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LEARNING TEACHING MATHEMATICS WITH INSTRUCTIONAL TECHNOLOGY: A PRAXEOLOGICAL ANALYSIS OF A GHANAIAN MATHEMATICS TEACHER EDUCATION COURSE

Farouq Sessah Mensah

Stockholm University, Sweden, farouq-sessah.mensah@su.se

This study explores how a mathematics teacher educator prepares pre-service teachers to integrate instructional technology into mathematics teaching, using the Anthropological Theory of the Didactic (ATD) as a framework. A qualitative case study at a Ghanaian university discusses the MTE's practices through video observations and a semi-structured interview. Findings reveal that representation of practice through modelling provided PSTs with practical examples of instructional integration, while decomposition allowed for refining lesson plans through structured analysis. However, the absence of approximations of practice due to time constraints limited experiential learning opportunities. The study emphasises the need for explicitly articulating meta-didactical logos and simulated teaching environments to bridge theory and practice.

Keywords: Instructional technology, anthropological theory of the didactic, mathematics teacher education, pedagogies of practice

Introduction

The demands of modern educational environments require high-quality mathematics instruction, of which instructional technology plays an integral part. I refer to instructional technologies (IT) as the intentional integration of various digital technologies into the classroom to increase the efficiency with which teachers and students can accomplish their tasks or the depth with which teachers and students can acquire new knowledge. There is a growing expectation that integrating IT in mathematics teaching and learning can contribute to meaningful learning (e.g., Hillmayr et al., 2020). However, integrating IT in teaching and learning does not always produce the desired learning outcomes (e.g., Player-Koro, 2013). The literature highlights the transformative power of IT when used meaningfully in teaching and learning (e.g., Hoyles, 2018). While it could provide meaningful learning opportunities for students, integrating IT into teaching practices also demands a lot of teacher effort. Some researchers have acknowledged the complexities of teaching mathematics with IT (e.g., Rakes et al., 2022). The challenges of integrating IT into teachers' daily routines remain unresolved despite many years of research and development. From the Ghanaian context, for example, studies on technology-enhanced learning (e.g., Buabeng-Andoh, 2019) have reported low levels of IT integration into core teaching practices. This low level of IT integration raises the question of how pre-service mathematics teachers (PSTs) are prepared for the complex work of teaching mathematics integrating IT.

Theoretical Consideration and Research Question

The study is grounded in the Anthropological Theory of the Didactic (ATD) (Chevallard, 1992), linked to the processes of teaching and learning mathematics. ATD considers doing, teaching, diffusing, creating, and transposing mathematics, as well as any other kind of knowledge, to be human activities taking place in institutional settings. ATD holds that any human activity can be understood as a praxeology. The ATD frames human activity as praxeology, comprising praxis and logos. Praxis involves tasks and the techniques used to solve them, while logos encompasses the theoretical discourse that explains and justifies these techniques. The ATD distinguishes between mathematical praxeologies (MP) and didactic praxeologies (DP) (Chevallard & Bosch, 2020). MPs involve types of tasks, techniques, technologies, and inherently mathematical theories. In contrast, DPs refer to praxeologies that support the teaching or learning of MPs, typically reflecting the activities of school teachers. In teacher education (TE), however, the focus shifts: the DP itself becomes the knowledge to be taught as PSTs learn how to teach. Arzarello et al. (2014) extend this by introducing meta-didactical praxeologies, which encompass the tasks, techniques, and justifying discourses related to teaching how to teach mathematics. In this context, mathematics education becomes the object of study. To avoid confusion, I will use the term meta-didactical praxeology when referring to praxeologies where didactical knowledge rather than mathematics is the central content to be taught and learned. From a praxeological perspective, the following research question guided the study: *How are meta-didactical praxeologies privileged when a mathematics teacher educator describes their teaching of IT integration in mathematics teaching?*

Method

The study is designed as a qualitative single-case study. It serves as a preliminary analysis, offering a nuanced, context-specific, and empirically rich account that lays the foundation for a broader, multi-case investigation. The design enables an initial critical reflection, analysis, interpretation, and deeper understanding of the research problem.

Case Context

The study was conducted in a teacher preparation course at a public university in Ghana. The course was designed to allow PSTs to develop skills in designing instructional materials, identifying various teaching and learning resources (TLRs), using manipulatives and using technology applications to meet students' diverse learning needs. The study focused on an MTE, referred to by the pseudonym Benjamin, as the principal participant, along with his 204 PSTs enrolled in the course.

Data Collection and Analysis of the Case

Data was collected through video observations and a semi-structured interview with Benjamin. Selected teaching episodes were drawn from the classroom observations, and the interview was audio-recorded and transcribed for analysis. The data was analysed by drawing on a reference model.

Reference Model

A reference model (RM) is a framework used by researchers to examine and respond to didactical situations within specific institutional contexts (see Wijayanti & Winslow, 2017). The RM served as the study's analytical framework. To answer the research question, I drew on Grossman et al. (2009) pedagogies of the practice of representation, decomposition, and approximation as reference models to analyse the meta-didactic praxis. The meta-didactic logos was determined inductively by looking for substantiations of the identified praxis. In the course, representation occurred when PSTs engaged with IT as part of their learning process or observed the MTE modelling its use. Decomposition involved PSTs critically analysing the use of various IT tools across different mathematical task situations. Approximation refers to PSTs actively enacting mathematics teaching with IT and reflecting on their experiences to develop their instructional practices.

Results and Discussions

Representation of Practice

The observation data showed how Benjamin, through the modelling of lessons, privileged different pedagogical and didactical strategies that integrate IT, providing PSTs with concrete examples of meaningful classroom application. This modelling showcased diverse pedagogical and didactical praxis of representing mathematical concepts. Benjamin indicated in the interview:

> Sometimes, I select three or four topics, such as solid mensuration, quadratic equations, and algebraic expressions. I then demonstrate the teaching process by enacting lessons based on these topics, supporting them with worksheets containing related activities for the students to engage.

The meta-didactic praxis of modelling teaching by integrating IT and hands-on activities suggests opportunities for PSTs to connect more deeply with the subject matter. This praxis reflects ATD's emphasis on immersing PSTs in real classroom dynamics where theoretical knowledge is contextualised and practised. Research also highlights the value of such modelling in enabling PSTs to internalise methods for making abstract concepts accessible and engaging (e.g., Ball & Cohen, 1999). Although implicit, the meta-didactic logos of representation reflected the need to foster active engagement and improve teaching efficiency through modelling.

Decomposition of Practice

Benjamin exemplified decomposition by guiding PSTs through designing and evaluating lesson plans. In dissecting the lesson planning process into its fundamental elements - objectives, instructional strategies, materials, and assessments, Benjamin facilitated a structured reflection that enabled PSTs to refine each component. Benjamin explained:

> So when they design their lesson plans, I check whether what they have designed can support the SHS students in learning the concept they want to teach through discussions.

The meta-didactical praxis of designing and evaluating lesson plans aligns with research in mathematics teacher education. It emphasises the importance of breaking down practice into core elements to support PSTs in developing targeted instructional skills (Ball & Cohen, 1999). The logos for the meta-didactical praxis of decomposition of practice was not evident from the data. While the decomposition of practice is critical for developing pedagogical content knowledge, it often remains implicit within instructional settings (Ball & Cohen, 1999). The absence of explicit logos for the meta-didactical praxis of decomposition highlights a broader challenge to IT integration in mathematics teacher education. The non-articulation of logos might hinder PSTs from fully grasping the nuanced decision-making processes involved in teaching with IT, which is essential for their development as reflective practitioners.

Approximation of Practice

The findings highlight a gap in implementing approximations of practice within the teacher education course. Although Benjamin recognised the value of having PSTs enact lessons they designed, time constraints prevented this crucial step from being integrated into the course. Benjamin acknowledged this limitation, noting:

> Yes, that is the unfortunate part we couldn't do... But it would have been very productive. Having them design and enact. Because designing a lesson is different from enacting it.

This statement highlights the distinction between lesson planning and the actual enactment of teaching, a crucial component of approximations of practice. In the context of mathematics teacher education, previous research has emphasised the importance of providing PSTs with opportunities to engage in approximations of practice. This approach allows PSTs to bridge the gap between theory and practice by actively participating in teaching scenarios that resemble real classroom situations (Grossman et al., 2009). The omission of this step, as highlighted in the interview, suggests a missed opportunity for PSTs to develop their pedagogical and didactical skills more deeply.

Concluding Remarks

This study highlights the role of representation, decomposition, and approximation of practice in preparing PSTs to teach mathematics using IT. Through modelling, Benjamin demonstrated IT integration with pedagogical and didactical strategies, providing PSTs with concrete examples of effective classroom practices. Decomposition of practice offered opportunities for PSTs to analyse and refine lesson planning. The meta-didactic logos underline the mata-didactic praxis of representation, and decomposition was not articulated. Approximations of practice were absent due to time constraints, reflecting a missed opportunity to bridge the gap between theoretical knowledge and practical application. The study suggests that MTEs explicitly articulate their meta-didactic logos and include approximations of practice through simulated teaching environments.

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