Should you optimize your portfolio?

On portfolio optimization: The optimized strategy versus the naïve and market strategy on the Swedish stock market

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Abstract

In this paper, I evaluate the out-of-sample performance of the portfolio optimizer relative to the naïve and market strategy on the Swedish stock market from January 1998 to December 2012. Recent studies suggest that simpler strategies, such as the naïve strategy, outperforms optimized strategies and that they should be implemented in the absence of better estimation models. Of the 12 strategies I evaluate, 11 of them significantly outperform both benchmark strategies in terms of Sharpe ratio. I find that the no-short-sales constrained minimum-variance strategy is preferred over the mean-variance strategy, and that the historical sample estimator creates better minimum-variance portfolios than the single-factor model and the three-factor model. My results suggest that there are considerable gains to optimization in terms of risk reduction and return in the context of portfolio selection.

Keywords: optimization; portfolio selection; portfolio strategy; estimation models; Sharpe ratio

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1. INTRODUCTION

How should an investor allocate capital among the possible investment choices? This is one of the most fundamental questions in finance. The major breakthrough came in 1952, in Harry Markowitz’s publication on the theory of portfolio selection, referred to as modern portfolio theory (Markowitz, 1952). By quantifying the return and risk of a security, Markowitz (1952) combines probability theory with optimization techniques to model investment under uncertainty. The mean-variance optimization method suggested by Markowitz (1952, 1959) is, however, sensitive to minor changes in the two input parameters (Best and Grauer, 1992); a feature that has a serious impact on the optimizer’s out-of-sample performance.

Portfolio optimization is commonly termed “error maximization” since it exaggerates estimation errors (Broadie, 1993). There is a consensus that stock returns are difficult to predict (Michaud, 1989), whereas return variance and covariance estimates are more accurate (Merton, 1980). As most predictions rely upon historical data, the forecasts are generally biased (Ballentine, 2013). While the expected return of a portfolio is biased upward, the variance of a portfolio is biased downwards (Frost and Savarino, 1988). A consequence could be overinvesting (underinvesting) in securities with favorable (unfavorable) estimation errors, and subsequent poor portfolio performance. For instance, it is well-known that an equally-weighted portfolio often outperforms the mean-variance optimal portfolio (DeMiguel et al., 2009b; Duchin and Levy, 2009).

Another issue is that the unconstrained optimizer does not necessarily produce well diversified portfolios (Black and Litterman, 1992). Imposing constraints does not make sense in a world where investment managers know the actual return and risk. As these are unknown in practice, imposing constraints can be justified. In many cases, it may be desirable to impose an upper bound constraint on portfolio weights as a method of hedging against estimation errors and making the portfolio more diversified (Green and Hollifield, 1992). Further, Jagannathan and Ma (2003) shows that high estimates of covariances are more likely to be caused by upward-biased estimation errors, and by imposing a nonnegative constraint, it is possible to further reduce the estimation error. However, as pointed out in Frost and Savarino (1988), if constraints are too severe, portfolio managers are unable to use valuable information about future performance when constructing portfolios.
Nevertheless, DeMiguel et al. (2009b) argues that no single portfolio strategy can, out-of-sample, consistently outperform an equally-weighted “naïve” portfolio strategy in terms of Sharpe ratio. This paper addresses this statement on the Swedish stock market. Thus, the research question posited by this study is: Do optimized portfolios outperform simple portfolios in terms of Sharpe ratio on the Swedish stock market?

In general, Swedes have a great deal of “home bias” when investing. The Swedish Investment Fund Association (2013) finds that, during 1998-2012, up to 90 percent of Swedish household’s direct savings involved investments in Swedish listed companies and that approximately 30 percent of the total net assets of equity funds were invested exclusively in Swedish companies. The simple benchmark strategies will thus consist of the naïve (equally-weighted) strategy discussed in DeMiguel et al. (2009b), and the popular market (value-weighted) strategy. Aiming to outperform these simple portfolio strategies, I create portfolio strategies using expected return and return covariance estimates from the classical historical model, the single-factor model (Sharpe, 1963) and the Fama-French three-factor model (Fama and French, 1992, 1993, 1996).

Since it is often very expensive, or sometimes even impossible, for an ordinary investor to short sell an asset (Bengtsson and Holst, 2002), and generally not permissible for most institutional investors (Kolm et al., 2013), I impose no-short-sales constraints on all portfolio strategies to provide realism to the study. To determine if the upper bound constraint has a positive out-of-sample implication on portfolio performance, all portfolios are tested with and without the upper bound constraint.

It is often argued that returns estimates are more important than return covariance estimates. However, it could be favorable to exclude all return predictions when optimizing because of the error-maximizing tendencies. The optimizer then focuses on minimizing portfolio variance and does not suffer from estimation errors in expected returns (Chopra and Ziemba, 1993). For instance, Behr et al. (2013) shows that the constrained minimum-variance strategy significantly outperforms the equally-weighed portfolio strategy. By adding the minimum-variance strategy, this study evaluates if either the mean-variance or the minimum-variance strategy can outperform the naïve and market strategy.

This study contributes to the existing literature on asset allocation by adding to the discussion of potential benefits of advanced modeling and optimization in contrast to simpler strategies,
where neither is required. As all portfolios are tested with and without an upper bound constraint, the study adds to Jagannathan and Ma's (2003) proposition that constraints can further hedge against estimation errors.

The remainder of this paper is structured as follows. Section two gives a brief introduction to portfolio selection and to previous research on portfolio optimization. Section three outlines my method. Section four contains the empirical results and performance analysis. Section five concludes. j

2. PORTFOLIO SELECTION

There are two essential characteristics of a portfolio according to Markowitz (1952): its expected return and a measure of the dispersion of possible returns around the expected return, the variance being the most tractable estimate (Farrell, 1976). The expressions for portfolio return and portfolio variance can be written as follows:\(^1\):

\[
E(R_p) = w^\prime \mu \\
Var(R_p) = w^\prime \Sigma w
\]

I define \(\mu\) as an \(N\)-vector of expected returns, \(\Sigma\) as the \(N \times N\) covariance matrix of returns and \(w\) as an \(N\)-vector of asset weights, where the \(i\)th weight \(w_i\) is the fraction of the total amount of invested capital in asset \(i\). \(N\) represents the total number of assets in the sample. The expected portfolio return is a weighted average of the expected returns of the individual securities, whereas portfolio variance is the weighted sum of the variance of the individual securities plus twice the covariance between the securities. Markowitz (1952) shows that an efficient portfolio is the portfolio with highest expected return given a level of variance or, equivalently, the portfolio with the lowest variance given a level of expected return.

2.1 MEAN-VARIANCE OPTIMAL PORTFOLIO

In the mean-variance optimal portfolio, the investor optimizes the tradeoff between the mean and the variance of portfolio returns. By definition, the fractions invested in each asset have to sum to one, \(w^\prime \iota = 1\), where \(\iota\) is an \(N\)-vector of ones. Hence, it is the budget constraint and the only binding one. By not including the risk-free asset, the unbounded mean-variance

\(^1\) The prime always denotes the transpose of a matrix or a vector.
optimization problem in its simplest form, is a parametric quadratic programming (PQP) problem (Scherer, 2004, p. 2):

$$\max_w \left\{ w'\mu - \frac{1}{2\lambda} w'\Sigma w \right\} \quad w'1 = 1 \quad (2.3)$$

This is also known as the maximum utility approach, where Equation (2.3) represents the quadratic utility function of a rational investor (Scherer, 2004, p. 2). By maximizing utility for various risk-tolerance parameters, $\lambda$, one can trace the efficient frontier. The higher risk-tolerance, the less weight is given to the portfolio variance, and the more aggressive the portfolio is. The optimal solution for $w$ is found by taking the first derivative with respect to portfolio weights and setting the term equal to zero.

$$\frac{\partial}{\partial w} \left( w'\mu - \frac{1}{2\lambda} w'\Sigma w \right) = \mu - \frac{1}{\lambda} \Sigma w = 0 \quad (2.4)$$

$$w^* = \lambda \Sigma^{-1}\mu \quad (2.5)$$

These are the absolute portfolio weights. To find the relative portfolio weights that fulfill the binding constraint, simply divide by the sum of the portfolio weights.

$$w^* = \frac{\Sigma^{-1}\mu}{\mu'\Sigma^{-1}\mu} \quad (2.6)$$

All optimization models considered in this study can be expressed as in Equation (2.3). The difference is how the strategies estimate expected returns ($\mu$) and return covariances ($\Sigma$). The optimal portfolio is the maximum Sharpe ratio portfolio (Michaud and Michaud, 2008, p. 30). Although financial economists acknowledge the benefits of Markowitz’s portfolio selection model, one of the main reasons why the mean-variance optimizer is not fully used in practical portfolio management is the fact that the mean-variance framework is unstable and sensitive to the input estimates (Ceria and Stubbs, 2006).

2.1.1 Criticism

Most researchers find that the unconstrained optimized portfolios perform poorly out-of-sample. Using Monte Carlo resampling techniques, Jobson and Korkie (1981a) was the first to show the limitations of the unconstrained mean-variance optimizer. The study shows that for a given set of 20 stocks, the unconstrained optimized portfolio has an average maximum Sharpe ratio of 0.08, while the naïve portfolio has an average maximum Sharpe ratio of 0.27.
Returns are by far the most central part of any return distribution for actual portfolio performance. If you are to have good results in any portfolio strategy, you must have good return estimates (Ziemba, 2010). Jorion (1985) argues that alternative estimators for expected returns should be explored to deal with estimation errors, in contrast to the covariance matrix that can be measured with relative precision. Errors in means are even more important at higher levels of risk tolerance. For instance, at a risk tolerance ($\lambda$) of fifty, errors in means are about eleven times as important as error in variances, and the average cash equivalent loss in expected returns (the loss in risk-adjusted return due to errors in estimates) is three times that for errors in variances and five times that for errors in covariances (Chopra and Ziemba, 1993).

On the other hand, Michaud (1989) claims that the fundamental problem is not the level of information in the input estimates, but the level of mathematical sophistication in the optimization algorithm. The mean-variance optimizer operates in such a manner that it enlarges estimation errors tremendously. For instance, Best and Grauer (1992) show that the elasticity of portfolio weights are 14,000 greater than any of the portfolio return variables, and an increase of 11.6 percent in any asset’s expected return would drive half the assets away from an equally weighted optimal portfolio. However, despite the drastic changes in the composition of the portfolio, the expected return and variance changes only by about 2 percent. Additionally, the optimizer does not distinguish between levels of uncertainty associated with the inputs. Differences in means are often not statistically significant, although they could have a colossal impact on the structure of the portfolio (Michaud, 1989).

As the flaws of the mean-variance optimizer piled up, a natural question emerged: Is it possible to mitigate the error maximization tendencies induced by expected returns? The answer was the minimum-variance approach.

2.2 Minimum-Variance Portfolio
One of the most successful portfolio strategies has been to exclude means in the optimizer (see e.g. Behr et al., 2013). Since the minimum-variance strategy does not require expected returns in the optimization process, the expression and solution for the optimal allocation can be thought of as a limited case of Equation (2.3), and can be written as (DeMiguel et al., 2009b):
If the investor ignores expected returns or restricts them such that they are identical across all stocks, we can replace the estimated return vector in Equation (2.6) with a vector of ones; thus solving for the minimum-variance portfolio. The fact that optimal portfolios favor securities with overestimated returns and underestimated variances makes it clear why excluding expected returns would reduce the error maximization property. As the covariance matrix has less impact on asset allocation and is easier to predict relative to returns, an investor need not expend a considerable amount of effort to attain estimates when following the minimum-variance strategy (Broadie, 1993).

Several studies show that the minimum-variance strategy outperforms other portfolio strategies (e.g. Behr et al., 2013; DeMiguel et al., 2009a; Jagannathan and Ma, 2003). Yet, DeMiguel et al. (2009b) finds that although the minimum-variance portfolio performs better than other strategies, it does not statistically outperform an equally-weighted portfolio. Perhaps troubling to practitioners, Green and Hollifield (1992) argue that minimum-variance portfolios with no restrictions will still produce extreme positive and negative weights. In the next section, I discuss the potential implications of imposing weight constraints to solve for some of the issues with optimization and the error maximizing tendency.

2.3 EFFECTS OF CONSTRAINTS
Much of the recent work on portfolio construction emphasizes the impact of constraints. Jagannathan and Ma (2003) and Frost and Savarino (1988) illustrate that certain constraints on the portfolio weights can be interpreted as a form of shrinkage estimation (e.g. Jorion, 1986). It is well known within both the finance and statistics literatures that shrinkage is a tradeoff between bias and variance (Tu and Zhou, 2011).

Prohibiting short sales and/or adding an upper bound constraint can be used as an ad hoc method of circumventing the worst effects of estimation errors (Board and Sutcliffe, 1994). For instance, Jagannathan and Ma (2003) shows that a minimum-variance portfolio subjected to a no-short-sales constraint is equivalent to modifying the covariance matrix of an unconstrained minimum-variance portfolio; the larger elements of the covariance matrix are shrunk towards zero.

\[
\min_w \{ w'\Sigma w \mid w' 1 = 1 \} \quad (2.7)
\]

\[
w_{\text{Min}}^* = \frac{\Sigma^{-1} 1}{1' \Sigma^{-1} 1} \quad (2.8)
\]
However, an implication of shrinkage is that it may induce specification errors. For instance, if the covariance between two assets is large, shrinkage would reduce the relationship. This exacerbates to the trade-off between the two types of errors. If the estimation error is larger (smaller) than the specification errors, prohibiting short sales would produce better (worse) out-of-sample results. By adding the no-short-sales constraint, I can modify the PQP problem to:

\[
\max_w \left\{ w' \mu - \frac{1}{2\lambda} w' \Sigma w \right\} \quad \text{subject to} \quad w' \lambda = 1, \quad w' \geq 0
\]  

(2.9)

Where \( w' \geq 0 \) is the no-short-sales constraint. Imposing upper bounds on portfolio weights have similar effects as the nonnegative constraint. In an unconstrained optimal portfolio, stocks with low covariance tend to receive high portfolio weights. As low covariance estimates are likely to be caused by downward biases, an upper bound constraint will reduce the estimation errors at the cost of potential specification errors, in which we have the same trade-off situation as with prohibiting short sales (Frost and Savarino, 1988). If short sales are already prohibited, an upper bound constraint is less likely to have a significant positive effect on out-of-sample performance even though it may reduce portfolio risk (Jagannathan and Ma, 2003). To add upper bounds in the optimization problem, we can rewrite Equation (2.9) as:

\[
\max_w \left\{ w' \mu - \frac{1}{2\lambda} w' \Sigma w \right\} \quad \text{subject to} \quad w' \lambda = 1, \quad 0 \leq w' \leq 2\%
\]  

(2.10)

Where \( 0 \leq w' \leq 2\% \) is the no-short-sales and one particular upper bound constraint. Recent studies find however that weight constraints can reduce out-of-sample portfolio performance (Clarke et al., 2002; Grinold and Kahn, 2000). Fan et al. (2012) and Jagannathan and Ma (2003) find that the no-short-sales constraint reduces the number of stocks in the optimal portfolio and causes the portfolio to be less diversified. The nonnegative strategy constructs portfolios with only 24 to 40 stocks from a 500-stock universe, depending on the covariance matrix estimate used.

Fan et al. (2012) suggests that by allowing for some short positions, the out-of-sample performance could improve. Guerard et al. (2010) verifies Fan et al. (2012) results by showing that the use of leverage, in a 130 percent long and 30 percent short context, can dominate a long-only strategy. Likewise, Chiou et al. (2009) show that too restrictive constraints on weights could have negative effects on out-of-sample Sharpe ratio when looking at international diversification.
2.4 SUMMARY
Although the potential benefits of optimization are well known within finance, the unconstrained mean-variance strategy is proven to perform poorly out-of-sample, largely due to its error maximizing properties. For the unconstrained mean-variance optimizer to perform up to its potential, accurate return estimates are necessary, which remains an unsolved puzzle.

One solution is to prohibit the portfolio of short selling and/or limiting the portfolio weights by an upper bound; another solution is to exclude returns completely. In contrast to return estimates, return covariance estimates are more precise. However, none of these solutions are optimal. Constraints limit the use of information in return and return covariance estimates; thus, hedging against estimation errors at the cost of introducing specification errors.

On the other hand, the minimum-variance portfolio ignores all information in return estimates and constructs, ex-ante, a portfolio that does not maximize utility. As this study evaluates both the mean-variance and minimum-variance strategy subjected to the no-short-sales and the upper-bound constraint, I can summarize the optimization problems as follows:

**Mean-Variance optimization**

No-short-sales constraint  
\[
\max_w \left\{ w'\mu - \frac{1}{2\lambda} w'\Sigma w \mid w' = 1, w^* \geq 0 \right\}
\]

No-short-sales and upper bound constraint  
\[
\max_w \left\{ w'\mu - \frac{1}{2\lambda} w'\Sigma w \mid w' = 1, 0 \leq w^* \leq 2\% \right\}
\]

**Minimum-Variance optimization**

No-short-sales constraint  
\[
\min_w \left\{ w'\Sigma w \mid w' = 1, w^* \geq 0 \right\}
\]

No-short-sales and upper bound constraint  
\[
\min_w \left\{ w'\Sigma w \mid w' = 1, 0 \leq w^* \leq 2\% \right\}
\]

3. METHOD
This section describes the data used in this study, the estimation models, the portfolio construction, and the performance measures. All estimations, optimizations, and analyzes are performed with MATLAB.
3.1 DATA
The data consists of weekly total returns on all nonfinancial and common domestic listed Swedish stocks on Nasdaq OMX Stockholm (OMXS) from the year 1992 to 2012. During this period, the Swedish stock market has been characterized by bull (2003 - 2007) and bear (1998 - 2002, 2008 - 2012) markets. All data, except the Treasury bill, used in this study are gathered from Datastream. The return on a 30-day Swedish Treasury bill is used as a proxy for the risk-free rate and is obtained from the Riksbank, Sweden's central bank.

Financial firms are excluded from the sample since high leverage does not have the same implications for these firms as for nonfinancial firms, where it is more likely to be an indicator of financial distress (Fama and French, 1992). Following Chan et al. (1999) and Jagannathan and Ma (2003), only common domestic listed stocks are included in the sample, as the market value of a firm that trades by depository receipts does not reflect its actual market value. To minimize survivorship bias in the sample, I include both listed and delisted firms.

The return variable is obtained from Datastream’s total return index. Since the database has issues with missing reverse share splits (Ince and Porter, 2006), all weekly returns above 200 percent are set as missing. Furthermore, since Datastream does not provide market value for delisted firms, the size and the book-to-market variables are created using only active firms on OMXS. Another issue is that Datastream does not provide market value for active firms prior to 2000. To create a variable that represents the market value of a firm prior to 2000, market values for different classes of shares are summarized into one that represents the market value of the company. The Datastream mnemonics and the procedures implemented to attain the variables can be found in Appendix A.

3.2 ESTIMATING THE INPUT PARAMETERS
In practice both expected returns and covariances are unobservable and need to be estimated through historical data or risk models. Estimating expected return by using average realized returns as a proxy suggests that information surprises cancel out over time, and can thus be thought of as an unbiased estimator of future returns. However, as Elton (1999) illustrates, this belief is often misplaced. Duchin and Levy (2009) shows that unstable empirical distribution increases the risk of the optimal portfolio strategy to underperform.

Nonetheless the classical method is to use historical return data. The historical sample estimator is an efficient estimator if and only if the parameters are constant through time.
(Jorion, 1985). The expected return is simply the mean return of the sample. Since no assumptions are made as to how and why stocks move together, the covariance is estimated directly (Elton and Gruber, 1973):

$$
\sigma_{ij} = \frac{1}{T-1} \left( \sum_{t=1}^{T} (R_{it} - \bar{R}_i)(R_{jt} - \bar{R}_j) \right) \tag{3.1}
$$

Where $\sigma_{ij}$ is the covariance between asset $i$ and $j$ and $\bar{R}_i$ is asset $i$’s mean return. An alternative approach is to assume a behavioral model of why securities move together. Chan et al. (1999) illustrates the difference in the ability of factor models to produce good estimates of the covariance matrix. Although no preferred model appears, the study concludes that a three-factor model produce, more than often, good enough estimates of the actual covariance matrix, and that the single-factor model only performs marginally worse than other high-dimensional models.

Ledoit and Wolf (2003) argues that the choice of number of factors in the risk model is ad hoc, and that there is no way of telling which model performs best without looking out-of-sample. Even if a particular factor model performs well in one sample, the results may not be applicable in another sample. Although asset pricing models have been under scrutiny with empirical results questioning the usefulness of the models, MacKinlay and Pástor (2000) show that casting them aside could be premature. In this study, I evaluate the single-factor model and the three-factor model’s usefulness within the context of portfolio selection. Ordinary least squares (OLS) is used to estimate the coefficients for both factor models. The proxy for the market index in the regressions throughout the study is the OMXS index.

3.2.1 THE SINGLE-FACTOR MODEL
The simplest factor model was proposed by Sharpe (1963), where he assumes that stocks move together because of a common response to changes in a market index. The single-factor “market” model can be written as:

$$
E(R_{it}) - R_f = \alpha_i + \beta_i [E(R_{mt}) - R_f] + \epsilon_{it} \tag{3.2}
$$

Where $E(R_i)$ is the expected return on asset $i$, $R_f$ is the return on the risk-free asset, $E(R_m)$ is the expected return on the market index, $\alpha_i$ is the abnormal return on asset $i$, $\beta_i$ is a measure of the responsiveness of asset $i$ to the market index and $\epsilon_i$ is the asset-specific return (residual term). As this model assumes that the correlation between stocks completely depends on each
should you optimize your portfolio?

Stock’s sensitivity to the market, the estimated covariance matrix, $\Sigma$, is (Ledoit and Wolf, 2003):

$$\Sigma = \sigma_m^2 \beta \beta' + \sigma^2$$

(3.3)

Where $\sigma_m^2$ is the variance of the market index, $\beta$ is the vector of estimated betas for the assets in the sample and $\sigma^2$ is the diagonal matrix of the residual variance.

3.2.2 THE THREE-FACTOR MODEL

The three-factor model suggested by Fama and French (1992, 1993, 1996) augments the single-factor model with size (market capitalization) and book-to-market equity (BE/ME) as systematic risk measures. The authors find that firms with low BE/ME tend to have low earnings on assets that persist for five years before and five years after the ratio is measured. They also find a negative relationship between size and average returns. Following Fama and French (1993), I create six portfolios formed upon market capitalization and book-to-market equity (BE/ME).

Each week, all firms in the sample are measured on market capitalization (MC), where the ones with market capitalization higher (lower) than the median are classified as Big (Small). All firms are also ranked according to BE/ME, where the highest (lowest) 30 percent of the sample are classified as High (Low) BE/ME, while the firms in the middle 40 percent are classified as Medium BE/ME. Table 1 illustrates the structure of the portfolios.

<table>
<thead>
<tr>
<th>Table 1. Fama-French portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td>High BE/ME (Top 30%)</td>
</tr>
<tr>
<td>Big Firms (Top 50% MC)</td>
</tr>
<tr>
<td>Small Firms (Bottom 50% MC)</td>
</tr>
</tbody>
</table>

Each week, assets are sorted on size and book-to-market ratio into six Fama-French portfolios: S/L, S/M, S/H, B/L, B/M, B/H. For instance, S/L consists of the stocks that are smaller than the medium firm in terms of market value and that have book-to-market ratios that are at the bottom 30 percent of all stocks on the market.

From the resulting six portfolios, two variables are created to mimic the risk factor in returns related to size (“small minus big” or “SMB”) and to book-to-market equity (“high minus low” or “HML”). SMB is the difference between the simple average return on the three small portfolios (S/H, S/M, S/L) and the three big portfolios (B/H, B/M, B/L), while HML is the difference between the simple average return on the two high BE/ME portfolios (B/H, S/H).
and the two low BE/ME portfolios (B/L, S/L). The variables are computed each week from the dataset. Thus, the three-factor model can be written as follows:

\[
E(R_{it}) - R_f = \alpha_i + \beta_i[E(R_{mv}) - R_f] + s_i E(SMB_t) + h_i E(HML_t) + \varepsilon_{it}
\] (3.4)

Where \( E(SMB) \) is the excess expected return related to size, \( E(HML) \) is the expected excess return related to book-to-market equity, and \( \beta_i, s_i, \) and \( h_i \) measures the responsiveness of asset \( i \) to the respective risk factors. The three-factor model assumes that the co-movement depends on all three risk factors rather than only on the market. Therefore, the estimated covariance matrix, \( \Sigma \), of asset returns is (Chan et al., 1999):

\[
\Sigma = B\Omega B' + \sigma^2
\] (3.5)

Where \( \Omega \) is the \( 3 \times 3 \) covariance matrix of factor returns, \( B \) is the \( N \times 3 \) matrix of factor betas and \( \sigma^2 \) is the diagonal matrix of the residual variance.

### 3.3 Benchmark Portfolios

I consider two groups of benchmark strategies for the analysis: the equally-weighted (naïve) and the value-weighted (market) strategy. The naïve strategy invests an equal amount of capital among the risky assets, while the market strategy invests in each security proportional to its market capitalization. These strategies have one important advantage over the mean-variance method, which is that they are not prone to estimation errors. As these strategies neither require forecast models nor optimization, they are comfortable and easily implementable strategies for investors (Duchin and Levy, 2009). Not surprisingly, studies reveal that individual investors use these strategies much more than expected utility theory would suggest (Benartzi and Thaler, 2001).

### 3.4 Portfolio Construction

I use the rolling horizon method of DeMiguel et al. (2009b) for comparison. First, starting at the end of January 1998, I use 260 weeks (60 months) of historical data to estimate the expected returns and the covariance matrices for all considered models. This is the in-sample (estimation) window. All stocks included in the in-sample window have been active for the previous five years, thereby excluding delisted firms in the in-sample window. I then form the two mean-variance portfolios and the two minimum-variance portfolios for each of the three models. Figure 1 shows the number of active firms in the estimation window from January 1998 to November 2012 and Table 2 lists all portfolio strategies in this study.
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Figure 1. Number of stocks in the estimation window

The figure shows the number of stocks in each estimation window from January 1998 to November 2012. The number of stocks increases from 166 to 265 between June 2002 and January 2006. From that point on, the number of stocks in each estimation window starts to stagnate.

Table 2. List of portfolio strategies.

<table>
<thead>
<tr>
<th>#</th>
<th>Description</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A. Mean-Variance Portfolios (MV)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Single-factor model with no-short-sales constraint</td>
<td>MV₁F,C</td>
</tr>
<tr>
<td>2</td>
<td>Single-factor model with no-short-sales and upper-bound constraint</td>
<td>MV₁F,D</td>
</tr>
<tr>
<td>3</td>
<td>3-factor-model with no-short-sales constraint</td>
<td>MV₃F,C</td>
</tr>
<tr>
<td>4</td>
<td>3-factor-model with no-short-sales and upper-bound constraint</td>
<td>MV₃F,D</td>
</tr>
<tr>
<td>5</td>
<td>Historical model with no-short-sales constraint</td>
<td>MV₉, C</td>
</tr>
<tr>
<td>6</td>
<td>Historical model with no-short-sales and upper bound constraint</td>
<td>MV₉, D</td>
</tr>
<tr>
<td>Panel B. Minimum-variance portfolios (Min)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Single-factor model with no-short-sales constraint</td>
<td>Min₁F,C</td>
</tr>
<tr>
<td>8</td>
<td>Single-factor model with no-short-sales and upper-bound constraint</td>
<td>Min₁F,D</td>
</tr>
<tr>
<td>9</td>
<td>3-factor-model with no-short-sales constraint</td>
<td>Min₃F,C</td>
</tr>
<tr>
<td>10</td>
<td>3-factor-model with no-short-sales and upper-bound constraint</td>
<td>Min₃F,D</td>
</tr>
<tr>
<td>11</td>
<td>Historical model with no-short-sales constraint</td>
<td>Min₉, C</td>
</tr>
<tr>
<td>12</td>
<td>Historical model with no-short-sales and upper bound constraint</td>
<td>Min₉, D</td>
</tr>
<tr>
<td>Panel C. Benchmark Portfolios</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Market (Value-weighted)</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>Naïve (Equally-weighted)</td>
<td>-</td>
</tr>
</tbody>
</table>

The table lists all considered strategies in this study and their abbreviation. Panel A lists the mean-variance strategies. Panel B lists the minimum-variance strategies. Panel C lists the benchmark strategies. The “C” indicates that the portfolio is subjected to the no-short-sales constraint, while the “D” indicates that the portfolio is subjected to both the no-short-sales and the upper bound constraint.
Second, after forming the portfolios, all strategies are tested in the following four weeks. This is the out-of-sample (event) window which I evaluate. To deal with changes in portfolio weights caused by fluctuations in stock prices, all portfolios are constantly rebalanced according to their strategy. If a firm is delisted in the out-of-sample window, remaining returns are set equal to zero. The naïve portfolio includes all assets that fulfill the criteria to be included in the in-sample window. The market portfolio, however, should be viewed as a portfolio that has a perfect correlation with the market index, and therefore includes all active assets on the market. Third, I roll the estimation and event window four weeks forward, and repeat this procedure until the end of the dataset is reached, which contains 194 out-of-sample periods.

I use a 60-month estimation window (or 260 weekly observations) in contrast to Bengtsson and Holst (2002)’s findings that the eight year in-sample window performs best on the Swedish stock market. A difference between this study and Bengtsson and Holst (2002) is that I use weekly observations rather than monthly since a greater in-sample window does increase the risk of including old and outdated information when estimating parameters. However, DeMiguel et al. (2009b) find that the result of a 60-month window is not very different from the 120-month window. Therefore, a 60-month window is regarded as sufficient.

A limitation of this study is that the effects of transaction costs are excluded. Rebalancing the portfolios each month would, in practice, be associated with high transaction costs, thus eroding the total gain in wealth. In theory, this would affect the optimized portfolios the most. Therefore, the results should be interpreted with a bit caution.

3.5 Measuring Performance
Following DeMiguel et al. (2009b)’s method of measuring portfolio performance, I evaluate the out-of-sample performance of all portfolio strategies using two measures: Sharpe ratio, \( SR \) and return-loss, \( RL \).

The most natural and standardized performance measurement is the Sharpe ratio (Sharpe, 1966), as it measures the excess return per unit of risk. The Sharpe ratio prevents portfolios to be measured solely on return, since high average returns are typically obtained by higher risk. Thus, the Sharpe ratio rewards low risk portfolios that earn high average returns. From the out-of-sample periods, I measure the average Sharpe ratios for each strategy, defined as the
sample mean, $\hat{\mu}$, of the out-of-sample excess return over the risk-free asset divided by the sample standard deviation, $\hat{\sigma}$.

$$SR = \frac{\hat{\mu} - r_f}{\hat{\sigma}}$$  \hspace{1cm} (3.6)

In order to assess whether the difference in Sharpe ratio performance is statistically significant, I follow Jobson and Korkie (1981b)’s method and compute p-values. The purpose of this study is to evaluate whether the optimized strategy outperforms the market and naïve strategy in terms of out-of-sample Sharpe ratio. Therefore, I can formulate following hypothesis:

$$H_0: SR_k \leq SR_b$$

Where $k$ is the optimized strategy and $b$ is the benchmark strategy. The null hypothesis is rejected at the 0.05 level of significance. The Sharpe ratio $Z$-statistics can be found by:

$$Z_{jk} = \frac{\hat{\mu}_k \hat{\sigma}_b - \hat{\mu}_b \hat{\sigma}_k}{\sqrt{\hat{\theta}}}$$  \hspace{1cm} (3.7)

Jobson and Korkie (1981b) assumes normal distribution with mean $\hat{\mu}_k \hat{\sigma}_b$, where $\hat{\mu}_k$ is the sample mean of strategy $k$ and $\hat{\sigma}_b$ is the standard deviation of strategy $b$. The variance, $\hat{\theta}$, is given by:

$$\hat{\theta} = \frac{1}{T} \left( 2\hat{\sigma}_k^2 \hat{\sigma}_b^2 - 2\hat{\sigma}_b \hat{\sigma}_k \hat{\sigma}_{b,k} + \frac{1}{2} \hat{\mu}_k^2 \hat{\sigma}_b^2 + \frac{1}{2} \hat{\mu}_b^2 \hat{\sigma}_k^2 - \frac{\hat{\mu}_k \hat{\mu}_b}{2 \hat{\sigma}_b \hat{\sigma}_k} \left( \hat{\sigma}_{k,b}^2 + \hat{\sigma}_k^2 \hat{\sigma}_b^2 \right) \right)$$  \hspace{1cm} (3.8)

The test statistics hold under the assumption that returns are distributed independently and identically over time with a normal distribution. However, this assumption is typically violated (DeMiguel et al., 2009b).

The return-loss is measured as a complement to the Sharpe ratio. It shows the additional return that would be needed for a strategy to perform as well as the benchmark portfolios in terms of Sharpe ratio (DeMiguel et al., 2009b). A positive return-loss value shows the additional return needed to perform as well as the benchmark strategy, while a negative value shows the loss in terms of Sharpe ratio of undertaking the benchmark strategy. The return-loss between the benchmark strategy $b$ and the optimized strategy $k$ is:

$$RL_{b,k} = \frac{\hat{\mu}_b}{\sigma_b} \times \sigma_k - \hat{\mu}_k$$  \hspace{1cm} (3.9)
4. EMPIRICAL RESULTS AND PERFORMANCE ANALYSIS

This section consists of the empirical results and analysis of the out-of-sample performance. I have constructed 12 optimized portfolios, each with a unique feature, in order to find a portfolio strategy that outperforms the naïve and market strategy. First, I present some descriptive statistics for each strategy (Table 3). Second, I analyze the out-of-sample Sharpe ratio and the return-loss (Table 4). Third, I test each portfolio against the two benchmark portfolios and present the p-value (Table 5).

4.1 DESCRIPTIVE STATISTICS

Reported in Table 3 are the monthly out-of-sample means, monthly standard deviations, and cumulative change in wealth by following a strategy from January 1998 to December 2012. Expressed in the parentheses are the annualized values. The standard deviation is annualized through multiplication by $\sqrt{T_2}$. The bolded figures are the best performers within each panel.

The first panel in Table 3 contains the empirical results of mean-variance optimal portfolios. Not surprisingly, the worst strategy when including returns in the optimization process is the historical model. This implies that the factor models produce better return estimates. The double constrained historical portfolio yielded on average approximately 8 percent annually, whereas the best strategy, nonnegative constrained three-factor portfolio, yielded 11.2 percent annually.

On the other hand, I find that the historical model constructs the best minimum-variance strategies. This explicitly infers that the historical sample estimate is better at predicting comovement among assets than either of the two behavioral models. The historical model’s nonnegative constrained minimum-variance portfolio outperformed all other optimized portfolios in all categories.

I find that the minimum-variance portfolios achieve lower risk than the mean-variance portfolios. Bengtsson and Holst (2002) find that the single-factor minimum-variance portfolio has a lower standard deviation than the historical sample estimate. As Table 3 presents, this observation does not correspond to the results of this study. On the other hand, the mean-variance strategy has the objective to maximize Sharpe ratio and should, in theory, yield higher returns than the minimum-variance strategy. While this is true for the factor models, the presence of the sample mean estimates reduces portfolio return. However, the increase in standard deviation is larger than the increase in mean return for the factor models.
Table 3. Portfolio return and risk

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Total change in wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MV$_{1F}$, C</td>
<td>0.0087</td>
<td>(10.93%)</td>
<td>0.0449</td>
</tr>
<tr>
<td>MV$_{1F}$, D</td>
<td>0.0077</td>
<td>(9.67%)</td>
<td>0.0456</td>
</tr>
<tr>
<td>MV$_{3F}$, C</td>
<td><strong>0.0089</strong></td>
<td>(11.19%)</td>
<td>0.0446</td>
</tr>
<tr>
<td>MV$_{3F}$, D</td>
<td>0.0076</td>
<td>(9.57%)</td>
<td>0.0454</td>
</tr>
<tr>
<td>MV$_{H}$, C</td>
<td>0.0081</td>
<td>(10.15%)</td>
<td><strong>0.0427</strong></td>
</tr>
<tr>
<td>MV$_{H}$, D</td>
<td>0.0065</td>
<td>(8.04%)</td>
<td>0.0459</td>
</tr>
<tr>
<td><strong>Panel B.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min$_{1F}$, C</td>
<td>0.0083</td>
<td>(10.39%)</td>
<td>0.0363</td>
</tr>
<tr>
<td>Min$_{1F}$, D</td>
<td>0.0077</td>
<td>(9.59%)</td>
<td>0.0403</td>
</tr>
<tr>
<td>Min$_{3F}$, C</td>
<td>0.0085</td>
<td>(10.69%)</td>
<td>0.0350</td>
</tr>
<tr>
<td>Min$_{3F}$, D</td>
<td>0.0083</td>
<td>(10.40%)</td>
<td>0.0399</td>
</tr>
<tr>
<td>Min$_{H}$, C</td>
<td><strong>0.0088</strong></td>
<td>(11.11%)</td>
<td><strong>0.0348</strong></td>
</tr>
<tr>
<td>Min$_{H}$, D</td>
<td>0.0078</td>
<td>(9.76%)</td>
<td>0.0395</td>
</tr>
<tr>
<td><strong>Panel C.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>0.0053</td>
<td>(6.59%)</td>
<td>0.0615</td>
</tr>
<tr>
<td>Naive</td>
<td><strong>0.0056</strong></td>
<td>(6.89%)</td>
<td><strong>0.0576</strong></td>
</tr>
<tr>
<td><strong>Panel D.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>True Optimal, C</td>
<td><strong>0.2356</strong></td>
<td>(1.166%)</td>
<td>0.0244</td>
</tr>
<tr>
<td>True Optimal, D</td>
<td>0.0846</td>
<td>(165%)</td>
<td><strong>0.0044</strong></td>
</tr>
</tbody>
</table>

This table displays the monthly out-of-sample mean return standard deviation and the cumulative change in wealth by following a strategy from January 1998 to December 2012. The annualized values are presented in the parentheses and expressed in percentages. The standard deviation is annualized through multiplication by $\sqrt{T \Delta}$. Panel A lists the mean-variance strategies, Panel B lists the minimum-variance strategies and Panel C lists the benchmark strategies. The “C” indicates that the portfolio is subjected to the no-short-sales constraint, while the “D” indicates that the portfolio is subjected to both the no-short-sales and the upper bound constraint. The bolded figures are the best performers within each panel.

The third panel reports the results of the benchmark strategies. The empirical results show that market strategy is the worst strategy in all three categories. Chan et al. (1999) finds that the market strategy yields less and is riskier than the minimum-variance strategy. However, they show that the naïve strategy yields more than one percent annually than the minimum-
variance strategy. I find that this is not true on the Swedish stock market, where the naïve strategy only performs slightly better than the market strategy. Both the naïve and the market strategy have a larger standard deviation than the optimized strategies; yet, their monthly mean return is approximately 0.3 percent lower. The difference in risk between the benchmark and the optimized strategies is striking.

Panel D in Table 3 shows the performance of the true, ex-post, mean-variance optimal portfolio in each month, i.e., the actual results of the optimal mean-variance strategy given zero estimation errors. Comparing these results with that of the ex-ante optimized strategies shows that the difference is substantial in the dataset. The standard deviation is lower for the double constrained true optimal portfolio since it cannot utilize all the information, thus yielding a more stable return over time compared to the nonnegative constrained true optimal portfolio.

Furthermore, the main argument for adding a weight constraint is that it reduces the effects of estimation errors by shrinking the extreme values towards zero. In contrast to Jagannathan and Ma (2003) - who find that, although not significant, imposing an upper bound constraint can reduce risk - I find that the upper bound constraint actually increase portfolio risk. This is true for each of the portfolio strategies I test. The risk reduction that occurs when excluding the upper bound constraint can be interpreted in three ways. First, the optimal no-short-sale portfolios are already diversified and an upper bound constraint is unnecessary. Second, the upper bound constraint of 2 percent is too severe; thus reducing the benefits of optimizing. Third, the specification errors outweigh the effects of estimation errors, implying that all three models produce good estimates.

The last column in Table 3 shows the actual change in wealth of each strategy over all time periods. The nonnegative constrained minimum-variance portfolio increased the most for each estimation model. For these portfolios and the benchmark, Figure 2 displays the change in wealth from January 1998 to December 2012. Assume $1 is invested in each of these strategies at January 1998. The optimized strategies follow each other to a great extent until the financial crisis in 2007. The historical minimum-variance strategy performed better than the factor models after August 2007 and peaked in 2011. The naïve strategy deviates from the optimized strategies in 2001, while the market strategy changes course at the end of 1999. The evolution over shorter intervals can be found in Appendix B.
Figure 2. Change in Wealth of $1 invested Jan 1998 - Dec 2012

![Figure 2](image_url)

The figure shows the monetary change in wealth of each nonnegative constrained minimum-variance strategy, the market strategy and naïve strategy from January 1998 to December 2012. Assume you invest $1 in the Historical no-short-sales constrained minimum-variance portfolio in January 1998. At the end of 2012 (2007), the portfolio would be worth approximately $7.2 ($6.2).

4.2 **Sharpe Ratio Performance**

In order to assess the magnitude of potential advantages of optimizing, it is essential to analyze the out-of-sample Sharpe ratio. Table 4 displays the monthly out-of-sample Sharpe ratios and return-loss relative to the market and naïve portfolio.

The nonnegative constrained three-factor portfolio obtained the highest Sharpe ratio of the mean-variance portfolios. The loss in return is approximately 0.5 percent a month, or 6.2 percent annually, by switching strategy to either the market or the naïve strategy. In general, the single-factor model and the three-factor model yield similar optimal portfolios, although the three-factor model performs slightly better. This coincides with Chen et al. (1999)’s results. By comparing the minimum-variance portfolio against the corresponding mean-variance portfolio, I find that the expected returns estimated by the single-factor model affects the portfolio outcome the least; an indication that it produces better return estimates than the three-factor and historical models.

The worst optimal strategy was the double constrained historical mean-variance strategy; the best strategy was the nonnegative constrained historical minimum-variance strategy. Again, this shows the poor estimating ability of returns by the historical model and the effects of estimation errors in expected returns. The key concern with the historical model is that it
overfits the comovement; hence, a poor estimate of return covariance. My result suggests that Swedish stocks, in general, move accordingly and that the historical sample estimate is a valid estimator of covariance when compared to the single-factor and three-factor model.

Table 4. Sharpe Ratio and Return-Loss (RL)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Sharpe Ratio</th>
<th>RL_{Market,k}</th>
<th>RL_{Naïve,k}</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MV_{1F}, C</td>
<td>0.1932</td>
<td>-0.0048</td>
<td>-0.0043</td>
</tr>
<tr>
<td>MV_{1F}, D</td>
<td>0.1692</td>
<td>-0.0038</td>
<td>-0.0033</td>
</tr>
<tr>
<td>MV_{3F}, C</td>
<td><strong>0.1990</strong></td>
<td><strong>-0.0050</strong></td>
<td><strong>-0.0046</strong></td>
</tr>
<tr>
<td>MV_{3F}, D</td>
<td>0.1684</td>
<td>-0.0037</td>
<td>-0.0033</td>
</tr>
<tr>
<td>MV_{H}, C</td>
<td>0.1893</td>
<td>-0.0044</td>
<td>-0.0040</td>
</tr>
<tr>
<td>MV_{H}, D</td>
<td>0.1408</td>
<td>-0.0025</td>
<td>-0.0020</td>
</tr>
<tr>
<td><strong>Panel B.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min_{1F}, C</td>
<td>0.2280</td>
<td>-0.0051</td>
<td>-0.0048</td>
</tr>
<tr>
<td>Min_{1F}, D</td>
<td>0.1898</td>
<td>-0.0042</td>
<td>-0.0038</td>
</tr>
<tr>
<td>Min_{3F}, C</td>
<td>0.2428</td>
<td>-0.0055</td>
<td>-0.0051</td>
</tr>
<tr>
<td>Min_{3F}, D</td>
<td>0.2075</td>
<td>-0.0048</td>
<td>-0.0044</td>
</tr>
<tr>
<td>Min_{H}, C</td>
<td><strong>0.2535</strong></td>
<td><strong>-0.0058</strong></td>
<td><strong>-0.0055</strong></td>
</tr>
<tr>
<td>Min_{H}, D</td>
<td>0.1972</td>
<td>-0.0044</td>
<td>-0.0040</td>
</tr>
<tr>
<td><strong>Panel C.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>0.0867</td>
<td>-0.0006</td>
<td>0.0006</td>
</tr>
<tr>
<td>Naïve</td>
<td><strong>0.0967</strong></td>
<td>-0.0006</td>
<td>-</td>
</tr>
</tbody>
</table>

This table displays the monthly out-of-sample Sharpe ratio and the return-loss relative to the market and naïve portfolio for each strategy listed in Table 2. Panel A lists the mean-variance strategies. Panel B lists the minimum-variance strategies. Panel C lists the benchmark strategies. The “C” indicates that the portfolio is subjected to the no-short-sales constraint, while the “D” indicates that the portfolio is subjected to both the no-short-sales and the upper bound constraint. The bolded figures are the best performers within each panel.

In general, adding the upper bound constraint or including returns when optimizing reduces the Sharpe ratio. The Sharpe ratio is on average about 0.04 and 0.045 lower for the upper bound constrained portfolios and mean-variance portfolios, respectively, compared to the no-short-sales constrained minimum-variance portfolios. Although this indicates that the
nonnegative minimum-variance strategy outperforms the other strategies, I find that the difference in Sharpe ratio is not statistically significant at the 0.05 level.

The differences in terms of out-of-sample Sharpe ratio between the optimized and the simple strategies deliver prominent results. While the worst optimized portfolio attained a Sharpe ratio of approximately 0.14, the market and naïve strategy obtained Sharpe ratios of 0.087 and 0.097, respectively. Behr et al. (2013) find that the nonnegative constrained minimum-variance strategy had a monthly out-of-sample Sharpe ratio of 0.1265, while the Sharpe ratio of the market and naïve strategy was 0.0754 and 0.0842, respectively. Although their results on the benchmark strategies are similar to that of this study, I find that the benefits of optimization are considerably larger on the Swedish stock market.

These results cast a serious doubt over the claim that the naïve portfolio cannot be beaten. To determine if the difference in performance is statistically significant, I implement Jobson and Korkie (1981b)’s method and compute the p-value. Table 5 reports the p-value of the Sharpe Ratio $Z$-test for each strategy. The null hypothesis is rejected at the 0.05 level of significance. One, two, or three stars (*) is printed if the null hypothesis is rejected at either the 0.05, 0.01 or 0.001 level, respectively.

Panel A consist of the test results of the mean-variance optimized strategies against the two benchmark strategies. All mean-variance portfolios except the double constrained historical portfolio have significantly higher Sharpe ratios than the market and naïve portfolio. This contradicts the results of DeMiguel et al. (2009b) who find that the naïve strategy significantly outperforms the mean-variance strategy. The best mean-variance strategy, the no-short-sales constrained three-factor portfolio, is significant at the 0.01 level (p-value 0.0078) when tested against the naïve portfolio.

I find that all minimum-variance portfolios have significantly higher Sharpe ratio than both benchmark strategies. It is interesting that the nonnegative constrained three-factor and historical portfolio are the best strategies against the market portfolio, while the double constrained three-factor and historical portfolio are the best strategies against the naïve portfolio. The apparent contradictive result stems out of differences in correlation between the optimized strategies and the benchmark strategies, since higher correlation yield stronger power in the Sharpe ratio $Z$-test. On average, an investor would increase their portfolio Sharpe ratio by 120 percent by switching from the simple strategy to the optimized strategy.
The results of the Sharpe ratio Z-test strongly suggest that an investor should optimize his or her portfolio.

### Table 5. Sharpe Ratio Z-test

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Panel A</th>
<th>Panel B</th>
<th>Panel C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_0: SR_k \leq SR_{Market}$</td>
<td>$H_0: SR_k \leq SR_{Naive}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$p$-value</td>
<td>$p$-value</td>
<td></td>
</tr>
<tr>
<td>MV$_{1F}$, C</td>
<td>0.0204*</td>
<td>0.0125*</td>
<td>MV$_{1F}$, C</td>
</tr>
<tr>
<td>MV$_{1F}$, D</td>
<td>0.0357*</td>
<td>0.0281*</td>
<td>MV$_{1F}$, C</td>
</tr>
<tr>
<td>MV$_{3F}$, C</td>
<td><strong>0.0163</strong></td>
<td><strong>0.0054</strong></td>
<td>MV$_{3F}$, C</td>
</tr>
<tr>
<td>MV$_{3F}$, D</td>
<td>0.0352*</td>
<td>0.0113*</td>
<td>MV$_{3F}$, C</td>
</tr>
<tr>
<td>MV$_{H}$, C</td>
<td>0.0345*</td>
<td><strong>0.0074</strong></td>
<td>MV$_{H}$, C</td>
</tr>
<tr>
<td>MV$_{H}$, D</td>
<td>0.1132</td>
<td>0.0145*</td>
<td>MV$_{H}$, D</td>
</tr>
<tr>
<td>Market</td>
<td>1.000</td>
<td>0.1000</td>
<td>Market</td>
</tr>
<tr>
<td>Naive</td>
<td>0.3895</td>
<td>1.000</td>
<td>Naive</td>
</tr>
</tbody>
</table>

This table reports the p-value of the Jobson and Korkie (1981b) Z-test. The null hypothesis is rejected at the 0.05 level of significance. One, two or three stars (*) is printed if the null hypothesis is rejected at either the 0.05, 0.01 or 0.001 level. Panel A lists the mean-variance strategies. Panel B lists the minimum-variance strategies. Panel C lists the benchmark strategies. The “C” indicates that the portfolio is subjected to the no-short-sales constraint, while the “D” indicates that the portfolio is subjected to both the no-short-sales and the upper bound constraint. The bolded figures are the best performers within each panel.

DeMiguel et al. (2009b) shows that constrained minimum-variance strategy outperforms the naïve strategy in four out of six samples, while only once significantly. Again, the market strategy is found to be the worst strategy among the three. However, the main results stems out of simulated data, where the authors find that the naïve strategy cannot be beaten. An
important difference between this study, Behr et al. (2013), and DeMiguel et al. (2009b) is the sample size. As Figure 1 illustrates, the sample size varies between 169 and 290 stocks in this study, while the largest sample in Behr et al. (2013) and DeMiguel et al. (2009b) consist of 100 and 50 stocks, respectively. The problem is that it becomes more difficult to justify normal distribution with smaller samples (Jagannathan and Ma, 2003). Duchin and Levy (2009) argues that loss caused by the employment of the naïve strategy is a function of the number of assets held in the portfolio. Therefore, while the naïve strategy may be irrational for institutional investors, it may not be as irrational for household investors, since they often choose among a few assets.

5. CONCLUSION

In this paper, I reexamine the statement that simple portfolio strategies outperform optimized portfolio strategies on the Swedish stock market. The portfolio optimizer has come into question due to the difficult task of estimating returns and return covariances ex-ante, in addition to the error maximizing tendencies of the optimizer. Several studies show that simple strategies, such as the naïve, yield higher Sharpe ratio than optimized strategy. In order to find the preferred procedure for attaining the ex-ante best optimal portfolio, I have provided three estimation models – the historical sample estimator, the single-factor model, and the three-factor model – with respect to estimating the input parameters of the optimizer. This aggregates to 12 unique optimized portfolios that have been tested on out-of-sample Sharpe ratio performance against the market and naïve strategy.

I find that 11 out of 12 optimized portfolios significantly outperform the market and naïve portfolio in terms of Sharpe ratio; thus, the results disputes the conclusion of DeMiguel et al. (2009b) that the naïve portfolio is preferred over the ex-ante optimal portfolio. Even in the presence of estimation errors, an investor would be noticeably better off by employing the optimized strategy rather than the market strategy or the naïve strategy on the Swedish stock market. The sizeable difference in performance between the optimized portfolios and the benchmark portfolios amounts to an average Sharpe ratio increase of 120 percent. Although I have excluded the effects of transaction costs on Shape ratio performance, my results suggest that investors should implement the optimized strategy rather than simple strategies, such as the popular passive market portfolio, on the Swedish stock market.
Although my primary focus in this paper has been to test optimized portfolios against simple portfolios, my findings suggest that the historical sample estimator is a better model for estimating the covariance matrix in the constrained minimum-variance portfolio compared to the factor models. However, the factor models produce better expected return estimates, and are therefore preferred in the constrained mean-variance portfolio. In contrast to previous studies, I find that the upper-bound constraint of 2 percent actually increases portfolio standard deviation in the presence of the no-short-sale constraint. In other words, I find that investors should not add the upper bound constraint in a realistic setting where short selling is often not viable, and therefore binding.

The difference between the true optimal portfolio and the ex-ante estimated optimal portfolio should raise some concern over the ability of the estimation models to produce accurate estimates. As the three-factor model and the single-factor model are outperformed, although not significantly, by the historical sample estimator with respect to estimating the return covariance matrix for the constrained minimum-variance portfolio, a better behavioral model of stock returns should be explored.

Future research should include transaction costs when testing optimized portfolio against simple portfolios, since the presence of these costs may have significant effects. Since the Sharpe ratio Z-test of Jobson and Korkie (1981) becomes less valid when returns have tails heavier than normal distribution, future research should also implement the preferred method of Ledoit and Wolf (2008) when evaluating relative Sharpe ratio performance.
6. REFERENCES


APPENDIX A: DATASTREAM VARIABLES

Table 6 displays all variables used in this study and their Datastream mnemonic.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Datastream mnemonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Value For Company</td>
<td>MVC</td>
</tr>
<tr>
<td>Market Value</td>
<td>MV</td>
</tr>
<tr>
<td>Total Return Index</td>
<td>RI</td>
</tr>
<tr>
<td>Common Equity</td>
<td>WC03501</td>
</tr>
</tbody>
</table>

This table displays all variables obtained from Datastream and the corresponding Datastream mnemonic.

Since Datastream does not provide the MVC variable prior to 2000 for active firms, the market value for the company is approximated by the total value of the outstanding classes of shares. For instance, if the market value for company X’s share A and B is 5 and 10, respectively, the cumulative market value for the company is approximated by $5 + 10 = 15$. The book-to-market variable is the company’s book-equity relative to their market equity. Thus, it is calculated through:

$$BM_t = \frac{WC03501_t}{MVC_t} \quad \text{or} \quad BM_t = \frac{WC03501_t}{MV_{A,t} + MV_{B,t}}$$
APPENDIX B: CHANGE IN WEALTH

Figure 3. Change in Wealth of $1 invested January 1998 - December 2002

The figure shows the monetary change in wealth of each nonnegative constrained minimum-variance strategy, the market strategy and naïve strategy from January 1998 to December 2002. Assume you invest $1 in the Market portfolio in January 1998. At the end of 2002 (2000), the portfolio would be worth about $1.03 ($1.8). Although the bear market in 2000 onward affected the Naïve portfolio, the optimized portfolios actually increased the wealth of their investors.

Figure 4. Change in Wealth of $1 invested January 2003- July 2007

The figure shows the monetary change in wealth of each nonnegative constrained minimum-variance strategy, the market strategy and naïve strategy from January 2003 to July 2007. Assume you invest $1 in the Naïve portfolio in January 2003. At July 2007, the portfolio would be worth approximately $3.8. The Naïve strategy outperformed the other strategies during this bull market period.
Figure 5. Change in Wealth of $1 invested August 2007 - January 2010

The figure shows the monetary change in wealth of each nonnegative constrained minimum-variance strategy, the market strategy and naïve strategy from August 2007 to January 2010. Assume you invest $1 in the Market portfolio in August 2007. At January 2010, the portfolio would be worth approximately $0.8. The Historical model survived the financial crises in 2007 better than the other strategies, while the Single-factor model and the three-factor model performed almost equally.

Figure 6. Change in Wealth of $1 invested January 2010 - December 2012

The figure shows the monetary change in wealth of each nonnegative constrained minimum-variance strategy, the market strategy and naïve strategy from January 2010 to December 2012. Assume you invest $1 in the Market portfolio in January 2010. At December 2012, the portfolio would be worth approximately $1.2, while the other strategies actually lost $0.1 in wealth.