Time Dependent Modelling and Simulation of the Corona Discharge in Electrostatic Precipitators
Abstract

Corona discharge is one of the crucial problems related with high-voltage equipment. This paper focuses on the physical and numerical modelling of corona discharge in an Electrostatic precipitator (ESP). The model is based on Maxwell's equations and the Finite element method (FEM) and is implemented with the COMSOL Multiphysics software. The simulation allows studying the electric charge distribution and the behaviour of the electric field inside the ESP. The work is focused primarily on time-dependent studies of the corona discharge.
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Chapter 1

Introduction to ESP

The Electrostatic Precipitator (ESP) is a collecting device that removes different particles (such as dust and aerosols) from the gas flow using the force of the electric field [1]. It is usually designed as a set of metal plates with metal electrodes stretched between them. A potential difference of several kilovolts (in industrial installations up to several tens of kilovolts) is created between the electrodes. The potential difference results in a strong electric field between the electrodes and plates. On the surface of the electrode a corona discharge occurs and in combination with the electric field it provides an ion current from the electrode to the plate. Polluted air is fed into the space between the plates. Dust particles from the polluted air passing through the filter become charged (ionized) under the influence of the ion current, and then get attracted to the plates and settle on them.

![Figure 1.1: Overview of an Electrostatic precipitator [1].](image)

The principle of operation of the Electrostatic Precipitator was proposed in 1824, and in 1907 Frederick Cottrell patented the first design.

ESPs continue to be excellent devices for control of many industrial particulate emissions, including flue gas from electricity-generating utilities (coal and oil fired). Electrostatic precipitators are generally able to remove more than 99% of the particles in exhaust, without having to force the air through a filter. They have lower pressure drop and therefore lower operating costs compared to fabric filters [2].

Setting up an ESP system is an expensive and complicated process. But what is even more important is keeping the system effectively working for as long as possible
(this period can last for 20-30 years). That is why the processes inside an ESP must be monitored. The problem here is that only few parameters can be adjusted or measured directly. It is only the output of the power supply that can be adjusted; it is also possible to measure the current on the plates. But this information in itself does not give the whole picture of what is happening inside the ESP.

The way for solving this problem is to have a real time virtual model of an ESP. It would become possible to see how the real system behaves at this particular moment when a certain voltage (or current) is applied, how its state changes in time etc., just by using the model with the same input parameters.

The separation of suspended particles from a flue gas by the electrostatic precipitation requires three fundamental steps [3]:

- electrical charging of suspended particles,
- collection of the particles on the surface in an electric field,
- removal of the precipitated material from the collecting electrodes.

However, for a complete description of the process more details must be considered. The expanded scheme of the processes and the relations between them are given in Figure 1.2 [4]. The blocks of the scheme can be expanded for including additional phenomena or combined and avoided for simplification depending on the demands for a particular problem. The mandatory step for building the model is the representation of the structure (geometry) of the ESP.

The goals of this work:

- Developing a model of the central process in ESP operation - the process of corona discharge - and analysis of data received during simulations.

- Recreation of the stationary analysis models described in [5], [6], [7] and performing simulation using COMSOL Multiphysics software.

- Improving and expansion of the existing methods of corona discharge analysis to use them for the time-dependent study of the process.

- Confirmation of the possibility of time-dependent analysis by means of modern hard- and software.

- Developing adequate models for time-dependent simulation as the first step towards the creation of real-time model of an ESP.

- One of the additional aims of the project is to develop the instructions for adaptation of the COMSOL Multiphysics software for the simulation of corona discharge in ESPs.
Figure 1.2: Schematic representation of the processes inside an ESP [4].
Chapter 2

Theoretical basics for modelling and simulation

2.1 Corona discharge theory and approximations

The basic theory and assumptions for describing and modelling corona discharge given in this section follow the book by H.J. White [3] unless other sources are stated.

Corona discharge is one of many forms of electrical conduction in gases. Electrical conductivity in gases differs from that in solids and liquids since gases normally contain no free electrons or ions. However, the conductivity of the gas increases greatly when the potential between the electrodes is high and the ionization occurs. The transition of gas from the insulating to conducting state is called an electric breakdown or gas discharge.

The main ionization mechanism in the corona discharge is the electron impact. According to Townsend [8] the equation for electron ionization is

\[ \frac{dn}{dx} = \alpha n \]

(2.1)

where \( dn \) is the increase of the number of electrons produced by \( n \) electrons which pass a distance element \( dx \) in the field. The coefficient \( \alpha \) is a function of the field strength and gas pressure and depends on the gas type. In the general case, when the field and therefore \( \alpha \) vary with \( x \), the integration of (2.1) gives

\[ \int_{x_0}^{x} \frac{dn}{n} = \int_{x_0}^{x} \alpha dx \]

(2.2)

or

\[ n = n_0 e^{\int_{x_0}^{x} \alpha dx} \]

(2.3)

Another property of the electrons is their ability to attach to many molecular and atomic species. Negative gas ions are the result of the electrons attachment. The electron attachment is a probability phenomenon described by a survival equation

\[ n = n_0 e^{-\eta x} \]

(2.4)

where \( \eta \) is a coefficient of attachment dependent both on the gas type and the electric field. Combining ionization (2.3) and attachment (2.4) for non-uniform fields the value of \( n \) becomes

\[ n = n_0 e^{\int_{x_0}^{x} (\alpha - \eta) dx} \]

(2.5)
The example of this situation is the negative corona in air. The electrons at the corona wire generate many additional electrons because of large $\alpha$ and small $\eta$. Some gases have little or no affinity for electrons which means that electrons do not attach to the molecules of these gases and their coefficient of attachment $\eta$ is zero. It is obvious that negative corona is impossible in gases with zero electron affinity.

Gas discharges may be divided into self-maintaining and nonself-maintaining [3] depending on the necessity of external ionization means for maintaining the process. The unipolar corona (negative), used in electrical precipitation, is a stable, self-maintaining gas discharge between an emitting electrode and a plate.

The first studies about corona discharges started early in the 20th century as a detrimental mode of breakdown in high voltage conductors.

The physical phenomena of corona discharge were carefully studied by Townsend [8], Loeb [9], Peek [10] and Kaptzov [11], both theoretically and empirically through experiments.

One of the most important gas discharge characteristics is the current voltage characteristic usually expressed in the form of a curve which shows the relation between the current and the voltage. Warburg [12] (1899), working with point-plane electrodes, discovered the important parabolic relation between the corona current $i$ and the voltage $U$ expressed by

$$i = AU(U - M).$$

A and $M$ are experimentally determined constants. A similar relationship was found to hold for concentric wire-and-cylinder electrodes by Almy [13] (1902). A theoretical derivation of the parabolic equation was given by Townsend [8] for concentric cylinder electrodes.

Since the time of early researches, corona discharges have been used for various applications, such as electrostatic precipitation, ozone generation, ionic wind devices, nitrogen laser, etc. Therefore, a comprehensive knowledge of all processes occurring in high voltage electric fields is important for using these processes and phenomena in electrotechnical apparatus.

Types of corona:

- **Positive corona.** The discharge is formed on the conductor, positively charged (e.g., during the positive half-wave voltage). It is generally found in areas with sharp corners. The visual appearance of this corona type is small region of a glow around a certain location. This is a relatively weak source of corona discharge, and it creates a slight sound.

- **Negative corona.** This type of corona typically occurs on the conductor, negatively charged, for example, during the negative half-wave voltage. This type of corona looks like a flame with constantly changing shape, direction and size. This corona is very sensitive to changes in the environmental parameters. Its appearance gives rise to an acoustic tone around the double-frequency (e.g. 100 Hz) of the electric field or a multiple of it.

The second type of corona discharge is used in electrostatic precipitation.
The main functions of electric field in ESPs can be divided into charging of particles and controlling their movement trajectories. Corona discharge is one of the most efficient ways to charge particles. The collecting devices use two forms of electrode systems: "needle-to-plane" and "wire-to-plane". The first is used when larger values of the electric field strength, spatial charge density and electric wind velocity must be concentrated in a small area, and the second where these quantities can be smaller, but more evenly distributed in a wide area. In an electrostatic precipitator the second type of electrode system, "wire-to-plane", is often used. Therefore, in this work, the "wire-to-plane" electrode system will be considered for future investigation.

The process of the electric corona discharge for an ESP includes the following factors:

- Applied voltage $U$ between collecting plane and emitting electrode (wire);
- Electrical field $E$ (described by the Poisson equation [14]);
- Ion space charge density, $\rho$;
- Fluid (moving gas, $v_{\text{air}}$);
- Particle tracking, particle charge, $\rho_{\text{particle}}$;
- Corona discharge.

2.1.1 Mathematical model

The mathematical model of the corona discharge has been derived under the following assumptions:

- The emitting electrode is a smooth round wire, with the corona discharge uniformly distributed over its surface;
- The Ion mobility is constant;
- The Thickness of the ionization layer can be neglected;
- The drift zone contains ions of one polarity;
- The electric field satisfies Kaptzov’s assumption [11] (explained in sec. 2.1.2).

The following set of equations for the electric field and the space charge density describes the process of a unipolar corona discharge.

Maxwell’s equations are used as a basis [15]:

\[
\nabla \cdot \varepsilon_0 E = \rho ,
\]

\[
\nabla \cdot B = 0 ,
\]

\[
\nabla \times \frac{1}{\mu_0} B = J + \frac{\partial \varepsilon_0 E}{\partial t} ,
\]
∇ × \(\mathbf{E}\) + \(\frac{\partial \mathbf{B}}{\partial t}\) = 0, \hspace{1cm} (2.10)

where \(\rho\) - space charge density, \(\varepsilon_0\) - electric permittivity \(\mu_0\) - magnetic permeability, \(\mathbf{J}\) - current density. Since the magnetic field \(\mathbf{B}\) is divergence free it can be represented by a vector potential \(\mathbf{A}\) so that

\[\mathbf{B} = \nabla × \mathbf{A}.\] \hspace{1cm} (2.11)

The space charge \(\rho\) has to satisfy the charge conservation law. Taking the divergence of (2.9) yields the continuity equation:

\[\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0.\] \hspace{1cm} (2.12)

It now follows from Maxwell’s equations (2.10) that there exists a potential \(\Phi\) so that

\[\mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}.\] \hspace{1cm} (2.13)

Assuming that \(\partial \mathbf{A}/\partial t\) is neglected in (2.13) the quasi stationary approximation can be made for time-dependent case meaning that

\[\mathbf{E} = -\nabla \Phi.\] \hspace{1cm} (2.14)

This is a low frequency approximation. Let \(a\) be a typical distance in the geometry and let \(c = (\varepsilon_0 \mu_0)^{-1/2}\) be the speed of light. Then a typical angular frequency is \(\omega_0 = c/a\) and a typical time is \(t_0 = a/c\). The quasi stationary approximation holds if the angular frequency \(\omega\) in \(\mathbf{E}\), which is the time Fourier transform of \(\mathbf{E}\), is much smaller than \(\omega_0\) or the time \(t\) after the source have been switched on is much larger than \(t_0\).

The combination of (2.7) and (2.14) gives the Poisson equation:

\[\nabla \cdot \mathbf{E} = \nabla \cdot (-\nabla \Phi) = -\nabla^2 \Phi = \rho/\varepsilon_0 ,\] \hspace{1cm} (2.15)

The current density \(\mathbf{J}\) is assumed to be [16]

\[\mathbf{J} = K \rho \mathbf{E} + \rho \mathbf{v}_{\text{air}} - D \nabla \rho ,\] \hspace{1cm} (2.16)

where \(K\) is the mobility of the charge carriers, which depends on atmospheric pressure and room temperature. It can be taken as a constant equal to \(K = 1.8 \sim 2.2 \cdot 10^{-4} \, \frac{m^2}{Vs}\) [16]. \(D\) is a diffusion constant in the order of \(10^{-5} \, \frac{m^2}{s}\) [17]. Thus the system of two coupled PDEs 2.12 and 2.15 with \(\mathbf{J}\) given by 2.16 describes the physics of the process.

The current density equation (2.16) contains three components via \(\rho\) and \(\phi\): the drift current \(K \rho \mathbf{E}\), the convection current \(\rho \mathbf{v}_{\text{air}}\), and the diffusion current \(-D \nabla \rho\). The typical gas velocity is about 15m/s [3]. The drift velocity of the ions is approximately \(K \cdot \mathbf{E} \approx 2 \cdot 10^{-4} \cdot 5 \cdot 10^6 = 10^3 m/s\) which is two orders of magnitude higher then the
velocity of the gas flow. Neglecting or keeping the convective component creates two models of the corona discharge problem: decoupled and fully coupled models [16].

The decoupled model for the static case when

\[ \nabla \cdot J = 0 \]  

is derived by combining (2.17) with (2.15) and (2.16):

\[ \nabla \cdot J = \nabla \cdot (K \rho E) - \nabla \cdot (D \nabla \rho) = 0 \]

\[ \nabla \cdot (K \rho E) = \nabla \rho \cdot K E + K \rho (\nabla \cdot E) = K \left( (\nabla \rho) (-\nabla \Phi) + \rho^2 / \varepsilon_0 \right) \]

\[ \nabla \cdot (D \nabla \rho) = D \nabla^2 \rho \Rightarrow \]

\[ K (\rho^2 / \varepsilon_0 - \nabla \rho \cdot \nabla \Phi) - D \nabla^2 \rho = 0. \]  

Neglecting the diffusion current in (2.18) gives

\[ \nabla \rho \cdot \nabla \Phi = \rho^2 / \varepsilon_0. \]  

The fully coupled model is derived the same way, but includes the convective current (diffusion current neglected for simplicity):

\[ \nabla \cdot J = \nabla \cdot (K \rho E) + \nabla \cdot (\rho \mathbf{v}_{\text{air}}), \]

\[ \nabla \rho \cdot (K E) + \rho (\nabla \cdot K E) + \nabla \rho \cdot \mathbf{v}_{\text{air}} + \rho (\nabla \cdot \mathbf{v}_{\text{air}}) = 0, \]

\[ \rho (\nabla \cdot K E) = K \frac{\rho^2}{\varepsilon_0} \]  

(2.15), \nabla \cdot \mathbf{v}_{\text{air}} = 0 \ (\text{flow continuity} \ [16]),

\[ \nabla \rho (K E + \mathbf{v}_{\text{air}}) = -K \rho^2 / \varepsilon_0. \]  

In this work the convective component is not considered and the diffusion component is sometimes used for better numerical convergence.

2.1.2 Boundary conditions

The set of equations (2.14 - 2.20), represented above, is supplemented by the following boundary conditions:

- The potential on the collecting plate is equal to zero;
- The potential on the emitting electrode is equal to the ESP’s operating voltage \( U \);
- The space charge density conditions are derived from Kaptzov assumption [11]: the electric field on the emitting electrode increases proportionally to the voltage until a certain value, called the corona onset level \( E_{\text{on}} \). After that the electrical field at the injecting electrode remains unchanged and equal to the onset value \( E_{\text{on}} \).
The corona onset level $E_{on}$ for cylindrical geometry is derived from Peek’s formula [3]:

$$E_{on} = 3 \cdot 10^6 \delta (1 + \frac{0.03}{\sqrt{\delta r}}).$$

(2.21)

Peek’s formula is a semi-empirical expression derived in [[10]]. Expression (2.21) is a modified version where $E_{on}$ is expressed in $\frac{V}{m}$, $r$ is the electrode radius expressed in m (instead of $\frac{kV}{cm}$ and m respectively in the original formula). The numerical coefficient $3 \cdot 10^6$ is considered to be expressed in volts per meter, while the second coefficient 0.03 has the units of $\sqrt{m}$. The unitless coefficient $\delta$ is found as

$$\delta = \frac{T_0}{T} \times \frac{P}{P_0}.$$  

(2.22)

In equation (2.22), $T_0$ - absolute room temperature; $P_0$ - normal atmospheric pressure; and $T$ and $P$ are the actual temperature and pressure of the air for which $\delta$ is to be calculated.

### 2.2 Numerical methods overview

Common numerical algorithms used for solving partial differential equations are the boundary element method (BEM), method of characteristics (MOC), the finite element method (FEM) and the hybrid FEM-BEM-MOC.

#### 2.2.1 The boundary element method (BEM)

The boundary element method (BEM) [7] is an important algorithm for the numerical solution of a number of physical problems. This method is used for solving partial differential equations (PDEs). When the corona discharge simulation is handled a very strong electric field near the emitting electrode is observed. The domain techniques used for this case should have a very fine discretization; as a result a very large number of nodes is needed. Using of the boundary element method leads to reducing the number of nodes as it requires discretization of the electrodes only. However, it is efficient only for small volumes near the electrode and needs to be combined with other methods for the complete domain solution. Therefore, the usage of the pure BEM method is significantly less efficient for the numerical treatment of corona discharge simulation than volumediscretization methods (finite element method, finite difference method and finite volume method).

#### 2.2.2 The method of characteristics (MOC)

The method of characteristics is a method that can be used for solving partial differential equations. The method is to reduce a partial differential equation to a family of ordinary differential equations along which the solution can be integrated from some initial data given on a suitable hypersurface. In practice, MOC is more suitable for hyperbolic partial differential equations and can be used for the space charge density problem. MOC can be used for the approximation when ion diffusion is neglected. Along a characteristic line (field line) defined as

$$\frac{\partial r}{\partial t} = KE,$$  

(2.23)
the charge density satisfies an ordinary differential equation [18] which can also be derived from (2.12 - 2.19)
\[
\frac{\partial \rho}{\partial t} = -K\rho^2/\varepsilon_0
\]  
(2.24)
as a combination of the current density and the charge conservation law. The Equation (2.24) has a simple solution [7]
\[
\frac{1}{\rho} = \frac{1}{\rho_0} + \frac{Kt}{\varepsilon_0}.
\]  
(2.25)
The characteristic lines are traced from those points on the corona electrode surface where the electric field is larger than the onset field, as only at those points the electric charge is injected. The charge from the initial point with value \(\rho_0\) on the corona electrode decays to the collecting electrode along the characteristic field line.

The methods described above (BEM and MOC) are given as a short overview and are not further used in this particular paper. All further computations and simulations involve the finite element method.

2.2.3 The finite element method (FEM)

The Finite Element Method (FEM) is a powerful numerical method for solving partial differential and integral equations arising in physical problems. In the finite element method the model is divided into separate parts - finite elements, connected to each other in nodes. The connected finite elements create a mesh of the studied model. The advantage of the method is that the shape of the treated area can be arbitrary and the mesh can be sparser in areas where special accuracy is not necessary. In the corona discharge problem, a fine mesh is used close to the emitting electrode, because a large gradient of the electric field is required. At the same time, a large mesh can be used in areas where the variation of the voltage is relatively small, and also in areas which are not of large interest for the particular calculation. Therefore, before the creation of the finite element mesh it is necessary to allocate estimated areas of concentration of voltage. For the ESP geometry, these are the electrodes [19].

From a mathematical point of view the finite element method is a generalization of the Rayleigh-Ritz-Galerkin method. Therefore, it can be applied to a wide class of PDEs. After the studied area is divided into smaller pieces, which may be triangles or rectangles, a test function is given within each finite element in a simple way - usually, a polynomial of the third or fourth degree. Boundary conditions are much easier to put along the sides of a triangle or rectangle than on the entire boundary. Each test function \(\varphi(x)\) is equal to zero in most of the region and is different from zero only in the neighbourhood of a node. The calculation accuracy is improved by a finer partition of the area [19].

A major problem is to investigate the accuracy of the solution obtained with the approximation by a polynomial function. In other words, it is necessary to identify how well the finite elements are constructed on the basis of computational simplicity, and if they would give a good approximation. It is intuitively clear that any sufficiently good function can be approximated arbitrarily closely by linear functions. The mathematical problem is to obtain the best estimate of the error and determining the rate of decrease of the error with increasing number of elements of the partition (or the degree of the polynomial in each element).
The basis of most CAD systems uses the finite element method. COMSOL Multiphysics software uses the finite element method for solving differential equations.

2.3 COMSOL Multiphysics

COMSOL Multiphysics is a very powerful system that allows to solve a wide class boundary value problems by means of the finite element method. Finite element analysis, which is used in the program, is run together with an adaptive mesh (if selected) and an error control using a variety of numerical solvers. COMSOL Multiphysics records all the steps, such as parametric data, the model geometry, mesh, various solvers, visualization and presentation of results during the simulation process. The structure of the program makes it very easy to change any of the parameters of the model. It only requires changing a corresponding node in the model tree and restart the sequence. The program stores and reappplies all other information and the data model. In COMSOL the user can independently set the equations that describe a process, and the boundary conditions are set in familiar mathematical form using the Dirichlet and Neumann forms. COMSOL Multiphysics allows creating triangular, quadrilateral, tetrahedral and hexahedral computational grid. The mesh generator works quite well with a complex geometry and has many features to improve the grid. The post-processor has all the necessary features and options for visualization and processing of numerical data.

The efficiency and accuracy of FEM and COMSOL Multiphysics software for the ESP modelling problems is proved in [20], [5], [14], [21] by comparing results of the simulation with experimental data and therefore will be used for modelling and simulation in this work.
Chapter 3

Modelling and simulation

Modelling and simulation was handled using the basic package of COMSOL Multiphysics software, version 4.2. As it was stated in Chapter 1 typical duct-type ESP is represented as set of metal plates with the electrodes between them. For the simplest case (with straight electrodes) most physics can be studied with 2D models [6]. The symmetry cell for this case is shown in Figure 3.1. The size of the electrodes and plates is chosen to match the experimental ESP setup based in the Alstom laboratory. The electrode curtain consists of 4 plates, each of 800 mm width. However, the complex profile of the plates is not taken into account in the model for simplification. The electrode (wire) diameter is 2.7 mm.

For both stationary and time dependent simulation the mesh is finer close to the electrode. The equations are entered using the COMSOL PDE interface (a part of the basic COMSOL package). Specific parameters for stationary and time-dependent solutions are described in corresponding sections.

![Figure 3.1: Basic 2D geometry for duct-type ESP.](image)

3.1 Stationary analysis

According to Section 2.1.1 for the stationary (time-independent) case the problem is stated by two PDEs: the Poisson equation (2.15) for the potential

$$\nabla^2 \Phi = -\frac{\rho}{\varepsilon_0}$$  \hspace{1cm} (3.1)

and the charge conservation equation in the form given by (2.19)
\[ \nabla \Phi \cdot \nabla \rho - \frac{\rho^2}{\varepsilon_0} = 0 \quad (3.2) \]

For solving the PDEs within the specified geometry, boundary conditions are needed. A simple Dirichlet boundary condition

\[ \Phi = 0, \quad (3.3) \]

is used for the potential on the plates, and on the electrode

\[ \Phi = U, \quad (3.4) \]

where \( U \) is the operating voltage of the ESP. However, the boundary conditions for the charge density bring the main complexity to the problem. Two approaches for setting up the boundary conditions were chosen here giving the possibilities to study different parameters. Both approaches are described in the following sections.

### 3.1.1 Direct implementation of Peek-Kaptzov model

The first approach uses Peek’s value for corona onset field strength \( E_{on} \) and Kaptzov’s assumption. Peek’s condition states that when the field strength exceeds the onset value the charge density at the discharge electrode becomes non-zero [10]. The Kaptzov hypothesis postulates that with the increase of the potential difference between the electrodes the electric field increases proportionally to the potential difference and only low discharge (dependent on residual ionization) with a very weak current takes place. When the onset voltage is reached the corona discharge starts and the field on the inner boundary of the “coronating” layer stays close to the corona onset value [11]. The charge density at the boundary increases in such a way that the charge supplied to the domain balances the electric field to stay at \( E_{on} \).

The onset field value is set as a Neumann boundary condition on the electrode for the Poisson’s equation (COMSOL node “Flux/Source” is used) giving the constant value of electric field on the boundary. The onset field value is calculated for the geometry described above using Peek’s formula for \( \delta = 1 \) and equals \( E_{on} = 5.7 \cdot 10^6 \text{V/m} \).

For further steps the following reasoning is needed. If the current density from the electrode value is given the current can be calculated by integration:

\[ I = \int_S J \cdot dS = \int_S J_n \cdot dS. \quad (3.5) \]

For convenience in the 2D model the current per meter of wire (electrode) \( I_w \) can be used instead. This can be achieved by integration over the electrode’s cross-section boundary \( L \):

\[ I_w = \int_L J_n \cdot dL. \quad (3.6) \]

The current density is

\[ J = \rho K E. \quad (3.7) \]

By integration of (3.7) taking into account that \( \rho \) and \( E_n = E_{on} \) are constants on the boundary the following result is obtained

\[ I_w = \rho K E_{on} \int_L dL = \rho K E_{on} \cdot 2\pi r_w, \quad (3.8) \]
where \( r_w \) is the electrode radius and the ideal cylindrical electrode is considered. Assuming that length (height) of the electrode is equal to the height of the plate, it is then possible to derive the current per square meter of the plate \( I_p \) by distributing \( I_w \) over the plate element with the horizontal (along the gas flow direction) length \( L_p \), which for two symmetric plates gives:

\[
I_p = \frac{2\pi r_w \rho KE_{on}}{2 \cdot L_p}.
\]

Now the boundary condition value for \( \rho \) can be derived from (3.9):

\[
\rho = \frac{2 \cdot I_p \cdot L_p}{2\pi r_w KE_{on}}.
\]

Given the current \( I_p = 500 \mu A \) and considering the previous reasoning the BC value for charge density on the electrode is \( \rho_0 = 4.6 \cdot 10^{-5} \frac{C}{m^3} \). Note that the symmetry cell used for further simulations includes only one plate and a half of the electrode.

The output of this simulation is the calculated voltage (potential) on the electrode which is needed to provide the predefined current. The distributions of the potential and charge densities are shown in figures 3.2 and 3.3 respectively. It is also possible to obtain the current-voltage characteristics by running a series of simulations of this type with different values of the current.

**Figure 3.2:** Potential distribution.

The result of the simulation is geometry-sensitive: the calculated electrode voltage (potential) increases with the increasing of the distance between the electrode and the
plate which is in conformity with the expression for onset voltage value given in [3]. This confirms the correct functioning of the model in the sense that, assuming the constant predefined plate current value, higher electrode potential is needed to produce the onset field $E_{on}$ and provide this current value for higher electrode-to-plate distance.

Figure 3.4 shows the values of the potential along a line from the electrode to the plate. Note that the physical model of the negative corona discharge is considered in all cases further in the paper. However, the potential and charge density values in all plots are positive. This is done to avoid possible mistakes with the minus sign.

The consistence of the model can be shown on the value of the current which can be calculated by different means (post-processing tools in COMSOL). The results obtained by the integration of the current over the cross-section boundary of the electrode, the integration over the plate line (with further division by the plate length) and the average value over the plate line all give the value of $500 \mu A/m^2$. This is equal to the pre-defined value. The current distribution on the plate is shown on figure 3.5

### 3.1.2 Injection law-based modelling

It is obvious that the approach given above with the direct implementation of Peek-Kaptzov model is only suitable for cylindrical geometry or for perfectly aligned duct-type structures (equidistant electrodes, precise positioning of the electrodes w.r.t. the plates). However, for more complicated geometries another approach must be chosen to include the impact of the discharge electrode surface and the electrode positions between the plates. For this purpose an injection law was proposed [22]:

\[
\rho(x_T) = \alpha(|E(x_T)| - E_{on}) \text{ for } |E(x_T)| > E_{on},
\]

(3.11)
where $\boldsymbol{x}$ is the coordinate vector $(x, y, z)$ (or $(x, y)$ for the 2D case); $\Gamma$ stands for the boundary of the electrode.

**Figure 3.4:** Potential along the wire-plate line.

**Figure 3.5:** Current profile on the collecting plate.
If the value of $\alpha$ is large the exact functional relationship becomes insignificant and expression (3.11) forms the Peek-Kaptzov condition. Being introduced to the system of coupled PDEs, it provides the dependence of the charge density on the local electric field at the discharge electrode surface. The main disadvantage of this formulation is that with a large $\alpha$ value the system becomes strongly non-linear, which leads to convergence problems. Studies of the convergence for different values of parameter $\alpha$ have been done in [5]. The value for $\alpha$ can be estimated by combining the current flow out from the straight electrode

$$i = 2\pi r_w K |E(x_T)| \rho(x_T)$$

(3.12)

with the injection law (3.11). Let

$$\delta = \frac{|E(x_T)| - E_{on}}{|E(x_T)|}$$

(3.13)

be the relative error in the Kaptzov assumption for $|E(x_T)| > E_{on}$. A combination of (3.12) - (3.13) yields that

$$\alpha = \frac{i}{2\pi r_w K |E(x_T)|^2} \cdot \frac{1}{\delta}.$$  

(3.14)

Therefore, the value of the parameter $\alpha$ that leads to the Peek-Kaptzov treatment should be chosen such that

$$\alpha > \frac{i_{max}}{2\pi r_w K E_{on}^2} \cdot \frac{1}{\delta} \text{ for } |E(x_T)| > E_{on},$$

(3.15)

where $i_{max}$ is the highest current to be handled (per meter of the discharge electrode) [5].

In order to run this type of simulation several changes must be done to the model described above. To improve the convergence for numerical solution the diffusion term should be kept in the continuity equation as in (2.18)

$$K \cdot (\nabla \Phi \cdot \nabla \rho - \frac{\rho^3}{\varepsilon_0}) + D \nabla^2 \rho = 0.$$  

(3.16)

The boundary condition for the charge density on the electrode is changed to the injection law. Peek’s value for the onset field strength is no longer explicitly set as a boundary condition. It is now obtained by the correct choice of $\alpha$ for the injection law (3.11) (an example will follow below). The potential on the electrode is now set as an ordinary Dirichlet BC according to (3.4).

Good convergence is achieved by using a 3-step calculation: solving separately the Poisson equation (3.1) (with $\rho = 0$); solving both PDEs (3.1) and (3.2) for $\rho$ only (using the values of $\Phi$ obtained from the first step in each point); solving (3.1) and (3.2) for both variables using the results of the previous two steps as initial values.

Figure 3.6 shows the explanation of how the injection law works for a non-cylindric wire (electrode). The electric field is “concentrated” near the sharper regions of the electrode area which means that these parts “coronate” more intensively.

This injection law approximation works also for non-perfect electrode alignment giving the uneven corona discharge boundary (see figure 3.7).
Figure 3.6: Electric field on the boundary of the electrode with elliptic cross-section.

Figure 3.7: Charge distribution for the ESP structure with misaligned electrodes. The model includes 2 electrodes and the profiled plates as in experimental set up.
As it was stated above for keeping the electric field near the electrode close to Peek’s value (according to the Kaptzov assumption) the value for $\alpha$ must be chosen wisely. This can be practically done by “ramping” the value of $\alpha$ for each calculation until the desired value is reached. This method provides good convergence and visibility. An example of $\alpha$ “ramping” is shown in figure 3.8. It can be seen that the value of the electric field approaches the Peek’s value with the increasing of $\alpha$.

![Electric field around the electrode for different values of $\alpha$ (each point on the $x$ axis corresponds to a point of the circular boundary of the electrode cross-section).](image)

**Figure 3.8:** Electric field around the electrode for different values of $\alpha$ (each point on the $x$ axis corresponds to a point of the circular boundary of the electrode cross-section).

Stationary studies are of great importance for modelling of ESPs and a variety of parameters can be investigated. However, a real ESP operates continuously in time which naturally leads to the necessity of studies in the time domain. And although stationary analyses using numerical methods has been done for a long time now, there are almost no reports about time-dependent simulations. Transient analysis is the natural next step from the stationary one and in the same time it is the first step towards the development of a real-time ESP model. Therefore the next section is dedicated to the modelling and analysis of corona discharge effects in the time domain.

### 3.2 Time-dependent analysis

Time-dependent studies are very important for understanding and explaining the processes occurring during the operation of ESPs, but they are also much more difficult to handle both in terms of correct theoretical formulation (including approximations) and in terms of higher requirements for hardware and time resources during the computation process. According to Section 2.1.1 the system of PDEs must be slightly changed to in-
clude time dependency. This concerns adding the time derivative of charge density into
the continuity equation (3.16) (see (2.12) and (2.16)) and setting up a time-dependent
boundary condition on the electrode for one of the variables.

3.2.1 Current-controlled model

Power sources used for energizing the industrial ESPs physically represent the current
sources. Therefore it is more natural to use current pulses then voltage pulses during
the simulation. Industrial ESP is usually fed by current pulses with the duration in the
range of milliseconds. The motivation for the pulsating power supply is described in many
that when the field structure in the outer domain of the discharge volume is distorted
by space charges, less voltage falls on the coronating layer. Therefore the ionization rate
decreases. The discharge process becomes stationary with comparably small current. The
voltage which drops on the discharge layer would not be enough for keeping the high rate
of the current. This explains why a pulsating current is used in the ESP for producing
corona discharges with high values of the current.

In the first simulation pulses of current have been set through the boundary condition
for the charge density. Figure 3.9 shows the pulses of charge density (equivalent to the
current) and voltage on the same point of the electrode boundary. Note that Peek’s onset
voltage value has been forced during the whole simulation time as a Neumann BC for
the Poisson equation. This model is therefore can be considered as the time-dependent
expansion of the model described in sec. 3.1.1.

Figure 3.9: Pulses of voltage and charge (current) on the electrode.

Figure 3.10 shows the pulses of voltage and current (charge) on the electrode in time
domain. From the delays in the current values calculated on the electrode and on the plate it can be concluded that the charge density spreads as a wave in the domain (see figure 3.10). A good visual representation of this wave can be obtained by making an animated plot of the charge density over a line from the electrode to the plate, which can be done by means of COMSOL tools. Unfortunately, there is no possibility to include such a plot into the paper, and a series of frame-by-frame plots would not be valuable in this case.

![Figure 3.10: Current and voltage change in time.](image)

Figure 3.10 shows the voltage shape on the electrode when the system is fed by the current pulses. From physical reasoning and the figure it can be concluded that the ESP system can be approximately represented by a parallel RC circuit shown in figure 3.12.
Figure 3.11: Current (charge) and voltage pulses.
The capacitance $C$ of the ESP is not constant and depends on voltage. The capacitance change for the time-dependent simulation is shown in figure 3.18. The calculation was done for the simulation in figure 3.11 using the expression for the energy $W$ inside the capacitor:

$$
W = \frac{1}{2} \varepsilon_0 \int_V E^2 dV = \frac{1}{2} C U^2,
$$

(3.17)

$$
C = \frac{\varepsilon_0 \int_V E^2 dV}{U^2}.
$$

(3.18)

For the case of voltage raised from 44kV to 130kV (3 times) the capacitance $C$ changed from 5pF to 8pF (per unit length of the electrode) or about 60%. Figure 3.13 shows the result of the calculation.

The variation of the capacitance can be estimated analytically. The calculation of the capacitance variation is done for the cylindrical geometry using the expressions for electric field, voltage and current given in [3] and follows the algorithm outlined by Leif Lindau [23]. It is worth to mention that the expressions (3.19) and (3.23) are taken directly from [3]. These expressions were derived semi-empirically and are valid only in Gaussian units. The estimation procedure given below leads to the capacitance ratio value, which is dimensionless, and therefore the conversion to SI unit system is not necessary.

Let $a$ be the radius of the electrode and $b$ the radius of the outer cylinder. Then the electric field depends on the given current $i$ and the radial coordinate $r$ [3]:

![ESP equivalent electric circuit.](image)
CHAPTER 3. MODELLING AND SIMULATION

\[ E(r) = \sqrt{\frac{2i}{K} + \left( \frac{E_{\text{on}}r_0}{r} \right)^2}, \]  

(3.19)

where \( r_0 \) is the radius of corona region which can be approximately taken as \( r_0 = a \). Expression (3.18) for cylindrical geometry and 2D calculation becomes

\[ C = \frac{2\pi \varepsilon}{U^2} \int_a^b r E^2(r) dr. \]  

(3.20)

Taking into account (3.19) the integration gives

\[ \int_a^b r \left( \frac{2i}{K} + \left( \frac{aE_{\text{on}}}{r} \right)^2 \right) dr = \frac{i}{K} (b^2 - a^2) + \left( E_{\text{on}}a \right)^2 \ln \frac{b}{a}. \]  

(3.21)

The onset field value can be expressed in terms of the onset voltage \( U_0 \) as

\[ E_{\text{on}} = \frac{U_0}{a \ln \frac{b}{a}}, \]  

(3.22)

see [3]

According to [3] the current is given by

\[ i = U(U - U_0) \frac{2K}{b^2 \ln \frac{b}{a}}. \]  

(3.23)

Combining (3.23), (3.22), (3.21) and (3.18) and taking into account that \( b \gg a \) gives

\[ C = 2\pi \varepsilon \frac{2U^2 - 2UU_0 + U_0^2}{U^2} \]  

(3.24)

or

\[ \frac{C}{C_0} = \left( \frac{U_0}{U} \right)^2 - 2\frac{U_0}{U} + 2. \]  

(3.25)

For \( U_0 = 44\text{kV} \) and \( U = 130\text{kV} \) (3.25) gives \( \frac{C}{C_0} = 1.44 \).

The analytical result of 44% variation in capacitance is close to simulation results, which confirms accuracy of the model. The possibility of describing the ESP by means of a simple equivalent circuit can be used for monitoring and designing a real-time controller for the ESP system.
Now when the values of the capacitance are computed it is possible to compute the capacitive current using the expression

\[ i = C \frac{dU}{dt}. \]

The values of the capacitive component of the current are given in Table 3.1 and compared to the values of the current, computed on the electrode and on the plate at the same moments of time. As it can be seen in figure 3.14 the values of the capacitive current and the current computed on the electrode have the same order of magnitude. This latest investigation raised a reasonable question if the currents which are described in this section are interpreted in a proper way. The most problematic is the current on the electrode which is calculated using the integration over the electrode boundary. It is not clear if this value can be interpreted as the input current. However, the shape of the currents and the values obtained during the simulation are similar to the experimental results obtained in the laboratory. The present work leaves the question open for further investigation of this problem during future research.
Table 3.1: Results of the capacitive current calculation.

<table>
<thead>
<tr>
<th>Time, s</th>
<th>Capacitance, F/m</th>
<th>Voltage derivative dU/dt, V/s</th>
<th>Capacitive current per meter of wire, A/m</th>
<th>Current on the electrode per meter of wire, A/m</th>
<th>Current on the plate per square meter of plate, A/m²</th>
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<tr>
<td>0.1201</td>
<td>4.78·10^{-12}</td>
<td>7.52·10⁷</td>
<td>3.60·10^{-4}</td>
<td>3.85·10^{-4}</td>
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<td>2.96·10^{-12}</td>
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<td>7.52·10⁷</td>
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<td>1.20·10^{-3}</td>
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</tr>
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</tr>
<tr>
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<td>0</td>
<td>2.29·10^{-4}</td>
</tr>
<tr>
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<td>-2.74·10^{-4}</td>
<td>0</td>
<td>4.67·10^{-4}</td>
</tr>
</tbody>
</table>

Figure 3.14: Current comparison.
3.2.2 Voltage-controlled model

The next step of the investigation is the adaptation of the stationary voltage-controlled model with the injection law (see sec. 3.1.2) for time-dependent simulations. The results of this simulation are shown in figure 3.15.

The introduction of the injection law (3.11) for time-dependent studies leads to an increased complexity of the model. One of the main conditions of mathematical modelling of the corona discharge is the Peek-Kaptzov condition, which states that the electric field at the electrode remains constant after reaching Peek’s value even when voltage is further increased. However, with direct implementation of the injection law, setting voltage pulses as the boundary condition for potential on the electrode results in a significant variation of the field from the onset value.

To solve this problem the coefficient $\alpha$ can be made time-dependent. One possibility is to raise the value of $\alpha$ during the time of the voltage pulse. It was confirmed by several simulations that this change of $\alpha$ does not affect the convergence in a bad way and gives a suitable result keeping Peek’s value preserved, which makes the simulation results more trustworthy by providing better agreement with the theory and experimental studies of corona discharge in an ESP.

Figure 3.16 shows the result of a simulation with two identical voltage pulses as an input. The increase of $\alpha$ during the first pulse reduces the electric field variation (onset value is $E_{on} \approx 5.835 \cdot 10^6$ V/m). This type of manipulation may seem to be just a numerical trick. However, it is justified by the Kaptzov assumption in the form given in
[5]. It says that when the voltage is raised above the onset value, the charge density at the boundary has to increase correspondingly in such a way that the charge supplied to the domain balances the electric field to stay at $E_{on}$. This also explains the fact that the charge density (and current) on the electrode is slightly higher during the first voltage pulse compared to the pulse with a lower $\alpha$ value.

![Graph for Voltage and Electric Field](image)

**Figure 3.16:** Effect of time-dependent parameter $\alpha$.

Another problem may occur when the voltage falls rapidly below the onset value (figure 3.15). Since the injection law sets the charge density to zero in this case, the field value at the electrode falls significantly instead of a slow decay to the stationary value for the lower voltage. However, it then rises to approach the physically condition value after a short time which proves a satisfying behaviour of the solver (see figure 3.17).

On the other hand, such a rapid decay of the voltage is unlikely in the real situation and the simulation shows appropriate results when the voltage decreases more slowly (see figures 3.18 - 3.19).
Figure 3.17: Electric field on the electrode (rapid voltage decrease).

Figure 3.18: Results of time-dependent simulation with the injection law (slow voltage decrease).
Calculations show that with dropping voltage from 90kV to 40kV the current decreases from 600\(\mu\)A/m\(^2\) exponentially with a time constant of about 1.3ms (see figure 3.15). The average voltage is \(\frac{90+45}{2} = 65\)kV. The electrode-plate distance of 0.2m gives a field of \(\frac{65000}{0.2} = 325000\)V/m (see figure 3.20). Considering the ion mobility value of \(K = 2 \cdot 10^{-4}\)m\(^2\)/Vs, the ion speed is \(v_{ion} = K \cdot E = 32500 \cdot 2 \cdot 10^{-4} = 65\)m/s. It takes then \(0.2/65 = 3\)ms to cover the full distance or 1.5ms to travel the average distance for the ion. Estimated value of 1.5ms is very close to the 1.3ms calculated time constant.
Then next step is to compare the results of simulations obtained from current- and voltage-controlled model. For this purpose the voltage pulse obtained in section 3.2.1 (figure 3.11) was approximately reconstructed and set as the input voltage and set as the BC for voltage on the electrode (in the voltage-controlled model described in this section). Figures 3.22 and 3.23 show the results of the simulation obtained from current- and voltage controlled models respectively.
Figure 3.21: Difference between plate and electrode currents.

Figure 3.22: Current-controlled simulation (1ms current pulse is set as the BC on the electrode).
The voltage in the voltage-controlled model had nearly the same shape as the voltage calculated in the first model; however, the values of the voltage were slightly higher, which was done for better convergence. The simulation showed that the voltage-controlled model requires more computational time and convergence problems may appear with the long simulation time. In the voltage-controlled case the current value is non-zero even before the pulse starts, but the value is small and is caused by the fact that the voltage is already higher than the onset level (slightly higher value chosen for convergence reasons). Although the current on the electrode is not a rectangular pulse, it grows and falls very fast compared to the voltage. Therefore it is concluded that two models with the opposite approach give very close and reasonable results.
Chapter 4

Conclusions and discussion

The experiments described in the paper show the transition from stationary to time-dependent analysis. The models for stationary analysis, previously described in [5], [6], [7] etc. and approved by the ESP scientific society, were successfully recreated and simulated by means of COMSOL Multiphysics software. Two models for stationary analysis with different approach were proposed (see sec. 3.1), both showing good agreement with the theory and experimental studies as expected.

Time-dependent simulations is a logical and very important step in ESP studies. It allows to expand the knowledge of ESP systems and test their performance under certain conditions, previously impossible by means of stationary studies.

Two possible models for time-dependent simulation of corona discharge in an ESP, the current-controlled and voltage-controlled model, were proposed in sections 3.2.1 and 3.2.2 respectively. Both models are in good agreement with current theoretical investigations and experimental analysis of ESP systems. However, the simulations with both models leave open questions about some of the results, see comments found in corresponding sections.

As to the difference in performance of two models, the computations are much faster and there is no convergence problems with the current-controlled model. It is also more convenient because the power supplies for real ESPs are current sources. On the other hand, the results obtained using this model can be misinterpreted (see sec. 3.2.1) and further research and laboratory experiments are needed for a complete approval of the model. The voltage-controlled model is more difficult to handle in terms of computation time and convergence, but it is more flexible and allows the study of the geometry features by using the injection law (3.11) for the charge density on the electrode.

It was shown that time-dependent simulations can be performed by means of modern simulation software on an ordinary PC with reasonable computation times.

It is obvious, however, that the presented models are far from real-time models and further research and experiments on the matter are necessary.

The instructions for adaptation of the COMSOL Multiphysics software to the simulation of corona discharge in ESPs, including the time-dependent simulation, were developed for internal use in ALSTOM AB.
Bibliography


