Abstract

X-ray computed tomography (CT) is a medical imaging modality that allows reconstruction of the internal structure of the human body from a large number of x-ray attenuation measurements. In recent years, the development of spectral CT, in which the energy dependence of the x-ray attenuation coefficient is utilized, has attracted considerable interest. This thesis is concerned with a spectral CT system based on a photon-counting silicon strip detector which is being developed in our group.

A computer model for photon counting spectral CT was developed and used for simulating the proposed CT system as well as a laboratory CT setup designed for testing purposes. The simulations were used to compare the performance of two reconstruction methods for spectral CT, image-based energy weighting and basis material decomposition. The study shows that the two methods perform equally well when the number of x-ray photons in each measurement is high, while basis material decomposition performs significantly worse than energy weighting for data with few photons in each measurement.

Also included in the thesis is a simulation study of the detrimental effects of electronic noise and threshold variations on image quality. It is shown that a proposed electronic noise reduction of 20% in the readout channels gives an improvement in mean SDNR of only 1.8%. In addition, a scheme for countering electronic noise contamination of the lowest energy bins due to threshold variations is discussed.

The final chapter of the thesis describes an experiment which demonstrates that the CT detector design studied here can indeed be used to obtain high-quality images.
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Chapter 1

Introduction

1.1 Computed tomography

Computed tomography (CT) is a medical imaging technique, in which x-ray beams are sent through the patient to be imaged in a large number of directions. From measurements of how much each x-ray beam is attenuated during the passage through the body, a computer is used to reconstruct an image of a two-dimensional slice or a three-dimensional volume inside the body of the patient. After a number of years in development, this technique was first used to scan a patient in 1971 [1], and its two principal inventors, Godfrey N. Hounsfield and Allan M. Cormack, were awarded the 1979 Nobel prize in Physiology or Medicine for the development of CT.

There is a number of different CT scanner designs, but in the present report, we shall confine ourselves the commonly used third-generation scanner, which is sketched in figure 1.1. In this type of scanner, the stationary patient is surrounded by a rotating circular frame (the so-called gantry). The x-ray source is mounted on one side of the gantry and an arced array of x-ray detectors is mounted on the opposite side, which means that each detector always has the same position relative to the source.

\[ \text{Figure 1.1: Sketch of a third generation CT scanner.} \]

\[ \text{Figure 1.2: Photo of a CT scanner.}^{1} \]

\[ ^{1}\text{The photograph comes from http://en.wikipedia.org/wiki/File:64_slice_scanner.JPG} \]
When working with CT, one should always keep in mind that x-rays are ionizing radiation and therefore hazardous for the human body. When designing CT systems, it is therefore very important to keep the radiation dose to the patient as low as possible. This is one of the reasons why the development of detectors sensitive enough to detect single photons, so-called photon-counting detectors, is attracting interest.

\section{Interaction between x-rays and matter}

In the x-ray energy range used in computed tomography, which is about 20-140 keV, three kinds of interactions between x-rays and matter can take place. Firstly, the photon may be Compton scattered by an electron, meaning that it transfers part of its energy to the electron and is deflected from its initial path, often by more than 90°. Secondly, the photon may be Rayleigh scattered, meaning that it interacts with an atom and is slightly deflected without losing any energy. Thirdly, the x-ray photon may be entirely absorbed and its whole energy transferred to an electron, a phenomenon referred to as the photoelectric effect. A notable feature of the photoelectric absorption spectrum is the presence of K-edges, very sharp rises in the absorption coefficient which appear where the photon energy is equal to the binding energy of the electrons in the innermost shell (K shell) of an atom.\cite{1, 2} The different contributions to the attenuation coefficient of iodine, which is used as a contrast agent for x-rays, is shown in figure 1.3.

In a macroscopic description, the effect of the above interactions is that the intensity of a monochromatic beam decreases along its path according to the Lambert-Beer law:\cite{1}
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\[ I = I_0 e^{-\int \mu(x) dx} \]  

(1.1)

In this formula, \( I \) is the transmitted intensity, \( I_0 \) is the intensity in the unattenuated beam, \( \mu \) is the linear attenuation coefficient and the integration is carried out over the path of the beam. It is by measuring this attenuation that one can reconstruct the interior structure of the imaged object. Instead of specifying the absorption coefficient directly, it is common to speak about the so-called CT number, which is given by

\[ 1000 \cdot \frac{\mu - \mu_{\text{water}}}{\mu_{\text{water}}} \]  

(1.2)

The numeric value one gets from this formula is said to be expressed in Hounsfield units (HU).

The detectors will also register scattered radiation. Since there is not much information to be gathered from it, however, one often seeks to filter out the scatter in some way.\[1\]

1.3 Tomographic reconstruction

The physical acquisition process in CT can be thought of as an approximation of the so-called two-dimensional Radon transform which is a mathematical operator that takes a function \( f \) on \( \mathbb{R}^2 \) and delivers as its output the integral of \( f \) over all straight lines in \( \mathbb{R}^2 \):

\[ (\mathcal{R}f)(\gamma) = \int_{\gamma} f(x, y) ds \]  

(1.3)

Here, \( \gamma \) denotes a straight line in \( \mathbb{R}^2 \). The space of all such lines is called sinogram space. In CT, \( f \) is taken to be the absorption coefficient \( \mu \), and by measuring the transmitted intensity \( I \) one can calculate the so-called projection \( p = -\ln \frac{I}{I_0} = \int_{\gamma} \mu(x, y) ds \), thereby sampling sinogram space.

The simplest situation to consider is when parallel beams are used for measuring the projections. In this case it is convenient to parameterize the sinogram space with an angle \( \theta \in [0, \pi) \) and a coordinate \( t \in \mathbb{R} \) as shown in figure 1.4. It is then possible to reconstruct the original function exactly by applying a filter in sinogram space and then letting \( f \) be the sum of the projections along all lines passing through \((x, y)\) (the so-called filtered backprojection method, FBP):\[1\]

\[ f(x, y) = \int_{0}^{\pi} h(t) * p(t, \theta) d\theta \]  

(1.4)

In this formula, \( * \) denotes convolution with respect to \( t \), and when the integration is carried out the integrand must be evaluated at the \( t \) value corresponding to \((x, y)\), i.e. \( t = x \cos \theta - y \sin \theta \). Furthermore, \( h(t) \) is the distribution whose Fourier transform is \(|\omega|\):

\[ h(t) = \int_{-\infty}^{\infty} |\omega| e^{i2\pi \omega t} d\omega \]  

(1.5)

or, more explicitly,\[3\]
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Figure 1.4: Parallel beam geometry for CT (adapted from a similar picture in [1]).

\[ h(t) = \begin{cases} \frac{1}{2\pi t^2} & t \neq 0 \\ \infty & t = 0 \end{cases} \quad (1.6) \]

\[ \int_{-\infty}^{\infty} h(t) dt = 0 \quad (1.7) \]

Computationally, the filtering can be performed by convolving with a discretized version of \( h(t) \), but it is typically faster to compute the Fourier transform \( P(\omega, \theta) \) of \( p(t, \theta) \) with respect to \( t \), multiply by \( |\omega| \) and transform back:[1]

\[ f(x, y) = \pi \left( \int_{-\infty}^{\infty} P(\omega, \theta)|\omega|e^{i2\pi\omega t} d\omega \right) d\theta \quad (1.8) \]

In a fanbeam geometry, when the the projections are acquired along beam paths diverging from a single point source, it is more convenient to parameterize the sinogram space using the angles \( \gamma \) and \( \beta \) shown in figure 1.5. When measuring the projections along such
beam paths, we get a different sampling of the sinogram space compared to the parallel beam case. In order to reconstruct the image, one can either interpolate the sinogram values in the points that would be sampled in a parallel beam geometry (a process known as rebinning) and use the formulas above, or use the fanbeam data directly in a modified FBP formula (here stated in the case of equiangular spacing between the detectors): \[ f(x, y) = \frac{2\pi}{L^2} \left( \int_{-\gamma_m}^{\gamma_m} q(\gamma, \beta) h''(\gamma' - \gamma) D \cos \gamma d\gamma \right) d\beta \] (1.9)

In this formula, L is the distance from the point source to \((x, y)\) and D is the distance from the point source to the isocenter, which is the point that the central ray in the fan passes through for all angles \(\beta\). \([-\gamma_m, \gamma_m]\) is the angular range covered by the detector array, \(\gamma'\) is the \(\gamma\) angle of the particular ray which goes through \((x, y)\) and \(h''(\gamma) = \frac{1}{2} \left( \frac{1}{\sin \gamma} \right)^2 h(\gamma)\) where \(h\) is the filter distribution defined above.

Despite being very widely used in CT scanners, the filtered backprojection algorithm has some severe limitations, such as problems handling noise and imperfections in the acquisition system and difficulties utilizing redundant data that may be available [1, 4, 5]. Because of this, a large class of reconstruction algorithms called iterative methods, which are superior to FBP in these respects, are attracting interest. However, these methods are typically more computationally complex than FBP, thereby putting large demands on the reconstruction computer hardware [1].

One important example of an iterative algorithm is the so-called maximum likelihood (ML) method, which incorporates the statistical nature of the noise in the projections. More specifically, one models the measured projections \(N = (N_1, ..., N_k)\) for all lines of measurement, as random variables depending on the unknown attenuation coefficients...
\( \mu = (\mu_1, \ldots, \mu_l) \) for all pixels in the image area. If the detector is ideal, the \( N_i \) should be modelled as Poisson distributed independent random variables. However, if there is scatter or cross-talk between the detector elements, the \( N_i \) will not be independent anymore, and this must be taken into account in the model. One can then construct the likelihood function \( P(N|\mu) \) which gives the probability of measuring precisely the data that has actually been measured, given a certain set of attenuation coefficients \( \mu \). The reconstructed image, i.e. the attenuation coefficients in all points, is then obtained by maximizing \( P(N|\mu) \) (or equivalently, \( \ln P(N|\mu) \)) with some iterative optimization scheme, for example the expectation maximization (EM) algorithm.[6]

Another class of iterative methods is iterative filtered backprojection (IFBP), in which the physical acquisition process is modelled as a matrix \( P \) mapping the vector of attenuation coefficients \( \mu \) to the set of projections \( n \). The reconstruction is then performed, in the simplest IFBP method, by picking a starting guess \( \mu_0 \) and then carrying out the iteration scheme \( \mu_{k+1} = \mu_k - \alpha Q(P\mu_k - n_{in}) \), until convergence.[5] Here, \( Q \) is a matrix representing filtered backprojection, \( n_{in} \) is the measured projection data and \( \alpha \) is a constant determining the step length. A detailed investigation of several IFBP methods can be found in [5].

From a theoretical point of view, it is of course interesting to have a mathematical proof that a certain reconstruction algorithm converges to the exact image. However, such proofs of exactness are not of any real value when developing reconstruction algorithms for practical use, since real-world CT scanner setups differ from the ideal situation in several ways.[4] Any real scanner can only perform finitely many projections, so the reconstruction algorithm has to be discretized in some way. Also, all measurements are to some extent disturbed by noise, scattered radiation, imperfections in the detector elements etc. Therefore, there will always be differences between the reconstructed image and the actual anatomical structure. If these differences are clearly visible, they are referred to as artifacts. These artifacts are of course undesirable, but in many situations the image can be used despite their presence if the physician studying the pictures is aware of them. Therefore, testing of algorithms must be done with real data and with a specific imaging task in mind.[4]

An important example is the so-called beam hardening artifacts. These artifacts are caused by the fact that x-rays with high frequencies are attenuated less than x-rays with lower frequencies in most materials. When passing through a strongly attenuating object, the x-ray beam is therefore shifted towards higher frequencies. Any materials traversed in the subsequent propagation of the beam therefore attenuate the beam less than they would have if the frequency spectrum had been unchanged. In the reconstructed image this is visible as shadows in the vicinity of strongly attenuating tissues like bone.[1]

### 1.4 Statistical decision theory

In order to study how image quality is affected by imperfections in the image acquisition hardware and by the choice of reconstruction method, we shall review the basics of statistical decision theory, as applied to imaging science. A much more comprehensive treatment can be found in [7]. Assume that we have two hypotheses \( H_0 \) and \( H_1 \) and that we know that exactly one of them is true. We want to use the image data to determine which one is true and which one is false. In the cases which we will be concerned with here, \( H_1 \) represents the presence of some feature, e.g. a tumor or a bone, which is different from the background tissue, while \( H_0 \) represents the absence of the feature. Assume that we acquire a number of real-valued measurements, each of which can be seen as the outcome of a random variable \( g_i \). Let

\[
\mathbf{g} = (g_1, g_2, \ldots, g_N)^T
\]
be a vector containing all these random variables. Note that if we want to use \( g \) to determine whether \( H_0 \) or \( H_1 \) is true, the probability densities \( p_{g|H_0}(g|H_0) \) and \( p_{g|H_1}(g|H_1) \) must be different. Let \( g^m = (g|H_m) \) be the expectation value of \( g \) under hypothesis \( m \) for \( m = 0, 1 \) and let \( K^m \) be the covariance matrix of \( g \), with entries \( K^m_{ij} = \langle (g_i - \bar{g}_i)(g_j - \bar{g}_j) \rangle |H_m| \).

In order to use the measured data \( g \) to decide whether it is \( H_0 \) or \( H_1 \) that is true, one forms a test statistic \( t = T(g) \) where \( T \) is some real-valued function of the data vector, possibly nonlinear. Then, \( t \) is compared to some threshold value \( t_c \), and the outcome of the test will be \( H_0 \) if \( t < t_c \) and \( H_1 \) if \( t > t_c \) (or vice versa if it is \( H_1 \) that corresponds to lower \( t \) values). The so-called discriminant function \( T \) and the decision threshold \( t_c \) should of course be chosen so that, if possible, this test identifies the correct hypothesis for most outcomes \( g \). However, it is usually impossible to find a discriminant function that takes entirely separate values under \( H_0 \) and \( H_1 \), and so there will always be a risk that the test makes some incorrect decisions, so-called false positives and false negatives.

There are several ways of comparing the performance of different tests. One approach is to assign costs to the different outcomes (true positive, false positive, true negative and false negative) and then say that the best test is that which minimizes the expected value of the cost. This is called the Bayes criterion. Another approach, which does not require that one assigns costs to the different outcomes, is the so-called Neyman-Pearson criterion which states that the best test is that which gives maximal true positive rate for a fixed false positive rate. It can be shown that [7] there is an ideal discriminant function which is optimal under both these criteria, namely the likelihood ratio

\[
A(g) = \frac{p_{g|H_1}(g|H_1)}{p_{g|H_0}(g|H_0)} \tag{1.11}
\]

The observer that decides between \( H_0 \) and \( H_1 \) according to (1.11) is called the ideal observer. In practice, the complete probability distribution function of \( g \) is seldom known, meaning that one has to use other, nonideal, observer models to assess image quality. It also worth noting that medical images are normally intended to be used by human observers, who might perform significantly worse than the ideal observer.

In the present study we shall measure image quality by the performance of the linear observer, i.e. the observer with discriminant function given by

\[
T(g) = w^T g \tag{1.12}
\]

where \( w = (w_1, \ldots, w_N)^T \) is a vector of weights. Furthermore, we will optimize this with respect to the squared signal-difference-to-noise ratio

\[
\text{SDNR}^2 = \frac{(T(g)|H_1) - (T(g)|H_0))^2}{V[T(g)|H_0] + V[T(g)|H_1]} = \frac{(w^T g^1 - w^T g^0)^2}{w^T K^0 w + w^T K^1 w} = \frac{(w^T (g^1 - g^0))^2}{w^T (K^0 + K^1) w} \tag{1.13}
\]

where \( \Delta g = g^1 - g^0 \), \( V[\cdot] \) denotes variance and the identity

\[
V[T(g)|H_m] = \langle (w^T g - w^T \bar{g})^2 \rangle |H_m| = \langle (w^T (g - \bar{g}))^2 \rangle |H_m| = \langle w^T (g - \bar{g}) (g - \bar{g})^T w \rangle |H_m| = w^T (g - \bar{g}) (g - \bar{g})^T |H_m| w = w^T K^m w
\]

has been used. Note that an additional factor of two is included in the definition (1.13) by some authors, since this gives a neater formula in the case when \( K^0 = K^1 \). We shall show
that whenever $K^0 + K^1$ is invertible, which is the case in most practical situations, (1.13) is maximized by choosing
\[ w = (K^0 + K^1)^{-1} \Delta g \]  
(1.14)
or some scalar multiple thereof. In order to show this, we note that $((K^0 + K^1)^{-1})^T = (K^0 + K^1)^{-1}$, since $K^m$ is symmetric and the inverse of a symmetric matrix is symmetric. Now, equation (1.13) with $w$ given by (1.14) yields
\[ \text{SDNR}^2 = \frac{(\Delta g^T(K^0 + K^1)^{-1} \Delta g)^2}{\Delta g^T(K^0 + K^1)^{-1}(K^0 + K^1)(K^0 + K^1)^{-1}\Delta g} = \frac{(\Delta g^T(K^0 + K^1)^{-1} \Delta g)^2}{\Delta g^T(K^0 + K^1)^{-1} \Delta g} = \Delta g^T(K^0 + K^1)^{-1} \Delta g \]  
(1.15)
We want to show that the above expression is an upper bound for (1.13). Note that $K^0 + K^1$ has a square root matrix $(K^0 + K^1)^{1/2}$ that is symmetric and invertible (See appendix B), meaning that $w^T \Delta g$ can be rewritten as
\[ w^T \Delta g = w^T(K^0 + K^1)^{1/2}(K^0 + K^1)^{-1/2} \Delta g = \left( (K^0 + K^1)^{1/2}w \right)^T \left( (K^0 + K^1)^{-1/2} \Delta g \right) \]
Interpreting the above expression as a scalar product and using the Cauchy-Schwarz inequality (see [8], theorem 6.2.1) then gives
\[ (w^T \Delta g)^2 \leq \|(K^0 + K^1)^{1/2}w\|^2 \cdot \|(K^0 + K^1)^{-1/2} \Delta g\|^2 \leq w^T(K^0 + K^1)^{1/2}(K^0 + K^1)^{1/2} w \Delta g^T(K^0 + K^1)^{-1/2}(K^0 + K^1)^{-1/2} \Delta g \]
\[ (w^T \Delta g)^2 \leq w^T(K^0 + K^1) w \Delta g^T(K^0 + K^1)^{-1} \Delta g \]
Dividing both sides by $w^T(K^0 + K^1)w$ gives
\[ \frac{(w^T \Delta g)^2}{w^T(K^0 + K^1)w} \leq \Delta g^T(K^0 + K^1)^{-1} \Delta g \]
\[ \text{SDNR}^2 \leq \Delta g^T(K^0 + K^1)^{-1} \Delta g \]  
(1.16)
with equality if and only if $(K^0 + K^1)^{1/2}w$ and $(K^0 + K^1)^{-1/2} \Delta g$ are linearly dependent, i.e. for some real-valued constant $k$,
\[ (K^0 + K^1)^{1/2}w = k \cdot (K^0 + K^1)^{-1/2} \Delta g \]
\[ w = k \cdot (K^0 + K^1)^{-1} \Delta g \]  
(1.17)
which is the required formula.

The conclusion is that the discriminant function of the linear observer which gives optimal SDNR is obtained by substituting (1.14) into (1.12):
\[ T(g) = \Delta g^T(K^0 + K^1)^{-1} g \]  
(1.18)
The above formula is called the Hotelling observer for the problem of discriminating between $H_0$ and $H_1$. It can be shown [7] that this observer is actually equivalent to the ideal observer for the case when $g$ is multivariate normal with equal covariance but different mean under
the two hypotheses. We shall use (1.18) and the corresponding optimal weight factors (1.14) in order to measure and optimize detectability in images.

It should be pointed out that the above discussion based on SDNR is relevant only in the limit where the structures to be detected in the image are uniform over very large areas. When investigating the detectability of smaller structures, one has to compare the signal strength with the noise level for each of the spatial frequencies present in the image. In other words, it becomes necessary to use linear systems theory to investigate how signal and noise propagates through the imaging system in Fourier space. When studying x-rays, the linear systems theory approach is complicated further by the fact that the number of photons in each measurement is few enough that the flux distribution cannot be treated as a continuous function but must be represented by a point process, which is a certain kind of stochastic process whose realizations are sums of delta functions. These ideas form the basis for a useful framework which is, however, beyond the scope of this thesis. The interested reader is referred to [7].

Even though SDNR is a crude measure of imaging system performance, it is useful since it is so easy to calculate while still giving some important information about the imaging performance, at least in large homogeneous regions of the images. It is therefore useful during the process of designing an imaging system, when it is desirable to have a figure of merit in order to make design decisions quickly, without having to analyze and model the complete system. This is the main reason why SDNR is used in the present study.

1.5 Spectral computed tomography

Traditional CT scanners, which use so-called energy integrating detectors, do not take advantage of the energy information in the x-ray beam. This means that even though the x-ray source emits x-rays in a broad spectrum, the detectors are not able to differentiate between photons of different frequency but delivers an output signal proportional to the total energy of the photons registered during the readout interval. This gives a monochromatic reconstructed image representing a weighted average of the attenuation coefficients at different frequencies.

However, there is an ongoing effort to develop CT scanners that take advantage of the spectral information present in the x-ray beam after the passage through the patient. There are several approaches to designing such scanners. Devices that employ two x-ray tubes operated at different voltages and mounted in different locations on the same gantry, together with two detector arrays (so-called Dual source CT), are already in clinical use.[9] Since the two tubes emit x-rays of different spectral content, this technique gives two images which together give some spectral information about the imaged tissues. One drawback of this approach is that it suffers from larger problems with scatter than single-source CT.[10] Another approach is kVp-switching, in which a single x-ray tube is used with an operating voltage that is rapidly switched between two levels. One problem with this approach is that the scan time increases, thereby increasing the risk of motion artifacts, since the patient is not exposed to the two energy spectra simultaneously.[11] Sandwich detectors, where the x-rays first hit one layer of detectors, then a filter altering the spectrum and finally a second layer of detectors, are another alternative.[12]

The above mentioned approaches are difficult to implement for more than two different energy levels.[11]. Therefore, energy resolving photon counting detectors, which can discriminate between several different energy levels in the incoming x-ray beam, are attracting interest.[13, 11]. It is this type of spectral CT that is the subject of the present report.
The spectral information can be used in different ways to improve the detectability in the reconstructed images. One straightforward way of doing this is to sum the images acquired at different energies with weight factors optimized for the imaging task in question, a method referred to as energy weighting.

Another way to use the spectral information is to perform so-called basis material decomposition, which gives the full energy dependence of the linear attenuation coefficient in each pixel. The key fact making this possible is the observation that the attenuation coefficient $\mu(E)$ is, to good accuracy, an element of a low-dimensional vector space of functions. More precisely, for all materials with low atomic number (i.e. that do not have K-edges in the energy range of interest) $\mu(E)$ can be expressed as a linear combination of only two basis functions, while if materials containing elements with higher atomic numbers (i.e. contrast agents or metal implants) are present in the body, the attenuation coefficients of these have to be included in the basis set as well. If the number of linearly independent energy measurements (i.e. energy bins in a photon counting detector) is equal to or greater than the number of basis functions, it is possible to reconstruct basis images containing the coefficients of the different basis functions in each pixel. The final image presented to the physician will then be a weighted sum of these images, with weight factors depending on the imaging task.

Both the above methods can be used in different ways to improve the detectability, depending on the imaging task. One possibility is to choose the weight functions so that the signal-difference-to-noise ratio between two kinds of tissue is maximized. Alternatively, one can cancel the contrast between two different kinds of tissue in order to maximize the visibility of a third tissue type, e.g. a tumor. Spectral CT is especially promising for tasks involving the use of contrast agents, for example Iodine and Gadolinium, since these substances have K-edges in the energy range relevant for CT, which gives their attenuation coefficients $\mu(E)$ very specific energy dependences which can be detected easily with energy resolving detectors. In ordinary CT, on the other hand, regions with contrast agents show up as having altered CT number in the reconstructed image but do not distinguish themselves from the surrounding tissue in any other way.

1.6 Photon counting semiconductor detectors

The detector design used in the systems which are simulated in this study is the photon counting semiconductor detector. It consists of a semiconductor wafer, typically silicon, cadmium telluride (CdTe) or cadmium zinc telluride (CZT), with a high voltage (a few hundred V) applied so that the semiconductor material is highly depleted. When a photon is absorbed in the semiconductor, a large number of electron-hole pairs are created. Since a certain amount of energy is required to create each electron-hole pair, the total number of pairs created will be proportional to the incoming energy. The bias voltage applied across the semiconductor causes the electrons and holes to diffuse away from each other and be collected on the electrodes on the opposite sides of the detector, and this process gives rise to a current pulse in the readout channel connected to the electrodes. The number of pulses, and thereby the number of incident photons, is then counted by the readout electronics by comparing the height of each pulse with a predefined threshold value. It is also possible to measure the photon energies by having more than one such threshold value. The whole spectrum of energies is thereby divided into several energy bins, and the number of photons in each bin is counted. This feature is referred to as pulse height analysis (PHA). (See figure 1.6.)

The most common of all semiconductors, silicon, is an obvious candidate for making such
1.6. PHOTON COUNTING SEMICONDUCTOR DETECTORS

Figure 1.6: An energy resolving photon counting detector compares the height of the electronic pulse generated by each photon to a number of thresholds $T_1, \ldots, T_N$, so that the registered photons are sorted into different energy bins. An energy integrating detector would only measure the integral of the signal, including noise, over the readout interval $[0, t_R]$. (Here, $t_R$ denotes the readout time.)

detectors because of the very developed processing technology which exists for this material.

One problem with this material, however, is that its low absorption at the photon energies used in CT causes many photons to go straight through the detector material without interacting, unless the detector is made very thick. Furthermore, the low atomic number of silicon means that photons have a large probability of undergoing Compton scattering in such detectors. This degrades the energy resolution, since a photon can undergo multiple Compton events in the detector before being absorbed, depositing part of its energy at each interaction, so that it is registered not as one photon but as several lower energy photons in the detector. This means that the true incident energy distribution cannot be read out directly but must be statistically inferred from the registered data.[15]

The problem with Compton scattering could be circumvented by using another material in the detector. In particular, CdTe and CdZnTe have attracted interest since these materials have a lower fraction of Compton scattering and a higher absorption efficiency than silicon.[11, 15] However, the technology for processing these materials is much less mature than that for silicon, causing impurities in the produced crystals. This leads to a low drift velocity of electrons and holes in the material, so that the detector cannot resolve the photons in time if the flux rate is too high. This is actually a serious problem for the flux levels in a typical CT scan.[11, 15] Another problem with CdTe and CdZnTe is the high probability of fluorescence compared to silicon. This means that some of the energy in the original incident photon does not contribute to the number of electron-hole pairs but is emitted as a secondary, fluorescent photon which may be absorbed in another detector element or escape the detector altogether.[16] In silicon, on the other hand, the K-shell emission line energies are comparatively low (1.7-1.8 keV)[17], and any fluorescent photons emitted at these energies are absorbed before they have time to escape the detector.
The problems with CdTe/CZT detectors mean that silicon is still a feasible material for photon counting detectors, despite the Compton scatter and low detection efficiency. Since both these problems become more severe at high energies, silicon is particularly useful for imaging modalities employing low energy x rays, like mammography where it is used commercially today in the Sectra Microdose Mammography system (Sectra Mamea AB, Solna, Sweden). Pediatric CT, where lower energies are used compared to adult CT, has been identified as another likely candidate for the application of silicon semiconductor detectors. [18]

There are also some other factors lowering resolution in both silicon and CdTe/CZT detectors. If the photon energy is deposited near the border between two detector elements, one part of the charge cloud may be deposited on one electrode and the other part on the neighboring electrode, meaning that the photon is registered as two lower-energy photons. [19] This is called charge-sharing. Furthermore, the diffusion of charge within one detector element can produce a current in a neighboring element by induction, causing a double-count. [19]
Chapter 2

Description of studied CT systems

In the present study, two different CT systems were studied. The first setup is a multislice photon-counting spectral CT scanner, primarily intended for pediatric imaging, which is being developed in our group. The detector used in this setup is the photon counting silicon strip detector proposed in [18] and further studied in [19]. This kind of detector consists of a wafer of n-doped silicon, 500 µm thick, with aluminum electrodes on each side. The low detection efficiency of silicon is countered by using the wafer in an edge-on geometry, meaning that the x rays hit the wafer from the side and traverse its full length, which is 32 mm, before leaving it.

In order to reduce the count rate, each detector element is subdivided into 16 depth segments, each connected to a different channel of the readout electronics, which consists of an ASIC performing pulse height analysis on the signal. The lengths of the depth segments have been chosen so that the count rate should be equal in each of them. 50 detector elements, each 0.40 mm wide, are located in an array, giving a total detector width of 20 mm. The orientation of the detectors is chosen so that these 50 detectors allow the simultaneous acquisition of 50 slices. A photograph is shown in figure 2.1. An arc-shaped detector array is built up by stacking a large number of these detectors, with 50 µm thick tungsten sheets located in between for scatter rejection. Apart from this, no scatter rejection grid is used. In order to facilitate cooling and make place for readout circuitry, the detectors are placed in two arrays, one located behind the other, so that one half of the detectors lie in the front array and the rest lie in the back array. See figure 2.2.

This CT system is designed to be used with a cone-beam helical acquisition scheme, but in the present study only single-slice fanbeam acquisitions have been simulated. (See figure 1.5.)

The second system studied here is a partial fanbeam geometry setup designed for laboratory experiments, shown in figure 2.3. This setup uses a smaller silicon strip detector wafer, originally designed for mammography and described thoroughly in [20]. It consists of an 500 µm thick wafer with 64 50 µm wide detector elements in a row, giving a total width of 3.2 mm. This detector is also used in an edge-on geometry, but in contrast to the previous system, the detector here is oriented in the plane of the slice being measured. In other words, only a single slice is measured at each time. The detector elements of this detector are not subdivided but consist of a single depth segment, connected to an ASIC carrying out pulse height analysis on the measured data.

In order to take a picture of an extended object with a single detector, the object to be imaged has to be translated stepwise through the area between the source and the detector, then rotated a small angle and translated again, and so on until the whole required
Figure 2.1: A prototype of the silicon diode developed for photon-counting CT. The photons hit the diode at the top of the figure and move downwards.

Figure 2.2: Drawing of the CT detector, showing the edge-on orientation of the diode wafers and the double-array stacking making place for readout electronics. The x-ray source is located at the center of curvature of the arcs. (a) Perspective view. (b) View along the y-axis.
data set has been acquired. For conceptual simplicity, however, we take the view during the simulations that the object is stationary and that the source and the detector are translated and rotated. See figure 3.1. Moreover, all simulations carried out in two dimensions since the image acquisition with this system is essentially two-dimensional, even though it is possible to acquire multislice images by translating the object in the direction normal to the slices and repeating the acquisition procedure.
Chapter 3

Simulation method

Two two-dimensional acquisition geometries, corresponding to the two measurement setups described in chapter 2, were modelled in this study: fanbeam geometry with an arc detector, as shown in figure 1.5, and partial fanbeam geometry as shown in figure 3.1. The phantom was represented as a pixel image and forward projected using a discretization of the Radon transform. It should be noted that this approach to simulating the acquisition is flawed in the sense that scatter in the object to be imaged is neglected. (It is however possible to model scatter crudely by adding a certain number of counts to each measurement afterwards, which is done here.) Simulating the photon transport through the object with a radiative transfer model would be better in this respect, though more computationally demanding. For the forward projection in the fanbeam case, the fanbeam function included in the MATLAB software package was used. For the partial fanbeam geometry a forward projector based on the so-called Joseph method was implemented.

3.1 The Joseph forward projection method in the partial fanbeam geometry

In this section we describe the Joseph forward projection method, first described in [21], and its implementation in the case of a partial fanbeam geometry. Let us assume that the discretized phantom is given as a matrix \((a_{ij})\) for \(i = 1 \ldots M, \ j = 1 \ldots N\) and introduce a coordinate system \((x,y)\) with its origin in the isocenter, as illustrated in figures 3.3 and 3.4. The coordinates should be defined so that the unit distance in both \(x\) and \(y\) direction is equal to the spacing of the discretization grid. The relationship between matrix indices and the coordinates \(x\) and \(y\) will then be

\[
\begin{align*}
  x &= j - j_c \\
  y &= i_c - i
\end{align*}
\]

or, equivalently,

\[
\begin{align*}
  j &= j_c + x \\
  i &= i_c - y
\end{align*}
\]

where \(i_c\) and \(j_c\) is the row and column index, respectively, of the point of the matrix representing the isocenter. Situations where the isocenter does not coincide with one of the discretization points can be dealt with by allowing \(i_c\) and \(j_c\) to assume noninteger values.

\[\text{The Mathworks inc., Natick, Massachusetts, USA}\]
Furthermore we will denote the rotation angle of the projection by $\phi$. More precisely, $\phi$ is the rotation angle of the line between the midpoint of the detector array and the source, relative to the $y$-axis.

In order to calculate the integral over the line from source to detector, we have to find the coordinates of the source $(x_s, y_s)$ and detector $(x_d, y_d)$ as well as the angle $\gamma$ between the projection line and the $y$-axis. In order to find out these values, one first has to calculate the relevant quantities for $\phi = 0$, i.e. the unrotated source coordinates $(x_u^s, y_u^s)$ and detector coordinates $(x_u^d, y_u^d)$ as well as the rotation angle $\alpha$ of the line between them relative to the $y$-axis. In the partial fanbeam geometry, we denote by $L_1$ the orthogonal distance from the source to the $x$-axis and by $L_2$ the orthogonal distance from the detector array to the $x$-axis, so that we have $y_u^s = L_1$ and $y_u^d = -L_2$. See figure 3.2. $x_u^s$ will assume a number of discrete values $x_{u,k}^s, \ k = 1 \ldots K$ where $K$ is the number of projections, and for each of these source positions, $x_u^d$ will assume a number of discrete values $x_{d,kl}^u = x_{s,k}^u + \Delta x_l, \ l = 1 \ldots L$, where $L$ is the number of detector elements in the detector array and $\Delta x_n$ is the displacement of detector element $n$ relative to the center of the detector array. The angle $\alpha$ for each projected ray is then given by $\tan \alpha = \frac{x_{d,kl}^u - x_{s,k}^u}{L_1 + L_2}$.

If one wants to take the finite size of the x-ray tube focal spot point into account, one may easily replace each source position with a number of closely spaced subspots and let the sought projection value be the average of the line integrals from all these subspots to the detector element. In the present study, however, the source is always taken to be a single point.

After having calculated the unrotated source position $(x_u^s, y_u^s)$ and the angle $\alpha$, one can calculate the angle of the rotated projection line relative to the $y$-axis as $\gamma = \alpha + \phi$, and
the rotated source coordinates \((x_s, y_s)\) and detector coordinates \((x_d, y_d)\) by multiplication of the unrotated coordinates \((x_s^u, y_s^u)\) with a rotation matrix \(R(\phi)\):

\[
\begin{align*}
(x_s) &= \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} x_s^u \\ y_s^u \end{pmatrix} \\
(y_s) &= \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} x_s^u \\ y_s^u \end{pmatrix}
\end{align*}
\]

(3.3)

Now that the projection line has been defined, we turn to the description of the Joseph forward projection method. The first step in this algorithm is to divide the data points \(a_{ij}\) into lines which are inclined more than \(\frac{\pi}{4}\) to the projection line. That is, into vertical lines for \(\gamma \mod \pi \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]\) (figure 3.3) and into horizontal lines otherwise (figure 3.4). For convenience, we assume without loss of generality that \(x_s < x_d\) in the first case (figure 3.3) and \(y_s > y_d\) in the second case (figure 3.4). (Otherwise, one just swaps position of the detector and source, an operation which does not affect the line integral.)

The projection line is then parameterized by \(y = y_s + \tan(\gamma - \frac{\pi}{2})(x - x_s)\) in the case of figure 3.3 and \(x = x_s + \tan\gamma(y_s - y)\) in the case of figure 3.4.

This parameterization is then used to find the intersection points between the projection line and the horizontal/vertical lines. For each intersection point, the function value is calculated by linear interpolation along the vertical lines in figure 3.3 or the horizontal lines in figure 3.4. If we consider the case of figure 3.3 and assume that the intersection point has coordinates \((x, y)\) and lies on column \(j\) between rows \(i_1\) and \(i_2 = i_1 + 1\), we get the interpolated function value at the intersection point from the expression

\[
a_{i_1j} + (i_c - y - i_1)(a_{i_2j} - a_{i_1j})
\]

(3.5)

The corresponding formula for the interpolation value in the case of figure 3.4, assuming that the intersection point \((x, y)\) lies on row \(i\) between columns \(j_1\) and \(j_2 = j_1 + 1\), is

\[
a_{ij_1} + (j_c + x - j_1)(a_{ij_2} - a_{ij_1})
\]

(3.6)

Finally, the interpolated values for all intersection points are summed together and multiplied with the distance between the intersection points, which is \(\frac{1}{\cos(\gamma - \frac{\pi}{2})} = \frac{1}{\sin \gamma}\) in the case of figure 3.3 and \(\frac{1}{\cos \gamma}\) in the case of figure 3.4. This yields an approximation to the integral over the projection line in units where the discretization grid spacing is 1, and so it has to be rescaled if another unit of length is used.

3.2 Data acquisition modelling

In all simulations, tungsten anode x-ray tube spectra from [22] were used. Before entering the object to be imaged, these were filtered through 0.1 mm copper and a bowtie filter made of aluminum, designed to give a relatively uniform postpatient flux. The thickness of the bowtie filter for the different detector pixels is shown in figure 3.6. A typical example of a postpatient x-ray spectrum is shown in figure 3.5.

In principle, it would be possible to calculate the postpatient spectrum by forward projecting a large number of monoenergetic phantoms, each containing the linear attenuation coefficient \(\mu(E_n)\) for one energy \(E_n\). In order to get realistic x-ray spectra, however, one would then have to choose the separation between these discrete energy levels to be quite small, on the order of 1 keV, meaning that more than 100 monochromatic phantom images have to be forward projected. Therefore, this method is inconveniently expensive from a
computational point of view, though certainly not in impossible to use. By exploiting the fact that the attenuation coefficient $\mu(E)$ can be expressed as a sum of a small number of basis materials, however, one can calculate the full postpatient energy spectrum with only one monochromatic forward projection step per basis function. This is the method used here.

The phantom was turned into a set of basis coefficient images by least-squares fitting the attenuation coefficient $\mu(E)$ (coming from [23] and [24]) to a sum of basis materials $\sum_{i=1}^{M} a_i f_i(E)$. Two basis functions were used: $1/E^3$ for photoelectric interaction and the Klein-Nishina function\[14\] $f_{KN}$ for Compton interaction, where

$$f_{KN}(\alpha) = \frac{1 + \alpha}{\alpha^2} \left[ \frac{2(1 + \alpha)}{1 + 2\alpha} - \frac{1}{\alpha} \ln(1 + 2\alpha) \right] + \frac{1}{2\alpha} \ln(1 + 2\alpha) - \frac{1 + 3\alpha}{(1 + 2\alpha)^2} \quad (3.7)$$

In this formula, $\alpha = E/E_e$ where $E_e$ is the rest energy of the electron. The normalization of these functions may be chosen rather arbitrarily. In the simulations carried out here, the photoelectric function was normalized so that its integral from 5 keV to 140 keV is 50, while the Compton basis was normalized so that its integral over the same energy range is 1.

Each basis image was forward projected with the Joseph method, and the postpatient spectrum for each projection line was calculated as

$$\Phi(E)e^{-\sum_{i=1}^{M} a_i f_i(r)df_i(E)} \quad (3.8)$$

where $\Phi(E)$ is the prepatient flux measured in photons per detector element and measurement. Taking into account the energy dependence of the detection efficiency $D(E)$, which
3.2 DATA ACQUISITION MODELLING

\[ D(E) = e^{-\mu_a t_a} \left( 1 - e^{\mu_a t_a} \right) \] (3.10)

where \( \mu_a \) is the linear attenuation coefficient and \( t_a \) thickness of the active silicon in the detector, and \( \mu_d \) and \( t_d \) are the corresponding quantities for the dead silicon oxide layer which the photons have to pass through in order to reach the active part. For the CT detector the values \( t_a = 30.4 \text{ mm} \) and \( t_d = 0.8 \text{ mm} \) were used, whereas the corresponding values for the mammography detector were \( t_a = 12.3 \text{ mm} \) and \( t_d = 0.85 \text{ mm} \).

Scatter in the object was modelled by adding a certain number of scattered photon counts to all detectors for each angle. The number of scattered photons was chosen to be a constant (the scatter-to-primary ratio) times the average number of counts in all detectors for that angle. For the CT setup, the scatter-to-primary ratio was taken to be 1.5 %, based on Monte Carlo ray tracing calculations carried out within the group. For the partial fan beam setup, scatter in the object was neglected, i.e. it was assumed that the collimating slit present in the setup allows perfect scatter rejection.

In order to simulate the spatial blurring effect of scatter in the detector, the measured intensity was convolved with a point spread function. In a real system, charge sharing will also contribute to the point spread function, but in the CT setup, charge-sharing can only occur in the out-of-slice direction because of the way the silicon wafers are oriented. As
Figure 3.4: Geometry for the Joseph forward projection, when the projection line is close to vertical.
3.2. DATA ACQUISITION MODELLING

Figure 3.5: Tungsten anode x-ray tube spectrum, after filtration through 2 mm Al and 0.1 mm Cu.

Figure 3.6: Aluminum bowtie filter thickness as a function of detector element position. The labelling of the x-axis in this figure corresponds to a scan with 1005 detector elements.
the simulations in this study are limited to single slices, the point spread function for the CT system, shown in figure 3.7, only models scatter. It was derived from the results of the Monte Carlo ray tracing simulations described in [15]. Note that scatter only occurs between detector elements with an even number of steps between them, because the wafers are placed in two arrays which are located too far apart for scatter to take place between them. (See figure 2.2.) This function sums to 1.04, reflecting the fact that each interacting photon results in 1.04 counts on average because of the possibility of multiple interactions. The sum of the off-center elements of this point-spread function is 0.025, meaning that 2.5% of the interactions occur outside the correct detector element. There will also be out-of-plane scatter, but since the simulations carried out here are two-dimensional, out-of-plane scatter is modelled by simply multiplying the number of counts with 1.05, where the out-of-plane scatter fraction 5% was derived from the ray tracing simulations in [15].

For the detector used in the partial fan beam setup, scatter can be neglected since only a single, very thin wafer is used. However, due to the narrow size of the detector strips, it is likely that charge-sharing has a significant effect. Since there is currently no reliable estimate of the magnitude of this effect, however, it has been neglected in the present simulations, meaning that the point-spread function of the system is assumed to be ideal, i.e. 1 in the detector element hit by the projection ray and 0 in all other pixels. Admittedly, this is a gross simplification making the simulations of this setup less trustworthy.

The nonideal energy response of the silicon detectors caused by Compton scattering was simulated with a model based upon that in [15]. The number of counts for different deposited energies was calculated from

\[ N_{dep}(E_{dep}) = \int_0^\infty f_{E_{dep}|E_{act}}(E_{dep}|E_{act}) N_{act}(E_{act}) dE_{act} \]  

(3.11)

Here, \( N_{dep}(E_{dep}) dE_{dep} \) is the number of interactions in a certain measurement with deposited energies between \( E_{dep} \) and \( E_{dep} + dE_{dep} \), while \( N_{act}(E_{act}) dE_{act} \) is the number of interactions with actual photon energies between \( E_{act} \) and \( E_{act} + dE_{act} \). Furthermore, \( f_{E_{dep}|E_{act}}(E_{dep}|E_{act}) \) is the conditional probability that energy \( E_{dep} \) is deposited in an interaction event given that the actual energy is \( E_{act} \). Since the deposited energy always comes from either a photoelectric or a Compton interaction, \( f_{E_{dep}|E_{act}}(E_{dep}|E_{act}) \) is given by

\[
 f_{E_{dep}|E_{act}}(E_{dep}|E_{act}) = p_{ph}(E_{act}) f_{E_{dep}|E_{act}}^{ph}(E_{dep}|E_{act}) + p_{Co}(E_{act}) f_{E_{dep}|E_{act}}^{Co}(E_{dep}|E_{act}) 
\]

(3.12)

where \( f_{E_{dep}|E_{act}}^{ph}(E_{dep}|E_{act}) \) and \( f_{E_{dep}|E_{act}}^{Co}(E_{dep}|E_{act}) \) are the conditional probability densities for photoelectric and Compton interactions and \( p_{ph} \) and \( p_{Co} \) are the probabilities of an interaction being due to photoelectric and Compton effect, respectively. These probabilities are calculated from the cross sections \( \sigma_{ph}(E_{act}) \) and \( \sigma_{Co}(E_{act}) \) for photoelectric and Compton interaction (taken from [23]) according to

\[
 p_i(E_{act}) = \frac{\sigma_i(E_{act})}{\sigma_{ph}(E_{act}) + \sigma_{Co}(E_{act})} 
\]

(3.13)

The distribution of deposited energies for photoelectric interactions \( f_{E_{dep}|E_{act}}^{ph}(E_{dep}|E_{act}) \) is assumed to be a Gaussian centered on \( E_{act} \) with standard deviation 1.49keV for all values of \( E_{act} \). The deposited energy in the Compton interactions is given by

\[
 E_{dep} = E_{act} \left(1 - \frac{1}{1 + \alpha(1 - \cos \theta)} \right) 
\]

(3.14)
Here, $\theta$ is the deflection angle and $\alpha = E_{\text{act}}/E_e$ is the ratio of actual photon energy to the electron rest energy. The probability distribution of the deflection angle $\theta$ is proportional to the Klein-Nishina differential cross-section

$$
\left( 1 + \cos^2 \theta + \frac{\alpha^2 (1 - \cos \theta)^2}{1 + \alpha(1 - \cos \theta)} \right) (1 + \alpha(1 - \cos \theta))^{-2} \sin \theta
$$

(3.15)

The probability of a Compton interaction depositing energy between $E_{\text{dep}}$ and $E_{\text{dep}} + dE_{\text{dep}}$ can be calculated by first calculating the corresponding $\theta$ and $\theta + d\theta$ and then using 3.15 to calculate the probability of an interaction with deflection angle between these values.

The number of counts registered in energy bin $i$ is then calculated as $\int_{T_i}^{T_{i+1}} N_{\text{dep}} dE_{\text{dep}}$, where $T_i$ is threshold number $i$. Up to this point, all calculated count numbers have been expectation values. As a final step, the Poisson statistics of the photon flux is simulated by drawing the number of counts randomly from the Poisson distribution with the calculated expectation value.
Chapter 4

Image reconstruction

In this section, two methods for reconstructing spectral CT images, energy weighting and basis material decomposition, are described and compared against each other. Both these methods include ordinary filtered backprojection as an intermediate step. For filtered back-projection of the fanbeam data in the simulations of the CT system, the MATLAB built-in function `ifanbeam` was used. The partial fanbeam data was first rebinned to parallel projections (that is, to sampling points on a grid which is equispaced in \( t \) and \( \theta \)) through linear interpolation and then backprojected with the `iradon` function in MATLAB. In both cases, reconstructions were performed with a Ram-Lak filter. In order to avoid infinities in the reconstruction algorithm, all photon starved bins were set to 0.5 counts before log normalization.

When displaying energy weighted images or weighted basis material images, the usual Hounsfield unit does not have any real meaning, since the quantity displayed in the image is not proportional to the linear attenuation coefficient. If one still wants to use the Hounsfield unit to specify display windows, for example, one can define a generalized CT number scale by letting -1000 HU be the value of air and letting 0 HU be the value of water in the weighted image. The display windows of all reconstructed images in this section are specified in generalized Hounsfield units defined in this way.

4.1 Energy weighting

The most straightforward way to use the energy information to improve image quality is simply letting the final image be a weighted sum of the images acquired by the different energy bins. The weight factors should be chosen so that the contrast-to-noise ratio between the tissue types which one wants to distinguish between is maximized. Note that this could be done either with the projection data, so-called projection-based weighting, or with the reconstructed images for each energy bin, so-called image-based weighting.

Note that if filtered backprojection is used for the reconstruction, the reconstructed image related to the log-normalized projection data \(-\ln(N/N_0)\) by a linear transformation. (Here, \(N\) denotes the registered number of counts and \(N_0\) denotes the number of photons in the unattenuated beam.) This means that image-based weighting can be done by weighting together the log-normalized projections and then carrying out a single reconstruction step, instead of carrying out one separate reconstruction step for each energy bin, which would be computationally expensive.

It was shown in [25] that there exists a tradeoff between beam-hardening artifacts and signal-to-noise ratio in projection-based energy weighting, since optimally weighted im-
CHAPTER 4. IMAGE RECONSTRUCTION

Images exhibit more severe beam hardening artifacts than images acquired with conventional energy-integrating detectors. However, it was pointed out in [26] that this tradeoff is eliminated by using image-based weights instead, since this gives images almost free of beam-hardening artifacts and the same level of signal-to-noise-ratio as projection-based weighting. Therefore, only image-based weighting has been used in the present study.

In the present study, the means and variances of the images, which are necessary in order to find the optimal weight function, are measured in regions of interest in the images.

4.2 Basis material decomposition

Unlike energy weighting which only gives one image as its output, basis material decomposition allows us to infer the attenuation coefficient $\mu(E)$ in all parts of the image volume and for all energies $E$ in the relevant range. As explained in section 1.5, this is made possible by the fact that the attenuation coefficient can be expressed as $\mu(E) = \sum_{i=1}^{M} a_i f_i(E)$ where $M = 2$ is enough for most situations although three or four basis functions $f_i$ may be required if there are metal objects or contrast agents present within the body. Let $\Phi(E)$ be the energy density of the photon flux incident on the object, meaning that the number of photons per pixel and readout time with energy between $E$ and $E + dE$ is $\Phi(E) dE$. Also let $D(E)$ be the detection efficiency of the detector as a function of incoming photon energy and let $S_j(E)$, the bin response function of bin $j$, be the expected number of counts in bin $j$ when one photon with energy $E$ interacts with the detector. (Ideally $S_j(E)$ should be 1 between the bin thresholds and 0 outside, but by using other bin response functions one can model various imperfections such as compton scatter, charge sharing and nonideal energy resolution.) For the projection along the line $\ell$, the expected number of counts in bin $j$, $j = 1, \ldots, N$ will be

$$\lambda_j = \int_{0}^{\infty} \Phi(E) e^{-\int_{\ell} a_i(r) f_i(E) d\ell} D(E) S_j(E) dE$$

(4.1)

Defining $A_i = \int_{\ell} a_i(r) d\ell$, we can rewrite this expression as

$$\lambda_j = \int_{0}^{\infty} \Phi(E) e^{-\sum_{i=1}^{M} A_i f_i(E) D(E) S_j(E) dE}$$

(4.2)

Performing basis material decomposition now amounts to estimating the basis coefficient projections $A_i$ from the registered counts $m_j$, $j = 1, \ldots, N$. If these are recovered, then the basis coefficients $a_i$ can be recovered using one filtered backprojection step for each basis function. The resulting images can then be weighted together to single image with optimal SDNR$^2$ for some specified imaging task. In the weighted basis material images found in section 4.3, the optimal weight factors have been calculated from regions of interest in the images.

In the present study, the maximum likelihood (ML) method was used to estimate the basis coefficient projections $a_i$. In order to find the ML estimator, we make the modelling assumption that $m_j$ are Poisson distributed and independent, even though this is unlikely to be strictly true in the presence of scatter and charge-sharing. For brevity we will write $A = (A_1, \ldots, A_M)$ and $m = (m_1, \ldots, m_N)$. The ML estimate of $A$ is then the argument that maximizes the likelihood function

$$\mathcal{L}(A) = f_m(m|A) = \prod_{j=1}^{N} \frac{\lambda_j(A)^{m_j}}{m_j!} e^{-\lambda_j(A)}$$

(4.3)
4.3. **COMPARISON OF ENERGY WEIGHTING AND BASIS MATERIAL DECOMPOSITION**

where \( f_m(m|A) \) denotes the conditional probability distribution of \( m \) given \( A \). This is equivalent to minimizing the negative log-likelihood function

\[
-\ln \mathcal{L}(A) = \sum_{j=1}^{N} \lambda_j(A) - m_j \ln \lambda_j(A) + \ln(m_j!)
\]

(4.4)

The last term can be dropped since it is independent of \( A \). Accordingly, the ML estimate is given by that \( A \) which minimizes

\[
\ell(A) = \sum_{j=1}^{N} \lambda_j(A) - m_j \ln \lambda_j(A)
\]

(4.5)

where \( \lambda_j(A) \) is given by (4.2).

The optimization problem (4.5) can be solved with any optimization algorithm. In the present study, the downhill simplex method [27] was used, in the implementation of [28] with slight modifications. This algorithm is probably not the fastest available algorithm for this problem but it is known for being easy to get working for most problems. For a detailed description, see appendix A.

The basis materials in this study were the same two functions, representing the photoelectric and Compton effects, that were used for the forward projection (see section 3.2).

### 4.3 Comparison of energy weighting and basis material decomposition

In order to compare energy weighting and basis material decomposition, both fanbeam scans with the CT scanner and partial fanbeam scans with the laboratory setup have been simulated. Detectability in the images is quantified by the single-pixel squared signal-difference-to-noise ratio, which is calculated by the formula

\[
\text{SDNR}^2 = \frac{(\bar{T}_{\text{obj}} - \bar{T}_{\text{bg}})^2}{\sigma_{\text{obj}}^2 + \sigma_{\text{bg}}^2}
\]

(4.6)

which is (1.13) in the special case of a single input channel. Here, \( \bar{T}_{\text{obj}} \) and \( \bar{T}_{\text{bg}} \) are the expectation values of the pixel value in the object and background part of the image, respectively, and \( \sigma_{\text{obj}}^2 \) and \( \sigma_{\text{bg}}^2 \) are the corresponding variances. In the present study these were estimated from the \( 17 \times 17 \) (for the thorax phantom) and \( 26 \times 26 \) (for the knee phantom) pixel regions of interest shown in figures 4.1 and 4.4.

The SDNR\(^2\) values have statistical uncertainties caused by the randomness of the pixel values which they are estimated from. In order to find the precision of these estimates, approximate 95% confidence intervals for SDNR\(^2\) were calculated by filling 100000 pairs of square images, of the same size in pixels as the regions of interest, with stationary Gaussian white noise of mean and variance equal to the estimated mean and variance in the regions of interest in the image. For each pair of square images, one is filled with noise with the mean and variance of the background region of interest and the other is filled with noise with mean and variance corresponding to the target region of interest. By calculating the SDNR\(^2\) for each such simulated image pair, the standard deviation of the estimator, and thereby the confidence interval, can be calculated. It should be noted that this analysis can only quantify the uncertainty stemming from the noise in the images, and not SDNR\(^2\) inaccuracies caused by artifacts in the image.
CHAPTER 4. IMAGE RECONSTRUCTION

Figure 4.1: (a) FORBILD thorax phantom. (b) close-up view of the central parts of the phantom, with the regions of interest for heart and background tissue shown.

It should also be kept in mind that the ability of an observer to detect an object depends not only on the single-pixel SDNR$^2$ but also on the area of the object.

For the fanbeam scans, the FORBILD thorax phantom [29] has been used. This phantom, which is shown in figure 4.1a, models the thorax as a water filled ellipse with axis lengths 200 mm and 400 mm respectively containing features modelling lungs, ribs, vertebra, heart and aorta. The phantom was discretized on a $512 \times 512$ pixel grid before forward projection, and the reconstruction was also made on a $512 \times 512$ grid. Figure 4.1b shows the regions of interest used for computing optimal weight functions and calculating SDNR$^2$.

Figure 4.2 shows the reconstructed image of a simulated fanbeam scan of the thorax phantom with 1005 detector elements and 1000 rotation angles covering $360^\circ$. The prepatient photon fluence was $2.86 \cdot 10^3$ and $9.80 \cdot 10^4$ photons per detector element and measurement behind the thickest and the thinnest part of the bowtie filter, respectively, which is in the range used in clinical CT examinations. A 120 kV x-ray tube spectrum was used, and the energy bin thresholds were set to 5, 15, 20, 35, 50, 60, 70 and 75 keV. The distance from the source to the center of rotation was 320 mm.

The energy weighted images are shown in figure 4.2a-b. The heart is only barely visible in the display window of figure 4.2a, but can be more clearly seen if the same image is displayed using a narrower display window as in figure 4.2b. Basis material decomposition was performed on the same simulated data, and the resulting photoelectric and Compton basis images are shown in figure 4.2c-d. The result of optimal weighting of these two basis images is shown, for two different display windows, in figure 4.2e-f. In the energy weighted image, SDNR$^2 = 0.46$ (95% confidence interval: 0.30-0.63) between heart and background, whereas SDNR$^2 = 0.40$ (95% confidence interval: 0.25-0.56) in the weighted basis material image.

In order to investigate how sensitive the two investigated reconstruction methods are to bad photon statistics, the same study as above was repeated for a simulated fanbeam scan with 1005 detectors and 5000 rotation angles covering $360^\circ$. The prepatient fluence was $5.71 \cdot 10^2$ and $1.96 \cdot 10^4$ photons per detector element and measurement behind the thickest and the thinnest part of the bowtie filter, respectively. All other parameters were the same as in the previous simulation. The result is shown in figure 4.3. The energy weighted image
4.3. COMPARISON OF ENERGY WEIGHTING AND BASIS MATERIAL DECOMPOSITION

Figure 4.2: Simulated scan of the thorax phantom with 1005 detector elements and 1000 rotation angles. (a),(b) Energy weighted image. (c) Photoelectric basis image. (d) Compton basis image. (e),(f) Weighted basis material image.
is shown with two different display windows in figures 4.3(a)-(b) while the weighted sum of the basis material decomposed images is shown for the same two windows in figures 4.3(c)-(d). SDNR$^2$ between the heart and the surrounding tissue is 0.40 (95% confidence interval: 0.25 - 0.56) in the energy weighted image and 0.60 (95% confidence interval: 0.42 - 0.80) in the weighted basis material image.

Evidently, the weighted basis material image exhibits severe streaking artifacts along those projection lines where the absorption in the phantom is the largest. This is a sign of that the maximum likelihood estimation of the basis coefficients becomes very unreliable when the number of registered counts is low, causing some estimated values of the basis coefficient projections $A_i$ to deviate very much from the true values. It should be noted that the severity of these streaks can be reduced very much by postprocessing of the sinogram. In order to show this, the two basis material sinograms were modified by forcing the basis coefficient projections to lie within the interval $-100 \leq A_i \leq 1000$ for both basis functions, so that all sinogram values falling outside this range were replaced by either -100 or 1000. (Note that the magnitudes of the basis projections $A_i$ depend on how the normalizations of the basis functions are chosen, which is specified in section 3.2.) These thresholds were selected by plotting histograms of the registered $A_i$ values and picking the cutoff values so that apparent outliers in the histograms would be eliminated. Then new reconstructions were made, and the resulting weighted basis image is shown in figure 4.3(e)-(f). Here SDNR$^2 = 0.39$ (95% confidence interval: 0.24 - 0.55), measured from the same regions of interest as before, meaning that the artifact correction method has affected the regions of interest, even though these are located outside the area with streak artifacts. As can be seen in these images, the streaks were greatly reduced by this method, even though they are still present and degrade the image quality. There is possibility, however, that they may be reduced even further by selecting the sinogram cutoff thresholds very carefully in order to optimize image quality.

A scan was also simulated in the partial fan-beam geometry of the laboratory setup. For this simulation, the knee prosthesis phantom shown in figure 4.4 was used. It is modelled after the real physical phantom shown in figure 6.1, and it consists of a water filled PMMA tube, 90 mm in diameter. In the center of the tube, a piece of PMMA, shaped as the plastic insert of a knee joint prosthesis, is located.

The simulation was made with a 120 kV x-ray tube spectrum and bin thresholds located at 5, 15, 20, 35, 50, 60, 70 and 75 keV. The prepatient photon fluence was $1.00 \times 10^4$ photons per detector and measurement. Data was acquired for 180 rotation angles covering 180$^\circ$ and 63 detector positions, which gives 4032 measured projection lines per rotation angle since the detector consists of 64 elements. The distance from the source to the isocenter and the distance from the isocenter to the detector were both 330 mm. The phantom was discretized on a $276 \times 276$ pixel grid before forward projection. Because of limitations in the implementation of the MATLAB built-in function ifanbeam, reconstructions had to be performed on a $4000 \times 4000$ pixel grid. They were then scaled down to $512 \times 512$ pixels before the region-of-interest measurements and weighting.

The resulting energy weighted image and weighted basis image are shown, for two different display windows, in figure 4.5. The SDNR$^2$ between the plastic insert and the surrounding water is 0.099 (95% confidence interval: 0.052 - 0.15) in the energy weighted image and 0.096 (95% confidence interval: 0.050 - 0.15) in the weighted basis material image.

In all the above simulations, a significant difference in SDNR$^2$ between the two methods has been found only in one case: the simulated thorax scan with 5000 angle measurements. Here, the weighted basis material image without streak artifact reduction has a higher SDNR than the corresponding energy weighted image, despite the presence of streak artifacts. As can be seen in figure 4.3(c)-(d), there is a quite large area surrounding the streak artifacts.
4.3. COMPARISON OF ENERGY WEIGHTING AND BASIS MATERIAL DECOMPOSITION

Figure 4.3: Simulated scan of the thorax phantom with 1005 detector elements and 5000 rotation angles. (a), (b) Energy weighted image. (c),(d) Weighted basis material image. (e),(f) Weighted basis material image where streak artifact reduction has been carried out by enforcing $-100 \leq A_i \leq 1000$ for $i=1,2$. 
Figure 4.4: Knee prosthesis phantom. The two squares mark the regions of interest for plastic and background and do not represent any features in the phantom.

Figure 4.5: Simulated scan of the knee prosthesis phantom with 4032 detector elements and 180 rotation angles.
artifacts where the CT number is artificially lowered. This is an effect of the filtering step preceding backprojection, acting on the corrupted projection values in the sinogram where the maximum likelihood basis material decomposition method has failed. Even though it is not clearly seen in figure 4.3(c)-(d), a careful investigation shows that this artifact is responsible for incorrectly inflating the SDNR\textsuperscript{2} value by affecting the two regions of interest differently.

In conclusion, it is evident from figures 4.3(c)-(d) that the basis material decomposition method stops working when the number of registered photons becomes too low, i.e. when the statistics becomes too poor. This indicates that energy weighting should be the more reliable method of the two in scans of thick or highly attenuating objects and scans with a large number of measured lines with few photons in each. Regarding the image quality outside the areas with streak artifacts, no significant differences in SDNR\textsuperscript{2} were detected between the two methods except in one case, where the SDNR\textsuperscript{2} estimate was incorrect due to the presence of artifacts. In order to find such a difference, if one exists at all, it would be necessary to get more narrow confidence intervals for SDNR\textsuperscript{2} by carrying out several simulations of each image.
Chapter 5

The effect of electronic noise and threshold variations on image quality

5.1 General remarks on electronic noise and threshold variations

In the process of designing a photon counting detector, it is important to understand how imperfections in the readout electronics affect the image quality. In this section, the two most important imperfections, electronic noise and threshold variations between pixels, are studied.

Figure 5.1 is a simple schematic of one input channel custom-made ASIC which was used for the experiments in the present study. In each channel, the electrical input signal from the silicon wafer is amplified in a charge sensitive amplifier (CSA) and passed through a shaping filter (SF) in order to get a suitable pulse shape. The pulse height is then compared to a number of thresholds and registered as a count in the appropriate energy bin. After a certain time, a reset signal forces the output of the filter to become zero, which allows a new pulse to be registered without interfering with the tail of the previous one.

One such readout channel is connected to each depth segment of the CT detector wafer described in 2.

The effect of electronic noise in this circuit is to alter the pulse height of each pulse randomly. Furthermore, a count may be registered even though no photon has been detected, if the background noise happens to reach above the lowest threshold at some point. It is sometimes stated that photon-counting detectors allow for complete rejection of electronic noise.

Figure 5.1: Block diagram of one input channel of the ASIC.
noise by placing the lowest threshold so high that this does not occur [11], but in this case, where it is desirable to register Compton interactions with very low deposited energy, one cannot get rid of electronic noise counts completely.

Another imperfection in the ASIC that has been observed is that all the input channels have different zero levels. Since the threshold levels are not set individually but are common to all the 160 channels in the ASIC, this means that the location of the noise floor and the peaks relative to the thresholds vary between channels. See figure 5.2. From a user's point of view, this is equivalent to the signal level being fixed and the thresholds varying between pixels. This phenomenon will therefore be called threshold variation in the following.

Threshold variations are detrimental to the quality of the image in several ways. They reduce energy resolution since it becomes impossible to define all the bin edges to be at the same energy, which is desirable in K-edge imaging. Also, they cause signal variations between neighboring detector elements. In circular and single-slice helical CT this gives rise to ring artifacts in the reconstructed image, since each detector element contributes to pixels located in a circle around the isocenter in the image. In multislice helical CT one detector element contributes to a helical set of image pixels meaning that the artifacts will be helical and visible as partial rings in each slice. Since both whole and partial rings are very disturbing for human observers and could possibly be mistaken for a pathological conditions, it is very important to keep the variations in response of different detector elements as small as possible and to apply software corrections to further reduce these inhomogeneities. Another effect of threshold variations is that they make rejection of electronic noise more difficult. Even if the mean threshold is chosen so that a good tradeoff between the number of noise counts and the number of missed real Compton counts is reached, some channels will be completely contaminated by noise while others will miss a good deal of the Compton events.

Measurements on the ASIC, performed with the input channels connected to the detector wafer, show that the standard deviation of the threshold offsets is 4.12 mV. They also show that the electronic noise level (i.e. the standard deviation of the noise signal) of each channel can be modelled as a number randomly drawn from a probability distribution with mean 1.58 mV and standard deviation 0.14 mV. In order to facilitate comparison to the threshold levels, these values will in the following be measured not as voltages in mV but as the equivalent photon energies in keV, calculated with the measured conversion factor 1.73 mV/keV. Note that the quite large threshold variation measured here applies to a prototype version of the ASIC and is expected to be greatly reduced in the final version.

In the detector model used for the simulations below, a maximum of 143 counts can be registered during one data acquisition interval, while the average number of real incident photons during such an interval is approximately 2. If, for example, the average lowest threshold for all channels is placed at 5 keV and the threshold spread standard deviation is \( \frac{4.12 \text{ mV}}{1.73 \text{ mV/keV}} = 2.38 \text{ keV} \), as measured, then the lowest threshold will be located below 0 keV, i.e. below the noise floor, in 1.8% of the channels. In these channels, the number of electronic noise counts in the lowest bin will be more than half the maximum number of counts, i.e. much more than the number of real counts. This example shows how extremely sensitively the noise level depends on the threshold.

5.2 Electronic noise and detectability

An important question is how much the electronic noise level affects the quality of the image. In this section, the detectability is evaluated for two different noise levels: the measured value
5.2. ELECTRONIC NOISE AND DETECTABILITY

\[ \sigma_n = \frac{1.58 \text{ mV}}{1.73 \text{ mV/keV}} = 0.91 \text{ keV} \] and the 20% lower value \( \sigma_n = 0.73 \text{ keV} \), which is believed to be an attainable value with further modifications to the ASIC.

The imaging task of detecting 0.5 cm bone in 25 cm soft tissue was chosen, meaning that we want to discriminate between the cases (1) the object case: 0.5 cm bone and 24.5 cm soft tissue, and (2) the background case: 25 cm soft tissue. In this study it is assumed that the detector is operating in pure photon-counting mode, since the thing of interest here is the advantage of being able to capture more low energy counts - i.e. counts from Compton interactions, which carry little energy information anyway. The conclusions that can be drawn for pure photon-counting mode should therefore also be valid, at least qualitatively, for energy weighting mode.

As a measure of signal detectability, the squared signal-difference-to-noise ratio of a single channel will be used. In pure photon counting mode, one channel only measures one value: the number of counts \( N \). In the terminology of section 1.4, both the data vector \( g^m \) and the covariance matrix \( K^m \) consist of a single element each, which is the number of counts \( N \) and the variance \( \sigma^2 \) of the number of counts, respectively. So equation (1.13) reduces to

\[ \text{SDNR}^2 = \frac{(N_{\text{obj}} - N_{\text{bg}})^2}{\sigma_{\text{obj}}^2 + \sigma_{\text{bg}}^2} \]  

(5.1)

where \( N_{\text{obj}} \) and \( \sigma_{\text{obj}}^2 \) is the expectation value and variance of the number of photon counts in one channel with the object present and \( N_{\text{bg}} \) and \( \sigma_{\text{bg}}^2 \) are the corresponding
CHAPTER 5. THE EFFECT OF ELECTRONIC NOISE AND THRESHOLD VARIATIONS ON IMAGE QUALITY

quantities when the object is absent. (The use of $\text{SDNR}^2$ can be justified by pointing out that $\text{SDNR}^2$ is proportional to the integral of the noise equivalent quanta (NEQ) over all frequencies, which should be a relevant quantity to consider when discussing detectability without taking spatial dependence into account. See [7], eq. 13.243.)

One has to keep in mind that the quantity studied here, which is the $\text{SDNR}^2$ for one of the 16 depth segments in a single detector element of the CT detector, is much too low for distinction between the two cases to be possible in practice. However, if one considers using a large number of independent detector channels, all with equal incoming photon distribution, the data vector $\mathbf{g}$ will have independent identically distributed elements, the covariance matrices $K^0$ and $K^1$ will be diagonal and the optimal weighting scheme will be flat weighting, i.e. just summing the counts from all different channels. This gives an $\text{SDNR}^2$ value which is just the expression in (5.1) multiplied by the number of channels. Therefore, studying the single-channel $\text{SDNR}^2$ is sufficient for performance comparison purposes.

For the simulations a 120 kVp spectrum was used, with a postpatient photon flux of 240 kcps per detector element in the background case. In the CT machine this will correspond to one of the lowest flux regions behind the patient - in most of the projection lines through the patient, more photons than this are transmitted. The data acquisition time is assumed to be 10 $\mu$s. The noise in the detector is modelled as follows: Every clock cycle (10 ns), a noise level is sampled from a Gaussian white noise distribution with zero mean and variance $\sigma_n^2$. (The slight variation in noise level between the detector elements is not included in the model.) The simulation program also checks if a photon is detected during that particular clock cycle, and if such an event occurs, the deposited energy of the event is drawn randomly from the appropriate distribution. The sum of the noise level and the deposited energy is checked against a threshold value. If it is above the threshold, a count is registered and the detector will be “dead”, i.e. register no counts, for 70 ns.

The simulation described above was performed for different threshold values in 1 keV steps from 0 keV to 6 keV. For each threshold value, $10^7$ acquisition intervals were simulated, and the mean and variance of the number of registered counts in one acquisition interval were computed. The result for two noise levels $\sigma_n$ is shown in figure 5.3. The red dashed curve corresponds to the measured noise level $\sigma_n = 0.91$ keV, while the green solid curve corresponds to the 20% lower value $\sigma_n = 0.73$ keV. As seen in the figure, a 20% noise reduction would change the optimal lowest threshold from 5 keV to 4 keV and thereby improve $\text{SDNR}^2$ by 3.7%, from 0.00156 to 0.00161. As a simple check of this result, note that the ideal (no electronic noise) $\text{SDNR}^2$, which is obtained by putting $\sigma_{\text{obj}}^2 = \overline{N}_{\text{obj}}$ and $\sigma_{\text{bg}}^2 = \overline{N}_{\text{bg}}$ in (5.1), is proportional to the number of photons, which increases with about 5% when the threshold is lowered from 5 keV to 4 keV. The ideal $\text{SDNR}^2$ is the blue dotted curve in figure 5.3.

So far, no threshold variations have been taken into account. In order to quantify the effect of these, we now assume that there are a number of thresholds and take the viewpoint that the threshold variations simply move all these thresholds uniformly by a certain displacement. We then introduce the following rule for rejecting electronic noise: the photon counts in any bin whose lower threshold, after adding the random offset, lies lower than a certain limit value, are discarded. (This limit value will be referred to as the “limit for throwing away bins” in the following.) In this way one can counter the electronic noise contamination problem by putting a number of small bins near the intended lowest threshold and throwing away those corrupted by noise. One should then rescale the remaining number of counts by a suitable factor in order to estimate the total number of real counts which would have been registered if the lowest energy events had not been thrown away. However, such a scaling leaves $\text{SDNR}$ unchanged and therefore it does not have any impact on the...
5.2. ELECTRONIC NOISE AND DETECTABILITY

As a measure of performance in the presence of threshold variations we take the average of 10000 SDNR\(^2\) values, interpolated from figure 5.3, with random threshold offsets drawn from a Gaussian distribution with mean 0 and standard deviation equal to the measured value \(4.12 \text{ mV} / \text{keV} = 2.38 \text{ keV}\). (The mean can be taken to be zero, since any nonzero mean threshold offset can be compensated for by moving all thresholds uniformly.) When the threshold falls below 0 keV, SDNR\(^2\) is taken to be 0, and when it falls above 6 keV it is assumed to be given by the ideal (no electronic noise) expression, since figure 5.3 suggests that the number of noise counts are few enough for the noise-free formula to be valid for thresholds located at approximately 5 keV and higher.

It is now an optimization problem to find the threshold configuration and the limit for throwing away bins that give the highest average SDNR\(^2\). The higher thresholds do not matter much since all nondiscarded energy bins are treated equal in pure photon counting mode, and they were set to 16, 20, 35 and 110 keV here. A rather coarse exhaustive search was carried out over the lower bin thresholds. In this search, the lowest bin threshold could be located at 2, 3, 4, 5 or 6 keV, and 0, 1, 2 or 3 small bins of size 1, 2 or 3 keV were inserted just above this lowest threshold. The limit for throwing away bins, which could be anything between 1 and 15 keV in 0.5 keV steps, was also included in the parameter search.

The outcome of this optimization is shown in table 5.1. Also, the dependence of SDNR\(^2\) on the limit for throwing away bins is shown in figure 5.4, for the two bin configurations that are optimal for each noise level. By lowering the noise by 20%, one can get 1.8% higher average SDNR\(^2\).

One conclusion which can be drawn from these simulations is that lowering the electronic noise by 20%, which is a substantial improvement from a readout electronics point of view,
results in a very small detectability improvement of 1.8%, with threshold variations taken into account. It also seems that the threshold variations deteriorate the detectability more for the lower of the two noise levels, so that the gain in performance one would get by lowering electronic noise 20% would become larger if the threshold variation could be decreased. One should keep in mind that even a gain in detectability of only a few percent could make a large difference to a human observer in some cases, when there is a feature in the image just on the verge of being distinguishable from the background.

Another conclusion suggested by these results would be that the problem with contamination of some channels by electronic noise, which is caused by the threshold variations, can be effectively countered by putting several small bins just above the lowest threshold and discarding those which get too much electronic noise. If this were not the case, SDNR$^2$ would be much lower when threshold variations are taken into account than when they are not, but SDNR$^2$ is only lowered by 0.5%, from 0.00156 to 0.00155, for the higher noise level and by 2.4%, from 0.00161 to 0.00158, for the lower noise level.

However, here it must be mentioned that it has recently come to the author’s attention that there is a discrepancy between the model used here and the behaviour of the real physical ASIC. In the real ASIC, the lowest threshold $T_1$ has a special status, since the registration of a pulse higher than $T_1$ triggers a procedure in the ASIC where the peak of the pulse is searched for during a certain number of clock cycles, whereafter the channel is unable to register any counts at all during a certain reset time. For channels where $T_1$ is located close to or below the noise floor due to the threshold variations, this cycle will be triggered very often. Since pulses with heights corresponding to the higher energy bins will be missed if they occur during the reset part of this cycle, there will be a decrease in detection efficiency in the higher bins which can probably approach tens of percent, if $T_1$ falls below the noise floor so that the ASIC registration cycle is triggered again as soon as it has finished. It is also likely that some of the events that occur during reset will be registered but with the wrong energy. In order to find out how large detrimental effect this has on the signal-difference-to noise ratio, a further study including more sophisticated modelling of the ASIC input channel would be necessary.

Despite this discrepancy between the model and the real ASIC, the results for the single-channel SDNR$^2$ presented here should be valid because the effects discussed in the previous paragraph do not make a difference to the model when the detector is operated in photon counting mode and no bins are discarded. In particular, the result that lowering noise by 20% gives a 1.8% gain in SDNR$^2$ is unaffected by the above discussion. However, the detrimental effect of threshold variations on SDNR$^2$ is likely to be larger than what is suggested by table 5.1 and figure 5.4. Therefore, the scheme described here, i.e. putting small bins near the lowest threshold and discarding some of them, is probably not as effective in countering the problem of contamination with electronic noise in the real ASIC as would be suggested by the above results.

One should also keep in mind that the ring artifacts are likely to be an even more important effect of threshold variations on image quality than this noise contamination.

<table>
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<tr>
<th>$\sigma_n$ (keV)</th>
<th>Optimal bin thresholds (keV)</th>
<th>Optimal limit (keV)</th>
<th>Maximal SDNR$^2$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>4, 6, 8, 10</td>
<td>4</td>
<td>0.00155</td>
</tr>
<tr>
<td>0.73</td>
<td>3, 5, 7, 9</td>
<td>3.5</td>
<td>0.00158</td>
</tr>
</tbody>
</table>

Table 5.1: Optimal lower bin thresholds and optimal limit for throwing away bins, with the corresponding SDNR$^2$ values, for the two different noise levels.
Figure 5.4: Average SDNR$^2$ for 1000 channels, with threshold variations taken into account, as a function of the limit for throwing away bins. Note that the two curves correspond to different bin configurations, optimized for the two different noise levels.
Chapter 6

Experimental results

The partial fan beam geometry laboratory setup described in section 2 and shown in figure 2.3 was used for taking images of a knee prosthesis phantom. This phantom consists of a knee joint prosthesis (femur 5500-F-401 4Lt, Tibia 5520B400 No SK 3HT, manufactured by Stryker Corporation, Kalamazoo, Michigan, USA) immersed in water and contained in a plastic tube, 90 mm in diameter. The knee prosthesis consists of an upper (femoral) and a lower (tibial) metal part made of vitallium (a metal alloy consisting of mainly chromium and cobalt) and a plastic insert (ultra-high molecular weight polyethylene) between the two metal parts. This phantom was chosen in order to test the effect of energy-discriminating computed tomography on metal artifacts.

The knee was scanned by translating it through the field of view in 61 steps, with a translation step length of 1.671 mm. For each of these translation steps, the knee was rotated 200° in a continuous rotation movement during which 8190 measurements were made, with an exposure time of 1.3 ms for each measurement. As the X-ray source, an X-ray Module XRS-160 MXR-160HP/11 from Comet AG, Flamatt, Switzerland was used, with 120 kV tube voltage and the current 5 mA. The eight bin thresholds were distributed between 0 and 120 keV so that the average number of counts in the different bins were

Figure 6.1: Knee prosthesis phantom
CHAPTER 6. EXPERIMENTAL RESULTS

Figure 6.2: CT images of the knee phantom, taken with the partial fan-beam-geometry laboratory setup in pure photon-counting mode. (a) Without ring artifact correction. (b) With ring artifact correction.

approximately the same. Since the experiment was carried out with a prototype version of the ASIC, the exact location of the energy thresholds is not known.

Out of the 8190 angular sampling points, 7390 sampling points covering 180° were used in the reconstruction. In order to decrease the relative statistical error of the measurements and reduce the computational load, the measurements were summed over in the angular direction in sets of 10, giving a total of 61 \cdot 64 = 3904 data points in the detector direction and 739 data points in the angular direction. In this early stage of the experiment, the reconstructions are made in pure photon counting mode, meaning that the counts from all bins are summed before reconstruction. The unattenuated number of counts $N_0$ for each detector element was measured from an air scan of 2000 measurements, each with 1.3 ms exposure time. Since each rotation takes seven minutes to acquire because of long readout times in the current hardware implementation and since a drift in the count rate over time was observed, one such calibration had to be made for each translation step. (The drift in count rate is believed to be caused by variations in the x-ray flux from the tube and probably also by threshold drifts caused by temperature changes in the ASIC.)

The projection $p = \int \mu ds$ corresponding to each measurement was then calculated as $p = -\ln \frac{N}{N_0}$, where $N_0$ is the unattenuated number of counts for the particular detector element that made the measurement. In this way, the difference in individual response of the detector elements caused by threshold variations is compensated for to a large extent. In order to avoid infinities, all photon starved bins were set to 0.5 counts before log normalization.

Finally, the projections were rebinned onto a grid equally spaced in $t$ and $\theta$ (3781 points in the $t$ direction and 739 points in the $\theta$ direction) through linear interpolation and then reconstructed with the MATLAB built-in function `ifanbeam`, using a Ram-Lak filter. The reconstruction was made on a $4000 \times 4000$ pixel grid, and then the image was rescaled to $400 \times 400$ pixels for display.

A reconstructed slice of the knee is shown in figure 6.2a. This image clearly shows the shape of the plastic insert, as well as the three rubber bands that hold the upper and lower parts of the knee together. This slice is located between the tibial and femoral metal parts, but a faint metal artifact is visible as a white shadow over the lower left part of the plastic.
insert, showing that the femoral metal part barely touches the measured slice.

The prominent ring artifacts in figure 6.2a are caused by inhomogeneities in the detector element responses, most likely caused by the threshold variations in the ASIC. In future studies, the plan is to use more than 2000 measurements of the unattenuated beam in order to improve statistics and to acquire several flatfield scans where the beam has been passed through different thicknesses of some homogeneous material. This approach has proven successful in scans of a small plastic object but has not been implemented for the knee image yet because the longer scan duration for the knee means that a larger number of calibrations are needed, thereby further prolonging the scan time.

It is possible to remove ring artifacts to some extent by postprocessing. Figure 6.2b shows an image reconstructed from the same data as figure 6.2a but with the following ring artifact reduction scheme applied: The sinogram of \( p \) values, after rebinning into parallel-beam geometry, was summed in the angular direction creating a sinogram profile. This profile was then smoothed by Gaussian filtering, and the sinogram was then renormalized by multiplying all projections corresponding to a certain \( t \) value by the smoothed sinogram profile value divided by the original sinogram profile value for that particular \( t \). Expressed in formulas, this means that the original sinogram value \( p(t_i, \theta_j) \) is replaced by the renormalized value \( \hat{p}(t_i, \theta_j) \) given by

\[
\hat{p}(t_i, \theta_j) = p(t_i, \theta_j) \frac{h(t_i) \ast \left( \sum_{j'} p(t_i, \theta_j') \right)}{\sum_{j'} p(t_i, \theta_j')}
\]

where \( \ast \) denotes convolution in the \( t \) direction and \( h(t_i) \) denotes the impulse response of the filter used to smooth the sinogram profile.

During the development of this filtering technique, it was observed that secondary artifacts occur in the outermost part of the cylinder if it is filtered too heavily, whereas the inner parts of the cylinder seem to be less sensitive to this problem. Therefore the part of the sinogram corresponding to the outer regions of the cylinder (\(-44.3 \, \text{mm} < t < -37.7 \, \text{mm}\) and \(35.8 \, \text{mm} < t < 44.7 \, \text{mm}\)) was filtered using a weak lowpass filter, with Gaussian shaped impulse response of standard deviation 0.26 mm. The part of the sinogram corresponding to the inner region of the cylinder (\(-37.7 \, \text{mm} < t < 35.8 \, \text{mm}\)) was filtered more strongly, with a Gaussian filter of standard deviation 0.78 mm. The outermost part of the sinogram (\(t < -44.3 \, \text{mm}\) and \(t > 44.7 \, \text{mm}\)) was not filtered at all. The locations of the boundaries between regions with different filtering were selected so that there would be no discontinuities in the smoothed sinogram profile.

Figure 6.2b was then obtained by filtered backprojection in the same way as described above. As can be seen from the image, this ring reduction method has successfully eliminated the rings in the central part of the image, although a few rings are still visible in the outer part of the cylinder where the weaker smoothing filter was applied.

Finally, it must be pointed out that this kind of postprocessing methods for ring removal are risky in the sense that they might remove important features in the image and, possibly, create secondary artifacts. They should therefore be avoided if it is technically possible to get rid of the detector inhomogeneities by calibration or hardware changes. The filtering technique described here is therefore intended to be used only during the development of the CT system, not in the final version.
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Appendix A

The downhill simplex method

In order to solve the optimization problem (4.5), the downhill simplex method [27] was used, in an implementation that is somewhat modified from that in [28]. This method, which should not be confused with the so-called simplex method used in linear optimization, makes use of a simplex which changes size and moves around in the parameter space until it reaches the optimal value of a nonlinear target function.

Assume that the optimization task is to minimize $f(x)$ where $x = (x_1, \ldots, x_N)$. As a starting guess, one must specify a starting simplex by its $N+1$ vertex vectors $x_1, \ldots, x_{N+1}$. In each iteration step, this simplex is then transformed into a new simplex by one of two basic operations: (a) replacement of the vertex with the highest (worst) function value by another point (figure A.1) and (b) contraction towards the vertex with the lowest (best) function value (figure A.2). We denote by $x_h$ the highest vertex, by $x_{nh}$ the next highest vertex and by $x_l$ the lowest vertex. Also, we calculate the center point $r_M$ of the side of the simplex opposite to $r_h$ by the formula $r_M = \sum_{i \neq h} r_i$ where the sum is taken over all vertices except $r_h$. Furthermore we let $\Delta x = x_h - x_M$ and then $x_a = x_M + \frac{\Delta x}{2}$, $x_b = x_M - \frac{\Delta x}{2}$, $x_c = x_M - \Delta x$ and $x_d = x_M - 3\Delta x$, as shown in figure A.1. The simplex is then updated according to the scheme in the algorithm below. This iteration is continued until the relative error

$$\frac{|f(x_h) - f(x_l)|}{\sqrt{|f(x_h)| + |f(x_l)|}}$$

falls below some predefined tolerance limit.
if $f(x_c) < f(x_i)$ then
  if $f(x_d) < f(x_c)$ then
    Replace $x_h$ with $x_d$.
  else
    Replace $x_h$ with $x_c$.
  end if
else if $f(x_i) \geq f(x_h)$ then
  if $f(x_a) < f(x_h)$ then
    Replace $x_h$ with $x_a$.
  else
    Contract towards $x_i$ by replacing $x_i$ with $\frac{x_i + x_l}{2}$ for all vertices except $x_l$.
  end if
else if $f(x_c) \leq f(x_h) < f(x_i)$ then
  if $f(x_a) < f(x_c)$ then
    Replace $x_h$ with $x_a$.
  else
    Replace $x_h$ with $x_c$.
    Contract towards $x_i$ by replacing $x_i$ with $\frac{x_i + x_l}{2}$ for all vertices except $x_l$.
  end if
else
  Replace $x_h$ with $x_c$.
end if

Figure A.1: Possible points to replace the highest vertex in the downhill simplex algorithm.
Figure A.2: Contraction towards the lowest vertex in the downhill simplex algorithm.
Appendix B

The square root of a symmetric positive definite matrix

In this section we will fill in some details justifying the calculations leading up to equation (1.17). Let $K^0$ and $K^1$ be symmetric, positive semidefinite matrices and assume that $K^0 + K^1$ is invertible, like in section 1.4. Then $K^0 + K^1$ is also symmetric and positive semidefinite. The symmetry implies that $K^0 + K^1 = PDP^{-1}$ for some matrix $P$ such that $P^T = P^{-1}$, where $D$ is the diagonal vector of eigenvalues of $K^0 + K^1$:

$$D = \begin{pmatrix}
\lambda_1 & 0 & \cdots & 0 \\
0 & \lambda_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \lambda_N
\end{pmatrix}$$  \hspace{1cm} (B.1)

(See [8], theorem 7.3.1.) It also follows that all $\lambda_i$ are real and strictly positive. (The possibility of some eigenvalue being zero is ruled out by the invertibility of $K^0 + K^1$.) Now $D^{1/2}$ is the diagonal matrix with entries $\lambda_i^{1/2}$ on the diagonal, and then $(K^0 + K^1)^{1/2}$ is defined as $PD^{1/2}P^{-1}$. The invertibility of $(K^0 + K^1)^{1/2}$ follows from this equation and the fact that all $\lambda_i^{1/2} > 0$. The symmetry of $(K^0 + K^1)^{1/2}$ follows from the equation $(K^0 + K^1)^{1/2} = PD^{1/2}P^T$. To conclude, $(K^0 + K^1)^{1/2}$ exists and is symmetric and invertible, as was assumed in section.
Bibliography


