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A conceptual framework for student-centered teaching development in a digital era

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Yvonne Liljekvist

Contents

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Abstract

In the context of teaching, there is often a clash of different generation cohorts, which have different attitudes to and skills in digital media and corresponding ways of teaching and learning, respectively. The current COVID19 situation works as an intensifying lens and shows the problems occurring in this context even more clearly. Many mathematics teachers feel urged to move away from the instruction- and lecture-centered teaching style and many universities are upgrading digitally, thus creating the conditions for innovative teaching.

Considering the change in learning theories and education curricula, we offer a general model called “Rooms of Learning” as a quick-to-see-and-interpret framework for the pre- and post-analysis of course design. It is backed by subject didactical frameworks that focus on interactions between the main actors involved in learning.

As starting points, we use the basic needs of today’s learners for autonomy, relatedness, and competence as well as the requirements of the present information society. Focusing on different levels of the learners’ autonomy, as the key factor for building up mathematical competence and life-long learning, we illustrate our practice-oriented model by analyzing different course layouts. Herein the strength of the model is demonstrated, as commonalities of the modern course designs successfully applied today are revealed. It suggests, when designing a course, care should be taken to ensure that the defined Peer teaching room and Self-studies room are given enough space.

Introduction

The current and further developing information society demands flexible, adaptive skills and abilities. The workforce is expected to "reason dynamically and critically, make informed decisions, solve unknown problems, and work collaboratively to address unanticipated priorities" (Lee & Hannafin 2016, p. 709, after ISTE 2015). With the new requirements on the labor market, the demands on schools are also changing. Although competence orientation has been anchored as a new direction in curricula and goals of many countries (e.g., KMK 2004; OECD 2018; Skolverket 2011), the critical voices calling for changes in the education system have not fallen silent. "Education needs to shift its objectives to life-long and life-wide learning because future employees will be knowledge workers who will be able to transfer learned knowledge to solve complex problems" (Mosca et al. 2019, p. 68). But it is not only the changing requirements of the professional fields that make this reform inevitable. Based on studies from around the year 2000, Jones et al. (2007) summed up: "It is evident from research that today's students are very different from those of yesterday, yet our system of education remains based on a traditional model. Surely as the student population evolves, so should our schools." (p. 2) Especially in our rapidly changing media world, student characteristics have changed. They want to be actively involved in the learning process (Lowyck 2014). Not only since the Covid19 restrictions, mathematics teachers have become aware that many students are only partially able to work in a self-directed and autonomous manner **(**Winsløw & Grønbæk 2013).

Every crisis also holds an opportunity. Many schools and universities are upgrading digitally, thus creating the conditions for modern and innovative teaching. But digital media alone do not make for good teaching. To understand the mismatch between the reality of traditional teaching and the needs of today's students, some of the underlying causes will be discussed in the next section. Finally, we present models that are developed to help planning, analyzing, and evaluating teaching.

From Baby Boomers to Generation Z

Groups of persons born within the same period are called birth cohorts, or generation for short. Although there are slightly different limits, each generation span lasts about 20 years (Cilliers 2017; Jones et al. 2007; Mohr & Mohr 2017). People from the same generation experience similar events and therefore grow up with similar characteristics and attitudes (Jones et al. 2007). This results in a common *Zeitgeist*[1], which shapes distinct values and views. Each generation is different and therefore exhibits different behavior in the workplace and also in education. Since learners and teachers are usually from different generations, a mismatch of characteristics, attitudes, and values can occur, which is called a *cohort problem*. Mohr and Mohr (2017) indicate capabilities regarding the cohort problem by saying: "Bridging the possible divide between older and younger generations can be stimulating and affords an opportunity to rethink who current students are and what they need and want as learners" (p. 85).

The generations participating in university are shown in Table 1 with the approximate corresponding years of birth.

Table 1 *Demographics by Generation (after Jones et al. 2007)*

Cohort	Year of Birth
The Baby Boomers	From 1946 to 1963
Generation X	From 1963 to 1980
Generation Y/ Millennials	From 1980 to 2000
Generation Z	From 2000 to 2020

In the last few years, universities have been filled almost exclusively by students from Generation Y. Some teachers belong to this generation themselves. However, most lecturing university members are Baby Boomers or Generation-Xers, who are now teaching primarily Generation Y and Generation Z students. Although the Internet had its beginnings in 1969, it was still a long way from what we understand by it

[1] "Zeitgeist, from the German word meaning the spirit of the time, denotes the cultural mood, taste and outlook of a particular era. It refers to the ethos of a group of people born within a certain span of years, usually representing one generation." (Jones et al. 2007, p. 2)

today, so both generations, Baby Boomers and Generation Xers, grew up without the Internet and mostly without computers. In contrast, Millennials grew up in the early stages of the publicly accessible Internet and therefore are referred to as the "Net Generation" (Jones et al. 2007, p. 3) or "Generation iY" (Elmore 2010). The habitual way in which technologies are handled means that they are sometimes described as "technology savvy" (Cilliers 2017, p. 190). Elmore (2010), representative of Generation X, also sees a negative side to this development. He claims they would be overwhelmed, overconnected, and overserved. This assessment shows possible dissonances, which are likely to occur when different generations interact with each other.

While Generation Y was described by older sources as "relaxed and confident with all form of technology" (Jones et al. 2007, p. 3), it represents - in terms of technology - only a transitional generation and precursor to Generation Z, the "digital natives" (Dauksevicuite 2016). Generation Z has been raised in a time in which the Internet has always existed. These students do not know a world without smartphones. While computers and smartphones were still helpful tools in working and business life for Generation X, for Generation Z they have long since become everyday objects that are used seamlessly in every area of life. People from Generation Z are highly active on social media. They gather and spread information by sharing their experiences online through posts with images, videos, and links (Relander 2014) to gain popularity and self-esteem (Zhitomirsky-Geffet & Blau 2017). Their heavy consumption of digital media[2]makes them increasingly dependent on digital media. They " 'live and breathe' technology" (Cilliers 2017, p. 190). Their need for social interaction is expressed by the fact that they are used to being in constant communication through digital media (Roberts, Yaya & Manolis 2014). Nevertheless, personal interaction remains a desired method of communication (Schawbel 2014).

According to Seemiller and Grace (2016), Generation Z ascribes to itself attributes like thoughtfulness, open-mindedness, or responsibility. They see themselves as world-changers and problem-solvers who are

[2] Shatto & Erwin (2016) showed that students from Generation Z use their smartphone up to 9 hours a day.

willing to stand up for problems, which have meaning to them. To participate in the ‘Fridays for future’ protests for a more responsible approach to the environment, thousands of young people accepted reprisals in schools to reach their goals (cf. Whittaker 2019; Beck 2019).

However, Mohr and Mohr (2017) highlight some contradictions between goals and predominant characteristics of Generation Z, such as the lack of focus, lack of creativity, and their preference to work alone vs. their goal of changing something and making a difference. As a consequence, Mohr and Mohr (2017) suggest that educators “need to help Gen-Z students reconcile these possible conflicts as they negotiate higher-education experiences” (pp. 87). Additionally, the learner attention span has shortened significantly in recent years (Hallowell & Ratery 2011). So overall, the learning process and the learning environment must be designed in such a way that Generation Z students with their needs and prerequisites can develop the necessary characteristics to achieve their laudable goals.

Change in learning theories and curriculum

Certainly, the Zeitgeist has a great influence on how learning processes and learning environments are designed. Dominant learning theories act as an 'educational Zeitgeist' that influences all decisions in the teaching process. Additionally, the development of computers in the last decades has contributed to a change in learning. "How-to" videos are becoming increasingly popular and learning apps for all kinds of subjects are widely spread (Moore et al. 2017; Mosca et al. 2019). Lowyck (2014) contrasts the interplay of dominant learning theories with the design of digital learning media. He states that, in their education, Generation Xers mainly came into contact with media in a behaviorist sense, such as simple question-answer-feedback loops. He sums up the development during this time following the results of Shute and Psotka (1996): “The interplay between behaviorist learning theory and technology ultimately resulted in inflexible [...] instruction” (Lowyck 2014, p. 5).

Neuroscience and cognitive psychology brought about a turnaround in education. Since 1980, Western governments together with cognitive psychologists have tried to set up “a new science of learning” (Sawyer

2006). “The Cognitive learning theory concentrates on the conceptualization of students’ learning processes. It focuses on the exploration of the way information is received, organized, retained, and used by the brain.” (Thompson et al. 1996, p. 11) After cognitivism, it is necessary for meaningful learning to occur that students “first attend to and select relevant verbal and non-verbal information for further processing in working memory. Then, students need to organize the multiple representations into a coherent mental model and integrate the organized information with their prior knowledge.” (Moreno & Mayer 2007, p. 313) Thus, the insights in brain research on information processing led to a shift from the generalizing behavioristic view of stimulus-response learning to a complex learning process, which takes place in the learner himself and is therefore highly individual.

Cognitive psychology principles have also strongly influenced the development of constructivist theories, which are widely accepted today (Lowyck 2014), thus describing the current ‘educational Zeitgeist’. “From a constructivist perspective, knowledge is not conceptualized as a body of information based on verified facts but, rather, as individually constructed by observation and experimentation.” (ibid., p. 8) Constructivism emphasizes the individual construction of knowledge as a result of interactions with the outside world. The early constructivism eventually developed into a socio-constructivist theory (cf. Lowyck 2014). It highlights the importance of interactions with others and recognizes that they are a key component of learning, so it is in line with Generation Z's strong desire for communication, which was highlighted in the previous section.

A shift in the curriculum: Competence-Orientation and Autonomy

In order to meet the demands of the modern information society, it has been recognized that the focus of mathematics learning in schools and universities must be shifted. Mathematics education should no longer be primarily thought of in terms of the mathematical content to be learned, but should instead be oriented towards *mathematical competence* (general concept) or *mathematical competencies* (context- and task-related) that need to be acquired (for a detailed account of this development and the basic terms see Niss & Højgaard, 2019). Therefore, mathematics education curricula entail standards to ensure the new orientation towards competence or skills (e. g., KMK 2004; OECD

2018; Skolverket 2011). Rychen and Salganik (2003, p. 43) define the construct of competence as "domain-specific cognitive dispositions that are required to successfully cope with certain situations or tasks, and that are acquired by learning processes." The goal of education is therefore that students build up competencies in various domains making them "active, responsible and engaged citizens" (OECD 2018, p. 4) and thus life-long learners. For life-long learning to happen, a certain "autonomy" (cf. Ryan & Deci 2000), and as a prerequisite also "agency", is necessary, even already in their own education (OECD 2018). Lee and Hannafin (2016) define "autonomy as one's self-endorsed will to make one's own decisions and take voluntary actions" (p. 715). It concerns "a sense of initiative and ownership of one's actions" (Ryan & Deci 2020, p. 2) and implies a sense of responsibility to participate in the world and influence the environment. However, autonomy should not be misunderstood as merely acting independently. It can also occur when people voluntarily and reflectively follow certain expectations, rules, constraints, or demands (cf. Lee & Hannafin 2016). Similarly, it occurs when students actively collaborate with the teacher in organizing their learning process.

In accordance with the above-presented background and concerning research findings, Ryan and Deci (2000, 2020) formulate in their Self-Determination theory three fundamental psychological needs: the need for competence, relatedness, and autonomy. "Competence concerns the feeling of mastery, a sense that one can succeed and grow" while "relatedness concerns a sense of belonging and connection. [...] Thwarting of any of these three basic needs is seen as damaging to motivation and wellness." (Ryan & Deci 2020, p. 2) Therefore, when designing a learning environment, these basic needs of students should be considered.

A model for learning environments

As described above, neither knowledge nor competence can be transferred directly from the teacher to the student. Some kind of mediator provided by the teacher and used by students is needed. Ulm (2010) names this mediator "learning environment", which is an "essential link between the teacher and the learner". In his model, he indicates that an ideal constructivist learning environment is based around an

open learning system that is student-centered, content-centered, task-centered, and community-centered with widespread media and technology availability (see Fig. 1).

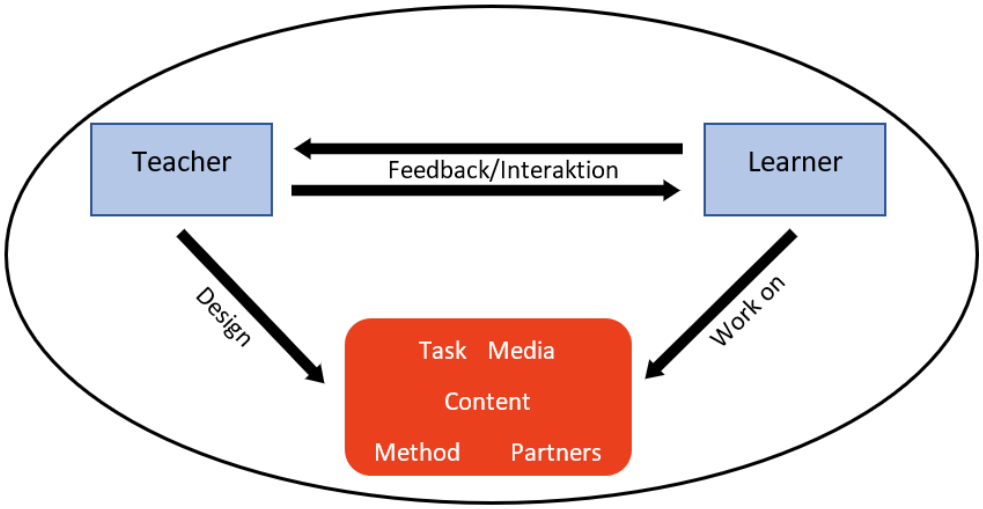


Fig. 1 *Learning environment (Own illustration after Ulm 2010 and Verschaffel et al. 2020)*

This corresponds well with the wish of Generation Z students to become active participants in the learning process (Mosca et al. 2019). Mosca et al. (2019) conclude from a literature review of Generation Z characteristics:

> An optimal learning environment should foster pedagogical components that encompass information, performance, situations, and hands-on experiences along with collaborative activities with other students, which allows for the exchange of knowledge. It is the exchange of knowledge and the interactive experience, coupled with hands-on reinforcement, that results in the greatest individual learning. (p. 67).

They emphasize the important aspect of student interactions with the content, as well as interactions between students, without which learning from a socio-constructivist perspective hardly can take place. This principle must be considered when designing the learning environment.

Overall, however, it must be noted that the term “learning environment” is used very broadly in literature. Learners and teachers can influence the learning environment from outside as in Ulm's (2010) def-

inition or be part of the learning environment. For instance, Verschaffel et al. (2020) describe teachers as one important element of the learning environment, activating students' in different instructional techniques, like classroom discussions, or small group work. The learning environment is hence looked upon as a place "in which the actions of the student or the teacher take place" (Brousseau & Gibel 2005, p. 16). The latter interpretation has the advantage that one can generally speak of a learning environment when looking at the design of a lesson or course. In this paper, we follow the more general approach to learning environments (see Fig. 1).

Designing and evaluating (constructivist) learning environments

Due to the high individuality of learning, many constructivist frameworks for planning teaching, in the sense of student-centered learning environments, focus on the activity of learners. The learning process is always based on the interactions of an individual student with the other two central actors of the learning process, the content, and the teacher. In the following, two models are presented and classified in Klafki's perspective chart for planning teaching (1985). Finally, a supplementary model for planning teaching is developed.

A theoretical framework with a focus on the mixture of interactions (Anderson 2003)

More than 40 years ago, Daniel and Marquis (1979) challenged educators to choose the right mix between self-study phases and interactive and collaborative teaching methods. Anderson (2003) takes up this demand 20 years later and underlines its relevance by outlining further developments and resulting possibilities in the digital field. In particular, he emphasizes the transformation of the "nearly ubiquitous (in developed countries) Net-based telecommunications system" (p. 1). He also emphasizes the importance of interactivity for the learning process. For him, learning results from three different interactions, which he takes over from Garrison and Shale (1990). They interpret "all forms of education (including that delivered at a distance) as essentially interactions between content, students, and teachers" (Anderson 2003, p. 2). As a consequence of these considerations, one obtains the didactic triangle known in different ways of representation, which contains interactions between the three major actors involved in the learning process (see Fig. 2). In this paper, the main focus will be on student-student interactions, student-teacher interactions, and student-content interactions, as the most important ones.

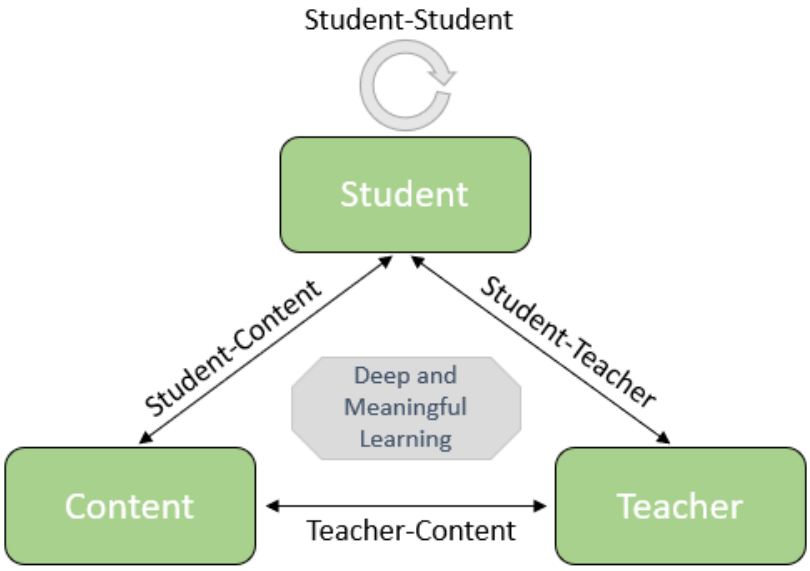


Fig. 2 *Interactions in the didactical triangle (after Anderson 2003)*

A practical framework for the design and evaluation of (online) learning modules

From the side of learning theories, Bradley (2011) approaches the construction of online learning units with a focus on interactions. He is interested in a framework for creating "self-standing lesson[s]" (p. 197) as part of an entire semester course. His framework (see Fig. 3) is based on Anderson's (2003) model and places the three types of student interactions at the center saying "the conduct of the course revolves around student interactions with the instructor, other students, and the course content" (Bradley 2011, p. 200). Bradley does not regard interactions as rigid and disjunctive areas but assumes that they flow into each other as soon as the learning module starts. For him, the primary question of course design is: "What is the best weighting of the student interactions?" (ibid., p. 200). He sees the answer to this question mainly influenced by the choice of valid learning theories, the available software and the content to be taught. Finally, he discusses numerous digital tools that can facilitate the implementation of an online course. However, Bradley does not address a further analysis of the target group or the design of the overall course structure itself. The resulting problem will be discussed in the next section using a model by Wolfgang Klafki.

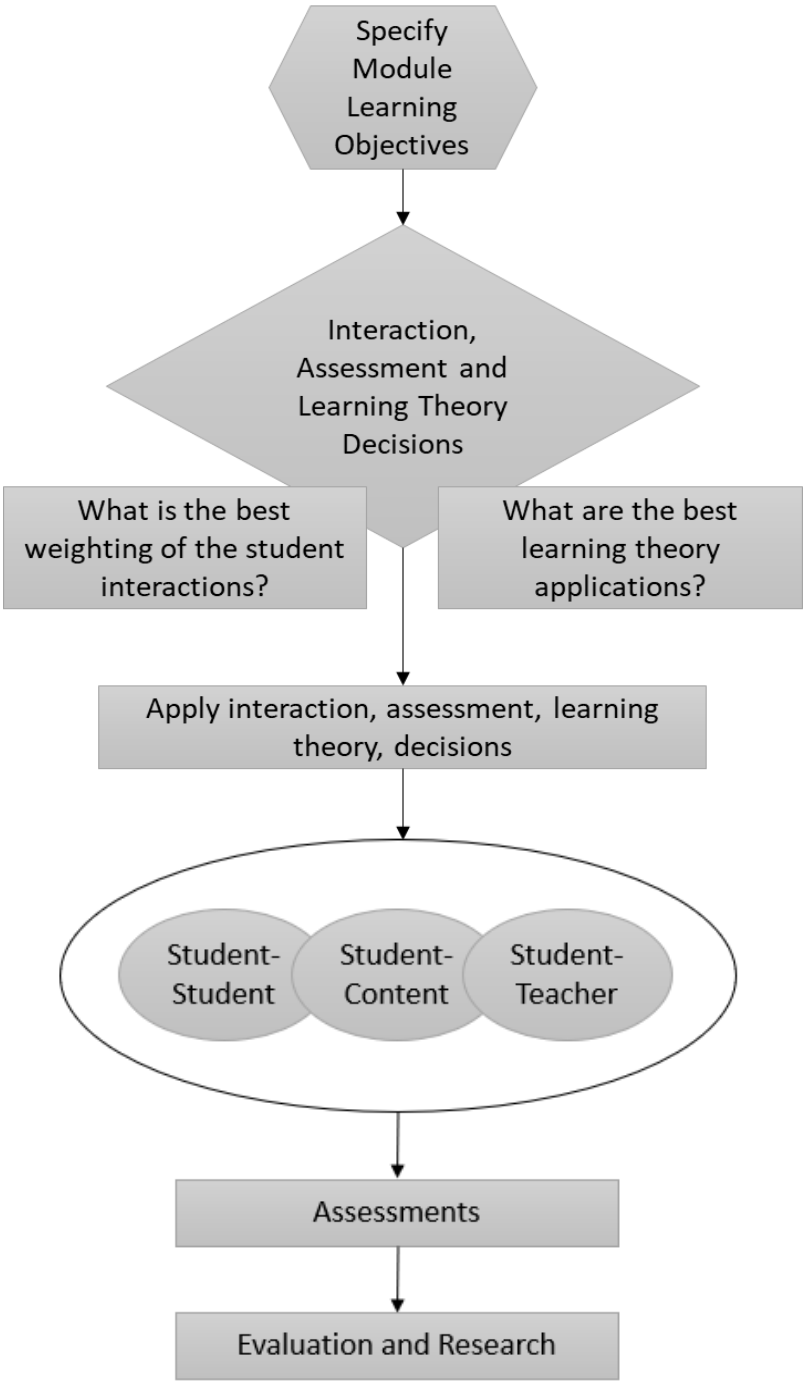


Fig. 3 *Conceptual Framework for Learning Module Design (after Bradley 2011)*

Klafki's (1985) detailed model for planning teaching

The German pedagogue Klafki (1985) used a still applicable "very general concept of didactics, in which he included the curriculum theory and the learning theory or methodology" (p. 194). As early as 1985,

Klafki saw the development from a "humanistic to a critical-constructive didactic" (ibid., p. 32) as necessary. He attributes the need for change to findings in the field of general education and psychology, which have also led to the replacement of behaviorist theories as the predominant ones by cognitive and constructivist learning theories. The principles of his "critical-constructive" didactics are not only represented in many of today's German curricula but can also be found implicitly and explicitly in other countries (cf. Köker & Störtländer 2017). Klafki calls his approach "*critical*" because his didactics are based on "the goal of empowering all children and adolescents [...] to a growing capacity for self-determination, co-determination, and solidarity in all dimensions of life, while at the same time taking seriously the fact that the reality of educational institutions often does not correspond to this goal" [3] (pp. 37–38, translated by the authors). He sees the term "constructive"[4] as a hint to the consistent practical application, the interest in action, design, and change, which is constitutive for this didactic concept (ibid., p. 38). He seeks a constant connection between theory and practice so that his theoretical explanations, including his model for planning teaching, which is the subject of the following, are directly applicable in practice. In his "Perspektivenschema zur Unterrichtsplanung" (Engl. perspective chart for planning teaching, see Fig. 4), Klafki addresses seven questions that are assigned to four didactical subareas. He regards the numbering as a chronological possibility, but by no means as binding.

[3] „Ziel der Befähigung aller Kinder und Jugendlichen [...] zu wachsender Selbstbestimmungs-, Mitbestimmungs- und Solidaritätsfähigkeit in allen Lebensdimensionen orientiert, zugleich aber den Tatbestand ernst nimmt, daß die Wirklichkeit der Bildungsinstitutionen jener Zielsetzung vielfach nicht entspricht."

[4] The term "constructive" should not be confused with the term "constructivist", even though Klafki's views are quite close to those of the socio-constructivist learning theory. He understands learning and teaching as a process of interaction in which the learners are to become increasingly independent (cf. Klafki 1985, p. 199). In addition, he emphasizes that learning is always a social process in which different actors with individual biographies are involved (cf. ibid., p. 200).

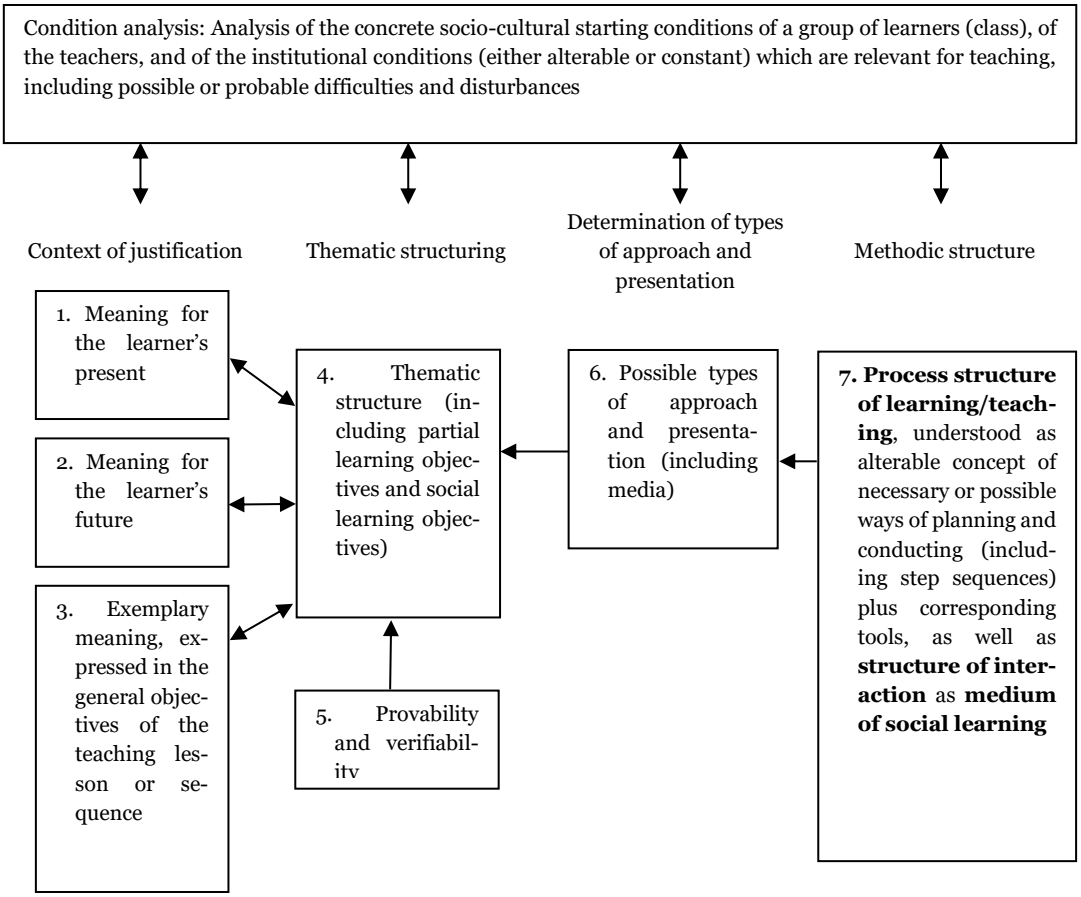


Fig. 4 *Klafki's Perspective chart for planning teaching (after Klafki 1985, p. 215)*

The arrows indicate that questions can influence each other (peeks on both sides) or just one side influences the other (peek on one side). Each of the questions is closely linked to the important "Bedingungsanalyse" (Engl. condition analysis), which Klafki added to his original model after criticism from numerous colleagues. Every decision in the teaching-learning process has to be reflected upon this analysis. It should be noted that Klafki's condition analysis for teaching also includes an analysis of the three major actors mentioned in the didactic triangle of Anderson and Garrison (1998), which is used by Anderson (2003) and Bradley (2011). However, neither of them does integrate such an analysis. According to Klafki (1985, pp. 208–227), a complete model of planning teaching has to include at least three functions that can be assigned to different levels (see Fig. 5). Therefore, the lower levels serve as the basis for the levels above.

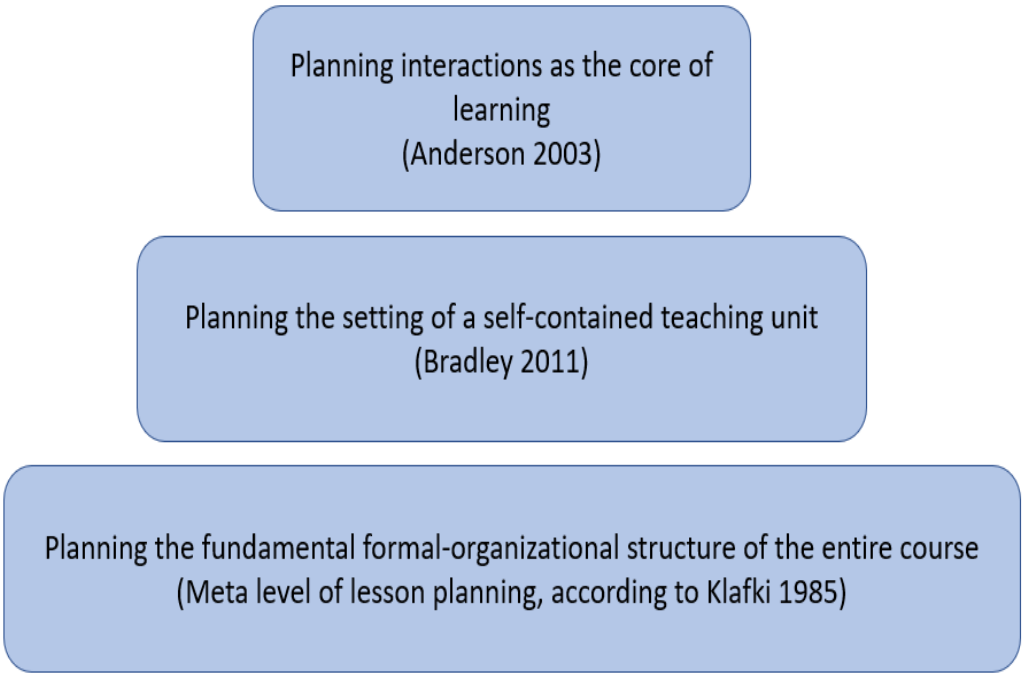


Fig. 5 *Hierarchy of planning teaching*

Concerning the teaching methodology, it is necessary to look at point 7 in more detail. It "focuses on how the moments identified by the previous questions can be translated into a *successive sequence of a teaching-learning process* or into *alternative possibilities of such sequences*. [...] At the same time, the question is always asked about the *forms of interaction* in which the sequence of the teaching-learning process can take place" [5] (Klafki 1985, p. 226, translated by the authors). At this point, the models of Anderson (2003) and Bradley (2011) come into play. Anderson (2003) specifically focuses on the smallest unit of learning, the interactions, while Bradley (2011) goes one step further and integrates Anderson's theory into his framework for the design of self-contained learning modules. Applying this to classroom

[5] „richtet sich darauf, wie die durch die vorangegangenen Fragen ermittelten Momente in eine *sukzessive Abfolge eines Lehr-Lern-Prozesses* bzw. in *alternative Möglichkeiten solcher Abfolgen* übersetzt werden können. [...] Gleichzeitig wird damit immer nach *Interaktionsformen*, in den sich die Abfolge des Lehr-Lern-Prozesses vollziehen kann, gefragt."

teaching, Bradley's model can be used to plan specific lessons or sequences. It is thus found on the second level of planning teaching (see Fig. 5).

Klafki (1985) himself implicitly speaks of a meta-level of planning teaching. Planning teaching requires a concept of meaning, in other words: the general aim of the teaching to be planned for and the *basic structure* of the teaching. Especially concerning teaching methodology, he sees the problem as being "whether the *organizational and executive forms* of teaching enable adequate learning" (1985, pp. 205–206, translated by the authors). This means that in addition to the important theories of Anderson (2003) and Bradley (2011), the right conditions on a meta-level must be given so that Anderson's interactions in Bradley's individual self-contained teaching units can have their full effect.

Framework for analyses and evaluation of course design

A study by Mosca et al. (2019) with over 14,000 faculty members shows that the necessary framework conditions are not always given. They found out: “students want to be actively engaged in the learning and benefit when instructors provide space for students to flexibly interact with others who are also allowed to problem-solve on their own” (p. 73), but much of the learning time is spent as a presentation from the front, so there is little time for discussion or group work. To create the right conditions for the use of versatile interactions, a model will be presented as a planning aid as well as for the analysis and evaluation of course structure.

Teaching and Learning Rooms

As *Teaching* and *Learning Rooms,* we generally understand the larger organizational room, in which the three types of interactions that contribute to learning can take place. In differentiation to the term learning environment, these rooms refer to a superordinate organizational basic structure of a teaching format (course), which in Klafki's sense is decisive for the further selection of the methodology. Experience-based we distinguish between three types of Teaching and Learning Rooms occurring (side by side, or in sequence) in mathematics courses, in which learning and teaching take place (see Fig. 6).

Fig. 6 Teaching and Learning Rooms

The *Teacher's teaching Room* is probably most widespread in traditional lecture-based teaching scenarios. In this room, the teacher is steering the learning process. Methods, content, tasks, and learning pace are all determined and controlled by the teacher. Therefore, from the student perspective, the degree of external structuring is high and

the “instructional locus of control” (Hannafin 1984) is external. In our context, external locus of control occurs when all learners follow a predetermined path established by the designer without being able to decide whether this path is appropriate for them. Internal locus of control is demonstrated in courses where the individual learner controls the path, pace, and/or contingencies of the instruction, typically by specifying choices among a range of designer-embedded options (cf. Hannafin 1984, p. 6). Thereby, instructional locus of control can be seen as a continuum ranging from full teacher control to full learner control. An example of teaching and learning situated in the Teacher’s teaching room is lecturing in a traditional way. Here, the learning process is determined by the teacher as he has to be the Initiator for almost all interactions.

The *Self-studies Room* can be described intuitively as the student's space in which self-regulated learning takes place and the responsibility for organizing the learning process is on the student’s side. If interaction with the teacher or other students is desired, the initiative always has to come from the individual learner. The level of autonomy is high and, accordingly, the degree of external structuring is low. Examples of activities in this room are preparation or follow-up of lectures or the search for additional material. Here we need to point out that follow-up lessons, based on questions and answers mainly may be characterized as part of the self-studies room. But teachers are also steering the content and tasks in these kinds of lessons, for instance by determining the workload, the list of suggested tasks, or examples.
Students often come together informally and form learning groups. Here they give up their autonomy gradually and enter the *Peer teaching room*. In this room, both teaching and learning take place, most of the time simultaneously. The degree of the external structuring is low and the initiative for occurring interactions is shared among the students.

Using this analytical tool, courses can be developed, analyzed, and evaluated with regard to their general structure. As stated above, the structure can either hinder or favor the use of specific interactions. For example, frequent student-student interactions are more difficult to implement in the *Teacher's teaching room* than in the *Peer teaching*

room. Thus, on the meta-level, the choice of the general course structure at the beginning of the planning process is an important factor for individual lessons and interactions. The characteristics of the different Learning and Teaching rooms are summarized in Table 2.

Table 2. Characteristics of Teaching and learning rooms

	Teacher's Teaching Room	Peer Teaching Room	Self-Studies Room
Instructional locus of control	External	Internal	Internal
Degree of external structuring	High	Low	Low
Initiator of interactions	Teacher	Different students	Individual student
Favored types of interactions	Student-Instructor	Student-Student	Student-Content
Responsible for organizing the learning process	Teacher	Several students	Individual student
Level of autonomy	Low	High	Very high

Graphical visualization of the course structure

The above statements as well as the presentation of the analysis of course design will be illustrated in the following. The closed ellipse symbolizes a whole course, and the parts are symbols for what relative importance it is given in relation to other aspects (e.g., where the mathematical content is made visible for the learner). The size of the ellipses is not proportional to the actual time on tasks. A lecture-based course

at university may have the following superordinate organizational basic structure (see Fig. 7):

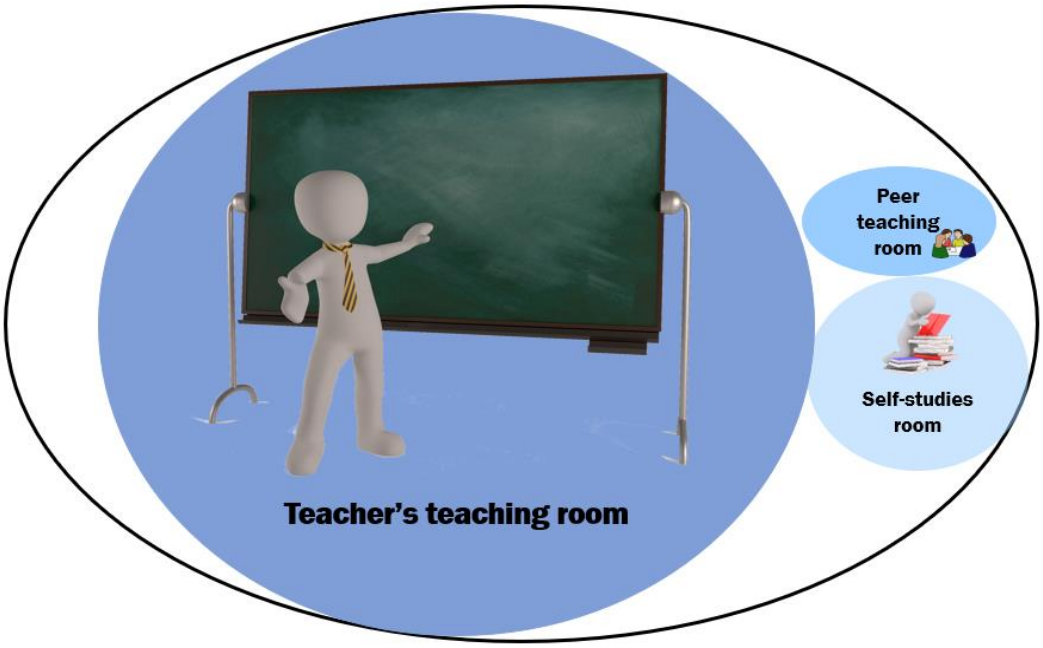


Fig. 7 *Visualization of a traditional lecture-based course's structure*

At the center of such a course is a lecturing teacher, who tries to transmit as much knowledge as possible. In this phase, the control throughout the lesson and the responsibility for interactions that take place lies on the teacher's side. A large part of the event is therefore spent in the *Teacher's teaching room*. The remaining time is filled with independent preparation and follow-up work in the *Self-studies room*. This phase of the learning process is characterized by the fact that the responsibility lies on each individual student. The student controls how much time he or she invests. Some students then decide to form learning groups where the responsibility for interactions that occur is divided among them. Hence, some of the learning processes are going on in the *Peer teaching room*.

If there are interactions in the Teacher's teaching room, these are mainly student-teacher interactions (e.g., questions and answers). It should be noted that student-student interactions and student-content interactions are very well possible, but the organizational structure of this room creates certain obstacles for these types of interactions. Students have become accustomed to the fact that they are rather passive

consumers (Lee & Hannafin 2016, p. 711). It is therefore questionable if the learning time in this room is used effectively because very little, or no interactions occur. Similarly, there will be hardly any teacher-student interaction in the other Teaching and Learning rooms. In return, student-content interactions will be more common in the Self-studies room, while student-student interactions will probably be the dominant type of interaction in the Peer teaching room.

Competence-oriented teaching without a competence-oriented course design

As shown above, there has been a shift over the last 15 years from solely focusing on the acquisition of disciplinary concepts to focusing more on competence and therefore on agency and autonomy. Nevertheless, lecture-based teaching remains the dominant teaching style, at least in university mathematics (cf. Petropoulou et al. 2016). However, many mathematics teachers are aware that without real interaction, a deeper understanding is not possible. These interactions are to be enforced by activation phases in the lecture or by fixed and partly obligatory exercises. However, the lack of autonomy on the part of the students inhibits their motivation and thus ultimately the learning success (Ryan & Deci 2020). Reinforced by the structural obstacles, students are far away from being "active, responsible and engaged" (OECD 2018, p. 4), which would be important for building up competence and the ability for life-long learning. It would make more sense to balance the lecture-based teaching style by implementing other teaching methods as well. The responsibility and control for the learning process must be given up partly and handed over to the students. Educators need to shift from their role of "sage on the stage" to that of "guide on the side" (Dlalisa & Govender 2020, p. 65). The increased autonomy of students is a characteristic of student-centered teaching and learning environments (see, e.g., Hoidn & Klemenčič, 2020), which will be illustrated in the following.

Analysis of alternative modern course layouts

> When preparing a lesson plan to engage students in the learning process, instructors need to create learning spaces that foster a student-centered environment. For the most part, the traditional classroom setting with seats/desks/tablets in a fixed lined up position facing the instructor at the front of the room does not give rise to student empowerment or an engaging environment. (Kolb 2015, cited in Mosca et al. 2019, p. 71)

The quote shows the necessity of making the entire learning environment, whose superordinate organizational structure is defined by the degree of using the Teaching-Learning rooms, interaction-friendly. This is exactly what contemporary approaches like blended learning, flipped classroom or problem-based learning try to do. They not only transfer a large part of the responsibility to the learner but also create an interaction-friendly environment, where autonomous and self-regulated learning is unavoidable. In addition, they increasingly use digital platforms and offerings with which students from Generation Z are well acquainted. When analyzing the approaches mentioned above, the strength of the model becomes clear. Common features of these teaching concepts that have proven themselves in recent years (see references below) are clearly depicted.

Blended Learning

At its simplest, blended learning is characterized by a combination of classroom and online phases. The advantages of synchronous classroom teaching and asynchronous online teaching can be exploited. However, by designing a blended learning course, such phases should not simply be arranged in a sequence isolated from each other. A blended learning design "represents a fundamental reconceptualization and reorganization of the teaching and learning dynamic, starting with various specific contextual needs and contingencies (e.g., discipline, developmental level, and resources) [...]. What makes blended learning particularly effective is its ability to facilitate a community of inquiry" (Garrison & Kanuka 2004, p. 97). Garrison and Kanuka then refer to three necessary components of the "community of inquiry" (social presence, cognitive presence, and teaching presence), which can be identified with the Teaching and Learning rooms. Of particular interest is the transfer of responsibility for the learning process to the learners themselves. In the online phase, the teacher provides material, but the

responsibility for dealing with the presented material now lies with the learner, so that this part of the learning process shifts from the Teacher's teaching room to the Self-studies room. For the visualized course structure, this results in a composition that is rather balanced between the Teacher's teaching room and the Self-studies room (see Fig. 8).

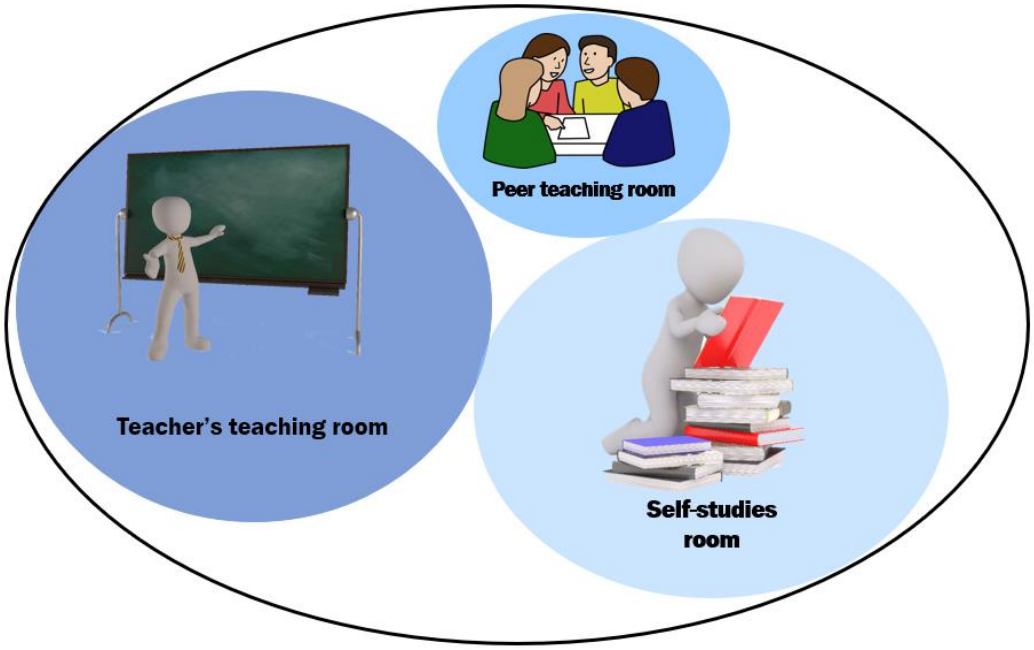


Fig. 8 *Visualization of a Blended learning course's structure*

Flipped classroom

The Flipped (or inverted) classroom method is a special case of Blended learning. A very simple definition is given by Lage et al. (2000), who say: "Inverting the classroom means that events that have traditionally taken place inside the classroom now take place outside the classroom and vice versa" (p. 32). This definition explains the terminology of this method but hits its core only superficially. Bishop and Verleger (2013) state in their meta-study that flipped classroom is mainly characterized through "interactive group learning activities inside the classroom, and direct computer-based individual instruction outside the classroom" (p. 4). The course structure shifts from a traditional structure to one that gives learners more autonomy and responsibility for interactions and their learning process. "Students may be ill-prepared to assume greater autonomy for their learning" (Lee & Hannafin 2016, p. 711), so "instructors need to learn how to relinquish control and support students to become the owner of their learning" (ibid., p. 711). This results in a course focused on peer teaching and self-studies. The different

rooms intersect each other as the students study the provided content in autonomy, and finally, discuss and apply it in a small collaborative group in the classroom, all supported by the teacher (see Fig. 9).

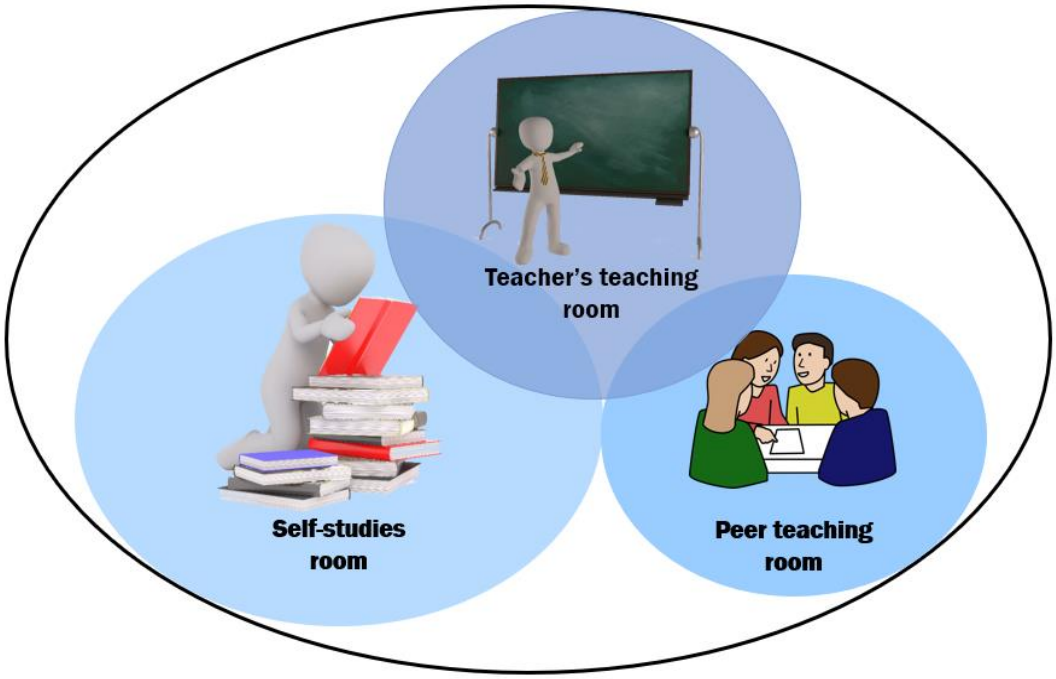
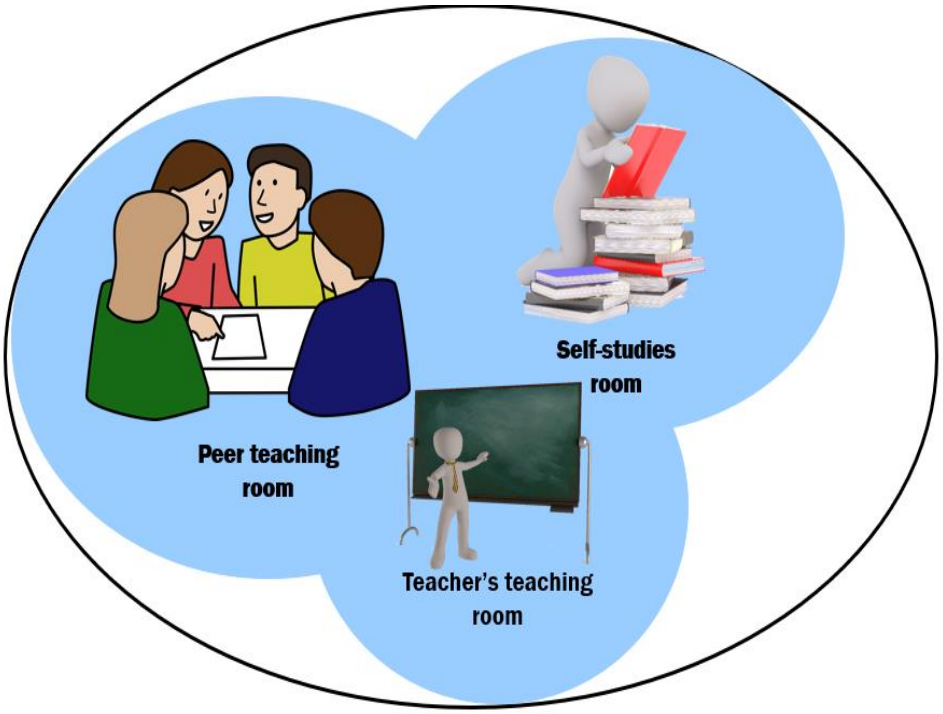


Fig. 9 *Visualization of a flipped classroom course's structure*

Problem-based Learning (PBL)

Problem-based learning "is an instructional (and curricular) learner-centered approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem" (Savery 2015, p. 7). Hmelo-Silver (2004) describes problem-based learning as a teaching method in which students have to solve a complex task where there is not only one correct way to solve it. She sees it as a decisive characteristic that students have to work cooperatively in small groups, but also have to learn in a self-regulated way to solve a problem. Therefore, problem-based learning seems to be a suitable method, especially with a competence-oriented approach. Barrows (1988) sees as an additional defining characteristic of PBL a "tutorial process", in which teachers accompany and support learners in the learning process. In PBL, all three Learning rooms even fade into each other. The illustrated course design is thus as follows (see Fig. 10):



Fia. 10 *Visualization of a Problem-based learnina course's structure*

Managing the balancing act between autonomy and support using learning tools

The advantages and disadvantages of modern course designs, which demand more autonomy from students, are discussed extensively in literature (cf. references in Lee & Hannafin 2016, pp. 711–712). While the concerns that can be attributed to the Teacher's teaching room and the Peer teaching room are limited, the main criticisms of the above modern formats tend to be found related to the Self-studies room. The key argument against self-regulated course designs is the organizational and cognitive overwhelming of the learners. Studies show that (math university) students are often incapable of working autonomously (Winsløw & Grønbæk 2013). Furthermore, the sheer endless flood of information on the Internet can easily lead to cognitive overload of learners (cf. Scheiter & Gerjets 2007, pp. 291–292). However, as already shown, numerous national education curricula urge the promotion of autonomy and agency, so that lifelong learning and self-regulation can become a reality. As a consequence, teachers have the task of managing this balancing act between demanded autonomy and given support. To give the reader an impression of what can be done in this difficult area of intersection between the Teacher's teaching room and the Self-studies room, three ongoing projects will be briefly presented which aim to foster and support self-regulated, autonomous learning (see Fig. 11). That implies that teachers need to take a supportive role also in the self-studies room. More details for the individual projects can be seen in the related references.

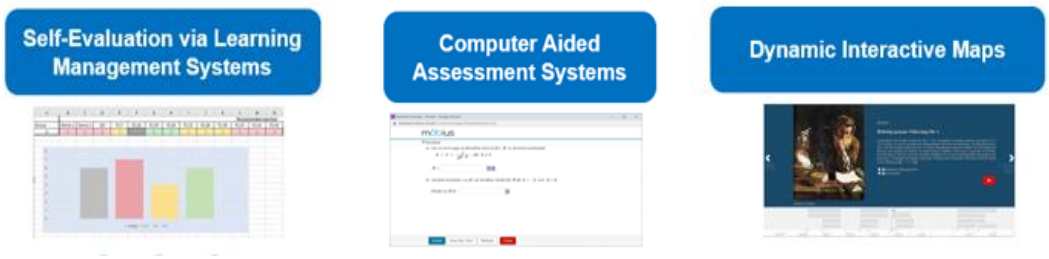


Fig. 11 *Ongoing projects for the intersection of Teacher's teaching room and Self-studies room*

In the first project (cf. Mellroth et al. 2019), students have the opportunity to use a self-evaluation app in which their weekly assignments are uploaded and evaluated anonymously. The group results in the app can be used as a reference point for teachers to choose additional formative resources and adapt courses to the needs of students.

The second project focuses on supporting the process of self-regulation. Using a computer-aided assessment (CAA) system, students can receive automated detailed feedback and individually tailored exercises. Through the assistance in the selection of exercises, as well as the automated corrections, the students are relieved in their learning process and thus the risk of cognitive overload is reduced. The first step in this project had a focus on the design of tasks where a dynamic mathematics program (GeoGebra) and a "CAA system" (Möbius) were combined (cf. Brunström et al. 2020). Knowing more about how different tasks work and what type of response one can expect from these tasks allows focusing more on the construction of feedback in the second step of the project.

The third project deals with the pre-structuring of information necessary for a course. In interactive digital maps (cf. Przybilla et al. 2021), content on a mathematical sub-area is presented, which is important for a course, for example in the course ”history of algebra”. In this way, students only have to search for and find important information for a problem within a predefined frame and not within the endless worldwide web.

Conclusion

Due to the findings of modern pedagogy and learning psychology, many mathematics teachers feel urged to move away from the instruction- and lecture-centered teaching style. The learning requirements of students from Generation Z further increase this pressure. Recent approaches of course designs have a strong focus on the learner's needs. They try to satisfy the need for autonomy, relatedness, and competence while shifting control for the learning process over to the learner. In this context, the formal-organizational structure of the course either favors or hinders the interactions that are essential for learning. When designing a course, care should be taken to ensure that the Peer teaching room and Self-studies room are given enough space. The use of digital media and learning tools facilitates efficient use of interactions within the rooms and helps to connect especially the Teacher's teaching room and the student's Self-studies room. The model of the Teaching-Learning rooms should be placed at the beginning of the planning process of lessons and lectures so that learning modules, interactions, and other specific measures can be taken in harmony with the superordinate structure of the course.

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Rooms of Learning

A conceptual framework for student-centered teaching development in a digital era

In the context of teaching, there is often a clash of different generation cohorts, which have different attitudes to and skills in digital media and corresponding ways of teaching and learning, respectively. The current COVID19 situation works as an intensifying lens and shows the problems occurring in this context even more clearly. Many mathematics teachers feel urged to move away from the instruction- and lecture-centered teaching style and many universities are upgrading digitally, thus creating the conditions for innovative teaching.

Considering the change in learning theories and education curricula, we offer a general model called "Rooms of Learning" as a quick-to-see-and-interpret framework for the pre- and post-analysis of course design. It is backed by subject didactical frameworks that focus on interactions between the main actors involved in learning.

As starting points, we use the basic needs of today's learners for autonomy, relatedness, and competence as well as the requirements of the present information society. Focusing on different levels of the learners' autonomy, as the key factor for building up mathematical competence and life-long learning, we illustrate our practice-oriented model by analyzing different course layouts. Herein the strength of the model is demonstrated, as commonalities of the modern course designs successfully applied today are revealed. It suggests, when designing a course, care should be taken to ensure that the defined Peer teaching room and Self-studies room are given enough space.

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