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Operationalizing teachers' instructional visions and self-efficacy: Methodological insights for planning ambitious mathematics instruction in co-teaching settings

Anna Bengtsson

Linnaeus University, Sweden

Abstract: Ambitious mathematics instruction has emerged as a response to persistent inequities in mathematics classrooms, where students who struggle are often excluded from meaningful engagement. This paper is methodological in nature and situated in the critical preparatory phase of an intervention within an Educational Design Research (EDR) project. Rather than presenting empirical results, the paper explores how two theoretical frameworks - teachers' instructional visions (Munter, 2014) and teachers' self-efficacy (Bandura, 1997) - can be operationalized as meta-design principles to inform the planning of an intervention in co-teaching settings. Teachers' instructional visions offer a normative orientation towards high-quality and inclusive instruction, while teachers' self-efficacy provides a pragmatic focus on teachers' perceived capabilities for action. Together, these frameworks suggest how interventions can begin from where teachers currently are and at the same time create opportunities for sustainable shifts in practice. Design principles such as talk moves and proactive support are discussed as concrete entry points that both align with teachers' existing instructional visions and support a reimagining of inclusion. That is moving from participation as presence towards participation as valuing diverse student thinking. By positioning these frameworks as complementary lenses in the design phase, this paper contributes a methodological proposal for how theory can be operationalized to bridge equity-oriented aspirations with the realities of teacher learning in professional development for ambitious mathematics instruction.

Keywords: Teachers' instructional visions, teachers' self-efficacy, ambitious mathematics instruction, educational design research

Contact: anna.bengtsson@lnu.se

1 Introduction

In recent years, two significant reform efforts have shaped the landscape of mathematics education in Sweden: the call for high-quality instruction and the push for inclusive education. High-quality mathematics instruction is commonly understood as teaching that emphasizes conceptual understanding, reasoning, and problem solving rather than focusing primarily on procedural fluency. Inclusive

education is grounded in the principle that all students, regardless of ability, background, or prior achievement, should participate meaningfully in shared learning environments (Bagger et al., 2024).

Ambitious mathematics instruction has emerged as a response to the fact that high-quality mathematics instruction is not equally available to all students. Research shows that students who already demonstrate advanced understanding often dominate reasoning-oriented classroom discussions, while those who struggle in mathematics are frequently excluded from such opportunities (Jackson et al., 2017). Instead, these students are often offered tasks with low cognitive demand, focused on repetition and procedure, rather than deep understanding (Bagger et al., 2024; Boaler, 2016).

From this perspective, ambitious instructional practice is not only about engaging students in rich mathematical reasoning and problem-solving, it is fundamentally about inclusion. It aims to create learning environments where all students, regardless of prior achievement, can engage in meaningful mathematical thinking and discourse. Equity is therefore not a separate goal but embedded in the very definition of ambitious mathematics instruction (Cobb & Jackson, 2021; Jackson et al., 2017). However, the enactment of AMI is complex, particularly in co-teaching contexts where teachers must navigate both their own and their colleagues' instructional orientations.

Alongside instructional approaches such as AMI, co-teaching has emerged as a structural strategy to foster inclusive learning environments, bringing together general and special education teachers in shared classroom practice. While co-teaching holds potential for addressing needs for all learners, it often manifests in a "one-teach, one-assist" arrangement (Gardesten, 2023). As a result, the full potential of co-teaching to support both academic and social inclusion remains unrealised.

In this educational context, the present study seeks to develop AMI. However, this paper is methodological in nature. Rather than reporting empirical findings, it examines how theory can be operationalized in the preparatory phase of an intervention aiming to develop AMI in co-teaching settings.

2 Background

This section examines two interrelated components: ambitious mathematics instruction and co-teaching in diverse classroom settings.

2.1 Ambitious mathematics instruction (AMI)

Ambitious mathematics instruction (AMI) is grounded in a set of principles that emphasize teaching as a responsive, dialogic practice where instruction is intentionally designed to build on students' thinking and support them in taking intellectual risks (Jackson et al., 2017; Kazemi et al., 2009; Lampert, 2001). Teachers guide whole-class discussions by using student contributions to advance mathematical ideas, helping students connect various strategies and deepen their conceptual understanding (Franke et al., 2007; Kazemi & Stipek, 2001).

Implementing AMI is challenging and even when teachers aim for high-quality instruction, sustainable change rarely occurs without structural support (Cobb & Jackson, 2021). A key insight from Cobb's and Jackson's (2021) framework is that AMI must be supported by a coherent system that aligns ambitious goals with instructional practices, high-quality materials, and teacher learning. Crucially, this system must also include proactive and goal-aligned support for students who may struggle to access complex content. This does not mean offering simplified tasks. Instead, support must be designed in advance to prepare students to participate meaningfully in whole-class instruction (Cobb & Jackson, 2021). Proactive support in this sense is helping students access key ideas and concepts beforehand so they can engage with rich mathematical discussions alongside their peers.

Munter (2014) introduced the notion of instructional vision - the underlying images teachers hold of what good mathematics instruction looks like - as a key factor in shaping their instructional decisions. The framework of teachers' vision was developed as a way of analysing teachers' learning trajectories when implementing AMI.

Inclusion in mathematics education has long been both a policy ambition and a pedagogical challenge. Research increasingly frames inclusion as an ethical and relational practice, rather than a matter of physical placement (Bagger et al., 2024). From this perspective, inclusion is constituted through everyday classroom interactions and the structures that enable participation and recognition.

Still, inclusion in mathematics is far from straightforward. Studies have shown that students in learning difficulties are less likely to be engaged in high-quality mathematical instruction that emphasizes reasoning and problem-solving (Bagger et al., 2024). Teachers' beliefs about students' capabilities strongly shape these instructional decisions (Cobb & Jackson, 2021). As Björklund Boistrup (2010) highlights, students in mathematical difficulties tend to receive feedback focused on

completion and effort rather than on their mathematical thinking. While such encouragement may be well-intentioned, it risks lowering expectations and reducing opportunities for these students to engage meaningfully in the mathematical practices of the classroom.

This tendency is linked to a broader tension in inclusive education: whether diversity is seen as a resource or a challenge. This is less a matter of teacher competence and more about differences in instructional routines. As Stigler and Hiebert (1999) argue, teachers often act with the best of intentions, but without being fully aware of the underlying cultural scripts and structural assumptions that shape their instructional decisions.

The Norwegian project Mastering ambitious teaching in mathematics (MAM) offers an example of how AMI can be developed through structured professional learning. Drawing on Kazemi et al. (2009) and Lampert (2001) the MAM project supports teachers to plan, enact, and reflect on lessons collaboratively using tools as rehearsal and video-reflections of lessons in their interventions. Moreover, talk moves (Kazemi & Hintz, 2014) are used to guide classroom discourse, enabling teachers to steer conversations toward mathematical goals while maintaining responsiveness to student thinking.

From this, AMI involves creating learning environments where diverse student trajectories are possible and valued. Achieving this may require a shift in teacher practice, where teachers are supported in developing proactive approaches that enable all students to participate meaningfully. In the next section, co-teaching is introduced as one approach to support inclusive and ambitious mathematics instruction in diverse classrooms.

2.2 Co-teaching

Co-teaching can take various forms: one-teach, one-assist, station teaching, parallel teaching, or team-teaching, each with different potentials depending on the context and level of collaboration (Friend et al., 2010). One-teach, one-assist is the most common model of co-teaching, however, any model reaches its full potential when teaching partners jointly plan, implement, assess, and reflect on instruction (Rexroat-Frazier & Chamberlin, 2019).

Akyuz and Stephan (2022) argue that willingness to participate and appropriate training are foundational, and that deliberate attention to the match between teaching partners can enhance the success of co-teaching initiatives. It is suggested that

collaboration is accompanied by a shared sense of purpose anchored in a shared vision of instruction and informed by research-based models (Rimpola, 2014).

Previous research highlights that teachers' self-efficacy can influence both the extent and nature of their engagement in such practices (Ekstam et al., 2017). Teachers with strong efficacy beliefs tend to be more active in and use collaborative approaches such as co-teaching.

Taken together, the nature of the professional partnerships and the specific intentions behind co-teaching arrangements all play a role. These conditions also intersect with larger questions about how AMI is envisioned.

3 Towards a methodological contribution

Previous research on ambitious mathematics instruction has highlighted both its potential and its complexity. Large-scale design-based projects, such as the MIST project in the United States and the Norwegian MAM project show that sustainable improvement in mathematics teaching requires substantial teacher learning. These studies emphasize that AMI cannot be achieved solely through new tasks or materials but necessitates changes in teachers' instructional practices and in their underlying conceptions of what constitutes high-quality teaching. Such teacher change is inherently challenging, particularly in co-teaching contexts where teachers must also negotiate a shared vision of instruction.

Intervention-oriented research has often focused on supporting teachers' learning trajectories and on tracing how their practices develop over time (Munter, 2014). While these approaches provide important insights into what changes occur, there remains a methodological need to also consider how theoretical constructs can be operationalized in the design phase to guide and scaffold teachers' learning. For example, Munter's (2014) framework of teachers' instructional visions has been used to document teachers' evolving perspectives in the implementation of AMI. In this paper, I propose to take a step further by using teachers' instructional visions, together with teachers' self-efficacy (Bandura, 1997), as meta-design principles in the preparatory phase of an intervention.

This positioning responds to two key challenges identified in previous research: the necessity of supporting shifts in how teachers understand AMI (Cobb & Jackson, 2021) and the importance of building coherence and shared understandings among co-teachers (Rimpola, 2014). Moreover, studies of teacher learning stress the need to meet teachers where they are, recognizing their current practices and resources as

starting points for growth (Munter, 2014). By integrating these insights, the methodological approach advanced in this paper aims to strengthen the design of interventions that are both equity-oriented and grounded in teachers' situated possibilities for action.

4 Aim and research question

Educational Design Research (EDR) offers a way to address these challenges by iteratively designing, enacting, and refining interventions in close collaboration with practitioners (Bakker, 2018). A critical part of EDR is the preparatory phase, in which theoretical perspectives are operationalized to inform the design of interventions. This paper contributes to that phase by exploring how two established theoretical constructs - teachers' instructional visions (Munter, 2014) and teachers' self-efficacy (Bandura, 1997) - can be used as meta-design principles to guide the planning of an intervention aimed at developing AMI in co-teaching settings.

The aim of this paper is methodological: to examine how theory can be operationalized in the preparatory phase of an intervention within an EDR project. Specifically, the paper addresses the following research question:

How can teachers' instructional visions and self-efficacy be operationalized as meta-design principles to inform the planning of an educational design research intervention for ambitious mathematics instruction in co-teaching settings?

5 Theoretical framework

As shown, making high-quality instruction available to all students requires more than structural solutions. It demands a shift in teachers' instructional methods, and thus in their underlying beliefs. Co-teaching offers one way to support such change, but its potential depends on how teachers engage in planning, teaching, and reflecting together, and, according to Rimpola (2014), on the extent to which they develop a common purpose grounded in research-based models. In this EDR study, research-based models must align with ambitious goals.

Against this background, the combination of two frameworks - teachers' visions (Munter, 2014) and self-efficacy (Bandura, 1997) offers a productive lens. Together, they provide insight into what teachers aspire to do and what they believe they are capable of doing in their specific contexts. In this paper, I do not propose a new theoretical framework. Instead, I combine Munter's (2014) construct of instructional

visions and Bandura's (1997) theory of self-efficacy as complementary perspectives. Instructional visions provide a normative orientation, pointing toward what ambitious mathematics instruction might look like. Self-efficacy provides a pragmatic orientation, highlighting teachers' perceived capabilities to act within their instructional settings. Together, these constructs are positioned as meta-design principles that can guide the planning of an EDR intervention by linking aspirational goals with teachers' situated realities.

In what follows, I outline each framework and describe how they are integrated as meta-design principles in this study to inform the design.

5.1 Teachers' visions

In this study, the framework for teachers' visions of high-quality mathematics instruction is adopted as a theoretical and pragmatic tool. The concept of vision in the context of an intervention refers to the images and ideas teachers hold about what AMI should look like. Drawing on the work of Munter (2014) the framework is developed to both characterize teachers' current perspectives and to trace their evolving understandings over time.

The framework stems from the MIST project (Means of Instructional Support for Teachers) and defines teachers' instructional visions as more than individual preferences; instead, they are understood as socially negotiated constructs that gain clarity and purpose through professional discourse. As Munter (2014) argues, effective professional development is rarely achieved without a shared and meaningful vision of high-quality instruction. Importantly, such a vision cannot be imposed or simply chosen. Rather, it develops as teachers engage collaboratively over time, making sense of teaching and learning together.

Munter (2014) describes teachers' visions as evolving images of ideal teaching that extend beyond their immediate practice and reflect the gap between current routines and future aspirations. This distinction makes visions especially useful in interventions, where the goal is not only to understand current practice but to support its development.

The framework operationalises visions of high-quality mathematics instruction across three core dimensions: the role of the teacher, classroom discourse, and mathematical tasks. The framework offers different levels within each dimension to capture variation in practice. For example, the role of the teacher ranges from efficiently delivering content at a basic level (Level 1) to more advanced roles where

the teacher actively facilitates student reasoning (Levels 3–4). Similarly, classroom discourse progresses from teacher-dominated talk toward more dialogic interactions, where students engage with each other to construct mathematical meaning. Finally, mathematical tasks vary in cognitive demand, spanning from routine procedural exercises to open-ended problem solving and authentic mathematical inquiry (Munter, 2014).

The framework does not prescribe change but supports a reflective practice culture. However, the framework, while not a trajectory for teacher learning in itself, can be paired with a theory of action (Munter, 2014) - such as self-efficacy - to guide planning toward more coherent and sustainable professional growth.

5.2 Teachers' self-efficacy

Bandura's (1997) theory of self-efficacy provides an action-oriented perspective that makes it possible to anticipate and scaffold teacher learning and change. Self-efficacy beliefs are teachers' perceived capabilities to organize and execute actions required to achieve desired instructional outcomes.

This framework is essential not only for structuring intervention cycles but also for making decisions about how and when to introduce new instructional practices. In line with Bandura's work, teacher self-efficacy is understood to be developed and strengthened through four core mechanisms: mastery experiences, vicarious experiences, verbal persuasion, and emotional/physiological states (Bandura, 1997). These mechanisms guide how professional learning activities are designed and implemented. Mastery experiences are supported through repeated opportunities for teachers to successfully plan and enact instruction. Vicarious experiences are made possible when teachers observe their colleagues managing complex instructional moments, such as co-partners facilitating student reasoning. Verbal persuasion is incorporated through peer feedback and ongoing coaching that sustains teachers' growing competence. Attention is also given to emotional and physiological states, by fostering a collaborative culture that reduces stress and builds confidence. This theoretical foundation supports the view that a change in instructional practice must be accompanied by a strengthened belief in both one's own ability and in students' capability to participate meaningfully.

As such, self-efficacy is a practical framework for supporting teachers' learning trajectories. It allows an intervention to be responsive to where teachers are currently positioned in their practice and what supports they may need to progress. The

framework thus aligns with Munter's trajectory-based model of instructional vision by offering insight into how teachers might move between levels, not just in what they envision, but in what they perceive as doable in their everyday context.

In this way, the frameworks of teachers' vision and self-efficacy enable a systematic approach to professional growth within an intervention and help connect the study's goals with the practical realities and potential of the participating teachers.

5.3 Meta design principles

The EDR study focuses on two interrelated design objects: (1) teacher practice, defined as the situated and responsive orchestration of instruction and, (2) the learning environment, as the socially and materially mediated context in which mathematical reasoning unfolds. These elements will be jointly designed and studied to add to the knowledge base about how such practices can be supported, co-developed and sustained over time.

Building on research around inclusive and ambitious teaching, the EDR study will work closely with mathematics teachers and special education teachers engaged in co-teaching partnerships to test and analyse design principles aimed at creating learning environments where diverse student trajectories are supported.

In the EDR study, the frameworks of instructional visions (Munter, 2014) and teacher self-efficacy (Bandura, 1997) will function as meta-design principles. As Bakker (2018) describes, meta-design principles operate at a higher level of abstraction than pragmatic, classroom-focused design principles. Rather than only guiding analysis, they will be actively integrated into the intervention to support teacher learning alongside the development of student-focused design principles.

Teachers' instructional visions, understood as evolving ideas about high-quality mathematics instruction, will be negotiated throughout the process and used as a tool for reflection, shared meaning-making, and professional alignment. By using Munter's interview protocols where teachers are asked to describe what they would look for in a mathematics lesson to judge it as high-quality, the framework aims to reveal their interpretive lens - the criteria they apply when evaluating instruction. By discussing and revisiting their visions, teachers will make their values and intentions visible, which creates engagement and a foundation for collective instructional goals.

Self-efficacy will guide how the design supports teachers in enacting AMI. Drawing on Bandura's (1997) sources of efficacy the study will structure the professional learning to build confidence through collaborative planning, teaching, and reflection.

Together, these frameworks ensure that the intervention remains grounded in both theory and teachers' professional realities. Although this paper primarily conceptualizes teachers' instructional vision and self-efficacy as meta-design principles, the design principles that emerge from these meta-principles – and the connection between them – will be explicitly taken up and elaborated to show how teachers' instructional vision and self-efficacy are operationalized as meta-design principles in the design principles.

6 From meta-design principles to design principles

The step from meta-design principles to specific design principles is not straightforward. Based on the literature on AMI and co-teaching, two design principles were selected: talk moves and proactive support. Talk moves and proactive support can both be seen as core components of AMI, as they provide teachers with structured tools and strategies that center student reasoning while also anticipating and addressing diverse learning needs. In this way, teachers are supported with clear instructional approaches that reduce the perceived risks of engaging all learners in ambitious tasks. Both talk moves and proactive support, therefore contribute to strengthening teachers' self-efficacy and offer opportunities for mastery experiences in enacting AMI. These design principles were chosen not as final prescriptions but as manageable entry points that connect teachers' current practices with the broader aspiration of inclusive AMI.

In this study, design principles are understood as practice-oriented tools that mediate between theoretical constructs and instructional practice (Bakker, 2018). Their primary purpose is to operationalize insights from prior research on AMI and co-teaching in ways that are grounded in teachers' current instructional visions and perceived self-efficacy. In this sense, design principles are more directly linked to instructional strategies and student learning. Given the documented challenges in shifting teacher perceptions about students' mathematical capabilities and in supporting diverse learning trajectories within inclusive classrooms (Cobb & Jackson, 2021), the design principles aim to support such change by meeting teachers where they are. The design principles are developed with the assumption that participating teachers in the EDR study do not initially operate at the higher levels of instructional vision. Instead, they aim to provide concrete and manageable entry points that support progression toward more inclusive and high-quality practices over time. In this way, the design principles work in parallel with the overarching meta-design

principles of vision and self-efficacy to scaffold teacher learning and co-teaching in context. Together, these layers of principles will support the goal of co-developing both teacher practice and the learning environment.

Two design principles have been identified based on prior research and initial analyses:

Talk moves

Talk moves have been used to scaffold mathematical discourse. These include revoicing, prompting elaboration, and framing disagreement as generative. Talk moves are seen not merely as verbal techniques but as pedagogical routines that support a collective culture of reasoning (Kazemi & Hintz, 2014). By focusing teacher attention on the form and function of student contributions (Munter, 2014) talk moves can shift teachers' instructional visions and build self-efficacy from procedural accuracy toward conceptual engagement.

For example, when a student asks, "Do you always have to divide by 3?", one teacher might see the function of the question as checking for correctness. Another teacher, however, might use the question as an opportunity to probe the student's reasoning and invite the class to consider why division by 3 might be appropriate in a given context.

Talk moves can support this shift in perspective by giving teachers concrete tools to respond to student utterances in ways that reveal and extend mathematical thinking, rather than simply evaluating correctness. By focusing on the substance of students' ideas, talk moves also offer teachers a language to acknowledge and build on contributions from all students, including those who struggle, without relying solely on praise for effort or engagement. This reorientation toward student reasoning is central to building AMI.

Proactive support

This includes the design of tasks and pre-lesson supports that anticipate conceptual difficulties and enable all students to enter the lesson on equitable footing. Such support is planned in advance and aligned with the lesson's mathematical goals (Cobb & Jackson, 2021).

For example, when students receive targeted support before a lesson, they are better prepared to engage actively in classroom discussions. A student who might typically remain silent now feels confident enough to contribute publicly. This visible

participation can strengthen the student's self-efficacy and challenge the teacher's assumptions about who is capable of conceptual reasoning. Over time, such shifts in participation can influence how teachers envision all students' potential, reinforcing a more inclusive approach to high-quality instruction.

These two principles are intentionally modest in scope to allow teachers to experience success and gain confidence in a co-teaching setting. They will be introduced early and refined iteratively through classroom enactment and joint reflection.

7 Further implications

An iterative process, where testing, reflecting, and learning from practice are central and structures the professional development work. The intervention process evolves around meta-design principles and design principles enacted through three iterative cycles. These principles provide a structured framework that guides teachers' engagement, supports AMI, and allows teachers to develop self-efficacy through concrete strategies. Within this process, various activities can be used to support teacher learning, such as analysing classroom episodes, video clips from their own practice, or peer observations, engaging in collaborative planning and rehearsal, or documenting reflections through fictive lesson observations, in logbooks or other artifacts. Crucially, these activities are not the contribution in themselves. Their purpose and impact arise only when they are consistently aligned with the meta-design principles of instructional vision and self-efficacy. Activities are meaningful when they are integrated, revisited, and adapted in light of the meta-principles, rather than treated as isolated exercises. The following three activities exemplify this:

Fictive classroom observation

Teachers begin by describing a fictive classroom observation they would recognize as high-quality. Using Munter's (2014) dimensions - mathematical tasks, classroom discourse, and the teacher's role - they sort and problematize these observations. This strategy makes their instructional visions visible, highlights shared or diverging perspectives, and identifies gaps between current practice and AMI. It establishes a collective lens for what matters in a mathematics classroom and sets the foundation for iterative reflection.

Rehearsals of lesson enactments

In subsequent cycles, teachers rehearse planned lessons with colleagues acting as students. These rehearsals provide a safe space to experiment with talk moves and proactive support, test instructional decisions, and receive peer feedback. By practicing in a controlled setting, teachers gain vicarious and mastery experiences that strengthen their self-efficacy while refining the implementation of design principles.

Video-based reflection

All enacted lessons are filmed and analyzed individually and collaboratively. Teachers examine student reasoning, participation, and the effects of their instructional choices. Video reflection allows for iterative evaluation of both teaching practice and design principles, fostering analytical skills and reinforcing confidence in their ability to lead ambitious and inclusive lessons.

Throughout the intervention, teachers' agency is central. They actively shape and refine the instructional principles, engage in problem solving, and collectively make sense of observed classroom dynamics. This iterative engagement ensures that both talk moves and proactive support are not merely prescribed strategies, but tools teachers can adapt to their own practice, aligning with AMI while simultaneously building self-efficacy and professional mastery.

8 Discussion

This study is situated within a context grounded in equity-oriented principles. Drawing on Cobb and Jackson's (2021) work, this paper argues that meaningful instructional change requires more than new methods or materials, it demands that teachers' own pedagogical resources, such as instructional visions and self-efficacy, be brought into the intervention process. This study contributes by showing how such resources can function as meta-design principles that guide the practical enactment of professional learning. Rather than assuming shared vision or readiness for collaboration, the intervention supports teachers in gradually building confidence and aligning instructional goals through structured activities such as video analysis and rehearsals of lesson enactments. In this way, the intervention responded to calls for shared purpose in co-teaching (Rimpola, 2014), while also addressing the cultural scripts that shape practice (Stigler & Hiebert, 1999). Embedding self-efficacy and instructional vision throughout the process allows for change to emerge from within

practice, sustainably and realistically.

A key contribution of this study lies in illuminating a specific shift in teachers' instructional visions: from viewing inclusion as ensuring participation, to valuing and responding to diverse mathematical thinking as a resource for collective learning. This reorientation requires that design principles meet teachers where they are and offer tangible strategies, such as talk moves and proactive support, that help teachers recognise students' contributions not only as correct or incorrect, but as openings for advancing mathematical understanding. This supports Cobb and Jackson's (2021) call for coherent alignment to sustain AMI. Rather than relying on praise or procedural success, teachers will be trained to use student thinking to shape lesson content and see questioning as a generative part of learning. In doing so, they may begin to reframe inclusion not as a challenge to manage, but as a commitment to designing for diverse learning trajectories. This represents a concrete enactment of relational and equity-oriented goals (Bagger et al., 2024; Kazemi & Hintz, 2014).

9 Conclusion and implications

This paper positions teachers' instructional visions and self-efficacy as central design elements in the development of inclusive high-quality mathematics instruction. By embedding these subjective resources into the early planning of an intervention, the work may contribute to a more situated and sustainable model of instructional design. A design that acknowledges the interpretive agency of teachers and the complexity of real classrooms.

The findings suggest that meaningful inclusion is closely tied to teachers' instructional vision about what counts as mathematical thinking and who is seen as capable of engaging in it. When teachers are supported in rethinking the function of student contributions and offered tools, they are more likely to create learning environments where diverse learning trajectories are not only possible but valued. Inclusion, then, is an evolving practice shaped by how teachers make sense of teaching.

If inclusion is to be realised not as a policy ambition but as the lived reality of mathematics classrooms, interventions must meet teachers where they are and offer meaningful tools for moving forward. By positioning teachers' instructional visions and self-efficacy as parallel and complementary perspectives - one providing normative direction and the other pragmatic orientation - the paper argues that

theory can serve as a methodological tool to bridge equity-oriented aspirations with teachers' perceived capabilities for action.

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