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Practical Application of Modern Portfolio Theory

Bachelor Thesis within Business Administration

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Abstract

There are several authors Markowitz (1991), Elton and Gruber (1997) that discuss the main issues that an investor faces when investing, for example how to allocate resources among the variety of different securities. These issues have led to the discussion of portfolio theories, especially the Modern Portfolio Theory (MPT), which is developed by Nobel Prize awarded economist Harry Markowitz. This theory is the philosophical opposite of traditional asset picking.

The purpose of this thesis is to investigate if an investor can apply MPT in order to achieve a higher return than investing in an index portfolio. Combining a strong portfolio that beats the market in the long-run would be the ultimate goal for most investors.

The theories that are used to analyze the problem and the empirical findings provide the essential concepts such as standard deviation, risk and return of the portfolio. Further, diversification, correlation and covariance are used to achieve the optimal risky portfolio. There will be a walk-through of the MPT, with the efficient frontier as the graphical guide to express the optimal risky portfolio.

The methodology constitutes as the frame for the thesis. The quantitative method is used since the data input is gathered from historical data. This thesis is based on existing theories, and the deductive approach aims to use these theories in order to accomplish a valid and accurate analysis. The benchmark that is used to compare the results from the portfolio is the Stockholm stock exchange OMX 30. This index mimics and reflects the market as a whole. The portfolio will be reweighed at a preplanned schedule, each quarter to constantly obtain an optimal risky portfolio.

The finding from this study indicates that the actively managed portfolio outperforms the passive benchmark during the selected timeframe. The outcome someway differs when evaluating the risk adjusted result and becomes less significant. The risk adjusted result does not provide any strong evidence for a greater return than index. Finally, with this finding, the authors can conclude by stating that an actively managed optimal risky portfolio with guidance of the MPT can surpass the OMX 30 within the selected timeframe.

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1 Introduction

This chapter aims to give an understanding and perspective of Modern Portfolio Theory. The background gives a historical view within the topic, which is followed by the problem discussion that will discharge into a narrowed down problem question that are of significance to fulfill the purpose of this thesis. Further, we present the delimitations, approach and as a final point the disposition of this thesis.

1.1 Background

Until the 1970's a bank savings account as a risk-free asset combined with a stock portfolio would be a great investment and the strategy that financial management advisors would recommend. The difference from now and then is the access to a broader variety of asset classes and more available information. From the information perspective it is also easier to combine these different assets into complex portfolio strategies. One has to understand that each asset must be judged on its contribution when it comes to risk and return, but the combination of a couple of stocks can provide a different risk and return for the portfolio in overall. (Bodie, Kane & Marcus, 2004).

There are several authors, for example Markowitz, (1991), Elton and Gruber, (1997) that discusses the main issues that an individual faces when investing, one issue is how to allocate the resources among alternative assets. All financial institutions have the same problem, the added difficulty and the complication needed to explicitly include the characteristics of the liabilities in the analysis. The structure of these problems is different, but we can still classify these to the portfolio theory.

There have been a lot of previous studies within the field of portfolio theory. One article written by Cowles (1933), examined the outcome from passive versus active managed portfolios. The result from this research was that the managed portfolio underperformed the passive benchmark. Cowles examined return but did not take into consideration risk, but the Modern Portfolio Theory (MPT) states that risk as well as return must be considered according to Elton and Gruber (1997). This makes the use of risk as an important factor when constructing a portfolio. Markowitz (1959) argues that risk can be minimized but not eliminated, and this without changing a portfolios' return.

Since the risk is such an important concept is has to be defined and according to Investopedia.com (2006) one interpretation is: "*The chance that an investment's actual return will be different than expected. This includes the possibility of losing some or all of the original investment. It is usually measured by calculating the standard deviation of the historical returns or average returns of a specific investment*".

MPT is the philosophical opposite of traditional asset picking. It is the creation of economists, who try to understand the market as a whole, rather than looking for what that makes each investment opportunity unique. The asset allocation problem is one of the fundamental concerns of financial theory according to Cohen and Natoli (2003). Asset allocation and risk are vital components in the MPT. Investments are described statistically, in terms of their expected long-term return rate and their expected short-term volatility. The volatility is equated with "risk", measuring how much worse than average an investment's bad years are likely to be. The goal is to identify the acceptable level of risk tolerance, and then find a portfolio with the maximum expected return for that level of risk (Elton & Gruber, 1997).

If the investor were to create the perfect investment, attributes to include would be high return coupled with no risk. The reality is as Elton and Gruber (1997) states, this kind of investment is almost impossible to find. Not amazingly, individuals spend a lot of time developing methods and theories that come close to the "perfect investment". But none is as popular, or as powerful, as the MPT.

It is important that industry professionals understand how to use that available theory to design portfolios that best align with a client's wishes and risk tolerances. It is also important that financial advisors understand what drives portfolio risk and return and how these forces can be manipulated for the maximum benefit. The MPT provides a solid theoretical foundation for building portfolios that are robust and closely aligned with an investors stated risk and return preferences.

MPT holds that diversification of assets may increase returns at given risk levels or at least provide the same results at a reduced risk level. Applications of the theory use volatility of returns implied by market price fluctuations as the composite of risks. It is most certainly the dominant theory in portfolio strategies. It is a theory on how risk-averse investors can construct portfolios in order to optimize market risk for expected returns, emphasizing that risk is an inherent part of higher reward.

The concept for investors when combining a less-risky portfolio is diversification according to Bodie et al. (2004). The adage "don't put all your eggs in the same basket" is easy to say but more difficult to actually perform in reality. The importance for diversification is of great value, and as a proof of this Harry Markowitz won the Nobel price in economics for his research within this field (Markowitz, 1991).

1.2 Problem discussion

There are thousands of different investments to choose from with different risk and return levels (Morningstar Investing Classroom, 2006). With so many different options, the investor may feel unsettled to take the right direction when picking stocks that fulfills the expectations.

There are several different areas to discover before allocating in specific stocks. How has it performed historically? How risky has it been? And what does it cost? When judging the performance of an asset it is important to make comparisons with similar assets in the same industry (Bodie et Al., 2004).

The return of an asset is commonly compared to an index. The benchmark index is the most frequent tool to estimate the fluctuation and trend on the stock market. In the long run a stock index will grow accordingly to the development of the general economy. What most investors strive for is to beat the index, generating a surplus return. Beating an index is often more difficult than one would assume. Looking only at historically high performance of an asset is not a guarantee for a future performance with a high return. The business cycle has a large impact on certain assets and less impact on others (Bodie et Al., 2004).

When investing in stocks the investor are forecasting that the time and money invested will grow and in the future become more valuable in relationship to the risk that the investor are willing to deal with. (Bodie et Al., 2004).

The Nobel price awarded economist Harry Markowitz is the initiator of MPT, a theory which provides mathematical tools to create an optimal risky portfolio that generates more return in relation to the risk. MPT advocates "magic diversification", which means that keep the rate of re-

turn on the same level but minimize the unsystematic risk. Hence, achieve a higher risk-adjusted return. The mathematical tools needed for the diversification are getting its input from statistical data from selected assets (Saunders & Cornett, 2006).

Researchers' debate on this topic has ever since been controversial, the debate about investing strategies is always under expansion and will probably forever be. This is one of the reasons that make this topic so interesting to focus on.

1.3 Research Questions

The discussion in the previous sections lies as the foundation for the following research questions:

- Does the OMX 30 index represent an optimal portfolio?
- Can Modern Portfolio Theory transform OMX 30 index into an optimal portfolio?
- Can an optimal portfolio present the investor with a higher return than the selected index?
- Can an optimal portfolio present the investor with a higher risk adjusted return?

1.4 Purpose

The purpose of this thesis is to investigate if an investor can apply modern portfolio theory in order to achieve a higher return than investing in an index portfolio.

1.5 Delimitations

The authors have neither the time[1], nor the resources to take every parameter into consideration when evaluating the performance of the portfolio, the perspective of this thesis is the performance outcome exclusively. Further, positive and negative leverage, tax–efficiency and transaction costs are disregarded in this thesis. Simply, the pure generated result from the portfolio is of interest.

No actual amount of money will be invested in the portfolio, instead the performance of the portfolio will be displayed in percentage, to be able to give the investor a clearer and more comprehensive conclusion of the findings.

The authors narrowed down the benchmark index to get a more evident result of the thesis. The market index that is used as the benchmark is OMX 30. This index gives an indication of the market movement in total. The historical data input is to some extent limited, consequently the thesis is narrowed down to a limited time frame of 10 consecutive years.

Global investing of asset allocation along with the downside of the different management styles will be discussed during the thesis, to give a detailed and reality-based description of the subject. This thesis is seen through the perspective of performance result solely, on a one dimensional level. Hence, the authors ignore the global investing viewpoint when conducting statistical analysis and evaluating the performance of the portfolio.

[1] The thesis is written during a period of 10 weeks

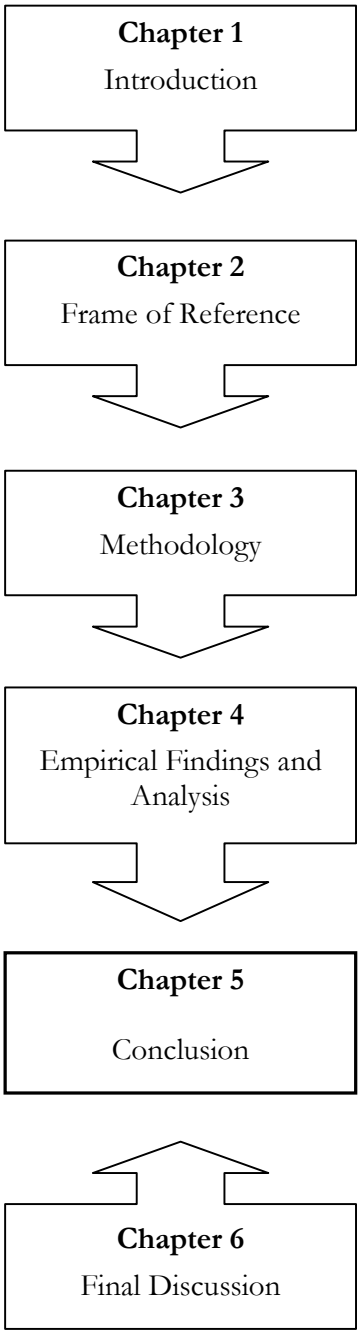
1.6 Pre-study & Approach

The authors want to create an optimal risky portfolio and compare its development with an index. The first step in this thesis was to collect knowledge about the topic. How to create an optimal risky portfolio, financial parameters and other factors with potential impact was of interest.

The achieved knowledge in the pre-study phase, created a solid base for the authors to advance with more specific studies in the subject. The pre-study was constituted by using databases in the university library, and its resources were used as a primary tool to find books, articles and supportive information.

By using specific keywords such as portfolio management, diversification, efficient frontier, Markowitz, MPT, asset allocation, risk and return, the authors have been able to find accurate information. The historical data input for the selected assets used to create the portfolio was gathered from OMX Group Stockholm, they keep records of historical stock prices. The data input from OMX Group Stockholm are then put into the mathematical tools provided by the theory and tested in order to analyze and discover major patterns. The result from the test will be the conclusion originated from the purpose statement which the thesis is based upon.

1.7 Disposition of the Thesis



The first chapter aims to give a clear picture of the background, problem, purpose and the delimitations. It states the reason why this subject is of interest.

Chapter two intends to present and describe relevant theories regarding the purpose of the thesis. The purpose of this chapter is to give the reader a clear picture of the chosen theories as well a good understanding.

The third chapter describes the relevance of the methodology, the chosen method, the approach and why the authors have chosen them. Information of the selected sample, in this case the selected index and the benchmark to make comparison.

In this chapter the findings are presented and analyzed. Graphs and figures illustrate the result of the findings.

In this chapter the authors present the conclusions from the analysis, in order to fulfill the purpose.

The last chapter of the thesis gives a discussion of the subject, what more or else that could have been done. Criticism and reflections that have emerged during the study.

2 Frame of Reference

This chapter will be the guide through the theory associated with the Modern Portfolio Theory and other theories that are connected to this topic. To illustrate the theory the authors will start by defining the fundamental concepts which lie as a foundation for the Modern Portfolio Theory, supported by other relevant theories.

2.1 Expected Return

Sharpe (2000) states that a portfolio's expected return is the weighted average of the expected return of the individual assets. Depending on the weight of an individual asset this asset will have a larger or smaller impact on the return of the portfolio. Alternative assets differ in their terms of expected return, but the expected return is only a part of the asset's future performance. What may influence the expected return is how volatile the asset is (Gibson, 2000).

There are different approaches to estimate the expected return of an asset. One approach is to estimate the probability of different return outcomes, opposed to making estimates based on historical data. To compose a portfolio, it is crucial to make estimates of the returns of assets included in the portfolio. If an accurate measurement of the return of each asset can be made, the return of the whole portfolio can be predicted with the same accuracy. Unfortunately it is not possible to state the rate of return of an asset with certainty. The objective is to make a prediction about each asset in order to produce predictions about the whole portfolio. Without estimations of the individual assets, it is impossible to make a prediction of the portfolio (Sharpe, 2000). The following equation shows the expected return of the portfolio, while the equation 2-2 shows the actual return of the portfolio.

$$E(r_p) = \sum_{i=1}^{N} X_i E_i$$

Equation 2-1 Expected Return of the Portfolio

Where:

$E(r_p) =$ expected return of the portfolio

$X_i =$ proportion of security i

$E_i =$ expected return of asset i

The actual return for portfolio is expressed in the following equation:

$$R_p = \sum_{i=1}^{N} X_i R_i$$

Equation 2-2 Return of the Portfolio

Where:

R_p = actual return of the portfolio

X_i = proportion of security i

R_i = actual return of security i

2.2 Standard Deviation

The standard deviation is the measurement of uncertainty associated with the asset, a measure of the dispersion of a set of data from its mean. The more spread the data is, the higher is the deviation (Sharpe, 2000).

As with the expected return, it is a measurement needed to estimate the standard deviation of the portfolio. The standard deviation is considered to be the risk measurement of an investment. It can be provided by logic estimation or by setting up a probability distribution.

The standard deviation of the portfolio's rate of return depends on the standard deviations of return for its component securities, their correlation coefficients and the proportions invested. It is calculated with the following equation:

$$\sigma_p = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j \rho_{ij} \sigma_i \sigma_j}$$

Equation 2-3 Standard Deviation of the Portfolio

Where:

$X_i\ X_j$ = proportions invested in each asset

ρ_{ij} = correlation coefficients between i and j

$\sigma_i \sigma_j$ = standard deviation of each asset

2.3 Portfolio Risk

The risk is as stated in the background chapter "*The chance that an investment's actual return will be different than expected. This includes the possibility of losing some or all of the original investment. It is usually measured by calculating the standard deviation of the historical returns or average returns of a specific investment*" (Investopedia.com, 2006).

The portfolio risk can be interpreted with the following equation:

$$\sigma^2 = \sum_N P(s)[r_N - E(r)]^2$$

Equation 2-4 Risk of the Portfolio

Where:

$E(r) =$ the expected return

$P(s) =$ the probability that the rate r occurs

$r =$ the return level

2.4 Diversification

Diversifying in several securities decreases the exposure to firm-specific factors, this leads to portfolio volatility continues to decrease. But even with a large number of assets, it is not possible to avoid all risk. All portfolios are affected by the macroeconomic factors that influence the market (Bodie et Al., 2004).

When allocating the assets it is important to understand how the uncertainties of the different assets interact. The key determinant of the risk from the portfolio is the extent to which the returns on the different tend to vary either together or in the opposite direction. Risk depends on the correlation between returns on the different securities in the portfolio. The performance of assets within a given portfolio tend to follow the market, if there is a recession or a growth the asset will move in a certain direction. If one asset goes up in a growth period, the chance that a similar asset will go up is almost certain. This is what correlation states and this is also why there have to be a variety of assets within the portfolio in order to provide a portfolio that can handle recessions and growth (Sharpe, 2000).

The problem is to measure the tendency of the returns from the different assets, if they move together or in the opposite direction. The measurements to solve these problems are the covariance and the correlation coefficient. The covariance[2] is calculated similar to the variance, but instead of measuring the difference of an asset from its expected value, it is measured to the extent of the returns from the different assets reinforce or offset each other.

[2] Further explanation of covariance is given in part 2.4

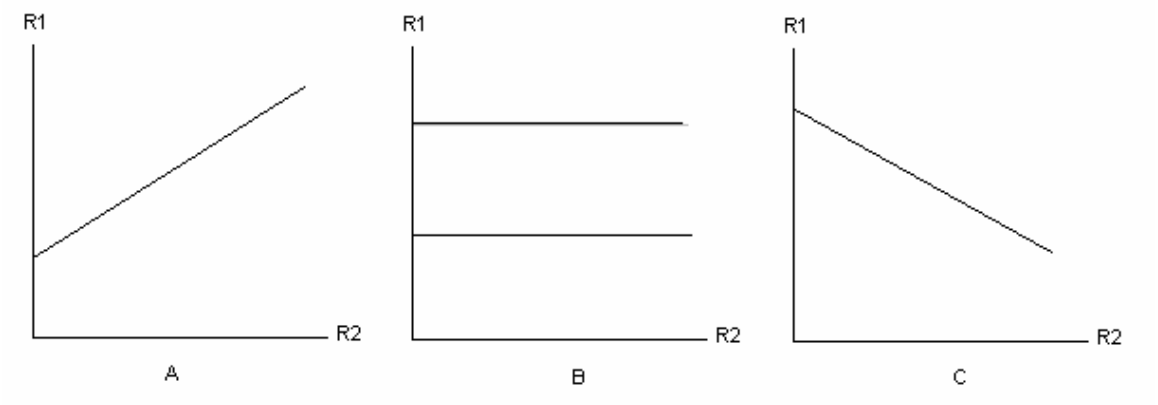


Figure 2-1 Correlation

Figure 2-1 illustrates the three extremes of correlation coefficients between two risky assets. The relationship in figure 2-1 (A) represents a case of perfect positive correlation, where the correlation coefficient is +1. This means that the returns of the two assets follow each other perfectly. If the return of one asset increases, the return of the other asset will increase with the same amount. Comprising a portfolio of two perfectly positively correlated assets will not have an effect of diversification (Sharpe, 2000).

The straight opposite of the positive correlation is figure 2-1 (C), representing a perfect negative correlation of -1. When the return of one asset increases, the return of the second asset will decrease with the same amount (Sharpe, 2000).

Effective diversification is finding risky assets with as low correlation as possible, as illustrated in figure 2-1 (B). Zero correlation results in that the assets interact independently of each other. One asset's movement does not affect another asset's direction, consequently efficient diversification is accomplished. A compounded portfolio with an overall low correlation is crucial for investors' that aims to diversify in order to eliminate the unsystematic risk (Sharpe, 2000).

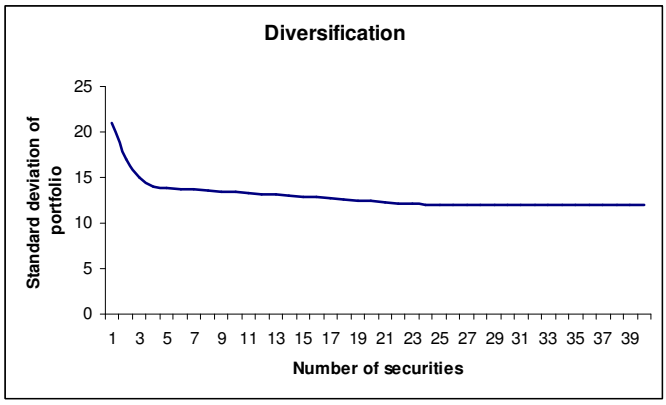


Figure 2-2 Diversification

As figure 2-2 shows, diversification is highly beneficial. A portfolio consisting of only five securities will have a portfolio risk only 14 percent higher than the most highly diversified

portfolio possible. When the portfolio consists of 10 assets the risk is only 7 percent higher than the most highly diversified portfolio. As we can see, the risk falls significantly with only a small degree of diversification. The more assets added to the portfolio, decrease the marginal benefit of the diversification. The largest advantage of the diversification is gained with the first five assets added to the portfolio. (Sharpe, 2000)

2.5 Covariance

What Markowitz did was to find a way to determine how risky the entire portfolio is (Markowitz, 1959). This might be his greatest contribution. He called it covariance, based on the already established formula for the variance of the weighted sum.

Covariance measures the direction of a group of stocks. Two stocks exhibit high covariance when their prices, for whatever reason, tend to move together. In opposition, low covariance describes two stocks that move in opposite directions. According to Markowitz (1959), the risk of a portfolio is not the variance of the individual stocks but the covariance of whole portfolio. The more they move in the same direction, the greater is the chance that economic shifts will drive them all down at the same time. By the same token, a portfolio composed of risky stocks might actually be a conservative selection if the individual stock prices move differently.

Hagstrom, (2001) stated that the appropriate action for an investor is to first identify the level of risk that can be comfortably handled, and then construct an efficient diversified portfolio of low-covariance stocks. The smaller the covariance between the two securities is, the more out of sync they are and the smaller is the volatility of a portfolio that combines them. The ultimate would be to find securities with as low covariance as possible. The covariance between securities rate of return is the weighted average of the product.

Covariance equals the product of the correlation coefficient and the standard deviations of the securities' rates of return. The equation is the following:

$$C_{jk} = \rho_{jk}\sigma_j\sigma_k$$

Equation 2-5 Covariance

Where:

ρ_{ij} = correlation coefficients

$\sigma_i\sigma_j$ = Standard deviation of each asset

2.6 Systematic and Unsystematic Risk

There are according to Ahmed (1998) two types of risk and these are systematic- or market risk and unsystematic- or diversifiable risk. The unsystematic risk between firms occurs because of differences in the corporate financial decision (Ahmed, 1998). Examples of an unsystematic risk may be R&D failures, unsuccessful marketing or losing major contracts, these all are unique events affecting the firm.

"Diversification provides substantial risk reduction if the components of a portfolio are uncorrelated. In fact, if enough are included, the overall risk of the portfolio will be almost (but not quite) zero!" – (Sharpe, 1985 in De Ridder, 1986 p. 225)

As indicated above, there is still a small portion of risk that cannot be eliminated, the systematic risk. Bodie et Al. (2004) writes about the uncertainties that creates this risk and states that it comes from conditions in the general economy, such as business cycle, inflation, interest rates, and exchange rates. This kind of risk can not be predicted with certainty and will affect the return from an asset.

Consider a portfolio with only one asset, if there is a change in the macroeconomic environment, the asset will go up or down. When all risk is placed in one asset, the risk is exclusively on that asset. With a diversification strategy, the unsystematic risk will decrease since the risk is spread over several asset classes.

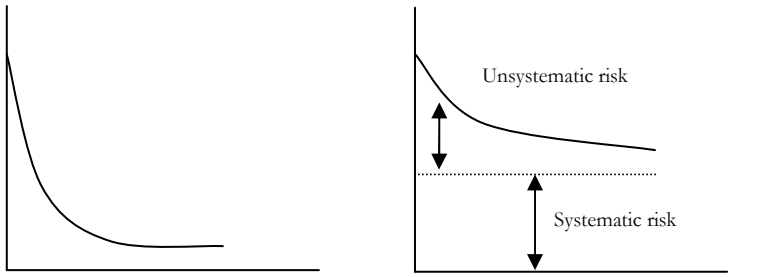


Figure 2-3 Systematic and Unsystematic Risk

The figure above illustrates this concept that the total risk is the combined risk of the unsystematic and the systematic risk. The unsystematic risk is the risk that can be lowered and minimized with a diversification strategy, but it can not lower the systematic risk even with a highly diversified portfolio (Bodie et Al., 2004). Risk tied to an individual asset is measured in its variance and standard deviation from the mean return. The variance is comprised of both systematic and unsystematic risk, hence the whole risk of an asset cannot be diversified away, only the risk associated with firm specifics.

2.7 Relation between Risk and Return

Risk is the force that ultimately drives the return of an investment up or down. An investor that takes on an investment is putting himself in a risky situation. Since risk drives the return it is naturally not something that should be avoided. However it has to be managed, and the long term potential of an investment depend on the investor being willing and prepared for risk. The risk has to be budgeted, and one has to decide what potential loss that can be afforded (Litterman, 2003).

Successful investments are those assets with the best risk reward relationship. An investor demand reward for making a risky investment. The concept of portfolio theory is to maximize the relationship between the risk and the reward (Bodie et Al., 2004).

2.8 Modern Portfolio Theory

The Modern Portfolio Theory (MPT) was developed by Harry Markowitz. He assumed that most investors want to be cautious when investing and that they want to take the smallest possible risk in order to obtain the highest possible return, optimizing return to the risk ratio. MPT states that it is not enough just to look at the expected risk and return of one particular stock. By investing in more than one stock, an investor can obtain the benefits of diversification, a reduction in the volatility of the whole portfolio (Markowitz, 1959).

O'Neill (2000) states that there are two aspects behind the theory and those are that history may repeat itself, which means that the use of historical data of securities is important and useful. The second aspect is that not all assets go up and down in tandem. These aspects are useful for a variety of investors because they may help the investor when making decisions.

According to O'Neill (2000) MPT has important practical applications such as, it reduce volatility in a portfolio of individual stocks. Until the time when MPT was invented by Markowitz, investors gave very little thoughts about managing a portfolio or to the concept of risk. Portfolios were constructed randomly. If an investor thought a stock was going up in price, it was added to the portfolio. No other thinking was required or done (Hagstrom, 2001).

Markowitz (1959) developed a mathematical procedure that would produce the set of theoretical best portfolios. Assume that the investor could line up a table of all the portfolios that have the same level of risk. While the risk of the different portfolios is the same, but with a different return, the choice of the best portfolio is simple, the one with the highest return. And vice versa, one would choose the one with the lowest risk. The theoretical best portfolio will have the least risk for a given expected return level and the highest expected return for a given risk level.

The essence of MPT is to seek optimization of the relationship between risk and return by composing portfolios of assets determined by their returns, risks, and covariance or correlations with other assets. MPT develops a framework where, any expected return is composed of various future outcomes and are thereby risky, and this relationship between risk and return can be optimized through diversification.

Each portfolio that satisfies these two conditions is called an efficient portfolio. No other portfolio will have a higher return at the same risk level (Markowitz, 1959). A portfolio is inefficient if it is possible to achieve higher expected return with no greater risk, or to reduce risk with the same level of expected return (Markowitz, 1991).

Elton and Gruber (1997) states that there are two principles from MPT that led to the formulation of the efficient frontier[3], which is, holding constant variance and maximizing expected return and holding constant expected return will minimize variance.

2.8.1 Efficient Frontier

The efficient frontier solves the question of how to identify the best level of diversification. The concept of an efficient frontier can be applied in a number of ways. In essence, an efficient frontier is a curve on a graph representing the relationship between return and risk for a set of portfolios. For a portfolio to be on the efficient frontier, the portfolio must maximize return for a given level of risk (Manganelli, 2002).

It is simple concepts that risk and return are linked together and that there is a relationship between them, and thus there could be a way to determine the degree of risk that would be required for various levels of return. According to Hagstrom (2001) it is hard to generate high returns without exposing yourself to some kind of risk.

Markowitz (1959) devised what he called the efficient frontier, a trade-off graph with the expected return on one axis and risk on the other axis. It is a curve representing all portfolios that maximize the expected return for a given level of risk. The efficient frontier is simply a line drawn from the bottom left to the top right where each point on that line represents an intersection between potential reward and its corresponding level of risk. The most efficient portfolio is the one that gives the highest return for a given level of portfolio risk.

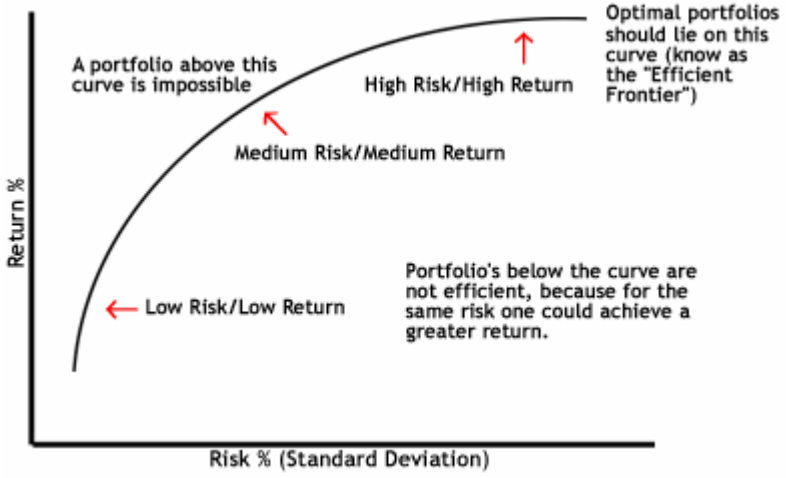


Figure 2-4 The Efficient Frontier

As illustrated in the figure above there are no portfolios above the efficient frontier, and all portfolios that lie below are inferior to those on the frontier. Each point located on the frontier represents a different efficient portfolio. When moving from the bottom left to higher right, the risk as well as the return increase.

[3] Further explanation of the efficient frontier is given in part 2.8.1

In order to achieve a tangent portfolio on the efficient frontier, each asset of the total portfolio has to be weighted in a certain manner. Simply using one single asset will not provide contact with the efficient frontier, nor will a portfolio with equally distributed fractions of each asset. The weighting process is of significance to accomplish a tangent portfolio on the efficient frontier. Historical data is analyzed and implemented and the generated outcome is examined by mathematical paraphernalia. De Bondt and Thaler (1985), states that each asset has a specific fraction of the overall portfolio. That is, a tangent portfolio that matches the efficient frontier.

For every level of return, there is one portfolio that offers the lowest possible risk, and for every level of risk, there is a portfolio that offers the highest return. Any portfolio that is on the upper part of the curve is efficient, that is, it gives the maximum expected return for a given level of risk. It is clear that for any given value of the volatility, the investor strives for a portfolio that gives the highest possible rate of return, i.e. a portfolio located along the efficient frontier. According to Manganelli (2002) a portfolio is on the efficient frontier when the portfolio maximizes return for a given level of risk.

Since distributed returns are not fixed over time the weights in the portfolio have to be reallocated. Macroeconomic factors affect the market and a specific industry sector fluctuates over time (De Bondt & Thaler, 1985).

An inefficient portfolio exposes the investor to a level of risk without a corresponding level of return. The goal for investors' is according to Markowitz (1991), to match portfolios to a level of risk tolerance while limiting or avoiding inefficient portfolios (Hagstrom, 2001).

2.8.2 Capital Allocation Line and Sharpe-ratio

The capital allocation line (CAL), shows the risk-return combination available by varying asset allocation, by choosing a different point on the CAL (see graph below). It plots the risk/return ratio by varying the allocation between risk-free assets to the risky assets. Bodie et Al. (2004) states that the slope of the line is equal the increase in expected return that an investor can obtain per unit of additional risk, that is, extra return per unit of risk.

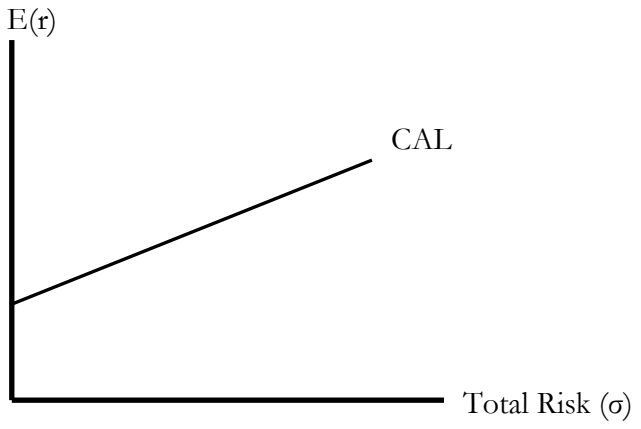


Figure 2-5 Capital Allocation Line

The CAL-slope can also be interpreted as the Sharpe-ratio which is used to compare the expected returns of an investment to the amount of undertaken risk. The Sharpe-ratio is according to Gregoriou (2004) more appropriate when analyzing an entire portfolio then for example a single security. It tells us whether the returns from a portfolio come from good investments or as a result of excess risk. The Sharpe-ratio is used to compare it to a benchmark or another index (Gregoriou, 2004). The higher the ratio is, the better its risk adjusted performance has been. It is calculated with the following equation:

$$S = \frac{E(r) - r_f}{\sigma}$$

Equation 2-6 Sharpe Ratio

Where:

$S =$ Sharpe ratio

$E(r) =$ Expected Return of the Portfolio

$r_f =$ Risk-free Rate

$\sigma =$ Volatility of the Portfolio

2.8.3 Optimal Portfolio

The optimization principle should follow when the investor know the relation between risk and return. As illustrated graphically bellow, the indifference curves represent the risk/return combination over which the investor is indifferent. Investors are better off with investments that are on the indifference curves that are situated further up to the right (Markowitz, 1991).

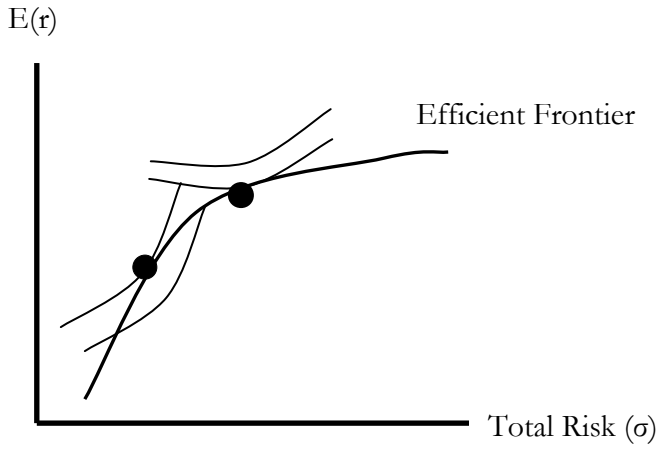


Figure 2-6 Optimal Portfolio

The figure above illustrate two conclusions regarding portfolio theory, the optimal portfolio for an investor are the ones on the higher indifference curve. The more risk averse an investor is, the lower will the optimal portfolio be on the efficient frontier. According to Goodall (2002), all investors will choose a portfolio that is the optimal risky portfolio. Which portfolio that is chosen as the optimal, depends on the investors decision rule[4]. However, Markowitz (1959) disregards the need to have all results evaluated by the individual investor with these indifference curves. The only portfolio that is optimal according to Markowitz (1991) is the one that is tangent between the CAL and the Efficient Frontier.

2.9 Asset Allocation

A variation of different assets will provide the investor with a variability of return in the investor's portfolio and reduce the risk. In order to achieve portfolio optimization the investor has to allocate the portfolio in different asset classes. The most beneficial way to allocate the assets is to let a global touch permeate the portfolio. Look at different national markets in order to find independency, hence reduce the risk (Litterman, 2003).

In order to attain optimization at a national level it is of great importance to look at different type of sectors and industries, this is supported by Markowitz (1959). The investor treats the national market as a global market with all its different industrial sectors where each sector symbolizes a national market. The diverse industrial sectors are to some extent uncorrelated and will provide a positive excess return and consequently should be added to the portfolio.

Independency and diversification is of significance to attain optimization, and allocating assets in different type of industries is what the investor practically does to achieve optimization. Search for the window of opportunity with mathematical tools, the statistical result gives the investor an indication of the suitability of the opportunity in the perspective of the investor's aversion to risk. When allocating the portfolio from a global point of view, it is important to be aware of that the transaction costs probably will rise to a great extent (Litterman, 2003).

2.10 Passive and Active Management

There are two distinguished types of styles to use when managing a portfolio, active and passive management. The two management styles are each others opposites, and the combination of the styles is almost infinite. An active management style advocates aggressiveness, buy and sell securities in order to beat the market, which typically is symbolized by some type of index (Litterman, 2003).

Grinold (1989) states in his research that "*all investors that have mean variance objectives will prefer to have the portfolio with the highest information ratio possible*". One thing that differs between the investors is their aversion to risk. An investor with high risk aversion will have a diplomatic outlook towards the information and will have lower value added and vice versa for the investor with low risk aversion.

[4] i.e. the investors taste for risk

Information ratio is the most important parameter for each investor that uses an active management style, since active managers search incorrectly priced securities, the available information is fundamental. There are some consequences that characterize active management. They sustain more search/information costs, portfolio turnover is higher – greater transactions cost, frequent trading leads to low tax efficiency – short term capital gains are fully taxed and greater risk leads to higher expected returns (Bodie et Al., 2004).

The passive management style implies that the portfolio is build up so it mimics a market index. This is a less risky alternative where you let go off the opportunity to beat the market, but on the other hand make sure that the portfolio does not under perform the market movement. The consequences that distinguish passive management are, low portfolio turnover – turnover only take place when the composition of the index changes, Low search/information costs, Low transactions cost and high tax efficiency – long term capital gains are taxed at lower rates (Bodie et Al., 2004).

2.11 Strategic Asset Management vs. Tactical Asset Management

According to Litterman (2003) the time horizon is a fundamental factor in strategic and tactical asset management. This theory consider how long period of time the investor will hold its portfolio before liquidating the investment. The strategic asset management style focus on long-term holdings of each investment, opposite tactical asset management focus at short-term investment objectives. Similar to active and passive management each investor's approach is individual and the selected management style will reflect the investor's criteria for investing.

Campbell and Viceira (2002) states, if holding a long term portfolio the investor constantly needs to readjust the assets, in order to keep up a consequent investment policy, since the market conditions fluctuates over time.

3 Methodology

This chapter will describe how the authors have approached the problem and gathered information to solve the purpose. Issues that are explained in this chapter are the different methods and approaches used in this thesis. The indexes used in our analysis are defined and how we have assembled the portfolio.

When writing a thesis, the chosen methodology helps the authors investigate and write a thesis that fits the specific needs and wants, and will provide the best to answer the specific questions. The choices of methods have to be done to reach the best possible conclusions. Which types of methods and approaches that are used are determined by the set purpose: "*The purpose of this thesis is to investigate if an investor can apply modern portfolio theory in order to achieve a higher return than investing in an index portfolio*".

Some types of choices that have to be made are whether to conduct a qualitative- or quantitative method, inductive- or deductive approach, whether to use primary or secondary data in your thesis.

3.1 Quantitative vs. Qualitative

In order to obtain the information needed in a research like this, there are two methods that can be used, a *qualitative*- and a *quantitative* method (Svenning, 2003). They have the same purpose, which is to create a better understanding of a phenomenon and how it all affects us. Which method to use depends on if there is a need of a total perspective, quantitative or a deeper understanding, the qualitative. Different fields of studies require different methods. Method chosen should be based on the theory used, the set purpose and what the authors wants to accomplish.

The quantitative method is based on the transformation on information into numbers in order to make an assumption and to get a conclusion (Holme & Solvang, 1997). With this method the researcher gathers information in the form of data from example databases, a large sample and with a statistical method concludes the problem. According to Holme and Solvang (1997) this method goes more wide than deep. This method is very structured and gives a straight forward result. The analysis can be used as a measure of the whole population, if it is done in a proper manner.

The qualitative method is more based on different patterns and to give a deeper understanding about the subject and the set problem. The patterns are analyzed and according to Holme and Solvang (1997) the goal is to find unique details. The information in this method is carefully picked, mainly from interviews and observations. It is more unstructured and unsystematic (Holme & Solvang, 1997). The disadvantage of the qualitative method is that the information gathered is often biased by the authors own opinions.

The focus of this thesis is on the valuation of the MPT, in the field of finance. In a research like this, mathematical and statistical models are used to explain relationships. The most valid method for us is to use the quantitative method, this because of the fact that we will almost exclusively use measurements and interpretations of numerical data in form of index prices. The use of historical data will be used to predict a possible future outcome. The data-set is interpreted in a way to give the best possible explanation. With this dataset we will analyze our set problem and the purpose with this thesis.

3.2 Deductive vs. Inductive Approach

To further investigate a problem or a situation, one has to decide which approach to use when attacking the problem. When doing the research it is very important to establish what is true or false when it comes to the theory. There are two ways or approaches that can be used to draw conclusions, what is true or false, these are deductive- and inductive approach.

The *deductive* approach is according to Eriksson and Wiedersheim-Paul (1999) based upon existing theory by using, adapting or developing that theory, and come up with a conclusion. The *inductive* approach aims to point out the empirical findings and to build up a new knowledge that will contribute to some new theories.

The research approach we will use is deductive, which means that we will use existing theory and common principles as a starting point, and then base this thesis from the foundations from the theory. This way will give a logical conclusion if it is logically connected (Eriksson & Wiedersheim-Paul, 1999). For this thesis, the information and the dataset that will be collected is determined by the theory, also how it should be interpreted, in order to produce an accurate analysis. While using existing theories, the author has greater chance to stay objective. However, to be finite to existing theories also hinders him from reaching new aspects to the problem (Patel & Davidsson, 1994).

The deductive approach aims to explain reality but also to predict the future. The deductive approach is often supplemented by empirical verification in form of data of stock prices or this case index prices. This is exactly what the purpose of this thesis is, to use historical data and with this try to predict or at least state what the result could be. Give a picture of what investors with the use of the MPT may accomplish.

3.3 Predicting the Future with Historical Data

To compose an optimal portfolio of risky assets it is crucial to make estimates of expected return and standard deviation of each asset that can be included in the portfolio. Theoretically the values are calculated by setting up a probability distribution of possible returns, and from that derive the standard deviation. (Sharpe, 2000)

The more practical method to get the estimates is to assume that the future will be like the past. The historical values of average return, variability and correlation can then be converted and used as the predictions for the future performance of a stock. In order to use this method for estimation the underlying distribution of return has to be stable over time. It is therefore more applicable to large stable stocks rather than small developing companies. (Sharpe, 2000)

There are some objections that can be made against this method. The historical data should reach a relatively long period back in time to give an accurate measurement. The risk associated with the companies current business activities may not be comparable to the past activities, and therefore the statistics are misguiding. (Sharpe, 2000)

3.4 The Research Approach

The purpose of this thesis is to find out if there is a possible gain when investing in the pattern of MPT. The authors have constructed a portfolio consisting of 10 indexes that are

compounded in a specific manner according to the theory[5]. The research is done with a data-set consisting of historical data from 10 indexes performance during the time period 1996-2006. The authors have analyzed the secondary data and then constructed a portfolio of these 10 different assets. The historical data of the different indexes is provided by OMX Group.

The theory also states that there should be a risk-free asset, used to get the CAL. The risk-free asset is a 3-month treasury note from Riksbanken (2006).

The benchmark the authors will compare the portfolio with is the Stockholm OMX 30. This index reflects the current status and tangible change in the market. This index consists of the 30 most traded companies and gives an estimation of the whole market, why this is a valid index to make comparisons with.

3.4.1 Indexes

The sub-indexes are the following:

SX 10 Energy

SX 15 Materials

SX 20 Industrials

SX 25 Consumer Discretionary

SX 30 Consumer Staples

SX 35 Health Care

SX 40 Finance

SX 45 Information Technology

SX 50 Telecommunication Services

SX 55 Utilities

[5] How the portfolio is assembled and constructed is explained in part 3.4.2

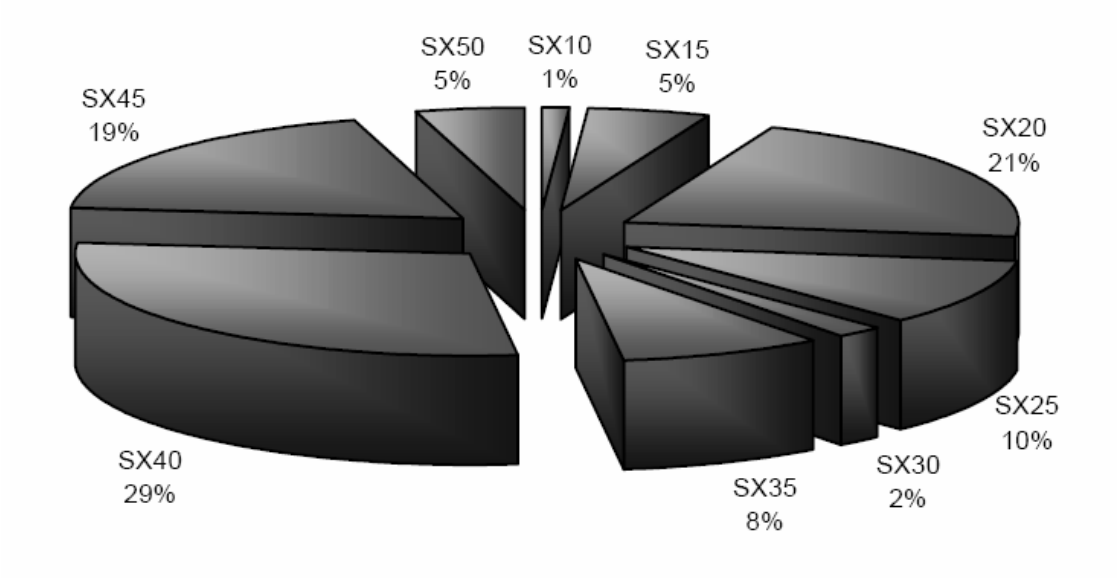


Figure 3-1 Allocation of the OMX Stockholm Benchmark

These indexes are part of the OMX Stockholm Benchmark (OMXSB). The OMXSB is constructed to reflect the different stocks on the Stockholm Stock Exchange. It consists of the 80-100 largest and most traded companies. The figure above illustrates the weights of the different indexes in the OMXSB[6]. The different indexes represent the major different industry sectors. These indexes were developed in order to meet the investors' more exact need for exhaustive information. The weight for each sector index is based upon the market value of the stocks that is compounded in each index adjusted for free float. The indexes are based upon GISC, Global Industry Classification Standard, produced by Morgan & Stanley Capital International Inc. (MSCI) and Standard & Poor's (S&P). The Global Industry Classification Standard (GICS) were developed to facilitate international comparisons under a worldwide standard, (OMX Stockholm Benchmark, 2006).

The indexes weights are based on the EU:s UCITS standards[7]. The authors have chosen these indexes because they are solid and easy to measure. The fact that these indexes are a huge part of Stockholm OMX makes the relevance and the analysis from the research accurate. Each asset is represented by a particular sector index and the different sectors embody a wide range of industries.

3.4.2 The Portfolio

The assets included in the portfolio are, as mentioned earlier, ten sub indexes to the OMXSB. To construct the optimal risky portfolio, expected return and standard deviation is estimated for each asset. The first set of estimations is based on the assets historical returns[8] for a five year period. Sharpe (2000), states that the accuracy of estimation, increases

[6] The figure is from 2005-09-01

[7] UCITS states that a fund or index may only invest in maximum of five percent in a specific stock

[8] See Appendix 1

with the extended time series. The authors have chosen to recalculate the statistics every quarter to achieve the updated values for the assets. Consequently the portfolio is weighted upon each statistical recalculation.

These 10 assets are weighted in the most efficient way, supported by MPT in order to achieve a tangent portfolio on the efficient frontier. Simply using one single asset will not provide contact with the efficient frontier, nor will a portfolio with equally distributed fractions of each asset. The weighting process is of significance to accomplish a tangent portfolio on the efficient frontier. The result is evaluated and the portfolio is corrected according to a weighting scheme (De Bondt & Thaler, 1985).

Sharpe (2000) defines the portfolio risk as the covariance of the assets included. The asset allocation in the portfolio strives to fulfill the most desirable tradeoff between risk and return, i.e. constructing a portfolio with the highest Sharpe ratio. The covariance is kept to a minimum for a given return. Since distributed returns are not fixed over time the weights in the portfolio has to be reallocated. Macroeconomic factors affect the market and a specific industry sector fluctuates over time. For that reason the portfolio is actively managed and each assets fraction is adjusted over time in order to continuously keep up the tangency with the efficient frontier.

3.4.3 Timeframe

The estimation period for each quarter is held constant to the preceding five years of return. For each quarter new estimates of the individual assets are made using the preceding five years of historical return. A new optimal risky portfolio is constructed every quarter during the five year period of investigation.

3.4.4 Evaluating the Portfolio

When the indexes are assembled in the portfolio, there will be performance tests according to what is declared in the portfolio theorem. The evaluation is done as the theory states. The yield from the portfolio will then be compared with an index (Stockholm OMX 30), this to see if the portfolio strategy used will surpass the chosen index.

When evaluating a portfolio's performance, one difficulty may be that the average return from the portfolio must be adjusted for risk. Without this risk adjustment it is hard to get a valid result. Performance measure must according to Bodie et Al. (2004) consist of both risk and return.

The simplest, easiest and most valid comparison is to compare the results with an index or a benchmark, since this index has the same characteristics as the one in this research. With this method it is easy to compare return and risk.

Risk adjustment models are used to get a valid result even if the risk is different. The Sharpe-ratio measure divides average portfolio excess return over a sample period by the standard deviation of returns over that period. The numerator is the incremental return the portfolio earned in comparison with an alternative investment in a risk-free asset, and the denominator is the increment in portfolio volatility compared with the risk-free alternative. Hence, the ratio measures the reward to volatility trade-off.

Rather than focus on risk-adjusted returns, investors often want to ascertain which decisions that results in superior or inferior performance. Superior investment performance depends on the ability to be in the right assets with the right weight or it may be defined as which industry that went well. The difference between a managed portfolio and a benchmark in terms of performance may be expressed as followed, asset allocation (weights within the portfolio), choice of market and the choice of risk-free asset (Bodie et Al., 2004).

The other different measurements that are used in the evaluation process are:

The standard deviation of the portfolio's rate of return is executed in order to outline the portfolio's volatility, and from that insight, aim to hold a portfolio with as low deviation as possible with the highest return possible.

The result from the portfolio is later on presented in the empirical findings and analysis chapter[9].

3.5 Critique of Chosen Method

In every thesis there are always some critiques from the chosen method, or the conclusions drawn from the research. There are two problems within the field of empirical research, the reliability and the validity. To give legality to the thesis there are some important measures to strengthen the trustworthiness of this thesis.

3.5.1 Validity

Validity relates to the thesis's ability to examine what is intended to be researched, find a connection between theory and empirical findings (Eriksson & Wiedersheim-Paul, 1999). Svenning (2003) states that the data collected must be precise and accurate with the set purpose, this to make the right interpretations and to give a valid analysis. According to Eriksson & Wiedersheim-Paul (1999) validity means how good a thesis's results could be applied to other groups, situations, contexts, theories and methods.

We use historical data of the chosen indexes from 1996-2006, this to get a long-run perspective but also most importantly to gain a valid research from it, a validity in our thesis. What else that has to be done to get a valid research is to get an extensive knowledge within the field, this through research.

3.5.2 Reliability

Reliability is a measure of how trustworthy the authors' conclusions and their interpretations are (Eriksson & Wiedersheim-Paul, 1999). This means that a similar research with the same data and the same method should give the same conclusions. This is more important with a quantitative method, since it tries to generalize. According to authors Saunders, Lewis and Thornhill (2003), there are three questions that can measure or estimate reliability in a research.

- Will the estimates give the same results on a different occasion?

[9] The result is given in chapter 4

- Will comparable observations be reached by other observers?

- Is it easy for another observer to understand how sense was made from the raw data?

To get a high reliability in this thesis, the authors have followed the method and used the theory in a proper way. The conclusion from other researches within the same field as ours will give the same result, given that the use of the same type or indexes and the same data will be used, from the same time-period. This means that the reliability from this thesis is high and accurate.

4 Empirical Findings and Analysis

The statistical study provides the fundamentals to fulfil the purpose: The purpose of this thesis is to investigate if an investor can apply modern portfolio theory in order to achieve a higher return than investing in an index portfolio. In this chapter the authors present and discuss the empirical findings obtained from the statistical analysis, the result is further analyzed and explained in terms of theoretical parameters and models. Finally, the result of the optimal risky portfolio's performance will be compared with the benchmark index, OMX30.

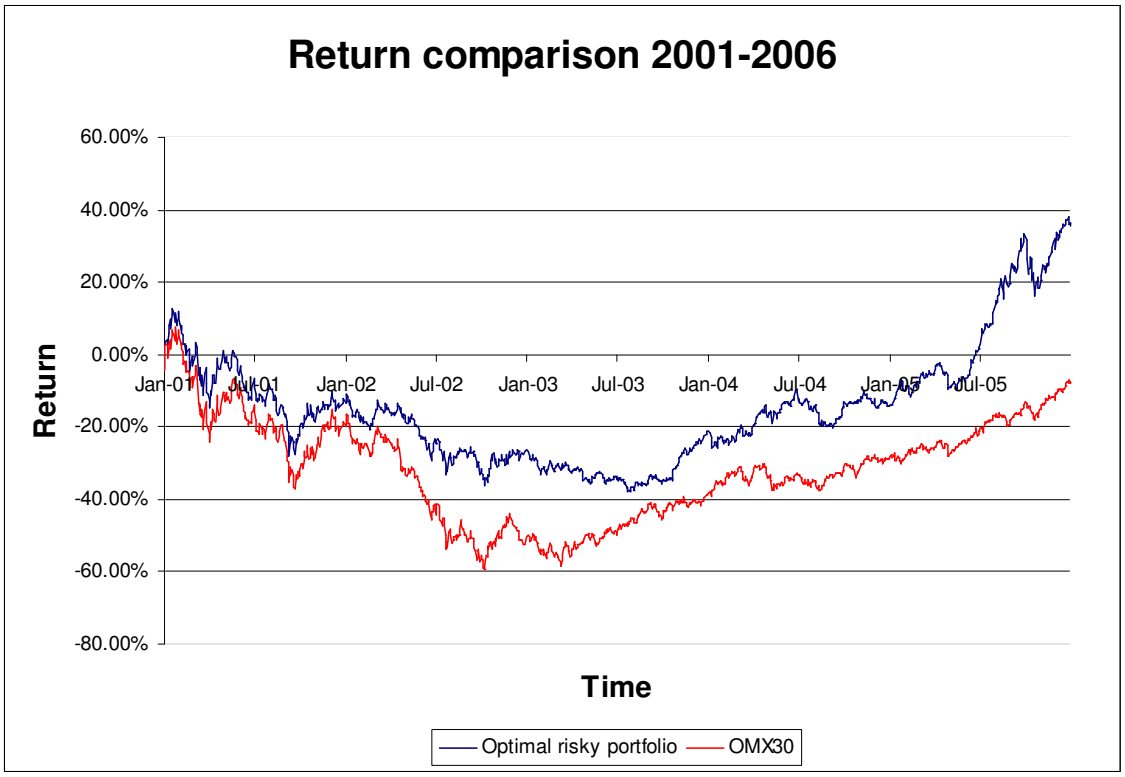


Figure 4-1 Return Comparison 2001-2006

This figure shows the performance of the two portfolios, the blue graph illustrates the development of the optimal risky portfolio created by the authors, and the red graph illustrates the performance of the benchmark index, OMX 30. The timeframe of the table is 5 years, 2001-2006.

4.1 Constructing the Portfolio

All the statistics are expressed in terms of daily return and standard deviation.

4.1.1 Historic Estimates

The statistical estimates of mean return and standard deviation based on historical return are presented in appendix 1.

4.1.2 Statistical Estimation for Each Asset

In figure 4-2 the assets are plotted according to their individual relationship between risk and return. The theory states that risk and return are linked together (Hagstrom 2001), and therefore the degree of risk is measured. Hagstrom (2001) also states that it is hard to generate high return without being exposed to the same level of risk. Seen in the graph below where this is shown graphically, the indexes that generates high return also show high risk in most cases.

The estimation is based on the preceding five year period of mean return and standard deviation. By investing solely in one of the ten assets, it is not possible to achieve a return more desirable than the asset itself. An investor would prefer a risk/return relationship situated as far to the upper left of the graph as possible, yielding a high return associated with low risk, however this is not possible due to the fact that high return almost always comes with high risk. By constructing a portfolio of the assets, the authors strive to reach a return more desirable than any of the assets themselves, a combined portfolio with a minimized risk and a more desirable return.

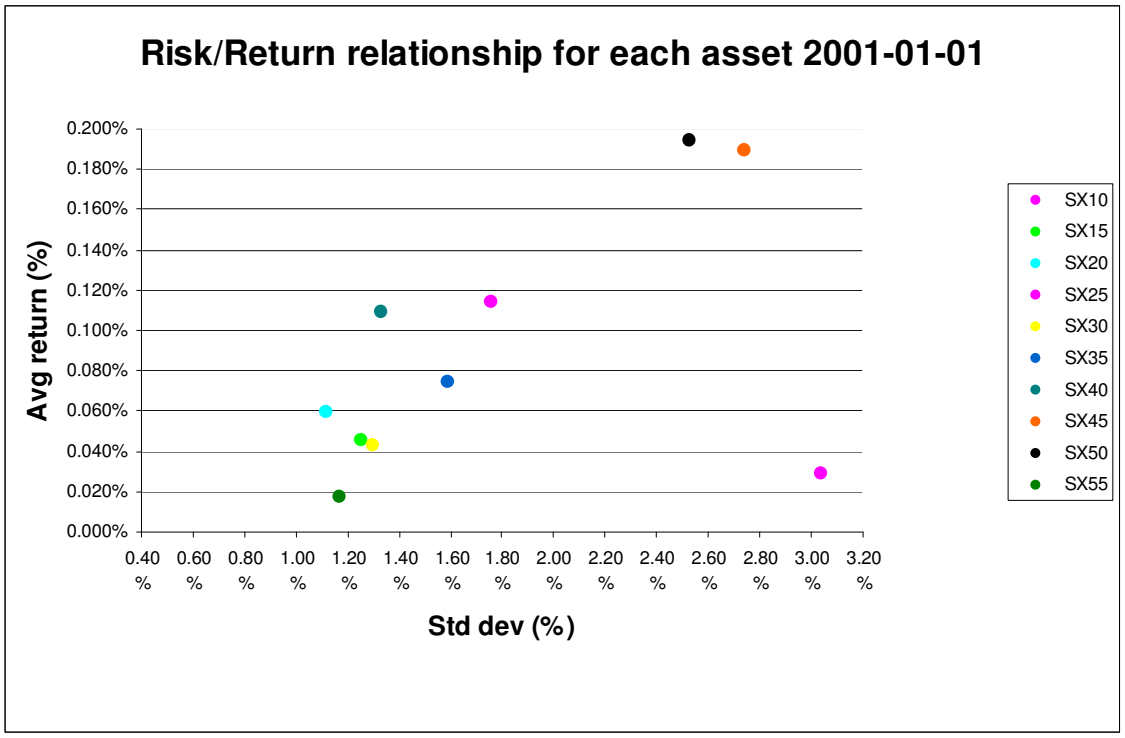


Figure 4-2 Risk/Return for Each Asset

The risk/return relationship of the individual assets plotted in figure 4-2 supports the theory of a positive correlation between risk and return. A higher return is associated with an elevated standard deviation i.e. risk. The outlier SX10 has the highest risk for a relatively low return. When composing an optimal portfolio, the SX10 asset will have be of no beneficial use.

4.1.3 The Efficient Frontier

The efficient frontier is constructed by holding a constant return while minimizing the standard deviation. In table 4-1 each return and its corresponding standard deviation is listed. The weight for achieving the maximum Sharpe ratio is given for each scenario. The covariance and bordered-multiplied covariance matrix are presented in appendix 3.

Mean return	Std dev	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
0.02%	1.06%	6%	0%	0%	0%	6%	0%	0%	0%	0%	88%
0.03%	0.79%	4%	18%	3%	0%	21%	0%	0%	0%	0%	54%
0.04%	0.69%	2%	16%	19%	0%	20%	7%	1%	0%	0%	36%
0.05%	0.68%	1%	13%	18%	3%	18%	8%	9%	0%	0%	30%
0.06%	0.69%	0%	10%	16%	6%	17%	9%	13%	1%	3%	26%
0.08%	0.76%	0%	5%	12%	9%	14%	10%	20%	4%	7%	19%
0.10%	0.88%	0%	0%	8%	12%	11%	11%	26%	8%	12%	12%
0.12%	1.05%	0%	0%	2%	16%	7%	11%	33%	11%	17%	4%
0.14%	1.23%	0%	0%	0%	18%	0%	8%	36%	16%	23%	0%
0.16%	1.50%	0%	0%	0%	14%	0%	0%	26%	25%	35%	0%
0.18%	1.85%	0%	0%	0%	7%	0%	0%	8%	36%	49%	0%
0.19%	2.04%	0%	0%	0%	3%	0%	0%	0%	41%	56%	0%

Table 4-1 Mean return/Standard deviation matrix 2001-01-01

The weightings are solved by testing every possible combination of assets resulting in the given return. The weights generating the lowest standard deviation for the given return is considered an efficient portfolio. Every efficient portfolio is then presented as a data point, resulting in an efficient frontier.

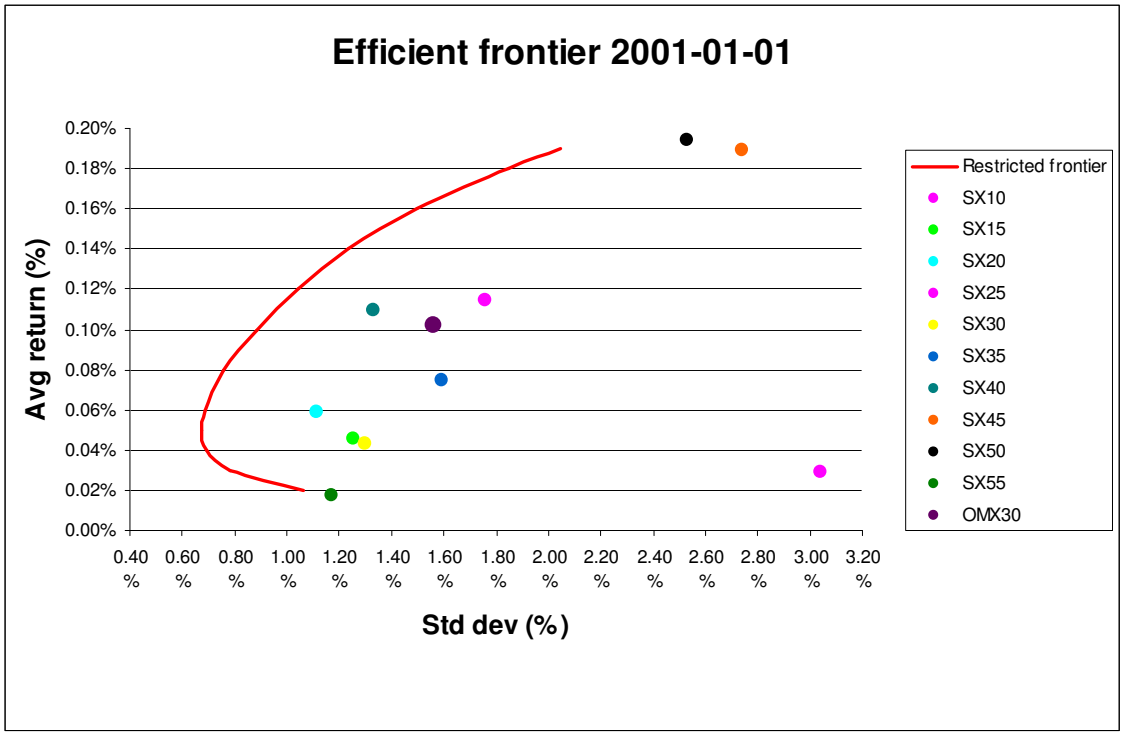


Figure 4-3 Efficient Frontier 2001-01-01

Using the data in table 4-1, the efficient frontier is plotted in figure 4-3. By combining the assets, a portfolio for every point on the efficient frontier can be comprised. As illustrated in the figure above, there are no portfolios situated above the efficient frontier and all those that lie below are inferior to those that are situated on the efficient frontier. Each point on the efficient frontier represents a different but efficient portfolio. By investing in an efficient portfolio, the investor achieves the highest possible return for the given risk.

The OMX30 index which is comprised by market value weights of each individual asset is, as illustrated in the graph, not an efficient portfolio. By weighting the assets differently from the index, a higher risk/reward ratio can be achieved.

4.2 The Optimal Risky Portfolio

For a given set of assets there is only one optimal risky portfolio. No other combination can provide the investor with a higher risk adjusted return. The optimal risky portfolio is the tangency point of the CAL and the efficient frontier, presented in figure 4-4. The tangency point represents the portfolio with the highest Sharpe ratio i.e. the optimal risky portfolio.

There are several combinations of portfolios on the efficient frontier, each with a unique weighted combination of assets. However, there is only one portfolio that can be considered optimal, it is the portfolio located on the tangent of the CAL and the efficient frontier (Markowitz, 1991). Figure 4-4 illustrates the optimal risky portfolio in 2001-01-01, the estimated return is 0.1378% and the standard deviation is 1.2111%. The Sharpe ratio is 0.0972, it measures the risk-adjusted performance of the portfolio. There are no other

combinations of the assets that will yield the investor with a higher risk adjusted return. Investing in any other way than the optimal risky portfolio would not constitute a rational behavior. A positive Sharpe ratio is crucial to get a beneficial result of the investment.

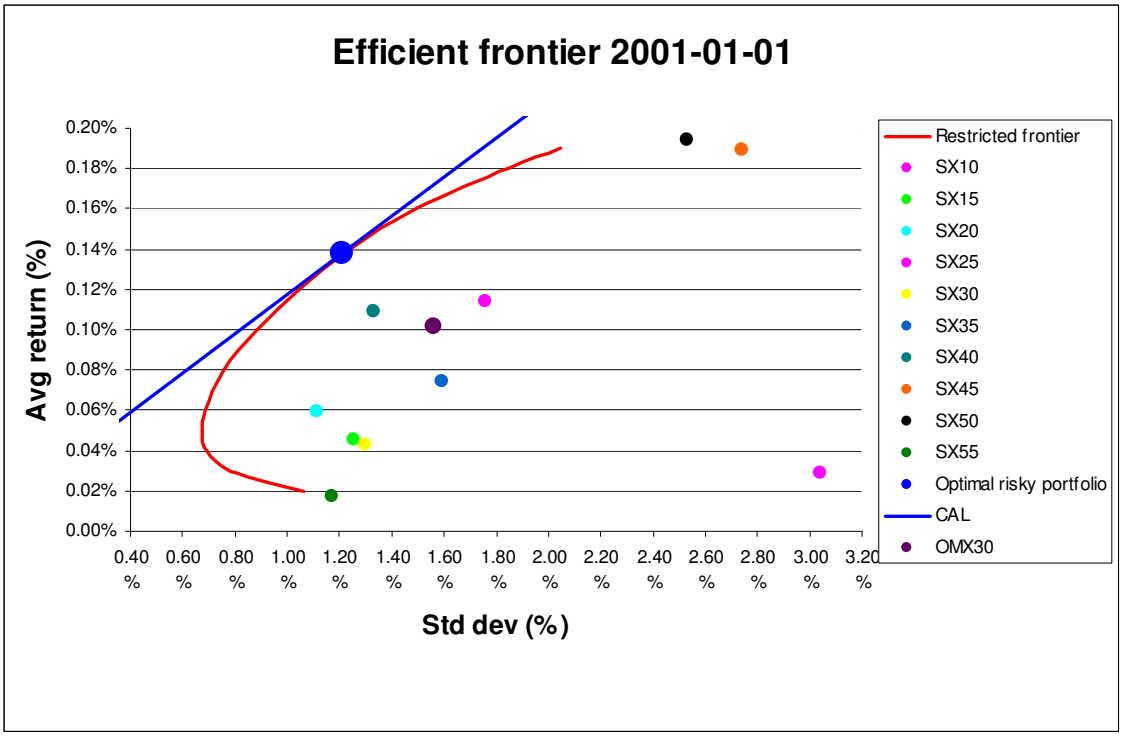


Figure 4-4 Optimal Risky Portfolio 2001-01-01

The optimal risky portfolio presented in figure 4-4 and table 4-2 represents the initial investment made at time T0, 2001-01-01.

Optimal Risky Portfolio	
Expected return	0.1378%
Standard deviation	1.2111%
Sharpe ratio	0.0972

Table 4-2 Optimal Risky Portfolio

In January 2001, the assets were allocated according to the weighting scheme presented in table 4-3.

	SX10	**SX15**	**SX20**	**SX25**	**SX30**	**SX35**	**SX40**	**SX45**	**SX50**	**SX55**
Weights	0%	0%	0%	18%	0%	9%	36%	15%	22%	0%

Table 4-3 Portfolio Weights

4.3 Reweighed Optimal Risky Portfolio

For each quarter new estimates of the individual assets are made using the preceding five years of historical return. A new optimal risky portfolio is constructed every quarter during the five year period of investigation. The asset allocation along with the quarterly actual return for each optimal portfolio is presented in appendix 2.

4.4 Performance Evaluation

The performance evaluation will be presented in two major blocks. The authors have chosen to begin the evaluation by presenting the actual return, annually and quarterly, for the optimal risky portfolio in comparison to the OMX30 index. The actual returns are not adjusted for the inherent risk associated with each investment. Further, the actual return of both the optimal risky portfolio and the OMX30 index will be adjusted for risk and presented as a set of Sharpe ratios for each quarter. There will be a discussion concerning the risk and non risk adjusted performance of the portfolio.

4.4.1 Non-risk Adjusted Performance

Annualized Return

In table 4-4 the annualized return for the optimal risky portfolio in comparison to the OMX30 index is presented.

Annualized Return					
	2001	2002	2003	2004	2005
Optimal risky portfolio	-12.41%	-16.47%	7.41%	8.87%	58.16%
OMX30	-18.74%	-41.74%	29.01%	16.59%	29.86%

Table 4-4 Annualized Return

Although both the optimal portfolio and the OMX30 index have a negative return in both 2001 and 2002, the optimal portfolio presents the investor with a relatively higher return. During 2003 and 2004 the OMX30 index have a higher return than the optimal portfolio. The reader should bear in mind that the superior return of the index during these years are offset by the weak performance during the beginning two years. Since the time span of the investigation is five years, the compounded return comparison is a more accurate illustration of the two investment alternatives.

Quarterly Return

The quarterly return for the optimal risky portfolio and the OMX30 index is presented in appendix 2.

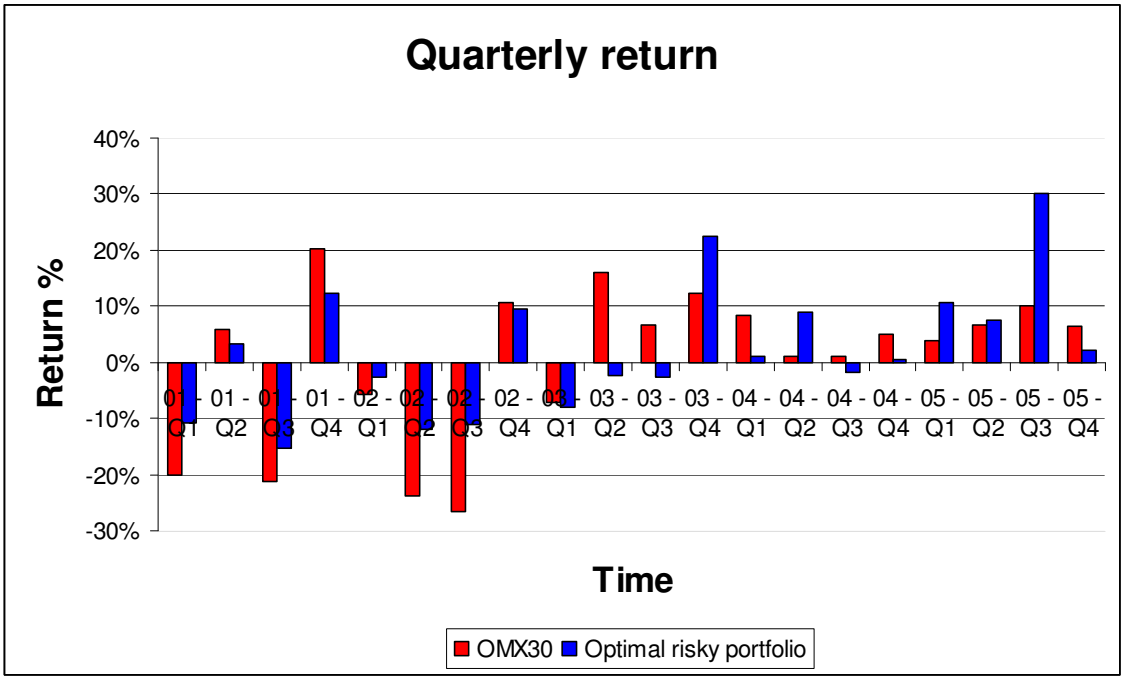


Figure 4-5 Quarterly Return

Figure 4-5 illustrates the return for each of the twenty quarters of the investigation period. In ten out of twenty quarters the optimal risky portfolio outperforms the OMX30 index. As in the case of the annual return, the simple quarterly return is not perfectly representative for the performance during the full five year period.

Cumulative Return

The compounded return represents the return yielded from the beginning of the investigated period and onwards. As presented in table 4-5, the optimal risky portfolio would for each year have been a considerably better investment than the OMX30 index.

Cumulative Return					
	2001	2002	2003	2004	2005
Optimal risky portfolio	-12.41%	-26.84%	-21.42%	-14.45%	35.30%
OMX30	-18.74%	-52.65%	-38.92%	-28.78%	-7.51%

Table 4-5 Cumulative Return

From the beginning of the measured period the optimal risky portfolio has a constantly higher compounded return. Even though it does not outperform the index in more than fifty percent of the quarterly returns, the compounded return is higher during the full period.

Figure 4-6 Return Comparison

4.4.2 Risk Adjusted Return

In appendix 4 the actual return, standard deviation and Sharpe ration for each quarter is listed for the OMX30 index and the optimal risky portfolio.

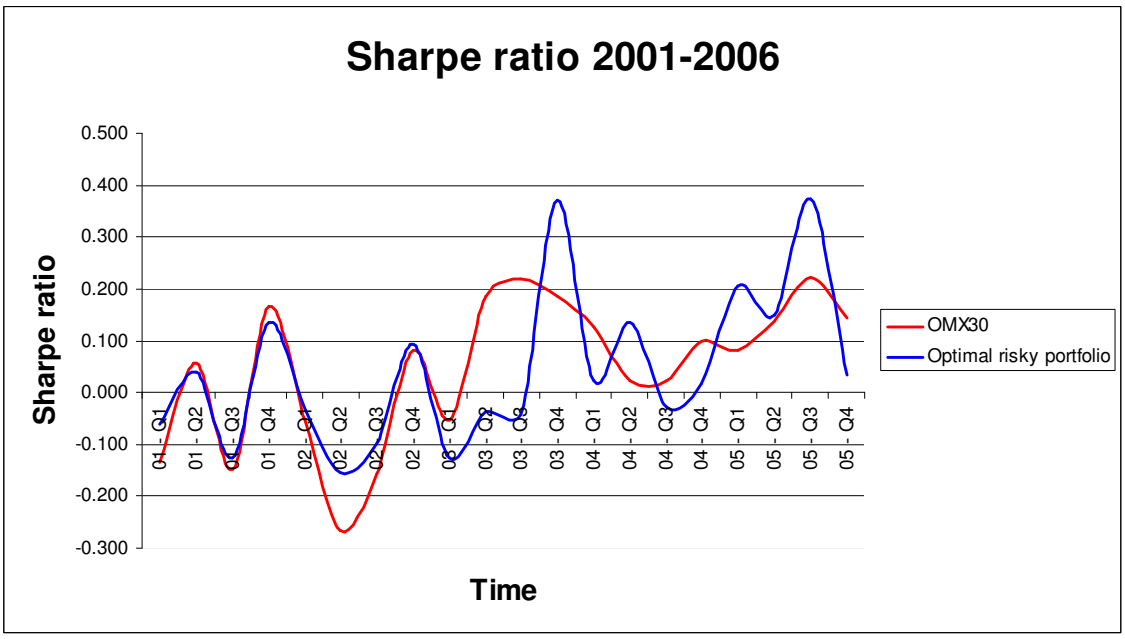


Figure 4-7 Sharpe Ratio

In figure 4-7 the Sharpe ratio for the two investment alternatives are plotted in comparison. The ratio varies over time, obviously being negative during periods of negative return, and correspond positive in periods of positive return. By analyzing the graph of Sharpe ratios, it is not possible to conclude that the optimal portfolio presents the investor with a constantly higher Sharpe ratio. During the first two years the ratios seem to vary in tandem with a slight advantage for the optimal portfolio. The following three years the optimal portfolio peeks during two quarters, but in between the OMX30 index has the higher ratio. There seems to be weak evidence for concluding an advantage for the optimal risky portfolio.

In order to extend and further investigate the risk adjusted return, the optimal portfolio return is adjusted to match the risk of the OMX30 index. The matrix of the risk adjusted return can be found in appendix 4.

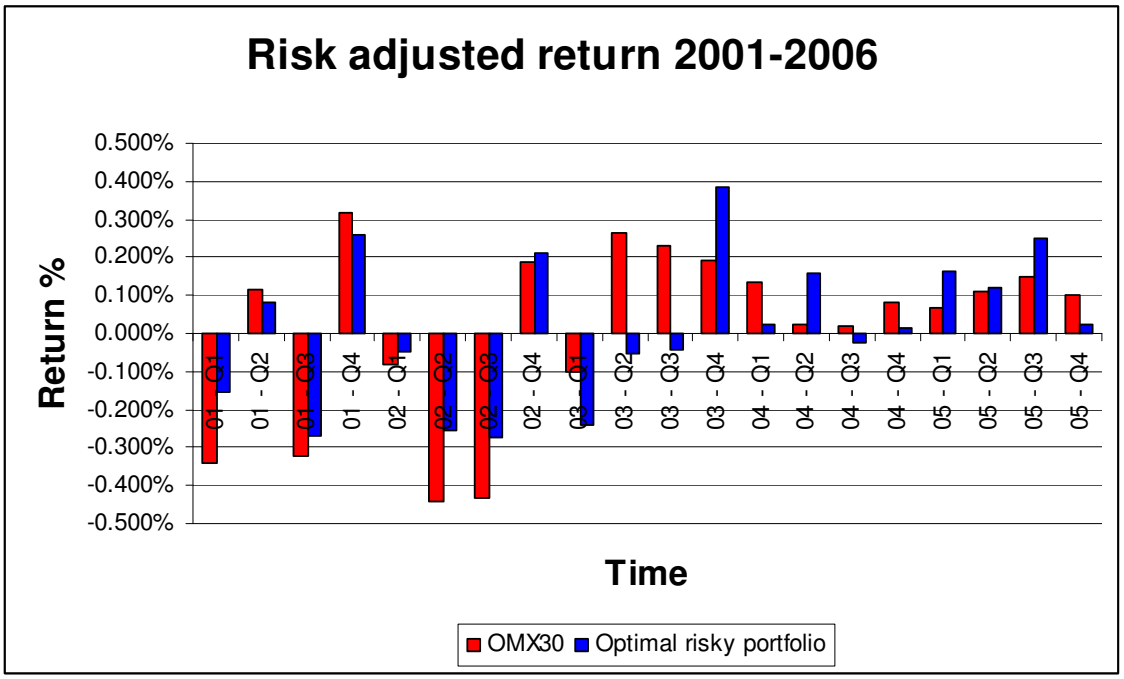


Figure 4-8 – Risk adjusted return

Figure 4-8 presents a return comparison based on the same risk level for both investment alternatives. The Sharpe ratio of the optimal portfolio is multiplied by the standard deviation of the OMX30 index. The returns are thereby perfectly comparable since they are associated with the same risk. By observing the return distribution one can see that there is no significant evidence for the optimal portfolio outperforming the OMX30 index.

4.5 Concluding Analysis

When analyzing the return comparison presented in figure 4-6, the optimal portfolio has been a better investment during the full five year period. Consequently, there is evidence that the return of the optimal portfolio during the investigated period far succeed the return of OMX30. During the first two years the portfolio withstands negative return stronger than the index. The third and partially fourth year, both actually and risk adjusted,

the index has a better development than the portfolio, followed by a strong fifth year for the portfolio.

The compounded non risk adjusted return shows that the portfolio is the better investment. However, when scrutinizing the results the evidence is not as clear. The quarterly returns presented in figure 4-5 prove that the portfolio return in ten out of twenty quarters is lower than the OMX30. When adjusting the return for risk, the portfolio has a higher risk adjusted return in eleven out of twenty quarters, figure 4-8. The Sharpe ratio in figure 4-7 varies significantly and can not be considered higher for the portfolio in comparison to OMX30.

For the five year investigation period the optimal portfolio has yielded a considerably higher return. When analyzing the periodical returns, and adjusting them for risk there are no obvious evidence that a portfolio weighted according to MPT presents the investor with a higher return.

5 Conclusion

In this chapter the authors gives a summary of the conclusions from the study. The findings are analyzed in relation to the purpose of the thesis.

This study has discovered that an optimal risky portfolio influenced by Modern Portfolio Theory actually provides the investor with a higher return during the timeframe the authors have chosen. Investing merely in a passive index can not be considered an efficient investment alternative. However, the research questions also calls for an analysis of the portfolios inherent risk, and its impact on the risk adjusted return. The analysis and conclusion is therefore more extensive than simply comparing the simple compounded return during the investigation period.

> *"The purpose of this thesis is to investigate if an investor can apply modern portfolio theory in order to achieve a higher return than investing in an index portfolio."*

When plotting the index and sub-indexes along with the efficient frontier, the authors can conclude that neither OMX30 index nor the sub-indexes were efficient investment alternatives. The individual assets can be weighed differently to achieve a portfolio with a higher Sharpe ratio, located on rather than below the efficient frontier. This portfolio should according to MPT generate the investor a higher risk adjusted return.

During the five year investigation period the optimal risky portfolio had a significantly higher compounded return than the passive index portfolio. Investing accordingly to MPT would have yielded an investor a cumulative return of 35.3 percent rather than a negative 7.52 percent return of OMX30 index. When analyzing the simple quarterly periodical return, the proof of the optimal portfolios superiority is not as strong since the index had a higher return in ten out of twenty quarters.

When adjusting the return of both investment alternatives for inherent risk, the authors could not find a consequent patter of higher risk adjusted return for the optimal portfolio. The advantage in Sharpe ratio for each quarter varied between the optimal portfolio and the OMX30 index. The risk adjusted return had the same periodical distribution as the simple quarterly return, with no significant advantage for the optimal portfolio.

To conclude, the optimal risky portfolio had over the full five year time span a significantly higher return than a passive index portfolio. However, the authors are disinclined to argue that investing accordingly to MPT, forecasting merely on historical data, would be a consequently better alternative than an index portfolio.

To further investigate the advantage of MPT, one would have to base the forecast on a stock picking approach rather than using only historical data. The authors believe a fundamental analysis of the individual assets included in the portfolio would to a great extent improve the performance of a diversified portfolio.

6 Final Discussion

In this final chapter, the authors reflect and comment on the obtained result. Further, the authors reflect upon the findings, and evaluate their performance in order to display what could have been done differently to improve the findings of the thesis. Finally, the optimal risky portfolio's development is revised from different perspectives in order to provide some leads for future research.

6.1 Reflections and Criticism

The time that the authors spent on this thesis has been very valuable. The knowledge of making a successful investment has broadened, since several theories and academic journals have been analyzed and interpreted. Further, all the practical work that a thesis demand have improved the authors analytical ability and skills in terms of constructing an academic report.

During the work on the thesis it became more obvious for the authors that an index is a very powerful measurement, and that it is hard for an investor to outperform the index. Depending on what perspective to use when analyzing the findings of the thesis, the result for the optimal risky portfolio will vary. By using return comparison between the two portfolios during the total timeframe, the optimal risky portfolio provides the investor with a significant greater return, consequently the compounded return for the optimal risky portfolio is also superior to the index portfolio. On the other hand the result is ambiguous when looking at the risk adjusted return, where the findings display the index portfolio and optimal risky portfolio as equally good.

In order to conclude the optimal risky portfolio as the superior one depends upon what perspective that is of interest. The findings do not provide a total picture that consistently recommends the optimal risky portfolio as the most successful. One important parameter that might have an impact on the result is the investor's wisdom of a certain asset, a specific asset class may be totally overvalued and should be excluded from the portfolio, but since the authors' optimal risky portfolio is based upon historical data, the individual investor's preferences are totally disregarded in this thesis.

Finally, the timeframe should be extended in order to achieve a more reliable result, the optimal risky portfolio is based upon a 5 year period of historical data. A short investigation period increase the risk for error, hence a longer timeframe will be preferred to lessen the risk for error and to accomplish a more trustworthy result.

6.2 Suggestions for Further Research

The authors consider this type of study extremely interesting and were pleased with the outcome of thesis. Although the authors would find it interesting to perform a similar test but with several ongoing portfolios compared to different index types to be able to achieve a conclusion statement that is more valid and reliable. Further, each parameter can be analyzed separately but still with the same objective to achieve a greater development than index, that would be a great complement to the authors study. The authors study is exclusively based upon historical data, adding a parameter that includes the investor's preferences would make this type of study even more interesting and would highlight the pros and cons with the human impact.

To construct an optimal risky portfolio, that is fully diversified the global perspective should be taken into consideration. It is acknowledged in research (Bodie et Al., 2004) that a globally diversified portfolio will provide the investor with even more valuable diversification opportunities, consequently it would be interesting to investigate if a portfolio with globally diversified assets would show a different pattern of the result.

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Appendix 1 - Periodical historical estimates

T-note Return	*31-Dec-00*	*31-Mar-01*	*30-Jun-01*	*30-Sep-01*	*31-Dec-01*	*31-Mar-02*	*30-Jun-02*	*30-Sep-02*	*31-Dec-02*	*31-Mar-03*	*30-Jun-03*	*30-Sep-03*	*31-Dec-03*	*31-Mar-04*	*30-Jun-04*	*30-Sep-04*	*30-dec-04*	*31-mar-05*	*30-jun-05*	*30-sep-05*
APR	4.02%	4.05%	4.04%	4.21%	3.69%	3.90%	4.27%	4.21%	3.90%	3.55%	3.16%	2.70%	2.72%	2.43%	2.00%	2.00%	1.99%	1.98%	1.85%	1.48%
Daily return	0.0201%	0.0202%	0.0202%	0.0210%	0.0185%	0.0195%	0.0214%	0.0211%	0.0195%	0.0178%	0.0158%	0.0135%	0.0136%	0.0122%	0.0100%	0.0100%	0.0100%	0.0099%	0.0092%	0.0074%

Portfolio

SX10	*31-Dec-00*	*31-Mar-01*	*30-Jun-01*	*30-Sep-01*	*31-Dec-01*	*31-Mar-02*	*30-Jun-02*	*30-Sep-02*	*31-Dec-02*	*31-Mar-03*	*30-Jun-03*	*30-Sep-03*	*31-Dec-03*	*31-Mar-04*	*30-Jun-04*	*30-Sep-04*	*30-dec-04*	*31-mar-05*	*30-jun-05*	*30-sep-05*
Average return	0.0029%	0.0040%	0.0058%	0.0043%	0.0030%	0.0043%	0.0041%	0.0019%	0.0029%	0.0012%	0.0048%	0.0083%	0.0156%	0.0142%	0.0138%	0.0140%	0.0149%	0.1734%	0.0198%	0.0219%
Variance	0.0092%	0.0097%	0.0099%	0.0102%	0.0116%	0.0117%	0.0116%	0.0119%	0.0120%	0.0118%	0.0119%	0.0107%	0.0100%	0.0097%	0.0095%	0.0094%	0.0091%	0.0834%	0.0081%	0.0077%
Standard deviation	3.04%	3.11%	3.15%	3.19%	3.40%	3.42%	3.41%	3.45%	3.46%	3.44%	3.45%	3.28%	3.16%	3.12%	3.09%	3.07%	3.02%	2.90%	2.84%	2.78%
Sharpe ratio	0.010	0.013	0.018	0.013	0.009	0.012	0.012	0.006	0.008	0.004	0.014	0.025	0.049	0.046	0.045	0.046	0.050	0.060	0.070	0.079

SX15	*31-Dec*	*31-Mar*	*30-Jun*	*30-Sep*	*31-Dec*	*31-Mar*	*30-Jun*	*30-Sep*	*31-Dec*	*31-Mar*	*30-Jun*	*30-Sep*	*31-Dec*	*31-Mar*	*30-Jun*	*30-Sep*	*30-dec*	*31-mar*	*30-jun-*	*30-sep*

Appendix 1 - Periodical historical estimates

	-00	*-01*	*-01*	*-01*	*-01*	*-02*	*-02*	*-02*	*-02*	*-03*	*-03*	*-03*	*-03*	*-04*	*-04*	*-04*	*-04*	*-05*	*05*	*-05*
Average return	0.046%	0.039%	0.041%	0.038%	0.045%	0.041%	0.035%	0.013%	0.030%	0.014%	0.018%	0.051%	0.053%	0.047%	0.037%	0.027%	0.017%	0.027%	0.037%	0.044%
Variance	0.016%	0.016%	0.016%	0.017%	0.017%	0.017%	0.017%	0.018%	0.018%	0.018%	0.018%	0.017%	0.015%	0.015%	0.015%	0.014%	0.014%	0.013%	0.012%	0.012%
Standard deviation	1.25%	1.26%	1.27%	1.30%	1.30%	1.30%	1.30%	1.34%	1.33%	1.34%	1.34%	1.32%	1.23%	1.21%	1.21%	1.19%	1.18%	1.13%	1.11%	1.10%
Sharpe ratio	0.0366	0.0314	0.0323	0.0289	0.0345	0.0315	0.0269	0.0095	0.0228	0.0108	0.0133	0.0391	0.0429	0.0391	0.0303	0.0229	0.0141	0.0238	0.0334	0.0400
SX20	*31-Dec-00*	*31-Mar-01*	*30-Jun-01*	*30-Sep-01*	*31-Dec-01*	*31-Mar-02*	*30-Jun-02*	*30-Sep-02*	*31-Dec-02*	*31-Mar-03*	*30-Jun-03*	*30-Sep-03*	*31-Dec-03*	*31-Mar-04*	*30-Jun-04*	*30-Sep-04*	*30-dec-04*	*31-mar-05*	*30-jun-05*	*30-sep-05*
Average return	0.059%	0.045%	0.046%	0.026%	0.034%	0.028%	0.015%	-0.014%	-0.003%	-0.019%	-0.008%	0.018%	0.026%	0.019%	0.014%	0.010%	0.008%	0.016%	0.026%	0.036%
Variance	0.012%	0.013%	0.014%	0.015%	0.016%	0.016%	0.016%	0.018%	0.019%	0.020%	0.020%	0.020%	0.018%	0.018%	0.018%	0.018%	0.018%	0.018%	0.018%	0.018%
Standard deviation	1.12%	1.15%	1.17%	1.22%	1.25%	1.26%	1.27%	1.34%	1.37%	1.40%	1.42%	1.40%	1.34%	1.34%	1.34%	1.34%	1.35%	1.34%	1.33%	1.33%
Sharpe ratio	0.0531	0.0395	0.0388	0.0215	0.0274	0.0218	0.0121	-0.0103	-0.0024	-0.0137	-0.0057	0.0127	0.0193	0.0140	0.0105	0.0077	0.0061	0.0118	0.0197	0.0272
SX25	*31-Dec-00*	*31-Mar-01*	*30-Jun-01*	*30-Sep-01*	*31-Dec-01*	*31-Mar-02*	*30-Jun-02*	*30-Sep-02*	*31-Dec-02*	*31-Mar-03*	*30-Jun-03*	*30-Sep-03*	*31-Dec-03*	*31-Mar-04*	*30-Jun-04*	*30-Sep-04*	*30-dec-04*	*31-mar-05*	*30-jun-05*	*30-sep-05*
Average return	0.114	0.108	0.112	0.087	0.096	0.090	0.067	0.041	0.046	0.036	0.036	0.039	0.026	0.032	0.014	0.016	0.004	0.019%	0.039	0.043

	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%		%	%
Variance	0.0031%	0.0033%	0.0035%	0.0037%	0.0038%	0.0038%	0.0039%	0.0041%	0.0041%	0.0041%	0.0041%	0.0039%	0.0036%	0.0035%	0.0034%	0.0033%	0.0032%	0.0026%	0.0025%	0.0024%
Standard deviation	1.76%	1.82%	1.86%	1.91%	1.94%	1.96%	1.96%	2.03%	2.02%	2.03%	2.02%	1.98%	1.89%	1.88%	1.85%	1.83%	1.79%	1.62%	1.57%	1.55%
Sharpe ratio	0.0650	0.0595	0.0602	0.0454	0.0493	0.0462	0.0342	0.0204	0.0229	0.0178	0.0180	0.0197	0.0135	0.0169	0.0078	0.0085	0.0021	0.0116	0.0247	0.0281
SX30	*31-Dec-00*	*31-Mar-01*	*30-Jun-01*	*30-Sep-01*	*31-Dec-01*	*31-Mar-02*	*30-Jun-02*	*30-Sep-02*	*31-Dec-02*	*31-Mar-03*	*30-Jun-03*	*30-Sep-03*	*31-Dec-03*	*31-Mar-04*	*30-Jun-04*	*30-Sep-04*	*30-dec-04*	*31-mar-05*	*30-jun-05*	*30-sep-05*
Average return	0.0043%	0.0055%	0.0061%	0.0069%	0.0059%	0.0065%	0.0076%	0.0070%	0.0065%	0.0062%	0.0058%	0.0070%	0.0073%	0.0079%	0.0080%	0.0074%	0.0076%	0.0085%	0.0089%	0.0090%
Variance	0.0017%	0.0017%	0.0017%	0.0017%	0.0017%	0.0016%	0.0017%	0.0018%	0.0018%	0.0019%	0.0018%	0.0018%	0.0018%	0.0017%	0.0017%	0.0017%	0.0016%	0.0016%	0.0015%	0.0015%
Standard deviation	1.30%	1.30%	1.30%	1.29%	1.29%	1.28%	1.30%	1.32%	1.36%	1.36%	1.36%	1.35%	1.33%	1.31%	1.29%	1.29%	1.27%	1.25%	1.22%	1.21%
Sharpe ratio	0.0330	0.0422	0.0468	0.0533	0.0457	0.0506	0.0589	0.0528	0.0482	0.0453	0.0430	0.0519	0.0551	0.0603	0.0623	0.0573	0.0594	0.0679	0.0730	0.0748
SX35	*31-Dec-00*	*31-Mar-01*	*30-Jun-01*	*30-Sep-01*	*31-Dec-01*	*31-Mar-02*	*30-Jun-02*	*30-Sep-02*	*31-Dec-02*	*31-Mar-03*	*30-Jun-03*	*30-Sep-03*	*31-Dec-03*	*31-Mar-04*	*30-Jun-04*	*30-Sep-04*	*30-dec-04*	*31-mar-05*	*30-jun-05*	*30-sep-05*
Average return	0.0074%	0.0066%	0.0071%	0.0067%	0.0057%	0.0055%	0.0032%	0.0009%	0.0019%	0.0000%	0.0015%	0.0035%	0.0024%	0.0018%	0.0028%	0.0022%	0.0011%	0.0020%	0.0018%	0.0013%
Variance	0.0025%	0.0026%	0.0025%	0.0026%	0.0026%	0.0026%	0.0025%	0.0028%	0.0028%	0.0029%	0.0030%	0.0029%	0.0026%	0.0026%	0.0025%	0.0025%	0.0024%	0.0022%	0.0021%	0.0021%

Standard deviation	1.59%	1.60%	1.59%	1.62%	1.61%	1.61%	1.59%	1.67%	1.68%	1.71%	1.72%	1.70%	1.62%	1.60%	1.58%	1.57%	1.56%	1.49%	1.45%	1.43%
Sharpe ratio	0.0468	0.0415	0.0442	0.0416	0.0352	0.0342	0.0203	0.0051	0.0114	-0.0002	0.0088	0.0208	0.0148	0.0113	0.0178	0.0141	0.0073	0.0135	0.0124	0.0088
SX40	*31-Dec-00*	*31-Mar-01*	*30-Jun-01*	*30-Sep-01*	*31-Dec-01*	*31-Mar-02*	*30-Jun-02*	*30-Sep-02*	*31-Dec-02*	*31-Mar-03*	*30-Jun-03*	*30-Sep-03*	*31-Dec-03*	*31-Mar-04*	*30-Jun-04*	*30-Sep-04*	*30-dec-04*	*31-mar-05*	*30-jun-05*	*30-sep-05*
Average return	0.109%	0.098%	0.094%	0.065%	0.064%	0.050%	0.033%	0.004%	0.008%	-0.013%	-0.009%	0.020%	0.021%	0.023%	0.022%	0.024%	0.019%	0.010%	0.003%	0.007%
Variance	0.018%	0.019%	0.020%	0.022%	0.023%	0.023%	0.024%	0.027%	0.029%	0.030%	0.030%	0.029%	0.026%	0.026%	0.025%	0.025%	0.025%	0.023%	0.022%	0.022%
Standard deviation	1.33%	1.38%	1.42%	1.47%	1.51%	1.52%	1.54%	1.66%	1.70%	1.73%	1.73%	1.70%	1.61%	1.60%	1.60%	1.59%	1.57%	1.53%	1.50%	1.49%
Sharpe ratio	0.0823	0.0709	0.0665	0.0440	0.0424	0.0331	0.0215	0.0022	0.0044	-0.0073	-0.0054	0.0120	0.0130	0.0141	0.0137	0.0149	0.0124	0.0064	0.0019	0.0047
SX45	*31-Dec-00*	*31-Mar-01*	*30-Jun-01*	*30-Sep-01*	*31-Dec-01*	*31-Mar-02*	*30-Jun-02*	*30-Sep-02*	*31-Dec-02*	*31-Mar-03*	*30-Jun-03*	*30-Sep-03*	*31-Dec-03*	*31-Mar-04*	*30-Jun-04*	*30-Sep-04*	*30-dec-04*	*31-mar-05*	*30-jun-05*	*30-sep-05*
Average return	0.189%	0.138%	0.143%	0.100%	0.115%	0.081%	0.001%	-0.069%	-0.017%	-0.045%	-0.028%	0.019%	0.004%	0.035%	0.015%	0.020%	-0.046%	0.081%	-0.051%	-0.032%
Variance	0.075%	0.086%	0.092%	0.100%	0.104%	0.107%	0.115%	0.130%	0.140%	0.147%	0.149%	0.148%	0.140%	0.141%	0.142%	0.143%	0.139%	0.103%	0.129%	0.125%
Standard deviation	2.74%	2.93%	3.04%	3.16%	3.23%	3.27%	3.39%	3.61%	3.74%	3.83%	3.86%	3.84%	3.74%	3.76%	3.77%	3.78%	3.73%	3.21%	3.59%	3.53%

Sharpe ratio	0.0690	0.0472	0.0469	0.0318	0.0356	0.0248	0.0003	-0.0190	-0.0045	-0.0117	-0.0073	0.0050	0.0011	0.0092	0.0041	0.0052	-0.0123	0.0253	-0.0143	-0.0090
SX50	*31-Dec-00*	*31-Mar-01*	*30-Jun-01*	*30-Sep-01*	*31-Dec-01*	*31-Mar-02*	*30-Jun-02*	*30-Sep-02*	*31-Dec-02*	*31-Mar-03*	*30-Jun-03*	*30-Sep-03*	*31-Dec-03*	*31-Mar-04*	*30-Jun-04*	*30-Sep-04*	*30-dec-04*	*31-mar-05*	*30-jun-05*	*30-sep-05*
Average return	0.1194%	0.1185%	0.1170%	0.1151%	0.1131%	0.1110%	0.0078%	0.0065%	0.0060%	0.0022%	0.0021%	0.0022%	0.0013%	0.0010%	0.0002%	-0.0001%	-0.0045%	0.1006%	-0.0041%	-0.0022%
Variance	0.0064%	0.0064%	0.0065%	0.0068%	0.0070%	0.0071%	0.0075%	0.0082%	0.0082%	0.0085%	0.0085%	0.0080%	0.0074%	0.0073%	0.0072%	0.0072%	0.0068%	0.0060%	0.0055%	0.0055%
Standard deviation	2.53%	2.52%	2.56%	2.61%	2.64%	2.66%	2.73%	2.86%	2.87%	2.91%	2.91%	2.83%	2.72%	2.70%	2.69%	2.68%	2.61%	2.45%	2.35%	2.34%
Sharpe ratio	0.0769	0.0732	0.0664	0.0578	0.0497	0.0414	0.0284	0.0228	0.0211	0.0076	0.0073	0.0078	0.0047	0.0038	0.0008	-0.0004	-0.0170	0.0432	-0.0176	-0.0093
SX55	*31-Dec-00*	*31-Mar-01*	*30-Jun-01*	*30-Sep-01*	*31-Dec-01*	*31-Mar-02*	*30-Jun-02*	*30-Sep-02*	*31-Dec-02*	*31-Mar-03*	*30-Jun-03*	*30-Sep-03*	*31-Dec-03*	*31-Mar-04*	*30-Jun-04*	*30-Sep-04*	*30-dec-04*	*31-mar-05*	*30-jun-05*	*30-sep-05*
Average return	0.0018%	0.0042%	0.0040%	0.0038%	0.0044%	0.0039%	0.0023%	0.0023%	0.0034%	0.0020%	0.0033%	0.0031%								
Variance	0.0014%	0.0015%	0.0016%	0.0016%	0.0018%	0.0017%	0.0019%	0.0021%	0.0023%	0.0023%	0.0024%	0.0024%								
Standard deviation	1.17%	1.24%	1.26%	1.25%	1.32%	1.32%	1.37%	1.45%	1.51%	1.53%	1.55%	1.56%								
Sharpe ratio	0.0150	0.0341	0.0317	0.0305	0.0332	0.0292	0.0166	0.0156	0.0224	0.0128	0.0213	0.0200								

Appendix 2 - Weights and actual quarterly return

30 December 2000 - 31 March 2001

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
Weights	0%	0%	0%	18%	0%	9%	36%	15%	22%	0%

Period start value	464.97
Period end value	414.19
Portfolio return	-10.92%
Compounded return	-10.92%
Index return	-20.15%
Compounded index return	-20.15%

31 March 2001 - 30 June 2001

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
Weights	0%	0%	0%	16%	11%	8%	27%	5%	22%	11%

Period start value	376.53
Period end value	389.24
Portfolio return	3.38%
Compounded return	-7.92%
Index return	5.77%
Compounded index return	-15.55%

30 June 2001 - 30 September 2001

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
Weights	0%	0%	0%	17%	16%	11%	23%	5%	18%	10%

Period start value	366.99
Period end value	311.01
Portfolio return	-15.25%
Compounded return	-21.96%
Index return	-21.19%
Compounded index return	-32.49%

30 September 2001 - 31 December 2001

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
Weights	0%	0%	0%	13%	32%	14%	6%	2%	21%	12%

Period start value	310.13
Period end value	348.07
Portfolio return	12.24%
Compounded return	-12.41%
Index return	20.37%
Compounded index return	-18.74%

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30 December 2001 - 31 March 2002

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
Weights	0%	0%	0%	19%	26%	10%	6%	5%	17%	17%

Period start value	343.97

Period end value	335.13
Portfolio return	-2.57%
Compounded return	-14.66%
Index return	-5.66%
Compounded index return	-23.33%

31 March 2002 - 30 June 2002

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
Weights	0%	0%	0%	21%	37%	12%	0%	1%	15%	14%

Period start value	336.76
Period end value	296.75
Portfolio return	-11.88%
Compounded return	-24.80%
Index return	-23.88%
Compounded index return	-41.64%

30 June 2002 - 30 September 2002

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
Weights	0%	0%	0%	18%	70%	0%	0%	0%	12%	0%

Period start value	331.32
Period end value	294.53
Portfolio return	-11.10%
Compounded return	-33.15%
Index return	-26.69%
Compounded index return	-57.22%

30 September 2002 - 31 December 2002

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
Weights	0%	0%	0%	4%	83%	0%	0%	0%	13%	0%

Period start value	285.47
Period end value	312.44
Portfolio return	9.45%
Compounded return	-26.84%

Index return	10.67%
Compounded index return	-52.65%

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30 December 2002 - 31 March 2003

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
Weights	0%	0%	0%	9%	67%	0%	0%	0%	9%	15%

Period start value	294.89
Period end value	271.75
Portfolio return	-7.85%
Compounded return	-32.58%
Index return	-7.18%
Compounded index return	-56.05%

31 March 2003 - 30 June 2003

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55

Weights	0%	0%	0%	9%	91%	0%	0%	0%	0%	0%

Period start value	281.29
Period end value	274.89
Portfolio return	-2.27%
Compounded return	-34.11%
Index return	16.10%
Compounded index return	-48.98%

30 June 2003 - 30 September 2003

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
Weights	6%	0%	0%	7%	70%	0%	0%	0%	0%	17%

Period start value	252.80
Period end value	246.14
Portfolio return	-2.64%
Compounded return	-35.85%
Index return	6.69%
Compounded index return	-45.57%

30 September 2003 - 31 December 2003

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
Weights	10%	28%	0%	0%	51%	4%	0%	0%	0%	7%

Period start value	222.39
Period end value	272.42
Portfolio return	22.50%
Compounded return	-21.42%
Index return	12.22%
Compounded index return	-38.92%

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30 December 2003 - 31 March 2004

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50
Weights	23%	29%	0%	0%	48%	0%	0%	0%	0%

Period start value	290.85
Period end value	294.21
Portfolio return	1.15%
Compounded return	-20.51%
Index return	8.49%
Compounded index return	-33.73%

31 March 2004 - 30 June 2004

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50
Weights	21%	23%	0%	0%	56%	0%	0%	0%	0%

Period start value	300.22
Period end value	327.39
Portfolio return	9.05%
Compounded return	-13.32%
Index return	1.14%
Compounded index return	-32.98%

30 June 2004 - 30 Sep-

tember 2004

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50
Weights	23%	11%	0%	0%	66%	0%	0%	0%	0%

Period start value	344.30
Period end value	337.99
Portfolio return	-1.83%
Compounded return	-14.91%
Index return	1.09%
Compounded index return	-32.25%

30 September 2004 - 31 December 2004

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50
Weights	28%	0%	0%	0%	72%	0%	0%	0%	0%

Period start value	356.94
Period end value	358.86
Portfolio return	0.54%

Compounded return	-14.45%
Index return	5.12%
Compounded index return	-28.78%

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30 December 2004 - 31 March 2005

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50
Weights	29%	0%	0%	0%	71%	0%	0%	0%	0%

Period start value	359.91
Period end value	398.02
Portfolio return	10.59%
Compounded return	-5.39%
Index return	3.84%
Compounded index return	-26.04%

31 March 2005 - 30 June 2005

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50
Weights	25%	0%	0%	0%	56%	0%	0%	2%	17%

Period start value	424.43
Period end value	455.84
Portfolio return	7.40%
Compounded return	1.61%
Index return	6.76%
Compounded index return	-21.04%

30 June 2005 - 30 September 2005

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50
Weights	29%	8%	0%	2%	61%	0%	0%	0%	0%

Period start value	436.06
Period end value	567.58
Portfolio return	30.16%
Compounded return	32.26%
Index return	10.08%

Compounded index return	-13.08%

30 September 2005 - 31 December 2005

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50
Weights	30%	13%	0%	2%	55%	0%	0%	0%	0%

Period start value	567.49
Period end value	580.56
Portfolio return	2.30%
Compounded return	35.30%
Index return	6.41%
Compounded index return	-7.51%

Appendix 3 - Portfolio efficient frontier

Asset	Return	Std dev
3 month treasury	0.020%	
SX10	0.029%	3.04%
SX15	0.046%	1.25%
SX20	0.059%	1.12%
SX25	0.114%	1.76%
SX30	0.043%	1.30%
SX35	0.074%	1.59%
SX40	0.109%	1.33%
SX45	0.189%	2.74%
SX50	0.194%	2.53%
SX55	0.018%	1.17%

Correlation matrix

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
SX10	1									
SX15	0.27195	1								
SX20	0.300161	0.705977	1							
SX25	0.236618	0.508694	0.568358	1						
SX30	0.167502	0.340158	0.353311	0.275326	1					
SX35	0.187555	0.391104	0.433925	0.323854	0.227464	1				
SX40	0.24341	0.558945	0.694501	0.536403	0.282324	0.451373	1			
SX45	0.230602	0.451103	0.52679	0.406702	0.259266	0.268379	0.535567	1		
SX50	0.165737	0.362842	0.439039	0.432497	0.219406	0.240879	0.466372	0.524024	1	
SX55	0.118997	0.096671	0.122485	0.065619	0.064952	0.079945	0.130422	0.08558	0.11135	1

Covariance matrix

	SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
SX10	0.000924									
SX15	0.000104	0.000157								

SX20	0.000102	9.88E-05	0.000125							
SX25	0.000127	0.000112	0.000112	0.00031						
SX30	6.61E-05	5.53E-05	5.12E-05	6.29E-05	0.000169					
SX35	9.06E-05	7.79E-05	7.7E-05	9.06E-05	4.7E-05	0.000253				
SX40	9.84E-05	9.31E-05	0.000103	0.000126	4.88E-05	9.54E-05	0.000177			
SX45	0.000192	0.000155	0.000161	0.000196	9.22E-05	0.000117	0.000195	0.00075		
SX50	0.000127	0.000115	0.000124	0.000192	7.2E-05	9.68E-05	0.000157	0.000363	0.000639	
SX55	4.24E-05	1.42E-05	1.6E-05	1.35E-05	9.88E-06	1.49E-05	2.03E-05	2.74E-05	3.3E-05	0.000137

Border-Multiplied Covariance Matrix for actual portfolio

		SX10	SX15	SX20	SX25	SX30	SX35	SX40	SX45	SX50	SX55
	Weights	0%	0%	0%	18%	0%	9%	36%	15%	22%	0%
SX10	0%	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000
SX15	0%	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000
SX20	0%	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000
SX25	18%	0.00000000000	0.00000000000	0.00000000000	0.00000969844	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000
SX30	0%	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000
SX35	9%	0.00000000000	0.00000000000	0.00000000000	0.00000145757	0.00000000000	0.00000208863	0.00000000000	0.00000000000	0.00000000000	0.00000000000
SX40	36%	0.00000000000	0.00000000000	0.00000000000	0.00000805028	0.00000000000	0.00000314365	0.00002322398	0.00000000000	0.00000000000	0.00000000000

SX45	15%	0.00000000000	0.00000000000	0.00000000000	0.00000522122	0.00000000000	0.00000159891	0.00001063964	0.00001699377	0.00000000000	0.00000000000
SX50	22%	0.00000000000	0.00000000000	0.00000000000	0.00000746586	0.00000000000	0.00000192964	0.00001245793	0.00001197406	0.00003072494	0.00000000000
SX55	0%	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000
	100%	0.00000000000	0.00000000000	0.00000000000	0.00003189338	0.00000000000	0.00000876083	0.00004632155	0.00002896783	0.00003072494	0.00000000000

Portfolio variance	0.0147%
Standard dev	1.2111%
Mean	0.1378%
Sharpe ratio	0.0972

Appendix 4 - Risk matrix

Risk matrix OMX 30

		2001	2002	2003	2004	2005
Q1	Return	-0.341%	-0.083%	-0.102%	0.135%	0.065%
	Std dev	2.518%	1.486%	1.911%	1.059%	0.795%
	Sharpe	-0.135	-0.056	-0.053	0.127	0.082
Q2	Return	0.116%	-0.440%	0.263%	0.026%	0.109%
	Std dev	2.059%	1.655%	1.418%	1.183%	0.789%
	Sharpe	0.056	-0.266	0.186	0.022	0.138
Q3	Return	-0.321%	-0.432%	0.229%	0.021%	0.148%
	Std dev	2.149%	2.772%	1.048%	0.944%	0.669%
	Sharpe	-0.149	-0.156	0.219	0.022	0.221
Q4	Return	0.317%	0.189%	0.191%	0.081%	0.099%

	Std dev	1.909%	2.304%	1.033%	0.827%	0.697%
	Sharpe	0.166	0.082	0.185	0.098	0.143

Risk matrix optimal risky portfolio

		2001	2002	2003	2004	2005
Q1	Return	-0.149%	-0.035%	-0.127%	0.024%	0.169%
	Std dev	2.476%	1.094%	1.006%	0.981%	0.828%
	Sharpe	-0.060	-0.032	-0.126	0.024	0.204
Q2	Return	0.073%	-0.203%	-0.034%	0.150%	0.119%
	Std dev	1.812%	1.315%	0.908%	1.110%	0.789%
	Sharpe	0.040	-0.154	-0.037	0.135	0.150
Q3	Return	-0.237%	-0.164%	-0.036%	-0.024%	0.406%
	Std dev	1.887%	1.650%	0.845%	0.878%	1.086%
	Sharpe	-0.125	-0.099	-0.043	-0.028	0.374

Q4	Return	0.198%	0.159%	0.333%	0.010%	0.044%
	Std dev	1.459%	1.721%	0.896%	0.642%	1.326%
	Sharpe	0.136	0.093	0.371	0.016	0.033

Risk adjusted return optimal risky portfolio

		2001	2002	2003	2004	2005
Q1	Return	-0.152%	-0.048%	-0.242%	0.025%	0.163%
	Std dev	2.518%	1.486%	1.911%	1.059%	0.795%
	Sharpe	-0.060	-0.032	-0.126	0.024	0.204
Q2	Return	0.083%	-0.255%	-0.053%	0.160%	0.119%
	Std dev	2.059%	1.655%	1.418%	1.183%	0.789%
	Sharpe	0.040	-0.154	-0.037	0.135	0.150
Q3	Return	-0.270%	-0.275%	-0.045%	-0.026%	0.250%
	Std dev	2.149%	2.772%	1.048%	0.944%	0.669%
	Sharpe	-0.125	-0.099	-0.043	-0.028	0.374

Q4	Return	0.259%	0.213%	0.384%	0.013%	0.023%
	Std dev	1.909%	2.304%	1.033%	0.827%	0.697%
	Sharpe	0.136	0.093	0.371	0.016	0.033