

CLOSED—FORM SOLUTION OF THE RESPONSE OF SINGLE PEDESTRIAN INDUCED LOAD FOR CLAMPED—CLAMPED BRIDGES

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Abstract. *This paper presents an analytical approximation to describe the vertical and horizontal vibrations of clamped-clamped beams subjected to a single pedestrian induced load. No intra—person variability is considered and only a single crossing is taken into account. This approximation is performed by choosing a suitable trigonometric function that well estimates the dynamic properties and mode shape of the system. Moreover, the approximation enables to find the relationship between the loading parameters such as the step frequency and velocity and, the dynamic properties of the system as the natural frequency and damping. Furthermore, approximation formulas are presented for peak displacements, velocities and accelerations in the damped, low damped and undamped cases. Finally, amplification factors and minimum design mass curves are defined using non dimensional parameters, aiming to contribute to a fast pedestrian bridge dynamic assessment for systems whose mode shapes reassemble the mode shape of a clamped—clamped beam, for both walking and running pedestrian load cases.*

1 INTRODUCTION

As high strength materials become more readily available, lightweight structures and slender structures are more commonly produced, having well aesthetic design whilst less material is being used. Consequently, new lightweight and lively structural systems may exhibit larger deformations than old heavy structures, making them susceptible to a dynamic excitation. This is the case for many newly constructed footbridges.

The behaviour of footbridges are heavily dependent on the natural frequency of the structure itself. In this way, the dynamic effects need to be considered due to the high levels of accelerations obtained when the frequencies associated with pedestrian loading are within the range of the fundamental natural frequencies of the structural system. Additionally, the pedestrians introduce both vertical and horizontal forces meaning that, structural systems may become vulnerable in different directions and serviceability criteria of lightweight structures of pedestrian bridges can be easily exceeded.

Different load case scenarios are considered in the design stage and assessment of footbridges. Consider the S etra design code for footbridges [1], in which by finding the equivalent number of pedestrians synchronized in a footbridge, gives a procedure to address the walking crowd scenario in order to assess serviceability levels of the system. However, no specific procedure is given for the running load event like marathons nor the cases in which a single or few runners excite the structural system while walking or standing pedestrians are present, exceeding the serviceability levels required. This is the case of [2], in which a single runner was able to surpass the acceleration limit. To this aim, fast and reliable tools for design and assessment aiming to address the running load case in footbridges are necessary as well as more research on the subject is required.

In the present work, the moving harmonic load problem is studied in the context of pedestrian bridge dynamics for a clamped—clamped beam system. The interest in this subject relies on the fact that, the symmetric mode shape of vibration of tied arch bridges and frame type bridges, which are typical structural topologies found in footbridges, can be approximated and reassembled to the first mode of vibration of a clamped—clamped beam. In this way, the aforementioned approximation is here presented focusing on the comparison between the influence of walking and running in the response of footbridges.

2 PROBLEM FORMULATION

Consider a theoretical model of a clamped—clamped beam subjected to a single time varying force $F(t)$, moving with a constant velocity v from the left side to the right-hand side of the continuous system shown in Figure 1.

In this work, it is considered $y(x, t)$ as the deflection of the beam along the vertical axis y , evaluated at the horizontal position x in the time t , m is the linear density of the beam, E is the elastic modulus of the beam, I the inertia of the system, c_e the external damping coefficient and c_i the internal damping coefficient as considered in [3]. The span length L of the clamped—clamped beam model is considered as the effective length of the typical symmetric mode shape of a tied arch bridge. The equation of motion of the system is set as:

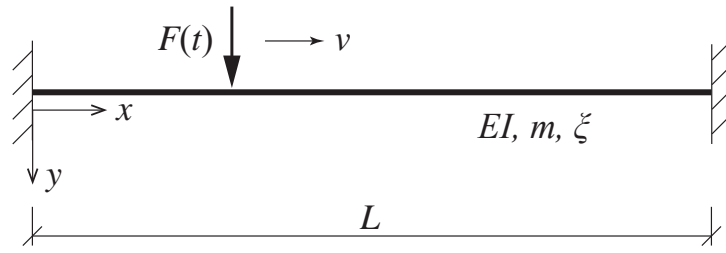


Figure 1: Clamped-clamped beam

$$m\ddot{y} + c_e\dot{y} + c_i I \dot{y}''' + EI y'''' = F(t)\delta(x - vt), \quad 0 \leq vt \leq L \quad (1)$$

In which dots ($\dot{}$) represent a time derivative, primes ($'$) indicate a spatial derivative respect to the horizontal coordinate x of the system, δ is the Dirac function. For a clamped—clamped beam, the boundary conditions are:

$$\begin{aligned} y(0, t) &= 0 \\ y(L, t) &= 0 \\ y'(0, t) &= 0 \\ y'(L, t) &= 0 \end{aligned} \quad (2)$$

Initial conditions considered as:

$$\begin{aligned} y(x, 0) &= 0 \\ \dot{y}(x, 0) &= 0 \end{aligned} \quad (3)$$

And the time varying force $F(t)$ defined as [1]:

$$F(t) = G_0 + \sum_{i=1}^3 G_i \sin(2\pi f_{step} i t) \quad (4)$$

Where G_0 denotes the weight of the pedestrian and $G_i = DLF_i \cdot G_0$, in which DLF_i refers to the dynamic load factor i of the pedestrian load in accordance to the activity performed by the pedestrian.

Let us consider, as an approximation, the first vibration mode ϕ_1 of the beam as a linear combination of assumed functions that satisfy boundary conditions as a "shape function" that well suits the problem as:

$$\phi_1(x) = 1 - \cos^2\left(\frac{\pi x}{L}\right) = 1/2\left(1 - \cos\left(\frac{2\pi x}{L}\right)\right) \quad (5)$$

Considering the modal superposition method, the response of the beam can be approximated by its first mode of vibration as:

$$\begin{aligned} y(x, t) &= \sum_{n=1}^m \phi_n(x) q_n(t) \cong \phi_1(x) q_1(t) \\ \dot{y}(x, t) &\cong \phi_1(x) \dot{q}_1(t) \\ \ddot{y}(x, t) &\cong \phi_1(x) \ddot{q}_1(t) \end{aligned} \quad (6)$$

Where q_n represents the generalized coordinate of the n th mode. Equation (6) applies under the hypothesis that the first mode of vibration very well suits and represents the system's behaviour under a moving harmonic load with an exciting frequency f_{step} closed to the natural frequency of the system f_1 . Substituting (6) in (1), multiplying the resulting equation by the assumed mode shape function ϕ_1 and integrating along the beam axis, one obtains:

$$\begin{aligned} m\ddot{q}_1(t) \int_0^L (\phi_1(x))^2 dx + \dot{q}_1(t) \left[c_e \int_0^L (\phi_1(x))^2 dx + c_i I \int_0^L (\phi_1(x))^{<4>} \phi_1(x) dx \right] + \\ EI q_1(t) \int_0^L (\phi_1(x))^{<4>} \phi_1(x) dx = \int_0^L F(t) \delta(x - vt) \phi_1(x) dx \end{aligned} \quad (7)$$

In which the natural frequency of vibration of the system becomes:

$$\omega_1^2 = \frac{EI \int_0^L (\phi_1(x))^{<4>} \phi_1(x) dx}{m \int_0^L (\phi_1(x))^2 dx} \quad (8)$$

In this way, the numerator and the denominator of equation (8) denote the modal stiffness and the modal mass of the first mode of vibration, respectively. Solving the integrals one gets:

$$K_1 = EI \int_0^L (\phi_1(x))^{<4>} \phi_1(x) dx = \frac{2\pi^4 EI}{L^3} \quad (9)$$

$$M_1 = m \int_0^L (\phi_1(x))^2 dx = \frac{3}{8} mL \quad (10)$$

And the natural frequency is then defined as:

$$\omega_1^2 = \frac{K_1}{M_1} = \frac{16\pi^4 EI}{3L^4 m} \quad (11)$$

Under the hypothesis of Rayleigh's damping in which $c_e = \alpha_e m$ and $c_i = \alpha_i E$, the damping coefficient ξ_1 is defined as:

$$\xi_1 = \frac{1}{2} \left(\frac{\alpha_e}{\omega_1} + \alpha_i \omega_1 \right) \quad (12)$$

And, equation (7) is reduced to the equivalent dynamic equation in the modal coordinates $q_1(t)$ as:

$$\begin{aligned} \ddot{q}_1(t) + 2\xi_1\omega_1\dot{q}_1(t) + \omega_1^2q_1(t) &= \frac{F(t)\phi_1(vt)}{m \int_0^L (\phi_1(x))^2 dx} \\ \ddot{q}_1(t) + 2\xi_1\omega_1\dot{q}_1(t) + \omega_1^2q_1(t) &= \frac{8F(t)\phi_1(vt)}{3mL} \end{aligned} \quad (13)$$

Substituting the assumed mode shape function ones gets:

$$\ddot{q}_1(t) + 2\xi_1\omega_1\dot{q}_1(t) + \omega_1^2q_1(t) = \frac{8}{3mL} F(t) \frac{1}{2} \left(1 - \cos\left(\frac{2\pi vt}{L}\right) \right) \quad (14)$$

By neglecting G_0 and considering a general harmonic component i in the load model presented in equation (4), it follows that:

$$\ddot{q}_{1,i}(t) + 2\xi_1\omega_1\dot{q}_{1,i}(t) + \omega_1^2q_{1,i}(t) = \frac{8}{3mL} G_i \sin(2\pi f_{step} i t) \frac{1}{2} \left(1 - \cos\left(\frac{2\pi vt}{L}\right) \right) \quad (15)$$

And, using the principle of superposition and trigonometric identities the general problem can be reduced to the summation of the threefold components:

$$q_{1,i}(t) = z_{1,i}(t) + z_{2,i}(t) + z_{3,i}(t) \quad (16)$$

In which $z_{1,i}(t)$, $z_{2,i}(t)$ and $z_{3,i}(t)$ correspond to the following threefold problems:

$$\begin{aligned} \ddot{z}_{1,i}(t) + 2\xi_1\omega_1\dot{z}_{1,i}(t) + \omega_1^2z_{1,i}(t) &= \frac{8}{3mL} \frac{G_i}{2} \sin(2\pi f_{step} i t) \\ \ddot{z}_{2,i}(t) + 2\xi_1\omega_1\dot{z}_{2,i}(t) + \omega_1^2z_{2,i}(t) &= -\frac{8}{3mL} \frac{G_i}{4} \sin\left(2\pi f_{step} i - \frac{2\pi v}{L} t\right) \\ \ddot{z}_{3,i}(t) + 2\xi_1\omega_1\dot{z}_{3,i}(t) + \omega_1^2z_{3,i}(t) &= -\frac{8}{3mL} \frac{G_i}{4} \sin\left(2\pi f_{step} i + \frac{2\pi v}{L} t\right) \end{aligned} \quad (17)$$

From equations (17), a beating phenomena can be identified as a consequence of the definition of the load in terms of harmonic components and the spatial wave that defines the mode shape of vibration. Finally, the solution of the aforementioned equations can be found in [3] and it is here adapted:

$$\begin{aligned}
z_{1,i}(t) &= \frac{G_i L^3}{4\pi^4 EI} \frac{1}{(1 - S_{1,i}^2)^2 + (2\xi_1 S_{1,i})^2} \left\{ (1 - S_{1,i}^2) \sin(\Omega_{1,i}t) - 2\xi_1 S_{1,i} \cos(\Omega_{1,i}t) + \right. \\
&\quad \left. e^{-\xi_1 \omega_1 t} \left[2\xi_1 S_{1,i} \cos(\omega_{d1}t) + \frac{S_{1,i}}{\sqrt{1 - \xi_1^2}} (2\xi_1^2 + S_{1,i}^2 - 1) \sin(\omega_{d1}t) \right] \right\} \quad (18) \\
z_{2,i}(t) &= -\frac{G_i L^3}{8\pi^4 EI} \frac{1}{(1 - S_{2,i}^2)^2 + (2\xi_1 S_{2,i})^2} \left\{ (1 - S_{2,i}^2) \sin(\Omega_{2,i}t) - 2\xi_1 S_{2,i} \cos(\Omega_{2,i}t) + \right. \\
&\quad \left. e^{-\xi_1 \omega_1 t} \left[2\xi_1 S_{2,i} \cos(\omega_{d1}t) + \frac{S_{2,i}}{\sqrt{1 - \xi_1^2}} (2\xi_1^2 + S_{2,i}^2 - 1) \sin(\omega_{d1}t) \right] \right\} \\
z_{3,i}(t) &= -\frac{G_i L^3}{8\pi^4 EI} \frac{1}{(1 - S_{3,i}^2)^2 + (2\xi_1 S_{3,i})^2} \left\{ (1 - S_{3,i}^2) \sin(\Omega_{3,i}t) - 2\xi_1 S_{3,i} \cos(\Omega_{3,i}t) + \right. \\
&\quad \left. e^{-\xi_1 \omega_1 t} \left[2\xi_1 S_{3,i} \cos(\omega_{d1}t) + \frac{S_{3,i}}{\sqrt{1 - \xi_1^2}} (2\xi_1^2 + S_{3,i}^2 - 1) \sin(\omega_{d1}t) \right] \right\}
\end{aligned}$$

Where $\Omega_{1,i}$, $\Omega_{2,i}$ and $\Omega_{3,i}$ are the loading frequencies defined as:

$$\begin{aligned}
\Omega_{1,i} &= 2\pi f_{step} i & (19) \\
\Omega_{2,i} &= 2\pi f_{step} i - \frac{2\pi v}{L} \\
\Omega_{3,i} &= 2\pi f_{step} i + \frac{2\pi v}{L}
\end{aligned}$$

The factors $S_{1,i}$, $S_{2,i}$ and $S_{3,i}$ are:

$$\begin{aligned}
S_{1,i} &= \frac{\Omega_{1,i}}{\omega_1} & (20) \\
S_{2,i} &= \frac{\Omega_{2,i}}{\omega_1} \\
S_{3,i} &= \frac{\Omega_{3,i}}{\omega_1}
\end{aligned}$$

And ω_{d1} represents the damped frequency of the system:

$$\omega_{d1} = \omega_1 \sqrt{1 - \xi_1^2} \quad (21)$$

Furthermore, magnification factors of the components of each solution in (18) can be defined as:

$$\begin{aligned} \Phi_{1,i} &= \frac{1}{(1 - S_{1,i}^2)^2 + (2\xi_1 S_{1,i})^2} \left\{ (1 - S_{1,i}^2) \sin(\Omega_{1,i}t) - 2\xi_1 S_{1,i} \cos(\Omega_{1,i}t) + \right. & (22) \\ & \left. e^{-\xi_1 \omega_1 t} \left[2\xi_1 S_{1,i} \cos(\omega_{d1}t) + \frac{S_{1,i}}{\sqrt{1 - \xi_1^2}} (2\xi_1^2 + S_{1,i}^2 - 1) \sin(\omega_{d1}t) \right] \right\} \\ \Phi_{2,i} &= \frac{1}{(1 - S_{2,i}^2)^2 + (2\xi_1 S_{2,i})^2} \left\{ (1 - S_{2,i}^2) \sin(\Omega_{2,i}t) - 2\xi_1 S_{2,i} \cos(\Omega_{2,i}t) + \right. \\ & \left. e^{-\xi_1 \omega_1 t} \left[2\xi_1 S_{2,i} \cos(\omega_{d1}t) + \frac{S_{2,i}}{\sqrt{1 - \xi_1^2}} (2\xi_1^2 + S_{2,i}^2 - 1) \sin(\omega_{d1}t) \right] \right\} \\ \Phi_{3,i} &= \frac{1}{(1 - S_{3,i}^2)^2 + (2\xi_1 S_{3,i})^2} \left\{ (1 - S_{3,i}^2) \sin(\Omega_{3,i}t) - 2\xi_1 S_{3,i} \cos(\Omega_{3,i}t) + \right. \\ & \left. e^{-\xi_1 \omega_1 t} \left[2\xi_1 S_{3,i} \cos(\omega_{d1}t) + \frac{S_{3,i}}{\sqrt{1 - \xi_1^2}} (2\xi_1^2 + S_{3,i}^2 - 1) \sin(\omega_{d1}t) \right] \right\} \end{aligned}$$

Then, the solution of the modal coordinates for a given harmonic component i can be written as follows:

$$\begin{aligned} q_{1,i}(t) &= \frac{G_i L^3}{4\pi^4 EI} \left(\Phi_{1,i} - \frac{\Phi_{2,i}}{2} - \frac{\Phi_{3,i}}{2} \right) & (23) \\ \dot{q}_{1,i}(t) &= \frac{G_i L^3}{4\pi^4 EI} \left(\dot{\Phi}_{1,i} - \frac{\dot{\Phi}_{2,i}}{2} - \frac{\dot{\Phi}_{3,i}}{2} \right) \\ \ddot{q}_{1,i}(t) &= \frac{G_i L^3}{4\pi^4 EI} \left(\ddot{\Phi}_{1,i} - \frac{\ddot{\Phi}_{2,i}}{2} - \frac{\ddot{\Phi}_{3,i}}{2} \right) \end{aligned}$$

And, the final solution becomes:

$$\begin{aligned} y(x, t) &= \frac{L^3}{4\pi^4 EI} \left(1 - \cos\left(\frac{2\pi x}{L}\right) \right) \sum_{i=1}^3 G_i \left(\Phi_{1,i} - \frac{\Phi_{2,i}}{2} - \frac{\Phi_{3,i}}{2} \right) & (24) \\ \dot{y}(x, t) &= \frac{L^3}{4\pi^4 EI} \left(1 - \cos\left(\frac{2\pi x}{L}\right) \right) \sum_{i=1}^3 G_i \left(\dot{\Phi}_{1,i} - \frac{\dot{\Phi}_{2,i}}{2} - \frac{\dot{\Phi}_{3,i}}{2} \right) \\ \ddot{y}(x, t) &= \frac{L^3}{4\pi^4 EI} \left(1 - \cos\left(\frac{2\pi x}{L}\right) \right) \sum_{i=1}^3 G_i \left(\ddot{\Phi}_{1,i} - \frac{\ddot{\Phi}_{2,i}}{2} - \frac{\ddot{\Phi}_{3,i}}{2} \right) \end{aligned}$$

3 ACCELERATION RESPONSE

This section considers the response at the mid point $x = L/2$ of the beam. In order to evaluate the acceleration response of the system, the second derivative of equations (22) are computed as the following:

$$\ddot{\Phi}_{j,i} = \frac{\omega_1^2}{(1 - S_{j,i}^2)^2 + (2\xi_1 S_{j,i})^2} \left\{ (1 - S_{j,i}^2) S_{j,i}^2 \sin(\Omega_{j,i} t) - 2\xi_1 S_{j,i}^3 \cos(\Omega_{j,i} t) + \right. \quad (25)$$

$$e^{-\xi_1 \omega_1 t} \cos(\omega_{d1} t) \left[2\xi_1 S_{j,i} (2\xi_1^2 - 1) - 2\xi_1 S_{j,i} (2\xi_1^2 + S_{j,i}^2 - 1) \right] +$$

$$\left. e^{-\xi_1 \omega_1 t} \sin(\omega_{d1} t) \left[4S_{j,i} \xi_1^2 \sqrt{1 - \xi_1^2} + (2\xi_1^2 - 1) (S_{j,i} / \sqrt{1 - \xi_1^2}) (2\xi_1^2 + S_{j,i}^2 - 1) \right] \right\}$$

Where $j = 1, 2, 3$. If low damping is assumed, equation (25) can be written as:

$$\ddot{\Phi}_{j,i} = \frac{\omega_1^2}{(1 - S_{j,i}^2)^2 + (2\xi_1 S_{j,i})^2} \left\{ (1 - S_{j,i}^2) S_{j,i}^2 \sin(\Omega_{j,i} t) - 2\xi_1 S_{j,i}^3 \cos(\Omega_{j,i} t) + \right. \quad (26)$$

$$\left. - e^{-\xi_1 \omega_1 t} \cos(\omega_1 t) \left[2\xi_1 S_{j,i}^3 \right] - e^{-\xi_1 \omega_1 t} \sin(\omega_1 t) \left[S_{j,i} (S_{j,i}^2 - 1) \right] \right\}$$

And, if no damping is considered one gets:

$$\ddot{\Phi}_{j,i} = \frac{\omega_1^2}{(1 - S_{j,i}^2)^2} \left\{ (1 - S_{j,i}^2) S_{j,i}^2 \sin(\Omega_{j,i} t) - \sin(\omega_1 t) S_{j,i} (S_{j,i}^2 - 1) \right\} \quad (27)$$

Considering a single harmonic i the response is approximated as:

$$\ddot{y}_i(t) = \frac{L^3}{4\pi^4 EI} G_i \left(\ddot{\Phi}_{1,i} - \frac{\ddot{\Phi}_{2,i}}{2} - \frac{\ddot{\Phi}_{3,i}}{2} \right)$$

Finally, by taking in ω_1^2 as a common factor from equation (25) and considering equation (11), the response due to a single harmonic i component can be written as:

$$\ddot{y}_i(t) = \frac{G_i/2}{3/8mL} \Theta_i(S_{j,i}, \Omega_{j,i}, \omega_1, \xi_1, t) \quad (28)$$

$$\ddot{y}_i(t) = \frac{G_i/2}{M_1} \Theta_i(S_{j,i}, \Omega_{j,i}, \omega_1, \xi_1, t)$$

Where Θ_i can be defined as the magnification factor of the dynamic response, that is a function dependent on the frequency ratio ($S_{j,i}$ between the input frequency $\Omega_{j,i}$ and the natural frequency ω_1 of the system, damping ratio ξ_1 and the loading time t).

3.1 ESTIMATION OF MAXIMUM RESPONSE

In order to evaluate the importance of the position of the load, parameters $\Omega_{2,i}$ and $\Omega_{3,i}$ from equation (19) can be written as the following:

$$\begin{aligned}\Omega_{2,i} &= \frac{2\pi v}{L}(i k - 1) \\ \Omega_{3,i} &= \frac{2\pi v}{L}(i k + 1)\end{aligned}\quad (29)$$

Where k denotes the number of cycles or steps applied to the system by the pedestrian. The aforementioned equation, when multiply by time t , it allows to evaluate the magnification factor as function of the non-dimensional position vt/L in accordance with $S_{j,i}$, where $j = 1, 2, 3$. Also, equations (29) can be written as follows:

$$\begin{aligned}\Omega_{2,i} &= 2\pi f_{step} \left(i - \frac{1}{k} \right) \\ \Omega_{3,i} &= 2\pi f_{step} \left(i + \frac{1}{k} \right)\end{aligned}\quad (30)$$

In this way, equations (30) indicate that, as the number of cycles increases the velocity term can be neglected, approaching the steady state response. In this regard, it is important to mention that the equations here exposed, will correctly approximate the response when the step frequency f_{step} is close to the natural frequency f_1 rather than exactly the natural frequency of the bridge. This is understood from the fact that, the differential equation (13) changes when perfect resonance is addressed. Furthermore, the system will achieve its maximum response when the $S_{j,i}$ parameter is close to the unity, meaning that the step frequency f_{step} is at resonance with the natural frequency f_1 . This is a consequence of the presence of the velocity term that originates the beating of the system response components as shown in equations (17) and (18). From equation (30), it can be seen that for very short span bridges the velocity term has a greater influence on the $S_{j,i}$ parameter. However, if the pedestrian bridge is short there will not be enough cycles to achieve high amplification in the system's response. Finally, considering the first harmonic component, the envelope of the magnification factor Θ_1 can be evaluated, as shown in Figure 2. The envelop is made by evaluating the magnification factor as a function of different frequency ratios $S_{j,i}$, respect to the non-dimensional position vt/L , given a number of cycles k and damping ratio ξ_1 . The maximum values of the magnification factor were taken and plotted in Figure 2, which correspond to the case of a resonant loading case, i.e. $S_{j,i} \cong 1$. This is a consequence of the presence of the velocity term. The obtained magnification factor curves, are consistent with results published by [4, 5, 6, 7, 8], taking into account the modal mass and modal stiffness of the clamped-clamped beam system. It can be seen from from Figure 2 that, the response is highly dependent of the damping ratio of the system; a small change in the damping characteristics of the system has the effect of drastically reducing its dynamic response, even reducing the order of magnitude of the expected accelerations. This exposes the disproportional trade-off between the dissipation capacities of the system and its corresponding response given a resonant input, in regards to a transient event.

Equation (26) has been validated since it produces the same magnification factor curves as the complete solution in equation (25), for the presented damping levels. Then, the maximum response due to the i th harmonic component of the pedestrian load can be estimated as:

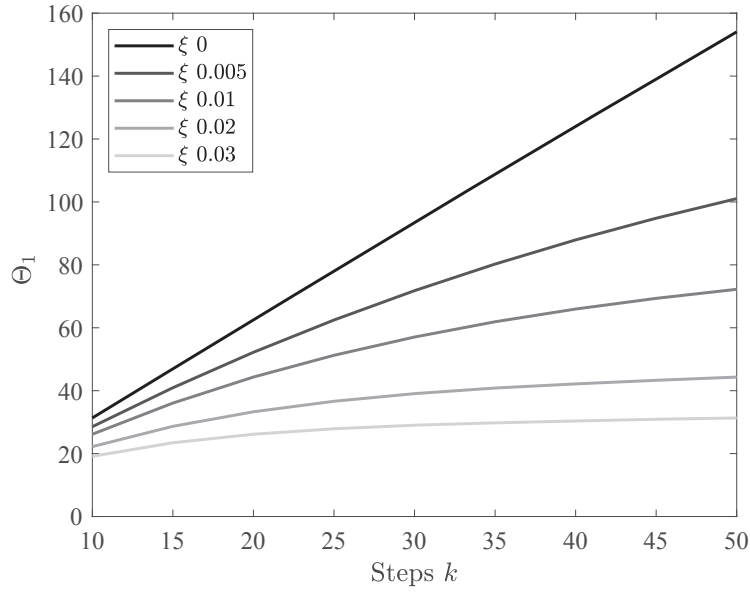


Figure 2: Magnification factor

$$\ddot{y}_i = \frac{G_i/2}{3/8mL} \Theta_i(k, \xi) \quad (31)$$

Finally, equation (31) can be normalized by taking into account the equivalent steady state response of the prescribed single crossing pedestrian \bar{y} , i.e. the steady state response produced by a harmonic load placed in the centre of the beam as:

$$\frac{\ddot{y}_i}{\bar{y}} = \Theta_i(k, \xi) \xi, \quad (32)$$

$$\ddot{y}_i = \Lambda \bar{y}$$

In which Λ is defined as $\Lambda = \Theta_i(k, \xi) \xi$. The parameter Λ represents the fraction of the transient structural response respect to the equivalent steady state response, quantifying how representative the transient event is in respect to the steady state. Results are shown in Figure 3, in which normal walking and normal running defined with a stride length of 0.75 m and 1.25 m as in [9] have been taken into account. In Figure 3, normal running has plotted with a dash line while normal walking with a continuous line. From the results it can be seen that, due to the difference of the stride length and its immediately consequence in the number of cycles applied to the system between the aforementioned activities, for a given bridge length L and a damping ratio ξ , it is easier to achieve an steady state response for normal walking than normal running. This effect is emphasized as damping increases.

3.2 MINIMUM MASS DESIGN CURVES

By the use of equation (31), it is possible to generate design curves for minimum mass needed for a single running and a single walking load cases in pedestrian bridges. Given a

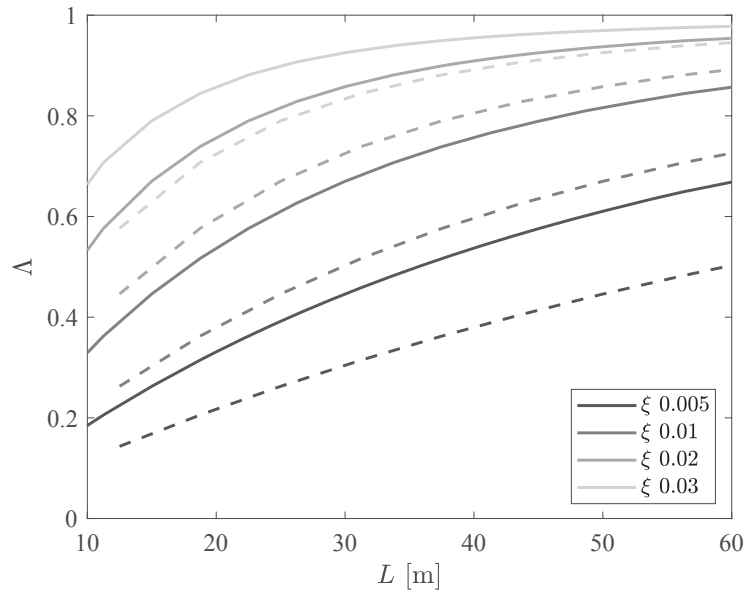


Figure 3: Δ (- - normal running — normal walking)

specific acceleration limit a_{limit} one gets:

$$m = \frac{G_i/2}{3/8 a_{limit} L} \Theta_i(k, \xi) \quad (33)$$

By selecting a specific stride length l_s as reference value and considering $L = k l_s$, the minimum linear mass needed can be defined as:

$$m = \frac{G_i/2}{3/8 a_{limit} k l_s} \Theta_i(k, \xi) = \frac{G_i/2}{3/8 a_{limit} L} \Theta_i(L/l_s, \xi) \quad (34)$$

It is considered that normal walking and normal running are defined with a stride length of 0.75 m and 1.25 m, respectively [9]. For a given a_{limit} , a general minimum mass design curve can be developed for any given value of a_{limit} , as shown in Figure 4, in which normal running has been plotted with a dash line and normal walking with continuous line. It can be seen in equation (34) that the minimum linear mass is inversely proportionally to the length L of the bridge system and proportional to the magnification factor $\Theta_i(k, \xi)$. In this way, by studying the minimum linear mass m it is possible to appreciate which effect dominates in the verification of the serviceability limit state.

It can be seen from Figure 4 that, under the same levels of damping, running is always more critical than walking, which exposes the need of addressing the running load case scenario for pedestrian bridges. The high differences between normal running and normal walking are explained by the combined effect of the differences in their corresponding dynamic load factors DLF_i and the different magnification factors Θ_i for a given bridge length L , i.e. the number load cycles applied. The importance of the aforementioned results lies on its generality, solved for

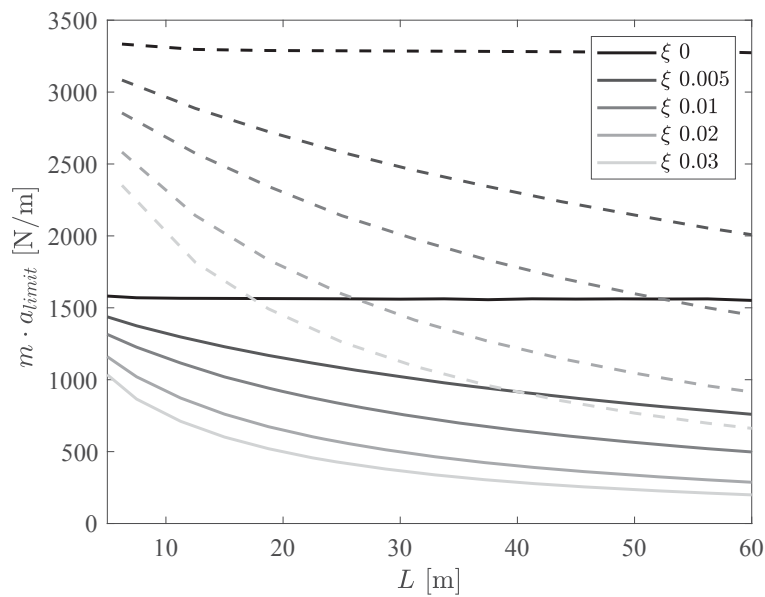


Figure 4: Minimum linear mass (- - normal running — normal walking)

any prescribed acceleration limit a_{limit} in regards to vertical vibrations. Finally, it is important to remark that the equations developed can be used for any type of bridge that reassembles the first mode of a vibration of a clamped—clamped beam, as found in tied arch bridges, portal frame bridges and frame type bridges.

4 CONCLUSIONS

This paper presents an analytical approximation to describe the vertical vibrations of clamped-clamped beams subjected to a single pedestrian induced load, and the following can be concluded.

- A magnification factor is presented and quantified in a closed—form equation. And, the low damping assumption has been introduced and validated for the single passing pedestrian load case scenario. The magnification factor exposes the disproportional trade-off between the dissipation capacities of the system and its corresponding dynamic response given a resonant input, which indicates how pedestrian bridges can be susceptible to experience a large acceleration response.
- It has been quantified how the number load cycles influence the transient response to reach its equivalent steady state response for both normal running and normal walking, in which due to the differences in the stride length, it is easier to achieve steady state under a normal walking regime load than normal running.
- By studying the minimum linear mass parameter, it has been demonstrated that running is more critical than walking for a given damping ratio, exposing the need to address its corresponding load case scenario in the design codes. To this aim, a design chart is given in the present paper in order to perform fast design and assessment of pedestrian bridges for a single crossing pedestrian load case scenario.

- The high differences between normal running and normal walking regarding the minimum mass parameter are explained by the combined effect of the differences in their corresponding dynamic load factors DLF_i and the different magnification factors Θ_i for a given bridge length L , i.e. the number load cycles applied. The importance of this result lies on its generality, since it has been solved for any prescribed acceleration limit a_{limit} in regards to vertical vibrations.

Finally, the moving harmonic load problem applied to clamped—clamped beams has been addressed and it is explained how it can be used for structures that reassemble the mode shape of the studied system such as frame-type bridges and tied arch bridges, exposing the utility and generality of the present work.

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