

COMPUTATIONAL AND ACCURACY BENCHMARKING OF SIMULATION AND SYSTEM-THEORETIC MODELS FOR PRODUCTION SYSTEMS ENGINEERING

Bachelor Degree Project in Industrial Engineering
Bachelor level 30 ECTS
Spring Term 2021

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Certification

This thesis has been submitted by Antonio José Ramos Calderón to the University of Skövde as a requirement for the degree of Bachelor of Science in Production Engineering.

The undersigned certifies that all the material in this thesis that is not my own has been properly acknowledged using accepted referencing practices and, further, that the thesis includes no material for which I have previously received academic credit.

A handwritten signature in black ink, appearing to read 'Antonio', with a long horizontal stroke extending to the left.

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Abstract

The modern industry has an increasing demand for simulation software able to help workers and decision-makers visualize the outputs of a specific process in a fast, accurate way. In this report, a comparative study between FACTS (Factory Analyses in ConcepTual phase using Simulation), Plant Simulation, and PSE (Production System Engineering) Toolbox is done regarding their capacity to simulate models with increasing complexity, how accurate they are in their outputs with different optimized buffer allocations, and how well they perform on the task of detecting the bottlenecks of a process. Benchmarking simulation software requires an experimental approach, and for gathering and organizing all the data generated using external programs like MATLAB, C, Excel, and R are used. A high level of automatization is required as otherwise the manual input of data would take too long to be effective.

The results conclude on major concordances among FACTS and Plant Simulation as the most used commercial DES (Discrete Event Simulation) software and a more mathematical-theoretical approach coming from PSE Toolbox. The optimization done in the report links to sustainability, with an enhanced TH improving the ecological, social and economic aspects, and to Lean philosophy using lean buffers that smooth and improve the production flow.

Keywords: DES, Benchmarking, Lean Buffer, Optimization, Simulation software, Performance measure, Plant Simulation, FACTS, PSE Toolbox.

Acknowledgments

I would like to direct my gratitude to my supervisor Amos Ng, for the wonderful guidance and knowledge given, without whom this thesis would not have delivered the same results.

Special thanks to Sunith Bandaru, for his role as an examiner at the University of Skövde.

Another thought goes to Simon Lidberg, for answering my questions when needed in Plant Simulation.

Finally, I want to thank my family, who always offered unconditional support through the completion of my academic studies.

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List of abbreviations

AGV	Automated Guided Vehicles
BL	Blockage of Machine
C	Machine Capacity
CI	Confidence Interval
DES	Discrete-event simulations
E	Efficiency of the line
FACTS	Factory Analyses in ConcepTual phase using Simulation
JIT	Just In Time
LT	Lead time
M	Machine
MTTR	Mean time to repair
N	Buffer allocation
p	Efficiency/probability of machine being up
PR	Production Rate
PSE	Production System Engineering
SCORE	Simulation-Based-CONstraint Removal
SD	Standard Deviation
SMO	Simulation-based multi-objective optimization
ST	Starvation of Machine
TH	Throughput
TNB	Total Number of Buffers
WIP	Work in process

1. Introduction

In this chapter, the background and the problem description will be presented.

1.1. Background

Accuracy and computational efficiency of different modeling methods for studying production system performance are crucial if the results are used for decision-making and implementation. Inaccurate results can lead to not only investment loss but more long-term financial and business consequences (e.g., insufficient capacity to meet the demand) to the manufacturing companies. While simulation is the most popular modeling method used in the manufacturing industry (Tempelmeier, 2003), mathematical modeling methods, like system-theoretic models developed by MIT (Gershwin, 1994) and Michigan University (Li and Meerkov, 2008) can also be applied to real-world production systems engineering. Cross-verifying the accuracy of different simulation software related to the accuracy of system-theoretic models when compared to discrete-event simulations (DES) has not been fully researched so far. This takes special relevance for realistic industrial production lines, for example, the automotive industry in Sweden.

In this study, three example modeling methods will be used in an experimental research to study how accuracy and computational efficiency can be cross-verified and analyzed:

- FACTS (Factory Analyses in ConcepTual phase using Simulation) is originally a Swedish research project supported by the Swedish automotive industry. FACTS Analyzer is its major software tool, and it is specifically designed for supporting factory design, analysis, and optimization during the conceptual design phase.
- Plant Simulation by Siemens helps create digital models of logistic systems (e.g., production) to explore the systems' characteristics and to optimize their performance.
- The PSE (Production System Engineering) Toolbox is focused on a more theoretical view of simulation, although it can also be used for industrial purposes.

The aim of the project is, therefore, to compare and cross-verify this simulation software introduced above in a designed experimental research framework.

1.2. Problem description

Over the years, many simulation projects have been conducted by students and production engineers in the industry using FACTS Analyzer. It has been developed as an efficient, conceptual simulation software for the optimization of production systems. Still, nowadays, its computational performance has not been verified against other software like Plant Simulation. It is of special interest the scalability of performance: how the computational efficiency is affected when the scale and/or complexity of the models gradually increase.

Furthermore, different simulation software will provide results and outputs in different ways. Part of this report will be focused on 'lean buffers' (introduced in chapter 3.2). Only PSE was able to work with the buffer allocation directly, making the combination of all mentioned simulation software above a must.

1.3. Aim and objectives

This project aims to design an experimental research framework for the benchmarking of different types of models/software for production systems engineering. The objectives to achieve this aim are:

- A brief literature review on system-theoretical modeling, simulation and simulation-based optimization.
- Design benchmarking experiments relating to the computational burden, lean buffer design, and bottleneck detection, considering the input and output data complexity as well as the model scalability.
- Conduct empirical simulation and optimization experiments on the benchmark models.

1.4. Extend and limitations

The report has a delimited extend, described in the next paragraphs:

- The parameters to cross-verify will be the throughput (TH), the work in process (WIP), the buffer allocation, the lead time (LT), the accuracy of the simulations, and the time needed to run them.
- Only simple lines with identical Bernoulli and exponential machines are going to be studied.

Of course, the simulation software also has some limitations themselves when it comes to experimenting. For example, it is not possible to include the Mean Time to Repair (MTTR) in PSE Toolbox using the Bernoulli distribution for machines, while Plant Simulation does not allow to introduce any desired line efficiency.

Due to license issues, experiments relating PSE Toolbox cannot contain more than 5 machines in the model.

1.5. Sustainability

Models that accurately describe the world or a system, using their parameters, are crucial to analyze sustainable development. Three aspects are generally used for describing such sustainable development: *economic*, *ecological*, and *social* sustainability (Kuhlman & Farrington, 2010).

Economic aspect: Nowadays, there is pressure to increase the efficiency of production systems due to the competition among different production networks. Simulating the material flow detailing

production, storage, and transport is key to develop better production systems, as it can lead to reduced TH and inventory. Enhanced productivity of existing production facilities by 15-20% can be achieved in real-life projects with a correct simulation (Bangsow, 2015) so choosing the right simulation software is crucial for this matter.

Ecological aspect: Englobes everything linked to the ecosystem. Using the appropriate simulation software will lower the waste and the time used, as stated before. A direct example of this can be seen in the term of the electricity used during the production.

Social aspect: Linked to the people involved in the production system. Workers will greatly benefit from the enhanced productivity, acquiring reduced shifts and labor accidents.

2. Theoretical framework

In this chapter, a frame of reference will be created related to simulation, benchmarking, and lean. Information from books and scientific papers will be used so people that do not belong to this field can understand the study. An introduction to the topics mentioned will be given as well as the scientific method used to discuss and validate the solutions obtained.

2.1. Manufacturing concepts

The next sub chapter is focused on a more in-depth explanation of key manufacturing concepts compared in this report in order to build up a consistent view of them:

- LT: Time between the customer placing and receiving the order. Excessive lead-time indicates delay in production (Dennis P, 2016). Similar to the cycle time but accounting for waiting times and transportation. Also referred a *residence time* by Li and Meerkov (2008).
- TH: It is the number of products that a company can produce within a specified period (Dennis P, 2016).
- WIP: Measures the number of parts in the process in each moment (Groover, 2015). WIP should be low to avoid large lead-time using lean buffers (see chapter 3.3).
- Bottlenecks: The Theory of Constraints claims that ‘No chain is stronger than its weakest link’ (Goldratt,2014). Minimizing constraints will allow gaining higher capacity in the production as some machines in a production line can affect the whole system performance. These machines or processes are called bottlenecks.

2.2. Lean philosophy

Nowadays, most of the production is directed by the “Lean” philosophy introduced by Toyota. It would be difficult to understand this report in depth without a proper introduction to some key aspects of Lean, like the WIP and the LT.

Bicheno, Holweg, Anhede, and Hillberg (2009) summarized the main characteristics of the Lean philosophy, some of them are:

- Simplicity: Complexity does not always facilitate the work.
- Waste: Eliminating or reducing the wastes (Muda): overproduction, waiting times, unnecessary transport, excess inventory, overprocessing, unnecessary movement, defects correction, and/or unused employee creativity. Waste increases costs and manufacturing time while not adding any value to the product or service.
- Flow: It is important to achieve a constant flow. The product should move as the demand moves.

- Pull: A pull-logic more oriented to the customer helps with overproduction. Just-in-time (JIT) production.
- LT: The shorter the lead time is, the easier it is to control the flow.
- Continuous improvements: Innovation and small or big improvements should be made continuously.
- Variation: Variation is present in almost every process and makes the task of following the Lean philosophy more difficult, so it the source of it should be treated.
- Standardized work: The method with the best results should be standardized in order to fight variation.

It is normal to see many companies, e.g., Volvo, are applying the Lean philosophy, Figure 1:

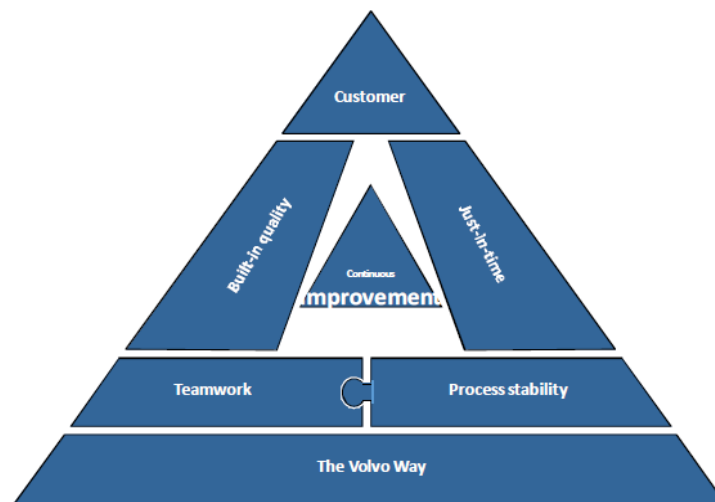


Figure 1. Volvo's Lean philosophy

2.3. Benchmarking

Benchmarking is a formal experiment (a technique of conducting a scientific investigation) often done to enable comparisons between different software configurations, where five key aspects must be considered (Rydgren Erik, 1997):

- Replication: Given the same influencing factors, the measurement must produce the same result every time.
- Representativity: At least enough to make a reasonable comparison between different tested objects.
- Local control: Uncontrollable influencing factors must be eliminated.
- Portability: Measurement using different software must have to be valid on another software to compare the results

- Simplicity: Easy implementation and data check.

2.4. Simulation

For a proper approach to simulation, it is necessary to first understand the concept of a system. A system could be described as a group of defined objects that interact or work independently towards a defined purpose. It is possible to regard a production line as a system where machines, components, and operators contribute towards a joined purpose (Banks et al., 2010). There are two types of systems: discrete event (objects change instantly) and continuous (objects change regarding time) systems (Law 2015).

“A simulation is the imitation of the operation of a real-world process or system over time” (Banks et al., 2010). Simulations have been proven to be a complete success, especially in the modern industry, as it allows to test systems and introduce changes in them without interfering with the real-world operations or make predictions of the future state of the system. This will require accuracy and validation will turn necessary. Most of the time, more accuracy will translate into more complexity of the model.

The time needed for the simulation run will totally depend on the complexity of the model itself. This deserves a special mention now as this report will also cross-verify the time needed for different simulation software to run the same model. To cope with this kind of complexity DES models are the best option.

It was previously hinted that DES is based on the principle that the state of the variable changes only at a discrete-time (Law, 2015). It is required to have an independent clock to record the discrete set of points in time (Law, 2015)

2.4.1. Simulation methodology

It is fundamental to reach a verified and valid simulation. The most recognizable simulation methodology is represented in Figure 2 (Banks et al., 2010). Briefly describing each step of Figure 2 will give a clearer sight of the simulation methodology.

Step 1 Problem formulation: Previous description of the problems existing.

Step 2 Setting of objectives and overall project plan: Results expected from the simulation. Here it is important to balance if it is appropriate to carry out the simulation (it is not appropriate when the system is too complex to simulate, too easy, more expensive than direct experimentation, there is not enough data, or the verification/validation of the model is impossible).

Step 3 Model conceptualization: A simple model needs to be developed. A classic flowchart will help to easily visualize the model.

Step 4 Data collection: One of the most time-consuming steps, getting the correct data sometimes is not even possible.

Step 5 Model translation: Here all the previous steps converge into simulation software (or just some programming language) as the amount of data is considerable.

Step 6 Verification: This steps' objective is to verify the computer program used is performing properly, usually using common sense with the input parameters and the logical flow of the system.

Step 7 Validation: The goal is to reduce the differences between the real system and the model calibrating it until it is "good enough".

Step 8 Experimental design: This step considers the length, number of runs, and replications needed.

Step 9 Production runs and analysis: The previously defined experiment is run, and an analysis of the results is performed.

Step 10 More runs?: More runs can be needed if the results are not fully satisfactory.

Step 11 Documenting and reporting: This step is very useful in case the simulation must be presented, either for validating the simulation or for other people that need to work around the project. It is recommended a to be carried out since the beginning of the process, explaining how the program operates (i.e., how to introduce data) and how the simulation reaches the objectives (progress documentation)

Step 12 Implementation: Finally, using the simulation model, with the correct documentation describing the whole process in hand.

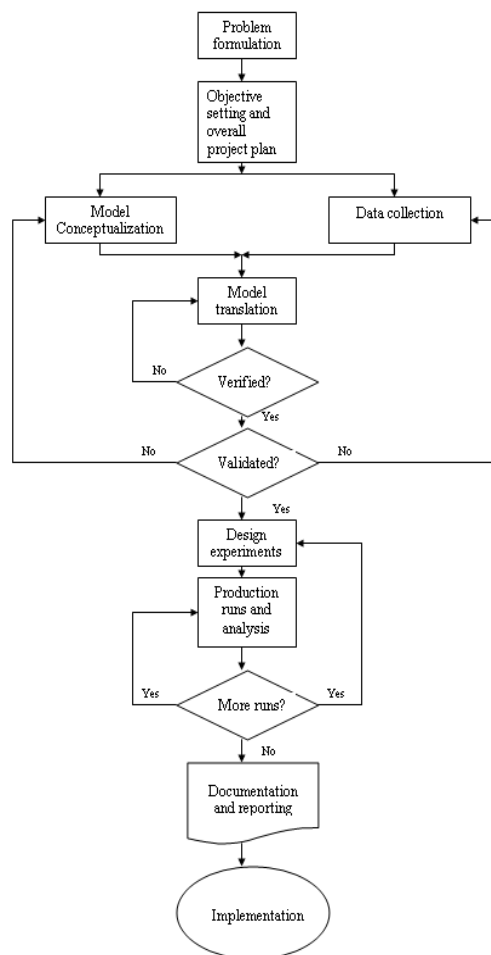


Figure 2. Steps in a simulation by Banks et al. (2010)

2.4.2. Simulation software

Using simulation software (pre-programmed software) risks being slower than using a traditional programming language, although it is easier to modify the inputs and visualize flaws in the simulation model (Law, 2015).

In this project 3 simulation software are used:

- FACTS: A toolset developed based on the concept of integrating model abstraction, input data management, and simulation-based optimization (Ng et al., 2007). Allows accurate system description and neutrality while keeping the simplicity. Thanks to the integrated optimization application, it is easy to focus on the improvement on specific tasks.
- Plant Simulation: To ensure realistic system models, Plant Simulation provides a programming language called SimTalk that enables the modification of objects using control structures and language constructs (conditions, loops, etc.) (Bangsow, 2015). This simulation software allows more complex simulation systems risking not being as simple to use as FACTS.
- PSE Toolbox: Consists of several functions for modeling, analysis, design, and continuous improvement of production systems. Each function consists of several tools (Li and Meerkov, 2008). The functions offered are: modeling, performance analysis, continuous improvement, bottleneck identification, lean buffer design, product quality, customer demand satisfaction, and simulations. PSE Toolbox will allow a more theoretical/mathematical approach to the simulations carried out.

There are many other simulation software available. The popularity of the different tools (Dias et al., 2016) showed other plausible simulation software that is more broadly used, like Arena or FlexSim. In the research paper, referenced Plant Simulation ranks 8 in popularity, while FACTS and PSE Toolbox do not even appear on the list. The main reason these programs are going to be used is because they can mix simplicity in the case of FACTS, high level of detail related to the real-world industry in Plant Simulation with fast and theoretical results in PSE Toolbox.

Three different programming languages are also used in this report for deeper cross-verification. The running time of the FACTS simulation model is obtained using MATLAB, and before that C is required to do previous checks. Eventually, R is used to automate some statistical calculations on the output data.

2.5. Simulation-based Multi-Objective Optimization

Mathematical optimization is the selection of the best element, regarding some criteria, from a set of available alternatives. In simple cases, an optimization consists of minimizing or maximizing an objective function.

When two or more criteria conflict with each other, a single best solution is not possible as it would deteriorate one of the criteria. This leads to several optimal possible solutions (trade-offs) known as Pareto solutions. The Pareto front is nothing but the most optimal trade-offs (better in some

objectives and not bad for any other) plotted in a chart (Deb, 2001). An example of a Pareto front can be seen in Figure 3 below:

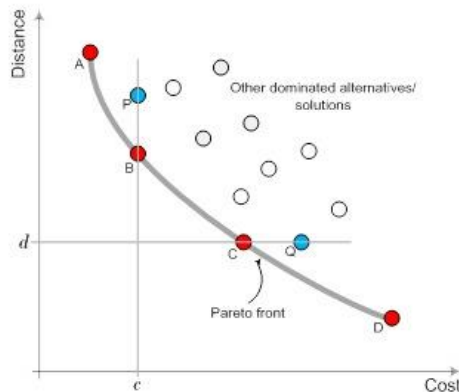


Figure 3. Example of Pareto front (Evoma, 2020)

To find out these trade-offs, simulation-based multi-objective optimization (SMO) is the best way to deal with the situation as the objectives and the decision variables are set into an iterative optimization algorithm. The algorithm runs the model several times, comparing each solution and selects the best one.

Understanding SMO is key as the optimal buffer allocation (lean buffers) is carried out using it in this project.

2.6. Combining Lean, simulation, and optimization

Although there is a lack of studies in which a framework has been implemented to be used in a standardized way, there is an increased interest in the combination of both Lean and simulation (Goienetxea et al., 2020)

Lean, simulation, and optimization have the same objective (helping in the design/improvement of systems), so they should start being used together on a regular basis, according to Goienetxea, Urenda Morris, Ng, and Oscarsson (2015). The authors defend that usually simulation engineers and lean managers do their jobs separately, when in fact, the simulation engineers should already be working Lean from the beginning. They also claim that first combining simulation and lean derives in covering different weaknesses and that lastly, adding optimization reduces the time needed to find optimal solutions.

2.7. System-theoretical modeling

Losing some fidelity of the original system that requires to be simulated can be a rewarding expense in order to set some standard models to which most of the production systems can be related to (Li & Meerkov, 2008).

This chapter focuses on how a production system can be reduced to these standard models defining five components as Li and Meerkov (2008) introduced.

2.7.1. Type of a production system

It defines the flow of the system. There are two types: serial production lines (Figure 4) and assembly systems. Only the first type is going to be studied in this case.

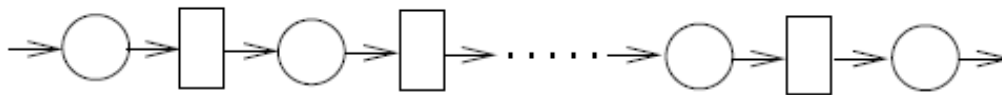


Figure 4. Serial production line (Li & Meerkov, 2008)

In the serial production line, machines (the circles in Figure 4) and the buffers (the squares) are in consecutive order. The parts flow also flow in a consecutive direction.

Machines or work cells can be anything up from an oven to a paint station. The material handling devices may be automated guided vehicles (AGV), conveyors, etc., referred to as buffers and their most important feature is their capacity, which will be studied in this chapter.

Serial lines can be also be closed, include quality inspection or rework, although those features are not going to be considered in this report.

2.7.2. Definitions for the mathematical models of machines

-Cycle time (τ): the time needed to process a part by a machine. It is going to be considered as constant. It is important to coordinate takt time (indicates the demand frequency) and cycle time to maintain efficient continuous flow (Dennis P, 2016)

-Machine capacity (C): the number of parts produced per unit of time by the machine. It corresponds to the inverted value of the cycle time.

In the case different machines in a line have the same cycle time, the time axis can be considered slotted or unslotted.

-Slotted time: all transitions take place at the beginning or the end of the time slot, which duration equals the cycle time. That is a synchronous system.

-Unslotted time: also called continuous time. Changes may occur at any moment. Asynchronous systems can be conceptualized as DES or as flow systems.

2.7.3. Machine reliability model

Li and Meerkov (2008) defined the machine reliability model as the probability mass functions or the probability density functions of the up and downtime of the machine in a slotted or unslotted time.

A Bernoulli machine implies that said machine has an independent status in all other cycles, obeying the Bernoulli reliability model.

Focusing on the slotted time case, such Bernoulli reliability model is found along with the Geometric reliability model (only identical Bernoulli machines and exponential machines are going to be studied to set a good delimitation to the project), where at the beginning of every time slot the machine can be found up or down depending on a chance experiment, according to which it up with probability 'p' and down with '1-p' independently of the machine's status in previous time slots. Although simple, this reliability model considered as a discrete-event system is practical and useful, for example, in assembly operations.

In the case of the continuous-time, only the exponential reliability model is going to be considered. The uptime and downtime probability density function of the machine in an exponential reliability model is given by the exponential distributions (Equation 1).

$$\begin{aligned} f_{t_{up}}(t) &= \lambda e^{-\lambda t}, & t \geq 0, \\ f_{t_{down}}(t) &= \mu e^{-\mu t}, & t \geq 0. \end{aligned} \quad \text{Equation 1. Exponential reliability model (Li \& Meerkov, 2018)}$$

The main drawback according to the authors is that the breakdown and repair rates are only constant, which does not describe the real world as accurately as possible.

2.7.4. Model validation

The process of assessing the accuracy of the mathematical model of a production system is called model validation, and for this purpose, the predictions of the model and the factory measurements are compared. A relatively simple way for considering the accuracy of the model is using the error formula.

PR is the production rate of the factory, and PR[^] the production rate predicted by the model; the value of the error is given by:

$$\epsilon_{PR} = \frac{|PR - \widehat{PR}|}{PR} \cdot 100\%, \quad \text{Equation 2. Error of the model (Li \& Meerkov, 2008)}$$

This error gives a measurement of the fidelity of the mathematical model and an acceptable value for it is between 5% and 10%. This error can also be worked out using the expression but with different parameters like the WIP, the starvation times of the blockage times of the machines.

3. Experimentation topics

3.1. *Computational burden*

The computational complexity theory focuses on arranging different computational problems regarding their resource usage, like time or memory usage, also regarded as 'space' (Goldreich, 2008). In this experimentation field, all the efforts of the report are to compare the time needed for each simulation software to run the very same model.

For obtaining the running times, both FACTS in combination with MATLAB and Plant Simulation are used, but also all the other outputs can be studied when the simulations are finished. It is essential to gradually increase the complexity of the models analysed to obtain a general idea of how the software behaves in lower and higher bounds.

The running times depend on computer hardware (the computer carrying out the operations), the number of machines or general complexity of the simulation, and the buffer sizes. For avoiding possible 'unfair' results obtaining the running times, all the simulations are done on the same laptop with an i7 processor in equal conditions.

3.2. *Accuracy / Lean buffer*

This field of the experiment covers how accurate the simulation software is using a specific buffer capacity, usually the lowest one possible.

The design of lean buffering refers to the smallest buffer capacity, which is necessary and sufficient to ensure the desired throughput of the system. For calculating the right buffer allocation (N) other inputs such as the efficiency of the line and the efficiency of the line must be considered. The efficiency of a line (E) is considered as its production rate divided by the largest production rate obtained when buffers are infinite. E must remain between 0 and 1 (Li & Meerkov, 2008).

Assuming that all Bernoulli machines (M) in the line (now a Bernoulli line) have identical efficiency (p) and that there are more than 3 of them (3 or less will not be taken into consideration in this report), with all buffers have identical capacity N and a desired E, the lean buffer capacity can be defined as in Equations 3 and 4:

$$\hat{N}_E(M > 3) = \left\lceil \frac{\ln \left\{ \frac{1-E-\hat{Q}}{(1-E)(1-\hat{Q})} \right\}}{\ln \left\{ \frac{(1-p)(1-\hat{Q})}{1-p(1-\hat{Q})} \right\}} \right\rceil$$

Equation 3. Estimated Lean buffer capacity in Bernoulli lines with more than 3 machines (Li & Meerkov, 2008).

$$\hat{Q} = 1 - E^{\frac{1}{2}[1+(\frac{M-3}{M-1})^{M/4}]} + \left(E^{\frac{1}{2}[1+(\frac{M-3}{M-1})^{M/4}] - E^{(\frac{M-2}{M-1})}} \right) \exp \left\{ -\frac{E^{\frac{1}{M-1}} - p}{(1-E)(1/E)^{2E}} \right\}$$

Equation 4. Estimated Q (Li & Meerkov, 2008).

Interestingly N for the desired E is constant for all $M \geq 10$, so the lean buffering that is appropriate for lines with 10 machines is also appropriate for any line with a larger number of machines (Li & Meerkov, 2008), as seen in Figure 5:

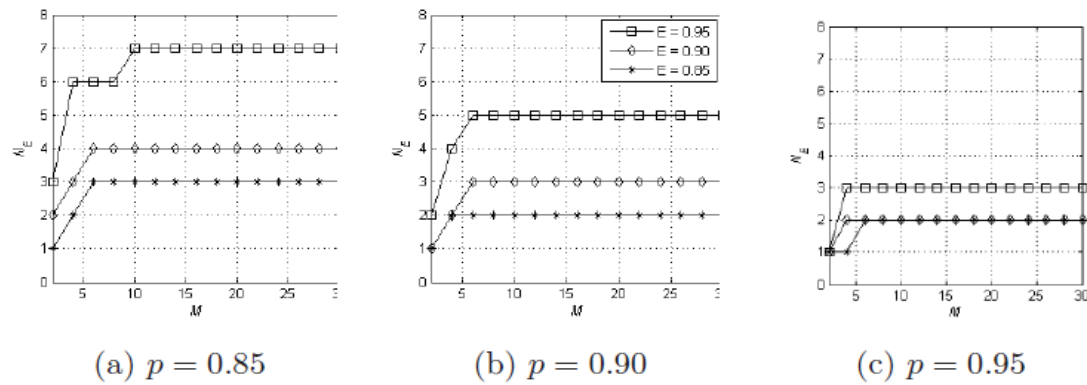


Figure 5. Lean buffering as a function of M . (Li & Meerkov 2008)

So it is possible to formulate the Rule-of-thumb for selecting lean buffering: In Bernoulli lines with identical machines and $M > 10$, the capacity of the Lean buffering can be selected as shown in Table 1:

	$E = 0.85$	$E = 0.90$	$E = 0.95$
$p = 0.85$	3	4	7
$p = 0.90$	2	3	5
$p = 0.95$	2	2	3

Table 1. Rule-of-thumb (Li & Meerkov, 2008)

Roser C., Nakano M., and Tanaka M. (2003) warn that adjusting the buffer allocation is one of the simplest ways to improve the performance of a manufacturing system as it is cheap and does not require modifying the system layout. Buffers reduce the possible starving time or blocking time of the machines of the system (definitions in the next sub-chapter, 3.3) by allowing them to have free spaces, which improves the system's TH and shapes the possible bottlenecks in the system. At the same time, WIP will increase carrying with it more costs for the inventory and slower answers to customer orders, deactivating the possibilities to produce following the Lean philosophy of JIT.

In this report, two ways for obtaining the Lean Buffer are followed: first using the optimization feature of FACTS with the aim of reducing the buffer capacities and second using the Lean Buffer feature of PSE Toolbox that works out the Lean Buffer using the mathematical considerations described in this chapter.

3.3. Bottlenecks

The objective of using this mathematical modeling is calculating performance measures of a given production system, and for its characterization it is essential to study the blockages and starvations percentages of the machines analysing how those machines and buffers are placed in the system. The definitions of these performance metrics are defined below:

-Blockage of machine i (BL $_i$): steady-state probability that machine i is up, buffer i is full, and machine $i+1$ does not take any part from the buffer.

-Starvation of machine i (ST $_i$): steady-state probability that machine i is up and buffer $i-1$ is empty.

For the case of serial lines in slotted time, these performance measures can be expressed as seen in Equation 5:

$$\begin{aligned}
 BL_i &= P[\{m_i \text{ is up at the beginning of the time slot}\} \cap \{b_i \text{ is full at the} \\
 &\quad \text{end of the previous time slot}\} \cap \{m_{i+1} \text{ does not take a part} \\
 &\quad \text{from } b_i \text{ at the beginning of the time slot}\}], \quad i = 1, \dots, M-1, \\
 ST_i &= P[\{m_i \text{ is up at the beginning of the time slot}\} \cap \{b_{i-1} \text{ is empty at} \\
 &\quad \text{the end of the previous time slot}\}], \quad i = 2, \dots, M.
 \end{aligned}$$

*Equation 5.
Performance
measure for
slotted time (Li &
Meerkov, 2008)*

For the case of serial lines in continuous time, these performance measures can be expressed as seen in Equation 6. Normally it is assumed m_1 is never starved and m_M is never blocked.

$$\begin{aligned}
 BL_i &= P[\{m_i \text{ is up at time } t\} \cap \{b_i \text{ is full at time } t\} \cap \{m_{i+1} \text{ does not take} \\
 &\quad \text{material from } b_i \text{ at time } t\}], \quad i = 1, \dots, M-1, \\
 ST_i &= P[\{m_i \text{ is up at time } t\} \cap \{b_{i-1} \text{ is empty at time } t\}], \quad i = 2, \dots, M.
 \end{aligned}$$

*Equation 6.
Performance
measure for
continuous time
(Li & Meerkov,
2008)*

To improve the bottlenecks, Goldratt (2014) presents five steps: Identify the constraints (weakest link of the chain), exploit the system's constraint, subordinate other processes to the constraint (having other processes supporting the already identified bottleneck), elevate the constraint (new machines? hiring people?) and repeating the process (there will be other processes being the new bottleneck now). There are 3 categories of bottlenecks: simple bottleneck, multiple bottlenecks, and shifting bottleneck (Roser, 2002).

In this report identifying the bottlenecks is done using the performance measures in PSE Toolbox (in the Bottleneck feature) and using the 'Shifting bottleneck' option of FACTS.

FACTS also offers another bottleneck detector called SCORE (Simulation-based- CONstraint Removal) that used SMO to identify and classify the bottlenecks and their causes. This method systematically improves the constraints normally maximizing the TH, confirming the Theory of Constraints detailed in Chapter 2.1 in the Bottleneck definition.

4. Literature review

This chapter introduces case studies by other authors with similar aims and methodologies that those considered in this project.

4.1. Comparative studies relating SMO

In the paper *A comparative study of production control mechanisms using simulation-based multi-objective optimization* (Ng, et al., 2012) a study is done comparing the different production control mechanism (PCMs) with their optimal parameter setting in a multi-objective context in FACTS, that generated the Pareto-optimal frontiers in the form of optimal trade-off curves. The total number of buffers (TNB) is set to be reduced along with the cycle time and the throughput is parallelly set to be maximized.

Comparing two Pareto fronts coming from SMO using two PCMs (A and B), there is an optimal configuration of A (A₁) which has higher throughput than the optimal configuration using PCM B (B₁). Similarly, by comparing A₂ and B₂, it can be said that for the same level of throughput (TP₂ in the paper), PCM A can achieve shorter Cycle time (or CT as in the paper)/WIP when compared with PCM B, as seen in Figure 6:

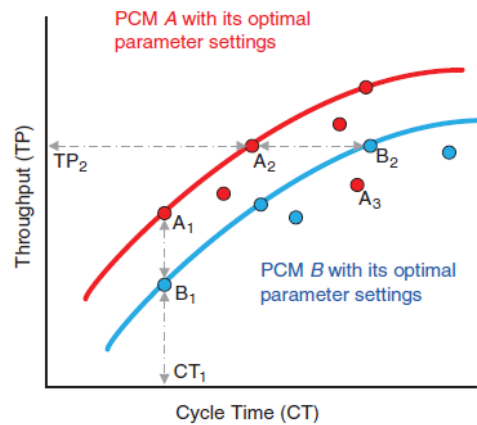


Figure 6. Comparing two PCMs with their Pareto-optimal setting in a cycle time-throughput plot

The paper compares the results of the four most popular PCMs, Push, Kanban, CONWIP, and DBR on an unbalanced serial flow using a dynamic replication analysis where FACTS requests for more replications if the computed error is higher than a certain level. The different PCMs are compared with each other while changing the bottlenecks of the system.

The conclusions of the paper are two: optimizing the trade-off between production rate and cycle time heavily depends on the PCM as DBR normally outperforms Push and other pull mechanisms like Kanban and CONWIP, and that a certain PCM could be desirable in a specific region but not in others. For example, Push performs better than Kanban if the target to increase the production rate, as seen in Figure 7 for the study case with 15 machines and the bottleneck on machine 12:

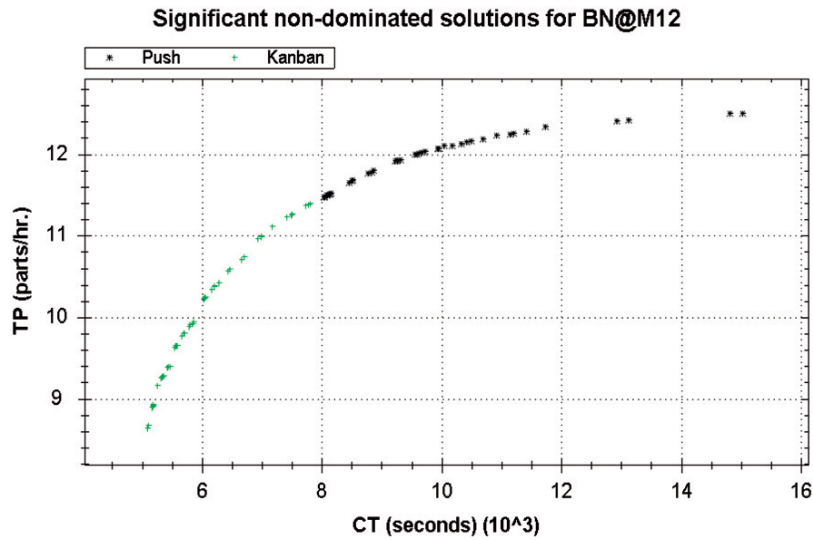


Figure 7: Optimal solutions comparing Push and Kanban models

4.2. Simulation-based optimization of Lean Buffers

Buffer allocation has been extensively studied as it is a well-known problem in the industry as the authors, Zhang, Matta, and Pedrieli (2016) acknowledge. Also, Weiss, Matta, and Stolletz (2018) described the allocation as a trade-off situation between TH, where blocking or starvation may occur, and WIP, as large buffers are more expensive.

A study deciding how to distribute the buffer capacity among the machines of the lines by Christian Urnauer, Eva Bosch, and Joachim Metternich (2019) dealt with the resequencing using a simulation model. It was decided to use simulation due to the high complexity of the system (Banks, 1998). Although the improvement achieved 'only' reached 0.21%, it opens the door to further simulation-based optimization of the buffers, especially in automated storage and retrieval systems. Some of the results can be seen in Figure 8:

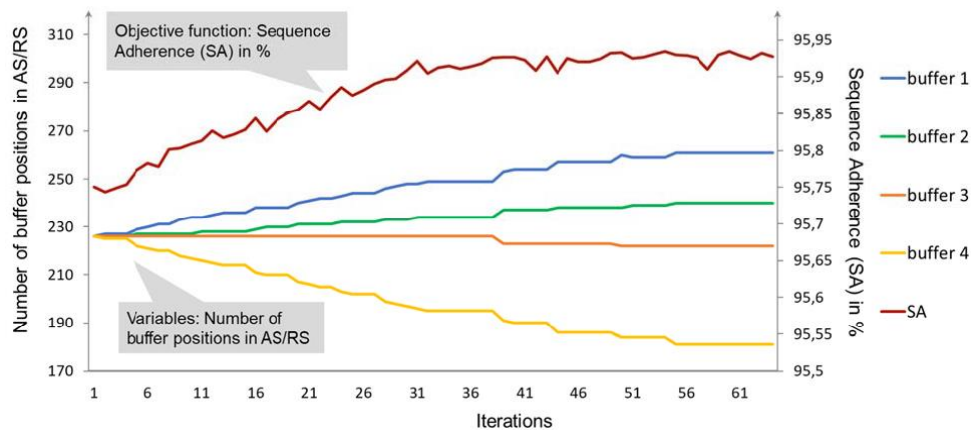


Figure 8. Simulation-based optimization results (Urnauer et al., 2019)

Pehrsson, Frantzén, Aslam, and Ng (2015) experiment on different complex models using FACTS and the results were very satisfactory. For example, in one of the cases, it was possible to maximize the

TH by 13.7% with a return of investments of only 2.4 months. The key was to decrease the processing time of one machine and increase the capacity of one buffer and two machines. In other simulation-based experiments, it was possible to reduce the total number of buffers by 44% while reaching the targeted TH.

4.3. Model validation

Robert G. Sargent (2010) discussed the verification and validation of simulation models and concluded that these eight steps are to be performed to validate a model:

- Agreement among the model developers and the model sponsors.
- Specify the accuracy for the out variables of interest.
- Test the assumptions of the simulation model.
- In each iteration, perform face validity on the conceptual model.
- In each iteration, explore the simulation model's behaviour using the computerized model.
- Make comparisons in at least the last iteration between the simulation model and system behaviour output data.
- Develop validation documentation.
- Schedule periodic reviews of the model's validity if the model is used on a regular basis.

4.4. Lack of cross-checked models

There are several projects, papers, final degree projects, etc. where SMO is applied, normally using a single simulation program and very seldom the accuracy of the models are cross-checked between different software or even different modeling formulations.

Going back to sub-chapter 4.1 and 4.2, the previous works by Goinetxea, Ruiz, Urenda, and Ng (2015) and Pehrsson, Frantzén, Aslam, and Ng (2015) are carried out using only one specific simulation software (FlexSim and FACTS, respectively).

The authors Aalto, Karttunen, and Ranta (2019) analyzed the estimations for a simple forest machinery model in Finland using both a spreadsheet and the simulation modeling software Anylogic. Validation showed that the spreadsheet announced lower estimations for some of the resources (i.e., harvesters and forwarders) than the mean values reported by the simulation, probably because the spreadsheet gave very theoretical results (too optimistic). The conclusion drawn from the authors is that although a simple spreadsheet can do decent work with simple models, but with increased complexity or for more realistic results, proper simulation software gives a better view or point and open the door to not so simple models that would bring great advantages to the study.

4.5. Summary of literature review

This chapter of literature review shows how SMO is used broadly nowadays and not only in the manufacturing world for obtaining for improving different kinds of systems with the near-optimal solutions obtained.

The counterpart is that all models need to be verified and validated, although the simulation models used are rarely cross verified with other simulation software, which may lead to improvable models and results, as the literature review shows.

To summarise the literature review, previous works and papers with objectives and methods that relate to the current project are studied, where Benchmarking different simulation software is still an unexplored area that has been proven to be another possible improvable foundation for future projects.

With the knowledge gain until now, it is possible to step forward the methodology to carry out the project.

5. Experimental methodology

In this chapter, the methodology used during the project is detailed and how it is going to be executed in the three different experimentation topics: computational burden, lean buffer, and bottlenecks.

5.1. Experimental research

Briony J. Oates (2006) shows six different approaches for research: survey, design and creation, experiment, case study, action research, and ethnography. An experiment is focused on the cause-effect relationships, testing hypothesis, and proving/disproving possible causal links between the different factors and the outcomes after the measurement.

The methodology that best suits this report is experimental research. Babbie (1998) defines it as is a study that strictly adheres to a scientific research design. Experimental research consists of a hypothesis, a manipulable variable, and measurable and comparable variables in a closed environment. After the data collection, the hypothesis can be supported or rejected. The aim of an experimental research is to check if it is possible to establish a correlation between a specific aspect of an entity and the variables studied.

The scientific method is characterized by being iterative and cyclical, where information is continuously revised (Gold-Frey, 2009). A possible scheme to follow in this method can be described as (Crawford, 1990):

1. Define the problem.
2. Gather information and resources.
3. Develop a hypothesis.
4. Test the hypothesis by carrying out experiments and collecting the results in a reproducible manner.
5. Analyse the data.
6. Draw conclusions serving as the starting point for other hypotheses.
7. Document the results.

The iteration of the method should start with the development of the hypothesis (3) and go all the way up to the conclusive step (6).

5.2. Quantitative and qualitative data. Statistical tools

Quantitative data references that data based on numbers (Oates, 2006), and it typically comes from experiments or surveys. Analyzing quantitative data seeks patterns in that data and drawing conclusions using tables, graphs, and/or different statistical tools. There are different types of

quantitative data: nominal, ordinal, interval, ratio, discrete and continuous data. Especially interesting in this report is the discrete data where each measurement gives a whole number and the continuous data where the measurement can always be given further accuracy. Good examples of these two groups are the WIP for the discrete data where it is simply impossible to have one piece and a half still being done and only whole numbers are going to be used (except for mean values and similar) while on the continuous data category some data like the running time can be found as it is always possible to keep adding seconds or milliseconds to the measurement. Some visual aid for quantitative data analysis used are tables and line graphs among others, to organize all the empirical data obtained through the experiments.

Statistics help this report to structure generic means and criteria for evaluating the results and coming to evidence-based conclusions. For describing the central tendency, the 'mean' value is used repeatedly. It is worked out by adding up all the values obtained and dividing them by the number of total cases.

For describing the distribution (how data is spread), the standard deviation (SD) is used in every single experiment done. It shows the average amount of variability in a data set. The larger or smaller the SD is, the smaller or larger the average distance each data value is from the mean. The SD can be calculated like the difference between each individual value and the mean, squaring each difference and summing all these squares together, then dividing the sum by the size of the sample -1 and finding the square root of the result. Most of the simulation software will provide the SD automatically. In some cases, the SD was calculated using MATLAB.

For further statistical study, 95% confidence intervals are used in the Computational benchmarking experiment. The confidence interval tells that it is possible to be 95% confident that the true mean is between the lower and upper limits. This also means that there is a 5% risk that the true mean (expected mean) value lies outside these limits. If there is a large variation (high SD), it results in a big difference between the lower and the upper limit. One way to decrease this difference is to run more replications. The t-distribution can be used when the standard deviation is unknown provided that the number of replications is above 30 or that the results are known to be normally distributed (Lövås, 2006). The results of a discrete event simulation model can typically be assumed to be normally distributed. Equation 7 describes how to calculate the confidence interval (standard error) based on the t-distribution:

$$\bar{X}(n) = \pm t_{n-1, 1-\frac{\alpha}{2}} \sqrt{\frac{s^2(n)}{n}}$$

Equation 7. Confidence interval based on t-distribution

The first element references the mean value in the simulation (TH, WIP, or the parameter to study). The second element of the equation stands for the t-distribution for the one-sided confidence interval $(1-\alpha/2)$ and the degrees of freedom $(n-1)$. 's' is the standard deviation of the replication means and 'n' is the total number of replications.

$$\bar{X}_1(n_1) - \bar{X}_2(n_2) \pm t_{f, 1-\alpha/2} \sqrt{\frac{s_1^2(n_1)}{n_1} + \frac{s_2^2(n_2)}{n_2}}$$

Equation 8. Welch CI

To check if two values are really ‘different’ from each other, a statistical test called the Welch confidence interval is used. Equation 8 shows how this CI is calculated:

The parameters are the same as exposed for the confidence interval explained above but regarding the two sets of data to compare. When the confidence interval is obtained, if it does not contain a 0 in it, it shows that the two means compared are different.

Ideally, statistical hypothesis tests should be done in every case, but in this project visual analysis was mainly performed due to the considerable amount of data.

The qualitative data includes all the non-numeric data such as words, images, and so on (Oates, 2006). In this empirical project, the qualitative data is not as relevant as the quantitative data.

5.3. Experiments set-up

In this preliminary part of the experiments, the warm-up time, the number of replications, and the simulation horizon are established using FACTS.

For the steady-state analysis, two different simulation models were used: the simplest model composed of 5 machines with 4 buffers and the most complex model that uses 200 machines and its 199 buffers to see if the difference among both extremes was enough not to set a standardized initial set.

Starting with the 5-machine model, the configurations settings in FACTS are as displayed below:

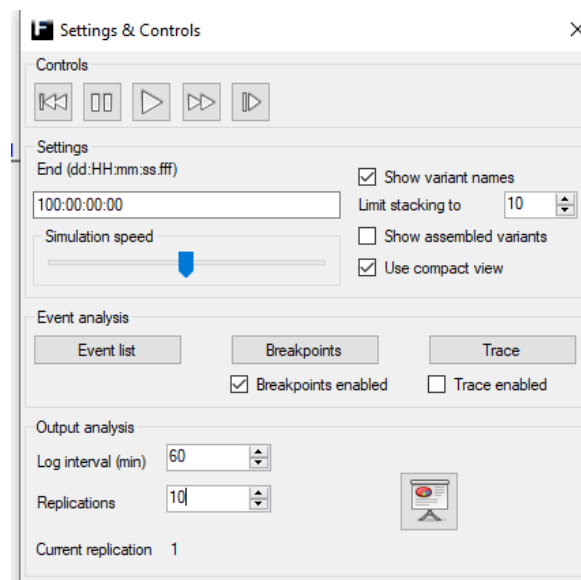


Figure 9. FACTS setting for calculating the steady-state

The simulations are not really intended to run for 100 days. The reason for this is to have a margin and check the simulation horizon.

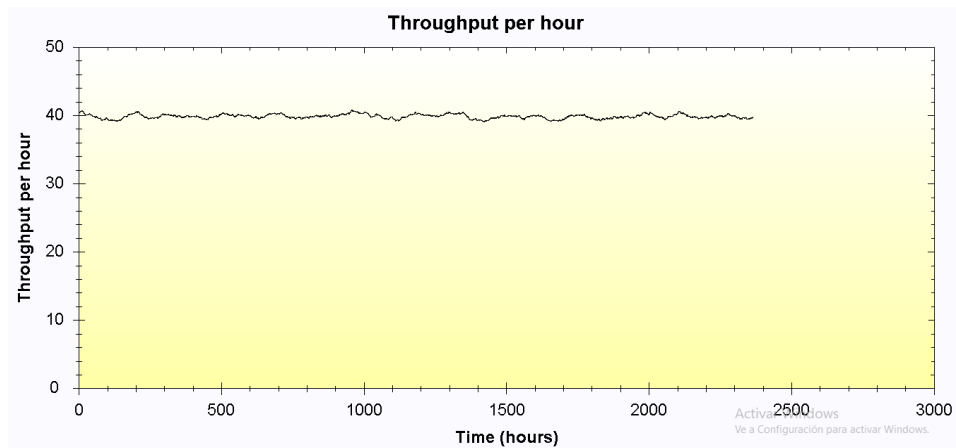


Figure 10. Hourly TH calculating the warm-up time of the 5-machine model

As can be appreciated in Figure 10 above, there is no clear need for a warm-up time as the TH follows a very flat line without big variations that may show an initial disturbance. The same conclusion can be obtained by analysing the WIP (Figure 11)

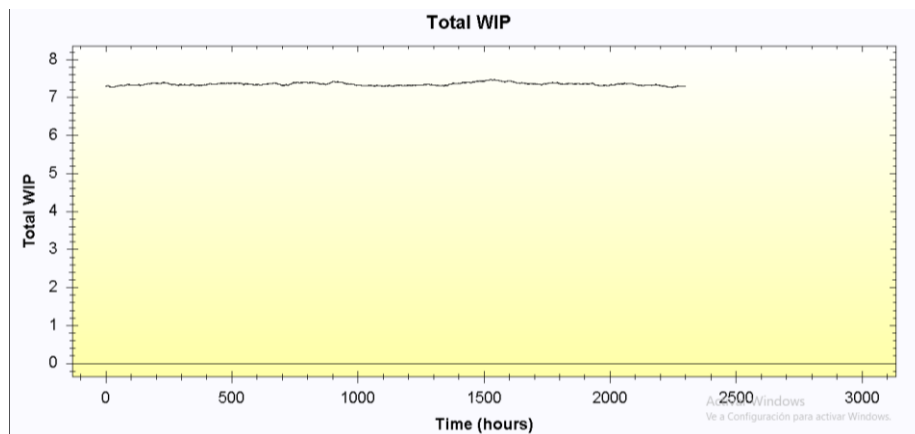


Figure 11. Hourly WIP calculating the warm-up time of the 5-machine model

After the good initial response of the first model, the 200-machine model shows a need for a warm-up time that is at least long enough to stabilize the TH and the WIP as seen in Figures 12 and 13:

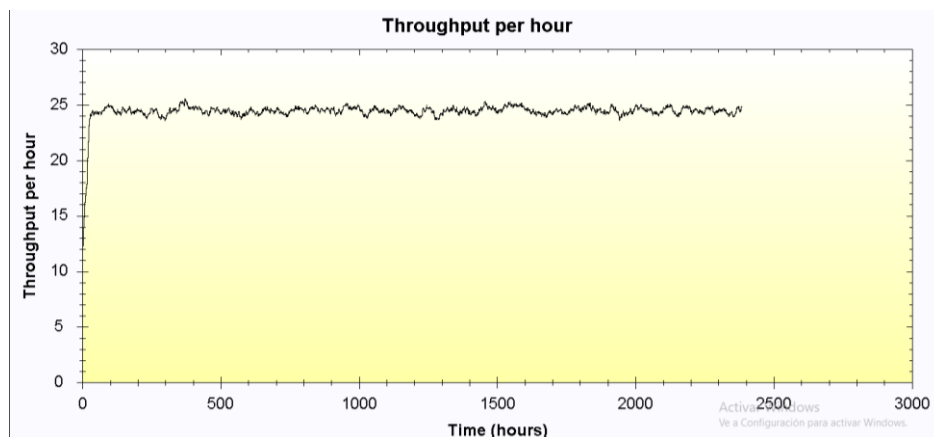


Figure 12. Hourly TH calculating the warm-up time of the 200-machine model

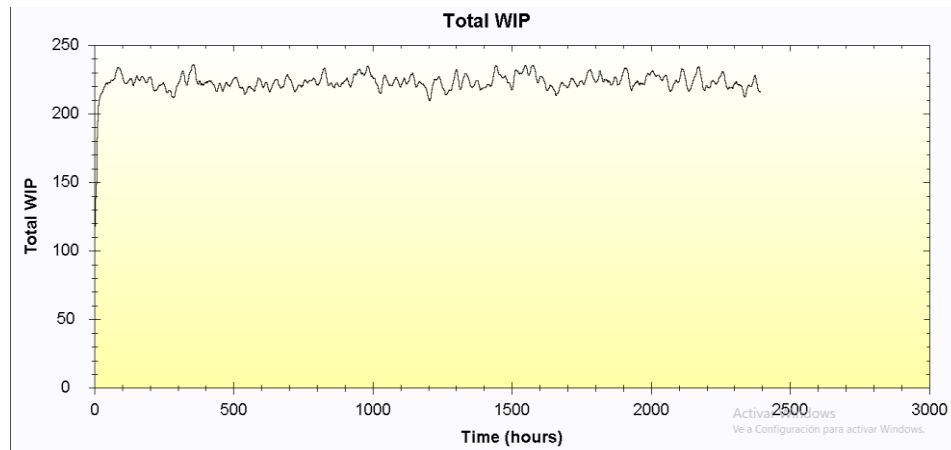


Figure 13. Hourly WIP calculating the warm-up time of the 5-machine model

It is clear that the most complex model starts with a very low TH and rapidly achieves to stabilize it around a value of 24. The WIP follows a very similar pattern stabilizing close to 230-240.

In the light of the results, a standardized warm-up time of 24 hours is set along with a simulation horizon of 6 days which is long enough to guarantee sufficiently precise estimates of steady-state behavior. Results prior to the first 24 hours of simulation may not be representative, especially in the models containing a higher number of machines.

The decision of how many replications should be considered is a balance of accuracy and simulation times. The number of replications is set to 10, so the computing times are not excessive but keep the results trustworthy. More replications could be considered but, in this report, the simulation horizon is long enough to smooth out temporary effects on the system.

5.4. Computational benchmarking tests

In the computational benchmarking tests, all the parameters mentioned previously, such as TH, WIP LT, running times needed for every replication/simulation, standard deviations, and buffer allocation are analysed in both FACTS and Plant Simulation. PSE Toolbox will not be considered here for three main reasons: it does not allow to record the standard deviation of the results or even set a number of replications, the LT cannot be studied, and the running times are impossible to obtain.

The analysis of the manufacturing outputs could be classified as a kind of accuracy test, although the prime aim of this experiment is to record and discuss the running times. The comparison of the TH and WIP is also done here as it is the only set of experiments that gives information about this data with more than 5 machines, which excludes PSE Toolbox for the limitations details in chapter 1.4.

For this initial set of tests, five different models are considered, ranging from 5 machines to 50 machines, 100 machines, 150 machines, and 200 machines with 4, 49, 99, 149, and 199 buffers, respectively, between every machine. All machines are identical with a constant processing-time of 1:00 minute (only Bernoulli distribution) and an availability of 90% with 5:00 minutes of MTTR. Buffers are also identical and do not present any dwell time or failure.

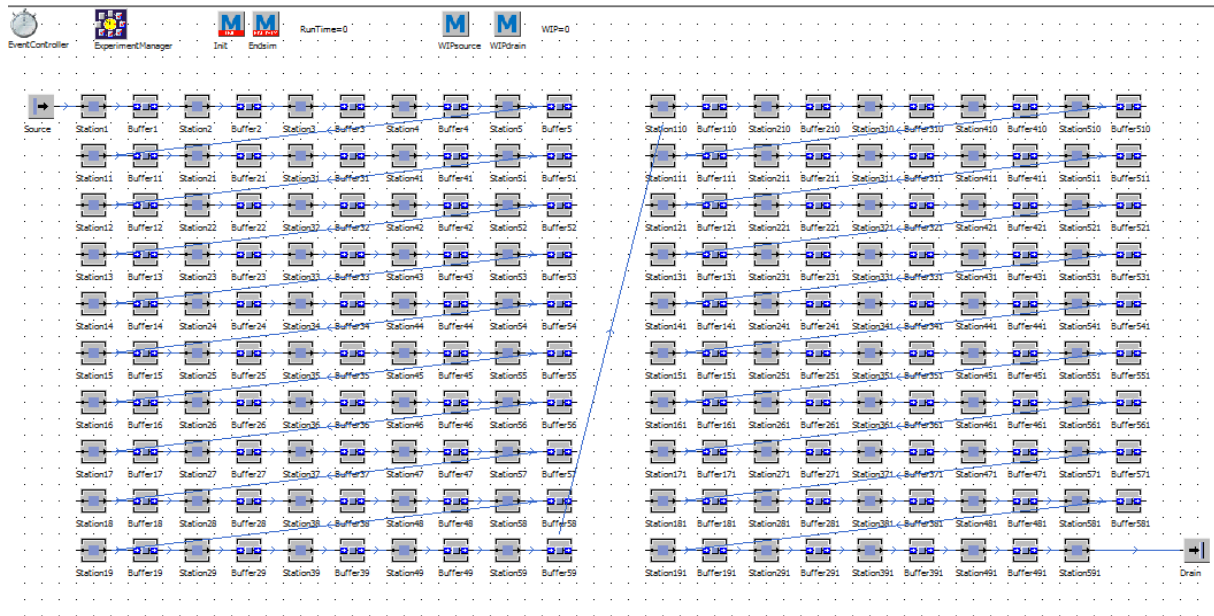


Figure 14. 100 machine model in Plant Simulation

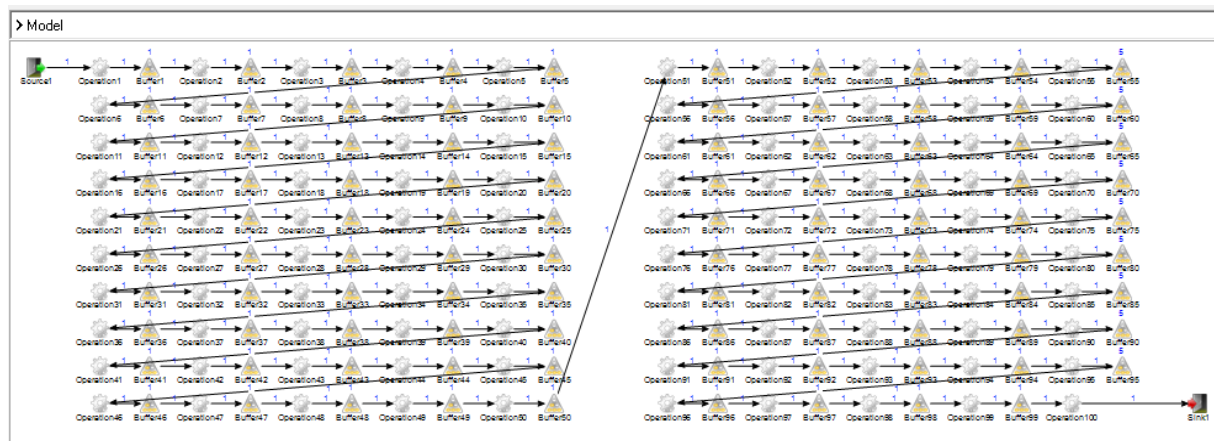


Figure 15. 100 machine model in FACTS

The buffer sizes range from a capacity of 1 to 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100. As this happens in each of the five models, automatization was key to obtain the results fast, safe, and in a reproducible manner. In the case of Plant Simulation, this can be easily done using the feature 'Experiment Manager' which allows selecting some attributes as inputs (only the different buffers capacities are needed now) and some others as outputs such as the TH, WIP, and LT. For FACTS, the matter of automatization requires more steps. A .exe called 'xsim-runner' obtained from FACTS developers is used for this experiment enabling a way of executing a FACTS model saved in '.xml' format and that has been enabled to have the buffer capacity as an optimization variable in the software. For modifying the buffer sizes automatically while obtaining all the outputs already mentioned a MATLAB script is developed as seen in Annex 1 Figure 86, asking only for the concrete model that needs to be studied as an input (number of machines) for the xsim-runner, along with the capacities mentioned. Modifying the script, more inputs for the x-sim-runner can be introduced, like the availability and the MTTR. An initial 'for' loop comprises another loop where the .exe is run for every solicited capacity and different seeds to ensure different results. After the initial loop is made 10 times, as 10 replications are needed, all the data is saved on an Excel table where it is read by a

final loop that calculates the mean values and the standard deviation of the 10 replications and writes it in a final Excel file. Before eventually using the MATLAB script, xsim-runner.exe is executed using C to ensure it does not crash when initialized, using at least two different models as seen in Figure 85 of Annex 1.

Running times consumed by each software gains special interest in this test as it is not considered again in any other experiment and the results can be of extreme relevance when choosing among Plant Simulation or FACTS for complex and long simulations. This result is recorded indeed in a different way for the different software. For Plant Simulation the running time is calculated using two 'methods' (See SimTalk in Chapter 2.4.2) both at the beginning and the end of the simulation that notes the time passed among them in a variable named 'RunTime' and that is selected as an output in the Experiment Manager. Another way to obtain the running times would be just waiting for the experiment manager to finish its task as when this happens it shows the time required for it but in this way, it would only be possible to obtain the total time of the 10 replications without the standard deviation for every single replication, but this information is useful anyways as it allows to check all times match. For FACTS, the running times are calculated using MATLAB and the 'tic-toc' function. A 'tic' is placed just before MATLAB runs the xsim-runner.exe in the script developed and a 'toc' is placed just after that, allowing to see the time passed while the .exe did his job. The value obtained is also written in the Excel tables and treated like any other output.

While the WIP is obtained directly from FACTS as an output along with TH and LT, in Plant Simulation two other methods need to be implemented, counting the parts in the source and the drain in a created variable named 'WIP'.

Finally, another MATLAB script reads all the data generated and plots the 95% confidence interval graphics for every model (20 figures in total) which allows an easier discussion of the results. The calculation for the 95% confidence interval is also coded in MATLAB using the mean values and the standard deviations worked out in the first MATLAB script.

All the data is compared and analyzed regarding the TNB (Total Number of Buffers), meaning the total sum of all the buffer capacities used in the model. The running times are eventually studied with a fixed N and the increasing number of machines of each model.

5.5. Accuracy / Lean buffer tests

In this set of tests, one of the main focuses of this project is considered: Is the model simulated accurate? For answering this question FACTS, Plant Simulation, and PSE Toolbox are set to simulate a simple model with only 5 equal machines with 1:00 minute of process time and 4 buffers with a capacity variable that may or may not be identical for all of them.

To continue, three different variations of the 5 machine models are studied: the 'base' model used in the computational test (machines have an availability of 90% and an MTTR of 5:00), another model where machines have a higher availability of 95% while staying the same in the rest of parameters, and one last model with still an availability of 90% but with a lower MTTR of 2:00. The three different models are studied following both a Bernoulli distribution first and an exponential distribution later,

allowing PSE to control the MTTR as when only using the Bernoulli distribution; the MTTR cannot be changed or set.

Nevertheless, the MTTR is not a direct input for the exponential distribution machines when using PSE Toolbox, and some mathematical efforts are needed. When using the exponential distribution, the software will ask for the number of machines (M), λ , μ , the processing time (τ), and the N . Considering the availability of the machine (A) as the result of the time the machine is working (mf) divided among ' mf ' plus the MTTR (Equation 9) while ' mf ' can be considered as the inversed value of ' λ ' (Equation 10) and the MTTR the inversed value of ' μ ' (Equation 11).

$$A = mf / (mf + MTTR)$$

Equation 9. Availability of a machine

$$\lambda = 1 / mf$$

Equation 10. How to obtain λ from ' mf '

$$\mu = 1 / MTTR$$

Equation 11. How to obtain μ from the MTTR

In this case, as the availabilities are already set to either 90% or 95% and the MTTR can only be 5:00 minutes or 2:00 minutes, the only values unknown are ' mf ' and λ that can be obtained from Equation 9 and 10 respectively. How the inputs are introduced in PSE for both Bernoulli and exponential distribution can be seen in chapter 6.2 along with the correspondent outputs.

An optimization for each of the models is carried out first using FACTS with the command to maximize the TH, minimize the WIP, and the LT only changing the buffer allocation using 5000 replications. The resulting plot will show all 5000 evaluations (or solutions in the multi-objective optimization terminology) in the TNB objective space and will give place to the already discussed Pareto front required for the experiments. All plots regarding the optimization will be discussed in the results section.

Once all the points from the Pareto fronts are obtained, 30 of them are selected, spacing the TNB slowly, so the curve (or the front) is still recognizable and fully working but easier to work with. It would not be a problem to work with all the possible solutions in Plant Simulation (FACTS already has its own results from the optimization itself) as the process can be automated using the 'Experiment Manager', but PSE Toolbox requires manual input, so a higher number of points would be unpractical. Eventually, all the 30 TNB solutions obtained are simulated in Plant Simulation and PSE for the 3 models described in both Bernoulli and exponential distribution. Only the TH and the WIP are analyzed in this test as PSE Toolbox does not support any LT output, and the running times are already covered. The results are 12 graphics showing how the three software simulate the same three different scenarios with the different distributions regarding the TH and the WIP versus the TNB.

Lastly, another set of tests based on the PSE Toolbox interpretation of the Lean Buffer definitions given by Li and Meerkov in 2008, a concept already developed in the report in Chapter 3.2, is applied to the 'base' case model (1:00 minute of process time, Bernoulli and exponential distributions and most importantly equal buffers) with an E increasing from 0.7 to 0.99. The results are compared to the ones coming from the FACTS optimization, FACTS using the lean buffer data coming from PSE Toolbox, and PSE Toolbox using its calculated lean buffer data using the PSE Toolbox function 'Performance analysis'. In short words, the N calculated by PSE Toolbox for either Bernoulli or

exponential distribution is exported to FACTS, Plant Simulation, and PSE itself in order to compare their simulation results with the proper optimization results from FACTS obtained in the previous chapter. The only outputs studied are the TH and the WIP, as the rest are not that relevant or impossible to obtain using PSE. Another PSE Toolbox function is used here for the first time along with the experimentation: the 'Simulation' function. Nor the authors or PSE Toolbox itself gives an explanation of the difference between this last function and the 'Performance analysis' used until now.

5.6. Bottleneck tests

For the last experiment, a comparison on how FACTS and PSE Toolbox detect and work with bottlenecks is done. FACTS handbook (Evoma, 2020) offers a guide to carry out such an experiment while also optimizing the model. In this case, the data offered by the handbook for the experiment is accepted but assigning the same processing time for each machine (32s) as PSE Toolbox cannot give independent values to all 5 different machines (Table 2). The experiment will be carried out following a Bernoulli distribution first and an exponential distribution later. The buffers are equal, with no times, and can have either a capacity of 1, 10, or 50 parts. This gives as a result 6 different scenarios: 3 Bernoulli models with the buffer sizes mentioned and 3 exponential models with the same buffer sizes. The outputs such as the TH or the WIP are not considered for this experiment as it already studied before and the focus is solely the bottleneck study.

Parameter	M1	N1	M2	N2	M3	N3	M4	N4	M5
τ [s]	32	1, 10 or 50	32	1, 10 or 50	32	1, 10 or 50	32	1, 10 or 50	32
Availability [%]	95	1,10 or 50	92	1,10 or 50	86	1,10 or 50	94	1,10 or 50	90
MTTR [s]	600	1,10 or 50	600	1,10 or 50	600	1,10 or 50	600	1,10 or 50	600

Table 2. Input for the bottleneck test

FACTS offers a feature when starting an experiment called 'Shift bottleneck' that displays two bar plots that are essential for this part of the report: the bottleneck chart that indicates the bottleneck itself (either shifting, in colour red, or sole, in colour blue) and the utilization charts that dive into further detail like the percentage of working time (green), failing time (red), waiting (grey) or starving as it is called in PSE Toolbox, and blocked (yellow). This last chart is primarily used if the bottleneck chart does not provide clear enough information of the bottleneck.

PSE Toolbox also allows a direct bottleneck study with the tool 'Bottleneck Identification'. In the case of the Bernoulli models, only the availability of the machines, along with their process time and the buffer sizes, are required. For the exponential distribution, the maths already explained in Equations 9, 10, and 11, are used, which once again leads to a controlled MTTR. Unlike FACTS, PSE also points out not only the bottleneck machine but also the buffer bottleneck.

6. Results and analysis

6.1. Computational benchmarking

In Appendix 1 all the results regarding TH, WIP LT, running times, their standard deviations, and buffer allocation is shown for both Plant Simulation in Table 7 and FACTS in Table 8. Even though a visual analysis can be done with the plots that are going to be discussed, already some valuable data can be obtained and processed from the table, for example, how the TH tends to decay every time the number of machines is increased but at the same time increases with higher buffer sizes. An initial thought could be that the more machines the model has, the higher the production rate should be, but it is indeed the other way around in most models and especially in these simple lines as the machine failures make downstream machines starve and upstream machines to block, as the buffer sizes also do. As the buffer sizes increase, so do the WIP and the LT as the parts now have more space to take along the processes. Especially interesting are the standard deviations of these last two parameters: while it remains low in every other parameter, it seems extremely high for both the WIP and the LT as they are very sensitive to variation, especially when the number of parts is elevated.

The biggest conclusions can be drawn by analysing the graphs. In every case, FACTS will be showed with a red colour and Plant Simulation will be blue, same pattern for the error bars. This colour pattern is set for the whole extension of the report, with different variations for the plots where the same software is used more than once.

6.1.1. TH analysis

Starting with the TH, all plots show very similar tendencies for both programs being FACTS always the one giving slightly higher values for all models except for the 5-machine model (Figure 16). The error bars are very similar, along with the almost identical TH curve. The biggest error bars are present in the 5-machine model, probably due to the instability such a small system provides. This proves both programs similarly work on the TH and that both can be equally trusted.

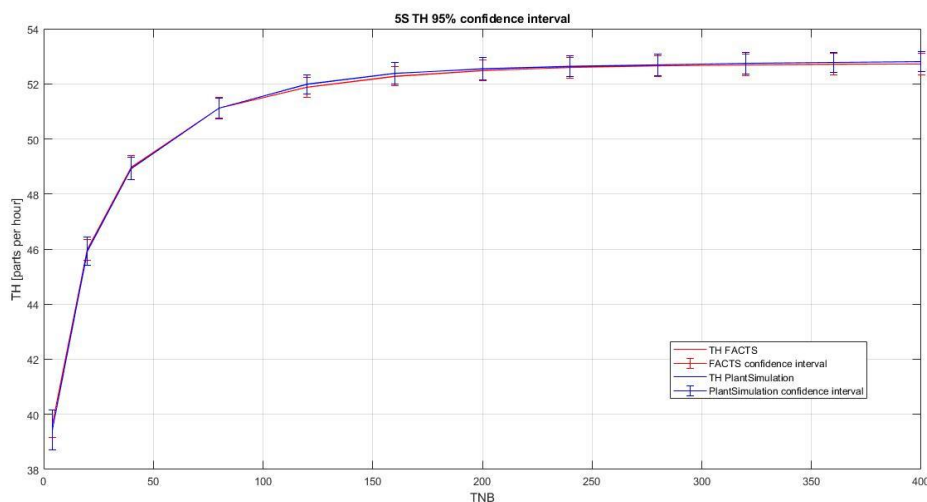


Figure 16. TH depending on the TNB for FACTS and Plant Simulation for 5 machines

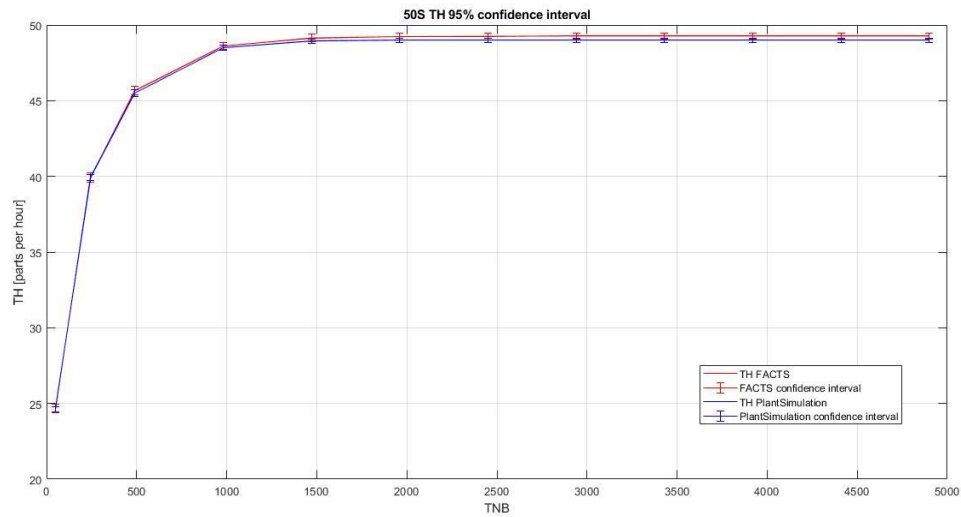


Figure 17. TH depending on the TNB for FACTS and Plant Simulation for 50 machines

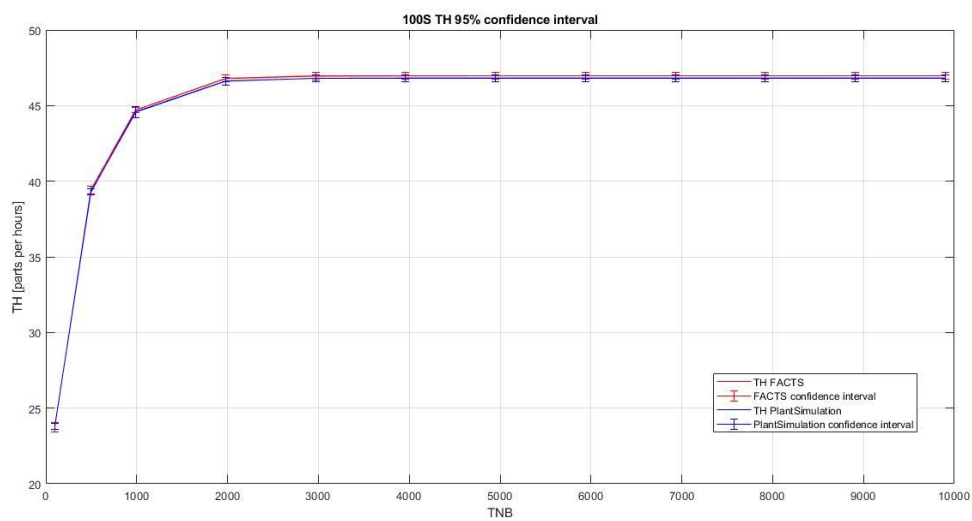


Figure 18. TH depending on the TNB for FACTS and Plant Simulation for 100 machines

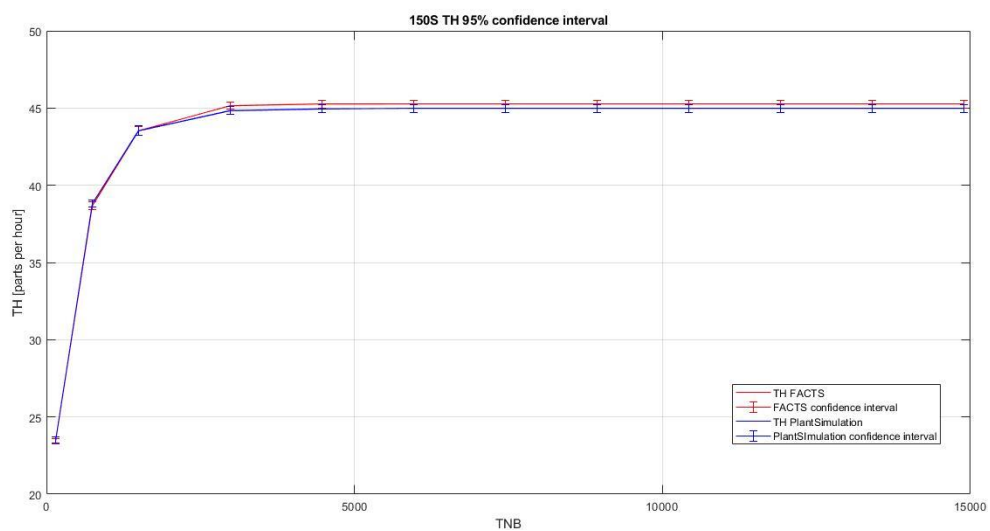


Figure 19. TH depending on the TNB for FACTS and Plant Simulation for 150 machines

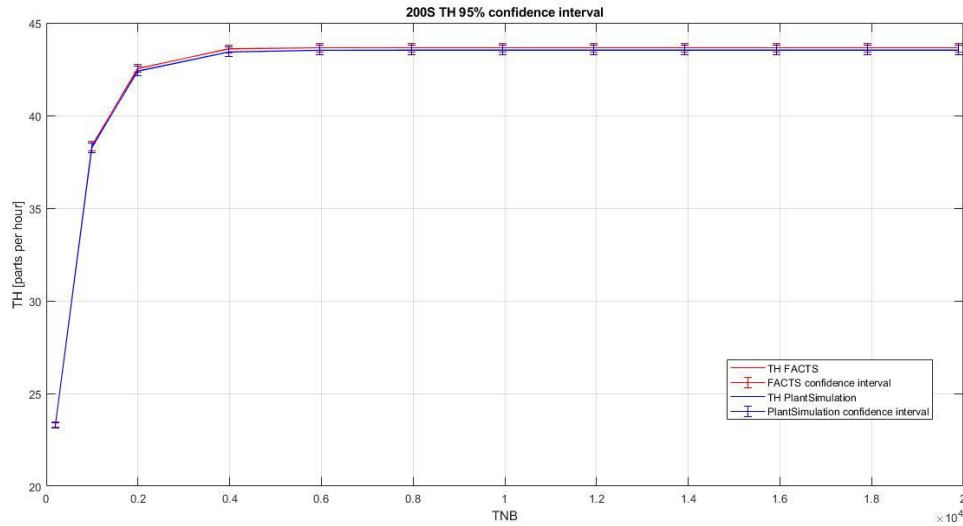


Figure 20. TH depending on the TNB for FACTS and Plant Simulation for 200 machines

6.1.2. WIP analysis

Moving into the WIP analysis, both software start to show notable differences. In the first plot (Figure 21), with the fewer machines both Plant Simulation and FACTS have similar results, Plant Simulation having a considerably higher range for the confidence interval. For the rest of the WIP results, Plant Simulation shows higher parts unfinished and with higher confidence intervals reaching a very different result especially in Figure 25 with the 200 machines where the difference shown is more than 400 parts. The higher variance Plant Simulation displays is linked to a larger WIP, as wider confidence intervals means higher mean values, although it is impossible to discern if any of the two programs is closer to reality. FACTS shows a lower WIP with very short confidence intervals which leads to the conclusion that studying this parameter with this program can result in more accuracy.

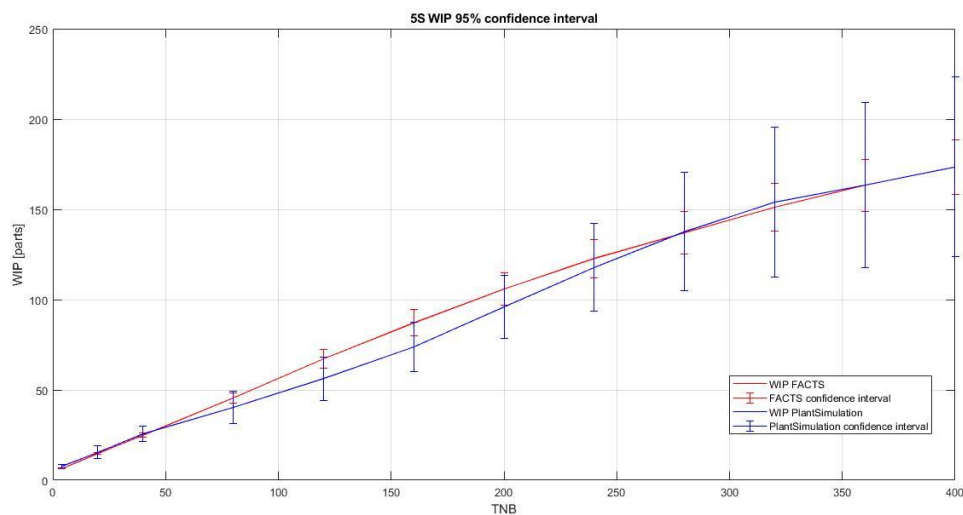


Figure 21. WIP depending on the TNB for FACTS and Plant Simulation for 5 machines

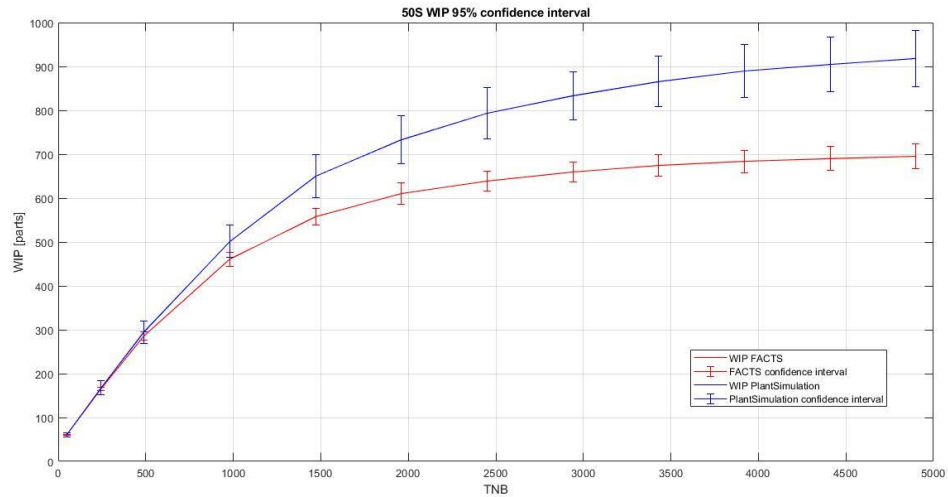


Figure 22. WIP depending on the TNB for FACTS and Plant Simulation for 50 machines

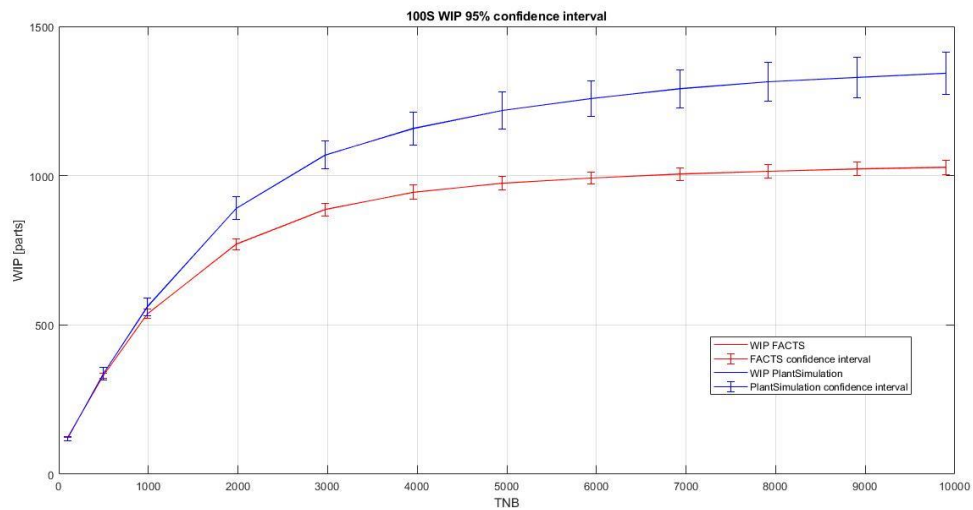


Figure 23. WIP depending on the TNB for FACTS and Plant Simulation for 100 machines

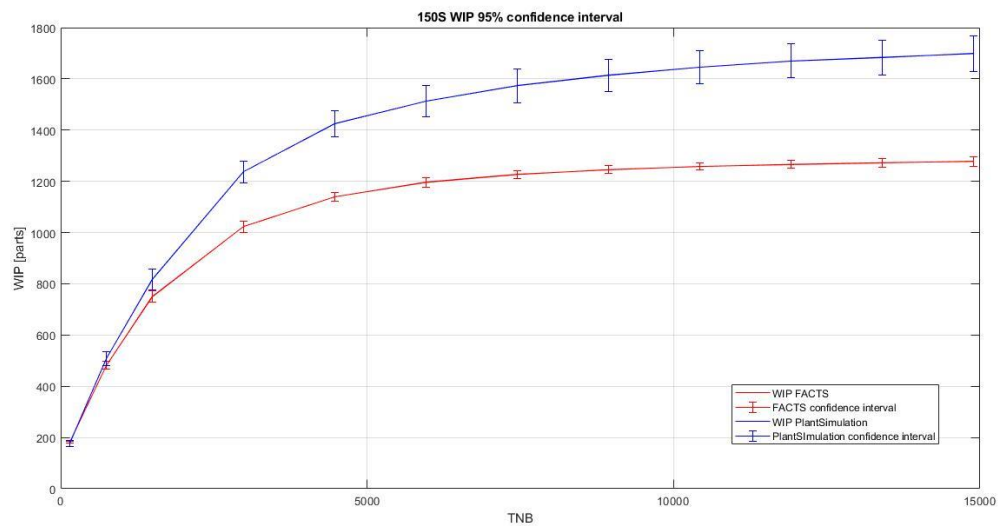


Figure 24. WIP depending on the TNB for FACTS and Plant Simulation for 150 machines

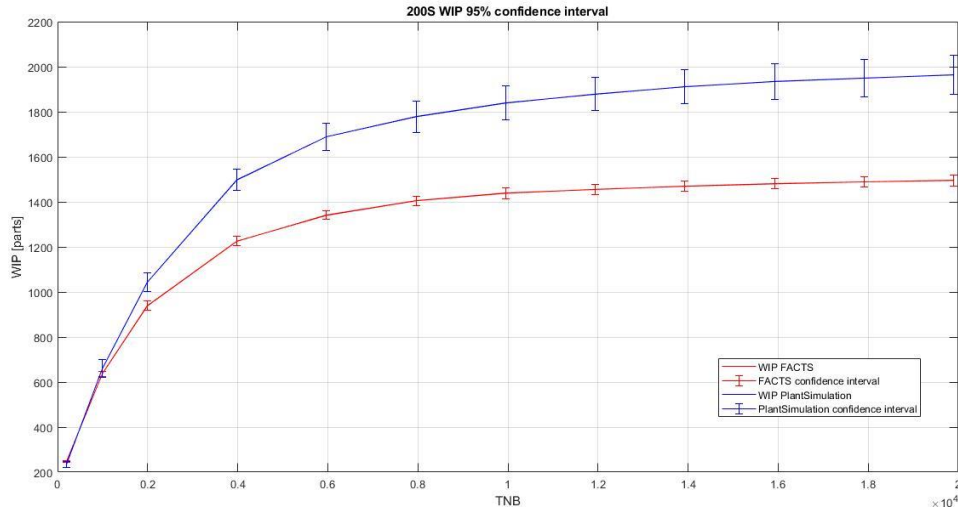


Figure 25. WIP depending on the TNB for FACTS and Plant Simulation for 200 machines

6.1.3. LT analysis

Hand by hand with the WIP comes the LT. Reinforcing the idea that a WIP study using FACTS could be more accurate, the LTs calculated by FACTS are higher and with shorter confidence intervals, proving the simulation is conservative when it comes to the number of pieces but realistic at the same time. Especially interesting is the case of Figure 26 with 5 machines, where large regions of the upper confidence interval of Plant Simulation at some points go higher than mean values of FACTS and even higher than the high values of its confidence interval giving the feeling of inconsistency from the LT from Plant Simulation.

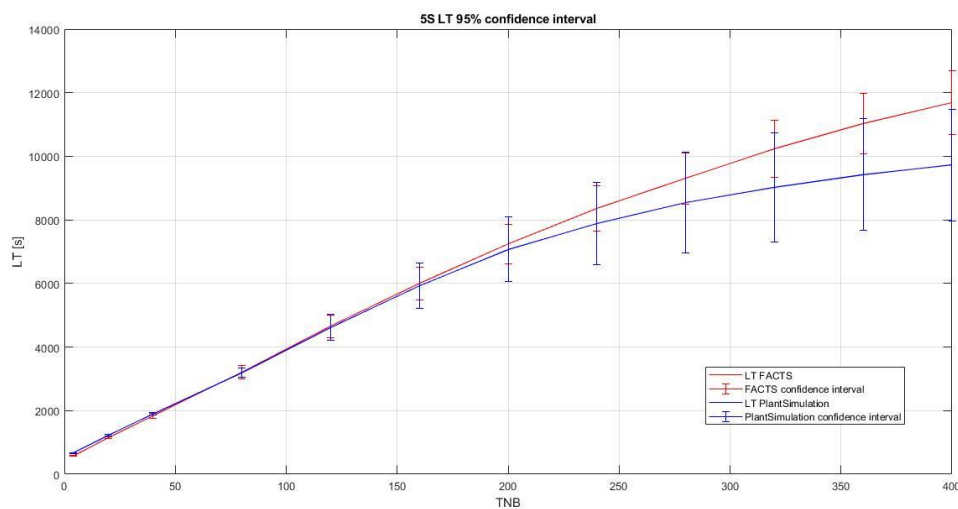


Figure 26. LT depending on the TNB for FACTS and Plant Simulation for 5 machines

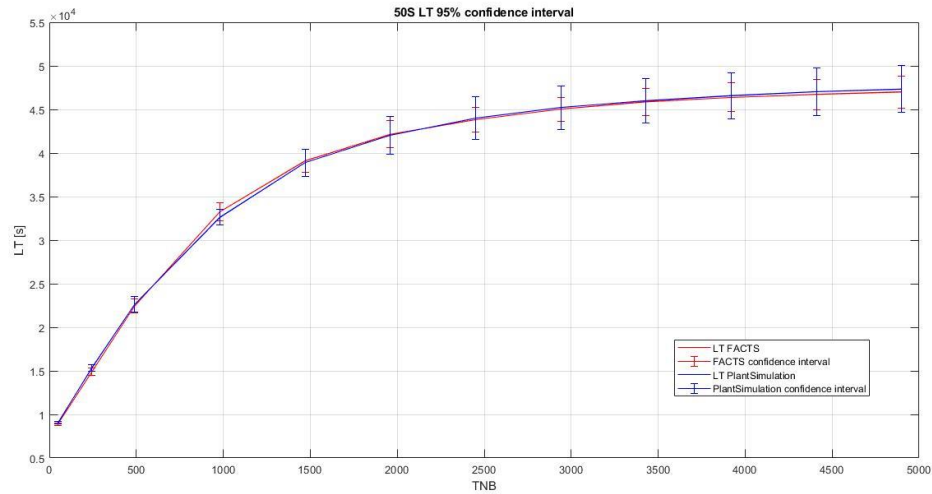


Figure 27. LT depending on the TNB for FACTS and Plant Simulation for 50 machines

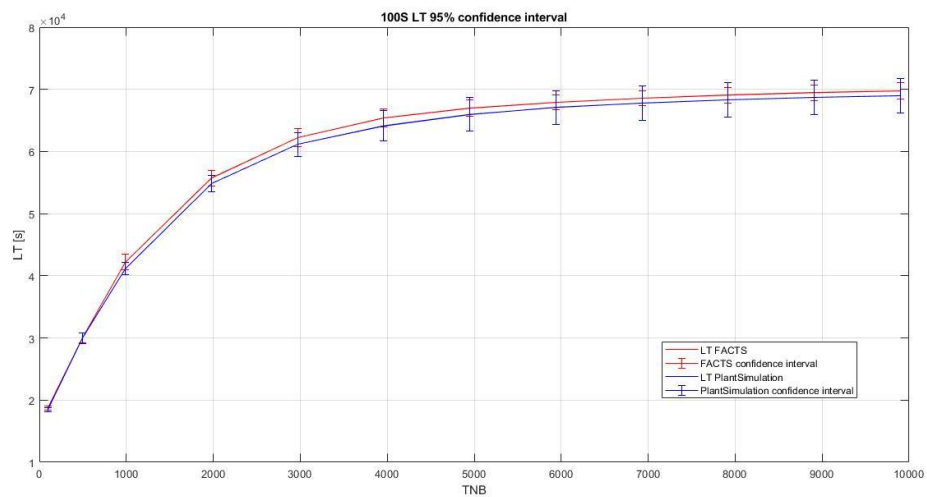


Figure 28. LT depending on the TNB for FACTS and Plant Simulation for 100 machines

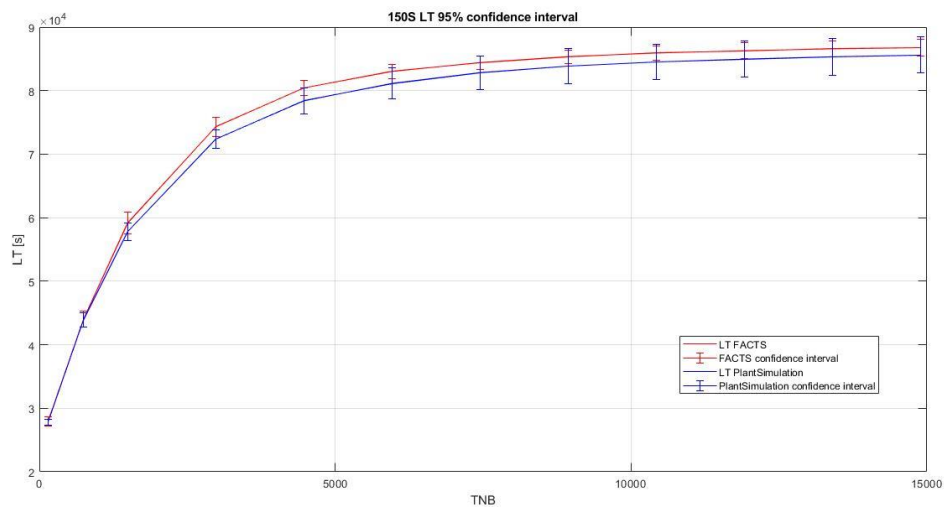


Figure 29. LT depending on the TNB for FACTS and Plant Simulation for 150 machines

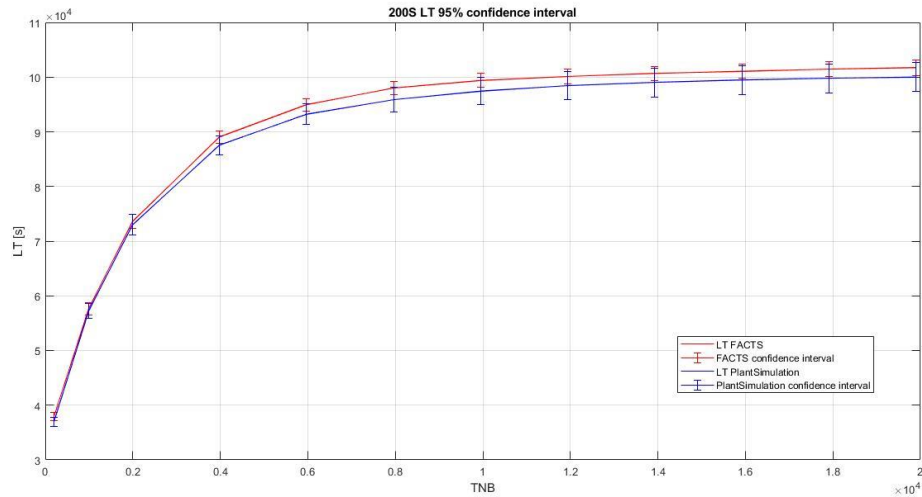


Figure 30. LT depending on the TNB for FACTS and Plant Simulation for 200 machines

6.1.4. Running times analysis

Lastly, the running times needed for each replication of the simulation are studied. The plots obtained from this experiment show a clear tendency: FACTS can simulate faster than Plant Simulation when it comes to models with a high number of machines and buffers. In the case with 200 machines (Figure 35) the difference among both programs can be up to 0.7 seconds in the case of a TNB of 280, being the time consumed by FACTS 2.26 seconds and 3.04 by Plant Simulation. The exception comes for smaller models like the one with 5 machines (Figure 31) where FACTS (0.18s) is not as fast as Plant Simulation (0.10s), being 0.08 seconds slower. Another aspect to consider is that the confidence intervals are higher for FACTS than for Plant Simulation for the first time in all the sets of experiments, probably due to the method used to quantify the mean values of the running times every replication by MATLAB, showing differences of up to +0.26s -0.26s in the case of the model of 150 machines (Figure 34).

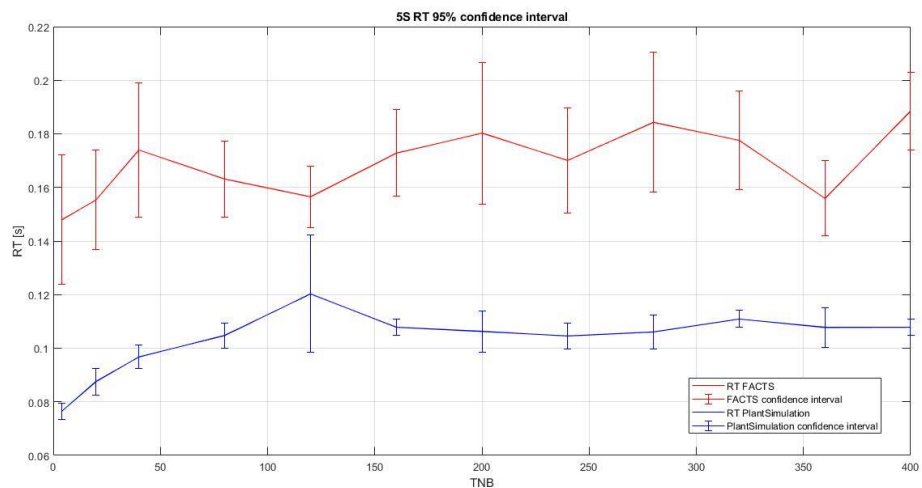


Figure 31. Running Time depending on the TNB for FACTS and Plant Simulation for 5 machines

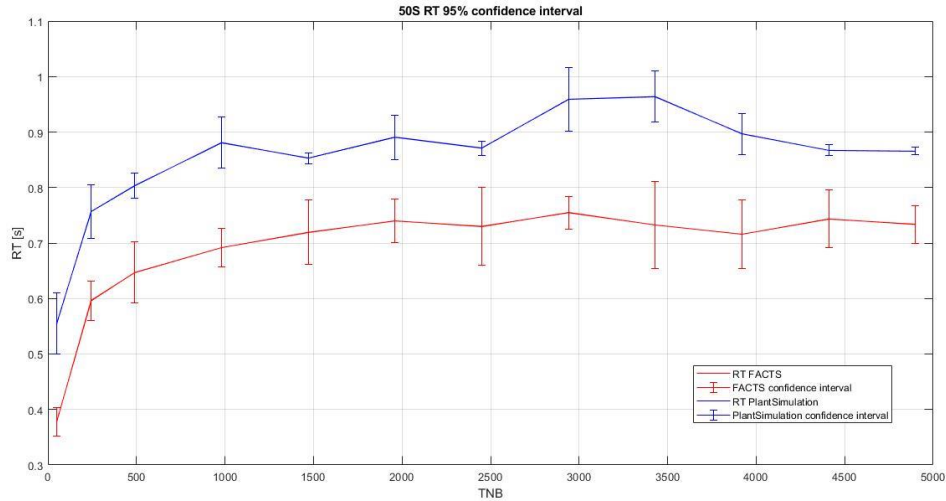


Figure 32. Running Time depending on the TNB for FACTS and Plant Simulation for 50 machines

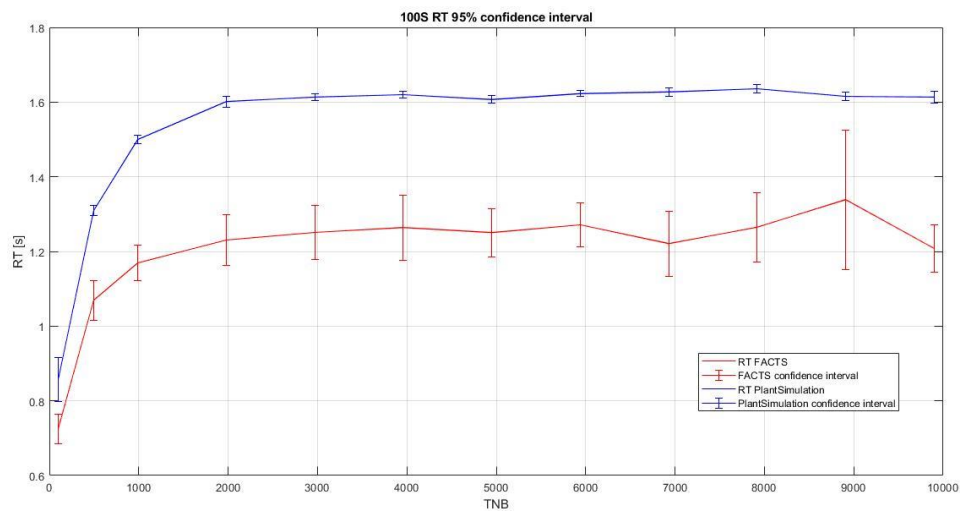


Figure 33. Running Time depending on the TNB for FACTS and Plant Simulation for 100 machines

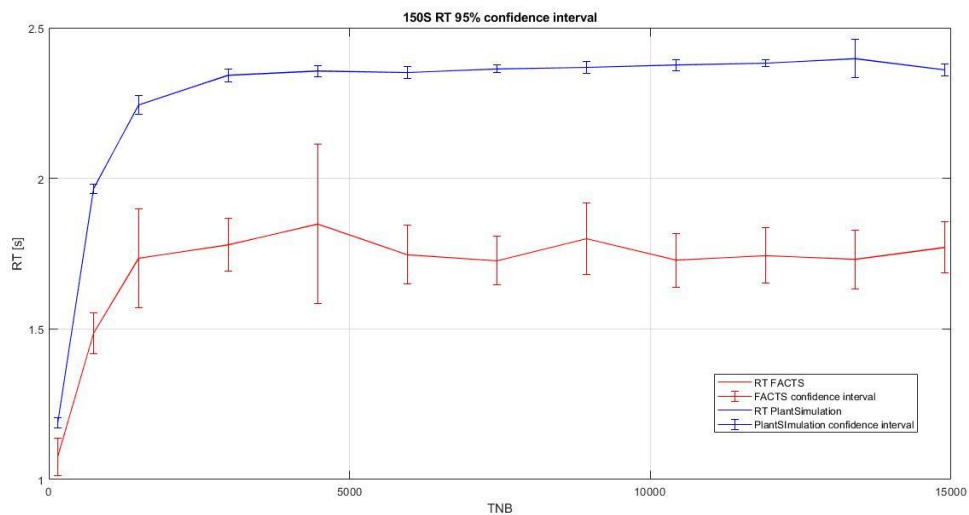


Figure 34. Running Time depending on the TNB for FACTS and Plant Simulation for 150 machines

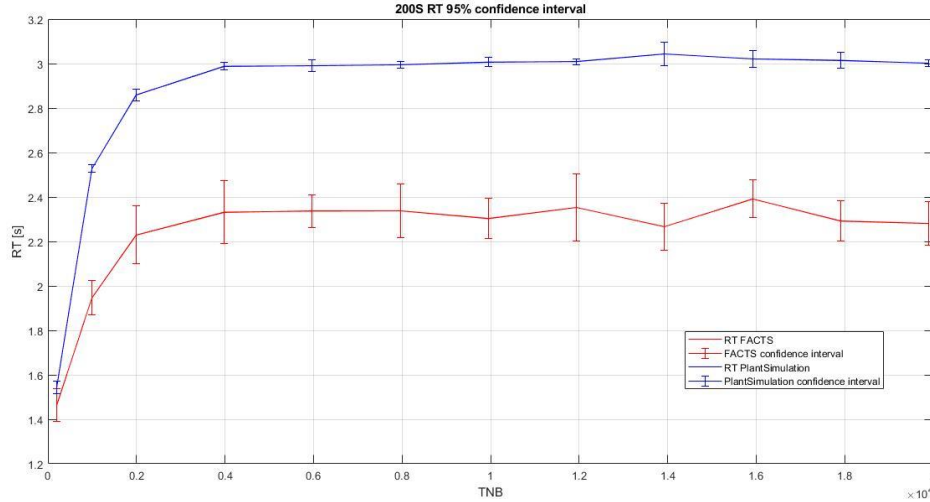


Figure 35. Running Time depending on the TNB for FACTS and Plant Simulation for 200 machines

Further analysing the running times relating to the number of machines, N being either 1 or 100 (Figure 36), the extreme scenarios, the plots confirm the ideas exposed initially. Another interesting conclusion is that Plant Simulation is more affected by the TNB, as for 200 machines, FACTS goes from 1.46s to 2.28s and Plant Simulation doubles it going from 1.5s to 3s for N 1 or 200 respectively.

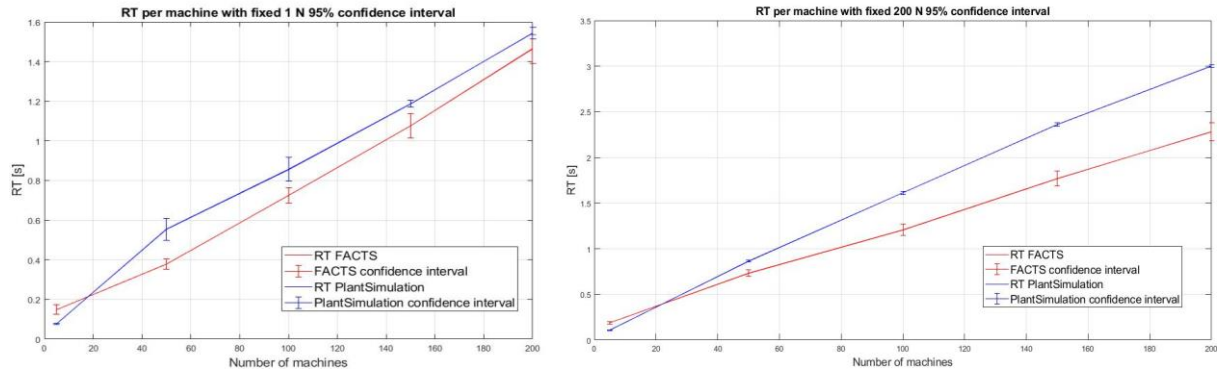


Figure 36. Running Time per number of machines with a fixed N

6.2. Accuracy / Lean buffer benchmarking

In this experiment, the optimization is done first so the simulation software can use the FACTS-optimized TNB, and then the results are analyzed for every model. The last part of the experiment regards the Lean Buffer feature of PSE Toolbox

6.2.1. Optimization

The optimization for each of the three models is carried out in FACTS with the objectives of minimizing the buffer capacity and maximizing the TH, using 5000 replications. Starting with the Bernoulli models, the base case comes first. The plot below shows all 5000 evaluations (or solutions in the MOO terminology) in the TNB objective space with the TH in the Y-axis.

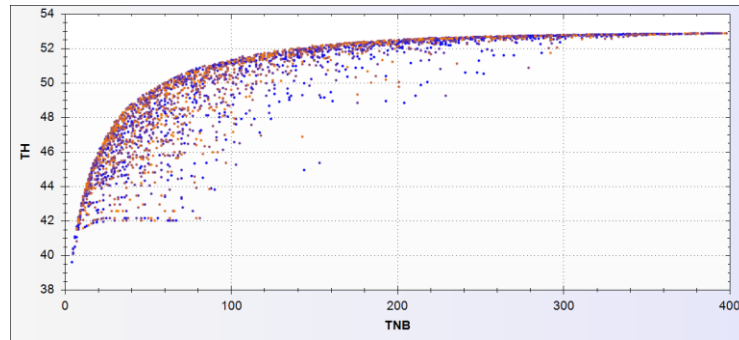


Figure 37. 5000 solutions for the Bernoulli base case, TH, and TNB

Applying the NDS filter, the Pareto front is obtained for the TH regarding TNB (Figure 38).

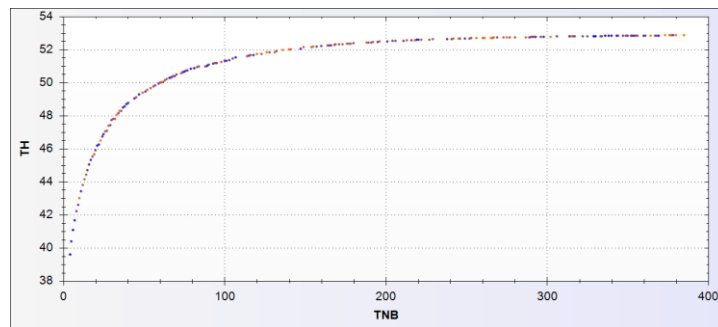


Figure 38. Pareto front for the Bernoulli base case, TH, and TNB

Following the same steps until now, the Pareto fronts are obtained for the remaining two models with the Bernoulli distribution and the same three same models but following the exponential distribution:

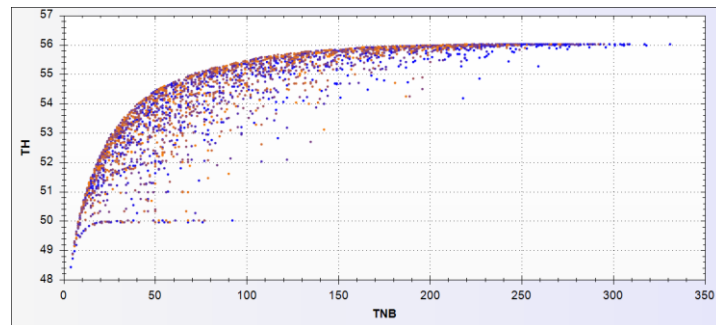


Figure 39. 5000 solutions for the Bernoulli 95% availability case, TH, and TNB

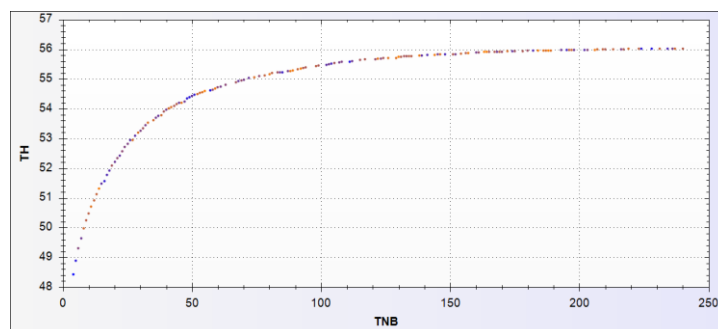


Figure 40. Pareto front for the Bernoulli 95% availability case, TH, and TNB

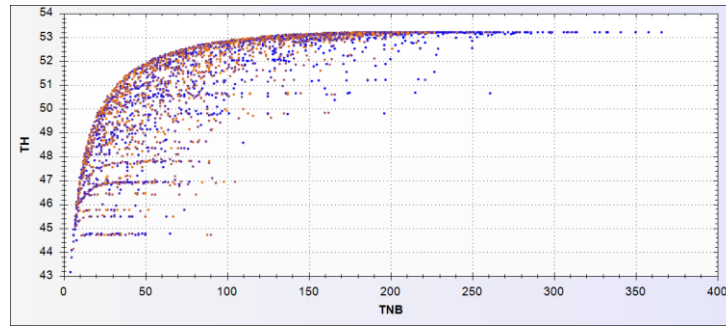


Figure 41. 5000 solutions for the Bernoulli 2 minutes MTTR case, TH , and TNB

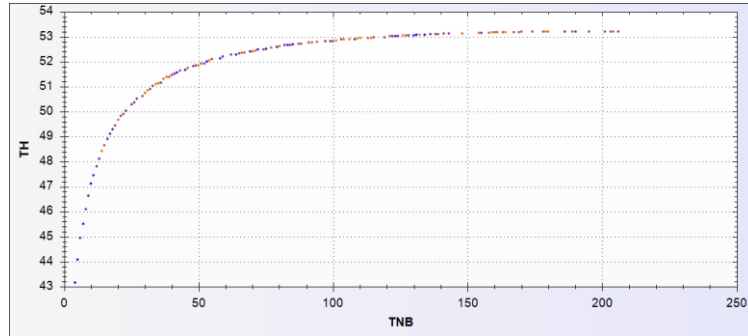


Figure 42. Pareto front for the 2 minutes MTTR case, TH , and TNB

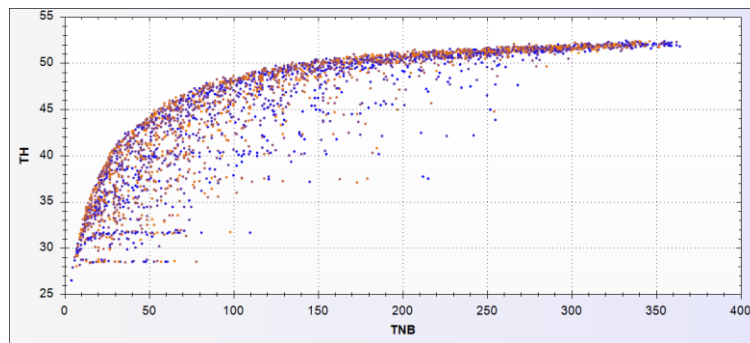


Figure 43. 5000 solutions for the exponential base case, TH , and TNB

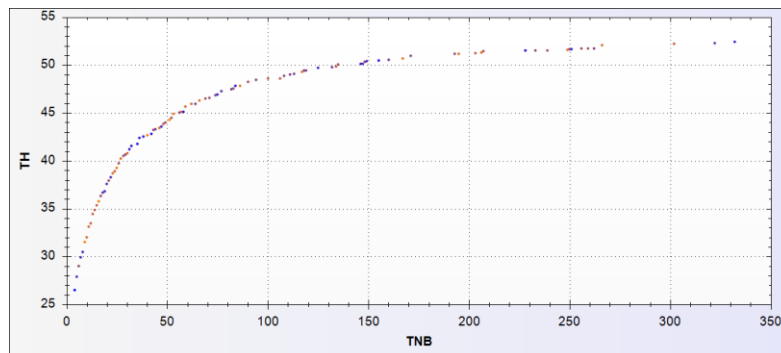


Figure 44. Pareto front for the exponential base case, TH , and TNB

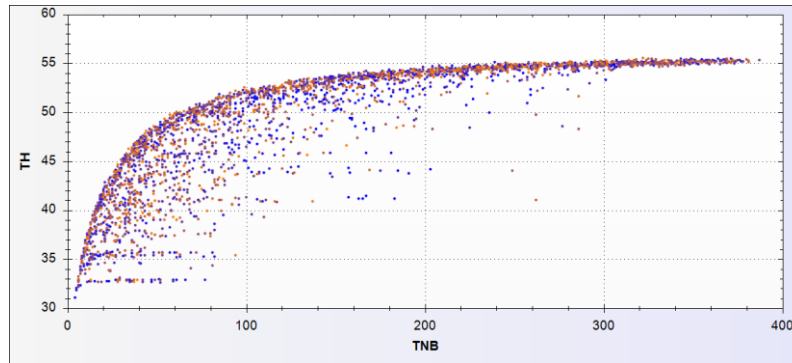


Figure 45. 5000 solutions for the exponential 95% availability case, TH, and TNB

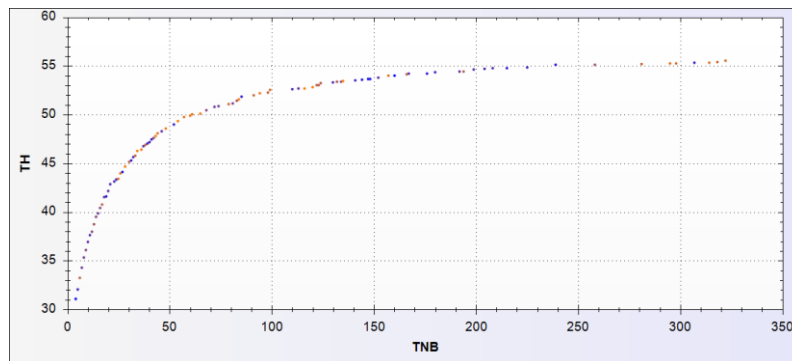


Figure 46. Pareto front for the exponential 95% availability case, TH, and TNB

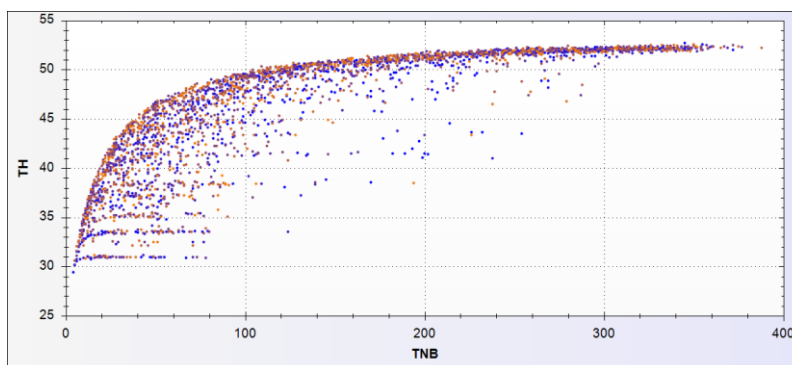


Figure 47. 5000 solutions for the exponential 2 minutes MTTR, TH, and TNB

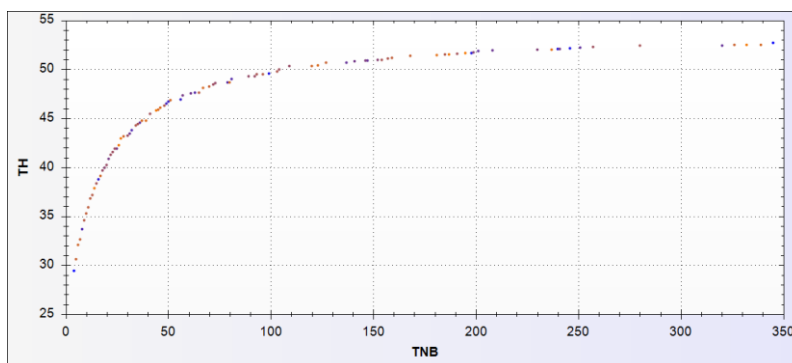


Figure 48. Pareto front for the exponential 2 minutes MTTR case, TH, and TNB

After all the Pareto fronts are obtained, the optimized TNB points calculated by FACTS are taken to a .CSV file where only 30 points are taken as explained in the design of this experiment and the simulation with Plant Simulation and PSE Toolbox (the optimization already gives the FACTS solutions for those points). The table of results for each model can be consulted in Appendix 2.

6.2.2. TH and WIP analysis with FACTS-optimized TNB, Bernoulli

The base case opens the door to the general tendency followed by the rest of the cases regarding the TH: while FACTS and Plant Simulation show an almost exact result, PSE Toolbox is more optimistic and gives the model a higher TH.

In this case, the maximum TH offered is 52.86 by FACTS, 52.79 by Plant Simulation, and 53.9 by PSE (Figure 49). In the same order, the WIP is evaluated like 139, 172, and 199 (Figure 50). The error bars for FACTS and Plant Simulation (it is not possible for PSE as it offers no standard deviation) in the WIP analysis overlap each other. The variance for this output is generally high, but it is important to check if the results are truly different from each using the Welch confidence interval (Table 3). There are zeros in every interval, which tells that the error bars give the right idea and the WIP results are not different from each other. The Welch statistical test can only be done for Plant Simulation ('PS' in the table) and FACTS as PSE Toolbox do not show any standard deviation ('SD' in the table). The R script used for calculating the Welch confidence interval can be seen in Figure 87 Appendix 2. It is used for the rest of the intervals along with the report and only requires changing the four first inputs, the value of facts, the mean value of Plant Simulation, and their standard deviations. It must be done for every different TNB point.

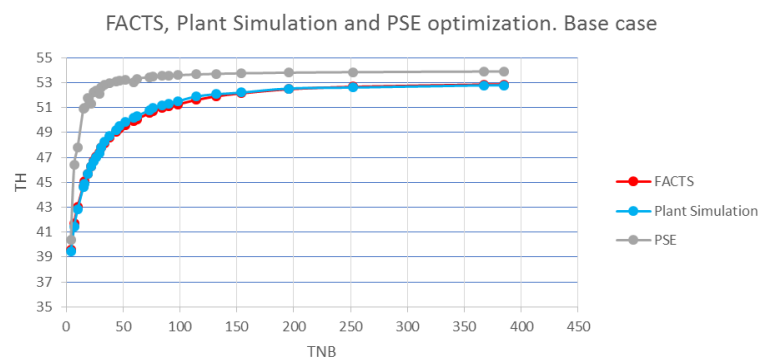


Figure 49. TH with optimized TNB by FACTS, Plant Simulation and PSE. Bernoulli base case

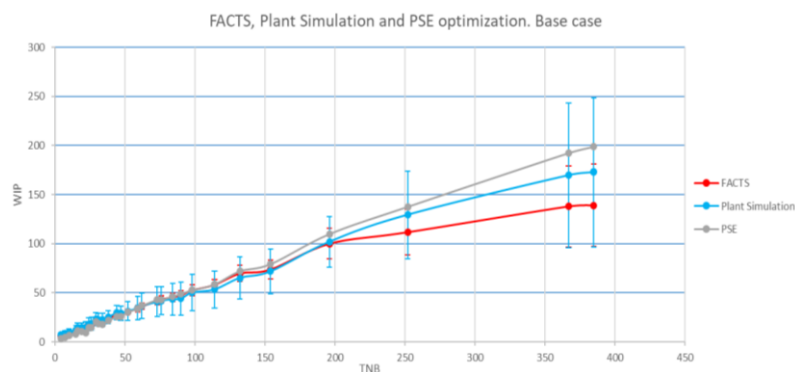


Figure 50. WIP with optimized TNB by FACTS, Plant Simulation and PSE. Bernoulli base case

TNB	FACTS WIP Mean	FACTS WIP SD	PS WIP Mean	PS WIP SD	WCImin	WCI _{max}
252	111,7175709	23,12368423	129,4	44,5800155	-51,86	16,48
367	137,9426843	41,1184644	169,6	73,7024046	-88,857	25,542
385	139,0941841	41,7726914	172,5	75,9579562	-92,205	25,393

Table 3. Welch CI for the three highest TNB points, Bernoulli base case

Moving to the 95% availability model, the results are expectedly higher in terms of TH and WIP as the system flow is higher and the machines fail less. The same pattern for both graphics is repeated, but the Welch statistical test is not carried out as the difference in the WIP is visibly low.

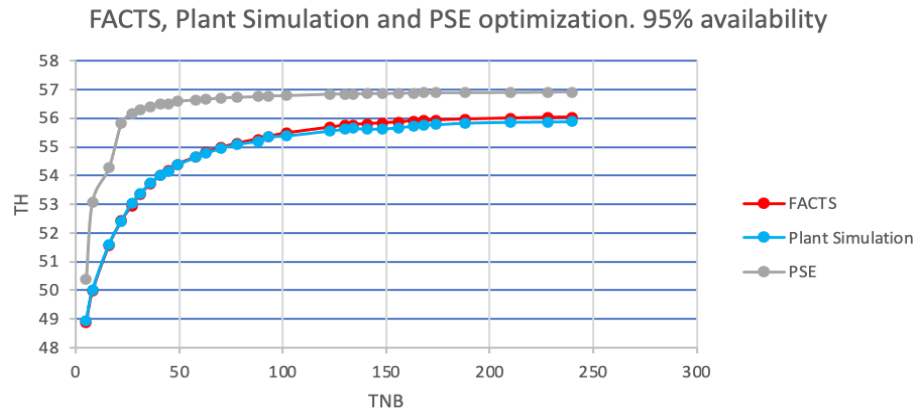


Figure 51. TH with optimized TNB by FACTS, Plant Simulation and PSE. Bernoulli 95% availability case

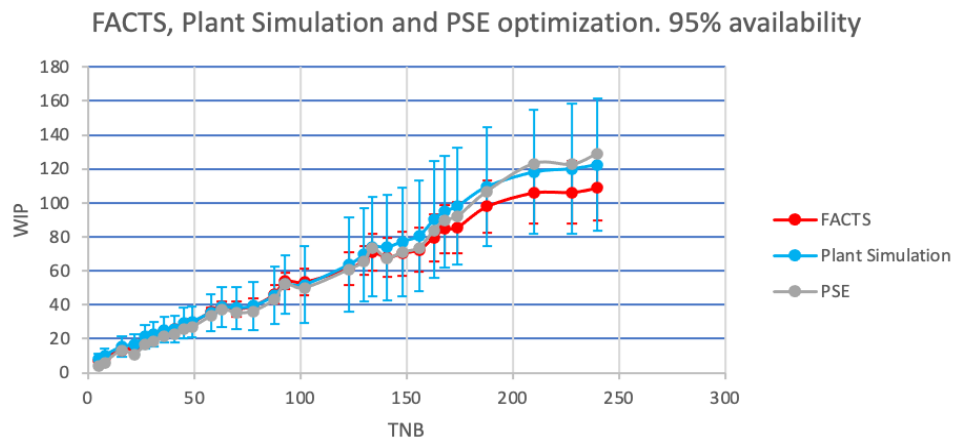


Figure 52. WIP with optimized TNB by FACTS, Plant Simulation and PSE. Bernoulli 95% availability case

Once again, the results for the case with 2 minutes of MTTR are very similar, but this time PSE gets more accurate results, as FACTS and Plant Simulation are not so penalized by the MTTR that is uncontrolled in PSE at the moment. The WIP values are now very similar for the three of the simulation software.

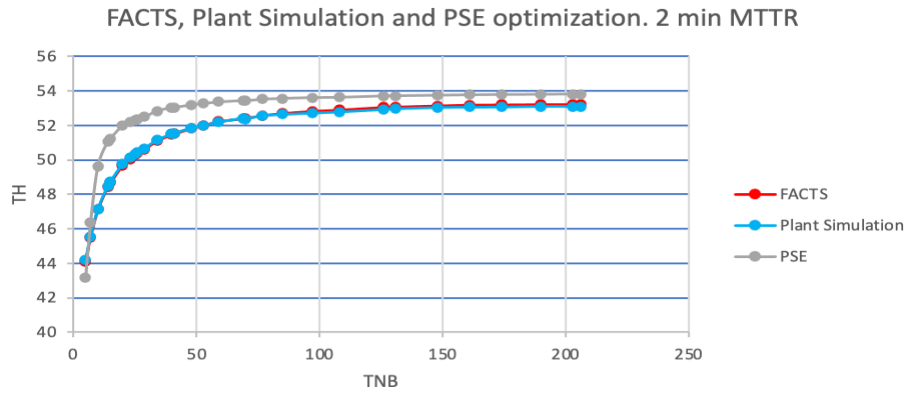


Figure 53. TH with optimized TNB by FACTS, Plant Simulation and PSE. Bernoulli 2 min MTTR

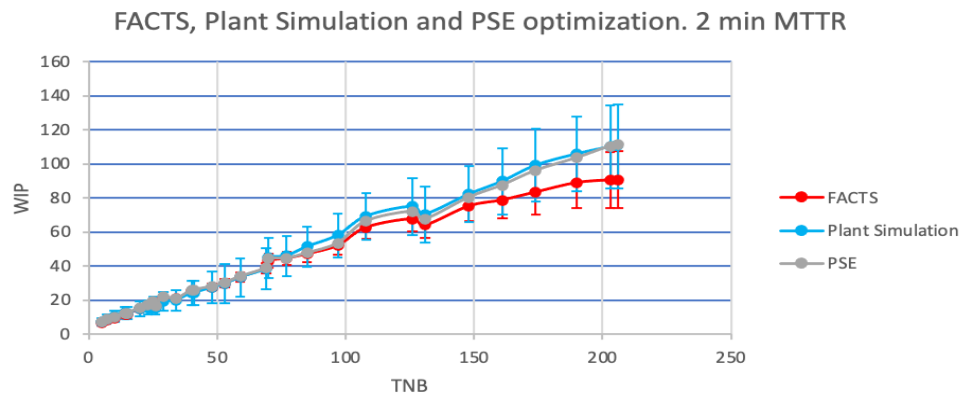


Figure 54. WIP with optimized TNB by FACTS, Plant Simulation and PSE. Bernoulli 2 min MTTR

6.2.3. TH and WIP analysis with FACTS-optimized TNB, exponential

Now that the MTTR can be controlled in PSE thanks to Equations 9, 10, and 11, PSE Toolbox presents a more accurate TH output even though it is higher than the other two commercial software.

In the base case, FACTS and Plant Simulation draw the plot almost identically for the TH while PSE starts with a higher value for low TNB to almost match the others later when the TNB is over 250 (Figure 55). The WIP seems very accurate for the three programs (Figure 56), being PSE Toolbox just in the middle this time. As the TNB is relatively high, the Welch statistical test is done (Table 4), reassuring the idea that effectively the WIP are similar between each other, as all the confidence intervals contain a 0.

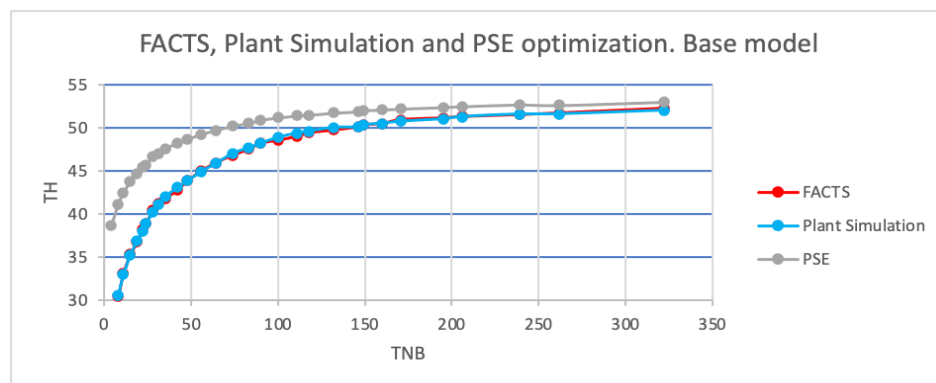


Figure 55. TH with optimized TNB by FACTS, Plant Simulation and PSE. Exponential base case

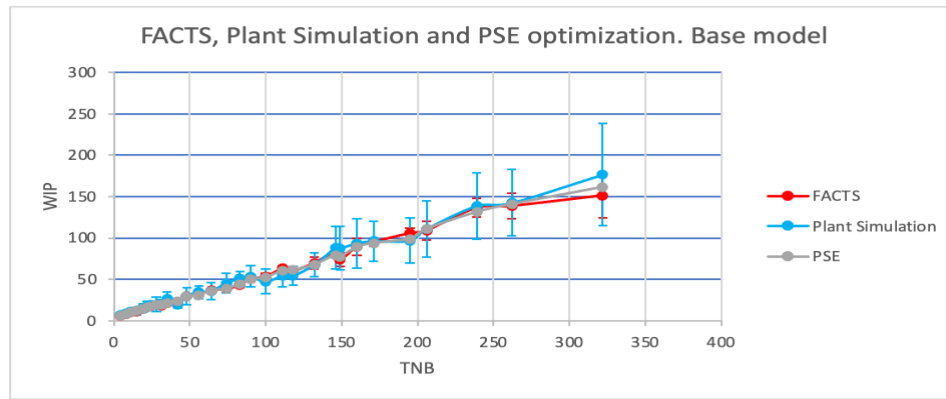


Figure 56. WIP with optimized TNB by FACTS, Plant Simulation and PSE. Exponential base case

TNB	FACTS WIP Mean	FACTS WIP SD	PS WIP Mean	PS WIP SD	WCImin	WCImax
239	136,660801	11,7033143	139	40,2850951	-31,7	27,026
262	138,840784	15,6944737	142,6	39,9699887	-32,426	26,906
322	151,424338	26,7362531	176,8	61,3384056	-71,3547	20,60207

Table 4. Welch CI for the three highest TNB points, exponential base case

Following the experiment, the exponential case with 95% of availability repeats the same pattern that the base case, this time being the TH for lower a TNB even higher (Figure 57). Analyzing the WIP results (Figure 58), they seem to be more apart from each other in the case of FACTS and Plant Simulation, although their error bars still overlap. The Welch confidence interval, [298, 322] shows the results are not different when the TNB is 258.

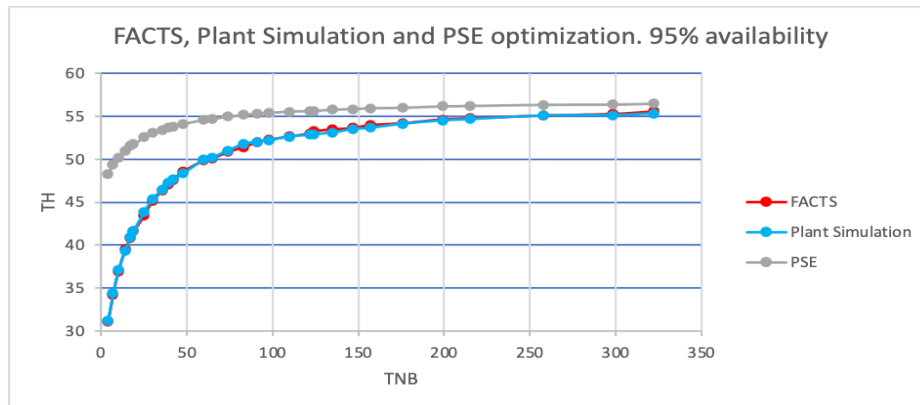


Figure 57. TH with optimized TNB by FACTS, Plant Simulation and PSE. Exponential 95% availability case

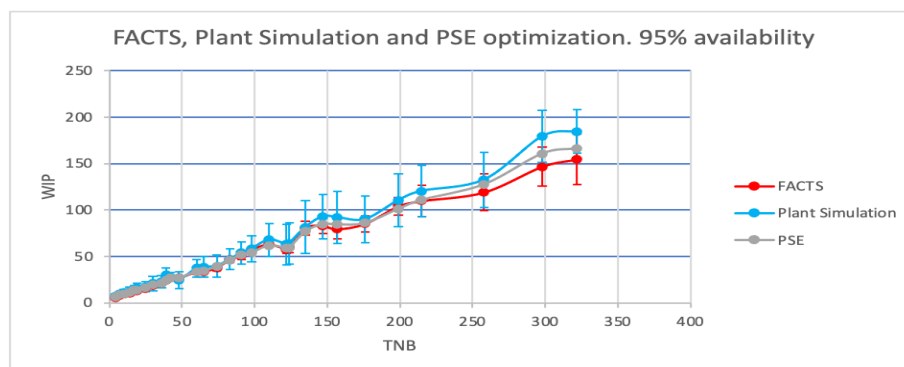


Figure 58. WIP with optimized TNB by FACTS, Plant Simulation and PSE. Exponential 95% availability case

TNB	FACTS WIP Mean	FACTS WIP SD	PS WIP Mean	PS WIP SD	WCImin	WCImax
258	119,038228	19,6459546	132,6	29,5040487	-37,36322	10,23922
298	146,765782	21,0524029	179,3	28,0952744	-55,99	-9,07
322	154,620723	27,0777653	184,7	23,3049828	-53,7359	-6,424

Table 5. Welch CI for the three highest TNB points, exponential 95% availability case

Finally, the exponential model with a 2 minutes MTTR is studied, following the same trend as before (Figure 59). The WIP values seem to differ from the TNB values of 257 and 332, being 160 and 187 for Plant Simulation and 130 and 159 for FACTS for those TNB points (Figure 59). The Welch confidence interval (Table 6) shows only for TNB 257 the results are truly different, even though the upper bound of the confidence is close to zero (1.71) for the 332 TNB point.

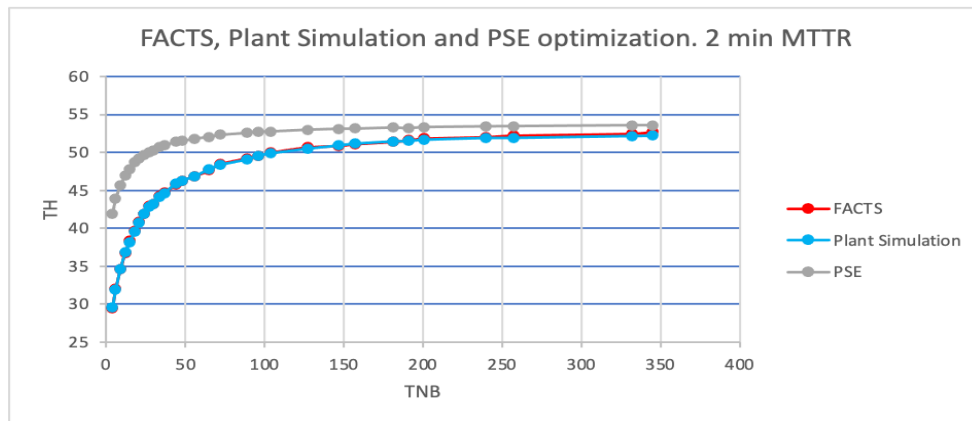


Figure 59. TH with optimized TNB by FACTS, Plant Simulation and PSE. Exponential 2 min. MTTR case

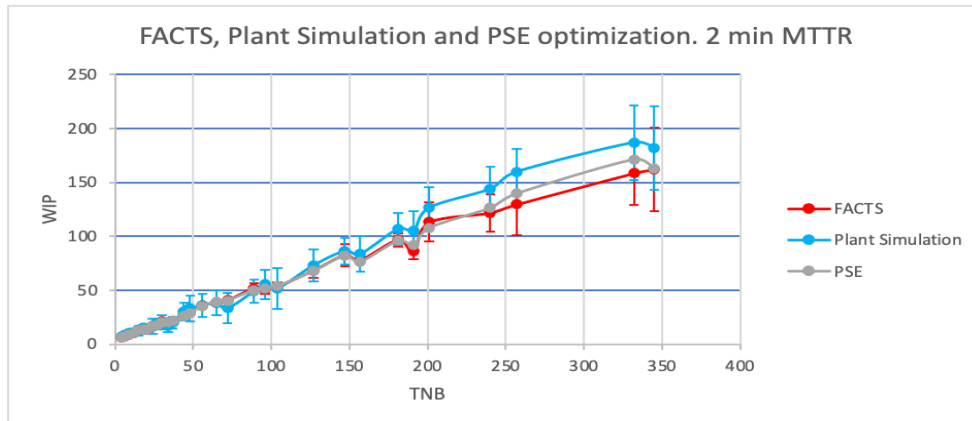


Figure 60. WIP with optimized TNB by FACTS, Plant Simulation and PSE. Exponential 2 min. MTTR case

TNB	FACTS WIP Mean	FACTS WIP SD	PS WIP Mean	PS WIP SD	WCImin	WCImax
257	129,569254	28,2664547	159,9	21,5275018	-54,056	-6,605
332	158,554362	29,4425444	187	34,4641521	-58,61	1,71
345	162,009461	38,5157272	182,1	38,7941863	-56,318	16,318

Table 6. Welch CI for the three highest TNB points, exponential base case

6.2.4. TH and WIP analysis with Lean buffer from PSE Toolbox TNB

Comparing the results coming from the Bernoulli base case simulations for the Lean buffer experiments, PSE Toolbox meanwhile remains to show higher TH values for the same TNB mainly because of the inability of modifying the MTTR (Figure 61). Another important conclusion coming from this plot is that the newly introduced ‘Simulation’ function from PSE gives very similar results compared to the other functions. But the most interesting aspect is how the curve of TH of FACTS coming from the refined FACTS-optimization done in the previous sub-chapters and the TH also coming from FACTS but using the TNB calculated by PSE Toolbox practically overlap each other (Figure 62). These two plots could be represented together in just one, but there would be too much data represented that would end in overlaps impeding to see the results clearly.

The WIP once again is very similar for both FACTS simulations, being the one using the lean buffer even more desirable as its curve is smoother, the WIP is lower (Figure 63). PSE, in this case, goes to very high levels in a straight line, unmatching FACTS, especially after the TNB reaches more than 200 but still close to the results coming from Plant Simulation.

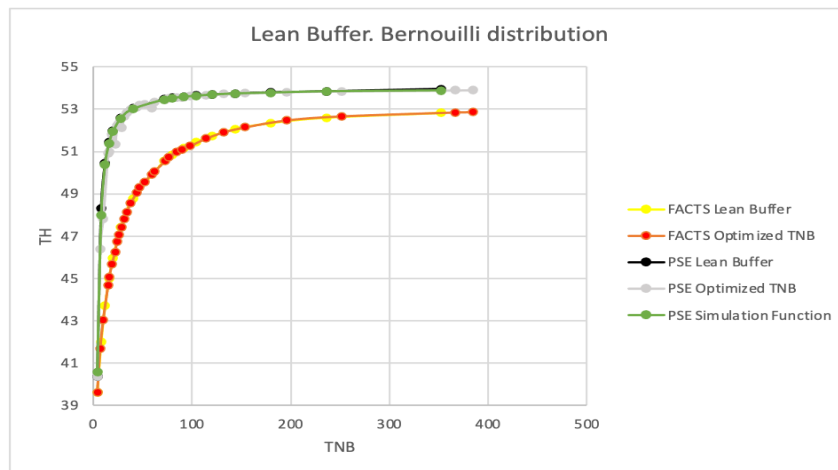


Figure 61. PSE TH results compared to FACTS using FACTS-optimized TNB and Lean buffer TNB. Bernoulli base case

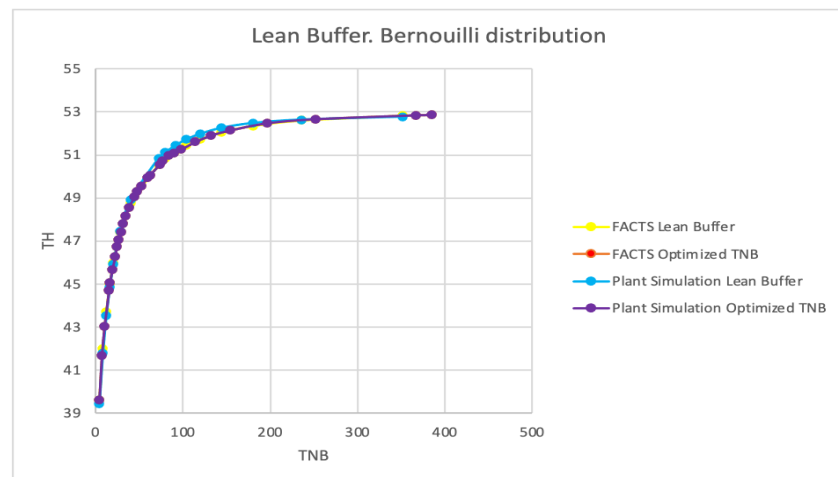


Figure 62. Plant Simulation TH results compared to FACTS using FACTS-optimized TNB and Lean buffer TNB. Bernoulli base case

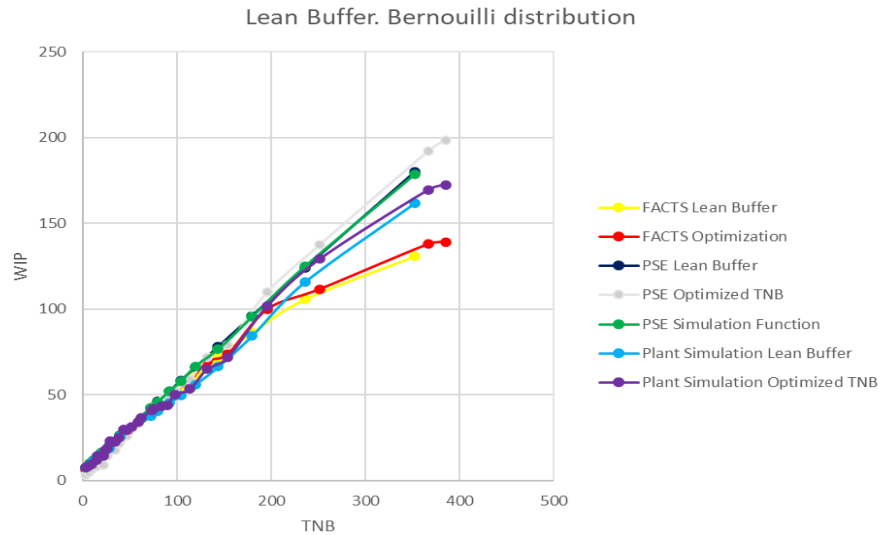


Figure 63. FACTS, PSE, and Plant Simulation WIP results using FACTS-optimized TNB and Lean buffer TNB. Bernoulli base case

Moving to the exponential simulations, the plots are quite similar, but TH tends to be lower for all cases. Even though PSE Toolbox was given a MTTR, its TH continuous to be higher than expected especially at lower TNB (Figure 64) where the E is low, while FACTS once again offers similar results for both the optimization points and the Lean buffer calculated by PSE. The WIP this time is similar for all cases, giving very good results for the three simulation programs using the Lean buffer calculated by PSE Toolbox.

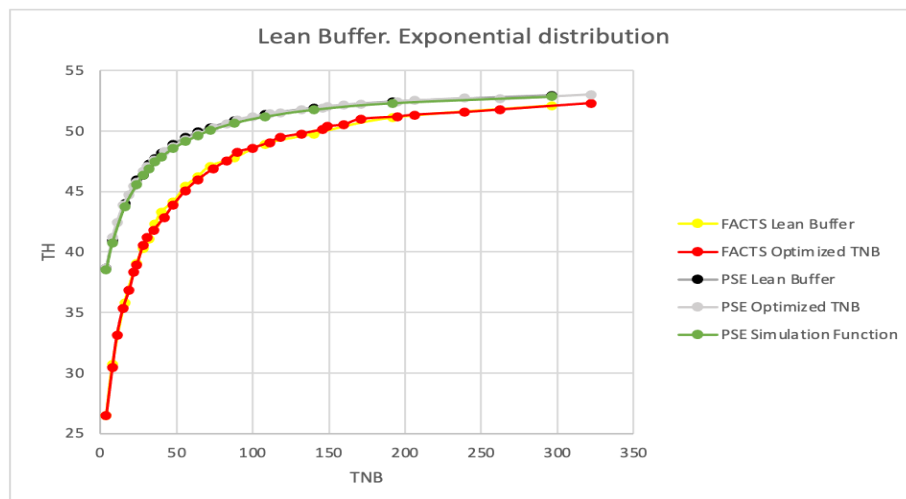


Figure 64. PSE TH results compared to FACTS using FACTS-optimized TNB and Lean buffer TNB. Exponential base case

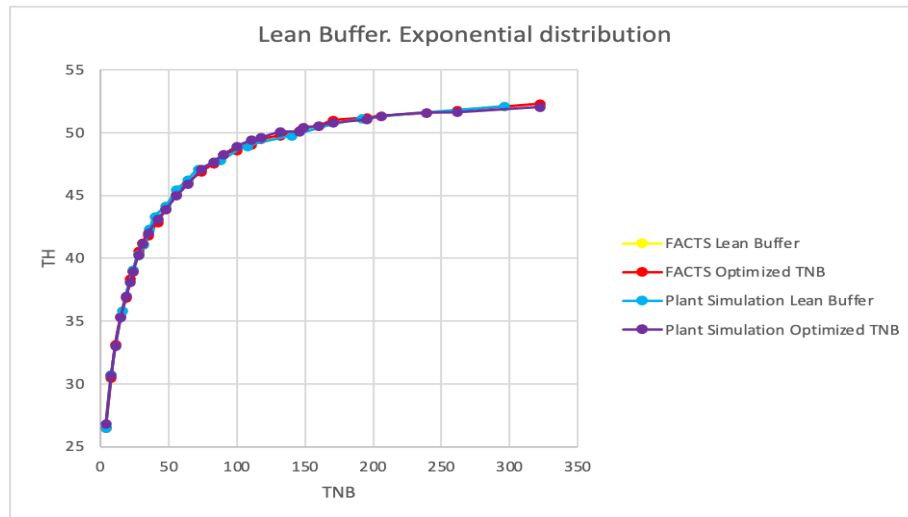


Figure 65. Plant Simulation TH results compared to FACTS using FACTS-optimized TNB and Lean buffer TNB. Exponential base case

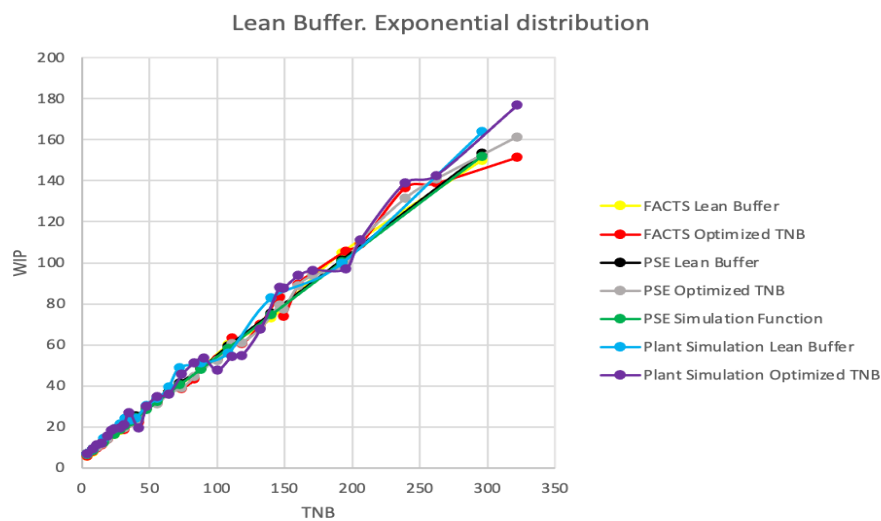


Figure 66. FACTS, PSE, and Plant Simulation WIP results using FACTS-optimized TNB and Lean buffer TNB. Exponential base case

The experiment proves the Lean Buffer feature from PSE Toolbox can be very timesaving if used correctly, as the data obtained from FACTS and Plant Simulation do not differ so much among the proper optimization and the Lean buffer, and when it does, it does it for the best. Eventually, using PSE Toolbox for real-life calculations of the TH in these conditions do not seem very useful, as the results are always overrated compared to FACTS or Plant Simulation.

6.3. Bottleneck

The results obtained from this experiment are composed of 12 graphs coming from FACTS (3 bottleneck bar plots for the 3 different buffer sizes using the Bernoulli distribution, other 3 using the exponential distribution, 3 utilization plots for every buffer size for Bernoulli, and other 3 for the exponential distribution) and 6 diagrams with results coming from PSE (again 3 for the Bernoulli distribution and another 3 for the exponential).

6.3.1. Bottleneck Bernoulli distribution

In the 3 different Bernoulli scenarios, the bottleneck is always identified as the third machine by both FACTS and PSE Toolbox. Analyzing the first case: Bernoulli distribution with N 1, the bottleneck is identified as operation 3 (Figure 67) by FACTS being it the bottleneck at least 50% of the time (sole bottleneck 18% of the time and shifting bottleneck 32% of the time). Further analyzing the utilization diagram (Figure 68), upstream the machine 3, operations 1 and 2 are blocked 30% and 75% of the time, respectively, that meaning they cannot send their parts to the next machine because machine 3 is blocked and starved 10% of the time (5% blocked 5% starved). The flow downstream to the source presents huge starving percentages, particularly high for machine 5 where there is no blocking time, and it is empty 25% of the time. PSE Toolbox provides a very similar analysis (Figure 69), with machine 1 being blocked 25.11% of the time, machine 2 blocked 19.44%, machine 4 starving 18.23% and blocked only 7% and machine 5 starving for parts 20.11%. Machine 1 and machine 5 present no starvation or blockage, respectively, as explained at the end of Chapter 3.4 in the report. PSE also clarifies buffer 4 is the 'buffer bottleneck' (BN-b). The disruption produced in machine 3 mainly occurs because of its failed time (15% of the time) given by its lower availability (86%).

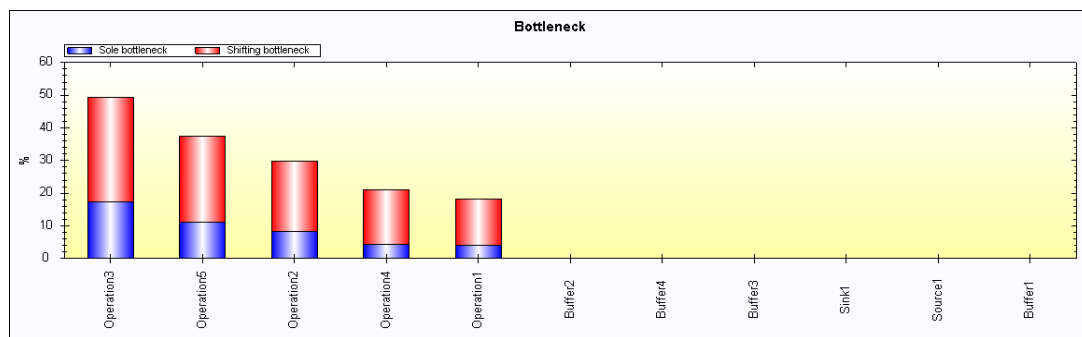


Figure 67. Bottleneck bar plot from FACTS. 5 machines Bernoulli distribution, N 1

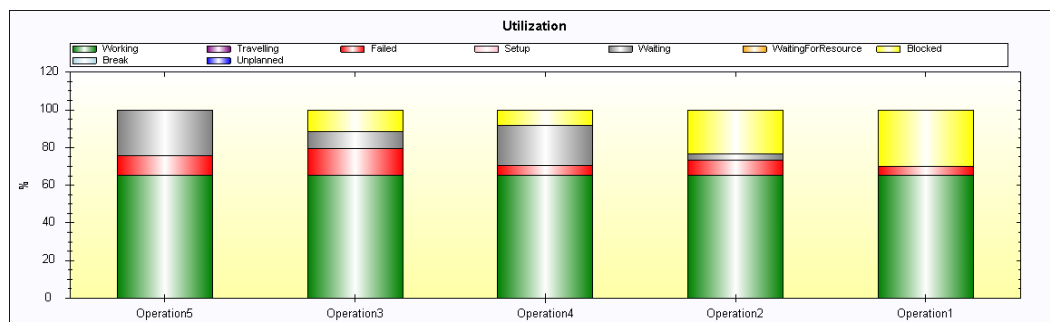


Figure 68. Utilization bar plot from FACTS. 5 machines Bernoulli distribution, N 1

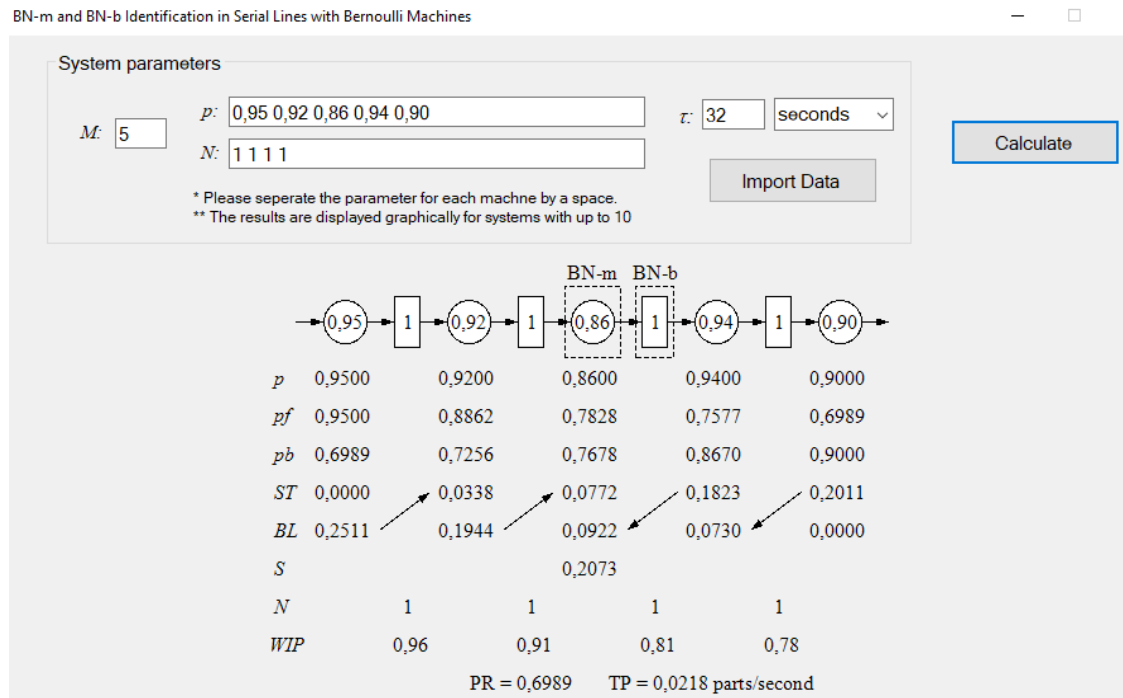


Figure 69. Bottleneck identification from PSE Toolbox. 5 machines Bernoulli distribution, N 1

This operation being the bottleneck becomes more obvious as the N increases. With the capacity set at 10, machine 3 is a bottleneck 54% of the time (Figure 70), and with 50 it is a bottleneck 76% of the time following FACTS (Figure 73) and PSE Toolbox later confirming it (Figure 72 for 10 machines and Figure 75 for 50 machines). The trend followed upstream and downstream the bottleneck is the same, being extreme in the last simulation with 50 machines where the operation 3 basically have no time where it is blocked or starved as it always working full capacity (82%) or failed (17%), machines 1 and 2 do not starve and are always blocked (13% and 9% respectively) or working with its failing times and machines 4 and 5 are starving (10% and 5% respectively) or working, indicating the problem machine 3 creates in the flow of parts (Figure 74). Data gathered by PSE Toolbox points in the same direction: machine 3 does not have time to block or starve as it is working at its full, machine 1 and 2 never starving and machines 4 and 5 never blocked. The percentages offered by each software is slightly different in this case as machines 1 and 2 are blocked for only 9% and 6% of the time and machines 4 and 5 are starving 8% and 4%. PSE now points out the second buffer as the BN-b in both cases.

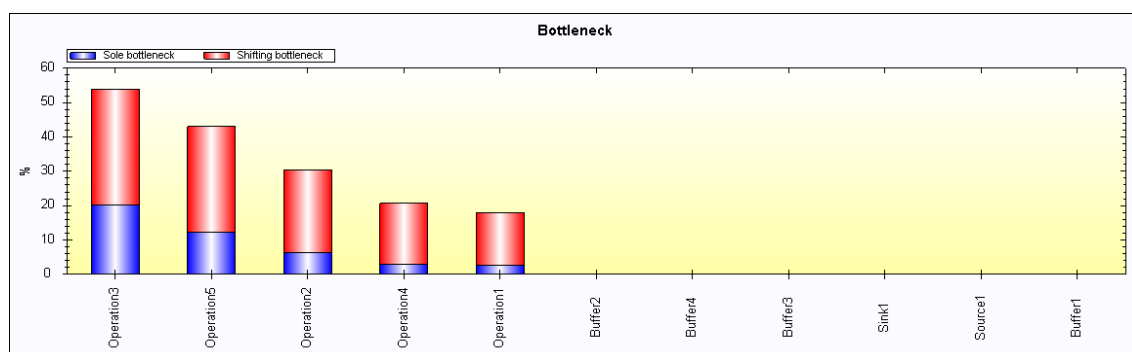


Figure 70. Bottleneck bar plot from FACTS. 5 machines Bernoulli distribution, N 10

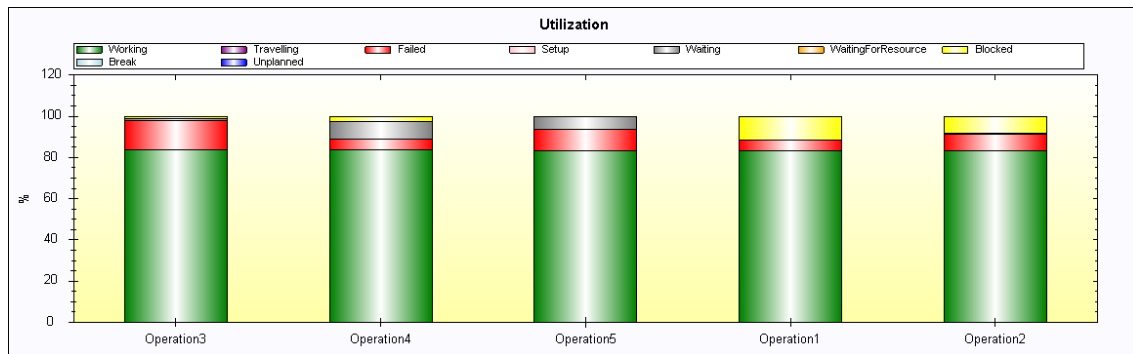


Figure 71. Utilization bar plot from FACTS. 5 machines Bernoulli distribution, N 10

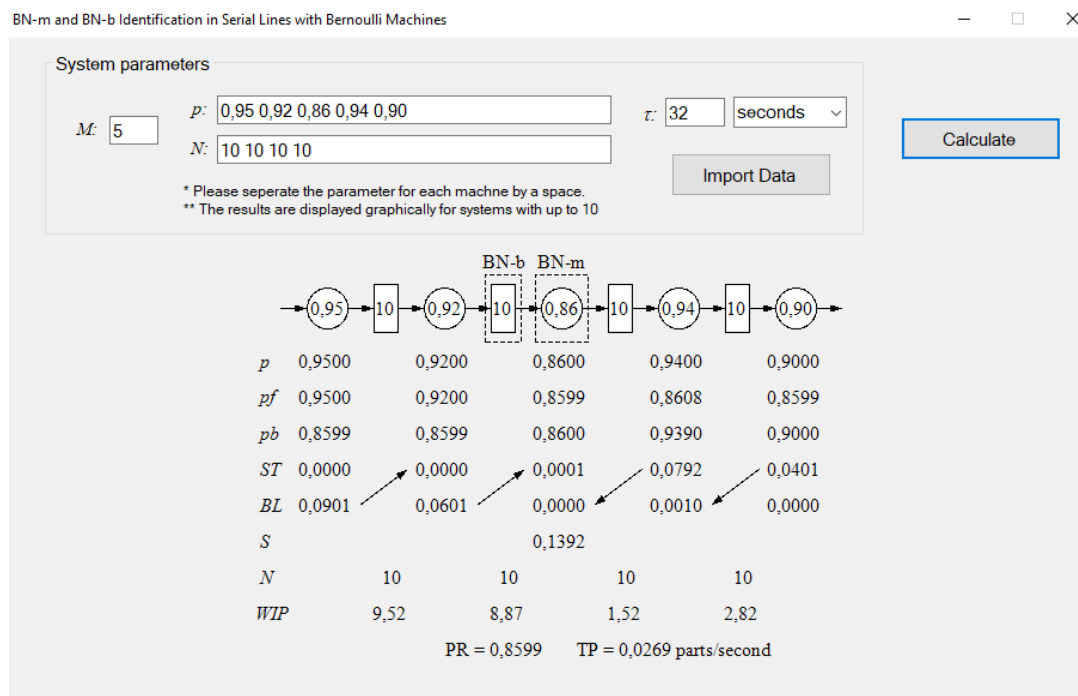


Figure 72. Bottleneck identification from PSE Toolbox. 5 machines Bernoulli distribution, N 10

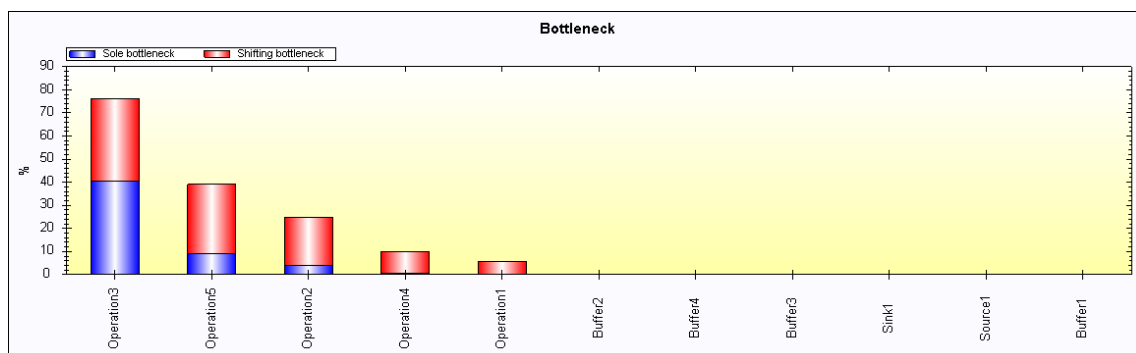


Figure 73. Bottleneck bar plot from FACTS. 5 machines Bernoulli distribution, N 50

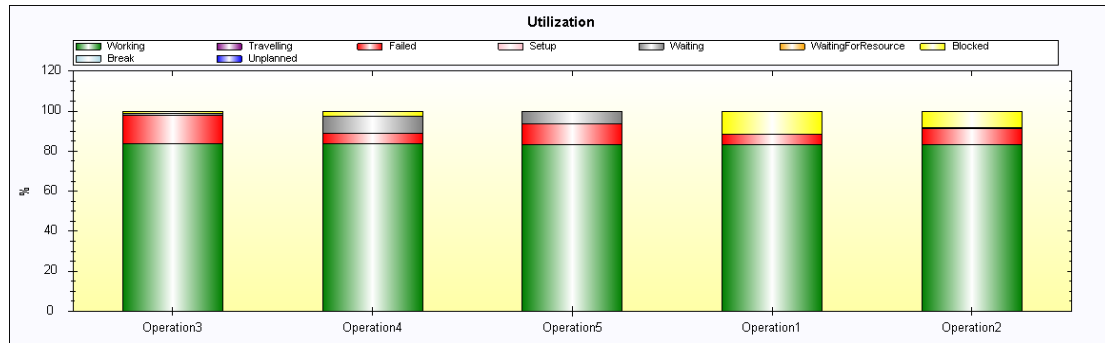


Figure 74. Utilization bar plot from FACTS. 5 machines Bernoulli distribution, N 50

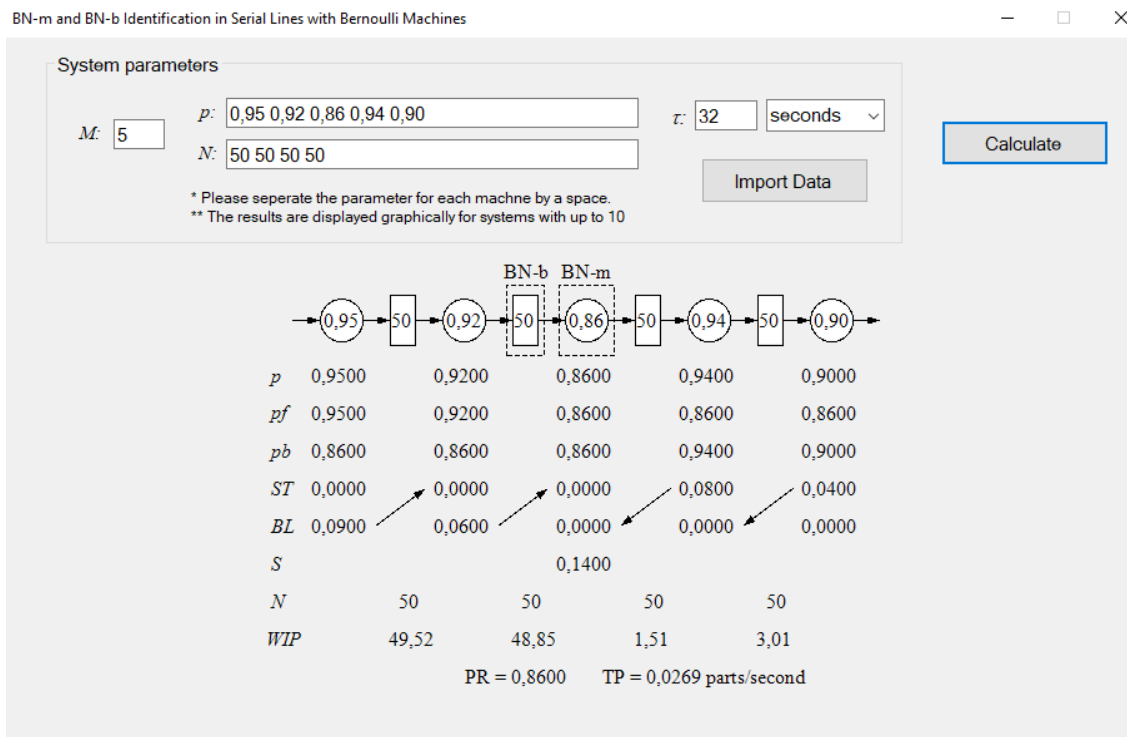


Figure 75. Bottleneck identification from PSE Toolbox. 5 machines Bernoulli distribution, N 50

6.3.2. Bottleneck exponential distribution

For the first time during the experimentation, PSE Toolbox shows a completely different result from FACTS.

Starting with the model with buffer allocation 1, the shifting bottleneck detection in FACTS gives machine 3 as the bottleneck as seen in Figure 76, followed by the fifth machine. The utilization chart is also clear (Figure 77), having operation 3 the same percentage of blockage and starvation, and showing machines upstream blocked (especially machine 1) and machines downstream starving for parts. Nevertheless, PSE Toolbox points out the second machine as a bottleneck instead (Figure 78), showing a very blocked machine 1 (25%) and starving machines downstream (16.9%, 14.16%, and 22% for machines 3, 4, and 5).

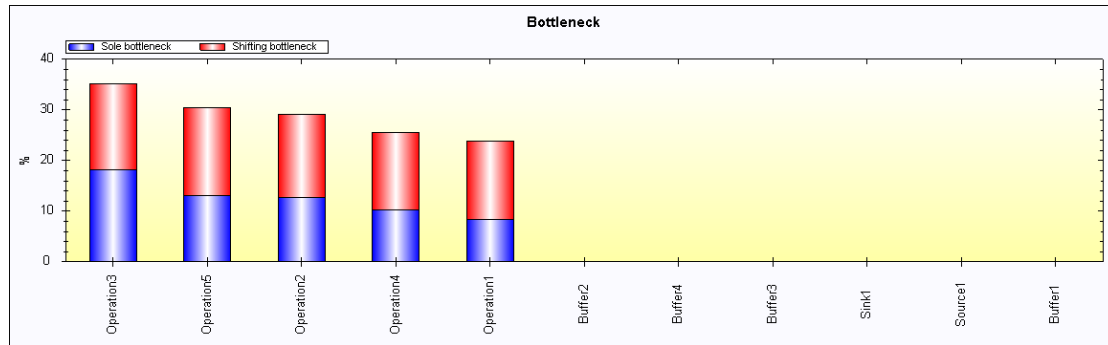


Figure 76. Bottleneck bar plot from FACTS. 5 machines Bernoulli distribution, N 1

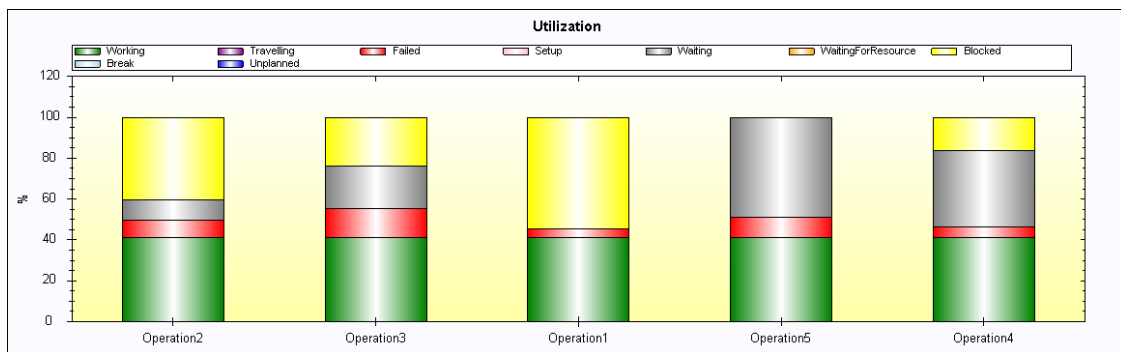


Figure 77. Utilization bar plot from FACTS. 5 machines Bernoulli distribution, N 1

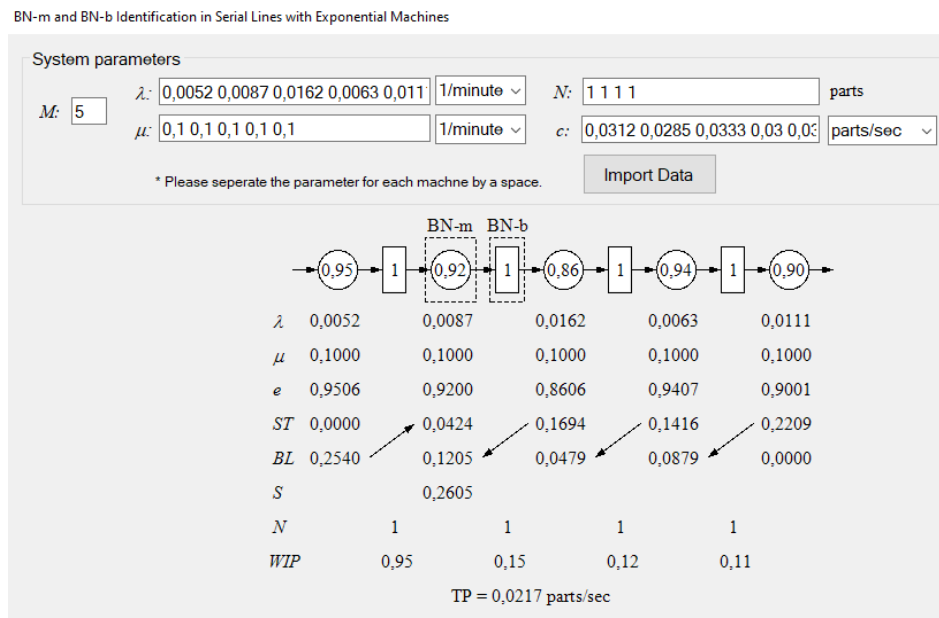


Figure 78. Bottleneck identification from PSE Toolbox. 5 machines exponential distribution, N 1

Following the study for the buffer sizes 10 and 50, the results are the same: FACTS calculates machine 3 as the main bottleneck while PSE Toolbox gives machine 2 this consideration. Analyzing the situation deeper, it can be seen how for bigger buffer allocation the bottleneck situation for machine 3 is clearer for FACTS with bigger starvation and blockage relative percentages and also increasing the total % of sole bottleneck or shifting bottleneck while for PSE Toolbox the situation deescalates with a lower % of blockages or starvations, being inconsistent with the results previously obtained with the Bernoulli bottleneck cases.

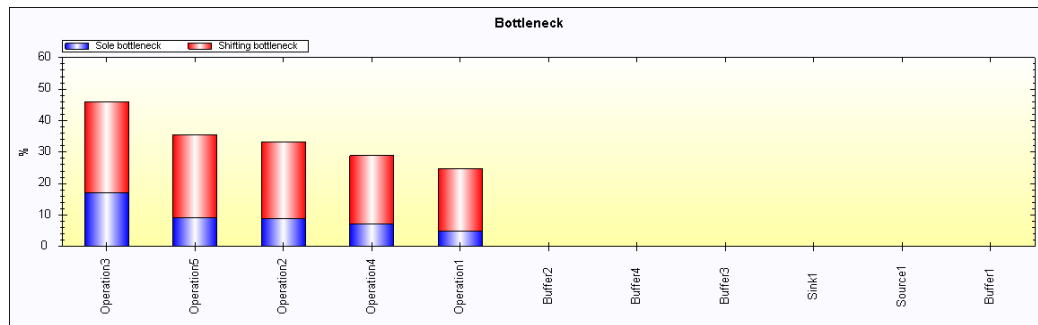


Figure 79. Bottleneck bar plot from FACTS. 5 machines exponential distribution, N 10

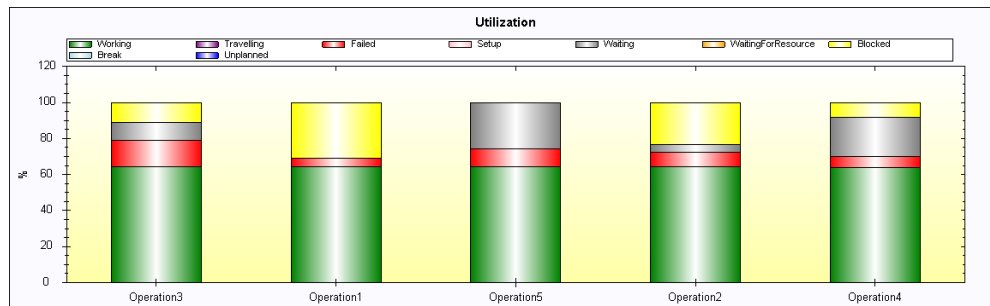


Figure 80. Utilization bar plot from FACTS. 5 machines Bernoulli distribution, N 10

BN-m and BN-b Identification in Serial Lines with Exponential Machines

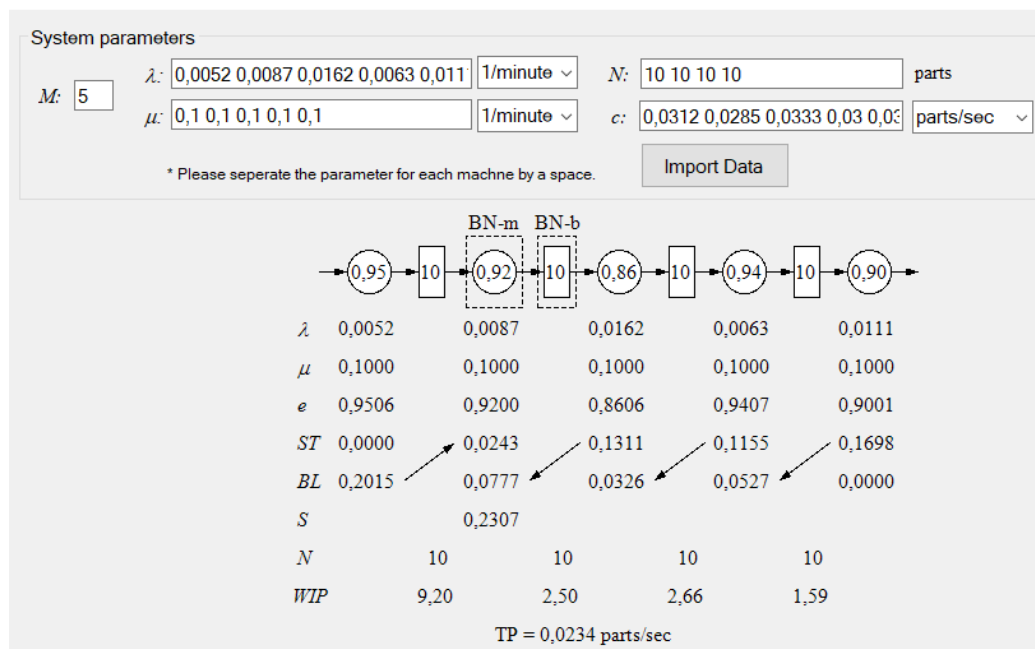


Figure 81. Bottleneck identification from PSE Toolbox. 5 machines exponential distribution, N 10

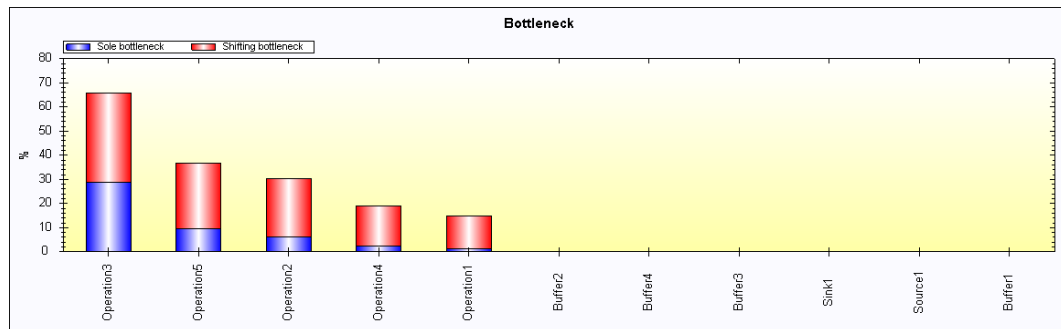


Figure 82. Bottleneck bar plot from FACTS. 5 machines exponential distribution, N 50

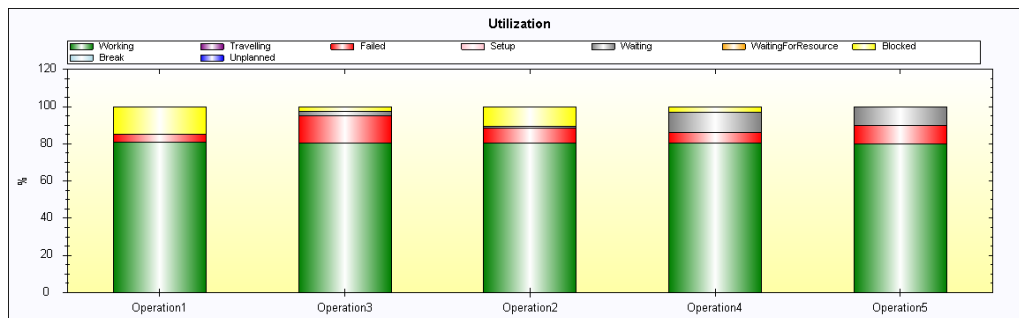


Figure 83. Utilization bar plot from FACTS. 5 machines Bernoulli distribution, N 50

BN-m and BN-b Identification in Serial Lines with Exponential Machines

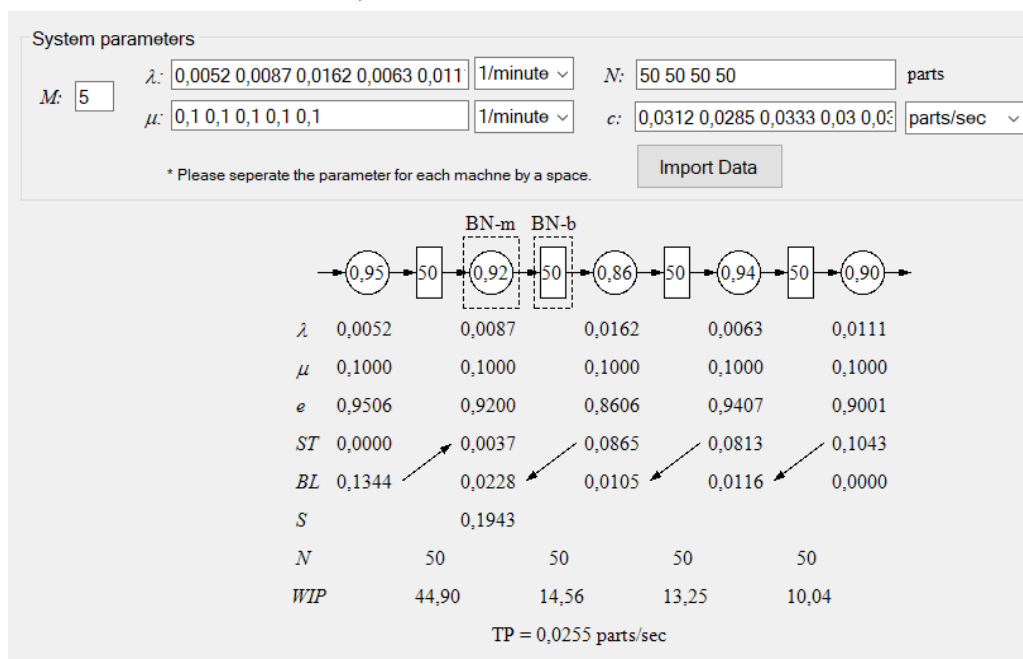


Figure 84. Bottleneck identification from PSE Toolbox. 5 machines exponential distribution, N 50

7. Conclusions and recommendations for future work

After all the experimentation is finished and all the data is processed and analysed, the results conclude that the three software can elaborate outputs that relate to the reality, with different approaches, but with major convergence overall.

It is proven FACTS can be faster when handling more complex and heavy models and Plant Simulation can perform better using smaller models in terms of speed. The outputs regarding the process production coming from both are very similar, with the mean values and error bars normally overlapping. For the buffer allocation experiments, FACTS proves its power to create simulation models with a TNB as low as possible while maximizing the TH, a skill that is also well managed by the Lean buffer function from PSE Toolbox and in a less amount of time, as the plots are practically identical. The bottleneck identification seems to be a more complex task to carry out, as Plant Simulation lacks such a feature, and FACTS and PSE Toolbox can reach the point of completely different results depending on the model. The role of this last simulation software even it does not seem very relevant to directly simulate real-life models, can be useful for a rapid and easy buffer allocation calculation that can be used in more industry-focused software. Any company willing to improve their production via simulations should take into consideration which software they are using, to avoid over-optimistic results but also to tackle the issues with the right approach.

All the optimizations were done along with the report directly link to the sustainability and to the Lean philosophy remarked in the second chapter. An optimized TH with low buffer sizes provides an enhanced economic profit while reducing some constraints that may lower the human interaction what would reduce the risk of injury in dangerous operations or granting them more resting hours, impacting on the sociological sustainability of the factory or process. Ecological sustainability is a logical outcome of this optimization as, for example, some factories would require less energy or some others may require less raw materials, thanks to very delimited bottlenecks and constraints. Eventually, this reduction of waiting times and lean buffers that provides 'just on time' concords with the Lean philosophy, as it produces a smoother flow of parts that reduces the waste, strengthens the pull logic, and shortens the LT.

This project would have been impossible if it were not for the possibility to automate FACTS files using xsim-runner.exe and MATLAB and the Experiment Manager of Plant Simulation, as the manual input of data would take an unbearable amount of time otherwise. PSE Toolbox lacks such an external or internal automatization option like the other programs that leads to longer times to experiment on it.

For future work, this project set the stone for further simulation in the same line using the same programs, as much of the scripts used can be reused. There are a lot of the PSE Toolbox functions that have not been tested like 'Lead time analysis and control', 'Customer demand satisfaction', 'Product quality' and 'Continuous improvement', ideas very linked to the Lean philosophy. Also, the 'General model of machines reliability' could be studied. Further studies could also experiment on different types of production systems such as closed lines and assembly systems. Another interesting line of study would be the introduction of artificially created bottlenecks and to check if the simulation programs would identify it correctly, especially PSE, and the use of the SCORE function from FACTS and its similitudes to its shifting bottleneck detection used here. Finally, a very useful future work would be doing the same experiments with the same methodology on real-life models.

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Appendix 1 Computational benchmarking data

```

C:\> Símbolo del sistema - xsim-runner.exe --model FACTS200S.xml

Microsoft Windows [Versión 10.0.19042.928]
(c) Microsoft Corporation. Todos los derechos reservados.

C:\Users\Antonio>cd\

C:\>cd FACTS benchmarking

C:\FACTS benchmarking>xsim-runner.exe --model FACTS5S.xml

C:\FACTS benchmarking>xsim-runner.exe --model FACTS200S.xml
  
```

Figure 85. xsim-runner.exe check on C

```

4 - delete 'Results.xlsx';
5 - delete 'MeanAndST.xlsx';
6
7 - NumOfStations = 5; %Options: 5, 50, 100, 150, 200
8 - NumOfBuffers = NumOfStations-1;
9
10 %%Outputs and running times for buffer capacities 1, 5, 10...100
11 for replication = [1: 1: 10] %replications
12     i=1;
13     for capacity = [1 5 10: 10: 100]
14         actual_x = [linspace(capacity, capacity, NumOfBuffers)];
15         dlmwrite('input.txt', actual_x, 'precision', 10);
16         fprintf('replication = %d\n', replication);
17         tic
18         system(['xsim-runner.exe --model FACTS', num2str(NumOfStations), '.S.xml --seed ' num2str(replication) ' --input input.txt --output_txt output.txt']);
19         toc
20         time = toc;
21         results = [readmatrix('output.txt') time];
22         writematrix(results, 'Results.xlsx', 'sheet', 1, 'WriteMode', 'append');
23         i=i+1;
24     end
25 end
26 %Results.txt gives 12 sheets. Each sheet corresponds to capacity 1,5,10,20...100. Each column has 10 values: one for each replication.
27 %Order of the columns: TH,LT,WIP and running times
28
29 %%For calculating the mean values and the standard deviations. If more/less sheets than 12 are used, increase or decrease the final number of the loop
30 for j = [1: 1: 12]
31     Mean = mean(readmatrix('Results.xlsx', 'sheet', j));
32     ST= std(readmatrix('Results.xlsx', 'sheet', j));
33     MeanAndST= [Mean ST];
34     writematrix(MeanAndST, 'MeanAndST.xlsx', 'WriteMode', 'append');
35 end
36 %First 4 columns showed in MeanAndST.xlsx are the mean values of TH,LT,WIP and running times and the other four columns are the standard deviation
37 %of each output in the same order
  
```

Figure 86. MATLAB script for modifying FACTS inputs.

Model	Buffer Capacity	TH	SD for TH	LT(s)	SD for LT	WIP	SD for WIP	Running time (s)	SD for RT
5S	1	39.4375	1.132967329	669	14	7.6	1.429840706	0.0765	0.004743414
	5	45.92333333	0.813502942	1227	28	15.5	5.602578771	0.0875	0.007975655
	10	48.9275	0.62489567	1900	64	25.6	6.752777206	0.096799999	0.006713174
	20	51.12083333	0.57903509	3195	245	40.3	13.99245829	0.104700001	0.007394442
	30	51.97833333	0.563953198	4617	632	56.2	19.0834425	0.120299999	0.034331881
	40	52.37416667	0.617809876	5922	1115	73.8	21.52414665	0.107800001	0.004871688
	50	52.55	0.631478766	7072	1605	96	27.5680975	0.106200001	0.012318007
	60	52.64	0.606571523	7883	2036	117.8	38.18027414	0.104599999	0.007560131
	70	52.69166667	0.602361607	8544	2480	137.7	51.91028586	0.106100002	0.010060374
	80	52.74166667	0.603896811	9028	2707	153.9	65.35450337	0.1109	0.004976612
	90	52.7775	0.591151967	9428	2781	163.4	71.75761671	0.107700001	0.011557102
	100	52.80166667	0.577254036	9727	2768	173.5	78.22084121	0.107799999	0.004871683
0									
50S	1	24.635	0.276937618	9042	173	60.4	8.694826048	0.5546	0.086928067
	5	39.9325	0.299548271	15320	554	167.7	26.26383233	0.7568	0.076679418
	10	45.50333333	0.343079734	22625	1455	294.5	39.03630789	0.802899998	0.035450506
	20	48.49666667	0.28426318	32635	1432	501	57.8734251	0.881100002	0.072327111
	30	48.93833333	0.223958495	38899	2468	649.9	77.11960408	0.852800001	0.015324636
	40	49	0.21566721	42032	3408	732.6	85.79717685	0.8904	0.063515881
	50	49	0.21566721	43995	3835	793.3	90.84669626	0.870799999	0.019752919
	60	49	0.21566721	45218	3923	833.5	86.11136201	0.959100001	0.090057328
	70	49	0.21566721	46010	4016	865.6	90.99841268	0.9639	0.071903408
	80	49	0.21566721	46598	4155	889.6	96.09393552	0.896599999	0.058456442
	90	49	0.21566721	47045	4261	904.4	97.7379717	0.867000003	0.015209642
	100	49	0.21566721	47348	4259	918.4	100.2565598	0.865500002	0.010814085
0									
100S	1	23.8075	0.328775553	18484	524	119.9	10.84691456	0.856600001	0.092947059
	5	39.28916667	0.290621142	29951	1320	335.5	33.29414363	1.309099999	0.021789138
	10	44.5675	0.571094493	41154	1503	560.4	47.52356141	1.499699999	0.018190659
	20	46.61416667	0.392228714	54845	2145	890.4	60.5295153	1.601199999	0.022324877
	30	46.8075	0.351066497	61168	3031	1069.1	74.08921049	1.613700001	0.015100004
	40	46.82083333	0.350996157	64146	3867	1157.6	88.68070064	1.619899999	0.014843257
	50	46.82083333	0.350996157	65975	4232	1218.3	97.4087037	1.607300003	0.015790642
	60	46.82083333	0.350996157	67086	4292	1258.5	92.92081456	1.623100001	0.013723056
	70	46.82083333	0.350996157	67798	4351	1290.6	98.29343823	1.6277	0.01787643
	80	46.82083333	0.350996157	68317	4421	1314.6	102.3340934	1.635600001	0.016554287
	90	46.82083333	0.350996157	68712	4458	1329.4	106.12278	1.6152	0.018195239
	100	46.82083333	0.350996157	68982	4417	1343.4	110.8183699	1.613700001	0.024458809
0									
150S	1	23.49333333	0.316003204	27842	749	177.4	19.50612439	1.187200002	0.026544721
	5	38.81666667	0.324916893	43885	1746	507.7	41.04211929	1.965099999	0.025335308
	10	43.52916667	0.503418101	57779	2141	816.3	63.54884386	2.2432	0.049532929
	20	44.84083333	0.392551208	72372	2355	1237.1	67.89608072	2.341599999	0.033287305
	30	44.95416667	0.390161608	78383	3147	1425.4	80.63663353	2.355700001	0.027374156
	40	44.97666667	0.389618363	81146	3824	1512.8	96.11775186	2.351099999	0.030223058
	50	44.97833333	0.390990354	82843	4144	1573.3	103.7015483	2.363600001	0.020871032
	60	44.97833333	0.390990354	83865	4280	1613.5	98.45613801	2.368299998	0.03030603
	70	44.97833333	0.390990354	84509	4410	1645.6	101.4781859	2.376000001	0.027968238
	80	44.97833333	0.390990354	84972	4492	1669.6	104.3787973	2.3822	0.016771663
	90	44.97833333	0.390990354	85334	4524	1684.4	107.0308367	2.397900002	0.010429177
	100	44.97833333	0.390990354	85588	4472	1698.4	110.5483303	2.3603	0.02967996
0									
200S	1	23.28333333	0.240594685	36955	1343	234.7	21.28666771	1.5434	0.044754144
	5	38.25583333	0.387518419	57202	2134	659.9	60.89599512	2.527500001	0.028249876
	10	42.40166667	0.438586394	72974	3014	1042.5	66.76867196	2.8588	0.04123591
	20	43.435	0.386097122	87531	2677	1497.9	72.25025952	2.988299999	0.028503606
	30	43.525	0.397891821	93239	3048	1689	96.95703516	2.989999999	0.043006464
	40	43.53666667	0.384924235	95869	3606	1777.7	110.2220486	2.9947	0.024399684
	50	43.53666667	0.384924235	97470	3885	1838.4	118.0717296	3.007000001	0.03145014
	60	43.53666667	0.384924235	98434	4037	1878.6	114.4369793	3.008500002	0.021077894
	70	43.53666667	0.384924235	99031	4134	1910.7	120.581047	3.043000001	0.08531901
	80	43.53666667	0.384924235	99457	4176	1934.7	126.1525091	3.0212	0.060359664
	90	43.53666667	0.384924235	99792	4182	1949.5	131.382605	3.0149	0.05743102
	100	43.53666667	0.384924235	100032	4127	1963.5	136.9965531	3.001100001	0.025154848

Table 7. Plant Simulation Computational benchmarking results

Model	Buffer Capacity	TH	SD for TH	LT	SD for LT	WIP	SD for WIP	Running Time	SD for RT
5S	1	39.6475	0.781035341	578.1282694	14.2245597	6.364821644	0.107348905	0.14787914	0.038031345
	5	45.9725	0.586697311	1153.105825	52.3141533	14.72916244	0.660488374	0.15537425	0.02903564
	10	48.96416667	0.691728578	1836.931056	122.2074084	24.99010671	1.597558974	0.17393496	0.039262499
	20	51.11833333	0.611676506	3209.705403	318.9772595	45.59102463	4.405833485	0.16316907	0.022346141
	30	51.87916667	0.564118043	4663.115347	582.2884696	67.19380468	8.231610317	0.15653376	0.018137964
	40	52.27916667	0.558923817	5999.334705	798.4286964	87.22771695	11.525868	0.17281263	0.025415088
	50	52.48333333	0.596491801	7243.601949	981.2972699	105.8759882	14.30391214	0.18021375	0.04154812
	60	52.59083333	0.588094984	8360.03542	1141.569522	122.6590631	16.80118382	0.1700871	0.030889917
	70	52.65	0.583782896	9307.529289	1267.08134	136.9712107	18.82756218	0.18433681	0.041146666
	80	52.68333333	0.608631332	10241.77042	1405.255435	151.1501578	21.05595695	0.1775457	0.029042842
	90	52.705	0.61998855	11035.03685	1503.473962	163.3167168	22.72044742	0.15593377	0.022081244
	100	52.72166667	0.62812429	11691.10183	1563.748226	173.4788214	23.85153583	0.18839676	0.022775666
50S	1	24.6475	0.442897481	8891.309348	238.789855	60.85061854	1.560348817	0.37771233	0.039805429
	5	39.91583333	0.503911398	14895.7913	632.5953578	165.419824	6.539778773	0.59594188	0.055628603
	10	45.65916667	0.454919101	22437.51302	1219.007843	285.8182675	15.55036742	0.64689144	0.087204683
	20	48.60083333	0.347554286	33255.6698	1658.132613	460.6175617	24.1534653	0.6916192	0.055057997
	30	49.14583333	0.381117792	39113.0515	2043.431257	557.7253648	30.67860689	0.71938176	0.091872464
	40	49.23333333	0.377348612	42153.36002	2442.070782	610.1810347	39.13413469	0.73968254	0.062451924
	50	49.255	0.357183772	43828.17482	2203.352712	638.9220522	35.92252294	0.73012945	0.109608466
	60	49.27083333	0.334102123	45034.32386	2148.102892	659.5840422	34.4991063	0.75446493	0.047301101
	70	49.27333333	0.330539215	45856.49982	2414.039294	674.230468	38.12229118	0.73251497	0.12340155
	80	49.27333333	0.330539215	46376.14946	2604.656443	683.6580839	40.53886625	0.71575532	0.096715312
	90	49.27333333	0.330539215	46719.18624	2737.507522	690.1573315	42.40897791	0.74344153	0.082553109
	100	49.27333333	0.330539215	46983.00941	2864.527746	695.4504288	44.51963136	0.7333327	0.05438867
100S	1	23.75416667	0.502159688	18721.51391	601.6387214	123.3825228	2.571165356	0.72462162	0.061287264
	5	39.41833333	0.371865501	30000.74097	1180.039444	329.6017212	11.90983548	1.06884287	0.082864903
	10	44.7025	0.296586808	42207.36768	1926.898264	537.5750753	23.38785017	1.1691922	0.075483477
	20	46.78333333	0.35224415	55713.96495	1948.95577	770.0709758	30.08223652	1.23079084	0.106363235
	30	46.945	0.360687786	62226.7625	2204.888279	886.092656	35.20614889	1.25101867	0.113140612
	40	46.96416667	0.371372371	65393.31814	2242.659153	944.8508476	37.83763392	1.2637122	0.136228956
	50	46.97	0.373178705	66957.53137	2058.114089	974.3024001	36.60801978	1.25012761	0.101938653
	60	46.97	0.373178705	67899.34468	1903.15762	992.1110403	32.52036323	1.27135484	0.09355336
	70	46.97	0.373178705	68567.73521	1941.137717	1004.825071	33.85834148	1.22065763	0.135999708
	80	46.97	0.373178705	69064.0149	1952.530808	1014.572195	34.43732519	1.26525863	0.146733344
	90	46.97	0.373178705	69458.86546	2020.063857	1021.99145	35.76204589	1.33838901	0.293392133
	100	46.97	0.373178705	69744.90143	2053.902193	1027.536734	36.61629322	1.20787047	0.100552714
150S	1	23.4175	0.300730848	27978.21106	1123.872544	182.0393051	6.681687609	1.07609529	0.09791624
	5	38.67916667	0.413119157	43990.72417	1994.911387	480.7980775	23.84149077	1.48573518	0.108092992
	10	43.52916667	0.423922927	59175.23521	2701.797507	749.8533924	36.15962431	1.7348022	0.257180077
	20	45.14166667	0.36565517	74293.14905	2438.889645	1022.965688	35.16686548	1.77847544	0.138312645
	30	45.26083333	0.406961983	80429.11322	1834.499233	1139.58831	27.97976343	1.8487151	0.416474322
	40	45.26416667	0.41107451	83011.65738	1757.274975	1195.515401	28.18650736	1.74637231	0.154321802
	50	45.26416667	0.41107451	84428.5298	1643.402127	1226.408362	24.75313512	1.72618616	0.127330098
	60	45.26416667	0.41107451	85367.56228	1674.450467	1245.905563	22.38165727	1.79941685	0.187905913
	70	45.26416667	0.41107451	85912.29383	1789.270566	1257.713816	22.89175145	1.72809684	0.141341488
	80	45.26416667	0.41107451	86293.24119	1900.985063	1266.441657	23.8836709	1.74343806	0.145321724
	90	45.26416667	0.41107451	86574.21147	2046.418191	1272.937059	26.24816536	1.73065569	0.153868919
	100	45.26416667	0.41107451	86754.70318	2151.49978	1277.509734	28.51300543	1.76986749	0.1336803
200S	1	23.32166667	0.24331177	37956.97073	1153.829849	246.4351855	7.714098562	1.46412478	0.116296393
	5	38.3575	0.385650205	57642.41345	1808.779544	637.2510718	18.55687769	1.94656983	0.12362995
	10	42.54333333	0.314436592	73599.13758	2087.975506	938.6286802	32.536945	2.22878913	0.205231591
	20	43.58583333	0.357655474	89026.56573	1780.732282	1225.450774	31.44354064	2.33193403	0.221660346
	30	43.65333333	0.366397207	94934.15913	1725.869823	1339.792155	29.73542389	2.33612901	0.117526686
	40	43.65333333	0.366397207	98024.47979	1958.531425	1404.688855	33.4350004	2.33741227	0.18978903
	50	43.65333333	0.366397207	99408.3098	2053.176871	1437.955527	36.29479135	2.30231905	0.142520445
	60	43.65333333	0.366397207	100128.7858	2070.004379	1455.564848	36.06886909	2.35257542	0.239480055
	70	43.65333333	0.366397207	100660.7714	2082.622365	1468.922368	35.34186668	2.26634561	0.168766813
	80	43.65333333	0.366397207	101093.633	2132.576001	1479.624674	35.27805164	2.39135189	0.134424781
	90	43.65333333	0.366397207	101453.1685	2224.476365	1488.753621	35.6978235	2.29099414	0.142354175
	100	43.65333333	0.366397207	101708.1739	2339.009233	1494.956493	37.25154592	2.28045393	0.15465345

Table 8. FACTS Computational benchmarking results

Appendix 2 Accuracy/Lean buffer benchmarking experiment results

TNB	Buffer 1	Buffer 2	Buffer 3	Buffer 4	TH	SD for TH	WIP	SD for WIP	TH	SD for TH	WIP	SD for WIP	TH	WIP
4	1	1	1	1	39.61	0.56703147	6.32402163	0.08494865	39.4375	1.13296733	7.6	1.42984071	40.38	3.35
7	1	2	3	1	41.685	0.58802085	7.80101126	0.11244704	41.4325	1.04921671	8.7	2.31180545	46.398	4.94
10	1	3	5	1	43.03	0.58451204	9.21170316	0.18819408	42.8275	0.98068578	9.7	3.591657	47.808	6.48
15	2	5	4	4	44.6966667	0.60859583	11.2984323	0.29934446	44.6108333	0.88369184	12.1	4.62961481	50.904	7.97
16	4	5	5	2	45.0541667	0.56215868	13.2860333	0.31798923	44.8833333	0.90309823	14.4	4.7656176	51	11.27
19	4	4	6	5	45.685	0.59001203	13.2571178	0.41194862	45.6708333	0.83574881	14.3	5.12184862	51.798	10.36
22	2	8	8	4	46.2608333	0.50463974	14.31243	0.53158687	46.2683333	0.83096701	14.3	6.01941304	51.33	9.31
24	7	5	8	4	46.7508333	0.53570725	17.1901221	0.5295727	46.7033333	0.78670668	17.7	6.61731735	52.242	14.88
26	4	11	6	5	47.0591667	0.50162468	17.4571921	0.715537	47.0291667	0.77247448	18.6	6.66999917	52.35	14.46
29	6	12	8	3	47.4208333	0.43619514	20.7322137	0.91000564	47.3116667	0.80330337	23	6.5149401	52.11	20.6
31	6	12	8	5	47.805	0.45844611	20.6101785	0.98193761	47.8016667	0.74456674	22.5	6.4678693	52.644	18.83
34	6	12	8	8	48.1525	0.50517461	20.6926466	1.03724932	48.2658333	0.71096407	22.5	6.43341969	52.83	18.04
38	9	12	8	9	48.5766667	0.49934834	23.5715883	1.14642305	48.6966667	0.64671773	25.1	6.77331365	52.968	21.55
44	13	12	10	9	49.0516667	0.47437093	27.8554566	1.41537433	49.2175	0.58524093	29.7	7.45430524	53.094	26.06
47	12	13	11	11	49.2933333	0.47716045	27.9167743	1.64725416	49.5291667	0.59464236	29.2	7.1460945	53.184	26.14
52	11	16	17	8	49.575	0.42819244	30.7587758	2.31167183	49.8333333	0.66672453	31.4	9.58239126	53.22	30.64
59	11	20	19	9	49.9333333	0.42464582	33.8153013	3.07617826	50.1941667	0.67423997	34.2	11.8771489	53.04	33.91
62	12	22	19	9	50.0633333	0.41681849	36.1791873	3.34095938	50.3041667	0.66661169	36.6	12.8166558	53.322	36.88
73	17	20	25	11	50.5608333	0.42810323	41.5540634	3.67400047	50.7991667	0.59924657	40.9	15.0513934	53.442	43.04
76	20	22	19	15	50.725	0.41677776	43.2993215	3.73652474	50.9625	0.57214468	42	14.3836327	53.496	43.12
84	20	21	28	15	50.9683333	0.44313697	45.4803361	4.18407904	51.2083333	0.58077481	43.5	16.1468951	53.544	46.22
90	20	21	34	15	51.0858333	0.47189613	47.5237162	4.57878078	51.3266667	0.59290819	44.1	16.6429832	53.562	49
98	22	33	19	24	51.255	0.42239032	52.531189	5.74319728	51.5008333	0.59565622	50.2	18.3230274	53.604	52.72
114	31	24	31	28	51.6083333	0.44298371	58.1402435	5.42089582	51.8925	0.53349896	53.4	18.9748371	53.664	58.7
132	31	43	30	28	51.9075	0.42765962	69.606971	8.08024467	52.0758333	0.6195061	65.1	21.4032708	53.712	72.1
154	29	47	50	28	52.1466667	0.46250125	73.7130205	9.68199675	52.2075	0.6318263	71.8	22.8755085	53.754	79.39
196	57	51	50	38	52.4775	0.37276184	99.9442602	15.4727791	52.5341667	0.66006512	101.9	25.6274592	53.808	109.9
252	62	66	79	45	52.6691667	0.40311384	111.717571	23.1236842	52.615	0.63965896	129.4	44.5800155	53.85	137.68
367	98	96	85	88	52.8516667	0.46715913	137.942684	41.1184644	52.79	0.5928301	169.6	73.7024046	53.898	192.26
385	100	100	90	95	52.8633333	0.47305808	139.094184	41.7726914	52.7975	0.58293703	172.5	75.9579562	53.904	198.67

Table 9. Bernoulli base case Accuracy benchmarking results

TNB	Buffer 1	Buffer 2	Buffer 3	Buffer 4	TH	SD for TH	WIP	SD for WIP	TH	SD for TH	WIP	SD for WIP	TH	WIP
5	1	1	2	1	48.8883333	0.892529	7.09783828	0.13203837	48.9441667	0.78537018	8.4	2.7968236	50.37	4.05
8	1	3	3	1	49.9733333	0.80714212	8.78275307	0.23906004	50.005	0.73056295	10.3	4.0290611	53.082	5.76
16	3	4	8	1	51.5708333	0.72139108	13.2717944	0.60159707	51.6025	0.69231953	15.3	6.1110101	54.264	13.2
22	3	5	9	5	52.4225	0.68967305	14.58419	0.74097062	52.4158333	0.65627844	16.8	6.01479657	55.824	10.56
27	6	10	5	6	52.9516667	0.75701248	19.1192971	0.8712038	53.0433333	0.66921734	21.3	7.00872472	56.166	16.38
31	8	7	11	5	53.3508333	0.68330454	21.1400569	1.19568132	53.3608333	0.6893821	22.6	7.24492159	56.286	18.73
36	10	10	9	7	53.7025	0.69205199	24.2170147	1.35582326	53.7525	0.67871122	25.4	7.530678	56.412	21.6
41	10	10	12	9	54.0183333	0.63764129	25.4249956	1.7729129	54.0216667	0.68291767	25.9	7.90850456	56.508	22.55
45	8	15	15	7	54.1916667	0.58949131	27.7390412	2.3412355	54.1433333	0.71081591	29.4	8.90942073	56.514	25.66
49	11	13	15	10	54.3908333	0.59075242	29.2945123	2.45443671	54.3716667	0.68483575	30	9.00617072	56.592	26.68
58	18	12	17	11	54.6483333	0.55131437	35.7566474	2.89577284	54.6366667	0.66080992	35.4	10.7414048	56.634	33.35
63	18	17	17	11	54.8133333	0.53890607	39.0324089	3.00981759	54.785	0.64922556	38.7	11.7761529	56.664	37.38
70	13	19	24	14	54.9725	0.50932735	37.6211108	4.39751497	54.9466667	0.65528714	38.2	12.5237552	56.706	35.25
78	13	23	22	20	55.1216667	0.49297848	39.2294541	4.51007088	55.0875	0.6226592	39.3	13.9128238	56.73	36.18
88	14	35	18	21	55.2708333	0.48637691	46.136116	5.42916006	55.2041667	0.63135962	45.4	16.8338284	56.754	43.19
93	28	23	22	20	55.3625	0.47297734	54.0135505	5.02775298	55.35	0.59975561	52.1	17.1752406	56.778	52.27
102	16	39	27	20	55.4816667	0.43797021	53.2344111	7.82691975	55.3883333	0.63799696	51.8	22.5082701	56.79	49.86
123	22	36	41	24	55.6825	0.39848188	61.2959492	9.84552699	55.5625	0.63323891	63.6	27.8416155	56.838	60.69
130	28	38	30	34	55.7508333	0.39374841	65.9122966	8.51860492	55.6191667	0.61364437	69.5	27.2610834	56.85	65.31
134	29	41	40	24	55.7741667	0.42076595	70.9210173	10.6088937	55.645	0.6407485	74.2	29.3855596	56.85	73.24
141	24	47	39	31	55.805	0.36805713	67.8277277	11.3877455	55.6233333	0.64766675	73.9	31.1214467	56.856	67.43
148	24	54	39	31	55.83	0.35347045	70.2808954	13.0167344	55.6325	0.65668981	77.2	32.0270719	56.862	70.92
156	27	48	46	35	55.865	0.34535721	72.4516383	13.1507992	55.6691667	0.64824133	80.6	32.8640296	56.868	73.52
163	35	48	45	35	55.9066667	0.36063644	79.6056079	13.7921367	55.735	0.66326447	90.2	34.3245944	56.88	83.91
168	42	48	41	37	55.9225	0.38127568	84.5871236	14.3127935	55.7725	0.66922023	95	32.8362943	56.886	89.93
174	42	48	47	37	55.9333333	0.37472212	85.5386052	15.0190554	55.7775	0.67239495	98	34.509258	56.886	91.99
188	56	48	49	35	55.9625	0.38546609	97.8797992	15.5079149	55.8325	0.67230773	109.6	35.0466356	56.892	106.81
210	61	61	52	36	55.9883333	0.38920225	105.906051	18.0439433	55.8641667	0.68544221	118.1	36.5435813	56.898	123.11
228	63	61	51	53	56.0225	0.38737901	106.238659	18.1672944	55.8758333	0.68643661	120.1	38.3969038	56.91	122.62
240	65	65	56	54	56.03	0.3832367	108.798689	19.3417069	55.8833333	0.68676869	122.5	39.0306148	56.916	129.07

Table 10. Bernoulli 95% availability case Accuracy benchmarking results

TNB	Buffer 1	Buffer 2	Buffer 3	Buffer 4	TH	SD for TH	WIP	SD for WIP	TH	SD for TH	WIP	SD for WIP	TH	WIP
5	1	1	2	1	44.0958333	0.41483393	6.91880618	0.10120791	44.1866667	0.53707024	7.3	2.16281709	43.164	7.39
7	1	3	2	1	45.5158333	0.411476	8.272579	0.17388154	45.5616667	0.57453954	8.7	2.71006355	46.38	8.75
10	2	3	3	2	47.1183333	0.38699539	9.72409936	0.23526937	47.145	0.5179822	10.5	3.37474279	49.644	10.26
14	3	4	4	3	48.4441667	0.35832364	11.7823523	0.35834965	48.5066667	0.47597852	12.5	3.77859468	51.06	12.3
15	3	4	4	4	48.6658333	0.32724651	11.8482025	0.37591713	48.735	0.4935979	12.4	3.65756446	51.234	12.32
20	5	5	6	4	49.68	0.30820067	15.1437752	0.52084174	49.7591667	0.46809388	14.8	4.31534729	51.996	15.69
23	4	8	7	4	50.0533333	0.30697147	16.5365261	0.81517093	50.1233333	0.47856524	15.7	4.16466619	52.2	16.97
25	6	8	7	4	50.2916667	0.32005979	18.6698463	0.80338441	50.3616667	0.46168305	17.3	4.69160006	52.326	19.42
26	4	8	7	7	50.3708333	0.29110395	16.5142707	0.83454313	50.4325	0.51185709	15.7	4.27005074	52.374	16.53
29	6	10	9	4	50.62	0.31334023	20.9575127	1.13495106	50.6725	0.43866116	19.4	5.64111888	52.482	22.09
34	6	10	10	8	51.1058333	0.27944085	20.7776632	1.2208824	51.155	0.49046778	19.9	6.17251974	52.842	20.98
40	11	10	9	10	51.475	0.27822187	25.4953904	1.20734534	51.505	0.45299415	24.4	7.07420981	53.022	26
41	11	10	9	11	51.5175	0.27742895	25.5318789	1.20273664	51.545	0.46191697	24.4	7.07420981	53.04	26.06
48	11	10	16	11	51.8316667	0.29906748	27.6522161	1.87027445	51.8533333	0.45708078	27.4	9.22797679	53.184	28.19
53	9	17	16	11	52.0033333	0.28782389	29.9485094	2.59816866	51.9875	0.47961935	29.6	11.3939555	53.262	30.21
59	12	18	14	15	52.225	0.27270366	33.5370168	2.55524073	52.1991667	0.47553495	33.4	11.0775047	53.358	33.94
69	18	15	17	19	52.4083333	0.25558575	38.9588629	2.92337461	52.4058333	0.45931797	38.6	12.0295931	53.436	39.04
70	15	26	17	12	52.4183333	0.3311456	43.1430646	4.00450874	52.3733333	0.41694065	44.9	11.6089046	53.424	44.45
77	20	18	22	17	52.5766667	0.26867875	44.6099731	3.82220682	52.5616667	0.4287003	46	11.9443152	53.514	44.64
85	18	24	25	18	52.7	0.28120113	47.1525682	4.58971659	52.6383333	0.43966528	51.4	11.7492317	53.562	47.72
97	19	28	31	19	52.8266667	0.28395357	52.0673499	5.44272499	52.7125	0.43191095	58.1	12.9739054	53.61	53.42
108	26	31	34	17	52.9025	0.30608666	62.6998554	6.41337619	52.7825	0.42164526	69.3	13.7844518	53.628	66.43
126	30	35	37	24	53.0566667	0.30311653	67.7489645	7.59909653	52.9191667	0.43002745	75.1	16.7229051	53.706	72.09
131	28	35	33	35	53.0758333	0.28903442	63.9873093	7.62491273	52.965	0.43448565	70.5	16.467139	53.718	67.66
148	40	33	40	35	53.145	0.29426619	75.1781248	8.85800891	53.035	0.43482294	82.2	16.4167936	53.748	80.32
161	40	43	40	38	53.1891667	0.30025838	78.768172	10.7932514	53.055	0.43498971	89.8	19.4753405	53.772	87.48
174	43	49	44	38	53.2033333	0.29110263	83.3947072	12.9353696	53.065	0.4280807	99.2	21.3479117	53.79	96.09
190	49	51	45	45	53.2183333	0.28360009	88.9646761	14.7762044	53.0983333	0.42576728	105.9	22.024633	53.802	103.55
203	48	60	50	45	53.2225	0.27894339	90.5186543	16.4441608	53.0966667	0.42550261	110.1	24.5558321	53.82	110.49
206	48	60	53	45	53.225	0.27641332	90.695268	16.6737286	53.0966667	0.42550261	110.4	24.5320652	53.82	111.46

Table 11. Bernoulli 2 minutes MTTR Accuracy benchmarking results

TNB	Buffer 1	Buffer 2	Buffer 3	Buffer 4	TH	SD for TH	WIP	SD for WIP	TH	SD for TH	WIP	SD for WIP	TH	WIP
4	1	1	1	1	26.4741667	0.45635854	5.77238871	0.11454807	26.775	0.74194231	6.9	1.85292561	38.664	6.5
8	1	3	3	1	30.4491667	0.56361447	7.97016039	0.21407903	30.6141667	0.81052157	9.5	2.46080384	41.202	8.5
11	2	4	3	2	33.145	0.40726807	9.79844871	0.22377973	33.0091667	0.85122869	11.3	3.0568684	42.456	10.12
15	2	6	4	3	35.3616667	0.46481445	11.3076906	0.19189104	35.2883333	0.76507685	12.2	4.68567557	43.848	11.87
19	4	7	3	5	36.8141667	0.77018166	14.0913006	0.4087507	36.9833333	0.78079771	15.7	5.16505351	44.742	14.17
22	5	9	4	4	38.3116667	0.45921968	16.8384957	0.6787635	38.0466667	0.74317055	18.4	5.23237783	45.45	16.58
24	8	6	5	5	38.8966667	0.57040163	17.9478779	0.5326607	38.9166667	0.7032223	19.1	4.84079883	45.756	17.74
28	7	9	7	5	40.5191667	0.73520437	19.7606953	0.35085185	40.245	0.68862226	19.7	8.85751407	46.68	19.59
31	7	8	7	9	41.2041667	0.560922	18.8508861	0.78345355	41.1508333	0.69031494	21	5.14241621	47.046	19.39
35	7	12	7	9	41.8	0.53194263	21.7880646	0.48819163	41.9958333	0.73156616	27.1	7.48999332	47.604	21.93
42	7	9	17	9	42.8375	0.28707362	22.3675772	0.97132577	43.0958333	0.77998942	19.7	4.29599297	48.288	23.37
48	14	8	18	8	43.8891667	0.54557776	30.1839058	0.78454465	43.8716667	0.78920377	30.2	10.2393576	48.726	29.5
56	14	9	21	12	45.07	0.74965218	31.6875166	1.80438844	44.9616667	0.82721084	34.8	7.8002849	49.296	31.32
64	18	14	13	19	45.9575	0.59684678	37.1726503	1.15003803	45.8741667	0.42274182	36	10.0774776	49.734	36.29
74	18	15	19	22	46.86	0.52657465	38.8692249	1.75390457	47.045	0.58420834	45.6	12.084885	50.25	39.2
83	22	17	19	25	47.5383333	0.66416429	43.2741344	3.02300292	47.6825	0.59430749	51.3	8.00069441	50.556	44.45
90	20	28	22	20	48.2416667	0.77770833	51.3233421	4.55126842	48.2183333	0.68415488	53.7	12.6846364	50.934	50.49
100	20	28	28	24	48.5816667	0.8297032	53.1167899	4.42189849	48.9025	0.69178435	47.7	14.6443011	51.204	52.5
111	28	28	29	26	49.0466667	0.36631296	63.4409972	2.75902022	49.4041667	0.64102601	54.4	12.7993055	51.432	60.73
118	28	28	29	33	49.4541667	0.7281726	60.5657866	5.30070182	49.6166667	0.7007932	54.7	11.4411538	51.522	61.19
132	32	28	39	33	49.7683333	0.48088139	70.0615164	7.26161778	50.0425	0.75001492	67.7	14.3530872	51.756	68.04
146	30	53	28	35	50.1325	0.63090971	83.3972037	6.78660121	50.0958333	0.68331357	88.2	25.8405366	51.912	79.58
149	32	41	41	35	50.3783333	0.4774579	73.9393273	8.05625833	50.36	0.76743922	87.7	26.5206419	52.038	77.47
160	38	50	39	33	50.5233333	0.42890904	89.4621289	9.96349819	50.5275	0.61621668	93.8	29.9176648	52.134	89.02
171	43	50	39	39	50.9741667	0.29646711	95.4570137	4.59895707	50.7675	0.64869825	96.4	24.1578145	52.236	93.9
195	43	42	71	39	51.1758333	0.45924068	105.839539	6.34967844	51.0416667	0.73085651	97.1	26.8181116	52.416	99.12
206	46	67	47	46	51.3291667	0.42155009	109.301333	11.2331051	51.3075	0.69660165	111.2	34.0907286	52.518	111.11
239	60	67	65	47	51.5491667	0.51039276	136.660801	11.7033143	51.5908333	0.5638882	139	40.2850951	52.716	131.43
262	77	67	39	79	51.7508333	0.31833018	138.840784	15.6944737	51.6225	0.74386431	142.6	39.9699887	52.65	140.85
322	78	74	90	80	52.2891667	0.65720191	151.424338	26.7362531	52.0325	0.55095946	176.8	61.3384056	53.028	161.49
					m _f	m _r	λ	μ						
					45.00	5	0.02222222	0.2						

Table 12. Exponential base case Accuracy benchmarking results and PSE inputs

TNB	Buffer 1	Buffer 2	Buffer 3	Buffer 4	TH	SD for TH	WIP	SD for WIP	TH	SD for TH	WIP	SD for WIP	TH	WIP
4	1	1	1	1	31.0525	0.45862869	5.93281823	0.07917467	31.1758333	0.62477712	7.3	2.16281709	48.27	6.75
7	2	2	2	1	34.2558333	0.44474035	8.14846431	0.07975811	34.3858333	0.62008624	9.6	2.22111083	49.332	8.58
10	2	3	3	2	36.9441667	0.42176602	9.25937647	0.09713331	37.0975	0.54949542	11.6	3.20416396	50.184	9.75
14	3	3	4	4	39.4858333	0.37857453	10.6928305	0.26028345	39.3225	0.77443777	12.4	4.52646539	50.952	11.29
17	4	5	5	3	40.7941667	0.60828957	13.6570884	0.27600285	40.925	0.67080889	14.5	4.16999867	51.558	13.61
19	4	5	4	6	41.6291667	0.40717428	13.1642529	0.21135341	41.6766667	0.64746419	16	5.39547135	51.744	13.69
25	4	8	6	7	43.4175	0.5573921	15.3796076	0.51862887	43.8116667	0.6516148	17.2	5.3913511	52.548	16.36
30	6	8	9	7	45.1983333	0.22131063	18.8786808	0.59638922	45.3816667	0.68727185	20.9	7.56380269	53.058	19.22
36	8	8	9	11	46.3866667	0.65236039	20.9594385	0.41482022	46.4916667	0.76238741	23	6.3420992	53.412	21.47
39	9	11	10	9	47.0491667	0.55178512	24.3729043	0.76950805	47.3025	0.63504945	30.1	7.21803297	53.688	24.4
42	11	10	13	8	47.605	0.34280974	26.8921657	0.86897327	47.6633333	0.64957488	28.1	4.67736868	53.844	26.55
48	9	11	17	11	48.5966667	0.47826526	26.8735217	1.46976787	48.385	0.74293795	24.6	9.1189668	54.126	26.99
60	13	16	16	15	49.8933333	0.78804532	33.294011	1.49406017	49.9383333	0.66537143	37.4	9.47745864	54.6	33.9
65	13	16	17	19	50.1066667	0.46580727	33.769497	1.15466411	50.22	0.62910626	38.8	10.9422728	54.708	34.58
74	13	20	24	17	50.8641667	0.33472049	37.6440564	3.12705923	51.0208333	0.73591979	39.7	12.2660326	54.966	39.15
83	20	20	24	19	51.4208333	0.55042422	46.5419546	3.03671089	51.7641667	0.74109997	47.1	10.7646541	55.158	46.04
91	22	26	24	19	51.9916667	0.44109852	50.6015256	3.95217968	52.0241667	0.76776743	54	11.8790198	55.302	51.98
98	21	27	31	19	52.29	0.4993947	57.0404425	2.86948113	52.2408333	0.78643446	58.4	13.7533834	55.41	54.19
110	26	29	35	20	52.6541667	0.2702665	63.0637516	3.52407095	52.63	0.65041772	68	17.8200885	55.554	61.97
122	25	26	33	38	53.0383333	0.56695526	57.630068	4.07007346	52.86	0.7182562	62.9	22.0577525	55.62	58.92
124	25	26	35	38	53.28	0.6221751	59.9147411	5.44627904	52.9033333	0.73864031	64.2	22.2301097	55.638	59.48
135	30	41	42	22	53.4825	0.78516777	80.7280376	7.48154584	53.1041667	0.63961613	81.5	28.4302265	55.776	77.09
147	42	41	34	30	53.6575	0.47345346	82.914152	7.92918926	53.5125	0.65108231	93.1	24.020593	55.854	84.95
157	30	54	42	31	53.985	0.54397372	79.3450833	10.2763299	53.6516667	0.71262642	92	27.844808	55.944	84.49
176	42	33	52	49	54.21	0.33579646	85.0723473	8.82287858	54.12	0.76859861	90	25.0776572	56.004	86.03
199	42	56	52	49	54.645	0.31274867	104.019069	9.4460356	54.5066667	0.72059432	110.5	28.4497608	56.16	101.39
215	42	72	52	49	54.7758333	0.38545007	109.77492	17.273465	54.6558333	0.72891186	120.4	27.4922211	56.208	110.94
258	51	71	76	60	55.1016667	0.58795523	119.038228	19.6459546	55.0741667	0.63409372	132.6	29.5040487	56.34	127.7
298	99	57	64	78	55.2808333	0.44793852	146.765782	21.0524029	55.1358333	0.6523941	179.3	28.0952744	56.37	160.83
322	73	94	77	78	55.5666667	0.33448874	154.620723	27.0777653	55.3258333	0.60829465	184.7	23.3049828	56.466	166.32
					mf	mr	λ	μ						
					95.00	5	0.01052632	0.2						

Table 13. Exponential 95% availability case Accuracy benchmarking results and PSE inputs

Buffer 1	Buffer 2	Buffer 3	Buffer 4	TH	SD for TH	WIP	SD for WIP	TH	SD for TH	WIP	SD for WIP	TH	WIP
1	1	1	1	29.43	0.40454974	5.95936893	0.0528356	29.5475	0.49342983	6.9	1.10050493	41.88	6.5
1	2	2	1	32.0516667	0.40324303	7.09659593	0.07833073	31.9791667	0.43069442	8.5	1.8408935	43.89	7.5
2	3	2	2	34.61	0.23500722	8.96840101	0.08671866	34.645	0.52776608	10.7	2.16281709	45.672	9.14
3	3	3	3	36.7933333	0.38228116	10.2867446	0.17655782	36.8175	0.45527515	11.6	3.06231575	46.95	10.5
4	5	3	3	38.3841667	0.23430519	12.7951519	0.20529471	38.1533333	0.71871155	12.4	4.32563419	47.838	12.7
4	5	6	3	39.6683333	0.2908003	14.1237706	0.28334755	39.6208333	0.57671176	15.3	2.26323269	48.708	13.83
4	5	6	6	40.855	0.48810227	13.6450021	0.44130769	40.7933333	0.52438677	14.4	4.35124503	49.182	14.08
6	7	6	5	41.9166667	0.38232557	17.207364	0.35765951	41.9216667	0.67780282	16.5	7.1063352	49.722	17.1
6	8	6	7	42.9408333	0.34481275	18.0360507	0.48164305	42.8933333	0.48690887	19.6	3.9777157	50.076	17.89
9	8	6	7	43.2483333	0.36001543	20.9429822	0.38114377	43.2691667	0.57941874	20.3	6.37791328	50.286	20.67
6	10	10	8	44.2825	0.3670337	20.1618578	0.43403059	44.1758333	0.51072525	17	5.55777733	50.772	20.44
6	12	10	9	44.745	0.3627782	21.1829299	0.9752997	44.6658333	0.46420398	20.3	5.35516366	50.964	21.67
10	12	13	9	45.7791667	0.39746981	27.2744946	0.59257493	45.9516667	0.57374124	30.1	8.43866761	51.432	25.85
13	12	10	13	46.3266667	0.67003547	29.1495817	0.42011807	46.3066667	0.38276528	33.3	11.8326291	51.522	28.87
17	13	16	10	46.9183333	0.46961371	36.0170871	2.24084672	46.9233333	0.42701592	36.1	10.9792734	51.84	35.32
17	21	12	15	47.6566667	0.46732427	38.4229521	1.61191726	47.8158333	0.42765962	38.3	11.3338235	52.074	39.69
17	17	23	15	48.495	0.57954058	40.7511206	1.82143753	48.3958333	0.4454719	33.5	13.7860638	52.332	40.51
17	27	30	15	49.2633333	0.63938628	51.9544247	4.67100956	49.1275	0.43256788	49.2	10.5703989	52.602	50.24
22	24	28	22	49.5266667	0.50336522	51.2452987	4.16573736	49.6158333	0.3057941	55.4	13.8980414	52.734	51.79
22	29	23	30	50.0166667	0.73669264	53.5789786	3.46499426	49.9316667	0.25054878	51.8	18.8903267	52.794	53.9
27	33	42	25	50.7025	0.41421594	68.3879118	6.67457592	50.5316667	0.31448567	73.2	14.5281336	53.016	67.84
35	44	38	30	50.9041667	0.38008385	82.6513908	10.4907303	50.96	0.27230158	86.5	12.3580833	53.148	81.95
36	33	42	46	51.1383333	0.41442359	78.2870982	2.78008725	51.1933333	0.2791024	83.8	16.7385118	53.166	76.73
38	59	42	42	51.4425	0.52592438	96.7645223	5.95844948	51.4925	0.34967909	107	14.5983256	53.298	95.79
38	44	63	46	51.6283333	0.44427427	86.021863	7.17884265	51.5341667	0.31740776	105.2	18.0849846	53.22	91.73
51	53	51	46	51.8758333	0.49931203	113.100147	18.1570282	51.7191667	0.28300771	126.6	19.0216251	53.37	107.87
57	68	58	57	52.0733333	0.50287445	121.805144	17.2841686	51.9525	0.29726246	144	20.81666	53.472	126.41
57	79	72	49	52.2608333	0.41978555	129.569254	28.2664547	51.9658333	0.3445889	159.9	21.5275018	53.502	139.81
77	86	97	72	52.51	0.63795584	158.554362	29.4425444	52.2133333	0.34816982	187	34.4641521	53.622	171.42
83	66	97	99	52.7391667	0.51909599	162.009461	38.5157272	52.2425	0.35042853	182.1	38.7941863	53.61	163.17
				mf	mr	λ		μ					
				18.00	2	0.05555556		0.5					

Table 14. Exponential 2 minutes MTTR case Accuracy benchmarking results and PSE inputs

TNB	Buffer 1	Buffer 2	Buffer 3	Buffer 4	TH	SD for TH	WIP	SD for WIP	TH	SD for TH	WIP	SD for WIP	E	TH	WIP	PSE Simulation Function	
																TH	WIP
4	1	1	1	1	39.61	0.567	6.32	0.0849	39.4375	1.13296733	7.6	1.42984071	0.7	40.38	7.85	40.596	7.85
8	2	2	2	2	42.015	0.585	8.43	0.1322	41.7858333	1.02485357	9.3	2.45175674	0.85	48.3	10.11	48	10.12
12	3	3	3	3	43.718	0.603	10.477	0.1983	43.5366667	0.92187242	11.5	3.65908307	0.9	50.46	12.19	50.394	12.17
16	4	4	4	4	44.987	0.613	12.479	0.307	44.8708333	0.8677455	13.7	4.64399254	0.95	51.438	14.22	51.384	14.2
20	5	5	5	5	45.977	0.603	14.443	0.427	45.9233333	0.81350294	15.5	5.60257877	0.96	51.996	16.22	51.942	16.23
28	7	7	7	7	47.415	0.553	18.295	0.676	47.4616667	0.70717443	19	6.89605362	0.97	52.602	20.24	52.566	20.26
40	10	10	10	10	48.759	0.52	23.931	1.14	48.9275	0.62489567	25.6	6.75277721	0.98	53.04	26.27	53.004	26.33
72	18	18	18	18	50.568	0.44	38.716	3.189	50.8516667	0.58220526	37.4	12.0572707	0.99	53.478	42.29	53.454	42.44
80	20	20	20	20	50.847	0.44	42.423	3.719	51.1208333	0.57903509	40.3	13.9924583	0.991	53.532	46.28	53.52	45.98
92	23	23	23	23	51.18	0.43	48.008	4.69	51.4558333	0.56362268	45.4	16.7012974	0.992	53.592	52.28	53.58	52.31
104	26	26	26	26	51.451	0.43	53.532	5.37	51.7225	0.55508383	49.6	18.6201802	0.993	53.64	58.29	53.628	57.68
120	30	30	30	30	51.734	0.4396	60.981	6.043	51.9783333	0.5639532	56.2	19.0834425	0.994	53.688	66.3	53.682	66.81
144	36	36	36	36	52.067	0.45	71.618	7.36	52.2508333	0.60512791	66.4	20.0676633	0.995	53.742	78.29	53.736	76.75
180	45	45	45	45	52.349	0.429	86.164	11.56	52.4741667	0.62612183	84.5	24.3413229	0.996	53.796	96.3	53.778	95.58
236	59	59	59	59	52.605	0.393	105.715	19.362	52.6341667	0.60825659	115.8	36.8414018	0.997	53.844	124.28	53.838	125.05
352	88	88	88	88	52.828	0.463	130.68	35.6705	52.7708333	0.59369436	161.6	70.3486555	0.998	53.958	180.06	53.88	178.53

Table 15. Bernoulli base case Lean buffer results

TNB	Buffer 1	Buffer 2	Buffer 3	Buffer 4	TH	SD for TH	WIP	SD for WIP	TH	SD for TH	WIP	SD for WIP	E	TH	WIP	PSE Simulation Function	
																TH	WIP
4	1	1	1	1	26.474	0.456	5.77	0.114	26.775	0.74194231	6.9	1.85292561	0.7	38.664	7.1444	38.556	6.58
8	2	2	2	2	30.72	0.35	7.94	0.09	30.7891667	0.788224	9.4	2.06559112	0.75	40.962	9.1827	40.758	8.58
16	4	4	4	4	35.82	0.855	12.381	0.33	36.0175	0.78175027	14.3	3.56058672	0.8	44.016	13.2336	43.728	12.54
24	6	6	6	6	39.0675	0.527	16.4	0.4989	39.0625	0.73323074	19.3	5.9076222	0.85	45.93	17.2655	45.564	16.56
28	7	7	7	7	40.285	0.908	18.5	0.6	40.4166667	0.64469583	21.3	6.91295081	0.86	46.38	19.273	46.332	18.55
32	8	8	8	8	41.073	0.434	20.43	0.513	41.5041667	0.61251417	24.3	8.05605362	0.87	47.238	21.2873	46.872	20.64
36	9	9	9	9	42.289	0.416	22.54	0.77	42.3241667	0.73194787	23.3	7.45430524	0.88	47.742	23.2957	47.442	22.64
40	10	10	10	10	43.317	0.599	25.13	0.535	43.1658333	0.81838868	24.4	8.97156	0.89	48.18	25.303	47.856	24.61
48	12	12	12	12	44.144	0.418	28.728	1.184	44.3283333	0.77852345	30.6	5.10337579	0.9	49.3	29.315	48.57	28.63
56	14	14	14	14	45.427	0.556	32.987	1.46	45.4041667	0.60613186	34	10.1214843	0.91	49.458	33.3243	49.128	32.56
64	16	16	16	16	46.21	0.458	37.07	2.088	46.2366667	0.7134748	39.4	8.92188321	0.92	49.908	37.3318	49.626	36.66
72	18	18	18	18	47.089	0.769	40.46	2.036	47.0383333	0.56762174	48.9	11.3377442	0.93	50.28	41.338	50.04	40.56
88	22	22	22	22	47.76	0.553	48.354	2.971	48.1	0.67835813	51.1	9.21894185	0.94	50.85	48.3475	50.652	48.26
108	27	27	27	27	48.922	0.618	60.07	4.046	49.2633333	0.72297183	56.1	7.63689873	0.95	51.354	59.3559	51.18	58.42
140	35	35	35	35	49.7475	0.87	72.998	3.633	50.2241667	0.70504629	82.9	22.7178344	0.96	51.9	75.365	51.756	74.71
192	48	48	48	48	51.085	0.413	104.84	8.383	51.2025	0.70906879	99.8	31.8147415	0.97	52.422	101.3737	52.302	100.5
296	74	74	74	74	52.094	0.61	150.13	14.553	51.9508333	0.5847529	164	56.4702478	0.98	52.95	153.3825	52.842	152.08
					mf		mr		λ		μ						
					45		5		0.02222		0.2						

Table 16. Exponential base case Lean buffer results and PSE input

```

RGui (64-bit) - [R Console]
Archivo Editar Visualizar Misc Paquetes Ventanas Ayuda

> Abar=111.71
> Bbar=129.4
> s1=23.123
> s2=44.58
> n1=10
> n2=10
> DF=( (s1^2/n1 + s2^2/n2)^2 )/( (s1^2/n1)^2/(n1-1) + (s2^2/n2)^2/(n2-1))
> qprob=0.05
> WCImax=Abar-Bbar+qt((1-qprob/2), df=DF)*(sqrt((s1^2/n1 + s2^2/n2)))
> WCImin=Abar-Bbar-qt((1-qprob/2), df=DF)*(sqrt((s1^2/n1 + s2^2/n2)))
> cat("Welch confidence Interval=[", WCImin, ",", WCImax, "]\n")
Welch confidence Interval=[ -51.86612 , 16.48612 ]
> |

```

Figure 87. R script example for calculating the Welch CI