On stochastic optimization for short-term hydropower planning

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Abstract

Renewable generation is the fastest growing energy resources in the past decade. Renewable energy sources, particularly wind power, provide clean and environmentally friendly energy to meet the system demand, meanwhile introducing huge levels of uncertainty in the system. On the one hand the deregulated electric power industry and on the other hand the intermittent nature of renewable energy sources cause highly volatile and uncertain electricity prices in different market places. This will create challenges for economical operation and planning of the flexible energy sources, particularly hydropower, which being a flexible energy source is the best option to balance wind power variation.

The main purpose of this work is to develop optimal short-term planning models for price taker hydropower producer working in the existing environment. Those models have to deal with the huge level of uncertainties the wind power introduces into the power system.

An optimization tool known as stochastic optimization is used to plan hydropower production under uncertainties.

The first model, which is used to make sensitivity analysis, is a two-stage stochastic linear programming problem. The uncertainties are handled by generating scenarios based on historical data. Profound sensitivity analysis is provided, in terms of volatility in day-ahead market prices and water inflow level as well as in terms of water opportunity cost and initial volume of the reservoir. Based on the comparison of the stochastic and corresponding deterministic problems, the result aims to show the impact of modeling the uncertainties explicitly. The results show that for the short-term hydropower planning problems the effect of considering price uncertainty in the stochastic model is higher compared with considering inflow level uncertainty.

The second model used in this work is a two-stage stochastic linear programming problem. The model generates optimal bids to day-ahead market considering real-time market price uncertainties. While simultaneously bidding to both markets, the results for most of the hours suggest two actions; either to bid the available amount of energy to upward regulation market or to bid the maximum capacity to day-ahead market and bring back the whole amount making down regulation. To make the bidding strategies more flexible and robust different approaches are modeled and assessed. Finally one of the approaches is suggested as the most applicable one.
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List of Symbols

Price modeling

$t$  index for time; hourly resolution;
$\lambda_t$  spot market prices Euro/MWh;
$r_t$  real time market prices Euro/MWh;
$a_t$  the continues part of the real time market price Euro/MWh;
$b^↓_t, b^↑_t$  discrete part of the real time market price Euro/MWh;
$\delta^↓_t, \delta^↑_t$  price difference (real time market price - spot market price) Euro/MWh;
$P$  order for the seasonal AR polynomial;
$p$  order for the AR polynomial;
$Q$  order for the seasonal MA polynomial;
$q$  order for the MA polynomial;
$\Phi_i$  coefficients for the seasonal AR polynomial, $i=1,...,P$;
$\phi_i$  coefficients for the AR polynomial, $i=1,...,p$;
$\Theta_i$  coefficients for the seasonal MA polynomial, $i=1,...,Q$;
$\theta_i$  coefficients for the AR polynomial, $i=1,...,q$;
$\varepsilon_t$  white noise;
$s$  length of the season;
$D$  Difference for seasonal model;
$d$  Difference for ARMA model;
$\Delta^d$  difference of order d;
$\Delta^D_s$  difference of order D with season length s;
$p^k_{ij}$  transition probability from state i to state j, when the process stayed in state i k steps

Hydropower modeling

Sets

$i$  index for possible bid prices $i = 1, \ldots, I$;
$j$  index for power plants $j = 1, \ldots, J$;
$n$  index for discharging segments $n=1, \ldots, N$;
$s$  index set for scenarios $s = 1, \ldots, S$;
$t$  index for planning periods (h) $t = 1, \ldots, T$;
$R_j$  the set of indices for power plants downstream hydropower plant $j$;
Parameters

- $p_s$: probabilities associated with the price scenarios;
- $I_{j,t}$: inflow level to each power plant at each scenario and time (HE);
- $G_j$: maximum power production at plant $j$ (MW);
- $ar{m}_j$: maximum reservoir content (HE);
- $m_{j,t}^i$: initial reservoir content (HE);
- $m_{j,t}^e$: reservoir content at the end of the planning period (HE);
- $\mu_{j,n}$: marginal production equivalent at plant $j$ segment $n$ (MWh/HE);
- $\gamma_j$: expected future production equivalent for plant $j$ (MWh/HE);
- $\bar{Q}_j$: maximum discharge level at plant $j$ (HE);
- $q_{j,n}$: maximum discharge level in plant $j$ at segment $n$ (HE);
- $\lambda_f$: expected future electricity price (Euro/MWh);
- $\lambda_{s,t}$: realized spot market price for each hour and scenarios (Euro/MWh);
- $r_{s,t}$: real time market prices for each hour and scenario (Euro/MWh);
- $p_i$: possible fixed bid prices (Euro/MWh);
- $\tau_j$: the delay time for the water between power plants;

Variables

- $G_{j,t,s}$: generation level at each power plant, hour and scenarios (MWh);
- $m_{j,t,s}$: content of reservoir $j$ at the end of hour $t$, for each scenarios (HE);
- $Q_{j,t,n,s}$: discharged volume for hourly bids, for each power plant, segment, hour and scenarios (HE);
- $S_{j,t,s}$: spillage from reservoir $j$ during hour $t$, for each scenarios (HE);
- $w_{i,t}$: hourly bid volumes to day-ahead market corresponding to the possible bid prices (MWh);
- $x_{t,s}$: dispatch level for each hour, scenario according to day-ahead market bids (MWh);
- $d_{sl}^t$: dispatch level for each hour, scenario according to up-regulating market (MWh);
- $d_{sk}^t$: dispatch level for each hour, scenario according to down-regulating market (MWh);
- $z$: objective function value (Euro);
Chapter 1

Introduction

The wind power share in the power system is growing notably [3]. Predictions show that this growth will keep on increasing in the future [3], [4]. In addition, the wind power prediction techniques are not capable enough to consider wind power production variations, and therefore huge unpredicted wind power production variations might happen in the power system in the near future. These variations must be handled in an optimal way. Some conventional generation units must be used to compensate wind power production variations in the power system and in this way, maintain the balance between production and consumption of power in the power system.

Sweden has a large amount of hydropower, hence, it is highly possible to balance wind power variations with hydropower. This can be done in the following way: in the windy periods the water is stored in reservoirs, which later can be used to produce power, when the wind is extremely low. Due to the fluctuations in the wind power, the need for balancing power will increase, which in turn will change real-time market prices (increase upward regulation prices and decrease downward regulation prices). Because of its flexibility and capability to increase production very fast, hydropower with reservoirs can provide balancing power and earn extra profit. Therefore, there is a value of improved planning tools, which allow the hydropower producer to provide even more flexibility.

Until recently the electricity markets were regulated: there was no competition between market players. The consumers could not choose the electricity supplier and had to simply buy electric power from the local retailer. In contrast, in the deregulated electricity markets, electricity is traded in a competitive way. Hence, the electricity market players have to make their decisions without having proper
information about the competitors’ actions. Therefore, after the deregulation of the electricity market, optimal short-term hydropower planning has become even more challenging, and an essential task for a hydropower producer.

With the decentralized competitive nature of current electricity market on the one hand and on the other continuous increases in wind power share in the power system, a challenge is created for a short-term hydropower producer to plan and operate hydropower in an optimal way. Since traditional operation rules were working according to centralized optimization (optimization of separate power plants owned by several producers that are coordinated legally or voluntarily), there is a need for new, advanced hydropower planning techniques capable of working under existing new requirements. The suggested research project aims to contribute to this knowledge gap.

The most important aspects that short-term hydropower producers should take into consideration while developing production plan under uncertainty are the following: day ahead commitment (production bidding to spot market), production plan according to day-ahead commitment, bidding strategy to regulating market, water inflow, and future water value (up to several days counted from the delivery day). All those above mentioned points are subject to uncertainty depending on unexpected market changes and unpredictable weather conditions. Multi-state stochastic programming is used in the literature to cope with these uncertainties.

There are many deterministic models to plan short-term hydropower production in the literature [9], [11], [14] and [15]. However, researchers’ interest in work under uncertainty is noticeable recently. Deterministic models do not have any random variables and have a known set of inputs which will result in a unique set of outputs. In those models there is no new information arriving over time, and decisions are made in advance for the whole planning horizon, even for long-term planning. In contrast, stochastic programming addresses optimization under uncertainty, and reflects the fact that new information about the uncertain data arrives as time evolves along the planning horizon. According to information flow, a multi-stage stochastic program is characterized by partition of decision variables into stages in a way that decisions made for one stage are not affected by the information arriving in following stages.

Stochastic programming has been used for long-term planning for several decades. With higher levels of non-dispatchable generation, stochastic tools will become necessary also for short-term planning. The following papers apply multi-stage stochastic optimization techniques to plan short-term hydropower production [16], [23], [24], [26], [46], [50], [54].

Up to now, most existing stochastic models for short-term hydropower plan-
1.1. SCIENTIFIC OBJECTIVES

The aim of this thesis is to develop short-term stochastic planning models for hydropower producers, capable to work under the existing conditions. Those models have to generate optimal strategy for the hydropower producer: to sell water to day-ahead market, to provide reserve power or to keep the water for the future usage.

1.2. Contributions

- An overview of the literature in the field of short-term hydropower planning in the power systems with large amount of wind power (chapter 2).

- A review and assessment of short-term hydropower planning deterministic and stochastic models (chapter 2).

- There are different type of uncertainties to deal with while planning short-term hydropower production. Since it is not possible to model all uncertainties at once, it is important to know which type of uncertainties has a significant impact on the results in order to model them first. In this work the profound sensitivity analysis has shown that for the short-term hydropower planning it is more important to model uncertainties related to day-ahead market prices than the water inflow level (chapter 5).
• A case is studied using stochastic bidding model under the uncertainties of day-ahead and real-time market prices. The results are discussed and commented (chapter 5).

• To make the regulating market bids more realistic different modeling approaches are tested and evaluated (chapter 5).

1.3 Published papers

• Y. Vardanyan, M. Amelin, "The state-of-the-art of the short-term hydropower planning with large amount of wind power in the system", EEM 2011, Zagreb, Croatia, May 25-27


• Y. Vardanyan, L. Söder, M. Amelin, "On different approaches for realistic hydropower bidding strategies to day-ahead market considering real-time market", submitted to IEEE Transactions on Sustainable Energy.

1.4 Outline

The thesis is outlined in the following way:

• *Chapter 2* provides background information and theory needed to understand all notions, methods and models discussed in this work. First, it describes electricity markets and presents trading of the electric power as a commodity. Then, it gives general understanding of the wind power and hydropower as renewable energy sources. Next, it introduces short-term hydropower planning and appropriate planning tools. Finally, stochastic optimization theory is described as an appropriate short-term planning technique for a hydropower producer according to current requirements.

• *Chapter 3* describes the existing forecasting models for day-ahead market and real-time market prices. For day-ahead market prices ARIMA model is used and for real time market prices the combination of SARIMA and Markov Chains are used. Those models are using historical time series to predict and then generate corresponding scenarios.
1.4. OUTLINE

- **Chapter 4** explains the stochastic models which are used in the case studies. The first model is a two-stage stochastic problem with recourse under the uncertainty of day-ahead market prices and water inflow level. The second model is a two-stage stochastic bidding program under the uncertainty of day-ahead and real-time market prices.

- **Chapter 5** introduces case studies, which uses real data taken from Nordic electricity market. First, the electricity market price forecasting models described in chapter 3 are conducted and corresponding parameters are estimated. Then, the two-stage stochastic bidding strategy developed in [23] and two-stage stochastic bidding model developed in [48] with some modifications are applied to real data. Finally, sensitivity analyses are provided for the first model and different modeling approaches are formulated for the second model.

- **Chapter 6** concludes this thesis, provides discussion on the models presented in the previous chapters and states the ideas for future work.
Chapter 2

Background

This chapter serves as a fundament for the remaining chapters. The description for the restructured electricity market is provided. The modeling issues related to hydropower and wind power are explained. Optimization theory for stochastic modeling is presented.

2.1 Electricity market

At the end of the last century, the old electricity market model stylized as vertically integrated monopoly with the structured franchise was replaced by restructured models in many countries. The restructuring of the electricity market typically means functional decoupling of the vertically integrated system. In the restructured settings the electric power is traded in a competitive way and electricity market players are forced to have more market oriented approaches [31].

In reality, the electricity markets are highly complicated because of market imperfections such as information asymmetry, market power and transaction costs. The competitive environment can handle these and make the system work smoother, although even in restructured electricity markets it is not possible to eliminate above mentioned imperfections.

To maintain the system reliable and to handle short-term market interactions, there is a strong need for an operator. The system operator coordinates the use of the transmission network, and thus controls the dispatch. The control of the dispatch means to adjust the use of the transmission network. Therefore, in an electricity market, the core focus is on the design of the transmission-dispatch interaction.
In general, the electricity market is a way to transfer electric power from producers to end users in an optimal way (Figure 2.1).

![Figure 2.1: Competitive electricity market structure.](image)

### 2.1.1 Electricity market players

Many independent players are involved in the electricity market [2]:

- system operator,
- producers,
2.1. ELECTRICITY MARKET

- customers,
- grid owners,
- retailers,
- balance providers.

System operator

For any electricity market the reliable operation is the main requirement; for any instant in time the production has to be equal to the consumption Figure 2.2. The system operator is the main responsible player for co-ordination of electricity trading with the physical transmission, economic and physical balancing. The big influence on the transmission network means capability to control dispatch, which is the principal means of adjusting the use of the network.

It is important to mention that the system operator has to be independent of the existing electric utilities. Otherwise, if the system operator competes in the energy market, it can take a decision in favor of its own benefits.

In Sweden Svenska Kraftnät has the role of system operator and is responsible for the reliable operation of the Swedish electricity system and ensures that in Sweden the production and import is always in balance with the consumption and export.

Producers

Producers refer to electricity market players who own and operate generation units. The generated power is fed to the grid, and producers have to pay for the benefits they get from being connected to the network.

Consumers

Anyone from a big industry to a small household, who takes the electric power from the grid and consumes it, is called consumer. The consumer has to have a contract with an electricity trader to consume electric power. Big consumers can buy power directly from the producers or the power exchange. However, a small consumer can have a contract with a retailer, who will purchase electric power on their behalf. In addition, consumers must have an agreement with the grid owner to be connected to his grid and pay a network fee.
Grid owners

Grid owners are market players, who are responsible to transfer electric power from the producer to a consumer through the national, regional and local network. For example, in Sweden the national grid is owned by the Svenska Kraftnät.

The role of the regional network is to transmit the electric power from the grid to the local network. Finally, the end users in each geographical location get electric power through the local network.

Retailers

Small consumers, such as householders, turn to middlemen called retailers to purchase electric power on their behalf. The main role of retailers, sometimes called traders, are to purchase the electric power from the power exchange or directly from the producer and sell it to the small consumers. The existence of the retailers in the power market guarantees larger freedom for the consumers. Consumers are free to choose their retailer. Thus, the competition increases, which might result not only in lower prices, but also better service. The retailers manage
Balance providers

The balance providers are financially responsible for the electricity that is used during the actual hour to compensate the deviation from the beforehand planned amount. It is not necessary that every single market participant be a balance responsible player, since it is possible to transfer this responsibility to another player. It is common that retailers take this responsibility and of course get paid for the service they provide.

2.1.2 Power exchange

The power exchange power pools consist of physical trading and financial trading.

Electricity trading

The physical trading of the electric energy takes place in the day-ahead market, in the intra-day adjustment market and in the real-time market. In addition, players can trade the electric power through the bilateral contracts.

The day-ahead market, also called spot market, is a market place where the players can trade power in hourly contracts for the 24 hours of the following day. Selling and purchasing bids has to be submitted before the day-ahead market is closed. After the clearing of the market, the system price for each hour of the coming day is known and all players get notified whether their bids are accepted or not.

The intra-day adjustment market is a market place where actors can put selling or purchasing bids when the spot market is cleared. The intra-day market is closed before the delivery hour; in the Nordic market the intra-day market is closed one hour before the actual delivery. Therefore, this market makes it possible for players to adjust their trading in the spot market, according to the revealed information over time. For example, if the player is a wind power producer, the forecast about the possible wind power generation for the specific hour is more accurate close to the delivery hour. Thus, this is a great chance for the wind power producer to participate in the intra-day market and decrease the possible imbalances.
The real-time market also called regulating market is a market place for reserve power. Producers either having reserves or being rejected from the spot market can trade electric power on this market. The time span to submit bids to the real-time market is different from market to market. For the Nordic electricity market, the bids to the real-time market can be submitted earliest 14 days and latest 30 minutes before the delivery. Then for each actual hour bids are arranged according to the price and form a staircase (Figure 2.3). At the end of each hour the regulation price is settled, according to the most expensive bid activated during the upward regulation or the cheapest bid activated during the downward regulation.

![Figure 2.3: Staircase formed from the bids submitted to the real-time market.](image)

Financial trading

The power exchange includes future markets where different financial instruments are traded such as futures and options, by which market players tend to hedge prices [65]. In the market, hedging against risks means that one tries to bound the risks related to the price volatility. By hedging, electricity market players gain some protection and control on the risk level. The basic types of the financial contracts, also referred to as financial derivatives, are futures, forwards and options.

**Forward contract** is an agreement between two parties to trade electric power at a specific time in the future with a specific price. The future price varies in the different period in the future. Two parties having a forward contract can be a
2.1. ELECTRICITY MARKET

company and a financial actor. The forward contract includes information about the traded power and defined forward price. According to the forward contract, the power trading company will get a specified price for the traded energy. This is risky for the financial actor, since the future electricity prices are not known. Depending on the future electricity price the financial actor either makes a loss or gains some profit.

Futures contract The structure of the futures contract is similar to the forward contract. The difference is that the futures are traded on exchanges. The exchange guarantees the contract performance and defines the product price, the delivery quantity and the location. Since the product is traded on the exchange and standardized it will gain more liquidity, which in its turn will lead to more accurate prices. Generally, the futures market is easier to analyze due to the high level of transparency.

Options Having an option contract the holder gains a right but not an obligation to buy or sell electric power in some time in the future. The buyer of an option pays for the right, not the obligation, to trade the electricity on the contracted date; hence it can be interpreted as an insurance against price changes. The main types of options are a put option and a call option.

An agreement between two parties to exchange an asset at a certain price (strike price), by a predetermined date is called a put option. The seller of the put option has the obligation to buy the asset at the strike price if the buyer exercises the option. However, the buyer has the right but not an obligation to sell the asset at the strike price by the future date.

A financial contract between two parties is referred to as a call option. The buyer has the right but not the obligation to buy an agreed amount of a commodity from the seller at strike price and at the certain date. It is the seller’s obligation to sell the commodity if the buyer exercises the option. To have this right, the buyer has to pay a fee called a premium.

Post-trading

A market place where the imbalances are settled is called a post market. In the post market players having positive imbalance sell the amount of extra energy to the system operator and players who have negative imbalance buy the imbalance amount from the system operator. Figure 2.4 shows the time table for Nordic electricity market.
2.2 Renewable Energy Sources

Renewable energy is extracted from the natural resources that are replenished continuously. Renewable energy sources are environmentally friendly and do not cause air pollution. Main characteristics related to renewable energy sources are that they have to be generated where the source is located and then have to be transported to the final customer. Wind and hydropower are the main renewable energy sources under interest of this work.

2.2.1 Wind Power

Wind power as an energy source has been developed since the 1970s [71]. Wind turbines transform kinetic energy in the wind to electric power (Figure 2.5). Wind turbines do not create air pollution, do not need any fuel transport that can harm the environment and do not leave waste behind. Recently, the wind power is a fast growing industry. The size of the wind turbines increases over years and the technology becomes more advanced; new design for the rotor blades capable to extract more power from the wind, cheaper control system, new power electronic equipment capable to use variable speed.

Wind Power and its Impact on Power System

The renewable energy impact on the power system, particularly the wind power
impact is huge and interests many researchers in the field. All those impacts mainly depend on the following factors: the penetration level of wind power, the generation mix in the power system and grid size [61]. Moreover, the impacts of wind power on the power system can be divided into short and long-term effects. The short-term effects are those which last minutes to hours, whereas the long-term effects have yearly timescale. Example of long-term effect can be the contribution of wind power to the system adequacy: its capability to meet peak load situations in a reliable way. Besides, wind power impact on the power system can be categorized into local or global effects according to the effect size scale.

Although wind power is introducing more uncertainty to the system which creates planning problems for all electricity players in the system, it is always important to emphasize and start from the benefits of wind power. First, installed wind power can increase the energy reliability (the ability of the generating system to meet the existing demand) and decrease the risk of energy deficit. This is concluded in [63] where the author analyzes the impact of wind energy on the energy reliability in power system where hydro is the dominated power source and in [6], where the author analyzes the wind power contribution to supply adequacy. However, the intermittent behavior of wind power can also decrease the energy reli-
ability in different circumstances. Second, wind power can provide voltage control and partially support to primary and secondary control, however it can cause power quality problems creating harmonics and voltage flicker [38].

Installed wind farms can reduce congestion problems on transmission lines and can also reduce transmission losses if the wind farm is installed close to the heavy consumption areas. For example, consider a two-area system where electricity is mainly produced in one area and is consumed in another area. If the wind farm is installed in the area where the consumption is higher, it will decrease the congestion on the transmission line which transfers electricity from the production area to the consumption area. Unfortunately renewable energy like wind power and solar power are produced where the source is and transferred to the final customer. Therefore sometimes it can even increase the congestion on the lines. A method for scheduling and operation of wind power storage in electricity market as an alternative to transmission line enforcement or wind curtailment is introduced in [37].

On the other hand, the quality of wind power forecast is not so high; hence large unforeseen variation of wind power production can occur in the system, which must be handled in a financially feasible way. In [64] the author presents a method which simulates potential wind speed outcomes using available wind speed forecast from several areas with the information about the forecast errors correlation in the neighboring areas. The impact of hourly wind power variations on the hourly load following reserve requirements is discussed in [32].

Beside above mentioned characteristics, wind power creates imbalance cost for the wind farm owners when they make in advance contract. This increases market prices, which in turn has effects on other players in electricity market. A new imbalance cost minimization method using stochastic programming is developed in [40], which generates optimal wind power production bids for a short-term power market. Finally, stochastic behavior of wind power production has a big impact on the demand for real time balancing power, as well as on market prices. Those issues are discussed in [49] and [52]. The experience of some regions with high penetration of wind power, describing the consequences if there will be further increase in wind power is presented in [66]. Table 2.1 summarizes wind power impacts on the power system.

**Reserve Requirement of Wind Integrated Power System**

Wind power integration to the power system is growing significantly. The risk related to keep short-term power balance (avoiding frequency deviations) can increase radically and the planning of the frequency control reserves (instantaneous,
Table 2.1: Wind power impact on power system

<table>
<thead>
<tr>
<th></th>
<th>Local</th>
<th>Global</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Short-Term</strong></td>
<td>1. wind farms can provide voltage support depending on the design,</td>
<td>1. increasing need for the real time balancing power imbalance,</td>
</tr>
<tr>
<td></td>
<td>2. wind power can cause power quality problems creating harmonics and</td>
<td>2. can also partially contribute to primary or secondary control,</td>
</tr>
<tr>
<td></td>
<td>voltage flicker,</td>
<td></td>
</tr>
<tr>
<td><strong>Long-Term</strong></td>
<td>1. wind farm can decrease/increase congestion on the transmission</td>
<td>1. wind power will contribute to the system adequacy,</td>
</tr>
<tr>
<td></td>
<td>line,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. can reduce/increase transmission losses,</td>
<td>2. wind farms can create a need for additional investment to enhance</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a transmission line,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. if conventional unit is replaced by wind farm additional reserve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>power will be needed.</td>
</tr>
</tbody>
</table>
CHAPTER 2. BACKGROUND

fast and slow) becomes more challenging because of the output uncertainty of the wind power due to its intermittency. The requirements for instantaneous, fast and slow reserves as well as the available capacity of corresponding reserve type when wind power is introduced to the power system is estimated in [42] and [62]. A new tool to analyze capacity shortage and reserve requirements is described in [7].

In addition, the amount of wind energy that can be integrated in the system highly depends on the available capacity of the conventional units. In order to increase the utilization of wind power and decrease the operation risks, there is a need to coordinate different types of generation units properly. According to [68], it is mutually beneficial for hydro and wind power producers to work together to supply stable electricity. However, the long-term system adequacy can be affected by such kind of synchronization. The impact of wind-hydro-thermal coordination on system reliability is discussed in [34], which concludes that wind-hydro cooperation can have positive or negative impacts on the system adequacy depending on the size of hydro reservoir and the number of hydro units assigned to cooperate with wind power.

**Congestion Management**

When the transmission network is not capable to transfer electric power according to the market desire, we get congestion. Usually wind power plants are installed in the remote areas to avoid obstacles and harvest wind power more efficiently. On the other hand the transmission network might not be strong enough in remote areas to be able to transfer extra energy.

However, if there is a hydropower plant with sufficient large reservoir installed nearby, it is possible to cooperate which can be mutually beneficiary for both players and leads to use the transmission network in a best possible way. The alternative solutions are to enhance the existing transmission line or to have battery storage for the wind power farm when the transmission line is congested. Both are costly and not always economically possible.

The way the hydropower and wind power owners can cooperate is the following: the wind power owner has priority to use available transmission capacity when it is windy. It means in those cases the hydropower owner can reduce hydropower generation according to physical limitations of power plants, and use stored water when wind is low and power prices are high. When the hydropower plant and wind farm are owned by the same player, the cooperation is straightforward. In the case with separate ownership different cooperating strategies are studied in [33]. A daily stochastic planning algorithm, under uncertainty of wind power forecast,
2.2. RENEWABLE ENERGY SOURCES

for a multi-reservoir hydropower system cooperated with a wind farm is presented in [41], in other words the algorithm tends to decrease wind energy curtailment in the cases when congestion occurs. The above mentioned planning algorithm in [41] is improved further in [39]. The new planning algorithm is a two-stage stochastic program with recourse under uncertainty of wind power forecast and power market prices.

2.2.2 Hydropower

A hydropower plant generates electric energy using the difference of potential energy between an upper and a lower water level. Then, the turbine, by transferring the potential energy of the water to electrical energy, generates electricity. The generated amount of energy on a hydropower plant mainly depends on the head height, turbine efficiency and discharge level (Figure 2.6).

![Figure 2.6: Hydropower station.](image)

Hydropower production

The efficiency of the hydro turbine mainly depends on the head height and the rate of water discharge. Head height in its turn depends on the water level in the reservoir, the tail-race level and the penstock losses. Thus, the power production from a hydro unit is mainly influenced by the reservoir characteristics and the water flow. In mathematical language, the power $H$ generated by a hydro unit is a function $f$ of the water discharge $Q$ and the water volume $M$.

$$H = f(Q, M), \quad (2.1)$$
This function is non-linear and non-concave and it is important to emphasize that \( f \) is different for each generation unit. Non-concave functions are problematic in optimization models, since it is difficult to prove the global optimality of the given solution. Methods applicable for this field are described in [27]. However, there are methods to approximate the non-linear function. One of those is piecewise linear approximation, according to which water volume is discretized and for each \( M^k \) are interpolated (Figure 2.7). Hence, the function become,

\[
H = f \mid_{M^k} (Q),
\]  

(2.2)

Another method to approximate (2.2) is suggested in [15] using meshing and triangulation, which is more applicable for the small systems.

In this work piecewise linear approximation is used Figure 2.7.

![Figure 2.7: Piecewise linear approximation,'x' approximation, '···' turbine H-Q characteristics. (Figure by Prof. Alberto Borghetti)](image)

Now having an approximation for the power generation function, other notions important for hydropower production can be defined. The production equivalent can be defined as the relation of generated power as a function of discharge and discharge by

\[
\gamma(Q) = \frac{H(Q)}{Q},
\]  

(2.3)

A measure which shows how much power production will change in terms of a
2.2. RENEWABLE ENERGY SOURCES

small change in discharge is referred to as the marginal production equivalent.

\[ \mu = \frac{dH(Q)}{dQ}, \quad (2.4) \]

Let the power generation as a function of discharge divide into \( j \) segments. Thus, the total power generation in a power plant for each hour \( t \) will be (2.5)

\[ H_t = \sum_j \mu_j Q_{j,t}, \quad (2.5) \]

In order to avoid binary variables, it is important to force the model to utilize the first segment completely, before it will start to discharge from second one. This can be achieved by having a piecewise linear model, where marginal production equivalents have decreasing manner.

**Short-term hydropower planning**

According to the time span, the hydropower planning can be categorized short, mid-term and long term planning. Short-term planning refers to daily planning, mid-term and long term planning refer to weekly and yearly planning respectively. The optimization techniques discussed in this work (deterministic and stochastic) are equally applicable for mid-term and long term hydropower planning. However, the objective might be different. Examples of optimization models in the long term planning context are the followings: [22], [53], [58] and [72]. In addition, optimization models suitable for mid-term planning problems can be seen [5], [30] and [59].

Mid-term and long term hydropower planning are out of the scope of this thesis. Short-term hydropower planning and the challenges short-term hydropower producer is facing currently are discussed in detail below.

On one side decentralized competitive nature of current electricity market and on the other side continuous increase in wind power integration to the power system creates a challenge for a short-term hydropower producer to plan and operate hydropower in an optimal way. Due to its flexible and fast generation hydropower is a very suitable power source to balance wind power variations [8]. Since traditional operation rules were working according to centralized optimization, there is a need for new, advanced hydropower planning techniques capable to work under existing requirements. There are a lot of deterministic models to plan short-term hydropower production in the literature. However, researchers’ interest to work under uncertainty is noticeable recently.
Deterministic models do not have any random variable and have a known set of inputs which will result in a unique set of outputs. In those models there is no information arriving over time and decisions are made in advance for the whole planning horizon even for long-term planning. In contrast, stochastic programming addresses optimization under uncertainty, and reflects the fact that new information about the uncertain data arrives as time evolves along the planning horizon. According to information flow, a multi-stage stochastic program is characterized by partition of decision variables into stages in a way that decisions made for one stage are not affected by the information arriving in following stages.

Recently published papers, which develop deterministic models for short-term hydropower planning are [9], [11], [14], [15] and [57]. Papers where authors handle data uncertainty by using stochastic optimization are [16], [19], [21], [23], [24], [26], [43], [46], [50], [54], [55] and [56].

Generally, the aim of those models is to increase the profit of a hydropower producer while considering physical and economic/legal restrictions. Note that, it is important to reflect in the planning model the spatial layout of the power plants located on the same river; otherwise it will cause unnecessary spillage and thus, not optimal usage of the water.

Table 2.2 is summarizing deterministic and Table 2.3 is summarizing stochastic short-term hydropower models. In the literature there are research groups who work with optimization and its application in power systems and have several consecutive models in the field of short-term hydropower planning. The short-term hydro planning models described below are representative models.

In [57] authors develop a deterministic optimization model for short-term hydropower scheduling for the third large power sector in Canada, using linear programming. An optimal short-term operating policy of hydroelectric power systems is suggested in [9]. The authors describe optimal daily operational strategy for a wind-hydro power plant considering that wind power forecast is characterized by some uncertainty: solve independent deterministic optimization problems for different scenarios sampled by Monte-Carlo simulation [14]. The operation efficiency of the hydropower plants with the small storage capacity highly depends on the head (head effect is negligible for the plants with a large reservoir). Therefore, in order to get more realistic short-term production plan, it is important to introduce head-dependency to the optimization model. In [11], MILP short-term hydropower production plan with head-dependent reservoir is introduced, while the authors in [15] formulate short-term hydropower planning model using nonlinear optimization and taking into account head-dependency.

Summarizing the thoughts in the literature in this field, the most important as-
Table 2.2: Short-term hydropower planning DETERMINISTIC models

<table>
<thead>
<tr>
<th>Title</th>
<th>Objective</th>
<th>Uncertainty</th>
<th>Solution Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>The B.C. hydropower short-term scheduling, Z.K. Shawwash et al [57]</td>
<td>To determine the optimal hourly generation and trading schedules in a competitive power market</td>
<td>–</td>
<td>Advanced algebraic modeling language and a linear programming package</td>
</tr>
<tr>
<td>An hourly discredited optimization algorithm, E.D. Castronuovo et al [14]</td>
<td>To identify the optimum daily operational strategy to be followed by wind and hydro generation units</td>
<td>–</td>
<td>-</td>
</tr>
<tr>
<td>A MILP approach for short-term hydro planning with head effect, A. Borghetti et al [11]</td>
<td>To find the optimal scheduling of a multi-unit pump-storage hydropower station</td>
<td>–</td>
<td>Commercial software</td>
</tr>
<tr>
<td>Nonlinear approach for short-term hydro planning with head effect, J.P.S. Catalao et al [15]</td>
<td>To solve the short-term hydro scheduling problem considering head dependency</td>
<td>–</td>
<td>Commercial software</td>
</tr>
<tr>
<td>Optimal short-term hydro scheduling of large power systems, A. Bensalem et al [9]</td>
<td>To determine the optimal short-term operating policy of hydroelectric power systems</td>
<td>–</td>
<td>Augmented Lagrangian method</td>
</tr>
</tbody>
</table>

pects that short-term hydropower producer should take into consideration while developing production plan under uncertainty are the followings: day-ahead commitment (production bidding to spot market), production plan according to day-
ahead commitment, bidding strategy to regulating market, water inflow, future water value (up to several days counted from the delivery day). All those above mentioned points are subject to uncertainty depending on unexpected market changes and unpredictable weather conditions. Multi-state stochastic programming is used in the literature to cope with these uncertainties. In [54] a hydro-electric unit commitment model subject to uncertain demand is developed. A two-stage stochastic model for hydro-thermal systems, under the uncertainty of the fuel price, electricity price and load is presented in [46]. The authors in [16] provide a probabilistic structure for a price taking producer to deal with market-clearing prices.
<table>
<thead>
<tr>
<th>Title</th>
<th>Objective</th>
<th>Uncertainty</th>
<th>Solution Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic hydro-electric unit commitment, A.B. Philpott et al [54]</td>
<td>To determine which turbine unit to commit in each half of the day</td>
<td>Demand</td>
<td>Approximated by solving a deterministic equivalent linear programming, then solving a stochastic dynamic programming recursion</td>
</tr>
<tr>
<td>Power management under uncertainty, N. Gröwe-Kuska et al [29]</td>
<td>To investigate the weekly cost-optimal generation of electric power in a hydro-thermal generation system</td>
<td>Demand and prices for fuel and delivery contracts</td>
<td>Lagrangian relaxation</td>
</tr>
<tr>
<td>Two-stage stochastic planning model, R. Nurnberg et al [46]</td>
<td>To develop short or mid-term cost-optimal electric power production plan</td>
<td>Demand and prices for fuel and delivery contracts</td>
<td>Lagrangian relaxation scheme, Solving the dual by a bundle subgradient method</td>
</tr>
<tr>
<td>Self-scheduling profit maximization problem, A.J. Conejo et al [16]</td>
<td>To obtain the optimal bidding strategy of a price-taker producer</td>
<td>Price</td>
<td>Commercial software</td>
</tr>
<tr>
<td>Short-term hydropower scheduling model, M. Olsson [50]</td>
<td>To manage the trade-off between energy and reserve markets</td>
<td>Real-time balancing market prices</td>
<td>Commercial software</td>
</tr>
<tr>
<td>Stochastic joint optimization of wind generation and pumped-storage units, J.G. Gonzalez, etc.</td>
<td>To investigate the combined optimization of a wind farm and a pumped-storage facility</td>
<td>Market prices, wind generation</td>
<td>Commercial software</td>
</tr>
<tr>
<td>Two-stage stochastic model , S.E. Fleten et al [23]</td>
<td>To determine optimal bidding strategy to day-ahead market</td>
<td>Spot market prices</td>
<td>Commercial software</td>
</tr>
<tr>
<td>Multi-stage mixed-integer linear stochastic model, S.E. Fleten et al [24]</td>
<td>To develop a short-term production plan for a price-taking hydropower</td>
<td>Spot market prices, water inflows</td>
<td>Commercial software</td>
</tr>
</tbody>
</table>
In [43], the authors consider a generator, which offers electric power to day-ahead market under the price uncertainty. In [55, 56] the authors use stochastic dynamic programming to derive bidding curves. A stochastic programming model for bidding to two-hour-ahead market is introduced in [19]. Stochastic optimization model under uncertainty of regulating market prices for price-taking hydropower producer is introduced in [50]. The authors in [23] and [24] develop two-stage stochastic optimization models for a price-taking hydropower producer. A hydropower producer, who does not have market power, takes market prices, which are settled by the other market participants and in general there is no relevant information how those participants will act in the power market. The first paper presents two-stage stochastic programming model for optimizing bidding strategies under the uncertainty of the day-ahead market prices, while the second paper introduces one-day production plan that keeps a balance between current profits and expected future profits.

There is an increasing interest in pumped-storage power units due to their characteristics to hedge to a wind farm, which is participating in the spot market. Those facilities are consuming electrical energy operating as a pump to store water and generating electrical energy operating as turbine for peak hours. Therefore, wind farm can take advantage cooperating with a pumped-storage unit to compensate positive and negative energy imbalances along the time. For those hours in which wind farm production exceeds dispatched volume to the spot market, the energy surplus is used to store water by pumping. However, if wind farm produces less energy compared with the dispatched volume, the energy shortage is produced by discharging stored water. J. G. Gonzalez and others formulate two-stage stochastic optimization problem to study joint optimization of a wind farm and a pumped-storage unit in a market environment [26]. Bidding strategy to day-ahead market for price taker hydropower producer considering the possibility to trade on the intra-day market has been explained in [21].

2.3 Stochastic Optimization

Linear programs for which some problem data are considered uncertain is referred as Stochastic program [10] (note that [10] is the main source for the whole section). Programs in which some recourse actions can be undertaken when the uncertainty is revealed, are called recourse programs (RP).
2.3. STOCHASTIC OPTIMIZATION

2.3.1 Two-stage Stochastic program with recourse

It is assumed, that the probabilistic distributions of the random variable are known. The values of the random variables will be available only after the random experiment. Let $\xi$ be the vector of the random variables. Furthermore, the decision variables are divided into two groups

- First-stage decisions: these decisions have to be taken before the experiment. The decision period is called first stage accordingly.
- Second-stage decisions: these decisions are taken after the experiment and the period is called second stage respectively.

The general form of the two-stage linear program with fixed recourse is given below

$$\min \quad z = c^T x + E\{q(s)^T y(s)\}$$

subject to

$$Ax = b,$$

$$T(s)x + Wy(s) = h(s),$$

$$x \geq 0, y(s) \geq 0.$$ (2.9)

Note that $x$ is the first stage variable, which does not depend on the realization of the random variable $s$. $c$, $b$, and $A$ are first stage vectors and matrices corresponding to $x$. $q$, $T$ and $h$ are problem data, which are known, when the random event $s$ is realized. Thus, every component of $q$, $T$ and $h$ is random variable. When the $s$ random event is realized the second-stage decision $y(s)$ is taken. It is important to see that, the way $y$ depends on $s$ is completely different from the dependence $q$, $T$ and $h$ have from $s$. For each $s$, $y(s)$ decision is taken in a way that constraints (2.8) and (2.9) are satisfied.

The objective function consists of deterministic part $c^T x$ and expected value of the second stage objective $[q(s)^T y(s)]$, which considers all realizations of the random event $s$. For each $s$ the value of $y(s)$ is the solution of the linear program.

An example of a two-stage stochastic programming problem, which defines the production plan of a company is introduced below.

**Example 2.1** Assume a small company produces and sells ice-cream. The demand (denoted $d$) and the price (denoted $p$) of the product highly depend on the next days weather. If it is sunny day, the demand and the price will be higher and if it is cloudy day the demand and the price will be lower. Thus, the price and
the demand are stochastic variables. The production cost (denoted $c$) per unit as well as the usage of raw material (denoted $a_j$, where $j$ is index for needed raw materials) are known. The raw material $j$ is limited (denoted $b_j$). Any surplus is suggested with 50% discount. There is a punishment cost (denoted $q$) associated with the deficit of the product due to the loss of customers. The aim is to plan the production for the company while minimizing the cost.

For that purpose we need to introduce variables:

- $x$ to denote the number of produced ice-creams,
- $y_s^+$ to denote the surplus of the product in scenario $s$,
- $y_s^-$ to denote the deficit of the product in scenario $s$.

where $s$ is index for scenarios associated with the $\pi_s$ probabilities. Note that $x$ does not depend on scenarios, since the decision is taken before the observation of the unknown variable (Figure 2.8). Thus, $x$ is first-stage decision variable and $y_s^+, y_s^-$ are second-stage decision variables.

\[
\begin{align*}
\text{Decision 1} \quad & \quad \text{Randomness} \quad & \quad \text{Decision 2} \\
\downarrow \quad & \quad \downarrow \quad & \quad \downarrow \\
x \quad & \quad P_s, d_s \quad & \quad y_s^+, y_s^-
\end{align*}
\]

Figure 2.8: Decision process

The two-stage stochastic production plan will have the following form:

\[
\begin{align*}
\min & \quad -cx + \sum_s \pi_s (p_s x - 0.5 p_s y_s^+ - q y_s^-), \\
s.t. & \quad a_j x \leq b_j, \\
& \quad x - y_s^+ - y_s^- = d_s, \\
& \quad x \geq 0, \\
& \quad y_s^+ \geq 0, \\
& \quad y_s^- \geq 0.
\end{align*}
\] (2.10)

(2.11)

(2.12)

(2.13)

(2.14)

(2.15)

Now assume we have three scenarios (normal day, cloudy day and sunny day), which are equally likely to happen. The prices and the demand for each scenario
2.3. STOCHASTIC OPTIMIZATION

are $p_s = (20, 30, 40)$ and $d_s = (8, 10, 12)$. The unit production cost is 10 kr, two materials are used and the consumption is given by $a_1 = 1$ and $a_2 = 2$. The availability of each raw material is $b_1 = 15$, $b_2 = 20$ and the deficit cost is assumed to be 15 kr. Using GAMS the optimal solution is found, which suggests to produce 10 ice-cream for the coming day. Objective value is 183.3 kr.

2.3.2 Multi-stage Stochastic program with recourse

There are decision problems containing a sequence of decisions which react to outcomes that evolve over time (Figure 2.9). These programs are called multi-stage stochastic decision programs.

Let assume we have $t = 1...H$ stages. Let $x_1, ..., x_H$ be the decision variables and $c_1, ..., c_H$ be the objective function coefficient for each stage accordingly. Then the general formulation of the multi-stage stochastic linear problem with recourse can be expressed in the following way.

\[
\begin{align*}
\min & \quad z = c^1 x^1 + E_{\xi^2}[\min c^2(s_2) x^2(s_2) + ... + E_{\xi^H}[\min c^H(s_H) x^H(s_H)]] \\
\text{s.t.} & \quad W_1 x^1 = h^1, \\
& \quad T_1(s_1) x^1 + W_2 x^2(s_2) = h^2(s_1), \\
& \quad \vdots \\
& \quad T_{H-1}(s_{H-1}) x^{H-1}(s_{H-1}) + W_H x^H(s_H) = h^H(s_1), \\
& \quad x^1 \geq 0, x^t(s^t) \geq 0, t = 1...H.
\end{align*}
\]  

where $c^1$ and $h^1$ are known vectors, and $\xi^t(s) = (c^t(s)^T, h^t(s)^T, T_{t-1}^t(s), ..., T_{m_t-1}^t(s))$ is a random vector for all $t = 2, ..., H$, and $W$ is a known $m_t \times n_t$ matrix.

2.3.3 The value of stochastic solution and perfect information

It is well known that stochastic problems are mostly big and computationally difficult to solve. Sometimes, people intend to solve deterministic versions of the stochastic problem by replacing all random variables by their expected values. Another approach is to solve several deterministic problems, each corresponding to one scenario and summing the objective values based on some heuristic rule. Whether these approaches are nearly optimal or they are completely inaccurate two theoretical concepts are used in the literature: The expected value of perfect information (EVPI) and the value of stochastic solution (VSS) [10].
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The expected value of perfect information

To measure the maximum amount the decision maker is willing to pay to get future information, the EVPI concept is used in the literature. Assume that for each scenario $\xi$ we want to find $\bar{x}(\xi)$ optimal solution to the stochastic program and the related optimal objective values $z(\bar{x}(\xi), \xi)$. Then, it is possible to calculate the expected value of the optimal solution, which in the literature is referred to as wait-and-see (WS) solution (2.21).

$$WS = E_\xi[\min_x z(x, \xi)] = E_\xi z(\bar{x}(\xi), \xi),$$  \hspace{1cm} (2.21)

The corresponding solution for earlier defined RP problem is also known as here-and-now. The brief expression for the RP problem is given in (2.22) with an optimal solution $x^*$.

$$RP = \min_x E_\xi z(x, \xi),$$  \hspace{1cm} (2.22)

Now we are ready to define EVPI, which is the difference between here-and-now and wait-and-see solution (2.23).

$$EVPI = RP - WS.$$  \hspace{1cm} (2.23)

From Example 2.1 we obtained the value for RP. Using the same example we can calculate WS. For that purpose we have to solve the same problem for each scenario separately. Let denote $z_s$ the objective value for each scenario $s$ (in this example $s=3$). Using GAMS, the objective values are obtained $z_s=80; 200 \text{ and } 270$ for $s = 1, 2, 3$ respectively. $EVPI = RP - WS = RP - \sum_s \pi_s z_s = 0$. 

![Figure 2.9: Decision process over time.](image-url)
The value of the stochastic solution

In order to assess the value of knowing and using distributions on the future outcomes, the VSS concept is used in stochastic optimization [10]. For that purpose we have to construct the expected value (EV) problem, or mean value problem which is obtained by replacing all random variables by their expected values (2.24).

\[
EV = \min_z(x, \xi), \tag{2.24}
\]

Let \( \bar{x}(\xi) \) be the optimal solution to (2.24) called expected value solution. VSS measures how inaccurate is the \( \bar{x}(\xi) \) decision in terms of the (2.22). Then, the expected value (EEV) is calculated using the EV solution (2.25).

\[
EEV = E_{\xi}(z(\bar{x}(\xi), \xi)), \tag{2.25}
\]

The VSS is simply the difference of the EEV results and the optimal value of the stochastic problem (2.26).

\[
VSS = EEV - RP. \tag{2.26}
\]

Using Example 2.1 first we have to take the mean values of prices and demand and than solve the mean value problem (EV). Lets denoted the optimal solution by \( \bar{x} \). Now we have to solve EEV formulated below to calculate VSS.

\[
\min - c\bar{x} + \sum_s \pi_s(p_s\bar{x} - 0.5p_s y_s^+ - qy_s^-), \tag{2.27}
\]

\[
s.t. \ a_j \bar{x} \leq b_j, \tag{2.28}
\]

\[
\bar{x} - y_s^+ - y_s^- = d_s, \tag{2.29}
\]

\[
x \geq 0, \tag{2.30}
\]

\[
y_s^+ \geq 0, \tag{2.31}
\]

\[
y_s^- \geq 0. \tag{2.32}
\]

Now from (2.26) VSS can be calculated. For this specific example \( VSS = 0 \). Note that the purpose of the example was to show the computation procedure.
CHAPTER 2. BACKGROUND

Scenario generation

As it was stated already, some parameters in the stochastic program have uncertain values. This means that instead of single values they have continuous distributions. Only trivial problems are possible to solve with continuous distributions [36]. Therefore, in most of the cases, the continuous distribution of the random variable is approximated by a discrete distribution. This discretization is known as a scenario tree or an event tree (Figure 2.10).

Therefore, the solution with the discrete distribution is only the approximation of the solution of the original problem. The accuracy of the approximation directly depends on the accuracy of the scenario tree. There are quite a few scenario generation methods like conditional sampling, moment matching, path-based methods and so on. There is no recommended method as the best one for all possible models. Different methods are good for different models. To assess how good the scenario generation method fits to the model, it has to satisfy stability requirement. The stability test requires that the optimal value of the objective function approximately remains the same, if several scenario trees are generated and the stochastic program is run with each of them [36].

Scenario reduction

To be able to get the best possible approximation of the original problem, many
2.3. **STOCHASTIC OPTIMIZATION**

scenarios have to be generated. However this can increase the problem size significantly and make the solution time unacceptably long. The most common approach is to generate large number of scenarios and reduce it in a way that the previous information about the stochastic process is maintained in a best possible way [28]. Figure 2.11 shows how 15 scenarios are selected out of 729 scenarios.

![Figure 2.11: The left figure is a ternary scenario tree. The right one is the reduced scenario tree with 15 served scenarios using the forward algorithm. (Figure by Prof. Nicole Gröwe-Kuska)](image)

The scenario reduction algorithms define a sub-scenario tree and reassign probabilities to the saved scenarios in a way that the probability distance between reduced probability measure $Q$ and the original probability measure $P$ is minimized.

The probability distance is measured considering the scenario probabilities and the distances of the scenario values. The described algorithm applies Kantorovich distance $D_k$, which can be calculated by (2.33).

$$D_k(P, Q) = \sum_{i \in J} p_i \min_{j \notin J} c_T(\xi^i, \xi^j),$$  \hspace{1cm} (2.33)

where $J$ is the fixed set of the deleted scenarios and $Q$ is the reduced set of scenarios and

$$c_T(\xi^i, \xi^j) = \sum_{t=1}^T | \xi^i_t - \xi^j_t |$$  \hspace{1cm} (2.34)

where $| \cdot |$ denotes some norm on $\mathbb{R}^n$.

The probabilities of the preserved scenarios $\xi^j$, $j \notin J$ of $Q$ is computed by (2.35).

$$q_j = p_j + \sum_{i \in J(j)} p_i,$$  \hspace{1cm} (2.35)

where

$$J(j) = \{i \in J : j = j(i)\}, j(i) \in \arg\min_{j \notin J} c_T(\xi^i, \xi^j), \forall i \in J.$$

(2.36)
According to (2.35) the new probability of a preserved scenario is equal to the old probability plus probabilities of all deleted scenarios close to it considering (2.34).

The most common reduction algorithms are backward reduction and fast forward selection [28]. The principle of generation of scenario tree from scenarios applying reduction method is demonstrated step by step in Figure 2.12.
Chapter 3

Electricity price modeling

In this chapter, the issues related to the modeling of the day-ahead and regulating market prices are described. The models are introduced and parameter estimation methods are presented. The ARIMA model is used to predict day-ahead market prices [12, 13] and the model which is a combination of ARIMA and Markov processes is applied to forecast regulating market prices [48].

3.1 Introduction

The electricity market experiences pronounced short-term volatility due to its peculiarities, first of all continuous load variations, increasing wind power integration to the power system associated with the impossibility to profitably store electric energy. Electricity price forecast is extremely important for all market players and in particular for generating companies, who must manage their units, and the associated economic risk, in a short, medium and long term. In general, price volatility depends on a large number of parameters such as: fuel prices (often related to currency exchange rates), availability of generating units, hydro generation production, demand elasticity and variations, network congestion and management rules of any specific electricity market.

3.2 Day-ahead market prices

It is assumed that electricity market players can exchange power on hourly bases for each hour of the following day on the day-ahead market. Selling and purchasing bids for a specific hour should be submitted the previous day, before
the spot market is closed (the exact closing time is different in different markets, e.g. in Nordic market it is before noon). Then, according to the price, selling bids are listed in ascending order and purchasing bids in descending order. The cross point of two curves decides the market price and the volume to be dispatched for each hour. Then, all electricity players are informed about the price and volume to be traded for each hour in the coming day.

Huge amount of uncertainties are being introduced to the power market because of the ongoing growth in the renewable energy sources like wind and solar power. The intermittent nature of these power sources increases the volatility of the day-ahead market prices.

Significant amount of work has been done in the literature related to the modeling and forecasting of the day-ahead market prices. A number of existing approaches can be categorized according to the following: cost based models, equilibrium models, fundamental models and quantitative models [69]. Statistical models like ARIMA and ARMA to forecast day-ahead market prices are applied in [17], [18], [73]. Generalized Autoregressive Conditional Heteroscedastic (GARCH) processes are used to forecast spot market prices in [25]. Moreover, Artificial Neural Networks (ANN), that are famous in their application to predict electricity load, found its wide range application in price forecasting [67]. In this thesis ARIMA technique is used to predict day-ahead market prices.

3.3 Day-ahead market price forecasting model: ARIMA

This section is written based on the book *Time Series Analysis: Forecasting and Control* by G.E.P. Box and G.M. Jenkins [12] and the book *Time Series: Theory and Methods* by P.J. Brockwell and R.A. Davis [13]. A number of time series behave in a way as if there is no fixed mean for them. However they show homogeneity in a way that, a part of the time series behaves quite the same as any other part. To describe the mentioned homogenous non-stationary behavior, models can be obtained by proposing fitting difference of the process to make it stationary. A specific class of stochastic processes which incorporates a wide range of non-stationary series, is provided by the ARIMA processes, which after differencing finitely many times, reduces to the ARMA process.

The process $\lambda_t$ is called an ARMA(p,q) process, if $\lambda_t$ is stationary and if for every $t$ the process can be expressed in the following form:

$$\phi(B)\lambda_t = \theta(B)\epsilon_t;$$

(3.1)
3.3. **DAY-AHEAD MARKET PRICE FORECASTING MODEL: ARIMA**

![Flowchart](image)

**Figure 3.1: Flowchart**

where $\lambda_t$ is the spot market price at time $t$, $\phi$ and $\theta$ are respectively $p$-th and $q$-th degree polynomials

$$
\begin{align*}
\phi(\lambda) &= 1 - \phi_1 - \ldots - \phi_p \lambda^p, \\
\theta(\varepsilon) &= 1 + \theta_1 + \ldots + \theta_q \varepsilon^q,
\end{align*}
$$

and $B$ is backward shift operator defined by $B^j \lambda_t = \lambda_{t-j}$. In addition, $\varepsilon_t$ is white noise sequence, which is independently and identically distributed with 0 mean and $\sigma^2$ variance: $\varepsilon_t \sim WN(0, \sigma^2)$.

To build a model which best fits to the historical time series usually requires to follow the following steps:
CHAPTER 3. ELECTRICITY PRICE MODELING

1. Identification,
2. Estimation,
3. Diagnostic checking or validation,
4. Prediction using the constructed model

The flowchart of the described procedure is provided in Figure 3.1

3.3.1 Identification

To produce stationarity, the underlying time series should be differenced as many times as needed (the inclusion of the factors of the form \((1 - B), (1 - B^{24})\) and \((1 - B^{168})\), having a hope that the process under study will be reduced to ARMA process. In addition, the logarithmic transformation of the original data might be needed to get more homogenous mean and variance. Then, after having the stationary process the second step is to identify the resulting ARMA process: the order of the \(\phi(B)\) and \(\theta(B)\) polynomials. The existing tools are autocorrelation (ACF) and partial autocorrelation (PACF) functions. If the partial autocorrelation function of an autoregressive process of order \(p\) cuts off, its autocorrelation function has a tail off after lag \(p\). Similarly, the partial autocorrelation function of a moving average process of order \(q\) has a tail off after lag \(q\), while its autocorrelation cuts off. A mixed process is suggested if both functions tail off. In successive trials the newly built model can be refined studying the ACF and PACF of the residuals [12].

3.3.2 Estimation

After completion of the identification process, there is a need to obtain efficient estimates of the parameters in the stochastic model. The Maximum Likelihood (ML) method is used for that purpose.

Let \(\eta\) be a vector, which represents the set of parameters. For an ARIMA model, \((\phi, \theta\) and \(\sigma)\) are those parameters and therefore the size of \(\eta\) is equal to \(p+q+1\). Let \(L(\eta \mid \lambda)\) be the likelihood function, in which \(\lambda\) is fixed and \(\eta\) is variable. Thus, the values of the parameters, which maximize the likelihood function, are called maximum likelihood estimates. In general, likelihood is a conditional function associated with the parameter values, the choice of \(\lambda\) observations and \(\epsilon\) residuals. It has the following form:

\[
\tilde{L}(\phi, \theta, \sigma_\epsilon) = -nln\sigma_\epsilon - \frac{\tilde{S}(\phi, \theta)}{2\sigma_\epsilon^2}; \tag{3.3}
\]
where

\[ S(\phi, \theta) = \sum_{t=1}^{n} \varepsilon_t^2(\phi, \theta | \bar{\lambda}, \bar{\epsilon}, \lambda) \]  \hspace{1cm} (3.4)

Likelihood and sum of squares function have a superscript bar to stress that they are conditional and depend on the choice of starting values. From (3.3) it is possible to see that the conditional likelihood only involves the data through the sum of squares function. Therefore, for any fixed value of \( \sigma_\varepsilon \) the contours of \( L \) on the space of \((\phi, \theta, \sigma_\varepsilon)\), are contours of \( \bar{S} \). As a result, the maximum likelihood estimates are the same as the least squares estimates. Thus, this makes it possible to study the behavior of the conditional likelihood by studying the conditional sum of square function.

### 3.3.3 Diagnostic checking or validation

The model validation is performed based on the diagnostic check. The most common way to do it is to judge the goodness of fit of the statistical model by studying residuals; by comparing the forecasted values from the fitted model with the observed values. The residuals has to reflect the properties of the white noise sequence \( \varepsilon_t \), if the fitted model is well adjusted; residuals must be independent, uncorrelated and normally distributed with 0 mean and constant variance. The ACF of the residuals together with the \( \pm 1.96/\sqrt{n} \) confidence bounds can be used to study how good the model can reflect the reality. \([-1.96; 1.96]\) is the confidence interval in which a measurement or trial falls corresponding to probability = 0.95. This means that 95% of the autocorrelation values will lie inside the bounds and only 5% of autocorrelation values can be expected to fall outside of the bounds.

Another widely used method to validate the ARIMA model is Ljung-Box test for residual autocorrelation. Ljung-Box test (which is also known as Ljung-Box Q-test) is given by the following formula:

\[ Q = n(n+2) \sum_{j=1}^{L} \frac{\hat{\rho}(j)^2}{n-j} \] \hspace{1cm} (3.5)

The test assesses the null hypothesis that a sequence of residuals does not show any autocorrelation for a fixed number of lags \( L \), in contrast to the alternative hypothesis that there are some nonzero autocorrelation coefficients. Under the null hypothesis the asymptotic distribution of \( Q \) is chi-square (\( \chi^2 \)) with \((L - p - q)\) degrees of freedom. Thus, the null hypothesis is rejected at a level \( \alpha \) if

\[ Q > \chi^2_{L-p-q}(1-\alpha) \] \hspace{1cm} (3.6)
The advantage of the Ljung-Box Q-test is that it takes the information from the correlations at different lags. However, the disadvantage is that sometimes they accept models which fail to reflect the reality.

In addition to the above-mentioned methods to check the model validity based on the ACF, there are many other tests which aim to check the ‘randomness’ of the residuals, for example the hypothesis that the residuals are independent and identically distributed series.

### 3.3.4 Prediction

In this step, the model is ready to forecast. The minimum mean square error forecast \( \tilde{\lambda}_t(j) \) for lead time \( j \) is the conditional expectation \( E[\lambda_{t+j}] \) of \( \lambda_{t+j} \), at origin \( t \). In order to calculate the conditional expectations, we note that for \( j > 0 \) the following is true,

\[
\begin{align*}
[\lambda_{t-j}] &= E[\lambda_{t-j}] = \lambda_{t-j} \quad \text{for} \quad j = 0, 1, 2, \ldots \\
[\lambda_{t+j}] &= E[\lambda_{t+j}] = \tilde{\lambda}_t(j) \quad \text{for} \quad j = 1, 2, \ldots \\
[\varepsilon_{t-j}] &= E[\varepsilon_{t-j}] = \varepsilon_{t-j} = \lambda_{t-j} - \tilde{\lambda}_{t-j-1} \quad \text{for} \quad j = 0, 1, 2 \\
[\varepsilon_{t+j}] &= E[\varepsilon_{t+j}] = 0 \quad \text{for} \quad j = 1, 2, \ldots
\end{align*}
\]  

(3.7)

In (3.7) the \( \lambda_{t-j} (j = 0, 1, 2, \ldots) \) have already occurred at origin \( t \), therefore are left unchanged.

The \( \lambda_{t+j} (j = 1, 2, \ldots) \) have not occurred yet, thus are replaced by their forecasts at origin \( t \) \( \tilde{\lambda}_t(j) \).

The \( \varepsilon_{t-j} (j = 0, 1, 2, \ldots) \) have already occurred and can be calculated from the \( \lambda_{t-j} - \tilde{\lambda}_{t-j-1} \) (3.7).

The \( \varepsilon_{t+j} (j = 1, 2, \ldots) \) have not occurred yet and are replaced by zeros.

The variance of the forecast error having lead time \( j \) at origin \( t \) is given by;

\[
V(j) = (1 + \sum_{l=1}^{j-1} \psi_l^2) \sigma_{\varepsilon}^2
\]  

(3.8)

Having the given information up to \( t \) and assuming that the distribution of forecast error is normal, the conditional probability distribution of a future value \( \lambda_{t+j} \) of the process will be normal with mean \( \tilde{\lambda}_t(j) \) and standard deviation given by;

\[
\sigma(j) = (1 + \sum_{l=1}^{j-1} \psi_l^2)^{1/2} \sigma_{\varepsilon}
\]  

(3.9)
3.4 Real-time market prices

The wind power production variations are huge, because the wind power forecast quality is comparatively low [20]. These variations must be balanced by conventional units. Due to the fluctuations in the wind power, the need for balancing power will increase, which in its turn will increase the real-time market prices. Hence, there is a need for modeling real-time market prices. In contrast with the day-ahead market prices, there are few attempts in the literature to model real-time market prices. An approach to model real-time market prices is presented in [60]. The model is built based on the existing dependency between the day-ahead market prices, the traded volumes in the real-time balancing market and the real-time balancing prices. Several approaches to model real-time market prices are presented in [48]. The first model does not contain any stochasticity and takes real-time balancing demands and day-ahead market prices as input signals. The second model treats prices as stochastic variables and is a combination of ARIMA and Markov processes. The third model is based on non-linear time series analysis including exogenous variables.

3.5 Real-time market price forecasting model

In this thesis the stochastic model based on the SARIMA and Markov processes, developed in [48] is used. The model reflects the existing strong correlation between day-ahead and real-time market prices and takes into consideration the specific characteristics of the hourly real-time market prices; for each hour one upward and one downward balancing price can be defined. However, there might be some hours, when only one type of balancing price is defined, or balancing price is not defined at all. Thus, there is a need to have a parameter $b_t \in 0, 1$ to represent the state of the real-time price for a specific hour [51].

$$b_t = \begin{cases} 
1 & \text{if real-time price is defined} \\
0 & \text{if real-time price is not defined} \end{cases} \tag{3.10}$$

The all possible states of $b_t$ for a specific hour are described below.

1. $(b_t^+, b_t^-)$ $(0, 0)$
2. $(b_t^+, b_t^-)$ $(0, 1)$
3. $(b_t^+, b_t^-)$ $(1, 0)$
4. $(b_t^+, b_t^-)$ $(1, 1)$

\[ \text{(3.11)} \]
The most common states happen in the Nordic market are the second and the third ones, according to which only upward or downward regulation happened during a specific hour; only one type of balancing price is defined. There are hours, when at the beginning of the hour there is a need to up-regulate and at the end of the hour there is a need to down-regulate, which means that both type of prices are defined and this corresponds to the fourth state. And the first state corresponds to those hours when neither type of balancing is performed and hence balancing power prices are not defined. In order to consider the undefined prices in the model, the balancing power prices are formulated in the following way:

\[
 r_t = \begin{cases} 
 a_t & \text{if } b_t = 1 \\
 \text{not defined} & \text{if } b_t = 0 
\end{cases} 
\]  
(3.12)

where \(a_t\) represents the continuous behavior of the prices, concerning only defined prices and \(b_t\) represents the discrete behavior of the undefined and defined prices.

The SARIMA process is used to model \(a_t\) and discrete Markov chain model is used to capture the stochastic behavior of \(b_t\).

### 3.5.1 SARIMA processes

#### Modeling

As it was mentioned already \(a_t\), for which all prices are defined, is modeled with the SARIMA process. Let’s denote \(\delta_t\) as a difference between spot market prices and real-time market prices for each hour (3.13). In order to reflect strong correlation between the spot market prices and the real-time market prices their difference, \(\delta_t\) is modeled

\[
 \delta_t = a_t - \lambda_t 
\]  
(3.13)

It is important to notice that the spot market prices serve as a bound for the real time market prices, since upward prices (corresponding to the traded up-regulation quantity in the market) are always greater than or equal to spot market prices and downward prices (corresponding to the traded down-regulation quantity in the market) are less than or equal to spot market prices. Thus, \(\delta_t^+ \in \mathbb{R}_+\) and \(\delta_t^- \in \mathbb{R}_-\).

Usually, in order to avoid to have negative stochastic variables, a logarithmic transformation is applied on \(\delta_t : s\). In addition, to manage the cases when \(\delta_t = 0\) arbitrary small number \((\nu \in \mathbb{R}_+\) is added to the price differences [51]. As a result,
3.5. REAL-TIME MARKET PRICE FORECASTING MODEL

\[
\ln(\delta^+ + \nu) \in \mathbb{R}_+ \text{ and } \ln(-\delta^- + \nu) \in \mathbb{R}_+ \text{ stochastic series will be introduced. The SARIMA } (p,d,q) \times (P,D,Q), \text{ model of those series can be expressed as in (3.14):}
\]

\[
\Phi(B^s) \phi(B) \Delta^d \ln(\delta_t + \nu) = \Theta(B^s) \theta(B) \epsilon_t
\]  

(3.14)

where \( \Delta^D = (1 - B^d)^D \), \( \Delta^d = (1 - B)^d \) and \( \Phi(B^s) \), \( \phi(B) \), \( \Theta(B^s) \) and \( \theta(B) \) are polynomials expressed as in (3.15), (3.16) and \( \epsilon_t \in N(0, \sigma^2) \) is white noise sequence.

\[
\Phi(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - ... - \Phi_p B^{ps},
\]

(3.15)

\[
\Theta(B^s) = 1 + \Theta_1 B^s + \Theta_2 B^{2s} + ... + \Theta_Q B^{qs},
\]

(3.15)

\[
\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - ... - \phi_p B^p,
\]

(3.16)

\[
\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + ... + \theta_q B^q.
\]

Parameter estimation

Like in ARIMA models the parameter estimation process consists of the following steps:

1. Preliminary identify the model orders: \( (p,d,q) \), \( (P,D,Q) \) and \( s \).
2. Estimate the model coefficients: all \( \Phi_s \), \( \Theta_s \), \( \phi_s \), and \( \theta_s \).

The model order estimation is based on the ACF and PACF functions. For the estimation of the model coefficients, the Maximum Likelihood method is used. The resulting model has to be validated, and if it is necessary the model has to be refined based on the residuals. For the detailed description see [48].

3.5.2 Discrete Markov chains

A type of stochastic processes is known as a discrete Markov chain. Markov chains found their applications in different areas and rich literature is available on the topic.

Modeling

\( b_t \) is a binary stochastic variable, and as it was stated already is modeled using discrete Markov chains.

Definition - A discrete time stochastic process is a Markov chain if, for \( t = 1,2,... \) and all states, the following takes place [70]

\[
P(b_{t+1} = i_{t+1} | b_t = i_t, b_{t-1} = i_{t-1}, ..., b_1 = i_1, b_0 = i_0) = P(b_{t+1} = i_{t+1} | b_t = i_t).
\]

(3.17)
Basically (3.17) states that the probability distribution of the state at time $t+1$ is dependent only on the realization of hour $t$ and does not take information from the whole history the chain passed through up to the state $i_t$ at time $t$. In this thesis it is assumed that for all states $i$ and $j$ and all $t$, $P(b_{t+1} = j \mid b_t = i)$ does not depend on $t$, which makes it possible to write,

$$P(b_{t+1} = j \mid b_t = i) = p_{ij},$$

(3.18)

where $p_{ij}$ is the probability that the system will be in a state $j$ at time $t+1$, based on the information that the current state of the system is $i$. The $p_{ij}$s are often referred to as the transition probabilities for the Markov chain. Given that the system has $s$ states, the transition probabilities will be expressed as the $s \times s$ transition probability matrix, which may be written as

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1s} \\ p_{21} & p_{22} & \cdots & p_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ p_{s1} & p_{s2} & \cdots & p_{ss} \end{pmatrix}$$

(3.19)

Knowing that at time $t$ the system is at state $i$, at time $t + 1$ the system has to be in one of the existing states. The following formula expresses the same idea.

$$\sum_{j=1}^{s} P(b_{t+1} = j \mid P(b_t = i)) = 1$$

(3.20)

$$\sum_{j=1}^{s} p_{ij} = 1$$

(3.21)

In addition the entries in $P$ matrix must be non-negative as they express the corresponding probabilities and the sum of the entries in each row must sum up to 1 (3.21). It is possible to give a graphical representation of a transmission matrix; each node represents a state and each arc represents the transition probability between the nodes which define that arc. The graph for three node transition matrix is drawn in Figure 3.2.

**Parameter estimation**

The main parameters, which have to be estimated in the discrete Markov model are entries in the transition matrix $p_{ij}^h$ where $i,j = 1,2,3,4$. Those parameters can
3.5. REAL-TIME MARKET PRICE FORECASTING MODEL

be estimated using historical market prices. Based on the price series, \((b_t^\uparrow, b_t^\downarrow)\) pairs are obtained, which show the state of the upward and downward prices at time \(t\). Let \(o_t\) be the variable which show the price state at time \(t\) as it is demonstrated below

\[
o_t = \begin{cases} 1, & \text{if } (b_t^\uparrow, b_t^\downarrow) = (0,0) \\ 2, & \text{if } (b_t^\uparrow, b_t^\downarrow) = (0,1) \\ 3, & \text{if } (b_t^\uparrow, b_t^\downarrow) = (1,0) \\ 4, & \text{if } (b_t^\uparrow, b_t^\downarrow) = (1,1) \end{cases} \quad t = 1, 2, ..., T \tag{3.22}
\]

Let \(O_{ij}^h\) be a set defined in the following way

\[
O_{ij}^h = o_t : o_t = j, o_{t-1} = i, ..., o_{t-h} = i, t = h + 1, ..., T \tag{3.23}
\]

then transition probabilities for \(i, j = 1, ..., 4\) can be calculated like in (3.24)

\[
p_{ij}^h = \frac{#O_{ij}^h}{\sum_{n=1}^4 #E_{in}}, \quad i, j = 1, ..., 4 \tag{3.24}
\]

3.5.3 Scenario generation

At this stage it is possible to generate a price scenario of length \(T\) using SARIMA process with Markov model and following to the corresponding steps stated below.

1. Using (3.14) generate \(\ln(\delta^\uparrow + \nu)^T_{t=1}\) and \(\ln(\delta^\downarrow + \nu)^T_{t=1}\).
2. Calculate \([\hat{a}^\uparrow]_{t=1}^T\) and \([\hat{a}^\downarrow]_{t=1}^T\) using (3.13)
3. Using Markov model generate $\tilde{b}_t^{\uparrow}$ and $\tilde{b}_t^{\downarrow}$
4. Generate $\tilde{r}_t^{\uparrow}$ and $\tilde{r}_t^{\downarrow}$ using (3.12).
Chapter 4

Hydropower modeling

In this chapter, the stochastic models used in the case studies are introduced. The first model is a two-stage stochastic problem for bidding to the day-ahead market. In this model the day-ahead market prices and inflow level are taken stochastic. The second model is a two-stage stochastic problem. The model aims to generate optimal bidding strategy to the day-ahead market considering the real-time market. Uncertainties related to the day-ahead market prices and real-time market prices are taken into account.

4.1 Introduction

To have a more realistic plan for the hydropower production, uncertainties have to be considered. For hydropower production planning problems, data uncertainties are related to market prices, start contents of reservoirs and water inflow level. Prices in different places of the electricity market are volatile because of unexpected market events and thus it is extremely hard to predict precisely.

For both models it is assumed that the hydropower producer is a price taker. Moreover, the hydropower producer does not have bilateral trading.

4.2 Planning model I

It is assumed that electricity market players can trade electric power on the day-ahead market for each hour of the following day. This means that bids for all hours of coming day to the day-ahead market are submitted at once. When the day-ahead market is cleared; day-ahead market prices for all hours of the next day are set at
once. Bids to the day-ahead market are submitted before the day-ahead market prices are cleared; thus for the first-stage decision making process the prices are unknown. However, the second-stage decision making process benefits from the additional information related to the cleared day-ahead market prices. The optimal bidding strategy is a two stage stochastic program. The first stage refers to day-ahead market bidding and the second stage refers to the delivery day generation commitment.

4.2.1 Objective function

The model aims to maximize the hydropower producer’s expected sale and production profit based on the optimal decision; to sell electric power to day-ahead market or to store for the future usage. The first term is the expected profit coming from the trading on day-ahead market and the second term is the expected profit from stored water.

\[
\text{Max } \sum_{s,k} P_s \pi_k \left[ \sum_t \left( \lambda^s_t x^{s,k}_t + \lambda_f \sum_{r \in R_j} \gamma_{m_{j,T}} \right) \right].
\]  

In (4.1) \(s\) and \(k\) are indices for day-ahead market and inflow level scenarios respectively with probabilities \(P\) and \(\pi\). \(\lambda^s_t\) is price in day-ahead market and \(\lambda_f\) is water opportunity cost. \(x^{s,k}_t\) is dispatch level to day-ahead market, \(m^{s,k}_{j,T}\) is the end content of the reservoirs and \(\gamma\) is expected future production equivalent for a plant.

4.2.2 Optimization constraints

The feasible region of the problem is defined by the following constraints.

**Bidding rule to day-ahead market**

The price-taker hydropower producer has to develop a bidding strategy to the day-ahead market and plan the next day generation according to the day-ahead market commitment. In order to model the bidding process, the possible bidding prices are fixed: some price points are selected (in this work equidistance) and the corresponding bid volumes are considered as variables. Let \(i\) be the index for the possible bid prices and \(\rho_i\) represent these prices. Then, a fundamental rule is applied to couple bid volumes \(w_{i,t}\) and dispatched volumes \(x_t\): for each hour if \(\rho_i \leq \lambda_t \leq \rho_{i+1}\) then \(x_t = \sum_{p=0}^{i} w_{i-p,t}\), where \(p = 1, 2, \ldots, i\). However, there is a data uncertainty related to the day-ahead market prices. Since the day-ahead market...
prices for each hour are known after the market is cleared, the hydropower producer is bidding under the uncertainty of day-ahead market prices. Hence, the coupling rule will have the following form:

\[ x_{s, k}^{t} = \sum_{i}^{\lambda_{i}} w_{i, p, t} \quad \text{if} \quad \rho_{i} \leq \lambda_{i}^{s} \leq \rho_{i+1}, \tag{4.2} \]

where \( w_{i, t} \) is hourly bid volumes corresponding to the possible bid prices.

**Generation constraint**

Constraints (4.3) bound hourly generation for each operated power plant according to its maximum generation capacity and marginal production equivalent. It is assumed that the marginal production equivalents have decreasing manner: \( \mu_{j, n} > \mu_{j, n+1} \), otherwise additional binary variables will be necessary.

\[ G_{j, t}^{s, k} \leq \sum_{n} \mu_{j, n} Q_{j, t, n}^{s, k}, \tag{4.3} \]

Here \( G_{j, t}^{s, k} \) is generation level and \( Q_{j, t, n}^{s, k} \) is discharged volume for hourly bids.

**Load balance constraint**

For each hour the quantity traded on the day-ahead market should be equal to the total output of the generation units.

\[ x_{t}^{s, k} = \sum_{j} G_{j, t}^{s, k}, \quad s \in S, \quad t \in T, \quad k \in K, \tag{4.4} \]

**Hydrological balance constraint**

It is necessary to have a constraint, which keeps the hydrological balance between reservoirs: for each hour the reservoir’s new contents \( m_{j, t}^{s, k} \) are equal to its old contents \( m_{j, t-1}^{s, k} \) plus water inflow \( I_{j, t}^{k} \) minus water outflow. The discharge and the spillage \( S_{j-1, t-\tau}^{s, k} \) from the upper reservoir flow in the reservoir located downstream with some delay time \( \tau \). This is reflected in (4.5).

\[ m_{j, t}^{s, k} = m_{j, t-1}^{s, k} - \sum_{n} Q_{j, t, n}^{s, k} - S_{j, t}^{s, k} + \sum_{n} Q_{j-1, t-\tau, n}^{s, k} + S_{j-1, t-\tau}^{s, k} + I_{j, t}^{k}, \quad s \in S, \quad t \in T, \quad k \in K. \tag{4.5} \]
Variable limit constraints

The following two constraints together put bounds on hourly discharge level and on hourly water storage level respectively.

\[ Q_{s,k}^{i,j,t} \leq \bar{q}_{j,n}, \]  

\[ m_{s,k}^{i,j,t} \leq \bar{m}_j, \]

all variables \( \in \mathbb{R}_+ \), \( \bar{q}_{j,n} \) and \( \bar{m}_j \) are maximum discharge level and maximum reservoir content respectively.

Optimization model I

The complete mathematical formulation of the two-stage stochastic optimization problem applicable for power plants located in a row is presented below.

\[
\text{Max} \sum_{s,k} P^i \pi^k \left[ \sum_t \left( \lambda_t x_t^{s,k} + \lambda_f \sum_{r \in R_j} y_{r,m_j^T} \right) \right]
\]

s.t.

\[
x_t^{s,k} = \sum_{p=0}^i w_{i-p,j} \quad \text{if} \quad \rho_i \leq \lambda_t^s \leq \rho_{i+1},
\]

\[
m_{s,k}^{i,j,t} = m_{s,k}^{i,j,t-1} - \sum_n Q_{s,k}^{i,j,t,n} - S_{s,j}^{i,j,t-1} - \sum_n Q_{s,k}^{i,j,t-1,n} + S_{s,j}^{i,j,t-1} + I_{j,t}^k, \quad s \in S, \ t \in T, \ k \in K,
\]

\[
G_{s,k}^{i,j,t} \leq \sum_n \mu_{j,n} Q_{s,k}^{i,j,t,n},
\]

\[
x_t^{s,k} = \sum_j G_{s,j}^{i,j,t}, \quad s \in S, \ t \in T, \ k \in K,
\]

\[
Q_{s,k}^{i,j,t} \leq \bar{q}_{j,n},
\]

\[
m_{s,k}^{i,j,t} \leq \bar{m}_j,
\]

all variables \( \in \mathbb{R}_+ \).
4.3 Planning model II

As it is stated above the second model is a stochastic bidding strategy to the day-ahead market while considering the real-time market. When the day-ahead market prices are cleared so called second level uncertainty related to the development of the real-time market takes place. Rules for submitting bids to the real time market are different from the one for the day-ahead market. The regulating market for a specific hour is closed only short before the delivery. Hence, the real-time market prices are settled according to the time progress. As a producer of a flexible power source, the hydropower producer has to make optimal decisions (to participate in the real-time market, or sell the whole electric power on the day-ahead market) under the uncertain data, which gradually reveals over time. Multi-stage stochastic programming technique is applicable in this context, however the problem is modeled using two-stages.

The first stage concerns the day-ahead bidding before the observation of the uncertain data, and the second stage contains electric trading on the real-time market and the actual production planning process. At the first stage prices are completely uncertain, whereas decision making process for the second stage takes advantage of the information revealed over time. Probability description about the uncertain data is approximated by a so called scenario tree. The set \((\lambda_s, r_{il}, r_{ik})_{s \in S, l \in L, k \in K, t \in T}\), with corresponding probabilities \(p_s, p_l, p_k\), generate this scenario tree. Note that, for simplicity, \(s\) is chosen to represent day-ahead market price scenarios, \(l\) and \(k\) for up and down regulation price scenarios respectively. The decision tree for a simple case (three spot price scenarios, each having two regulating price scenarios: note that regulating market scenarios are representative for both up and down regulation price scenarios), corresponding to the described decision making process for each is shown in Figure 4.1. Dark spots on the tree represent the producer’s decisions and the light spots represents optimization variables calculated after each stage.

4.3.1 Objective function

The objective of the optimization problem is to maximize the whole profit consisting of sales on the day-ahead market, on regulating market and opportunity cost of stored water. Let’s denote the expected income corresponding to the sold volume on the day-ahead market \(z^{da}\), the expected income corresponding to the sold volume on the up-regulating market \(z^\uparrow\), the expected cost corresponding to the bought volume on the down-regulating market \(z^\downarrow\) and finally the expected value of
CHAPTER 4. HYDROPOWER MODELING

First stage
Bidding to day-ahead market
\( w_{i,t} \)

Spot prices are cleared
\( \lambda_{st} \)
Dispatched volumes
\( x_{st} \)

Second stage
Get info of real-time prices
\( \lambda_{lt}, \lambda_{kt} \)
Actual dispatch
\( \delta_{slt}, \delta_{skt}, m_{l,t}^{s,l,k}, Q_{l,t}^{s,l,k}, s_{l,t}^{s,l,k}, G_{l,t}^{s,l,k} \)

Figure 4.1: Decision tree for each hour.

the stored water at the end of the planning period \( z_{future} \). Thus, the complete form of the objective function will be

\[
z = z^{da} + z^{\uparrow} - z^{\downarrow} + z_{future},
\]

(4.15)

The expected sales profit on the day-ahead market for the producer can be formulated as in (4.16).

\[
z^{da} = \sum_{s} p_{s} \sum_{t} \lambda_{st} x_{st},
\]

(4.16)

The corresponding expected profit and cost related to the real-time market trading can be formulated as in the (4.17) and (4.18) respectively.

\[
z^{\uparrow} = \sum_{s,l} p_{s} p_{l} \sum_{t} r_{l,t}^{\uparrow} d_{slt}^{\uparrow},
\]

(4.17)

\[
z^{\downarrow} = \sum_{s,k} p_{s} p_{k} \sum_{t} r_{k,t}^{\downarrow} d_{skt}^{\downarrow},
\]

(4.18)
4.3. PLANNING MODEL II

In the objective function, the \( z^\downarrow \) term is a cost. If down regulating has happened, the hydropower producer is buying the specific amount of energy from the TSO (Transmission System Operator) with the down-regulating market price, instead of producing it. The gain of the hydropower producer while doing down-regulation is to save the corresponding amount of water to be sold for coming hours with more favorable prices.

The expected profit from the stored water can be formulated as in (4.19).

\[
z_{\text{future}} = \lambda_{\text{future}} \sum_{s,l,k} \sum_{j \in R_j} \gamma_{j} m_{j,t}^{s,l,k}
\]  

(4.19)

In (4.19) hydrological coupling is considered corresponding to the structure of the system. The dispatched water from the upper generation unit flows to the station located downstream. \( R_j \) is the set of stations located downstream \( j \). Generally, the value of the stored water, in other words the future market prices are highly uncertain. However in this model it is assumed deterministic.

4.3.2 Optimization constraints

The constraint for the bidding rule to day-ahead market, the generation constraint, the hydrological balance constraint and variable limit constraints are the same as in model I. Thus, they are listed without further explanation.

\[
x_{s,t} = \sum_{p=0}^{i} w_{i-p,t} \quad \text{if} \quad \rho_t \leq \lambda_{s,t} \leq \rho_{t+1};
\]  

(4.20)

\[
m_{j,t}^{s,l,k} = m_{j,t-1}^{s,l,k} - \sum_n Q_{j,t,n}^{s,l,k} - S_{j,t}^{s,l,k} + \sum_n Q_{j-1,t-\tau_j,n}^{s,l,k} + S_{j-1,t-\tau_j}^{s,l,k} + I_{j,t}, \quad s \in S, \ t \in T,
\]  

(4.21)

\[
G_{j,t}^{s,l,k} \leq \sum_n \mu_{j,n} Q_{j,t,n}^{s,l,k},
\]  

(4.22)

\[
Q_{j,t,n}^{s,l,k} \leq \bar{q}_{j,n},
\]  

(4.23)

\[
m_{j,t}^{s,l,k} \leq \bar{m}_j,
\]  

(4.24)

Moreover, all variables \( \in \mathbb{R}_+ \).
CHAPTER 4. HYDROPOWER MODELING

Load balance constraint

For each hour, the sum of the quantity traded on the day-ahead market, the quantity traded on the upward regulating market \((d_s^{↑}, l, t)\) and the quantity traded on the downward regulating market \((d_s^{↓}, k, t)\) should be equal to the total output of the generation units. Note that here the quantity traded on the downward regulating market has a negative sign, since in this case the producer buys this quantity from TSO.

\[
x_s, t + d_s^{↑}, l, t - d_s^{↓}, k, t = \sum_j G_s^{i, j, k}; \tag{4.25}
\]

Restrictions on the down-regulated quantity

The quantity which is traded on the real-time market is bound. The amount of energy traded on downward balancing market can not be larger than the amount of energy traded on the day-ahead market. To bound the down regulating quantity additional constraint is needed (4.26). (4.26) states that for a producer participating on the day-ahead market, the possible down-regulating quantity is bound by the amount of power sold on the day-ahead market. For up regulating quantity the maximum capacity of the production units together with the load balance constraint will serve the bound.

\[
d_s^{↓}, k, t \leq x_s, t; \tag{4.26}
\]

Optimization model II

Putting all together the complete mathematical model has the following form:

\[
\text{Max} z = z^{da} + z^{↑} - z^{↓} + z^{future} \tag{4.27}
\]

subject to

\[
x_s, t = \sum_{p=0}^{i} w_{i-p, t} \quad \text{if} \quad \rho_t \leq \lambda_{s,t} \leq \rho_{t+1}; \tag{4.28}
\]

\[
G_s^{i, j, k} \leq \sum_n \mu_{j,n} Q_s^{i, j, k}; \tag{4.29}
\]

\[
d_s^{↓}, k, t \leq x_s, t; \tag{4.30}
\]

\[
x_s, t + d_s^{↑}, l, t - d_s^{↓}, k, t = \sum_j G_s^{i, j, k}; \tag{4.31}
\]

\[
m_{s, t}^{i, j, k} = m_{s, t-1}^{i, j, k} - \sum_n Q_{j, n}^{i, j, k} - S_{j, t}^{i, j, k} + \sum_n Q_{j-1, t-τ_j, n}^{i, j, k} + S_{j-1, t-τ_j, n}^{i, j, k} + I_{j, t}, \quad s \in S, \ t \in T, \tag{4.32}
\]
4.3. PLANNING MODEL II

\[ Q_{j,t,n}^{l,k} \leq \bar{q}_{j,n}, \quad (4.33) \]

\[ m_{j,t}^{x,l,k} \leq \bar{m}_j, \quad (4.34) \]

all variables \( \in R_+ \).
Chapter 5

Case study

In this chapter hydropower planning models are applied on a real case. The models are tested by studying a three reservoir test system. The first model is an optimal bidding strategy to day-ahead market. In this model, the uncertainties related to day-ahead market prices and water inflow are taken into account. Profound sensitivity analysis is provided, in terms of volatility in day-ahead market prices and water inflow level as well as in terms of water opportunity cost and initial volume of the reservoir. The second model is an optimal bidding strategy to day-ahead market considering the real-time market under the day-ahead and real-time market price uncertainties. To make the model results more realistic different formulation approaches are suggested and then commented on the results.

5.1 Introduction

For the case study three cascaded reservoirs are studied. The upper reservoir is larger, which is followed by two smaller reservoirs. Every reservoir has local inflow. In addition, each reservoir has a power station which contains a turbine, which by transferring the potential energy of the water to electrical energy generates electricity. Afterwards, the water flows downstream and is stored in the lower plant’s reservoir, until released through the lower plants turbine. The data for the initial reservoirs’ content and the inflow level are according to Swedish hydropower plants Figure 5.1. Table 5.1 summarizes the maximum storage capacity, the maximum design flow, the maximum production capacity for each power plants and water delay time.
Figure 5.1: Hydro system layout located on the Ume river.

Table 5.1: Data for Power Plants

<table>
<thead>
<tr>
<th></th>
<th>$\bar{m}_j$ (HE)</th>
<th>$\bar{Q}_j$ ($m^3/s$)</th>
<th>$\bar{G}_j$ (MW)</th>
<th>$\tau_j$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>305856</td>
<td>340</td>
<td>95</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>1392</td>
<td>310</td>
<td>50</td>
<td>0.5</td>
</tr>
<tr>
<td>III</td>
<td>4008</td>
<td>330</td>
<td>90</td>
<td>2</td>
</tr>
</tbody>
</table>

5.2 Sensitivity analysis to price and inflow uncertainty

This study aims to provide profound sensitivity analysis of short-term hydropower planning for a price-taker hydropower producer in a pool-based electricity market, under uncertainty related to the day-ahead market prices and water inflow to the reservoirs. ARIMA time series analysis tool, explained in chapter 3, is used to forecast day-ahead market prices and generate scenarios based on that. A price-taker hydropower producer is small and does not have an influence on the market prices. Moreover, it is assumed that there is no correlation between spot market price scenarios and inflow scenarios.

Inflow data corresponds to week 16 in 2008. For the spot market price prediction, the data is taken from Nord Pool website from April 20 to June 23, 2010.
5.2. SENSITIVITY ANALYSIS TO PRICE AND INFLOW UNCERTAINTY

5.2.1 Spot market price model

As it is explained in chapter 3 to build a model which best fits the historical time series requires to follow the following steps:
1. Identification,
2. Estimation,
3. Diagnostic checking or validation,
4. Prediction using the constructed model

1. To produce stationarity the underlying time series should be differenced as many times as needed (the inclusion of the factors of the form \((1 - B), (1 - B^{24})\) and \((1 - B^{168})\), having a hope that the process under study will be reduced to ARMA process. In addition, the logarithmic transformation of the original data might be needed to get more homogenous mean and variance. Then, after having the stationary process the second step is to identify the resulting ARMA process: the order of the \(\phi(B)\) and \(\theta(B)\) polynomials. The existing tools are autocorrelation (ACF) Figure 5.2 and partial autocorrelation (PACF) Figure 5.3 functions of the original data. For example, the terms \((1 - \phi_{24}B^{24})\) and \((\theta_{72}B^{72})\) are included in forecasted model. The first term corresponds to AR (Auto Regressive) model, since there is an exponential decline in ACF and a peak in PACF at value 24. In the same way, the second term corresponds to MA (Moving Average) model, since there is a peak in ACF and decline in PACF at value 72. In successive trials the newly built model can be refined studying the ACF and PACF of the residuals.

![Figure 5.2: ACF.](image)

2. After completion of the identification process, there is a need to obtain efficient estimates of the parameters in the stochastic model. Maximum Likelihood
(ML) method is used for that purpose. Table 5.2 displays the estimated parameters. The estimated value for the white noise standard deviation is $\sigma_\varepsilon = 0.1053$

Table 5.2: Estimated parameters for the spot market price model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>0.5968</td>
<td>$\theta_5$</td>
<td>0.01769</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.418</td>
<td>$\theta_7$</td>
<td>0.00627</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>0.2235</td>
<td>$\theta_8$</td>
<td>0.003232</td>
</tr>
<tr>
<td>$\phi_{48}$</td>
<td>0.03418</td>
<td>$\theta_{21}$</td>
<td>0.015077</td>
</tr>
<tr>
<td>$\phi_{72}$</td>
<td>0.06269</td>
<td>$\theta_{47}$</td>
<td>0.04521</td>
</tr>
<tr>
<td>$\phi_{96}$</td>
<td>0.05646</td>
<td>$\theta_{66}$</td>
<td>0.0134</td>
</tr>
<tr>
<td>$\phi_{168}$</td>
<td>0.08474</td>
<td>$\theta_{192}$</td>
<td>-0.0150</td>
</tr>
<tr>
<td>$\phi_{192}$</td>
<td>0.0085</td>
<td>$\theta_{215}$</td>
<td>-0.0042</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0.1354</td>
<td>$\theta_{241}$</td>
<td>0.0055</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>0.0082</td>
<td>$\theta_{264}$</td>
<td>-0.0027</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>0.0503</td>
<td>$\theta_{288}$</td>
<td>0.0083</td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>-0.0806</td>
<td>$\theta_{337}$</td>
<td>0.0238</td>
</tr>
<tr>
<td>$\theta_5$</td>
<td>0.0512</td>
<td>$\theta_{480}$</td>
<td>-0.0282</td>
</tr>
</tbody>
</table>

3. The model validation is performed based on the diagnostic check, which
5.2. **SENSITIVITY ANALYSIS TO PRICE AND INFLOW UNCERTAINTY**

Tests whether the hypothesis about the residuals is true: residuals must be uncorrelated, normally distributed with 0 mean and constant variance. For that purpose ACF and PACF plots of the residuals are studied. Besides, the Ljung-Box test is applied. After the validation step, the model can be used to predict.

![Figure 5.4: Real and forecasted prices.](image)

4. In this step, the built model is ready to forecast the next day spot market prices. The constructed final model to predict spot market prices in the Swedish electricity market is the following:

\[
\begin{align*}
(1 - \phi_1 B^1 - \phi_2 B^2)(1 - \phi_{23} B^{23} - \phi_{38} B^{48} - \phi_{72} B^{72} - \phi_{96} B^{96} \\
- \phi_{168} B^{168} - \phi_{192} B^{192}) \log \lambda_t &= (1 - \theta_1 B^1 - \theta_2 B^2 - \theta_3 B^3 \\
- \theta_4 B^4 - \theta_5 B^5 - \theta_6 B^6 - \theta_7 B^7 - \theta_8 B^8)(1 - \theta_{21} B^{21} \\
- \theta_{47} B^{47} - \theta_{66} B^{66} - \theta_{192} B^{192} - \theta_{215} B^{215} - \theta_{241} B^{241} \\
- \theta_{264} B^{264} - \theta_{288} B^{288} - \theta_{337} B^{337} - \theta_{480} B^{480} - \theta_{504} B^{504}) \epsilon_t
\end{align*}
\]

Figure 5.4 demonstrates the forecasting results. All those steps are implemented in MATLAB using the GARCH package.

The future value \(\hat{\lambda}_{t+j}\) of the process is normally distributed with mean \(\hat{\lambda}_t(j)\) and \(\sigma(j)\) expressed in (3.9) (section 3.3). Thus, day-ahead market price scenarios are generated as a normal distribution with the mean equal to the forecasted values and constant standard deviation, which is calculated based on the historical data. Simulation scenarios are plotted in Figure 5.5.
5.2.2 Sensitivity analysis

As it was stated above, the random variables in the stochastic problem have continuous distribution. To make the problem solvable, the continuous distribution is discretized and approximated by a scenario tree. This scenario tree should be assessed based on the decision it generates: how close is the approximated solution to the 'true' solution. For that reason the stability requirement has to be fulfilled. For this problem in-sample stability is tested: first, several trees with different number of scenarios are generated, then the optimization problem is solved for each tree and finally solutions are compared [35]. Based on the simulation results, ten scenarios seem sufficient to get stable results Table 5.3. Thus, ten scenarios are generated for the day-ahead market prices and ten scenarios for the water inflow level to each reservoir respectively.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4226871</td>
</tr>
<tr>
<td>15</td>
<td>4227111</td>
</tr>
<tr>
<td>20</td>
<td>4227504</td>
</tr>
</tbody>
</table>

The two-stage stochastic programming problem is formulated in GAMS. CPLEX
5.2. SENSITIVITY ANALYSIS TO PRICE AND INFLOW UNCERTAINTY

12.2 solver is used to solve the resulting mixed integer linear programming model and the execution time is 4.48 seconds. Based on the solution the bidding curve for each hour has been drawn. Figure 5.6 and Figure 5.7 demonstrate the results. The bidding curve in Figure 5.6 corresponds to the first, fifth and sixth hours and the curve in Figure 5.7 corresponds to the remaining hours. According to the bidding curves; bidding volume is less than total maximum capacity of the three power plants only in the first, fifth and sixth hours. For those hours, scenario combinations for day-ahead market prices and inflow level implied it was not profitable to bid with the maximum capacity. In the remaining hours it bids with the maximum capacity (235 MWh).

![Figure 5.6: Bidding curve for the first, fifth and sixth hours.](image)

![Figure 5.7: Bidding curve for the remaining hours.](image)
CHAPTER 5. CASE STUDY

The solution (Fig. 5.6 and Fig. 5.7) shows that with this condition there is no bottleneck. A bottleneck may be defined as the part of the system which either causes a forced generation of the electricity while the prices are not so attractive or causes water spillage. The possible explanation is nearly the same maximum discharge level in all power plants. Having huge difference in the maximum reservoir capacity, first and second power plants have nearly the same maximum discharge levels. Thus, the water inflow is nearly equal to the water outflow. However, the bottleneck occurs in the system when the initial level of reservoirs is considered more than 80 % of its maximum reservoirs respectively. In this case the model provides more flexible solution, it starts to bid at 35 Euro/MWh, when water opportunity cost is 50 Euro/MWh. Apparently, there is a forced release of water under the low market prices to avoid later spillage.

5.2.3 Sensitivity analysis in terms of the water opportunity cost

The sensitivity of the stochastic model is tested in terms of water opportunity cost. When it is taken as 55 Euro/MWh instead of 50 Euro/MWh, it only bids for a few hours. When the water opportunity cost is 60 Euro/MWh, it only bids for one hour. As expected, the solution is very sensitive to the changes in the water future value. Thus, it is logical to conclude that the introduction of the uncertainty related to the water opportunity cost might have a strong effect on the solution.

5.2.4 Sensitivity analysis in terms of VSS: Initial case

To assess the possible gain from the stochastic model: the effect of including the uncertainties in the model, the VSS quantity is calculated (chapter 2). Table 5.4 demonstrates the calculation result (first row).

Table 5.4: Calculation results for different cases

<table>
<thead>
<tr>
<th>Case</th>
<th>SP</th>
<th>EEV</th>
<th>VSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>4226871</td>
<td>4223197</td>
<td>3674</td>
</tr>
<tr>
<td>High price vol.</td>
<td>4243480</td>
<td>4222098</td>
<td>21382</td>
</tr>
<tr>
<td>Low price vol.</td>
<td>4222352</td>
<td>4222301</td>
<td>51</td>
</tr>
<tr>
<td>High inflow vol.</td>
<td>4226709</td>
<td>4223040</td>
<td>3669</td>
</tr>
<tr>
<td>Low inflow vol.</td>
<td>4226610</td>
<td>4222933</td>
<td>3677</td>
</tr>
<tr>
<td>Higher inflow</td>
<td>4340100</td>
<td>4336491</td>
<td>3609</td>
</tr>
<tr>
<td>Lower inflow</td>
<td>4169313</td>
<td>4165625</td>
<td>3688</td>
</tr>
</tbody>
</table>
According to the figures in the Table 5.4, VSS = 3674 Euro for the base case. This is the gain we get using the stochastic model rather than the expected value problem for only one day. Thus, the motivation using the stochastic approach will be stronger if we consider yearly profit.

Moreover, to check the sensitivity of the stochastic model towards price volatility and water inflow level volatility, the model is run with the high/low volatile price scenarios and high/low volatile inflow scenarios: for the high volatile cases the standard deviation is taken as double compared with the base case and for the low volatile cases standard deviation is taken as half of the base case. Then VSS values are calculated for each case and are shown in Table 5.4. The comparison in the VSS values shows that noticeably bigger changes happen when different price volatility levels are considered. Small changes in VSS values occur when the models are run with high volatile water inflow levels and comparatively low volatile water inflow levels (Figure 5.8). These results motivate us to investigate further whether it has a big effect to consider water inflow level uncertainty. For that purpose the cases are considered for which the mean of the water inflow level is varied: first the mean is taken twice as big as in the base case, and then the mean is taken half of the mean in the base case. Again the results demonstrate slight changes in the VSS values (last two rows in Table 5.4 and last two bars in Figure 5.8).

Figure 5.8: VSS results for different cases.

These comparison results, with the VSS values summarized in Table 5.4, in-
CHAPTER 5. CASE STUDY

dicates that the effect including price uncertainty in the model is higher compared with the effect of considering water inflow level uncertainty.

5.2.5 Sensitivity analysis in terms of VSS: general case

To generalize the findings, all these scenarios are studied for a case where three cascaded reservoirs with nearly identical maximum storage capacity is considered. The calculated results are summarized in Table 5.5 and also drawn in Fig. 5.9. Again no observable changes in VSS values occur in terms of the cases considered for inflow level. Thus, the conclusion stated in the previous section is not altered.

Table 5.5: Calculation results: General case

<table>
<thead>
<tr>
<th>Scenario</th>
<th>SP</th>
<th>EEV</th>
<th>VSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>2874921</td>
<td>2871231</td>
<td>3690</td>
</tr>
<tr>
<td>High price vol.</td>
<td>2891528</td>
<td>2870132</td>
<td>21450</td>
</tr>
<tr>
<td>Low price vol.</td>
<td>2870386</td>
<td>2870335</td>
<td>51</td>
</tr>
<tr>
<td>High inflow vol.</td>
<td>2874769</td>
<td>2871074</td>
<td>3695</td>
</tr>
<tr>
<td>Low inflow vol.</td>
<td>2874667</td>
<td>2870967</td>
<td>3700</td>
</tr>
<tr>
<td>Higher inflow</td>
<td>2988203</td>
<td>2984513</td>
<td>3690</td>
</tr>
<tr>
<td>Lower inflow</td>
<td>2817368</td>
<td>2813658</td>
<td>3710</td>
</tr>
</tbody>
</table>

Figure 5.9: VSS results for different cases: General case
5.2. SENSITIVITY ANALYSIS TO PRICE AND INFLOW UNCERTAINTY

5.2.6 Sensitivity analysis in terms of VSS: small reservoir case

In order to create a situation where there is an impact of inflow level uncertainty in the model, all those cases are considered where three cascaded small reservoirs are studied. The results are demonstrated in the Table 5.6 and Fig. 5.10.

<table>
<thead>
<tr>
<th>Case Description</th>
<th>SP</th>
<th>EEV</th>
<th>VSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>217274</td>
<td>209293</td>
<td>7981</td>
</tr>
<tr>
<td>High price vol.</td>
<td>233997</td>
<td>206108</td>
<td>27889</td>
</tr>
<tr>
<td>Low price vol.</td>
<td>213502</td>
<td>213358</td>
<td>144</td>
</tr>
<tr>
<td>High inflow vol.</td>
<td>216829</td>
<td>208821</td>
<td>8008</td>
</tr>
<tr>
<td>Low inflow vol.</td>
<td>217032</td>
<td>208975</td>
<td>8057</td>
</tr>
<tr>
<td>Higher inflow</td>
<td>326230</td>
<td>306788</td>
<td>19442</td>
</tr>
<tr>
<td>Lower inflow</td>
<td>159207</td>
<td>155201</td>
<td>4006</td>
</tr>
</tbody>
</table>

From both Table 5.6 and Fig. 5.10 it is possible to see that now there is noticeable change in VSS values for the cases of higher inflow and lower inflow (where mean inflow level is taken double/half as big as in the base case respectively). However, even in this specific structure, which was designed on purpose to see the impact of the inflow level uncertainty, changes in VSS values for inflow level for above mentioned cases are less significant compared with the changes in VSS.
values in high/low price volatility cases. All these scenarios tend to emphasize the previously drawn conclusion: the impact of including the price uncertainty in the model is higher than that of inflow level uncertainty. In contrast with the day-ahead market prices, within short time period the changes in the inflow level cannot be sharp to have a strong effect. This might be because of physical characteristics of the inflow: weather changes and soil structure. The weather changes from hour to hour can not be very big (in any season), which means that the difference in the amount of the melted water that reaches the reservoir from the hour to hour can not be large. On the other hand the specific features of the soil structure do not allow sharp increase in inflow level directly after the heavy rain; it only increases gradually.

5.3 Day-ahead bids considering real-time market: Different approaches to limit real time bids

The study aims to test the stochastic hydropower planning model II stated in chapter 4 and comment on the results. According to the results of this model, two extreme cases tend to dominate in the decision making process; either the maximum amount is offered to up regulating market or the maximum amount is bid to day-ahead market and put an offer on down regulating market. For a hydropower producer this is not a realistic case. Since day-ahead market sales are more definite, the hydropower producer will bid the base amount to the day-ahead market and only a small part will be offered to the real-time market. Otherwise it can cause unwanted spillage if the storage capacity of the hydro system is small.

This section formulates different approaches which force the optimization model to give more realistic solutions. Each approach is a modification of the base optimization model, that creates bounds on the traded quantity on the real-time market. Every approach is assessed and discussed based on the results.

Corresponding models are used to forecast and generate input scenarios for the day-ahead and real-time market prices. Time series used for the day-ahead and real-time price forecast is historical data for day-ahead and real-time market prices correspondingly for 2011, October-November published on Nord pool. First the ARIMA model (described in chapter 3) is used to forecast day-ahead market prices and to generate scenarios based on that. Then SARIMA time series analysis tool together with Markov chain (described in chapter 3) is used to forecast real-time market prices and to generate corresponding price scenarios. Again the GARCH
package in MATLAB is used for this purpose. Finally, after having all input data, the optimization model is built in GAMS.

5.3.1 Spot market price model

As it was mentioned above, spot market prices October-November 2011, are used to build a model to forecast day-ahead market prices. Figure 5.11 shows the spot market prices for the year 2011. Note that this model is different from the one built in section 5.2.1. Again those four steps explained in the chapter 3 should be completed. The identification process, as it was explained in section 5.2.1, is carried out studying the ACF and PACF of the historical time series shown in Figure 5.12 and Figure 5.13 respectively.

A preliminary model is built and the corresponding parameters are estimated according to the methods described in chapter 3. Then the model is refined studying the ACF (Figure 5.14) and PACF (Figure 5.15) of the residuals; whether they resemble a white noise. The horizontal lines represent the interval \( \pm 1.96/\sqrt{N} \), according to which in order a sequence be a white noise only 5% of the values are expected to go beyond the lines. According to Figure 5.14 and Figure 5.15, less than 5% of the values fall outside the interval; hence the sequence is white noise.

In addition, the Ljung-Box statistics is applied, according to which, if \( H = 0 \) the fact that the residuals resemble white noise series can not be rejected. If according to ACF and PACF the residuals series do not fulfill the requirements to be a white noise, or if according to Ljung-Box statistics \( H = 1 \), the model orders (p,q) are adjusted and the whole procedure is repeated again.
The constructed ARIMA model ready for spot market price prediction is the following.

\[
(1 - \phi_1 B^1)(1 - \phi_{17} B^{17} - \phi_{23} B^{23} - \phi_{48} B^{48} - \phi_{72} B^{72} - \phi_{167} B^{167}) \log \lambda_t = (1 - \theta_1 B^1 - \theta_2 B^2 - \theta_3 B^3 - \theta_4 B^4 - \theta_5 B^5)(1 - \theta_{18} B^{18} - \theta_{45} B^{45} - \theta_{68} B^{68}) \epsilon_t
\]

Figure 5.16 shows the forecasting results. For this model the estimated value for \( \sigma_e = 0.1162 \).
5.3. DAY-AHEAD BIDS CONSIDERING REAL-TIME MARKET: DIFFERENT APPROACHES TO LIMIT REAL TIME BIDS

![Sample Autocorrelation Function](image1)

Figure 5.14: ACF for the residuals: day-ahead market.

![Sample Partial Autocorrelation Function](image2)

Figure 5.15: PACF for the residuals: day-ahead market.

5.3.2 Regulating market price model

The time series used to predict regulating market prices is real-time prices for 2011 taken from the Nord Pool website. To show the existing correlation between day-ahead and real-time market, prices are plotted (Figure 5.17). As it can be seen from Figure 5.17, the day-ahead market prices serve a bound for real-time market
prices: upward regulated prices are always greater than day-ahead market prices and downward regulated prices are always less than day-ahead market prices. Another important thing possible to see from the Figure 5.17 is that, it is uncommon to have both upward and downward regulation happening within one hour. Most of the cases either upward or downward regulation is taking place.

As it is described in chapter 3, the SARIMA process is used to model continuous behavior of the real-time market prices and discrete Markov chain is used to
5.3. DAY-AHEAD BIDS CONSIDERING REAL-TIME MARKET: DIFFERENT APPROACHES TO LIMIT REAL TIME BIDS

capture the direction of the real-time balancing market prices.

SARIMA model

To keep the correlation between day-ahead and real-time market prices, differences in real-time and day-ahead market prices are calculated and then logarithmized. Then, model orders are set and parameters are estimated based on the methods presented in chapter 3. SARIMA models for upward and downward regulation prices are presented below:

• The upward regulated prices are modeled with a SARIMA \((2,0,1) \times (2,0,1)_{24}\) process expressed by the following polynomials:

  \[
  \Phi(B) = 1 + 0.053B^{24} - 0.024B^{48}, \\
  \Theta(B) = 1 + 0.09B^{24},
  \]

  \[
  \phi(B) = 1 + 0.27B - 0.63B^2, \\
  \theta(B) = 1 - 0.98B,
  \]

  \[(5.3)\]

  For this case the estimated value for \(\sigma_\varepsilon = 0.2105\).

• The downward regulation prices are modeled with the SARIMA \((3,0,2) \times (1,0,1)_{23}\) process expressed by the following polynomials:

  \[
  \Phi(B) = 1 - 0.013B^{23}, \\
  \Theta(B) = 1 - 0.006B^{23},
  \]

  \[
  \phi(B) = 1 - 0.43B - 0.26B^2 + 0.052B^3, \\
  \theta(B) = 1 + 0.15B - 0.033B^2,
  \]

  \[(5.5)\]

  The estimated value for \(\sigma_\varepsilon = 0.1922\)

  The residuals are generated for those models, then ACF and PACF are calculated. The results for the upward regulation model are plotted in the Figure 5.18 and the Figure 5.19 accordingly. Again the horizontal lines represent the interval \(\pm 1.96/\sqrt{N}\). Since according to Figure 5.18 and Figure 5.19 less than 5% of the values fall outside the interval, the sequence is white noise. In addition, the same conclusion is confirmed by the Ljung-Box statistics.
Markov model

To have a Markov model to describe the discrete behavior of the system for next 24 hours, first the probabilities of the transition matrix have to be estimated. The method to estimate the transition probabilities are presented in chapter 3. To
recall, all possible states of $b_t$ for a specific hour were:

1. $(b_t^+, b_t^-) (0, 0)$
2. $(b_t^+, b_t^-) (0, 1)$
3. $(b_t^+, b_t^-) (1, 0)$
4. $(b_t^+, b_t^-) (1, 1)$

The resulting transition matrix is presented below:

$$
P = \begin{pmatrix}
0.6396 & 0.1762 & 0.1816 & 0.0054 \\
0.1052 & 0.8259 & 0.0414 & 0.0276 \\
0.1452 & 0.0456 & 0.7805 & 0.0269 \\
0.0645 & 0.4516 & 0.4839 & 0
\end{pmatrix}
$$

The first term in the matrix means that there is 63.96% chance that the system will stay in state 1, if the system has been in state 1 in the previous hour. Having the transmission probability matrix estimated, it is possible to predict different scenarios for the state of the system for coming 24 hours.

An example of a scenario tree is provided in Figure 5.20.

![Figure 5.20: An example of spot, upward and downward scenarios. Spot market prices are represented by the continuous line, upward regulated prices are shown by '⋄' and downward regulation prices are indicated by '⊿'.](image)

5.3.3 Optimal bids for day-ahead and regulating markets

The models described in the previous section are used to predict market prices and generate scenario tree which is used to represent day-ahead and regulating market prices in the optimization model II presented in chapter 4. The optimization
model is programmed in GAMS and the CPLEX solver is used to solve it. All assumptions are listed below:

- The hydro system in Fig. 5.1 and system data summarized in Table 5.1 are used in the optimization model.
- Initial reservoir content is considered 36% full.
- The future electricity price is considered deterministic setting 50 Euro/MWh and is estimated taking the average value of futures and forward contracts.
- The hydropower producer is a price taker, hence does not have any market power.
- Water delay time is considered.
- Ten day-ahead market price scenarios are generated for each hour according to the described model in section 5.3.1.
- Five upward regulated and five downward regulated price scenarios are generated using the method described in section 5.3.2. The model takes all combinations of the day-ahead, upward regulated and downward regulated prices. In this way there will be some combinations when for example upward regulated price will be less than day-ahead market price. However, the nature of optimization problem (for maximization case) will take it as no regulation has happened. The motivation behind this is to have more combinations; 250 scenarios instead of 50. More precise alternative for this is to generate 10 day-ahead market price scenarios and then for each day-ahead market scenario to generate 5 upward regulation and 5 downward regulation scenarios. This is tested for the same problem; the same system is used only for 30 scenarios (ten day-ahead market scenarios each of them having three upward regulated and three downward regulated price scenarios). The results have the same profile as for the first case.

The simulated result gives bidding volumes to different market places for each hour, which can be seen from Figure 5.21. According to Figure 5.21 for most of the hours the following happens:

- The maximum amount is bid in upward regulated market;
- The maximum amount is bid to day-ahead market and offered to be bought back in downward market.
5.3. DAY-AHEAD BIDS CONSIDERING REAL-TIME MARKET: DIFFERENT APPROACHES TO LIMIT REAL TIME BIDS

Figure 5.21: Traded volumes in spot and regulating markets for a specific scenario. Spot market traded quantities are represented by continues line. Upward traded quantities are shown by ’○’ and downward traded quantities are indicated by ’△’.

Similar results have been observed for example in [48]. The solution is optimal, however the solution is not realistic: in reality the hydropower producer will hardly act in this way. It is expected that the hydropower producer for each hour will bid the main volume to spot market and only small quantity will be offered to the up or down regulating market. This behavior is dependent on the special characteristics of day-ahead market and real-time market. Sales on day-ahead market are definite; when a bid is accepted on day-ahead market, the quantity mentioned on the bid must be provided. In contrast to day-ahead market, sales on real-time market are uncertain. Bids submitted to the regulating power market in this work concern as an energy volume in MWh. These bids are activated by the System Operator, when the system needs regulating power. Thus, the amount of regulating power the actor will buy or sell totally depends on the duration the bid has been active within that hour. For example, it might happen that there is a need to provide upward regulation power only for the first ten minutes in the delivery hour. This indicates that only $\approx 17\%$ of the allocated volume will be used in the up regulating market. The remaining volume has to be stored for future usage. If this happens in several consecutive hours, it can cause spillage for a system with a small storage capacity.

In addition, the model structure does not consider the next days trading. It is possible that, if the model had been simulated for a couple of days, the results could have been more flexible to avoid spillage. Since for a short time period the storage capacity will support to store water, the increase in planning period from one day to couple of days will increase the model size dramatically. Thus, the model becomes
non-applicable for the large systems.

The different modifications of the base model are described below.

**Trivial approach**

The first approach is very trivial. It states that the maximum bidden amount to the up/down regulation can not be higher than 20% of the maximum capacity. For that purpose the following constraints should be added to the optimization problem introduced in chapter 4.

\[ d_{sl}^i = 0.2 \sum_j G_j; \quad (5.9) \]

\[ d_{skb}^i = 0.2 \sum_j G_j; \quad (5.10) \]

The result, drawn in Figure 5.22, is as naive as the approach itself; whenever it is beneficial to participate in upward/downward regulation market, it bids the allowed amount (20%) and the remaining amount is bid to the day-ahead market.

![Figure 5.22: Traded volumes in spot and regulating markets: trivial case.](image)

**Limit the end reservoir content approach**

The second approach is fixing the reservoirs content at the end of the planning period. Let \( m_{desired}^j \) be the desired end reservoir content, \( \Gamma \) be the cost of deviation and \( V_{sl}^j \) be the absolute deviation from the end content of reservoirs. (5.11) is
expressing the objective function of this approach.

\[ \max z = z^{da} + z^+ - \sum_{slk} p_s p_l p_k \sum_j TV_{slk} \]  

(5.11)

Moreover, the constraint should be added to capture the deviation (5.12).

\[ V^j_{slk} = m^j_{alt} - m^j_{desired}; \]  

(5.12)

Results give more flexible solution, but it is still not very realistic (Figure 5.23).

![Figure 5.23: Traded volumes in spot and regulating markets: Fixing the end content of reservoirs case.](image)

According to Figure 5.23, up/down regulation volume is less than the maximum capacity for most of the hours. For example, from hour two to hour eight there is down regulation; for the 2nd and 8th hours it bids maximum capacity to day-ahead market and brings back by offering down regulation. While for other hours in the mentioned interval down regulated volume is less than maximum capacity, which was traded on day-ahead market. During several hours it again directly bids to the upward regulating market with the maximum capacity. Generally speaking, overall results get better, but the improvement is not sharp.

**Non-linear approach**

In the third approach we have assumed that real-time market prices are highly influenced by the bidden volume. The real-time market prices are least beneficial (low for upward regulation and high for downward regulation), when the bidden
volume is equal to the maximum capacity and improving gradually while bidden volume is decreasing. To express this idea in mathematical language the following nonlinear convex constraints are added to the model.

\[
\begin{align*}
    r_{up}^{s,l,t} &= \frac{r_{up}^{s,l,t+1}}{(d_{up}^{s,l,t} + 1)} \\
    r_{down}^{s,k,t} &= \frac{r_{down}^{s,k,t}}{(\zeta d_{down}^{s,k,t} + 1)}
\end{align*}
\]  

In equations (5.13) and (5.14), \( r_{up}^{s,l,t} \) and \( r_{down}^{s,k,t} \) are new variable prices in the real-time market for each scenario and hour and \( \zeta \) is a scalar which helps to keep the \( r_{down}^{s,k,t} \) prices in the desired interval. With the introduction of variable real-time market prices the objective function becomes nonlinear. A local optimum for the optimization problem is also a global optimum if all nonlinear constraints and objective function are convex. A multidimensional function is convex if Hessian matrix is positive semidefinite [27]. Alternatively, the function is convex if all eigenvalues of Hessian matrix are greater than or equal to zero. It was already mentioned that equations (5.13) and (5.14) are convex. However, the objective function is not convex since not all of the eigenvalues are greater than or equal to zero. Thus, we can not claim for the global optimality of the solution. Figure 5.24 shows the results. As one can see the results are more realistic in this case: for every hour the base volume is traded in the day-ahead market and only a small amount is bid to upward or downward regulated market.

![Figure 5.24: Traded volumes in spot and regulating markets: Nonlinear case.](image-url)
The main drawback of this approach is that it is not easy to prove the global optimality of the solution. Moreover, it takes 10 hours to solve, which is very long for the small system like this. Thus, it is clear that the model is not applicable for large systems; for example for a river system with 50 hydropower plants.

**Discretized approach**

The last approach is the advanced version of the trivial approach. In this case we assume, that available traded amount to upward/downward market has upper bound (20% of the maximum capacity). However, upward/downward market prices are discretized and different percentage of the available amount is bid according to that. For example, if upward/downward market prices are not very attractive it bids only 10% of the available quantity. It increases the bidding percentage when prices get better and finally it bids all volume only for the case when the prices are the most beneficial. The results are drown in the Figure 5.25.

![Figure 5.25: Traded volumes in spot and regulating markets: discretized case.](image)

In this case also the base quantity is traded in the day-ahead market. Small amount is traded on the upward/downward market. Hence, this approach also gives a realistic solution. The model is linear and solving time is very short. Table 5.7 summarizes different approaches.

From all cases above it is obvious that the most promising and applicable approach is the fourth one: it gives flexible results, keeps the optimization problem linear and it is fast to solve.
Table 5.7: Summary of the results

<table>
<thead>
<tr>
<th>Approaches</th>
<th>Model type</th>
<th>Results</th>
<th>Solution time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>Linear</td>
<td>Optimal but not realistic</td>
<td>12 seconds</td>
</tr>
<tr>
<td>Trivial</td>
<td>Linear</td>
<td>Optimal but not interesting; naive</td>
<td>9 seconds</td>
</tr>
<tr>
<td>Fixed end reservoir</td>
<td>Linear</td>
<td>Optimal but little improvement</td>
<td>9 seconds</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>Nonlinear</td>
<td>Locally optimal but realistic</td>
<td>10 hours</td>
</tr>
<tr>
<td>Discretized</td>
<td>Linear</td>
<td>Optimal and realistic</td>
<td>9 seconds</td>
</tr>
</tbody>
</table>
Chapter 6

Conclusion

This chapter concludes the thesis. It highlights all parts which need further clarification and provides an overview of the future work.

6.1 Summary

The continuous growth of renewable energy sources creates huge production variation in the electric power system, which has to be handled in an optimal way. It is possible to compensate these variations using hydropower, since it is very fast for hydropower plants to change their production. Thus, the main aim of this work is to develop advanced planning tools for a price taker hydropower producer, which will allow allocating the hydropower production in the better way and increasing the profit.

A profound literature review has been done in the field of short-term hydropower production. The assessment of short-term hydropower planning deterministic and stochastic models has been provided. The first study considered a model, which is an optimal bidding strategy to the day-ahead market. The case has been studied based on the Swedish hydropower plants. Two-stage stochastic programming has been used to deal with uncertainties related to the day-ahead market prices and water inflow levels. The solution clarifies what is profitable for the price-taker hydropower producer; to sell the electric power to the spot market or to store the water for the future production.

To cope with the uncertainties, ten representative scenarios have been generated for the prices and inflow levels. The comparison of the optimal value of the stochastic program with the expected result of the expected value problem
shows the consequence of considering uncertainties in the problem. Moreover, deep sensitivity analysis has been provided, which shows that for the short-term hydropower planning problems the effect of considering price uncertainty in the stochastic model is higher compared with considering inflow level uncertainty.

The model used in the second case study is an optimal bidding strategy to the day-ahead market while considering real-time market. The results of this model are not realistic. They tend to follow two dominant trends: either to participate only in upward regulating market or sell the whole capacity in day-ahead market and put an offer to bring back in downward regulation market. Therefore, the purpose of this study was to suggest some modifications of the base model, which would provide more robust results. Different modeling approaches have been tested and evaluated.

The first and the second approaches are not improving the solution sharp. The third approach gives more realistic solution, however the optimization problem becomes nonlinear and takes long time to solve it. This means that the model is not applicable for large systems. Moreover, the objective function is non-convex and it is hard to show that the local optimum is also the global optimum. The forth approach gives reasonable solution and keeps the optimization model linear. This makes it possible to solve within very short time and hence apply for a large system. Thus, the fourth approach is suggested as the most promising one.

6.2 Discussion

6.2.1 Hydropower planning model

The head height is the difference in meters between the water surface of the reservoir and the water level downstream of the reservoir. As the water content moves up in a reservoir, the production rate increases because the head height increases too. The head effect is not considered in the planning models to avoid non-linearity. For the case of non-linear optimization models it is always necessary to use appropriate methods to prove that the local optimum is also the global optimum. On the other hand, in the short-term planning the change in the reservoir content can not be very big (unless the reservoir is not very small). Therefore, the impact of the head effect on the short-term hydropower planning is small.

The discharged water from the upper reservoir has some delay time until it reaches to the reservoir located downstream. The delay time is modeled throughout this work.
6.2. DISCUSSION

The start and stop costs for the plants are neglected and not modeled in this thesis, since start and stop costs are very small for hydropower plants. However, it is very easy to model them by introducing binary variables. The resulting model will be mixed integer linear stochastic program. In this case, it is difficult to be sure that the optimum solution will be in one of the extreme points in the feasible region. It might be inner point from the feasible region. This will reduce the chances that the optimal discharges will appear in the best efficiency points in the discharge-generation curve. Hence, another approximation method might be useful to apply in this case.

6.2.2 Solution techniques

The optimization problems described in chapter 4 are solved using CPLEX solver capable to solve linear programming problems. No effort has been made to study the structure of the models and apply any solution technique appropriate for these specific problems to increase computational efficiency. However, it might be beneficial for the multi-stage stochastic models to use appropriate solution technique to shorten the solution time. One of the most widely used methods is to apply some type of decomposition on the hydropower planning models [44].

The increase in computational efficiency will allow us to introduce another type of uncertainty into the model in addition to the existing ones and still be able to solve it. For example, it will give possibility to include the uncertainty related to the future market prices or the intra-day market.

6.2.3 Scenario tree generation

Scenarios are generated in MATLAB. GAMS has a package called SCENRED [1] which allows the user to generate a scenario tree and then to reduce the size of the tree directly in the GAMS at any desired percentage level. The reduction algorithm in SCENRED determines scenario subset and gives optimal weight to the saved scenarios.

SCENRED is not used in this work. The application of SCENRED will be straightforward in model I. However the application of SCENRED in the second model requires further studies, since for each hour a number of scenarios for regulating market prices are bundled by a day-ahead market price scenario.

6.2.4 Bidding curves

Simple bidding curve model is used in hydro planning models to bid day-ahead
market and regulating market, which are linear and cannot alter the linearity of the optimization model.

More advanced bidding strategies for the regulating market are presented in [47]. A producer who participates in upward regulating market is willing to sell power and a producer who is participating in downward regulating market, is willing to buy power. Hence bidding strategies to upward regulating market will have increasing manner and to downward regulating market will have decreasing manner. The bidding functions for upward and downward regulating markets in [47], which exhibit these properties are non-linear. However, they are concave and convex respectively, which makes the situation better. It is easier to deal with an optimization problem, which has nonlinear but concave/convex functions.

6.2.5 Regulated quantity MW vs. MWh

Bids submitted to the regulating power market in model II described in chapter 4 concern as an energy volume in MWh. However, in general bids submitted to the regulating power market consists of the price and the amount of power in MW. The bids to the regulating market can be activated by the System Operator, when the system needs regulating power. This indicates that the amount of regulated power the actor will buy or sell totally depends on the duration the bid has been active within that hour. For example, it might happen that there is a need to provide upward regulation power only for the first ten minutes in the delivery hour. Hence, the time when the particular bid is activated can be considered as a stochastic process. It is easy to notice that this will make the problem size double. This might require some changes in scenario tree generation and reduction methods.

6.2.6 Markov model

Markov chains are used to model the discrete behavior of the real-time balancing market prices. Based on the historical price series the transmission probability matrix is calculated. These probabilities are calculated assuming that the system will change its state (let’s say from state one to state two) after staying in the previous state one hour.

According to [47], in order to validate that the Markov model is an appropriate method to model discrete behavior of the regulating power market prices, one has to study the number of steps the system stays in the same state before going to another state. The number of state might be considered as a stochastic variable. If this stochastic variable has exponential distribution, the Markov model is suitable
to model the process [45]. These validation steps are not accomplished in this thesis.

### 6.2.7 Intra-day market

A short description of the intra-day market (Elbas) is provided in chapter 2. After spot market clearing, the adjustment of the actual generation is taking place by trading electric power in intra-day market. The trading of the energy for each specific hour is stopped an hour before the actual delivery. Bids submitted to the intra-day market contain information about the bid type (sell or buy), price Euro/MWh and volume MWh for a specific hour. Each participant places its anonymous bids to a web based trading system. The trading system allows participants continually to follow the situation in the market. The continuous trading process on the market makes it even harder for the participant to predict the electricity price precisely and include in a bid, which will increase the probability that the bid will be accepted and will provide high profit.

The existence of the intra-day market decreases the volume needed to trade in the real-time market. Thus, the price changes in the real-time market for upward and downward regulation would not be sharp.

In both models, the intra-day market is not considered. Currently traded volume on an intra-day market is quite small compared with a day-ahead market. However, this profile will change in the future, when the renewable energy share will increase, which in turn will increase the uncertainties in the market and the needed volume to adjust those uncertainties. The consideration of the adjustment market will capture future market reality better and might have a significant impact on the decision making process. Thus, to model the future electricity market, the intra-day market must be introduced in the planning models. This will increase the model complexity and the computational time significantly. Therefore, some solution methods have to be applied, which might increase the computational efficiency and shorten the solution time.

### 6.2.8 Future price for electric power

In this thesis the future price for electric power is assumed deterministic and is estimated studying financial contracts. However, future electricity price is a stochastic variable. According to the sensitivity analysis introduced in chapter 5, the bidding strategy is very sensitive towards the changes in future electricity price. Small changes in the future electricity prices cause big changes in the model...
results. Thus, it is very important to have the best estimator for the future electricity prices. One way to get this is to consider the future electricity prices as stochastic
variables.

6.3 Future work

- To develop optimal bidding strategy to the day-ahead market, considering both adjustment and real-time markets. Since in the adjustment market accepted bids are paid according to the price stated in the bid (in contrast to the day-ahead market), it is very important to develop high quality forecast for adjustment market prices and to have a bidding strategy based on that forecast. In addition, it might be interesting to study the alternative rules to provide balancing power and its impact on the planning. Whether the hydropower producer is free to use the water or he is required to provide reserve margins. Finally, it is important to study the impact of the introduction of the new market rules on the planning models.

- To introduce wind power to the stochastic model. Up to now we have just considered wind power impact on the system (more volatile prices). It might be interesting to add wind power output to the model as an uncertain variable. Then to study a balance responsible player with a production portfolio of both wind and hydropower and introduce imbalance settlement cost in the planning problem. This will increase the complexity of the problem a lot.

- To study and apply techniques which will increase computational efficiency of the problems under study. Multi-stage stochastic models for the planning of the power systems are large-scale and contain internality constrains, which makes it a demanding task to solve. Thus, it is necessary to investigate scenario reduction algorithms and scenario tree construction for large-scale problems. In this way, it might be possible to reduce the computation time of the problem without altering the accuracy of the solution. In addition, another way to increase computational efficiency is to discover proper formulation of the problem, e.g. proper formulation of a constraint(s), or to provide good starting points for a particular solver.

- It is not trivial to generate input data for developed models (day-ahead market and real-time market price scenarios). To make the model useful for the industry, there should be efficient methods to generate appropriate scenarios.
6.3. **FUTURE WORK**

In addition, it might be interesting, if it is possible to test developed models and scenario generation methods on real data from the Swedish hydropower producers.
Bibliography

[1] *GAMS/SCENRED*.


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