LARGE SCALE YIELDING FATIGUE CRACK GROWTH

- A Literature Survey

Pär Ljustell

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Department of Solid Mechanics
KTH School of Engineering Sciences
Royal Institute of Technology
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Abstract

In this literature survey report an overview of different issues connected to fatigue crack growth under large scale yielding is given. Nonlinear measures as the $J$-integral and the Crack Tip Opening Displacement ($CTOD$) are reviewed and the requirements for path independence of the $J$-integral are highlighted. The extension of the measures to the fatigue situation, i.e. $\Delta J$ and $\Delta CTOD$ are also reviewed from a theoretical as well as an experimental point of view.

Crack closure is an important issue to consider when conducting fatigue life assessments. The development of the crack closure behaviour under constant amplitude loading under both small scale and large scale yielding is reviewed. Crack closure under variable amplitude loading and large scale yielding becomes very complex and is only discussed briefly.

It was found that the operational definition of $\Delta J$, i.e. $\Delta J_D$ (where index $D$ designates an $J$-value based on the load deformation path of the actual geometry) and also the $\Delta CTOD$ are able to correlate the fatigue crack growth under both small scale yielding and large scale yielding. The most common measure used as correlating parameter is $\Delta J_D$. However, the main issue today is the lack of an simple engineering method based either on $\Delta J$ or $\Delta CTOD$. 
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1 Introduction

This report considers fatigue crack growth in metals, with focus on the Low Cycle Fatigue regime. Fatigue crack growth is possible to occur when a structure or a component is loaded with an in time varying load. If no measures are taken this might lead to break down, with expensive consequences. Today a large number of all failures in load carrying structures originates from the fatigue phenomenon.

Fatigue crack growth is usually divided into two phases. The first phase is called the initiation period and is associated with rather large a scatter. Here the crack initiates and grows to a size detectable with today’s methods. With non-destructive testing it is possible to detect crack-sizes down to $\sim 0.5$ mm. The initiation part of the total life can be as large as 90% and above and naturally the propagation part is only a negligible part of the total life. This uneven distribution between initiation and growth of the fatigue crack normally arises when the load amplitude is low. This kind of fatigue crack growth is called High Cycle Fatigue, HCF due to the high number of load cycles required until failure, normally $> 10^6$ cycles.

Methods for calculating the number of cycles to initiation but also the crack growth rate exists today for HCF applications. One of the challenges today lies within extending the fatigue crack growth theories in order to cover the growth rate of cracks smaller than $\sim 0.5$ mm. I.e. assessment of the fatigue crack growth rate when the crack in the order of 50 to 100 microns and thereby decrease the need for initiation estimates. This is desirable due to the rather large scatter in the initiation assessment and consequently the reliability of the predictions would increase.

Normally the crack propagation share of the total life extends when the load amplitude increases and at very high load levels the total life merely consists of crack propagation. This part of the fatigue crack growth theory is called Low Cycle Fatigue, LCF and the fatigue life is in the order of 10 to $10^4$ cycles. In this phase linear theory is not applicable and some other measure that takes the nonlinearity into account is desirable.

Applications and areas of interests for these theories where the crack growth is driven by large plastic deformation are: nuclear power industry, offshore industry, transport industry (air, sea and land based systems) among others.

The purpose with this literature study was to find information regarding the fatigue crack growth process during extensive plastic deformation and possible parameters able to characterize the process. Also, experimental methods applicable to the nonlinear test situation were sought for.
1.1 Objective

The objective of this literature study was to find answers to the following questions:

- Is the Crack Tip Opening Displacement (CTOD) an unambiguous parameter? *i.e.* is the crack tip opening displacement the same for a mechanically short crack [1] as for a long crack at the same crack growth rate?

- Is $\Delta J$ an appropriate measure for estimates of the fatigue crack growth rate under large scale yielding? Also, is $\Delta J$ a suitable parameter for safety assessment of engineering applications?
2 Fatigue Crack Growth Mechanisms

Basically there are two crack growth mechanisms suggested. The first is Neumann’s coarse slip model [2]. This model suggests the crack to grow along a single slip system. Along with the crack growth the material work hardens and after a while a secondary slip system activates, \textit{i.e.} the crack growth continues along that plane. The process will continue by alternating slip and a crack surface will result with striation marks. The second model is based on the blunting and resharpening mechanism [3, 4]. Here the crack growth is controlled by slip along two slip planes approximately 45° to the crack plane at the same time, thus blunting the crack tip. Upon unloading reversed slip takes place and the crack is resharpened.

Tomkins [5] investigates the growth mechanisms of a fatigue crack under both regular and high loads. He concludes that the striation pattern observed is a consequence of a localized shear process. They do not necessarily correspond to the amount of crack growth in each load cycle but they reflect the process giving rise to both crack extension and blunting of the crack tip. At high crack growth rates it’s possible to activate several slip planes at the same time resulting in a striation pattern with several finer striations in-between larger and coarser striations. Fig. 1 shows the striations measurements on a Type 304 stainless steel presented in [5]. Also shown is the macroscopic fatigue crack growth rate.

As obvious from Fig. 1, the fine and the coarse striations tend to coincide at higher crack growth rates. This has also been observed in other materials and the crack growth rates then follow the coarser striation spacing which can consists of several fine striations according to Tomkins.
Figure 1: Comparison between the crack growth rate, the striation spacing and calculated $CTOD_{\text{max}}/2 (\delta/2)$. The material used is a Type 304 stainless steel and a load ratio of $R = 0.5$. Source [5].
3 Characterizing Crack Tip Measures

3.1 Linear Elastic Stress Field

Consider a crack tip in an infinite plane body. Williams [6] is first to study the solution of the elastic stress field surrounding the crack tip. Fig. 2 shows a polar coordinate system defined at the crack tip and the different stress components can be represented by

\[ \sigma_{\theta\theta} = \frac{\partial^2 \phi}{\partial r^2}, \quad \sigma_{rr} = \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2}, \quad \sigma_{r\theta} = \frac{1}{r} \frac{\partial \phi}{\partial r} - \frac{1}{r} \frac{\partial^2 \phi}{\partial r \partial \theta} \]  

where \( \phi \) is the Airy stress function. Equilibrium is automatically satisfied since the stresses are expressed through the Airy function. To ensure compatibility, when expressed in terms of the Airy stress function, the biharmonic equation has to be satisfied

\[ \nabla^2 (\nabla^2 \phi) = 0, \quad \nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}. \]  

In order to solve Eq. 2 boundary conditions are needed. The crack is assumed to be traction-free and the boundary conditions becomes

\[ \sigma_{\theta\theta} = \sigma_{r\theta} = 0 \quad \theta = \pm \pi. \]  

The solution to Eq. 2 is assumed to be of separable type

\[ \phi = r^{\lambda+1} f(\theta), \]  

where \( f(\theta) \) is the angular function describing the variation in the \( \theta \)-direction. Insertion of Eq. 4 into Eq. 2 leads to an eigenvalue problem with the solution
\[
\phi = \sum_{n=1,3,\ldots} r^{1+\frac{n}{2}} \left[ C_n \left( \cos \frac{n-2}{2} \theta - \frac{n-2}{n+2} \cos \frac{n+2}{2} \theta \right) 
+ D_n \left( \sin \frac{n-2}{2} \theta - \sin \frac{n+2}{2} \theta \right) \right] 
+ \sum_{n=2,4,\ldots} r^{1+\frac{n}{2}} \left[ C_n \left( \cos \frac{n-2}{2} \theta - \cos \frac{n+2}{2} \theta \right) 
+ D_n \left( \sin \frac{n-2}{2} \theta - \frac{n-2}{n+2} \sin \frac{n+2}{2} \theta \right) \right],
\]  \tag{5}

where \( \lambda = n/2 \). Physically admissible values for \( \lambda \) in order to satisfy the criterion of finite displacements and bounded strain energy at the crack tip \((\phi < \infty \text{ when } r \to 0)\) give

\[
\lambda = \frac{1}{2}, 1, \frac{2}{3}, \ldots \tag{6}
\]

Substitution of Eq. 5 into Eq. 1 and differentiation give the stress distribution around the crack. Assuming that the singular term will dominate the stress field in the vicinity of the crack tip all but the first term will be truncated. Further rewriting the constants \( C_1 \) and \( D_1 \) to \( K_I/\sqrt{2\pi} \) and \( K_{II}/\sqrt{2\pi} \), respectively, gives

\[
\sigma_{rr} = \frac{K_I}{\sqrt{2\pi}r} \left[ \frac{5}{4} \cos \frac{\theta}{2} - \frac{1}{4} \cos \frac{3\theta}{2} \right],
\]

\[
\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi}r} \left[ \frac{3}{4} \cos \frac{\theta}{2} + \frac{1}{4} \cos \frac{3\theta}{2} \right], \tag{7}
\]

\[
\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi}r} \left[ \frac{1}{4} \sin \frac{\theta}{2} + \frac{1}{4} \sin \frac{3\theta}{2} \right],
\]

for the Mode I case. The anti-symmetric Mode II part of the solution is derived in an analogous way. The anti-plane shearing mode called Mode III, is more straightforward to derive.

The so called stress intensity factors \( K_I, K_{II} \) and \( K_{III} \) are measures of the strength of the singularity. The strains and stresses increase in proportion to stress intensity factors and Eq. 7 describes a first order approximation of the field quantities near the crack tip. Fig. 3 shows the radial and angular variation of the stress components for a Mode I load.
The stress intensity factor is not only used as a parameter describing the strength of the singularity at the crack tip, but also used as a parameter describing the onset of crack growth or fracture. The critical value in Mode I is then termed “fracture toughness” and labeled $K_{\text{Ic}}$.

3.2 The $J$-integral and the $\Delta J$-integral

The $J$-integral and $\Delta J$-integral are two measures used in the elastic-plastic nonlinear fracture mechanics field. This means that the measures are capable of characterizing the state at the crack tip even if the plastic zone is not confined to a very small region compared to the in-plane dimensions. Still the plastic zone may not interact with any in-plane dimension for the theory to be applicable.

The $J$ contour integral is presented in 1968 by Rice [8]. He idealizes the elastic-plastic nonlinear constitutive behaviour to a nonlinear elastic behaviour. Rice then shows that the nonlinear energy release rate $J$, can be written as a path independent line integral, Eq. 8,

$$J = \int_{\Gamma} \left( w dy - T_i \frac{\partial u_i}{\partial x} ds \right)$$

Here are $w$ the strain energy density, $T_i$ are the components of the traction vector, $u_i$ are the displacement vector components and $ds$ is the length increment along the contour $\Gamma$, Fig. 4.
The following equation needs to be satisfied in order for the contour integral to be strictly path independent,

$$\sigma_{ij} = \frac{\partial W}{\partial \epsilon_{ij}}.$$  \hspace{1cm} (9)

The requirement of elastic material response, linear or nonlinear, may be relaxed to at least approximately include elastic-plastic materials under the condition of monotonic loading. The response of the two different constitutive models is approximately identical (apart from a small change of the Poisson’s ratio) as long as no unloading occurs. In three dimensions this may not be true because unloading may occur due to stress redistribution in the elastic-plastic material. Still, the assumption of nonlinear elastic behaviour may be a good assumption in cases where a high triaxiality exists at the crack tip and no global unloading occurs.

In the case of a growing crack elastic unloading occurs in the wake behind the crack tip. The size of the path independent zone is then expected to be proportional to the growth increment of the crack.

The $J$-value can also be calculated as a deformation-J designated $J_D$. The deformation $J_D$ is basically calculated as an integration of the load-deformation path. In an elastic-plastic material, however, a history dependence exists that is removed when the integration is performed. The $J$-value from the contour integral and the (deformation) $J_D$ may differ somewhat dependent on the amount of crack growth in the elastic-plastic material. The path independence is lost for the contour integral, since the integral tends to zero as the contour shrinks towards the crack tip.

The situation becomes a lot more complicated in the case of fatigue. The $J$ contour integral is only path independent for linear and nonlinear-elastic material and deformation plasticity. However, when unloading occurs in an

![Figure 4: Path around a crack tip. Source [9].](image-url)
elastic-plastic material, deformation plasticity theory no longer describe the actual behaviour of the material. Thus, highly path dependent values result in the situation of unloading and are therefore questionable to use in the fatigue situation. Despite the loss of generality several researchers have tried to use the $\Delta J$ when correlating the crack growth rate with different degrees of success. Below the definition of $\Delta J$ is presented, first presented by Lamba [10] in 1975 in connection to stress concentration factors.

The material in front of the crack tip experiences a cyclic stress-strain range and consequently the process is characterized by $\Delta \sigma_{ij}$ and $\Delta \epsilon_{ij}$. Fig. 5 shows a cyclic stress-strain loop where the initial value is indicated with number 1 and final value with number 2. The range of the $J$-integral is then defined as [10, 11, 12]

$$\Delta J = \int_{\Gamma} \left( \psi \left( \Delta \epsilon_{ij} \right) dy - \Delta T_i \frac{\partial u_i}{\partial x} ds \right).$$

(10)

Here $\Gamma$ is the integration path around the crack tip. Further, $\Delta T_i$ and $\Delta u_i$ are the changes in traction and displacement between the initial point 1 and the final point 2. In addition, $\psi$ corresponds to the strain energy density and is defined in an analogous way by

$$\psi \left( \Delta \epsilon_{ij} \right) = \int_{\epsilon_{ij}^1}^{\epsilon_{ij}^2} \sigma_{ij} d\left( \Delta \epsilon_{ij} \right) = \int_{\epsilon_{ij}^1}^{\epsilon_{ij}^2} \left( \sigma_{ij} - \sigma_{ij}^1 \right) d\epsilon_{ij}.$$  

(11)

Figure 5: Stress strain curve. Source [9].

Usually only the loading part of the cyclic loop is included in the integration of the strain energy density rather than the whole loop. If the initial point 1 is located at zero stress and strain then $\Delta J = J$. Eq. 10 is a generalization of Eq. 8 to include the situation when the stress and strain at point 1 is not at zero. The requirements for $\Delta J$ to be path independent are analogous to those for the original $J$ mentioned above, i.e. $\sigma_{ij} = \partial W / \partial \epsilon_{ij}$.
As in the case of monotonic loading it is also possible to calculate the $\Delta J_D$ from the cyclic load-displacement curve. Eq. 12 shows the general form,

$$\Delta J_D = \eta \frac{Bb}{V_{\text{max}} - V_{\text{min}}} \int \Delta P \, d(\Delta V)$$

where $\eta$ is dimensionless constant, $B$ the specimen thickness and $b$ the uncracked ligament. $P_{\text{max}}$, $P_{\text{min}}$, $V_{\text{max}}$, and $V_{\text{min}}$ are the maximum and minimum load and displacement, respectively, during the specific load cycle as shown in Fig. 6.

Eq. 10 and Eq. 12 do not necessarily give the same value under equal conditions. This may happen when excessive plastic deformation occurs in the geometry. Still, the state at the crack tip may be characterized by Eq. 12 meaning that the path independence is lost in Eq. 10.

### 3.3 Crack Tip Opening Displacement

An easily interpreted measure is the so called Crack Tip Opening Displacement, $\text{CTOD}$. Originally this parameter was developed by Wells [13] who discovers that several structural steels could not be characterized by LEFM, i.e. $K_{\text{IC}}$ was not applicable. He also discovers while examining the fracture surfaces that the crack surfaces moves apart prior to fracture. Plastic deformation precedes the fracture and the initially sharp crack tips are blunted. The plastic deformation increases with increasing fracture toughness and Wells proposes the opening at the crack tip as a fracture toughness parameter.

The $\text{CTOD}$ has no unique definition. Fig. 7 shows three of the different definitions of the $\text{CTOD}$. The most common one, Rice [8] is measured as the
distance between two lines separated by 90° lines extending from the crack tip and intersecting the crack surfaces behind the tip, shown in Fig. 7(a). Others define the CTOD at the elastic-plastic boundary or at a distance beyond the plastic zone where the crack face is no longer deformed. In connection with fatigue often the range of the crack tip opening displacement is used, i.e. $\Delta \text{CTOD}$ or $\Delta \delta$.

### 3.3.1 The Irwin Approximation

When the crack tip is plastically deformed due to loading the crack behaves as it is slightly longer than the actual crack length. This is shown by Irwin [15]. Using this information it is possible to estimate the CTOD in the limit of Small Scale Yielding (SSY). Assuming an effective crack length of $a + r_y$, where $r_y$ is the length of the plastically deformed material in front of the crack tip. The plastic zone correction according to Irwin [15] is

$$r_y = \frac{1}{2\pi} \left( \frac{K_I}{\sigma_y} \right)^2. \quad (13)$$

Eq. 13 combined with the elastic solution for the displacement of the crack surface in plane stress,

$$u_y = \frac{\kappa + 1}{\mu} K_I \sqrt{\frac{r_y}{2\pi}}, \quad (14)$$

gives the $\text{CTOD}$ for a stationary crack in the limit of SSY, Eq. 15,
CTOD = 2u_y = \frac{4}{\pi} \frac{K_I^2}{\sigma_Y E} \ . \quad (15)

Eq. 15 is based on a linear elastic solution for the stress field in front of the crack tip, i.e. it is a first order estimate of the plastic zone size. Due to yielding the stress must redistribute in order to satisfy equilibrium. But the redistribution is not taken into account and equilibrium is not satisfied. By assuming a non hardening material giving a constant stress in the yield zone, \( \sigma_Y \), a simple force balance is stated in Eq. 16 in order to fulfill equilibrium,

\[ \sigma_Y r_p = \int_0^{r_p} \sigma_y dr = \int_0^{r_p} \frac{K_I}{\sqrt{2\pi r}} dr. \quad (16) \]

Here \( r_p \) is the plastic zone size, \( \sigma_y \) the elastic stress distribution stated in Eq. 7. Integrating and solving for the plastic zone size, \( r_p \) gives,

\[ r_p = \frac{1}{\pi} \left( \frac{K_I}{\sigma_Y} \right)^2. \quad (17) \]

This new estimate of the plastic zone size is twice as large as the first order estimate, i.e. Eq. 13.

3.3.2 The Strip Yield Model

The strip yield model proposed by Dugdale [16] among others estimates the size of the yield zone ahead of a Mode I crack in a thin plate (plane stress) of elastic perfectly plastic material. Two elastic solutions are superimposed; one through crack in an infinite plate under remote tension and a through crack with closure stresses at the tip, Fig. 8. The model assumes a long, slender plastic zone (with near-zero height) at the crack tip with length \( r_p \), i.e. the total crack length is \( 2a + 2r_p \).

The stress over \( r_p \) is equal to \( \sigma_Y \) and since the stresses are finite in the yield zone there cannot be a stress singularity at the crack tip. This is accomplished by choosing the plastic zone length such that the stress intensity factors from the remote tension and closure stress cancel each other. This leads to a plastic zone size of,

\[ r_p = \frac{\pi}{8} \left( \frac{K_I}{\sigma_Y} \right)^2. \quad (18) \]

The CTOD from the strip yield model can be derived at the crack tip by superposition of the crack surface displacements (in the same way as for the stress intensity factors). The CTOD becomes
Figure 8: Superposition of two load cases that form the basis for the Dugdale model.

\[
CTOD = \frac{8 \sigma_Y a}{\pi E} \ln \left( \frac{1}{\cos \left( \frac{\pi \sigma a}{2 \sigma_Y} \right)} \right). \quad (19)
\]

A series expansion of the logarithmic term in Eq. 19 and truncating all but the first two terms gives (zero would be obtained if only one term is included),

\[
CTOD = \frac{\pi^2 \sigma^2 a^2}{8 \sigma_Y} = \frac{K_1^2}{\sigma_Y E}. \quad (20)
\]

The CTOD from the strip yield model differs only slightly from Eq. 15 given by the Irwin approach.

These measures (Eqs. 13 to 20) apply for SSY conditions and monotonic loading. The cyclic crack tip opening displacement, \( \Delta CTOD \) can be estimated by a substitution of the applied load by the load range and a multiplication of the yield stress level by a factor of 2. This applies for \( R = 0 \) and no crack closure, cf. Rice [17].

If the line integral, Eq. 8 is applied to the strip yield model the following relationship between the crack tip opening displacement is obtained,

\[
CTOD = \frac{J}{\sigma_Y}. \quad (21)
\]

In the late 1970’s Budiansky and Hutchinson [18] present a comprehensive analysis of the \( \Delta CTOD \) and crack closure under steady-state crack growth conditions and SSY. The crack is assumed to grow in a semi-infinite thin plate and the load is specified as a constant value of \( \Delta K_{\text{nom}} \), thus assuming
a constant thickness of the plastic zone left behind the crack. The effect of different \( R \)-ratios on the ratio \( \Delta K_{\text{eff}} / \Delta K_{\text{nom}} \) is included and a unified basis for correlating experimental crack growth rates at different load ratios is presented.

Budiansky and Hutchinson formulate the problem as a boundary-value problem and solve it by use of standard Muskhelishvili potentials. An approximate solution to the cyclic plastic stretch at the crack tip for positive load ratios is presented, given in Eqs. 22 - 23,

\[
CTOD_{\text{max}} - CTOD_R \approx 0.73 \frac{(K_{\text{max}} - K_{\text{open}})^2}{E \sigma_Y} \tag{22}
\]

and

\[
CTOD_{\text{max}} - CTOD_R \approx 0.54 \frac{(K_{\text{max}} - K_{\text{clos}})^2}{E \sigma_Y}. \tag{23}
\]

Here \( CTOD_{\text{max}} \) is the displacement at the crack tip at maximum load and \( CTOD_R \) is the residual displacement due to the plastic stretch, i.e. \( \Delta CTOD = CTOD_{\text{max}} - CTOD_R \). Also, \( K_{\text{open}} \) and \( K_{\text{clos}} \) are the corresponding opening and closure levels at the crack tip given from the analysis.

Quite recently Rose and Wang [19] present an extension of the Dugdale model to be applicable in the large scale yielding (LSY) region. The model is consistent with self-similar fatigue crack growth, where the plastic zone and the plastic wake thickness increase linearly with increasing crack length. The solution is obtained by use of Mushelishvili potentials and the Riemann-Hilbert problem.

One fundamental assumption in the model is that the \( \Delta CTOD \) can serve as a unifying correlating parameter for crack growth rates within and beyond the limits of SSY. This assumption is not new and has been used earlier, but this model seems to be the first one applicable to a study of the influence of plasticity induced crack closure within and beyond the limits of SSY. Wang et al. [20] later present an extension of the model to be applicable for the plane strain case under LSY conditions. Two plastic constraint parameters are introduced, one for tension yielding and the other for compression yielding. The two parameters are determined by finite elements computations.

### 3.3.3 HRR Singularity

The relationship between \( J \) and \( CTOD \) is also investigated by Shih [21]. He evaluates the displacements at the crack tip given by the HRR singularity and related these to \( J \) and flow properties. The displacement near the crack tip is given by Eq. 24 according to the HRR solution,
Here \( \tilde{u}_i \) is a dimensionless function of \( \theta \) and \( n \), where \( n \) is the strain hardening exponent. Shih evaluates the CTOD at the 90° interception as defined by Rice [8] and obtains,

\[
CTOD = d_n \frac{J}{\sigma_0},
\]

(25)

where \( d_n \) is a dimensionless constant given shown in Fig. 9.

Figure 9: The dimensionless constant \( d_n \) for plane stress and plane strain. Source [9].

Shih shows that the relationship Eq. 24 applies well beyond the validity of LEFM for stationary cracks.

Rice [17] presents the CTOD for a quasi-statically growing crack in a perfectly plastic material loaded in Mode I,

\[
CTOD = \frac{4(1-\nu)}{\mu} \sigma_Y r \ln \left( \frac{R_t}{r} \right).
\]

(26)

Here \( R_t \) is the plastic zone size. The interesting feature about Eq. 26 is the singularity. The strains exhibit a logarithmic singularity while the stresses are bounded, resulting in no energy flow to the crack tip. Fig. 10 shows the crack tip surface displacements for the elastic and logarithmic singularity.
Figure 10: The crack tip shapes for stationary elastic and an elastic perfectly plastic growing crack tip.

3.4 CTOA, $\Delta$CTOA

The Crack Tip Opening Angle (CTOA) has no clear definition. Fig. 11 shows the usual operational definition of the measure. The measure is traditionally used for crack growth under strictly increasing loading but may also be a decent measure for fatigue crack growth. The measure $d$ in Fig. 11 is usually between 0.25 - 1 mm.

Figure 11: The definition of the crack tip opening angle. Source [22].
4 Small, Intermediate & Large Scale Yielding

4.1 Small Scale Yielding

Linear Elastic Fracture Mechanics, LEFM predicts infinite values of the stresses and strains at the crack tip. However very few materials exhibit this behaviour, instead a nonlinear behaviour develops governed by the nonlinear part of the stress-strain curve.

Assume that the nonlinear behaviour only occurs in a very limited spatial region. Suppose further that outside this region at a radius \( r \) the stresses and strains can be described with the singular solution given by LEFM. All information known at the crack tip is then passed through the singular field and the stress intensity factor can be used as a characterizing parameter in this region. The SSY requirement states that the zone defined by radius \( r \) should be small compared to characteristic dimensions of the crack body. This means that LEFM may still be applicable in certain situations while the SSY conditions are not necessarily fulfilled. Assessment of the applicability of LEFM can be judged by use of nonlinear fracture mechanics.

The ASTM E647 [23] standard recommends that any characteristic dimension in the CT-specimen should be larger than

\[
W - a = \frac{4}{\pi} \left( \frac{K_{\text{max}}}{\sigma_Y} \right)^2,
\]

for LEFM to be applicable. Here \( K_{\text{max}} \) is the maximum stress intensity factor and \( \sigma_Y \) is the yield stress of the material. \( W \) is the total ligament length and \( a \) is the crack length.

4.2 Intermediate Scale Yielding

This load region is characterized by a plastic zone size at the crack tip that is about the same as the characteristic dimensions of specimen, still the plastic zone should not interact with the outer boundaries of the body. As stated above, the stress intensity factor may be a possible characterizing measure even in this region but will not always work. A better measure would be the \( J \)-integral or the \( \overline{CTOD} \), which are able to account for the nonlinear deformation.

The ASTM E1820 [23] standard recommends that any characteristic dimension \( l \) in the body should be larger than

\[
l = 25 \left( \frac{J_{\text{lc}}}{\sigma_Y} \right),
\]
for the $J$-integral to be applicable. Here $J_{lc}$ is the nonlinear fracture toughness. Eq. 10 and Eq. 12 should give about the same result as long as Eq. 28 is fulfilled.

4.3 Large Scale Yielding

When extensive plasticity occurs and the size of the yield zone becomes large compared to characteristic dimensions in the body the single parameter description breaks down. The size of the $J$-dominated zone becomes strongly dependent on the configuration of the test specimen, which means that the distributions of stresses and strains in the crack tip vicinity are not unique. The state may still be characterized by Eq. 12 but the values are very geometry dependent and Eq. 10 would not give the same result.
5 Crack Closure

Crack closure or crack opening, depending on whether if the level is measured or calculated at the loading or unloading path, is first reported by Elber [24] in the early 1970’s. The phenomenon has since then provided a rationale for different observed behaviours such as: retardation of the fatigue crack growth rate due to overloads, the load ratio effect on the growth rate, some of the observed fatigue threshold effects and ”small crack” growth behaviour to mention a few.

Elber [24, 25] observed a change of the compliance during loading and interpreted this as a gradual change of the crack length due to closure of the crack faces behind the crack tip, even if a positive global load is applied. An effective stress intensity factor range $\Delta K_{\text{eff}}$ is suggested to be used as the crack driving parameter instead of the nominal stress intensity factor range $\Delta K_{\text{nom}}$. The effective stress intensity factor range is defined as,

$$\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{closure}},$$

where $K_{\text{closure}}$ is the stress intensity factor level as the crack opens. Elber finds that the use of $\Delta K_{\text{eff}}$ eliminates the $R$-ratio dependence in fatigue data and all fatigue data coincide to a common line in double-logarithmic diagram. He also states that the crack closure phenomenon accounts for acceleration and retardation effects in variable amplitude crack growth. Others [26] show the same result, eliminating the $R$-ratio dependence in fatigue data by use of an effective stress intensity factor range, but this is not universally observed.

Another indirect definition of the crack closure level is the one stating that the difference between crack propagation data at different $R$-values is due to crack closure, i.e. the crack closure level is estimated from fatigue crack growth rate data. The crack closure level is solved for by enforcing the crack growth rate to be the same in the actual test as in the crack closure free experiment. This definition eliminates the need for accurate crack closure measurements during the actual test, but requires a fatigue crack growth rate experiment where no crack closure is present. Also, with the two different definitions mentioned above it is possible to assess the accuracy of the other crack closure measuring techniques cf. [27].

Crack closure effects may have several different origins. The common base is some change of length of the crack faces behind the crack tip or some obstacle between the crack faces. The origin to this may be due to corrosive environment that causes the newly created crack faces to oxidize/corrode. Another cause is the plastic deformation formed due to the growth of the crack leaving a wake of deformed material behind the crack tip. This plastic
stretch will vary if the load amplitude is variable causing the closure level to change with crack length. This type of crack closure is called plasticity induced crack closure. A third type may occur if a Mode I growing fatigue crack is subjected to a Mode II overload, a mismatch between the crack surfaces is created, called roughness induced crack closure. Due to the relative displacement in the crack plane retardation effects occur that may affect the crack growth rate several millimeters ahead [28, 29]. For a summary of proposed mechanisms, see for instance Suresh [1].

The majority of the conducted research on crack closure is focused on the behaviour during SSY conditions, which is natural since the incorporation of the closure level into the fatigue crack growth law is straightforward by use of the stress intensity factor range. Under LSY conditions no theoretically justified and simple engineering type of parameter exist. This is probably one of the reasons for the less amount of research work conducted in the LSY region compared to the SSY region.

5.1 Crack Closure under SSY

5.1.1 Crack Closure Measuring Techniques

Many experimental studies of fatigue crack closure measurements have been reported since the early 1970’s where various techniques have been used to measure the closure level. Some of the techniques used are the compliance method, ultra-sonic measurements, replica, Scanning Electron Microscope (SEM) and the PD technique. The most common method is the compliance technique, which is also recommended in ASTM E647 [23]. For a comprehensive review of different techniques for crack closure measurements see for instance Stoychev and Kujawski [30].

The principle of the compliance method is to attach a strain-gauge or clip-gauge somewhere on the test specimen. When the load is applied the crack starts to open and the compliance changes as the crack opens up. Finally when the crack is fully opened and the load continues to increase a linear response of the compliance is obtained as long as the maximum load is restricted to the SSY region. Thus, the crack closure level is obtained as the point when curve starts to deviate from the linear relation; Fig. 12(a) shows a typical applied load-displacement curve. Due to the difficulty of extracting the true crack closure level from the load-displacement curve, ASTM E647 recommends a subtraction of the slope of the fully open crack from all points in the load-displacement curve. The resulting curve is shown in Fig. 12(b); a typical compliance offset curve resulting from such a procedure. The crack closure level is defined as the point of 1%, 2% or 4% compliance offset from
the fully open crack compliance (depending on the noise in the raw load-displacement data).

![Diagram](image)

Figure 12: (a) Typical applied load-displacement curve. (b) Typical compliance offset curve.

However, the accuracy of the compliance method has been questioned and investigated by several researchers since the results are not always consistent with the outcome from other measuring techniques [30, 31]. According to Stoychev and Kujawski [30] the measured crack closure level is about the same as obtained by the Crack Mouth Opening Displacement (CMOD) and Back Face Strain (BFS) method, whereas the Near-Crack-Tip-Gauge (NCTG) and Acoustic Emission (AE) methods result in somewhat higher values. The difference is explained by plane strain versus plane stress arguments. The CMOD and BFS method measure the bulk behaviour of the specimen whereas the NCTG and AE in general capture the near-face crack closure behaviour where a higher crack closure level is present. Kujawski and Stoychev [32] also study the scatter in the opening load by use of a number of different curve fitting procedures to load-displacement data obtained from CMOD clip-gauge. The result, Fig. 13 shows quite a large difference in obtained crack closure level between the methods and also sometimes high scatter within the different methods.

Another method for crack closure measurement is the electrical Potential Drop technique (PD). The idea is to pass a current, Direct Current (DC) or Alternating Current (AC), through the specimen and measure the potential difference across the mouth. The potential drop across the crack increases with increasing crack length. Thus the technique is well suited for crack length measurements. Several researchers have also used the method for crack closure measurements. The idea is that the current passes through the
Fig. 13: Normalized crack closure level obtained by the compliance method through different curve fitting methods. Source [32].

contact between the crack surfaces as long as contact exists. Consequently, when the contact is released and the crack opens changes in the potential-drop signal takes place until the crack is fully opened and the potential-drop value remains constant. However, a number of studies indicates problems with the PD technique. The problems are mostly connected to fatigue crack propagation in vacuum where the formation of an oxide layer on the crack surfaces is absent.

Wilkowski and Maxey [33] present a review paper on the PD technique for measuring crack initiation and growth of cracks loaded both under monotonically increasing load, and cyclic load (fatigue) and static loading (creep crack growth). Tab. 1 summarizes some of the points made in [33].

Wilkowski and Maxey conclude that the use of DCPD is increasing and is in general the better method for crack initiation and length measurement.

Bachmann and Munz [34] use both the PD and the compliance technique to evaluate the techniques in vacuum and air on a Ti-6Al-4V titanium alloy. The authors conclude that crack closure measurements may provide erroneous results in both vacuum and air due to the uncertain development and stability of the oxide layer, Fig. 14. Pippan et al. [35] perform experiments on ARMACO iron and conclude that the PD technique is useful for both crack length and crack closure measurements in air, but if the contact resis-
<table>
<thead>
<tr>
<th>Table 1: Comparison between the DCPD and ACPD methods.</th>
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<tr>
<td><strong>Advantages with DCPD</strong></td>
</tr>
<tr>
<td>Good for detecting and measure fatigue crack growth of 2D cracks and subsurface nucleated cracks.</td>
</tr>
<tr>
<td>Good for detecting crack growth under monotonically increasing loads. Possible to separate crack tip blunting from crack growth.</td>
</tr>
<tr>
<td>In general simpler equipment is needed for DCPD compared to ACPD measurements.</td>
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<tr>
<th><strong>Disadvantages with DCPD</strong></th>
<th><strong>Disadvantages with ACPD</strong></th>
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<tbody>
<tr>
<td>Because of the low resistance of most metallic specimens, the voltage across the crack can be in the microvolt range. Resulting in either the use of high current input or the need for very sensitive voltmeter with reduction of noise and drift.</td>
<td>Electromagnetic induced voltages may appear in the measuring wires if the wires for the current input are close. This problem may be circumvented by use of a phase-sensitive detector.</td>
</tr>
<tr>
<td>Thermal EMF is important to consider if temperature differences appear and dissimilar metals are used in the wire connections.</td>
<td>Hard to detect the point of crack growth under monotonically increasing load.</td>
</tr>
<tr>
<td>Not as good as DCPD for crack length measurements due to the &quot;skin effect&quot;.</td>
<td></td>
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tance is low (as in vacuum) this technique is not working, as compared to the compliance technique, which is working in both air and vacuum, but may be less accurate, Fig. 15.

Figure 14: Load versus compliance and load versus potential-drop range for a test in vacuum. Source [34].

Figure 15: Load versus potential-drop signal at the end of a fatigue growth experiment in vacuum, continuing with the change of the potential during exposure to air and the first few load cycles in air. Source [35].

Shih and Wei [36] study crack closure in fatigue of Ti-6Al-4V titanium alloy in dehumified argon that was further purified by a titanium sublimation pump in order to circumvent the effect of the insulating oxide layer formed on the fracture surface. Both the PD and compliance techniques are used and the results show that both techniques give the same result and that the
crack closure alone cannot account for the stress ratio effect on crack growth. Fig. 16 shows the load versus potential-drop data from [36].

Spence et al. [37] conduct fatigue crack growth experiments on IMI 834 titanium alloy at +350°C in laboratory air. The result shows that because of the severe oxidization at 350°C the measured potential-drop data are not reliable. A solution to the problem is to lower the temperature to +20°C and propagate the crack a few microns while conducting the potential-drop measurement. Fig. 17 shows the result.

5.1.2 General Trends of Crack Closure in Constant Load Amplitude Testing and SSY

James and Knott [38] conduct fatigue crack growth experiments on Q1N (HY80) steel in order to assess the crack closure levels and the extent of the short crack regime. The crack length is measured by the PD technique and the crack closure levels are measured by BFS and CMOD techniques. The test geometries are different four-point bend specimens with a thickness of 25.4 mm. The fatigue crack growth experiments are performed under a load shedding scheme, which as the authors note probably influences the measured crack closure level, especially in the threshold region at low R-values.

Fig. 18(a) shows the stress intensity factor level at crack opening, $K_{\text{open}}$, normalized by the maximum stress intensity factor level, $K_{\text{max}}$, as a function of the stress intensity factor range, $\Delta K$. As the $\Delta K$ decreases the closure data for all $R$-values increases being less marked for the higher load ratios. An estimate of the load shedding effect shows that the true normalized crack
opening level is about 0.15-0.20$K_{\text{max}}$ and that the increase shown in Fig. 18(a) is primarily a load shedding effect. This means that the crack closure data should be quite constant and independent of the $\Delta K$. Fig. 18(b) shows some dependence of the $K_{\text{open}}/K_{\text{max}}$ ratio on the $K_{\text{max}}$-level.

Marci [39] uses a different approach to experimentally obtain the crack opening level. The technique relies on the fatigue propagation behaviour, i.e. whether the crack can propagate or not. Two different materials are used: Al 7475-T7351 and the Nickel base alloy Nicrofer 5219 Nb annealed and the specimens is of CT type with thicknesses 12.5 mm and 5 mm respectively. The fatigue crack length measurements are conducted in air by the PD technique. Fig. 19 shows the experimental results of the normalized crack opening level versus $K_{\text{max}}$ and the load ratio $R$.

Fig. 19(a) shows that the normalized crack closure level is independent of the $K_{\text{max}}$-level, but also shows rather high crack opening levels high up in the load ratio levels for both tested materials, see Fig. 19(b). Marci also presents empirical functions for the normalized crack closure level as a function of the load ratio. A review of some of the presented empirical functions the literature can be found in Kumar [40].

Sehitoglu [41, 42] investigates the fatigue crack growth behaviour from sharp and blunt notches. The crack opening and closure levels are measured by replica at different load levels and crack lengths. The experimentally obtained crack opening levels are compared with the analytical model presented by Budiansky and Hutchinson [18] based on the Dugdale model and a rather good agreement is obtained, see Fig. 20.
Figure 18: (a) Crack closure data from measurements on long through-cracks. (b) Replot of some of the data from Fig. 18(a) indicates some dependence on the $K_{\text{max}}$-level. Source [38].
Figure 19: (a) Normalized crack closure level versus $K_{\text{max}}$ for Al 7475-T7351. (b) Normalized crack closure level versus the load ratio for both materials. Source [39].

Figure 20: Experimental and analytical normalized crack opening levels versus the load ratio $R$. Source [41].
Sehitoglu [41] also concludes that the crack opening and closure level may differ significantly depending on the load ratio, \( R \). Generally the crack opening level is higher than the crack closure level.

Ljustell and Nilsson [43] investigate the effects of different constant amplitude load schemes on the experimentally obtained fatigue data. The crack closure levels are estimated as the difference between crack closure free fatigue data and fatigue data containing crack closure, i.e. no explicit measurements of the closure level are done. CT-specimens with thicknesses in the range 3 mm to 10 mm are used. The experiments are conducted at +400°C and the compliance technique is utilized to measure the CMOD. The material is a nickel-based super alloy, Inconel 718. Fig. 21 shows the normalized crack closure level versus \( \Delta K \) and the load ratio \( R \).

![Figure 21: Experimental data of the crack closure level versus the stress intensity factor range and the load ratio \( R \). Source [43].](image)

The results show that the normalized crack closure/opening level is rather independent of the stress intensity factor range, similar to the results obtained in [38, 39]. Also, the crack opening/closure dependence of the load ratio seems to be well described by the crack opening function presented by Newman [44], here shown as the solid line in Fig. 21(b).

### 5.1.3 Numerical Aspects on Simulations of Crack Closure under SSY

There are, as mentioned above, several different types of crack closure. Roughness and oxide/corrosion/fretting induced crack closure for instance are different types that contribute to retardation of the crack growth rate in certain situations. The problem with these types is the uncertainty in and sometimes lack of reliable physical models describing the process. This is probably one
of the reasons for the rather few papers investigating and quantifying the
effects of these mechanisms. Recently Parry et al. [45] perform the first
combined roughness and plasticity-induced crack closure analysis based on
finite element technology.

In the 1970’s the development and implementation of the finite element
method into numerical computer codes started. This gave the possibility
to analyze crack closure induced by plastic deformation at the crack tip.
The models were very small and simple at the early stages. But along with
the immense development of the computer capacity larger and more complex
models are now being solved. Solanki et al. [46] recently present an extensive
review paper on simulations of plasticity induced crack closure by the finite
element method. Below are different aspects of crack closure simulations
reviewed.

5.1.4 Mesh refinement

The element size along the crack path is important for several reasons. Suf-
ficiently many elements must be used to resolve the high stress gradients in
front of the crack tip; also the element size determines the simulated crack
growth rate. The first investigation of crack closure by finite elements is
conducted by Newman [47]. He conducts crack propagation simulations with
Constant Strain Elements, (CST), under plane stress, on a Middle Tension
specimen, (MT). The results show that a much finer mesh is needed in order
to obtain convergence when low nominal stress levels are used as opposed to
high stress levels. The constitutive model used assumes elastic perfectly plas-
tic behaviour. McClung and Sehitoglu [48, 49] present two extensive studies
on the effects of mesh refinement on a MT-specimen with a hole. They use
a bi-linear elastic-plastic constitutive model with kinematic hardening. Both
plane stress and plane strain computations are conducted with four noded
isoparametric elements and they suggest that the refinement should be based
on the number of elements in the forward plastic zone. The reference plastic
zone size is of the Irwin type for a center cracked panel under plane stress
and the criterion $\Delta a/r_p \leq 0.10$ is presented, where $\Delta a$ is the element size in
front of the crack tip and $r_p$ the forward plastic zone size given by

$$r_p = \left( \frac{K_I}{\sigma_Y} \right)^{\frac{2}{3}}.$$  (30)

They also note that a sufficient refinement of the cyclic plastic zone may be
important. Additional results show that the crack must propagate through
the initial forward plastic zone in order to develop a stabilized plastic wake
and thereby steady state crack opening value. Dougherty et al. [50] conduct
Figure 22: The numerically simulated crack closure level versus element size for the CT- and MT-specimen. Source [52].

mesh refinement studies under plane strain and obtain a similar condition as [48, 49] with a multi-linear constitutive description and isotropic hardening. The element type used is a four node quadrilateral. Dougherty et al. also evaluate the eight node quadrilateral elements but obtain a saw-tooth pattern in the residual stresses and displacements along the crack plane. They attribute the behaviour to the shape functions of elements and decide to use the four node elements. Solanki et al. [51] perform finite element mesh refinement studies on the CT and MT geometry using an elastic perfectly plastic material description and compare both plane strain and plane stress. The four node quadrilateral element type is used. Convergence is obtained for both geometries under plane stress but only for the MT under plane strain. They also conclude that 3-4 elements are required in the reversed plastic zone for the MT-specimen under plane strain and the same number of elements in both geometries under plane stress. Newman [52] presents a review paper on the advances in finite element modeling of crack closure under fatigue conditions that includes an overview of the results obtained in [51] see Fig. 22. The smallest element in Fig. 22 corresponds to a size of 2.7 μm.

Gonzalez-Herrera and Zapatero [53] present a detailed study on the convergence of the crack opening/closure level as the mesh refinement increases towards approximately 150 to 200 elements in the forward plastic zone as given by the Dugdale, Eq. 18. They use the CT geometry and perform analysis with an elastic almost perfectly plastic constitutive law. Initially, both isotropic and kinematic hardening constitutive laws are evaluated and differences in the crack opening/closure level are observed when the ratio between the plastic slope, $H$ and the elastic modulus $E$ becomes $H/E > 0.04$. The results show no convergence of the crack opening/closure level and the main
conclusion is that the obtained level is dependent on the element size. The recommended procedure to obtain a converged value is to use at least three different element sizes and linearly extrapolate the converged value to zero element size. Jiang et al. [54] obtain the same non-convergent behaviour of the crack opening level with increasing mesh refinement. The same geometry and conditions as Solanki et al. [51] are used.

5.1.5 Node Release Schemes

One of the limitations when simulating fatigue crack growth with the finite element method is the relatively large discrete jumps corresponding to one element in front of the crack tip. Several different methods are reported in the literature and the difference is basically when to release the crack tip node in the load cycle and when to register the crack opening/closure after the node release. The origin to this problem is related to how and when a real fatigue crack extends under the load cycle. One maybe natural way is to assume growth at maximum load and this scheme is suggested by Newman [47]. A similar technique is used by Fleck [55] and Blom and Holm [56]. Others release the node at minimum load and obtain similar results as when the release is done at maximum load. Alizadeh et al. [57] conduct a systematic analysis of the four most common schemes and the results are presented as Fig. 23 and Fig. 24.

![Figure 23: Different load release schemes for numerical estimation of the crack closure level. Source [57].](image-url)
Figure 24: Comparison of the crack closure results from different node release schemes. Source [57].

An elastic perfectly plastic constitutive law is utilized together with an element size of 0.1 mm, which is very large compared to what Solanki et al. [51] and Gonzalez-Herrera and Zapatero [53] recommend. Nevertheless the results obtained by Alizadeh et al. [57] are in general consistent with results obtained by others, the node release scheme has negligible influence on the crack opening/closure level, cf. [48, 49, 51].

5.1.6 Constitutive Dependence of Crack Closure

The constitutive law has a major impact on the response of finite element analyses. The most commonly used constitutive law when studying crack opening/closure by numerical methods is the elastic perfectly plastic law. The reason is probably that this is the simplest law, which minimizes the initial efforts of conducting and fitting the parameters in the law. Also, the hardening rule has no effect on the constitutive response for a perfect hardening material. This allows for parametric studies with a minimum of parameters. The major drawback is the limited possibility to describe important features like mean stress relaxation and ratchetting.

Budanski and Hutchinson [18] using a strip yield model and Sehitoglu et al. [58] using the finite element method show that an increase in the amount of hardening leads to an increase of the crack opening level. Pommier and
Bompard [59] find in their parametric study that isotropic hardening lowers the effective part of the fatigue cycle, while kinematic hardening or the Bauschinger effect is found to increase the effective part. This is explained by reverse plastic deformation taking place at unloading in a material dominated by kinematic hardening, thereby drastically reducing the compressive residual stresses ahead of the crack tip. These numerical results were later verified by an experimental study and the results are concluded to be satisfactory, Pommier [60].

Jiang et al. [54] conduct a finite element study on a MT-specimen under plane stress. Three different constitutive models are used: elastic perfectly plastic, bi-linear stress strain relationship with kinematic hardening and a kinematic hardening model developed by Jiang and Sehitoglu [61] capable of displaying ratchetting and cyclic stress relaxation. The results shown in Fig. 25(a)-(c) indicate a decreasing dependence of the element size as a more realistic constitutive description is used.

5.1.7 Constraint Effects

The 2D plane strain state often used in finite element simulations is an idealization of the condition prevailing in the interior of a 3D fatigue crack. The material transport is totally constrained in the thickness direction, which has led to a debate of the presence of crack closure under this condition. Very contradictory results are reported in the literature. Fleck [55] conducts a 2D plane strain finite element study of crack closure in a MT-geometry. He concludes that, in general, crack closure does not occur under steady state conditions. Contrary to the conclusion obtained by Fleck [55], Blom and Holm [56], McClung and Sehitoglu [48, 49] conclude that crack closure does indeed occur under plane strain conditions. McClung et al. [62] propose an explanation for crack closure under plane strain. The material along the crack faces is stretched in the in-plane direction supplying the vicinity of the crack tip with material causing crack closure to occur. Later studies conducted by Solanki et al. [46, 51] and Roychowdhury and Dodds [63] show that the proportional term in the series expansion of the stress intensity factor solution, the $T$-stress has significant effect on the crack closure level under plane strain conditions. The MT-specimen and CT-specimen often used in both experimental and numerical fatigue studies have very different $T$-stresses defined as [9],

$$T = \frac{\beta K_1}{\sqrt{\pi a}},$$  \hspace{1cm} (31)
Figure 25: (a) Crack closure level obtained from an elastic perfectly plastic material description. (b) Crack closure level obtained from a bilinear elastic plastic material description. (c) Crack closure level obtained from a more sophisticated constitutive description. Source [54].
where $a$ is the crack length, $\beta$ the biaxiality ratio and $K_I$ is the stress intensity factor. The ratio is equal to -1 and 0.425 for the MT and CT-specimen, respectively. This means that a compressive stress acts along the tangential direction of the crack faces in the CT-specimen contrary to a tensile stress in the CT-specimen. Fig. 26 obtained from [51] shows the effect on the crack closure level under plane strain of an externally induced $T$-stress applied as tractions along the crack face, varying from compressive to tensile. As the $T$-stress becomes tensile, Solanki et al. [51] state that the crack front becomes fully open except for the element just behind the crack tip, which would indicate negligible closure. This may be an explanation to the problem of finding a converged value of the crack closure level in CT-geometries (bending type of geometries) even if extremely refined meshes are used, cf. [53].

Figure 26: The crack closure level versus element size and $T$-stress level. Source [51].

The effect of the $T$-stress on the crack opening level under plane stress conditions is found to be very small under SSY-conditions [51]. Roychowdhury and Dodds [63] also show that the $T$-stress has a very small influence on the variation of the opening level in the plane stress region.

It is well known that the triaxial stress state in front of the crack tip will vary with specimen thickness, distance from the crack tip and the load level. A full three dimensional analysis is the only way to capture the relation between these parameters in the same model.
5.1.8 Analytical Model; the Strip Yield Model

Based on the Dugdale model [16] several researchers have developed semi-analytical codes in order to simulate the crack opening/closure behaviour in constant and variable amplitude loading situations (strip yield models). The models leave plastically deformed material in the wake of the advancing crack. Perhaps one of the most famous models is developed by Newman [44]. He presents a general crack opening stress equation for constant amplitude loading, Eqs. 32 - 37. The equation is a function of stress ratio, stress level $S_{\text{max}}$ and a three dimensional constraint parameter, $\alpha$. The equation is based on a curve fit to simulated crack opening levels by a two dimensional semi-analytical model, Newman [64], based on the Dugdale model. The constraint parameter is included in order to account for the three dimensional effects, which are not possible to simulate naturally in a two dimensional model. That is, the material yields when the stress is $\alpha \sigma_0$, where $\sigma_0$ is the average of the yield stress and the ultimate tensile strength. In compression the material is assumed to yield when the stress is equal to $-\sigma_0$. The constraint parameter is equal to 1 for plane stress and equal to 3 for pure plane deformation. The equations are,

\[
\frac{S_0}{S_{\text{max}}} = A_0 + A_1 R + A_2 R^2 + A_3 R^3 \quad \text{for} \quad R \geq 0 \quad (32)
\]

and

\[
\frac{S_0}{S_{\text{max}}} = A_0 + A_1 R \quad \text{for} \quad -1 \leq R < 0 \quad (33)
\]

where $S_0$ is the crack opening stress and $S_0 \geq S_{\text{min}}$. The coefficients are:

\[
A_0 = (0.825 - 0.34\alpha + 0.05\alpha^2) \left[ \cos \left( \frac{\pi S_{\text{max}}}{2\sigma_0} \right) \right]^\frac{1}{\alpha} \quad (34)
\]

\[
A_1 = (0.415 - 0.071\alpha) \frac{S_{\text{max}}}{\sigma_0} \quad (35)
\]

\[
A_2 = 1 - A_0 - A_1 - A_3 \quad (36)
\]

\[
A_3 = 2A_0 + A_1 - 1. \quad (37)
\]

The value of $\alpha$ is chosen as a curve fitting parameter in order to collapse experimental fatigue crack growth rates at different load ratios into a single curve. But the constraint parameter is also dependent on the stress level and specimen thickness since the constraint effect will vary with applied load and
geometry. Newman points out that the precise effect of the three dimensional constraint on the crack opening stresses is still not known.

The crack opening stress equations are based on an elastic perfectly plastic constitutive description. The geometry used is the same as Dugdale used; a Center Crack Tension panel, CCT specimen. Fig. 27(a) shows variation of the normalized crack opening stresses versus the load ratio for different applied stress levels. Fig. 27(b) illustrates the dependence of constraint on the crack opening stress and Fig. 28 shows the normalized crack opening stresses as a functions of stress level for several different load ratios.

Figure 27: (a) The crack closure level under plane stress versus the load ratio for different load levels. (b) The crack closure level versus the load ratio for different constraint factors. $\alpha = 1$ for plane stress and $\alpha = 3$ for plane strain. Source [64].

Fig. 28 gives that the maximum stress has a major affect on the crack opening stress at low load ratios and especially under plane stress conditions. At a load ratio equal to 0.7 no effect can be observed.

## 5.2 Crack Closure and Crack Length Measurement under ISY and LSY Conditions

### 5.2.1 Crack Closure and Length Measuring Techniques

The number of experimental studies based on the PD method is limited in the area of crack closure under ISY and LSY conditions compared to the amount of research conducted under SSY conditions.
Andersson et al. [65] conduct LCF experiments and measure the crack length and the crack closure level with the DCPD technique. The calibration curve is constructed from known crack length increments versus increase in potential-drop value obtained in another study by Andersson et al. [66], but the function is not reported in either paper. The potential-drop signal is recorded during the entire load cycle and the crack closure level is extracted at the first point on the linear part of the upper part of the curve, shown in Fig. 29.

Fig. 29 shows an increase in the potential-drop signal even though the crack is fully open. Andersson et al. attribute this behaviour to the plastic deformation, or the change in conductivity due to plasticity.

In another study by Andersson et al. [31] the possibility to use DCPD method for crack closure measurements is evaluated by comparison with in situ scanning electron microscope (SEM) and compliance based closure simulations and measurements. The compliance measurements are done by placing a strain-gauge close to the crack tip. Numerical simulations of the change of the potential-drop signal over a load cycle are also conducted by use of a 2D FE-model with an elastic-plastic constitutive model. The result shows that the compliance based simulation of the crack closure level gives
Figure 29: The normalized potential-drop signal versus the strain level obtained from an experiment. Source [65].

About 10 – 20% lower values than obtained by potential-drop simulations, which is consistent with the measurements. The crack closure level obtained from the potential-drop measurements are very close to the ones obtained with SEM, Fig. 30. Andersson et al. conclude that the DCPD technique is a reliable technique for measuring the crack closure level under LSY conditions.

Figure 30: The crack closure level obtained from the potential-drop signal (marked with an arrow) and measured with a scanning electron microscope. Source [31].

Some researchers have tried to survey the ability of PD technique to measure the crack length under ISY and LSY conditions, often in connection with creep. Saxena [67] employs the DCPD method to measure the crack length under monotonic creep crack growth at high temperature. Saxena observes differences between potential-drop measured crack length and di-
Arguments are presented pointing towards the presence of plastic and creep deformation at the crack tip resulting in crack tip blunting, tunneling and change of geometry. Saxena suggests an arbitrary linear relation that corrects the difference observed in the experiments. Wilkowski and Maxey [33] pointed out that the potential-drop signal will change with plastic deformation (material resistivity change) and geometry change due to plastic deformation. A schematic picture of the dependence of the plastic deformation and toughness of the material on the potential-drop calibration curves is presented and is shown in Fig. 31.

Figure 31: The effect of plasticity on the potential-drop signal, schematically. Source [33].

The authors (Wilkowski and Maxey) also suggest that an initial correction for the plastic deformation and resistivity change on the reference voltage is all that is needed before the fatigue experiment (steady state) can be conducted and the correct crack lengths would be measured. Merah et al. [68] show in their paper that a calibration curve obtained from a replica made of aluminum can be used in the presence of high inelastic deformation in a steel material at room temperature. The important point is to choose the reference voltage corresponding to the same steady state as the actual experiment is to be conducted at. Merah et al. also measure the crack length
directly by an optical system and image processing in order to detect the crack length. However, at an elevated temperature of 600°C the calibration curve is found not to be directly applicable. The authors (Merah et al.) suggest a correction due to the increase in plastic deformation and crack tip blunting at the elevated temperature. Fig. 32 shows the potential ratio versus cycle number; point A is the initial ratio and point B is the ratio after a few cycles (~100) chosen as the reference ratio.

Merah [69] also presents a review paper on ACPD and DCPD techniques as non-destructive detection tools for preventive maintenance.

Härkegård et al. [70] recently use the DCPD technique for fatigue tests under LSY conditions. Fatigue cracks are initiated and propagated up to a crack length of 1-1.5 mm under strain controlled conditions. No problems are reported regarding the measurements.

Several researchers use the compliance based method to measure the crack closure level during fatigue crack growth under ISY and LSY conditions. Under SSY conditions the majority places a strain-gauge close to the crack tip and record the load cycles. This method is not possible to use under LSY conditions since the plastic deformation is very large in the vicinity of the crack tip. Dowling and Begley [11] conduct LSY fatigue tests on CT-specimens. Shallow threaded holes are made on both sides of the crack plane and knife edges are attached in order to measure the deflection along the crack line with a clip-gauge. Comparison between side clip-gauge and load line clip-gauge is done in order to assess the possibility to extract the crack opening level from the load line displacement versus load data. No significant difference is observed and Fig. 33 shows the point of crack closure in a typical load cycle. Dowling and Begley also state that this is just a first order estimation of effect of crack closure and no significance should be putted into its details.

Tanaka et al. [71], El-Haddad and Mukherjee [72], Jolles [73] and several others all use the same method (load line displacement versus applied load) as Dowling and Begley for estimating the crack opening and closure level and the crack length is almost always measured by a visual system. Fig. 34 shows the load line displacement versus applied load presented by Jolles.

5.2.2 General Trend of Crack Closure under Constant Load Amplitude Testing and ISY/LSY

A systematic experimental study of the crack closure behaviour under LSY fatigue has not been published yet (to the author’s knowledge), although some measurements have been conducted. Iyyer and Dowling [74] conduct fully reversed strain controlled LCF tests on smooth specimens and measure
Figure 32: The potential drop signal versus cycle number under two fatigue tests. Source [68].
Figure 33: Estimation of the cyclic $J$-integral, $\Delta J_D$. Source [11].

Figure 34: The load versus the load line displacement from a LSY fatigue test. Source [74].
the crack opening and closure levels. The qualitative trends show that the relative crack opening and closure level decrease with increase in strain range, with the crack closure level decreasing even more than the opening level. Also, the closure levels are negative meaning that the crack is still open at negative global loads when gross plastic deformations prevail. However, limitations in their experimental equipment make the specific quantitative values approximative. McClung and Sehitoglu [75] present results that are consistent with the results obtained in [74]. Strain controlled and fully reversed, constant and variable block loading, experiments with nominal strain ranges between 0.001 and 0.007 are conducted. The crack lengths vary between 0.1 to 1.5 mm initiated from 50 μm deep notches. Crack opening and closure levels are measured with replica technique. The normalized crack opening level is observed to decrease with increasing strain amplitude for constant amplitude histories. The crack closing level is also observed to decrease with strain amplitude and becomes significantly lower than the opening levels at high strain. Fig. 35 shows the crack opening dependence presented in [76].

![Figure 35: Experimental and numerical data on the crack closure level under load ratio $R = -1$ versus the normalized load level. Source [76].](image)

Andersson et al. [65] present an overview of the potential-drop measured crack closure level shown here in Fig. 36.

The experiments are conducted under strain control and the crack length is measured with DCPD technique as mentioned above.

Despite the rather few experimentally published results regarding crack opening and closure under ISY and LSY they seems to be quite consistent.
The crack opening level is less affected than the closure level with increasing strain range, but still a strong dependence exists. The crack opening level decreases with increasing strain range and negative global crack opening levels occur as the nominal stress approaches the yield stress level. Fig. 36 also shows an independence of the strain ratio at very high compressive strain ranges.

5.2.3 Crack Closure under Variable Load Amplitude Testing and ISY/LSY

A systematic study of the crack closure effects at LSY and simple variable amplitude load is lacking. McClung and Sehitoglu [75] present some data on fatigue crack growth rates of small cracks (0.1 to 1.5 mm) subjected to strain controlled simple variable amplitude block loading. The conclusion from the study is that an overload in the LSY region may result in an acceleration of the fatigue crack growth rate in certain types of blocks. If the same load block would be applied within the SSY region a retardation effect would have been expected. This is probably due to the overall difference in strain level, which is the major difference between the two regions (SSY and LSY).
5.2.4 Numerical Aspects on Simulations of Crack Closure under ISY/LSY

In the ISY and LSY regions one can expect plasticity induced crack closure to be the dominating crack closure mechanism. But still very few papers are published and results regarding the FEM, crack closure, LSY. No systematic study has been conducted.

5.2.5 Mesh Refinement

The idea with different mesh refinement criteria is to resolve the plastic strain field in front of the crack tip sufficiently at SSY load levels in order to obtain steady state of, for instance, the numerically simulated crack closure level. The element size is then calculated as the ratio between some load parameter and a material parameter as for instance in Eq. 30. If the load level is in the ISY or LSY region one can suspect by looking at the criteria obtained for the SSY region that the element size requirement can be relaxed when applying higher loads. No papers are published investigating this, but Newman indicates this in [47].

5.2.6 Constitutive Dependence of Crack Closure

As stated above, the constitutive model has a major influence on the simulated crack closure level. When LSY prevails, the material is plastically deformed out to the boundaries of the geometry. This means that the overall response of the geometry is dependent on how well the constitutive model represents the real material behaviour, thus the local behaviour at the crack tip becomes dependent of the deformation of the whole geometry. Most metallic material shows some degree of mean stress relaxation and/or ratchetting depending on the boundary conditions. The local steady state stress loop typically becomes \( R = -1 \) if some degree of strain control exists in the boundary condition independent of the global strain ratio. This effect becomes even more pronounced as strain/load range increases. Simple constitutive models as elastic perfectly plastic and bilinear elastic-plastic combined with a kinematic and/or isotropic hardening rule do not capture the mean stress relaxation or ratchetting effect, which may be very important in the ISY and LSY regime.

5.2.7 Constraint Effects

As gross plastic deformation occurs around the crack tip it can be expected that the constraint effect will decrease. The stress state, which is near plane
deformation in the interior of the crack tip and plane stress on the surface under SSY conditions, will change towards plane stress through the specimen, cf. McClung and Sehitoglu [75].

5.2.8 Numerical Results of Crack Closure Simulations under ISY/LSY

If the number of publications on experimental measurements of the crack closure level is small, the situation is at least better regarding the number of numerical results in the area.

Lalor and Sehitoglu [77] study the crack opening and closure level of a growing fatigue crack from a notch and the results are said to be independent of the notch affected region. A 2D FE model is used and both plane strain and plane stress are studied under load control. Two different constitutive models are used; both bilinear elastic-plastic but with different ratios between the hardening modulus $H$ and elastic modulus $E$. Kinematic hardening is employed for both models. Fig. 37 shows the crack opening results for plane stress and plane strain.

![Figure 37: The crack opening levels obtained numerically for a crack growing out from a hole in two different materials. Source [77].](image)

Lalor and Sehitoglu conclude from their analysis that the difference between the crack opening and closure level becomes more significant outside the SSY regime; the closure level decreases more compared to the opening level. McClung and Sehitoglu [75] use the modified Dugdale model by Newman to estimate the crack opening level and compare with the experimental results shown in Fig. 35. In order to at least approximately take the strain-hardening into account an average of the yield stress and ultimate tensile strength is used as yield limit in the model. Fig. 38 shows the general behaviour of Newman’s crack opening model.

As McClung and Sehitoglu point out, crack closure seems to have a more significant effect in high strain fatigue than in fatigue under SSY. From Fig.
one can see that Newman’s results of the opening levels show weak dependence of the maximum stress level at plane strain, but vary widely with maximum stress under plane stress, which is similar to what Lalor and Sehitoglu show. Also, a more pronounced dependence of the load ratio seems to exist at higher maximum stresses. McClung and Sehitoglu conclude that the effective stress range ratio increases with increasing strain amplitude for constant amplitude cycling at fixed stress ratio. Further the crack closure levels are significantly lower than the opening levels at large strain amplitudes.

A few years later McClung [78] presents an overview of the effect of stress amplitude, stress ratio and strain hardening on the crack opening stress under ISY and LSY, see Fig. 39. McClung also points out from the FEM analysis that the crack depth had no significance on the crack opening level.

A clear difference exists in the crack opening and closure level when going from the SSY regime to the ISY and LSY regime. In the SSY regime the crack opening/closure level is rather independent of the load range applied under constant amplitude loading and plane strain assuming a constant load ratio. This is not the case when raising the load range (under constant
Figure 39: The crack closure level estimated numerically for two different materials and load ratios versus the normalized load level. Source [78].

load ratio). The crack opening level starts to decrease and the crack closure level decreases even more. When the nominal stress approaches the yield limit negative crack opening/closure levels are observed as shown both experimentally and numerically in Fig. 35. In Fig. 35 also the prediction of Newman’s crack opening model is included, which is a SSY model, however here approximately extended to the ISY/LSY regime.

5.2.9 Analytical Method; Extended Dugdale Model

Rose and Wang [19], as mentioned in section ”The strip yield model”, present an extension of the Dugdale model into the LSY region under the condition of self similarity. Fig. 40 shows the normalized residual stretch behind the crack tip of a crack growing at a constant load ratio for different applied load levels and load ratios under plane stress conditions.

Fig. 40 shows a decrease in the normalized residual stretch level as the load level is increased at $R < 0.3$. When the applied load level approaches the yield limit and $R = -1$ only very small normalized residual stretches seems to exist. At $R > 0.3$ the residual stretch level seems to be rather independent of the load level. The result seems to be quite in line with the results presented by [14, 78, 79], i.e. high load levels and low load ratios give very low crack
Figure 40: The normalized residual stretch at the crack tip versus the load ratio for different load levels. Source [19].

Rose and Wang also compare the model with Newman’s crack closure model [44] here presented in Fig. 41. From Newman’s model the crack opening level is obtained and through Eq. 38, which is applicable under SSY conditions, is the nominal $\Delta CTOD$ calculated,

$$\Delta CTOD = \frac{\Delta K_{eff}^2}{E\sigma_0}.$$  \hspace{1cm} (38)

Fig. 41 shows that Newman’s model is unconservative compared to the present model presented by Rose and Wang and the difference increases as the ISY and LSY regions are approached.
Figure 41: Normalized cyclic crack tip opening range versus normalized applied load level. Source [19].
6 Fatigue Crack Growth Investigations & Analysis Based on $\Delta J$

Nonlinear fatigue crack growth experiments and investigations are first conducted and published in the middle of the 1970’s, with Dowling and Begley [11] as the first to investigate the ability of $\Delta J$ to characterize the state at the fatigue crack tip during elastic-plastic conditions. The question they wanted to answer was: “Does the $J$-integral concept have meaning relative to the changes that occur in the crack tip stress and strain fields during the loading half of one fatigue cycle?” The $\Delta J_D$ is calculated from Eq. 12 by integrating the load displacement curves from the experiments. Crack closure occurs under the experiments and is measured by a clip-gauge attached to the side of the specimen. As the crack extends the clip-gauge is moved in order to be as close to the crack tip as possible. The displacement from the side clip-gauge is plotted versus the load line displacement and the nonlinearity is interpreted as an approximation of the crack closure level. This crack closure level is compared with the crack closure level estimated directly from the load displacement data taken from the load line. No significant difference is observed and the crack closure level from the clip-gauge at the load line is used. The other type of test, the load controlled experiments, show no signs of crack closure. Fig. 33 shows the operational definition of $\Delta J_D$.

In order to use as much of the specimen ligament as possible the maximum load/displacement level is decreased during the test. Otherwise rupture of the specimen would occur quite early during load control or on the other hand the crack would stop when the displacement is the controlled parameter. The load/displacement ratio is zero in all experiments.

The material used is A533B, which is a pressure vessel steel, and the specimens fulfilled the specifications according to ASTM E 399-73. The crack length is measured by a low magnification traveling microscope and the crack growth rates are calculated from a seven point interpolation formula of the crack length versus cycle number data. Fig. 42(a) shows the results from the displacement controlled experiments and Fig. 42(b) shows the results from the load controlled experiments. Fig. 42(a) also includes SSY fatigue data.

Fig. 42(a) shows a very clear linear correlation between $\Delta J_D$ and the fatigue crack growth rate under elastic-plastic conditions (solid circles). Also, the gross plasticity fatigue data are in agreement with the straight line extrapolation on a log-log plot of the LEFM data. Fig. 42(b) shows significant deviation from a straight line observed in the tests. Dowling and Begley explain this as an effect of incremental plasticity or increase in mean $J$ while $\Delta J_D$ remains approximately constant. This indicates that ratchetting could
be a problem in experiments if the mean load level is positive and large. A least squares curve fit to the displacement controlled experimental data is presented by Dowling and Begley and given below

$$\frac{da}{dN} = C (\Delta J)^\gamma,$$

(39)

where $C$ and $\gamma$ are regarded as material constants. As Dowling and Begley [11] note, the geometric independence is not tested since the same geometry is used in all experiments. Dowling presents further experiments [79] on center crack specimens on the same material and testing conditions as in [11]. Also tested and presented in [79] are fatigue data from large CT-specimens allowing for high crack growth rates under condition of LEFM. Fig. 43(a) shows the result from [79] and Fig. 43(b) shows the results from both [79, 11].

Dowling [79] concludes that $\Delta J_D$ is a valid geometry and size independent correlation parameter for elastic-plastic fatigue crack growth. This since the same fatigue crack growth rate is obtained from two different geometries under the same $\Delta J_D$ level. Also the fatigue crack growth rate obtained from
the large size linear elastic tests correlate with the rates obtained from the smaller specimen under nonlinear conditions. Dowling further concludes that an extrapolation of the LEFM fatigue data is in agreement with the fatigue crack growth rates obtained at elastic-plastic conditions in a log-log plot.

A few years later Brose and Dowling [80] investigate the in-plane size effects on the fatigue crack growth rate by testing CT-specimens with varying sizes differing by a factor of 16. The tests where gross plasticity occurs are conducted with the same method as in [11], i.e. under displacement control to a sloping line on a load versus deflection plot. This means that the deflection limit was increased as the load dropped due to crack growth. The other tests under linear elastic conditions are conducted at load control. The load ratio and displacement ratio are always zero. Fig. 44 shows the results from the tests. Brose and Dowling conclude that the cyclic $\Delta J_0$ based crack growth rate agrees well with data obtained under linear conditions. They also conclude that the different size criteria tested vary widely in the amount of plasticity they allow but provide comparable correlations of crack growth rate. The material used is AISI Type 304 stainless steel.
The same year as Brose and Dowling present their data (1979), Mowbray [81] publishes a paper that includes both experiments and analysis. Mowbray uses a somewhat different experimental approach compared to earlier investigations [11, 79, 80]. He uses a compact-type stripe specimen shown in Figure 44.
in Fig. 45. The specimen is found to give constant growth rates over the ligament under simple load control. The load ratio is 0.1 in all tests and the crack length is measured by visual means.

![Figure 45: Design of specimen that Mowbray uses. Source [81].](image)

Fig. 46(a) and 46(b) show the results from the tests. The cracks are propagated approximately 10 mm at each load range and the load frequency is varied in each test from 10 to 0.01 Hz. Displacement across the knife edges is measured with a clip-gauge and the crack opening levels are estimated from the load displacement curve. The material used is a chromium-molybdenum-vanadium steel.

The estimation of $\Delta J_D$ is based on integrating the area under the load displacement curve above the crack opening level. Fig. 47 shows the fatigue crack growth rate from the elastic-plastic tests as solid markers, also fatigue crack growth rates from LEFM fatigue tests (on different geometries) are included as open markers.

The fatigue crack growth rate tends to move away from the linear trend at very high rates. Mowbray relates this behaviour to the low toughness of the material compared with A533B used by Dowling and Begley [11]. Mowbray concludes that the overall results support the Dowling and Begley hypothesis that the crack growth rate is controlled by the $\Delta J_D$ and presented material constants according to Eq. 39.

The mean stress level seems to have a clear effect on the fatigue crack growth rate as Dowling and Begley [11] observe, shown in Fig. 42(b). This is further investigated by Tanaka et al. [71] in the early 1980’s. The materials used are two different low carbon steels with moderate hardening and overall similar mechanical properties. The specimens are of CT type and Center Cracked Plates (CCP) with different dimensions. The tests are performed
in a closed-loop servo-hydraulic testing machine under load controlled conditions with load ratio, $R$, between -1 and 0.7. The crack length is measured with traveling microscope with $\times 100$ magnification. The load displacements are recorded several times during the tests. The displacement is measured at the loading point in the CT-specimen and at the center of the crack in the CCP specimens. The crack closure levels are estimated from the load displacement record.

For the CT-specimen $\Delta J_D$ is evaluated from the load displacement curve above the crack closure point by use of the Merkle and Corten equation [82]

$$\Delta J_D = \frac{1 + \alpha \eta A^*}{1 + \alpha^2 Bb}, \quad (40)$$

where $\eta = 2$ for a CT-specimen and $A^*$ is the area under the load displacement curve, $\alpha$ is given by

$$\alpha = 2 \left[ \frac{1}{2} + \frac{a}{b} + \left( \frac{a}{b} \right)^2 \right]^{\frac{1}{2}} - 2 \left[ \frac{1}{2} + \frac{a}{b} \right]. \quad (41)$$

For the CCP specimen, $\Delta J_D$ was evaluated from
Figure 47: Fatigue crack growth rates from linear elastic and elastic-plastic tests. Source [81].

$$\Delta J_D = \frac{\Delta K_1^2}{E} + \frac{S^*}{Bb}, \quad (42)$$

where $\Delta K_1$ is the maximum stress intensity factor minus the stress intensity factor at the crack closure point, and $S^*$ is the energy enclosed by the loading curve and a secant line from the maximum load to the closure point shown below in Fig. 48.

Tanaka et al. observe that cyclic creep, i.e. ratchetting or accumulated plastic deformation, starts to occur when the back face of the CT-specimens yields. The same thing applies for the CCP specimen but the affect is not as clear as for the CT-specimen. Fig. 49(a) and 49(b) shows the fatigue crack growth rate for the two different specimens. "BY" means back face yielding and "GY" means general yielding. The results show rather clearly that a deviation from the linear trend occurs when general yielding takes place in
the specimens under constant load, i.e. $\Delta J_D$ is unable to account for the accumulated plastic deformation.

The fracture surfaces of all tested specimens are macroscopically flat and no shear lips are developed. Also, the fracture surfaces are examined with SEM, which shows that the initial deviation from the linear trend in growth rate is not due change in growth mode. The growth mode changes when rapid acceleration of the growth rate occurs well up in the general yielding zone. The rapid growth mode is due to void growth and coalescence while the more moderate growth rates create striation patterns on the fracture surface.

In order to approximately account for the contribution of the cyclic ratchetting to the growth rate a division of the maximum $J$-value is done. It is assumed that the total deformation can be decomposed into a cyclic component and an accumulated monotonic component. The $J_{D,max}$ is evaluated from the load displacement curve according to

$$J_{D,max} = \frac{1 + \alpha}{1 + \alpha^2} \frac{P_{max} t_e^{max}}{Bb} + 2 \frac{1 + \alpha}{1 + \alpha^2} \frac{P_{max} t_p^{max}}{Bb},$$

(43)

Here $t_e^{max}$ and $t_p^{max}$ are the elastic and plastic components of the displacement, Fig. 50, at the load line shown.

The relation between the crack growth increment per cycle and $J_{D,max}$ is shown in Fig. 51 for the CT-specimens. The results for load ratio $R = 0.1$ show that the growth rate seems to be rather independent of the maximum load level. Also, three different stages can be seen in the fatigue crack growth rate according to Tanaka et al. [71]. The first consists of stable fatigue growth
Figure 49: (a) The fatigue crack growth rate from the CT-specimens. (b) The fatigue crack growth rate from the CCP specimen. Source [71].

Figure 50: Schematic with the variables used in Eq. 40 marked. Source [71].
in the region below back face yielding, the second is the plateau near general yielding of the specimen and the third is the acceleration stage due to tearing mode growth contribution.

Figure 51: The fatigue crack growth rate versus $J_{D,max}$ for different load levels. Source [71].

In the mid 1980's Jollès [73] investigates the effects on the fatigue crack growth rate of the load gradient used in the tests. A few years earlier Hutchinson and Paris [83] theoretically show that the region of non-proportional strains due to crack extension will not have any significant effect on the validity of $J$ as a characterizing parameter as long as the crack growth increment is small in comparison to the region of proportional strains that controls the singular field. Hutchinson and Paris state this in an equation,
\[ \omega = \frac{b \, dJ}{J \, da} \gg 1, \quad (44) \]

where \( b \) is the uncracked ligament. Jolles uses CT-specimens of one size and conducts three tests under load control (increasing load at the crack tip) and seven tests under displacement control (decreasing load at the crack tip). The load ratios are between 0.1 and 0.3 and the displacement ratios are between -0.04 and 0.3. The material used is A533B and the crack length is measured with a low power traveling microscope.

The crack opening load is assumed to be equal to the crack closure load and is evaluated graphically from the load displacement record. Evaluation of \( \Delta J_D \) is done by integration of the load displacement record above the crack opening point. Fig. 52 shows the fatigue crack growth rates versus the \( \Delta J_D \) for all ten tests.

The non-dimensional \( J_D \) gradient varies between -2.85 to 4.53 in the experiments. The filled markers in Fig. 52 correspond to load control and the other to displacement control. Jolles concludes from the results that the relation between the fatigue crack growth rate and \( \Delta J_D \) is valid for non-dimensional \( J_D \) gradients down to -2.85. Also noted is that consideration of the crack closure level is necessary when analyzing the fatigue data.

In the second half of the 1980’s the regular PC became more widespread and consequently allowing for more numerical analyses and also to control the experiments in more sophisticated ways. Lambert et al. [12] conduct fatigue crack propagation tests on CT-specimens made of AISI 316L stainless steel. Side grooves are used to avoid curved crack front, but the result shows an increase in crack growth rate at the sides when large \( \Delta J_D \) is applied. The crack length is measured by use of the electric PD method and no problems are reported regarding influence of the large plastic deformations on the potential-drop signal. Optical crack length measurements are not possible due to the side grooves according to the authors. The tests are either conducted under constant \( \Delta J_D \) or increasing \( \Delta J_D \). The \( \Delta J_D \) at cycle \( n \) is calculated from the load displacement curve by use of Eq. 40 and the displacement is adjusted accordingly. The crack length is taken from cycle \( n-1 \). The crack closure level is corrected for in \( \Delta J_D \) during the tests but the procedure was not reported in detail.

Lambert et al. report influence of the load ratio \( R \) on the fatigue crack growth rate, indicating an effect of the maximum load level since the crack closure level already is taken into account. The authors use Eq. 45 proposed in [84] together with Eq. 39 to compensate for the load ratio effect. Note \( \Delta J_{D,eff} \) in Eq. 45 is not compensating for the crack closure level but for the load ratio (or equivalent the maximum load level),
Lambert et al. conduct a fractographic investigation of the fracture surfaces and both striations and void growth are observed. Void growth is found below the monotonic fracture toughness level.

Fig. 53 shows the fatigue crack growth rate versus $\Delta J_D$ (open markers) and $\Delta J_{D,\text{eff}}$ (solid markers). The solid line corresponds to fatigue crack growth rates obtained under SSY conditions.

The authors conclude from the investigation that $\Delta J_D$ seems to give somewhat lower fatigue crack growth rates compared to extrapolated fatigue crack growth rates from the SSY region. Also, the fatigue crack growth rates based
on different load ratios can be correlated with $\Delta J_{D,\text{eff}}$ in the range 0.05 to 0.5 mm/cycle.

In the beginning of the 1990's Banks-Sills and Volpert [85] report a combined experimental and numerical study on the cyclic $J$ integral, $\Delta J$. The idea is compare the experimental results with numerical analysis. The experiments consist of constant amplitude fatigue tests with two different load ratios $R = 0.05$ and 0.5 under load control. The load levels are rather low and are considered to be in the SSY regime. Five CT-specimens of aluminum 2024-T351 are used and the CMOD is measured with a displacement gauge at the load line. All tests are conducted in room temperature with a sinusoidal waveform and mainly at a frequency of 10 Hz. The load displacement data are periodically recorded by a personal computer. The crack length

Figure 53: The fatigue crack growth rates obtained by Lambert and Bathias. The solid line corresponds to LEFM data. Source [12].
measurements are conducted visually with traveling microscope with a maximum magnification of \( \times 80 \). The crack opening point is evaluated from the load displacement record, but the procedure is not reported in detail.

Elastic-plastic finite element simulations are carried out for a single crack length for both load ratios \( R = 0.05 \) and 0.5. Comparisons are made between \( \Delta J_D \) calculated both from the experimental load displacement curve and Eq. 40 and the numerical load displacement curve. Also, \( \Delta J \) calculated as a path independent integral is integrated into the comparison.

The results show a small difference, \(< 10\%\), between the numerical results of \( \Delta J \) and the experimentally obtained estimations, \( \Delta J_D \). The difference is pointed out to be related to the simple constitutive law included into the finite element simulations. Bank-Sills and Volpert conclude that \( \Delta J \) may be considered as a crack growth parameter to correlate fatigue crack growth data in the SSY region. The authors (Bank-Sills and Volpert) also point out that the line integral is a possible way to evaluate \( \Delta J \) in more complex geometries.

Rosenberger and Ghonem [86] investigate the anomalous behaviour of small fatigue cracks in a nickel-base superalloy by use of deformation based \( \Delta J_D \), similar to Eq. 40. They conclude that the anomalous crack growth is not related to the physical smallness of the crack, nor is it an intrinsic material property but is created by the precracking technique. The \( \Delta J_D \) parameter is able to consolidate the growth of small and long crack growth data according to the Rosenberger and Ghonem.

In the middle of the 1990’s Joyce et al. [87] investigate the general material properties including the elastic-plastic fatigue properties of a cast stainless steel. The elastic-plastic fatigue experiments are conducted under load control with load ratios \( R = -1, 0 \) and 0.3. The crack length is measured by compliance estimates, but Joyce et al. point out from earlier work that side grooves are essential to maintain a straight crack front. Further, the crack length measurements by the compliance method badly underestimate the crack length if the crack front tunnels, either forward or backward. Some problems are encountered with measuring the crack length in the tests with load ratios \( R = 0 \) and 0.3. Crack length measurements of the crack front on the fracture surface reveal a longer crack compared to the result of the compliance measurement.

The crack closure level is estimated by finding the point on the loading portion of the fatigue cycle corresponding to the slope of the initial unloading part of the same cycle. This is done numerically with a PC. Fig. 54 shows the closure load versus cycle number for all \( R = -1 \) tests. The results show that the closure load becomes rather constant (after \(~25\) cycles) or somewhat decreasing with increasing cycle number and \( \Delta J_D \). Near the end of the test a
steep increase of the closure load is observed that is related to the approach of the tensile load capacity according to the authors (Joyce et al.).

![Figure 54: Measured crack closure levels from tests conducted by Joyce et al. [87].](image)

The cyclic $J$-integral is estimated by use of Eq. 40 and consideration of the crack closure level. Fig. 55 shows the fatigue crack growth rate versus $\Delta J_D$ and in Fig. 56 SSY fatigue data are included together with a correlation equation for austenitic stainless steel from the standard text book of Rolfe and Barsom [88].

![Figure 55: Fatigue crack growth rates from test conducted by Joyce et al. [87].](image)
A fractographic investigation is conducted to reveal the micro mechanisms of fracture. The result shows striations marks on the fracture surface and no ductile tearing occurs until the very last few cycles in each experiment.

Joyce et al. conclude from the investigation that LSY fatigue crack growth using $\Delta J_D$ is consistent with SSY fatigue data typical for this type of material. Further, taking the crack closure effects into consideration is essential to accurately determine the crack driving force in elastic-plastic crack growth. The fractographic investigation reveals striation pattern on the fatigue crack surface and ductile tearing occurred only during the last few cycles.

Lu and Kobayashi [89] conduct experimental work on a low alloy steel with yield stress of 448 MPa and an ultimate tensile strength of 548 MPa. The tests are conducted under load control with load ratio $R = 0.05$ and 0.6. They observe two transitions in the fatigue crack growth rate. The fractographic investigation reveal striations at the crack surface when $J_{\text{max}} < J_{\text{IC}}$, and when $J_{\text{max}} > J_{\text{IC}}$ the micro mechanism change to ductile tearing with void growth and coalescence.

In 1996, Skallerud and Zhang [90] present a numerical study of ductile tearing and fatigue under cyclic plasticity of a welded plate. The total crack growth rate is assumed to be a sum of the fatigue crack growth rate and the tearing crack growth rate.

A 3D finite element model of a flat plate with a nearly circular embedded crack is modeled and subjected to a controlled nominal strain corresponding to the conditions used in earlier experiments on the same geometry. A
bilinear kinematic hardening constitutive model is used in the cyclic analyses. The cyclic $\Delta J_D$ is calculated by use of Eq. 12, taking the stress-strain curve from the FE model at different crack lengths. Minimal crack closure is detected in the finite element analysis and the whole load range is used in Eq. 39. The fatigue crack growth rate is obtained by extrapolation of the crack growth rate obtained under SSY conditions. The ductile tearing part is modeled with a modified Gurson-Tvergaard constitutive model accounting for nucleation, growth and coalescence of voids. As voids nucleate, growth and finally coalescence in front of the crack tip the crack extends in a stable manner.

The numerically calculated fatigue crack growth rates correlates rather well with the experimental ones when the ductile crack growth part is active in the numerical analyses. At lower loads, when only fatigue crack growth is active, an underestimate of the fatigue crack growth rate is obtained. Skallerud and Zhang relate the mismatch to the assumption of a homogeneous material in the weld in the numerical analyses. In reality a spatial difference exists in both fatigue crack growth characteristics and stress-strain behaviour. Also, the constitutive assumption of a bilinear kinematic hardening law is a simplification of the real behaviour.

Approximately 25 years after the first publication of research in the field of nonlinear elastic-plastic fatigue crack growth McClung et al. [91] conclude at the end of the 1990’s that it is rather clear that ”$\Delta J$ appears to be the parameter of choice for characterization of elastic-plastic fatigue crack growth”. McClung et al. present in [91] the first steps of a practical methodology towards an engineering treatment of the elastic-plastic fatigue problem. Most of the earlier studies have focused on to show experimentally that $\Delta J_D$ or $\Delta J$ is an appropriate parameter for characterizing fatigue crack growth under linear and nonlinear conditions. Very few of the investigations actually try to predict the fatigue crack growth rate by either numerical methods or other estimations schemes of $\Delta J$. The reason for this is probably connected to the non-existing computer resources and software needed for the predictions at that time. In [91] McClung et al. propose to use the reference stress method [92], a strategy developed by Ainsworth and colleagues at the former Central Electricity Generating Board (CEGB). Later the reference stress method is implemented into the R6 procedure for structural integrity assessment. An overview of different analytical flaw assessment methods can be found in [93].

The reference stress method requires three basic inputs: a $K$ solution, a description of the elastic-plastic constitutive behaviour and an estimate of the limit load for the cracked member assuming elastic perfectly plastic material. Then the $J$ estimate can be written as
\[ J = \frac{K_i^2}{f}, \]  
\[ \text{(46)} \]

where \( f \) is a function of the limit load, yield stress and constitutive equation. 
McClung \textit{et al.} [94] generalize Eq. 46 to the fatigue situation where the effective cyclic \( J \)-value is estimated by

\[ \Delta J_{\text{eff}} = \Delta J_{\text{eff}}^e + \Delta J_{\text{eff}}^p. \]
\[ \text{(47)} \]

The elastic term is written as

\[ \Delta J_{\text{eff}}^e = \frac{U^2 [\Delta K_1 (c^\Delta)]^2}{E'}, \]
\[ \text{(48)} \]

where \( U = \frac{\Delta K_{\text{max}} - \Delta K_{\text{open}}}{\Delta K_{\text{max}} - \Delta K_{\text{min}}} \) is the effective stress intensity factor range ratio. The crack opening level is obtained from Newmans crack closure equation, Eqs. 32 - 37. For plane strain \( E' = \frac{E}{(1-\nu^2)} \) and \( E' = E \) for plane stress, and \( c^\Delta \) is the crack length corrected for the cyclic plastic zone size. The plastic term is estimated by use of the reference stress method as

\[ \Delta J_{\text{eff}}^p = \mu V \alpha U \Delta J_0 (c) \left[ \frac{\Delta P}{2P_0 (c)} \right]^{n-1}. \]
\[ \text{(49)} \]

\( \Delta P \) is the range of the applied primary load, \( V \) is a dimensionless structural parameter and \( \mu = 1 \) for plane stress and \( 1 - \nu_p^2/1 - \nu_e^2 \) for plane strain, where \( \nu_e \) and \( \nu_p \) are the elastic and plastic values of Poissons ratio and \( c \) is the actual crack length. \( P_0 \) is an optimized characteristic yield load [95] for a cracked structure of yield strength \( \sigma_Y \). The form of Eq. 49 applies to materials following the Ramberg-Osgood constitutive relationship.

The method is implemented into the fracture mechanics analysis program NASGRO and the structural parameter \( V \) and the optimized yield load \( P_0 \) are determined from FE calculations of \( J \) for the surface crack and corner crack geometry [91]. The approach is validated against experimental data from surface and corner cracked geometries. The tests are conducted both under SSY and LSY conditions with load ratios \( R = 0, 0.1 \) and -1. Fig. 57 shows the correlation of the fatigue crack growth rates based on \( \Delta J_{\text{eff}} \) for the different geometries under both SSY and LSY conditions. McClung \textit{et al.} conclude that it is “possible to perform accurate elastic-plastic fatigue crack growth rate life predictions based on Paris crack growth constants from baseline SSY tests”.

In 2005, Sherry \textit{et al.} [96] present experiments and numerical analyses on the interaction between the fatigue crack growth and the ductile crack
Figure 57: Fatigue crack growth rates versus $\Delta J_{\text{eff}}$. Source [94].

growth. The material used is an austenitic stainless steel, 316L(N) and all experiments are conducted in ambient air. Sherry et al. conclude from the study that the tearing reduces the fatigue crack growth rate by up to a factor of 50%. The reduction is likely to result from residual compressive stresses in front of the crack tip and a mismatch of the fracture surfaces due to ductile crack growth. However, when the maximum stress intensity range becomes $\Delta K > 60$ MPa$\sqrt{m}$ directly calculated from the applied load levels, a drastic increase in crack growth rate is observed. The explanation according to the authors is that the peak driving force is within the elastic-plastic regime close to or above the initiation toughness.

The same year (2005) Tanaka et al. [97] present tests on thin-walled tubular specimens made of low-carbon steel. A circular notch is cut out in the tubular specimens. The specimens are loaded by a cyclic uniaxial tension-compression combined with cyclic torsion with and without superposed static and cyclic axial loading. The tests are conducted under load control and the crack length measurements are done by using a digital video microscope and plastic replicas. The crack growth directions follow the plane on which the total range of the normal stress including the compressive component of the stress is at maximum for both combined mode and single mode loading.

The crack opening level is measured by a specially designed extensometer continually measuring the load displacement loop. Fig. 58 shows the development of the crack opening level versus crack length.
Both the elastic crack tip parameters, $\Delta K$ and $\Delta K_{\text{eff}}$ based on the load level information, and the elastic-plastic $\Delta J_D$ based on load-displacement loop corrected for the crack closure level are calculated. Fig. 59(a) shows the fatigue crack growth rate based on elastic assumptions. The solid line corresponds to fatigue crack growth under SSY conditions and $R = 0$ for carbon steel given in maintenance code of JSME Standard (JSME S NA1-2000) according to the Tanaka et al. A clear tendency of under prediction of the fatigue crack growth rate is noted when the nominal stress intensity factor range is used. This even applies for the effective stress intensity factor range. Fig. 59(b) shows that the fatigue crack growth data can be approximated by $\Delta J_D$. The solid line is a least squares regression line to the data and not a prediction. This is the main problem for the application to engineering problems as the authors state, “Some computational methods of the $J$ estimation need to be developed for design purposes”.

As evident from the reviewed papers above $\Delta J_D$ is a unifying parameter for the fatigue crack growth under both SSY and LSY conditions. The work by Banks-Sills and Volpert [85] is probably one of the few on how to compare $\Delta J_D$ (Eq. 40) obtained experimentally with $\Delta J$ (Eq. 10) obtained numerically. The difference is rather small between the different results and Banks-Sills and Volpert point out this may be a possible way to evaluate $\Delta J$ in complex geometries. However, one should remember that the experiments are conducted under SSY conditions. But the problem seems to remain that no simple predictive engineering method is presented. McClung et al. [94]
Figure 59: (a) The fatigue crack growth rate versus the effective stress intensity factor range. (b) The fatigue crack growth rate versus $\Delta J_D$. Source [97].

present a suggestion to an engineering method, but it seems not to have been adopted by the community, yet.

Another problem is the ratchetting that takes place under load controlled experiments at high load ratios. The accumulated plastic deformation seems not to be accounted for by $\Delta J_D$ cf. [11, 71] and Figs. 42(b) and 49. Tanaka et al. [71] propose a solution based on the load-displacement curve and the results becomes rather independent of the maximum load level.
7 Fatigue Crack Growth Investigations & Analysis Based on $\Delta CTOD$

Crack tip blunting as a fatigue crack growth mechanism was first proposed by Laird and Smith [3] in year 1962. Direct observations of the crack tip opening profile are made. Later Newmann [98] and Kikukawa et al. [99] make further quantitative observations of the fatigue crack growth rate in connection to the $CTOD$ parameter. In 1980 Tomkins [5] investigates the connection between the fatigue crack growth rate under both SSY and LSY conditions and the micromechanical processes taking place at fatigue crack tip. The cracked surfaces are studied with aid of SEM and different types of striations are found. Tomkins concludes that there is a clear link between $CTOD$, the striations on the cracked surface and the fatigue crack growth rate, but no identity exists between them. Fig. 1 taken from [5] suggests that the fatigue crack growth rate is much smaller than $CTOD_{\text{max}}$ (marked as $\delta/2$ in the figure and calculated theoretically as half of Eq. 20 with the yield stress substituted with the flow stress) at low growth rates but approaches about half of the $CTOD_{\text{max}}$ at high growth rates.

In 1984 Tanaka et al. [14] present a thorough investigation on the crack tip blunting as a fatigue crack growth mechanism. Fatigue experiments are conducted on center crack specimens under load or displacement control. The displacement is completely reversed and controlled through a gauge placed on the specimen. The strain range $\Delta \varepsilon$ varies from 0.125 to 1.0% resulting in quite high growth rates. Load controlled experiments with load ratio $R = -1$ is used to obtain data at lower growth rates. The crack length is measured with a travelling microscope. $\Delta J_D$ is evaluated from the load-displacement record and correction is made for the crack opening level. Three different materials are used: oxygen-free high-conductivity copper, very low carbon steel and stainless steel Type 304.

The fracture surface is observed with a scanning electron microscope and special measurements of the striations pattern are done. The crack opening displacement is measured with an optical microscope with $\times100 - 400$ magnification. A number of pictures are taken at the specimen surface around the crack tip at loading and unloading. Careful examinations give that the crack tip blunts when the tensile load is applied. As the load is increased the crack becomes more blunted, controlled by the shear band near the crack tip spreading in two directions. Tanaka et al. point out that as the shape of blunted crack tip develops, the position of the shear band seems to suggest that the alternating shear mechanism rather than the simultaneous shear mechanism is active.
The fractures surfaces are covered by striations at crack growth rates above \(\sim 0.2 \ \mu m/cycle\). Dimples are found among striations on the fracture surface of the Type 304 stainless steel at rates higher than 3 \(\mu m/cycle\). No dimples are found on the fracture surfaces of copper or low carbon steel below 50 \(\mu m/cycle\). Fig. 60 shows the measured striation spacing versus macroscopic fatigue crack growth rates. The bars indicate the range of the measured values.

![Figure 60: Measured striation spacing versus the fatigue crack growth rate. Source [14].](image)

The figure shows a clear 1:1 coupling between the striation spacing and the fatigue crack growth rate at rates between 2 \(\mu m/cycle\) and 50 \(\mu m/cycle\) for both copper and the low carbon steel. In the case of the stainless steel the maximum rate where the striation spacing is measured is 8 \(\mu m/cycle\). Thus the void growth seems not to affect the equality between the striation spacing and the fatigue crack growth rate at the measured growth rates. This result is quite in line with the results obtained by Tomkins [5] on the same material shown in Fig. 1.

Tanaka et al. [14] evaluate the CTOD in three different ways in order to investigate if proportionality exists to the fatigue crack growth rate. The conclusion is that no proportionality exist and the fatigue crack growth rate must be expressed as
\[ \frac{da}{dN} = A (\Delta CTOD)^p. \] (50)

Where \( \Delta CTOD \) is correlated to \( \Delta J_D \) through the following equation, simply by assuming that a variant of Eq. 21 is applicable in the fatigue situation,

\[ \Delta CTOD = B \left( \frac{\Delta J_D}{\sigma_{Y,cyc}} \right)^q. \] (51)

The parameter \( q \) is equal to one for the low carbon steel and larger than one for copper.

Tanaka et al. present the fatigue crack growth rate versus \( \Delta J_D \) shown in Fig. 61(a) below. The results exhibit quite large a scatter according to Tanaka et al. and different slopes for the different materials tested. The same experiments are also evaluated by measuring the \( \Delta CTOD \) 250 \( \mu m \) in-situ behind the crack tip, in Fig. 61(b) expressed as \( \Delta \Phi 250_{\mu m} \).

Figure 61: (a) Fatigue crack growth rates versus \( \Delta J_D \). (b) Fatigue crack growth rates versus \( \Delta CTOD \) measured 250 \( \mu m \) behind the crack tip. Source [14].
The results in Fig. 61(b) show that the variance between the different materials is very small based on $\Delta CTOD$ measured 250 $\mu$m behind the crack tip compared to the results based on $\Delta J_D$. An explanation for this was not given by Tanaka et al.

McClung and Sehitoglu [75] conduct an extensive study of the closure behaviour and fatigue crack growth rates of small fatigue cracks (0.1 to 1.5 mm in length). The crack closure part is discussed earlier in section "General Trend of Crack Closure under Constant Load Amplitude Testing and ISY/LSY". Both constant amplitude and variable strain amplitude block tests are conducted. The constant amplitude tests have strain amplitudes ranging from 0.001 to 0.007 and the variable tests have one major cycle within each block with strain range of 0.005 and several ($10^2$, $10^3$ or $10^4$) small cycles. The fatigue data are evaluated by use of both $\Delta J_D$ and $\Delta CTOD$ of which the $\Delta CTOD$ data for the constant amplitude tests are reported here. The crack lengths are measured by the replica technique.

McClung and Sehitoglu use a series expansion of the result of Dugdale Eq. 19, here shown in Eq. 52. The expression is modified to fit the fatigue situation with cyclic loading,

$$\Delta CTOD_{eff} = \frac{1.25 \pi a}{2\alpha \sigma_0} \left[ \frac{(\Delta \sigma_{eff})^2}{E} + U \Delta \sigma_{eff} \Delta \epsilon_p \right]. \quad (52)$$

The coefficient 1.25 is a free edge correction factor, $\alpha$ is a constraint parameter equal to 1 for plane stress and 3 for plane strain. $U$ is the effective stress range defined as $\Delta \sigma_{eff}/\Delta \sigma$. The crack opening levels are obtained from replica measurements. Fig. 62 shows the fatigue crack growth rate versus $\Delta CTOD_{eff}$ for all the tests. A good correlation is obtained and the data fall within a scatterband of $\times 3$.

Measurements of the $\Delta CTOD$ are also reported. The measurements are taken on the side surface of the specimens with the aid of engraved fine polishing marks 1 to 5 $\mu$m apart and at an angle of 45° to the specimens load axes. The marks are continuous across the crack when it is closed and offset by a distance proportional to the $CTOD$ when the crack is open. Fig. 63 shows measured $\Delta CTOD$ versus computed $\Delta CTOD$ by Eq. 52.

It seems to be generally accepted that the micro mechanisms acting in the case of a short crack are the same as for a long crack. When applying a tensile load the shear bands develop on two main slip systems accompanying the creation of the blunting form of the crack tip. At unloading the crack tip will resharpen under reversed plastic deformation at the tip. Davidson [100], McClung and Davidson [101], Nisitani et al. [102] all report experimental investigations on the blunting shape for both short and long cracks.
Figure 62: Correlation of constant amplitude fatigue crack growth. Source [75].

Davidson [100] concludes that the "fast growth of small fatigue cracks at high stresses is no more anomalous than the slow growth of large fatigue cracks at low stresses". The load levels used in [100] corresponds to SSY. McClung and Davidson [101] present an extensive study comparing numerically obtained strains and displacements at the crack tip with experimentally measured data on commercial 7075 aluminum. Fig. 64 shows the normalized $CTOD_{eff}$ measured by stereo imaging techniques near the crack tip for both small and large cracks. The maximum applied stresses are about 85% of the yield stress for the small cracks. Also included are numerical calculations of the $CTOD_{max}$ for both small and large cracks. The numerical calculations are based on propagation of the crack through the mesh by changing the boundary conditions. The material is modelled as linear elastic-plastic with low kinematic hardening. The small crack data seem to lie somewhat above the average level of the large crack growth even though the difference is not as accentuated as the numerical analysis gives. McClung and Davidson note that the essential difference between large and small cracks does not seem to be the $CTOD$ behind the crack tip.

Nisitani et al. [102] compare the crack opening displacement near the
Figure 63: Measured versus predicted crack tip opening range. Source [75].

Figure 64: Numerical and experimental data on the normalized $CTOD_{eff}$ versus the normalized distance behind the crack tip. Source [101].

low cycle fatigue crack tip (in Fig. 65 shown as $\delta$) measured with SEM at the same crack growth rate for different crack lengths between 0.598 to 1.198 mm. The notations R and L in Fig. 65 indicate right and left hand sides of a crack in center crack panel. Nisitani et al. point out that the crack opening displacements near the crack tips are very similar at the same fatigue crack growth rate. Thus, the crack opening displacement near the crack tip is a
fundamental factor controlling the fatigue crack growth rate.

Figure 65: The crack tip shape for different crack lengths at the same load level. Source [102].

Wang and Rose [103] try to use the crack tip plastic blunting as a scaling parameter for correlating growth rates of short and long cracks. The parameter is estimated by finite element calculations that account for large deformation and non-proportional straining at the crack tip. The finite element model is only loaded with a monotonically increasing load and the Ramberg-Osgood constitutive model is used. Wang and Rose end up with a closed form expression for the crack tip blunting as a function of the stress intensity factor, applied far field plastic strain and material parameters as yield stress and hardening exponent. Fig. 66(a)-(d) show the improvement of the fatigue crack growth predictions based on the blunting parameter, $\Delta b$.

Although not as popular as the $\Delta J_D$, $\Delta CTOD$ seems to be a parameter able to correlate both linear and nonlinear fatigue data. The appealing feature with $\Delta CTOD$ is the apparent interpretation of the parameter. But, it is also more expensive in terms of measuring or calculating with the aid of the FEM compared to $\Delta J$ or $\Delta J_D$.

The measured striations spacing seems to have a clear coupling to the fatigue crack growth rates, Figs. 1 and 60. However, as Tomkins [5] points out, no identity exists for sure between the fatigue crack growth rate and the striation spacing although Tanaka et al. [14] later observes a 1:1 relation.

Very few papers are published regarding measurements and comparison of the crack tip shapes for short and long fatigue cracks at the same load levels. The results that Nisitani et al. [102] present shown in Fig. 65 are very interesting. They show that there seems to be no major difference between the shape of short and a long crack at the same growth rate.
Figure 66: (a) and (b). The fatigue crack growth rate versus the stress intensity factor range. (c) and (d). The fatigue crack growth rate versus the crack tip blunting parameter, $\Delta b$. Source [103].
8 Discussion

The experimental and numerical results presented in [19, 64, 74, 75, 76, 77, 78] all indicate a decreased crack opening/closure level with increased load level from SSY to LSY levels. The effect of not considering the crack closure/opening level is obvious; very non-conservative assessments may be the result.

The effect of the load level on the crack closure level is larger than on the opening level. There is essentially no difference between these in the SSY region but as the load is increased the closure level decreases even more than the opening level [64, 74, 75, 77]. The dependence of the crack closure on the load ratio seems also to be more pronounced in the LSY region, cf. Newman [64], Fig. 38.

Papers regarding crack closure under variable amplitude loading and LSY are lacking. Retardation effects due to overloads in the SSY region may act as accelerators for the fatigue crack growth rate when the load levels are increased towards the ISY/LSY region. This applies especially for negative load ratios since they may result in negative crack opening levels, cf. McClung and Sehitoglu [75].

Several different techniques exist for measuring the crack opening/closure level. The most common techniques in the LSY region are the compliance and the PD method. The compliance technique is basically used in papers presented early (1970’s) while usage of the PD technique is introduced later. The compliance method (Crack Mouth Opening Displacement, Back Face Strain) gives somewhat lower closure/opening values compared to Near-Crack-Tip-Gauge and the Acoustic Emission methods in the both SSY and LSY region, [30, 31]. The PD method gives about the same crack opening level as the Scanning Electron Microscope method under LSY conditions, Andersson et al. [31]. This may be an explanation to the increased popularity of the PD technique in later years. The drawback with the PD method is the need for an oxide layer to develop, on the freshly cracked surfaces, in order to function as a crack closure measurement method.

The most common technique for crack length measurements under LSY is some kind of visual system. However, the PD technique seems to be applicable even in situations of LSY and fatigue, [33, 68]. The requirement is to make an initial correction for the plastic deformation and resistivity change on the reference voltage, Wilkowski and Maxey [33]. Also, the continuation of the test must be conducted under approximately steady state conditions in order for the potential drop signal not to drift away.

When it comes to simple variable amplitude loading or random loading and LSY the situation becomes a lot harder to handle. At least if the test
method is based on in situ measurements of for instance the crack length. Crack closure measurements with a clip-gauge of the CMOD are not reliable even under SSY conditions [27] and are thus not expected to work in the LSY region. The same kind of problems may be expected with the BFS method. The PD method may work in the variable amplitude situation, but problems with distinguishing the crack closure point may appear as large plastic deformation takes place in certain cycles, Fig. 29.

Crack length measurements with the PD technique are probably not possible due to the overloads deforming the specimen and thereby destroying the steady state condition needed for this method to work. Methods based on the compliance are more probable to work if the compliance is measured on the elastic unloading part on the load cycle.

Some aspects on the numerical treatment of fatigue crack growth under LSY conditions are the following. The element size requirement in the SSY region will most probably automatically be satisfied in the case of LSY since the plastic zone is large compared with the characteristic size of the body. However, other parameters like the CTOD may be necessary to consider when studying element size requirements.

The requirement on a good description of constitutive behaviour of the material is expected to increase as the load level is increased from the SSY to the LSY region. This is due to the plastic deformation, which is not confined to a small region as in the SSY region, spreading out towards the boundaries of the body. Thus, the local behaviour at the crack tip may become dependent on the deformation of the whole geometry and thereby the constitutive description is very important. Also, simple constitutive models like elastic perfectly plastic or bilinear elastic-plastic combined with a kinematic and/or isotropic hardening rule do not capture the mean stress relaxation or ratchetting effect, which may be very important when conducting numerical simulations of fatigue crack growth at LSY conditions.

Several researches show [14, 81, 88] experimentally that $\Delta J_D$ is an unambiguous parameter correlating the fatigue crack growth rate under SSY, ISY and LSY conditions. The fatigue crack growth rate falls onto a single line plotted in a double logarithmic diagram versus $\Delta J_D$. However, the parameter is not suitable for all situations. For instance, fatigue crack growth at high load ratios, load control and LSY is a situation shown to be difficult for $\Delta J_D$ to correlate due to ratchetting effects, [71, 79]. While $\Delta J_D$ has shown to correlate fatigue data under both SSY and LSY one cannot expect $\Delta J$ calculated as a line integral to do the same. If the body is plastically deformed out to the boundaries i.e. LSY, the behaviour locally at the crack tip becomes dependent on the overall behaviour of the geometry and thereby result in path dependent $\Delta J$ values. However, in the SSY [86] and
ISY region both parameters are expected to give similar results. No papers are published comparing the both parameters (\(\Delta J_D\) and \(\Delta J\)) under ISY and LSY conditions.

The theoretical base for use of \(\Delta J\) in elastic-plastic materials is not without doubts. The definition of the path independence of the \(J\)-integral relies on linear elastic, nonlinear-elastic or deformations plasticity. Also, it is very hard to interpret the physical meaning of \(\Delta J\).

Despite the theoretical doubt about \(\Delta J\) as a correlating parameter, McClung et al. [91] based on the quite successful correlations presented by the research community, conclude that characterization of elastic plastic fatigue crack growth should be based on \(\Delta J\).

The other nonlinear elastic-plastic parameter \(\Delta CTOD\), not as frequently used as \(\Delta J\), is another possibility for correlating fatigue crack growth. This parameter has a very clear physical interpretation, easy to relate and compare with physical mechanisms taking place in the fracture process zone in contrast to \(\Delta J\). However, more sophisticated equipment is needed in order to measure the \(\Delta CTOD\) in experiments compared with \(\Delta J_D\).

The results that Tanaka et al. [14] and McClung and Sehitoglu [75] present shown in Figs. 61(b) and 62 respectively, show that \(\Delta CTOD\) correlates very low (\(10^{-9}\) m/cycle) to very high (\(10^{-5}\) m/cycle) fatigue crack growth rates. The fatigue crack growth rates of different materials are almost gathered by \(\Delta CTOD\) into a single line as shown in Fig. 61(b).

The results that McClung and Davidson [101] and Nisitani et al. [102] present indicate the \(CTOD\) parameter to be an unambiguous parameter. That is, the \(CTOD\) value is the same for a long as for a short crack at the same fatigue crack growth rate, Fig. 65. However, it should be noted that the crack length difference is quite small in the experiments Nisitani et al. present. Further investigations should be conducted.

The striation pattern is a sign left on the fracture surface from the fracture process. The measurements conducted in [14], Fig. 60 show a clear 1:1 coupling between the fatigue crack growth rate and the striations. Tomkins [5] obtains results quite in line with the ones presented by Tanaka et al. [14].

The suitability of the two nonlinear parameters \(\Delta CTOD\) and \(\Delta J\) as engineering parameters in assessments is problematic. There is no "easy" way today to make predictive estimates. The work that McClung et al. [91] present in the direction of developing an engineering approach to the nonlinear fatigue crack growth problem is very important. More effort should be put into that direction. Which of the two parameters should one choose? No clear answer exists. \(\Delta J\) maybe requires somewhat less work compared to \(\Delta CTOD\) to calculate by FEM, but on the other hand very hard to interpret physically.
9 Conclusions

- Experiments indicate that the $\Delta CTOD$ parameter seems to be an unambiguous parameter able to correlate fatigue crack growth data under LSY of both short and long cracks.

- The $\Delta CTOD$ parameter is able to correlate fatigue crack growth from the SSY to the LSY region into a single line in double logarithmic diagram.

- Experiments show that $\Delta J_D$ is an appropriate measure for fatigue crack growth under LSY. The parameter is able to correlate fatigue crack growth from the SSY to the LSY region into a single line in double logarithmic diagram.

- Engineering methods for fatigue crack growth under ISY/LSY are scarce. $\Delta J$ and $\Delta CTOD$ are possible parameters to use. However, more research efforts need to be focused on the development of an engineering method.

- The crack opening/closure level is decreasing with increasing load level. The closure level is decreasing more compared with the opening level and may become negative at high load level and negative load ratios.

- Realistic constitutive models incorporating the mean stress relaxation effects and the ratcheting effects are important when conducting simulations of fatigue crack growth under LSY.

- The electric potential drop method is a possible method to measure the fatigue crack growth rate and crack closure level under LSY. However, certain conditions must be fulfilled: an oxide layer seems to be necessary in order to the crack closure measurements to work, the reference voltage must be taken from a steady state condition.
10 Future Work

The following is a list of issues where further research efforts are needed. The list is not complete in any way.

- Continuation of the initial efforts on the development of an engineering approach to handle fatigue crack growth under ISY/LSY based on $\Delta J$ or $\Delta C T O D$.

- Further theoretical development of the $J$-integral in line of trying to justify the use of $\Delta J$ in fatigue crack growth situations.

- Systematic experimental measurements of crack opening/closure under constant amplitude loading and ISY/LSY are missing.

- Systematic experimental measurements of crack closure under variable amplitude loading and ISY/LSY are missing.

- Predictions of fatigue crack growth rates based on $\Delta J$ calculated as a line integral. Very few investigations are found actually trying to predict the growth rates of a fatigue crack growing under ISY/LSY conditions.
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