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MARIE SJÖBLOM

PROMOTING MATHEMATICAL DIALOGUE

Students' and teachers' listening, questioning and participation

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By working together, each might learn something
about the world of the other
Wagner (1997, p. 16)

Deep in the forest there is an unexpected clearing that can be reached only
by someone who has lost her way
Tranströmer (1978)

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PREFACE

A thesis is always written within a context, with different opportunities and limitations for what to see and what to work with. Therefore, I would like to invite you to my context and describe it, before I write and you read the actual thesis.

My PhD journey

This PhD project has lasted a long time – eight years – during which I have been working simultaneously as a PhD student, a mathematics teacher, and with school development. Having this time line and these three different roles has made it possible for me to reason about and understand the problematique I would like to discuss in this thesis from different positions. My focus has been mathematical dialogue across all eight years, involving students and teachers, both in classrooms settings and in a professional development group setting.

For the first four years, I conducted what I call the first sub-study, which also led to a licentiate thesis (Sjöblom, 2015). Here, the focus was on students and student-to-student interactions. I wanted to understand what affected students' mathematical dialogue and investigate it in cooperation with a teacher, since the teacher had knowledge of the students that I (as a researcher coming from the outside) did not. Together, we used educational design research, a form of cooperation between teachers and researchers in which theory plays an important role, and we worked in three cycles to design activities and interpret the results. This gave us insights about students' engagement in mathematical dialogue and what affects their participation, in relation to listening and questioning.[1]

Over the next four years, I conducted the second sub-study, which built on learnings from the first, but also included a new educational design research project together with a group of four teachers, as part of their professional development work, and with a focus on the role of teachers in promoting mathematical dialogue. Together, the teachers and I wanted to learn more about what teachers can do when it comes to promoting student listening, questioning, and how to make all students participate in mathematical dialogue. Again, three cycles were conducted, this time mainly from the viewpoint of the teachers, but also including a professional development setting in which we tried to understand the complexity of how teachers can develop an awareness of and refine their actions in relation to mathematical dialogue.

This PhD thesis includes four articles, two from each sub-study. The first two were written after my licentiate thesis was finished, and extend the thinking of that thesis, focusing on mathematical dialogue from a student viewpoint. Articles 3 and 4 focus more on the teacher viewpoint and build on data from the second sub-study. These articles and the text in this thesis tie together the student and teacher viewpoints on mathematical dialogue.

Many PhD projects have to settle for investigating either what is happening in a classroom setting or in a professional development setting, focusing either on the interaction in the classroom or how teachers can develop their teaching together. The last eight years have given me opportunities to do both, moving back and forth between the two settings. I see that as a strength, since it is hard to separate students' and teachers' participation in mathematical dialogue as they are inevitably intertwined. Another intertwinement is connected to research and practice – when wanting to do research together with teachers, it is not a question of conducting "research on" teachers; rather, they are an important part of the research process. These two intertwinements have made the research processes in my PhD project more complex, but have also enabled learnings about mathematical dialogue that we would have missed if we had tried to avoid the intertwinements. Complex research processes are sometimes needed when trying to understand complex phenomena.

[1] In this thesis, the expression "mathematical questioning" is equivalent to "asking mathematical questions".

With these words in the preface, I would like to welcome you to the world and context in my thesis, with different actors/viewpoints (students, teachers, school developers, researchers) and different settings (the classroom and the professional development group) and making it both a possibility and a challenge to understand more about the intertwinements and complexity when it comes to promoting participation in mathematical dialogue.

Acknowledgements

Writing a thesis within a context is not only a writing process or a learning process; it is also a process in which you get the opportunity to meet, cooperate, share thoughts with, and get inspiration and support from others. Therefore, I would like to acknowledge and say thank you to some of the people who have been part of my PhD journey.

First, I would like to thank the students and teachers who have been part of my studies, and the school leaders who enabled the projects. Without you, it would not have been possible to implement this PhD. Thank you for openheartedly letting me be part of your classrooms and professional development group. I have learnt so much from you.

I would like to thank my supervisors, Clas Olander and Paola Valero in my PhD, and Tamsin Meaney and Maaike Hajer in my licentiate project. You have challenged and supported me at the same time, making me grow both as a researcher and as a human being. This process has changed me in a variety of positive ways, some visible, some invisible, and our cooperation has been an important factor in that. Thank you for your support, for writing together with me, and for believing in me.

I would also like to thank Lisa Björklund Boistrup for careful readings of my almost-finished thesis, and Jeppe Scott for being discussant at my 90 percent seminar. Your support and feedback were very valuable for me when improving the text, giving me both knowledge and energy that helped me reaching the goal in this long journey.

I have had the opportunity to cooperate with other groups and individuals during my PhD journey, that have supported me in different ways. In Malmö stad, I have been working with the inspiring people at FoU/Pedagogisk Inspiration. Special thanks to everyone in the research group for always being there for me, and to Jens Ideland for lending me the picture on my cover page. At GVF in Malmö stad I had the opportunity to cooperate with senior lecturers, lead teachers, and school leaders who inspired my work. Special thanks to Edward Jensinger for enabling me to be both a PhD student as well as a senior lecturer in the municipality at the same time, believing that I could do it. I also would like to say thank you to the doctoral students and colleagues at NMS and in my research school, as well as the LIT research program, whom I have had the benefit of working together with. Although COVID-19 made it hard to see all of you every day, being part of our community made it so much easier and funnier to go to work. Another group to thank is my nod-grupp, working with constructing tasks for national tests in mathematics, who have supported and believed in me.

I would also like to thank my family and friends, both outside and inside the academic world. You know who you are, so I will not write your names. You have given me the opportunity to think about things other than writing and research once in a while, which gave me energy to continue. Thank you for making me laugh and for reminding me of what is most important in life. Especially Sofie, you ;

Erica, David, and Jennifer – you are and will always be the most important people for me. Thanks for having patience with me and for making me smile every day. I love you.

Marie Sjöblom, Malmö, November 2021.

1. INTRODUCTION

Problematizing participation in mathematical dialogue

In Swedish mathematics classrooms, students have different opportunities for participating in mathematical dialogue. This is a problem that can be analyzed from different viewpoints connected to different groups of actors. *Firstly*, it is a problem for students who do not receive equal opportunities for participating, and hence do not have equal opportunities for learning. *Secondly*, it is a problem for teachers, who need to work with this inequity and create teaching that reaches out to all students. *Thirdly*, it is a problem for those working with professional development, finding ways of supporting teachers to support students. *Fourthly*, it is a problem for researchers, as there are still unsolved questions regarding this issue.

The field of mathematics education has concluded that opportunities for participating in mathematical dialogue are important, and researchers have used different theories and methods to study interaction and participation in mathematics classrooms. However, there are still gaps in research within this area; see Chapter 2. One of these gaps concerns how the problem can be explored simultaneously from all four of the above-mentioned viewpoints. This is important as the four groups of actors are connected to each other and contribute with different inputs when trying to understand and solve the problem. Although research has been conducted on students, on teachers, and on professional development, more research is needed on how a research-based setup of professional development can combine the students' and teachers' viewpoints to understand the problem. This will make it possible to study the complex connection between what teachers do, both in classrooms and in their professional development work, and students' opportunities for participating in mathematical dialogue. In summary, this thesis is about mathematical dialogue and how teachers, by paying attention to how students work with questioning and listening, and by being aware of how they themselves interact with students, can refine their actions to promote students' participation in mathematical dialogue.

Researching participation in mathematical dialogue

Students' opportunities to participate in mathematical dialogue and how they interact both with each other and with their teachers, are inevitably connected to teachers' actions. Likewise, teachers' actions must be connected and related to what happens within the mathematical dialogue and students' interactions. The core of this project was the connection between the students and teachers and their interaction in a classroom setting; see Figure 1.1. In order to try to understand more about it, a professional development setting was created in which teachers reflected, discussed, and analyzed what was happening in the classroom setting. Both of these settings were included within educational design research projects, in which a researcher (me) cooperated with the teachers. By moving back and forth between the settings, all four viewpoints (those of students, teachers, professional developers, and researchers) were connected.

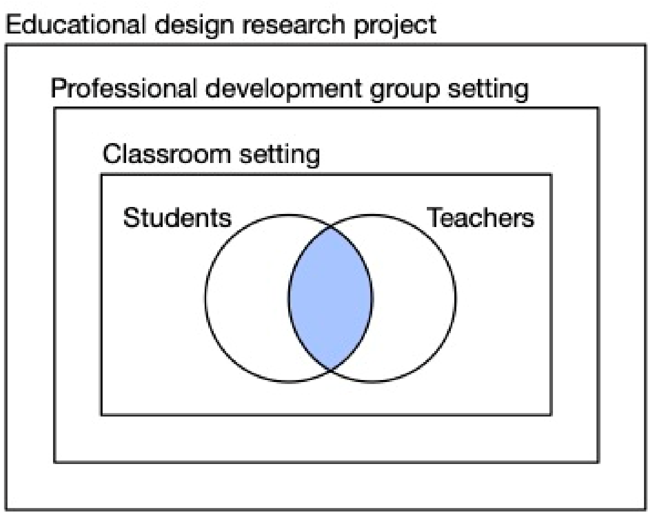

Figure 1.1 Setup

However, researching mathematical dialogue generally was too large a scope for one thesis and also for a professional development project, so the focus areas and analyses were limited to the following aspects of mathematical dialogue: listening, questioning, and participation in small group problem-solving work.

To understand more about these aspects of mathematical dialogue, two different sub-studies were conducted. The first emphasized mathematical dialogue in student-to-student interaction, especially how students ask each other questions and listen to each other. This resulted in a licentiate thesis (Sjöblom, 2015). Thereafter, the data material also gave rise to two articles with focus on students, which are part of this PhD thesis. In the second sub-study, the emphasis changed to mathematical dialogue from a teacher's viewpoint, and how teachers can promote equitable participation opportunities for students through listening and questioning. This was done both in the classroom setting and the professional development group setting, to follow how teachers' awareness on mathematical dialogue changed, supporting them to refine their teaching. The second sub-study resulted in two more articles with focus on teachers.

Aim and research questions

The aim of this thesis is to explore how questioning and listening can promote participation in mathematical dialogue. This is done to contribute to understandings on both teachers' and students' engagements in mathematical dialogue and provide insights on collaboration between research and teacher professional development in two settings: the classroom and a teacher professional development group. The following three research questions (RQ) are addressed:

1. What aspects of students' questioning and listening do teachers need to pay attention to when promoting students' participation in mathematical dialogue?
2. What aspects of teachers' questioning and listening are important when teachers promote students' participation in mathematical dialogue?
3. How can teachers, in cooperation with researchers, develop an awareness and refine their teaching in relation to students' listening, questioning, and participation in mathematical dialogue?

Outline of the thesis

This thesis includes four articles, in which Article 1 and Article 2 contribute to answer Research Question 1, Article 3 and Article 4 contribute to answer Research Question 2, and all articles together with the discussion in the wrapping contribute to answer Research Question 3.

The articles are:

1. Sjöblom, M. (2018). Developing mathematical reasoning by using questions in a multilingual mathematics classroom. *Nordic Studies in Mathematics Education 23*(3–4), 61–79.
2. Sjöblom, M., & Meaney, T. (2021). "I am part of the group, the others listen to me": Theorising productive listening in mathematical group work. *Educational Studies in Mathematics 107*(3), 565–581.
3. Sjöblom, M., Valero, P., & Olander, C. (resubmitted to *Journal of Mathematics Teacher Education after major revision*). Teachers' noticing to promote students' mathematical dialogue in group work.
4. Sjöblom, M. (submitted to *Educational Studies in Mathematics*). Promoting equitable participation opportunities in mathematical dialogue through mathematical questioning.

I wrote Article 1 myself. Article 2 was a co-production with Professor Tamsin Meaney, in which I did the data collection with her support as my supervisor in my licentiate project, and we conducted the writing and the review process together, doing equal parts of the work. Article 3 was a co-production with my supervisors in my PhD project, Professor Paola Valero and Professor Clas Olander. I was responsible for the data collection and did the main part of the writing, with solid support and feedback on the text from the others. We have received this article back with major revisions and cooperated in the review process to resubmit it. I wrote Article 4 myself, with support from my supervisors and it has been submitted to a journal.

The thesis contains six chapters. This chapter (Chapter 1) introduces the thesis, describing the problem, aim, and research questions. Chapter 2 discusses what mathematical dialogue is and relates the problem to previous research. Chapter 3 discusses how theories are used to understand the problem, and also how different theories are related to each other and how they can be coordinated. Chapter 4 describes the methodology used to understand the problem, including the data generation, analysis processes, and ethical considerations. Chapter 5 summarizes the results of the articles and the findings in relation to the research questions. Chapter 6 contains a concluding discussion and problematization of the contributions of this thesis, including a meta-reflection on how different actors can collaborate to understand more about the complexity of mathematical dialogue.

2. LITERATURE REVIEW: LISTENING, QUESTIONING, AND PARTICIPATION IN MATHEMATICAL DIALOGUE

While there has been a lot of research conducted on communication in mathematics classrooms, it has not always been consistent in its conclusions, which could be a sign of the complexity of the subject. It is possible to approach the interaction between students and teachers from different viewpoints, with different theories, investigating different aspects and finding results either strengthening or contradicting each other. Many studies are small and context-specific, which makes it hard to generalize; hence, it is something of a puzzle to try and understand what has been done when it comes to mathematical communication, how previous research studies relate to each other, and what remains to be investigated.

In this chapter, the concept of “mathematical dialogue” is discussed in order to delimit what kind of mathematical communication will be investigated further in this thesis. Thereafter, a literature review is conducted, in which previous research is discussed, compared and problematized, in order to situate and identify where this thesis can contribute within the field of mathematics education. Since the existing research within these areas is comprehensive, the literature review will be limited to the areas concerning mathematical dialogue that are mentioned in the research questions; that is, questioning, listening, and participation. This is done in three sections, connected to each of the three research questions, studying mathematical dialogue in the classroom setting from a student viewpoint (RQ1) and a teacher viewpoint (RQ2), and when using the professional development setting, studying how teachers, in cooperation with researchers, can work to develop an awareness and refine their teaching when it comes to mathematical dialogue (RQ3).

Research on mathematical dialogue

Communication in mathematics has been considered important for mathematics learning, both in research as well as in syllabuses and policy documents from around the world. For instance, the National Council of Teachers of Mathematics has identified communication as an important feature since 1989 (Kosko & Gao, 2017), and the Swedish syllabuses emphasize, for all age-groups in schools, the importance of students communicating and reasoning in mathematics (Swedish National Agency for Education, 2011). Connected to OECD and PISA, mathematical literacy is seen as the students' capacities to "analyse, reason, and communicate ideas" in mathematics (OECD, 2013, p. 24).

Emphasis on mathematical dialogue in previous research

The emphasis on mathematics learning as a dialogical process that occurs in interaction between people, and not as a monological individual process, has grown since the social turn, with sociocultural perspectives on learning (Lerman, 2000). For instance, the amount of research regarding language and interaction in mathematics is comprehensive. However, not all parts of mathematical communication have been researched sufficiently, and there are questions related to language and participation that remain unanswered. For instance, Morgan, Craig, Schuette and Wagner (2014) called for more research regarding "what are the linguistic competences and knowledge required for participation in mathematical practices?" (p. 851). Resnick, Libertus and Schantz (2019) claimed that there is not yet enough evidence that all students can potentially benefit from dialogic teaching. From an equity/participation perspective, the NCTM Research Committee (2015) identified achieving equity in mathematics education as one of the current grand challenges of the field. One part of this challenge can be promoting equitable participation opportunities in mathematical dialogue.

In mathematical communication, teachers and students cooperate and their actions are closely intertwined with each other. Hufferd-Ackles, Fuson, and Sherin (2004) showed that when teachers change their teaching, this affects students' mathematical conversations, and Franke, Kazemi and Battey (2007), claimed "Teaching is relational. Teachers, students and subject matter can only be understood in relation to one another" (p. 227). Also, Radford (2016) pointed to the importance of viewing teaching and learning as a common activity that students and teachers engage in together.

The focus of the present thesis is not on the broad concept of mathematical communication or language, but is instead limited to a special kind of communication, namely *mathematical dialogue*. This does not mean any kind of mathematical communication. Hennessy, Calcagni, Leung and Mercer (2021) claimed that mathematical dialogue is not only talk or conversation, but that it "offers opportunities for active student participation and agency as learners elaborate ideas and make their understanding and reasoning explicit" (p. 2).

A tentative description is that mathematical dialogue is "a certain kind of communication that helps students to develop certain mathematical abilities that support mathematical learning". However, what this means differs depending on what definition of mathematical dialogue is used. Wegerif (2019) emphasized the need to be explicit about what is meant by dialogue and dialogic teaching in order to investigate it.

Definition of mathematical dialogue used in this thesis

In order to investigate further mathematical dialogue, Alrø and Skovsmose's (2004) definition is a starting point in this thesis. Their definition can be seen both as part of previous research, but also part of the theoretical and methodological choices discussed in Chapter 3 and Chapter 4. However, in order for readers to relate it to the literature review, the definition is discussed here already, and I will return to it in coming chapters.

I chose Alrø and Skovsmose's (2004) definition because it relates not only to language and communication as such, but also to what is quality in mathematical conversations, and social aspects of communication that are important for mathematics learning. Alrø and Skovsmose (2004) found eight dialogic acts that are important for students to develop mathematical abilities and create opportunities for mathematics learning. These eight dialogic acts are used in this thesis to identify elements of conversations that signal quality in mathematical communication. The eight dialogic acts, written in italics in this thesis, are: *getting-in-contact* (preparing for interaction), *locating* (understanding the problem), *identifying* (finding the mathematics in the problem), *advocating* (examining ideas), *thinking aloud* (making perspectives and thoughts visible), *reformulating* (clarifying and rephrasing), *challenging* (questioning ideas), and *evaluating* (looking back at the problem). By using all of the dialogic acts, it is possible to obtain a conversation with focus on mathematics, in which students listen to each other and ask each other questions.

In addition to the research of dialogic acts, Faustino and Skovsmose (2020), came up with eight non-dialogic acts that should not occur in mathematical dialogue; namely: "ignoring, distorting, confronting, ridiculing, disqualifying, excluding, stigmatising, and lecturing" (p. 13).

However, simply using the dialogic acts or avoiding the non-dialogic acts is not enough for a conversation to become a mathematical dialogue. It also has to do with the purpose of the conversation, which Alrø and Skovsmose (2004) connected to three characteristics describing a mathematical dialogue. Firstly, to be a dialogue, the conversation must be seen as a mathematical *inquiry process* in which students are actively participating. They need to have a will to solve some kind of mathematical problem, to gain knowledge or get new experiences. In the inquiry process, students can use different dialogic acts and, in an open-ended process, together try to find out what they can learn.

Secondly, since what is happening within a mathematical conversation is unpredictable, mathematical dialogue always involves *running a risk* of some kind. Alrø and Skovsmose (2004) claimed that risks make participants "open to inquiry, and learning but openness also makes them vulnerable" (p. 122), as they might, for instance, show insecurity or lack of knowledge when plunging into the unknown in a mathematical inquiry process. This means that running risks can give both positive and negative consequences.

Thirdly, mathematical dialogue is about *maintaining equality*. According to Alrø and Skovsmose (2004), in dialogue there is "no demonstration of power" (p. 124). This is complicated, however, as in my interpretation, teacher–student relations in a classroom generally do contain some kind of power relationship, at least when it comes to mathematical knowledge and the fact that teachers continuously assess students and grade them towards the end of the course. According to Alrø and Skovsmose (2004), in mathematical dialogue, equality means that dialogue cannot be forced upon anyone, but instead that people can be invited to take part, and need to accept that invitation in order for a dialogue to take place. Also, equality does not mean that everyone has to agree on everything, but rather that everyone's opinions and thoughts count.

In this thesis, connected to the last characteristic of Alrø and Skovsmose's (2004) definition of mathematical dialogue about *maintaining equality*, the term *establishing equitable participation opportunities* will be used. This is a limitation to the bigger issue with maintaining equality, but it can be grasped within the work in this PhD, and with focus on listening, questioning, and participation.

If the mathematical conversation contains the three characteristics of mathematical dialogue mentioned by Alrø and Skovsmose (2004), this implies the opportunity to create an environment in which students can talk to each other in a common inquiry process and discuss the mathematics together in depth, both with and without the teacher. The mathematical dialogue can give students opportunities to develop their mathematical knowledge. In this thesis, since the project is a collaboration with teachers, this mathematical knowledge will be connected to the mathematical abilities that their students need to develop according to the Swedish syllabus for upper secondary school (Swedish National Agency for Education, 2011) on (1) handling concepts, (2) handling procedures, (3) problem solving, (4) modelling, (5) reasoning, and (6) communicating mathematics.

In summary, *mathematical dialogue*, building on the research of Alrø and Skovsmose (2004), is defined in this thesis as a conversation about mathematics, containing dialogic acts and the three characteristics making an inquiry, running a risk and maintaining equality/establishing equitable participation opportunities, which supports the development of the mathematics syllabus abilities on concepts, procedures, problem solving, modelling, reasoning, and communication as defined by the Swedish syllabus (Swedish National Agency for Education, 2011).

With this definition of mathematical dialogue in mind, now previous literature is discussed as a point of departure for this PhD study, starting in the classroom setting.

Research on students' questioning and listening

The first research question in this thesis is about what aspects of students' questioning and listening teachers need to pay attention to when promoting students' participation in mathematical dialogue.

Overview

When it comes to communication in mathematics, most research studies point in the same direction, about it being crucial for students to explain, reason and justify in mathematics (Brandt & Schütte, 2010). Communication – in small groups, for instance – is considered to contribute to collective mathematics learning (Alexander, 2020; Ametller, 2019; Cobb, Stephan, McClain, & Gravemeijer, 2001; Walshaw & Anthony, 2008a), at least when the group work is operational. In much of this research, questions can be considered central because they are important in mathematical inquiry processes (Alrø & Skovsmose, 2004). However, as we will see, there are other studies that have problematized mathematical interaction and dialogue, showing that it is not always beneficial for mathematics learning. Most research on questions has also been done from a teacher's viewpoint, so there is a need for more research from the student's viewpoint (Kemmerle, 2013).

When focusing on students, research on mathematical dialogue often poses questions about which students need to communicate in mathematics, pointing to it being extra-important for certain groups. For instance, Dominguez (2011) and van Eerde, Hajer and Prenger (2008) claimed that multilingual students need to actively use language in mathematics. Other studies have investigated multilingualism as a resource in dialogue (Adler, 2001; Setati, 2005, 2008; Moschkovich, 2002, 2007) and multilingual students' opportunities for learning mathematics (Källberg Svensson, 2018). Although multilingual matters are important, they have not been put in the foreground in the present thesis. Rather, it has been assumed that all students, regardless of languages, could benefit from actively taking part in mathematical dialogue. As most classrooms in Sweden contain students with a mixture of first languages, with 30 percent of students in upper secondary school having a foreign background[2] (Swedish National Agency for Education, 2020), no extra attention has been paid to multilingual issues. Rather, the point of departure has been how teachers can meet the dialogic needs of heterogenous groups of students. So, what does research then show about how students work with questioning and listening in mathematical dialogue?

Students' questioning

When studying mathematical dialogue, one of the more common areas to focus on relates to students' and teachers' use of questions as a tool for improving the conversation (see, for instance, Alexander, 2020; Alrø & Skovsmose, 2004; Drageset, 2014; Milani, 2012; Sahin & Kulm, 2008). Questions have also been connected to mathematics learning. For instance, Esmonde (2009) claimed that when students receive answers to their questions, this helps their

[2] Here, foreign background means that the students are either born abroad or have two parents born abroad.

learning processes, and Mercer (1995, p. 10) stated that interaction gives students the opportunity to "check, refine and elaborate" on what they know.

However, asking questions and contributing in mathematical dialogue is not always easy and previous research has problematized how questions are used in dialogue. For instance, Fuentes (2009) concluded that it takes time for students to learn to work together in groups, especially if they are not used to collaborative working methods. According to the Swedish School Inspectorate (2010), in Sweden too much time is spent individually when students perform tasks in their textbooks, which means that students do not have enough opportunities to discuss mathematics. Hence, in the Swedish context it is unsurprising that group work is sometimes problematic.

So what could be the problems when it comes to questioning? Fuentes (2009) concluded that there are aspects of group work that might affect interaction negatively; namely, lack of communication between all students in a group, poor communication patterns, and norms that impede students' learning. To understand how students worked with questions, Fuentes (2009) identified eight question/comment-response pairs (see Chapter 3 for more information), building on the research of Dekker and Elshout-Mohr (2004), to analyze how students interacted with each other. From a student's viewpoint, this seems to be an important part of previous research to build onto, as it identifies what can be problematic and what aspects teachers need to understand more about and try to change when it comes to students' use of questions.

Other research studies support Fuentes' (2009) findings. For instance, Hansson (2011), showed that multilingual students do not always receive the support they need to actively participate in mathematical dialogue, which means that the dialogue does not function. Barnes (2005) wrote about students being outsiders in groups and therefore not being given the same opportunities to participate as other students. Together, these research studies have identified a need for teachers to understand reasons for why students are not included or choose not to participate in mathematical dialogue.

Students' listening

A first step for understanding more about students' work with mathematical questioning could be related to another subject close to mathematical dialogue: listening. Listening is often seen as a presumption for being able to ask questions, but the role of listening in mathematical dialogue is under-researched, and what research there is has often been conducted from a teacher viewpoint

(Hintz & Tyson, 2015). For instance, Reeder and Abshire (2012) and Carpenter and Fennema (1992) highlighted the importance of teachers listening to students. The present thesis also includes the students' viewpoint when it comes to listening, in order to deepen the understandings about students listening to each other, and what teachers need to do to promote listening.

When it comes to students' listening, there is a common divide between passive and active listening. Here, passive listening is defined as students not interacting with others about what they hear, but still hearing what is said (Otten, Herbel-Eisenmann & Steele, 2011). Active listening is defined as students interacting with each other to gain meaning (Kosko, 2014) and through "asking questions and giving non-verbal support while finding out what the other is getting at" (Alrø & Skovsmose, 2004, p. 62). Again, there is a connection here between listening and questioning. However, the division between passive and active listening is not unproblematic, since it is hard to measure or evaluate how actively students actually are listening. As such, according to Kosko (2014), listening is "an unobservable action" (p. 216), and therefore a complex factor in mathematical dialogue.

Two reasons why it could still be worth investigating how actively students listen are because the definition of mathematical dialogue used in this thesis assumes some kind of active participation from the students, and also because there is much research connecting listening to learning. When students actively listen and take part of discussions, this might affect their opportunities for learning. Otten et al. (2011) concluded that when students listen actively, "opportunities for productive learning within mathematical discussions are increased" (p. 4). Kosko (2014) claimed that when students were silent and passive, they were disengaged from learning. Asami-Johansson (2021) found a connection between students' interaction and their problem-solving success. When students did not listen to each other, their possibilities for formulating plans in mathematical problem solving were smaller (Artz & Armour-Thomas, 1992). Barnes (2005) claimed that students who participated less, learned less. Hence, it is interesting for teachers to consider how students are listening when trying to promote them to participate in mathematical dialogue.

However, students' mathematical dialogue is not only affected by how active students are in their listening, but also by the purpose of listening. The purpose of listening in mathematical dialogue is under-researched and differs for different students and different classrooms. Webel (2010) found that some students only care about listening to the teacher, while others also listen to classmates, depending on what they find important – the procedural explanation, the justification of thoughts, or the answer to mathematical tasks. Therefore, it could be important for teachers to investigate their students' reasons for listening and how this affects participation.

Active or passive listening can be related to individual students, claiming that a student is listening either actively or passively. In addition, affecting the inquiry process and who dares to run risks are social aspects that affect who is being listened to in group work and who is allowed to ask questions. Lack (2010) found that there is a social risk involved in giving incorrect solutions to mathematical tasks. Similarly, Bishop (2014) concluded that who students are with regard to mathematics affects how they talk and interact with others. Hence, as Horn (2017) concluded, it seems important to weigh the possible positive learning outcomes of group work against the social risks that students might be exposed to.

How students work with questioning and listening is closely related to teachers' actions concerning the same topics. The next section focuses on research from a teacher's viewpoint.

Research on teachers' questioning and listening

The second research question in this thesis concerns what aspects of teachers' questioning and listening are important when teachers promote students' participation in mathematical dialogue.

Overview

As summarized above, most teachers are aware of research showing that students should be active participants in mathematical dialogue. Matematiklyftet (the Mathematics Boost), a governmental professional development program that builds on research literature and that 75 percent of Swedish mathematics teachers have attended (Swedish National Agency for Education, 2016), emphasizes the importance of mathematical dialogue and active participation to develop mathematical abilities in the syllabus (Swedish National Agency for Education, 2011). However, the best way to work with this and what actions teachers need to take are not easy issues to address. Not all research studies point in the same direction, and not all types of interaction or communication are beneficial (Mercer & Sams, 2006). Deen and Zuidema (2008) concluded that even though students have opportunities to talk when they are arranged to work in small groups, "group work proves to be not a sufficient condition for learning" (p. 171). Hence, it is not only the student-to-student interaction when it comes to questioning and listening that is complex, but also teachers' roles in it and how to support students' work. These issues need to be addressed in a variety of settings in which students, as well as students together with teachers, can interact with each other (Webb, Franke, Ing, Torrou, Johnson & Zimmerman, 2019).

When it comes to teachers' ways of promoting mathematical dialogue, there is a gap in research for secondary schools, as most studies are conducted in compulsory schools (Staples, 2008; Walshaw & Anthony, 2008b). Hennessy et al. (2021) claimed that there is not much large-scale evidence on how teacher–student dialogue connects to student learning, and found that only two studies had been conducted on the topic, both in primary schools. Especially important for further research within this area could be aspects of teachers' work with promoting small group work that leads to mathematical dialogue. Ehrenfeld and Horn (2020) concluded that teachers' routines for monitoring students' work in small groups are under-researched. Also, Sfard (2015) claimed that what makes learning mathematics in small group powerful and durable remains an open question, while van de Pol, Mercer, and Volman (2019) saw a need to research more how different kinds of adaptive support, such as scaffolding, support students' learning. Before looking into what parts of these gaps in research the present study can contribute to, a summary is provided of what is already known connected to teachers' questioning and listening in mathematical dialogue.

Teachers' questioning

Questions are a common tool that teachers use in mathematics classrooms to start and to promote mathematical dialogue (Alexander, 2020; Alrø & Skovsmose, 2004; Bingham, 2005; Hufferd-Ackles et al., 2004). However, not all kinds of questions are part of a mathematical conversation in which students need to formulate their thoughts and where students are actually invited to mathematical dialogue. Instead, many questions are part of an IRE pattern within the classroom in which the teacher initiates (I) a conversation by asking a question. This is responded (R) to by a student, and thereafter the teacher evaluates (E) the answer. According to Alrø and Skovsmose (2004), this kind of conversation often takes the form of quizzing, in which students try to guess what answer the teacher is looking for, which is not in line with their way of defining mathematical dialogue. Also, Aizikovitsh-Udi and Star (2001) problematized how efficient teachers really are at asking mathematical questions, concluding that many questions are closed, and that only a small proportion demand students' higher-order thinking. Hofman (2015) expressed a need for more research on how teachers can sustain and support subject-based discussions in small groups, and also emphasized the importance of content-specific questions.

Mason (2000) problematized the purpose of teachers' questioning and concluded that questions have the potential to provoke a shift in students' attention. According to Mason (2000), there are three types of mathematical questions: asking as focusing attention, asking as testing comprehension, and asking as enquiring. Another division was made by Boaler and Brodie (2004), who identified nine types[3] of mathematical questions, asked from a teacher's viewpoint. Enoch (2013) divided teacher question into four categories: dripping, embedded, telling, or sustain focus. There are both differences and similarities between these ways of viewing questions. For example, while Mason (2000) has more general question types that can be applied in different kinds of mathematical dialogue, Boaler and Brodie (2004) are more specific; for instance, their category "generating discussion" only concerns whole-class discussions. Another difference is that Boaler and Brodie (2004) also included non-mathematical questions in their "establishing context" category. Similarities can be found in Mason's (2000) category about focusing attention, which is similar to Boaler and Brodie's (2004) "orienting and focusing" category or Enoch's (2013) category about sustaining focus.

It can be concluded from the previous literature that categorizing questions and teachers' use of them to promote students' participation in mathematical dialogue is a complex issue. Categorizations also seem to be context-specific and can be made differently depending on what kind of mathematical dialogue is in focus. In the present thesis, the context specificity will be acknowledged in letting teachers start investigating questioning in their specific classrooms, with their specific students, and identify what kind of mathematical questions can promote mathematical dialogue there.

[3] Gathering information, inserting terminology, exploring mathematical meanings and/or relationships, probing, generating discussion, linking and applying, extending thinking, orienting and focusing, establishing context.

Teachers' listening

In order for teachers to support and promote mathematical dialogue, they not only need to ask mathematical questions, but it is also important that they listen with intent to understand students' reasoning (Reeder & Abshire, 2012). Carpenter and Fennema (1992) stated that teachers can learn better about students' problem-solving processes when they listen more and explain less. However, research is scarce within this area. Lim, Lee, Tyson, Kim H-J., and Kim J. (2020) concluded that more research is needed on "how teachers (and students) listen to students, understand student thinking, respond with appropriate language, support students to listen to one another's ideas, and engage students in mathematics discussion and reasoning" (p. 394).

Apart from listening to students, teachers can also actively make students listen to each other, by having them focus on why they need to listen; for instance, to understand each other's explanations, explain their solutions or ask for help (Mueller, Yankelewitz & Maher, 2014). Teachers can also function as role models by modelling behavior and being curious about students' mathematical thinking, for instance through asking questions to amplify their ideas and help them develop creative thinking (Mrayyan, 2016).

Theories and categorizations have been used for interpreting teachers' work with listening. One categorization was made by Davis (1996, 1997), and included three categories: *evaluative* – listening for something; *interpretive* – hearing, responding and trying to understand other's thoughts; and *hermeneutic* – a shared project between the people in the interaction. One reflection I make is that all of these kinds of listening are needed in mathematical dialogue; otherwise, it will not fulfil the requirements for being a dialogue. However, all three types are not needed all the time, and different kinds of listening might be needed depending on what kind of participation teachers want to promote. By investigating the role of listening, including what different types of listening there are, and combining this with questioning, this thesis can contribute with deepened knowledge about what teachers need to understand more about when it comes to promoting mathematical dialogue.

Research on mathematical dialogue in the classroom and in a professional development setting

The third research question is about how teachers, in cooperation with researchers, can develop an awareness and refine their teaching in relation to students' listening, questioning and participation in mathematical dialogue.

Research together with teachers

Research together with teachers can be done in a classroom setting or in a professional development setting, or in both. A lot of research has focused on either the classroom or the professional development setting, and in each setting it is often only one participation structure (for instance, whole-class discussions or small group work) that is considered (Webb et al., 2019). In this thesis, the idea is to go back and forth between the two settings in order to follow how teachers' awareness and refinement of their teaching are developed in different participation structures, but with focus on small group work. Also, developing mathematical dialogue cannot be seen as a top-down process. Rather, according to Warwick and Cook (2019), all stakeholders in the classroom have much to both gain and contribute with when it comes to developing mathematical dialogue.

According to Enoch (2013), research encourages teachers to plan for discussions and small group work. However, there has not been much research on how these plans are connected to what is actually happening within the mathematics classroom, which justifies moving back and forth between the two settings in this thesis. Webb, Franke, De, Chan, Freund, Shein and Melkonian (2009) claimed that more research is needed regarding how teachers can promote efficient student collaborations; for instance, how to intervene in student group discussions. There is also a need for more research on professional development approaches that help build teacher capacity and improve pedagogical practices (Hardman, 2019).

Of course, this can be researched further in a variety of ways, but this thesis emphasizes the connection between teachers' professional development work and what is happening in the classroom. One method that could be used here, which will be thoroughly explained in Chapter 4, is teacher noticing (Mason, 2002; Sherin, Jacobs & Philipp, 2011). Teacher noticing often involves three steps – teachers *attending* to what is happening in certain aspects of, for instance, mathematical dialogue; *interpreting* it; and then deciding how to *act/respond* (Jacobs, Lamb, Philipp & Schappelle, 2011; Kazemi, Elliott, Mumme, Carroll, Lesseig & Kelley-Petersen, 2011). By using this structured way of trying to understand mathematical dialogue, it is possible to develop an awareness of factors, such as listening, questioning and participation in mathematical dialogue. However, what has not been done so often, and where this thesis can contribute to the research field, is to connect teacher noticing to questions about equity (Dindyal, Schack, Choy & Sherin, 2021; Santagata, König, Scheiner, Nguyen, Adleff, Yang & Kaiser, 2021). Another area where research is scarce concerns the third step in noticing, about acting/responding to what is found about the mathematical dialogue. Many studies on teacher noticing have only focused on attending and interpreting, but in order to develop new awareness and refine teaching, the third step is important. This is done in the present study by using educational design research; see Chapter 4 for more details. For instance, Dindyal et al. (2021) and Santagata et al. (2021) concluded that very few studies have focused on all three aspects of EDR, equity, and teacher noticing.

One common way in which teachers get time to discuss their teaching and develop an awareness is in so-called professional learning communities.

Professional learning communities

There are many different definitions of professional learning communities (PLCs), most of which are about teachers working together in a structured way to develop their teaching. PLCs are about improving conditions for supporting students' and teachers' learning, and there is a lot of research evidence about positive outcomes and effects (Cordingley, Bell, Thomason & Firth 2005; Harris, 2014; Hattie, 2008; Swedish National Department of Education, 2016; Timperley, 2011). However, PLCs need to be connected to the context in which teachers work. For instance, Fullan (2008) claimed that learning needs to be built into the culture of an organization and that it is important to "avoid superficial learning and instead embed philosophies and principles of learning in the specific context that needs improvement" (p. 87).

In this thesis, Stoll and Seashore's (2007) definition of a professional learning community is used, as it focuses not only on the learning of individual teachers, but on

> (1) professional learning; (2) within the context of a cohesive group; (3) that focuses on collective knowledge, and (4) occurs within an ethic of interpersonal caring that permeates the life of teachers, students and school leaders. (p. 3)

In PLCs, research and scientific models are often connected to the professional development processes. Many of these build on cyclic models, in which teachers are working together to develop their teaching, in order to support students' learning. According to the Swedish curriculum, education should build on scientific grounds (Swedish National Agency for Education, 2011). One model that is commonly used in Swedish schools is Timperley's (2011) model on teacher inquiry and knowledge-building cycle to promote important outcomes for the students. In the model, the starting point is the needs of the students and what they require to meet important goals. From this, the knowledge and skills that teachers need to develop are identified and worked with to refine the teaching. Students are then engaged in new learning experiences, and the impact of the changed actions is evaluated toward outcomes in student learning. Then the cycle starts over again. In this process, students and teachers are actively connected to each other, which makes it possible to build the research process relationally (Franke et al., 2007). Another benefit of working with a model like Timperley's cycle (2011) is that it gives a structure for the professional development work, which support "focus, collaboration, reflective professional inquiry, leadership and group and individual learning", which are seen as prerequisites for teacher professional development work to be efficient (Brown & Portman, 2018).

However, simply putting teachers together in a group to discuss their teaching does not mean that it will affect students' learning. PLCs do not always reach their goals. One example of this is Matematiklyftet (the Mathematics Boost). A report from 2020 showed no improvements in students' achievement for teachers who had done the program compared to those who had not (Lindvall, Helenius, Eriksson & Ryve, 2021), although other reports showed a positive influence on teaching (Österholm, Bergqvist, Liljekvist and van Bommel, 2016). There seems to be a need to delve deeper into the discussions and connect them more clearly to the specific students who teachers have in their classroom. There is no one-size-fits-all-solution.

When planning research in connection to PLCs, an important question is how teachers are involved in the research processes.

Doing research "on" or "with" teachers

Making teachers work in PLCs, on scientific grounds, can be made with or without cooperation with researchers. However, researching how teachers can develop an awareness and refine their teaching assumes that they are active participants in one way or another. Superfine (2019) suggested that design-based research can make research relevant for teachers, and "engage teachers as collaborators in the design and implementation processes" (p. 1). According to Erickson (2014), when teachers are co-designers and co-implementers it is easier to understand questions about what works in a specific context and that general prescriptions cannot be used for specific situations. He even claimed that we should consider replacing "'research-based practice' with 'practice-based research'" (p. 6).

As mentioned in the literature review, much of the research is context-specific, and when trying to understand development processes it is not fruitful to ask "what works?", but rather "what works, for whom, and under what circumstances?" (Bryk, Gomez, Grunow & LeMahieu, 2018, p. 38). Here teachers have special knowledge, since they are on the inside, with expert knowledge about their context that researchers coming from the outside lack. According to Bereiter (2014), it is when researchers work together with teachers that we can understand how and why teachers make changes in their practice and overcome the gap that exists between research and practice.

One way for researchers to cooperate with teachers is educational design research (Barab, 2006; Barab & Squire, 2004: Cobb & Gravemeijer, 2008; McKenney & Reeves, 2012); see Chapter 4 for more details. This kind of research involves the teachers in different ways, for instance by making them part of the design- and/or analysis-processes. Several EDR studies have been oriented towards mathematics education. For instance, Smit and van Eerde (2011) studied mathematics teachers' ways of scaffolding language in multilingual mathematics classrooms and claimed that design research is a "fruitful way to promote and trace the development of a mathematics teacher's expertise" (p. 899). Swain and Swan (2009) conducted a nine-month study on integrating research-based principles into the teaching of numeracy with post-16 learners, with focus on teachers' learning. There are also examples of educational design research being more oriented towards students' results in mathematics; for instance, Verschaffel, De Corte, Lasure, Van Vaerenbergh, and Bogaerts and Ratinckx (1999) analyzed fifth-graders' work with solving mathematical application problems, while Diefes-Dux, Zawojewski and Hjalmarson (2010) analyzed university-level students' work with open-ended mathematics problems.

However, conducting research together with teachers is not unproblematic. Areljung, Leden and Wiblom (2021) discussed ownership in research projects as being both about risks and benefits, as well as rights and obligations. Hence, it seems important to build a cooperation that finds a balance between different actors.

Summary: implications from previous research affecting this PhD study

This chapter shows that several areas connected to mathematical dialogue are under-researched, especially when it comes to students in upper secondary school (Hennessy et al., 2021; Staples, 2008; Walshaw & Anthony, 2008b); and how teachers can promote small group work (Enoch, 2013; Sfard & Kieran, 2001; Webb et al., 2009) in a way that includes all students and promotes participation. There are still unanswered questions regarding matters such as listening or questioning from a student viewpoint (Hintz & Tyson, 2015; Kemmerle, 2013), how communicating in mathematics affect learning (Sfard, 2015), or how teachers can work with monitoring and scaffolding students' work (Ehrenfeld & Horn, 2020; van de Pol et al., 2021). Methodologically, in previous research it is uncommon for teachers to work with noticing of classroom interaction connected to equity issues by using EDR (Dindyal et al., 2021; Santagata et al., 2021), so this is something that the present thesis can contribute with knowledge about. This is preferably done without viewing research as a dichotomy, focusing either on student viewpoints or teacher viewpoints; either on the classroom setting or the professional development setting. Rather, this thesis can contribute with knowledge that can only be found when moving back and forth between classroom and teacher professional development meetings, connecting everyone who is part of the mathematical dialogue and building bridges between different viewpoints.

3. THEORIES TO UNDERSTAND MATHEMATICAL DIALOGUE

In research, theories can be used in different ways for different purposes. In this thesis, building on educational design research, EDR (McKenney & Reeves, 2012), theories have been used in the design of the research process, in analyzing and interpreting the data, and, as a result, contributing to theoretical understandings in the field of mathematics education.

This chapter starts by describing the theories used in the PhD project, including justifications of why the theories were connected to the project. This is followed by a discussion of how the different theories were related and coordinated to each other (Prediger, Bikner-Ahsbahs & Arzarello, 2008; Wedege, 2010) in the process of trying to understand more about mathematical dialogue.

Theories on three different levels

When aiming to explore how questioning and listening can promote participation in mathematical dialogue, theories were connected to the research project on three different levels; see Figure 3.1.

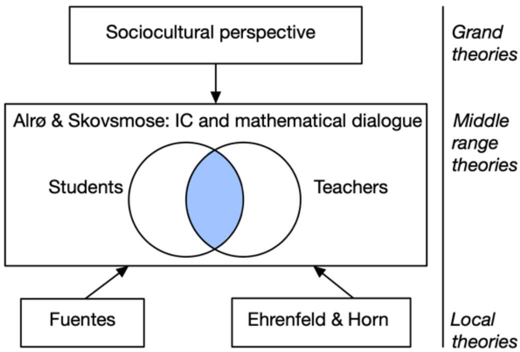

Figure 3.1 Theories on three different levels

Firstly, it was important to situate this PhD project on a grand theory level to show how the research in this study connects to the education research field generally. Secondly, it was important to find a middle range theory to define what quality can be in mathematical dialogue. Thirdly, it was important to find local theories that further could illuminate aspects or facets of mathematical dialogue connected to questioning, listening and participation.

Grand theories: Situating this thesis within a paradigm

Mathematical dialogue can be viewed from many different perspectives. Regardless of whether someone takes a behavioristic, cognitive, constructivist, socio-cultural, critical, or "something else" perspective, conversations can be used to understand or analyze different aspects of mathematics education. To situate my research within one of these aspects, with its different principles, certain ways of reasoning, adopted concepts, and preferred methodologies can help place my research within the mathematics education research field and answer different kinds of research questions (Radford, 2008; Wedege, 2010).

This thesis could have been placed within a socio-political or critical perspective with focus on how all students can obtain equal opportunities to participate in mathematical dialogue, discussing equity and justice. One justification to do so is that the middle range theory used to define quality in mathematical dialogue (Alrø & Skovsmose, 2004) originates from this perspective. Another placement to think about is related to Mason's (2002) concept of teacher noticing, which is often used in constructivist studies.

However, what articulates most elements used in this thesis is the socio-cultural perspective. This is also a common choice in much research literature when it comes to studying mathematical dialogue (Ametller, 2019). Radford (2016) argued that socio-cultural theories have a common denominator in that human beings are "consubstantial with the culture in which they live their lives", meaning that what they "think, do, feel, imagine, hope, and dream is deeply entangled in their culture" (p. 188). In this PhD study, this is connected to the culture in the mathematics classroom and how this affects the mathematical dialogue. When outlining the social turn in mathematics education, Lerman (2000) described this as "theories that see meaning, thinking, and reasoning as products of social activity" (p. 8). With the definition chosen in this thesis of mathematical dialogue (consisting of activities and conversations about mathematics, containing dialogic acts and the three characteristics of making an inquiry, running a risk, and maintaining equality), it is clear that mathematical dialogue is a social activity created by students and teachers together, affected by the culture in their classrooms and professional development group, as well as the abilities in the mathematics syllabus.

Situating the study within the socio-cultural paradigm does not mean using all the traditional concepts, such as those about artefacts or mediating tools by Vygotsky (1978) or communities of practice by Lave and Wenger (1991). Rather, the socio-cultural perspective will be used as a background, in which the activity of mathematical dialogue is put in the center as an arena for teaching and learning in mathematics. Lerman (2006) found four categories of socio-cultural research at the PME conferences from 1978 to 2005, namely cultural psychology, ethnomathematics, sociology, and discourse. With a starting point in Lerman's (2006) research, Radford (2016) stated that a common factor between these different categories of socio-cultural research was to understand the role of "culture, history, and society in conceptions of mathematics and in ideas about the students' learning" (p. 189). Although this PhD study is only a small qualitative study, it has similar goals regarding understanding mathematical dialogue. Through activities connected to mathematical dialogue, it is possible to see how culture, history, and society affect teachers' ways of promoting mathematical dialogue. In the present study, it is the interaction between students and teachers that are foregrounded. The ideas that learning and meaning making are made in a common reflexive inquiry process frames both what I and the teachers in the study did together, but also what was promoted in the classroom. In the promotion processes, the focus was not on cognitive processes or constructs of mathematical concepts as such, but rather the mathematical dialogue as part of the culture, history, and society in mathematics classrooms.

Situating this research project within a sociocultural perspective also affected what theories to work with on the other levels. In the center of Figure 3.1, regarding middle range theories, the key concern was about finding a theory that could define and help understanding quality in mathematical dialogue.

Middle range theories: Using the IC-model to understand quality in mathematical dialogue

In all parts of this PhD study, Alrø and Skovsmose's (2004) Inquiry Co-operation (IC) model has played an important role. By considering what dialogic acts might be part of a mathematical conversation, the model has helped define what quality in mathematical dialogue can be.

As described in Chapter 2, a mathematical dialogue is an inquiry process in which the viewpoints of the participants of a conversation can be explored. This presumes active participation, running risks, as well as thoughts about maintaining equality (Alrø & Skovsmose, 2004). In a mathematical dialogue amongst students, or even together with a teacher, all these aspects are not always present, but the IC-model can still provide direction regarding what to strive for in mathematical dialogues.

The Inquiry Co-operation model

Alrø and Skovsmose (2004) created the IC-model as a way to understand quality in mathematical dialogue. It was originally developed to understand how a teacher and a group of students together can explore mathematical problems, and what is important in the dialogue to create possibilities for mathematics learning. In the model there are eight dialogical acts, summarized in the following list (Alrø & Skovsmose, 2004):

1. *Getting-in-contact* is about students listening to each other "in a relation of mutual respect, responsibility and confidence" (p. 101)
2. *Locating* is about finding out new things through inquiring questions while "examining possibilities and trying things out" (p. 102)
3. *Identifying* is about trying to "crystallise mathematical ideas, meaning being able to identify a mathematical principle or algorithm" (p. 104)
4. *Advocating* is about "stating what you think and at the same time being

willing to examine your understandings and pre-understandings" (p. 106) and "a trying out of suggestions for proving" (p. 106)

5. *Thinking aloud* is about "expressing one's thoughts, ideas and feelings during the process of inquiry" (p. 107)
6. *Reformulating* is about "repeating what has just been said" (p. 108)
7. *Challenging* is about attempting to "push things in a new direction or to question already gained knowledge or fixed perspectives" (p. 109)
8. *Evaluating* is about "correction of mistakes, negative critique, constructive critique, advice, unconditional support, praise or new examination" (p. 109)

In this PhD project, the IC-model is used in all four articles, and has guided what is important when it comes to promoting mathematical dialogue. It is chosen as it connects to questioning, listening and participation. When *getting-in-contact* everyone is invited to participate and when trying to get in touch with each other, inviting questions, like "have you understood the task?", or "are we ready to start?" are common. This also assumes that students listen as well as request listening from each other. When *locating*, *identifying* and *advocating*, mathematical questions of different kinds are used to understand the mathematics and the problem discussed, and in the dialogue, students have to listen to each other to create a common problem-solving process. When *reformulating*, although not always phrased as a question, the one doing the rewording at the same time poses an indirect question of whether he or she has understood what already has been said, which also always presumes that the one reformulating has listened to what the others have said before. *Challenging* is about questioning the mathematical thinking of others or yourself, which often is done by asking questions. In order to be able to ask challenging questions, listening to others' reasoning is required. Finally, *evaluating* the found solution or mathematical thinking is often done by first listening and thereafter questioning the reasoning.

Using the IC-model in research

The IC-model has been used in other studies on mathematical dialogue, for different reasons. For instance,

- Malasari, Herman and Jupri (2020) used it in relation to mathematical literacy proficiency.

- Weng and Jankvist (2017) used it to study mathematics teachers' conversations with mathematically gifted students.
- Likwamve (2018) used it to explore the nature of dialogue from a teachers' viewpoint in a calculus course.

What is common between these former studies, and this PhD study as well, is the focus on the eight dialogic acts. In all studies they were central in the analysis processes of the mathematical conversations, trying to identify which dialogic acts were present in the dialogues.

What however is different in this PhD study, is that the IC-acts are used as a way to view quality that affected what was designed. With quality here, is meant that students and teachers are using all the different IC-acts in their discussions, since then they will use listening and questioning in their dialogues, which the literature review in Chapter 2 shows is important in relation to students' learning processes and participation (Esmonde, 2009; Mercer, 1995). This PhD study is not only about understanding mathematical dialogue, but also about changing it, as the teachers tried deliberately to make all students within the groups to *get-in-contact*. They wanted students to deliberately use questions, in order to find the quality that the IC-acts bring forward, and they also asked questions of their own, for instance to *challenge* students' thinking, or make them deepen their *advocating* processes. Another difference is that sometimes the IC-acts are used to analyze the mathematical dialogue also when the teacher is not present.

The use of the IC-model can be connected to Sfard and Kierans' (2001) claim about learning the art of communication, and Fuentes' (2009) problematization about difficulties in mathematical dialogue when it comes to lack of communication between all students in a group, poor communication patterns, and norms that impede students' learning. By using the IC-acts as a way of defining quality in mathematical dialogue, it is possible to see what parts of the communication that are not functional, and then try out different techniques that could help. Also, it could support the analysis of how actively students are listening (Kosko, 2014). Otten et al. (2011) concluded that learning was more productive when students listened actively. Since all IC-acts are connected to listening, promoting them, could also promote the active listening amongst students. Using Davis (1996,1997) categorization of listening, student might be working with evaluative listening, while *locating* or *identifying* the mathematics within the task, *interpretive* listening, while *getting-in-*

contact, and *hermeneutic* listening while *advocating*. Hence, the quality in students' listening and questioning processes, could be connected to how they and the teachers work with IC-acts.

Regarding participation, the IC-model is more difficult to use, as it assumes active participation from the people entering the dialogues. However, by looking at who is not participating, and when there are no IC-acts in the analysis, when there are no questions asked, this could be interpreted as a sign of the quality of the mathematical dialogue being reduced.

The research questions in this study relates to listening, questioning and participation. Although the IC-model do say something about all these aspects of mathematical dialogue, the model does not always give enough details, hence, the local theories in the third level of Figure 3.1 are needed.

Local theories: finding theoretical models and concepts to understand aspects about questioning, listening and participation

Certain facets of mathematical dialogue are highlighted in this thesis, to better understand the complexity and to explain and organize the findings, when the IC-model was not enough to explain what was happening. In this section, two extra local theories are added: Fuentes' (2009) framework of analyzing student communication (used in Articles 1 and 2), and Ehrenfeld and Horn's (2020) framework of initiation-entry-focus-exit and participation (used in Article 3). However, no extra local theory was found on student listening. Instead, as will be shown in Article 2, in addition to the previous research literature presented in Chapter 2, a new local theory was needed to explain the listening in relation to the IC-model and promoting mathematical dialogue. This is further discussed in Chapter 5.

Fuentes' framework for analyzing student communication

Although the IC-model could be connected to questions in mathematical dialogue, it does not work as a tool for understanding details concerning what types of questions are asked and how questions are responded to.

Fuentes (2009) developed a framework on how to analyze student communication in which questions and responses were tied together, building on and extending the research of Dekker and Elshout-Mohr (2004). She used the framework so that teachers could identify problems in mathematical interaction and learn how to act to prevent them (Fuentes, 2013). In the same way as the IC-model, Fuentes' (2009) framework relates to both questioning and listening. Fuentes (2009) claimed that students needed to listen carefully to each other so that they could critique anything they considered incorrect in the mathematical reasoning, in order for them to reach further in their mathematics learning.

Fuentes' (2009) framework contains eight question/comment-response pairs (QCR pairs) to classify questions between students; see Table 3.1.

Table 3.1. Fuentes' (2009) framework for analyzing student communication

Question/Comment	**Response**
1. A asks B to show work	1. B shows own work
2. A asks B to explain work	2. B explains work
3. A criticizes B's work	3. B justifies own work
4. A rejects B's justification	4. B reconstructs own work
5. A asks B to evaluate work	5. B evaluates A's work
6. A suggest a strategy to the group	6. The group tries the strategy
7. A asks B a content question	7. B answers A's question
8. A asks B a clarification question	8. B answers A's question

In this PhD study, Fuentes' (2009) framework was used in Articles 1 and 2, both as inspiration in the design phases on what kind of questions students needed to work with, but also to analyze when students did not listen to each other.

As listening is hard to analyze (Kosko, 2014), the idea is that some conclusions can be drawn about students' listening through investigating how students answer each other's questions. Answering a comment/question, or reacting to it, assumes that students have listened to each other, at least to some degree. For instance, when a student suggests a strategy (QC6) and all students then jointly try the strategy out (R6), it can be assumed that the students have listened to the suggested strategy. The QCR pairs can also closely be connected to IC-acts and illuminate details about IC-acts. For instance, working with QCR2 and QCR6, relates to students' *advocating*, QCR5 concerns *evaluating*, and QCR3 and QCR4 refer to *challenging*. Hence, Fuentes' (2009) framework can be used to further understand details about IC-acts.

Ehrenfeld and Horn's framework on teacher moves

Article 3 focuses on teachers' actions when visiting students working with mathematical dialogue in small groups. The IC-model was used mainly as a basis for what kind of acts to work with and promote, but it did not give a structure for how to do that. Webb et al. (2009) claimed that more research is needed on how teachers can promote efficient student collaborations, such as how to intervene in student group discussions. As a way of understanding teachers' moves coming into and participating in students' mathematical dialogues, the framework by Ehrenfeld and Horn (2020) was used to organize the findings in Article 3. The framework contains five teacher moves (written in bold in this thesis), namely:

1. **Initiation** of conversations with students
2. **Entry** into student conversations
3. **Focus** of the interactions
4. **Exit** from student conversations
5. **Participation** patterns

Many of the teacher moves, include both listening and questioning. For instance, **initiating** or **entering** student conversations, were often done by listening to what the students said, and then posing a question. To steer the **focus** of the interactions, or affect who was **participating** in the conversations, teachers often listened to students' conversations to find out what they focused on and reflected on the participation patterns. This gave teachers an opportunity to affect focus and participation through the use of questions. The last move, **exiting**, is about what happens when teachers leave a conversation; for instance, if they give students an open question or a closed question or no question at all. **Exiting** student participation has not been emphasized in this thesis, not because it is not important, but simply because it was not what the teachers chose to work with.

The framework of Ehrenfeld and Horn (2020) might be critiqued for only focusing on teachers' moves and actions as such. However, using the framework for organizing data on classroom dialogue might help explain more about listening, questioning and participation. Lim et al.'s (2020) call for more research on how teachers listen to, understand, or engage with students in reasoning could be examined using the framework. Also, the framework makes it possible to connect teachers' moves to IC-acts and quality in mathematical dialogue. For instance, **initiating** or **entering** student conversations is often done by teachers first listening, and then asking questions of various kinds, for instance to *challenge*, *evaluate*, or help students with *locating/identifying*. Further, a common way of working with **focus** or **participation** is to engage students in IC-acts using questions, which could be connected to *getting-in-contact*, or all the other IC-acts depending on what questions are asked.

Using the theories on the three levels together

The theories in this PhD study have been used in a pragmatic way, to try to understand the complexity of mathematical dialogue and how to promote participation. This means using different parts of the theories to explain or organize certain data related to the research questions. According to Prediger et al. (2008), connecting theories might contribute to "increasing explanatory, descriptive, or prescriptive power" (p. 169).

However, simply mixing theories in a non-systematic way that is not reasoned for or justified might lead to questions about whether it is even possible to combine them. Therefore, in this section, a problematization about how the theories can be used together is made. This was previously done (Sjöblom, 2014) for the IC-model and Fuentes' (2009) framework, but in this thesis, all three levels in Figure 3.1 are included.

Coordinating theories in this PhD project

Prediger et al. (2008) gave examples of different strategies to connect theories in mathematics education research; namely: *ignoring other theories, understanding others, making understandable, contrasting, comparing, combining, coordinating, synthesizing, integrating locally,* and *integrating globally.* When having different theories on different levels, several of these strategies could be applied. Here, the focus is on *coordinating* them, as it can give "a networked understanding of an empirical phenomenon or a piece of data" (Prediger et al., 2008, p. 10). Another benefit of *coordinating* is that it makes it possible to view data from different viewpoints (Prediger et al., 2008), which is useful in this study since, even if the IC-model gave a lot of answers, the local theories gave new insights that complemented the data from new angles.

When working with *coordinating* in this study, the arguments of Wedege (2010) are used. Wedege combined Prediger et al.'s (2008) networking strategies with Radford's (2008) morphology of theories, in which theories can be seen as triplets $T=(P,M,Q)$, where P is a system of principles, M is a methodology supported by P, and Q is a set of research questions. When *coordinating* theories, Wedege (2010) found it important to analyze how the triplets of the different theories were interrelated, so that no contradictions were found. Wedege (2010) saw *coordinating* as:

> the term *coordinating* is used when a conceptual framework is built by well fitting elements from different theories: elements e.g. from the basic principles P, are chosen and put together in a more or less harmonious way to investigate a certain research problem. (p. 67)

So, what can then be said about the triplets *T*, for the theories in this PhD study?

Coordinating theories: System of principles

When analyzing the basic principles, *P*, of the theories, it is possible to find several common features. Radford (2008), saw *P* as "implicit views and explicit statement that delineate the frontier of what will be the universe of discourse and the adopted research perspective" (p. 319).

In this thesis, when situating the PhD study within the sociocultural perspective, the basic principles are, in a way, already defined, in that mathematical dialogue is an activity that happens between human beings with the intentions of teachers and students working together, and through cooperation and interaction creating possibilities for learning.

A common principle amongst the theories is that they all relate to quality in mathematical dialogue. Through the different IC-acts and definition of dialogue, the IC-model provides a way of viewing quality in conversations, while the analyses of Fuentes (2009) and Ehrenfeld and Horn (2020) looked more at the actions of the participants in the dialogue. All theories build on assumptions that these ways of viewing mathematical dialogue say something about the quality of the interaction between the participants in the dialogue.

A second common principle is that all theories assume that students and teachers are active participants in the mathematical dialogue. However, they do this from different viewpoints. The IC-model can be applied both to teacher–student and student–student dialogues, assuming that participants want to be part of a common process. In Fuentes' (2009) framework, when someone makes a question or a comment, a response is anticipated. In Ehrenfeld and Horn's (2020) framework, the different moves that teachers work with are made in relation to participation of teachers in students' mathematical dialogues, with the intention of also making students active participants.

A third common principle is that questions are important in all theories, albeit in different ways. All IC-acts involve questions, and so do the QCR pairs in Fuentes' (2009) framework. In mathematical dialogue, questions are a prerequisite in the inquiry process and can also be seen as a tool for teachers to maintain equality through involving all students in mathematical dialogue. Also, Ehrenfeld and Horn (2020) included questions in the teacher moves.

Coordinating theories: Methodology

All theories used in this thesis can be applied to a variety of methodologies *M*. Instead of looking broadly at all possibilities, this section considers the ability to use them in educational design research, EDR (McKenney & Reeves, 2012), in combination with teacher noticing (Mason, 2002; Sherin et al., 2011). Further discussions on noticing and EDR are found in Chapter 4.

Since all theories assume the common principles *P* about quality in mathematical dialogue, the active participation of teachers and students, and the focus on questions, they also share similarities regarding what methods to use in the analysis processes. Choosing EDR and using teacher noticing provides an opportunity to follow both what is happening within mathematical dialogues in the classroom setting and what is happening within teachers' professional development setting, and refinements of actions and awareness through a cyclic process. Here, all theories can be used, both as a grounding, with different inputs, for what to design in the cycles, but also for the analysis processes. Mathematical dialogue, teacher noticing, or EDR cycles can all be supported or explored by IC-acts, QCR pairs, or teacher moves in Ehrenfeld and Horn's (2020) framework.

Another similarity when it comes to methodology is that the analysis using all theories can be steered towards certain aspects of the mathematical dialogue that include the focus areas in the research questions; namely, questioning, listening, and participation.

Coordinating theories: Set of research questions

When identifying what research questions, *Q*, can be asked in relation to the theories in this thesis, as well as EDR methodology, there are similarities. Although the theories focus on different aspects of mathematical dialogue, they can all be used to deepen the understanding about teachers' and students' engagement in these dialogues. All theories can be related to both students and teachers, and also to follow the development processes of actions to promote opportunities for participation in mathematical dialogue.

Summary: Possibilities for coordinating the theories

When comparing the triplets $T = (P,M,Q)$ for the theories presented in this chapter, there are a lot of similarities regarding P, M, and Q. There are no obvious contradictions. Rather, the theories seem to complement each other and can be used both in the design process of the research study and in the analysis of data to highlight different theoretical aspects to understand the complexity of promoting participation in mathematical dialogue.

4. METHODOLOGY

From a methodological viewpoint, mathematical dialogue can be studied in a variety of ways. An important component in answering the research questions in this thesis is the movement back and forth between the classroom setting and the professional development setting. Therefore, it was necessary to choose a methodology that supported this and that could be done in cooperation with teachers in order to closely follow their development processes. The choice fell on educational design research, EDR (McKenney & Reeves, 2012).

This chapter starts with a short description of EDR, followed by a justification of why this methodology is applicable in this research study. Thereafter, the concept of noticing is further introduced as a way of organizing the work in the EDR cycles. With EDR and noticing as the methodological cornerstones, I describe the study, divided in two sub-studies, with regard to data collection, data analysis, and the participants – students, teachers, and researcher/school developer (me). The chapter ends with a discussion about the ethical considerations and quality of the study.

Educational design research

EDR is a way of conducting research that is designed to "explore, rather than mute, the complex realities of teaching and learning contexts, and respond accordingly" (McKenney & Reeves, 2012, p. 15). In EDR, both the process of learning and how learning can be supported in the classroom are studied (Barab & Squire, 2004: Cobb & Gravemeijer, 2008). McKenney and Reeves (2012) defined EDR as

> A genre of research in which the iterative development of solutions to practical and complex educational problem also provides the context for empirical investigation, which yields theoretical understanding that can inform the work of others. (p. 7)

The cyclic nature of EDR

EDR is an iterative process, with three phases in each cycle: analysis/exploration, design/construction and evaluation/reflection (McKenney & Reeves, 2012); see Figure 4.1.

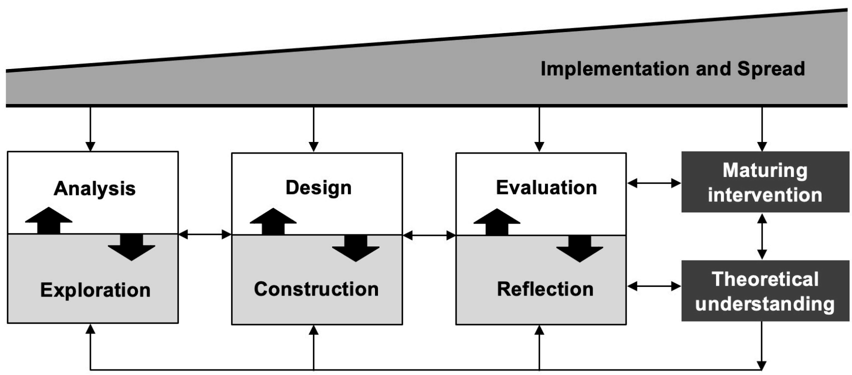

Figure. 4.1. Educational design research (McKenney & Reeves, 2012, p. 77)

In the analysis/exploration phase, the problems or focus areas are identified and diagnosed; in the design/construction phase, solutions to the problems are designed and carried out; and in the evaluation/reflection phase, the design choices in the intervention are evaluated and reflections are made.

In EDR, theories are used both to understand what is happening within the cycles, but also to theoretically justify design choices (McKenney & Reeves, 2012). The results of an EDR study are both practical, in that they should produce a maturing intervention that can be used in mathematics classrooms, and theoretical, in that they contribute to theoretical understandings within the field of mathematics education.

EDR in mathematics education research

EDR methodology can be used within a range of subjects and is not explicit to mathematics education. However, there is research literature specifically on EDR within mathematics classrooms. For instance, Gravemeijer and van

Eerde (2009) wrote about how EDR could contribute to building a knowledge base for teachers and teaching in mathematics education. They concluded that, in EDR, teachers often need to gain new knowledge for students to reach their learning goals, and hence they viewed it as dual design research, with two experiments going on simultaneously – one with students and one with teachers. Gravemeijer and van Eerde (2009) considered it important that teachers are placed on equal footing with the researchers, claiming

> Ideally, teachers and researchers experience the project as a collective effort in which they together analyze video footage, student work, and other data to decide on the next steps. Note that the key principles here are – just as with the students – ownership and understanding. (p. 523)

Jaworski (2004) discussed educational design research in relation to cooperation and how teachers (as insiders) can work together with researchers (as outsiders) in research projects, both to improve knowledge in the mathematics education field and to improve practice. Jaworski problematized the role of the teachers in design research, for instance questioning whether teachers are seen as partners in design or mere implementers. She suggested the concept of co-learning (building on Wagner, 1997) as a way of collaboration between teachers and researchers, in which everyone is involved, both in action and reflection. According to Wagner (1997), "by working together, each might learn something about the world of the other" (p. 16).

Reasons for choosing EDR in this study

To be able to connect research with development of practices, Van den Akker, Gravemeijer, McKeenye, and Nieveen (2006) claimed that research needs to be interventionist, iterative, process-oriented, utility-oriented, and theory-oriented. EDR opens up possibilities to meet all these properties. In this thesis, there are three main reasons for choosing EDR, which now is discussed.

The first reason, which was mentioned in the introduction of this chapter, is the possibility and need to involve teachers in the research processes. When conducting research in classrooms, the findings are inevitably dependent on the relations between specific teachers and specific students. The teachers are specialists within their own classrooms, as they are the ones who know their students and what they concretely need in their learning. Hence consciously including them in all three phases of the EDR process opens doors to knowledge that would otherwise have been closed.

The second reason has to do with the cyclic nature of EDR. In order to answer research questions on teachers refining their actions to promote opportunities for students' participation, teachers need to consciously design and try out different strategies over time, and the results need to be analyzed depending on what is happening within each cycle. The cyclic process in EDR provides a structure that enables this.

The third reason is connected to how theories are used in EDR. Theories on different levels (see Figure 3.1) affect all three phases in the EDR process. The sociocultural perspective connects to why mathematical dialogue and interaction between teachers and students are important. The IC-model and concept of mathematical dialogue (Alrø & Skovsmose, 2004), which is at the center of this study, help identify what to focus on in the cycles. The local theories (Ehrenfeld & Horn, 2020; Fuentes, 2009) provide details about the interaction. Both the middle range and local level theories are used to understand what is happening in the analysis/exploration phase, to make informed choices in the design/construction phase, and to assess what happened within the evaluation/reflection phase. Therefore, choosing EDR as a methodology in this thesis provides a scientific basis for the project, in which theories are used in all steps of the research process. It also provides expectations to find both practical and theoretical understandings, which suits a cooperation project between teachers and researchers. The primary goal of the teachers could be to reach new knowledge together and improve their teaching to better understand and support their students. The researcher's primary goal might be to reach new knowledge that contributes to the field of mathematics education, both in a theoretical and in a practical way. By choosing EDR, the needs of teachers and researchers can be met and inspired by each other.

Teacher noticing

One way of working and organizing data in the EDR cycles is to use teacher noticing (Mason, 2002; Sherin et al., 2011), which is a recently developed way of working together with teachers and is used to accomplish a "movement or shift of attention" (Mason 2011, p. 45). It can be defined in a variety of ways. In this study, the definition by Jacobs et al. (2011) and Kazemi et al. (2011) is used, where noticing consists of three parts: *attending* to what is happening, *interpreting* it, and deciding how to *act/respond*. All three steps can be done both directly in the classroom, but also in a common process together in a professional development group, meaning that teachers attend in the classroom to what is happening, interpret it spontaneously when it happens, and act/respond in the moment. In professional development meetings, the three parts of noticing are deepened so that teachers together decide (in relation to the needs of their students) what to specifically attend to, then interpret the classroom interactions together with support of video recordings, and then plan for how to act/respond by designing lesson activities.

Two sub-studies

This thesis builds on the results of two sub-studies containing two separate EDR processes.

Data generation and data collection

In the first sub-study, data generation and collection were performed during the fall semester 2013, with one teacher, in an upper secondary school in Sweden. Three EDR cycles were conducted. The focus was more on student-to-student interaction and less on the teacher. This resulted in a licentiate thesis (Sjöblom, 2015) in January 2015, and thereafter in Articles 1 and 2. Table 4.1 summarizes the data in the first sub-study.

Table 4.1 Data collection in Sub-study 1

Cycle	Data collected	Time
1, 2, 3	25 lesson observations	32 hours
1	Audio recordings, 12 students in 3 groups, Task 1	111 minutes
1	Audio recordings, 12 students in 3 groups, Task 2	120 minutes
1	Interviews, 9 students in 4 groups	59 minutes
2	Audio recordings, 11 students in 3 groups	144 minutes
2	Interviews, 9 students in 4 groups	39 minutes
3	Audio/video recordings, 7 students in 2 groups	155 minutes
3	Interviews, 5 students in 2 groups	28 minutes
3	Final interviews, 6 students in 2 groups	95 minutes

In the first sub-study, data were generated and collected mainly during mathematics lessons, and the recordings focused on what happened within the student groups. The parts of the first sub-study that are included in the PhD study relate mainly to the first research question, about what aspects of students' questioning and listening teachers need to pay attention to when promoting students' participation in mathematical dialogue.

The results in Articles 1 and 2 were then used as input in the second sub-study; for instance, the importance of listening and mathematical questions found in the first sub-study affected the initial directions of the second sub-study. This was done even if the articles were not published when the second sub-study started.

In the second sub-study data generation and collection were performed during one school year, from August 2018 to June 2019 with four teachers in an upper secondary school in Sweden. Again, three EDR cycles were conducted. The focus was on teachers' work with promoting student interaction and participation in mathematical dialogue in small groups. The EDR cycles resulted in Articles 3 and 4. Table 4.2 summarizes the data in the second sub-study.

Table 4.2 Data collection in Sub-study 2

Cycle	Data collected	Time
1	In classrooms: Video recordings of 11 group discussions	824 minutes
1	In teacher group: Audio recordings of 16 meetings	706 minutes
2	In classrooms: Video recordings of 8 group discussions	472 minutes
2	In teacher group: Audio recordings of 6 meetings	280 minutes
3	In classrooms: Video recordings of 17 group discussions	1099 minutes
3	In teacher group: Audio recordings of 8 meetings	385 minutes

The video recordings were made in two classrooms with two of the teachers in the group. In the first cycle, because of a parental leave, one of the other teachers taught the third-year class, and hence participated in the video recordings. There were several reasons why not all four classrooms were video-recorded. The group of teachers considered it sufficient to analyze video materials from two classrooms. From a research point of view, this was also true for the EDR study, as it resulted in more hours of material than could be analyzed anyway. Also from a student viewpoint and an ethical viewpoint, it seemed unnecessary to ask more students to participate and then not use the material.

However, all four teachers worked with the designed lessons in their own classrooms and tried out the decided strategies, and hence were well prepared both for discussions as well as the analysis process.

As can be seen in Tables 4.1 and 4.2, a comprehensive amount of data has been generated and collected in the two sub-studies. This is because it is hard to decide before-hand which tracks could be the most important to follow up on, and because rich data material makes it possible for teachers and researchers together to follow up on the questions in focus. Also, in the second sub-study, recording both classroom interactions and teacher meetings made it possible to connect what happened with mathematical dialogue in small group work, to teachers' discussions both before and after the designed lesson activities.

In both sub-studies, not all parts of the extensive data collection were analyzed. Instead, the analysis was directed towards the parts that the teachers and I wanted to focus on. This means that there are many minutes of recordings that have not yet been used for the analysis and instead are saved for later, either to go deeper into the analysis or to analyze from new viewpoints in articles that comes after the work with this PhD thesis.

About the students

The students in both sub-studies, were 16–19 years old and attending university preparatory programs. The first sub-study included students in one first-year class, and the second sub-study included students in two classes in their first and third years of upper secondary school. Choosing students from these exact years was not a conscious plan in the research process; instead, it was explained by which classes the participating teachers taught. In both sub-studies, students were used to mathematics teaching that consisted of the teacher explaining a mathematical topic, concept, or technique, and then conducting individual work with tasks in their mathematics books.

In the cycles, the teachers and I designed lesson activities to make students work together in small groups consisting of three or four students who were seated around tables so that they could cooperate with each other. Sometimes they were only given one sheet of paper with the task, which meant they had to sit closely together. Every group was either audio-recorded (Cycles 1 and 2 in Sub-study 1) or video-recorded (Cycle 3 in Sub-study 1 and all cycles in Sub-study 2), with one camera in each group and one audio recorder on the middle of the table for better sound recording.

The intentions in both sub-studies were to use heterogenous student groups. With a starting point in which students volunteered for the project, the teachers decided what groups to use. The groups were changed during both sub-studies, as part of the teachers' work in the classrooms. There were some groups that did not function well, as students did not cooperate. Rather than keeping groups together for the sake of the research study, for ethical reasons the teachers changed students within the participant groups in the same way they changed the non-participant groups.

Students' backgrounds differed. In the first sub-study there were many students who had Swedish as their second language, with at least nine different languages spoken in the classroom (Sjöblom, 2015). Given that language does not always need to be the first priority when it comes to promoting student interaction (see Article 1), what languages the students spoke were not in focus in the second sub-study, in which most students had Swedish as their first language. No comparisons or analyses depending on language have been conducted in this thesis.

About the teachers

The teachers in both sub-studies volunteered to participate in the projects and were responsible for conducting the teaching in the classroom, designing tasks, and planning the lesson activities together with the researcher (me), and taking part in the analysis work. In the first sub-study, there was one teacher. However, as the focus was on students in the first sub-study, the work with this teacher is not thoroughly described in the articles, only the outcomes that affected the student-to-student interaction when it comes to questioning and listening. For further details about the cooperation with this teacher, see Sjöblom (2015).

In the second sub-study, there was a group of four teachers, two males and two females, of different ages and different experience backgrounds when it comes to mathematics teaching. They worked together in a professional learning community (Stoll & Seashore, 2007) at the same school, and met weekly for one-hour professional development time. One of the teachers, a lead teacher, led the group on those occasions when they met without me.

About me as a researcher and school developer

In both sub-studies, my role as researcher was to cooperate with the teachers, support them, and discuss with them what kind of tasks and lesson activities could be designed to make students participate in mathematical dialogue. The IC-model (Alrø & Skovsmose, 2004) was introduced as a way of thinking about quality in mathematical dialogue, although I experienced that the teachers were, to a large degree, more interested in the actual practical work than the theories behind. I also planned how the data could be analyzed and decided together with the teachers about the analysis processes.

In the second sub-study, apart from working as a researcher, I also had the role of leading the teachers' professional development work. That meant arranging and leading meetings with the teachers for approximately one hour a week, a total of 30 meetings. Having two roles simultaneously was somewhat problematic. The role of a researcher differs from that of a process leader, in that the former often wants to hear the thoughts from the teacher group without affecting their processes, while the latter is responsible for challenging the thinking of the group and making them move forward in their processes. One solution was to use summaries of what teachers had said, and ask them a lot of questions, to make them as active participants as possible, and not steer their work too much.

Analysis processes

The analysis processes were planned differently in the two sub-studies and in relation to the three research questions. This section describes the overall analysis processes. In addition, for each article, certain parts of the data collection were put in focus, which is described further in the articles.

Analysis process related to the first research question

The first research question was mainly answered by analyzing data from the first sub-study. The analysis was conducted in two steps. The first was made together with the teacher during the three EDR cycles, as we continuously discussed what happened in the classroom and how we could work with promoting student interaction and meet the students' needs in the coming cycles. Here, information from the interviews was also included, listening to students' voices about mathematical dialogue.

In the second step, which was the main part of the analysis, I as a researcher transcribed all recordings and coded the material using the IC-acts (Alrø & Skovsmose, 2004) and QCR pairs (Fuentes, 2009). Thereafter, the parts that focused on mathematical questioning and listening connected to mathematical problem-solving in small groups from a student viewpoint were highlighted and the results were discussed and problematized in Articles 1 and 2. For more detailed information about the data collection and analysis in the first sub-study, see Sjöblom (2015, 2018) and Sjöblom and Meaney (2021).

Articles 1 and 2 were written a long time after the cooperation with the teacher was finished and the results were never discussed with him. One reason for this was that the teacher ended his employment at the school soon after the EDR process was finished.

Analysis process related to the second research question

The second research question was mainly answered by analyzing data from the second sub-study. The analysis was again conducted in two steps, with a focus on teachers promoting participation in mathematical dialogue. The first step included the teachers: we watched the video recordings together and used the results when planning the next cycle. This was done by working with teacher noticing, both in the classroom setting and in the professional development group setting. In the analysis, teachers discussed certain episodes from students' group work, using the recordings to understand the interactions. This made it possible to connect the teachers' discussions to what happened within the classroom and made teachers active participants in the analysis process.

Not all parts of the data material were analyzed. Rather, in line with Jordan and Henderson (1995), the data material was skimmed through to identify situations that needed more attention, so-called hot spots, which were "sites of activity for which videotaping promises to be productive" (p. 43), and these were then analyzed in greater depth. The teachers and I decided together what to look for, in connection to the themes in the research questions about listening, questioning, and participation.

The way in which the analysis was conducted and which episodes teachers watched were planned differently in the three EDR cycles. In the first cycle, I showed episodes connected to how students listened, how they asked questions, and what happened when teachers visited student groups. This was done because listening was in focus in the first cycle, and also because interesting situations occurred when students asked each other questions, for instance more students were included in the dialogues, and that I found it interesting and somewhat surprising that there was almost no interaction between teachers and student groups. In the second cycle, each teacher watched one video recording of a student group (approximately 30 minutes long) with a focus on questions, and the teachers summarized for each other what they had found interesting. In the third cycle, I selected clips that showed how teachers acted when visiting student groups. Special interest was paid to what questions teachers asked and how this affected the mathematical dialogue. I also selected clips in which teachers did or did not talk to all students in the groups in order to understand more about the difference in mathematical dialogue depending on how many students were included and participated in the dialogue.

The second step of the analysis was again conducted by me after the data collection was finished, by using the analysis in the first step, and adding extra local theories (see Figure 3.1) to deepen the understanding of special facets of the teachers' work with promoting student participation in mathematical dialogue. In Article 3, Ehrenfeld and Horn's (2020) framework was applied to structure the work, while in Article 4 the three characteristics of mathematical dialogue (Alrø & Skovsmose, 2004), as well as the three steps in teacher noticing (Jacobs et al., 2011; Kazemi et al., 2011), were used. For more details about this, see Articles 3 and 4.

There is an agreement that the teacher group will receive the articles (Articles 3 and 4) when they are published, so that the discussion within the teacher group can continue.

Analysis process related to the third research question

The third research question was analyzed, looking back at both two sub-studies. This was conducted both by working with the data material, but also by reflecting on the learnings from the two sub-studies and the results in the four articles.

Because the focus in the second sub-study was on developing teachers' awareness and refining their teaching in relation to students' listening, questioning and participation in mathematical dialogue, the main material used in the analysis process was the 30 meetings within the professional development group. All of these meetings were transcribed and coded with 63 different codes, so that interesting parts of the data material easily could be identified and used in the analysis processes. For instance, the codes that were mainly used in writing Articles 3 and 4 were "questions" (169 occurrences, divided into 12 subcodes), "in-out"[4] (61 occurrences), "active" (55 occurrences), "listening" (50 occurrences), "equity" (35 occurrences), and "talk to everyone" (27 occurrences).

[4] Here "in-out" was code for when teachers visited student groups and went in and out of their mathematical dialogue.

With a starting point in the coding, the articles were written and a reflection process started regarding what affected how teachers can develop an awareness and refine their teaching when cooperating with researchers. Here, three areas were highlighted as important: the cyclic nature of EDR, conducting the study both in the classroom setting and in the professional development setting, and deliberately listening to students' and teachers' voices in EDR and making them part of the research process. These three areas will be further discussed as results and answers to the third research question in Chapter 5.

Ethics

Both sub-studies have been planned in accordance with the guidelines by the Swedish Research Council about good research practice (2017),[5] and ethical choices has been made throughout the projects.

Ethical considerations

The participating teachers and students were asked to sign consent forms if they wanted to participate in the research studies, see appendix A and B for the second sub-study, and Sjöblom (2015) for the first sub-study. The students who did not want to take part in the studies still worked with the designed mathematics lesson activities, as this was part of their teachers' teaching. However, they were not recorded when working in groups, and when video recordings were made the non-participating students were placed behind the cameras so that they could not be seen. Sometimes – such as in whole-class discussions – their voices were accidently audio-recorded, but none of this was included in the transcripts.

Students and teachers were informed that they could withdraw their consent forms at any time, but none have done so yet.

In the second sub-study, an ethics review request was made at the ethical review board in Lund. The result was that no ethics review was needed, since the research study did not include the processing of sensitive personal data. No ethics review was needed in the first sub-study either.

All student names and teacher names (when used) have been changed to new names in order to anonymize the data.

[5] For the first sub-study there was an older version of the same document, with similar content when it comes to the ethical considerations discussed here.

Ethical difficulties

The main ethical dilemma of the second sub-study[6] was related to the cooperation with the teachers, and who has power to decide what to do in researcher–teacher collaborations. This resulted in at least three difficulties.

A first difficulty referred to the choice of what to focus on. The PhD project was suggested from the start to include an overall focus on listening and questioning in mathematical dialogue in the first two cycles when the teacher group agreed to participate. It was decided from the start that the teachers would decide on the content of the third cycle after conducting the first two cycles. Although the overall focus was listening and questioning, the designed lesson activities were made in relation to the teachers' students, and only inspired by the first sub-study. I tried to be responsive to what the teacher group wanted to do with the previous knowledge and only use suggestions that the teacher group thought could be applied to their students, bearing in mind that all classrooms are different.

A second difficulty referred to the dilemma mentioned in the section about my role as researcher, and being both the process leader and researcher in the project. This was solved by talking openly about the power relations in the project, reminding the participating teachers that this was a joint project and that it was important for me what they thought and what they wanted to try in their classrooms. We also discussed that we might have different areas of interest, with the teachers being more focused on the learning processes of their students and the classroom practice, and while I was also interested in those areas, I had an interest in applying different theoretical frameworks and concepts to the research process.

A third difficulty was related to how the IC-model (Alrø & Skovsmose, 2004) was used within the project. Here, as a researcher, I wanted to use it as a theoretical basis for design choices in the EDR process, while the teachers did not actively use a language including the dialogic acts much in their professional development meetings. However, they did talk a lot about the different acts, without referring to the exact words. In one meeting, the focus of the discussion was the IC-acts and, from a starting point in literature (Sjöblom, 2015), we discussed them together and gave examples of how they could be part of mathematical dialogues. I subsequently returned to these acts, while

[6] This discussion relates only to the second sub-study, as it is mainly the articles from the first sub-study (Articles 1 and 2) that are included in the PhD thesis. For further discussions about ethical considerations in the first sub-study, see Sjöblom (2015).

the teachers still did not include them often in their conversations. Also in the analysis process, it was mainly I who referred back to the IC-acts. A similar challenge was discussed by Hamza et al. (2018), who conducted a researcher–teacher collaboration project in science education. Referring to Richardson's (1992) dilemma about agenda-setting in teacher professional development, Hamza et al. (2018) also wanted to use a theoretical framework that teachers initially found unnecessary or did not use. Hamza et al. (2018) chose to take over power and helped teachers implement the theoretical framework, and they succeeded in making the teacher group use the framework themselves. Although it would have been interesting in my project to include the teachers more on a theoretical level, I chose not to use a similar solution in our project, since I did not want to disturb the pronounced collaboration about this being a joint project. Instead, the analysis process was divided into two steps, where the different theoretical frameworks were applied by me alone in the latter step. The same is true for the notion of teacher noticing. Although we worked with the three steps – attending, interpreting and acting/responding (Jacobs et al., 2011; Kazemi et al., 2011) – they were not named like this in the teacher group. Instead, we just talked about the EDR cycles as such. The reason why teacher noticing was not a spoken concept was that I did not know about this research field until after the project with the teachers was finished, so it is mainly used as a way of organizing the findings of the EDR cycles in Articles 3 and 4.

Discussion on the quality of the PhD study

What constitutes quality in research is an important question. Tracy (2010) specified what quality can be in qualitative research, using eight criteria: *(1) worthy topic, (2) rich rigor, (3) sincerity, (4) credibility, (5) resonance, (6) significant contribution, (7) ethics,* and *(8) meaningful coherence*. In this section, the quality of this thesis is discussed in relation to these criteria.

The first criterion is *worthy topic*, which means that the research should be "relevant, timely, significant, interesting, or evocative" (Tracy, 2010, p. 840). This PhD study can be considered relevant from both teachers' and researchers' viewpoints. The teachers chose to participate since promoting student interaction and mathematical dialogue were something they wanted to develop as teachers. Also, the Swedish school inspectorate (2010) has identified the importance of teachers making students actively take part in mathematical discussions in order to develop mathematical abilities. From a research viewpoint, research on mathematical dialogue in upper secondary school – such as when it comes to listening, and promoting interaction – is scarce, so this thesis makes a timely and relevant contribution within this area.

The second criterion is about *rich rigor*, which means that the research "is marked by rich complexity of abundance" (Tracy, 2010, p. 240). The present PhD project builds on a rich and comprehensive data material from both the classroom and teachers' professional development meetings. However, according to Tracy (2010), it is not the amount of data that is important, but rather its use. In the analysis, not all data have been included because of the limited time within the PhD project. Instead, the analysis process and selection of relevant data have focused on specific questions, related to listening, questioning, and participation, in order to dig deeper in these specific areas. More data could be analyzed in the future in relation to what has already been done, to further increase the quality of the analysis.

The third criterion is *sincerity*, which can be achieved through "self-reflexivity, vulnerability, honesty, transparency, and data auditing" (Tracy, 2010, p. 841). In the present thesis, the processes both in the classroom and in the group of teachers have been described in a transparent way, although not entirely, since only certain parts of the data material have been analyzed deeply. The role of the teacher and my role as researcher have been discussed transparently, and the pros and cons of the theoretical choices of frameworks have been problematized. Also, the difficulties from an ethical viewpoint connected to power relations in teacher-researcher collaborations have been discussed openly.

The fourth criterion concerns *credibility*, which refers to the "trustworthiness, verisimilitude and plausibility of the research findings" (Tracy, 2010, p. 842). In this thesis, trying to understand mathematical dialogue, both in the classroom setting and in the professional development setting, the movement back and forth between the settings, makes it possible to follow the teachers' processes of developing awareness from two different viewpoints. By following the teachers' noticing processes in both settings, the research setup adds to the credibility of the results.

The fifth criterion is *resonance*, which refers to "research's ability to meaningfully reverberate and affect an audience" (Tracy, 2010, p. 244). This thesis intends to be important both to researchers as well as to teachers, although the format of a thesis, and scientific articles in English, might not be the optimal form for all mathematics teachers in Sweden to read (although many mathematics teachers do read and enjoy research when they have time). However, the results could be presented in more available formats later, and then have more impact in mathematics classrooms. Another source that could lead to resonance is the fact that although this thesis is connected to mathematics education, it could be used for research and professional development projects in other school subjects. There are parts that are specific for mathematics education, but for instance with other choices of theoretical frameworks, the research setup could be applied and transferred to other subjects, research areas, and contexts.

The sixth criterion concerns *significant contribution*, which relates to "the current climate of knowledge, practice and politics" (Tracy, 2010, p. 845). The articles included in this thesis contribute to unanswered questions within the mathematics education field and also problematize and learn about challenges in practice and in teachers' mathematics classrooms.

The seventh criterion is *ethical considerations*, such as "procedural, situational, relational, and existing ethics" (Tracy, 2010, p. 847). This thesis follows the guidelines of the Swedish research council (2017), and when writing the articles, and as already described under sincerity, it was important to be transparent in the research process. Also, ethical decisions have been made throughout the two sub-studies, for instance considering what groups are best for students when working together, prioritizing relations amongst students higher than the research study setup.

The eighth criterion concerns *meaningful coherence*, meaning that research studies should "(a) achieve their stated purpose; (b) accomplish what they espouse to be about; (c) use methods and representation practices that partner well with espoused theories and paradigms; and (d) attentively interconnect literature review with research foci, methods, and findings" (Tracy, 2010, p. 848). This thesis includes chapters about purpose, results, methods, and literature review. However, as with all research texts, it is up to the readers to decide how meaningful the coherence is.

5. RESULTS

In this chapter, results from the two sub-studies are presented in relation to the research questions:

1. What aspects of students' questioning and listening do teachers need to pay attention to when promoting students' participation in mathematical dialogue?
2. What aspects of teachers' questioning and listening are important when teachers promote students' participation in mathematical dialogue?
3. How can teachers, in cooperation with researchers, develop an awareness and refine their teaching in relation to students' listening, questioning, and participation in mathematical dialogue?

A PhD can often be seen as a journey. My PhD journey is illustrated in Figure 5.1.

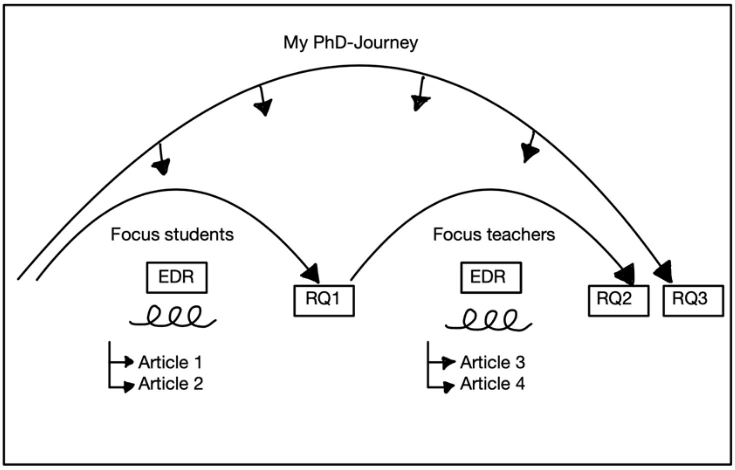

Figure 5.1 PhD journey

My PhD study was divided into two sub-studies, one with a greater focus on the students and Research Question 1, which resulted in Articles 1 and 2; and one that focused more on the teachers and Research Question 2, which resulted in Articles 3 and 4. Looking back at both sub-studies in the journey, reflecting on EDR as the methodology, moving back and forth between the classroom setting and the professional development setting and listening to students' and teachers' voices in research made it possible to find answers to Research Question 3.

The results chapter follows the structure of this journey.

RQ1: What aspects of students' questioning and listening do teachers need to pay attention to when promoting students' participation in mathematical dialogue?

For teachers to understand what they need to do to promote students' participation in mathematical dialogue, it is essential that they can identify what parts of students' listening and questioning they need to understand more about. Without this knowledge, it is hard for them to change their actions and teaching to meet the needs of their students.

In the first sub-study, it was not clear from the start that it was important to focus on listening when it comes to mathematical dialogue. Rather, this was something that was found during the EDR cycles. The same was true for questioning, and how students' questions had the opportunity to change the mathematical dialogue. Another result from the first sub-study, is how closely tied together listening and questioning are, and by studying both aspects at the same time, it is possible for teachers to understand more about the complexity of promoting participation.

In Articles 1 and 2, the IC-model (Alrø & Skovsmose, 2004), and Fuentes' (2009) framework for analyzing student communication were the main theoretical frameworks. These theories made it possible to analyze listening and questioning from different viewpoints, but also produced unexpected results. For instance, in Article 1, it was found that students' linguistic backgrounds do not always need to be taken into consideration as a first priority when it comes to promoting student participation in mathematical dialogue, and that other factors might be more important. In line with Fuentes (2009), this study found a lack of communication between all students in a group, poor communication patterns, and norms that impede students' learning, to be more important than language. These factors affected both questioning and listening and hence participation. In Article 2, a new framework on productive listening was developed, as the already existing frameworks on, for instance, active-passive listening (Kosko, 2014; Otten et al., 2011), or the division in evaluative, interpretive or hermeneutic listening (Davis 1996, 1997), were not sufficient to explain the role of listening in mathematical dialogue.

Below is a short summary of the results presented in the articles in relation to the first research question.

Summary of Article 1: Developing mathematical reasoning by using questions in a multilingual mathematics classroom

Article 1 focuses on students' use of questions in small group work. At first, with support in previous research, it was assumed that students' linguistic backgrounds might play a role in what happened in the mathematical dialogues (Dominguez, 2011; van Eerde, Hajer & Prenger, 2008). However, it was clear after the first cycle that even though the environment in the first sub-study was multilingual, what needed to be in focus was not students' different languages, but rather how to make students interact and ask each other questions, regardless of what languages were used. This does not mean that language is not important, only that other issues in classroom interaction might be even more important when the aim is to explore how questioning and listening can promote participation in mathematical dialogue.

In the first sub-study, students were given different support means to promote their interactions (Sjöblom, 2015), namely problem-solving strategies (building on Polya, 2014), questions lists (to make students write down and observe their own questions), and communicative roles (such as chairperson, questioner, summarizer, and accountant). The support means were rarely used in the first cycle, but in later cycles they were made more concrete and students became more acquainted to them; then, the transcripts showed how students used suggested formulations about questions from the support means that aided the mathematical dialogue.

Across the three EDR cycles, students changed from not seeing the point of asking questions in Cycle 1, where many of the students concentrated on finding the right solution and enforcing their own strategy rather than listening to each other, to being more aware of the importance of working together. In the third cycle, students worked more with joint *advocating* and more often questions/comments were followed by responses, when analyzing the QCR pairs (Fuentes, 2009). In interviews, students claimed that they had started asking more questions, and that they were more interested in how others reasoned, wanting to understand each other's thinking.

An unexpected conclusion in Article 1 is that although student interaction is studied in a multilingual setting, it is not automatically what languages students speak that needs to be in focus when the aim is to explore how questioning and listening can promote participation in mathematical dialogue. Rather, the results suggest that what students need when starting to ask questions was related to the norms within the groups about asking questions. For instance, getting support from their teacher in paying attention to the role of questions was important. This was done through support means and emphasized in whole-class discussions, when groups were asked about what questions they used during group work.

A conclusion related to the first research question is that teachers need to pay attention to questioning with a starting point in their own students and their own context. Previous research can be useful and provide valuable input about what might be important, and also be used as a help to analyze mathematical dialogue, but it is important to remember that all classrooms are different and for teachers to start with the needs of their own students and the norms in their own classrooms.

Summary of Article 2: "I am part of the group, the others listen to me": Theorising productive listening in mathematical group work

Article 2 focuses on theories about students' listening in mathematical dialogue, and what is missing in the existing theoretical frameworks when it comes to understanding listening as part of small group problem solving work. For teachers to promote students' participation in mathematical dialogue, listening proved to be a key factor. Previous research has generally focused on different definitions of listening (Davis, 1996, 1997; Kosko, 2014; Otten et al., 2011), but also the purpose of listening (Mueller et al., 2014; Webel, 2010), or social aspects connected to listening (Cobb et al., 2001; Boaler, 2011; Lack, 2010). However, there is a lack of research when it comes to combining these factors to understand the complexity of listening. Since listening in many forms is hard to analyze, given that it is an unobservable action (Kosko, 2014), Article 2 focuses on developing a new framework on understanding listening, which includes observable features connected to students' ability to *show willingness to listen* and *request listening* from others.

The framework was named the *productive listening framework*, and was developed while evaluating how Alrø and Skovsmose's (2004) IC-model and Fuentes' (2009) framework for analyzing student communication could be used to understand students' listening. In these two frameworks, listening and questioning are tied closely together, and were therefore considered a good basis for analyzing listening. However, during the cyclic EDR process and analysis in the first sub-study, it became apparent that there was more to listening than these two frameworks could reveal. Connected to the literature review on listening, and the empirical material of the first sub-study, in order for teachers to promote listening they need to consider students' purposes for listening, which could be about understanding the problem better, identifying key information to determine a solution strategy or identifying what works or what needs improving with the solution strategy. Teachers also need understandings about social aspects to do with listening, such as group members respecting other's contributions, and also knowing that their own contributions will be respected.

By combining purposes for listening and social aspects of listening, a new definition – that of productive listening – was developed, and this is the main contribution of Article 2. In the definition, students both show willingness to listen and request listening from others. This builds on expectations about listening to other's thinking, as well as expectations about asking clarifying questions.

Following the results in Article 2, and in relation to the first research question, it is suggested that teachers need to pay attention to productive listening within small group work. By being aware of the purposes of listening as well as social aspects of listening, teachers can learn how to better promote student participation in mathematical dialogue.

Conclusions from the first sub-study used in the second sub-study

Since this PhD study consists of two sub-studies with two separate data generations/collections and EDR processes, it seemed wise to include some of the results and learnings from the first sub-study when designing the second one. As the first sub-study focused on students' listening and questioning, these two areas were chosen to also inspire the first two cycles in the second sub-study. The direction of the third cycle in the second sub-study was not decided until after the first two were conducted, as the group of teachers should have ownership and participate in the decisions of what to work with, depending on the needs of their students.

In the second sub-study, the linguistic backgrounds of students were not considered the most important issue, since the results (presented in Article 1) suggested that other issues might be more important to prioritize to meet the needs of the students in order to promote participation in mathematical dialogue. Also, the classrooms in the second sub-study were not as multilingual as in the first sub-study, so it was hard to find results that connected to students' first languages.

Another result from the first sub-study that was taken into account when planning for the second sub-study was the importance of questions – both that students' questions to each other mattered for inclusion, but also that the students' questions were sometimes not enough. Hence, how teachers could interact with students working in small groups, and add their own questions as part of creating equitable participation opportunities in the mathematical dialogue was considered important.

The results from Article 2 concerning the productive listening framework were not clearly included in the second sub-study, as the article was not written when the second data collection was made. However, the thoughts about the importance of listening were included when planning the second sub-study.

In the second sub-study, the emphasis was no longer on student-to-student interaction and students' actions when it comes to questioning and listening, but rather on teachers' actions in promoting and including all students in mathematical dialogue. This led to the second research question, with focus on teachers' actions.

RQ2: What aspects of teachers' questioning and listening are important when teachers promote students' participation in mathematical dialogue?

The second sub-study emphasized what was important in teachers' questioning and listening when it comes to promoting all students to participate in mathematical dialogue.

This was done in three cycles in a new EDR project, in which the first cycle focused on listening, the second on questioning, and the third on teachers' moves when visiting students working together in small groups, trying to promote equitable participation opportunities. Whereas the orientation of the first two cycles was inspired by the first sub-study (Sjöblom, 2015), it was only the overall focus area that was decided in this way. What to do in the classrooms in detail, how to design mathematical activities and tasks, and how to evaluate the work were connected to the specific students and teachers participating in the second sub-study. The direction of the third cycle was decided by the teachers.

In the second sub-study, there was also an overall focus on students' inclusion in mathematical dialogue, as teachers were concerned with giving all students equitable learning opportunities (Esmonde, 2009), through making all of them participate in the mathematical dialogue.

In Articles 3 and 4, the theory used to understand the complexity of teachers' work with promoting student participation in mathematical dialogue was mainly the IC-model (Alrø & Skovsmose, 2004), as this provided a basis for what quality in mathematical dialogue is. After finishing the EDR project, the "initiation-entry-focus-exit and participation" framework by Ehrenfeld and Horn (2020) was added to specifically look at what teachers did while visiting the students working in small groups in Article 3. When problematizing how to create equitable participation opportunities through the use of mathematical questioning in Article 4, the concept of mathematical dialogue of Alrø and Skovsmose (2004) was used.

In the second sub-study, as a complement to the EDR cycles, the concept of teacher noticing (Mason, 2002; Sherin et al., 2011) was added as a way of organizing the results and having a method to make visible how teachers' awareness developed when it comes to promoting questioning, listening, and participation in mathematical dialogue. Although teacher noticing is rarely connected to EDR (Santagata et al., 2021), this seemed crucial in order to understand how to concretize the third step in teacher noticing, which is about acting/responding.

Below is a short summary of the results presented in the articles in relation to the second research question.

Summary of Article 3: Teachers' noticing to promote students' mathematical dialogue in group work

Article 3 focuses on how teachers can refine their strategies for purposefully engaging students in mathematical dialogue when students are working in groups and teachers enter ongoing group conversations.

In the analysis of the three EDR cycles in Sub-study 2, special attention was paid to what happened before, during, and after teachers visited student group discussions. In the first cycle, the focus was on listening. Here it was found that questions made student listen to each other, but also that students did not always ask the questions they needed to ask. Instead, they kept quiet, or went into some kind of "presentation mode" when teachers visited groups. When teachers did ask questions, or students asked each other questions, this made it possible to change the dialogue into a more welcoming environment in which more people were involved. Therefore, attention was directed toward questions in the second cycle. Here, teachers wanted students to become more aware of, for instance, how- and why-questions in mathematical dialogue. Teachers tried to initiate conversations, but often without mathematical content. Questions like "how are you doing?" or "are you finding something out?" were asked. It was not always that students wanted to show what was problematic to them. In the third cycle, the group of teachers, turned the focus to their own actions, as facilitators of students' mathematical dialogue. By deliberately preparing and asking questions with mathematical content to all students in the groups, and making sure they did not leave the group until everyone had said something, they intended to change the participation patterns.

After the EDR project was finished, the IC-model analysis (Alrø & Skovsmose, 2004), which was conducted together with the teachers, was complemented by a second analysis using Ehrenfeld and Horn's (2020) framework in Article 3. There are several connections between the frameworks, in that **initiating/entering** conversations is largely about *getting-in contact*, while **focus** is about making students aware of the mathematical content in the tasks (*locating, identifying, advocating*) and, depending on how teachers act to do that, *reformulating* or *challenging* might be used. When it comes to **participation**, again *getting-in-contact* is an important IC-act to use. In the second analysis, Ehrenfeld and Horn's (2020) framework was used to systematically identify instances of teachers' work with engagements in students' discussions, to understand more of the complexity concerning how teachers can promote participation in mathematical dialogue. One difference the teachers found in comparison to Ehrenfeld and Horn's (2020) study was the importance of including all students in the mathematical dialogue. While Ehrenfeld and Horn (2020) did not emphasize this, the teacher group found that sometimes, when students were quiet and did not participate, this was a sign that they could not understand or follow the conversations, and by asking questions to everyone within a group, teachers could better understand and support all students.

The results were discussed in the article by following teachers' noticing processes in two different settings – the classroom and the teacher professional development group. By attending to certain aspects of the mathematical dialogue in the classroom, interpreting the results within the professional development group and deciding together how to act/respond, it was possible to make visible how teachers refined their actions and learnt more about how to promote mathematical dialogue. The results indicated that teachers became more aware of the importance of explicit instructions, as well as their own role as facilitators of students' discussions through asking mathematical questions to all students within the groups.

The main contribution of Article 3 in relation to the second research question is the use of the IC-model (Alrø and Skovsmose, 2004), together with the initiation-entry-focus-exit and participation framework by Ehrenfeld and Horn (2020), to understand more about how teachers can promote mathematical dialogue through listening and questioning. Also, practical learnings about what happens before, during, and after teachers enter mathematical dialogues – for instance, the importance of asking questions to all students in a group and be specific and prepared about what mathematical content to address while visiting student groups – could be considered results of the EDR cycles with importance for the second research question.

Summary of Article 4: Promoting equitable participation opportunities in mathematical dialogue through mathematical questioning

The focus in the fourth article was on how teachers can develop and promote equitable participation opportunities in mathematical dialogue through mathematical questioning. This article connects back to the aspects of inclusion and student participation, which address the problem stated in the beginning of this thesis about students not having equitable learning opportunities because their participation in small group work and mathematical dialogue differs.

All three EDR cycles in the second sub-study related to participation and questions in mathematical dialogue in different ways. The first cycle revealed that questions, both from teachers and students, can change the mathematical dialogue, and an interest in the teacher group about the difference between how- and why-questions was raised. This can be compared to Mason's (2000) research about questions changing students' attention in certain directions. In the second cycle, the focus was on how teachers can make students more aware of the roles of participation and questions in mathematical small group work. Through a meta-discussion in the classroom with the students, teachers wanted to make students discuss when and why they asked questions. However, it proved difficult to change students' attitudes when it comes to questioning in a short time line; therefore, the focus in the third cycle was back on teachers' strategies on asking questions to promote participation of all students in mathematical dialogue.

The results in the three cycles were organized using the three elements of teacher noticing – attending, interpreting, and acting/responding (Jacobs et al., 2011; Kazemi et al., 2011). The elements were then connected to Alrø and Skovsmose's (2004) concept of mathematical dialogue when it comes to *making an inquiry*, *running a risk*, and *maintaining equality*. In the analysis, these characteristics of mathematical dialogue were used and problematized to better understand the complexity of teachers promoting students' participation. The third characteristic was especially interesting, with the addition not only focusing on maintaining equality, but on establishing equitable participation opportunities through mathematical questioning.

The main contributions of Article 4 with regard to the second research question are the importance of listening and questioning in mathematical dialogue and how teachers can support the *making an inquiry* by asking not only general questions about how the group is doing, but also asking specific pre-thought mathematical questions. Also, *risk taking* is something that applies to both teachers and students. The teachers felt that it was a risk that they would disturb the inquiry process by asking questions, since it might interrupt the students and hinder them from thinking on their own. At the same time, the data indicated that not asking questions was also a risk, since it could contribute to students being lost in group work for a long period of time. Students seemed to think it was a risk to ask questions, since this could reveal that they did not know how to solve mathematical tasks and therefore did not (yet) have the right mathematical knowledge. If teachers and students were not taking the risks mentioned, this meant that, sometimes, *equality* was not maintained and equitable participation opportunities were not established. This meant that students did not get the same learning opportunities (Esmonde, 2009; Mercer, 1995). Hence, asking questions and including all students in the mathematical dialogue was a way of promoting participation. Summarizing the results in relation to the second research question, there were changes across the three EDR cycles in terms of teachers' awareness and strategies regarding mathematical questioning, what kind of focus and intent questions had in mathematical dialogue, and what kind of responsibility teachers considered they had in asking questions.

There are also methodological contributions in Article 4. The combination of using teacher noticing with EDR is uncommon in previous research (Santagata et al., 2021). The same is true regarding research on teacher noticing and equity (Dindyal et al., 2021; Santagata et al., 2021). Therefore, the present study serves as an example of how EDR can contribute to the third step in teacher noticing, about how acting/responding can be realized systematically within a teacher group and in the mathematics classroom in relation to creating equal opportunities for students to participate in mathematical dialogue.

RQ3: How can teachers, in cooperation with researchers, develop an awareness and refine their teaching in relation to students' listening, questioning and participation in mathematical dialogue?

Using results from the two sub-studies, it was possible to look back at the two EDR processes to find answers to the third research question. The findings are connected mainly to three areas: using the cyclic nature of EDR, combining students' and teachers' viewpoints by conducting the studies both in the classroom setting and in the professional development setting, and deliberately listening to students' and teachers' voices in EDR and making them part of the research process.

Using the cyclic nature of EDR as a method to refine teachers' teaching

Cyclic processes are commonly used, both in research and professional development when it comes to understanding complex situations (McKenney & Reeves, 2012; Timperley, 2011). In this PhD study, the cyclic nature of EDR enabled and supported two different processes simultaneously in line with the goal of EDR; namely, producing both practical as well as theoretical results (McKenney & Reeves, 2012).

Firstly, from a practical viewpoint, the cyclic processes made it possible for the teachers to work with student interaction and learn more about listening, questioning, and participation in mathematical dialogue. The processes also helped them, through the different phases of analysis/exploration, design/construction, and evaluation/reflection (McKenney & Reeves, 2012), to systematically understand how their own actions affected the students' interactions, and step by step they chose what was most important to do in relation to their students' learning processes. The EDR cycles were complemented by the concept of teacher noticing (Mason, 2002; Sherin et al., 2011) in the second sub-study, as a way of organizing the EDR cycles that illuminated how teachers' awareness developed. This was done by following what the teachers attended, how they interpreted the results, and thereafter chose to act/respond to the results. The EDR cycles also supported teachers to produce practical material and lesson activity plans to use in the future, which are not only interesting for teachers, but also from a researcher's viewpoint because they exemplify and make visible teachers' development processes.

Secondly, from a theoretical viewpoint, the cyclic EDR processes made it possible to follow the work of the teachers, and understand more about how teachers can refine their teaching. By applying different theoretical frameworks and concepts to the data (Alrø & Skovsmose, 2004; Ehrenfeld & Horn, 2020; Fuentes, 2009), four articles were written, which are the main theoretical contributions of this study. The articles problematize and complement the existing theories, and also add a new one in the productive listening framework. When the empirical results were not consistent with previous studies using the theories, the discussions in the articles could be seen as a complement and a post in the mathematics education debate that help deepen theoretical understanding.

One result of this study, related to the third research question, is that the EDR methodology contributes both practically and theoretically to how teachers' awareness can change and how their teaching can be refined when it comes to promoting participation in mathematical dialogue. Without the cyclic process of EDR, it would not have been possible to reach any of these results.

Combining the classroom setting and the professional development setting in EDR to change teachers' awareness

One prerequisite for developing awareness and refining teaching in relation to students' participation in mathematical dialogue was to analyze the teachers' actions in relation to what happened in students' small group work. In line with Franke et al. (2007), it is important not to view students and teachers as separate entities, but rather in relation to each other. Many research studies do focus on either students or teachers, and although going back and forth between the classroom setting (which focused mostly on the students) and the professional development setting (which focused mostly on the teachers and their actions) resulted in a lot of data, it also opened up the possibility to understand more. Without the movement between the two settings, it would not have been possible to explore the research questions and follow how teachers' awareness developed.

In the cyclic processes, teachers' and students' actions complemented each other and were intertwined in the analysis. The concept of teacher noticing helped to organize this, by applying the steps of noticing on two different levels.

Firstly, teacher noticing in the classroom setting made it possible for teachers to attend to certain aspects of students' mathematical dialogue spontaneously as it happened; they interpreted it there and then decided how to act in the moment when talking to the students. This part of teacher noticing was very closely connected to understanding mathematical dialogue from the students' viewpoints.

Secondly, the teacher noticing processes were deepened within the teacher professional group setting. This part of the analysis also started with the students' work in mathematical dialogue, but the focus was now on attending and interpreting certain aspects of the video recordings so that the teacher group could decide together on how to act/respond in the coming lessons. Hence, this relates closely to understanding mathematical dialogue from the teachers' viewpoints.

Combining the first and second levels of noticing was made as the teachers moved back and forth between the two settings. In this process, it was possible to understand the intertwinement between students and teachers and each setting informed the other. This created a movement in which students' actions affected teachers plans that again affected students' actions, and so on. Meanwhile, in relation to the third research question, it was possible to follow how teachers' awareness, when it comes to promoting students' participation in mathematical dialogue, gradually developed.

Deliberately listening to students' and teachers' voices in EDR and making them part of the research process

As a researcher with a wish to understand a specific part of the mathematics education field, it is easy to come and plan a research study that is conducted on students and teachers. However, for teachers to not just become research objects, but to actually gain both practical and theoretical insights and refine their awareness as well as their teaching, it is important for them to be active participants. Kieran, Krainer, and Shaughnessy (2013) discussed the importance of viewing teachers as key stakeholders in research, and analyzed how to enable an intersection between research and practice. Also, Gravemeijer and van Eerde (2009), Jaworski (2004) and Wagner (1997) emphasized the importance of working together with teachers.

Van den Akker et al. (2006) viewed EDR as a way of increasing the relevance of research for practice. In the present PhD study, this could be seen, for instance, in the attempts to connect the IC-model (Alrø & Skovsmose, 2004) to the reality in the mathematics classrooms. It could also be seen when teachers were involved in a structured way with EDR as a research methodology in their professional development work.

To make teachers active in the research project, it was considered important for them to be involved in all parts of the EDR cycles in order to work "with" them, rather than conduct research "on" them (Superfine, 2019). Being part of a research project could benefit their professional development when they worked together in their professional learning community with problems that were relevant to their context and the challenges within their classrooms. As Bryk et al. (2018) wrote, it is not about finding out "what works", but rather "what works, for whom, and under what circumstances" (p. 38).

However, there are benefits in both directions here. It is not only practice that can learn from research, but also the other way around. Kieran et al. (2013) claimed that it is the teachers that are the true experts of their own classrooms and know their students. By listening carefully to the teachers, more fine-grained designs could be made that actually connect to students' learning needs and the classroom challenges and possibilities. Including teachers in the research project and listening to both their voices and those of their students made it possible to dig deep into specific situations with specific students and specific teachers. Without their expertise and way of inviting research to take part of their reality, the results of this study would not have been reachable and the third research question would have been hard to answer. Without viewing teachers as active participants in the research process, it would not have been possible to follow their ways of developing an awareness and refining their teaching in relation to students' listening, questioning, and participation in mathematical dialogue.

6. CONCLUDING DISCUSSION

For me, it is important that a PhD thesis can answer the question of what the thesis contributes to the field of research. In this thesis, intending to view mathematical dialogue from the viewpoints of students, teachers, school developers and researchers (see Figure 1.1), it seems important, in this last chapter, to view contributions not only for mathematics education research, but also in a wider meaning. This concluding chapter starts with a summary of three main contributions of the PhD study in relation to the aim and the research questions, and thereafter the contributions are problematized and discussed. Also, a meta-reflection is made about the contributions in relation to the four viewpoints. At the end of the chapter, suggestions for further research are made. Finally, there is a last return to the problem formulation, looking back at all that has been written in the thesis.

Contributions of this research study

The aim of this thesis was to explore how questioning and listening can promote participation in mathematical dialogue. Through working with the three research questions and their answers accounted for in Chapter 5, the three main contributions are related to understandings of teachers' and students' engagements in mathematical dialogue and what part theories play in these understandings.

A first contribution of this study, which is connected to exploring listening, is the *productive listening framework* presented in Article 2. This framework contributes to how listening relates both to questioning and participation in mathematical dialogue. Although the framework is written from a student viewpoint, there are learnings for teachers as well, connected to teachers' promotion of productive listening amongst all their students.

A second contribution of this study, which is connected to exploring questioning, written from a student viewpoint in Article 1 and from a teacher viewpoint in Article 4, is the results of the EDR cycles about *teachers' and students' use of questions in mathematical dialogue*, and how these questions affect and promote participation.

A third contribution is how the results of the EDR cycles are connected to *use of theories to understand mathematical dialogue* in the field of mathematics education. The theories are connected both to students' and teachers' viewpoints and to all parts of the aim, considering listening, questioning, and participation. This contribution includes the justification of what theories to use to better understand mathematical dialogue, as well as the coordinating of theories in Chapter 3.

These three contributions are now further discussed and problematized in the following sections.

Contribution 1: The productive listening framework

A first contribution of this PhD study is the productive listening framework presented in Article 2, and how listening relates to questioning and participation in mathematical dialogue. Studying listening has proven difficult because it is hard to observe (Kosko, 2014). However, combining listening with questioning makes it possible to gain a deeper understanding about some parts of the listening processes and how they relate to promoting equitable participation opportunities in mathematical dialogue. In the productive listening framework, listening is related to two observable features: students' abilities to show willingness to listen and to request listening from others. This is often connected to asking questions, such as clarifying questions. By analyzing the questions asked in mathematical dialogue, this can say something about listening and participation. However, as this thesis builds on small, classroom-specific sub-studies, it is important to see whether the productive listening framework also holds in other classrooms and study further which social and socio-mathematical norms (Cobb et al., 2001) affect the mathematical dialogue there.

In this PhD study, there has not been time to apply the productive listening framework to data from the second sub-study, but it would have been interesting to do so, as productive listening is not only something that can be applied to students, but also to teachers, and their interactions within student groups.

For instance, teachers themselves can show a willingness to listen to their students in order to understand more about their mathematical thinking (Carpenter & Fennema, 1992; Reeder & Abshire, 2012), which also happened in the third cycle of the second sub-study, where teachers stayed a little longer when visiting the student groups. They can also request listening. This is usually not a problem for teachers, as students are used to listening to teachers. However, teachers can not only request listening for themselves, but also request that students listen to each other, for instance by connecting what different students are saying, and asking quiet students questions about what other students have said. This can have effects when promoting equitable participation opportunities.

Mueller et al. (2014), suggested that teachers need to promote students' listening. Having worked on this PhD project, I would suggest that teachers can work with observable actions connected to listening that help include all students. This means promoting not only listening, but also productive listening. To function as role models (Mrayyan, 2016), and by both request listening and show willingness listen, this can also affect how students work with productive listening.

This PhD study is an educational design research study that intends to contribute both to theoretical results as well as practical results (McKenney & Reeves, 2012). The productive listening framework could be seen as a theoretical contribution, but how teachers work with questions to promote productive listening and participation could also be seen as a practical contribution. Here, the theoretical and practical results complement each other and can be viewed as a way of connecting school development work with research.

Contribution 2: Teachers' and students' use of questions in mathematical dialogue

A second contribution of this PhD study is the empirical results of how both teachers and students use questions in mathematical dialogue and how these questions affect and promote participation. Article 1 problematizes questioning in mathematical dialogue from a student viewpoint, while Article 4 does the same, mainly from a teacher viewpoint. Hence, mathematical questions were the focus of the start of this PhD process as well as the end of it.

However, it is not always possible or even desirable to separate students' and teachers' questions from each other, as they are closely intertwined. In the first sub-study, the teacher tried to make students more aware of mathematical questions by having them reflect about what questions they had asked each other during group work. This was difficult for the teacher because the students did not, at least not initially, see the point in asking questions, and sometimes even considered it to be a weakness to ask questions, as it could simultaneously be a sign of needing help (Sjöblom, 2015). The same was true for the second sub-study, in which the teachers tried in the second cycle to make students more aware of the role of questions in mathematical dialogue; see Article 4. Again, students did not always see the point of asking questions, and sometimes preferred to be quiet when they felt that they could not contribute to the discussions. This might be connected to social risks with asking questions (Horn, 2017; Lack, 2010). After the meta-discussion about questions with the students in Cycle 2, when teachers listened for questions amongst students in the coming group discussions, students' use of both how- and why-questions were scarce. Hence, the teachers concluded that it was easier for them to change their own actions than to change students' actions. The complexity of mathematical questioning cannot be solved within an EDR cycle; rather, it is a long-term process that is challenging both for school developers and researchers. Although the EDR methodology, the cyclic setup and the two settings made it possible to try out different aspects of questioning, both from a student and a teacher viewpoint, it could not grasp the long-term challenge of, for instance, changing how students view questions. Again, in line with Fuentes (2009) and Sfard and Kieran (2001), mathematical dialogue is not something that always happens by itself; it is a learning process that takes time.

One similarity between students' and teachers' questions found in the two sub-studies is that both open up for participation in mathematical dialogue. When students ask each other questions, quiet students are sometimes included in the discussions, and in the second sub-study teachers consciously used questions in order to include all students and have all of them participate in the mathematical dialogue. This can also lead to more equal learning opportunities (Esmonde, 2009), as well as avoiding students being outsiders (Barnes, 2005). Hence, mathematical questions are not only a parameter to consider when it comes to mathematics or mathematical dialogue, but also when it comes to more equal opportunities to participate in mathematical activities, which could be extended to more equal opportunities for learning in mathematics. Hence, the problematizing of questioning, both from a student and a teacher viewpoint, need to be connected to problematizing equity and participation. A consequence of this reasoning is that researchers can benefit from investigating questioning, equity, and participation simultaneously, and teachers can benefit from considering how to promote them in their teaching simultaneously. This thesis contributes with a way of investigating this, using teacher noticing as well as EDR methodology to combine mathematical dialogue with equity issues.

Contribution 3: Use of theories to understand mathematical dialogue

A third contribution of this PhD study is the conclusions about how theories are used to contribute to knowledge on mathematical dialogue in the field of mathematics education. This includes the coordinating of theories in Chapter 3, as well as how different theories are used in the four articles. In a way, this could be considered a contribution mainly within the research viewpoint, but as teaching and school development should build on scientific grounds (Swedish National Agency for Education, 2011), both teachers and school developers should be invited to take part in this kind of contribution. In this section, in addition to what has been written in previous chapters and in the articles, the use of theories is problematized.

Problematizing theories

The use of theories and how they help answer the research questions have already been covered in the articles and the theory and result sections of this thesis. However, what might not be clear from the description in Chapter 3 is that although using the theories is described as a straight-forward process, it has been a challenge to know which theories to use and how to use them together. During this PhD project, a great deal of effort has been made to understand how the three levels of theories relate to each other, and it has not been obvious what theories to use, especially not on a local level. Depending on the choices of theories, some possibilities opened up, while others were closed. The theories also have their pros and cons, and some issues will be problematized further in this chapter as a contribution and discussion of existing theories. Finally, to include the teacher and school developer viewpoints even more, an additional issue that will be problematized is how the theories affected the work with the teachers in the two sub-studies.

Problematization of the IC-model and the concept of mathematical dialogue

The IC-model has been used to identify what quality can be in mathematical conversations, which Alrø and Skovsmose (2004) connected to the concept of mathematical dialogue. Using the characteristics of dialogue – being an inquiry process, running a risk, and maintaining equality – or the different IC-acts in the analysis presumed that students were active participants in the discussions and that the conversations focused on mathematics. However, as discussed in Alrø and Skovsmose (2004), and in the articles of this PhD study, this is not always the case. Students sometimes wish to work alone, or do not participate in mathematical dialogue, or zoom out from the topic, or just want to pursue their own strategy and do not care about what others think. All these situations can be seen as obstacles in the mathematical dialogue, but also make it impossible to analyze the quality of the dialogue using the IC-acts.

Alrø and Skovsmose (2004) and the present PhD project have both concluded that an issue with the IC-model is that it is rare for all dialogic acts to actually be used; therefore, the model might be more a wish for how a mathematical dialogue should be rather than a picture of a real mathematical dialogue.

Another issue, which is also partly problematized in Article 4, concerns the third characteristics of mathematical dialogue, maintaining equality (Alrø & Skovsmose, 2004). In the present study, the focus was not really on the maintenance, but rather on the establishment of equitable participation opportunities. In all recordings from the classrooms, it was the teacher that was in charge, steering the conversations, and always changing the courses of action when visiting the students' small group discussions. That could have several explanations; for instance, the fact that the teacher usually leads the work in the classroom, or that the teacher is grading students' mathematical knowledge towards the end of the course, which the data suggested made students less willing to admit when they were stuck and ask questions. By working with *establishing equitable participation opportunities* as a way of *maintaining equality*, this difference in power is acknowledged, but may also be challenged to be changed. By asking questions and listening to the answers, students can be considered to be competent sense-makers (Dunleavy, 2015), which can be a step towards working to maintain equality. Depending on what lesson activities are planned, how the classroom climate is framed, and how far the teachers and students have come in their development of why questions are asked and answered, this kind of research project can be seen as an ongoing process and idea about how to create equitable learning opportunities.

Problematization of Fuentes' framework

Fuentes' (2009) framework for analyzing student communication has certain pros and cons. Focusing on questions/comments and responses makes parts of the complexity in interaction visible. However, when the framework is combined with the results on listening, it is clear that it does not take into account the quiet students in the dialogue that might be listening but, for various reasons, do not participate. The framework presumes that there is a conversation between two students, A and B, while it might be the case that Students C and D are listening, and reflecting, and hence, could have new thoughts that they might have use for later on in the dialogue. One contribution from reflecting on this is the results that point to the importance of considering questioning and listening simultaneously, and connecting them when promoting participation.

Problematization of Ehrenfeld and Horn's framework

Ehrenfeld and Horn's (2020) framework focuses on teacher's actions. By combining this with what is happening in the mathematical dialogue, it is possible to say something about the connection between what teachers do and what happens in the student groups.

In this thesis, one conclusion is that, when working together with teachers in an EDR project, it is important to not only understand what is concretely happening in the classroom setting, but to also include the teacher professional development setting to understand why teachers act the way they do, and how their awareness can change when it comes to questioning, listening, and participation in mathematical dialogue. Hence, applying a framework like Ehrenfeld and Horn's (2020) after the EDR process was finished gives an extra dimension to the processes. However, it does not answer all questions. Another dimension that seems to be missing, and which affects the outcomes, is teachers' incentives regarding why they act the way they do. This project may have even needed a third setting – an interview setting – to further investigate incentives.

Problematization of using theories together with teachers in EDR projects

In both sub-studies in this PhD project, the theories were introduced to the teachers, and I as a researcher occasionally returned to what IC-acts (Alrø & Skovsmose, 2004) were used in the mathematical dialogue, and I sometimes mentioned Fuentes' framework (2009) or productive listening (Sjöblom & Meaney, 2021) in the second sub-study.

However, in both sub-studies, the teachers did not actively use the frameworks much when talking about the students' mathematical dialogues or their own actions when it comes to interacting with students. In the second sub-study, when asked about this, teachers claimed that they needed to be more familiar with the theory, and work more actively with the IC-acts to understand them, so perhaps the setup did not sufficiently support teachers' understanding of the IC-model. This could also reflect some kind of contraposition between what might be in focus from a teacher's viewpoint and from a researcher's viewpoint in an EDR study (Hamza et al., 2018; Richardson, 1992). For instance, theories might be more important for researchers to understand the complexity of the problem they want to address, while real actions within the classrooms are more important when teachers try to understand the complexity of listening, questioning, and participation. I could have made greater effort in this project to make these two ways of understanding the complexity coincide. It could be interesting to investigate further how doing so affects teachers' participation in EDR projects and feeling that they own the theories, and what we as researchers need to do to ensure that teachers are included more and function as key stakeholders (Erickson, 2014; Kieran et al., 2013; Superfine, 2019). This issue is also problematized in the coming meta-reflection, in which the four viewpoints in the thesis are tied together.

Meta-reflection on how collaboration between different groups can contribute to understanding complex phenomena

Contributing to all four viewpoints – students, teachers, school developers, and researchers – can seem like a big commitment, but since they are all tied together within the setup of this research project, it is not impossible. In this meta-reflection, the four viewpoints are first discussed separately and thereafter connected.

Understanding viewpoints of students, teachers, school developers and researchers

At the center of the two EDR processes has been what has happened in the classroom setting, in the mathematical dialogue between students and teachers, and therefore the two first viewpoints are automatically included. The discussion about the mathematical dialogue in the professional development group setting in the second sub-study opens the way for reflections that can be applied when it comes to the school developer viewpoint and how projects can be planned to deliberately develop an awareness and refine teaching on specific content areas. From a researcher's viewpoint, depending on what research one is interested in, theoretical or methodological aspects can be connected to different areas such as mathematics education, interaction or equity issues, so this viewpoint is also included.

The three contributions that are discussed in this chapter tie together the research questions and the aim of exploring how questioning and listening can promote participation in mathematical dialogue. In a way, these contributions are already covered in the articles and result chapters as a product of answering the three research questions. However, they alone do not tell the entire story of what this thesis contributes. As stated earlier, what does make this thesis interesting in my view is that it tries to connect the different viewpoints and the different settings in the same study. The insights from this process, and the reflections about the research setup and how it affects different actors within the project, are learnings that have not (yet) been described explicitly in a written article. Therefore, in this subsection, a meta-reflection will follow on how collaboration between different actors/viewpoints can contribute to understanding complex phenomena, such as promoting mathematical dialogue. This meta-reflection can also be seen as a post in the debate on teacher–researcher collaboration (Erickson, 2014; Kieran et al., 2013; Superfine, 2019). It can also be a way of avoiding viewing mathematical dialogue as a top-down development process (Warwick & Cook, 2019), and show the potential of all actors to contribute to and gain from the process.

This PhD study is an educational design research study that intends to contribute to the field of mathematics education. At the same time, it is a professional development project about teachers' practice in mathematics classrooms, making it possible for the teachers to develop an awareness and refine their teaching in relation to students' participation in mathematical dialogue. Simultaneously, it is also a series of designed lesson activities that the students have experienced, as a part of their mathematics learning. Depending on who you are in this project, the answers to the following three questions differ:

- *What* do you do?
- *Why* do you do it?
- *How do you think* about it?

These three questions will now be discussed from different viewpoints, first separately and then together. The intention is to make the answers to the questions visible and identify the interrelations between different viewpoints in order to better understand the complexity of collaborating on mathematical dialogue.

Students' viewpoint

From a student viewpoint, this PhD study has a starting point in *what* students do (or not do) when it comes to mathematical dialogue. Understanding students' actions has been essential for *what* the other actors should do. The results of the study identify what students can do with each other and with the teacher in mathematical dialogue. However, the *why* question and *how to think* question have not been properly investigated. There are signs of *why* students ask questions, for instance to understand each other's thinking (see Article 1). There are also signs about *how to think* about questions in the meta-discussion in Cycle 2 in Sub-study 2, when students for instance chose to be quiet when they do not know what to do, or when they view it as individual students' responsibility to participate (see Article 4). I have left it to future research to investigate these questions further when it comes to students. However, these questions are still important to think about, as the answers have implications for the other three actors, as students' and teachers' learning processes often are related (Gravemeijer & van Eerde, 2009). Hence, when trying to build a collaboration between different actors in mathematical dialogue, it seems important for teachers to try to understand how their specific students are viewing mathematical dialogue.

Teachers' viewpoint

From a teacher viewpoint, all three questions – *what* teachers do, *why* they do it, and *how they think* about it – are well covered in the articles and in this thesis as answers to the research questions. In the structured work in the EDR cycles, teachers decided, together with the researcher (me), *what* to do in their classroom, and justified in their discussions both *why* they did it and *how they thought* about it. By being active participants in the noticing processes (Mason, 2002; Sherin et al., 2011), they connected their actions to those of the students in the classroom setting; they also worked together with the school developer/researcher in the analysis process in the professional development setting. Hence, the teachers could be seen as some kind of interconnector that carried expertise knowledge from the classroom setting about their students and the teaching situations to the professional development setting, and also in the opposite direction, they carried back designed lesson activities and thoughts from the analysis to the classroom.

Researcher and school developers' viewpoint

The researcher and school developer were the same person (me) in this study, and I was involved in both students' and teachers' *what*, *why*, and *how to think* questions in a variety of ways, trying to follow and support the processes in both the classroom and in the professional development setting. However, as the researcher I also had my own agenda when it came to these questions. *What* to do was about creating sub-studies that could deepen the understandings about mathematical dialogue both from a student and a teacher viewpoint. It was also about planning a professional development program for the teachers, and writing research articles and a thesis. *Why* was related both to the problem formulation in Chapter 1 about students not having equal learning opportunities and a wish to change that together with the teachers. *How to think* about this was addressed in the analysis processes, both together with the teachers during the EDR cycles, but also afterwards, with the help of theories and the structure of teacher noticing (Jacobs et al., 2011; Kazemi et al., 2011). The answers to the *what*, *why*, and *how to think* questions did partly coincide with those of the teachers, but also had other elements, mainly related to the mathematics education research discourse outside both the classroom setting and the professional development setting.

Connecting the viewpoints and collaborating to visualize the complexity

As can be concluded in the previous subsections, different actors have similarities and differences in terms of their answers to the *what*, *why*, and *how to* *think* questions. This is one reason why understanding the phenomena of mathematical dialogue is so complex. However, drawing the chart of answers to the questions from the viewpoints of the different actors helps make the complexity visible and therefore understandable.

In my view, what makes this PhD study interesting is that in order to understand the complexity of mathematical dialogue, the different viewpoints have been connected through teachers' movements back and forth between the classroom setting and the professional development setting, and through collaboration between different actors. Here, "collaboration" is a key word, and as a continuation of this meta-reflection, three aspects of collaboration connected to mathematical dialogue will now be discussed to further problematize the complexity: common student goals, clarity of intensions and responsibilities of different actors in EDR projects.

Firstly, one way to understand the complexity of analyzing mathematical dialogue is to agree on what goal concerning mathematical dialogue to focus on. Although different viewpoints and actors add to the complexity, as the *what* of different groups differ, they are also intertwined, since what one actor does affects the actions of the others. For instance, for students to develop mathematical reasoning, teachers need to plan for activities that promote mathematical dialogue, and researchers and school developers, in order to support the teachers, work with practice-based research supporting their development processes, and contribute with understandings on mathematical dialogue. When different actors collaborate on the same student goal, the different *whats* become interrelated. Teachers' movement back and forth between the classroom and the professional development setting also makes it possible to actively reflect on the goals and collaborate on how to reach them.

Secondly, clarity of intentions connected to different actors is important to make visible the complexity both of *why* mathematical dialogue is important and *how to think* about it. For instance, in a classroom setting, students need to know *why* teachers want them to work with mathematical dialogue; likewise, teachers need to know *why* students view matters such as questioning, listening, and participation in mathematical dialogue the way they do in order to plan their teaching. In a professional development setting, when teachers collaborate with researchers or school developers, an ongoing, open communication about mathematical dialogue, *why* to work with it, and *how to think* about it is essential for understanding the complexity. Again, talking together, listening to each other's thoughts, and collaborating are keys for understanding the complexity.

Thirdly, it is also interesting to interrelate responsibilities and acknowledge different roles that actors have in mathematical dialogue without building a hierarchy about some actors being more important than others. As Areljung et al. (2021) wrote, cooperation between different actors includes risks and benefits, rights and obligations. For instance, in the classroom setting, to maintain equality or establish equitable participation opportunities, teachers and students need to together investigate the mathematics through mathematical dialogue. They have different roles and responsibilities within the classroom, but also commonalities, such as a focus on students' learning processes. Similarly, although teachers have different roles to that of school developers or researchers in professional development projects or research projects, by acknowledging differences in responsibilities within the projects, different groups have a lot to learn from consciously combining their viewpoints and work together (Jaworski, 2004; Wagner, 1997). Potari, Sakonidis, Chatzigoula, and Manaridis (2010) stated that the roles of teachers and researchers can be seen as separate, but at the same time not incompatible, and that the different actors can be seen as partners in an inquiry process about teaching and learning mathematics. My interpretation of this is not only that teachers can learn from researchers, but also that researchers have a lot to learn from teachers and students. This is also what happened in my PhD project.

Collaboration might not always be easy, which can add to the complexity, but without it and using the different viewpoints of different actors, it seems even harder to understand mathematical dialogue. By collaborating and deliberately building bridges between the viewpoints, and by systematically building bridges between the settings, it was possible to understand more about the complex phenomena of mathematical dialogue, which leads back to Wagner's quote (1997, p.16), "by working together, each might learn something about the world of the other".

Discussion about further research

In this chapter, the meta-reflection on collaboration in the PhD study highlights the importance of research and practice collaborating and learning from each other. Taking this into account, the suggested ways forward will have questions related to all four viewpoints – students, teachers, school developers and researchers – in mind.

In a discussion about further research, it also seems important to problematize the choices made in this project, and reflect on whether they could have been made in another way. For instance, the choices of theories affected the results. With other theories, used in other ways, the results would probably have been different. For example, instead of emphasizing the socio-cultural perspectives of mathematical dialogue, more critical perspectives that were the basis of the research of, for instance, Skovsmose, could be used to investigate further how mathematical dialogue can be connected to equity and justice.

There are also methodological choices that affected the results that could be further developed. For instance, the setup with me being both a school developer as well as a researcher simultaneously could be further problematized. When building a collaboration between researchers and teachers, it is not possible to simply be an observer on the outside, without taking part in the discussions. However, it could have been possible to add another person as school developer to avoid the problems that occurred having two roles at the same time. Another methodological question worth reflecting on is how noticing and EDR fit together and complement each other.

In this PhD study, certain aspects of teachers promoting mathematical dialogue have been emphasized, namely listening, questioning, and participation. Other aspects could be added to this list, depending on what needs students have, what questions teachers ask, and what questions researchers or school developers would like to investigate. Also, the issues analyzed in this thesis could be examined more thoroughly. In this study, only some of the data have been used. It would be interesting to further combine what teachers decide to do in professional development meetings with what is actually happening in the classrooms. It would also be interesting to see in greater depth how questions that teachers ask when visiting small groups evolve across the cycles and how they affect the interaction after the teachers have left, or to compare the results from the classrooms in this study with results in other classrooms. From a student viewpoint it could be rewarding to dig deeper into the *why* and *how to think* questions, perhaps to understand when students find participation relevant.

There are other aspects of the student–teacher interaction and of the student–student interaction that could be further studied when it comes to mathematical dialogue. For instance, it could be interesting to take the productive listening framework developed in the first sub-study and relate it to the second sub-study, and see what would happen, and if the socio-mathematical norms from the first classroom were also present in the new classrooms. Another approach could be to involve teachers in the further study of productive listening, since this framework was developed from a student viewpoint. What can teachers do to better promote productive listening? And what can researchers do to better understand productive listening?

Revisiting the problem in the last section of the thesis

At the beginning of this PhD thesis, the problem formulation was articulated: "In Swedish mathematics classrooms, students have different opportunities for participating in mathematical dialogue". This thesis has focused on some aspects that are important when trying to change students' opportunities for participation, namely listening and questioning in small group work.

All of the contributions in this thesis intend to help us better understand the complexity of the problem formulation. The results point to the fact that questions, from both students and teachers, play an important role to include everyone in mathematical dialogue. They also indicate that questioning and listening are closely tied together, at least when it comes to productive listening. Further, for teachers to develop their awareness of how to promote mathematical dialogue, the use of deliberate questions and moves to include all students are again important. All this needs to be made in a way that uses and coordinates mathematics education theories in a systematized way that combines the viewpoints of students, teachers, school developers, and researchers.

The conclusions in this thesis might not apply in all classrooms. It was never my goal to generalize a small qualitative study and claim that it is universal. However, the way in which the teachers in this study have investigated and tried to actively change their own and their students' practice within the classroom setting and the professional development setting could be one way of making visible opportunities to understand the complexity of mathematical dialogue. The movement back and forth between the settings could also be used when studying other complex research questions.

The journey in my PhD study could, perhaps wrongly, be interpreted as a linear process, in which all the right questions were asked to produce all the right answers. Of course, the complexity of a research project, and actually understanding what is important when it comes to promoting students to participate in mathematical dialogue, makes it impossible to create a linear journey. Also, as EDR builds on making decisions about what to do in the coming cycle depending on what happened in the former, it is impossible to actually plan for what will happen before it happens.

The three research questions were not formulated until after the two substudies had been conducted, after looking at the results and trying to understand what conclusions could be made. Figure 5.1, which describes the PhD process, was not made until this PhD thesis was written, in an attempt to summarize what had been done. In my licentiate thesis (Sjöblom, 2015), I included the following quote by Tranströmer (1978): "*Deep in the forest there is an unexpected clearing that can be reached only by someone who has lost her way*." This is also what happened during my PhD journey. Allowing myself to get a bit lost (often together with the teachers) when it came to data, designing, analysis, or theories made it possible to find results and think thoughts that otherwise might have been missed.

SAMMANFATTNING

I svenska gymnasiematematikklassrum har elever olika möjligheter att delta i matematiska diskussioner. Detta är ett problem inte bara för eleverna som får olika möjligheter till lärande, utan också för lärare som behöver planera för en likvärdig undervisning, och för matematikdidaktiska forskare och skolutvecklare som försöker kartlägga och stötta hur lärare kan arbeta med att främja matematisk dialog.

I den här avhandlingen undersöks hur lärare kan skapa likvärdiga möjligheter för alla elever att delta i matematisk dialog genom att fokusera på tre aspekter: lyssnande, frågande och deltagande. Avhandlingen har gjorts genom två separata delstudier på två gymnasieskolor – en höstterminen 2013 med fokus på elev-elev-interaktion, och en läsåret 2018-2019 med fokus på fyra lärares arbete, både i klassrummet och i en kollegialt lärande-grupp. I båda undersökningarna gjordes omfattande videoinspelningar av elevers gruppdiskussioner i matematik, samt i den andra delstudien även ljudinspelningar av lärarnas kollegialt lärande-diskussioner.

Syftet med avhandlingen är att utforska hur frågande och lyssnande kan främja deltagande i matematisk dialog. Detta görs för att bidra till förståelse kring lärares och elevers interaktion i matematisk dialog och för att skapa kunskap kring forskning och skolutveckling i två miljöer – klassrum och kollegialt lärande-grupper. Följande tre forskningsfrågor ställs:

1. Vilka aspekter i elevers frågande och lyssnande behöver lärare uppmärksamma för att främja elevers deltagande i matematisk dialog?
2. Vilka aspekter i lärares frågande och lyssnande är viktiga när lärare främjar elevers deltagande i matematisk dialog?

3. Hur kan lärare, i samarbete med forskare, utveckla en medvetenhet och förbättra sin undervisning i relation till elevers lyssnande, frågande och deltagande i matematisk dialog?

Forskningsfrågorna har undersökts med hjälp av educational design research (EDR), som är en forskningsmetodologi i vilken lärare och forskare tillsammans arbetar i en cyklisk process med att förstå och utveckla det som händer i klassrummet. I den cykliska processen finns tre faser – analys/utforskning, design/konstruktion och utvärdering/reflektion – som syftar till att generera både praktiska resultat som är tillämpbara i matematikklassrummet och teoretiska resultat som bidrar till matematikdidaktisk forskning (McKenney & Reeves, 2012).

I avhandlingen har teorier på tre olika nivåer använts.

- På en övergripande nivå finns en sociokulturell syn på lärande, där elever och lärare i klassrummet, och lärare tillsammans med varandra och mig som forskare i en kollegialt lärande-grupp, utvecklar ny kunskap tillsammans genom interaktion och samtal.
- På en mellannivå används modeller för vad en matematisk dialog kan vara. Alrø och Skovsmose (2004) menar att matematisk dialog består av tre delar: att delta i en undersökande process, att våga ta risker, och att skapa och upprätthålla likvärdighet. Detta har använts tillsammans med deras IC-modell, som genom åtta dialogiska handlingar definierar kvalitet i matematisk dialog.
- På en tilläggsnivå har två teoretiska modeller använts för att analysera specifika aspekter i matematisk dialog: vilka handlingar lärare använder sig av när de besöker elever som grupparbetar i matematik (Ehrenfeld & Horn, 2020), och vilka typer av frågor som ställs och besvaras i matematisk dialog (Fuentes, 2009).

Den första forskningsfrågan besvaras i två artiklar med fokus på studenters lyssnande, frågande och deltagande. Artiklarna bygger på tre EDR-cykler från den första delstudien.

I artikel 1 används IC-modellen (Alrø & Skovsmose, 2004), tillsammans med Fuentes modell (2009) för att analysera hur elever ställer frågor och får svar. Genom att använda de två modellerna samtidigt synliggörs frågande ur två olika synvinklar, och både vilka dialogiska handlingar eleverna använder och vilka frågor som ställs och besvaras kan analyseras. Ett resultat är att även om klassrummet i den första delstudien var flerspråkigt, så var det inte hänsyn till elevernas språkbakgrund som var den viktigaste faktorn för att få igång den matematiska dialogen, utan snarare fokus på att få alla att delta aktivt oavsett språk. Eleverna gick från att inte vilja ställa frågor till varandra i den första cykeln, till att vara mer nyfikna på varandras tänkande i den tredje. Slutsatsen är att lärare inte bör tro sig veta vilka behov som finns kring matematisk dialog, utan öppet undersöka och stödja vad elever behöver när det kommer till lyssnande, frågande och deltagande.

Artikel 2 fokuserar på lyssnande och inleds med en litteraturöversikt som sammanfattar olika definitioner av lyssnande, syfte med lyssnande samt sociala aspekter av lyssnande. Det är dock svårt att i EDR-cyklerna förstå vilken typ av lyssnande som behöver främjas i matematisk dialog och varför. Eftersom IC-modellen och Fuentes modell (2009) inte tillräckligt bra visar på komplexiteten i elevers lyssnande, utvecklas istället i artikeln ett nytt ramverk som kallas ”produktivt lyssnande”. Detta innehåller två synliga aspekter av lyssnande: hur elever visar villighet att lyssna på andra, och hur elever ber andra att lyssna på dem själva.

Den andra forskningsfrågan besvaras i två artiklar med fokus på hur en grupp med fyra lärare arbetar med att främja lyssnande, frågande och deltagande i matematisk dialog. Artiklarna bygger på tre EDR-cykler från den andra delstudien. Genom att arbeta med så kallad ”noticing” (Mason, 2002; Sherin et al., 2011) i EDR-cyklerna, uppmärksammar lärarna (i samarbete med mig som forskare) speciella delar av klassrumsinteraktionen, tolkar vad de ser, och sedan väljer de hur de ska svara/agera utifrån sin tolkning. Detta görs både spontant direkt i klassrummet, men sedan fördjupat genom gemensam videoanalys i kollegialt lärande-gruppen. Ytterligare ett analyssteg görs av mig som forskare efter EDR-cyklerna är avslutade, då fler teorier läggs på datamaterialet för att synliggöra resultaten.

I artikel 3 fokuseras hur lärare kommunicerar med elevgrupper för att främja matematisk dialog. Detta görs genom videoanalys av vad som händer före, under och efter lärare besöker elever som grupparbetar i matematik. EDR-cyklerna fokuserar på tre olika delar av matematisk dialog: lyssnande, frågande och att främja deltagande. Genom att kombinera de dialogiska handlingarna i IC-modellen med lärares handlingar vid grupparbeten (Ehrenfeld & Horn, 2020), framgår det hur lärarna blir mer explicita i sina instruktioner och samtal med eleverna och att de tar rollen som facilatorer av elevers matematiska dialoger genom att ställa matematiska frågor som inkluderar alla i samtalet.

Artikel 4 gör en djupdykning i hur lärare kan utveckla sitt sätt att använda frågor för att främja elevdeltagande i matematisk dialog. I alla EDR-cyklerna var frågor viktiga. I den första cykeln såg lärare hur frågor fick tysta elever att bli inkluderade i dialogen. I den andra cykeln skapade lärarna en metadiskussion i klassrummet med eleverna för att synliggöra olika sorters matematiska frågor, när de ställs och vem som ansvarar för dem. Här framgick att elever ofta är tysta och låter bli att fråga när de känner sig osäkra. I den tredje cykeln fokuserade lärarna på sina egna frågor, och gick från att ställa allmänna ”hur går det”-frågor i den första cykeln till att ställa förberedda varför-frågor kring specifikt matematiskt innehåll i den tredje cykeln, och där även säkerställa att alla elever i grupparbetet var aktiva i dialogen genom att inkludera dem med hjälp av frågor.

Den tredje forskningsfrågan besvaras genom att reflektera över resultaten i de två delstudierna. Genom att lärare och forskare samarbetade och arbetade med noticing i de cykliska EDR-processerna, skapades en struktur för att utveckla och främja matematisk dialog. Och genom att gå fram och tillbaka mellan de två miljöerna – klassrummet och kollegialt lärande-gruppen – var det möjligt att synliggöra hur lärares medvetenhet kring att främja matematisk dialog växte fram och utvecklades. I denna process var det också viktigt att analysera och förstå den matematiska dialogen ur elev-, lärare-, skolutvecklings- och forskningssynvinkel samtidigt, eftersom dessa otvetydigt påverkar varandra.

Avhandlingens bidrag till det matematikdidaktiska forskningsfältet kan sammanfattas i tre punkter: 1) Ramverket för produktivt lyssnande. 2) Resultaten om hur lärare och elever kan använda matematiska frågor för inkludering i matematisk dialog. 3) Slutsatserna kring hur teorier har använts och kombinerats i EDR-cyklerna för att bättre förstå matematisk dialog.

Sist i avhandlingen görs en metareflektion kring hur olika aktörer – elever, lärare, skolutvecklare, forskare – kan och bör samarbeta eftersom deras olika synvinklar kompletterar varandra. Genom att bygga broar mellan olika aktörer kan förståelsen fördjupas gällande komplexiteten kring matematisk dialog.

APPENDIX A: CONSENT FORM FOR STUDENTS

Till berörda elever på XXX (skolans namn)

Samtycke till medverkan i en forskningsstudie om interaktion i matematik

Under läsåret kommer ett forskningsprojekt att genomföras på XXX (skolans namn). Projektet bedrivs av forskare vid Malmö universitet i samarbete med lärare på skolan och under läsåret 18/19 kommer en studie att genomföras i din klass.

Projektets syfte är att förstå mer om hur lärare kan stötta interaktion mellan elever. Vi vill få mer kunskap om hur elever tillsammans kan utveckla sin resonemangs- och kommunikationsförmåga i matematik. Vi vill låta eleverna pröva olika metoder för att öka elev-elev-interaktionen. Vi kommer att videofilma i klassrummet och inspelningarna kommer sedan att användas i dels i forskning med fokus på interaktion och dels för att utveckla undervisningen. Några intervjuer, där ljudet spelas in, kommer också att genomföras.

Allt arbete inom projektet kommer att ske i enlighet med GDPR (dataskyddsförordningen). Inspelningar kommer att förvaras på sätt som innebär att obehöriga inte kan få tillgång till dem. De personer som medverkar på videoinspelningarna kommer att vara anonyma i den rapportering som kommer ut av projektet. Namn kommer att ändras till fiktiva namn i de texter som publiceras av projektet. Om bilder från videoinspelningarna används vid rapporteringar kommer även de att anonymiseras så att personerna inte är möjliga att känna igen.

Malmö 2018-08-14

Kontaktpersoner (ansvariga forskare) vid frågor eller funderingar:

Marie Sjöblom	Clas Olander
Malmö universitet	Malmö universitet
marie.sjoblom@mau.se	clas.olander@mau.se
XXX (telefonnummer)	XXX (telefonnummer)

Deltagandet i forskningsprojektet är frivilligt och medverkande kan när som helst välja att avbryta sitt deltagande. Meddela i talongen nedan om du vill delta eller inte.

_ _

☐ Ja, jag vill delta i forskningsstudien. Inspelningarna får användas i projektets forskning med fokus på interaktion i matematik.
☐ Nej, jag vill inte medverka i videoinspelningar och intervjuer.

Datum: ______________________________

Namn (texta!): ______________________________

Underskrift: ______________________________

APPENDIX B: CONSENT FORM FOR TEACHERS

Till berörda lärare på XXX (skolans namn)

Samtycke till medverkan i en forskningsstudie om interaktion i matematik
Under läsåret kommer ett forskningsprojekt att genomföras på XXX (skolans namn) Projektet bedrivs av forskare vid Malmö universitet i samarbete med matematiklärare på skolan under läsåret 18/19.

Projektets syfte är att förstå mer om hur lärare kan stötta interaktion mellan elever. Vi vill få mer kunskap om hur elever tillsammans kan utveckla sin resonemangs- och kommunikationsförmåga i matematik. Vi vill låta eleverna i några matematikgrupper pröva olika metoder för att öka elev-elev-interaktionen. Vi kommer att videofilma i klassrummet och inspelningarna kommer sedan att användas i dels i forskning med fokus på interaktion och dels för att utveckla undervisningen. Några intervjuer, där ljudet spelas in, kommer också att genomföras. Även lärarsamtal i kollegialt-lärande-grupper kommer att spelas in.

Allt arbete inom projektet kommer att ske i enlighet med GDPR (dataskyddsförordningen). Inspelningar kommer att förvaras på sätt som innebär att obehöriga inte kan få tillgång till dem. De personer som medverkar på videoinspelningarna kommer att vara anonyma i den rapportering som kommer ut av projektet. Namn kommer att ändras till fiktiva namn i de texter som publiceras av projektet. Om bilder från videoinspelningarna används vid rapporteringar kommer även de att anonymiseras så att personerna inte är möjliga att känna igen.

Malmö 2018-08-14

Kontaktpersoner (ansvariga forskare) vid frågor eller funderingar:

Marie Sjöblom	Clas Olander
Malmö universitet	Malmö universitet
marie.sjoblom@mau.se	clas.olander@mau.se
XXX (telefonnummer)	XXX (telefonnummer

Deltagandet i forskningsprojektet är frivilligt och medverkande kan när som helst välja att avbryta sitt deltagande. Meddela i talongen nedan om du vill delta eller inte.

– –

☐ Ja, jag vill delta i forskningsstudien. Inspelningarna får användas i projektets forskning med fokus på interaktion i matematik.
☐ Nej, jag vill inte medverka i videoinspelningar och intervjuer.

Datum: ______________________________

Namn (texta!): ______________________________

Underskrift: ______________________________

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In Swedish mathematics classrooms, students have different opportunities to participate in mathematical dialogue, and therefore also different opportunities to learn. By moving back and forth between two settings – the upper secondary mathematics classroom and the professional development group – this thesis aims to explore how questioning and listening can promote participation in mathematical dialogue.

The cyclic structure of educational design research (EDR) helps to visualize the development processes when teachers work with noticing and move back and forth between the two settings. The results point to the importance of listening and questioning from both students' and teachers' viewpoints when it comes to creating equitable participation opportunities.

At the end of the thesis, a meta-reflection is made on how collaboration between different actors – students, teachers, school developers and researchers – can build bridges and deepen the understandings of the complexity of mathematical dialogue.

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