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Young students' mathematical argumentation in social interaction

Video-based observations of student-student interaction during everyday work in the mathematics classroom

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Abstract

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Previous research indicates that students benefit from engaging in mathematical problem-solving activities together with peers. The aim of this thesis was to increase the knowledge of how social interaction can contribute to shaping young students' mathematical argumentation.

The analysis was based on a dialogical perspective on communication. In particular, an ethnomethodological approach was applied to the analysis of students' social interaction while engaging in discussions about solutions to mathematical tasks. Students' contributions to interaction were analysed using Conversation Analysis and multimodal analysis. In addition, the contents of students' explanations, justifications and generalisations were analysed according to procedures of qualitative content analysis.

The empirical material consisted of video recordings of naturally occurring interaction during mathematics lessons in two grade-6 classrooms (i.e., among students who are 11–12 years old). Findings were presented in four studies. Study I indicated that the mathematical argumentation among students working in the same classroom can orient towards very different social and sociomathematical norms. Study II focused on students' use of different types of justifications, showing that their general arguments consistently built on (and agreed with) results of preceding examinations of particular examples. In Study III, students' strategies of handling differing proposals were analysed, which showed that students often solicited explanations of peers' proposals by commenting on or asking questions about them without explicitly criticising them. Moreover, when students conceded to someone else's proposal and rejected their own, concessions and rejections were marked by affect-laden and/or embodied acts, indicating an urgency to display a change of state. In addition, marking their concessions may be part of students' ways of displaying independent epistemic access to the mathematical task as well as to the differing proposal. Focusing on students' methods of co-constructing general arguments, Study IV confirmed the importance of having access to and building on others' arguments. In addition, Study IV showed how the use of linguistic resources can indicate that students have identified regularities and/or transferred known mathematical facts into a new context.

The detailed analysis of students' argumentation while engaging in mathematical problem solving with peers emphasised the reflexive relation between "social" and "mathematical" aspects of interaction in the mathematics classroom. The analysis also exemplified how young students' use of justifications can be a first stage in developing an understanding of formal mathematical proof.

Keywords: small group interaction, elementary-school students, mathematical problem solving, multimodal interaction analysis, qualitative content analysis, emergent proving

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List of Studies

This thesis is based on the following studies, which are referred to in the text by their Roman numerals.

I. Fredriksdotter, H., Norén, N. & Bråting, K. (2021). A pre-study of grade 6 students' orientation to social and sociomathematical norms during mathematical problem solving in groups. In Y. Liljekvist, L. Björklund Boistrup, J. Häggström, L. Mattsson, O. Olande & H. Palmér (Eds.), *Proceedings of MADIF12. The twelfth research seminar of the Swedish Society for Research in Mathematics Education, Växjö, 2020* (pp. 21–30). SMDF. http://matematikdidaktik.org/wp-content/uploads/2021/03/MA-DIF12_dokumentation.pdf

II. Fredriksdotter, H., Norén, N. & Bråting, K. (2022). Investigating grade-6 students' justifications during mathematical problem solving in small group interaction. *Journal of Mathematical Behavior, 67,* 100972. https://doi.org/10.1016/j.jmathb.2022.100972

III. Fredriksdotter, H. & Norén, N. (2023). Displaying epistemic independence during mathematical problem solving in small group interaction. *Research on Children and Social Interaction, 7*(2), 214–237. https://doi.org/10.1558/rcsi.25468

IV. Fredriksdotter, H. (manuscript). Grade-6 students' generalising practices during mathematical problem solving with peers.

In the co-authored studies, Fredriksdotter was responsible for collecting and transcribing the data. Fredriksdotter also suggested topics for the studies, and had the primary responsibility for writing the articles. The co-authors contributed with methodological and analytical suggestions, as well as with constructive comments on the texts.

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Tack!

No man is an island skrev John Donne i början av 1600-talet. Under min tid som doktorand har det verkligen varit tydligt att jag är allt annat än ”hel och fullständig” i mig själv och jag vill därför uppmärksamma ett antal personer som på olika sätt har bidragit till att jag nu har kommit i mål.

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det enda jag vill säga
glimmar utom räckhåll
som silvret
hos pantlånaren

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[1] April och tystnad (ur Sorgegondolen)

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Hanna Fredriksdotter
Blåsenhus (21:224), april 2024

1 Introduction

In mathematics education research, several studies have shown that students' achievements, as well as their attitudes towards the subject, generally improve when they have the opportunity to work together with peers (e.g., Capar & Tarim, 2015; Cobb et al., 1997; Kyndt et al., 2013; Qin et al., 1995; Rabel & Wooldridge, 2013; Sjöberg, 2006; Sjöblom, 2022; Webb et al., 2023; Wyndhamn, 1993; Yackel et al., 1991). This thesis adds to previous research by analysing aspects of young students' small group interaction during everyday work in the mathematics classroom. In particular, the focus is on how processes of social interaction can shape students' argumentation, in terms of formulating explanations, justifications and generalisations, while engaging in mathematical problem-solving activities with peers.

In previous studies, attention has been drawn to the lack of research supporting teachers in how to implement problem-solving activities in their classroom practices (e.g., Zimmermann, 2016). The complexity of classroom interaction has also been emphasised; for example, studies have indicated that teachers find it difficult to involve students in group-based activities when teaching mathematics (e.g., Robertson & Graven, 2019), which means that such approaches have been 'rarely observed in the classroom' (Hardman, 2020:152). In addition, it has been argued that 'the relationship between the social and the mathematical in interaction requires further research' (Ingram, 2020:1). Moreover, a specific dilemma entailed with initiating small group work is that teachers no longer have 'total control' over the content of their students' discussions (Emanuelsson & Sahlström, 2008:220). Thus, previous studies indicate that teachers would need support in how to manage student-student interaction in the mathematics classroom.

For a long time, the didactics of mathematics 'neglected the human parts of its discipline by concentrating on the subject matter side via the focus on the curriculum' (Straesser, 2007:165–166). One way to emphasise the human side of the teaching and learning of mathematics is to represent didactic situations using a triangle as shown in Figure 1.

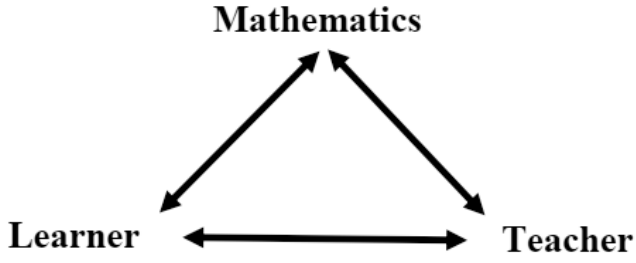

Figure 1: The didactic triangle (Straesser, 2007:165).

Over time, the representation of the didactic triangle shown in Figure 1 has also been problematised. For example, it has been argued that, in addition to analysing the teacher's and the learner's understanding of mathematics, as well as the relationship between the teachers and learners, it is crucial to raise questions about what it means to do mathematics and to discuss how learning environments that support students' mathematical development could be created (Schoenfeld, 2012). The focus of this thesis is young students' use of explanations, justifications and generalisations, which is a topic that relates to the learner's encounters with the subject matter. However, I am not primarily interested in the individual learner's understanding of mathematics; rather, my research interest concerns how young students' mathematical argumentation may be shaped by processes of social interaction during group work.

The empirical material in my studies primarily consists of a series of video recordings of Grade-6 students' discussions in small groups during everyday work in the mathematics classroom[2]. My focus on students' interaction does not mean that I disregard the teacher's role; rather, the video recordings show interaction that takes place outside the teacher's immediate influence, which reveals aspects of classroom interaction that can be difficult for teachers to observe themselves. Thus, in addition to contributing to previous research on students' small group work, this thesis can support teachers' didactic choices when initiating mathematical problem-solving activities.

In this thesis, the analysis of students' interaction during small group work is based on a *dialogical theory of communication*, which emphasises humans' interdependency when making sense of situations and activities in which they participate (Linell, 2009:12–14, 47; 2022:372–373). To illustrate my research interest, my empirical material and my methodological approach, I present below a short excerpt from the transcriptions of my video recordings, representing the initial part of a discussion between Ali and Björn about the task 'Ice cream' (see the mathematical tasks in Appendix A, an explanation of the

[2] In Sweden, ten years of schooling is compulsory. Children start their first year in Preschool class ('förskoleklass') at the age of 6, which is followed by Grades 1–3 in 'lågstadiet', Grades 4–6 in 'mellanstadiet' and Grades 7–9 in 'högstadiet'; that is, students in Grades 6 are 11–12 years old.

transcription symbols in Appendix B and the transcription in Swedish in Appendix C, Excerpt 1). The task was to find the number of ways in which two scoops of ice cream could be combined, depending on the number of flavours to choose from. The teacher instructed the students to work individually for a few minutes; the students were then instructed to compare their own suggested solution with a classmate's solution. Discussing the number of ways that two scoops of ice cream can be combined based on three flavours, both Ali and Björn suggested the multiplication 'three times two' to calculate the number of combinations. However, both of them also displayed uncertainty and requested explanations from each other.

```
1.   Ali:      can you explain it I don't understand.
2.   Björn:    it is (0.6) three times two?
3.             I guess.
4.   Ali:      yeah I know (.) but,
5.   Björn:    look here eh or,
6.             I don't really know [either.
7.   Ali:                          [no well I,
8.             I ↑know that it is three times eh times two¿
9.             but (.) explaining it is a bit harder.
10.  Björn:    but explain yours first.
11.  Ali:      okay.
```

In previous research, the activity of *mathematical problem solving* has been described as a series of processes involving understanding the task, devising a plan, executing relevant procedures, and 'looking back' to evaluate the result (e.g., Hanna & Knipping, 2020; Polya, 1991:5–23; Qin et al., 1995). However, it is not obvious what type of tasks should be regarded as *mathematical problems* as the same tasks can be a routine exercise for some students and call for efforts from others; thus, 'being a "problem" is not a property inherent in a mathematical task. Rather, it is a particular relationship between the individual and the task that makes the task a problem for that person' (Schoenfeld, 1985:74). When reading the excerpt above, we may question the students' problem-solving skills since the execution of the multiplication $3 \cdot 2$ is an incorrect strategy. On the other hand, it is important to note that both of the students displayed uncertainty by saying 'I don't understand' (line 1), 'I guess' (line 3) and 'I don't really know' (line 6). In addition, Björn's prosody when saying 'three times two' (line 2) was try-marked (see Schegloff, 2007:238–239). These displays of uncertainty indicate that the task was not treated as a 'routine exercise'; thus, the mathematical task 'Ice cream' can be regarded as a 'mathematical problem' for Ali and Björn.

Moreover, it is equally important to note that, despite claiming to know that 'three times two' was correct (lines 4, 8), Ali added 'explaining it is a bit harder' (line 9), with his emphasis on 'explaining' indicating that he treated his own knowledge claim as insufficient. In other words, it did not seem as if

it was enough to simply state what procedure to execute; instead, Ali showed that he was ready to explain and justify his calculation. In addition, the process of explaining appears as a shared responsibility since both of the students requested explanations (lines 1, 10). In particular, the request that Ali should explain his solution 'first' (line 10) indicates that Björn also was ready to contribute with his own explanation later on.

Although this excerpt is very short, it shows that mathematical problem solving can be quite a complex activity, not only mathematically but also interactionally, for example in terms of negotiating obligations to claim and justify knowledge. Thus, detailed analyses of student-student interaction can contribute to teachers' planning of students' group activities. Next, I present a background (section 1.1), which is followed by a presentation of the aim and the research questions of this thesis (section 1.2), and of the outline of the 'kappa' (section 1.3).

1.1 Background

This section comprises three subsections. In subsection 1.1.1, parts of the mathematics syllabus of the Swedish compulsory school are related to research on *the activity of mathematical problem solving*. This is followed by subsection 1.1.2, which refers to research on *the complexity of working in small groups*, and subsection 1.1.3, which presents parts of discussions in previous studies regarding *methodological aspects of analysing group work*.

1.1.1 The activity of mathematical problem solving

In the Swedish curriculum (Skolverket[3], 2023a), students' participation in mathematical reasoning is treated as a key element. For example, the mathematics syllabus states that the teaching should give the students opportunities to develop their mathematical knowledge 'in order to formulate and solve problems, and also reflect over and evaluate selected strategies, methods, models and results', and to develop abilities to 'argue logically and apply mathematical reasoning' (Skolverket, 2018:55). In addition, the so-called knowledge requirements[4] state that students should participate in mathematical argumentation by applying and following reasoning about methods and results; in particular, students are required to 'describe', 'account for' and 'discuss' their strategies, and to engage in reasoning about the plausibility of their results (Skolverket, 2018:60–65).

[3] The Swedish National Agency for Education.

[4] The knowledge requirements are individually formulated for each subject and represent the national requirements regarding the level of knowledge that students are expected to attain (Skolverket, 2018).

According to the knowledge requirements for Grade 3, 6 and 9 (Skolverket, 2023a), students are not expected to formulate *proofs* in a formal sense. Nevertheless, young students' use of explanations and justifications while engaging in mathematical problem solving with peers can serve as a precursor to formulating mathematical proofs (Sowder & Harel, 1998). Previous research (in Sweden and internationally) has also emphasised general benefits of letting students engage in mathematical problem-solving activities. For example, teaching experiments involving upper-secondary school students have shown that those students who were required to develop their own strategies for solving mathematical tasks achieved better results when writing tests, even if they were not always successful in solving similar tasks during lessons (Liljekvist, 2014:40–41). Analyses of discussions during whole-class teaching have also shown that mathematical problem-solving activities offer ample opportunities for students to learn and apply mathematical concepts (see an overview of previous research in Larsson, 2015) and to become familiar with classical rules, as well as to explore their own strategies (Zimmermann, 2016).

The curriculum for Swedish schools does not provide any information about how to arrange the classroom work during mathematical problem-solving activities. In other words, the mathematics syllabus does not specify whether students should 'describe', 'account for' and 'discuss' their strategies during individual work, in small group interaction, or during teacher-led whole-class discussions. On the other hand, the overall goals and guidelines of the curriculum state that all who work in the school should contribute to students' interacting with one another (Skolverket, 2018).

Next, I present findings of previous studies of small group work in the mathematics classroom, indicating *the complexity of working in small groups*.

1.1.2 The complexity of working in small groups

In mathematics education research, there is a growing body of studies indicating that various aspects of social interaction during whole-class teaching, as well as during group-based activities, can contribute to both improved learning outcomes and increased social and emotional wellbeing of students (see an overview of previous research in Hardman, 2020). A recent example is Sjöblom's (2022:72) analysis of interaction in a multilingual upper-secondary classroom, which indicated that factors related to the overall structure of the discussions seemed to have had a greater impact on students' communication than their linguistic background. The students' mathematical dialogue was supported when the teachers instructed students to take on 'communicative roles (such as chairperson, questioner, summarizer, and accountant)' (ibid.). Similarly, Webb et al.'s (2023) analysis of third-graders' participation during mathematics lessons, which showed a positive correlation between the students' interaction and their mathematical achievements; that is, students

who engaged in peers' ideas made apparent progress in terms of their individual development of problem-solving strategies.

However, several studies have also identified challenges related to collaborative activities in the mathematics classroom. Balacheff (1988:222) stated that students' interaction can 'provide an obstacle when pupils with very different conceptions are brought together', while Ahlberg (1992:191–196), applying a phenomenographic approach when analysing fourth-graders' discussions during mathematical problem solving, noted that some students focused more on which peer had formulated the solution than on the content of the solution. In comparison, Artzt and Armour-Thomas' (1997:71) teaching experiment indicated that high-ability secondary-school students were not necessarily successful during small group work because of the possible influence of 'other dynamics that extend beyond the mere ability level of group members operating in a group'. Similarly, Kilhamn et al. (2019:68), analysing sixth-graders' discussions, concluded that mathematical argumentation, for example, involving 'formulating mathematical expressions, identifying variables and working with structural aspects of equations', does not 'emerge spontaneously', while both Rabel and Wooldridge (2013) and Dahl et al. (2018) emphasised that group discussions are more or less productive and beneficial to elementary students, depending both on their mathematical abilities and on their 'type of talk' (see Mercer & Littleton, 2007:58–59). In comparison, Wood and Kalinec (2012:120) addressed the dilemma of students' tendencies to engage in extensive off-task behaviour during small group work; their case study of discussions among a group of fourth-grade students showed that only around 10% of the students' utterances concerned 'mathematical objects'. On the other hand, studies have also shown that students are able to integrate off- and on-task behaviour during collaborative activities (e.g., Thornborrow, 2003). In some of these studies, off-task behaviour was interpreted as a 'warming up' to collaboration, for example by gaining peers' attention and recruiting them to the collaborative activity (Langer-Osuna, et al., 2020:518), or as a 'mental break' between the attempts to approach the mathematical task (Harel & Lesh, 2003:373).

In addition to indicating the complexity of working in small groups, several previous studies have discussed methodological aspects of analysing students' work in small groups in the mathematics classroom. Parts of these discussions are presented in the next subsection.

1.1.3 Methodological aspects of analysing group work

In a number of studies, researchers' methods and selection of empirical material have been specifically pointed out as a potentially limiting factor when analysing various processes during classroom activities. For example, much research within this field has primarily investigated teacher talk during whole-class discussions, whereas fewer studies have focused on students'

interaction in small groups during everyday work in the mathematics classroom (see Gardner, 2019; Hardman, 2020; Zahner & Moschkovich, 2013). It has also been argued that many studies on students' collaboration in the mathematics classroom have been 'product focused' in the sense that the main focus has been on the outcome of the collaborative activity rather than on the processes of 'communication or collective meaning-making' during the students' on-going work (Seidouvy & Schindler, 2020:412). One example is Klang et al.'s (2021:7) intervention study, which showed that social acceptance and friendship were significantly associated with positive effects on fifth-graders' collaboration in the mathematics classroom. However, the authors also pointed out that they would have needed other kinds of empirical material, such as video recordings, to enable the analysis of 'dialogic relationships that arose in group discussions' (ibid.). This statement is in agreement with Gardner's (2014:610) argument that video recordings are essential when analysing situations 'where one can assume that gestural and other visual practices will play a central role'. Moreover, the mathematical sophistication of students' solutions has been found to improve when they are asked to not only present their arguments in writing but also formulate them orally (e.g., Ayala-Altamirano & Molina, 2021; Stylianides, 2019), which underlines 'the importance of multiple research methods' (Stylianides, 2019:177).

Summary

Activities that allow students to engage in mathematical problem solving and discussions of their solutions are regarded as key elements of the teaching and learning of mathematics. In addition, previous research has identified both supporting and challenging factors related to students' interaction in the mathematics classroom but there still appears to be a knowledge and research gap regarding processes that occur when students engage in mathematical problem-solving activities with their peers during everyday classroom work.

Next, I present the aim and research questions, as well as the rationale, of this thesis.

1.2 Aim and research questions

The aim of this thesis is to further the knowledge of how social interaction can contribute to shaping young students' mathematical argumentation during mathematical problem solving with peers. Two research questions guided the analysis:

- How is young students' mathematical problem solving socially and interactionally organised?
- What characterises the content of young students' explanations and justifications when discussing solutions during mathematical problem solving with peers?

In this thesis, 'young students' refers to students who are 11–12 years old. I find this age group interesting as students at this level of education have had an opportunity to acquire basic arithmetic skills but (usually) have not yet encountered advanced mathematics, such as the teaching of formal algebra or proofs.

The rationale of this thesis is that increased knowledge of students' social interaction in the mathematics classroom can support teachers' didactic choices when encouraging young students to work in small groups during mathematics lessons. Moreover, increased knowledge of the content of young students' mathematical argumentation (in terms of using explanations and justifications) can indicate what parts of the students' proving processes need the teacher's support. Thus, analysing various aspects of Grade-6 students' use of explanations and justifications in small group interaction contributes to increased knowledge of students' general mathematical development and adds to previous research on the role of proving in the teaching and learning of mathematics during the early school years.

1.3 Outline of the 'kappa'

As stated in Chapter 1, the analysis of students' small group work was based on a dialogical theory of communication (see Linell, 2001). Furthermore, in Studies I and III, the theoretical framework of *ethnomethodology* was applied to the analysis of student-student interaction during mathematics lessons. The choice of theoretical and methodological approaches affected the selection of previous studies on which this thesis is based. Thus, the theoretical framework is presented in Chapter 2, before the presentation of previous research in Chapter 3.

Chapter 4 presents the methodology of the studies. This chapter includes a description of my empirical material, my methods of analysis, and a discussion of ethical considerations entailed with recording, transcribing and analysing video recordings.

Chapter 5 summarises the four studies of the thesis. This chapter is followed by Discussion (Chapter 6) and a summary in Swedish (Chapter 7).

2 Theoretical framework

My overall approach to the analysis of students' social interaction in the mathematics classroom aligns with the interdisciplinary perspective of dialogism, which has been described as 'a family of somewhat loosely linked theories and traditions across many disciplines' (Linell, 2009:400). In this chapter, section 2.1 presents the fundamental principles of *a dialogical theory of communication*. This section also includes a theoretical background of *the social turn in mathematics education research* (subsection 2.1.1), an introduction to *an ethnomethodological approach* (subsection 2.1.2), and a presentation of the notion of *social and sociomathematical norms* (subsection 2.1.3).

In section 2.2, I present *theoretical aspects of mathematical argumentation*, which includes a subsection (2.2.1) in which *a theoretical background to proving* is presented, as proving in this thesis is regarded as a particular case of mathematical argumentation.

2.1 A dialogical theory of communication

The perspective of dialogism has several theoretical 'roots'; for example, it 'insists on the inherently sociocultural nature of discursive activities and dialogue' and emphasises that participants in talk-in-interaction are supported by (and negotiate with) both the other participants and the cultural artefacts that are available in the context in which the interaction takes place (Linell, 2001:47). According to a *dialogical perspective on communication*, social interaction is characterised by *sequentiality*, *joint construction* and *act-activity interdependence*.

The principle of *sequentiality* implies that the interactional significance of communicative acts (such as verbal utterances, silence, embodied actions, the use of artefacts, etc.) is 'intrinsically dependent on their positioning in the sequence'[5] (Linell, 2001:70). This means that verbal utterances not only represent the speaker's current contribution to interaction but also have responsive and initiatory aspects, in that they both respond to previous utterances and make further contributions relevant (Linell, 2009:181, 2022:373; Schegloff, 2007:16). This, in turn, implies that a participant's contribution to communication cannot be fully understood if it is taken out of

[5] See a description of the concept of *sequence* in Chapter 4, subsection 4.2.2.

its context since it derives parts of its meaning from the position in the event in which it occurs (Linell, 2009:187). One example of how the principle of sequentiality was applied to the analysis of my video recordings is illustrated by Excerpt 2 in Appendix C, which shows an extract from a discussion about the task 'Hockey stick' (Appendix A). The aim of the task was to calculate a percentage change in the price of an item, and one of the students suggested a solution by presenting a series of procedures that she had executed. One of her peers responded to the suggestion by turning her attention to the wording of the task, stating 'but it says like this' and reading the text aloud. This action achieved resistance to the suggested procedures because of the use of the particle 'but', as well as by its position in this particular sequence, indexed with the deictic resources 'it' and 'this'.

According to the dialogical principle of *joint construction*, communication is regarded as something that participants do together (Linell, 2001:74). The joint construction, or *co-construction* (see Jacoby & Ochs, 1995), of communication is accomplished through participants' use of previous utterances to make new actions relevant (Linell, 2001:76, 207). Below, an extract from Excerpt 3 in Appendix C exemplifies how this principle was applied to the analysis of my video recordings. The extract shows the joint construction of part of the solution to the task 'Shake hands' (Appendix A) about the number of handshakes that certain persons can make. Having performed empirical examinations, the students agreed that five persons can make ten handshakes. They then tried to find a general 'rule' for calculating the number of handshakes, using the result for five persons as a particular example. Following Sonja's demonstration of not understanding, Linn and Boris co-constructed an explanation of their calculations.

```
18.  Sonja:    I don't really get it (.) it was,
19.            but I get it now.
20.  Linn:     but look here,
21.  Boris:    but she had like five persons
22.            ((writes, Figure 2)) there.
23.  Linn:     and then I did,
24.  Boris:    one,
25.  Linn:     it was one times,
26.  Boris:    four.
27.  Sonja:    yeah?
28.  Boris:    cos one can do [four persons.
29.  Sonja:                   [one can do four.
30.  Boris:    and then times five (.) is twenty?
31.            then one should divide it by two
32.            to get the right answer.
```

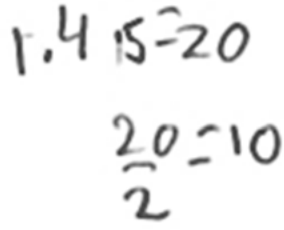

Figure 2: Extract from Boris' notes.

Although Sonja claimed understanding (line 19), Linn and Boris apparently treated her utterance as a request for more information as both of them started explaining the procedures of calculating the number of handshakes among five persons (lines, 20–26, 30–32). Their explanation was partly presented in the form of 'collaboratively built turns' (Sidnell, 2010:167–171) in that they completed each other's utterances; for example, Boris completed Linn's utterance 'it was one times' (line 25) by saying 'four' (line 26).

In many cases, the joint construction of communication is overt, in that 'conversationalists typically borrow words from each other' and use the other's utterances as 'the basis for frequent repetitions of key words' (Linell, 2001:75). This practice, which Goodwin (1990:177–188) denoted *format tying*, is also exemplified by the extract above. While Boris was justifying the multiplication $1 \cdot 4$, stating 'cos one can do four persons' (line 28), Sonja repeated part of Boris' utterance (in overlap with his justification) by saying 'one can do four' (lines 29). In this case, Sonja's repetition indicated agreement, even though the practice of reusing someone else's words can accomplish 'moves that counter as well as moves that affiliate' (Goodwin, 2006:182). Thus, regarding communication as a joint construction does not imply an assumption that participants always agree; both disagreements and agreements are co-constructed (Jacoby & Ochs, 1995).

The third dialogical principle, denoted *act-activity interdependence*, emphasises that participants' communicative acts are embedded in certain activities that can often 'be seen as representing some general type or as belonging to a particular genre' (Linell, 2001:87) at the same time as the activity is in itself constituted by these acts (Ingram, 2021:16). This principle has also been defined as a reflexive relation between 'speech and context'; that is, the *reflexivity* of participants' social actions and the context in which these actions occur is emphasised, since they 'continuously define and shape each other' (Duranti, 1991:136). The act-activity interdependence is particularly obvious when the communication is task oriented, such as in institutional talk, as participants' contributions to communication in task-oriented interaction are not only shaped by the immediate contextual resources (such as prior utterances) but also by the surrounding situation (Drew & Heritage, 1992; Heritage, 2004). Thus, the students' video recorded communication was

shaped by the surrounding situation of participating in an activity in the mathematics classroom; at the same time, the students' communicative acts contributed to constituting the activity of 'a mathematics lesson'.

In studies of task-oriented classroom communication, several characteristic features have been identified that contribute to shaping (and constraining) the interaction between teachers and students (see Clarke & Chan, 2020; Gardner, 2014). Previous research has shown that the teacher usually does most of the talking during teacher-led whole-class interaction[6], whereas the opportunities for students to select themselves as speaker are much more restricted (McHoul, 1978). As well as having the right to initiate and close sequences, the teacher usually controls the topic (Gardner, 2014). Furthermore, the teacher is typically positioned as the most knowledgeable person, which is particularly apparent when teachers ask questions to which they already know the answer (Heritage, 2012; Mehan, 1979). Another common feature in teacher-led communication is the so-called IRE-sequence (Mehan, 1979). During such sequences, the teacher Initiates interaction by raising a question to which the student Responds, and then the teacher closes the sequence by Evaluating the student's answer (ibid.). However, several analyses of classroom interaction have shown that the use of the IRE-sequence can be quite complex (e.g., Radford et al., 2011; Roth & Gardener, 2012). For example, participants in ordinary conversation usually minimise silence to less than one second (Jefferson, 1989), whereas Ingram and Elliott's (2014; 2016) analyses of interaction in secondary-school mathematics classrooms showed that teachers and students frequently allowed an extended 'wait time' (i.e., silence) between the teacher's Initiation and the student's Response. The teacher's silence may be interpreted as a negative assessment, but Ingram and Elliott (ibid.) showed that an extended wait time can also contribute to longer and more elaborate responses. Moreover, students sometimes extend the R-turn by elaborating a peer's answer (Ingram et al., 2019), whereas the teacher's way of handling students' incorrect answers can lead to extensions of the E-turn (Ingram et al., 2015; Tainio & Laine, 2015).

Several research traditions align with dialogist perspectives but approaches that focus on '*research into authentic discourse*' have been of particular importance to the framework of dialogism (Linell, 2001:50). These approaches also align with the so-called *social turn* in mathematics education research, which I present in the next subsection.

[6] According to Einarsson and Hultman's (1984:82) classroom studies, the teacher usually produced about 2/3 of all utterances during lessons.

2.1.1 The social turn in mathematics education research

Since the 1980s, theoretical frameworks for 'interpreting the *social* origins of knowledge and consciousness' have been increasingly frequent in mathematics education research (Lerman, 2000:23). The application of theories that acknowledge social activity as the base for thinking and reasoning, which 'goes beyond the idea that social interactions provide a spark that generates or stimulates an individual's internal meaning-making activity', is referred to as the *social turn* in mathematics education research (ibid.).

Several studies have contributed to the social turn, with Vygotsky's work as a 'key element' (e.g., Hanna & Knipping, 2020; Lerman, 2000). Vygotsky's theory of learning emphasises communication and participation in cultural practices as essential factors in children's development. According to this perspective, the connection between the social (external) practices and the individual's intellectual (internal) activity is considered to be mediated by the use of a variety of semiotic resources (Cobb et al., 1997:271; Lerman, 2000; Linell, 2001:47; Wyndhamn, 1993:26). Research in this tradition has therefore explored the use of semiotic resources as key aspects of students' mathematical development (e.g., Wathne & Carlsen, 2022).

Another key aspect of Vygotsky's theory of learning is the role of the teacher, who is not necessarily a person in a formal position; the teacher might just as well be an older relative or a more knowledgeable peer (Lerman, 2000). Still, the Vygotskian perspective on learning presupposes a rather distinct asymmetry between the interactants. This is particularly apparent in the notion of the zone of proximal development (ZPD), which represents the difference between the individual's developmental level and the 'potential development as determined through problem solving under adult guidance or in collaboration with more capable peers' (Vygotsky, 1978:86). The ZPD has also been described as an activity setting in which 'people gain access to cultural tools through participation as some kind of apprentice or novice' (Wyndhamn, 1993:25). For example, if the setting is the mathematics classroom, the teacher provides scaffolding in terms of the kind of support that the 'novice' needs to progress through the mathematical activity (ibid.). However, in recent studies, the asymmetry implied by the notion of the ZPD has been problematised. Instead of assuming asymmetry between participants, the ZPD can be regarded as an interactional achievement in which all participants have an opportunity to contribute, regardless of their social roles of being a 'teacher' or a 'student' (Roth, 2020a; Roth & Radford, 2010). This enables research on practices where the roles of 'teacher' and 'novice' are interactionally achieved, rather than determined a priori by the setting of the activity.

Vygotsky's work has contributed to the development of several theoretical frameworks, some of which have been used in mathematics education research. One example is *activity theory*, which aims to integrate the 'individual' and the 'collective' (as well as 'body and mind'), meaning that all processes

are analysed in terms of 'intersection of dimensions' in actions and activities (Roth, 2020b). Thus, activity theory implies a multidisciplinary approach since participants' social interaction and use of various mediating tools have to be analysed as integrated parts of their participation in the activity (Davydov, 2008:208–209; Linell, 2001:47). This theory has been used, for example, in studies of young students' development of algebraic thinking, which emphasised the importance of using mediating tools (such as various types of manipulatives) (e.g., Eriksson, 2021; Wettergren, 2022). Furthermore, according to the theoretical framework of *commognition* (which is a term that combines the concepts of *communication* and *cognition*), the individual's thinking can also be analysed in terms of a communicative process. This means that thinking is not regarded as a 'self-sustained process separate from [acts] of communication' but as an '*individualized form of the activity of communicating*' (Sfard, 2008:81–83; Sfard, 2020). In other words, the individual's cognition and his/her communication with others are not regarded as essentially separate processes, but rather as different manifestations of the same phenomenon since 'thinking' is conceptualised as communicating with oneself (ibid.). Consequently, this view implies that learning is not seen primarily as a matter of the individual's 'acquisition' of some knowledge but as a process that depends on how communication between 'the newcomer' and members of the community is interactionally organised; that is, according to such a *participationist* view, learning is a process involving activities during which the 'newcomer' (or 'apprentice') is moving 'toward full participation in the sociocultural practices of a community' (Lave & Wenger, 1991:29). For example, the framework of commognition has been applied to the analysis of discussions among biology students who engaged in an activity involving mathematical modelling, as an 'initiation into the discourses of mathematics' (Viirman & Nardi, 2019:236), and the participationist view has been applied to studies of mathematics teachers' professional development (e.g., Rupnow, 2016).

The development of the above-mentioned views of learning were partly based on a dialogical perspective on communication (see Davydov, 2008; Lave & Wenger, 1991; Sfard, 2008; Vygotsky, 1978). Yet another theoretical framework that belongs to the 'family' of dialogism is that of *ethnomethodology* (Linell, 2001:50), which is not (yet) commonly applied in mathematics education research (see Ingram, 2018). According to the theoretical framework of ethnomethodology 'the most commonplace activities of daily life' should be analysed as 'phenomena in their own right' (Garfinkel, 1967:1). In ethnomethodological studies, participants' actions while engaging in everyday activities are regarded as representations of their methods of making these activities 'accountable' (Garfinkel, 1967:vii). When making activities accountable, participants orient towards making the 'sense and purpose' of the activity understandable and recognisable for both themselves and others (ten Have, 2012:104). Thus, analyses using an ethnomethodological

approach are based on participants' perspectives on what is meaningful in a particular situation (i.e., an *emic* perspective), rather than on external categories (i.e., an *etic* perspective) (Broth & Keevallik, 2020; ten Have, 2012).

The aim of this thesis is to increase the knowledge of how social interaction can contribute to shaping young students' mathematical argumentation. Using ethnomethodology's emic perspective on students' meaning making practices during mathematical problem solving with peers, video recordings of young students' everyday work in the mathematics classroom are analysed. The next subsection further introduces fundamental principles of ethnomethodology.

2.1.2 An ethnomethodological approach

The primary interest of ethnomethodological studies is participants' *social actions*; that is, although 'language use cannot be isolated from language structure' (Linell, 2001:36), the focus is neither on language as such, nor on the individual participants per se, but rather on 'what participants are doing with their utterances or gestures' in interaction (Ingram, 2018:1065). This interest in social action as an interactive phenomenon builds on both the view that language is in itself a social phenomenon (Linell, 2001:86) and the idea that communicative acts achieve different 'things', not only depending on participants' choice of words but also on the way their utterances are produced, for example, in terms of prosody or pauses (Ingram, 2018), and on the local positions of the utterances in the unfolding of events in time (Heritage, 1984a).

One of the fundamental principles of ethnomethodology is that communicative acts are contextually oriented (Heritage, 1984a:242; Ingram, 2021:16)[7]. Contributions to communication are *context-shaped* since participants 'address themselves to preceding talk and, most commonly, the immediately preceding talk' (Heritage, 2004:105). In addition, all communicative acts are *context-renewing* in that the 'current' action forms the immediate context for a relevant 'next' action (Heritage, 1984a:242, 2004). Thus, participants' communication relies on 'detailed indexical understandings of what might be happening now, what just happened, and what will likely happen next' (Jacoby & Ochs, 1995:174). Here, the use of the word 'understandings' does not imply that the individual's cognitive processes are analysed; instead, it refers to the way that participants' responses display how they have taken up prior contributions (Linell, 2001:79; Schegloff, 2007:15). In this way, communication contributes to *intersubjectivity*, which represents a 'taken-as-shared' sense of the understanding of an object or event (Cobb & Bauersfeld, 1995:295). Intersubjectivity has also been described as a representation of a 'convergent knowledge of the world' (Sidnell, 2010:12), which, in turn, implies that participants 'act as if they are thinking of the same thing' (Voigt, 1995:172).

[7] The principle of contextuality resonates well with the dialogical principles of *sequentiality* and *act-activity interdependence*, which were introduced in section 2.1.

Furthermore, participants' responses to others' contributions to communication not only represent their understandings of the prior communicative acts, but also contribute to the very constitution of the activity itself (Garfinkel, 1967:3–4; Ingram, 2021:16)[8]. For example, a situation or an object cannot be considered inherently 'pedagogical' or 'mathematical'; instead, situations and objects become pedagogical and/or mathematical in the interaction of those who participate in an activity that occurs in a particular local context (Gejard, 2018:56-57). In other words, what matters is what activities and arguments participants themselves treat as mathematically relevant (see Ingram, 2021:106–107), which is an aspect of taking a participants' (i.e. an emic) perspective on mathematical activities and objects.

Yet another fundamental ethnomethodological principle is that no detail of interaction 'can be dismissed, *a priori*, as disorderly, accidental or irrelevant' (Heritage, 1984a:241). Instead, communicative acts represent participants' procedures of making their own actions *accountable* in the sense of being both understandable to the individual participant and relevant to the particular situation (Garfinkel, 1967:1). The principle of accountability also implies that speakers can be 'held accountable' for their actions, in that they are expected to 'explain or justify what they say and why (they think that) they do it' (Linell, 2001:79). For example, during discussions about mathematical tasks, students are expected to provide explanations and justifications that are regarded as mathematically acceptable (Ingram, 2021:106–108; Yackel & Cobb, 1996), which indicates how the principle of accountability applies to argumentation in the mathematics classroom.

The overarching principle of ethnomethodology is that *social interaction is orderly* in that participants' actions display observable and recognisable patterns (Garfinkel, 1967). For example, when participants contribute to conversation, they also project some relevant next action to be accomplished by someone else. In addition, participants who respond to others' utterances contribute to the accomplishment of the activity, since the recipients' responses both display their own understanding of the situation and allow others to see whether the recipients have responded in the expected way (e.g., Heritage, 1984a). Thus, while engaging in discussions in the mathematics classroom, participants co-construct a publicly recognisable order, which they use 'as a structure to construct their own actions and to interpret the actions of others' (Ingram, 2018:1066). This order has also been referred to as *norms*, which represent patterns that are 'usually seen but unnoticed' (ibid.) since '[much] of what is actually reported is not mentioned' (Garfinkel. 1967:3). In the mathematics classroom, interaction has also been characterised by a double set of norms, both general *social* norms and *sociomathematical* norms (Yackel & Cobb, 1996). These concepts are introduced in the next subsection.

[8] See also the introduction of the concept *act-activity interdependence* in section 2.1.

2.1.3 Social and sociomathematical norms

During the 1990s, Cobb and Yackel (1996, 1998; Yackel & Cobb, 1996) developed the *emergent perspective* on mathematical meanings in the social context of the mathematics classroom. This 'interpretive framework' drew on principles of constructivism, symbolic interactionism and ethnomethodology, and was also informed by findings of empirical classroom studies (Cobb et al., 1992a, 1992b; Yackel & Cobb, 1996). The aim was to coordinate social perspectives on classroom processes with psychological perspectives on individuals' participation in these processes (Cobb & Yackel, 1996, 1998).

According to the emergent perspective, the individual student's mathematical development depends on his/her participation in 'the interactive constitution of taken-as-shared mathematical meanings'; that is, the individual's processes of argumentation and 'the taken as shared basis for communication' are considered to be reflexively related (Yackel & Cobb, 1996:460–461). Furthermore, Yackel and Cobb (1996:461) treated these taken-as-shared mathematical meanings as *sociomathematical norms*, which represent what counts as 'mathematically normative' in interaction. In all classrooms, general *social* norms refer to the expectations that participants have for one another, and for the activity in which they are participating (e.g., Stephan, 2020). However, in the mathematics classroom, the interactional patterns are not only represented by such general norms. Instead, when participating in the teaching and learning of mathematics, participants' interaction orients towards norms that constitute a 'taken-as-shared' sense of when and how to contribute to mathematical activities and argumentation (Cobb & Yackel, 1998:170). For example, it is a general social norm that new contributions to discussions should be different from previous ones, but in the mathematics classroom the current contribution has to be *mathematically* different from previous arguments, which is a sociomathematical norm (Cobb & Yackel, 1998:169; Yackel & Cobb, 1996:461). Similarly, what is regarded as a mathematically 'sophisticated', 'efficient' or 'elegant' argument depends on the sociomathematical norms of the classroom (ibid.). As shown by classroom studies, norms are not 'predetermined criteria introduced into the classroom from the outside' (Yackel & Cobb, 1996:474). Instead, social and sociomathematical norms emerge from social interaction and are subject to negotiation between participants; thus, they are developing (as well as changing) over time (e.g., Hershkowitz & Schwarz, 1999; McClain & Cobb, 2001; Partanen & Kaasila, 2015; Stephen, 2020; Yackel & Cobb, 1996).

It has been argued that it is fairly easy for teachers to establish social norms (Stephen, 2020), possibly because such norms are present in all classroom communication and draw on students' participation in everyday social interaction. Sociomathematical norms, on the other hand, are more specific since they are co-constructed by participants in mathematical activities. However, studies have also indicated that students' perception of the sociomathematical

classroom norms can differ greatly from the teacher's intentions, which can cause some tension during classroom discussions (e.g., Levenson et al., 2009; Wester, 2015). Thus, the interactive construction of social and sociomathematical norms in classroom interaction can both support and restrain students' argumentation during mathematical problem-solving activities (Yackel & Cobb, 1996, Cobb & Yackel, 1998:166).

Next, I present some theoretical aspects of argumentative processes, with a particular focus on *mathematical argumentation*.

2.2 Theoretical aspects of mathematical argumentation

In this thesis, *mathematical argumentation* is (in accordance with Krummheuer, 1995) seen as a particular case of social interaction, in which participants strive to make sense of others' contributions, as well as to account for their own actions. However, there is a range of descriptions of argumentation in the mathematics classroom (see Staples & Conner, 2022). For example, it has been described as 'mathematical arguments that students and teachers produce in mathematics classrooms' (Sriraman & Umland, 2020), which emphasises the arguments as such, rather than the processes of arguing. Another approach is to describe mathematical argumentation as 'any written or oral discourse conducted according to shared rules, and aiming at a mutually acceptable conclusion about a statement' (Durand-Guerrier et al., 2021), which focuses on a strive for agreement. Yet another approach, which emphasises the argumentative process, is to regard mathematical argumentation as 'the process of making mathematical claims and providing evidence to support them' (Rumsey et al., 2022).

According to more general approaches to argumentation, the argumentative process has been described as 'a means for trying to lead people to accept [or acknowledge] what you say', which indicates an assumption that people do not already do so (Baker et al., 2020:77–78). Argumentation has also been described as part of dialogue, functioning as 'a communicative and interactional act complex aimed at resolving a difference of opinion' (van Emeren et al., 2014:5–7). The 'arguer' is then expected to put forward suggestions for which (s)he can be held accountable, and that the 'addressee' can respond to as 'a rational judge who judges reasonably' (ibid.). This does not mean that the concept of 'argumentation' necessarily refers to disagreements; instead, it is regarded as a process of social interaction in which 'two or more individuals engage in a dialogue where arguments are constructed and critiqued' (Nussbaum, 2011:84).

A well-established operationalisation of the concept 'argumentation' is Toulmin's (1958) general model of argumentative processes, which focuses on elements and stages of the process, rather than on the outcome. In addition,

this model does not assume that argumentation implies that there is disagreement among the participants. According to this model, argumentation involves the formulation of a *claim*, which is the part of the argument 'whose merits we are seeking to establish' (Toulmin, 1958:97ff). This is followed by references to *data*, which are facts that can be used to support the claim. The data can also be justified by the use of a *warrant* which, in turn, is supported by the *backing*. The certainty of the claim is indicated by a *qualifier*, whereas the *rebuttal* represents a challenge or refutation of the claim.

Although Toulmin's model is general, it has also been used in studies of mathematical argumentation. For example, the model was used in analyses of students' whole-class discussions (e.g., Knipping, 2008; Krummheuer, 2007), in a study of pre-service teachers' participation in a mathematics lesson study (Groth & Follmer, 2021), and in analyses of teachers' responses to and discussions about students' (oral and written) mathematical reasoning (Metaxas et al. 2016; Nardi et al., 2012; Zhuang & Connor, 2022). In these studies, Toulmin's model was primarily used as a tool to categorise arguments.

Research based on a sociocultural perspective on students' development and learning has emphasised the 'reciprocal relationship between individual thinking and the collective intellectual activities of groups', which is particularly apparent when students interact in argumentation during group work (Mercer, 2009:180ff). Similarly, when an ethnomethodological approach to mathematical argumentation has been applied, it has enabled an understanding of the argumentative process as an element of social interaction 'when cooperating individuals [try] to adjust their intentions and interpretations', and account for their contributions 'by verbally presenting the rationale of their actions' (Krummheuer, 1995:229, 232). In particular, Krummheuer (1995:232) emphasised that mathematical argumentation does not take the form of a monologue but rather is a joint accomplishment by participants engaging in face-to-face interaction.

Explaining and justifying solutions are characteristic elements of argumentation in the mathematics classroom (e.g., Zhuang & Connor, 2022). In a number of studies, *proving* has also been treated as a phenomenon that is closely related to argumentation (see Durand-Guerrier et al., 2021) even though distinctions have been made between the processes of argumentation and proving (e.g., Reid & Knipping, 2010:155). In the next subsection, I present some theoretical aspects of the activity of proving, regarded as a particular case of mathematical argumentation.

2.2.1 A theoretical background to proving

Since the notion of proof was first introduced[9], various meanings and functions of the activity of proving have been presented. In addition, questions have been raised if it is possible to reach a consensus on one unique definition of what constitutes a mathematical proof (e.g., Balacheff, 2008; Hanna, 2000; Reid & Knipping, 2010). According to Bell (1976:24), the aim of the proving process is to *verify* (or *justify*) the truth of an assertion, to *illuminate* (or *explain*) an argument, and to *systematise* results in terms of organising them 'into a deductive system of axioms, major concepts and theorems, and minor results derived from these'. Problematising the view that verification is the primary goal of mathematical proving, de Villiers (1990:18) elaborated five functions of proofs. de Villiers (ibid.) regarded *discovery* (in terms of discovering or inventing new results) and *communication* (representing the transmission of mathematical knowledge) as equally important functions of proving as verification/justification, illumination/explanation and systematisation. Hanna (2000:8) then expanded the 'list' of previously suggested definitions of mathematical proving by adding the processes of '*construction* of an empirical theory, *exploration* of the meaning of a definition or the consequences of an assumption [and] *incorporation* of a well-known fact into a new framework and thus viewing it from a fresh perspective'. Nevertheless, Hanna (ibid.) still emphasised that activities of explaining and justifying mathematical arguments are the most fundamental elements of proving.

A particular characteristic of formal proving is that the argumentation involves *generalising*. Like the concept of proving, generalising has been defined in various ways. For example, Mason (1996:65) stated that generalising involves '*seeing a generality through the particular* and *seeing the particular in the general*'. Applying this definition of generality, teachers can encourage students (at all levels of education) to formulate general arguments, while engaging in mathematical problem-solving activities, by emphasising that the goal of the proving process is to justify solutions in ways that satisfy 'a "for all" argument'; that is, an argument that neither concerns only isolated cases nor accepts any exceptions (Harel & Sowder, 2007:809).

Traditionally, students were not introduced to formal proving before upper-secondary school level (and then it commonly occurred in relation to the teaching of geometry), whereas the teaching of mathematics at lower levels of education usually focused on performing calculations and algorithms (Knipping, 2008; Stylianides, 2007). Thus, the introduction of formal proving was rather abrupt, which could have contributed to students experiencing difficulties in engaging in processes of proving, as well as in understanding

[9] Greece is commonly considered the geographical origin of proofs; in particular, Thales (around 600 years BCE) is often cited as the first mathematician to formulate a proof. However, mathematics was developed in a wide range of cultures over a long period of time – and many of the original sources have been lost (Reid & Knipping, 2010:3ff).

the content and characteristics of mathematical proofs (ibid). On the other hand, activities involving 'justification' have sometimes been described as closely associated with the proving process, even though the relationship between justifying and proving is not quite clear (Staples & Conner, 2022). For example, the concept of proof has been defined as 'the means for justifying knowledge in mathematics' (Hemmi, 2006:16), which indicates that proving and justifying are similar processes. However, the activity of justifying has also been described as a practice 'on-the-way-to-proof' (Staples & Conner, 2022:5), which indicates that 'justification' is a precursor to 'proof'.

Furthermore, formulations similar to definitions of what constitutes formal proofs have been used to describe activities in the elementary-school classroom. For example, Ayala-Altamirano and Molina (2021:361) applied Bell's definition of proofs when defining Grade-4 students' justifications as a result of a social process in which mathematical knowledge is 'explained, verified and systematized'. In addition, the process of proving has been described rather informally as 'a way of communicating ideas with others' and as a method of providing 'a clear explanation of why something is the case' (Schoenfeld, 1994:74, 76). Emphasising the content and the proving activity, rather than the form of the mathematical proof, Hanna (1989:20) stated that proofs represent a type of argument that is 'needed to validate a statement' and suggested that such arguments could be formulated in different ways as long as they are 'convincing'. Other ways to present the activity of proving more informally involve descriptions of processes that bring 'conviction' (Bell, 1976:24) or establish the 'truth for a person or a community' (Harel & Sowder, 2007:806).

As well as describing the processes of proving, studies have explicitly emphasised the social and contextual aspects of the proving process (e.g., Harel & Sowder, 2007; Shinno & Fujita, 2022). Bell (1976:24) defined proving as an 'essentially public' activity that requires 'classroom explanations to have meaning for the pupil rather than be formal rituals'. Similarly, Stylianides (2007:292) argued that proving implies using mathematical arguments that apply statements and modes of communication that are 'appropriate and known to, or within the conceptual reach of, the classroom community'. Furthermore, arguing that students' development of new knowledge 'does not take place in a vacuum' but is based on what they already know, as well as on the social context in which the learning process occurs, Harel and Sowder (2007:807) proposed a *comprehensive perspective* on proving that accounts for several factors (i.e., not only the students' cognitive development) that can support students' participation in mathematical argumentation and proving. A key element of the comprehensive perspective was to avoid all 'a priori' determinations of what constitutes proofs for the individual. Instead, they emphasised that 'what is offered as a convincing argument must be accepted by others' during the proving process (Harel & Sowder, 1998:244; 2007:808).

Such an interpretation of proving enables the use of this concept as a representation of any activity that involves acts of justification, rather than of processes that strictly relate to the narrower notion (and requirements) of formal proofs (Sowder & Harel, 1998). However, despite noting the contextual aspects of proving, both Harel and Sowder (1998:277; 2007:807) and Stylianides (2007) emphasised that the overall goal of the teaching of mathematics is to support the students' gradual development towards an understanding of proofs that is in line with what is accepted in the mathematics community.

Summary

In this thesis, the overarching perspective is represented by a *dialogical theory of communication*. In particular, an *ethnomethodological approach* (involving an *emic* perspective) was applied to the analysis of students' social interaction in the mathematics classroom. This means that students' communicative acts, while engaging in mathematical problem-solving activities, were regarded as both *context-shaped* and *context-renewing*.

In accordance with the ethnomethodological approach, students' social interaction in the mathematics classroom was regarded as orderly. This order was considered to be constituted by both *sociomathematical* and *social* norms, constructed socially and interactionally in and through the classroom discourse. In addition, the students' contributions to mathematical argumentation were analysed in terms of procedures of making their actions *accountable* in relation to their peers and their teacher.

Furthermore, *mathematical argumentation* was regarded as a particular case of social interaction, during which students strived to account for their own actions, as well as to make sense of others' contributions. In addition, the activity of *proving* (including acts of justifying arguments in an informal manner) was regarded as a special case of mathematical argumentation.

3 Previous research

As noted in Chapter 1, beneficial and challenging aspects of students' interaction in the mathematics classroom have been identified, but previous studies have also discussed methodological aspects of analysing students' group work (see subsection 1.1.3). I have used an ethnomethodological approach in the analysis of student-student interaction, which affected the selection of studies that formed the basis for this thesis. In the first section (3.1), I focus on *ethnomethodological studies in the mathematics classroom*; that is, I present previous studies that have applied ethnomethodological approaches, such as conversation analysis (CA) and multimodal analysis, in research on interaction during work on mathematical tasks in educational settings.

The second section (3.2) presents studies that have investigated aspects of the way in which students explain and justify solutions, which also involve elements of generalising and proving. In this subsection, I focus on *proving and generalising in school mathematics*; that is, I primarily present studies that have investigated students' argumentation in educational settings below the university level.

3.1 Ethnomethodological studies in the mathematics classroom

This section contains two subsections. In subsection 3.1.1, I present studies focusing on *teacher-led interaction in educational settings*, while subsection 3.1.2 focuses on previous research on *student-student interaction in the mathematics classroom*.

3.1.1 Teacher-led interaction in educational settings

In addition to passing on knowledge and 'coming to know', processes of negotiating meaning are key concerns in all classroom interaction (e.g., Heller, 2017; Ingram, 2020). In one of the earliest studies that applied the methodological framework of Conversation Analysis (CA) in the field of mathematics education research, Forrester and Pike (1998:337ff) analysed how 'locally produced meanings' were negotiated during the teaching and learning of estimation and measurement in Grade-5 and -6 classrooms. They

noted that teachers usually introduced the classroom work by saying that they would start by doing 'some estimating', indicating that estimation precedes measurement. The teachers also associated the activity of measuring with correctness and using a ruler, whereas estimation was explained as thinking 'roughly'. The students' discussions during small group work apparently reflected the teacher's talk; for example, the students talked about 'estimating' in terms of 'guessing', while 'measuring' was associated with finding 'the right answer' (ibid.). Similarly, Roth and Gardener (2012) showed how teachers' talk influenced Grade-2 students' mathematical argumentation during discussions about geometric shapes, for example by using formal concepts when asking follow-up questions or reformulating parts of the students' descriptions of the shapes that they had investigated. On the other hand, Mushin et al. (2013:427) showed that teachers' use of informal language (such as saying 'put all the big ones together', instead of 'put all of the same size together'), while comparing geometric shapes, supported primary-school students who had language difficulties. Thus, while negotiating meaning, teachers can contribute to students' development of mathematical problem-solving abilities, as well as their formal mathematical language, by 'working to re-establish shared understanding' of the mathematical content and by handling troubles related to 'word choice, grammar, pronunciation or natural language' (Barwell, 2023:540ff).

Processes of negotiating meaning could also involve teachers' treatment of their students' requests for help. Analysing secondary-school students' help-seeking behaviour during mathematics lessons, Koole (2012) and Koole and Elbers (2014), found that teachers' responses to students' requests for help did not always correspond to the students' displays of trouble. For example, the teacher sometimes started explaining before establishing the student's actual problem, or formulated an explanation that did not quite address the student's question; that is, although the student could be expected to be most knowledgeable about his/her own difficulties, the teacher's explanation sometimes 'oriented to a specific problem' that the student was 'invited to align to', rather than responded to what the student had asked about (Koole, 2012:1914). In contrast, Svahn and Melander Bowden's (2021) multimodal analysis of students' help seeking during homework support sessions at secondary-school level, arranged by a voluntary organisation, showed that the formulation of the problem was jointly achieved by the student and the homework 'tutor'[10]. This may have depended on the educational setting; although the homework support sessions had pedagogical goals, the setting was quite different from teaching in ordinary classrooms (ibid.), in which the teacher not only controls most of the interaction but is often also positioned as the most knowledgeable person (Gardner, 2014; Heritage, 2012). During the homework support

[10] The majority of the tutors who were enrolled by the voluntary organisation were university students with an interest in mathematics, but without teacher training.

session, the students worked individually and initiated interaction by calling for the tutor's support. The student then waited for the tutor to sit down next to him/her in a position in which materials such as worksheets, notes and textbooks were visually accessible to both of them (Svahn & Melander Bowden, 2021). The articulation of the problem then became a joint project, with both the student and the tutor using all the available materials to co-construct a question (ibid.).

Students' displays of 'understanding' has been found to be an interactional object that is quite different from displays of 'knowing' (see Koole, 2010). Moreover, an implicit element of students' help-seeking behaviour is the use of 'epistemic disclaimers', such as claims of not knowing or not remembering (Ingram, 2020). Students' epistemic disclaimers have generally been analysed in terms of an 'undisturbing' feature of classroom interaction since they do not challenge the teacher's epistemic status, whereas students' knowledge claims have usually been treated as problematic since they potentially challenge the teacher's position (Heller, 2017:159–160). However, analysing teacher-student interaction in secondary-school mathematics classrooms, Ingram (2020) showed that, when students claimed not to know, the teacher often still treated the student as knowledgeable, for example by repeating the question (which gave the student more time to answer), or handled the situation as an issue of 'not remembering' rather than 'not knowing'. The teachers may have acted in this way because the students' claims of 'not knowing' could suggest that their lack of knowledge was the teacher's fault, since the 'institutional role of the teacher includes responsibility for ensuring students have access to the knowledge they need' (Ingram, 2020:8). In contrast, when a student claimed not to remember, the teacher immediately redirected the question to someone else (ibid.).

Teachers' ways of handling incorrect answers are yet another aspect of their methods of responding to students' help-seeking behaviour. Analysing teachers' treatment of sixth- and seventh-grade students' responses both Tainio and Laine (2015) and Ingram et al. (2015) found that teachers generally avoided negative assessments. Tainio and Laine (2015) identified seven types of evaluative responses to students' incorrect answers that teachers used instead of explicitly assessing an answer as incorrect; these types of responses included reformulating the question (which allowed the student to self-correct or elaborate his/her answer), or formulating a new question based on the student's response. Moreover, teachers sometimes treated a student's incorrect answer as a mistake, rather than as evidence of a more fundamental misunderstanding, which Tainio and Laine (ibid.) interpreted as the teacher's way of conveying that producing an incorrect answer is a natural element in mathematical discussions. In addition, this may have been a way to encourage students to continue participating in the classroom activity (ibid.).

The studies referred to above indicate that students' active participation is a key factor of the processes of meaning making and 'coming to know'.

However, the opportunity to participate during mathematics lessons does not always lead to fruitful discussions. Comparing interaction in upper-secondary classrooms in Sweden and the USA, Emanuelsson and Sahlström (2008) found that the teachers in the USA controlled most of the discussion, whereas the students' participation was restrained, which usually led to a coherent presentation of the mathematical content. In contrast, the Swedish students were given more opportunities to participate, which seemed to cause a 'watering-down of the mathematical complexity of the question and answer' (Emanuelsson & Sahlström, 2008:212). For example, when the teacher in the Swedish classroom initiated a discussion of linear equations with different gradients, the students' contributions primarily concerned the visual aspects of the graphs that corresponded to the equations. However, Emanuelsson and Sahlström (2008:215) emphasised that this was not necessarily a consequence of participation as such but rather a result of how various communicative resources were used by the participants.

As can be seen in the studies reported in this subsection, previous ethnomethodological research on students' participation in mathematical activities have often concerned the teaching and learning of mathematics in school settings. However, analysing preschool children's block play, Gejard and Melander (2018) showed that play can also offer opportunities to engage in mathematical activity, for example, when children and teachers use geometric concepts while discussing and comparing shapes that the children built during informal play. Similarly, Creider (2012:40) noted how a teacher, engaging in play with plastic pegs in the informal setting of a three-year-old child's home, might have helped the child to 'think mathematically about their own play', for example by talking about stacking pegs on top of each other in terms of an activity that involved 'measuring'.

3.1.2 Student-student interaction in the mathematics classroom

A common trait in previous research on classroom interaction that has applied ethnomethodological approaches is that the studies have primarily focused on teacher-student interaction (see an overview of previous studies in Gardner, 2019). This is also the case in most ethnomethodological studies on social interaction during mathematics lessons. In other words, there is somewhat of a research gap regarding ethnomethodological studies on student-student interaction in the mathematics classroom (Kämäräinen et al., 2019). Thus, Kämäräinen et al.'s (ibid.) analyses of secondary-school students' epistemic management during mathematical problem-solving in small group interaction have made important contributions to this field of research.

Analysing students' epistemic management during small group work, Kämäräinen et al. (2019) showed that the characteristics of the interaction depended on the students' epistemic status (see Heritage, 2014). The interaction among peers varied depending on if the student who initiated conversation

was positioned as 'more knowledgeable' or 'less knowledgeable'. When the interaction was initiated by a student who was positioned as more knowledgeable (i.e., K+), the dialogue usually resembled the IRE-sequence[11]. In contrast, when the discussions were steered by a less knowledgeable student (i.e., K-), more so-called 'wh-questions'[12] were raised. In addition, the discussions were then often characterised by disputes during which the K- student challenged not only the mathematical arguments but also the K+ student's epistemic position (Kämäräinen et al., 2019). Moreover, a second study by Kämäräinen et al. (2021:312) showed how students for the most part used 'mitigated and persuasive ways of directing each other' when initiating a joint activity, or transitioning between activities, in that they used proposals designed as interrogatives or declaratives rather than imperatives. These ways of initiating and steering group work were interpreted as the students' 'endeavour to maintain egalitarian social relationships' with peers (ibid.).

A specific aspect of interaction in the mathematics classroom is that students are not only expected to explain their arguments, but also strive to *prove* their solutions, in terms of formulating a general justification. In the next section, I present studies on *proving and generalising in school mathematics.*

3.2 Proving and generalising in school mathematics

In this section, subsection 3.2.1 focuses on previous research on the teaching and learning of *proving in school mathematics*. Subsection 3.2.2 then presents studies on *generalising in school mathematics*.

3.2.1 Proving in school mathematics

As noted in Chapter 2 (subsection 2.2.1), the relationship between *justifying* and *proving* is not quite clear (see an overview of previous research in Staples & Conner, 2022). However, it has been suggested that a fundamental difference between justifications and proofs is that the former do not have to be 'complete', which implies that students at all levels of education can engage in activities involving justifying mathematical arguments (Thanheiser & Sugimoto, 2022:35ff). In addition, the act of justifying a solution has been described as 'a means for students to explain their reasoning' (ibid.), which also indicates that there is a close relationship between *explanations* and *justifications* (see also Cobb et al., 1992a).

[11] See a description of the IRE-sequence in Chapter 2, section 2.1.

[12] A 'wh-question' is a type of question used to require some kind of information (rather than only a 'yes' or 'no' answer) and is usually initiated by 'why', 'who', 'which', 'where', 'when' or 'how'.

Regardless of definitions, the concept of *proof* has been treated very differently over time in the curricula of various countries (see Hanna, 2000; Healy & Hoyles, 2000; Hemmi, 2006:14ff; Hersh, 2009). For example, when the notion of proof was first incorporated into the syllabus for school mathematics in the USA during the 1960s, the focus was initially on the teaching and learning of axiomatic structures (Hanna & Knipping, 2020). During the 1980s the view of what should be taught changed quite radically; more informal presentations of proofs, as well as arguments involving 'a mixture of natural language and formulae', were then accepted 'in practice' (Hanna & Knipping, 2020:2). Still, formal proving is generally considered a challenging part of the teaching and learning of mathematics at all levels of education (e.g., Flegas & Charalampos, 2013; Healy & Hoyles, 2000; Hersh, 2009; Morris, 2009). For example, previous research on teaching and learning about mathematical proofs at upper-secondary school level has shown that students (as well as many teachers and teacher students) tend to treat the results of empirical examinations of a limited number of examples as 'proofs', while the fact that one counterexample is sufficient to refute a general statement is not always accepted (Reid & Knipping, 2010:59ff).

Despite the increased attention in recent decades on the role of mathematical proofs in school curricula in several countries (see Flegas & Charalampos, 2013; Shinno & Fujita, 2022; Stylianides et al., 2017) most of the studies on students' work with proofs concern the teaching of mathematics in upper-secondary school and at university level (e.g., Campbell et al., 2020; Hanna & Knipping, 2020; Reid & Knipping, 2010:60). In other words, although some studies have investigated elementary- and secondary-school students' abilities to formulate and use elements of proofs while justifying solutions to mathematical tasks, there is still a research and knowledge gap regarding the role of proving at lower levels of education (Flegas & Charalampos, 2013; Shinno & Fujita, 2022). It could be that this gap reflects a low incidence of teaching about proofs in school mathematics. Summarising studies of teachers' perceptions and beliefs regarding proving, Reid and Knipping (2010:80) noted that teachers reported a rather restricted view of the role of proofs in that they singled out 'verification' and 'logical thinking' as the primary functions of a mathematical proof. In addition, Brunner and Reusser (2019:750) suggested that many teachers 'may not have established a regular practice of proving in their mathematics classrooms'. Wathne and Brodahl's (2019:16) study implicitly confirmed previous findings; according to their study, which was based on questionnaires and interviews, many schoolteachers reported that they were inexperienced with proving, possibly also indicating that these teachers were not fully aware of 'the importance of teaching reasoning'.

The view of what should be taught and what teaching methods should be used is related to what is treated as mathematically relevant, which in turn depends on the sociomathematical (and social) norms of the classroom (Yackel & Cobb, 1996). The changes during the 1980s regarding views about

the content of school mathematics, as reported by Hanna and Knipping (2020:1–2), corresponded to changes in the perception of the learning process, in the sense that learning was no longer regarded primarily as a consequence of passively receiving information from the teacher, but as a consequence of students' active participation in the creation of 'meaningful connections between prior knowledge and new knowledge'. Previous findings regarding the use of various semiotic resources implicitly align with this view of learning, in that tasks and work methods that encourage the active use of a variety of modalities have been found to support students' argumentation and proving[13]. Analysing mathematical tasks solved by fifth-graders, Richardson et al. (2010) concluded that the students' reasoning was supported by tasks that allowed them to answer why-questions. Such tasks were characterised by being open-ended, promoting predictions and encouraging conjecturing. In addition, Ayala-Altamirano and Molina (2021:372) identified a difference in the level of sophistication in fourth-grade students' written and oral arguments about functional relationships; for most of the students, the level of sophistication of their mathematical argumentation was higher 'orally than in writing'. Interviewing sixth-graders, Flegas and Charalampos (2013) also noted that references to empirical justifications were useful when students constructed algebraic or geometric proof. Similarly, Shinno and Fujita's (2022:3344) teaching experiment among Grade-5 students indicated that visual mediators 'often shaped students' ways of proving'. They also noted that the students' work on examples initially seemed to be part of the individual student's problem-solving process, but became a more explicit element of the students' joint proving when they engaged in dialogue with their peers.

3.2.2 Generalising in school mathematics

A common requirement for mathematical arguments to be considered formal proofs is that they fulfil the properties of generality in the sense that they involve 'for all' arguments (e.g., Harel & Sowder, 2007:809). Similar to research on young students' proving, studies focusing on students' abilities to generalise have noted that this is a challenging aspect of the teaching and learning of mathematics (e.g., Ellis, 2011; Ellis et al., 2021; Jurow, 2004). In addition, many studies on generalising have been conducted in 'laboratory settings', designed as teaching experiments or based on students' responses during 'clinical interviews' (Ellis et al., 2021:1420). Students' general arguments have then often been analysed as 'an individual, cognitive construct' (Ellis, 2011:310). Previous research has also identified connections between students' generalising and their practices of justification. For example, Lannin (2005:235) argued that generalisation 'cannot be separated from justification'

[13] This shift in approach to mathematics education also aligns with the 'social turn' within mathematics education research (see Lerman, 2000).

and Ellis (2007:196) stated that 'engaging in acts of justification may be as likely to influence students' abilities to generalize as the other way around'. In addition, Ellis (2011:337) emphasised that it is not only the teacher who can take on the role of 'an orienting guide' when students engage in processes of justifying and generalising; peers can also play such a supportive role.

Research on young students' generalising abilities has primarily concerned aspects of the teaching of early algebra (e.g., Blanton et al., 2019; Strachota, 2020, 2023; Urena et al., 2019, 2022) including students' work on patterns (e.g., Lannin, 2005). For example, Blanton et al.'s (2019:212) longitudinal teaching experiments that involved problem-solving activities in Grades 3–5 showed that students who had received teaching about 'early algebraic concepts and practices' were more successful when formulating general solutions and using formal mathematical representations than students who had received arithmetic-focused instruction. Analysing young students' reasoning about functional relationships as part of an early algebra intervention, Strachota (2020) emphasised the importance of teachers encouraging students to share their ideas during whole-class discussions (which enables peers to build on each other's suggestions) as a way to support students' generalising. Also focusing on young students' abilities to identify and formulate functional relationships, Urena et al.'s (2019) interviews with fourth-graders showed that the students were able to represent such relationships both verbally and numerically, even though they had not received any previous instruction about functional thinking. Using concrete objects to provide a visual representation of (for example) the relationships between numbers can then contribute to the teacher's verbal support of young students' generalising (Morris, 2009).

Previous studies have also indicated that the development of the ability to generalise can be supported by the use of natural (or informal) language (Ayala-Altamirano et al., 2022; Shinno & Fujita, 2022; Strachota, 2023; Urena et al., 2022). However, Blanton et al. (2019:212–213) argued that natural language is not necessarily 'easier' for students who have various kinds of language difficulties. Instead, formal algebraic notation should be treated as 'a valuable tool by which young children represent and reason with algebraic ideas' since a verbal (informal) formulation of mathematical ideas can also be quite complex. Urena et al.'s (2019) interviews with fourth-graders implicitly confirmed these arguments. During individual interviews, several students who had not previously been taught about algebraic symbols still managed to represent functional relationships using formal notation when they were supported by a teacher, which indicates that this representation is well within the reach of young students. On the other hand, the meaning of certain symbols that are introduced at an early age (such as the equal sign) may still be somewhat unclear to students at higher levels of education. Thus, students need support in terms of time and 'cognitive space' when formal mathematical symbols are introduced, in order to make sense of these types of representations (Blanton et al., 2019:212–213).

Based on the results of a teaching experiment involving two Grade-5 students' work on patterns, Lannin et al. (2006) discussed the factors that influenced the students' choice of strategies when solving their mathematical task, including 'social factors' in terms of interaction with the teacher and other students. However, their analysis primarily concerned the influence of task structure, whereas the 'social factor' was only briefly mentioned in terms of its implication on teaching, in that classroom interaction should 'encourage the sharing of strategies and discussing the advantages and limitations' (Lannin et al., 2006:25).

In comparison, some of the studies investigating secondary-school students' generalising focused more on students' problem-solving activities in small group interaction. Analysing Grade-8 students' work on tasks involving functional relationships and algebraic tasks, Jurow (2004) and Koellner et al. (2008) emphasised the students' need of the teacher's support when formulating general arguments. Jurow (2004:296) explicitly stated that engaging in collaborative activities is not sufficient if students are to see connections between tasks or formulate generalisations since they need 'guided reflection and multiple scaffolded opportunities to talk about, write about, and otherwise represent what is general in and across situations'. Similarly, Koellner et al. (2088) noted that the teachers' choice of tasks, as well as their questions and prompts, had a great influence on the students' generalisations. However, both of these studies also identified crucial elements of student-student interaction that could contribute to the students' generalising. Koellner et al. (2008) noted the importance of giving students time to share their strategies and build on each other's ideas, which could also include dealing with peers' misunderstandings, while Jurow (2004:290ff) identified the crucial practices of 'linking', which implies that students generalise by comparing and making connections between situations, and 'conjecturing', meaning that students describe and explain these situations.

According to Jurow (2004), the practice of conjecturing implies identifying and explaining patterns. Working on mathematical tasks about patterns can also serve as an introduction to algebra, and to the identification of functional relationships, which also involves generalising (Radford, 2010). Analysing eight-graders' work on patterns, as part of a longitudinal study of students' transition from arithmetic to algebra, Radford (2010:46ff) identified four levels of generality, which he denoted *arithmetic*, *contextual*, *factual* and *symbolic* generalisations. An *arithmetic* generalisation implies that the student identifies the numerical relationship between consecutive figures in a pattern, while *contextual* generalisations represent students' formulations in natural language (such as 'add the first figure with the next figure'). Compared to arithmetic and contextual generalisations, formulations of *factual* and *symbolic* generalisations resemble formal notation; *factual* generalisations build on the students' determination of the number of elements in any figure

of the pattern, while *symbolic* generalisations imply the use of formal algebraic symbols. Also analysing eight-graders' discussions about patterns, Varhol et al., (2021) noted that students working in small groups managed to formulate generalisations at all levels (even though not all of the students were able to generalise according to all four levels on their own).

In addition to identifying the four levels of generalisation, Radford (2010:56) emphasised the importance of having the opportunity to use a variety of semiotic resources while trying to find the structure of a pattern; for instance, combining verbal talk with gesturing and 'perceptual activities' (such as drawing) seemed to help students to 'refine their awareness of the general'. Radford (2010:57) also argued that it is not sufficient to simply share solutions if the goal of the teaching is for students to attain 'deeper layers of generality'; instead, students need to see the objects of knowledge from the perspective of others, since the individual's understanding of an object of knowledge is supported by their encounters with others' understandings of the same object.

However, students' communication might also be affected by the setting of the problem-solving activity. For example, Varhol et al. (2021) combined Radford's (2010) framework of generality with the *inquiry-cooperation model* developed by Alrø and Skovsmose (2003), according to which there are eight types of communicative acts: *getting in contact*, *locating*, *identifying*, *advocating*, *thinking aloud*, *reformulating*, *challenging* and *evaluating*. Varhol et al.'s (2021) detailed analysis of students' discussions during small group work revealed that the most common type of interaction was *advocating*, which represents a way of providing ideas in a manner that invites examination, or suggesting something that could serve as a response to a wh-question[14]. In comparison, there were very few examples of *challenging* (ibid.); that is, there were not many instances of attempts to 'push things in a new direction or to question […] fixed perspectives' (Alrø & Skovsmose, 2003:109). These patterns of interaction may have been particularly apparent as Varhol et al. (2021) analysed group work that took place in a special room (instead of in the students' classroom). In other words, students' interaction during the observations of the small group work might have oriented towards a social norm that did not necessarily reflect the norms of their ordinary classroom work (see Yackel & Cobb, 1996).

[14] See a description of 'wh-questions' in subsection 3.2.1.

4 Methodology

The empirical material of this thesis consisted primarily of video recordings of students' small group interaction during mathematics lessons that took place in two Grade-6 classrooms. After each observed lesson, the students' notes on worksheets were also collected. The first section (4.1) describes the collection of the *empirical material*; in particular, this section presents information about the work in the two classrooms in which the lessons were observed (subsection 4.1.1), about the mathematical tasks that were assigned to the students during the observed lessons (subsection 4.1.2), and how the video recordings were conducted (subsection 4.1.3).

The second section (4.2) presents the *methods of analysis* that were used during the investigations of the empirical material, with the first subsection (4.2.1) describing how the video recordings were transcribed. This subsection is followed by a presentation of the methods applied to the analysis of the students' social interaction (subsection 4.2.2) and of the contents of the students' argumentation (subsection 4.2.3).

In the third section (4.3), I reflect on ethical considerations related to recording, transcribing and analysing the video recordings.

4.1 Empirical material

The empirical material was collected in autumn 2018 as part of a sub-study within the project *Inkludering genom lärande i grupp*[15], which is described on the web page of Uppsala University (2023; see summary information in Swedish about the project in Appendix D). This project involved both quantitative and qualitative studies, with the quantitative study designed as an experimental intervention; that is, some schools were selected as an 'experiment group' and other schools as a 'control group' (see Klang et al., 2021:4). The project leader, Associate Professor Nina Klang, suggested that one of the schools belonging to the 'control group' would also be a suitable setting for a qualitative study, which included video recordings of mathematics lessons. Henceforth, I will refer to this school as 'the focus school'. In the next subsection, I give some background information about the classroom work at the focus school.

[15] Translated to English, the title of the project is *Inclusion through group learning*.

4.1.1 Classroom work in Grade 6 at the focus school

The focus school[16] is a municipal 'Fk–6' school, which means that students attend 'Förskoleklass' (i.e. Preschool class) and Grades 1–6. The school was founded in 1970 and can be described as a 'mainstream school'. According to information presented by Skolverket (2023b), around 450 students were enrolled at the focus school during the 2018–2019 school year. All classes comprised around 25 students. The gender distribution was approximately 55% boys and 45% girls, which corresponded to the distribution of boys and girls in the entire municipality. By the end of the school year, around 80% of the Grade-6 students in the entire municipality attained at least E in all subjects (which is equivalent to a 'pass'). The corresponding results at national level was 74%. However, the results at different schools in the municipality varied greatly – from 51% to 99% – with the students' results at the focus school belonging to the upper tier; approximately 90% of the Grade-6 students at the focus school attained at least E in all subjects.

The teachers had been in charge of their classes since Grade 4 and often applied various types of collaborative working methods[17]. The teacher started each observed lesson by providing general information about the content of the mathematical task. However, the teacher did not give specific instructions about how to solve the task since one of the aims of the lessons was to give the students the opportunity to explore, explain and justify their own problem-solving strategies. Once the teacher had informed the class about the assigned task, each student was given a worksheet with the task written on it. The students were instructed to work individually for a few minutes; they were then instructed to work together (in dyads and/or in groups). In addition, the teacher instructed the students to take on various roles, such as 'chairperson' or 'secretary', which is in line with the principles of cooperative learning (Johnson & Johnson, 2008).

After working in dyads and groups, the students were summoned by the teacher to a whole-class discussion about their solutions. There was then a focus on presenting a variety in strategies of solving the tasks. The teacher selected students to present their work using the 'no hands up' method, meaning that the teacher gives the floor to students on a random basis, rather than letting the students themselves choose whether or not to participate, for example, by raising their hands to show that they have an answer (Wiliam, 2018:92–93). In addition, the 'no hands up' method emphasises each student's individual accountability of the group's solution, which is in line with the

[16] Information about the focus school comes from the current web-page of the municipality in which it was located.

[17] According to personal communication with teachers at the focus school, several colleagues shared an interest in classroom collaboration. Even before participating in the project *Inkludering genom lärande i grupp*, teachers had participated in in-service training on collaborative working methods.

principles of cooperative learning (Johnson & Johnson, 2008). On several occasions, the teacher also paused the whole-class discussion and asked the small groups to continue talking about specific aspects of the task or the presented solutions.

In the next subsection, I present the content of the mathematical tasks that were assigned to the students during the observed lessons. I also describe how the specific tasks were selected.

4.1.2 Selecting the mathematical tasks

Before selecting the mathematical tasks that the students would solve during the observed lessons, the class teachers were consulted in order to find topics and types of tasks that would be suitable for their students[18]. The teachers wanted their students to work on combinatorics and tasks concerning percentages and fractions. Combinatorics was a completely new topic for the students and the teachers considered small group work a useful working method when introducing their students to combinatorial ideas. In contrast, the students were quite familiar with the topic of percentages and fractions and the teachers considered the small group work suitable for the students' revision of this topic. The selected tasks resemble teaching materials that are presented in textbooks that focus on mathematical problem solving (e.g., Hagland et al., 2005; Larsson, 2007), as well as tasks used in the so-called national tests in mathematics (PRIM-gruppen, 2022).

The selected tasks (Appendix A) could be regarded as 'closed' as it is possible to identify correct answers to them. On the other hand, it is possible to approach the tasks using different strategies, which makes them 'open'. In addition, the tasks were formulated in a way that would allow the students to engage in various types of argumentation, such as explaining their solutions, justifying their results and formulating general arguments.

Next, I describe how the video recordings were conducted.

4.1.3 Conducting the video recordings

A total of 17 lessons (lasting an average of 55 minutes) were recorded[19]. Two cameras per group (i.e., a total of four cameras in the classroom) were attached to tripods and arranged to capture as much as possible of the students' facial expressions, gestures and use of artefacts, as well as their talk, in a 'relatively unobtrusive' manner (see Heath et al., 2010:40). In addition, a fifth camera was used to capture the whole-class interaction. During each lesson, two groups were observed, resulting in 34 recordings of group work sessions.

[18] Maria Larsson (Senior Lecturer at Mälardalen University) also contributed with important suggestions regarding the selection and formulation of the mathematical tasks.
[19] Nina Klang recorded 7 lessons, I recorded 10 lessons.

In order to maintain the students' interest in the tasks, the teachers chose various ways of arranging the groups. During nine lessons, the students were instructed to form groups of three or four immediately after working individually, and during eight lessons, the students were instructed to work in dyads between the individual work and group work. These arrangements enabled a potential of 16 recordings of sessions in dyads. However, not all students followed the teacher's instruction to work in dyads; instead, I noted just eleven occasions when students worked in dyads before working in groups (see Appendix E for details about the recordings). On the other hand, parallel talk in dyads sometimes occurred during the group work and, in some groups, parts of the discussions were primarily steered by two students.

Video recordings facilitate the analysis of a number of factors that play important roles in social interaction, such as prosody, gestures, facial expressions and the use of artefacts, as well as the contents of participants' verbal talk (Gardner, 2014; Heath et al., 2010; ten Have, 2007). In other words, compared to other types of empirical material (such as interviews, questionnaires or teacher/student self-reports), video recordings show 'more' of the classroom interaction, in that they capture participants' simultaneous actions (Moschkovich, 2019:67). Having access to video recordings also makes it possible to pause and play back the recordings, which allows for a detailed analysis of the multimodal interaction among participants. The same parts of the recordings can also be analysed multiple times, be shared with colleagues, or considered from various standpoints (Heath et al., 2010). However, it is important 'to have a clear sense of what is required of the data before you begin to record' (Heath et al., 2010:37), which could be a limiting factor when using video recordings. In addition, the position and direction of the cameras impose limitations on what part of the activity can be recorded; in that sense, video recordings might capture 'less' of the classroom interaction, which makes information from supporting materials (such as field notes) an important supplement (Moschkovich, 2019:67).

It should be noted that the setting of the collection of the empirical material enabled recordings of activities that largely resembled the naturally occurring interaction of the two Grade-6 classrooms at the focus school. The teachers participated in selecting mathematical tasks that were suitable for their students; in particular, the tasks involved topics that the teachers had already decided to let their students work on. Furthermore, the teachers regularly let their students work in groups, and often used the 'no hands up' method (see Wiliam, 2018). On the other hand, the cameras and microphones, and the presence of the researcher, may have influenced the students' behaviour (Heath et al. 2010). One observable effect was that students occasionally directed their attention to the technical equipment as the cameras and microphones were positioned near (or on) the students' desks. It is also possible that some students worked extra hard in order to look good on camera, while other students may have downplayed their participation so as not to be seen in the recording.

However, I observed no major differences in the level of activity among students belonging to different groups (i.e., between those students whose work was recorded and those students whose work was not recorded).

4.2 Methods of analysis

In this section, I present the methods applied to the analysis of the video recordings of the students' interaction in dyads and small groups. In the first subsection (4.2.1), I present how I approached the process of transcribing the recordings. In the following subsection (4.2.2), I introduce principles and procedures of *Conversation Analysis (CA)* and *multimodal analysis*, which were applied to the analysis of students' social interaction; then (in subsection 4.2.3) I introduce the method of *qualitative content analysis* that was applied to the analysis of students' argumentation.

4.2.1 Transcribing the video recordings

The process of transcribing video recordings is a first stage of the analysis (Broth et al. 2020). Transcribing is entailed with several choices regarding what information to include and how to structure the layout of the transcript (Moschkovich, 2019:68ff; Ochs, 1979; ten Have, 2007). My approach to transcribing the video recordings was to initially transcribe most of the recorded classroom discussions verbatim and orthographically; that is, while watching the video recordings, I transcribed most of the students' utterances but I left out detailed information about prosody, pauses and gestures. However, I only noted (without transcribing) those parts of the conversations that occurred before the teacher initiated the problem solving-activity, or after the teacher's closing of the whole-class discussion; for example, I omitted students' questions (and the teacher's answers) about today's menu in the school cafeteria or what equipment to bring to the PE lesson on the following day. Furthermore, I did not transcribe sequences during classroom work involving potentially ethically sensitive situations, such as a teacher reprimanding a student, or students' conversations about quarrels among their classmates. These sequences were not transcribed because the way participants handled social conflicts was not the focus of this thesis.

In order to select sequences for further analysis, my initial investigation of the recordings resembled the method of 'an "unmotivated" examination', which means that the examination is 'not prompted by prespecified analytic goals' (Schegloff, 1996:172). However, as my studies were guided by two overarching research questions, regarding how young students' mathematical problem solving is socially and interactionally organised, and what characterises young students' explanations and justifications when discussing solutions with peers, I soon focused on sequences in the video recordings that would

support discussions of those questions in a general sense. During this stage of the analysis I also shared and discussed excerpts with my supervisors, as well as with experienced conversation analysts during data sessions in a Conversation Analysis 'lab' (see Broth et al., 2020:54).

The excerpts that were selected for further analysis were transcribed in more detail. Jefferson's (2004) conventions for transcribing audio recordings were used, representing not only prosodic aspects of the verbal utterances but also the interactional organisation in terms of (for example) overlapping turns. In some cases, Mondada's (2023) conventions for transcribing multimodality in video recordings were used, representing the students' directions of gaze, gestures and handling of artefacts. To some extent, the level of detail in the transcriptions was varied depending on the context in which they were to be published or presented. However, the level of detail was primarily determined by the focus of the analysis (see Linell, 2022:114; Ochs, 1979:44; Moschkovich, 2019:68; ten Have, 2007:93ff).

One particular challenge was how to translate utterances in Swedish into English. Translating from one language to another usually involves some interpretation as it is not always possible to directly translate talk in such a way that the translation conveys the meaning of the utterance in the original language (Moschkovich, 2019). When translation into English was required, I inserted a line with an idiomatic translation after each line written in Swedish.

4.2.2 Analysing social interaction

The aim of this thesis concerns how social interaction can contribute to shaping young students' mathematical argumentation. Some previous studies analysing students' discussions in the mathematics classroom (e.g., Dahl et al., 2018; Rabel & Wooldridge, 2013; Robertson & Graven, 2019) have categorised students' talk according to the categories established by Mercer and Littleton (2007:58–59). These categories are *disputational* talk, which is characterised by challenges, 'unproductive' disagreements and individual decision making; *cumulative* talk, in which the participants uncritically accept and build on others' arguments; and *exploratory* talk, which is characterised by the participants' critical but constructive engagement in the discussion. Other studies of classroom talk (e.g., Sjöblom, 2022; Varhol et al., 2022) applied Alrø and Skovsmose's (2003:45ff) *inquiry co-operation model*, which is a more detailed way of categorising contributions to mathematical discussions[20].

[20] As presented in Chapter 3 (section 3.2), the inquiry co-operation model includes eight types of communicative actions: *getting in contact*, *locating*, *identifying*, *advocating*, *thinking aloud*, *reformulating*, *challenging* and *evaluating*.

In this thesis, students' mathematical argumentation is regarded as a social process that involves meaning-making practices that cannot always be analysed according to predetermined categories of talk. Thus, instead of applying a set of external (i.e., etic) categories to the analysis of students' argumentation, I applied the principles and procedures of *Conversation Analysis (CA)* and *multimodal analysis* to emphasise the participants' (i.e., the emic) perspective (Broth & Keevallik, 2020; ten Have, 2012), which are methods not frequently used in mathematics education research (see Ingram, 2018).

Applying CA when analysing the sequences of students' mathematical argumentation enabled a detailed analysis of the students' behaviour, as well as of their own methods of interpreting 'the behaviour of others' (Heritage, 1984a:241). In addition, the inductive approach of CA and multimodal analysis aligns with the ethnomethodological principle of avoiding a priori assumptions (ibid.) regarding what social actions students achieve in their communicative acts during small group work. That is, the emic perspective also means avoiding a priori assumptions of what the students treated as relevant elements of mathematical argumentation. For example, when discussing the task 'Chocolate' (Appendix A; Appendix C, Excerpt 4), which involved suggesting the price of a box of chocolates and then calculating a percentage change of that price, some students not only strived to solve the actual task but also engaged in discussions about what prices they considered 'reasonable'. From a mathematical point of view, discussing 'reasonable' prices is irrelevant; however, the students' discussion indicated an effort to establish and maintain accountability[21] by suggesting a price that reflected the actual situation of buying a box of chocolates, even when this suggestion caused some difficulty in the calculation. Thus, sequences in the video recordings during which the students referred to (seemingly) non-mathematical topics could not be dismissed a priori as irrelevant elements of their mathematical argumentation.

According to the dialogical principle of sequentiality[22], the interactional significance of communicative acts depends on their position in a sequence (Linell, 2001:70). In CA research, a *sequence* is described as a *series of turns* that are 'not haphazard but have a shape or structure, and can be tracked for where they came from, what is being done through them, and where they might be going' (Schegloff, 2007:2–3). Moreover, sequences have been described as 'the vehicle for getting some activity accomplished' (ibid.). A minimal sequence is the *adjacency pair*, which consists of only two turns, placed one after the other (denoted the 'first' and the 'second' pair part) and uttered by different speakers (Heritage, 1984a:246; Sacks et al., 1974). Furthermore, the 'first pair part' projects a relevant 'second pair part', which is noticeably absent if it is not produced (Schegloff, 2007:16, 20). One example of such a sequence that frequently occurred in my video recordings

[21] See an introduction to the concept of *accountability* in Chapter 2, subsection 2.1.2.
[22] See an introduction to the concept of *sequentiality* in Chapter 2, section 2.1.

was the adjacency pair 'question-answer', for example, regarding specific parts of the mathematical tasks, such as 'what's your answer' – 'three'.

In ordinary conversation, some types of sequences make specific second pair parts relevant. For example, if the first pair part is a greeting, the relevant response is to return the greeting. Similarly, when a student suggested a solution to the mathematical task, the peer's relevant response seems to have been to also suggest a solution, for example, 'my result is six' – 'I think it is three'. One of the findings of CA research is that second pair parts can be equally relevant without being 'symmetrical alternatives' in the sense of being equally valued (Schegloff, 2007:58–59). In cases in which different second pair parts are equally relevant, the action that facilitates the accomplishment of the activity that was initiated by the first pair part is regarded as a *preferred* response (Sidnell, 2010:77). Here, the notion of 'preference' does not refer to participants' 'psychological states'; for example, if you are invited to a party the preferred response is to accept the invitation, regardless of whether you actually want to attend this party (or not) since accepting the invitation 'promotes' the activity (ibid.). Preferred responses are usually produced in a 'straightforward' manner, whereas dispreferred responses are often delayed, for example, by initial silence or some kind of verbal hedge (such as 'uh' or 'well') and/or accounted for by excuses or explanations (Hutchby & Wooffitt, 2008:47; Sidnell, 2010:78–79). For example, during the observed mathematics lessons, students who did not suggest a result usually accounted for not doing so by referring to a lack of time, or by saying 'I didn't get it'. Thus, when peers asked a direct question, or initiated discussion by suggesting a solution, these utterances were treated as first-pair parts to which the preferred response was to also present some kind of result.

Furthermore, in ordinary conversation, if a relevant response is absent due to problems of hearing, speaking or understanding, participants commonly apply a set of practices that is referred to as *repair*, which contributes to maintaining intersubjectivity, despite momentarily interrupting the interaction (Kitzinger, 2014; Sacks et al., 1974; Sidnell, 2010:110ff). CA research has shown that, in most cases, repair is 'self-initiated' in that the person who uttered the *trouble-source* (or *repairable*) stops what (s)he is saying in order to make some kind of adjustment or correction (Kitzinger, 2014:230ff). Repair can also be 'other-initiated', for example, when someone other than the speaker of the trouble-source asks a clarifying question, which (typically) is followed by the first speaker's *repair solution* (ibid.). Repair practices were common during the observed mathematics lessons; in addition, they were often 'other-initiated' by a peer's questions, such as 'what does that mean' or 'why do you do it like that', or utterances such as 'I don't really understand'.

As stated in Chapter 2 (subsection 2.1.2), studies that use ethnomethodological approaches focus primarily on social action, rather than on participants' language use, since 'the meaning of utterances in contexts is not inherent in the (linguistic) meanings of the constituent linguistic items' (Linell, 2001:98). However, even though the focus of this thesis was not on analysing specific linguistic aspects of mathematical argumentation, it was relevant to also note students' use of certain words while explaining and justifying their solutions. For example, using the Swedish pronoun *man* ('one'), instead of *vi* ('we') or *jag* ('I'), when explaining a procedure indicated a claim to generality, since *man* is a generalising pronoun (Teleman et al., 1999:241). Furthermore, the use of the Swedish modal particle *ju*, which can be approximately translated as 'you know', was interpreted as an indication that the argument referred to information that was treated as shared knowledge (Heinemann et al., 2011; Lindström, 2008:221).

In addition to analysing the students' verbal utterances, their embodied actions and references to objects in the classroom were analysed in accordance with the principles of *multimodal interaction analysis* (Broth & Keevallik, 2020:23ff). One example is the analysis of speakers' use of gaze, as gazing at someone can be a way of selecting that person to be the next speaker, or to solicit some kind of response (such as an assessment), whereas the withdrawal of a gaze might indicate resistance (Hayashi, 2014:170; Rossano, 2014:316, 323). Analysing video recordings also enables investigations of the potentially 'intimate relationship' between verbal utterances, gestures and the handling of various types of objects (Goodwin, 2018:221ff). When speakers use deictic terms, such as 'this' or 'that', it can be difficult to fully grasp what they are talking about if only their verbal contributions are analysed. Instead, the action can be analysed as an *environmentally coupled gesture*, which ties together language and gestures with some kind of environmental feature (ibid.). In addition, environmentally coupled gestures presuppose others' attention to specific structures in the local environment (Goodwin, 2007:55). While discussing solutions, the students frequently referred to notes on both their own and others' worksheets. Figure 3 represents an example of how such a reference could be analysed as an environmentally coupled gesture. This figure shows a capture of Darin and James who discussed the task 'Queue' (Appendix A), about the number of queues that certain numbers of persons can form. Presupposing Darin's attention, while pointing at his worksheet, James stated that two persons who are forming a queue can stand 'like this and then they can stand like that', which is an utterance that would have been difficult to interpret without seeing what James was pointing at.

Figure 3: Capture from my video recordings. The illustration represents a situation analysed as an 'environmentally coupled gesture' (Goodwin, 2007:55).

Compared to analytical approaches involving the use of external categories, the inductive approach of CA may seem somewhat arbitrary due to its explicit focus on the analysis of empirical material (ten Have, 1990). However, when participants in interaction respond to someone else's contribution they not only display their own understanding of the previous action; they also allow the speaker to see whether the recipient understood their contribution in the way they expected them to. In CA, this is a phenomenon commonly referred to as the *next-turn proof procedure* (Hutchby & Wooffitt, 2008:13; Sidnell, 2014:79). The idea of the next-turn proof procedure is based on the ethnomethodological principle that social interaction is orderly[23], first and foremost for the participants, but also for the observing analyst (ten Have, 1990:25). Thus, the next-turn-proof-procedure not only supports the progression of interaction; in addition, it is one of the tools of CA that ensures that the analysis is not primarily based on the analyst's assumptions but rather reflects the '*participants' orientations* which are displayed in their own conduct' (Sidnell, 2014:79).

The approaches of CA and multimodal interaction analysis are useful when analysing how various interactional phenomena come about (see Emanuelsson & Sahlström, 2008), but the analysis of contents of arguments requires other analytical frameworks. Thus, in order to analyse contents of students' mathematical argumentation, I applied principles and procedures of *qualitative content analysis*, which are presented in the next subsection. Furthermore, by using two different approaches to the analysis of the video recordings, the distinction between the processes and the products of the students' argumentation was emphasised.

[23] See a presentation of ethnomethodological principles in Chapter 2, subsection 2.1.2.

4.2.3 Analysing contents of mathematical argumentation

Applying the procedures of *qualitative content analysis* is a common approach when analysing texts. Several aspects of the content can be the object of analysis, including the 'underlying meaning structures' of communication (May-ring, 2015:365–367). For example, applying these procedures when analysing my empirical material facilitated an analysis of argumentative relations between various types of justifications, as presented in Study II.

According to Mayring (2015:370), the methodological approach of qualitative content analysis is not to be seen as something 'that always remains the same'. Instead, it needs to be adjusted to the particular context of the empirical material that is being analysed (Krippendorff, 2013:355). In Studies I and II, the analysis of the students' arguments was conducted according to procedures of *deductive category assignment*, which means that a set of external categories was used to identify various types of acts of justification (Mayring, 2015). In contrast, the analysis presented in Study IV was conducted according to the principles of *problem-driven content analysis*, which means that the analysis 'derives from epistemic questions', rather than being based on external categories (Krippendorff, 2013:357ff). Below, I present these two ways of using qualitative content analysis in separate subsections.

Applying the approach of 'deductive category assignment'

The external categories used in Study I and Study II originated in the classes and subclasses of the *taxonomy of proof schemes* (Harel & Sowder, 1998; 2007). According to Harel and Sowder (2007:808ff), the process of proving is constituted by two subprocesses; *ascertaining*, which is the process employed by individuals or communities to remove their own doubts about the truth of an assertion, and *persuading*, which represents the processes employed by individuals/communities to remove the doubts of others. The processes of ascertaining and persuading are not separate; together, these processes constitute the proof scheme (Harel, 2007; Harel & Sowder, 2007).

Harel and Sowder (1998; 2007) developed the taxonomy of proof schemes on the basis of empirical observations. During a series of teaching experiments involving university students in the US, Harel and Sowder (ibid.) collected empirical material comprising video recordings and field notes of whole-class interaction and students' discussions in small groups, clinical interviews with individual students, and students' solutions to homework tasks and written tests. They then developed the taxonomy, based on recurring patterns in this material. In particular, a priori definitions of the meanings and functions of mathematical proofs were then explicitly avoided; instead, Harel and Sowder (1998:244) focused on the students' demonstrations of their own conceptions of 'what constitutes proofs [and] refutations'. Furthermore, the taxonomy was not developed in terms of a hierarchical structure. Although justifications can differ in levels of sophistication, processes of argumentation can involve

demonstrations of more than one proof scheme (Harel & Sowder, 1998, 2007; Sowder & Harel, 1998:679). This means that proof schemes are not mutually exclusive since 'people can simultaneously hold more than one kind of scheme' (Harel & Sowder, 1998:244).

Harel and Sowder's taxonomy comprises three main classes, which include two or three subclasses each. In some presentations of the taxonomy, further subclasses are included (e.g., Harel & Sowder, 1998:245); moreover, the notation of the main classes and subclasses has been altered over time (see Harel, 2007). Table 1 presents a combination of the way in which the classes are denoted in Harel and Sowder (1998; 2007), Sowder and Harel (1998) and Harel (2007). Henceforth, I use the notation without parentheses in Table 1, except for the third subclass of the main class 'Externally based', which I refer to as the 'Non-Referential Symbolic' proof scheme.

Table 1: The main classes and subclasses of the taxonomy of proof schemes (Harel 2007; Harel & Sowder 1998, 2007; Sowder & Harel, 1998)

(External Conviction) Externally based			**Empirical**		**(Deductive) Analytic(al)**	
Authori-tarian	**Ritual**	**(Non-referential) Symbolic**	**(Inductive) Examples-based**	**Perceptual**	**Transfor-mational**	**Axiomatic**

Students who demonstrate an *authoritarian proof scheme* refer to some kind of authority, such as what the teacher has told them, or what they have read in their textbook (Harel & Sowder, 2007:809). The *ritual proof scheme* implies that students refer to the appearance of the argument (for example, 'the two-column proof' in geometry); these students might also reject a correct justification if 'it does not look like a proof' (Harel & Sowder, 1998:246). Demonstrations of a *non-referential symbolic proof scheme* involve manipulations 'with the symbols or the manipulations having no potential coherent system of referents' (Harel & Sowder, 2007:809). One example is the incorrect treatment of fractions, such as in this addition: $\frac{a}{b}+\frac{c}{d}=\frac{a+c}{b+d}$ (Sowder & Harel, 1998:672). Treating examinations of empirical examples as a mathematical proof implies a demonstration of an *examples-based proof scheme* (Harel & Sowder, 2007:809) while a demonstration of a *perceptual proof scheme* involves references to perceptions, for example, when determining whether a quadrilateral is a rectangle (Sowder & Harel, 1998:672). Demonstrations of a *transformational proof scheme* are characterised by 'generality', 'operational thought' and 'logical inference'. 'Generality' implies that the justification fulfils a 'for all' argument; that is, the justification neither concerns isolated cases nor accepts any exceptions. 'Operational thought' means that the student is

able to anticipate the outcome when examining 'goals and subgoals', while 'logical inference' implies an awareness that 'justifying in mathematics must ultimately be based on logical inference rules' (Harel & Sowder, 2007:809–810). Students who understand and refer to the 'accepted principles' (i.e., axioms) of mathematics demonstrate an *axiomatic proof scheme* (Harel & Sowder, 2007:810).

Before the students' acts of justification were categorised according to these external categories, *anchor samples* were formulated and 'cited as typical examples to illustrate the character of those categories' (Mayring, 2015:377). For example, students' justifications that were based on drawing pictures or writing tables were categorised as *examples-based*, while referring to calculations that contradicted results of empirical examinations were categorised as *non-referential symbolic* justifications[24]. Furthermore, in order to establish the reliability of the categorisation of students' acts of justification in terms of proof schemes, the co-authors of Study II analysed the material independently; thereafter, intercoder reliability was calculated using Miles and Huberman's (1994:64) formula, which indicated agreement in 85% of the cases[25].

Applying the approach of 'problem-driven content analysis'

As stated in Chapter 2 (subsection 2.2.1), Harel and Sowder (2007:807) emphasised that the goal of teaching about proving is to support students' understanding of formal proofs, even though the taxonomy of proof schemes does not in itself represent a hierarchical structure. A subgoal of developing an understanding of proofs is to develop the ability to *generalise*, which is one of the characteristics of the transformational proof scheme (Harel & Sowder, 2007:809). In Study IV, the focus was on students' formulation and use of general arguments, while engaging in mathematical problem solving with peers. Instead of using external categories, students' processes of generalising were then analysed according to principles of *problem-driven content analysis* (Krippendorff, 2013).

In order to develop stable categories for coding, which is one of the critical elements of problem-driven content analysis (Krippendorff, 2013:357ff), Mason's (1996:65) definition of the ability to generalise as '*seeing a generality through the particular* and *seeing the particular in the general*' was operationalised by combining findings of previous research on students' generalising (Ellis, 2011; Ellis et al., 2021; Harel & Tall, 1991; Jurow, 2004; Urena et al, 2019; Yeap & Kaur, 2008). Based on previous studies, generalising was

[24] In Study II, anchor samples are presented in more detail.

[25] Reliability = Number of agreements/(Number of agreements + Number of disagreements). According to Miles and Huberman (1994), intercoder reliability is rarely more than 70% after the initial coding. Thus, agreement in 85% of the cases indicates a high level of agreement. The disagreements primarily concerned the categorisation of students' incorrect arguments and these cases were discussed until all co-authors agreed on the categories.

interpreted as processes involving the *identification of regularities* and/or *transferring of argumentation into new contexts*.

Students' identification of regularities can be exemplified by the group discussion on the task 'Shake hands' (Appendix A; Excerpt 3 in Appendix C) introduced in Chapter 2, section 2.1. The task was to determine the number of handshakes that a certain number of persons can make and thereafter to formulate a general 'rule' for calculating the number of handshakes depending on the number of persons. At first, the students empirically showed that three, four and five persons can make three, six and ten handshakes, respectively. Based on these results, and using the number of handshakes among five persons to confirm the number of handshakes of ten persons, the students formulated a generalisation, implying that the number of handshakes equals half of the product when the total number of persons is multiplied by the number of handshakes that one person can make. With n representing the number of persons, the students' generalisation can be algebraically formulated $n(n\text{-}1)/2$.

Another example from the video recordings that represents both the identification of regularities and the transferring of argumentation into new contexts, can be seen in Excerpt 5 in Appendix C. Discussing the second part of the task 'Schools' (Appendix A), about finding a number that satisfies certain conditions within the interval 300–310, one of the students referred to numbers in the interval 0–10 instead, which implies the identification of regularities in properties of numbers in these two intervals. In addition, the percentage 25% was equated with ¼ (and 50% with ½), indicating an ability to transfer knowledge and argumentation about fractions into the context of percentages. In order to increase the reliability regarding the interpretations of students' generalising, empirical examples (such as Excerpts 3 and 5) were presented in seminars, inviting colleagues' reviews and comments.

In the next section, I present and reflect on ethical considerations related to the processes of recording, transcribing and analysing video recordings.

4.3 Ethical considerations

Schools belong to the type of setting that is often referred to as 'closed access groups' (Hornsby-Smith, in Heath et al., 2010:15) in that there are 'barriers to physically entering the setting as well as to undertaking the research' (Heath et al., 2010:15). As mentioned above (in section 4.1) the empirical material of this thesis was collected as part of a sub-study in the project *Inkludering genom lärande i grupp*[26], which was approved by the Swedish Regional Ethics Board (Dnr. 2017/371). In connection with registering an interest in the project, the teachers at the focus school gave their consent to participate in various types of studies.

[26] *Inclusion through group learning*

Furthermore, the students' legal guardians received written information about the project before the video recordings commenced. The information letter also included a consent form (Appendix F). All the guardians gave their consent for their children to participate in the video recordings. In contrast, the students were not asked to give their written consent; instead, the students' consent was treated as 'provisional' and something that needed to be continuously negotiated (Flewitt, 2006:31). In addition, the project leader and I visited the focus school and met with the teachers and the students, in order to give them the opportunity to ask questions regarding practical matters, such as how the video recordings would be conducted, as well as about general issues regarding the purpose of the project. The students were then explicitly informed that their participation in the video recordings was voluntary and that they could withdraw their consent at any time during the observed lessons. In addition to telling the students that their participation was voluntary, they were informed that neither their personal names nor the name or location of the focus school would be disclosed. In all the transcripts, fictitious names have been used that correspond to the names of the teachers and students in terms of gender; that is, girl/boy names have been used as pseudonyms, but these fictitious names do not represent, for example, the students' ethnicity, since issues related to their personal identity and socio-economical background were not the focus of this thesis.

Rather than making a priori assumptions about the influence of the video recordings on the participants, this issue can be addressed empirically; that is, while analysing video recordings, it is also worth noting how participants orient to the filming (Heath et al., 2010). Many of the students demonstrated an interest in the technical equipment and volunteered to help arrange the microphones on their desks and set up the tripods for the cameras, which provided an opportunity to talk with the students informally and sense whether or not they agreed to be filmed. Thus, the practicalities of conducting video recordings served as a 'tool' for investigating the students' attitudes to being observed. The video recordings also showed that several students treated the observations in a playful manner, for example by occasionally making funny noises into the microphone or acting as if they were hosting a TV talk show.

Analysing video recordings 'inevitably' involves the researcher's own comprehension of the situation being analysed (ten Have 2007:33). However, it has also been argued that it is 'not necessarily desirable' to fully disregard one's own previous experiences; instead, having some 'membership knowledge' can facilitate the understanding of what the interactants are doing (Ingram, 2021:17). My previous experience of teaching mathematics to students in Grades 6–9 helped me understand parts of the observed students' communicative acts during their discussions about solutions to mathematical tasks. Nevertheless, my background as a mathematics teacher may also have contributed to certain assumptions about students' actions that could have been interpreted in other ways.

5 Summary of Studies I–IV

This section contains a summary of Studies I–IV. The summary also reflects on how the four studies relate to each another.

Study I: A pre-study of grade 6 students' orientation to social and sociomathematical norms during mathematical problem solving in groups

As the title indicates, Study I served as a pre-study to the subsequent studies, in that only a small part of the video recordings was analysed. The focus of the pre-study was to combine an analysis of mathematical and social aspects of Grade-6 students' mathematical problem solving in small group interaction. That is, Study I considered both of the two research questions of this thesis.

The methodology of Study I was based on an ethnomethodological perspective on interaction (Heritage, 1984a), as well as on qualitative content analysis using the main classes of Harel and Sowder's (1998; 2007) taxonomy as external categories. Thus, the pre-study also served as a pilot study regarding methodological approaches. In addition, the analysis of the video recordings in Study I was informed by the *emergent perspective* (Yackel & Cobb, 1996), which shares part of the focus of ethnomethodological studies in that contributions to interaction during everyday activities are regarded as participants' ways of making these activities meaningful. However, the emergent perspective specifically focuses on mathematical activities, and applies the concept of *sociomathematical* norms (in addition to the notion of general social norms) in the analysis of classroom interaction.

Two questions guided the analysis in Study I: *What characterizes explanations and solutions that students consider mathematically acceptable?* and *What social and sociomathematical norms do students orient to, when engaging in mathematical problem solving?* The empirical material comprised sequences from video recordings of two groups of four students each, working in the same classroom and discussing the task 'Queue' (Appendix A).

In both of the observed groups, students justified solutions in a way that aligned with the *analytic* proof scheme (Sowder & Harel, 1998), in that they involved elements of generality. According to these solutions, four persons

can form 24 queues; moreover, students argued that this number can be calculated by the multiplication 'six times four', which was justified by the argument that regardless of which one of the four persons 'is in first place all the time', the other three can 'move about so there are six different ways'.

However, the calculation $4 \cdot 6$, and the 'analytic' (i.e., general) justification of this multiplication, was treated very differently in the two groups. In one of the groups, a student built on their joint solution and intuitively formulated the factorial function, stating that the number of queues that four persons form can be calculated as 'one times two is two (.) times three is six (.) six times four is twenty-four'. It is not possible to determine whether all of the students in this group really understood the solution but the calculation was openly appreciated; the peers talked about it as something 'cool', 'smart', 'magic' and 'awesome' and even called the teacher's attention in order to demonstrate their solution to her. The students' appreciation of the calculation that resembled the factorial function indicated that they were not satisfied by only solving the mathematical task; instead, their discussion seemed to orient towards a sociomathematical norm that *solutions should be presented on a general level.*

In contrast, the interaction within the other group primarily oriented towards a social norm of equality. Two students suggested the calculation $4 \cdot 6$ while the other two interpreted the wording of the task in such a way that each person had to change places to 'make a new queue'. Thus, they argued that four persons could only form four queues. The students then turned to the teacher and asked for her support in resolving the potential dilemma of having presented two different solutions. Moreover, in addition to following the teacher's recommendation to present both of their solutions, the students praised each other's work and, instead of addressing the difference in the level of sophistication of their solutions, they agreed that the difference between their results only was a matter of 'how you think about it'.

According to Levenson et al. (2009) and Wester (2015), students' perceived norms can differ greatly from the teachers' intended norms. However, it is interesting to note that the discussions observed in Study I occurred among groups of students who had belonged to the same class for several terms; still, their interaction oriented towards quite different norms. Thus, the findings of Study I show that there can be great differences not only between the students' and teachers' views of the classroom norms, but also between different students' perceptions of these norms, which could contribute to different learning opportunities for students working in the same classroom. As well as indicating that CA (and other ethnomethodological approaches) could offer a methodological contribution to analyses of students' mathematical problem-solving activities in small groups, the findings presented in Study I indicate that aspects of classroom interaction that might be regarded as 'social' could have a major impact on the mathematical content of students' discussions.

Study II: Investigating grade-6 students' justifications during mathematical problem solving in small group interaction

Study II built on Study I, in that it focused on investigating the students' use of various types of justifications while discussing solutions to the mathematical task 'Queue' (Appendix A). Like in Study I, Harel and Sowder's (1998; 2007) taxonomy of proof schemes was used as an analytical tool to categorise students' actions while justifying their arguments. However, by including a quantitative element when using the content-analytical method of deductive category assignment (Mayring, 2015:376ff), Study II primarily focused on the second research question of this thesis.

The analysis in Study II was guided by three questions: *What types of justifications do grade-6 students use when solving mathematical problems in small groups? How are justifications related, with regard to local ordering and agreement, when they occur in immediate proximity to one another?* and *What types of justifications are treated as most influential on the proving process?* The empirical material comprised video recordings of four groups of four students each and included their discussions on the task 'Queue' (Appendix A) during the entire lesson.

The quantitative element of the analysis indicated that the frequency of the various types of justifications that the observed Grade-6 students used corresponded well with the frequency and type of arguments that previous studies have shown to be commonly used by adult students. In other words, the findings presented Study II imply that the qualities of young students' justifications and adult students' proving are not necessarily very different; instead, young students' acts of justification could be regarded as precursors to processes of formal proving. Thus, the notion of *emergent proving* was suggested in Study II, in line with findings of research within the field of 'early algebra'. In the same way that the introduction of algebraic ideas in the early school years has been shown to facilitate the transition to formal algebra at higher levels of education (e.g., Blanton et al., 2015; Kieran et al., 2016), mathematical problem-solving activities involving students' justifications of solutions could facilitate the transition to working on formal proofs in upper-secondary school.

Previous studies that have used Harel and Sowder's (1998; 2007) taxonomy as an analytical tool have focused on individual students' achievements and combinations of proof schemes. Study II contributes to previous findings by presenting an analysis of young students' abilities to use and combine various types of justifications while discussing solutions to mathematical tasks together with peers. The analysis of the students' ways of combining proof schemes showed that all *transformational* justifications followed and agreed with *examples-based* justifications, whereas the local ordering between

non-referential symbolic and examples-based justifications was inconsistent; that is, there were instances where a non-referential symbolic justification followed the examples-based justification, and the other way around. Similarly, in some cases, the non-referential justification and the examples-based justification agreed, but there were also instances in which they disagreed.

Furthermore, non-referential symbolic justifications involving calculations seemed to have been treated by the students as more influential on the argumentative process, compared to references to results of examinations of empirical examples, even in cases when the calculation was incorrect and contradicted the empirical results. These findings imply that teachers should not only encourage students to share and discuss solutions; they should also emphasise the importance of consistency when using various types of arguments to justify a solution. In particular, students should be informed about the significance of counterexamples, as a support for students' development of their mathematical problem-solving skills as well as a preparation for formal proving at higher levels of education.

Study III: Displaying epistemic independence during mathematical problem solving in small group interaction

The analysis presented in Study II showed that calculations and empirical examples were often treated quite differently; in many cases, students seemed to treat their results of calculations as more reliable than their empirical examinations. In Study III, the analysis furthered the investigation presented in Study II by focusing on students' ways of handling situations in which differing solutions were proposed. Unlike Study II, the analysis in Study III focused on the first research question of the thesis. Using the principles and procedures of CA (e.g., Sacks et al., 1974; Schegloff, 2007; Sidnell, 2010) combined with elements of multimodal interaction analysis (e.g., Goodwin, 2006; Goodwin, 2007; Hayashi, 2014) in the investigation of young students' accounting practices when handling differing proposals, Study III aimed to increase the knowledge of the interactional organisation of mathematical problem solving.

The empirical material of Study III consisted of sequences during which students presented and discussed differing proposals. This situation occurred during 18 of the 34 recorded sessions of group work. For reasons of space in the study, representative examples were only selected from sessions in which students worked on combinatorial tasks. Students' discussions about solutions to combinatorial tasks were initiated by short proposals of results, whereas their discussions about tasks regarding percentages and fractions were more often initiated by elaborate descriptions of procedures for solving the task.

A characteristic feature of all the sessions in which students proposed differing solutions was that they did not explicitly tell their peer that (s)he was wrong. Instead, peers commented on or asked questions about the proposal. Although these comments and questions could be rather challenging, the students' treatment of their peers' suggestions corresponded to findings of previous research, which have shown that accounts are often unsolicited and that self-correction is preferred (Davidson, 1984; Houtkoop-Steenstra, 1990; Robinson, 2016). Moreover, when the student who had made the proposal responded to others' comments and questions, (s)he also explained and justified the solution; thus, peers' comments and questions contributed to the co-construction of more elaborate explanations and justifications. In Study III, this practice was denoted *soliciting explanation of someone else's proposal*. In addition to soliciting explanations and justifications, some students quickly accepted a peer's proposal. In cases in which a student abandoned his/her own solution, without further commenting on it, the practice was denoted *conceding to someone else's proposal*. When students not only conceded to peers' proposals but also explicitly rejected their own solutions, the practice was denoted *rejecting one's own proposal*.

In contrast to Study II, Study III had no quantitative element. However, it is interesting to note that the practices of conceding to someone else's proposal and explicitly rejecting one's own, were (almost) equally frequent. These two practices were identified in eight and seven sessions, respectively. There were also three sessions in which the students did not resolve the dilemma of having presented differing proposals. Instead, these sessions ended with the students (more or less reluctantly) accepting that two different solutions had been presented; Study I exemplified a situation in which the students agreed that it was relevant to present two very different solutions.

Another apparent feature was that students in many cases conceded to a peer's proposal before their peer had finished explaining it, which indicates a degree of urgency on the part of the conceder to display a change of state. In addition, students' reports about their own mistakes were then often marked by affect-laden acts. It could be that the practice of displaying a change of state in an explicit manner was a strategy of avoiding others' interpretation that the concession was due to 'mindless compliance' (Kotthoff, 1993; Waring, 2007); that is, students' accounting practices when handling situations in which differing proposals were presented might have been their way of displaying independent epistemic access to both the mathematical task and to their peers' solutions. Thus, the findings presented in Study III add to the discussion in Study I regarding the social norm of equality. In the same way that students' mathematical argumentation can be affected by their efforts to nurture social relations within their group, the students' 'maths talk' can contribute to maintaining or (re)establishing an identity of being an able and independent problem solver.

Study IV: Grade-6 students' generalising practices during mathematical problem solving with peers

In Study IV (which is still a manuscript), the focus was on students' formulation and use of general arguments while discussing solutions to mathematical tasks. Like Study I, the intention of the fourth study was to approach both of the research questions of this thesis. However, the more specific questions of Study IV were: *How is young students' generalising sequentially organised during discussions about solutions to mathematical problems?* and *What linguistic resources do young students use when formulating general arguments?*

The theoretical framework of Study IV built on Mason's (1996:65) description of generalising as '*seeing a generality through the particular* and *seeing the particular in the general*'. Based on previous research on students' generalising (e.g., Ellis, 2011; Ellis et al., 2021; Harel & Tall, 1991; Jurow, 2004; Urena et al., 2019; Yeap & Kaur, 2008) the concept of generalising was operationalised as processes of *identifying regularities among cases* and/or *transferring of argumentation into new contexts*. Moreover, the students' generalising was considered a *joint construction*; thus *other-oriented* linguistic formulations indicating generality were noted, even in cases in which it was mainly one student who directed the discussion (Linell, 2001:85ff).

The linguistic resources that indicated a process of *identifying regularities among cases* included students' use of the general pronoun *man* ('one'), statements about 'all' cases and/or descriptions of procedures that should 'always' be performed. Processes of *transferring of argumentation into new contexts* were noted in students' use of the modal particle *ju* (approximately 'you know'), which often indicates that some information is treated as shared knowledge (Heinemann et al., 2011). In addition to the use of linguistic formulations, students' references to examples were analysed; in particular, the ordering of references to empirical examples and formulations of generalisations was noted.

Three practices of generalising were identified: *Generalising after examining examples*, *Generalising before examining examples* and *Generalising by referring to mathematical facts*. The first two practices particularly emphasise the temporal relationship between students' references to empirical examples and their use of linguistic generalisations (that is, whether the students' references to examples occurred before or after their verbal formulation of the general argument). The third practice emphasises students' references to mathematical facts in a way that indicates that these facts were treated as common or shared knowledge.

The analysis of the students' generalising practices confirms results of previous studies that have shown how the formulation of general arguments can be supported by the use of a variety of semiotic resources (e.g., Urena et al., 2019; Yeap & Kaur, 2008). However, the analysis of the video recordings also indicated that peers' questions contributed to more thorough examinations of examples, as well as to more elaborate justifications. Thus, students should be encouraged to both raise and respond to critical questions and reviews of arguments and justifications since these elements of mathematical argumentation can also support students' generalising. Moreover, teachers should encourage students' attention to linguistic resources that indicate that certain mathematical facts are treated as shared (i.e. general) knowledge.

6 Discussion

The aim of this thesis was to increase the knowledge of how social interaction can contribute to shaping young students' argumentation when engaging in mathematical problem solving with peers, with a particular focus on students' use of explanations and justifications when discussing solutions. Student-student interaction during everyday classroom work was analysed using multimodal video analysis, which enabled detailed investigations of students' argumentative processes when engaging in mathematical problem-solving activities.

Findings presented in Studies I–IV indicate that 'the social' and 'the mathematical' (see Ingram, 2020) are reflexively related during students' mathematical problem solving with peers. These findings are discussed in section 6.1. In addition, the notion of *emergent proving* is elaborated in the first section of this chapter.

The analysis of student-student interaction occurring outside of the teacher's immediate control can also inform teachers' didactic decisions when implementing collaborative working methods during mathematics lessons. Thus, in section 6.2, I discuss this thesis' potential implications for teaching.

The last section of this chapter (6.3) presents suggestions for further research. This section also includes reflections on the methodology of the thesis.

6.1 Young students' mathematical argumentation – shaped by social interaction

In this section, I discuss how findings presented in Studies I–IV contribute to resolving the two main research questions of this thesis: *How is young students' mathematical problem solving socially and interactionally organised?* (discussed in subsection 6.1.1) and *What characterises the content of young students' explanations and justifications when discussing solutions during mathematical problem solving with peers?* (discussed in subsection 6.1.2).

6.1.1 How is young students' mathematical problem solving socially and interactionally organised?

Various factors related to the overall structure of discussions have been found to affect students' participation in mathematical activities (e.g., Dahl et al., 2018; Kilhamn et al., 2019; Klang et al., 2021; Langer-Osuna et al., 2020; Lannin et al., 2006; Rabel & Wooldridge, 2012; Sjöblom, 2022; Svahn & Melander Bowden, 2021). Students' mathematical argumentation can be supported by working on open-ended questions, as well as by using a variety of communicative resources such as verbal talk and visual mediators (e.g., Ayala-Altamirano et al., 2022; Flegas & Charalampos, 2013; Morris, 2009; Shinno & Fujita, 2022; Radford, 2010; Richardson et al., 2010; Stylianides, 2019; Urena et al., 2019). In addition, the teacher's approach to the mathematical task and choice of words during instruction is often reflected in the students' own talk when working on the task (Barwell, 2023; Creider, 2012; Forrester & Pike, 1998; Mushin et al., 2013; Roth & Gardener, 2012). Furthermore, the students' own contributions (such as sharing information) can also provide support for their peers' argumentation (Ellis, 2011; Koellner, et al., 2008; Lannin et al., 2006; Radford, 2010; Strachota, 2020; Webb et al., 2023). This implicitly emphasises the contextual aspects of proving, in that arguments that are presented as a mathematical proof have to be meaningful for the students themselves (see Bell, 1976; Hanna & Knipping, 2020; Harel & Sowder, 1998, 2007; Sowder & Harel, 1998; Stylianides, 2007).

In many respects, the findings presented in this thesis are consistent with previous research. Studies I–IV exemplified how students' small group work facilitated the use of a variety of communicative resources, such as 'natural' talk and references to written information, pictures and tables on both individual and shared worksheets, which also served as a base for students to share information and build on each other's arguments. For example, as shown in Studies I and III, peers' comments and questions led to more explicit and elaborate explanations of a suggested solution; in addition, excerpts in Study IV exemplified how participants' joint empirical examinations contributed to the formulation and verification of general arguments. Moreover, the students' relatively frequent use of transformational (i.e., general) justifications, as reported in Study II, can have been connected to the interactional organisation of working in dyads and small groups since this particular educational setting provided systematic opportunities for students to co-construct arguments (see Jacoby & Ochs, 1995; Linell, 2001) by building on each other's contributions to the discussion (Koellner, et al., 2008; Lannin et al., 2006; Radford, 2010; Strachota, 2020).

Several previous studies have also reported on the potential challenges related to students' participation in mathematical activities (e.g., Ahlberg, 1992; Artzt & Armour-Thomas, 1997; Balacheff, 1988; Emanuelsson & Sahlström, 2008; Kilhamn et al., 2019; Wood & Kalinec, 2012). These studies implicitly

indicate that students' argumentation is not only steered by the mathematical content of their contributions but is also influenced by participants' interactional work. For example, studies have shown that teachers rarely use explicitly critical assessments when evaluating students' incorrect responses, which could be part of their emotion work (e.g., Ingram, 2020; Ingram et al., 2015; Tainio & Laine, 2015). On a similar note, Varhol et al. (2021) discussed how secondary-school students tended to 'advocate' rather than 'challenge' suggestions, and Kämäräinen et al. (2021) noted how elements of secondary-school students' initiation of group work seemed to also contribute to establishing and maintaining 'egalitarian social relationships' within the group. Similarly, Studies I–IV indicated that students' mathematical argumentation is constituted by interactional phenomena that are intertwined. Below, these findings are discussed.

Study I presented an excerpt from a discussion during which the difference between two mathematical solutions was handled only as a case of differences in 'how you think about it', although one of the solutions was based on an unreasonable interpretation of the conditions of the task. In addition, Study III showed how students commented on and asked questions about their peers' proposals, without explicitly criticising them. In contrast to Study I, the analysis presented in Study III was not based on the concepts of social and sociomathematical norms (see Yackel & Cobb, 1996). Nevertheless, like the analysis in Study I, part of students' interaction presented in Study III can also be interpreted in terms of an orientation towards a social norm of equality. For example, students who solicit explanations without explicitly criticising others' suggestions may be orienting towards a norm that everyone's proposals should be presented during group work[27]. On the other hand, the practice of initiating mathematical argumentation by presenting differing proposals in the form of an adjacency pair, in which peers presented 'proposal B' immediately after presenting 'proposal A' without initially commenting on either of them, also seemed to have facilitated the students' talk about their different solutions to the task. In other words, students who do not explicitly criticise peers' differing proposals may be orienting not only towards normative practices of social interaction, but also towards a sociomathematical norm that contributions to mathematical argumentation should differ mathematically from previously presented proposals.

The analysis of student's argumentation on a turn-by-turn basis, as reported in Study III, also revealed that students' concessions to a peer's proposal often occurred before their peer had finished explaining his/her proposal, which indicates an urgency to display a 'change of state' (see Heritage, 1984b). In addition, students' concessions were then often achieved by affect-laden and/or embodied resources. It is possible that this was part of students'

[27] Soliciting explanations without criticising them may also be a strategy of maintaining 'egalitarian social relationships' within the group (see Kämäräinen et al., 2021).

strategies to avoid others' interpretation that the concession was a result of submissiveness or 'mindless compliance' (Kotthoff, 1993; Waring, 2007); that is, students' urgent and affect-laden concessions can have been a way to display independent epistemic access (see Heritage, 2014) to their peers' solutions, as well as to the content of the mathematical task. Thus, in addition to focusing on solving the assigned task, the students' mathematical argumentation seems to play an important role in their interactional work, for example by contributing to (re)establishing and/or maintaining a social identity of being an independent and competent participant in mathematical problem-solving activities.

6.1.2 What characterises the content of young students' explanations and justifications when discussing solutions during mathematical problem solving with peers?

Many of the previous studies on generalising have concerned students' work on patterns and tasks involving functional relationships, or aspects of teaching and learning (early) algebra (e.g., Blanton et al., 2019; Jurow, 2004; Koellner et al., 2008; Lannin, 2005; Radford, 2010; Strachota, 2020; Urena et al., 2019; Varhol et al., 2021). In comparison, Studies I–IV exemplified how engaging in mathematical problem solving with peers during everyday classroom work on other topics than algebra can also be a way of introducing young students to the requirements of proving by using general arguments (see Ellis, 2011; Ellis et al., 2021; Harel & Sowder, 1998, 2007; Jurow, 2004; Radford, 2010). By examining particular examples during work on tasks about combinatorics, and about percentages and fractions, students were able to formulate generalisations about what procedures that should 'always' be conducted, and what conditions are relevant to 'all' cases (Study I, II and IV).

In particular, the consistency of the use of examples-based justifications as a basis for transformational justifications, as reported in Study II, emphasises the importance of agreement between results of empirical examinations and general claims. Although young students' explanations, justifications and generalisations should not be treated as proofs in a formal sense, these findings make it relevant to discuss young students' mathematical argumentation in terms of a preliminary stage of proving (see Durand-Guerrier et al., 2021; Harel & Sowder, 1998, 2007). Thus, the notion of *emergent proving* was proposed in Study II, which emphasises that young students' generalising based on examinations of empirical examples can be considered a precursor to formal mathematical proving.

In Study II, the concept of *emergent proving* was related to research on early algebra since studies within this field have shown that it is generally beneficial to introduce algebraic ideas from the earliest school years (e.g. Blanton et al., 2015; Kieran et al., 2016). On a similar note, the discussion in

Study II suggested that an increased focus on young students' use of justifications during everyday classroom work on mathematical tasks could be one way of addressing the discontinuity that typically characterises the teaching and learning of proofs. Thus, in Study II, the notion of 'emergence' primarily referred to the temporal relationship between young students' justifying practices and adult students' formal proving. In other words, general justifications formulated in the context of school mathematics may be considered part of students' development towards their future understanding of mathematical proofs.

Moreover, acknowledging the social aspects of proving (see Bell, 1976; Harel & Sowder, 2007), and drawing on the *emergent perspective*, as formulated by Yackel & Cobb (1996:460ff), the concept of 'emergence' can also refer to processes occurring due to the reflexive relation between individual students' contributions to interaction and the joint negotiation of 'taken-as-shared mathematical meanings' in a particular educational setting. For example, Study IV showed how students co-constructed generalisations, in terms of justifications that fulfil 'for all' arguments, by sharing information and/or treating parts of the applied mathematical facts as shared knowledge, and holding each other accountable for their arguments and suggestions of how to solve the mathematical task. Thus, the notion of *emergent proving* can also represent students' joint construction of mathematical argumentation.

According to the mathematics syllabus (Skolverket, 2018:55) the teaching should give students opportunities to 'develop their ability to argue logically and apply mathematical reasoning', and help them develop their mathematical knowledge 'in order to formulate and solve problems, and also reflect over and evaluate selected strategies, methods, models and results'. Next, I discuss possible didactic implications of findings presented in Studies I–IV.

6.2 Implications for teaching

Previous studies have shown that, in many cases, students prefer asking a peer for help instead of turning to their teacher (Sjöberg, 2006) and that teachers who respond to students' help-seeking behaviour sometimes provide a solution that the student is invited to align with, even when the suggested solution does not quite correspond to the student's question (Koole, 2012; Koole & Elbers, 2014). During the observed small group interaction that was analysed in Studies I–IV, students' questions about their peers' proposals often contributed to clarifying their solutions. Thus, when responding to their students' requests for help, a useful strategy could be for teachers to involve the students' peers when answering questions and explaining solutions.

When working in small groups, students should be encouraged to share information to enable peers to build on each other's arguments (e.g., Koellner, et al., 2008; Lannin et al., 2006; Radford, 2010; Strachota, 2020). Findings

presented in Studies I, II and IV indicated that several students were quite capable of not only explaining their own solutions but also formulating general justifications. In other words, when engaging in mathematical problem solving in small group interaction outside the immediate control of their teacher, several students demonstrated an understanding of generality by identifying regularities among cases and indicating that known mathematical facts were applied in new contexts (see Ellis, 2011; Ellis et al., 2021; Harel & Tall, 1991; Jurow, 2004; Mason, 1996; Urena et al., 2019; Yeap & Kaur, 2008).

Furthermore, excerpts presented in Study III indicated that students who challenged others' proposal and arguments, although not explicitly criticising them, also contributed to more elaborate arguments. Thus, when teachers instruct students to share information and suggestions in dyads and small groups, they should also make the general social classroom norms explicit. For example, teachers can emphasise that processes of 'argumentation' does not necessarily imply disagreeing but rather is a part of social interaction during which participants strive to make sense of others' contributions, as well as to account for their own actions (Krummheuer, 1995; Nussbaum, 2011), which can also involve critical reviews of each other's arguments. For example, Study I showed how some students repeatedly stated that two different solutions were equally correct; if these students had been supported in reviewing each other's justifications they would also have been supported in discussing the differences in the level of sophistication of the two solutions. Thus, teachers can contribute to students' mathematical argumentation by discussing and modelling how to challenge each other's proposals, and address incorrect solutions, in ways that are not face-threatening.

Findings presented in Studies I–IV can also support the argument that teachers should make the whole-class discussion 'move beyond show and tell' (Stein et al., 2008). One way to develop the practice of whole-class discussions is to raise 'why'-questions (Richardson et al., 2010), in order to emphasise the importance of not only explaining but also justifying strategies and solutions. Similarly, teachers can make the sociomathematical norms of the classroom explicit by exemplifying what arguments should be considered mathematically acceptable (see Yackel & Cobb, 1996). For example, when teachers sum up the students' work during a whole-class discussion, this discussion should not only consider 'different solutions' in terms of differences in how they are represented, but also point out how various strategies differ with regard to the level of mathematical sophistication (see McClain & Cobb, 2001; Stein et al., 2008: Yackel & Cobb, 1996).

As noted above (in section 6.1), young students' mathematical argumentation should not be equated with formal proving. However, when teachers encourage students to elaborate on explanations and justifications in ways that are understood and accepted by their peers (see Bell, 1976; Harel & Sowder, 2007; Stylianides, 2007), they not only support students during their local

activity of mathematical problem solving but also contribute to the students' future use of formal proofs. Study IV presented part of a discussion among students who initiated their discussion by proposing a solution based on a general claim; the students then jointly examined empirical examples to confirm their initial claim. On the other hand, Study II showed how, during the initial part of their discussion, some students proposed a general solution in terms of a multiplication. These students continued to argue for their multiplication, even when they empirically discovered a particular example that contradicted the calculation.[28] Although the results of only a few empirical examinations should not be treated as a formal proof (Reid & Knipping, 2010), students need to acknowledge that a general argument has to be adjusted if it contradicts the results of examinations of particular examples, in line with the function of counterexamples in formal proving. Thus, according to findings presented in Studies II and IV, one particular aspect of formal proving that can be transferred into an explicit instruction to students engaging in mathematical problem solving at lower levels of education is that *the general argument has to agree with all examples*. In other words, students should be informed that general claims, formulated as 'for all' arguments, do not allow any exceptions at all (see Harel & Sowder, 1998, 2007), which is yet an example of how teachers can support students' processes of *emergent proving*.

6.3 Suggestions for further research

In contrast to research on language education, ethnomethodological approaches have not yet been very common within mathematics education research (Ingram, 2018). One limitation of these approaches concerns how much of the classroom interaction that can be captured in video recordings (see Moschkovich, 2019). Nevertheless (as indicated in Study I), it seems worthwhile to use Conversation Analysis (CA) and multimodal analysis in future studies of social interaction in the mathematics classroom. In particular, the detailed analysis of student-student interaction during classroom work, rather than interventions or interviews in laboratory settings, revealed key aspects of processes of teaching and learning of mathematics in everyday school settings that can be difficult for teachers to identify themselves.

Study IV exemplified students' processes of co-constructing general arguments in small group interaction while working on other topics than algebra, which adds to previous findings regarding students' development of their generalising abilities. In addition, findings presented in Study I and III add to

[28] In contrast to Study I, the analysis on Study II was not based on the framework of sociomathematical norms; however, students' tendency to adhere to an incorrect procedure possibly indicates an orientation towards a sociomathematical norm that mathematical arguments should involve calculations.

previous CA studies on interaction in the mathematics classroom since this thesis focused on student-student interaction rather than teacher-led classroom interaction. One potential focus of future research is to further the analysis of students' practice of proposing differing solutions in the shape of adjacency pairs (discussed in Study III). For example, a question to investigate is whether this practice is representative for Swedish classrooms in general or if it is rather an aspect of mathematics-specific interaction. In addition, a follow-up question could be to analyse the (potentially) reflexive relation of teachers' instructions and students' interactional practices while engaging in mathematical problem-solving activities with peers. The application of ethnomethodological approaches can then be further developed by involving teachers in formulating research questions related to their teaching practices during everyday classroom work to be investigated using CA and multimodal analysis.

Moreover, the notion of *emergent proving*, proposed in Study II and elaborated in subsection 6.1.2, should be a topic for further research. By involving teachers, and continuing the analysis of students' mathematical argumentation during every day classroom work, it would be possible to develop a program for teaching that supports students' emergent proving. Such a program should both account for students' mathematical progression over the school years and consider how to facilitate interaction in ways that support students' local co-construction of general arguments during work on mathematical tasks. In particular, this program would have the potential to introduce the notion of proving in the earliest school years, as a support to students' development of their understanding of formal proofs at higher levels of education.

7 Sammanfattning på svenska

Bakgrund, syfte och forskningsfrågor

Tidigare forskning har visat att elevers matematikutveckling generellt gynnas av arbete med problemlösning och samarbete med klasskamrater. Det finns dock studier som tyder på att lärare inte i någon större utsträckning tillämpar grupparbete under matematiklektioner (Hardman, 2020; Robertson & Graven, 2019; Zimmermann, 2016) samt att det saknas analyser av processer som uppstår i elevers interaktion under pågående samarbete (Seidouvy & Schindler, 2020)

I många länders styrdokument för undervisning på grundskolenivå hanteras elevers utveckling av sin problemlösningsförmåga, samt förmågan att delta i matematiska resonemang, som centrala element i undervisningen. Detta gäller även för svensk skola; grundskolans kursplan i matematik anger bland annat att undervisningen ”ska bidra till att eleverna utvecklar förmågan att argumentera logiskt och föra matematiska resonemang” (Skolverket, 2023a). Vidare hör det till kunskapskraven för årskurs 3, 6 och 9 att eleverna kan välja och använda strategier och metoder för att lösa olika typer av uppgifter, samt att de ”beskriver”, ”redogör för” och ”samtalar” om både sina tillvägagångssätt och sina resultat (ibid.). Däremot framgår det inte under vilka arbetsformer som dessa aktiviteter ska äga rum. Läroplanen anger alltså inte om elevernas beskrivningar och redogörelser ska formuleras muntligen eller i skrift; det anges inte heller om samtalen ska ske under lärarledd helklassundervisning eller inom ramen för samarbete med klasskamrater.

Den här avhandlingens syfte var att öka kunskapen om hur social interaktion kan bidra till elevers matematiska argumentation (i form av att förklara och motivera lösningar samt formulera generella argument) under gemensamt arbete med problemlösning. I de fyra delstudierna analyserades videoinspelningar av samarbete (inom ramen för ordinarie klassrumsarbete) mellan elever i årskurs sex. Analysen utgick från två forskningsfrågor:

- Hur organiseras elevers arbete med matematisk problemlösning, socialt och interaktionellt?
- Vad kännetecknar elevers sätt att förklara och motivera lösningar under diskussioner i samband med matematisk problemlösning tillsammans med klasskamrater?

Teoretiskt ramverk

Avhandlingens studier utgick från ett *dialogiskt* perspektiv på kommunikation (se Linell, 2001, 2009, 2022) vilket har beskrivits som ”en löst sammanhållen familj” av teoretiska riktningar och traditioner (Linell, 2009:400, min översättning). Ett grundläggande antagande inom detta perspektiv är att det råder ”ömsesidiga *beroenden*” (*interdependencies*) mellan en själv och andra, samt att människor har en förmåga att ”*tillsammans med andra* skapa ordning och mening” (Linell, 2022:372).

Mer specifikt tillämpades en *etnometodologisk* ansats (Garfinkel, 1967) i analysen av elevernas sociala interaktion, vilket hittills inte har varit en särskilt vanlig ansats inom matematikdidaktisk forskning (Ingram, 2018). Denna ansats innebar att elevers handlingar analyserades med utgångspunkt i ett deltagarperspektiv (*emic perspective*) i stället för att relateras till externa kategorier (Broth & Keevallik, 2020:37).

En av etnometodologins grundprinciper är att bidrag till interaktion är både kontextformade (*context-shaped*) och kontextförnyande (*context-renewing*) (Heritage, 1984a:242). Till exempel är ett verbalt yttrande ”bakåtpekande”, genom att det knyter an till tidigare yttranden, samtidigt som det är ”framåtpekande”, genom att det möjliggör kommande yttranden (Linell, 2009:181; 2022:373). Detta innebär i sin tur att ett kommunikativt bidrag inte kan förstås fullt ut om det tas ur sitt sammanhang. Vidare kan relationen mellan handlingar och aktivitet sägas vara *reflexiv* i bemärkelsen att kontexten påverkar deltagares handlingar, samtidigt som dessa handlingar bidrar till att konstruera själva aktiviteten (Linell, 2001). Detta är särskilt tydligt när interaktion sker i institutionella sammanhang eftersom interaktionen då oftast är målinriktad (Drew & Heritage, 1992; Heritage, 2004).

En överordnad etnometodologisk princip är att interaktion kan beskrivas som ordnad (eller organiserad). Denna ordning är dock inte något som deltagare i interaktion så att säga ”påtvingas” utifrån utan ordningen samkonstrueras av interaktionsdeltagare i och med att deras handlingar uppvisar igenkännbara mönster (Garfinkel, 1967) vilka även kan benämnas *normer* (Ingram, 2018). Vissa normer som uppstår under lektionsarbete är generella för alla klassrum; det är till exempel en generell *social* norm att ett nytt bidrag till en diskussion ska skilja sig från tidigare bidrag. Under interaktion i matematikklassrummet uppstår dock inte enbart sociala normer utan även ”sociomatematiska normer” (*sociomathematical norms*) (Yackel & Cobb, 1996). Dessa normer utgör interaktionella mönster som är specifika för matematikundervisning, såsom att nya bidrag till matematisk argumentation måste skilja sig på ett matematiskt sätt från tidigare bidrag.

Vidare betraktas matematisk argumentation i den här avhandlingen som en särskild typ av social interaktion, där deltagare strävar efter att både förstå andras bidrag till interaktionen och visa förklaringsansvar för (*account for*)

sina egna kommunikativa bidrag (Krummheuer, 1995). Att delta i argumentation behöver således inte innebära att man är oense med andra deltagare; argumentation kan istället ses som en social process inom vilken olika argument gemensamt konstrueras och granskas. Vidare betraktades bevisföring (*proving*) i avhandlingen som ett särskilt fall av matematisk argumentation, där formuleringen av generella argument ses som ett av bevisföringsprocessens mest karaktäristiska drag (Durand-Guerrier et al., 2021).

Tidigare forskning

Ett flertal av de tidigare matematikdidaktiska studier som har tillämpat etnometodologiska metoder, såsom samtalsanalys (*Conversation Analysis, CA)* och multimodal analys (*multimodal interaction analysis*) har analyserat olika aspekter av lärarledd klassrumsinteraktion. Studier inom detta fält har till exempel analyserat hur språkliga aspekter av lärares instruktioner kan avspeglas i elevers eget tal om matematikuppgifter (Barwell, 2023; Forrester & Pike, 1998; Mushin et al., 2013; Roth & Gardener, 2012), samt hur det fortsatta klassrumsarbetet kan påverkas av lärares sätt att involvera elever i diskussioner (Emanuelsson & Sahlström, 2008) eller bemöta elever som ber om hjälp (Ingram, 2020; Koole, 2012; Koole & Elbers, 2014; Svahn & Melander Bowden, 2021).

Vidare visade Tainio och Laine (2015), tillika med Ingram m.fl. (2015), att lärare vanligen undviker att ge negativa omdömen när elever ger felaktiga svar, vilket kan vara del av lärarens emotionella arbete (*emotion work*). Liknande drag har också upptäckts i högstadieelevers diskussioner; Varhol m.fl. (2021) påtalade att elever tenderade att förespråka (*advocate*) snarare än att utmana (*challenge*) klasskamraters lösningsförslag, medan Kämäräinen m.fl. (2021) diskuterade hur elever initierade problemlösningsaktiviteter på ett sätt som indirekt bidrog till att bevara jämlika relationer inom gruppen. Kämäräinen m.fl. (2019) visade också att elevers smågruppsdiskussioner fick olika förlopp beroende på om den elev som initierade och ledde diskussionen var positionerad som mer eller mindre ”kunnig” (*K+* eller *K-*); diskussioner ledda av en K- elev innehöll fler utmaningar och kritiska frågor medan diskussionen oftare liknade IRE-strukturen[29] (Mehan, 1979) när den leddes av en K+ elev.

Tidigare studier tyder på att elevers matematiska argument blir utförligare när de uppmuntras att ta del av och bygga vidare på varandras argument (t.ex. Strachota, 2020), samt att ställa och besvara ”varför-frågor” (Richardson m.fl., 2010). Elevers argumentation gynnas även när de får uttrycka sig både muntligen och i skrift och då använda sig av ”naturligt tal” (Ayala-Altamirano

[29] Under lärarledda diskussioner är IRE-strukturen vanligen förekommande. I står för lärarens initiering (*Initiation*) av samtalet, vilket ofta sker i form av att läraren ställer en fråga, R står för elevens svar (*Response*) och E står för lärarens utvärdering (*Evaluation*) svaret (Mehan, 1979).

& Molina, 2021; Ayala-Altamirano m.fl., 2022; Shinno & Fujita, 2022; Stylianides, 2019; Urena m.fl., 2019), samt av att ha tillgång till olika slags visuellt stöd såsom konkret material (Morris, 2009; Shinno & Fujita, 2022).

Ett särskilt ”fall” av matematisk argumentation är bevisföring. I grundskolans kursplan i matematik (Skolverket, 2023) används inte begreppet ”bevis”; traditionellt har bevis nämnts först i kursplaner för motsvarande gymnasienivå, vilket kan ha bidragit till att bevisföring av många (både lärare och elever/studenter) uppfattas som en utmanande del av matematiken (Hanna & Knipping, 2020; Knipping, 2008; Stylianides, 2017). Synen på vad som menas med bevis och bevisföring har dock varierat över tid (se Bell, 1976; Balacheff, 2008; de Villiers, 1990; Hanna, 2000; Hemmi, 2006; Reid & Knipping, 2010; Zhuang & Connor, 2022) men ett karaktärsdrag som i de flesta studier förknippas med formella bevis är att det byggs upp av generella argument (t.ex. Harel & Sowder, 2007). Generalisering kan i sin tur beskrivas som ”*seeing a generality through the particular* and *seeing the particular in the general*” (Mason, 1996:65) vilket kan förstås som förmågan att se relationer mellan vad specifika exempel visar och vad som är giltigt i allmänhet.

Liksom bevis och bevisföring har förmågan att formulera generella argument i många fall beskrivits som en utmaning (se Ellis, 2011; Jurow, 2004); dock har analyser av elevers generalisering ofta genomförts i form av undervisningsexperiment, eller intervjuer i ”labb-miljö” (Ellis m.fl., 2021), medan det har gjorts färre studier av elevers förmåga att generalisera under ordinarie klassrumsarbete. Det är inte heller helt klarlagt hur handlingar som ”förklarar”, ”motiverar” respektive ”bevisar” ett argument relaterar till varandra (Staples & Conner, 2022; Thanheiser & Sugimoto, 2022). Beroende på hur aktiviteten ”bevisföring” definieras kan det vara möjligt att se paralleller mellan yngre elevers arbete med att förklara och motivera lösningar och studenters bevisföring inom ramen för matematikundervisning på högre nivå. Genom att beskriva bevisföring i termer av handlingar som innebär att elever förklarar och förmedlar idéer (Schoenfeld, 1994), eller ordnar och bekräftar resultat (Ayala-Altamirano & Molina, 2021), samt betraktar dessa handlingar som bidrag till elevers meningsskapande i matematikklassrummet (Bell, 1976; Harel & Sowder, 2007; Stylianides, 2007), är det möjligt att se yngre elevers arbete med att förklara och motivera lösningar som ett förstadium till (senare) arbete med formulering av formella bevis.

Empiriskt material

Det empiriska material som analyserades i avhandlingens fyra delstudier bestod av videoinspelningar av elevers samarbete under ordinarie lektionsarbete i två klassrum i årskurs 6. Efter varje lektion samlades även elevernas anteckningar in.

Skolan där lektionerna videofilmades är en kommunal Fk-6 skola belägen i en av Sveriges större kommuner. Videoinspelningarna gjordes under hösten

2018, vilket innebär att eleverna var 11–12 år gamla vid tiden för observationerna. Matematikuppgifterna som användes under de observerade lektionerna (Appendix A) valdes ut i samråd med klasslärarna, som önskade att eleverna skulle arbeta med områdena kombinatorik, samt bråk och procent. Uppgifterna formulerades på ett sätt som liknar vedertagna läromedel (Hagland et al., 2005; Larsson, 2007) samt uppgifter i det nationella provet i matematik för årskurs 6 (PRIM-gruppen, 2022).

Videoinspelningarna ägde rum inom ramen för projektet *Inkludering genom lärande i grupp* (Appendix D; Uppsala universitet, 2023) som hade etikprövats av en regional etiknämnd (Dnr. 2017/371). Totalt 17 lektioner (som varade i genomsnitt ca 55 min) observerades. Under varje lektion videofilmades arbetet i två grupper, vilket resulterade i 34 inspelningar av gruppArbeten (se information om lektionerna i Appendix E). Innan observationerna påbörjades hade elevernas vårdnadshavare informerats skriftligen, och även givit skriftligt samtycke till barnens medverkan i studien (Appendix F). Elevernas samtycke hanterades däremot som ”provisoriskt” och kontrollerades muntligen inför varje lektionsobservation (se Flewitt, 2006; Heath m.fl., 2010).

Analys av elevers sociala interaktion

Elevernas sociala interaktion undersöktes med utgångspunkt i principer för samtalsanalys (*Conversation Analysis, CA*) (Heritage, 1984a; Hutchby & Wooffitt, 2008; Schegloff, 2007; Sidnell, 2010; ten Have). Elevers bidrag till diskussioner om matematikuppgifter analyserades därför inte i form av enstaka yttranden utan ställdes konsekvent i relation till den samtalssekvens (*sequence*) inom vilken bidraget gjordes. Vidare antogs ett deltagarperspektiv, i enlighet med avhandlingens etnometodologiska ansats (se avsnittet Teoretiskt ramverk), vilket innebar att det inte gjordes några antaganden *a priori* angående vad som kunde betraktas som relevant i elevernas diskussioner (se Heritage, 1984a). Under analysen av elevernas diskussioner betraktades samtalssekvenser som ordnade serier av talturer (*series of turns*) där det så kallade närhetsparet (*adjacency pair*) utgör en minimal sekvens. Ett sådant ”par” består av endast två på varandra följande turer som yttras av olika personer, där den första turen relevantgör en efterföljande tur, som är märkbart frånvarande (*noticeably absent*) om den uteblir (Sacks m.fl., 1974; Schegloff, 2007). Utöver elevernas verbala yttranden analyserades deras kroppsspråk (Hayashi, 2014; Rossano, 2014) och hantering av artefakter, framför allt i form av hänvisningar till anteckningar på arbetsblad (Goodwin, 2007).

Analys av elevers matematiska argumentation

För att även möjliggöra analys av elevers sätt att förklara och motivera lösningar till matematiska uppgifter analyserades innehållet i elevernas matematiska argumentation utifrån principer för kvalitativ innehållsanalys. Studie I och II presenterade analyser genomförda i enlighet med principerna för

deductive category assignment (Mayring, 2015), vilket innebär att ett innehåll kategoriseras i enlighet med en uppsättning yttre kategorier. De kategorier som användes i Studie I och II utgick från Harel och Sowders (1998, 2007) *taxonomy of proof schemes*. Elevernas sätt att förklara och motivera sina lösningsförslag analyserades i termer av att vara ”externt baserade” (*externally based*), ”empiriska” (*empirical*) eller ”analytiska” (*analytic*).

I Studie IV tillämpades istället principer för *problem-driven content analysis* (Krippendorff, 2013), där ett första steg är att formulera kategorier. Fokus för Studie IV var att undersöka elevers användning av generella argument. Analyskriterierna baserades därför på Masons (1996) definition av generalisering (se avsnittet Tidigare forskning) som operationaliserades genom att kombinera resultat av tidigare studier (Ellis, 2011; Ellis m.fl., 2021; Harel & Tall, 1991; Jurow, 2004; Urena m.fl., 2019; Yeap & Kaur, 2008). I Studie IV betraktades generalisering som en process som innebär att elever *identifierar regelbundenheter* och/eller *överför argument till en ny kontext*.

Studie I

Den första delstudien fungerade som en ”förstudie” i och med att endast en liten del av det empiriska materialet analyserades. Studie I presenterade en analys av korta utdrag ur två gruppers diskussion om uppgiften ”Busskön” (Appendix A). Inom båda grupperna föreslogs lösningar som elever motiverade i enlighet med ett *analytic proof scheme* (Sowder & Harel, 1998) eftersom de formulerade och tillämpade (korrekta) generella argument. Samtalen inom de två grupperna orienterade dock mot helt olika normer.

I den ena gruppen möttes den generella lösningen med uppskattning, och efter det att eleverna hade löst den angivna uppgiften formulerade de även principen för fakultet på ett informellt sätt. Elevernas interaktion orienterade därför mot en sociomatematisk norm att ”lösningar till matematiska uppgifter ska formuleras på ett generellt sätt”.

I den andra gruppen presenterades två olika lösningar och eleverna tillkallade läraren för att få hjälp att hantera detta potentiella dilemma. Läraren föreslog att gruppen skulle presentera båda lösningarna för resten av klassen, vilket följdes av ett samtal mellan eleverna som orienterade mot en social norm om jämlikhet. Trots att lösningarna som hade presenterats inom gruppen byggde på olika antaganden (vilket ledde till att lösningarna uppvisade olika nivå av matematisk elegans) underströk eleverna att båda lösningarna var ”lika rätt” och att den enda skillnaden mellan lösningarna var ”hur man tänker”.

Enligt Levenson m.fl. (2009) och Wester (2015) kan elevers och lärares uppfattningar om vilka sociomatematiska normer som råder i ett klassrum skilja sig åt i ganska hög grad. Studie I visade att det kan råda stora skillnader även mellan elevers uppfattningar om (och orientering mot) sociala och sociomatematiska normer i det gemensamma matematikklassrummet.

Studie II

Analysen som presenterades i Studie II byggde vidare på Studie I genom att alla videofilmade gruppers arbete kring uppgiften ”Busskön” (Appendix A) analyserades. Studiens huvudfokus var att kategorisera elevers sätt att motivera sina lösningar, där Harel och Sowders (1998, 2007) taxonomi användes som analytiskt ramverk. Elevers sätt att relatera olika typer av motiveringar till varandra analyserades också, vilket visade att elevers formulering av generella motiveringar alltid byggde på, och överensstämde med, resultat av konkreta undersökningar.

Studien innehöll även ett kvantitativt inslag, vilket visade att frekvensen av olika motiveringstyper i elevernas argumentation stämde överens med vad tidigare studier har visat angående hur vuxna studenter (såsom studenter inom lärarutbildning) ofta motiverar sina lösningar. Analysen som presenterades i Studie II gör det alltså relevant att argumentera för att delar av grundskoleelevers arbete med gemensam problemlösning har likheter med formell bevisföring på en högre utbildningsnivå. Yngre elevers sätt att förklara och motivera lösningar kan därför ses som ett förstadium till matematiska bevis; således föreslogs begreppet *emergent proving* som benämning för yngre elevers arbete med att förklara och motivera lösningar.

Studie III

Utöver att exemplifiera hur elevers generella argumentation utgick från konkreta undersökningar visade Studie II att elevers sätt att motivera sina lösningar hanterades på väldigt skilda sätt, inom olika grupper. I vissa fall förekom det till exempel att elever vidhöll att en beräkning var tillförlitlig, även då resultatet av beräkningen motsades av vad ett konkret exempel visade. Fokus för Studie III var därför att analysera hur samarbetande elever hanterade situationer där olika lösningar föreslogs.

Ett genomgående drag hos de analyserade grupparbetena var att lösningarna vanligen presenterades direkt efter varandra, i form av ett slags närhetspar, utan att något av förslagen initialt kommenterades. Ytterligare ett gemensamt drag var att explicit kritik av andras lösningsförslag inte förekom. Förklaringar efterfrågades (det vill säga, kommentarer och frågor om andras lösningar yttrades i hög grad) men ingen elev sa explicit att en klasskamrats lösning var felaktig.

I majoriteten av diskussionerna där olika lösningar presenterades ”gav sig” den ena parten, och anslöt sig till klasskamratens lösningsförslag. I samband med medgivandet av den andres lösning förekom det även att den som ”gav sig” explicit förklarade varför hens egen lösning var felaktig. Dessutom gjordes medgivandet ofta innan klasskamraten hade berättat klart om sitt alternativa lösningsförslag, vilket tyder på att det var angeläget att uppvisa ett ändrat ställningstagande (*change of state*) (Heritage, 1984b). När en elev ”gav sig”, och anslöt till en annans lösning, markerades också medgivandet tydligt,

till exempel genom att eleven talade med hög röst. Detta kan möjligen ha varit ett sätt för eleven att uppvisa att hen självständigt hade ändrat sitt ställningstagande. På så sätt kan den matematiska argumentationen också ha varit en del av elevens interaktionella arbete, genom att bidra till att bibehålla (eller återupprätta) en identitet som självständig och kompetent deltagare i problemlösningsaktiviteten.

Studie IV

Liksom Studie III byggde Studie IV vidare på analysen i avhandlingens andra delstudie. Fokus för den fjärde delstudien kan beskrivas som en fortsättning av analysen av elevers användning av generella argument för att motivera en lösning. Med utgångspunkt i operationaliseringen av Masons (1996:65) definition av generalisering som processer där elever *identifierar regelbundenheter* och/eller *överför argument till en ny kontext* analyserades elevers metoder för att gemensamt åstadkomma generalisering.

Studier har visat att elevers förmåga att generalisera gynnas av att dela information med andra, samt av att använda olika slags semiotiska resurser (Strachota, 2020; Urena et al., 2019; Yeap & Kaur, 2008). Studie IV bekräftade till stora delar vad som har framkommit i tidigare forskning, men bidrog även med ytterligare kunskap om elevers formulering av generella argument. Framför allt exemplifierade Studie IV i detalj hur elevers frågor och kommentarer kring andras lösningsförslag bidrog till en mer utvecklad argumentation.

Utöver att analysera det matematiska innehållet i elevernas argumentation noterades elevers användning av vissa språkliga resurser; till exempel noterades övergången från att använda personliga pronomen till det generaliserande ”man” (Teleman m.fli. 1999). Även elevers användning av modalpartikeln ”ju” noterades, då denna partikel kan antyda att information hanteras som gemensam (Heinemann m.fl., 2011), vilket i sin tur kan tyda på att elever tillämpar kända matematiska fakta i en ny kontext.

Sammanfattande diskussion

Avhandlingens bidrag till det matematikdidaktiska forskningsfältet består av fördjupad kunskap om elevers argumentation under gemensamt arbete med problemlösning. Tidigare forskning har visat att lärares och elevers interaktion i matematikklassrummet inte enbart involverar (eller styrs av) ett strikt matematiskt innehåll; både lärare och elever kan även göra ett visst ”emotionellt arbete” genom att (till exempel) undvika att uttrycka sig kritiskt mot andras bidrag till klassrumsdiskussionen, även då ett bidrag är matematiskt inkorrekt. Studie I och III visade exempel på gruppsamtal där explicit kritik mot andras lösningsförslag saknades, vilket kan tyda på en orientering mot en social norm om jämlikhet. Praktiken att presentera lösningsförslag på ett sätt som liknar ”närhetspar” (se Studie III) bidrog samtidigt till att olika lösningar ”lades på bordet” för senare granskning. Att elever inte explicit kritiserade vad någon annan föreslog kan därför även tolkas som en orientering mot en socio-

matematisk norm att nya bidrag till matematisk argumentation ska skilja sig matematiskt från tidigare bidrag.

Elevers sätt att tydligt uppvisa en ändrad ståndpunkt vad gäller det egna lösningsförslaget (vilket diskuterades i Studie III) kan också tolkas som ett sätt att använda matematiska argument för att återupprätta och/eller bibehålla en social identitet av att vara självständig och kompetent som problemlösare. Detta tyder i sin tur på att det finns en reflexiv relation mellan ”sociala” och ”matematiska” faktorer i elevers gemensamma arbete i matematik-klassrummet, vilket lärare bör ta hänsyn till när elever instrueras att samarbeta. Genom att visa hur både egna och andras lösningsförslag kan granskas på ett konstruktivt sätt under gemensamt arbete med matematisk problemlösning kan lärare synliggöra klassrummets sociala normer. Läraren kan då samtidigt bidra till en orientering mot en sociomatematisk norm om att målet med matematisk argumentation inte enbart är att presentera olika lösningsförslag utan även att delta i klassrumsdiskussioner på ett sådant sätt att lösnings-förslagen kan formuleras på ett mer utförligt och matematiskt elegant sätt.

Studie II exemplifierade elevers förmåga att formulera generella argument med utgångspunkt i undersökningar av konkreta exempel under arbete med en specifik uppgift (”Busskön”, Appendix A) medan Studie IV hade ett mer över-gripande fokus på elevers generalisering. Analysen som presenterades i dessa två studier bekräftade värdet av att utgå från och hänvisa till konkreta exempel, samt vikten av att de generella argumenten stämmer överens med vad exemplen visar. Både Studie I och Studie IV antyder också att yngre elevers arbete med att förklara och motivera lösningar under gemensamma problemlösningsaktiviteter har ett ”släktskap” med det som gäller för mate-matisk bevisföring på en högre utbildningsnivå; till exempel är användningen av ”mot-exempel” en vedertagen princip för formella matematiska bevis. Att arbeta med problemlösning inom grundskolans matematik kan därför fungera som ett första steg till förståelse av matematisk bevisföring inom högre utbildning.

Studier om ”tidig algebra” har visat att införandet av aspekter av ”algebra-iskt tänkande” redan på lågstadienivå bidrar till att underlätta elevers förståelse av formell algebra under senare delar av sin utbildning (Blanton m.fl., 2015). Med inspiration av ”tidig algebra”-forskning föreslogs i Studie II att begreppet *emergent proving* skulle kunna införas som en benämning på elevers stegvisa utveckling mot förståelse av formell bevisföring. En svensk motsvarighet av detta begrepp skulle till exempel kunna vara att ”arbeta med proto-bevis”. Begreppet ”emergence” kan dock även användas för att beskriva processer som uppstår i interaktion mellan människor. *Emergent proving* understryker därför också att yngre elevers arbete med att förklara och moti-vera en lösning är något som sker i interaktion med andra, vilket möjligtvis skulle kunna översättas till ”sam-bevisning” på svenska.

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Appendix A: Mathematical tasks

In Appendix A, the wording of the mathematical tasks used during the video recordings are shown in the order that they were presented to the students during the observed lessons. The text the line in each table were written on the back of the worksheets; these parts of the tasks were treated (by teachers and students) as 'extra' tasks to be solved if time allowed.

Clothes	**Kläder**
Lisa has three pairs of jeans and four shirts. She always wears jeans and a shirt. In how many ways can she dress?	Lisa har tre jeans och fyra tröjor. Hon använder alltid jeans och tröja. På hur många olika sätt kan hon klä sig?
One day it is cold and Lisa wants to wear a jacket too. She has two jackets. In how many ways can she dress now? Try to find a rule for the number of different ways to dress.	En dag är det kallt och Lisa vill också ha en jacka på sig. Hon har två jackor. På hur många sätt kan hon klä sig nu? Försök finna en regel för antalet olika sätt att klä sig.

Queue	**Busskön**
There is a queue at the bus stop. In how many ways can: a) 2 persons form a queue? b) 3 persons form a queue? c) 4 persons form a queue?	Det är kö vid busshållplatsen. På hur många olika sätt kan: a) 2 personer stå i kö? b) 3 personer stå i kö? c) 4 personer stå i kö?
d) Try to find a rule for the number of different ways to form a queue.	d) Försök finna en regel för antalet olika sätt att stå i kö.

Dice	**Tärningar**
Some classmates roll two identical dice and add the two numbers that show up. What are the most frequent sums they get if they use: a) two six-sided dice?	Några klasskamrater kastar två likadana tärningar och adderar de två talen som kommer upp. Vilka är de vanligaste summorna de får om de använder: a) två sexsidiga tärningar?
b) two ten-sided dice? c) two twenty-sided dice? d) Try to find a rule for the most frequent sum when you roll two dice, depending on the number of sides on the dice.	b) två tiosidiga tärningar? c) två tjugosidiga tärningar? d) Försök finna en regel för vilken summa som är vanligast när man kastar två tärningar, beroende på antal sidor på tärningen.

Ice cream	**Glass**
Kalle is buying ice cream and chooses between different flavours. He wants two scoops of ice cream. In how many ways can he choose his ice cream if there are: a) 3 flavours? b) 4 flavours?	Kalle ska köpa kulglass och kan välja mellan olika smaker. Han vill ha två kulor. På hur många sätt kan han välja sin glass om det finns: a) 3 smaker? b) 4 smaker?
c) More flavours (you choose the number)? d) Try to find a rule for the number of ways to choose ice cream.	c) Ännu flera smaker (ni bestämmer hur många)? d) Försök finna en regel för antalet sätt att välja glass.

Hockey stick	**Ishockeyklubban**
A hockey stick costs 1500 SEK. There is a sale and the stick is sold for half the price. Thereafter the price is raised again, by 50%. What does the stick cost then? Show how you reached your result!	En ishockeyklubba kostar 1500 kr. Det blir rea och klubban säljs för halva priset. Sedan höjs priset igen, med 50%. Vad kostar klubban då? Visa hur ni kom fram till ert svar!
Explain why the hockey stick does not cost 1500 SEK after the raise.	Förklara varför ishockeyklubban inte kostar 1500 kr efter höjningen.

Chocolate	**Chokladaskarna**[30]
 Many boxes of chocolates are sold before Christmas Eve. On Christmas Day a shop sold their boxes of chocolates with a 20% discount. One week later, on New Year's Day, there was another discount of 50% of the price. Give several examples of what a box of chocolates can cost initially, and after the final sale on New Year's Day.	 Inför jul säljs många chokladaskar. På juldagen sålde en affär sina chokladaskar med 20 % rabatt. En vecka senare, på nyårsdagen. var det 50 % rabatt på reapriset. Ge flera olika exempel på vad en chokladask kan kosta från början och efter sista rean på nyårsdagen.
Explain why the total discount is not 70%.	Förklara varför den totala rabatten inte är 70 %.

Schools	**Skolorna**
Elm School a) 300 students attend Elm School. One third of the students attend grade 6. How many students attend grade 6? b) 50% of the students who attend grade 6 at Elm School go to school by bus. How many students in grade 6 go by bus? *Beech School* a) More than 300 but less than 310 students attend Beech School. 25% of the students attend grade 6. How many students attend grade 6? b) You are told that 50% of the students in grade 6 at Beech School go to school by bus. In what way does this affect your answer to task a)?	*Alskolan* a) På Alskolan går det 300 elever. En tredjedel av eleverna går i sexan. Hur många elever går i sexan? b) 50% av eleverna som går i sexan på Alskolan åker buss till skolan. Hur många elever i sexan åker buss? *Bokskolan* c) På Bokskolan går det fler än 300 men färre än 310 elever. 25% av eleverna går i sexan. Hur många elever går i sexan? d) Du får veta att 50% av eleverna i sexan på Bokskolan åker buss till skolan. På vilket sätt påverkar det ditt svar i uppgift a)?
Cedar School More than 300 but less than 330 students attend Cedar School. 20% of the students attend grade 6. 25% of the students in grade 6 go to school by bus. How many students attend grade 6?	*Cederskolan* På Cederskolan går det fler än 300 men färre än 330 elever. 20% av eleverna går i sexan. 25% av eleverna i sexan åker buss till skolan. Hur många elever går i sexan?

[30] The picture is a copy from the national test for grade 9, given in 2008 (PRIM-gruppen, 2022).

Lions	Lejon
In a zoo there are more than 15 and less than 25 lions. How many lions can there be if: a) exactly 20% are males and the rest is females? b) exactly 25% are males and the rest is females? c) exactly 50% are females, exactly one third is young males and the rest is adult males? d) Explain why it cannot be any number of lions between 15 and 25.	En djurpark har fler än 15 och färre än 25 lejon. Hur många lejon kan det vara om: a) exakt 20 % är hanar och resten är honor? b) exakt 25 % är hanar och resten är honor? c) exakt 50 % är honor, exakt en tredjedel är unga hanar och resten är vuxna hanar? d) Förklara varför det inte kan vara vilket antal lejon som helst mellan 15 och 25.

Shake hands	Skaka hand
There is a party. Everyone who is at the party shakes hands. How many handshakes in total are done if there are: a) 3 persons at the party? b) 4 persons at the party? c) 5 persons at the party?	Det är fest. Alla som är på festen skakar hand med varandra. Hur många handskakningar blir det totalt om det är: a) 3 personer på festen? b) 4 personer på festen? c) 5 personer på festen?
d) 10 persons at the party? e) 100 persons at the party? f) Try to find a rule for the number of handshakes, depending on the number of persons at the party.	d) 10 personer på festen? e) 100 personer på festen? f) Försök finna en regel för hur många handskakningar det blir, beroende på antal personer på festen

Appendix B: Transcription symbols

Symbol	Use
. , ? ¿ a:	Full stop indicates “falling” prosody. Comma indicates “continuing” prosody. Question mark indicates (more or less) “rising” prosody. Colon indicates prolongation of the immediately prior sound.
(0.6) (.)	Numbers in parenthesis indicate a pause in tenths of seconds. A dot in parenthesis indicates a very short but hearable interval.
[	Square brackets indicate overlapping talk.
↑	Arrow up indicates a pitch peak within an utterance.
word	Underlining indicates emphasis.
((word))	Double parenthesis contains transcriber’s descriptions.

(Jefferson, 2004:24ff)

Appendix C: Excerpts in Swedish

Excerpt 1

1.	Ali:	kan du förklara’re ja förstår inte de.
2.	Björn:	de e (0.6) tre gånger två?
3.		antar ja.
4.	Ali:	a ja vet (.) men,
5.	Björn:	kolla eh eller,
6.		ja vet inte heller [riktit.
7.	Ali:	[nej asså ja,
8.		ja ↑vet att de e tre gånger eh gånger två¿
9.		men (.) förklara de e lite svårare.
10.	Björn:	men förklara din först.
11.	Ali:	åkej.

Excerpt 2

1.	Anna:	ettusenfemhundra delat på två e sjuhundrafemti,
2.		sjuhundrafemti delat i två
3.		de skulle va hälften plus.
4.	Charlie:	vah,
5.	Anna:	å då e're sjuhundrafemti,
6.		hälften av [sjuhundrafemti,
7.	Bertil:	[asså a::
8.	Anna:	de e trehundrasjuttifem å då bli're
9.		sjuhundrafemti plus trehundrasjuttifem
10.		e ettusenetthundratjufem.
11.		alltså e svaret ettusenetthundratjufem.
12.	Bertil:	ja.
13.	Charlie:	asså ja pallar inte (.) men äh,
14.	Dinah:	men de står ju så här
15.		((läser problemet högt)) Anna.
16.	Anna:	((ropar på läraren))
17.	Lärare:	((pratar med andra elever))
18.	Charlie:	de e säkert Anna som har rätt nu.
19.	Bertil:	a eller hur.
20.	Dinah:	men titta?
21.		eh [((läser problemet högt))
22.	Anna:	[men de måste va
23.		[så. ((ser på sina anteckningar))
24.	Charlie:	[yes.

Excerpt 3

1.	Linn:	va e nitti delat me två.
2.	Boris:	förtifem.
3.	Sonja:	eh ah exakt.
4.	Linn:	a då bli’re förtifem.
5.	Boris:	varför då¿
6.	Linn:	för att så här (.) förra,
7.		då tänkte ja att de va tjugi för ja gjorde
8.		så hära ((points at Figure 1)) för varje?
9.		men då bli’re eh (.) va heter’e (.) de blir eh
10.		asså de blir flera gånger om.
11.		den där grejen.
12.		å då bli’re liksom om man tar alla gånger,
13.		nie handskakningar per person?
14.	Boris:	mm?
15.	Linn:	då bli’re nitti å sen delar man de för ja,
16.		ja hade tjugi så dela ja de på två
17.		å då blev de tie.
18.	Sonja:	ja fattar inte riktit (.) de va,
19.		men ja fattar nu.
20.	Linn:	men kolla så här,
21.	Boris:	men hon hade så här fem personer
22.		((writes, Figure 2)) så.
23.	Linn:	å så gjorde ja,
24.	Boris:	en,
25.	Linn:	de va ett gånger,
26.	Boris:	fyra.
27.	Sonja:	a?
28.	Boris:	för en kan gö[ra fyra personer.
29.	Sonja:	[en kan göra fyra.
30.	Boris:	å sen så gånger fem (.) blir tjugi?
31.		sen ska man ju dela de på två
32.		för å få’re rätta svaret.
33.	Sonja:	mm?
34.	Boris:	de e de hon menar.
35.	Sonja:	aha smart.
36.	Boris:	å då om hon har en person gånger nie
37.		så bli’re nitti.
38.		eller liksom alla personer gånger nie.
39.		å sen dela de så bli’re förtifem.
40.	Sonja:	ja de e smart.

Figure 1: Linn's representation of ten persons' handshakes

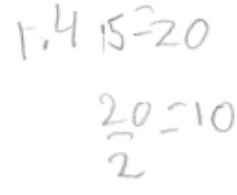

Figure 2: Boris' calculation of five persons' handshakes

Excerpt 4

1.	Carl:	asså ja tänkte inte rimilit ja skrev ba,
2.		asså hundra skulle va rimlit i verkliheten.
3.		max hundra va.
4.	Doris:	((nods))
5.	Bella:	nej.
6.	Doris:	en sån där aladdinask.
7.	Carl:	vi kan börja me typ sjutti kroner för de e,
8.		sjutti kroner e nåt som e rimlit.
9.		skulle ni köpa en chokladask för runt
10.		tvåhundrafemti spänn.
11.	Bella:	[nej,
12.	Doris:	[nä eller inte jag.
13.		vaddå kan man ta sjutti (.) vänta [sjutti
14.	Carl:	[a
15.		skriv ba sjutti.
16.	Doris:	delat me fem?
17.		lika me en åssså två åssså fyra då bli're fjorton
18.		å sen fjorton (.) bli're inte sju kroner.

Excerpt 5

1.	Jack:	va tog (.) du tog trehundraåtta.
2.	Daniel:	först (.) ja testade dom två som
3.		man kunde dela på fyra¿
4.		de e ba åtta å fyra av eh fram ti tie.
5.		asså du kan ju vahetere noll ti tie
6.		de e bara fyran å åttan du kan dela på fyra¿
7.	Jack:	a: a,
8.	Daniel:	ässå to- eh utan att de blir ett decimaltal,
9.		ässå började ja me trehundraåtta¿
10.		då bli're sjuttisju.
11.	Jack:	såru testa're bara.
12.	Daniel:	ja dom där två ja eh ja gjorde eh,
13.		asså ja kollade att,
14.		de e ju ba dom två som går¿
15.	Jack:	((nods)) a,
16.	Daniel:	först testa ( ) trehundraåtta¿
17.		de blev sjuttisju å de går inte
18.		för sen måste ju man dela're på två.
19.		((points at the worksheet))
20.	Jack:	((nods))
21.		a de måste va jämnt.
22.	Daniel:	a så då så därför så tog ja trehundrafyra
23.		istället för då bli're sjuttisex¿
24.		å därför så fick man veta att de va rätt
25.		eftersom att de blev jämnt å då kan ja
26.		dela sen också.

Appendix D: Information about *Inkludering genom lärande i grupp*

This appendix contains a copy of the information on the first page of the website presenting the project *Inkludering genom lärande i grupp* (https://www.edu.uu.se/forskning/pedagogik/ps/kooperativt_larande/om-projektet/)

Inkludering genom lärande i grupp

Arbetar du som lärare i matematik och/eller svenska i årskurs 5?
Vill du bidra till forskning i pedagogik och få en kostnadsfri utbildning och handledning?

Det pågår en ständig debatt om hur klassrumsmiljöer kan göras mer inkluderande. Det talas mycket om inkludering och ordet finns med i internationella styrdokument. Dock finns det få studier som handlar om hur man kan arbeta inkluderande. Vi vill utvärdera om arbetssättet kooperativt lärande har betydelse för att skapa inkluderande klassrumsmiljöer.

Arbetssättet kooperativt lärande handlar om att strukturera grupparbete i klassrummet så att alla elever är lika delaktiga och arbetar utifrån ett gemensamt mål. Genom att flera elever är aktiva under lektionen, skapas större möjligheter för lärande. Studier av arbetssättet kooperativt lärande i USA, Australien, Italien och Hong Kong visat att elever har bättre skolprestationer och kamratrelationer om lärare använder sig av kooperativt lärande. Vi vill utvärdera om kooperativt lärande gör skillnad i svenska klassrum.

Appendix E: Information about recordings

The tables present an overview of the length of each lesson, the sizes of the groups that were observed, the total number of students present in the classroom and the type of problem that students solved.

Date Group	**Approximate duration**	**Group sizes**		**Total nr. of students**	**Type of problem**
181015 6A1	50 min.	Two dyads =>	Four	22	Combinatorics (Clothes)
181015 6A2	-"-		Three	-"-	Combinatorics (Clothes)
181015 6B1	55 min.	Two dyads =>	Four	24	Combinatorics (Clothes)
181015 6B2	-"-	Two dyads =>	Four	-"-	Combinatorics (Clothes)
181016 6A1	55 min.		Four	23	Combinatorics (Queue)
181016 6A2	-"-	Two dyads =>	Four	-"-	Combinatorics (Queue)
181016 6B1	55 min.	Two dyads =>	Four	24	Combinatorics (Queue)
181016 6B2	-"-	Two dyads =>	Four	-"-	Combinatorics (Queue)
181017 6A1	50 min.		Four	23	Combinatorics, Probability (Dice)
181017 6A2	-"-		Four	-"-	Combinatorics, Probability (Dice)
181017 6B1	80 min.	Two dyads =>	Four	24	Combinatorics, Probability (Dice)
181017 6B2	-"-	Two dyads =>	Four	-"-	Combinatorics, Probability (Dice)
181019 6A1	50 min.		Three	22	Combinatorics (Ice cream)
181019 6A2	-"-	Two dyads =>	Four	-"-	Combinatorics (Ice cream)
181019 6B1	70 min.	Two dyads =>	Four	24	Combinatorics (Ice cream)
181019 6B2	-"-	Two dyads =>	Four	-"-	Combinatorics (Ice cream)

Date	Approximate duration	Group sizes		Students present	Type of problem
181022 6B1	60 min.		Three	23	Percentages, Fractions (Hockey stick)
181022 6B2	-”-		Four	-”-	Percentages, Fractions (Hockey stick)
181023 6A1	45 min.		Four	22	Percentages, Fractions (Hockey stick)
181023 6A2	-”-		Four	-”-	Percentages, Fractions (Hockey stick)
181023 6B1	50 min.		Four	22	Percentages, Fractions (Chocolate)
181023 6B2	-”-		Four	-”-	Percentages, Fractions (Chocolate)
181024 6A1	45 min.		Four	22	Percentages, Fractions (Chocolate)
181024 6A2	-”-		Four	-”-	Percentages, Fractions (Chocolate)
181024 6B1	45 min.		Four	21	Percentages, Fractions (Schools)
181024 6B2	-”-		Four	-”-	Percentages, Fractions (Schools)
181025 6A1	45 min.		Four	22	Percentages, Fractions (Schools)
181025 6A2	-”-		Three	-”-	Percentages, Fractions (Schools)
181025 6B1	50 min.		Four	23	Percentages, Fractions (Lions)
181025 6B2	-”-		Three	-”-	Percentages, Fractions (Lions)
181026 6A1	55 min.		Three	23	Percentages, Fractions (Lions)
181026 6A2	-”-		Four	-”-	Percentages, Fractions (Lions)
181026 6B1	65 min.		Three	21	Combinatorics (Shake hands)
181026 6B2	-”-		Four	-”-	Combinatorics (Shake hands)

Appendix F: Information to legal guardians

This appendix contains a copy of the letter and consent form that was distributed by the teachers to the legal guardians of the students.

Att arbeta med matematisk problemlösning genom kooperativt lärande

Projektet ”Inkludering genom lärande i grupp”

Vi undrar om du och ditt barn kan tänka er att delta i en studie om matematisk problemlösning genom kooperativt lärande. Under våren 2018 deltog lärare och elever i klassen i projektet ”Inkludering genom lärande i grupp”. Denna studie är en fördjupande studie inom ramen för projektet.

Syftet med studien är att undersöka hur elevers grupparbeten utvecklas över tid under den period som arbetssättet implementeras i ditt barns klassrum. Vi vill särskilt studera kunskapsutbyte och kommunikation i grupperna när elever deltar i övningar kring matematisk problemlösning. Undersökningen kommer att genomföras under höstterminen 2018. Under klassrumsbesöken genomförs observationer, videoinspelningar samt intervjuer med elever.

Ditt och ditt barns deltagande är frivilligt och ni kan närsomhelst avbryta er medverkan. Vi skickar här även med lättillgänglig information om projektet avsedd för ditt barn. Vi är måna om barnets egen rätt att bestämma om hans/hennes medverkan i projektet och kommer att vara uppmärksamma på om och när han/hon inte vill delta i intervjuer eller (video)observationer av grupparbete.

Fältanteckningar och intervjuer kommer att avidentifieras vid analys och rapportering av studiens resultat. Videoinspelningar kommer att förvaras på en extern hårddisk som krypteras och kräver lösenord för tillträde. Hårddisken kommer att förvaras i ett kassaskåp i Uppsala Universitets lokaler. Koder i form av siffror och bokstäver kommer att användas för att skydda ert barns identitet. Ingen obehörig kommer att få ta del av data om enskilda forskningspersoner.

Godkännande från vårdnadshavare

☐ Jag har fått information om projektet ”Inkludering genom lärande i grupp – fallstudier av grupparbete” och …

☐	Jag samtycker till att mitt barn deltar i projektet
☐	Jag samtycker till att mitt barn deltar i projektet, men mitt barn ska INTE filmas i grupparbeten
☐	Jag ger INTE mitt samtycke till att mitt barn deltar i studien

Barnets namn …….

Underskrift…….

Namnförtydligande……

Datum och ort ……..

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