Intelligent Reflecting Surfaces for MIMO Communications in LoS Environments

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***Abstract*—In line-of-sight (LoS) environments, point-to-point (P2P) multiple-input multiple-output (MIMO) channel matrix turns out to be rank deficient such that spatial multiplexing becomes unattainable. In this paper, we propose the deployment of distributed intelligent reflecting surfaces (IRSs) to act as artificial scatterers and synthesize a sort of multi-path propagation such that additional degrees of freedom are created. We show that given the far-field deployment of the IRS, it simply resembles a full-duplex relay with a single effective reflection coefficient. However, to maximize the channel capacity both the effective reflection coefficients of all IRSs and the transmit covariance matrix should be jointly optimized, which is a non-convex optimization problem. Thus, we develop an alternating optimization algorithm to iteratively find a sub-optimal solution. Moreover, we propose different schemes to enhance the composite channel power which would result in an improvement to the achievable rate. Our simulation results show that the deployment of distributed IRSs with P2P MIMO systems in LoS environments increases the rank of the channel matrix, and improves the achievable rate by making spatial multiplexing possible.**

***Index Terms*—Intelligent reflecting surface, MIMO, line-of-sight.**

I. Introduction

An Intelligent Reflecting Surface (IRS) is a thin planar array that consists of multiple reflecting elements, each of which is connected to a tunable chip such as PIN diode, and can induce a controllable phase shift to the incident signal [1]. IRS is nearly a passive unit as it only leverages the reflection on its elements, and it does not consume any power for transmission [2]. Furthermore, IRSs can actively control the propagation channel, by accurately tuning the phase shifts of their reflecting elements to fit a specific need. IRSs can be used to increase the coverage area for users with blocked direct links or introduce new paths to enrich the wireless channel [3]. Recently, IRS is introduced as a promising technology for several communication applications such as unmanned aerial vehicle (UAV) communications [4], physical-layer security [5], and non-orthogonal multiple access (NOMA) [6].

The current research in IRS-aided communications is focused on the deployment of a single IRS for single-input single-output (SISO) and multi-user multiple-input multiple-output (MIMO) systems [7]–[9]. However, the deployment of distributed IRSs could offer a promising solution for one of the fundamental bottlenecks in point-to-point (P2P) MIMO communications, where the propagation environment plays an important role in their performance. In the case of rich scattering environments, the channel is commonly modeled as independent and identically distributed (i.i.d.) Rayleigh fading, where the corresponding channel matrix has full rank. Thus, spatial multiplexing of independent data streams on the same time-frequency resource is possible. On contrary, in the case of Line-of-Sight (LoS) environments, where no scatterers or reflectors exist, which is common in many rural scenarios, the corresponding channel matrix becomes rank-deficient which prevents spatial multiplexing, and the benefits of MIMO systems diminish to merely an array gain [10].

In the literature, few studies discuss the deployment of IRS with P2P MIMO communications such as [11] and [12] in both studies, a single IRS is used. In [11] a Rician fading channel is assumed, whereas the authors in [12] assumed LoS channel but proposed an achievable rate maximization algorithm for a special system when the transmitter and receiver each have two antennas.

To the best of our knowledge, this is the first work that discusses the deployment of distributed IRSs with P2P MIMO communications in LoS environments. Specifically, we propose the use of randomly distributed IRSs to act as artificial scatterers and synthesize a multi-path propagation. Thus, the rank of the composite channel matrix increases, such that the spatial multiplexing of independent data streams becomes possible. We show that given the far-field deployment of the IRS, it simply resembles a full-duplex relay with a single effective reflection coefficient that not only amplifies the reflected signal on it without any noise amplification but also results in a phase shift compensation. Furthermore, we develop an alternating optimization algorithm that iteratively optimizes the transmit covariance matrix and the effective reflection coefficients of all IRSs. Moreover, we propose several schemes to enhance the composite channel power either by iteratively maximizing it using an alternating algorithm, by maximizing its lower-bound, or by maximizing the power of the channel matrix that results from the transmitted signal reflection on each IRS individually.

II. System Model

In this paper, we consider one-dimensional uniform linear arrays (ULAs) of n_t and n_r antennas at the transmitter and receiver, respectively. The distance between the transmit and receive arrays is denoted by d_o and assumed to be much longer than the array lengths such that the transmit and receive ULAs are in the far-field. Thus, using the plane wave propagation model and given that solely a LoS link exists between the transmitter and the receiver, the channel matrix is defined as [10]

$$\mathbf{H}_o = \alpha_o \mathbf{r}_o \mathbf{t}_o^H, \quad (1)$$

where $\mathbf{H}_o \in \mathbb{C}^{n_r \times n_t}$, $\alpha_o = \frac{\lambda_c}{4\pi d_o} e^{-jkd_o}$ is the complex channel gain for the LoS link, $k = 2\pi/\lambda_c$ is the wavenumber, while λ_c is the wavelength, and $\mathbf{t}_o$ and $\mathbf{r}_o$ denote the transmit and receive response vectors, respectively, which are defined as [10]

$$\begin{aligned} \mathbf{t}_o &= [1, e^{-jk\Delta_t \sin(\phi_{t_o})}, ..., e^{-jk(n_t-1)\Delta_t \sin(\phi_{t_o})}]^T, \\ \mathbf{r}_o &= [1, e^{-jk\Delta_r \sin(\phi_{r_o})}, ..., e^{-jk(n_r-1)\Delta_r \sin(\phi_{r_o})}]^T, \end{aligned} \quad (2)$$

where ϕ_{t_o} and ϕ_{r_o} denote the angle of departure (AoD) and the angle of arrival (AoA) at the transmit and receive arrays, respectively, both angles are measured relative to the boresight of its ULA, while Δ_t and Δ_r denote the antenna separations in the transmitter and receiver, respectively. It is clear from (1) that the channel matrix is a rank-one matrix with a unique singular value, $\delta_1 = |\alpha_o|\sqrt{n_t n_r}$. However, the number of degrees of freedom can be increased by inserting a set of randomly distributed IRSs to the environment. In this scenario, the distributed IRSs will act as artificial intelligent scatterers that could enrich the propagation environment and synthesize a sort of rich scattering environment such that the rank of the composite channel matrix increases.

Consider L passive distributed IRSs each having M lossless reflecting elements, each of which reflects the incident signal with a unit gain reflection coefficient denoted by $e^{j\varphi_{lm}}$ with $\varphi_{lm} \in [0, 2\pi]$, $\forall\, l \in \mathcal{L}$ and $\forall\, m \in \mathcal{M}$ where $\mathcal{L} = \{1, 2, ..., L\}$, and $\mathcal{M} = \{1, 2, ..., M\}$. In this paper, we ignore the multi-hop links that involve more than one IRS, as a consequence of its substantial path loss in comparison to direct LoS link and single reflection links [13]. Then, the composite channel matrix can be written as follows

$$\mathbf{H} = \mathbf{H}_o + \sum_{l=1}^{L} \mathbf{H}_l = \mathbf{H}_o + \sum_{l=1}^{L}\sum_{m=1}^{M} \alpha_{lm} e^{j\varphi_{lm}} \mathbf{r}_{lm} \mathbf{t}_{lm}^H, \quad (3)$$

where $\mathbf{H}_l \in \mathbb{C}^{n_r \times n_t}$ is the two-hop channel matrix which results from the signal reflection on the lth IRS, $\alpha_{lm} \in \mathbb{C}$ is the channel gain of the reflected signal on the mth reflecting element in the lth IRS, $\mathbf{t}_{lm}$ and $\mathbf{r}_{lm}$ are the transmit and receive response vectors for the wave impinges on and reflects from the mth reflecting element in lth IRS, respectively, which are defined as [9].

$$\begin{aligned} \mathbf{t}_{lm} &= [1, e^{-jk\Delta_t \vartheta_{lm}}, ..., e^{-jk(n_t-1)\Delta_t \vartheta_{lm}}]^T, \\ \mathbf{r}_{lm} &= [1, e^{-jk\Delta_r \mu_{lm}}, ..., e^{-jk(n_r-1)\Delta_r \mu_{lm}}]^T, \end{aligned} \quad (4)$$

where $\vartheta_{lm} = \sin(\phi_{t_{lm}})\sin(\theta_{t_{lm}})$, while $\phi_{t_{lm}}$ and $\theta_{t_{lm}}$ denote the elevation and azimuth AODs from the transmit array to the mth reflecting element in the lth IRS, respectively. Similarly, $\mu_{lm} = \sin(\phi_{r_{lm}})\sin(\theta_{r_{lm}})$, while $\phi_{r_{lm}}$ and $\theta_{r_{lm}}$ are the elevation and azimuth AOAs from the mth reflecting element in the lth IRS to the receive array, respectively.

It was shown in [10] and [14] that in multi-path propagation, the number of degrees of freedom of the MIMO channel is not only limited by $\min(n_t, n_r)$ but also by the number of paths that have distinct spatial angles at both the transmit and receive arrays. Thus, in this paper for the sake of rank improvement, we assume that all the IRSs exist far away from both the transmitter and receiver such that the paths created by the reflections on different IRSs are very likely to be distinct at both the transmitter and receiver sides.

As consequences of the IRSs' far-field operation, which occurs whenever the distances from the IRS to the transmitter and receiver are much longer than the maximum dimension on the IRS [15], the AODs and AOAs in (4) become approximately independent of m; hence, the transmit and receive response vectors turn out to be approximately equal constant vectors over all the reflecting elements in each IRS; i.e., $\mathbf{t}_{lm}$ and $\mathbf{r}_{lm}$ approximately equal to $\mathbf{t}_l$ and $\mathbf{r}_l$, respectively. Similarly, the magnitude of the complex channel gain in (3) becomes independent of m. Nevertheless, its phase shift still depends on the particular element in the IRS due to the sensitivity of the phase shift to small distance variations; i.e., $\alpha_{lm} = a_l e^{-j\psi_{lm}}$, while the phase shift $\psi_{lm} = k(d_{lm}^t + d_{lm}^r)$ depends on the exact distances from the transmitter to the reflecting element and from the reflecting element to the receiver which are denoted by d_{lm}^t and d_{lm}^r, respectively. Whereas, $a_l \in \mathbb{R}$ is the attenuation coefficient of the two-hop link from the transmitter to receiver through the lth IRS. In this paper, we assume a square shape reflecting element of aperture area equals $\lambda_c^2/4$. Furthermore, we rely on the plate scattering-path loss model and the radiation pattern of reflecting element introduced in [16]. Therefore, the attenuation coefficient of the lth IRS can be defined as

$$a_l = \frac{\lambda_c^2}{16\pi d_l^t d_l^r}\left(\cos(\beta_l^t)\cos(\beta_l^r)\right)^q, \quad (5)$$

where $q \approx 0.285$, d_l^t and d_l^r are the distances from the central element of the lth IRS to the transmitter and receiver, respectively, β_l^t and β_l^r denote the angles between the normal to the lth IRS and the vector connecting it to the transmitter and receiver, respectively. Thus, the composite channel matrix given far-field operation of the IRSs can be written as

$$\mathbf{H} = \mathbf{H}_o + \sum_{l=1}^{L} a_l \gamma_l \mathbf{r}_l \mathbf{t}_l^H, \quad (6)$$

where $\gamma_l = \sum_{m=1}^{M} e^{j(\varphi_{lm} - \psi_{lm})}$ represents the effective reflection coefficient of the lth IRS such that $|\gamma_l| \leq M$ where equality holds only if all the reflected paths from the lth IRS are co-phased at the receiver, and $\arg\{\gamma_l\} \in [0, 2\pi]$ where $\arg\{\gamma_l\}$ denotes the phase of γ_l. It is important to note that given the far-field deployment of the IRSs in LoS environment, each IRS simply resembles a full-duplex relay that not only amplifies the reflected signal on it without any noise amplification by controlling the magnitude of γ_l but also makes phase shift compensation based on the phase of γ_l. Furthermore, the composite channel matrix in (6) is a summation of multiple rank one matrices which would result in a composite channel matrix of a higher rank given that the IRSs create multi-path with distinct spatial angles at the transmitter and receiver.

In this paper, we assume perfect channel state information (CSI) at both the transmitter and receiver which could be done using the techniques proposed in [17], [18], while our main goal is to tune the IRSs to maximize the composite channel capacity defined as [10]

$$C = \log\det\left[\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{Q}\mathbf{H}^H\right], \tag{7}$$

where σ^2 is the noise variance, and $\mathbf{Q} \in \mathbb{C}^{n_t \times n_t}$ is the covariance matrix of the transmitted signal which satisfies $\mathrm{tr}(\mathbf{Q}) \leq P$, while P is the total transmitted power. In order to maximize the channel capacity, both the effective reflection coefficients of all IRSs $\{\gamma_l\}_{l=1}^{L}$ and the covariance matrix $\mathbf{Q}$ have to be jointly optimized. Therefore, the capacity maximization problem becomes

$$\begin{aligned} \text{(P1)} \quad & \max_{\gamma_1,\gamma_2,\ldots,\gamma_L,\mathbf{Q}} \quad \log\det\left[\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{Q}\mathbf{H}^H\right] && (8)\\ & \text{s.t.} \quad |\gamma_l| \leq M,\ \forall\, l \in \mathcal{L} && (9)\\ & \quad \mathrm{tr}(\mathbf{Q}) \leq P. && (10)\end{aligned}$$

Because of the non-concave objective function over the effective reflection coefficients of IRSs, (P1) turns out to be a non-convex optimization problem. Therefore, in the next section we discuss several sub-optimal solutions that could improve the achievable rate.

III. Distributed IRSs-Aided Point-to-Point MIMO

In this section firstly, we develop an alternating optimization algorithm to sub-optimally solve (P1). Secondly, we propose different approaches to enhance the composite channel power which will improve the achievable rate.

A. Alternating Optimization For Rate Enhancement

In this algorithm (P1) is divided into two sub-problems. In the first sub-problem, we solve for the optimum transmit covariance matrix while all the effective reflection coefficients of the IRSs are set as constants. In the second sub-problem, we solve for a single effective reflection coefficient while the transmit covariance matrix and the rest of effective reflection coefficients are set as constants. It was shown in [11] that by iteratively solving both sub-problems a locally optimal solution for (P1) can be obtained.

It is important to note that this alternating optimization algorithm is originally proposed in [11] for single P2P IRS aided MIMO communication in Rician fading channel, the main differences being that in our problem we optimize over multiple IRSs. Moreover, the far-field deployment of the IRSs in LoS environment adds the magnitude of the effective reflection coefficient as an additional optimization variable to our problem.

In the first sub-problem, since all the effective reflections coefficients $\{\gamma_l\}_{l=1,}^{L}$ are set as constants, (P1) becomes a convex problem. Thus, the optimum covariance matrix can be obtained based on the singular value decomposition of the composite channel matrix. In particular, define $\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$, where $\mathbf{U} \in \mathbb{C}^{n_r \times D}$ and $\mathbf{V} \in \mathbb{C}^{n_t \times D}$ are unitary matrices, D is the rank of composite channel matrix, and $\mathbf{\Lambda} = \mathrm{diag}\{\delta_1, \delta_2, ..., \delta_D\}$, where δ_i is the ith singular value of the composite channel matrix. Then, the optimum covariance matrix is given by [10]

$$\mathbf{Q}^{\text{opt}} = \mathbf{V}\mathrm{diag}\{p_1, p_2, ..., p_D\}\mathbf{V}^H, \tag{11}$$

where p_i denotes the power allocation of the ith data stream, which is optimally computed using the water-filling algorithm as follows [10]

$$p_i = \left(\mu - \frac{\sigma^2}{\delta_i^2}\right)^+, \tag{12}$$

where $(x)^+ = \max(x, 0)$, and μ is chosen to satisfy the total transmit power constraint as follows $\sum_{i=1}^{D} p_i = P$.

The second sub-problem tackles the optimization of a single effective reflection coefficient denoted by γ_k while the transmit covariance matrix and the rest of effective reflection coefficients denoted by $\{\gamma_l\}_{l=1,l\neq k}^{L}$ are set as constants. In this scenario, the composite channel matrix in (6) can be re-written as follows

$$\mathbf{H} = \mathbf{H}_f + a_k\gamma_k\mathbf{r}_k\mathbf{t}_k^H, \tag{13}$$

where $\mathbf{H}_f = \mathbf{H}_o + \sum_{l=1,l\neq k}^{L} a_l\gamma_l\mathbf{r}_l\mathbf{t}_l^H$. Moreover, define the eigenvalue decomposition of the transmit covariance matrix as $\mathbf{Q} = \mathbf{U}_Q\mathbf{\Sigma}\mathbf{U}_Q^H$, where $\mathbf{U}_Q \in \mathbb{C}^{n_t \times n_t}$ is a unitary matrix whose columns are the eigenvectors, and $\mathbf{\Sigma} \in \mathbb{C}^{n_t \times n_t}$ is a diagonal matrix of the non-negative real eigenvalues. Then, the term $\mathbf{H}\mathbf{Q}\mathbf{H}^H$ in (P1) can be written as

$$\begin{aligned}\mathbf{H}\mathbf{Q}\mathbf{H}^H &= \left(\mathbf{H}_f + a_k\gamma_k\mathbf{r}_k\mathbf{t}_k^H\right)\mathbf{Q}\left(\mathbf{H}_f + a_k\gamma_k\mathbf{r}_k\mathbf{t}_k^H\right)^H\\ &= \left(\mathbf{H}_f^{'} + a_k\gamma_k\mathbf{r}_k\mathbf{t}_k^{'H}\right)\left(\mathbf{H}_f^{'} + a_k\gamma_k\mathbf{r}_k\mathbf{t}_k^{'H}\right)^H\end{aligned} \tag{14}$$

where $\mathbf{H}_f^{'} = \mathbf{H}_f\mathbf{U}_Q\mathbf{\Sigma}^{\frac{1}{2}}$, and $\mathbf{t}_k^{'H} = \mathbf{t}_k^H\mathbf{U}_Q\mathbf{\Sigma}^{\frac{1}{2}}$. By substituting (14) in (8), the objective function of (P1) can be written as

$$f_k = \log\det\left[\mathbf{A}_k + \gamma_k\mathbf{B}_k + \gamma_k^*\mathbf{B}_k^H + |\gamma_k|^2\mathbf{C}_k\right], \tag{15}$$

where

$$\begin{aligned}\mathbf{A}_k &= \mathbf{I}_{n_r} + \frac{1}{\sigma^2}\mathbf{H}_f^{'}\mathbf{H}_f^{'H},\\ \mathbf{B}_k &= \frac{a_k}{\sigma^2}\mathbf{r}_k\mathbf{t}_k^{'H}\mathbf{H}_f^{'H}, \quad \text{and} \quad \mathbf{C}_k = \frac{a_k^2}{\sigma^2}\mathbf{r}_k\mathbf{t}_k^{'H}\mathbf{t}_k^{'}\mathbf{r}_k^H.\end{aligned} \tag{16}$$

It is important to note that $\mathbf{A}_k$, $\mathbf{B}_k$, and $\mathbf{C}_k$ are independent of γ_k. Hence, the second sub-problem of (P1) can be written as

$$\begin{aligned}\text{(P1-2)} \quad & \max_{\gamma_k} \ \log\det\left[\mathbf{A}_k + \gamma_k\mathbf{B}_k + \gamma_k^*\mathbf{B}_k^H + |\gamma_k|^2\mathbf{C}_k\right] && (17)\\ & \text{s.t.} \quad |\gamma_k| \leq M. && (18)\end{aligned}$$

To study the impact of the effective reflection coefficient's magnitude on the objective function, (17) can be re-written as

$$\begin{aligned} f_k &= \log\det\left[\mathbf{A}_k + |\gamma_k|\mathbf{D}_k + |\gamma_k|^2\mathbf{C}_k\right] \qquad (19)\\ &= \log\det\left[\mathbf{A}_k\right] + \log\det\left[\mathbf{I}_{n_r} + |\gamma_k|\left(\mathbf{A}_k^{-1}\mathbf{D}_k + |\gamma_k|\mathbf{A}_k^{-1}\mathbf{C}_k\right)\right],\end{aligned}$$

where $\mathbf{D}_k = b_k\mathbf{B}_k + b_k^*\mathbf{B}_k^H$, while $b_k = e^{j\arg\{\gamma_k\}}$. It is important to note from (16) that $\mathbf{A}_k$ is the summation of an identity matrix plus a positive semi-definite matrix. Thus, $\mathbf{A}_k$ is a full rank positive definite matrix. Also, since $\mathbf{C}_k$ is a rank one, positive semi-definite matrix, the matrix $\mathbf{A}_k^{-1}\mathbf{C}_k$ has a single non-negative eigenvalue as shown in corollary III.4.6 [19]. Furthermore, because $\mathbf{D}_k$ is a hermitian matrix, the matrix $\mathbf{A}_k^{-1}\mathbf{D}_k$ has real eigenvalues as shown in corollary 3 [20]. Consequently, given the aforementioned matrices properties, the real eigenvalues of $\left(\mathbf{A}_k^{-1}\mathbf{D}_k + |\gamma_k|\mathbf{A}_k^{-1}\mathbf{C}_k\right)$ become monotonically increasing in $|\gamma_k|$ as shown in theorem VIII.4.5 [19]. Thus, the objective function in (17) is monotonically increasing in $|\gamma_k|$, and the optimum magnitude of the effective reflection coefficient is

$$|\gamma_k^{\text{opt}}| = M. \tag{20}$$

The optimum magnitude of each effective reflection coefficient is simply the maximum possible value, which is a logical finding because this will maximize the reflected power from a particular IRS to the receiver. Now, given the optimum magnitude of the effective reflection coefficient is set, the objective function in (17) can be re-written as

$$f_k = \log\det\left(\mathbf{G}_k + Mb_k\mathbf{B}_k + Mb_k^*\mathbf{B}_k^H\right) \tag{21}$$

where $\mathbf{G}_k = \mathbf{A}_k + M^2\mathbf{C}_k$. It was shown in [11] that the optimum phase shift for the effective reflective coefficient that maximizes the previous objective function is

$$b_k^{\text{opt}} = \begin{cases} e^{-j\arg\{\lambda_k\}} & \text{if } \text{tr}(\mathbf{G}_k^{-1}\mathbf{B}_k) \neq 0 \\ 1 & \text{otherwise} \end{cases}, \tag{22}$$

where, $\lambda_k \in \mathbb{C}$ is the single non-zero eigenvalue of the rank one matrix $\mathbf{G}_k^{-1}\mathbf{B}_k$. Then, the optimum effective reflection coefficient of the kth IRS is

$$\gamma_k^{\text{opt}} = \begin{cases} Me^{-j\arg\{\lambda_k\}} & \text{if } \text{tr}(\mathbf{G}_k^{-1}\mathbf{B}_k) \neq 0 \\ M & \text{otherwise} \end{cases}, \tag{23}$$

The alternating algorithm starts with randomly generated phases for all IRSs; then it iteratively computes the optimum effective reflection coefficients and the transmit covariance matrix until the objective value converges. The overall algorithm is as shown in Algorithm 1.

Algorithm 1 :Alternating optimization for rate maximization

1: **Input** : $\mathbf{H}_o, \{\mathbf{r}_l\}_{l=1}^L, \{\mathbf{t}_l\}_{l=1}^L, \{a_l\}_{l=1}^L, \sigma^2$.
2: **Output** : $\gamma_1^{\text{opt}}, \gamma_2^{\text{opt}}, ..., \gamma_L^{\text{opt}}, \mathbf{Q}^{\text{opt}}$.
3: Randomly generate the phases of the effective reflection coefficients of all IRSs.
4: Obtain the transmit covariance matrix based on (11).
5: **for** $k = 1 \to L$ **do**
6: Compute $\mathbf{G}_k$ and $\mathbf{B}_k$ based on (16).
7: Update γ_k^{opt} based on (23).
8: **end**
9: Update the covariance matrix $\mathbf{Q}^{\text{opt}}$ based on (11).
10: If convergence occurred stop; else go to step 5.

Finally, after the convergence occurred in Algorithm 1, the reflecting elements in the IRSs should be tuned to achieve the optimum effective reflection coefficient of each IRS. This can be done simply as

$$e^{j\varphi_{lm}} = e^{j\left(\psi_{lm} + \arg\{\gamma_l^{\text{opt}}\}\right)} \quad \forall\, l \in \mathcal{L} \quad \& \quad m \in \mathcal{M}. \tag{24}$$

B. Enhancement of Composite Channel Power

In this subsection, instead of jointly optimizing the effective reflection coefficients and the transmit covariance matrix to maximize the channel capacity, we aim to enhance the composite channel power using different approaches which will result in an improvement to the achievable rate. One simple method is to maximize the power of the channel matrix that results from transmitted signal reflection on each IRS individually which is defined for the lth IRS as follows

$$\|\mathbf{H}_l\|^2 = \text{tr}\left(\mathbf{H}_l\mathbf{H}_l^H\right) = a_l^2|\gamma_l|^2 n_t n_r, \tag{25}$$

where $\text{tr}\left(\mathbf{r}_l\mathbf{t}_l^H\mathbf{t}_l\mathbf{r}_l^H\right) = n_t n_r$. The power of each IRS' channel matrix is maximized whenever $|\gamma_l| = M$, which occurs when all the paths reflected by IRS are co-phased at the receiver to result in $\|\mathbf{H}_l\|^2 = a_l^2 M^2 n_t n_r$. This can be done by tuning the phase shift of each reflecting element in each IRS as follows

$$e^{j\varphi_{lm}} = e^{j\psi_{lm}} \quad \forall\, l \in \mathcal{L} \quad \& \quad m \in \mathcal{M}. \tag{26}$$

It is clear that the previous solution is independent of the phase shift of the effective reflection coefficient because the power of the channel matrix from each IRS is maximized individually. Thus, for the sake of better composite channel power, the effective reflection coefficients should be jointly optimized to maximize the composite channel power as

$$\text{(P2)} \qquad \max_{\gamma_1, \gamma_2, \dots, \gamma_L} \|\mathbf{H}\|^2 \tag{27}$$

$$\text{s.t.} \qquad |\gamma_l| \leq M, \; \forall\, l \in \mathcal{L}. \tag{28}$$

where (P2) is a non-convex optimization problem because the composite channel power is non-concave over the effective reflection coefficients of IRSs. However, using an alternating optimization algorithm, similar to that of (P1), which solves for a single effective reflection coefficient while the rest are set as constants would converge to a sub-optimal solution. Then, the composite channel power, given that $\{\gamma_l\}_{l=1, l\neq k}^L$ are set to constants can be written as follows

$$\begin{aligned}\|\mathbf{H}\|^2 &= \|\mathbf{H}_f\|^2 + a_k\gamma_k \text{tr}\left(\mathbf{r}_k\mathbf{t}_k^H\mathbf{H}_f^H\right) + a_k\gamma_k^* \text{tr}\left(\mathbf{H}_f\mathbf{t}_k\mathbf{r}_k^H\right) \\ &+ \|\mathbf{H}_k\|^2 = \|\mathbf{H}_f\|^2 + 2a_k \text{Re}\left(\gamma_k^*\mathbf{r}_k^H\mathbf{H}_f\mathbf{t}_k\right) + a_k^2|\gamma_k|^2 n_t n_r,\end{aligned} \tag{29}$$

where $\mathbf{H}_f$ is defined in (13), and $\text{tr}\left(\mathbf{H}_f\mathbf{t}_k\mathbf{r}_k^H\right) = \mathbf{r}_k^H\mathbf{H}_f\mathbf{t}_k$. Then, the optimum effective reflection coefficient of the kth IRS that maximizes the composite channel power is as follows

$$\gamma_k^{\text{opt}} = Me^{j\arg\{\mathbf{r}_k^H\mathbf{H}_f\mathbf{t}_k\}} \tag{30}$$

The overall alternating algorithm starts with randomly generated phases for all IRSs; then iteratively computes (30) until the composite channel power converges. The overall algorithm is summarized in Algorithm 2.

Algorithm 2 :Alternating optimization for composite channel power maximization

1: **Input** : $\mathbf{H}_o, \{\mathbf{r}_l\}_{l=1}^{L}, \{\mathbf{t}_l\}_{l=1}^{L}, \{a_l\}_{l=1}^{L}$.
2: **Output** : $\gamma_1^{\text{opt}}, \gamma_2^{\text{opt}}, ..., \gamma_L^{\text{opt}}$.
3: Randomly generate the phases of the effective reflection coefficients of all IRSs.
4: **for** $k = 1 \rightarrow L$ **do**
5: Update γ_k^{opt} based on (30).
6: **end**
7: If power convergence occurred stop; else go to step 4.

After the convergence of the composite channel power in Algorithm 2, the reflecting elements in the IRSs should be tuned to achieve the optimum effective reflection coefficient of each IRS as shown in (24).

Finally, another low complexity solution for enhancing the composite channel power is possible. This solution aims to maximize the lower bound (LB) of the composite channel power which is defined as

$$\|\mathbf{H}\|^2 = \sum_{m=1}^{n_r}\sum_{n=1}^{n_t}\left|[\mathbf{H}_o]_{m,n} + \sum_{l=1}^{L} a_l \gamma_l r_{l,m} t_{l,n}^*\right|^2$$
$$\geq \left|\sum_{m=1}^{n_r}\sum_{n=1}^{n_t}[\mathbf{H}_o]_{m,n} + \sum_{l=1}^{L} a_l \gamma_l \sum_{m=1}^{n_r}\sum_{n=1}^{n_t} r_{l,m} t_{l,n}^*\right|^2, \quad (31)$$

where $[\mathbf{H}_o]_{m,n}$, $r_{l,m}$, and $t_{l,n}$ are the element in mth row and nth column of $\mathbf{H}_o$, the mth element in $\mathbf{r}_l$, and the nth element in $\mathbf{t}_l$, respectively. Thus, the effective reflection coefficient for the lth surface that maximizes the LB of the composite channel power is

$$\gamma_l^{\text{opt}} = Me^{j(\arg\{\sum_{m=1}^{n_r}\sum_{n=1}^{n_t}[\mathbf{H}_o]_{m,n}\} - \arg\{\sum_{m=1}^{n_r}\sum_{n=1}^{n_t} r_{l,m} t_{l,n}^*\})}. \quad (32)$$

Similar to the previous approaches, the phase shifts of the reflecting elements in each IRS are computed as in (24). Eventually, after enhancing the composite channel power either by optimizing the IRSs individually, jointly, or depending on the LB composite channel power maximization, the optimum transmit covariance matrix can be obtained as shown in (11).

IV. Simulation Results

In this simulation, we study the capacity gains of IRSs-aided P2P MIMO systems in LoS environments. The simulation parameters are presented in Table I. We assume that the transmit and receive arrays are placed parallel to the x-axis in the $x - y$ plane. Furthermore, all the IRSs are assumed to be parallel to the $y - z$ plane, and placed randomly inside a sphere that is centred at the origin and has a radius of $75m$, while maintaining the far-field assumption [15] such that d_l^t & $d_l^r > 10\sqrt{A_{irs}} = 7.5m, \forall\, l \in \mathcal{L}$ where A_{irs} is the surface area of the IRS. Since the IRSs are distributed randomly, we compute the 10% outage capacity over 10^4 realizations of IRSs positions. Moreover, in Algorithm 1 convergence is achieved whenever the increment in the objective function is less than 10^{-3}. Whereas, in Algorithm 2 the convergence is

TABLE I
Simulation Parameters

Parameter	Value
Carrier frequency	3 GHz
Receiver noise figure	9 dB
System bandwidth	10 MHz
Gain of transmit and receive antennas	3 dBi
Reflecting element aperture area	$\lambda_c/2 \times \lambda_c/2$
Number of reflecting elements per IRS	225
Area of each IRS	0.56 m^2
Antenna separation in Tx and Rx	$\lambda_c/2$
Number of transmit antennas	4
Number of receive antennas	4
Location of Transmitter ULA	$[0, -50, 0]$ m
Location of Receiver ULA	$[0, +50, 0]$ m

satisfied whenever the increment in the objective function is less than 0.1% of the initial channel power. We compare the performance of Algorithm 1, Algorithm 2, the LB channel power maximization scheme in (32), the co-phasing scheme in (26), and the scenario without IRSs.

In Fig. 1 the outage capacity given the deployment of 12 IRSs versus the signal-to-noise-ratio (SNR) of the direct LoS link is shown. Obviously, the four schemes result in a dramatic reduction in the transmit power compared to the system without IRSs, while Algorithm 1 achieves the best performance, then Algorithm 2, after that the LB channel power maximization scheme, and finally the co-phasing scheme. Furthermore, it is clear from the slope of the outage capacity at high SNR that the four schemes achieve a spatial multiplexing gain in comparison to the scenario without IRSs, which demonstrates that the deployment of IRSs in LoS environment improves the rank of channel matrix and makes spatial multiplexing possible.

In Fig. 2 we compare the outage capacity versus the number of IRSs given that the SNR of the direct LoS link is 25 dB. As the number of IRSs increases, the wireless channel becomes richer with more multi-path components, so the rank of the composite channel matrix and the power gain of the formulated eigenchannels improve hence, the outage

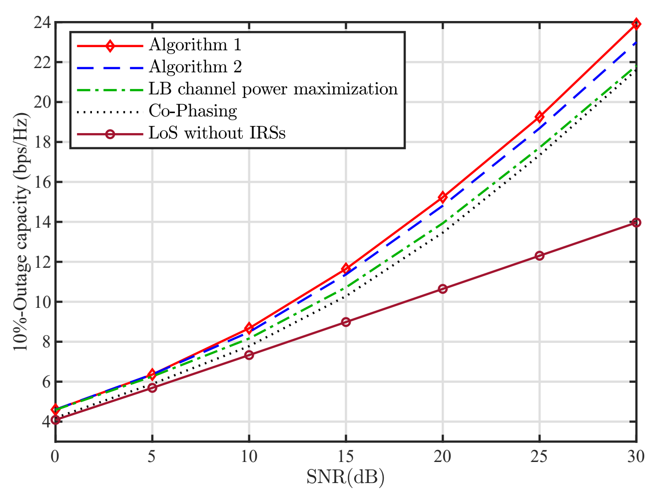

Fig. 1. 10% outage capacity versus the SNR of direct LoS link while using 12 IRSs for 4×4 MIMO system.

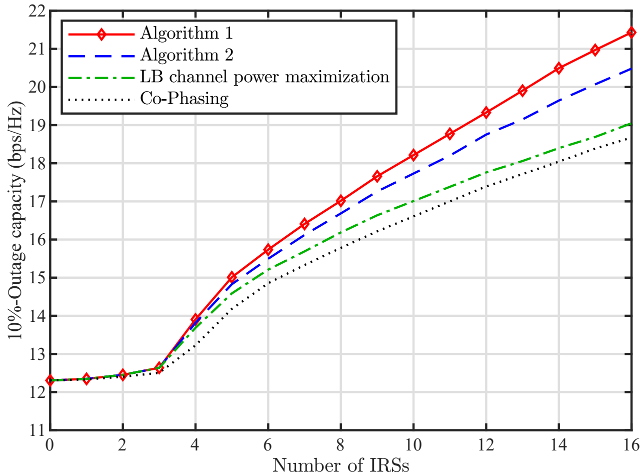

Fig. 2. 10% outage capacity versus the number of IRSs for 4×4 MIMO system. The SNR of direct LoS link equals 25 dB.

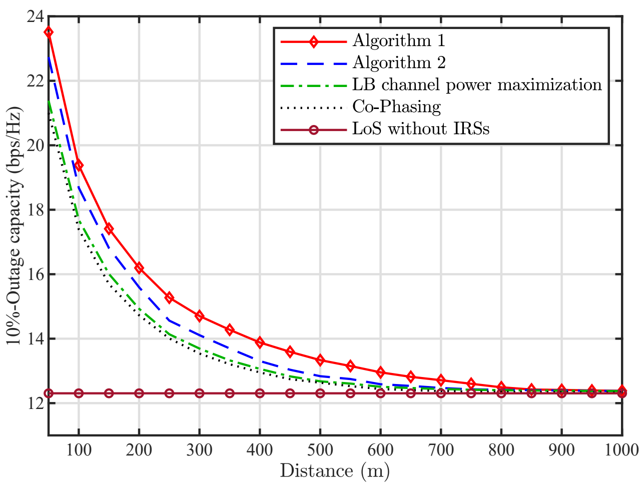

Fig. 3. 10% outage capacity versus the distance between transmitter and receiver using 12 IRSs for 4×4 MIMO system. The SNR of direct LoS link is 25 dB.

capacity increases. However, when the number of the IRSs is relatively small their impact on the performance becomes limited because the power of their reflected paths is negligible compared to that of direct LoS link and the composite channel matrix becomes dominated by the direct LoS channel matrix.

In Fig. 3 we compare the outage capacity when 12 IRSs are deployed versus the distance between the transmitter and receiver. In this comparison, the SNR of the direct LoS link is set to 25 dB for all distances. This will quantify the effect of IRS path loss on the performance of the IRSs-aided MIMO system. As seen, the outage capacity of all approaches decreases sharply with longer distances. This is attributed to the unfavourable plate scattering attenuation model of the IRS which is inversely proportional to the multiplication of IRS's distances to the transmitter and receiver as shown in (5).

V. Conclusion

We discussed the deployment of distributed IRSs with P2P MIMO communications in LoS scenarios. We developed an alternating optimization algorithm that enhances the achievable rate by iteratively optimizing the transmit covariance matrix and the effective reflection coefficients of all IRS. Moreover, we propose several schemes to enhance the composite channel power either by iteratively maximizing it using an alternating algorithm, by maximizing its lower-bound, or by maximizing the power of the channel matrix that results from the signal reflection from each IRS individually. We demonstrated that distributed IRSs can synthesize artificial multi-path propagation which increases the rank of the channel matrix, and improves the achievable rate by making spatial multiplexing possible.

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