propellant bar and uniform ablation of it.
However, experiments show a concave tendency in the ablation process, suggesting that heat transport concentrates in the middle of the surface, causing non-uniform ablation along the propellant surface. Furthermore, once the Teflon has been ionized into a plasma, complexities in its structure quickly emerge due to the low particle densities and high flow speeds. This causes conditions of non-equilibrium and non-uniformity in the exhaust stream, resulting in difficulties for experimental characterization and modeling. [3]

- **Low mass utilization.** While PPTs provide high values of specific impulse due to the sufficiently high temperatures of the plasma, the electron temperature is limited by inelastic processes and radiation. In turn, this limits the energy acquired by heavy particles due to heat transfer from the electrons, which limits the exhaust speed associated with electrothermal operation. In fact, heating of heavy particles by plasma electrons cannot provide more than about 25 % of the kinetic energy of the heavy particles. In addition, since the amount of energy needed to evaporate the propellant represents only a small fraction of the energy required to dissociate, ionize and accelerate the resulting vapor, high temperatures at the surface during the discharge pulse may cause sufficiently high temperatures below the surface for decomposition or phase transitions of the propellant due to the Teflon being at a temperature above its sublimation point. Thus, the propellant surface may continue to evaporate long after the discharge pulse has been completed and thereby providing mass that cannot experience acceleration to high speeds by electromagnetic and gasdynamic forces, so called late-time ablation. Combined with the pressure from the PPT, this can result in additional loss of material in the form of low-speed macroparticles that do not contribute significantly to the thrust production. Experiments suggest that as much as 40 % of the mass loss of the PPT can be due to such macroparticles. [3]

Some of these issues are inherent and has solutions yet to be found, while others can be solved by geometrical and/or electrical adjustments of the thruster channel.

### 1.2.5 PPT classification and design choices

There are many types of thrusters that fall under the PPT category. Their most significant differences in terms of the physical layout are summarized below [16]. In this work we focus on a solid propellant PPT of rail-type, where the propellant is accelerated between the two electrodes as described in section 1.2.3. By attempts at optimizing the propellant surface area exposed to the discharge, a variety of propellant feed mechanisms has been developed in the past and this study will investigating different types of feeding as well as electrode geometry.
1.3 Objectives

1.3.1 Design suggestions for improved thruster performance

The thruster efficiency of a PPT is mainly influenced by the geometrical and the electrical parameters. The electrical parameters includes the initial discharge energy, pulse frequency, resistance, inductance and capacitance and will not be left much consideration in this particular work. Instead, focus lies on the thruster geometry and the aim is to determine how the thruster performance depends on the geometrical design parameters listed below (cf. Figure 1.4) and how modifications of them may improve the PPT operation.

1) Dimensioning of electrodes, in terms of:
   - Interelectrode spacing (distance between anode and cathode), $h$
   - Electrode width, $w$
   - Electrode length, $l$
   - Electrode thickness, $t$
   - Electrode flare angle $\alpha$
   - Electrode shape: rectangular/tongue

\[
\begin{array}{|c|c|}
\hline
\text{Electrode geometry} & \text{Propellant state of matter} \\
\text{- Rail} & \text{- Solid (typ. Teflon)} \\
\text{  - Shape: rectangular/tongue} & \text{  - Liquid (typ. water doped with salt)} \\
\text{  - Angle: parallel/flare} & \text{  - Gas (typ. ammonia gas)} \\
\text{- Coaxial} & \\
\text{- Z-pinch} & \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Propellant feeding mechanism} & \\
\text{- Breech: Propellant fed into acceleration chamber from rear-end} & \\
\text{- Side: Propellant injected from two opposite sides perpendicular to the} & \\
\text{  thruster length extension} & \\
\text{- Oblique: Propellant fed from sides at diagonal angles} & \\
\hline
\end{array}
\]

Figure 1.5: PPT classification scheme.

Figure 1.6: Electrode configurations under consideration.
2) Propellant geometry

- Propellant area exposed to the discharge spark, $A$, and energy density, $E_0/A$
- Propellant bar height, $h_{\text{prop}}$ (we generally assume $h_{\text{prop}} = h$)
- Propellant bar width, $w_{\text{prop}}$
- Propellant bar length, $l_{\text{prop}}$

3) Investigation of propellant feeding mechanism

The propellant feeding mechanism is an important design choice for optimal mass utilization. There are several different ways to feed the propellant into the thruster channel and the current study investigates the three following types:

- Breech feeding: The earliest PPT versions were breech-fed, feeding the propellant from the breech of the thruster in the forward x-direction along the acceleration of the plasma.
- Side feeding: The side-fed design choice that was later developed incorporates two pieces of propellant entering from each side in the opposite y-directions and are fed towards the spark plug perpendicular to the acceleration of the plasma.
- Combination feeding: Another option under investigation is a three-directional feeding mechanism, in which breech feeding would be combined with side feeding. To the author’s knowledge such a solution has not been implemented previously.

The goal is to develop a technique for systematic investigation of these parameters with the aim to find optimums to be used for fabrication of an improved PPT prototype, for which the specific impulse and generated thrust can be increased so that the required propellant mass is reduced and the lifetime of the satellite system can be extended. There also exists contradictory results regarding how geometrical parameters affect the thruster performance, which needs some clarification. The result of the work is aimed to function as design guidelines for an experimental optimization of the micro-PPT.

\[^3\text{Since the spacing between the electrodes will essentially be equal to the height of the propellant bar, for simplicity we set } h_{\text{el}} = h_{\text{prop}} = h. \text{ In reality, there will be a small incision in the anode to provide resistance for the feeding mechanism which in practice means that the effective electrode spacing will be slightly smaller than the propellant height. However, this remains unsignificant for our theoretical purpose.}\]
1.3. Objectives

Figure 1.7: Propellant feeding types under consideration as seen from above. The 
\( z \)-axis points in the outward direction, the gray areas represent the propellant bars 
and the brown areas represent the cathode. Note that the point of origin differs 
between the different feeding types as it is used to define the electrode length 
available for plasma sheet acceleration.

1.3.2 Mission requirements

The current thruster system for the VELOX nanosatellite is designed for 3-6 months 
of station-keeping, but the PPT under development should be able to extend this 
lifetime. In order to lower launch costs and increase the sensitivity of the sensors 
because of the higher resolution that can be achieved as less ground is covered for a 
given viewing angle, the satellite should be placed LEO with an orbital altitude of 
300-600 km. However, placing satellites in LEO also implicates some disadvantages;
for instance, lowering of the altitude will decrease the lifetime of the satellite because 
of the increased drag it is exposed to. [1]

At an orbital altitude of 300 km the PPT must deliver enough thrust to com-
penstate for atmospheric drag of 45 \( \mu \text{N} \). To obtain a lifetime \( \tau \) at a firing rate \( f \) 
requires the relation between the specific impulse and the total propellant mass 
stored on-board the satellite to be

\[
I_{sp} = \frac{T}{m_{bit}gf} = \frac{T\tau}{m_{tot}g} \tag{1.6}
\]

where we have defined the mass ablated during one pulse pulse, or mass bit as

\[
m_{bit} = \frac{(dm/dt)}{f} = \frac{m_{tot}}{\tau f} \tag{1.7}
\]

This quantity is needed in order to determine how much propellant mass that is 
needed for the desired period of station keeping; however, it is difficult to predict 
accurately. Meanwhile, a rough estimation of the total propellant mass to be used
can be done to a certain order of magnitude based on the dimensions of the thruster. The propellant mass to be stored on-board the satellite can then be expressed as

\[ m_{\text{tot}} = \rho_{\text{Teflon}} V \]  

(1.8)

where \( \rho_{\text{Teflon}} = 2200 \text{ kg/m}^3 \) is the density of Teflon and \( V \) the volume of the propellant bar/bars, which for micro-PPTs will be in the order of \( V = h w_{\text{prop}} l_{\text{prop}} \sim 10^{-2} \times 10^{-2} \times 10^{-2} = 10^{-6} \text{ m}^3 \). The propellant mass should thus be of the order \( m_{\text{tot}} \sim 10^{-3} \text{ kg} \).

While there are no strict mass limitations on the thruster, it is desirable to keep the mass as low as possible, including the mass of the propellant. However, as we see from eq.(1.6), lower masses require higher specific impulses to maintain a lifetime of 6 months, as more efficient utilization of the propellant is needed. As the specific impulse for an electric thruster typically lies within the range of 1000-2500 s [1] and even lower for miniaturized models due to the limited available power, it is not realistic to expect efficiencies above this. In practical applications, the specific impulse of VELOX is expected to be around 500 s. The total propellant stored on-board the satellite is set to 30 g, which is believed to be sufficient to extend the lifetime at this value of specific impulse.

The current PPT prototype contains a PPU which voltage multiplier takes an input of 11.9 V DC from the solar cell power supply and converts it to a 1488 V DC output with a power consumption of about 1 W [1][2]. Along with this, the requirements on the thruster design are summarized in table 1.1.

Table 1.1: PPT requirements.

<table>
<thead>
<tr>
<th>Expected lifetime</th>
<th>Discharge voltage</th>
<th>Discharge energy</th>
<th>Propellant mass</th>
<th>Min. thrust level</th>
<th>Typ. frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;6 months</td>
<td>1.4 kV</td>
<td>2 J</td>
<td>~30 g</td>
<td>45 µN</td>
<td>1 Hz</td>
</tr>
</tbody>
</table>

Apart from these, another highly essential requirement on the design of the thruster is that it needs to be sized down to a dimension of 1U in order to fit on the nanosatellite. Out of this volume, half is allocated to the electrical parts, including the PPU, ESU and power supply, while the other half is allocated to the thruster head and discharge chamber. Thus, the base of the design presenter here is a volume of 100 x 100 x 50 mm³. This limited volume allocated for the thruster imposes constraints on the thruster dimensions which differ depending on the electrode configuration and propellant feeding type. These geometrical constraints are summarized in Table 1.2, where the maximum values specified in the last column refers to the total dimensions of the thruster discharge chamber.
1.3. Objectives

Table 1.2: VELOX thruster dimension constraints. We assume the length of the feeding springs to be $l_{spring} = 10$ mm. The $w_{prop}$ term appearing in the length expression is added only for the combination-fed design. The decrease in available volume due to the spark plug and casing has not been accounted for here.

<table>
<thead>
<tr>
<th>Height</th>
<th>Rectangular</th>
<th>$\alpha = 0$</th>
<th>$\alpha &gt; 0$</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tongue</td>
<td>$h_0 + 2t$</td>
<td>$h_0 + 2l \sin \alpha + t \cos \alpha$</td>
<td>$h_0 + 2l \sin \alpha$</td>
<td>50 mm</td>
</tr>
<tr>
<td>Width</td>
<td>$w_0 + 2(l_{prop} + l_{spring})$</td>
<td>$w_0 + 2(l_{prop} + l_{spring})$</td>
<td>$w_0$</td>
<td>100 mm</td>
</tr>
<tr>
<td>Breech</td>
<td>$w_0$</td>
<td>$w_0$</td>
<td>$w_0$</td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>$w_{prop} + l$</td>
<td>$w_{prop} + l \cos \alpha$</td>
<td>$w_{prop} + l \cos \alpha + l_{prop} + l_{spring}$</td>
<td>100 mm</td>
</tr>
<tr>
<td>Comb./Breech</td>
<td>$(w_{prop}) + l + l_{prop} + l_{spring}$</td>
<td>$(w_{prop}) + l \cos \alpha + l_{prop} + l_{spring}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The design suggestions for optimal thruster performance that this study aims to find should be established with consideration to the specified mission requirements, but should also hold in a general case.
Chapter 2

Analytical PPT model

2.1 Plasma dynamics and thrust generation

Despite being a simple construction, the operation of the PPT includes many complex interactions resulting from electromagnetic and electrothermal processes. Ever since the technology was first introduced, many experimental studies have been conducted on PPTs to clarify the underlying physics which lead to production of impulse bits in the thruster. An increased understanding of the plasma acceleration, discharge process and the energy transformation would assist in the comprehension of essential parameters that may be used to control the physics of the thruster and to optimize its function as well as solving the issues related to scaling. While the knowledge of these mechanisms remains incomplete and any microscopic details of the interactions are too complex to be identified precisely, parts of the process have been investigated to a certain degree and we will here try to summarize the basic operation principles of the PPT in order to formulate a mathematical model describing the thruster dynamics.

The thrust delivered by the PPT is caused by the actions of the electromagnetic part of the Lorentz force generated by the coupling of the magnetic field to the current density within the plasma and the gas-expanding force resulting from the creation of high-pressure plasma from the ablated material, as described in section 1.2.3 [10]. Therefore, this force can conveniently be divided into an electromagnetic and an electrothermal component:

\[ T = T_{EM} + T_{ET} \]  \hspace{1cm} (2.2)

In most applications, when compared to the electromagnetic component the electrothermal contribution

\[ T_{ET} = f \sqrt{\frac{8(\gamma - 1)}{\gamma^2(\gamma + 1)}} m_{bit} E_0 \]  \hspace{1cm} (2.1)

where \( \gamma \) is a dimensionless material constant (\( \gamma_{Teflon} = 1.3 \)), is sufficiently small to be neglected. [16] [17]
Chapter 2. Analytical PPT model

The high specific impulse of the PPT is a result of the high exhaust velocities of the particles accelerated by the Lorentz force, while the bulk of the ablated material is accelerated by the gas-expanding force and travels at a slower velocity. The proportion of the thrust that is generated by each force differs between different PPTs. [10]

In order to analyze the electromagnetic acceleration, we study the behavior of charged particles in an electromagnetic field. The Lorentz force law

\[
F_L = q \left( E + v \times B \right) = \int \left( \rho E + j \times B \right) dV
\]

(2.3)
defines the force acting on a charge \( q \) moving with velocity \( v \) in an electric field \( E \) and a magnetic field \( B \). \( \rho = \frac{dq}{dV} \) is the charge density and \( j = \rho v \) is the current density. As can be realized from equation (2.3) and seen in Figure 1.4, both the velocity of the particle and the magnetic field are perpendicular to the Lorentz force. The magnetic force on a stationary charge or a charge moving in parallel to the magnetic field is zero, meaning that the work done on the charges is zero. Thus, the Lorentz force does not change the magnitude of the charged particle’s velocity, but the direction of motion, so particles cannot gain kinetic energy from this force. Consequently, the main contribution to the increase in kinetic energy is by the electric field. [18]

### 2.2 General electromechanical model

As a result of the complex electromagnetic and thermal phenomena involved in the acceleration of the propellant, there are still no physical models to accurately model the PPT working mechanism and predict the behavior of the thruster. The process of energy transfer from discharge to solid propellant and mass ablating is not completely theoretically understood, but depends on the relative position of the discharge; therefore, a self-consistent solution of equations for plasma dynamical transport and electrical circuitry is required to understand the behavior of the PPT [3]. Consequently, in order to attempt to describe the PPT system mathematically, an equation regarding the electrical behavior as well as the dynamic motion of the plasma is required. In the following section, such a mathematical description is derived for a general rail-type PPT.

#### 2.2.1 Part I: Electrical model

The pulsed operation of the PPT further adds to the modeling complexity. However, because this type of thruster traditionally operates with an oscillatory waveform, closely resembling that of a simple LCR series circuit as shown in the Figure 2.1 below, when making a theoretical idealization the arc current layers can for simplicity be modeled as elements of such a circuit.
2.2. General electromechanical model

Figure 2.1: LCR circuit model of PPT electrical characteristics.

(a) Thruster equivalent LCR circuit model.  
(b) Further simplified LCR circuit model.  
\[ R = R_C + R_e + R_p, \quad L = L_C + L_e + L_p \]

We assume that the capacitance \( C \) is initially charged to a voltage \( V_0 \) and that  
the initial inductance available immediately after closing the circuit and determined  
by external circuit geometry, electrical connections and the inherent inductance of the capacitors, is \( L_0 \). Further, the initial resistance \( R \) is partly contributed by the  
external circuit hardware, including the capacitor, transmission lines and discharge chamber, and partly by the discharge itself.  

Application of Kirchhoff’s voltage law around the circuit yields  
\[ V = V_0 - \frac{1}{C} \int_0^{t'} i(t)\,dt = iR + \frac{d}{dt}(Li) = iR + L \frac{di}{dt} + i \frac{dL}{dt} \]  

(2.4)

When the system inductance is considered static in time, the current waveform and  
the solution to equation (2.4) is simply the LCR time response\(^2\) for the impulsive input [12]. However, as the inductance of the plasma will change as the current  
sheet moves throughout the thruster channel, a time dependence has to be taken  
into account.

The power delivered to the circuit by the capacitance is thus  
\[ P = iV = i^2 R + Li \frac{di}{dt} + i^2 \frac{di}{dt} = i^2 R + \frac{d}{dt} \left( \frac{1}{2} Li^2 \right) + \frac{1}{2} i^2 \frac{dL}{dt} \]  

(2.6)

\(^2\)While the discharge current will have a damped oscillatory response with varying character  
depending on the values of the electrical parameters, it is here assumed that the response is  
underdamped as that type of response generally has a higher occurrence frequency compared to  
the overdamped and critically damped cases [1]

\[ i(t) = -\frac{V_0}{L_0 \sqrt{\frac{1}{L_0 C} - \frac{R^2}{4L_0^2}}} e^{-\frac{R}{2L_0} t} \sin \left( \sqrt{\frac{1}{L_0 C} - \frac{R^2}{4L_0^2}} t \right) \]  

(2.5)
The first two terms on the right hand side represent the rate of resistive heat generation and the rate of change of energy stored in the magnetic field, while the last term represents the rate of work done on the moving current sheet, which is something that can be optimized. [15]

2.2.2 Part II: Dynamical model

Depending on the details of the heat transfer and acceleration process, the PPT can operate in two different modes. When the heat transfer is sufficient for the propellant surface to provide new electrically conducting material, allowing the discharge path to remain adjacent to the surface, the mode of discharge operation is of a stationary "ablation-arc" type. On the other hand, for insufficient heat transfer, the discharge is forced to follow the accelerated particles at high velocities, causing the PPT to operate in a propagating or "slug" mode.

The essential operation principle of the PPT is based on the mass to accelerate through the self-applied magnetic field. Since the plasma sheet under acceleration is the only movable element, its position and velocity determines the discharge current giving rise to this field, which makes the general dynamical problem nonlinear. Moreover, the complexity of the problem is raised by the uncertainty regarding the microscopic details of the acceleration process. As a result, heuristic electromechanical models have been developed to represent the coarse dynamical features of the acceleration process for solid propellant thrusters, in order to enable an analytically approach to this class of problems.

In the simplest theoretical model, the so-called slug model, developed in 1960 [15], one assumes idealized conditions in which the thruster is taken to be operating in slug mode so that the discharge of the PPT and the narrow current sheets are taken as discrete elements moving throughout the thruster channel. This allows us to describe the behavior of the system and derive some of its important properties starting from basic principles.

The slug model describes the acceleration of the plasma along the electrodes based on the following assumptions:

- The total propellant mass is ablated at once in the initial discharge at the minimum inductance configuration and is accelerated as a single layer which mass remains constant throughout the acceleration. When all ablated mass is assumed to be accelerated simultaneously, there will be a constant change in the inductance of the plasma sheet with regards to position when the magnetic field within the circuit is assumed to be constant.

- The resistance of the resonant circuit is constant over time.

- The magnetic field is zero outside of the circuit and constant and uniform between the electrodes in the direction of the y-axis (as shown in Figure 1.4).

- The current density within the plasma layer is uniform and constant and any displacement current is neglected.
In general, an acceleration may be described by Newton’s second law: a force \( \mathbf{F} \) applied to a constant mass \( m \) causes an acceleration \( \ddot{x} \). In describing the dynamic behavior of the plasma sheet, such an acceleration is caused by the Lorentz force acting on the discharge in a magnetic field induced by the current flowing through the electrodes. The magnetic field can be described as a superposition of all occurring contributions approximated by Biot-Savart’s law; however, the contribution from the current sheet itself can be neglected since it acts only as a force tending to narrow the plasma column and does not contribute to the acceleration. The antiparallel current flow in the electrodes causes both resulting magnetic field vectors to be oriented equally [11]. In accordance with the slug model, the mass ablated per pulse is assumed to be constant and the electrodes may be approximated as infinitely long, as the acceleration of the plasma sheet has been completed before reaching the electrode end. Assuming a uniform acceleration of the mass bit, Newton’s law for the system will describe the conservation of momentum and the motion of the plasma sheet ablated in one pulse, according to

\[
\sum \mathbf{F}(t) = m_{\text{bit}} \frac{d^2 \mathbf{x}_p}{dt^2} = \int (\mathbf{j} \times \mathbf{B}) dV_p
\]  

(2.7)

where \( V_p \) is the volume of the plasma sheet and \( \mathbf{x}_p(t) \) is the displacement of the current sheet from its initial position [12]. Further, as the exhaust velocity of the accelerated particles is directed along the length of the thruster (positive x-direction) the resulting magnetic field will be (cf. Figure 1.4)

\[
\mathbf{B} = B \hat{y}
\]  

(2.8)

In analogy with the slug model, we also assume a uniform current density within the plasma layer such that\(^3\)

\[
\mathbf{j}(t) = -\frac{i(t)}{w} \hat{z}
\]  

(2.9)

Thus, equation (2.7) applied to the system simplifies to the following expression for the electromagnetic thrust component [6]

\[
T_{\text{EM}} = i(t) \mathbf{\hat{x}} \int_0^h \int_0^w \int_{x_p(t)}^{x_p(t)+\delta} \frac{1}{w} B dx dy dz
\]  

(2.10)

Further, modeling of the inductance can be done by defining the magnetic field within the circuit by the total magnetic flux through a surface in the xz-plane,\(^3\)

\(^3\)The current density arises from the distribution of the current \( i(t) \) across the plasma surface area \( w \delta \) in the z-direction. However, we assume the thickness of the sheet to be infinitesimal \( \delta \to 0 \) so we may also assume that the current is only distributed across the width of the electrodes, neglecting the plasma sheet thickness.
Chapter 2. Analytical PPT model

obtained by integrating over the electrodes, such that

$$\Phi(t) = (L_0 + \Delta L)i(t) = L_0i(t) + \int_0^h \int_0^{x_p} B dz dx$$  \hspace{1cm} (2.11)$$

where $B$ is the magnetic field between the electrodes and $dz dx$ is the surface element on a closed path around the current $i$ that the circuit encircles.

2.2.3 Part III: Combined model as non-linear ODE system

We now wish to combine the dynamical model with the electrical model using a one-dimensional approach in which the PPT system is approximated as an electromechanical device constituted by an electrical circuit interacting with a dynamical system [4]. The general dynamics of the system can then be found by combining equation (2.4) and equation (2.10)

$$\begin{cases} m_{bit} \ddot{x}_p(t) - i(t) \int \int \int \frac{1}{w} B dz dy dz = 0 \\ i(t)[R + L' \dot{x}_p(t)] + \dot{i}(t)L + \frac{1}{C} \int_0^{\dot{t}} i(t) dt - V_0 = 0 \end{cases}$$  \hspace{1cm} (2.12a, 2.12b)$$

where the inductance gradient $L' = \frac{dL}{dx}$ is derived from $\frac{dL}{dt} = \frac{dL}{dx} \frac{dx}{dt}$ and describes the change in inductance per unit length along the acceleration channel.

As the current $i(t)$ is assumed to be invariant with respect to geometry, comparing with equation (2.11) the geometric change in inductance can be defined through integration of the magnetic field over the volume between the electrodes and can be investigated theoretically through [8]

$$\Delta L(x_p, y) = \frac{1}{i(t)} \int_0^h \int_0^{x_p} B dz dx$$  \hspace{1cm} (2.13)$$

The surface $dz dx$ depends on the position $x_p$ form the propellant surface and the change in inductance will thus vary accordingly. Further averaging across the width of the electrodes to yield the average change in inductance as a function of the position of the plasma sheet is necessary to obtain processable values [13]

$$\Delta \bar{L}(x_p) = \frac{1}{i(t)} \int_0^w \int_0^h \int_0^{x_p} \frac{1}{w} B dz dy$$  \hspace{1cm} (2.14)$$

Thus, equation (2.12a) simplifies to

$$m_{bit} \ddot{x}_p - \Delta \bar{L}(x_p)[i(t)]^2 = 0$$  \hspace{1cm} (2.15)$$

The model can be further simplified depending on the particular thruster geometry. When referring to different electrode geometries we will in what follows use
2.3 Parallel rectangular electrode geometry

2.3.1 Model simplification

For simplicity, the discharge channel geometry can be approximated as a quasi-infinite width ($w >> h$), one-turn solenoid composed of perfectly conducting sheets, each with a uniform current density \[ i \]. In such simple, sufficiently symmetric geometries the magnetic field generated by a slowly alternating electric current can be described using Ampere’s circuital law, which in its simplest form reads

\[
\sum B \parallel \Delta l = \mu i
\]  

(2.16)

and implicates that for any closed loop path, the sum of the length elements times the magnetic field in the direction of these elements is proportional to the electric current enclosed in the loop with the permeability as proportionality constant. In the case of the PPT, the self-induced magnetic field is directed along the thruster width and, in accordance with the slug model, the integration over the plasma sheet has been done under the assumption that no magnetic field outside of the solenoid exists, so that

\[
B(x, t) = \begin{cases} 
\mu_0 \frac{i(t)}{w} \hat{y}, & 0 < x < x_p(t) \\
0, & x > x_p(t) + \delta
\end{cases}
\]  

(2.17)

where, due to the diamagnetic properties of the plasma, $\mu = \mu_0$ is the vacuum permeability. The magnetic field throughout the current sheet will also have a dependence on the position relative to the current sheet position and can be found from application of Ampere’s law to a surface $S$ passing through the plasma sheet (see Figure 2.2). Using the magnetic field inside the solenoid as a boundary condition so that the magnetic field within the current sheet will increase linearly with the distance from the plasma propellant surface, we thus obtain the full-range magnetic field description \[ 4 \]

\[
B(x, t) = \begin{cases} 
\mu_0 \frac{i(t)}{w} \hat{y}, & 0 < x < x_p(t) \text{ (inside solenoid)} \\
\mu_0 \frac{i(t)}{w} \left(1 - \frac{x - x_p(t)}{\delta}\right) \hat{y}, & x_p(t) \leq x \leq x_p(t) + \delta \text{ (throughout plasma sheet)} \\
0, & x > x_p(t) + \delta \text{ (outside solenoid)}
\end{cases}
\]  

(2.18)
Chapter 2. Analytical PPT model

Figure 2.2: Application of Ampere’s Law throughout the plasma sheet.

Inserting this into equation (2.10) yields the electromagnetic force experienced by the electrode

\[ T_{EM} = h \frac{i(t)}{\delta} \hat{x} \int_{x_p(t)}^{x_p(t)+\delta} \mu_0 \frac{i(t)}{w} \left( 1 - \frac{x-x_p(t)}{\delta} \right) dx = \mu_0 h \frac{[i(t)]^2}{\delta w} \hat{x} \int_0^{\delta} \left( 1 - \frac{x}{\delta} \right) dx = \mu_0 \frac{[i(t)]^2}{w} \hat{x} \]  

(2.19)

From this we can construct the dynamic equation for the system

\[ m_{bit} \ddot{x}_p = \mu_0 \frac{h}{2} [i(t)]^2 \hat{x} \]  

(2.20)

Comparing with equation (2.11), the inductance contribution from parallel plate electrodes is thus

\[ L_{PE}(x_p(t)) = \frac{1}{\pi \mu_0} \iint_{\text{electrodes}} B(x,y) \cdot dS = \frac{w}{h} \int_0^{x_p(t) + \delta} \left[ \int_0^{x_p(t) + \delta} \int_0^{x_p(t)} \left( 1 - \frac{x-y}{\delta} \right) dy dx \right] = \mu_0 \frac{h}{w} (x_p(t) + \frac{\delta}{2}) \]  

(2.21)

Consequently, the inductance gradient will be [15]

\[ L'_{PE} = \frac{dL_{PE}}{dx_p(t)} = \mu_0 \frac{h}{w} \]  

(2.22)

For infinitesimal plasma sheet thickness \( \delta \to 0 \), parallel electrodes and a planar current sheet that remains perpendicular to the electrodes, the inductance model reduces to\(^4\)

\[ L = L_0 + \left( \frac{dL}{dx} \right) x = L_0 + \mu \frac{h}{w} x_p(t) = L_0 + L'_{PE} x_p(t) \]  

(2.23)

\(^4\)We here consider the simplest expression, but there have been several attempts to improve the inductance model for parallel plate PPTs, by for instance taking into account the finite width and non-zero thickness of the electrodes. [3]
2.3. Parallel rectangular electrode geometry

The dynamics of the system can now be described by

\[
\begin{align*}
\left\{ m_{\text{bit}} \ddot{x}_p(t) - \frac{\mu_0}{2} \left( \frac{h}{w} \right) \left[ i(t) \right]^2 &= 0 \\
\left[ L_0 + \mu_0 \left( \frac{h}{w} \right) x_p(t) \right] \dot{i}(t) + \left[ \mu_0 \left( \frac{h}{w} \right) \dot{x}_p(t) + R \right] i(t) + \frac{1}{C} \int_0^t i(\tau) d\tau - V_0 &= 0
\end{align*}
\]

accompanied by the trivial initial conditions

\[
x_p(0) = 0, \quad \dot{x}_p(0) = 0, \quad \int_0^{t=0} i(\tau) d\tau = 0, \quad i(0) = 0
\]  

We now wish to solve for \( i(t) \) and \( x_p(t) \) and therefore write the system as a set of four first order differential equations by introducing the state space variables

\[
x_1(t) = x_p(t) \\
x_2(t) = \int_0^t i(\tau) d\tau \\
x_3(t) = \dot{x}_p(t) \\
x_4(t) = i(t)
\]

This allows the system to be written in the simpler form:

\[
\begin{align*}
\dot{x}_1(t) &= x_3(t) \\
\dot{x}_2(t) &= x_4(t) \\
\dot{x}_3(t) &= \ddot{x}_p(t) = \frac{\mu_0}{2m_{\text{bit}}} \left( \frac{h}{w} \right) [i(t)]^2 = \frac{\mu_0}{2m_{\text{bit}}} \left( \frac{h}{w} \right) [x_4(t)]^2 \\
\dot{x}_4(t) &= \dot{i}(t) = \frac{V_0}{L_0 + \frac{\mu_0 h}{w} x_1(t)} - \frac{1}{C} x_2(t) - \mu_0 \left( \frac{h}{w} \right) x_3(t) + R \end{align*}
\]

2.3.2 Thruster performance evaluation parameters

While some characteristic parameters are commonly used to define the performance of a propulsion system, there exist variations in the definitions of these parameters. Such definition disagreements mainly arise from the various internal efficiency factors that need to be taken into consideration.

In this section important parameters for evaluating the thruster performance of a PPT are to be identified and brief descriptions of them as well as a definitions adapted in this particular study are listed below [3][15]. In this case, these parameters are defined in such a way as to be calculated from the results of solving the system of differential equations (2.27).
• **Effective exhaust velocity**, $v_e$

We start by defining the exhaust velocity, which is here the velocity of the accelerated particles at the end of the acceleration channel, at which the position of the plasma sheet is equal to the length of the electrodes according to

$$v_e = \dot{x}_p(t')$$

(2.28)

$$t' : x_p(t') = l$$

(2.29)

where $t'$ is the time it takes for the plasma sheet to reach the end of the electrodes and $\dot{x}_p$ is the state space variable $x_3(t)$ in the equation system (2.27) representing the plasma velocity at instant $t$.

The exhaust velocity is a central quantity from which the other performance parameters relevant in this study can be derived. As seen in section 2.2.3, the exhaust velocity depends on the inductance gradient as well as the current and therefore it is hardly surprising that there is also a dependence of thruster efficiency on exhaust velocity. High exhaust velocities result in lower fuel consumption and thus increase the payload portion of the spacecraft. Further, the maximum exhaust velocity has been shown to have a considerably stronger dependence on the geometric configuration of the thruster than on the discharge energy and it is therefore a highly relevant performance evaluation parameter for the purpose of this study [8].

• **Average thrust**, $T_{avg}$

Another important quantity is the thrust produced during each pulse. By using equation (2.10) we can define the average thrust generated from electromagnetic acceleration during the pulse duration as

$$T_{avg} = f \int_{0}^{t'} \left( i(t) \int_{0}^{h} \int_{0}^{w} \int_{x_p(t)}^{x_p(t)+\delta} \frac{1}{w} B dx dy dz \right) dt$$

(2.30)

where $f$ is the pulse frequency, which for parallel electrodes simplifies to (cf. equation (2.19))

$$T_{avg} = f \frac{\mu_0 h}{2w} \int_{0}^{t'} [i(t)]^2 dt = f \frac{L'}{2} \int_{0}^{t'} [i(t)]^2 dt$$

(2.31)

• **Impulse bit**, $I_{bit}$

The pulsating nature of the thrust force produced facilitates the need to measure it as an impulse. The impulse bit describes the momentum generated by the thruster during one pulse by accelerating the mass $m_{bit}$ to the average
exhaust velocity \( v_e \) and can be defined through the relation between the thrust and the exhaust velocity

\[
I_{\text{bit}} = m_{\text{bit}} v_e = \int_0^{t'} T_{\text{EM}} = \int_0^{t'} \left( \int_0^h \int_0^w \int_{x_p(t)}^{x_p(t)+\delta} \frac{1}{w} B dx dy dz \right) dt \quad (2.32)
\]

Carrying out this integral for a parallel plate PPT, the electromagnetic impulse bit acting on the plasma sheet can be evaluated as

\[
I_{\text{bit}} = \frac{1}{2} \frac{h}{w} \mu_0 \int_0^{t'} [i(t)]^2 dt \quad (2.33)
\]

This integral can be solved numerically using a trapezoidal summation, by using the fact that the state space variable \( x_4(t) \) in equation (2.27) equals the discharge current.

- **Specific impulse**, \( I_{sp} \)
  The specific impulse measures how efficiently a propulsion system converts propellant into thrust and is defined as

\[
I_{sp} = \frac{1}{m_{\text{bit}}} \int_0^{t'} T_{\text{EM}} dt = \frac{v_e}{g} \quad (2.34)
\]

- **Thruster efficiency**, \( \eta \)
  Without consideration to a possible thermal component, the electrical efficiency is simply defined as the percentage of the total energy flowing in the change of the inductance. It is a measure of the efficiency of energy conversion and describes the fraction of the energy available in the capacitor bank that can be converted into directed\(^5\) kinetic energy of the exhaust stream of plasma particles and thereby into thrust. For an electrical thruster this energy will be the kinetic energy of the ejected particles and thus the thruster efficiency translates into

\[
\eta = \frac{E_{\text{kin}}}{E_0} = \frac{1}{2} \frac{m_{\text{bit}} v_e^2}{E_0} = \frac{1}{2m_{\text{bit}} E_0} \left( \int_0^{t'} T_{\text{EM}} dt \right)^2 \quad (2.35)
\]

where the energy of the capacitor can be calculated as

\[
E_0 = \frac{1}{2} CV_0^2 \quad (2.36)
\]

This expression does not take into account that the exhaust particles may have different masses and spatial and time distribution of velocities and hence

---

\(^5\)The term 'directed' serves to imply that velocity distribution losses, wall drag and losses due to the exhaust beam spreading are included.
different energies. It is also worth noting that while the PPTs thruster efficiency is in general seen to increase with input power, a clear relationship with energy has not been experimentally observed [16].

**A note on maximum efficiency.** Another highly influential factor in each PPT design is the required power, which is a function of discharge energy $E_0$ and pulse frequency $f$ according to

$$P = fE_0$$  \hspace{1cm} (2.37)

Since all energy initially stored in the capacitor is delivered to the circuit in a single pulse of current during time $t'$ we can express the useful energy output according to

$$E_0 = \int_{0}^{t'} P \, dt \stackrel{(2.6)}{=} \int_{0}^{t'} \left( [i(t)]^2 R + \frac{1}{2} [i(t)]^2 \frac{dL}{dt} \right) dt$$  \hspace{1cm} (2.38)

The middle term in equation (2.6) representing the rate of change of energy stored in the magnetic field will disappear in the integration since the current is zero between pulses, i.e. at $t = 0$ and $t = t'$. The integral over the last term representing the work done on the moving arc sheet is here regarded as completely convertible to kinetic energy of the plasma layer, i.e. it neglects dynamical losses, while the integral over the rate of resistive heat is regarded as total loss. Consequently, the kinetic energy can be defined as

$$E_{\text{kin}} = \frac{1}{2} \int_{0}^{t'} [i(t)]^2 \frac{dL}{dt} \, dt$$  \hspace{1cm} (2.39)

and the electrical efficiency can thus be expressed as

$$\eta = \frac{1}{2E_0} \int_{0}^{t'} [i(t)]^2 \frac{dL}{dt} \, dt$$  \hspace{1cm} (2.40)

We now define a current $i_0$ as a reference current, which equals the amplitude of the sinusoidal current which would have arisen in the circuit if there would be no resistance and the inductance had remained constant at $L = L_0$, so that $E_0 = \frac{1}{2}L_0i_0^2$. In reality, $L$ increases monotonically from $L_0$ and the resistance is finite, resulting in the condition $i < i_0$ throughout the pulse. Consequently, we have that

$$\eta = \frac{\int_{0}^{t'} i_0^2 \frac{dL}{dt}}{L_0} < \frac{\int_{0}^{t'} i_0^2 \frac{dL}{dt}}{L_0} = \frac{\int_{0}^{t'} \frac{dL}{dt}}{L_0} = \frac{\Delta L}{L_0}$$  \hspace{1cm} (2.41)

where $\Delta L$ is the total increase in circuit inductance achieved during the current pulse. For a discharge dominated by electromagnetic acceleration, the maximal electrical efficiency has a value greater than or near unity [17][27].
From this we can conclude that maximizing the efficiency would require minimizing the initial inductance, which can for instance be achieved by designing the thruster to allow the capacitors to be mounted as close as possible to the discharge chamber so that losses due to electrical connections are avoided. Enhanced efficiency can also be achieved from an increased value of the inductance gradient obtained by suitable selection of the electrodes dimensions.

2.4 Flared and non-rectangular electrode geometries

The rectangular electrodes are usually parallel, but for some PPT designs the thruster performance can be enhanced by angling the electrodes away from the thrust axis, thereby providing a nozzle effect. The result of several previous studies on optimizing the performance of miniaturized PPTs has been that the primary consideration is the electrode design, in terms of shape and flare angle. Such optimizations have shown to have a greater value for miniaturized electrode configurations than optimizing circuit parameters, such as the system resistance and inductance. Meanwhile, due to the lack of complete knowledge of the discharge process and the complexity added by flared geometries, improvements resulting from modifications of electrode shape and flare angle are difficult to analyze analytically. As a consequence, a mathematical model describing the relation between thruster performance and the flare angle is at present not completely known.

The same is true for electrode shapes other than the standard rectangular geometries. The two different shapes of electrodes under investigation here can be described in terms of modifications of electrode width $w$ and thickness $t$:

- Rectangular: $t, w$ constant throughout the thruster channel.
- Tongue-shaped: both $t(x)$ and $w(x)$ is decreasing with $x$.

![Complex electrode geometry with flared electrodes of increasing separation and decreasing width along the x-axis.](image)
In the case of rectangular parallel electrodes it is evident that the inductance gradient \( L' = \mu_0 \frac{h}{w} \) will influence all parameters used to determine the thruster performance by its relation to the generated thrust, as can be seen in section (2.3.2). This should also hold for other electrode geometries, which can be easily realized by analyzing the representation of the inductance gradient.

The inductance gradient represents the energy available to the plasma and thereby determines the velocity of the particles within the plasma. Therefore, by maximizing the inductance gradient the highest possible amount of energy is ensured to be available to accelerate the ablated plasma sheet and in this way the change in inductance throughout the length of the thruster will directly affect the efficiency. For parallel geometries the inductance is linear with plasma position and time

\[
L \propto x_p(t)
\]  

which makes the inductance gradient constant with respect to plasma position, as it only depends on the aspect ratio, which is constant for this type of geometry. Conversely, this does not hold for flared electrode geometries, where the inductance is not linear in \( x_p \) and thus not constant with changes in plasma position over time. For this reason, the inductance gradient cannot be expressed analytically in a simple way and neither can the other performance parameters. Moreover, the current density increases with decreasing electrode width. For tongue-shaped electrode geometries there will be an implicit time dependence on the electrode width \( w[x_p(t)] \) and the interelectrode spacing \( h[x_p(t)] \), which complicates the solution of the ODE system defined in section 2.2. As a result, the current variable cannot be solved for and as this variable is crucial in defining the performance parameters, there will be no sufficiently simple analytical solution for these parameters for complex geometries. Instead, an average of the inductance change throughout the thruster channel is determined.

For parallel electrodes the inductance can be derived by adding self-inductance and mutual inductance according to Lenz’s law. However, if we introduce more complex channel geometries the integration of the magnetic field over the volume of the electrodes will be more difficult to perform analytically. In addition to the self-inductance from both the plasma and the electrodes when considered as conductors with rectangular cross-section, inclined inductors also show a dependence on the flare angle which is displayed in the mutual inductance between the inclined electrodes themselves, as well as a mutual inductance between the plasma sheet and the electrodes. Adding these extra terms due to the inclination of the electrodes according to Lenz’s law, the total inductance can be expressed as [12]

\[
L = 2L_{\text{electrode}} + L_{\text{plasma}} - 2L_{M^{\text{incl}}} - 2L_{M^{\text{el-plasma}}} \quad [\mu H]
\]  

(2.43)

where the mutual inductance terms are evaluated as

\[
L_M = 0.004 \cos \beta \left[ \left( \sqrt{\frac{h_0^2}{2-2\cos \beta}} + l \right) \tan^{-1} \left( \frac{l}{r+h_f} \right) - \sqrt{\frac{h_0^2}{2-2\cos \beta}} \tan^{-1} \left( \frac{l}{r+h_0} \right) \right]
\]  

(2.44)
2.4. Flared and non-rectangular electrode geometries

$h_0, h_f$ and $r$ are inclined electrode geometry parameters defined in Figure 2.4 and

$$\beta = \begin{cases} 2\alpha, & \text{for } L_M^{\text{incl}} \\ \alpha, & \text{for } L_M^{\text{el-plasma}} \end{cases}$$

(2.45)

since the plasma sheet is assumed to be homogeneous with cross-section equal to the electrode geometry $h_0 w_0$.

![Figure 2.4: Flared geometry parameters.](image)

The self-inductance of the electrodes is found by integrating the magnetic field over their respective volume. Meanwhile, the mutual inductance if found by integrating the magnetic field generated by the current in one electrode over the volume of the other electrode. According to Lenz’s law, the mutual inductance is subtracted from the self-inductance in order to obtain the total inductance and consequently, decreasing it will lead to gains in inductance. For this reason, an increased electrode separation should lead to an increased total inductance as the mutual inductance between the electrodes decreases proportional to the distance between them. Another method to achieve the same, although smaller, effect could be to decrease the cross-section of the electrodes in order to obtain a reduced volume of integration. For flare angles beyond $45^\circ$, the negative mutual inductance between the plasma sheet and the electrodes becomes the dominant parameter and therefore leads to a decrease of the overall inductance and the inductance gradient for this geometry [12]. For electrodes of non-rectangular cross section, such as tongue geometries, the determination of the inductance gradient is even more complex and cannot be performed analytically in completeness due to the divergence of the expressions in the limit where the cross-sectional area of the electrode tip approaches zero. Instead, the change in inductance is averaged over the geometry in the manner presented below.

We are now interested in determining how the inductance changes with the distance from the propellant surface for a general electrode geometry. We approximate
Chapter 2. Analytical PPT model

Each current filament of the electrodes as infinitely long and study the interaction at each point in the area between the electrodes, see Figure 2.5.

Figure 2.5: Upstream view of magnetic field for one current filament in the electrode.

The magnetic field will always be perpendicular to the distance vector $r_P$ between the current filament and the point of investigation $P(y, z)$. Using Ampere’s law, the component of the magnetic field between the electrodes can be expressed

$$
B_y = \frac{\mu_0 j}{2\pi r_P} \cos \theta = \begin{cases} 
-\frac{\mu_0 i}{2\pi w} \left( \frac{z}{(y-y')^2 + z^2} \right) & \text{for the anode} \\
-\frac{\mu_0 i}{2\pi w} \left( \frac{h-z}{(y-y')^2 + (h-z)^2} \right) & \text{for the cathode}
\end{cases}
$$

(2.46)

where $j$ is the current density resulting from the distribution of the total current $i$ across the width $w$ and $\theta$ is the angle between the magnetic field vector $B$ and the $y$-axis along the width of the electrodes. Thus, we can write

$$
B_y^{\text{tot}}(y, z) = -\frac{\mu_0 i}{2\pi w} \int_0^w \left( \frac{z}{(y-y')^2 + z^2} + \frac{h-z}{(y-y')^2 + (h-z)^2} \right) \, dy' = \\
= \{ k = y - y', \, dk = -dy' \} = \frac{\mu_0 i}{2\pi w} \int_0^w \left( \frac{z}{k^2 + z^2} + \frac{h-z}{k^2 + (h-z)^2} \right) \, dk =
$$
2.4. Flared and non-rectangular electrode geometries

\[
\begin{align*}
2.47 \quad &= \frac{\mu_0 i}{2\pi w} \left[ \arctan \left( \frac{k}{z} \right) + \arctan \left( \frac{k}{h - z} \right) \right]_0^w = \frac{\mu_0 i}{2\pi w} \left[ \arctan \left( \frac{y - y'}{z} \right) + \arctan \left( \frac{y - y'}{h - z} \right) \right]_0^w = \\
&= \frac{\mu_0 i}{2\pi w} \left( - \arctan \left( \frac{y}{z} \right) + \arctan \left( \frac{y - w}{z} \right) - \arctan \left( \frac{y}{h - z} \right) + \arctan \left( \frac{y - w}{h - z} \right) \right) = \\
&= -\frac{\mu_0 i}{2\pi w} \left( \arctan \left( \frac{y}{z} \right) + \arctan \left( \frac{w - y}{z} \right) + \arctan \left( \frac{y}{h - z} \right) + \arctan \left( \frac{w - y}{h - z} \right) \right)
\end{align*}
\]

for the total component of the magnetic field across the electrode width. By using equation (2.14), we wish to solve for the average inductance change, or inductivity

\[
\Delta \bar{L}(x_p) = \frac{\mu_0}{2\pi} \int_0^{h(x)} \int_0^{w(x)} \int_0^{x_p} \left[ \arctan \left( \frac{y}{z} \right) + \arctan \left( \frac{w - y}{z} \right) + \arctan \left( \frac{y}{h - z} \right) + \arctan \left( \frac{w - y}{h - z} \right) \right] dz dy dx
\]

As we wish to investigate the effect of flared and tongue-shaped electrodes, the spacing between the electrodes and their width will be a function of the axial coordinate \( x \). Based on Figure 2.3 we get

\[
\begin{align*}
h(x) &= h_0 + 2 \tan(\alpha) x \\
w(x) &= w_0 \left( 1 - \frac{x}{l} \right) + w_{\text{tip}} \frac{x}{l}
\end{align*}
\]

where \( h_0 \) and \( w_0 \) are the base spacing and width respectively, \( w_{\text{tip}} \) is the width of the tip of the electrode, \( l \) is the length of the electrode measured from the propellant surface and \( \alpha \) is the flare angle between the \( x \)-axis and each of the electrodes. In the case of tongue-shaped electrodes \( \left( \frac{w_{\text{tip}}}{w_0} \rightarrow 0 \right) \) the change in inductance goes towards infinity due to the singularity of the magnetic field in the tip of the electrode. [13]

By use of the mathematical software Mathematica, the integral (2.48) was performed analytically over the \( y \)- and \( z \)-axis with the limits defined by \( w(x) \) and \( h(x) \) respectively and by applying the requirement \( h, w \in \mathbb{R}_{>0} \). The remaining integral will be of the form

\[
\Delta \bar{L}(x_p) = \frac{\mu_0}{2\pi} \int_0^{x_p} f(x, w(x), h(x)) dx
\]

where

\[
\begin{align*}
f(x, h(x), w(x)) &= \frac{2h(x)}{w(x)} \left[ \pi - 2 \arctan \left( \frac{h(x)}{w(x)} \right) + \frac{h(x)}{w(x)} \ln|h(x)| \right] - \\
&\quad - 2 \ln|w(x)| + \left( 1 - \frac{|h(x)|^2}{|w(x)|^2} \right) \ln(|h(x)|^2 + |w(x)|^2)
\end{align*}
\]

with \( h(x) \) and \( w(x) \) defined as in equation (2.49), and can now be determined numerically for a given distance to the propellant surface \( x_p \). [13]
Chapter 3

Optimization

3.1 Optimization model

To reduce the complexity of the optimization problem, the electrical parameters have not been considered and the boundary of the problem have been set to the geometrical properties of the discharge chamber alone, despite the fact that the thruster efficiency is largely affected by the electrical characteristics of the propulsion system.

3.1.1 Optimization objectives

The aim of the optimization is to find a solution that allows the PPT to work as efficiently as possible while meeting the specified mission requirements. In summary, the main goals are:

Simple electrode geometries:  
- Maximized efficiency $\eta$
- Maximized specific impulse $I_{sp}$
- Maximized delivered thrust $T_{avg}$

Complex electrode geometries:  
- Maximized inductivity $\Delta L$

while meeting the specified mission requirements.

Parallel, rectangular electrodes

In the case of parallel, rectangular electrodes the objectives listed above also implies maximized exhaust velocity as it is proportional to the specific impulse, as well as maximized impulse bit as it is linearly related to the average thrust produced during
one pulse through the frequency. In fact, as we consider a typical pulse frequency of 1 Hz, the value of the impulse bit equals that of the average thrust.

Not only the efficiency and the specific impulse are related to the exhaust velocity, but in fact, all performance parameters defined in section 2.3.2 can be shown to be dependent on this quantity. Thus, the simplest objective is to maximize $v_e$. In addition, the thrust delivered by the PPT increases with the acceleration of the plasma particles, which depends on the current flowing through the system. The objective should therefore compromise maximization of the discharge current as well as the exhaust velocity.

In addition, a third requirement has been added to the objective function. For any solution to the system of differential equations defined in equation (2.27) the data presented in the graph of Figure 3.1 can be generated, where the red dotted line marks the thruster exit at the end of the electrodes.

Figure 3.1: Plasma sheet velocity and acceleration.

As mentioned, the plasma acceleration is directly proportional to the current which has a peak after which it will level off. This peak represents the point of maximal thrust generation. As the acceleration of the plasma particles will decrease after this point, their velocity saturate at the maximum achievable exhaust velocity. Once these two maximums are achieved, the discharge can advantageously be terminated. To ensure that the maximum exhaust velocity is reached before the end of the electrodes, i.e. that the red dot will not be placed somewhere on the slope of the velocity graph before the saturation point, the distance difference between the velocity at the end of the electrodes and the point where the maximum exhaust velocity is first reached is part of the minimization problem and incorporated
3.1. Optimization model

into the objective function. The optimization problem will therefore correspond to maximizing the function

\[
 f_{\text{obj}} = W_1[v_p(h_0, w_0; t)]_{\text{max}} + W_2 \frac{h_0}{w_0} \int_0^t [i(h_0, w_0; t)]^2 dt - W_3 \left[ v_p(h_0, w_0; t) \right]_{\text{ist max}} - v_e \tag{3.1}
\]

where \( v_e = v_p(t') \) and \( t' \) indicates the time when the exhaust stream has reached the end of the electrodes, which makes the objective function depend also on the electrode length \( l \). \( W_i \) is just a weight assigned in order to give the same order of magnitude to all three terms to ensure that they have equal amount of influence. Since the second terms defines the average thrust produced during one pulse, \( W_2 = C(\mu_0^2 f) \), where \( C \) is another weighting constant, so that the other constants are incorporated into the weight \( W_2 \). For the orders of magnitude we are considering here, we set \( W_1 = 1 \), \( W_2 = 100 \) and \( W_3 = 10 \), which has been determined empirically by statistics from several iterations of solving the system for different values of the input parameters.

**Flared, non-rectangular electrodes**

In the case of more complex geometries, where the inductivity is the only performance parameter that can be determined analytically, the objective function is instead given by (cf. equation (2.50))

\[
 f_{\text{obj}}(h(x), w(x), l, \alpha) = \frac{\mu_0}{2\pi} \int_0^l \frac{2h(x)}{w(x)} \left[ \pi - 2 \arctan \left( \frac{h(x)}{w(x)} \right) + \frac{h(x)}{w(x)} \ln[h(x)] \right] - \\
 - 2 \ln[w(x)] + \left( 1 - \frac{[h(x)]^2}{[w(x)]^2} \right) \ln([h(x)]^2 + [w(x)]^2) dx \tag{3.2}
\]

and the optimization problem will consist of finding values of \( h_0, w_0, l \) and \( \alpha \) that maximizes the change in inductance within the given constraint subset.

3.1.2 Standard form formulation

In order to find a solution to the optimization problem, models relating constraints and objectives to the design variables were developed. The independent variables of this optimization problem are values of the design parameters that should be carefully chosen to fit the optimal value of the objectives, which defines the thruster performance.

The current and exhaust velocity can both be determined by solving the system of differential equations specified in section 2.2. It can be seen that apart from the
parameters \{x(1),x(2),x(3),x(4)\} to be determined by this solution, the system also contains the variables \(h_0, w_0, m_{\text{bit}}, C, V, L_0\) and \(R\), where

\[
m_{\text{bit}} = \frac{m_{\text{tot}}}{f \tau}, \quad V = \sqrt{\frac{2E_0}{C}}
\]

Thus, the solution to the objective function (3.1) will depend on these. As we are only concerned with optimizing the geometrical parameters, a solution to the optimization problem will be found for a fixed set of circuit parameters \(C, V, L_0\) and \(R\), and the mass bit will be determined by the mission requirements for VELOX, which sets the total propellant mass \(m_{\text{tot}}\), pulse frequency \(f\) and expected lifetime \(\tau\). As the length of the electrodes \(l\) will define the exhaust velocity, we are left with \(h_0, w_0\) and \(l\) as variable parameters subject to the optimization. Consequently, we wish to find the vector of design variables

\[
x = \begin{pmatrix} h_0 \\ w_0 \\ l \\ (\alpha) \end{pmatrix}
\]

that minimizes \(y(x) = -f_{\text{obj}}(x)\) subject to

\[
\begin{pmatrix} (h_0)_{\text{min}} \\ (w_0)_{\text{min}} \\ l_{\text{min}} \\ (\alpha_{\text{min}}) \end{pmatrix} \leq x \leq \begin{pmatrix} (h_0)_{\text{max}} \\ (w_0)_{\text{max}} \\ l_{\text{max}} \\ (\alpha_{\text{max}}) \end{pmatrix}
\]

where the angle appearing in parenthesis is only added for flare geometries and

\[
c(x) \leq 0, \quad c_{\text{eq}}(x) = 0
\]

where \(c\) is a vector of inequality constraints, to ensure that the design variables are contained within a certain range, and \(c_{\text{eq}}(x)\) is vector of equality constraints that can be added to ensure that the design parameters have a certain desired value. Since we impose no such constraints on the parameters here, we set \(c_{\text{eq}} = 0\). In this particular case, the upper and lower bounds for the design variables will have a limited influence on the final solution of the problem, as they are instead more carefully set by the nonlinear constraints defined by the vector \(c(x)\). As each design variable depends on at least one of the others, this will simplify the formulation of the optimization problem.

### 3.1.3 General constraints

In general, the vector of nonlinear inequality constraints is defined as \(c(x) = (c_1, c_2, c_3, c_4, c_5, c_6, c_7)\). A solution will be constrained to a certain aspect ratio
range $a_{\text{min}} - a_{\text{max}}$ and energy density range $(E_0/A)_{\text{min}} - (E_0/A)_{\text{max}}$ and the four first elements of $\mathbf{c}$ ensures that these ranges are not exceeded by setting

$$c_1 = \frac{h_0}{w_0} - a_{\text{max}}, \quad c_2 = a_{\text{min}} - \frac{h_0}{w_0} \quad (3.7a)$$

$$c_3 = \frac{E_0}{A} - \left( \frac{E_0}{A} \right)_{\text{max}}, \quad c_4 = \left( \frac{E_0}{A} \right)_{\text{min}} - \frac{E_0}{A} \quad (3.7b)$$

where the exposed propellant surface area $A$ depends on the propellant feeding type according to

$$A = \begin{cases} h_0w_0, & \text{breach feeding} \\ 2h_0w_{\text{prop}}, & \text{side feeding} \\ 3h_0w_0, & \text{combination feeding} \end{cases} \quad (3.8)$$

### 3.1.4 Design specific constraints

The remaining three elements of the constraints vector ensures that the maximum dimensions of the thruster chamber are not exceeded. As a result, they differ between different designs, and depend on the feeding, flare angle and electrode shape. We therefore term $c_5, c_6$ and $c_7$ design specific constraints.

$c_5$ is related to the maximum thruster height, which is independent on the feeding mechanism, but dependent on both electrode shape and flare angle. $c_6$ defines the maximum thruster width, which depends on the feeding, but is not related to the shape of the electrodes or their flare angle. Meanwhile, $c_7$ limits the thruster length and depends on both the feeding mechanism as well as the flare angle, but is independent on the electrode shape. These constraints are defined in Table 3.1 below for different combinations of feeding, electrode shape and flare angle and their determination is clarified by Figure 3.2. The cell color indicates constraints that remain the same for different such combinations.

<table>
<thead>
<tr>
<th>$c_5$</th>
<th>$c_6$</th>
<th>$c_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Breach</strong></td>
<td><strong>Breach</strong></td>
<td><strong>Breach</strong></td>
</tr>
<tr>
<td><strong>Rectangular</strong></td>
<td><strong>Tongue</strong></td>
<td><strong>Rectangular</strong></td>
</tr>
<tr>
<td>$h_0 + 2t - 0.05$</td>
<td>$w_0 - 0.1$</td>
<td>$l_{\text{prop}} + l - 0.09$</td>
</tr>
<tr>
<td>$h_0 - 0t - 0.05$</td>
<td>$w_0 + 2l_{\text{prop}} - 0.08$</td>
<td>$w_0 + l + l_{\text{prop}} - 0.09$</td>
</tr>
<tr>
<td><strong>Comb.</strong></td>
<td><strong>Comb.</strong></td>
<td><strong>Comb.</strong></td>
</tr>
<tr>
<td>$w_0 + 2l_{\text{prop}} - 0.08$</td>
<td>$w_0 + 2l_{\text{prop}} - 0.08$</td>
<td>$w_0 + l + l_{\text{prop}} - 0.09$</td>
</tr>
</tbody>
</table>

The maximum thruster dimensions are defined by $h_{\text{max}} = 50 \text{ mm}$, $w_{\text{max}} = 100 \text{ mm}$ and $l_{\text{max}} = 100 \text{ mm}$. In Table 3.1 the feeding springs have been accounted for and their approximate length of 10 mm has been subtracted from the maximum
allowed dimensions according to the relevant propellant feeding type. Naturally, for \( c_7 \) the flared case transforms into the parallel case as the flare angle is set to \( \alpha = 0 \).

**Figure 3.2: Relation between total thruster dimensions and design variables.**

(a) Width of breech-fed design.  
(b) Width of side-fed and combination-fed designs.  
(c) Height and length of rectangular design.  
(d) Height and length of tongue-shaped design.

In Figure 3.2(c) and Figure 3.2(d) the distance\(^1 \) \( d \) is defined as

\[
d = \begin{cases} 
  l_{\text{prop}}, & \text{breech feeding} \\
  w_{\text{prop}}, & \text{side feeding} \\
  l_{\text{prop}} + w_{\text{prop}}, & \text{combination feeding}
\end{cases} 
\]

\[ (3.9) \]

### 3.1.5 Fix constants

The fix constants are variables that are treated as fixed with the purpose of investigating the influence of the remaining independent variables on the objectives for any given level.

\(^1\)The piece of the electrode covering this part may not be accurately depicted in Figure 3.2 for all feeding types, but this has no effect on the determination of the geometrical constraints.
## 3.1. Optimization model

### Propellant width

As we can see, the components $c_6$ and $c_7$, defining the limits if the thruster width and length, will be determined by the dimensional constraints of the propellant bar, which are dependent on the type of feeding mechanism that is used. The initial volume of one propellant bar can be expressed as

$$V_{\text{bar}} = h_0 w_l_{\text{prop}}, \quad w = \begin{cases} w_0, & \text{breech/comb.} \\ w_{\text{prop}}, & \text{side} \end{cases}$$  \hspace{1cm} (3.10)$$

where for the combination-fed design $w = w_0 = w_{\text{prop}}$ because of symmetry, i.e. all three propellant bars should be of equal volume and dimensions to ensure uniform mass distribution and ablation of the propellant surface. In the side-fed design we also require the two propellant bars to be symmetric, but in general $w_0 \neq w_{\text{prop}}$. The initial total volume of all propellant stored onboard the satellite is

$$V_{\text{tot}} = \frac{m}{\rho_{\text{Teflon}}} = n V_{\text{bar}}$$  \hspace{1cm} (3.11)$$

where $n=1, 2, 3$ for the breech-, side- and combination-fed configurations respectively.

By combining equation (3.10) and equation (3.11) it can be concluded that

$$l_{\text{prop}} = \frac{m}{n \rho_{\text{Teflon}} h_0 w}$$  \hspace{1cm} (3.12)$$

Thus, in the breech- and combination-fed configurations where $w = w_0$, the length of the propellant bar can be expressed directly in known quantities and components of the vector of design variables, $x$, if it is assumed that the mass of the total propellant stored onboard, $m$, is known. Thus, the propellant width will be set automatically by the optimization.

Conversely, in the case of side-feeding $w = w_{\text{prop}}$, which makes the optimization problem more complicated. In this case, the propellant bar length will be limited by the thruster maximum width and in addition, the electrode width and allowing enough space for the propellant feeding spring must be considered. The height of the propellant bar will be set by the electrode spacing, which leaves the propellant bar width as the critical parameter for determining the maximum propellant mass and the adjustable parameter to set the propellant surface exposed to the discharge and hence also the $E_0/A$ ratio. For this reason, in the case of side-feeding, the propellant width has been left as a variable that needs to be set manually prior to performing the optimization.

### Electrode thickness

As is evident from Table 3.1, the maximum thruster height is set by the value of the electrode thickness, which must be chosen with the aim of keeping the mass low
and the electrical resistance low enough to avoid losses while maintaining enough mechanical strength and durability. The electrodes must also be sufficiently robust to prevent severe corrosion, as thinner electrodes are more susceptible to erosion caused by the sparking of the igniter, which is a life limiting factor for PPTs. Further, an increase in inductance gradient, which is related to the efficiency, is associated with a decrease in electrode cross-sectional area and thereby affected by the electrode thickness. However, the relative gains for implementing thin electrodes are negligible for both parallel and flared geometries [11] [12].

Because of these special requirements on the electrode thickness, it is not part of the optimization variables and will instead be set manually by considerations for each specific design. The fact that the anode will be ∼1 mm thinner than the cathode in some areas, because of the incision that allows for propellant blocking on the electrode shoulder as part of the feeding mechanism, must also be taken into account.

**Circuit parameters**

The required breakdown voltage for the arc formation across the Teflon surface is around 1.4 kV DC and the design is such that the igniter will continue to spark until the voltage across the electrodes reaches this value and will then start the discharge. Because of the relation between the capacitance $C$ and resulting energy (2.36), the following constraint is imposed on these quantities based on the mission requirements (see Table 1.1)

$$V = \sqrt{\frac{2E_0}{C}} = 1.4 \text{ kV}$$

(3.13)

Because of the limited available power on-board miniaturized satellites, the initial capacitor energy should be in the joule level, which means that the capacitance must be of the order $10^{-6}$ F in order to produce a voltage of the above magnitude. For the VELOX nanosatellite, the discharge energy is typically 2 J, which requires the capacitance to be 2.1 µF.

In previous studies on micro-PPT development, the values of the overall inductance and resistance are in the order of hundreds of nH and tens of mΩ respectively [11] and based on this and tuning to meet the desired thrust level, the values presented in Table 3.2 were used in this investigation.

<table>
<thead>
<tr>
<th>Pulse frequency $f$</th>
<th>Discharge energy $E$</th>
<th>Capacitance $C$</th>
<th>Initial inductance $L_0$</th>
<th>Resistance $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Hz</td>
<td>2 J</td>
<td>2.1 µF</td>
<td>30 nH</td>
<td>40 mΩ</td>
</tr>
</tbody>
</table>

However, the optimization will allow for easy adjustment of the electrical input so that other solutions can readily be explored.
3.1. Optimization model

The remaining fix variables are set according to the values presented in Table 3.3.

Table 3.3: Fix parameter settings.

<table>
<thead>
<tr>
<th>Aspect ratio range</th>
<th>Energy density range</th>
<th>Electrode thickness $t$</th>
<th>Propellant width $w_{\text{prop}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5-2</td>
<td>1-5 J/cm$^2$</td>
<td>2 mm</td>
<td>10 mm</td>
</tr>
</tbody>
</table>

The ranges presented here have been chosen with care to the particular circumstances for VELOX as well as to previous optimization results found in the literature. By specifying the ranges for the aspect ratio and $E_0/A$ ratio, a more narrow interval for the results to fall in between can be chosen. In this study, relatively large intervals have been selected in order to investigate which boundary the optimization will favor. The $E_0/A$ ratio has been set generously as, in terms of influencing the optimization outcome, it only affects the dimensions of the propellant bars.

3.1.6 Computational implementation

In summary, the parameters to be specified manually prior to performing the optimization are:

- Total propellant mass $m$
- Expected thruster lifetime $\tau$
- Frequency $f$
- Capacitance $C$
- Discharge energy $E_0$
- Total initial inductance $L_0$
- Total initial resistance $R$
- Base electrode thickness $t$
- Aspect ratio range $a_{\text{min}}$ and $a_{\text{max}}$
- Energy density range $(E_0/A)_{\text{min}}$ and $(E_0/A)_{\text{max}}$
- Time span for differential equation solver $t_{\text{optim}}$
- Propellant width $w_{\text{prop}}$ in case of side-feeding
- Propellant feeding type: breech-, side- or combination-fed
- Electrode shape: rectangular or tongue-shaped
Specifying the above parameters will result in optimized values of:

- Base electrode spacing and propellant bar height, $h_0$
- Base electrode width $w_0$
- Electrode length $l$ (found through constraints)
- Base aspect ratio $a = \frac{h_0}{w_0}$
- Propellant area exposed to the discharge $A = nh_0w$
- Energy density $\frac{E_0}{A} = \frac{E_0}{nh_0w}$
- Propellant length $l_{prop} = \frac{m}{\rho_{Teflon}nh_0w}$
- Flare angle $\alpha$ (for flare geometries)

This was implemented in Matlab by the function script $\text{PPT}\_\text{Optim}$ which takes the fixed parameters specified above as input and delivers optimal values for the geometrical parameters $h_0$, $w_0$ and $l$ based on the objective function by solving the ODE defined by the model in section 2.3. Another script $\text{PPT}\_\text{Optim}\_\text{complex}$ was implemented in order to find values of $h_0$, $w_0$, $l$ and $\alpha$ that maximizes the inductivity for more complex geometries. From the values of the output parameters found by running each script the remaining design variables $a$, $A$, $E_0/A$ and $l_{prop}$ can be calculated from the relations in the list above.

$\text{PPT}\_\text{Optim}$ uses two standard Matlab functions in combination to perform this task: $\text{ode45}$, which is a differential equation solver and $\text{fmincon}$, which finds the minimum of a constrained nonlinear multivariable scalar function starting at an initial estimate. The system (2.27) was treated with $\text{ode45}$ in order to obtain numerical solutions and the objective function (3.1) was maximized using $\text{fmincon}$. $\text{PPT}\_\text{Optim}\_\text{complex}$ is, despite its name, less complex and uses $\text{fmincon}$ only to maximize the integral in the objective function (3.2). The source codes for both scripts can be found in appendix A.1.

Table 3.4: Summary of input and output for the optimization scripts developed in Matlab.

<table>
<thead>
<tr>
<th></th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{PPT}_\text{Optim}$</td>
<td>$m$, $\tau$, $f$, $E$, $C$, $L_0$, $R$, $t$, $w_{prop}$, $l_{optim}$, $a_{min}$, $a_{max}$, $(E_0/A)<em>{min}$, $(E_0/A)</em>{max}$, feed (=1,2,3)</td>
<td>$h_0$, $w_0$, $l$</td>
</tr>
<tr>
<td>$\text{PPT}_\text{Optim}_\text{complex}$</td>
<td>$m$, $E$, $l$, $w_{prop}$, $a_{min}$, $a_{max}$, $(E_0/A)<em>{min}$, $(E_0/A)</em>{max}$, feed (=1,2,3), shape (=1,2)</td>
<td>$h_0$, $w_0$, $l$, $\alpha$</td>
</tr>
</tbody>
</table>
3.2 Design evaluation tool

In order to systematically investigate how variations in the different design variables influence the thruster performance, an evaluation program was designed in Matlab. To make it user friendly and facilitate usage in future PPT optimizations by offering great flexibility to meet different requirements, it was implemented in the form of a graphical user interface (GUI) as seen in Figure 3.3 below.

Figure 3.3: Graphical user interface for evaluation of PPT performance.

The GUI allows the user to study how changes in both the geometrical and electrical design variables affect the performance parameters both numerically and graphically, for different types of feeding and electrode shapes, by incorporating the optimization scripts PPT_Optim and PPT_Optim_complex.

3.2.1 GUI components

The adjustable parameters in the GUI refer to the geometrical and electrical design variables that can be varied either using the corresponding slider or directly entered in the textbox below the slider. These are found in the Geometrical parameters and Electrical parameters panels. All adjustables can be deleted and set to zero and all initial settings restored by the Reset button in the upper left corner.

We distinguish between three different kinds of geometrical adjustables in accordance with the optimization model described in section 3.1. These are listed in Table 3.5 below.
### Table 3.5: Geometrical adjustables

<table>
<thead>
<tr>
<th>Fix</th>
<th>Optimizable</th>
<th>Constraining</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$, $w_{\text{prop}}$, feed, shape</td>
<td>$h, w, l, a, A, \alpha$</td>
<td>$l_{\text{prop}}, E_0/A$</td>
</tr>
</tbody>
</table>

The optimizable adjustables consist of the set of parameters that directly affect the performance of the thruster, i.e. they are either included in the differential equation defined by the model for simple geometries or in the expression for the inductivity in the case of complex geometries. Directly included in this model are $h$ and $w$ and as the exposed propellant area $A$ and aspect ratio $a$ are defined in terms of these quantities they are also part of the optimizable parameters. In the case of complex geometries the flare angle $\alpha$ also belongs to this set. Additionally, the electrode length is optimizable as it defines the exhaust velocity and determines when to terminate the discharge (see equation (3.1)). The propellant length and $E_0/A$ ratio are merely constraining parameters in this model, as their value will have no influence on the performance as evaluated here, but merely serves to constrain the other parameters.

### 3.3 User manual for micro-PPT performance evaluation GUI

Due to the complexity added to the GUI by considering different design configurations related to different optimization models, the use might not be entirely intuitive, which facilitates the need for some instructions on how to best handle the tool. In this section such guidance will be supplied.

#### 3.3.1 Mission requirement settings

The GUI can be used to study the performance of any micro-PPT that fulfills the dimensional requirements that the program is designed for. Apart from such general usage, the toggle button *Use VELOX-I constraints* in the *Mission requirements* panel head offers the option to make the functionality of the GUI specific for VELOX by adjusting the settings to match the particular constraints of this mission.

The user can specify the total propellant mass to be stored on-board the satellite as well as the its expected lifetime. Along with the pulse frequency, which can be adjusted under the *Electrical parameters* panel, this will determine the average mass bit needed as a parameter to solve the equation system defined in (2.27). Clicking the toggle button *Use VELOX-I constraints* once, will automatically set the propellant mass and expected lifetime to correspond to the VELOX requirements \{$m=30 \text{ g}, \tau=6 \text{ months}$\}. Another click on this button will deactivate this function.
3.3.2 Electrical parameter settings

The electrical parameters are not part of the optimization, but needs to be set as input variables in order to perform it. This is done under the Electrical parameters panel.

3.3.3 Geometrical parameter settings

Main settings

Under the main settings panel the user can specify all main criteria for the study in terms of the propellant feeding mechanism and electrode shape, as well as the fix geometrical adjustables that are needed to perform the optimization. These choices modifies the constraints for the optimization and the manual settings and determines how the results should be calculated.

Activating the flare geometry option allows the user to investigate the performance for different flare angles. When this option is activated, or when the 'Tongue' choice is made under the Electrode shape popup menu, only the average change in inductance can be calculated under the results panel and investigated under the Graphical study panel, to avoid inaccurate graphs caused by data produced from the model, which is invalid for complex geometries. The propellant width can only be adjusted for side feedings and the slider is disabled for other feeding types.
**Set by optimization or Manual parameter selection**

The user is given two choices to investigate the geometrical parameters of the thruster design. Ticking the radio button *Set by optimization* and specifying the allowed range for the aspect ratio and the $E_0/A$ ratio, in order to define the constraints (see section 3.1.2), will allow for an automatic optimization of the electrode spacing, electrode width and electrode length by clicking the *Optimize* pushbutton under the *Simple geometry* panel. This will call the function PPT.Optim for the specified electrical parameters and the values inputted in the *Main settings* section. The *Optimize* pushbutton under the *Complex geometry* panel will instead call the function PPT.Optim_complex and in addition to returning optimized values of $h, w$ and $l$, also give an optimized value of $\alpha$. In order to include the value of $\alpha$ in any subsequent calculations or graphical studies, the *Use* checkbox under the flare angle adjustable must be ticked.

Figure 3.7: *Set by optimization* selection.
Both optimizations will automatically generate values for the aspect ratio, propellant length, exposed propellant area and \( E/A \) ratio, calculated from the generated values of the electrode spacing, electrode width and electrode length. When pushing any of the *Optimize* pushbuttons an information dialog will be displayed as long as the optimization is running. Once it has stopped, another information dialog box is displayed notifying the user of the optimization outcome, i.e. whether the function converges to a solution or not during the maximum number of function evaluations.

As previously mentioned, the optimization model used by \textit{PPT Optim} is not valid for complex electrode geometries. Meanwhile, the GUI has been designed in such a way as to allow to user to perform this optimization under such circumstances as well. However, for any attempt at such an optimization, a message will be displayed to notify the user of the decrease in validity for these kind of electrode geometries as the optimized parameters may not be accurate. Note that for this type of usage, the flare angle and the electrode shape will only help modifying the constraints limiting the size of the thruster as compared to parallel, rectangular electrodes according to Table 3.1.

The *Plot results* pushbuttons appearing under the *Simple geometry* panel displays the position, velocity and acceleration of the plasma sheet as function of time under the *Graphical study* panel in order to allow the user to easily study the optimization outcome in terms of the solution to the differential equation defined by the model. A red dot is also added to each graph to see when the end of the electrodes is reached.

Figure 3.8: Plotted results from the the optimization or manual variable selection.

The GUI also offers the option to manually specify all geometrical parameters and study the results, without using the optimization. This is done under the
**Chapter 3. Optimization**

Manual parameter selection section by first ticking the radio button at the top of this panel, which gives the user the option to freely choose the desired value of the parameters under study within the range specified under each corresponding slider. While the optimization model is specific for VELOX, the manual selection can be used to evaluate the performance of a general micro-PPT of the same dimensional order.

Firstly the user needs to choose among the primary parameters (primaries): electrode spacing \( h \), electrode width \( w \), aspect ratio \( a \) and exposed propellant area \( A \) by ticking the corresponding radio button. These are the parameters directly affecting the thruster performance as they are part of the model found in section 2.3. In addition, they directly depend on each other according to

\[
\begin{align*}
h &= wa = \frac{A}{nw} = \sqrt{\frac{aA}{n}}, \\
w &= \frac{h}{a} = \frac{A}{nh} = \sqrt{\frac{A}{na}} \\
a &= \frac{h}{w} = \frac{nh^2}{A} = \frac{A}{n^2w^2}, \\
A &= nhw = n\frac{h^2}{a} = nw^2a
\end{align*}
\]  

and therefore they cannot all be specified independently simultaneously. Thus, any two chosen parameters will be mutually exclusive to the other remaining two and it is up to the preference of the user which should be specified. Consequently, as the user selects two primaries, the other two will be disabled and fixed by the values of the selected ones. As long as any two primaries are selected by the radio buttons any adjustments in these will set the other two directly and will also limit the constraint parameters \( l, E_0, A \) and \( l_{prop} \) found to the right of the Primary parameters panel. The constraint parameters will therefore be disabled until the user specifies two primaries.

In contrast to the primaries, the constraint parameters do not directly affect the PPT performance, but instead constrains or are constrained by the primary parameter settings. The manual selection panel allows the user to rank the importance of the parameters to be specified in order of preference for a particular design, since by adjusting the most important the others will automatically follow. A *Plot results* pushbutton to study the outcome is also available under this panel.

The boundaries defined in Table 1.2 are implemented in the GUI and a warning dialog is displayed when the user uses any input parameters that violate them. As soon as one parameter in the Manual parameter selection panel is updated by the user, either the current values or the maximum values of all other parameters appearing under this panel will also be updated accordingly. What is updated depends on whether the options to tick the *Use maximum volume* checkbox has been used, which helps the user adjust the parameters in such a way that all the allocated thruster volume capacity is ensured to be used up. Not ticking this checkbox instead allows the user the freedom to change the parameters within the constraints for the maximum set by the primaries, as specified in Table 3.1. In this

\[ \text{For side-fed propellant designs where } w_{\text{prop}} \neq w, \text{ these relations will be less straightforward.} \]
case, each time one primary is adjusted, all maximums of the other parameters are changed accordingly.

Figure 3.9: Manual parameter selection panel.

3.3.4 Calculation of results

Under the Results panel the calculation of the performance parameters can be handled. In the default mode the Calculate pushbutton calculates the average thrust, exhaust velocity, impulse bit, specific impulse, inductance gradient and thruster efficiency according to the model specified in section 2.3.2 and displays them in the Results panel. Exceptions to this occur when the Activate flared geometry radio button is chosen, the 'Tongue' choice selected in the Electrode shape pop-up menu, or the Use checkbox under the Complex geometry is checked. Under such conditions this button will instead calculate the average change in inductance, as the model for calculating the other performance parameters is not valid for these geometries. Any attempt to numerically evaluate the performance for for flare, non-rectangular geometries is met with a messages warning about the inaccuracy in the results of such calculations. The default time span to be used by the differential equation solver is set to 10 µs, but it can be changed under Time span.
Figure 3.10: Calculation of results for simple electrode geometries.

Figure 3.11: Calculation of results for complex electrode geometries.

3.3.5 Printing of results

The GUI offers the option to save all data neatly in a textfile by using the *Print* pushbutton in the upper left corner, which writes all parameter values to a textfile.
with a name specified by the user, which is then saved in the current directory.

Figure 3.12: Save datafile with all adjustable parameters and resulting performance characteristics.

Figure 3.13: Data saved as textfile in current directory.
3.3.6 Graphical study

In the Graphical study panel any of the performance parameters can be chosen from the Dependent variable pop-up menu and studied as function of an adjustable to be specified in the Independent variable pop-up menu. The range of the adjustable is specified below the latter menu and the resulting graph can be studied in either of the three plot windows. To be able to temporarily save the plots on the screen and compare with new ones, the Activate history checkbox can be ticked. The Clear pushbutton appearing above each plot window can be used to erase all graphs from that window and the Save pushbutton can be used to save the plots to the current directory as high resolution Portable Network Graphics files.

Figure 3.14: Plotted data for different independent and dependent parameters to assist in the performance evaluation.

Figure 3.15: Activate history to facilitate comparison in performance when changing parameter values.
Chapter 4

Result analysis

In this chapter the results from the design evaluation tool are used to explore the established analytical models and to further develop theoretical reasoning found in the literature. The chapter has been divided into two major parts, where the first concerns simple electrode designs for which all performance parameters can be determined from the differential equation model (2.27) and in this part a comparison between the different propellant feeding types is conducted. In the second part of the chapter, flared non-rectangular electrode designs are instead analyzed in greater detail, without regards to the propellant feeding type. Since for such complex geometries only the average change in inductance can be calculated, this quantity will be the base of our analysis.

Unless otherwise stated, all data displayed in the graphs result from calculations using the values of the fix parameters showed in Table 3.2, Table 4.1, Table 4.3 and Table 4.5.

4.1 Simple electrode geometry

4.1.1 Propellant feeding mechanism and propellant dimensions

There are several reasons to implement multi-directional feeding mechanisms, depending on the particular mission at hand. The purpose for increasing the number of exposed propellant surface areas can for instance be to

1. increase the total amount of propellant to extend the lifetime of the satellite or to obtain a larger exposed propellant area for an increased mass bit

2. maintain the same amount of propellant while expanding the exposed propellant area, meaning that the length of the propellant bars can be shortened and more conveniently positioned for a compact structure
3. obtain a more uniform ablation by maintaining the same exposed propellant area divided into several smaller surfaces (increase $V_A/A$ ratio, see below)

The design evaluation tool is based on case number 2, where the propellant mass remains fixed for all three feedings under investigation, so in this case the aim of the multi-directional feeding would primarily be to increase the amount of exposed propellant surface area. For a certain electrode spacing $h$, electrode width $w$ and propellant width $w_{prop}$ we have

$$A = \begin{cases} 
hw & \text{for breech feeding} \\
2hw_{prop} & \text{for side feeding} \\
3hw & \text{for combination feeding}
\end{cases} \quad (4.1)$$

where in the case of the combination-fed design we assume $w = w_{prop}$ for simplicity and symmetry reasons, leading to uniform mass ablation.\(^1\) For such fix values of $h$ and $w$, feeding the propellant from several sides will increases the surface area the arc covers as the total propellant surface area is the sum of all surfaces exposed to the discharge spark. Increasing the exposed area will cause a larger amount of propellant to be ionized with each pulse, which will possibly increase the electromagnetic thrust component. However, as the volume near the propellant surface will also increase, the pressure will decrease accordingly and thereby reduce the electrothermal thrust contribution instead. Meanwhile, because the pressure will also increase as more particles are released and because the electromagnetic acceleration contributes to the thrust production with a significantly higher amount compared to the electrothermal thrust component, increasing the propellant surface area $A$ may likely result in larger efficiency gains than reducing the volume of the acceleration channel in the immediate vicinity of the propellant surface $V_A$.

The results of the investigation of the three different feeding types based on the analytical model are displayed in the subsequent pages. The values of the performance parameters only differ slightly between the feeding types, with the multi-directional feeding designs being marginally superior to the breech-fed with regards to the level of performance parameters. In terms of the design parameters, the largest difference is seen in the exposed propellant area and propellant length, which are increased and decreased respectively with the number of propellant bars. This behavior is in accordance with how the optimization problem has been formulated according to case 2 above, assuming an equal amount of propellant for each feeding type. If we would instead assume case 3, where $A$ is kept constant, the $V_A/A$ ratio would decrease with the number of exposed surface areas, i.e. be lowest for combination feeding and highest for breech feeding. A low $V_A/A$ ratio would

\(^1\)Otherwise we would have $A = hw + 2hw_{prop}$ for combination feeding.
likely enhance the total thrust production by increasing both the electrothermal and electromagnetic thrust components.

However, an increase in exposed propellant surface area must be accompanied by an increase in discharge energy in order to maintain a high $E_0/A$ ratio. This energy density parameter is important to achieve a high specific impulse, something that is clarified in section 4.1.3, and there is a minimum ratio below which the propellant may not be ablated at all. For the VELOX system, the limited available power constrains the discharge energy to a maximum of 2 J, which means that the exposed propellant area cannot be too large if the $E_0/A$ ratio should be kept at a sufficiently high level for proper mass ablation. Therefore, increasing the number of propellant surfaces exposed to the discharge at a fixed amount of propellant mass may decrease the efficiency and constitutes a disadvantage of the implementation of multi-directional feeding types. As confirmed by the simulation program, when compared to side-fed and combination-fed designs, the breech-fed design allows for a higher value of the $E_0/A$ ratio, while maintaining a high aspect ratio.

Further, a combination-fed design is also accompanied by disadvantages such as additional feeding springs that would decrease the available thruster length as compared to side-fed designs, meaning that the electrodes would be shortened, which may influence the performance negatively. Meanwhile, side-fed PPT versions have the benefit of reducing the electrical connections, as the capacitor bank can be connected directly at the breech of the thruster, which is not possible for breech- and combination-fed versions. This increases the efficiency by minimizing the initial inductance and thereby reducing parasitic losses from the energy storage system resulting in maximized energy transfer from the capacitor to the thruster discharge. While this can also be achieved by increasing the capacitance of the capacitor bank in order to keep the inductance low, that would also mean that heavier capacitors would have to be used, which would further add to the mass of the thruster. However, since the mission requirement for VELOX does not specify a maximum mass, that may not be an issue in this particular case.

The type of feeding also influences the mass ablation rate. In the breech-fed design, the $\mathbf{j} \times \mathbf{B}$ force vector points away from the propellant surface so that when the discharge occurs it will move off the surface, reducing the ablation rate. Conversely, in side-fed PPT versions, the $\mathbf{j} \times \mathbf{B}$ force vector is instead parallel to the propellant surface so that the magnetic field lines are normal to it, which provides a channel for the electrons to reach the propellant for sustained ablation rate [3]. Any possible consequences of the combination of directions of this force vector when the two types of feedings are used simultaneously in the combination-fed design is not known at present, since it has not been implemented in any previous PPT versions. Nevertheless, one possible issue could be that the ablation rate at the side areas could exceed that of the breech area, which would lead to non-uniformities in the ablation process. Such an hypothesis must be verified experimentally and if proven to be correct, the issue could be solved by dimensioning the propellant bars to balance out the differences in ablation rate.
Breech feeding

Figure 4.1: Plasma sheet velocity and acceleration for the breech-fed propellant feeding mechanism. The red dotted line marks the end of the electrodes.

Table 4.1: Optimized design parameters for the breech-fed mechanism.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>19.7433 mm</td>
</tr>
<tr>
<td>w</td>
<td>9.8717 mm</td>
</tr>
<tr>
<td>a</td>
<td>(2)</td>
</tr>
<tr>
<td>A</td>
<td>(1.949 cm²)</td>
</tr>
<tr>
<td>l</td>
<td>20.0173 mm</td>
</tr>
<tr>
<td>l_{prop}</td>
<td>69.9663 mm</td>
</tr>
<tr>
<td>E₀/A</td>
<td>1.0262 J/cm²</td>
</tr>
</tbody>
</table>

Table 4.2: Performance parameters resulting from the optimization for the breech-fed mechanism.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>47.4455 µN</td>
</tr>
<tr>
<td>vₑ</td>
<td>12298.3507 ms⁻¹</td>
</tr>
<tr>
<td>I_{bit}</td>
<td>47.4455 µNs</td>
</tr>
<tr>
<td>I_{sp}</td>
<td>1254.9337 s</td>
</tr>
<tr>
<td>L'</td>
<td>2.5133 nH(mm)⁻¹</td>
</tr>
<tr>
<td>η</td>
<td>14.5881 %</td>
</tr>
</tbody>
</table>

Figure 4.2: CAD drawing of the breech-fed results from the optimization.
4.1. Simple electrode geometry

Side feeding

Figure 4.3: Plasma sheet velocity and acceleration for the side-fed propellant feeding mechanism. The red dotted line marks the end of the electrodes.

Table 4.3: Optimized design parameters for the side-fed mechanism.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>19.3972 mm</td>
</tr>
<tr>
<td>w</td>
<td>9.6986 mm</td>
</tr>
<tr>
<td>a</td>
<td>(2)</td>
</tr>
<tr>
<td>A</td>
<td>(3.7625 cm²)</td>
</tr>
<tr>
<td>l</td>
<td>33.0575 mm</td>
</tr>
<tr>
<td>l_{prop}</td>
<td>36.2427 mm</td>
</tr>
<tr>
<td>E₀/A</td>
<td>0.53156 J/cm²</td>
</tr>
</tbody>
</table>

Table 4.4: Performance parameters resulting from the optimization for the side-fed mechanism.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>49.8925 µN</td>
</tr>
<tr>
<td>vₑ</td>
<td>12932.9637 ms⁻¹</td>
</tr>
<tr>
<td>I_{bit}</td>
<td>49.8925 µNs</td>
</tr>
<tr>
<td>I_{sp}</td>
<td>1319.6902 s</td>
</tr>
<tr>
<td>L'</td>
<td>2.5133 nH(mm)⁻¹</td>
</tr>
<tr>
<td>η</td>
<td>16.1325 %</td>
</tr>
</tbody>
</table>

Figure 4.4: CAD drawing of the side-fed results from the optimization.
Combinational feeding

Figure 4.5: Plasma sheet velocity and acceleration for the combinational-fed propellant feeding mechanism. The red dotted line marks the end of the electrodes.

Table 4.5: Optimized design parameters for the combinational-fed mechanism.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>15.8919 mm</td>
</tr>
<tr>
<td>w</td>
<td>7.946 mm</td>
</tr>
<tr>
<td>a</td>
<td>(2)</td>
</tr>
<tr>
<td>A</td>
<td>(3.7883 cm$^2$)</td>
</tr>
<tr>
<td>l</td>
<td>29.2768 mm</td>
</tr>
<tr>
<td>$l_{\text{prop}}$</td>
<td>35.9959 mm</td>
</tr>
<tr>
<td>$E_0/A$</td>
<td>0.52794 J/cm$^2$</td>
</tr>
</tbody>
</table>

Table 4.6: Performance parameters resulting from the optimization for the combination-fed mechanism.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>49.6754 $\mu$N</td>
</tr>
<tr>
<td>$v_e$</td>
<td>12876.7233 ms$^{-1}$</td>
</tr>
<tr>
<td>$I_{\text{bit}}$</td>
<td>49.6754 $\mu$Ns</td>
</tr>
<tr>
<td>$I_{\text{sp}}$</td>
<td>1313.9514 s</td>
</tr>
<tr>
<td>$L'$</td>
<td>2.5133 nH(mm)$^{-1}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>15.9925 %</td>
</tr>
</tbody>
</table>

Figure 4.6: CAD drawing of the combination-fed results from the optimization.
4.1. Simple electrode geometry

4.1.2 Electrode dimensions

The electromagnetic acceleration contributes substantially to the thrust and can be enhanced in order to improve the efficiency. In this section we discuss methods for such improvements based on geometrical considerations for the thruster electrodes.

**Electrode length**

When we refer to the electrode length we mean the total length available for the plasma particles to be accelerated, which is here defined as the distance from the outermost edge of the propellant to the electrode tip at the thruster exit (see Figure 1.6). The electrode length is directly related to the electromagnetic efficiency component through the change in inductance, which indicates the crucial importance of the electrode length for an optimal design. For parallel electrodes we have (cf. equation (2.41))

$$\eta_{\text{max}} = \frac{\Delta L}{L_0} = \frac{L'}{L_0}$$

which would imply that the thruster efficiency would benefit from using longer electrodes. In general, elongating the electrodes will enable continued energy transfer between the capacitor and the current sheet and the longer the acceleration time of the particles within the electric field, the more kinetic energy they will gain.

The theoretical result from the performance evaluation program also displays an increased performance with electrode length, which can be seen in Figure 4.7. Meanwhile, it shows a saturation point beyond which the electrode length has very little effect on the efficiency, thus there must exist an upper bound to the electrode length. In fact, there is a point beyond which the gains may be reversed, which means there is an optimal electrode length for each design [27]. However, there is a lot of ambiguity regarding this matter and various studies have resulted in different conclusion regarding the influence of electrode length on PPT performance [14]. The main studies found on this issue were performed using different discharge energies, and their contradictory results may be caused by a possible correlation between electrode length, discharge energy, and performance, which yet remains unknown. Additionally, some of the differences might be due to variations in thruster geometry, utilized capacitors and thermal effects [11]. Our aim is to use a general reasoning to clarify this matter.

From equation (2.3) it is evident that the Lorentz force will act perpendicular to the direction of motion of the accelerated particles. Because of this, after the electrothermal acceleration this direction will be slightly bent to a circle with gyroradius

$$r_g = \frac{m_q v_\perp}{|q|B}$$

called the Larmor radius. This describes the radius of gyration of circular motion for a charged particle in the presence of a uniform magnetic field $B$, where $m_q$ is the mass of the particle of charge $q$ and $v_\perp$ is the velocity component perpendicular
to the direction of the magnetic field. The magnitude of the velocity will increase because of impact from the electric field and the direction will be changed by the magnetic field, and since the strengths of these fields will vary with time so will the Larmor radius. When the Larmor radius is smaller than the electrode length, the direction of motion of the particles will be reversed, causing mass loss that will weaken the thrust. Another effect of using two long electrodes is that the friction with the surrounding sidewalls will eventually slow down the propellant and also the discharge may not reach the end of the electrodes by the time the capacitor is fully drained. In addition, this would cause unaccelerated propellant to be stationary in front of the oncoming current sheet, which would block the exhaust stream and reduce the efficiency \([18]\). On the other hand, if the electrodes are too short, there will not be sufficient time for the particles to reach high velocities, which leads to a low accelerating effect. Further, the propellant may be expelled when there is still energy stored in the capacitors, which may decrease the efficiency. \([18]\)

(a) Specific impulse vs. electrode length

(b) Thruster efficiency vs. electrode length

![Graphs showing specific impulse and thruster efficiency vs. electrode length for different feeding types.](image)

Figure 4.7: Influence of electrode length on performance parameters for different feeding types.

It section 3.1.1, the importance of the electrode length for determining the optimal point of discharge termination has already been discussed briefly, as it is incorporated into the optimization objective function. As we have seen, the electrode length determines where the exhaust velocity is to be measured and as the velocity of the plasma sheet will saturate after the current peaks, as soon as the saturation point is reached the discharge should be terminated to avoid energy losses. As a matter of fact, the PPT efficiency will most likely reach its maximum before the capacitor is fully discharged. Current will still flow even after the maximum efficiency has been reached, but the current sheet will slow down due to the Lorentz force being smaller than the drag force from the mass accumulation. After the time of maximal efficiency, any extra energy the current sheet might gain from propagating further down the electrodes will be cancelled out by the decrease in efficiency. Therefore, in terms of directed kinetic energy, it would be beneficial to terminate the discharge and reduce the electrode length to
match the position of the current sheet at the time when the efficiency is optimal, thereby ending the mass accumulation and further energy addition [27]. The region of suitable electrode length is depicted in Figure 3.1, where it corresponds to the highlighted area marking the time where the discharge should be terminated for optimal efficiency.

In figure 4.8 we see a comparison between the average thrust and the instantaneous thrust. The instantaneous thrust describes the force generated by the acceleration of the plasma mass during each unit of time and corresponds to the electromagnetic thrust component. The average thrust is simply the the instantaneous thrust averaged over the duration of the discharge pulse. As the plasma sheet acceleration is determined by the current generated by the capacitor discharge, by looking at Figure 4.8 it can be realized that a design with too short electrodes, where the capacitor has not been completely discharge when the plasma reaches the end of the channel, corresponds to a scenario in which the peak of the blue curve has not been overcome so that the dot that marks the electrode length will be placed somewhere along the current barrier. Too long electrodes would instead mean that this point would be well past the thrust saturation point, where the green curves flattens in Figure 4.8.

Figure 4.8: Thrust vs. electrode length for different feeding types. The black dashed line represents the minimum thrust level defined by the mission requirements and the colored dots indicate the electrode length for each feeding type.

The results of the evaluation program show that the optimization has managed to find a value of the electrode length where the saturation point of the average thrust is just above 45 $\mu$N for all types of feedings considered, which is the minimum mission requirement for the thrust production. For each feeding type this thrust level is obtained before reaching the end of the electrodes as determined by the optimization, but without maintaining it for too long which avoids losses. Further, as can be seen in the graphs in Figure 4.1, 4.3 and 4.5, the electrode length is close
to optimal as the discharge is terminated as soon as the highest exhaust velocity is reached, as specified by the objective function. Such an optimal value can be found for each specific PPT design and needs to be experimentally verified for a complete optimization. In the theoretical case, the resistance and inductance can be tuned in the simulation program to obtain the desired thrust level.

**Interelectrode spacing, electrode width and their relative aspect ratio**

The interelectrode aspect ratio is defined as the ratio of the electrode spacing, $h$, to the width of the electrodes, $w$. While current research shows contradictory results regarding the the influence of various aspects of the electrode geometry on the specific impulse and thrust-to-power ratio, the aspect ratio is in general considered to be directly related to the thruster performance through

$$\frac{h}{w} \propto \frac{T}{P}$$

meaning that by increasing the aspect ratio at a given power level an increase in the generated thrust will be achieved. The analytical model confirms such an increase in performance with higher values of the aspect ratio as can be seen in Figure 4.9.

![Figure 4.9: Aspect ratio influence on performance parameters. The black dashed line defines the limits set by the mission requirements.](image)
4.1. Simple electrode geometry

For all different feeding types, the optimization also settled at the highest possible aspect ratio in the specified range, in this case $a = 2$. Since the aspect ratio is directly proportional to most of the performance parameters, these will therefore differ minimally between the feeding types, despite the fact that the differences in other, less influential, design variables are larger. Even with a wider aspect ratio range than what has been used in this study, starting from $w >> h$, higher aspect ratios will be favored by the optimization. In previous micro-PPT designs using discharge energies in the joule level, $h$ is normally of the order of one to some centimeters [26] and the optimization gives values of $h$ around 1.5-2 cm for all feeding types, as the value tends to go towards the maximum to fulfill the objectives. This is in accordance with theoretical predictions, where all performance parameters will reach a higher value with higher aspect ratios. However, this straightforward relation does not uncover the entire situation, for instance it does not provide an upper bound to the aspect ratio.

By studying the definitions of the parameters related to the thruster performance (see section 2.3.2) it can be realized that for parallel electrode geometries all of these increase with the aspect ratio through its relation to the inductance gradient. Meanwhile, an extensive value of the aspect ratio could cause non-uniformities to arise in the electromagnetic field, which would reduce the acceleration process efficiency. In addition, as confirmed by experiments, increasing the electrode separation at constant electrode width results in an increase of the system resistance and inductance, while the peak current decreases [12]. Since the resistance of the plasma is inversely proportional to the current integral, an increased resistance will reduce the electromagnetic impulse bit

$$I_{\text{bit}} = \frac{L'}{2} \int [i(t)]^2 dt$$

Furthermore, the choice of the electrode spacing $h$ is not only determined by the inductance per unit length parameter but also by the available energy and by the capacitor voltage. An increase in $h$ will result in a decrease in the electric field between the electrodes, hence an excessive value of $h$ might negatively influence the performance.

Moreover, a high aspect ratio means that the propellant bar height will be large compared to its width as it must also be dimensioned in accordance, and as the spark plug is located in the cathode this may lead to an increased problem of non-uniform ablation of the propellant, since the distance between the points farthest and closest to the discharge initiation will increase.

Additionally, increasing the interelectrode spacing gives rise to an increase in the exposed propellant area and as a consequence, the quantity of ablated particles will increase thus enhancing the pressure. The increased pressure will cause an increase in the gas-expanding force if the chamber volume is kept constant, while an increased spacing accompanied by an increased chamber volume will instead decrease the pressure which will weaken the gas-expanding force. Therefore, the
electrode spacing has an optimal value and must be balanced between the ablated particle quantity and the chamber volume for maximum efficiency.

Figure 4.10: Influence of interelectrode spacing and electrode width on performance parameters.

![Graphs showing influence of interelectrode spacing and electrode width on performance parameters.](image)

Similar to the electrode spacing, increasing the electrode width leads to an increase in the exposed propellant surface area as well as the chamber volume. The increase in the exposed area will increase the quantity of particles at discharge and the pressure will be adjusted depending on the variableness of the chamber volume. Consequently, there is also an optimal electrode width for each PPT design.

Figure 4.11: Influence of electrode geometry on specific impulse. The colored dots indicate the point where the plasma sheet has reached the end of the electrodes for each feeding type respectively.

![Graphs showing specific impulse vs. interelectrode spacing and electrode width.](image)

Figure 4.11 makes it evident that, in theory, the specific impulse is monotonically increasing with interelectrode spacing. Moreover, experimental data from standard-size PPTs has indicated a monotonic trend toward lower thrust-to-power ratio and
higher specific impulse as the electrode spacing is increased. In fact, the increase in thrust-to-power ratio afforded by decreasing $h$ is accompanied by a much stronger drop in specific impulse. The same studies shows that the efficiency will have a maximum value at a certain spacing after which it will quickly decrease and that as $h$ was decreased the discharge became more oscillatory. Conversely, other experimental studies indicate that increasing the distance between the electrodes will in fact decrease the exhaust velocity, while increasing the impulse bit [8].

Figure 4.12: 3D-plots of the exhaust velocity as function of various primaries.

As is illustrated in Fig. 4.12, the exhaust velocity is increasing with aspect ratio irrespective of electrode length. According to experimental studies, changes in PPT performance are not solely determined by the propellant surface area or aspect ratio, but also of how the aspect ratio is obtained. For configurations where aspect ratio may influence performance by maximizing the accelerating capability of the electrodes, thereby decreasing the energy required to produce a given impulse bit, increases in electrode separation rather than decreases in electrode width produce enhanced performance for the same propellant surface area [12]. This behavior may arise due to the increase in inductance gradient being associated with an increase in electrode separation. As can be seen in Figure 4.10, the thruster efficiency peaks for a certain electrode width in the lower ranges. Consequently, increasing interelectrode separation at such an optimal width is a good method to achieve enhanced performance. However, because of the dimensional constraints regarding the thruster height imposed on the PPT design for this particular mission, a decrease in electrode width may be more feasible in order to maintain a high aspect ratio.

4.1.3 $E_0/A$ ratio

The thruster performance cannot be assessed only in terms of the electrode configuration, the ratio of the discharge energy to exposed propellant surface area has also been shown to be a critical design parameter that should be included in the performance evaluation.

The discharge energy is by itself directly related to the thruster performance. For instance, a gain in discharge energy corresponds to an increase in discharge
current as well as in magnitudes of the electric and magnetic fields, which would enhance the electromagnetic acceleration. Further, numerous experimental observations confirm an essentially linear relationship between discharge energy and thrust as well as specific impulse, impulse bit and mass loss per pulse due to the fact that the plasma electron density is proportional to the discharge energy. [14][18]

The propellant surface area also plays a critical role in determining the thruster performance. Previous studies have shown that decreasing the propellant surface area exposed to the discharge can improve the thrust-to-power ratio [19]. For miniaturized PPTs, the energy density increases as this ratio is increased, i.e. as the surface area is decreased for a given discharge energy. Since the ratio of energy to ablated mass is directly proportional to the $E_0/A$ ratio, the increased energy density that follows from miniaturization can be a possible advantage by improving the propellant utilization efficiency. Meanwhile, an increase in energy density will result in higher thruster operating temperature, thereby increasing late-time ablation, a factor which may reduce the efficiency of micro-PPTs as compared to larger models. In addition, it may also cause charring of the propellant, as too high energy density can lead to excessive sparking [1]. In constrast, studies have shown that an increased energy density can actually prevent charring of the propellant surface up to a certain limit, after which no meaningful difference can be obtained [11].

When plotted on a logarithmic scale, the specific impulse as a function of the $E_0/A$ ratio can sufficiently accurately be approximated by a straight line so that

$$I_{sp} = C \left( \frac{E}{A} \right)^D$$

where $C$ and $D$ are constants specific for each individual PPT design, that can easily be obtained by a least-square interpolation of experimental data [22]. In general $C_{side} > C_{breech}$ and $D_{side} < D_{breech}$. However, this semi-empirical relation implies that the same impulse will be produced at constant energy for propellant surfaces of equal area, irrespective of the aspect ratio; something that is contradicted by experimental data. Moreover, increasing the electrode spacing for a fixed electrode width in order to achieve maximum aspect ratio would correspond to minimum energy density when the discharge energy is held constant. Since maximum impulse bit is obtained at the highest aspect ratio, a lower energy input would be required for high aspect ratio configurations in order to achieve higher impulse bits. Thus, while equation (4.6) predicts an increasing specific impulse with $E_0/A$ ratio, such an increase would also be accompanied by a decrease in impulse bit. [12]

One of the main reasons for exposing a larger propellant surface to the discharge spark is to increase the mass bit, which describes the propellant mass ablated during a pulse. This quantity is linear with the impulse bit while being inversely proportional to the specific impulse and as it scales with both exposed propellant area and energy, it is an interesting quantity to look at in order to find an optimum value for the $E_0/A$ ratio. According to experiments, increasing the available propellant surface area leads to an increase in the mass bit and also indicates that
4.1. Simple electrode geometry

the electrode width and the height of the fuel affect the ablation differently [6]. The energy deposited in the mass acceleration is defined by the velocity change per mass bit and for low energies this value reduces significantly with $h$, whereas it remains roughly constant at higher energies. A possible interpretation of this result is that higher mass ablation at lower energies reduces the acceleration energy, while for higher discharge energies there is excess energy at smaller electrode separations and even though mass is ablated, the velocity change for each particle remains approximately the same. The general conclusion is that the thruster efficiency per mass bit is higher for lower discharge energies and lower electrode separations. [8]

Figure 4.13: Effect of increasing the propellant mass on the thruster performance.

(a) Specific impulse vs. propellant mass.  (b) Thruster efficiency vs. propellant mass.

In studies where the propellant surface area has been increased with the aim of investigating the effect of introducing more propellant mass into the discharge while keeping all other parameters constant, the increase in mass per discharge and thereby thrust-to-power ratio lead to a reduction of both the specific impulse and the efficiency for the standard-sized PPT under experiment. This is supported by the results from the analytical model developed here, which are displayed in Figures 4.13 and 4.14. These results suggest that the introduction of more propellant mass, all other parameters held constant, reduces the efficiency at a given discharge energy [14]. This is an interesting result, especially when considering multi-directional propellant feeding, where it could be an issue due to the increased propellant surface area.

Despite numerous studies showing that the specific impulse, and thereby exhaust velocity, increases with $E_0/A$ ratio, when studying a generous energy density range 0.1-5 J/cm$^2$, the optimization carried out by the design evaluation program will favor lower values of the $E_0/A$ ratio. However, this can be explained by the fact that in the evaluation program the $E_0/A$ ratio functions only as a constraint for the geometrical parameters, due to the known experimental findings of its relation to the thruster performance. Nevertheless, the connection between the energy density and thruster performance is not part of the analytical model developed here.
Figure 4.14: Effect of introducing more propellant mass on energy density for different feeding types at various discharge energy levels. The mass increase is done by increasing the area exposed to the discharge while keeping all other parameters constant according to the values in Table 4.1, Table 4.3 and Table 4.5.

4.2 Complex electrode geometries

4.2.1 Electrode shape and flare angle

Several investigations support the conclusion that electrode geometry can largely impact the thruster performance in terms of impulse bit, mean exhaust velocity and efficiency [12][13]. The inductance gradient can be increased by increasing the spacing and decreasing the width of the electrodes along the axial direction resulting in improved acceleration because of the increased amount of energy available to accelerate the plasma.

In addition, tapering electrode width combined with tapering electrode thickness is believed to lead to an even greater increase in inductance further downstream of the acceleration chamber, compared to regions near the propellant surface, implying that tongue-shaped electrodes would contribute to a stronger increase in inductance gradient. Incorporating electrodes of a flared design with the tongue-shaped geometry would thus combine both improvements.

The superior performance that is displayed for the diverging geometries can be explained on the basis of more efficient electromagnetic plasma-sheet acceleration.
through decreased natural plasma expansion and decreased sheet canting, i.e., propagation of the plasma sheet with highly canted angles relative to the electrode normal. Furthermore, the nozzle effect of flared electrodes will result in a reduced plasma density that may benefit electromagnetic acceleration for some PPT designs [3].

In support of this, experiments confirm that increasing the total flare angle by 20° results in a typical gain in impulse bit of 45% when the operating parameters are held constant [12]. Nevertheless, similar to the other design variables, care must be taken as for each design there exists an optimal flare angle beyond which negative effects may outweigh the benefits. Several previous studies confirm the increment of impulse bit and exhaust velocity when the flare angle is increased from 0° to 20°, but declare a decrease for further increases from 20° to 30° [6] [23]. Other studies show that the positive effects may even be reduced for flare angles beyond 10° [8].

Additional observations indicate that, compared to electrodes of simple geometries, tongue-shape electrodes give higher impulse bits, higher mean exhaust velocity, higher efficiency and lower mass bits which means higher specific impulse under the same operating conditions [13] [14]. According to further studies, the coupled impact of tongue-shaped and flared electrodes was found to significantly improve the impulse bit with as much as 42% compared to a rectangular parallel electrode configuration [7].

There is also a significant difference in thruster performance between rectangular and tongue-shaped electrodes when compared at the same flare angle. Present modelling indicates that the implementation of tongue-shaped electrodes would enhance the performance beyond improvements achieved with the use of rectangular flared electrodes alone and typically doubles the observed impulse bit produced as well as having the benefit of improving the propellant utilization. In accordance with this theory, it is evident from Table 4.7 that implementing tongue-shaped electrodes will increase the inductivity by a significant amount as compared to rectangular electrodes. In fact, for the breech-fed design the optimization shows an increase in inductivity of 48% between rectangular and tongue-shaped electrodes, while the combination-fed option shows a slightly more humble increase of 43%. In the case of combination feeding the inductivity is improved by as much 56% by implementing tongue-shaped electrodes. This holds even though the optimization yields a flare angle of roughly the same value for both the rectangular end tongue-shaped geometry when comparing each feeding type.

According to the results from the evaluation program, the combination-fed tongue-shaped design is also the one that will result in the highest inductivity under the given conditions. However, the results for the side-fed version relies on the manual settings of the propellant width, causing inaccuracies in the comparison. The realization of the complex design that results in the highest inductivity, in terms of the electrode dimensions and propellant bar dimensions specified in the green column in Table 4.7, is displayed in Figure 4.15.
Table 4.7: Results of optimization for different test configurations. The green column marks the configuration that gives the highest inductivity.

<table>
<thead>
<tr>
<th></th>
<th>Rectangular electrodes</th>
<th></th>
<th>Tongue-shaped electrodes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>1.0964</td>
<td>1.8367</td>
<td>1.5586</td>
<td>1.0825</td>
</tr>
<tr>
<td>$l_{prop}$ [mm]</td>
<td>70.8833</td>
<td>32.3254</td>
<td>34.7568</td>
<td>69.692</td>
</tr>
<tr>
<td>$E_0/A$ [J/cm$^2$]</td>
<td>1.0396</td>
<td>0.54741</td>
<td>0.50977</td>
<td>1.0221</td>
</tr>
<tr>
<td>$\Delta L$ [nH]</td>
<td>17.2376</td>
<td>29.3213</td>
<td>28.4795</td>
<td>25.5557</td>
</tr>
</tbody>
</table>

Figure 4.15: CAD drawing of a combination-fed, tongue-shaped PPT dimensioned according to the optimization outcome and with a flare angle of 12.2°.

The integral (2.50) was also evaluated numerically for a wider range of values of the flare angle $\alpha$ and width of the electrode tip $w_{\text{tip}}$ in order to investigate their influence on the inductivity. The results are displayed in the Figure 4.16 below.
As expected, the highest inductivity is achieved for larger flare angles and smaller electrode width ratios. The parallel lines in 4.16b indicate that the increase in inductivity with flare angle is constant with regards to electrode width ratio. In Figure 4.16d, a slight convergence towards smaller electrode end widths can be observed, indicating that the difference in inductivity between different flare angles is greatest for rectangular electrode configurations.

Influence of aspect ratio on inductivity

In section 4.1.2 it was discussed how the electrode spacing and aspect ratio influences the thruster performance parameters, such as the impulse bit, by being directly proportional to the inductance gradient. In contrast, for flared geometries increasing the electrode separation produces no significant increase in impulse bit. Instead, increasing the aspect ratio from 1 to 3 typically decreases the impulse bit by at least one-third [12]. The opposite is true in the analytical investigation and Figure 4.17 illustrates the theoretical result that the inductance gradient increases
with higher electrode separation and lower electrode widths, indicating that an increase in aspect ratio would benefit the increase in amount of available energy.

(a) Rectangular, parallel electrodes.  
(b) Rectangular electrodes with 20° flare angle.

Figure 4.17: Influence of base interelectrode spacing and base electrode width on inductivity for 30 mm electrode length.

Figure 4.18: 3D-plot of inductance change for tongue-shaped electrodes with 20° flare angle and 30 mm electrode length.

Despite this, both analytical and experimental studies have shown that implementing flared tongue-shaped electrodes produces higher thruster efficiency and improved propellant utilization compared to parallel electrode geometries, regardless of the aspect ratio. The same studies on breech-fed PPTs has shown that the
performance of parallel electrode configurations display a stronger dependence on aspect ratio compared to that of flared geometries [12].

Results from the design evaluation tool show very little change in the dependence of inductivity on aspect ratio by flaring the electrodes, as can be seen in Figure 4.17. As expected, for tongue-shaped electrodes the increase in inductance is higher over all, but the dependence on aspect ratio is even further decreased and the inductivity is relatively constant for all combinations of electrode spacing and width displayed in Figure 4.18. Increasing the flare angle simply corresponds to a higher increase in inductance gradient compared to that achieved by increasing the electrode separation. The most important parameter for effective electrode configuration and hence optimal thruster performance is not the aspect ratio itself, but the inductance gradient. For complex geometries the linear relation between the inductance gradient and the aspect ratio through the constant factor $\mu_0$ that is displayed for simple geometries, is no longer valid. In such cases, for a fixed available thruster height, flaring the electrodes will reduce the maximal base spacing of the electrodes $h_0$, thus decreasing the aspect ratio for a fixed width. In the same manner, low aspect ratios will allow for a higher flare angle as the base electrode spacing is decreased. It is therefore interesting to note that the when performing the complex optimization using the design evaluation tool, lower aspect ratios will be favored. As can be seen in Table 4.7 the values of the aspect ratios resulting from the optimizations are widely spread, but did not settle at the highest value, in contrast to the case of simple electrode geometries (cf. Table 4.1, Table 4.3 and Table 4.5). In fact, for the aspect ratio range 1-3, the optimal value will instead settle near the lowest bound for all complex configurations, which clearly indicates that implementing flared electrodes is a superior method for performance enhancements compared to the use of geometries with high aspect ratios.

**Influence of electrode length on inductivity**

Apart from the improvements in performance parameters that can be obtained directly from flaring the electrodes, it will also increase their maximal allowed length as compared to parallel electrodes, which should in theory increase the thruster efficiency. Making use of the maximal allocated volume, we can express the flared electrode length relative the unflared $l$ according to

$$ (l_{\text{flared}})_{\text{max}} = \frac{l_{\text{max}}}{\cos \alpha} \quad (4.7) $$

which makes it obvious that

$$ (l_{\text{flared}})_{\text{max}} > l_{\text{max}} \quad (4.8) $$

Since the inductivity increases throughout the thruster channel, an increased electrode length is desirable for enhanced performance, as can be seen in Figure 4.19. Provided that the same reasoning regarding the electrode length as in section 4.1.2
holds also for complex geometries, such an increased maximum electrode length will allow greater flexibility for finding an optimal value of this design variable. The fact that an increased electrode length will decrease the maximum interelectrode spacing and thereby the aspect ratio for a thruster of fixed dimensions should not impose any problem for complex geometries, since high aspect ratios are not necessarily desirable for this kind of design, as explained in the previous section.

(a) Rectangular electrodes, \( w_{\text{tip}} = w_0 \).
(b) Tongue electrodes, \( w_{\text{tip}} \to 0 \)

Figure 4.19: 3D plot of inductivity as function of electrode flare angle and electrode length for base interelectrode spacing \( h_0 = 30 \text{ mm} \) and base electrode width \( w_0 = 15 \text{ mm} \).

4.3 Summary of design guidelines

In theory, many performance quantities are monotonically increasing with some independent variable, such as the dependence of efficiency on aspect ratio. While this means that the aspect ratio can advantageously be increased until the thruster volume limit is reached, this is not always the most beneficial solution in terms of obtaining an optimal efficiency, despite theoretical predictions. This facilitates the need for upper limits to be set on some of the design variables, that sometimes contradict the analytical model. Such limits are usually found empirically and the illustration below is an attempt at summarizing some of the most important limits derived from observations and uniting them with theoretical evidence.

In addition, the need for an optimization scheme arises because changes in some design variables in order to reach a higher performance may result in changes in some other design variables which affects the performance in the opposite manner. Therefore, changes in the thruster design resulting in enhancements of some performance aspects must also be judged by their subsequent effects on other aspects, resulting in trade-offs between these factors. Facilitating decisions about the order of priority of such design trade-offs also motivates the need for developing guidelines which unite theoretical and experimental results.

A summary of the main design guidelines resulting from this study is presented in Table 4.8 and Figure 4.20 below. As the resulting change in performance from...
introduction of more propellant mass and expansion of the exposed propellant area to obtain a higher mass bit is a factor surrounded by a lot of ambiguity, this aspect has been left out in Table 4.8, despite being the most direct difference between the various feeding types. In addition, the comparison depends on whether an equal amount of propellant is considered or on whether the introduction of extra propellant bars is done with the purpose of storing more propellant on-board the satellite (see section 4.1.1). Nevertheless, compared to the multi-directional feeding types, the breech-fed design is more likely to allow for a higher value of the $E_0/A$ ratio, while maintaining a high aspect ratio.

Table 4.8: Summary of positive and negative effects on the thruster performance resulting from the choice of propellant feeding type.

<table>
<thead>
<tr>
<th>Feeding type</th>
<th>Positive effects</th>
<th>Negative effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Breech</td>
<td>• Only one feeding spring reduces thruster mass</td>
<td>• Feeding spring at breech decreases max. thruster length, elongates electrical connections</td>
</tr>
<tr>
<td></td>
<td>• Reduced electrical connections decreases initial inductance</td>
<td>• Force vector directed away from propellant surface reduces ablation rate</td>
</tr>
<tr>
<td>Side</td>
<td>• Smaller individual surface areas leads to uniform ablation</td>
<td>• Additional feeding spring increases thruster mass</td>
</tr>
<tr>
<td></td>
<td>• Force vector directed parallel to propellant surface leads to sustained ablation rate</td>
<td>• Additional feeding springs decreases max. thruster length, increases thruster mass, elongates electrical connections</td>
</tr>
<tr>
<td>Comb.</td>
<td>• Smaller individual surface areas leads to uniform ablation</td>
<td></td>
</tr>
</tbody>
</table>

It is worth noting that no negative aspects of implementing tongue-shaped electrodes have been found in previous experimental studies and there is a solid theoretical foundation in support of this. However, in the complex geometry case, one of the trade-off factors is between the increase in flare angle or maintaining a high aspect ratio. The expense of increasing the flare angle is that the aspect ratio will be reduced as the base intelectrode spacing must be decreased. Another example of a trade-off for a general geometry is the increased $E_0/A$ ratio which, according to experimental studies, leads to increased mass bit, increased thrust and impulse bit, but decreased specific impulse. How such trade-off factors should be handled varies depending on the particular mission. For example, the mission carried out by VELOX may benefit from a higher thrust level afforded by a decrease in specific impulse.
Figure 4.20: Summary of design considerations. The positive and negative effects are marked in green and yellow respectively. The blue boxes indicate that there are ambiguous results regarding whether the effect of certain adjustments improves or reduces the PPT performance. The red arrows mark the direction of adjustment that is generally considered to lead to enhanced performance.
Chapter 5
Discussion

This chapter is dedicated to final comments regarding future work aimed at optimizing the thruster performance, known error sources for the conducted study and a summary of the work that has been carried out.

5.1 Future work

There are some issues that will not be addressed in this work, but that constitute future work related to miniaturization of PPT systems. These will briefly be discussed in what follows.

5.1.1 Design of discharge initiation system

The method used for initial ignition and the number of discharges delivered by an ignition system is highly significant to the thruster performance and is particularly critical for the lifetime of the thruster, but is also one of the factors that contribute to the complexity of the PPT. There are few commercial ignition system sufficiently miniaturized and with suitable performance for micro-PPTs. For the VELOX PPT, a spark-plug type ignition system needs to be developed to make it useful for an extended operation, as the spark plug in the current PPT design is not designed for space conditions. Instead of a vacuum-safe igniter model, this prototype uses an igniter that discharges air to create the feeder ions to discharge the main electrodes. In order to perform vacuum testing on the PPT and for it to eventually work in space, a semiconductor based igniter is needed.

The discharge initiation must be such that neutral or charged particles are injected into the interelectrode spacing from an external source other than the electrodes, which creates the need for an igniter plug. The complete discharge initiating system should consist of a pulse frequency control circuit which triggers an ignition circuit that provides energy for the initial discharge and a surface igniter...
Chapter 5. Discussion

plug. Such a plug is basically any structure that can ionize the propellant surface partially, thereby introducing electrons that drop the vacuum resistance within the interelectrode region and generate a small initial discharge that releases the main thruster capacitor. [19]

The method used to initiate the discharge is affecting the mass and volume of the thruster. One major issue for micro-PPTs is that the spark plug design used for discharge initiation in macroscale PPT systems are too heavy, power consuming and spacious for miniaturized models. Apart from the small dimensions of the electrodes constituting a primary challenge in the design of spark plugs for micro-PPTs, the location of the ignition must also be carefully chosen as it influences the thruster performance by impacting the impulse bit, mass consumption per pulse, specific impulse and thruster efficiency. Typically, the spark plug is inserted into the cathode and while some studies have shown that the distance to the propellant surface should be minimal for optimal performance [17], other experiments shown that this distance can advantageously be increased to an optimum value which needs to be found experimentally for each specific system design. [14].

Normally, spark plug igniters have a concentric design with a central electrode connected to a high voltage supply surrounded by a layer of semiconductor material. For miniaturized PPTs, the propellant length will be severely decreased and the a spark plug that would scale accordingly would be extremely small and difficult to manufacture. This problem can be solved by implementing a rectangular spark plug for which the discharge will be localized at the beginning of the main propellant bar, thus ensuring that the discharge sweeps along the entire propellant surface. [26]

All PPTs that have ever been used in space have used semiconductor switches to create a pulsed discharge across the propellant surface, which has the main advantage of a low starting voltage [3]. Meanwhile, given the limited reliability and lowered lifetime caused by carbon deposition and erosion, in recent years several other discharge initiation methods have been developed. Such alternative methods could be considered for VELOX, if there are enough beneficial contributions. Among these methods are laser assisted ignition, where IR monochromatic radiation is used to induce under-voltage ignition, which has promising results at low electrode voltages. Further, the issue of the downsizing of the spark plug has been addressed by the development of self-triggering micro-PPTs, where this problem is overcome by completely eliminating the spark plug from the design. Instead, the ignition is triggered by auto-initiation, where high vacuum pressure works as a catalyzer for the ignition through the air gap and high voltage is applied directly to the propellant as the discharge self-ignites when the charge voltage exceeds the surface breakdown voltage. The breakdown drains the capacitor until it recharges and self-ignites again, enabling a pulsed behavior [16]. This allows for the size of the thrusters to be reduced and for arrays of thrusters to be manufactured using MEMS techniques, which can provide tremendous control authority over the satellite positioning [25]. Additionally, because it solves the problem of erosion of the cathode and spark plug, the removal of the spark plug could largely benefit the
5.1. Future work

lifetime of the PPT. However, although the self-triggering design is simpler and up to five times lighter than the triggered design, inherent shot-to-shot variations in the discharge energy are expected to increase variations in the impulse bit. Such shot-to-shot variations of the thrust produced can also be caused by asymmetric discharge initiation, which could be avoided by having several spark plugs in operation and fired simultaneously [27].

5.1.2 Scaling effects of miniaturization

Despite the clear advantages of the miniaturization of satellites, such as decreased mass and manufacturing costs, it also implicates many design-related issues. The efficiency issues facing PPTs are further increased as the system is miniaturized, due to a scaling effect caused by reducing the propellant surface area and the dimensions of the thruster electrodes. The understanding of the influence of electrode geometry on performance is still incomplete, in particular for low energy operation PPTs and scaling laws evaluated for standard-sized PPTs in the past might not be valid for miniaturized models and require renewed investigation. In fact, extreme miniaturizations may severely impact the physics of the ablation and acceleration mechanism [11]. For instance, the increasing ratio of electrode area to plasma volume might result in a more efficient acceleration process because of higher ionization, but also increases the potential of losses due to frozen flow and viscous boundary losses. Another issue that has been a common problem among the various attempts to miniaturize PPTs in recent years is the carbonization of the Teflon surface that primarily arises due to an incomplete coupling of the electric energy into the propellant surface [34]. Since the details of the energy transformation as well as the exact sequence of events in the discharge process are still being researched, the reason for this problem is not yet completely understood, but needs to be solved in order to successfully miniaturize PPTs. Performance scaling laws could provide the required knowledge and bring clarity in the matter of how the thrust, efficiency, and specific impulse scale with various operating conditions, which is important to enable matching of a given thruster to mission requirements.

5.1.3 Experimental optimization

This work is based on a purely analytical optimization and many important factors affecting the thruster performance are not visible in the theoretical results. Therefore, it is of high importance for the results to be experimentally verified. Additionally, in order to determine whether the unique suggestion of a three-directional feeding mechanism is feasible requires an experimental comparison to conventional breech-fed and side-fed PPTs and constitutes another subject of optimization.

While an analytical evaluation is useful to theoretically determine the importance of the various design parameters, due to the lack of knowledge about the complex processes involved in the PPT operation, it appears to be crucial for each micro-PPT design to be optimized experimentally. The present work has provided
some guidelines for the PPT development that can be used to experimentally finalize the particular design for the VELOX thruster.

5.1.4 Other unexplored design possibilities

Apart from the design parameters investigated in this study, there are a number of other parameters that can be modified in order to enhance the PPT performance. Some of these are briefly mentioned below.

• **Applied magnetic field.** In addition to the self-induced field, a separate magnetic field can be applied in parallel in order to enhance the acceleration caused by the Lorentz force. This field can be supplied by electro- or permanent magnets afforded by an increase in thruster mass and volume.

• **High frequency-operation.** The most common standard operational frequency is 1 Hz and very little published experimental data is available for higher firing frequencies. Despite disadvantages of increasing the pulse frequency, such as higher electrode operational temperatures and increased erosion, it could increase the peak thrust and add flexibility of the impulse bit by enabling the use of frequency modulation. However, while the thrust will increase with frequency, increasing the frequency will also require an increase in input power and consequently, the thrust-to-power ratio may not be improved. [14]

• **Alternative materials.** The dominating electrode and fuel materials among solid propellant PPTs are copper and Teflon respectively, but other materials could be investigated for improved performance.

• **Propellant temperature.** The temperature of the propellant can impact the performance by affecting the pressure and thereby the amount of particles that reach the acceleration channel. Extending equation (2.20) to include also a gas dynamic force term $hwnkT$, where $n$ is the particle number density, $k$ is Boltzmann’s constant and $T$ is the plasma temperature, shows that the thrust produced will increase linearly with plasma temperature and also that, in contrast to the electromagnetic thrust component, the thrust from the gas expansion will increase with electrode width [21].

• **Simulation programs.** As the complicated processes occurring under PPT operation cannot be investigated by purely empirical means, there have been efforts to apply computer assisted tools such as magnetohydrodynamic codes in order to quantify discharge and flow parameters during the pulsed operation. Modeling the microscopic behavior of the plasma through numerical simulations can help yield a better understanding of the accelerating mechanism. [20] [24]
5.2 Error analysis

5.2.1 Model errors

The slug model uses many simplifications and suffers from severe limitations due to approximations that can generate errors. The main sources of error are summarized below.

**Constant resistance**

The resistance is considered to be constant, which is obviously an idealization. Further, the value of the resistance is a factor that largely affect the thruster performance and as it is decreased the current becomes more intense, which means that the thrust and efficiency will both increase. The results of the calculations in the analytical model are very sensitive to changes in the resistance parameter, which is an issue as it is basically arbitrarily set. While the evaluation program offers great flexibility in adjusting the input to more accurate values, should such be available, in order to give reasonable results this quantity must be kept low and set in the order of $10^{-2} \, \Omega$.

**Simplified magnetic field model**

The slug model assumes a constant change in inductance with time and a constant magnetic field within the circuit, which is an idealization even with respect to simple electrode geometries, but even more so for flared, non-rectangular electrodes. Further, the self-induced magnetic field model contains other simplifications that do not accurately describe the true situation for PPTs. For instance, in contradiction with the model, we typically have $h > w$, and in contrast to the one-turn solenoid approximation, the magnetic field will exist even outside of the approximated solenoid causing fringe effects. In addition, while the conductivity of the electrodes is very high and can quite accurately be approximated as infinite without sufficient loss of accuracy, the conductivity of the plasma sheet will be finite.

**Neglected plasma sheet dimension**

The finite conductivity of the plasma is also due to the fact that the plasma sheet in reality has a third dimension in terms of its thickness $\delta$, which has been neglected in the modeling of the thruster. In addition, such a macroscopic continuum model in which the stream of plasma particles is modeled as a one-dimensional uniform current layer with particles moving at the same velocity and can be localized at the same x-position simultaneously, is obviously a crude idealization. In reality the exhaust stream consists of a great amount of various types of particles moving at different velocities.
Mass utilization issues

The type of uniform idealization described under the previous point also neglects the particle mass distribution and the fact that different types of particles will be accelerated differently. For instance, the maximum velocity reachable by the exhaust stream is related to the charge of the particles. During the initiation of the electromagnetic acceleration stage, electrothermally accelerated particles enter into the electric field and as they travel further away from the propellant surface, the gas-expanding effect becomes weaker and the neutral particles can no longer be accelerated, since they are not affected by the electric field. The ions are much heavier than the electrons and thus contribute substantially to the thrust. Moreover, although the charged particles have less mass than the neutral particles, they will achieve high velocities in the electromagnetic acceleration stage and as a result, make a larger contribution to the momentums. In fact, research has shown that neutral particles constitute about 90% of the propellant mass, so that only the remaining 10% of the consumed propellant is converted to plasma and efficiently accelerated by electromagnetic forces. Consequently, the existence of neutral particles constitutes an important reason for the low efficiency of the PPT and many research efforts concentrate on decreasing the amount of neutral particles. [3]

The mass ablated per pulse and how it is efficiently accelerated is clearly a crucial component in determining the performance of the PPT. As the initiation of the discharge is the most difficult part to model accurately, the mass bit is possibly one of the greatest sources of error in the modeling. Essentially, all we can say about the mass bit is that it increases with exposed propellant area as well as with energy, and the mass ablated per pulse can in general be expressed as a function of the electrode spacing and discharge energy [8]

\[ m_{\text{bit}} = f(h)E_0 \]  

(5.1)

where \( f(h) \) is a linear function of \( h \). The behavior changes with the circuit properties of the thruster as well as the geometry and must be determined empirically for each design.

The design evaluation tool is programmed to use the maximum mass bit possible to obtain the desired lifetime of the satellite with the set propellant mass. Using this value in calculating the performance parameters therefore results in other values than can be expected in practical situations; in particular, it will significantly increase the thrust level as compared to experiments. Furthermore, the program treats the mass bit as invariant with respect to the thruster geometry, which causes errors in the comparison between design variations.

5.2.2 Neglection of electrical parameters

In this work only the geometrical parameters have been given deeper consideration; however, in order to perform a complete optimization, the electrical parameters needs to be taken into account. As an optimization of the electrical parameters
was not of interest in this study, they have essentially been arbitrarily set. The discharge energy and capacitance have been set in relation to each other with the aim to meet the breakdown voltage requirement of 1.4 kV. The circuit inductance and resistance have large effects on the optimization outcome; yet, when set in the evaluation tool, these quantities are pure estimations. However, the GUI offers great flexibility in investigating the results for other values of these parameters and they can easily be adjusted to more accurate values, in cases where there is a smaller uncertainty of the true value.

Further circuit characteristics are here assumed to be independent on thruster geometry. When comparing the different types of feeding, the initial inductance has been set equal and the decrease in this quantity for the side-fed mechanism has not been accounted for in the optimization process, but is a factor that will greatly benefit this design choice in terms of efficiency. Compared to combination fed-designs, the inductance should be lowered for side feeding due to the reduced electrical connections. An even more significant reduction in initial inductance should be observed for side-fed design when compared to breech-fed, as the latter requires a longer propellant bar in order to preserve the propellant mass requirement and thereby the electrical connections are also elongated.

5.2.3 Optimization errors

As a consequence of the $E_0/A$ ratio only being a constraining parameter in the optimization model, and not related to the thruster performance, the optimization fails to find feasible solutions for lower bounds of the $E_0/A$ ratio below 1 in the case of breech-fed design and 0.5 in the multi-fed design versions. The difficulty in finding solutions of higher energy density as the number of propellant bars is increased arises because the amount of propellant is fixed independently of the feeding type, which means that the exposed propellant surface area will increase as bars are added and thus the $E_0/A$ ratio will decrease for the fixed discharge energy considered here. In reality, as pointed out in section 4.1.3, the energy density is a highly influential factor on the thruster performance and $E_0/A$ ratios in the ranges found by the optimization are normally too low to obtain an efficient propellant stream. Moreover, the propellant may not be ablated at all if the $E_0/A$ ratio is too low.

The value of $w_{\text{prop}}$ has a great effect on the performance parameters and as it is manually set in the case of side-feeding it obstructs the comparison between other feeding types. Other sources of error related to the model formulation and the evaluation program are the possible inaccuracies of the mathematical solvers and the fact that the volume of the spark plug and casing has not been accounted for, which will in practical consideration reduce the available thruster height by a substantial amount.

In this work, the three-directional feeding has been investigated under the assumption to increase the propellant surface exposed to the discharge, as the primary aim has been to investigate the aspect ratio. However, for a complex
geometry with less dependence on high aspect ratio for increased performance, the propellant surface could instead be divided into three smaller surfaces; something that would likely improve the uniformity of the propellant ablation process, which in turn would benefit the mass efficiency.

5.2.4 Design variable boundaries

Analytical models fail to predict bounds to the design parameters where negative effects may exceed the gains in performance enhancement factors and the results from the design evaluation tool suffers from this issue. This means that many negative effects of adjusting the geometrical parameters are not noticeable in a theoretical analysis and it is for this reason that the design guidelines in section 4.3 have been established.

5.2.5 Incomplete theoretical knowledge

Some relations between the main design parameters of the PPT are known as semi-empirical as they have only been proven experimentally. In spite of the techniques about fifty-year heritage, there is lots of ambiguity surrounding exactly how the different design parameters are affecting the performance of the PPT. This is mainly due to the lack of theoretically thorough understanding of the energy transformation process, which involves many physical processes, such as the Lorentz forces, self-induced magnetic fields, Joule heating, neutral- and ion collisions and radiation losses, making accurate modeling of the PPT a complex issue. As a result, there is still no physical model to predict the behavior of the thruster in completeness, which causes inaccuracies in the analytical model developed here.

In addition, a remaining research gap is the lack of agreeing theoretical expressions for some of the performance parameters causing errors in the theoretical investigation. The development of robust analytical expressions to correctly describe the operation is therefore important in order to get good agreement with experimental data, which is often too design specific. In particular, for flared and tongue-shaped electrodes there has mostly been published experimental data confirming efficiency improvements due to such geometries, while little effort has been done to describe these improvements analytically.

5.2.6 Lack of experimental data

Despite the difficulties in accurately modeling the dynamics of the PPT, there is some empirical evidence that can be used to develop models describing how the geometrical properties impact the performance. However, there are few studies on other factors, including erosion data for instance, and there is also a lack of experimental data comparing different electrode geometry variations under the same operating conditions [16]. The same applies to the investigation of the propellant feeding mechanism. As the combination feeding has not been implemented yet,
there is no experimental data and no empirical relations describing the effect of this feeding mechanism. It has therefore been difficult to make any definite predictions regarding how such a mechanism would influence the thruster performance.

5.3 Summary and conclusion

The design evaluation tool developed to explore the established analytical model of the thruster dynamics has been used to calculate geometrical parameters leading to optimal thruster performance for different electrode geometry configurations when the electrical parameters are fixed. As the program is based on highly idealized models and a large amount of the complex underlying physics will not be visible, the resulting performance values, such as specific impulse and thruster efficiency, will take on a lot higher values than in true practical situations. It is therefore important to be aware of the limitations of the simulations and the fact that the evaluation tool does not necessarily provide accurate values comparable to experiments. However, as a relative measure for comparisons between different design options, the tool can be of great use as assistance in a future experimental optimization.

The strongest result from this study is that implementing tongue-flared electrodes is the most significant design choice for optimal thruster performance of miniaturized PPTs. The enhancement caused by such designs likely exceeds any improvements that can be reached by modifications of other design variables. Despite the analytical model not being extended to cover such complex geometries in terms of the relation to impulse bit, specific impulse and thruster efficiency, enough experimental evidence from other studies are available in support of this conclusion. In addition, the increase in inductivity caused by implementation of diverging electrode geometries has been investigated to a sufficient extent and the relation between the change in inductance throughout the thruster channel and the energy available for the particles to be accelerated is a well-established fact. This constitutes enough motivation for suggesting tongue-flared geometries as the major design choice for micro-PPTs. Further, the complex design optimization carried out using the design evaluation tool confirms the previous experimental result that for micro-PPTs, implementing complex electrode geometries is a superior method for performance enhancements compared to increasing the interelectrode aspect ratio.

The main known areas where the theoretical predictions fail to describe the true behavior of the thruster and where empirical data and experience from observations must instead be used in order to obtain the highest performance, has been pointed out. In such situations, the guidelines that have been established can facilitate design decisions by clarifying important trade-off factors. However, emphasis must be put on the significance of experimental verification for each particular PPT design prior to finalizing any design choices.
Appendix A

A.1 Source codes for Matlab-implementation of optimization

A.1.1 Simple electrode geometry

```matlab
function optim_param = PPT_Optim(m, tau, f, E, C, L0, R, t, wprop, feed, shape, t_optim, alpha, min_a, max_a, min_EA, max_EA)  

    global exitflag;
    mu0=1.2566370614*1E-6;
    mb=1/(f*tau);
    V=sqrt(2*E/C);

% Upper and lower parameter bounds
lw_h=10E-3; up_h=1E-2; % t
lw_w=10E-3; up_w=1E-2; % t
lw_l=1E-2; up_l=1E-2; % t

% Set up optimization
objective = @(p) objFcn(p);
optimoptions = optimset('Algorithm','interior-point');
p0 = [lw_h; lw_w; lw_l];
lb = [up_h; up_w; up_l];
ub = [up_h; up_w; up_l];
[optim_param,~,exitflag] = fmincon(objective,p0,[],[],[],[],lb,ub,[],optimoptions);  

% Define optimization function
function f_obj = objFcn(p)  
    tSpan = [0, t_optim];
    x0 = zeros(4,1);  
    ode_options=odeset('RelTol',1e-7,'Stats','on');
    [time,v] = ode45 @(t,x)updateFcn(t,x,p),tSpan,x0,ode_options;
    [~,ind_l]=min(abs(v(:,1)-p(3)));
    f_obj=-max(v(:,3))-100*(p(1)/p(2))*trapz(time,v(:,4).ˆ2)+10*abs(max(v(:,3))-v(ind_l,3));
end

function [dx_dt] = updateFcn(~,x,p)  
    % p(1)=h, p(2)=w, p(3)=l
    dx_dt=zeros(4,1);
    dx_dt(1)=x(3);
    dx_dt(2)=x(4);
    dx_dt(3)=0.5*(mu0/m blatant).*(p(1)/p(2)).*(x(4))2;
    dx_dt(4)=(V-(x(2)/C)-(mu0*(p(1)/p(2))).*(x(3)+R).*(x(4)))/(L0+mu0*(p(1)/p(2))).*(x(1))1;
end

% Define non-linear constraints
function [c, ceq]=constraints(p)  
    % Determine propellant area based on feeding type
    if any(feed==[1 3])
    A=p(1)*p(2)*feed;
    elseif feed==2
    A=p(1)*wprop*feed;
    end
    lprop=m/(2200*A);
    % Aspect-ratio
    c(1)=(p(1)*p(2))2-max_a;
    c(2)=min_a=(p(1)/p(2));
    % E/A-ratio
    c(3)=(E/A)-max_EA;
    c(4)=min_EA-(E/A);
```

A.1. Source codes for Matlab-implementation of optimization

%Design specific constraints:
 Thruster height
 if (alpha==0 && any(shape==[1 2])) || (alpha==0 && shape==1)
     c(5)=p(1)+2*(t*cosd(alpha)+p(3)*sind(alpha))-0.05;
 elseif shape==2 && alpha
     c(5)=p(1)+2*p(3)*sind(alpha)-0.05;
 end

 Thruster length and width
 if feed==1
     c(6)=p(2)-0.1;
     c(7)=p(3)*cosd(alpha)+1prop-0.09;
 elseif feed==2
     c(6)=p(2)+2*1prop-0.08;
     c(7)=p(3)*cosd(alpha)+wprop-0.1;
 elseif feed==3
     c(6)=p(2)+2*1prop-0.08;
     c(7)=p(3)*cosd(alpha)+1prop-0.09;
 end
c eq=0;
end
A.1.2 Complex electrode geometry

```matlab
function optim_param = PPT_optim_complex(m,E,t,wprop,feed,shape,min_a,max_a,min_EA,max_EA)
    global exitflag;
    mu0=1.2566370614*1E-6;
    low_h=5E-3; up_h=3E-2;
    low_w=5E-3; up_w=10E-2;
    low_l=1E-2; up_l=10E-2;
    low_alpha=0; up_alpha=pi/6;

    % Set up optimization
    objective = @(p) objFcn(p);
    optim_options = optimset('Algorithm', 'interior-point');
    p0 = [0;0;0;0];
    lb = [low_h; low_w; low_l; low_alpha];
    ub = [up_h; up_w; up_l; up_alpha];
    [optim_param,~,exitflag] = fmincon(objective,p0,[],[],[],[],lb,ub,@constraints,optim_options);

    % Define optimization parameter
    function optim_param = objFcn(p)
    if shape==1
        we=p(2);
    elseif shape==2
        we=1E-7;
    end
    fun=@(x)((2*(p(1)+2*tan(p(4)).*x)./(p(2)*(1-x/p(3))+we*x/p(3))).*(pi-2*atan((p(1)+2*tan(p(4)).*x)./(p(2)*(1-x/p(3))+we*x/p(3)))+((p(1)+2*tan(p(4)).*x)./(p(2)*(1-x/p(3))+we*x/p(3)))*log(p(1)+2*tan(p(4)).*x)+2*log(p(2)*((1-x/p(3))+we*x/p(3)))+1-((p(1)+2*tan(p(4)).*x).^2./(p(2)*(1-x/p(3))+we*x/p(3)).^2)).log(p(1)+2*tan(p(4)).*x)+(1-2+[(p(1)+2*tan(p(4)).*x).^2./(p(2)*(1-x/p(3))+we*x/p(3)).^2]);
    DeltaL=(mu0/(2*pi))*integral(fun,0,p(3));
    optim_param=-DeltaL;
    end

    % Define non-linear constraints
    function [c, ceq]=constraints(p)
    % Determine propellant area based on feeding type
    if any(feed==[1 3])
        A=p(1)*p(2)*feed;
    elseif feed==2
        A=p(1)*wprop*feed;
    end
    lprop=m/(2200*A);
    % Aspect-ratio
    c(1)=(p(1)/p(2))-max_a;
    c(2)=min_a-(p(1)/p(2));
    % E/A-ratio
    c(3)=(E/A)-max_EA;
    c(4)=min_EA-(E/A);
    if shape==1
        c(5)=p(1)+2*(t*cos(p(4))+p(3)*sin(p(4)))-0.05;
    elseif shape==2
        c(5)=p(1)+2*p(3)*sin(p(4))-0.05;
    end
    % Thruster length and width
    if feed==1
        c(6)=p(2)-0.1;
        c(7)=p(3)*cos(p(4))+lprop-0.09;
    elseif feed==2
        c(6)=p(2)+2*lprop-0.08;
        c(7)=p(3)*cos(p(4))+wprop-0.1;
    elseif feed==3
        c(6)=p(2)+2*lprop-0.08;
        c(7)=p(2)*p(3)*cos(p(4))+lprop-0.09;
    end
c eq=0;
end
```

```
Bibliography


[35] Intini Marques R. A Mechanism to Accelerate the Late Ablation in Pulsed Plasma Thrusters [PhD thesis], Southampton: University of Southampton; 2009.