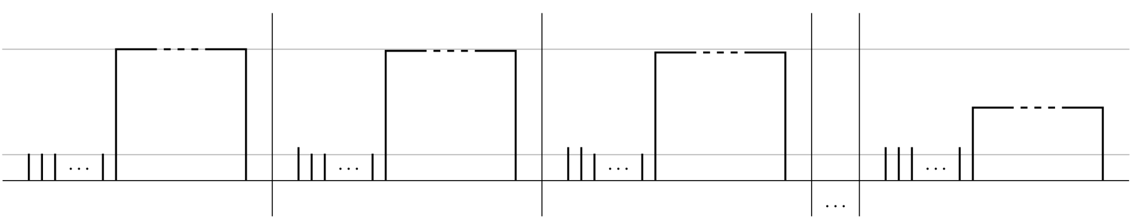
Stockholm University

On the possibility of limited weighing of lives

Daniel Ramöller



Doctoral Thesis in Practical Philosophy at Stockholm University, Sweden 2020

On the possibility of limited weighing of lives

Daniel Ramöller

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Abstract
This thesis discusses the possibility of limited moral trade-offs between different people's welfare. In chapter 2, I introduce the two central limited trade-off conditions. First, according to minimal infinite superiority, significantly benefiting one person matters more than slightly benefiting each of any number of better-off people. Second, according to minimal finite superiority, significantly benefiting many people matters more than slightly benefiting one person. I consider both axiological and deontic interpretations of these conditions. However, I explain why none of the simple classic moral principles—the simple total and the maximin principles—satisfy both conditions. Furthermore, in chapter 3, I strengthen several proved impossibility results according to which no moral theory satisfies weak interpretations of these central trade-off conditions and several other seemingly plausible minimal conditions. I show that giving up structural axiological and deontic conditions is not a satisfactory solution to these paradoxes. In chapter 4, I discuss the modification of a background assumption of these impossibility results on the measurement of welfare. I show that, given a modification that allows for lexicographically ordered welfare components, a total principle can satisfy all the conditions of the impossibility results. However, I argue that such a modification is not entirely satisfactory because it does not apply in certain instances of the paradoxes. In chapter 5, I discuss a further weakening of minimal infinite superiority. However, I show that a suggested possibility result based on this modification is not valid and that further moral conditions, such as minimal finite superiority, need to be modified. Moreover, I argue that these modified conditions and the principle suggested in the possibility proof—a minimax complaint principle—do not capture the basic idea of the two central limited trade-off conditions sufficiently well. In chapter 6, I argue that other principles suggested for the task—the total claim principles—share the same fate as the simple total principles or the minimax complaint principle. In chapter 7, I propose new principles that take their structural roots from voting theory. I show that, in contrast to the other principles discussed, these principles give plausible verdicts where the other principles failed. Finally, in chapter 8, I consider possible objections levelled against this proposal, and I suggest solutions and avenues for future research.

Keywords: *Aggregation, Axiology, Deontic Morality, Distribution, Ethics, Impartiality, Morality, Social Choice, Trade-offs, Values, Welfare.*

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Finally, I thank my family, especially my parents, Ingeborg and Hans-Joachim Ramöller, for their unfailing support throughout my life. And my partner, Lara Haasper, for her love, companionship, and probing questions of why the topic of my thesis is important.

Contents

Conditions and principles

This section provides a reference to conditions and principles discussed in the thesis. Note that this list contains only conditions and principles explicitly defined in the thesis. For example, many of the derived "two-option" versions of the deontic conditions and many strong versions of principles are omitted.[1]

Absolute Comparability: welfare can be measured and compared, both intra- and interpersonally, on an absolute scale (p. 16).

Acyclicity: for any outcomes x_1, x_2, …, and x_n, if x_1 is morally better than x_2, x_2 is morally better than x_3, …, and x_{n-1} is morally better than x_n, then x_n is not morally better than x_1 (p. 33).

Alpha: if an alternative is permissible in a given feasible set, then it is also permissible in any subset containing the alternative (p. 118).

Archimedeaness: for any real numbers r and r', if r' is positive, then there is some positive integer k such that k times r' is greater than r (p. 21).

Beta: if two alternatives are permissible relative to a given feasible set, then the one is permissible if and only if the other is permissible in any superset of the given feasible set (p. 120).

Binary DTRC: same as DTRC where claims are binary claim, i.e. an individual's claim to an alternative is relative to her lower welfare level over a pair of feasible alternatives, and except for 5 which is replaced by

B5. You should choose an alternative that satisfies the greatest sum of strength-weighted, relevant claims in pairwise comparison with all other feasible alternatives.

(p. 154).

Binary Independence: for any two feasible sets X and Y, and any alternatives x and y in both X and Y, x is at least as good as y relative to X if and only if x is at least as good as y relative to Y (p. 210).

Condorcet loser criterion: if an alternative is dominated by every other alternative, then it is impermissible (p. 180).

[1] See section 5.2 for an example of a "two-option" condition and footnote (9) for the definition of a strong principle.

Condorcet winner criterion: if a feasible alternative x dominates every other feasible alternative, then only x is permissible (p. 178).

Completeness: for any real numbers r and r', either r is at least as great as r' or r' is at least as great as r (p. 33).

Continuity: the relation $\succeq$ is continuous if and only if, for all $x \in O$, $\{y \in O : y \succeq x\}$ and $\{y \in O : x \succeq y\}$ are closed sets (relative to u) (p. 110).

Deontic Geometric Total Global Claims Principle: there exists $0 < q < 1$ such that, for any $x \in X \subseteq O$,

$$x \in C(X) \iff \text{for all } y \in X,\ v_q^*(x,X) \geq v_q^*(y,X).$$

(p. 168).

Deontic Geometric Total Global Complaints Principle: there exists $0 < q < 1$ such that, for any $x \in X \subseteq O$,

$$x \in C(X) \iff \text{for all } y \in X,\ v_q(x,X) \leq v_q(y,X).$$

(p. 165).

Deontic Lenimax Global Complaint: for any alternatives x, x is permissible if and only if there is no feasible alternative with a smaller maximum priority global complaint, or, in cases of ties, a smaller second-largest priority global complaint, etc. (p. 122).

Deontic Maximax Condorcet: an alternative is permissible if and only if it maximizes the maximum strength of reason to choose it over any alternative in a pairwise choice (p. 193).

Deontic Minimal Infinite Superiority: for any welfare levels w and w' such that w is lower than w', and positive welfare difference b, there is a positive welfare difference b' such that b' is smaller than b such that, for any numbers of people m', and alternatives x and y, if

1. one person is at or below w in x and better off by at least b in y compared to x;
2. at least m' people are at or above w' in y and each is better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

and x and y are feasible, then x is impermissible (p. 18).

Deontic Minimal Infinite Superiority*: "there is a harm small enough such that no number of such very minor harms to people who will in any case have good lives can outweigh curing one young person's terminal illness [...]."[2] (p. 146).

Deontic Minimax Condorcet: an alternative is permissible if and only if it minimizes the maximum strength of reasons to choose any alternative over it in a pairwise choice (p. 187).

Deontic Minimax SD Condorcet: an alternative is permissible if and only if it is in the smallest set reached by iteratively eliminating all alternatives not in the Deontic Minimax Condorcet set (p. 191).

Deontic Pigou-Dalton: for any alternatives x and y, positive welfare difference b, and persons i and j, if

1. i is at least as well off as j in y;
2. i is better off by b in x compared to y;
3. j is better off by b in y compared to x;
4. everyone else is at the same welfare level in x and y,

and x and y are feasible, then x is impermissible (p. 125).

Deontic Replication Invariance: for any feasible alternatives x and y, and positive integer k, if x is never permissible when y is feasible, then a k-replication of x is never permissible if a k-replication of y is feasible (p. 69).

Deontic Simple Total Principle: a feasible alternative is permissible if and only if no feasible alternative has a greater sum total of welfare (p. 22).

Deontic Strong Finite Superiority*: "you ought to save a large number of people from complete disability rather than one from premature death [...]."[3] (p. 146).

Deontic Strong Pareto: for any alternatives x and y, if everyone's welfare level in x is at least as high as in y and one person's welfare level in x is higher than in y and x and y are feasible, then y is impermissible (p. 125).

Deontic Strong Welfare Impartiality: the permissibility of an alternatives is invariant under permutation of its welfare distribution (p. 213).

Deontic Two-Option Minimal Finite Superiority: for any alternative x, there are positive welfare differences b and b' such that b is larger than b' such that, for any alternative y, if

1. one person is better off by at least b in y compared to x;
2. one person is better off by at most b' in x compared to y;

[2] Voorhoeve (2014: 65); but he does not explicitly label this condition.
[3] Voorhoeve (ibid.: 65); but he does not explicitly label this condition.

3. everyone else is at the same welfare level in x and y,

and only x and y are feasible, then x is impermissible (p. 130).

Deontic Two-Option Minimal Infinite Superiority: for any welfare levels w and w' such that w is lower than w', and positive welfare difference b, there is a positive welfare difference b' such that b' is smaller than b such that, for any numbers of people m', and alternatives x and y, if

1. one person is at or below w in x and better off by at least b in y compared to x;
2. at least m' people are at or above w' in y and each is better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

and only x and y are feasible, then x is impermissible (p. 118).

Deontic Ranked Pairs Condorcet: an alternative is permissible if and only if it is undominated in the Ranked Pairs-ranking of the feasible set (p. 194).

Deontic Ranked Pairs SD Condorcet: an alternative is permissible if and only if it is in the smallest set reached by iteratively eliminating all alternatives not in the Deontic Ranked Pairs Condorcet set (p. 200).

Deontic Ranked Pairs SD Condorcet*: an alternative is permissible if and only if it is permissible according to Deontic Ranked Pairs SD Condorcet among the remaining alternatives of the feasible set after eliminating all alternatives that are impermissible according to Deontic Strong Pareto (p. 203).

Deontic Ultra Minimal Finite Superiority: there is a population size n' such that, for any alternative x, there are welfare differences b and b' such that b is larger than b' such that, for any alternative y, if

1. the population is larger than n';
2. everyone, except for one, is better off by at least b in y compared to x;
3. the one person is better off by at most b' in x compared to y,

and x and y are feasible, then x is impermissible (p. 69).

Deontic Weak Minimax Binary Complaint: for any alternatives x and y, if the maximum binary complaint against x is smaller than that against y, and x and y are feasible, then y is impermissible (p. 117).

Deontic Simple Lexitotal Principle: a feasible alternative is permissible if and only if no feasible alternative has a greater sum total of welfare (p. 96).

Deontic Weak Welfare Impartiality: the permissibility of an alternative is invariant under permutation of the welfare profiles over feasible alternatives (p. 214).

Domain Richness: for any logically possible welfare distribution with a finite number of individuals and non-negative welfare levels, there is an alternative that generates that distribution (p. 49).

DTRC:

1. Each individual whose well-being is at stake has a claim on you to be helped. (An individual for whom nothing is at stake does not have a claim.)
2. Individuals' claims *compete* just in case they cannot be jointly satisfied.
3. An individual's claim is *stronger*:
   a) the more her well-being would be increased by being aided; and
   b) the lower the level of well-being from which this increase would take place.
4. A claim is *relevant* if and only if it is sufficiently strong relative to the strongest competing claim.
5. You should choose an alternative that satisfies the greatest sum of strength-weighted, relevant claims.[4]

(p. 147).

Extreme Strong Infinite Superiority: there are positive welfare differences b and b' such that, for any welfare levels w and w', number of people m', and outcomes x and y, if

1. one person is at w in x and better off by at least b in y compared to x;
2. m' people are at w' in y and better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x (p. 99).

Geometric Total Complaints Principle: there exists $0 < q < 1$ such that, for any alternative $x \in O$,

$$v(x) = v_q(x) := \sum_{i=1}^{n} q^{i-1} c_{(i)}(x).$$

(p. 165).

Global DTRC: same as DTRC where claims are global claims (p. 150).

Global DTRC': Global DTRC reformulated in terms of global complaints, in particular,

5'. You should choose an alternative that minimizes the sum of strength-weighted, relevant complaints.

(p. 159).

Infinite Difference Superiority: for any alternatives x and y, numbers of people m and m', welfare levels w and w', and welfare differences b and b' such that b is greater than b', if

1. m people are at w in x and each is better off by b in y compared to x;
2. m' people are at w' in y and each is better off by b' in x compared to y;

4 Voorhoeve (2014: 65).

3. everyone else is at the same welfare level in x and y,

then y is better than x (p. 116).

Infinite Priority: for any alternatives x and y, welfare levels w and w' such that w is lower than w', welfare differences b and b', and numbers of people m and m', if

1. m people are at w in x and better off by b in y compared to x;
2. m' people are at w' in y and better off by b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x (p. 24).

lexical superiority: a sequence of numbers, $(a_1, a_2, a_3, \ldots)$, is *lexically superior* to another, $(b_1, b_2, b_3, \ldots)$, if and only if an element in the former sequence is greater than the element with the same index in the latter sequence and all elements with a lower index are equal in both sequences (p. 26).

Minimal Finite Superiority: for any alternative x, there are positive welfare differences b and b' such that b is larger than b' such that, for any alternative y, if

1. one person is better off by at least b in y compared to x;
2. one person is better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x (p. 25).

Minimal Infinite Superiority: for any welfare levels w and w' such that w is lower than w', and positive welfare difference b, there is a positive welfare difference b' such that b' is smaller than b such that, for any numbers of people m', and alternatives x and y, if

1. one person is at or below w in x and better off by at least b in y compared to x;
2. at least m' people are at or above w' in y and each is better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x (p. 17).

Non-Anti Pigou-Dalton: for any alternatives x and y, positive welfare difference b, and person j, there is a number of people m such that if

1. j is at least as well off as the m people in y;
2. at least m people are better off by b in y compared to x;
3. j is better off by b in x compared to y;
4. everyone else is at the same welfare level in x and y,

then y is better than x (p. 61).

No Prohibition Dilemmas: for any set of feasible alternatives, at least one feasible alternative is not impermissible (p. 69).

Pigou-Dalton: for any alternatives x and y, positive welfare difference b, and persons i and j, if

1. i is at least as well off as j in y;
2. i is better off by b in x compared to y;
3. j is better off by b in y compared to x;
4. everyone else is at the same welfare level in x and y,

then y is better than x (p. 60).

Priority: for any alternatives x and y, welfare levels w and w' such that w is lower than w', welfare differences b and b' such that b is at least as great as b', and numbers of people m and m' such that m is at least as great as m', if

1. m people are at or below w in x and better off by at least b in y compared to x;
2. m' people are at or above w' in y and better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x (p. 22).

Replication Invariance: for any alternatives x and y, and any positive integer k, x is at least as good as y if and only if a k-replication of x is at least as good as a k-replication of y (p. 51).

Same Number Replication Invariance: for any alternatives x and y with the same number of people, and any positive integer k, x is at least as good as y if and only if a k-replication of x is at least as good as a k-replication of y (p. 66).

Same-People Independence: for all $M \subseteq N$ such that $M \neq \emptyset$ and, for all $x, y, x', y' \in O$, if,

1. for all $i \in M$, $u_i(x) = u_i(y)$ and $u_i(x') = u_i(y')$;
2. for all $j \in N - M$, $u_j(x) = u_j(x')$ and $u_j(y) = u_j(y')$,

then

$$x \succeq y \iff x' \succeq y'.$$

(p. 111).

Schwartz Choice Connection: a feasible alternative is permissible if and only if it is in the union of all minimal undominated sets (p. 72).

Strong Infinite Superiority: for any welfare levels w and w', there are positive welfare differences b and b' such that b is greater than b' such that, for any number of people m', and alternatives x and y, if

1. one person is at or below w in x and better off by at least b in y compared to x;

2. m' people are at or above w' in y and better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x (p. 98).

Strong Pareto: for any alternatives x and y, if everyone's welfare level in x is at least as high as in y and one person's welfare level in x is higher than in y, then x is better than y (p. 125).

Strong Welfare Impartiality: the value of an alternative is invariant under permutation of its welfare distribution (p. 212).

Super Asymmetry: for any feasible sets X and Y, and alternatives x and y in both X and Y, if x is better than y relative to X, then y is not better than x relative to Y (p. 211).

Super Transitivity: for any feasible sets X, Y and Z, and alternatives x in both X and Z, y in both X and Y, and z in both Y and Z, if x is better than y relative to X and y is better than z relative to Y, then x is better than z relative to Z (p. 211).

Super Ultra Minimal Infinite Superiority: there are welfare levels w and w' such that w is lower than w', and positive welfare differences b and b' such that b' is smaller than b such that, for any alternatives x and y, if

1. the worst-off in x are at or below w in x;
2. one worst-off in x is at or below average and w in y and better off by at least b in y compared to x;
3. the best-off in x are the best-off in y and at or above w' in y and better off by at most b' in x compared to y;
4. everyone else is at the same welfare level in x and y,

then y is better than x (p. 55).

Super Ultra Minimal Infinite Superiority*: there are a number of people m, welfare levels w and w' such that w is lower than w', and positive welfare differences b and b' such that b' is smaller than b such that, for any alternatives x and y, if

1. the worst-off in x are at or below w in x,
2. at least m worst-off in x are at or below average and w in y and better off by at least b in y compared to x;
3. the best-off in x are the best-off in y and at or above w' in y and better off by at most b' in x compared to y;
4. everyone else is at the same welfare level in x and y,

then y is better than x (p. 59).

Ultra Minimal Finite Superiority: there is a population size n' such that, for any alternative x, there are welfare differences b and b' such that b is larger than b' such that, for any alternative y, if

1. the population is larger than n';
2. everyone, except for one, is better off by at least b in y compared to x;
3. the one person is better off by at most b' in x compared to y,

then y is morally better than x (p. 63).

Ultra Minimal Finite Superiority*: for any number of people m', there is a population size n' such that, for any alternative x, there are welfare differences b and b' such that b is sufficiently larger than b' such that, for any alternative y, if

1. the population is larger than n';
2. everyone, except for m' people, is better off by at least b in y compared to x;
3. the m' people are better off by at most b' in x compared to y,

then y is morally better than x (p. 67).

Undominanted Choice Connection: a feasible alternative is permissible if and only if it is undominated (p. 68).

Unlimited Finite Superiority: for any alternative x, number of people m', and positive benefits b and b', there exists a number of people m such that, for any alternative y, if

1. at least m people are better off by at least b in y compared to x;
2. at most m' people are better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x (p. 21).

Weak Geometric Priority Total Principle: there is some $0 < q < 1$ such that

$$v(x) = v_q(x) := \sum_{i=1}^{n} \phi_i\big(u_{[i]}(x)\big)$$

with $\phi_i(u) = q^{i-1}u$, for $i \in \mathbb{Z}_{>0}$ and $u \in \mathbb{R}$ (p. 77).

Weak Lenimax Complaint: for any alternatives x and y, if the maximum complaint against x is smaller than that against y, or, in cases of ties, second-largest complaint against x is smaller than that against y, etc., then x is better than y (p. 114).

Weak Minimax Complaint: for any alternatives x and y, if the maximum complaint against x is smaller than that against y, then x is better than y (p. 113).

Weak Minimax Priority Binary Complaint: for any alternatives x and y, if the maximum priority binary complaint against x is smaller than that against y, then x is better than y (p. 117).

Weak Minimax Simple Binary Complaint: for any alternatives x and y, if the maximum simple binary complaint against x is smaller than against y, then x is better than y (p. 115).

Weak Simple Total Principle: for any alternatives x and y, if the sum total of welfare in x is greater than in y, then x is better than y (p. 21).

Weak Leximin: for any alternatives x and y, if the worst-off person in x is better off than the worst-off person in y, and in case of ties, the second worst-off in x is better off than the second worst-off in y, etc., then x is better than y (p. 24).

Weak Leximin Lexitotal Principle: for alternatives x and y, if

1. the sum total of welfare in x is greater than in y,

or

1. the sum total of welfare in x and y is equal, and the worst-off person in x is better off than the worst-off person in y, and in case of ties, the second worst-off in x is better off than the second worst-off in y, etc.,

then x is morally better than y (p. 94).

Weak Maximin: for any alternatives x and y, if the worst-off in x is better off than the worst-off in y, then x is better than y (p. 26).

Weak Simple Lexitotal Principle: for any alternatives x and y, if the sum total of welfare in x is greater than in y, then x is better than y (p. 88).

Weak Simple Priority Total Principle: for any alternatives x and y, if the sum total of non-relational finitely priority-weighted welfare in x is greater than in y, then x is better than y (p. 23).

Weak Welfare Impartiality: the (relative) value of an alternative is invariant under permutation of the welfare profiles over feasible alternatives (p. 214).

Welfarism: the moral goodness of alternatives is determined solely on the basis of their welfare distributions (p. 49).

1

Introduction

1.1 Minimal infinite superiority

Consider Scanlon's famous case of

Jones and the Transmitter:

Suppose that Jones has suffered an accident in the transmitter room of a television station. Electrical equipment has fallen on his arm, and we cannot rescue him without turning off the transmitter for fifteen minutes. A World Cup match is in progress, watched by many people, and it will not be over for an hour. Jones's injury will not get any worse if we wait, but his hand has been mashed and he is receiving extremely painful electrical shocks.[1]

Scanlon claims that one should save Jones and, more generally, that

(1.1) "[it is impermissible] to save a larger number of people from minor harms rather than a smaller number who face much more serious injuries".[2]

Similarly, Temkin claims that

(1.2) "virtually all agree that, other things equal, it would be worse if fifty people suffered from AIDS, quadriplegia, severe psychosis, or being deaf, dumb, and blind, than if virtually any number of people suffered from a minor nosebleed, a slight cold, a sprained finger, or a short mild headache".[3]

These examples may seem rather unrealistic. But policy makers and organizations may face similar scenarios. Consider this further example concerning the moral evaluation of health care policy.

Suppose that if an international agency like the World Health Organization (WHO) invests its resources in curing one illness, it will effectively prolong 1,000 people's lives by fifty years. Alternatively, if WHO invests its resources in attacking another illness that affects 2 million people, it will prolong their lives by one month. Although in the first case, WHO would be keeping people alive a total of 600,000 extra months, and in the second case it would be keeping people alive a total of 2 million extra months [...], I think many would agree that the first outcome would be better than the second.[4]

[1] Scanlon (1998: 235).
[2] Scanlon (ibid.: 238).
[3] Temkin (2012: 33).
[4] Temkin (ibid.: 79).

To see that even single individuals acting on their own might face similar choices, consider a single individual wondering whether to give money to charity A or charity B. Assume that giving the money to charity A would prolong the life of one person by fifty years while charity B would prolong the lives of 2,000 people by one month.

I hasten to add that these are still highly stylized cases and in reality many additional factors have to be considered, including uncertainty and possible side-effects, including burdens on third parties and repeated occurrence of such cases.[5] However, the evaluations in such stylized cases are relevant for many more complex realistic cases.

These moral judgements, such as (1.1) and (1.2), are also vague, imprecise and not general in several respects. Throughout the thesis, I will discuss several different general conditions that make the basic idea more precise. One way to make the basic idea more precise and general (but still rough) is what I call a version of *minimal infinite superiority*

(1.3) significantly benefiting one person matters more than slightly benefiting each of any (finite) number of (otherwise) similarly situated or better-off people.[6]

One can make this even more precise by distinguishing between *deontic* moral claims, i.e. claims about what is *(im)permissible*, and *axiological* moral claims, i.e. claims about moral value, and in particular what is morally *better (or worse)* than something else.[7] Accordingly, there are two versions of (1.3). On a deontic version, roughly,

(1.4) if significantly benefiting one person is feasible, then slightly benefiting each of any (finite) number of (otherwise) similarly situated or better off people is *impermissible*.

On an axiological version, roughly,

(1.5) significantly benefiting one person is *better* than slightly benefiting each of any (finite) number of (otherwise) similarly situated or better off people.

Consider again the philosophers' claims above. Scanlon's claim, (1.1), is an instance of (1.4). In Jones and the Transmitter, Jones is badly off and rescuing him would benefit him significantly while each viewer, who is better off than Jones we can

[5] For the discussion of side effects of speed limits, see e.g. Ridge (1998) and Norcross (1998b). For a discussion of repeated occurrence of cases and dynamic choice, see e.g. Temkin (2012: 79-80) and Gustafsson (2014).

[6] For discussion of this principle (or similar ones), see e.g. Taurek (1977), Parfit (1978), Brink (1993), Scanlon (1998: sec. 5.9), Temkin (2012: ch. 3), Norcross (1998a), Ridge (1998), Carlson (2000), Crisp (2003), Kamm (2008), Dorsey (2009), Fleurbaey, Tungodden and Vallentyne (2009), Fleurbaey and Tungodden (2010), Adler (2012: sec. 5.IV), Voorhoeve (2014), Gustafsson (2014) and Lazar (2018): ch. 9. I will state a more precise version in subsection 2.1.1 and further variations in chapter 3. As I explain in section 2.5, my terminology differs from that used in the literature.

[7] I mean "moral value", i.e. what is good taking the effects on every person into account, also called "general value" or "social value" in contrast to, for instance, "personal value", i.e. what is good for a single person. For simplicity, I will often skip the addition of "moral" since it is clear from the context of the discussion.

suppose, benefits from watching the World Cup only slightly. Hence, according to (1.4), it is impermissible to benefit the viewers.

This interpretation of Scanlon assumes a *comparative account* of harms and benefits. On this account a person is *harmed* in alternative x compared to alternative y if and only if the person would be worse off in x than in y.[8] Correspondingly, the person is *benefited* in y compared to x. So, minimal infinite superiority could be expressed equivalently in terms of harms. I do not assume that this comparative account of harm is the correct account of harm as the term "harm" is commonly used.[9] Rather, I use "harm" as a technical term. But I assume that what "harm" picks out has both deontic and axiological relevance, i.e. it has relevance for what is (im)permissible and morally better. Sometimes, I am also using the terms "loss" and "gain". Again, these are to be understood symmetrically in the following way. If a person has a certain magnitude of loss in an alternative x as compared to an alternative y, then this person has the same magnitude of gain in y as compared to x. The magnitude is just the absolute value of the difference.

Temkin's claim, (1.2), is axiological. However, it is weaker than (1.5) in two respects. First, there are *fifty* people rather than *one* person who would be badly off and would benefit significantly by not being so severely impaired. However, I think most people would also make the stronger claim. Indeed, Temkin himself later suggests a stronger principle that drops this restriction.[10] I will return to the weaker claim in chapter 3.

Secondly, Temkin's claim only holds for *virtually any* rather than *any* number. However, many people, Temkin himself included, are inclined to drop this qualification.[11]

Minimal infinite superiority might be (partly) justified in different ways. Some appeal to the intuitive attractiveness of the principle itself. Some appeal to its intuitively attractive implications in particular cases. And some appeal to the fact that it is implied by moral theories they favour independently. For example, contractualists justify the principle by arguing that this is an implication of contractualism, since it is, arguably, a principle that cannot be reasonably rejected based on implications for each *single* individual.See e.g.[12] On the other side, some reject minimal infinite superiority because it is ruled out by conditions, principles, or theories they accept—most famously by classic total act utilitarianism.

So, it seems important from many perspectives to answer the question: Is there a defensible general moral principle that satisfies minimal infinite superiority? This

[8] See e.g. Parfit (1984: 69) and Kavka (1982: 96).

[9] There are other ways to understand harms and benefits. For example, according to a *status quo ante account* of harm, roughly, a person is benefited (or harmed) if and only if she is better off (or worse off) *than before*. However, then it could not be claimed that there is always a reason to benefit (and not to harm) people. For example, a surgeon who amputates a patient's leg, so that an infection of the leg does not kill the patient, might turn out to *harm* the patient rather than to benefit her since before the operation the patient might have been better off than after the operation. But, surely, there is a reason for the surgeon to save the patient's life in this way.

[10] Temkin (2012: 67–8).

[11] Temkin (ibid.: n. 14).

[12] Scanlon (1998: ch. 5, sec. 9) and Parfit (2003: 372).

is the main question of the thesis. As it turns out, it is, maybe surprisingly, difficult to find such a principle. As far as I know, no such general principles have been discovered. There are some suggestions but they are, as I will show, unclear and, at best, extremely partial or severely restricted. This thesis is an attempt to make some more progress on discovering such principles. Furthermore, in order to answer the question, we need a precise statement of minimal infinite superiority.

1.2 Methodology

The general philosophical method I employ is to check whether certain intuitive moral conditions and principles have problematic implications. On closer inspection, these moral intuitions may turn out to be vague and ambiguous. Of the different ways to make them precise, some might turn out to be less prone to problematic implications than others.

There are two more specific methods. The first method is the *axiomatic method*. One shows that certain intuitive moral axioms, or conditions, that moral theories are supposed to satisfy are (in)consistent with each other, so called *(im)possibility results*.[13] In order to logically derive these results, the conditions need to be precisely stated, so that the logical relations between them become clear. One of these conditions, I will assume, is minimal infinite superiority. It will be shown that several seemingly plausible versions of this condition together with other seemingly plausible conditions are inconsistent. One possible reaction in face of these impossibilities is scepticism, i.e. giving up on finding plausible moral theories. Another is completely giving up at least one of these conditions and accepting a moral theory or principle—I will use these terms interchangeably—that satisfies the other. Instead of scepticism or completely giving up some condition, I take the impossibility results to suggest that we should try to moderately modify some of those conditions.

The second method is the *case-implication critique*.[14] One shows that a certain moral principle has clearly counter-intuitive implications when applied to specific cases, so called *counter-examples*. In particular, I will focus on counter-intuitive implications that clash with the conditions, such as minimal infinite superiority. I will try to find theories that avoid such implications. Again, to derive such results, it is necessary that the principles and conditions are precisely stated.

1.3 Numbering

It might be helpful to make explicit some numbering conventions of the thesis. In each chapter, a more informal discussion and results are followed by a more formal discussion and results in sections numbered by letters.

[13] Maybe most famously, this method has been employed in *Arrow's impossibility theorem* in social choice, see e.g. Arrow (1950).

[14] The term is from Sen (1980: 197).

For greater clarity, many claims and their implications are numbered and cross-referenced with brackets, e.g. "(2.3)" refers to the third such claim or implication in chapter 2.

Cross-references to other numbered elements, e.g. sections and tables, are explicitly qualified, e.g. "section 2.3" and "table 2.3" refers to the third section and table, respectively, in chapter 2.

1.4 Overview

Below is an overview of the chapters and section of the thesis. In the thesis a number of moral principles and conditions are discussed. Tables 1.1 and 1.2 also provides an overview of their relations.

2 Minimal Infinite and Finite Superiority

2.1 Background assumptions

For simplicity, the discussion is restricted to effects on welfare only. Furthermore, for the most part, it is assumed that welfare satisfies Absolute Comparability according to which, roughly, people's welfare can be represented by real numbers unique across different people.

2.2 Minimal Infinite Superiority and box-diagrams

A first precise statement of a version of the *minimal infinite superiority* condition is introduced according to which, roughly, significantly benefiting one person matters more than slightly benefiting each of any (finite) number of (otherwise) similarly situated or better-off people. This condition can be distinguished into *axiological* and *deontic* versions both of which are discussed in the thesis.

2.3 The simple total principles and Unlimited Finite Superiority

According to the *total principles* all that matters is the sum total of transformations of individual welfare levels. The *simple* version of the total principles, according to which all that matters is the sum total of welfare, violates minimal infinite superiority. Instead, many claim that there should be *priority for the worse-off* people. However, the classic version of the total principles with *non-relational finite* priority for the worse-off does not satisfy minimal infinite superiority either.

2.4 The maximin principles, Infinite Priority, and Minimal Finite Superiority

Infinite priority for the worse-off which implies, roughly, that one should maximize the minimum welfare level, for short *maximin*, satisfies minimal infinite superiority but instead violates another seemingly plausible trade-off condition which I call *minimal finite superiority* according to which, roughly, significantly benefiting one is better than slightly benefiting another person.

2.5 “Lexicality”, mere “superiority” and “non-aggregative priority”

The terminology commonly used in the literature for what I call “minimal infinite superiority”, i.e. “lexicality”, (mere) “superiority”, and “non-aggregative priority (for the worst-off)”, is inadequate because the same terminology is often used for different ideas in the discussion. I briefly explain my choice of terminology, taken from mathematics.

2.6 Restrictions

For the most part of this thesis, only outcomes are considered that have *fixed finite* populations, i.e. every outcome has the same finite population, and that occur with *certainty*.

2.A Basics

The basics for formal statements of claims and their technical derivations are introduced.

2.B The trade-off conditions and the simple classic principles

The simple classic principles and conditions are precisely stated and propositions about their logical relationships proved.

3 The Moderate Trade-offs Paradox

Several of the impossibility results by Fleurbaey, Tungodden and Vallentyne (2009) are discussed and strengthened. These results are particularly interesting because their underlying conditions are extremely minimal.

3.2 Background assumptions

The three background assumptions for the impossibility results are introduced and discussed, in particular Absolute Comparability. While Absolute Comparability seems a strong assumption, it is in fact a weak assumption in the context of the impossibility results because it makes more theories possible.

3.3 The First Moderate Trade-offs Paradox

An axiological impossibility result, Result 1*, a variant of a result by Fleurbaey, Tungodden and Vallentyne, is proved that shows that minimal infinite and finite superiority together with other seemingly plausible structural conditions, Acyclicity and Replication Invariance, are inconsistent. The proof involves the application of these conditions to a sequence of alternatives. I call the resulting judgements in the sequence the *First Moderate Trade-offs Paradox*.

3.4 The Third Moderate Trade-offs Paradox

An axiological impossibility result, Result 3, is proved that shows that weaker versions of minimal infinite and finite superiority together with other seemingly plausible conditions, e.g. a weak variant of priority for the worse-off, called *Pigou-Dalton*, are inconsistent. It involves the application of these conditions to a further sequence of alternatives. I call the resulting judgements in the sequence the *Third Moderate Trade-offs Paradox*. I discuss possible objections to the conditions of the impossibility results and strengthen the result in several ways to a Result 3*.

3.5 The Deontic Third Moderate Trade-offs Paradox

Deontic analogues of the axiological moderate trade-offs paradoxes and results are derived.

3.6 Rejecting Acyclicity or No Prohibition Dilemmas

It is suggested that common rejections of structural assumptions on betterness or permissibility cannot solve the paradoxes because the rejections are not sufficient for intuitive verdicts in the paradox sequence.

3.A Proof of Result 3*

The general proof of Result 3* is presented.

3.B The Weak Geometric Priority Total Principle

A *relational finite* priority total principle, which I call the *geometric priority total principle*, does not solve the paradoxes because it violates a version of minimal finite superiority.

3.C Derivation of the deontic from the axiological

It is shown how analogous deontic results can be derived from the axiological results.

4 Absolute Comparability reconsidered and the lexitotal principles

4.1 Absolute Comparability reconsidered

The background assumption of Absolute Comparability is questionable not because it grants too much but too little information. Importantly, it rules out lexicographically ordered welfare components.

4.2 The lexitotal principles solve the Third Moderate Trade-offs Paradox

Once Absolute Comparability is rejected, the simple total principles based on lexicographically ordered welfare components, and which I call the *lexitotal principles*, can solve the Third Moderate Trade-offs Paradox.

4.3 Implications for other results

Versions of the lexitotal principles solve the paradoxes involved in the other impossibility results.

4.4 Objections

It is unclear whether the lexitotal principles can give sufficient priority to the worse-off. Furthermore, the scope of the solution to the paradox is severely limited because not all applications of the paradox can be solved by an appeal to lexicographically ordered welfare components.

4.A The Simple Lexitotal Principle fulfils the conditions of Result 3*

The lexitotal principles are formally stated and it is proven that they satisfy all the conditions of Result 3*.

4.B Characterization theorem by Blackorby et al.

The lexitotal principles also challenge another impossibility result implied by one of Blackorby, Bossert and Donaldson (2005) theorems that characterize the classic total principles.

5 Trade-off conditions reconsidered and the minimax complaint principles

5.1 Non-aggregation and the minimax binary complaint principles

Some accept a version of what I call *minimax complaint* according to which, roughly, a greater individual complaint against an alternative matters infinitely more than any number of smaller individual complaints against another alternative. I am using *complaint* against an alternative and *claim against* an alternative interchangeably. These principles are non-aggregative, i.e., roughly, welfare of different individuals cannot be aggregated, or combined (e.g. additively) so that the aggregate can be morally weighed against the welfare of other individuals. There are different minimax complaint principles that specify complaints in different ways. First, a simple version of minimax complaint, which I call *minimax simple complaint* according to which the worst-off has the greatest complaint implies maximin and, hence, does not satisfy minimal finite superiority. Another version, which I call *minimax binary complaint* that bases complaints on the welfare shortfall in pairwise comparisons of alternatives does not solve the moderate trade-offs paradoxes.

5.2 Deontic Minimal Infinite Superiority reconsidered and Alpha rejected

One promising way to resolve the paradoxes is to claim that the deontic versions of minimal infinite superiority are over-generalizations and should be restricted. One version restricts minimal infinite superiority to apply only in cases with only two alternatives. However, in order to solve the paradoxes, additionally we need to reject a particular kind of context independence called *Alpha*, according to which if an alternative is permissible in a given feasible set, then it is also permissible in any subset containing the alternative, in order to solve the moderate trade-offs paradoxes.

5.3 The global complaint principles solve the moderate trade-offs paradoxes

A moral principle which I call *Deontic Lenimax Global Complaint* fulfils this "two-option" version of minimal infinite superiority and solves the moderate trade-offs paradoxes. Deontic Lenimax Global Complaint bases complaints not merely on pairwise comparisons but on comparisons to the set of feasible alternatives, so called *global complaints*.

5.4 Counterexamples to the proof of Result 7

However, other trade-off assumptions have to be restricted as well. I show that Deontic Lenimax Global Complaint violates both weak variants of Pigou-Dalton and minimal finite superiority. I suggest that these violations are defensible in the moderate trade-off paradoxes. However, contrary to a claim by Fleurbaey, Tungodden and Vallentyne, Deontic Lenimax Global Complaint does not satisfy all the conditions of possibility Result 7. But Deontic Lenimax Global Complaint makes implausible claims regarding minimal finite superiority in another sequence of alternatives, which I call the *Symmetric Ultra Minimal Finite Superiority Sequence*.

5.A The minimax and maximax binary claim principles

The minimax binary claim principles are formally stated and it is proved that they satisfies all the substantive conditions of the impossibility results.

5.B The lenimax and leximax global claim principles

The minimax global complaint principles are formally stated and the results from the informal part are proved. There is also an implausible asymmetry between global complaints, or claims, *against* an alternative and global claims *to* an alternative. Furthermore, a principle that appeals to these latter claims, which I call Deontic Leximax Global Claim, leads to implausible results in a similar symmetric case.

6 The total claims principles

6.1 The Deontic Total Relevant Claims Principle (DTRC)

Voorhoeve (2014) suggests a compromise principle between a total principle and minimax complaint which, roughly, maximizes the sum total of great enough claims. I call this principle the *Deontic Total Relevant Claims Principle* (DTRC),

6.2 The Two-Level Restriction

At least one of the restrictions under which Voorhoeve specifies the relevant complaints total principle is too severe. It restricts the application of the principle to cases where for each individual there are only two possible welfare levels.

6.3 Global DTRC

Relaxing this limitation, the global claim version of DTRC makes the same implausible claims in a symmetric minimal finite superiority sequence as does the global version of minimax complaint discussed in the previous chapter.

6.4 Binary DTRC

The binary complaint version of DTRC does not solve the moderate trade-offs paradoxes and an additional paradox, due to Parfit (2003), which I call the *Strong Finite Superiority Paradox*.

6.5 On the possibility of DTRC

The difficulties DTRC faces are discussed in light of the (im)possibility results presented by Fleurbaey, Tungodden and Vallentyne (2009).

6.6 Complaints

Switching from global claims *to* to claims *against* an alternative (or complaints) solves the symmetric minimal infinite superiority sequence but there are symmetric analogues in which this principle gives an analogously implausible verdict to the global claim to version.

6.A The simple total claims principles

The simple total principles can be restated in terms of claims (or complaints), which I call the *simple total claims principles*. Hence these principles do not solve the paradoxes discussed.

6.B The geometric total claims principles

Another complaints total principle suggested by Carlson (2000), which I call the *Geometric Total Complaints Principle*, does not solve the paradox in its binary version and makes the same implausible claim in the symmetric minimal finite superiority sequence in its global version as the other global principles previously discussed.

7 The maximum claim Condorcet principles

7.1 Simplified paradoxes and sequences

For clarity, the moderate trade-offs paradoxes and symmetric sequences can be simplified when searching for maximum claim principles.

7.2 The deontic Condorcet methods

I introduce the family of maximum claim Condorcet principles that are modelled on *Condorcet methods* from voting theory. These voting methods are designed to solve structurally similar problems. First, both the *Condorcet winner criterion* and the *Condorcet loser criterion* give plausible verdicts in the symmetric infinite and finite sequences, respectively. Second, they solve the *Condorcet paradox*, a paradox similar to the deontic moderate trade-off paradoxes which involves dominance cycles in pairwise choices. However, I show that none of the maximum claim Condorcet principles that are based on mere dominance, i.e. *most* reasons to choose in a pairwise choice, solves the deontic moderate trade-offs paradoxes.

7.3 Three variants of strength of reasons

Instead, I consider Condorcet methods relying on the *strength* of reasons to choose in a pairwise choice. There are at least three variants of these methods, relying on, respectively, the strength of opposing, winning, or marginal reasons.

7.4 Deontic Minimax SD Condorcet

I show that one such method, which I call *Deontic Minimax SD Condorcet*, solves the deontic moderate trade-offs paradoxes and gives the intuitive verdict in the symmetric finite and infinite sequences. However, this principle still has some shortcomings, e.g. creating an asymmetry between appeals to claims *to* alternatives and others that appeal to claims *against* alternatives.

7.5 Deontic Ranked Pairs SD Condorcet

I introduce yet another deontic variant of a voting method, which I call *Deontic Ranked Pairs SD Condorcet*, that avoids these shortcomings.

7.6 A problem: Deontic Strong Pareto

However, there is serious doubt that Deontic Ranked Pairs SD Condorcet, as it stands, is a plausible principle because it violates the very plausible Deontic Strong Pareto condition according to which an alternative is impermissible if another alternative is at least as good for everyone and better for someone. A solution is a simple extension of Deontic Ranked Pairs SD Condorcet, that first eliminates all alternatives that are impermissible according to Deontic Strong Pareto.

7.A Lexical Ranked Pairs Condorcet

An alternative possible solution is a deontic ranked pairs principle based on lexicographically ordered claim distributions that does not directly appeal to Deontic Strong Pareto.

8 Further problems: axiology, impartiality, and non-aggregation

8.1 Axiology reconsidered

I suggest that the Condorcet ranking, underlying Deontic Ranked Pairs SD Condorcet, provides an axiology with betterness rankings relative to a feasible set or comparison class that solves the axiological versions of the moderate trade-offs paradox.

8.2 Impartiality as permutation

Any principle that implies a version of strong infinite superiority, including the maximum claim principles, violates the condition, which I call *Strong Welfare Impartiality*, according to which the moral status of alternatives is not invariant under permutation of *welfare distributions*. However, there is a weaker condition, which I call *Weak Welfare Impartiality*, that only claims invariance under permutation of people's *welfare profiles* in feasible sets. I suggest that this condition does sufficiently capture impartiality. And the maximum claim principles satisfy it.

8.3 Beyond non-aggregation

Non-aggregation, and so the maximum claim Condorcet principles, might seem implausible to many because they satisfy only too limited finite superiority conditions. Instead, one could base Condorcet principles on the strength of reasons provided by Carlson's geometric complaints total principle. However, Voorhoeve (2014)'s DTRC cannot be easily adjusted in a similar way.

Table 1.1 Conditions satisfied (•) and violated (–) by the total principles, Leximin and the lexitotal principles.

	Total Principle				Lexitotal Principle	
	Simple	Simple Priority	Geometric Priority	Leximin	Simple	Leximin
Continuity	•	•[a]	?	–	–	–
Replication Invariance	•	•	–	•	•	•
Strong Pareto	•	•	•	•	•	•
Strong Welfare Impartiality	•	•	•	•	•	•
Pigou-Dalton	–	•	•	•	–	•
Minimal Finite Superiority	•	•	?	–	•	•
Ultra Minimal Finite Superiority*	•	•	–	–	•	•
Minimal Infinite Superiority	–	–	?	•	•[b]	•
Super Ultra Minimal Infinite Superiority	–	–	•	•	•	•

[a] Given the priority-weight function is continuous.
[b] Given the number of lexically ordered welfare components is infinite.

Table 1.2 Deontic conditions satisfied (•) and violated (–) by the claim principles

	Deontic Leximax Priority Claim		DTRC[a]		Deontic Condorcet[b]		
	Binary	Global	Binary	Global	Minimax	RP[c]	Pareto RP[d]
Alpha	•	–	•	–	–	–	–
No Prohibition Dilemmas	–	•	–	•	•	•	•
Condorcet winner criterion	•	–	•	–	•	•	•
Condorcet loser criterion	•	–	•	–	–	•	•
Deontic Strong Pareto	•	•	•	•	–	–	•
Deontic Strong Welfare Impartiality	–	–	–	–	–	–	–
Deontic Weak Welfare Impartiality	•	•	•	•	•	•	•
Deontic Pigou-Dalton	•	–	•	–	–	–	–
Deontic Two-Option Pigou-Dalton	•	•	•	•	•	•	•
Deontic Two-Option Finite Superiority*	–	–	•	•	–	–	–
Deontic Minimal Finite Superiority	•	–	•	–	–	–	–
Deontic Two-Option Minimal Finite Superiority	•	•	•	•	•	•	•
Deontic Minimal Infinite Superiority	•	–	•	–	–	–	–
Deontic Two-Option Minimal Infinite Superiority	•	•	•	•	•	•	•

[a] Deontic Total Relevant Claims Principle.
[b] Based on maximal claims.
[c] Ranked Pairs Condorcet.
[d] Pareto Ranked Pairs Condorcet.

2

Minimal Infinite and Finite Superiority

2.1 Background assumptions

Before I delve into the discussion of moral conditions and principles, I shall state a couple of background assumptions to put the discussion on solid grounds.

2.1.1 Welfare

I do not take a particular view on what *welfare* (or well-being), i.e. what is non-instrumentally or ultimately good *for* a person, consists in. For example, it can be understood hedonically, i.e. in terms of pleasure and pain, as desire satisfaction, or containing items from an "objective list", e.g. including knowledge.[1] To decide what welfare consists in is an interesting question but not one the current thesis is concerned with. Rather the thesis is about the evaluation of the distribution of welfare whatever it might exactly consist in.

However, to simplify, some of the particular examples I discuss make rather uncontroversial assumptions. Any plausible theory will acknowledge, e.g. that additional pain and life shortening (for otherwise healthy people) normally makes people's lives go worse. If you do not agree, the examples could be changed to fit your own ideas about what makes people worse off.

Unless otherwise stated, for simplicity, by welfare I mean *lifetime* welfare, i.e. welfare in a person's *life as a whole*. It might be suggested that the welfare of parts of a life can matter in their own right, perhaps because a person is not seen as a unified subject but more like a set of stages (mini-persons) of a person.[2] In this thesis I am ignoring this complexity. Many of my results can be used in the search for an acceptable theory that takes person-stages seriously.

2.1.2 Other things equal

I focus on welfare and restrict cases and principles *implicitly* with an "other things equal" clause. This means that, in the type of cases considered, no other relevant aspects than those solely based on welfare make a moral difference, e.g. neither

[1] This classic taxonomy is due to (Parfit [1984: 4]).
[2] See e.g. Parfit (ibid.: part 3).

desert nor special duties, and that the principles I consider have implications only in such cases.

For example, in Jones and the Transmitter, Scanlon's claim might be more controversial if Jones is an evil person who deserves to suffer pain. By assuming other things equal, such factors are assumed not to be present in this case. More generally, it might often help to consider rescue cases where you can aid some or other innocent strangers at no (or negligible) costs to yourself and third parties. In such cases, it seems plausible that the welfare considerations are the only morally relevant factor.

I make these restrictions for simplicity, but I do believe that there are many cases in which welfare considerations are the most important even if other things are *not* equal. Importantly, this does not mean to accept welfarism, i.e. only welfare matters morally. Moreover, much of what I discuss is directly relevant even for moral considerations that are not, or not solely, based on welfare as long as they exhibit similar structures, e.g. when weighing different people's strength of claim rights.[3]

2.1.3 Absolute Comparability

For the cases and principles under discussion, it is important to make explicit and precise assumptions of intra- and interpersonal comparisons of welfare. For example, the rough version of minimal infinite superiority, (1.3), makes a statement about how *well-off people are* and *how much people benefit* as well as *comparing these measurements across different people*. For simplicity and for the most part, I shall assume that welfare is measurable on an absolute scale and fully interpersonally comparable, for short

> **Absolute Comparability:** welfare can be measured and compared, both intra- and interpersonally, on an absolute scale,

i.e., roughly,

(2.1) welfare levels can be *represented* by single real numbers, so called *utilities*, that are *unique* across different people.[4]

This implies that welfare (of the same person and different persons) can be meaningfully compared like real numbers, including their levels, differences, and sums. This gives us the means for meaningful and precise statements of many moral principles and conditions. Absolute Comparability is a strong assumption. I will further discuss this assumption in sections 3.2 and 4.1. Briefly, it is an unrealistic assumption but the idea is that we are going to discuss and test different theories under the most

[3] See e.g. Thomson (1990).

[4] For a formally precise definition see subsection 2.A.7.

advantageous conditions. It will also simplify our discussion.[5]

It seems quite obvious that interpersonal comparisons, at least imprecisely, are sometimes possible. However, even this view is not uncontroversial and some neo-classical economists have furthermore denied *any* interpersonal comparisons.[6] There are interesting discussions about how exactly these measurements and comparisons of people's welfare can be done. However, this is not part of the current thesis. I will merely explore the theoretical implications *given* these assumptions.

2.2 Minimal Infinite Superiority and box-diagrams

Here is a precise statement of minimal infinite superiority. According to (axiological)

> **Minimal Infinite Superiority:** for any welfare levels w and w' such that w is lower than w', and positive welfare difference b, there is a positive welfare difference b' such that b' is smaller than b such that, for any numbers of people m', and alternatives x and y, if
>
> 1. one person is at or below w in x and better off by at least b in y compared to x;
> 2. at least m' people are at or above w' in y and each is better off by at most b' in x compared to y;
> 3. everyone else is at the same welfare level in x and y,
>
> then y is better than x.

Arguably, this is a strong interpretation of minimal infinite superiority, (1.5). I will stick to this (simple) interpretation in this chapter. However, I will turn to a weaker interpretation in subsection 3.4.1.

Another upshot of Absolute Comparability is that cases can be represented by box diagrams.[7] I will use them throughout for illustration. Consider figure 2.1. There are two alternatives x and y represented on the left and right of the thin vertical dividing line. Two welfare levels are marked by horizontal lines, level 45 and 30. There are one, person 1, and m' other people in both alternatives. The height of the other vertical lines and boxes represent their welfare levels while their width represents the number of people. So,

(2.2) in x, one person is at welfare level 16 and m' other people at 81;

[5] Many if not all of the problems we shall discuss are of relevance even if welfare is only measurable on some weaker scale, such as a ratio-scale, see subsection 2.A.7 for precise definition. However, limiting the measurability introduces formal complexities that are distracting from the main focus of the thesis. All results can probably be obtained with at least ratio-scale measurability. But I don't attempt to provide a proof of this in the thesis. For example, Bossert and Weymark (2004), see in particular their Theorem 13.8, show that, under ratio-scale measurability and additional common conditions, moral betterness is *characterized* by a very limited class of functions. However, note also that the possibility results discussed in this and later chapters in the thesis violate at least one of these common conditions, namely continuity (see section 4.B) or anonymity (see section 8.2). Hence, they are not themselves bound by this class of functions.

[6] Perhaps most famous is the denial by Arrow (1951: 9).

[7] The same is true under some weaker assumption, such as Ratio Comparability (subsection 2.A.7).

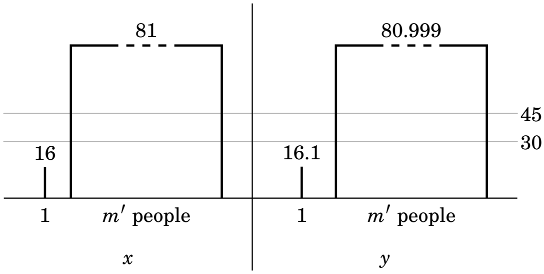

Figure 2.1 Illustration of Minimal Infinite Superiority

(2.3) in y, the one person is at welfare level 16.1 and m' other people at 80.999.

Next, consider again Minimal Infinite Superiority. Assume that

(2.4) the welfare levels w and w' are 30 and 45, respectively;

(2.5) the benefits b and b' are 0.1 and 0.001, respectively.

So, the assumption is that the difference between b and b' is great enough. Then, according to Minimal Infinite Superiority by (2.2) and (2.3),

(2.6) y is better than x

—independently of how large m' is. The broken lines at the top of the boxes represent that its width, representing the number of people m', is not necessarily in proportion and could be much larger.

As a concrete example, assume that the utilities in figure 2.1 represent lifetime in years. So, that a 16-year-old person has an increase in lifetime by more than a month (0.1 years) is better than that any number of almost 81-year-old people have an increase in lifetime by a couple of hours (0.001 years).

Analogously, according to the deontic counterpart to Minimal Infinite Superiority

Deontic Minimal Infinite Superiority: for any welfare levels w and w' such that w is lower than w', and positive welfare difference b, there is a positive welfare difference b' such that b' is smaller than b such that, for any numbers of people m', and alternatives x and y, if

1. one person is at or below w in x and better off by at least b in y compared to x;
2. at least m' people are at or above w' in y and each is better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

and x and y are feasible, then x is impermissible.

Consider again figure 2.1. Again under the assumption of (2.4) and (2.5), according to Deontic Minimal Infinite Superiority by (2.2) and (2.3),

(2.7) if x and y are feasible, then x is impermissible.

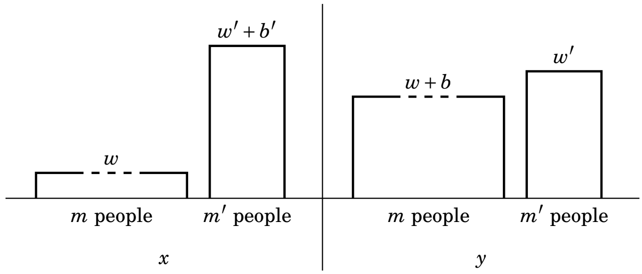

Figure 2.2 Illustration of the simple comparison

As Minimal Infinite Superiority, most conditions to be discussed in this thesis involve only simple comparisons of the following structure, see figure 2.2: for welfare levels w and w', positive welfare differences b and b', and numbers of people m and m',

(2.8) benefiting at least m people at or below w by at least b matters more than benefiting at most m' people at or above w' by at most b'.

For simplicity, a number or variable for a person or the number of people will denote the number of the *same* person or people across outcomes. So, the person denoted by "1" is the same person in x and y, and the "m people" denote the same people in x and y. Variables with the same number of dashes relate to the same people.

Again, there is an axiological and a deontic version of the comparison. In axiological terms, that

(2.9) at least m people at or below w benefit by at least b is better than at most m' people at or above w' benefit by at most b',

more precisely, for any outcomes x and y, if

B1. at least m people are at or below w in x and better off by at least b in y compared to x;

B2. at most m' people are at or above w' in y and better off by at most b' in x compared to y;

B3. everyone else is at the same welfare level in x and y,

then y is better than x.[8] And, in deontic terms, if B1 to B3 and x and y are feasible, then x is impermissible. For clarity, I have matched the m people, w welfare level, and b benefit and the m' people, w' welfare level, and b' benefit. That is also why I do not use the variable m in the statement of Minimal Infinite Superiority.

2.3 The simple total principles and Unlimited Finite Superiority

Figure 2.3 provides an overview of the central logical relations between principles and conditions in this section and the next section.

[8] See section 2.B for a precise statement.

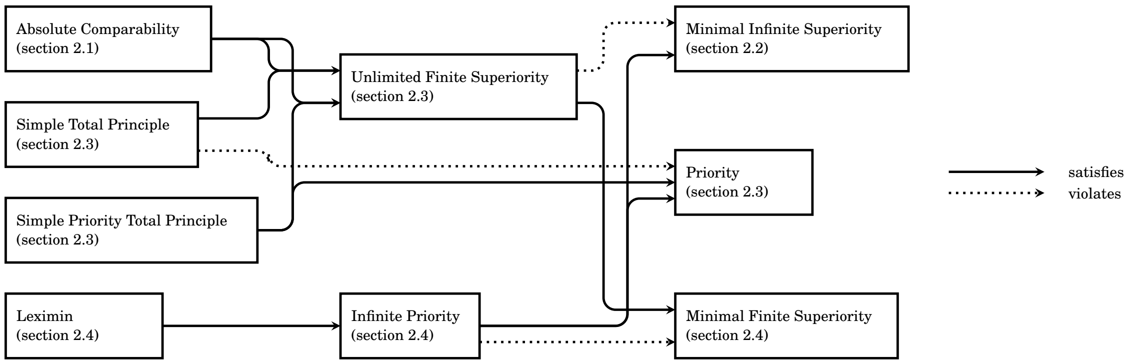

Figure 2.3 Central logical relations in chapter 2

Some classic principles violate Minimal Infinite Superiority. For example, according to the

Weak Simple Total Principle: for any alternatives x and y, if the sum total of welfare in x is greater than in y, then x is better than y.[9]

This principle implies that, roughly,

(2.10) there is some (finite) number of people such that slightly benefiting each of them is better than significantly benefiting one person.

And, more precisely and more generally, it implies

Unlimited Finite Superiority: for any alternative x, number of people m', and positive benefits b and b', there exists a number of people m such that, for any alternative y, if

1. at least m people are better off by at least b in y compared to x;
2. at most m' people are better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x.

To see this, consider again figure 2.1. In this case,

(2.11) the sum total of welfare in x is greater than in y if and only if m' times 0.001 is greater than 0.1.

More generally, consider figure 2.2. In this case,

(2.12) the sum total of welfare in x is greater than in y if and only if m' times b' is greater than m times b.

And the real numbers satisfy

Archimedeaness: for any real numbers r and r', if r' is positive, then there is some positive integer k such that k times r' is greater than r,[10]

i.e. no matter how large r and how small r', if r' is positive, then r' added sufficiently often to itself will be greater than r. Therefore, for any number of people m and positive benefits b and b',

(2.13) there is a number of people m' such that m' times b' is greater than m times b.

Therefore, according to the Weak Simple Total Principle,

(2.14) there is a number of people m' such that x is better than y.

[9] This principle is weak in the sense that it states only a sufficient condition.
[10] Archimedeaness is also called the "Archimedean property".

Therefore, the Weak Simple Total Principle implies Unlimited Finite Superiority, and hence, violates Minimal Infinite Superiority.

Similarly, Deontic Minimal Infinite Superiority is violated by the

Deontic Simple Total Principle: a feasible alternative is permissible if and only if no feasible alternative has a greater sum total of welfare.

Again, by (2.13) and the Deontic Simple Total Principle,

(2.15) there is a number of people m' such that if x and y are feasible, then x is permissible.

Therefore, the Deontic Simple Total Principle violates Deontic Minimal Infinite Superiority.

Note that Minimal Infinite Superiority, condition 1, states that the one person is *worse off* than the m people. But according to the Weak Simple Total Principle this fact is irrelevant. All that matters are the number of people and their benefits. For this reason, many people reject the Weak Simple Total Principle and instead believe that "benefiting people matters more the worse off these people are".[11] The basic idea is that worse-off people should get priority over better-off people. Benefits to the worse off should have extra weight in moral considerations. For example, a given benefit should be had by a worse-off person rather than a better-off person. More generally, a given benefit to a given number of people matters more the worse off these people are. More precisely, according to *Priority for the Worse-off*, for short

Priority: for any alternatives x and y, welfare levels w and w' such that w is lower than w', welfare differences b and b' such that b is at least as great as b', and numbers of people m and m' such that m is at least as great as m', if

1. m people are at or below w in x and better off by at least b in y compared to x;
2. m' people are at or above w' in y and better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x.[12]

Priority should be distinguished from the fact that worse-off people often benefit more from *resources* than better-off people. For example, often a poor person benefits more than a rich person from an additional 100 dollars because the former can buy essential resources that give the person a greater benefit, e.g. food for survival, while the latter already has all the essential goods and can only derive a smaller benefit from it. According to the Weak Simple Total Principle greater benefits matter

[11] Parfit (1997: 213). According to Parfit this, so called, "Priority View" should be understood in non-egalitarian (or non-relational) terms, i.e.. the priority-weight of an individual's welfare is independent of other people's the welfare level (which is compatible with additive separability, see e.g. Fleurbaey (2015: 206)). On this understanding, it does not matter that one is worse off than another. However, I will understand it as being silent on the debate on "equality vs. priority". See, e.g. Persson (2001) for an early discussion of the "Relational Priority View".

[12] Brown (2005: 204) calls this the "Core Prioritarian Thesis".

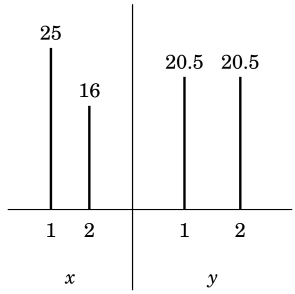

Figure 2.4 Illustration of Priority

more. Hence, "priority" for the worse-off in *resources* can be captured by the Weak Simple Total Principle and often the poor person should get the additional 100 dollar. However, the Weak Simple Total Principle cannot capture Priority, i.e. priority for the worse-off in getting *welfare* benefits. Priority is illustrated in figure 2.4 where

(2.16) one person, person 1, is at welfare level 16 in x and better off by 4.5 in y compared to x;

(2.17) another person, person 2, is at welfare level 20.5 in y and better off by 4.5 in x compared to y.

This is the marginal case of Priority where

(2.18) b and b' are equal;

(2.19) m and m' are equal.

Therefore, according to Priority,

(2.20) y is better than x.

However, Priority does not imply Minimal Infinite Superiority. To see this, note first that Priority is implied by the

> **Weak Simple Priority Total Principle:** for any alternatives x and y, if the sum total of non-relational finitely priority-weighted welfare in x is greater than in y, then x is better than y,

where *priority-weighted* welfare means that the contribution of a person's welfare to the overall value of an alternative gets weighted by a greater factor the worse off a person is; a priority-weight is *finite* if this factor is a finite number; a priority-weight is *non-relational* if it depends only on the person's welfare level and, in particular, not on others' welfare levels.[13]

Consider again figure 2.1. According to the Weak Simple Priority Total Principle, the increase by 0.001 in the welfare of the m' better-off people in y gets less weight

[13] This principle is sometimes referred to as "prioritarianism".

than the increase by 0.1 of the worse-off person in x. However, by the weight being finite and non-relational and Archimedeaness,

(2.21) there is a number of people m' such that m' times fixed finitely weighted 0.001 is greater than fixed finitely weighted 0.1,

and, more generally,

(2.22) there is a number of people m' such that m' times fixed finitely weighted b' is greater than m times fixed finitely weighted b.

Therefore, according to the Weak Simple Priority Total Principle,

(2.23) there is a number of people m', such that x is better than y.

So, the Weak Simple Priority Total Principle, as the Weak Simple Total Principle, implies Unlimited Finite Superiority and, hence, violates Minimal Infinite Superiority.

Considering the Weak Simple Priority Total Principle, makes clear why in Unlimited Finite Superiority the all-quantification over x, i.e. "for any alternative x", precedes the existential-quantification over m, i.e. "there exists a number of people m". According to the Weak Simple Priority Total Principle, for example, the number of better-off people m needed so that a given benefit to them outweighs a greater benefit to a worse-off person depends on the welfare levels at which the worse- and better-off are. And how well-off people are is determined by alternative x in the formulation of Unlimited Finite Superiority.

2.4 The maximin principles, Infinite Priority, and Minimal Finite Superiority

Instead, one could claim that benefiting people matters *infinitely* more the worse off these people are. More precisely, according to *Infinite Priority for the Worse-off*, for short

Infinite Priority: for any alternatives x and y, welfare levels w and w' such that w is lower than w', welfare differences b and b', and numbers of people m and m', if

1. m people are at w in x and better off by b in y compared to x;
2. m' people are at w' in y and better off by b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x.

One principle that implies this is the Weak Lexicographic Maximize the Minimum Principle, for short

Weak Leximin: for any alternatives x and y, if the worst-off person in x is better off than the worst-off person in y, and in case of ties, the second worst-off in x is better off than the second worst-off in y, etc., then x is better than y.

Consider again figure 2.1 where

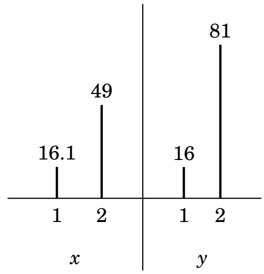

Figure 2.5 Illustration of Minimal Finite Superiority

(2.24) the worst-off in y is better off than the worst-off in x.

Therefore, according to Weak Leximin,

(2.25) y is better than x.

And, more generally, Weak Leximin implies Minimal Infinite Superiority.

Is this then a defensible principle? This is very doubtful because Infinite Priority violates another seemingly extremely plausible condition, roughly,

(2.26) significantly benefiting one is better than slightly benefiting another person.

More precisely, according to

> **Minimal Finite Superiority:** for any alternative x, there are positive welfare differences b and b' such that b is larger than b' such that, for any alternative y, if
>
> 1. one person is better off by at least b in y compared to x;
> 2. one person is better off by at most b' in x compared to y;
> 3. everyone else is at the same welfare level in x and y,
>
> then y is better than x.

Consider its illustration in figure 2.5. There are two people,

(2.27) one person is at welfare level 49 in x and (significantly) better off by 32 in y compared to x.

(2.28) one person is at welfare level 16 in y and is (slightly) better off by 0.1 in x compared to y;

Next, consider again Minimal Finite Superiority. Assume that, for x in figure 2.5,

(2.29) the welfare differences b and b' are 0.1 and 32, respectively.

Therefore, according to Minimal Finite Superiority,

(2.30) y is better than x.

Again, as a concrete example, assume that the utilities in figure 2.5 represent lifetime in years. So, according to Minimal Finite Superiority, that one 45-year-old person has an increase in lifetime of 32 years is better than that a 16-year-old person has an increase in lifetime by about a month (0.1 years).

However, to the contrary, according to Weak Leximin,

(2.31) x is better than y

since the worst-off person in y is worse off than the worst-off in x. And this is also the case for any other principle that implies the Weak Maximize the Minimum Principle, for short

Weak Maximin: for any alternatives x and y, if the worst-off in x is better off than the worst-off in y, then x is better than y.

However, note that this principle does not on its own *satisfy* Minimal Infinite Superiority, since it is silent in cases that do not affect the welfare of the worst-off.

Are there any other views that imply Minimal Infinite Superiority but don't violate Minimal Finite Superiority or any other seemingly extremely plausible condition? Maybe there is some more moderate principle that implies neither of the extremes, Unlimited Finite Superiority and Infinite Priority, that achieves this. As the next chapter shows, maybe surprisingly, the answer is no.

2.5 "Lexicality", mere "superiority" and "non-aggregative priority"

A remark on terminology. What I mean by "infinite superiority" is what others call "lexicality",[14] "superiority",[15] and "non-aggregative priority for the worse-off".[16] But, first, *lexicality* is ambiguous since it also refers to lexicographical superiority, for short

lexical superiority: a sequence of numbers, $(a_1, a_2, a_3, \ldots)$, is *lexically superior* to another, $(b_1, b_2, b_3, \ldots)$, if and only if an element in the former sequence is greater than the element with the same index in the latter sequence and all elements with a lower index are equal in both sequences.

For example, the sequence $(3,1,4,\ldots)$ is lexically superior to $(3,1,2,\ldots)$ because the element in the former sequence with index three, namely 4, is greater than the element with the same index in the latter sequence, namely 2, and all elements with a lower index, namely with index one and two, are equal in both sequences. The ambiguity of lexicality is particularly problematic since both meanings are often used in the same context. For example, the lexical superiority relation is used in the

[14] Broome (2004: 23-4) and Carlson (2000: 248).

[15] Arrhenius and Rabinowicz (2005: 129) and Arrhenius (2005: 291).

[16] Fleurbaey, Tungodden and Vallentyne (2009), Fleurbaey and Tungodden (2010) and Raz (2011: 207).

statement of Weak Leximin.[17] And, in chapter 4, I will discuss another principle that satisfies Minimal Infinite Superiority which involves lexical superiority. But, as I will also show later, lexical superiority is not necessary to satisfy minimal infinite superiority (see section 3.B).

Second, mere *superiority* is not very informative since normally it just means a higher evaluation and needs a specification.[18]

Finally, in the current context, *non-aggregation* usually means that, roughly,

(2.32) welfare of different individuals cannot be aggregated, or combined (e.g. additively) so that the aggregate can be morally weighed against the welfare of other individuals.

Note that both Minimal Infinite Superiority and Minimal Finite Superiority do not rule out non-aggregation. As seen above, some total and hence aggregative principles, like the Weak Simple Total Principle and the Weak Simple Priority Total Principle, violate Minimal Infinite Superiority, and some principles that are non-aggregative satisfy Minimal Infinite Superiority. However, not only non-aggregative principles, and even some total principles, imply Minimal Infinite Superiority, as the next chapters show.

Furthermore, Fleurbaey, Tungodden and Vallentyne (2009: 269-70) claim that a condition like Minimal Infinite Superiority "combines priority for the worst off with a limitation on aggregation that has been much discussed in the literature".[19] However, to the contrary, the condition does not imply "priority for the worse-off", as the next chapters show as well.

My justification for choosing the term "infinite superiority" is that it is analogous to a similar term in mathematics. In mathematics, for positive elements x and y, we say that x is *infinitely greater* than y (with respect to addition) if and only if, for all positive integers k, x is greater than k copies of y added together, i.e., for all $k > 0$, $x > kx$. Note that "infinitely greater than" is a relative expression in contrast to "infinitely great". In particular, it need not involve absolutely infinite quantities. Furthermore, in mathematics, the *Archimedean property* is said to state that no element is infinitely greater than another.

The notion of infinite superiority behaves in an analogous way. According to (axiological)

infinite superiority: a benefit b at level w is *infinitely better* than a benefit b' at a level w' (with respect to interpersonal aggregation) if and only if, for all positive integers k, that one person benefits by b at level w is better than that each of k people benefit by b' at level w'.

[17] See, in particular, Broome (2004: 28) who explicitly defines "lexical" in the sense I call "infinite superiority" and claims that, unlike Weak Leximin, Weak Maximin is not "lexical" which, as seen in the last section, is not true in this sense. Instead Weak Maximin, and unlike Weak Leximin, is not lexical in the sense that it does not involve lexical superiority as defined above.

[18] According to the Oxford Dictionary, "superior" means

Higher in degree, amount, importance, quantity, etc.; of greater value; of a higher grade or quality; greater, better, finer (than other persons or things of the same type). *Superior, Adj., n., and Adv.* (2019)

[19] See Fleurbaey, Tungodden and Vallentyne (2009: 258).

2.6 Restrictions

2.6.1 Population

Unless otherwise specified, I assume a

> **fixed finite population:** there is a fixed finite set of people existing in the outcomes,

i.e. no populations are considered whose members are infinitely many, as in so called "infinite ethics", or vary across outcomes, as is common in so called "population ethics".[20] I will almost entirely dodge these additional hard questions. However, in the following I will make a couple of comments on the relation between minimal infinite superiority and these areas of ethics.

As for infinite ethics, I believe that a version of minimal infinite superiority is plausible even for an *infinite* number of people, i.e.

(2.33) significantly benefiting one person matters more than slightly benefiting each of an infinite number of (otherwise) similarly situated or better off people.

And some moral principles I discuss can be easily extended to imply (2.33). But I will not discuss this issue further here.

As for population ethics, many choices affect who will come into existence, so called "different people choices". However, I focus on choices that do not affect who will come into existence, so called "same people choices". This is, arguably, a more severe limitation. But I am inclined to think that moral intuitions in same people choices are more reliable and, furthermore, that the problems these kinds of choices raise are hard enough for the scope of this thesis. And I am inclined to think that some of the lessons learned from same people cases may be fruitful for tackling different people cases (and vice versa). However, since the scope of this thesis is limited I will not be able to discuss the problems in population ethics at any length.

However, it is worth mentioning that there seems to be a close relationship between the problems I discuss and a family of cases from population ethics. One set of problems in population ethics concerns a subcategory of different people choices: choices which also affect the number of future individuals, so called "different number choices".[21] Consider Parfit's infamous

> **Repugnant Conclusion:**
>
> For any possible population of at least ten billion people, all with a very high quality of life, there must be some much larger imaginable population whose existence, if other things are equal, would be better, even though its members have lives that are barely worth living.[22]

The outcomes in the Repugnant Conclusion are illustrated in figure 2.6.

[20] The classic on population ethics is Parfit (1984: Part 4 Future Generations). That the population is *finite* is needed only for some views I discuss, for example those that appeal to the sum total of welfare in a simple fashion. For more on this Nelson (see e.g. 1991) and Vallentyne (1993).

[21] Parfit (1984: 356).

[22] Parfit (ibid.: 388).

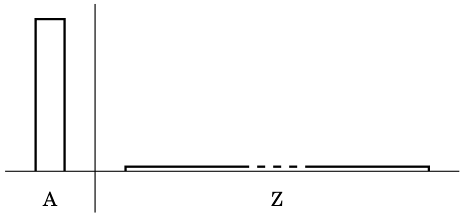

Figure 2.6 Illustration of the Repugnant Conclusion

(2.34) In outcome A, there exist at least 10 billion people with an arbitrarily high welfare level and they would not exist in the other outcome;

(2.35) in outcome Z, there exist a much larger number of different people with an arbitrarily low but positive welfare level and they would not exist in the other outcome.

As its name suggests, Parfit finds this conclusion repugnant, or even seemingly obviously false. The Repugnant Conclusion can be avoided by merely denying it by claiming that Z *not* better than A. However, many people, including Parfit, are inclined to make the stronger, opposite claim to the Repugnant Conclusion that

(2.36) A is *better* than Z.[23]

For example, Temkin claims that

> [m]ost people firmly believe that the Repugnant Conclusion is, indeed, repugnant. They believe that an outcome, A, of at least 10 billion people, all with a very high quality of life, would be better than an outcome, Z, with a large population all of whom have lives that are barely worth living, no matter *how many* people live in Z.[24]

Note that the rough axiological version of minimal infinite superiority, (1.5), implies (2.36), if we accept the (controversial) assumptions that

(2.37) people are benefited by existing with a positive welfare level, if they would not exist otherwise; and are benefited more so, the higher their welfare level;[25]

(2.38) people, existing in one of two outcomes, are otherwise similarly situated, if none of them would exists in the other outcome.[26]

By (2.34), (2.35), and (2.37),

(2.39) A is significantly benefiting people and Z is slightly benefiting people.

[23] Parfit (ibid.: 395).
[24] Temkin (2012: 35).
[25] See Parfit (1984: appendix G).
[26] Note that we already assume that non-welfare aspects don't make a moral difference (see subsection 2.1.2), so people are otherwise similarly situated in an outcome, i.e. apart from how well-off they are in that outcomes, if their state is the same in the other outcome, e.g. if they do not exist in the other outcome.

By (2.34), (2.35) and (2.38),

(2.40) The people who exist in A and the people who exist in Z are otherwise similarly situated.

Therefore, by (2.39), (2.40) and a repeated application of (1.5), once for each person in A, we get (2.36), i.e. A is better than Z.

2.6.2 Uncertainty

Another severe restriction for direct practical application is that uncertainty is excluded from the discussion. So, assume that each alternative leads with certainty to a particular outcome. For simplicity, I will *identify* an alternative with its outcome.

However, in reality an alternative has several possible outcomes each with a particular probability (although, arguably, these probabilities are sometimes neither determinate nor well-defined). I will at least give a hint on how the basic idea can be extended to cases of uncertainty. Two common opposing positions on how to deal with uncertainty are the *ex ante* approach and *ex post* approach. For simplicity, I will focus on the axiological versions of these approaches that can be represented by value functions. On the

***ex ante* approach:** an alternative is better than another if and only if the moral value of the *expected* utility distribution of outcomes of x is greater than that of y,

i.e. it, first, takes the probabilities into account by calculating the *expected utilities* for each person, i.e., for each person, the sum over the product of the probabilities of each possible outcome and the utility of the person in that outcome, and, second, applies the moral value function to these expected utilities. On the

***ex post* approach:** an alternative x is better than another alternative y if and only if the *expected* moral value of the utility distributions of outcomes of x is greater than that of y,

i.e. it, first, applies the moral value function to the outcomes and, second, takes the probabilities into account.

These positions have very different implications with respect to Minimal Infinite Superiority. Consider a probabilistic variation of Jones and the Transmitter from above. In the

Transmitter Lottery: either in

x**:** one viewer, chosen by a fair lottery among the viewers, gets her hand mashed and receives extremely painful electrical shocks for one hour;

or in

y**:** each viewer is for certain inconvenienced and her amusement interfered with.

So, in contrast to the original case, a fair lottery will determine who will be the unfortunate "Jones". Should this make a difference to which alternative is better (or permissible)?

According to the *ex post* position, there is no such difference. According to this position, we have to consider that, in x, in *each* outcome there will be someone who is tortured. Therefore, according to Minimal Infinite Superiority,

(2.41) y is better than x.

However, arguably, according to the *ex ante* position it makes a difference. In x, given the number of viewers is large enough, each viewer faces an extremely small probability of being tortured. Hence,

(2.42) for each viewer, the *expected* utility is *higher* in x than in y.

Minimal Infinite Superiority is silent in this case. But it might be appealed to

***Ex ante* Pareto:** if everyone has a higher expected utility in x than in y, then x is better than y.

Therefore, by (2.42),

(2.43) x is better than y.

And there are other possible approaches.[27] Unfortunately, space is too limited to discuss uncertainty in this thesis.

2.A Basics

Here are some basics used throughout the formal parts in formal statements and technical derivations. This section can be skipped by those familiar with the basic formal statements in social choice theory.

2.A.1 Logical connectives

Denote as follows the logical connectives.

$\neg$	negation, i.e. "$\neg p$" means "not p"
$\wedge$	conjunction, i.e. "$p \wedge q$" means "p and q"
$\vee$	disjunction, i.e. "$p \vee q$" means "p or q"
$\implies$	(material) conditional, i.e. "$p \implies q$" means "p only if q"
$\iff$	(material) equivalence, i.e. "$p \iff q$" means "p if and only if q"
$:\iff$	equivalence by definition, i.e. "$p :\iff q$" means "p if and only if q by definition"
$:=$	equality by definition, i.e. "$p := q$" means "p equals q by definition"

[27] See e.g. Fleurbaey and Voorhoeve (2013)

2.A.2 Sets

Denote *sets* by capital letters, A, B, C, etc. $A \subseteq B$ denotes that A is a *subset* of B. $a \in A$ denotes that a is an *element* of A. $\mathbb{Z}$ is the set of *integers*, i.e.

$$\mathbb{Z} := \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}.$$

$\mathbb{Z}_{>0}$ is the set of *positive integers*, i.e.

$$\mathbb{Z}_{>0} := \{i \in \mathbb{Z} : i > 0\} = \{1, 2, 3, \dots\}.$$

$\mathbb{Z}_{\geq 0}$ is the set of *non-negative integers*, i.e.

$$\mathbb{Z}_{>0} := \{i \in \mathbb{Z} : i \geq 0\} = \{0, 1, 2, 3, \dots\}.$$

$\mathbb{R}$ is the set of *real numbers*. $\mathbb{R}_{>0}$ and $\mathbb{R}_{\geq 0}$ are the sets of *positive* and *non-negative real numbers*, respectively. For $a, b \in \mathbb{R}$, $[a, b]$ is the *closed interval* between a and b, i.e.

$$[a, b] := \{r \in \mathbb{R} : a \leq r \leq b\},$$

and $]a, b[$ is the *open interval* between a and b, i.e.

$$]a, b[:= \{r \in \mathbb{R} : a < r < b\}.$$

$A \times B$ is the *binary Cartesian product* of A and B, i.e.

$$A \times B := \{(a, b) : a \in A, b \in B\}.$$

In particular, A^2 is the binary Cartesian product of A, i.e.

$$A^2 := A \times A := \{(a_1, a_2) : a_1, a_2 \in A\}.$$

More generally, for $n \in \mathbb{Z}_{>0}$, A^n is the n-ary Cartesian product of A, i.e.

$$A^n := \{(a_1, \dots, a_n) : a_1, \dots, a_n \in A\}.$$

In particular, $\mathbb{R}^n$ is the *n-dimensional (real) space*, i.e.

$$\mathbb{R}^n := \{(v_1, \dots, v_n) : v_1, \dots, v_n \in \mathbb{R}\}.$$

2.A.3 Relations

Let R be a (binary) *relation* on a set D, i.e. $R \subseteq D^2$. So, $(x, y) \in R$, for short xRy, means that x is R-related to y. For example, let D be the set of all bodies and $T \subseteq D^2$ be the binary relation "taller than". So, for two bodies x and y, xTy means "x is taller than y". To take another example, $> \subseteq \mathbb{R}^2$ is the binary relation "is greater than". So, for two numbers a and b, $a > b$ denotes "a is greater than b".

There are a number of properties of relations that will be relevant.

Irreflexivity: R satisfies *Irreflexivity* if, for all $x, x \in D$, x is not R-related to itself, i.e.

$$\neg(xRx).$$

Asymmetry: R satisfies *Asymmetry* if, for all $x, y \in D$, if x is R-related to y, then y is not R-related to x, i.e.

$$xRy \implies \neg(yRx).$$

Transitivity: R satisfies *Transitivity* if, for all $x, y, z \in D$, if x is R-related to y and y is R-related to z, then x is R-related to z, i.e.

$$xRy \wedge yRz \implies xRz.$$

Acyclicity: R satisfies *Acyclicity* if, for all $x_1, \ldots, x_m \in D$, if x_1 is R-related to x_2, $\ldots$, and x_{m-1} is R-related to x_m, then x_m is not R-related to x_1, i.e.

$$x_1 R x_2 \wedge \ldots \wedge x_{m-1} R x_m \implies \neg(x_m R x_1).$$

Completeness: R satisfies *Completeness* if, for all $x, y \in D$, x is R-related to y or y is R-related to x, i.e.

$$xRy \vee yRx.$$

Note that Acyclicity implies Asymmetry for the case of $m = 2$. Here are a couple more logical relations between these properties.

Proposition 2.1. *If R satisfies Asymmetry, then R satisfies Irreflexivity.*

Proof. Assume that R satisfies Asymmetry. And, for contradiction, that R does not satisfy Irreflexivity, i.e. for some $x \in D$,

(2.44) xRx.

Therefore, by Asymmetry,

(2.45) $\neg(xRx)$.

So, there is a contradiction. Hence, R satisfies Irreflexivity if R satisfies Asymmetry. □

Proposition 2.2. *If R satisfies Irreflexivity and Transitivity, then R satisfies Asymmetry.*

Proof. Assume that R satisfies Irreflexivity and Transitivity. For contradiction, assume that R violates Asymmetry, i.e.

(2.46) $xRy \wedge yRx$.

Therefore, by Transitivity,

(2.47) xRx.

But this contradicts Irreflexivity. Hence, R satisfies Asymmetry if R satisfies Irreflexivity and Transitivity.

□

Proposition 2.3. *If R satisfies Asymmetry and Transitivity, then R satisfies Acyclicity.*

Proof. Assume

(2.48) $x_1Rx_2 \wedge \ldots \wedge x_{m-1}Rx_m$.

Therefore, by (iterated application of) Transitivity,

(2.49) x_1Rx_m.

Therefore, by Asymmetry,

(2.50) $\neg(x_mRx_1)$.

Hence R satisfies Acyclicity. □

2.A.4 Functions

Let B^A denote the set of B-valued functions on A (or "functions from A to B"), i.e.

$$B^A := \{f \subseteq A \times B : \text{ for all } a \in A, \text{ there is exactly one } b \in B \text{ such that} (a,b) \in f\}.$$

Let $f \in B^A$. Denote $(a,b) \in f$ by $f(a) = b$. For example, $f \in \mathbb{R}^{\mathbb{R}_{\geq 0}}$ such that, for all $x \in \mathbb{R}_{\geq 0}$, $f(x) = \sqrt{x}$ is square root function.

Let $D \subseteq \mathbb{R}$, and ϕ be a real-valued function on D, i.e. $\phi \in \mathbb{R}^D$. ϕ is (weakly) *increasing* if and only if, for all $x, y \in D$,

$$x < y \implies \phi(x) \leq \phi(y).$$

ϕ is *strictly increasing* if and only if, for all $x, y \in D$,

$$x < y \implies \phi(x) < \phi(y).$$

ϕ is (weakly) *concave* if and only if, for all $x, y \in D$, and $r \in [0,1]$,

$$\phi(rx + (1-r)y) \geq r\phi(x) + (1-r)\phi(y).$$

ϕ is *strictly concave* if and only if, for all $x, y \in D$, with $x \neq y$, and $r \in]0,1[$,

$$\phi(rx + (1-r)y) > r\phi(x) + (1-r)\phi(y).$$

Proposition 2.4. *For all $\phi \in \mathbb{R}^{\mathbb{R}}$, $w, w' \in \mathbb{R}$, and $b \in \mathbb{R}_{>0}$, if*

1. *ϕ is strictly concave;*

2. $w < w'$,

then

$$\phi(w+b) - \phi(w) > \phi(w'+b) - \phi(w').$$

Proof. Let $\phi \in \mathbb{R}^{\mathbb{R}}$ strictly concave, $w, w' \in \mathbb{R}$, $b \in \mathbb{R}_{>0}$ with $w < w'$, and

(2.51) $r := 1 - \dfrac{b}{w' + b - w}$.

Then, by $w < w'$ and $b > 0$,

(2.52) $r \in \,]0,1[$

and

(2.53) $w + b = rw + (1-r)(w'+b)$;

(2.54) $w' = (1-r)w + r(w'+b)$.

Then, by ϕ is strictly concave,

(2.55) $\phi(w+b) > r\phi(w) + (1-r)\phi(w'+b)$;

(2.56) $\phi(w') > (1-r)\phi(w) + r\phi(w'+b)$.

Then

$$\begin{aligned}\phi(w+b) - \phi(w) &> r\phi(w) + (1-r)\phi(w'+b) - \phi(w)\\ &= (r-1)\phi(w) + (1-r)\phi(w'+b)\\ &= \phi(w'+b) - ((1-r)\phi(w) + r\phi(w'+b))\\ &> \phi(w'+b) - \phi(w').\end{aligned}$$

This concludes the proof.[28]

□

Proposition 2.5. *For* $\phi \in \mathbb{R}^{\mathbb{R}}$, $w, w' \in \mathbb{R}$, $b, b' \in \mathbb{R}_{>0}$, *and* $m, m' \in \mathbb{Z}_{>0}$, *if*

1. ϕ *is strictly increasing and strictly concave;*
2. $w < w'$*;*
3. $b \geq b'$*;*
4. $m \geq m'$*,*

then

$$m \cdot (\phi(w+b) - \phi(w)) > m' \cdot (\phi(w'+b') - \phi(w')).$$

[28] The idea of the proof is from Adler (2012: fn 82).

Proof. Follows from proposition 2.4 and that, by ϕ strictly increasing,

$$\phi(w+b) > \phi(w+b'). \qquad \square$$

Proposition 2.6. *For $\phi \in \mathbb{R}^{\mathbb{R}}$, $w, w' \in \mathbb{R}$, $b \in \mathbb{R}_{>0}$, if*

1. *ϕ is (weakly) concave;*
2. *$w < w'$,*

then

$$\phi(w+b) - \phi(w) \geq \phi(w'+b) - \phi(w').$$

Proof. Analogous to the proof of proposition 2.4. $\square$

2.A.5 Moral betterness and permissibility

Let O denote the set of possible *outcomes*. So, $x \in O$ means that x is an *outcome*. Let $\succ$ denote the (strict) moral *betterness* relation, i.e. $\succ \subseteq O^2$. Let $x, y \in O$. So, $x \succ y$ means that x is morally better than y. Denote x is *not* morally better than y, i.e. $\neg(x \succ y)$, by $x \not\succ y$. Assume throughout that $\succ$ satisfies Asymmetry, i.e.

(2.57) for all $x, y \in O$, $x \succ y \implies y \not\succ x$.

And, by proposition 2.1, $\succ$ satisfies Irreflexivity, i.e.

(2.58) for all $x \in O$, $x \not\succ x$.

There are three derived relations.[29] First, let $\sim$ denote the moral *(exactly) equally as good as* relation. x is (exactly) equally as good as y if and only if any z is better or worse than x if and only if it is correspondingly better or worse than y, i.e., for any $x, y \in O$,

$$x \sim y :\iff \forall z((x \succ z \iff y \succ z) \wedge (z \succ x \iff z \succ y)).$$

Actually, Broome (2004: 21) uses a slightly more complex definition: x is (exactly) equally as good as y if and only if *x is neither better nor worse than y and* any z is better or worse than x if and only if z is correspondingly better or worse than y, i.e.

(2.59) $$x \sim y \iff x \not\succ y \wedge y \not\succ x \wedge \forall z((x \succ z \iff y \succ z) \wedge (z \succ x \iff z \succ y))$$

However, given Irreflexivity, this definition can be reduced to the definition given above.

Proof. For contradiction, assume

[29] Following Broome (2004), I use the moral betterness relation as the fundamental relation but nothing substantial hinges on this choice here. Another common choice, e.g. in the economics literature, is the "at least as good as" relation.

(2.60) $x \sim y$;

(2.61) $x \succ y$.

Therefore, by (2.59) and (2.60),

(2.62) $\forall z(x \succ z \iff y \succ z)$.

Therefore, by (2.61),

(2.63) $y \succ y$.

But this contradicts Irreflexivity. The analogue holds for $y \succ x$ by the symmetry of $\sim$. □

Turn now to the second derived relation. Let $\succeq$ denote the moral *at least as good as* relation. x is at least as good as y if and only if x is better than y or x is equally as good as y, i.e.

$$x \succeq y :\iff x \succ y \lor x \sim y$$

Finally, let # denote the moral *incommensurability* relation. x is incommensurable with y if and only if x is neither better nor worse than nor equally as good as y, i.e.

$$x \# y :\iff x \not\succ y \land y \not\succ x \land x \not\sim y.$$

Let $X \subseteq O$ denote a set of *feasible outcomes*. So, $x \in X$ means that x is *feasible* (in X). Let $C(X) \subseteq X$ denote the set of morally *permissible outcomes* (in X). So, $x \in C(X)$ means that x is morally *permissible* (in X).

2.A.6 Population

Let $N(x)$ denote the *population* of outcome x, i.e. the set of people alive in x. So, $i \in N(x)$ means that i is an *individual* in X. Unless otherwise stated, assume a *fixed finite* population, i.e., for all $x \in O$, $N(x)$ is a finite number and, for all $y \in O$, $N(x) = N(y)$. For simplicity, for $n \in \mathbb{Z}_{>0}$ let

$$N(x) = N := \{1, \ldots, n\}.$$

In particular, n is the *population size*, i.e. the number of people alive.

2.A.7 Individual betterness and Absolute Comparability

For $i \in N$, $\succeq_i \in O^2$ is i's *individual at least as good as* relation. So, for $x, y \in O$, $x \succeq_i y$ means that x is *at least as good for i as y*.

For $i \in N$, $u_i \in \mathbb{R}^O$ is i's (representing) *utility function*, i.e., for all $x, y \in O$,

(2.64) $x \succeq_i y \iff u_i(x) \geq u_i(y)$.

For utility functions $u_1, \dots, u_n \in \mathbb{R}^O$, $\boldsymbol{u} := (u_i)_{i\in N} := (u_1, \dots, u_n) \in (\mathbb{R}^n)^O$ is the *utility distribution function*, i.e., for $x \in O$, $\boldsymbol{u}(x)$ is the *utility distribution* of x, i.e.

$$\boldsymbol{u}(x) = (u_i(x))_{i\in N} = (u_1(x), \dots, u_n(x)) \in \mathbb{R}^n.$$

There are different types of intra- and interpersonal comparability: for all $x, y \in O$ and $i, j \in N$, comparability of

***levels*:** $u_i(x) \geq u_j(x)$;

***units*:** $u_i(x) - u_i(y) \geq u_j(y) - u_j(x)$;

***ratios*:** $u_i(x)/u_i(y) \geq u_j(x)/u_j(y)$;

***zeros*:** $u_i(x) = 0 = u_j(x)$.[30]

Unless otherwise stated, assume

Absolute Comparability: for welfare distribution functions $\boldsymbol{u}, \boldsymbol{u}' \in (\mathbb{R}^n)^O$, $\boldsymbol{u}$ and $\boldsymbol{u}'$ contain the same information if, for all $x \in O$,

$$\boldsymbol{u}'(x) = \boldsymbol{u}(x),$$

i.e. $u_i'(x) = u_i(x)$ for all $i \in N$. Trivially, all different types of comparability are meaningful on Absolute Comparability. As mentioned above, I assume an Absolute Comparability for simplicity. Another, more common, assumption is that welfare is measurable on a ratio-scale and fully interpersonally comparable, for short

Ratio Comparability: for welfare distribution functions $\boldsymbol{u}, \boldsymbol{u}' \in (\mathbb{R}^n)^O$, $\boldsymbol{u}$ and $\boldsymbol{u}'$ contain the same information if there exists a increasing linear transformation between them, i.e. there exists $a \in \mathbb{R}_{>0}$ such that, for all $x \in O$,

$$\boldsymbol{u}'(x) = a\boldsymbol{u}(x),$$

i.e. $u_i'(x) = a u_i(x)$ for all $i \in N$.[31]

Proposition 2.7. *Ratio Comparability renders the intra- and interpersonal comparability statements of* levels, units, ratios, *and* zeros, *meaningful,*

i.e. their truth or falsity is unchanged by the transformation.

Proof. Let $x, y \in O$, $i, j \in N$, and $a \in \mathbb{R}_{>0}$,
levels:

$$u_i(x) \geq u_j(y)$$

[30] Comparability of levels and units is also called ordinal and interval (or difference) comparability, respectively.

[31] See e.g. Blackorby, Bossert and Donaldson (2005: 113-4). The classification of scales (or levels) of measurement is by Stevens (1946); for an extended discussion see Roberts (1985: 64-5).

$$\iff \quad au_i(x) \geq au_j(x);$$

units:

$$\begin{aligned} & u_i(x) - u_i(y) \geq u_j(y) - u_j(x) \\ \iff \quad & au_i(x) - au_i(y) \geq au_j(y) - au_j(x); \end{aligned}$$

ratios:

$$\begin{aligned} & u_i(x)/u_i(y) \geq u_j(x)/u_j(y) \\ \iff \quad & au_i(x)/au_i(y) \geq au_j(x)/au_j(y); \end{aligned}$$

zeros:

$$\begin{aligned} & u_i(x) = 0 = u_j(x) \\ \iff \quad & au_i(x) = 0 = au_j(x). \quad \square \end{aligned}$$

2.B The trade-off conditions and the simple classic principles

Here is a precise statement of the simple general form of many conditions, see figure 2.2. It can be formalized using a relation $B \subseteq O^2 \times (\mathbb{Z}_{>0} \times \mathbb{R} \times \mathbb{R}_{>0})^2$ with

(2.65) $B\big(x, y, \langle m, w, b\rangle, \langle m', w', b'\rangle\big) :\iff$ for all $M, M' \subseteq N$,

B1. $|M| \geq m$, and, for all $i \in M$,

$$u_i(x) \leq w \text{ and } u_i(y) - u_i(x) \geq b;$$

B2. $|M'| \leq m'$, and, for all $i \in M'$,

$$u_i(y) \geq w' \text{ and } u_i(x) - u_i(y) \leq b';$$

B3. for all $i \in N - (M \cup M')$,

$$u_i(x) = u_i(y).$$

With this the axiological version of the simple general form can be concisely stated as, for $x, y \in O$, $m, m' \in \mathbb{Z}_{>0}$, $w, w' \in \mathbb{R}$, and $b, b' \in \mathbb{R}_{>0}$,

$$B\big(x, y, \langle m, w, b\rangle, \langle m', w', b'\rangle\big) \implies y \succ x,$$

and the deontic version as, for $X \subseteq O$,

$$B\big(x, y, \langle m, w, b\rangle, \langle m', w', b'\rangle\big) \wedge x, y \in X \implies x \notin C(X).$$

So, all conditions discussed so far can be concisely stated.

Minimal Infinite Superiority: for all $w, w' \in \mathbb{R}$, and $b \in \mathbb{R}_{>0}$, there is $b' \in \mathbb{R}_{>0}$ such that $w < w'$ such that, for all $m' \in \mathbb{Z}_{>0}$, and $x, y \in O$,

$$B\big(x, y, \langle 1, w, b\rangle, \langle m', w', b'\rangle\big) \implies y \succ x.$$

Deontic Minimal Infinite Superiority: for all $w, w' \in \mathbb{R}$, and $b \in \mathbb{R}_{>0}$, there is $b' \in \mathbb{R}_{>0}$ such that $w < w'$ such that, for all $m' \in \mathbb{Z}_{>0}$, and $x, y \in O$,

$$B\big(x, y, \langle 1, w, b\rangle, \langle m', w', b'\rangle\big) \wedge x, y \in X \implies x \notin C(X).$$

Unlimited Finite Superiority: for all $x \in O$, $m' \in \mathbb{Z}_{>0}$, $w, w' \in \mathbb{R}$, and $b, b' \in \mathbb{R}_{>0}$, there exists $m \in \mathbb{Z}_{>0}$, such that, for all $y \in O$,

$$B\big(x, y, \langle m, w, b\rangle, \langle m', w', b'\rangle\big) \implies y \succ x.$$

Priority: for all $w, w' \in \mathbb{R}$ such that $w < w'$, $b, b' \in \mathbb{R}_{>0}$ such that $b \geq b'$, $m, m' \in \mathbb{Z}_{>0}$ such that $m \geq m'$, and $x, y \in O$,

$$B\big(x, y, \langle m, w, b\rangle, \langle m', w', b'\rangle\big) \implies y \succ x.$$

Infinite Priority: for all $w, w' \in \mathbb{R}$ such that $w < w'$, $b, b' \in \mathbb{R}_{>0}$, $m, m' \in \mathbb{Z}_{>0}$, and $x, y \in O$,

$$B\big(x, y, \langle m, w, b\rangle, \langle m', w', b'\rangle\big) \implies y \succ x.$$

Minimal Finite Superiority: for all $x \in O$, $w, w' \in \mathbb{R}$, there exist $b, b' \in \mathbb{R}_{>0}$ such that, for all $y \in O$,

$$B\big(x, y, \langle 1, w, b\rangle, \langle 1, w', b'\rangle\big) \implies y \succ x.$$

Let us end with a precise statement of the simple classic principles and show how they satisfy or violate the conditions.

Some moral principles can be represented by a (real valued) *value function*, $v \in \mathbb{R}^O$, i.e., for $x, y \in O$, $v(x) \in \mathbb{R}$ is the *value* of x, and if the value of x is greater than the value of y, then x is better than y, i.e.,

(2.66) for all $x, y \in O$,

$$v(x) > v(y) \implies x \succ y.$$

Analogously, for the deontic case: if the value of outcome x is greater than the value of outcome y and x and y are feasible, then y is impermissible, i.e.,

(2.67) for all $x, y \in O$,

$$v(x) > v(y) \wedge x, y \in X \implies y \notin C(X).$$

According to the

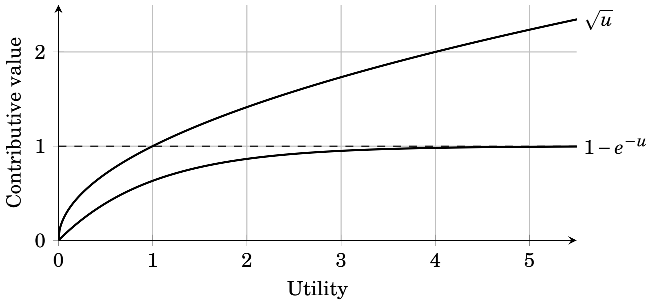

Figure 2.7 Priority-weight function $\sqrt{u}$ and $1-e^{-u}$

Weak Simple Total Principle: for all $x \in O$,

$$\begin{aligned} v(x) &= \sum_{i \in N} u_i(x) \\ &:= u_1(x) + u_2(x) + \ldots + u_n(x). \end{aligned}$$

Therefore, according to the Weak Simple Total Principle and by (2.66), for all $x, y \in O$,

$$\sum_{i \in N} u_i(x) > \sum_{i \in N} u_i(y) \implies x \succ y,$$

and, according to the Deontic Simple Total Principle and by (2.67), for all $x, y \in O$,

$$\sum_{i \in N} u_i(x) > \sum_{i \in N} u_i(y) \wedge x, y \in X \implies y \notin C(X).$$

Next, for $U \subseteq \mathbb{R}$, according to the

Weak Simple Priority Total Principle: there exists a strictly increasing and strictly concave $\phi \in \mathbb{R}^U$ such that, for all $x \in O$,

$$v(x) = \sum_{i \in N} \phi(u_i(x)).$$

Some common simple examples of this principle need a restriction of the domain U of ϕ, i.e. on possible utilities. For example, the square root function is restricted to *positive* utilities, i.e. $U = \mathbb{R}_{>0}$ and $\phi(u) = \sqrt{u}$ for all $u \in U$figure 2.7. Figure 2.7 shows the square root function as the strictly increasing and strictly concave function $\phi(u) = 1 - e^{-u}$ with upper bound (of 1).

The relationship between, on the one hand, the Weak Simple Total Principle and the Weak Simple Priority Total Principle and, on the other, the conditions shows

Proposition 2.8. *For all $\phi \in \mathbb{R}^U$, $x, y \in O$, $m, m' \in \mathbb{Z}_{>0}$, $w, w' \in \mathbb{R}$, and $b, b' \in \mathbb{R}_{>0}$, if*

1. *ϕ is (weakly) increasing and (weakly) concave;*
2. $m \cdot \big(\phi(w+b) - \phi(w)\big) > m' \cdot \big(\phi(w'+b') - \phi(w')\big)$*;*

3. $B\big(x, y, \langle m, w, b\rangle, \langle m', w', b'\rangle\big)$,

then

$$\sum_{i\in N} \phi(u_i(y)) > \sum_{i\in N} \phi(u_i(x)).$$

Proof. Let $\phi \in \mathbb{R}^U$, $x, y \in O$, $m, m' \in \mathbb{Z}_{>0}$, $w, w' \in \mathbb{R}$, and $b, b' \in \mathbb{R}_{>0}$ and 1 to 3. It is to be shown that

$$\sum_{i\in N} \phi(u_i(y)) > \sum_{i\in N} \phi(u_i(x)). \tag{2.68}$$

First, note that

$$\sum_{i\in N} \phi(u_i) = \sum_{i\in M} \phi(u_i) + \sum_{i\in M'} \phi(u_i) + \sum_{i\in N-(M\cup M')} \phi(u_i).$$

By B3, for all $i \in N-(M\cup M)'$, $u_i(x) = u_i(y)$, hence this is equivalent to

$$\sum_{i\in M} \big(\phi(u_i(y)) - \phi(u_i(x))\big) > \sum_{i\in M'} \big(\phi(u_i(x)) - \phi(u_i(y))\big). \tag{2.69}$$

Consider the left side of this inequation. By B1, for all $i \in M$, $u_i(y) \geq u_i(x) + b$, and hence, by ϕ is (weakly) increasing,

$$\text{for all } i \in M,\ \phi(u_i(y)) \geq \phi(u_i(x) + b). \tag{2.70}$$

Therefore,

$$\sum_{i\in M} \big(\phi(u_i(y)) - \phi(u_i(x))\big) \geq \sum_{i\in M} \big(\phi(u_i(x) + b) - \phi(u_i(x))\big). \tag{2.71}$$

By B1, for all $i \in M$, $u_i(x) \leq w$, and hence, by ϕ is (weakly) concave and proposition 2.6,

$$\text{for all } i \in M,\ \phi(u_i(x) + b) - \phi(u_i(x)) \geq \phi(w + b) - \phi(w). \tag{2.72}$$

Therefore,

$$\sum_{i\in M} \big(\phi(u_i(y)) - \phi(u_i(x))\big) \geq m\big(\phi(w + b) - \phi(w)\big). \tag{2.73}$$

Symmetrically, by B2,

$$\sum_{i\in M'} \big(\phi(u_i(x)) - \phi(u_i(y))\big) \leq m'\big(\phi\big(w' + b'\big) - \phi\big(w'\big)\big). \tag{2.74}$$

Therefore, by $m \cdot \big(\phi(w + b) - \phi(w)\big) > m' \cdot \big(\phi\big(w' + b'\big) - \phi\big(w'\big)\big)$,

$$\sum_{i\in M} \big(\phi(u_i(y)) - \phi(u_i(x))\big) > \sum_{i\in M'} \big(\phi(u_i(x)) - \phi(u_i(y))\big). \tag{2.75}$$

This concludes the proof. □

That the Weak Simple Total Principle satisfies Unlimited Finite Superiority follows for $\phi(x) = x$, and hence

$$m \cdot \big(\phi(w+b) - \phi(w)\big) = m \cdot b.$$

That the Weak Simple Priority Total Principle implies Unlimited Finite Superiority and Priority follows by ϕ is strictly concave and strictly increasing, and proposition 2.5.

Note that the Weak Simple Priority Total Principle only involves *non-relational* finite priority for the worse-off, see footnote (11). And principles that involve *relational* finite priority actually capture Minimal Infinite Superiority, as will be shown in section 3.B.

Next, according to

Weak Maximin: for all $x \in O$,

$$v(x) = \min_{i \in N} u_i(x).$$

However, not all principles can be expressed with value functions, e.g. Weak Leximin. First, define

lexical superiority: for $\boldsymbol{v} = (v_1, \ldots, v_n), \boldsymbol{v}' = \big(v'_1, \ldots, v'_n\big) \in \mathbb{R}^n$,

$$\begin{aligned} \boldsymbol{v} >_{\text{lex}} \boldsymbol{v}' :\Longleftrightarrow\ & \text{there exists } j \in N \text{ such that, for all } i < j, \\ & v_i = v'_i \text{ and } v_j > v'_j. \end{aligned}$$

Next, for $\boldsymbol{v} \in \mathbb{R}^n$, let

$$\boldsymbol{v}_{[]} := \big(v_{[1]}, \ldots, v_{[n]}\big) \in \mathbb{R}^n$$

be a reordering (or permutation) of $\boldsymbol{v}$ with $v_{[i]} \le v_{[i+1]}$ for $i = 1, \ldots, n-1$. In particular,

$$\boldsymbol{u}_{[]}(x) = \big(u_{[1]}(x), \ldots, u_{[n]}(x)\big) \in \mathbb{R}^n$$

is an ordered list of people's utilities in x re-ordered from the lowest to the highest utility According to

Weak Leximin: for any $x, y \in O$,

$$\boldsymbol{u}_{[]}(x) >_{\text{lex}} \boldsymbol{u}_{[]}(y) \implies x \succ y,$$

i.e., if there exists $j \in N$ such that, for all $i < j$, $u_{[i]}(x) = u_{[i]}(y)$ and $u_{[j]}(x) > u_{[j]}(y)$, then $x \succ y$. That Weak Leximin cannot be represented by a real-valued function shows

Proposition 2.9. *lexical superiority cannot be represented by a real-valued function, i.e. there is no $f \in \mathbb{R}^{\mathbb{R}^n}$ such that, for all $\boldsymbol{v}, \boldsymbol{v}' \in \mathbb{R}^n$*

$$\boldsymbol{v} >_{lex} \boldsymbol{v}' \iff f(\boldsymbol{v}) > f\big(\boldsymbol{v}'\big).$$

Proof. For contradiction, assume that the lexicographic ordering can be represented by $f \in \mathbb{R}^{\mathbb{R}^2}$, i.e.

(2.76) for all $x_1, x_2, y_1, y_2 \in \mathbb{R}$,

$$(x_1, x_2) >_{\text{lex}} (y_1, y_2) \iff f(x_1, x_2) > f(y_1, y_2).$$

It holds that,

(2.77) for all $x_1 \in \mathbb{R}$, $(x_1, 1) >_{\text{lex}} (x_1, 0)$.

Therefore, by (2.76),

(2.78) for all $x_1 \in \mathbb{R}$, $f(x_1, 1) > f(x_1, 0)$.

Furthermore, it holds that

(2.79) for all $x_1, y_1 \in \mathbb{R}$, $[x_1 > y_1 \implies (x_1, 0) >_{\text{lex}} (y_1, 1)]$.

Therefore, by (2.76),

(2.80) for all $x_1, y_1 \in \mathbb{R}$, $[x_1 > y_1 \implies f(x_1, 0) > f(y_1, 1)]$.

Between any two distinct real numbers there is a rational number. Therefore, by (2.78),

(2.81) for all $x_1 \in \mathbb{R}$, there is $r_{x_1} \in \mathbb{Q}$ such that $f(x_1, 1) > r_{x_1} > f(x_1, 0)$.[32]

Define a function $r \in \mathbb{Q}^{\mathbb{R}}$ such that,

(2.82) for all $x_1 \in \mathbb{R}$, $r(x_1) := r_{x_1} \in \mathbb{Q}$.

Therefore, by (2.80) and (2.81),

(2.83) for all $x_1, y_1 \in \mathbb{R}$, $x_1 > y_1 \implies r(x_1) > f(x_1, 0) > f(y_1, 1) > r(y_1)$.

Therefore, by contraposition,

(2.84) for all $x_1, y_1 \in \mathbb{R}$, $r(x_1) = r(y_1) \implies x_1 = y_1$,

i.e. r is injective.[33] But this is a contradiction because the real numbers are *uncountable* (infinite), i.e. there is *no* injective function from the real numbers to the natural numbers, and the rational numbers are only *countable* (infinite), i.e. there *is* an injective function from the rational numbers to the natural numbers.[34] □

[32] $\mathbb{Q}$ is the set of *rational numbers*, i.e.

$$\mathbb{Q} = \left\{ \frac{p}{q} : p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\}.$$

[33] So, an injective function is a one-to-one function—not to be confused with a surjective function, i.e. a one-to-one *correspondence*.

[34] The proof is based on Debreu (1959: 72-3).

Note that

$$u_{[1]}(x) = \min_{i \in N} u_i(x). \quad (2.85)$$

So, the value function of Weak Maximin can be restated as

$$v(x) = u_{[1]}(x).$$

3

The Moderate Trade-offs Paradox

3.1 Overview

In "On the Possibility of Nonaggregative Priority for the Worst Off", Fleurbaey, Tungodden and Vallentyne (2009) show that there is no moral theory that satisfies both minimal infinite and finite superiority together with other seemingly plausible conditions.[1] In section 3.2, I introduce their three background conditions. In section 3.3 and section 3.4, I present two paradoxes, that I call the First Moderate Trade-offs Paradox and Third Moderate Trade-offs Paradox, respectively, illustrating a proof of the axiological impossibility results, and show how they can be strengthened. In section 3.5, I explain how these results can be translated into analogous deontic impossibility results. In section 3.6, I argue that one common type of response to such results, merely giving up structural conditions on the moral betterness relation or permissibility, does not solve the paradox.

Table 3.1 provides an overview of the impossibility and possibility results by Fleurbaey, Tungodden and Vallentyne—3, 5 and 7—my variations of some of their results—1*, 3*, 5*, and 8*—and a corollary of a theorem by Blackorby, Bossert and Donaldson (2005)—4.22*. Additional background conditions for all results are, at least for simplicity, Welfarism, Domain Richness, and Absolute Comparability, to be discussed in section 3.2. In this chapter, Result 1* to Result 5*, are discussed. The other results are discussed in later chapters. There are several impossibility results discussed in the literature sometimes under the heading of "spectrum arguments".[2] However, the impossibility results by Fleurbaey, Tungodden and Vallentyne are particularly important since they assume solely minimal substantive trade-off conditions—compatible with non-aggregation—and only two minimal structural conditions, Acyclicity or No Prohibition Dilemmas and Replication Invariance—the latter of which can be dropped as will be shown in subsection 3.4.3.

[1] See also Fleurbaey and Tungodden (2010)'s cousin paper "The Tyranny of Non-Aggregation versus the Tyranny of Aggregation in Social Choices" containing a number of closely related formal results and proofs.

[2] See e.g. Temkin (2012: ch. 9).

Table 3.1 Overview of the results and their conditions. “A” and “D” indicate axiological and deontic results, respectively. “✗” and “✓” mark conditions of impossibility and possibility results, respectively. “~~✓~~”, “[✓]”, and “~~✓~~?” mark a condition that the principle suggested as a possibility proof does not satisfy, satisfies instead, and likely does not satisfy (but is not proved in this thesis), respectively.

	Results							
	1* (A)	3 (A)	3* (A)	5 (D)	5* (D)	7 (D)	8* (D)	4.22* (A)
Alpha/Binary Independence							✗	
Transitivity								✗
Acyclicity/No Prohibition Dilemmas	✗	✗	✗	✗	✗	✓	✗	
Continuity								✗
Replication Invariance	✗	✗		✗		~~✓~~?	✗	✗
Same-People Independence								✗
Strong Pareto						✓		✗
Strong Welfare Impartiality								✗
Pigou-Dalton		✗		✗		~~✓~~		
Non-Anti Pigou-Dalton			✗		✗			
Two-Option Pigou-Dalton						[✓]		
Two-Option Non-Anti Pigou-Dalton							✗	
Minimal Finite Superiority	✗							
Ultra Minimal Finite Superiority		✗		✗		~~✓~~		
Ultra Minimal Finite Superiority*			✗		✗			
Two-Option Ultra Minimal Finite Superiority*						[✓]	✗	
Minimal Infinite Superiority	✗							
Super Ultra Minimal Infinite Superiority		✗		✗				
Super Ultra Minimal Infinite Superiority*			✗		✗			✗
Two-Option Minimal Infinite Superiority						✓		
Two-Option Super Ultra Minimal Infinite Superiority*							✗	

3.2 Background assumptions

First, according to

> **Welfarism:** the moral goodness of alternatives is determined solely on the basis of their welfare distributions.[3]

Welfarism is a very strong assumption but, as the authors note, it is just a simplifying assumption for their results. For example, it could be removed if an *other things equal* clause is added to the conditions. Alternatively, the set of cases could be restricted to those where welfare is the only, or at least the most, important consideration.[4]

Second, according to

> **Domain Richness:** for any logically possible welfare distribution with a finite number of individuals and non-negative welfare levels, there is an alternative that generates that distribution.[5]

According to this assumption, for example, we should not only consider alternatives that one might face in practice. In particular, we should consider possible alternatives that involve a huge number of people and extremely high welfare levels which might be practically or psychologically impossible.

I think that even cases that are not practically possible and are hard to imagine in detail can serve as test cases for moral principles. But, it is important to note that one can reject Domain Richness and still prove the result because one needs only sequences of alternatives that generates the paradoxes. And I am inclined to think there are examples of the conditions that do not necessarily involve extremely high welfare levels in order for the conditions to hold and the result to follow. I will use such an example below.

At least for the sake of argument, we can assume both Domain Richness and Welfarism in this chapter.

The final assumption is Absolute Comparability.[6] Fleurbaey, Tungodden and Vallentyne (2009: 259) assume that welfare is "fully measurable and interpersonally comparable". However, they do not spell out the assumption in detail.[7] They claim to assume Absolute Comparability only "for the sake of argument". To defend this claim, they write,

> [Absolute Comparability] may seem like a strong assumption, but in the present context it avoids burdening our analysis with informational constraints. The assumption that benefits are so measurable and comparable does not entail that such information is relevant for the moral assessment of options. The assumption is simply that such information is available. This ensures that no principle of moral goodness is ruled out merely on the grounds that it presupposes that benefits are measurable or comparable in ways

[3] Fleurbaey, Tungodden and Vallentyne (2009: 259) use an assumption similar to Welfarism involving what they call "benefits" which is a generalization of welfare. For example, benefits might be *opportunities* to welfare. For the sake of simplicity, I will stick to welfare.

[4] See subsection 2.1.2.

[5] See Fleurbaey, Tungodden and Vallentyne (ibid.: 260).

[6] Introduced in the previous chapter, subsection 2.1.3.

[7] However, in personal communication with Arrhenius, Fleurbaey and Tungodden confirmed that they indeed have an absolute scale in mind.

that they are not. Our main argument is that no moral theory satisfies all of certain seemingly plausible conditions—even if benefits are fully measurable and interpersonally comparable. In this context, the measurability and comparability assumptions should make it easier to find a satisfactory theory and thus strengthen the significance of our conclusions.[8]

The last claim holds because the assumption of Absolute Comparability rather than a more limited form of measurability and interpersonal comparability makes possible the formulation of more theories. For example, assume, instead of Absolute Comparability, that there is only *ordinal* intra- and interpersonal comparability of welfare, i.e. only welfare levels, but not differences, are comparable both within and across people. Ordinal comparability makes it meaningful to talk about worse-off and better-off (and hence worst-off and best-off) people but not about differences between welfare levels and sums of welfare. Therefore, it makes meaningful the formulation of Weak Leximin (and Weak Maximin), because it refers only to the worse off (and worst off) people, but not the Weak Simple Total Principle (and the Weak Simple Priority Total Principle) because it refer to sums of welfare (and sums of priority-weighted welfare). In contrast, Absolute Comparability makes the formulation of the latter theories meaningful as well.[9] Their impossibility results hold even under assumptions weaker than Absolute Comparability but the results are not weakened by the assumption. I will postpone further discussion of this assumption until chapter 4 when we have a better understanding how it features in the moderate trade-offs paradoxes. Absolute Comparability applies only to *welfare*, to what is good *for* people, but not to what is *morally* good. For example, this does not rule out theories that imply that there are *morally incommensurable* alternatives, i.e. alternatives that are neither better nor equally good.[10]

3.3 The First Moderate Trade-offs Paradox

According to a variation of Fleurbaey, Tungodden and Vallentyne's simplest impossibility result,

> **Result 1*:** no moral betterness relation satisfies Minimal Infinite Superiority, Minimal Finite Superiority, Replication Invariance and Acyclicity.[11]

The proof consists in showing the inconsistency between seemingly plausible moral betterness rankings implied by the conditions of Result 1* in a sequence of alternatives. I call the resulting judgements in the sequence the First Moderate Trade-offs Paradox. Consider the

[8] Fleurbaey, Tungodden and Vallentyne (2009: 259).

[9] See also Blackorby, Bossert and Donaldson (2005: 114).

[10] See subsection 2.A.5 for a formal definition of incommensurability.

[11] This is result is in one way stronger than their "Result 1" since the "Weak Pareto" condition in their result is redundant. That this condition is redundant in Result 1 can be seen from the fact that their Result 2 does not employ this condition and that only weaker minimal infinite and finite superiority conditions are assumed, see Fleurbaey, Tungodden and Vallentyne (2009: 265). I will not discuss their *Result 2* because the principles discussed later on satisfy the stronger conditions of Result 1* or violate the even weaker conditions of 3.4 below.

First Moderate Trade-offs Paradox Sequence: illustrated in figure 3.1,

i.e., in x_0, m people are at welfare level 16, and m at 81; in x_1, person 1 is at 16.1, $m-1$ are at 16, and m at 80.999; in x_2, persons 1 and 2 are at 16.1, $m-2$ are at 16, and m at 80.998; ...; in x_m, m are at 16.1, and m at 49. So, in each step the m better-off people lose the same amount of utility, .001. Hence the sequence involves two times m, i.e. 64 000, people.[12] But the number of people involved can be arbitrarily increased, e.g. by decreasing the amount of utility lost by the m better-off people in each step.

It will be shown that there is a sequence of successively morally better alternatives from one alternative to the same, called a *cycle*. More precisely, there is a cycle involving x_0, x_1, x_2, ..., and x_m. But this violates

Acyclicity: for any outcomes x_1, x_2, ..., and x_n, if x_1 is morally better than x_2, x_2 is morally better than x_3, ..., and x_{n-1} is morally better than x_n, then x_n is not morally better than x_1.

This condition is a *structural* condition on betterness making a judgement about betterness independent of non-moral claims. Fleurbaey, Tungodden and Vallentyne (2009) assume Acyclicity as a general assumption of practical rationality.[13] In section 3.6, I will discuss rejecting Acyclicity as a way to solve the paradox. However, it turns out that even if Acyclicity is rejected and hence the inconsistency avoided, the paradox lacks a satisfactory solution.

Under assumptions from before, (2.4) and (2.5), according to Minimal Infinite Superiority,

(3.1) x_i is better than x_{i-1}, for $i = 1, \ldots, m$.

The ranking of the first and last alternative in the sequence, x_0 and x_m, also involves an additional structural condition. According to

Replication Invariance: for any alternatives x and y, and any positive integer k, x is at least as good as y if and only if a k-replication of x is at least as good as a k-replication of y,

where a *k-replication* of an alternative x is an alternative with, for each person in x, $k-1$ added people at the same welfare level as that person. In particular, the population of the k-replication of x will have k-times as many people as x. For example, consider table 3.2. In each alternatives x and y, there are 2 people; in x, person 1 is at welfare level 16.1 and person 2 at 49; in y, person 1 at 16 and person 2 at 81. In each 3-replication of x and y, there are 3 times the people in x (and y), i.e. there are 6 people; two added people for each person in x and y. Note that the replication has to be symmetric in the 3-replication of x and y, e.g. the welfare person 1 has in x and y is the same that person 3 has in x and y.[14]

Table 3.3 illustrates Replication Invariance. Consider an island, called Island 1,

[12] There are 64 000 people because there are $m = 32000$ (= (81 − 49)/.001) steps in the sequence.

[13] And it is, given Asymmetry, weaker than Transitivity. For the proof of this claim see proposition 2.3, in subsection 2.A.3.

[14] See Fleurbaey, Tungodden and Vallentyne (ibid.: 264). Dalton (1920: 357) proposed this condition first in the context of income inequality, see Blackorby, Bossert and Donaldson (2005: 88).

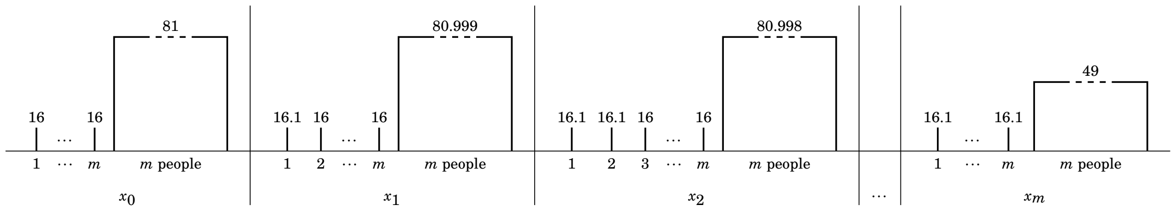

Figure 3.1 Illustration of the First Moderate Trade-offs Paradox

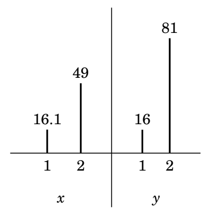

Figure 3.2 Illustration of Minimal Finite Superiority (repeated figure 2.5 on page 25)

Table 3.2 Example of welfare distributions of a k-replication ($k = 3$)

Person	x	y	3-replication of x	3-replication of y
1	16.1	16	16.1	16
2	49	81	49	81
3	–	–	16.1	16
4	–	–	49	81
5	–	–	16.1	16
6	–	–	49	81

Table 3.3 Illustration of Replication Invariance

	Island 1	Island 2	⋯	Island k
number of people	n	n	⋯	n
ranking	x is better than y			
number of people	$k \cdot n$			
ranking	k-replication of x is better than k-replication of y			

with n people. Assume that, if only the people on the island exist,

(3.2) alternative x is morally better than y.

Now assume that there were $k-1$ other islands, Island 2 to k, which each has the same number of people as Island 1. Therefore, by (3.2) and Replication Invariance, if the people on all the islands exist, then to have the welfare distribution of x on each of the k islands is morally better than to have the welfare distribution of y on each of the k islands, i.e.

(3.3) the k-replication of x is morally better than the k-replication of y.

All the simple classic principles satisfy Replication Invariance.

Consider again the alternatives from figure 2.5 (repeated in figure 3.2). Under the same assumptions as before, according to Minimal Finite Superiority, (2.30) y is better than x. Next, note that

(3.4) x_m and x_0 are m-replications of x and y, respectively.

Therefore, by Replication Invariance,

(3.5) x_0 is better than x_m.[15]

Thus, by (3.1) and (3.5), we arrive at the

First Moderate Trade-offs Paradox: there is a number of people m such that

1. x_1 is better than x_0, x_2 is better than x_1, …, x_m is better than x_{m-1};

[15] The results hold also with a slightly weaker version of Replication Invariance where "morally at least as good as" is replaced by "morally better than".

2. x_0 is better than x_m.

Therefore,

(3.6) there is a number of people m such there is a betterness cycle involving x_0, x_1, x_2, ..., and x_m.

But (3.6) violates Acyclicity. This concludes the illustration of the proof of impossibility Result 1*. The central element in the general proof of Result 1* is to show that the benefit to the better-off people in x_m compared to x_0 can be arbitrarily large in order to function as benefit b in Minimal Finite Superiority. This can be achieved by choosing the level of better-off people in x_0 high enough, the number of people m large enough, and, hence, the number of steps in the sequence large enough, so that along the sequence a large enough difference in welfare is achieved. In this way, the benefit to the better-off people in x_m compared x_0 can be arbitrarily large.

It might be objected that the conditions involved in impossibility Result 1* are too strong. However, as is shown next, there are impossibility results with weaker conditions.

3.4 The Third Moderate Trade-offs Paradox

According to Fleurbaey, Tungodden and Vallentyne (2009)'s impossibility result which involves very weak infinite and finite superiority conditions,

> **Result 3:** no moral betterness relation satisfies Super Ultra Minimal Infinite Superiority, Ultra Minimal Finite Superiority, Pigou-Dalton, Replication Invariance and Acyclicity.[16]

The proof consists in showing the inconsistency of seemingly plausible moral betterness rankings implied by the conditions of Result 3 in a sequence of alternatives. This inconsistency I call the Third Moderate Trade-offs Paradox. The basic idea is as follows. For convenience, I break the paradox down into two subsequences of alternatives.[17] Consider the

> **Third Moderate Trade-offs Paradox Sequence:** illustrated in figure 3.3,

i.e., in z_0, the m and the $m \cdot m''$ people are at welfare level 16, and everyone else is at 81; in z_m, the m people are at 25, the $m \cdot m''$ people are at 16, and everyone else is at 49; in $z_{m+m \cdot m''}$, the m and the $m \cdot m''$ people are at 16.1, and everyone else is at

[16] See Fleurbaey, Tungodden and Vallentyne (2009: 270) where the conditions Super Ultra Minimal Infinite Superiority and Ultra Minimal Finite Superiority are called "Super Ultra Minimal Nonaggregative Priority" and "Ultra Minimal Aggregation". I have justified my different labelling in section 2.5.

[17] This differs from the original proof of Result 3 by Fleurbaey et al. insofar as they use a double step sequence of application of Super Ultra Minimal Infinite Superiority and Pigou-Dalton where I separate those steps into two different successive sequences. Unfortunately, their original article does not contain the proof and the web link to their proof provided therein is not accessible anymore. I am working in part on the basis of a proof sent to me by Fleurbaey in personal communication and Fleurbaey and Tungodden (2010)'s cousin paper.

49.Here and in the following, nothing hinges on this specific choice of numbers but their choice will make calculations simpler later on. A general version is presented in section 3.A.

Here and in the following, nothing hinges on the choice of these specific numbers but their choice will make calculations simpler later on. A general version is presented in section 3.A.

It will be shown that, given the conditions of Result 3 except for Acyclicity, there are two sequences consisting of successively morally better alternatives: from z_0 to z_m and from z_m to $z_{m+m\cdot m''}$. These subsequences are not shown in figure 3.3 but the omission is marked by the ellipsis in between these alternatives. Finally, it is shown that z_0 is better than $z_{m+m\cdot m''}$. Hence, there is a betterness cycle from z_0 to z_0. But this violates Acyclicity.

In subsections 3.4.1 to 3.4.3, I illustrate each of the two subsequences and the direct ranking of z_0 and $z_{m+m\cdot m''}$. Each employs one of the *substantive* trade-off conditions, i.e. they make a moral claim dependent on non-moral claims, and discuss each condition in turn: Super Ultra Minimal Infinite Superiority, Pigou-Dalton, and Ultra Minimal Finite Superiority. It can be shown that the result is tight, in the sense that if any of the conditions is dropped, there is a betterness relation that fulfils all the other conditions.[18] However, as I will show below, still some substantive conditions can be weakened while preserving the paradox. In particular, the ranking of z_0 and $z_{m+m\cdot m''}$ involves a further structural condition, Replication Invariance, which I will argue is dispensable when strengthening another substantive condition accordingly. In subsection 3.4.4 follows an illustration of the proof of Result 3.

3.4.1 The Infinite Superiority Sequence

Now, we construct a sequence of successively better alternatives from z_0 to z_m by applying a weaker version of minimal infinite superiority,

> **Super Ultra Minimal Infinite Superiority:** there are welfare levels w and w' such that w is lower than w', and positive welfare differences b and b' such that b' is smaller than b such that, for any alternatives x and y, if
>
> 1. the worst-off in x are at or below w in x;
> 2. one worst-off in x is at or below average and w in y and better off by at least b in y compared to x;
> 3. the best-off in x are the best-off in y and at or above w' in y and better off by at most b' in x compared to y;
> 4. everyone else is at the same welfare level in x and y,
>
> then y is better than x.[19]

Figure 3.4 illustrates Super Ultra Minimal Infinite Superiority. There are two

[18] Cf. Fleurbaey and Tungodden (ibid.: 406-7).

[19] See Fleurbaey, Tungodden and Vallentyne (2009: 269-70). See also Fleurbaey and Tungodden (2010: 403)'s "Minimal Non-Aggregation" for a formal but slightly weaker statement (with "at least as good as" instead of "better than").

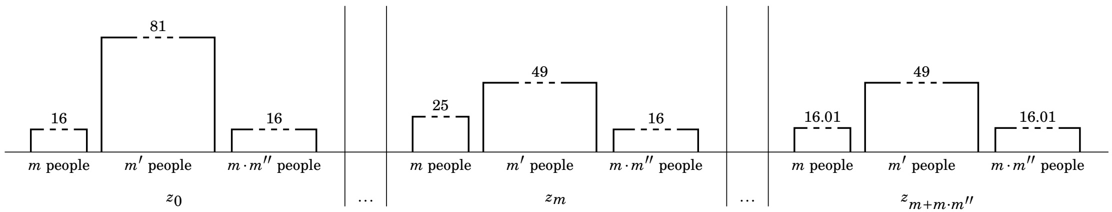

Figure 3.3 Illustration of the Third Moderate Trade-offs Paradox Sequence

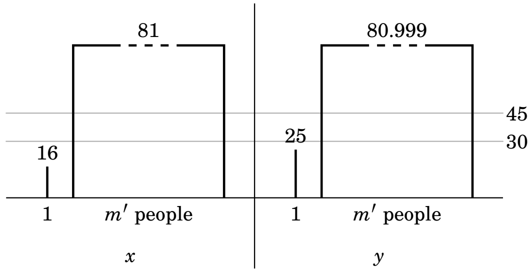

Figure 3.4 Illustration of Super Ultra Minimal Infinite Superiority

alternatives, x and y, and one worst-off and a number, m', of best-off people. Assume that the welfare levels and benefits mentioned in the condition are as follows:

(3.7) the low enough welfare level, w, is 30;

(3.8) the high enough welfare level, w', is 45;

(3.9) the great enough benefit size for one worst-off, b, is 9;

(3.10) the small enough benefit size for every best-off, b', is 0.001;

(3.11) m' is large enough such that the average level is arbitrarily close to the best-off level.

Therefore, according to Super Ultra Minimal Infinite Superiority,

(3.12) y is morally better than x.

That Super Ultra Minimal Infinite Superiority is compatible with a *great enough benefit size*, b, for the worst-off can be understood as the benefit for the worst-off be "nontrivial".[20]

Super Ultra Minimal Infinite Superiority is violated by the simple total principles, i.e. the Weak Simple Total Principle and the Weak Simple Priority Total Principle, and fulfilled by the maximin principles, i.e. Weak Maximin and Weak Leximin.

Now, consider the first subsequence, the

Infinite Superiority Sequence: illustrated in figure 3.5,

i.e., in x_0, m people are at welfare level 16, and m' at 81; in x_1, one at 25, $m-1$ at 16, and m' at 80.999; in x_2, two at 25, $m-3$ at 16, and m' at 80.998; ...; in x_m, m at 25, and m' at 49.

Assume the same as for the illustration of Super Ultra Minimal Infinite Superiority, i.e. that (3.7) to (3.11) are fulfilled for each successive pair of alternatives. Note that as long as the best-off are above the higher level, i.e. 45, and the other factors are kept the same, this condition can be reapplied each time with a different individual of the m worst-off people. Therefore, according to Super Ultra Minimal Infinite Superiority,

[20] See Fleurbaey, Tungodden and Vallentyne (2009: 269).

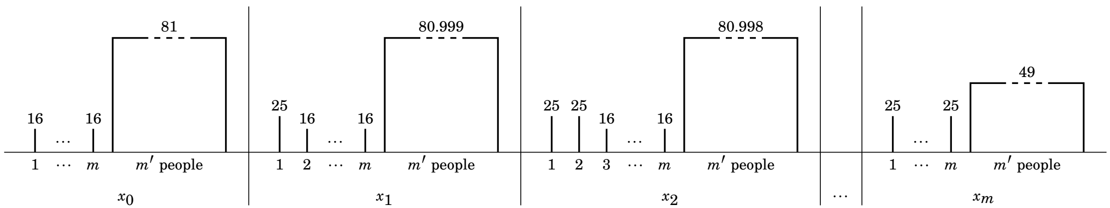

Figure 3.5 Illustration of the Infinite Superiority Sequence

(3.13) x_i is morally better than x_{i-1}, for all $i = 1,\dots,m$.

Finally, consider the first and last element of the paradox sequence, z_0 and z_m. The welfare distributions are the same as x_0 and x_m, except that additionally there are the $m \cdot m''$ people whose welfare is unaffected. And we can construct for each alternative x_i in the Infinite Superiority Sequence an alternative z_i by adding these extra m' unaffected people to each alternative. These additions of unaffected people do not matter for the ranking according to Super Ultra Minimal Infinite Superiority due to its condition 4. Therefore, by Super Ultra Minimal Infinite Superiority,

(3.14) z_i is morally better than z_{i-1}, for all $i = 1,\dots,m$.

Objections

It might be objected that Super Ultra Minimal Infinite Superiority is too strong, since it is sufficient that *one* worst-off person benefits significantly.[21] Instead according to the weaker

Super Ultra Minimal Infinite Superiority*: there are a number of people m, welfare levels w and w' such that w is lower than w', and positive welfare differences b and b' such that b' is smaller than b such that, for any alternatives x and y, if

1. the worst-off in x are at or below w in x,
2. at least m worst-off in x are at or below average and w in y and better off by at least b in y compared to x;
3. the best-off in x are the best-off in y and at or above w' in y and better off by at most b' in x compared to y;
4. everyone else is at the same welfare level in x and y,

then y is better than x.

However, it is easy to see how the Infinite Superiority Sequence can be modified such that in each step not only one but some large enough number of people benefits significantly.[22]

Anyway, I think that Super Ultra Minimal Infinite Superiority is plausible enough and will not discuss Super Ultra Minimal Infinite Superiority* below.

[21] See e.g. Temkin's claim involving fifty people in chapter 1.

[22] Fleurbaey, Tungodden and Vallentyne (2009: 266, n. 5) mention this possible weakening of Super Ultra Minimal Infinite Superiority. However, it does not strengthen their result since it is implied by the combination of Super Ultra Minimal Infinite Superiority together with Replication Invariance. In contrast, I argue, in subsection 3.4.3, that Replication Invariance is dispensable. Hence Super Ultra Minimal Infinite Superiority* strengthens this alternative result.

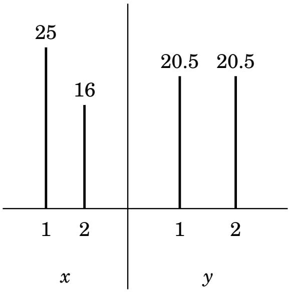

Figure 3.6 Illustration of Pigou-Dalton

3.4.2 The Pigou-Dalton Sequence and the Non-Anti Pigou-Dalton Sequence

Now, we construct a sequence of successively better alternatives from z_m to $z_{m+m \cdot m''}$ by applying a condition according to which, roughly,

(3.15) benefiting one person matters more than benefiting equally another person who would be better off than the first person would be with the benefit.

More precisely, according to (axiological)

Pigou-Dalton: for any alternatives x and y, positive welfare difference b, and persons i and j, if

1. i is at least as well off as j in y;
2. i is better off by b in x compared to y;
3. j is better off by b in y compared to x;
4. everyone else is at the same welfare level in x and y,

then y is better than x.[23]

Figure 3.6 illustrates Pigou-Dalton. There are two alternatives, x and y, and two people, person 1 and 2. In y relative to x,

(3.16) one worse-off person, person 1, gains 4.5, as much as one better-off person, person 2, loses;

(3.17) they don't reverse relative positions;

(3.18) everyone else is at the same welfare level in x and y.

Therefore, according to Pigou-Dalton,

(3.19) y is morally better than x.

[23] See Fleurbaey, Tungodden and Vallentyne (2009: 270).

As a concrete example, assume that the utilities in figure 3.6 correspond to lifetime in years. So, according to Pigou-Dalton, that a 16-year-old person has an increase in lifetime of 4.5 years is better than that a 20.5-year-old person has an increase in lifetime by 4.5 years.[24]

Pigou-Dalton is even weaker than Priority, since it applies only in the case of benefiting two people, equal benefits, and when the worse- and better-off do not reverse their rank. It would be accepted by many who are inclined to accept some (relative or absolute) priority for the worse-off over the better-off. Both the Weak Simple Priority Total Principle and Weak Leximin satisfy Pigou-Dalton. The Weak Simple Total Principle and Weak Maximin neither satisfy nor violate Pigou-Dalton, since they are silent when the total sum of welfare is equal and the worst-off is unaffected, respectively.[25]

Next, consider the second subsequence, the

Pigou-Dalton Sequence: illustrated in figure 3.7,

i.e., in y_0, one person is at welfare level 25, and m'' people at 16; in y_1, two at 20.5, and $m''-1$ at 16; in y_2, two at 18.25, one at 20.5, and $m''-2$ at 16; in y_3, four at 18.25, and $m''-3$ at 16; ...; in $y_{m''}$, $1+m''$ at 16.1. In each step, only one worst-off person is better off by as much as one best-off person is worse off and without them reversing positions. Therefore, according to Pigou-Dalton,

(3.20) y_i is morally better than y_{i-1}, for all $i = 1, \ldots, m''$.

Next, consider z_m and $z_{m+m\cdot m''}$. In z_m, for each of the m people at level 25, there are m'' people at welfare level 16, and additionally there are the m' people whose welfare is unaffected. Hence, the Pigou-Dalton Sequence can be applied m times, once for one and m people of the m and m times m'' people, respectively, which gives a chain of m sequences consisting of m'' elements such that

(3.21) z_i is morally better than z_{i-1}, for all $i = m+1, \ldots, m+m\cdot m''$.

Objections

Some people have doubts about even this very limited priority for the worse-off. However, Pigou-Dalton can actually be replaced in the proof by an even weaker condition that replaces “a worse-off person” by “some number of worse-off people”. More precisely, according to

Non-Anti Pigou-Dalton: for any alternatives x and y, positive welfare difference b, and person j, there is a number of people m such that if

1. j is at least as well off as the m people in y;
2. at least m people are better off by b in y compared to x;

[24] This assumes that extra life years have the same welfare impact on younger and older people. If we drop the simplifying assumption, the number of years have to be adjusted accordingly.

[25] Remember that Pigou-Dalton refers only to the *worse*-off (but not the *worst*-off) and that the Weak Simple Total Principle and the Weak Maximin are only sufficient conditions for betterness.

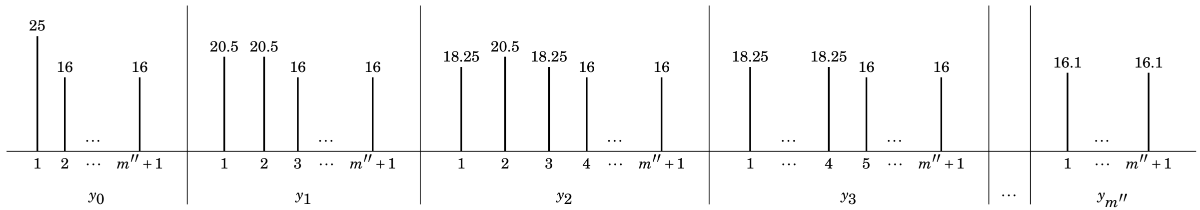

Figure 3.7 Illustration of the Pigou-Dalton Sequence

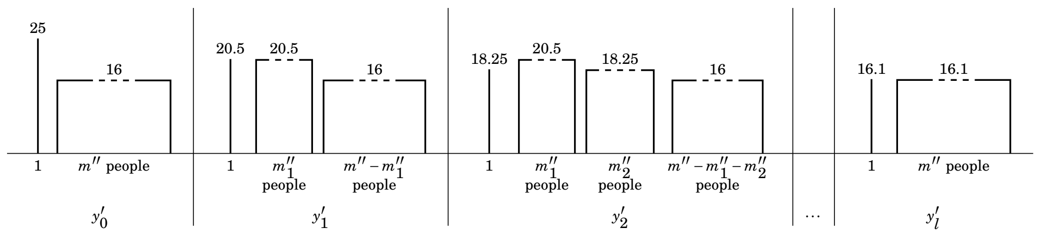

Figure 3.8 Illustration of the Non-Anti Pigou-Dalton Sequence

3. j is better off by b in x compared to y;
4. everyone else is at the same welfare level in x and y,

then y is better than x.[26]

Again, as a concrete example, assume that utilities correspond to lifetime in years of persons. So, according to Non-Anti Pigou-Dalton, that a large enough number of 16-year-old people has an increase in lifetime of 4.5 years is better than that a 20.5-year-old person has an increase in lifetime by 4.5 years.

In this way an appeal to priority for the worse-off is not necessary, since other aspects could be appealed to. For example, that an (arbitrary) *large* number of worse-off people are each better off by as much as *one* better off person would otherwise be, and there is an (arbitrary) *great* increase in the total sum of welfare in y compared to x. Hence, even the Weak Simple Total Principle satisfies Non-Anti Pigou-Dalton.

With this, consider the

Non-Anti Pigou-Dalton Sequence: illustrated in figure 3.8,

i.e., in y_1', one person is at 25, and m'' people at 16; in y_2', $1+m_1''$ at 20.5, and $m''-m_1''$ at 16; …; in y_l', $1+m''$ at 16.1. Here the m'' people are the sum of the m_1'', m_2'', …, m_l'' people.

The sequence follows the pattern of the Pigou-Dalton Sequence, figure 3.7, where y_i is replaced by y_i', for $i = 0,\dots,m''$, with the exception that, in each step, an *arbitrary large* number of people (rather than one person) are each better off by as much as *one* person is worse off. Therefore, according to Non-Anti Pigou-Dalton,

(3.22) y_{i+1}' is morally better than y_i', for all $i = 1,\dots,l-1$.

So, Result 3 can be strengthened by replacing Pigou-Dalton with the weaker Non-Anti Pigou-Dalton.

3.4.3 Replicated Ultra Minimal Finite Superiority

Let us now show that z_0 is better than $z_{m+m\cdot m''}$ by applying

Ultra Minimal Finite Superiority: there is a population size n' such that, for any alternative x, there are welfare differences b and b' such that b is larger than b' such that, for any alternative y, if

1. the population is larger than n';
2. everyone, except for one, is better off by at least b in y compared to x;
3. the one person is better off by at most b' in x compared to y,

then y is morally better than x.[27]

Figure 3.9 illustrates Ultra Minimal Finite Superiority. There is one person singled out and a number, n', of other people. Assume that

[26] Non-Anti Pigou-Dalton is similar to “The Non-Elitism Condition” by Arrhenius (2000).
[27] See Fleurbaey, Tungodden and Vallentyne (2009: 268).

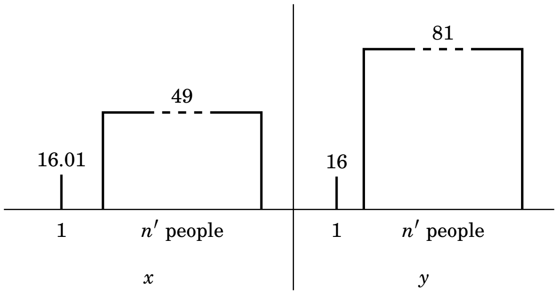

Figure 3.9 Illustration of Ultra Minimal Finite Superiority

(3.23) n' is large enough;

(3.24) the large enough benefit size, b, is 32 (from 49 to 81);

(3.25) the small enough benefit size, b', is 0.1 (from 16 to 16.1).

Therefore, according to Ultra Minimal Finite Superiority,

(3.26) y is morally better than x.

Again, as a concrete example, assume that the utilities in figure 3.9 correspond to lifetime in years. So, according to Ultra Minimal Finite Superiority, that each of a large enough number of 49-year-old people has an increase in lifetime of 32 years is better than that one 16-year-old person has an increase in lifetime by about a month (0.1 years).

The simple total principles, i.e. the Weak Simple Total Principle and the Weak Simple Priority Total Principle, satisfy Ultra Minimal Finite Superiority. But the maximin principles, i.e. the Weak Maximin and the Weak Leximin, violate it.

The ranking of z_0 and $z_{m+m\cdot m''}$ also involves Replication Invariance. Consider

Replicated Ultra Minimal Finite Superiority: illustrated in figure 3.10,

i.e. z_0 and $z_{m+m\cdot m''}$ of the Third Moderate Trade-offs Paradox Sequence where

(3.27) $m' = \left(m + m \cdot m''\right) \cdot n'$.

Assume the same as for the illustration of Ultra Minimal Finite Superiority, i.e. (3.23) to (3.25). Note that

(3.28) $z_{m+m\cdot m''}$ and z_0 are a $\left(m + m \cdot m''\right)$-replication of x and y, respectively.

Therefore, by (3.26) and Replication Invariance,

(3.29) z_0 is morally better than $z_{m+m\cdot m''}$,

i.e. that $\left(m + m \cdot m''\right) \cdot n'$ people are significantly better off by 32 is morally better than that $\left(m + m \cdot m''\right)$ people are very slightly better off by 0.001.

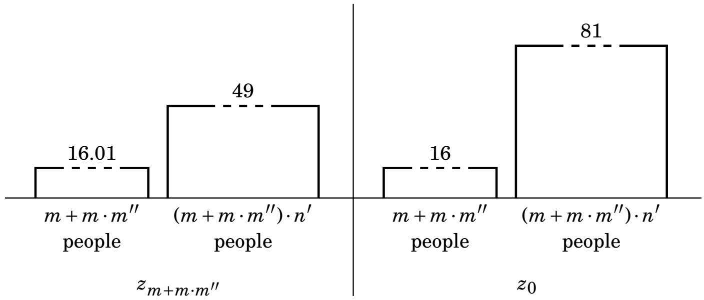

Figure 3.10 Illustration of Replicated Ultra Minimal Finite Superiority

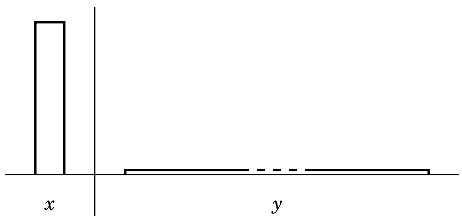

Figure 3.11 Illustration of the Repugnant Conclusion

Objections

There is at least one problem with Replication Invariance, familiar from population ethics. For example, many reject the Repugnant Conclusion, illustrated in figure 3.11, i.e. x is an alternative with a population of ten billion people each of which is at a very high welfare level, and y is an alternative with a much larger population with lives barely worth living.[28] People who reject the Repugnant Conclusion claim that

(3.30) y is *not* morally better than x.

However, consider figure 3.12 which is an illustration of a scaled down version of the Repugnant Conclusion (in subsection 2.6.1). It might seem much less clear whether

[28] The Repugnant Conclusion was introduced in subsection 2.6.1.

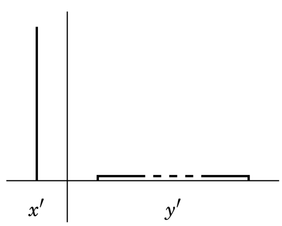

Figure 3.12 Illustration of a scaled down version of the Repugnant Conclusion

an alternative, x', with a population of *very few* people, in fact, only one person is not worse than an alternative, y', with a large population with lives barely worth living. The idea might be that a population with *too few* people is not valuable enough, even if each person's life has a very high quality. And hence much more people with lives worth living, even barely so, have a greater value even if this value is never greater than *sufficiently many* people with very high quality of life. Hence, it might be claimed,

(3.31) y' *is* morally better than x'.

Next, note that

(3.32) x and y are a ten billion-replication of x' and y', respectively.

Therefore, by Replication Invariance and (3.31),

(3.33) y *is* morally better than x.

But this contradicts the rejection of the Repugnant Conclusion, (3.30). Therefore, this might be a counterexample to Replication Invariance.

There are several ways in which Replication Invariance might be restricted to meet this objection. One restriction would be to assume that the populations under consideration are already big enough, so that it is ruled out that there are *too few* people.[29]

Another possibility is to restrict the condition to a fixed number of people in x and y, as in the island example above. According to this

> **Same Number Replication Invariance:** for any alternatives x and y with the same number of people, and any positive integer k, x is at least as good as y if and only if a k-replication of x is at least as good as a k-replication of y.[30]

And in Replicated Ultra Minimal Finite Superiority there are only the same number of people in z_0 and $z_{m+m \cdot m''}$. So, replacing Replicated Ultra Minimal Finite Superiority with Same Number Replication Invariance is another way to strengthen the impossibility result. A further strengthening is to restrict the condition to a fixed population, i.e. not only the same number but the same people. This will also satisfy the restriction to a fixed population in the alternatives compared.[31]

Furthermore, Replication Invariance is needed only because Ultra Minimal Finite Superiority applies only when only *one* person would benefit slightly. However, it is not clear why this feature is essential in Ultra Minimal Finite Superiority. So, a more direct approach is to strengthen Ultra Minimal Finite Superiority accordingly while giving up on Replication Invariance (and Same Number Replication Invariance), so that the result is directly implied by one condition rather than via the combination of two. According to

[29] See also Broome (2004: 13.4) for the idea of restricting the range of cases to those where the populations are large enough.

[30] The condition could be further restricted, since the Third Moderate Trade-offs Paradox involves only the *same* people.

[31] See section 2.6. While Fleurbaey, Tungodden and Vallentyne (2009) do not explicitly assume a fixed population, it is actually assumed in Fleurbaey and Tungodden (2010).

Ultra Minimal Finite Superiority*: for any number of people m', there is a population size n' such that, for any alternative x, there are welfare differences b and b' such that b is sufficiently larger than b' such that, for any alternative y, if

1. the population is larger than n';
2. everyone, except for m' people, is better off by at least b in y compared to x;
3. the m' people are better off by at most b' in x compared to y,

then y is morally better than x.

Next, assume again the same as for the illustration of Ultra Minimal Finite Superiority, i.e. (3.23) to (3.25) and, furthermore, that

(3.34) m' is sufficiently larger than $m + m \cdot m''$.

Then, by Ultra Minimal Finite Superiority*, again (3.29) z_0 is better than $z_{m+m\cdot m''}$.

And since Ultra Minimal Finite Superiority and Replication Invariance together imply Ultra Minimal Finite Superiority*, replacing the former two by the latter is another strengthening of Result 3.

As each of the simple classic principles satisfy or violate Ultra Minimal Finite Superiority and Replication Invariance, they equally satisfy or violate Ultra Minimal Finite Superiority*.[32]

I briefly note that Fleurbaey, Tungodden and Vallentyne (2009: 276-9) consider another weakening of Minimal Finite Superiority that applies only if people are below some "threshold" (or "poverty line"). And they show that there are principles that satisfy this condition and all other conditions discussed so far (except for the other minimal finite superiority conditions). These principles imply that any benefit (no matter how small) to those below the threshold matters infinitely more than any benefit (no matter how large) to those above the threshold. However, they are sceptical of the plausibility of such a principle.[33] I tend to agree. These views have been discussed at length elsewhere.[34] Since I do not think I have to add anything to what others have said, I do not discuss such views in the remainder of the thesis.

3.4.4 Proof: the Third Moderate Trade-offs Paradox

Finally, all three subsequences are chained in the Third Moderate Trade-offs Paradox Sequence (on page 54) for the

Third Moderate Trade-offs Paradox: there are numbers of people m, m', and m'' such that

1. (3.14), z_i is better than z_{i-1}, for $i = 1, \ldots, m$.

[32] Furthermore, Fleurbaey and Tungodden (ibid.: 404-5) show, how a principle, I call the Weak Geometric Priority Total Principle, satisfies all of their conditions except for Replication Invariance. Correspondingly, in section 3.B, I show how it also violates Ultra Minimal Finite Superiority*. Fleurbaey and Tungodden (2010: 405) and Fleurbaey, Tungodden and Vallentyne (2009: 266) call the principle "Geometric Gini".

[33] Fleurbaey, Tungodden and Vallentyne (2009: 279).

[34] See e.g. Crisp (2003), Brown (2005), and Adler (2012: ch. 5) for a related recent discussion.

2. (3.21), z_i is better than z_{i-1}, for $i = m+1, \ldots, m+m \cdot m''$.
3. (3.29), z_0 is morally better than $z_{m+m \cdot m''}$.

Therefore,

(3.35) there are numbers of people m, m', and m'' such that z_1 is better than z_0, z_2 is better than z_1, ..., and z_0 is better than $z_{m+m \cdot m''}$.

But (3.35) violates Acyclicity. This concludes the illustration of the proof of impossibility Result 3, and the stronger

> **Result 3*:** no moral betterness relation satisfies Non-Anti Pigou-Dalton, Acyclicity, Super Ultra Minimal Infinite Superiority*, and Ultra Minimal Finite Superiority*.

The additional central element in the general proof of Result 3 and Result 3*, in comparison to Result 1* (see section 3.3), is to show that the benefit to the worse-off people in x_0 compared to x_m can be arbitrarily small in order to function as benefit b' in Ultra Minimal Finite Superiority. And this is where the additional subsequences involving Pigou-Dalton and Non-Anti Pigou-Dalton, respectively, come into play (see subsection 3.4.2).

3.5 The Deontic Third Moderate Trade-offs Paradox

One possible reaction to Result 3 (and Result 3*) is to accept the *axiological* impossibility but claim that analogous *deontic* conditions can be defended. However, Fleurbaey, Tungodden and Vallentyne (2009) show that the axiological impossibility Result 3 has a deontic analogue,

> **Result 5:** no theory of moral permissibility satisfies Deontic Super Ultra Minimal Infinite Superiority, Deontic Ultra Minimal Finite Superiority, Deontic Pigou-Dalton, No Prohibition Dilemmas and Deontic Replication Invariance.[35]

There is a simple way to get from axiological to deontic conditions, namely by using the

> **Undominanted Choice Connection:** a feasible alternative is permissible if and only if it is undominated,

i.e. there is *no* feasible alternative that is better than it.

Here is how we can derive a deontic principle from Ultra Minimal Finite Superiority (on page 63). First, note that the Undominanted Choice Connection is equivalent to

(3.36) a feasible alternative is *im*permissible if and only if there is a feasible alternative that *is* better than it.

[35] See Fleurbaey, Tungodden and Vallentyne (2009: 270).

Therefore, by Ultra Minimal Finite Superiority,

> **Deontic Ultra Minimal Finite Superiority:** there is a population size n' such that, for any alternative x, there are welfare differences b and b' such that b is larger than b' such that, for any alternative y, if
>
> 1. the population is larger than n';
> 2. everyone, except for one, is better off by at least b in y compared to x;
> 3. the one person is better off by at most b' in x compared to y,
>
> and x and y are feasible, then x is impermissible[36]

So, while the axiological version deemed one alternative, y, that stands in the relation to another alternative, x, described by 1 to 3 morally *better* than the other, the deontic version deems the other alternative, x, *impermissible* if the one, y, is feasible. In the same way, the other substantive conditions can be transformed into their deontic counterparts.

Furthermore, by the Undominanted Choice Connection and Replication Invariance,

> **Deontic Replication Invariance:** for any feasible alternatives x and y, and positive integer k, if x is never permissible when y is feasible, then a k-replication of x is never permissible if a k-replication of y is feasible.[37]

Therefore, analogous to the axiological conditions implying the Third Moderate Trade-offs Paradox, the deontic conditions imply the

> **Deontic Third Moderate Trade-offs Paradox:** for the feasible set of alternatives $z_0, \ldots, z_{m+m \cdot m''}$, there are numbers of people m, m', and m'' such that
>
> 1. z_{i-1} is impermissible, for $i = 1, \ldots, m$;
> 2. z_{i-1} is impermissible, for $i = m+1, \ldots, m+m \cdot m''$;
> 3. $z_{m+m \cdot m''}$ is impermissible.

This is because where an alternative is judged worse than another by an axiological condition, the analogue deontic condition judges the former alternative impermissible. Therefore, the deontic conditions together imply

(3.37) there are numbers of people m, m', and m'' such that, in the set of feasible alternatives $z_0, \ldots, z_{m+m \cdot m''}$, *all* alternatives are impermissible.

But this violates

> **No Prohibition Dilemmas:** for any set of feasible alternatives, at least one feasible alternative is not impermissible.[38]

[36] Fleurbaey, Tungodden and Vallentyne (ibid.: 272). For a precise statement of the derivation see section 3.C.

[37] See Fleurbaey, Tungodden and Vallentyne (ibid.: 271). For a precise statement of the derivation see section 3.C.

[38] See Fleurbaey, Tungodden and Vallentyne (ibid.: 272). For a precise statement of the derivation see section 3.C.

This is a structural condition on permissibility, and the deontic analogue to Acyclicity, implied by the Undominanted Choice Connection under the assumption that the feasible set of alternatives is finite.[39]

This concludes the illustration of the proof of deontic impossibility Result 5. Analogously, Result 1* and Result 3* can be transformed into deontic impossibility results. For example, according to

> **Result 5*:** no theory of moral permissibility satisfies Deontic Super Ultra Minimal Infinite Superiority*, Deontic Ultra Minimal Finite Superiority*, Deontic Non-Anti Pigou-Dalton, and No Prohibition Dilemmas.

Objections

The Undominanted Choice Connection may seem controversial. However, remember that the scenarios under consideration are restricted by the *other things equal* clause.[40] So, by assumption, there are no morally relevant differences other than how people's welfare is affected. Restricted in this way the connection seems defensible.

For example, it might be morally *better* that you rescue someone from a burning house even though you suffer severe burns. But it might be morally *permissible* not to, since the cost to *yourself* is so great. The Undominanted Choice Connection rules out such actions "beyond the call of duty", or supererogation, according to which

(3.38) a permissible act can be *worse* than another act.

But this would violate the Undominanted Choice Connection. However, in the cases we consider there is no such difference. That there is a greater cost to *yourself* in one but not the other alternative and that this difference matters morally is ruled out by the *other things equal* clause.

Furthermore, while the Undominanted Choice Connection is *sufficient to derive* the deontic from the axiological conditions, it is *not necessary* to assume it. This is because the deontic conditions are seemingly plausible on their own without any appeal to axiological considerations. So, even the extreme position that rejects any axiological considerations still faces the deontic paradoxes.

3.6 Rejecting Acyclicity or No Prohibition Dilemmas

One way to resolve the inconsistency of the impossibility results above is to reject the structural conditions Acyclicity and No Prohibition Dilemmas, respectively. But, intuitively, this is not sufficient for a satisfying verdict in the Third Moderate Tradeoffs Paradox, at least for those inclined to believe in moderate trade-offs, as I suggest next.

Fleurbaey, Tungodden and Vallentyne state that

[39] For a precise statement of the derivation see section 3.C.
[40] See subsection 2.1.2.

We strongly believe that Acyclicity and No Prohibition Dilemmas are basic requirements of moral goodness and practical moral permissibility, so we do not see the weakening or the removal of these requirements as an appealing way out of the impossibility.[41]

To the contrary, some have questioned Acyclicity, notably Schwartz (1972), Temkin (1996) and Rachels (1998).[42]

However, *even if* Acyclicity is rejected, I think this does not help much for resolving the First Moderate Trade-offs Paradox or the Third Moderate Trade-offs Paradox. Consider the Third Moderate Trade-offs Paradox Sequence. Assume that the utilities in figure 3.3 correspond to lifetime in years. The sequence starts, see figure 3.5, with a couple of hours life-shortening for each of m' 81-year-old people. However, along the sequence, the same people's lives are shortened more and more so that they finally end up with 32 year less in $z_{m+m\cdot m''}$ compared to z_0. When I consider this sequence carefully, I have the following intuitions. Both extreme verdicts that z_0 is the best alternative, as implied by the maximin principles, and that $z_{m+m\cdot m''}$ is the best alternative, as implied by the simple total principles, are counter-intuitive. To the contrary, both alternatives seem inferior to some alternative along the sequence. It is implausible to claim that the sequence does not get off the ground, with the second alternative being better than the first, but even more implausible that the last alternative is no worse than any alternative or that it is even the best alternative. The additional small burden on the many best-off people for the sake of the worst-off seems better at first but not so all the way to the end of the sequence. In particular,

(3.39) in the beginning of the sequence, successive alternatives are better than their predecessor, starting with the second alternative being better than the first alternative;

(3.40) the first is better than the last alternative;

(3.41) there is a non-dominated alternative better than both the first and the last alternative,

i.e. there is an alternative that is *not* worse than any other alternative and better than the first and the last alternative (see figure 3.13). And I am inclined to think that, on careful consideration, many people who are inclined to accept moderate trade-offs, i.e. both minimal finite and minimal infinite superiority, share these intuitions. It is only that the Third Moderate Trade-offs Paradox Sequence shows that embracing both conditions is harder than might have been expected. But dropping Acyclicity alone won't explain this, because there is a betterness cycle in the Third Moderate Trade-offs Paradox Sequence and hence (3.41) is violated, i.e. there is no undominated alternative.

Note that the ranking of alternatives sketched in figure 3.13 as a smoothed curve is only a simplification. Possible rankings of alternatives that match the

[41] Fleurbaey, Tungodden and Vallentyne (2009: 274).

[42] The explicit target in Temkin (1996) and Rachels (1998) is Transitivity. However, they actually argue against Acyclicity as well (Temkin 1996: 180; Rachels 1998: 73). Their arguments rely on sequences in many aspects similar to the Third Moderate Trade-offs Paradox Sequence, but I will not discuss them in detail here.

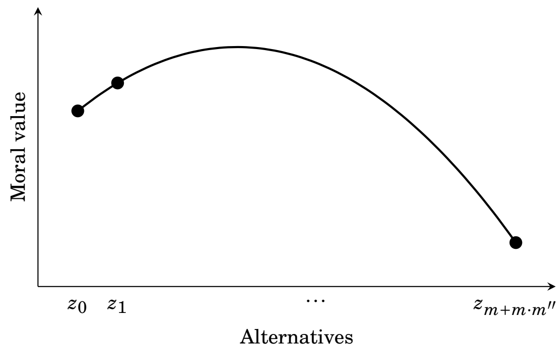

Figure 3.13 Moral value of alternatives in the Third Moderate Trade-offs Paradox Sequence

intuitions above, (3.39) to (3.41), may involve gaps, for example, due to moral incommensurability. And, as the next chapter discusses, perhaps the value of alternatives cannot be represented on a single number scale.

It might be replied that at least the *deontic* paradox can be satisfactorily resolved without violating No Prohibition Dilemmas. For it might be claimed that *all* feasible alternatives are permissible when there is a cycle among *all* of them.[43] For example, Schwartz (1972) argues for an alternative to the Undominanted Choice Connection that does not presuppose Acyclicity in order to satisfy No Prohibition Dilemmas. First, define an

undominated set: an *undominated set* is a non-empty subset of the feasible set of alternatives that are *not* worse than any alternative not in the set.

Second, define a

minimal undominated set: a subset, S, of the feasible set is a minimal undominated set if and only if

1. S is an undominated set;
2. *no* proper subset of S is an undominated set. [44]

Finally, according to, what I call, the

Schwartz Choice Connection: a feasible alternative is permissible if and only if it is in the union of all minimal undominated sets.[45]

We don't need to consider all the details of the Schwartz Choice Connection. For our purposes it suffices to consider its application to the Third Moderate Trade-offs Paradox Sequence. According to the Third Moderate Trade-offs Paradox,

[43] See, e.g. Schwartz (1972) and Ross (2015).
[44] A minimal undominated set is sometimes called a *Schwartz set component*.
[45] The union is used because the minimal undominated set is not necessarily unique. The union of all minimal undominated sets is sometimes called the *Schwartz set*.

(3.42) there is a betterness cycle among *all* feasible alternatives.

Therefore,

(3.43) for every strict subset S of all feasible alternatives, there is an alternative not in S that is better than one alternative in S.

Hence, no strict subset of the feasible alternatives is an minimal undominated set. Therefore,

(3.44) the only set that is an minimal undominated set is the set of *all* feasible alternatives.

Therefore, by the Schwartz Choice Connection,

(3.45) *all* feasible alternatives are *permissible*.

However, intuitively, this judgement is not satisfactory either. The Third Moderate Trade-offs Paradox Sequence does not seem like a sequence where "everything goes". In particular, the extremes z_0 and $z_{m+m \cdot m''}$ seem impermissible. Analogous to (3.39) to (3.41), it is intuitive that

(3.46) the first alternative is impermissible;

(3.47) the last alternative is impermissible;

(3.48) there is a permissible alternative.[46]

But the claim that all alternatives are permissible is incompatible with (3.46) and (3.47).

Furthermore, some people reject No Prohibition Dilemmas and claim that there are so called *moral dilemmas* in which *all* feasible alternatives are *im*permissible. However, the Third Moderate Trade-offs Paradox Sequence does not seems like a moral dilemma either. To claim that all alternatives *in the Third Moderate Trade-offs Paradox Sequence* are *im*permissible does not help since this is incompatible with (3.48).

Above I have focused the discussion on the Third Moderate Trade-offs Paradox. However, these arguments can be applied to solutions of the First Moderate Trade-offs Paradox. In particular, in the First Moderate Trade-offs Paradox it is intuitive that (3.39) to (3.41) and (3.46) to (3.48). In any case, the moral principles that will be discussed in the remaining chapters that resolve the Third Moderate Trade-offs Paradox also resolve the First Moderate Trade-offs Paradox by implying these claims.

3.A Proof of Result 3*

Result 3*: no moral betterness relation satisfies Non-Anti Pigou-Dalton, Acyclicity, Super Ultra Minimal Infinite Superiority*, and Ultra Minimal Finite Superiority*.

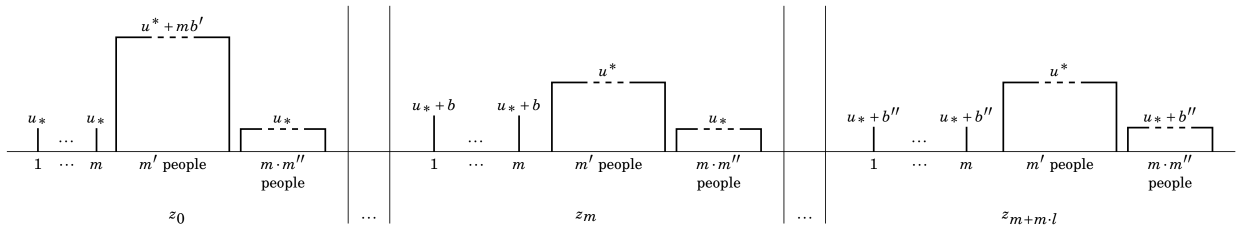

Figure 3.14 The Third Moderate Trade-offs Paradox Sequence

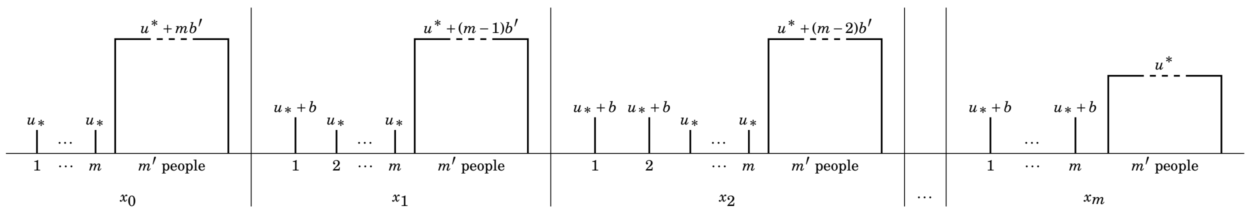

Figure 3.15 The Infinite Superiority Sequence

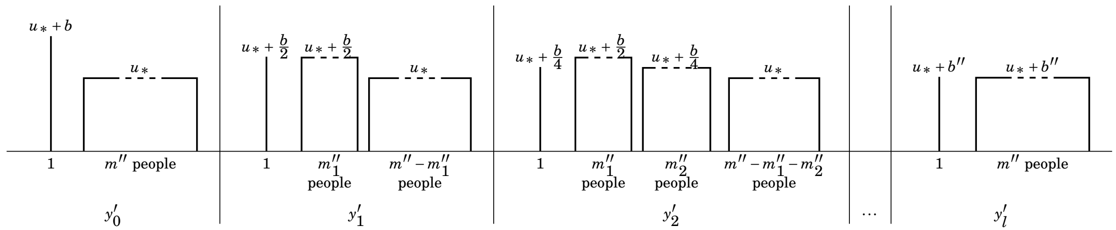

Figure 3.16 The Non-Anti Pigou-Dalton Sequence

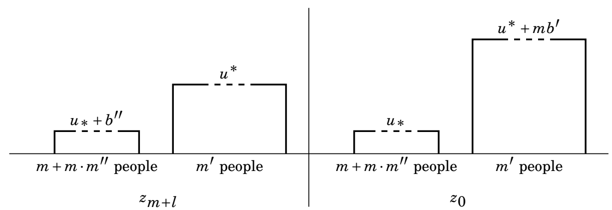

Figure 3.17 Replicated Ultra Minimal Finite Superiority

The proof is illustrated in figures 3.14 to 3.17.

Proof. The proof is brief since most details have already been presented above. First, consider the Infinite Superiority Sequence. For any

(3.49) welfare levels $u_*, u^* \in \mathbb{R}$ such that u_* and u^* are low and high enough, respectively;

(3.50) benefits $b, b' \in \mathbb{R}_{>0}$ such that b and b' are large and small enough, respectively;

(3.51) numbers of people $m, m', m'' \in \mathbb{Z}_{>0}$,

by Super Ultra Minimal Infinite Superiority,

(3.52) x_i is morally better than x_{i-1}, for all $i = 1, \ldots, m$.

Therefore, since the m'' additional people's welfare is unaffected,

(3.53) z_i is morally better than z_{i-1}, for all $i = 1, \ldots, m$.

Next, consider the Non-Anti Pigou-Dalton Sequence. For any

(3.54) benefit $b'' \in \mathbb{R}_{>0}$ such that b'' is small enough;

(3.55) number of people $m'' \in \mathbb{Z}_{>0}$ such that m'' is large enough,

by Non-Anti Pigou-Dalton,

(3.56) y'_i is morally better than y'_{i-1}, for all $i = 1, \ldots, l$.

Therefore, by repeated application and since the m' additional people's welfare is unaffected,

(3.57) z_i is morally better than z_{i-1}, for all $i = m+1, \ldots, m + m \cdot l$.

Finally, consider Replicated Ultra Minimal Finite Superiority. For any

(3.58) benefit $b'' \in \mathbb{R}_{>0}$ such that b'' is small enough;

(3.59) numbers of people $m, m', m'' \in \mathbb{Z}_{>0}$ such that m is large enough and m' is larger enough than both m and m'',

by Ultra Minimal Finite Superiority*,

(3.60) z_0 is morally better than $z_{m+m\cdot l}$,

Therefore, by (3.53), (3.57) and (3.60),

(3.61) z_1 is better than $z_0, \ldots$ and z_0 is better than $z_{m+m\cdot l}$.

But this violates Acyclicity. This concludes the proof. □

[46] Note that this also follows from the Undominanted Choice Connection and (3.39), (3.40), and (3.41).

3.B The Weak Geometric Priority Total Principle

According to the

Weak Geometric Priority Total Principle: there is some $0 < q < 1$ such that

$$v(x) = v_q(x) := \sum_{i=1}^{n} \phi_i\big(u_{[i]}(x)\big)$$

with $\phi_i(u) = q^{i-1}u$, for $i \in \mathbb{Z}_{>0}$ and $u \in \mathbb{R}$.

So,

$$\begin{aligned} v_q &= \sum_{i=1}^{n} q^{i-1} u_{[i]} \\ &= u_{[1]} + q u_{[2]} + q^2 u_{[3]} + \ldots + q^{n-2} u_{[n-1]} + q^{n-1} u_{[n]}. \end{aligned}$$

According to the Weak Geometric Priority Total Principle, the worst-off person's welfare gets the greatest weight of 1, the second worst-off gets the second greatest weight of q, the third worst-off gets the third greatest of q^2, etc. Hence, this principle gives priority to the worse-off where the priority weight of a person's welfare level depends the number of people worse off than this person.[47] Hence, this is a version of a *relational* priority total principle. Note that the Weak Geometric Priority Total Principle attaches *finite priority weight* to benefits to the worse off people. However, in contrast to the Weak Simple Priority Total Principle, these priority-weights are *variable* depending on the number of people. And, in contrast to the Weak Simple Priority Total Principle, the Weak Geometric Priority Total Principle fulfils Super Ultra Minimal Infinite Superiority.[48] The basic idea behind the Weak Geometric Priority Total Principle relies on the limit of the geometric series.

Proposition 3.1 (Limit of the Geometric Series)**.** *For any $q \in \mathbb{R}$ with $0 < q < 1$,*

$$\sum_{i=0}^{\infty} q^i = 1 + q + q^2 + q^3 + \ldots = \frac{1}{1-q}.$$

Proof. See introductory calculus textbooks. □

[47] The value function of the Weak Geometric Priority Total Principle is equivalent to the value function of "Geometric Gini", see Fleurbaey and Tungodden (2010), according to which, there is $r > 1$ such that

$$v = v_r := \sum_{i=1}^{n} r^{n-i} u_{[i]} = r^{n-1} u_{[1]}(x) + r^{n-2} u_{[2]} \ldots + r^2 u_{[n-2]} + r u_{[n-1]} + r u_{[n]}.$$

The equivalence follows for $q = 1/r$, and $v_q = v_r / r^{n-1}$. However, my statement seems clearer since the relationship to the classic statement of the geometric sequence (see proposition 3.1) is more obvious which also simplifies the proofs below.

[48] Fleurbaey and Tungodden (ibid.: 404).

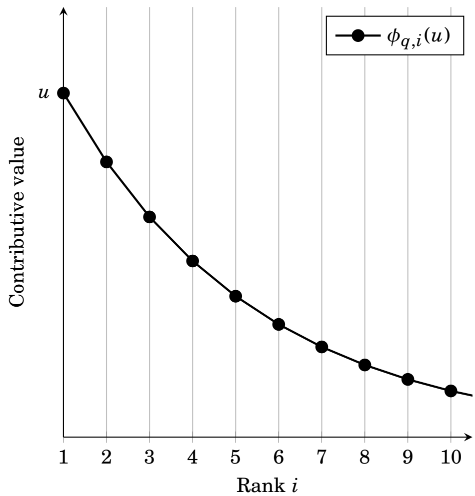

Figure 3.18 Priority-weight function $\phi_{q,i}(u) = q^{i-1} \cdot u$

Next, it is shown that the Weak Geometric Priority Total Principle satisfies Super Ultra Minimal Infinite Superiority but violates Ultra Minimal Finite Superiority*. Note that Weak Geometric Priority Total Principle satisfies Super Ultra Minimal Infinite Superiority without appeal to lexical superiority.

Proposition 3.2. *Weak Geometric Priority Total Principle satisfies Super Ultra Minimal Infinite Superiority.*

Proof. Follows immediately from proposition 3.3. □

Proposition 3.3. *According to the Weak Geometric Priority Total Principle, a benefit of one utility to the worst-off compensates a sacrifice of* $(q-1)/q$ *utility for each of everyone else—no matter how many people there are.*[49]

The compensation is illustrated in figure 3.19.

Proof. Consider a utility distribution where the worst-off person gets a benefit $b \in \mathbb{R}$ and everyone else loses a benefit $b' \in \mathbb{R}$, i.e. the contributive value is

$$v = \left(u_{[1]} + b\right) + \sum_{i=2}^{n} q^{i-1}\left(u_{[i]} - b'\right) = \sum_{i=1}^{n} q^{i-1} u_{[i]} + b - b' \sum_{i=2}^{n} q^{i-1}. \tag{3.62}$$

First, we are going to find an upper bound for the total priority-weight of the benefits of everyone other than this worst-off.

$$\sum_{i=2}^{n} q^{i-1} = \sum_{i=1}^{n-1} q^{i}$$

[49] Fleurbaey and Tungodden (2010: 404).

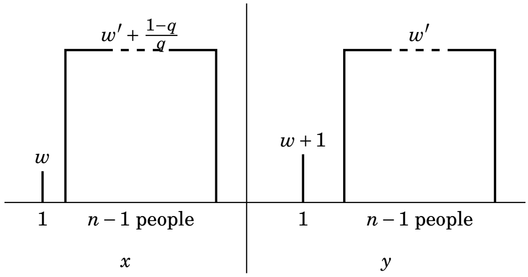

Figure 3.19 Illustration of proposition 3.3

$$
\begin{aligned}
&< \sum_{i=1}^{\infty} q^i \\
&= \sum_{i=0}^{\infty} q^i - 1 \\
&= \frac{1}{1-q} - 1 && \text{[by proposition 3.1]} \\
&= \frac{q}{1-q}.
\end{aligned}
$$

Therefore, by (3.62), for $b' = (1-q)/q$, the contributive value of these benefit is smaller than 1, i.e.

$$b' \sum_{i=2}^{n} q^{i-1} < 1.$$

Therefore, by (3.62), a benefit of one utility for the worst-off, i.e. $b = 1$, can compensate the sacrifice of $(1-q)/q$ for each of everyone else. □

Proposition 3.4. *Weak Geometric Priority Total Principle violates Ultra Minimal Finite Superiority*.*

Proof. Follows immediately from proposition 3.5. □

Proposition 3.5. *For all $x, y \in O$, and $q, \varepsilon, u \in \mathbb{R}$, with $q > 1$, and $\varepsilon > 0$, there exists a positive integer m such that, for all $n > m$,*

$$v_q(x) > v_q(y),$$

with $u(x) = (\varepsilon, \ldots, \varepsilon)$ and $u(y) = (\underbrace{0, \ldots, 0}_{m \text{ times}}, u, \ldots, u)$.

The welfare distributions of proposition 3.5 are illustrated in figure 3.20. Roughly, proposition 3.5 holds because, with increasing n, the value of x becomes greater (tends towards a fixed positive number) while, with increasing m, the value of y becomes arbitrarily close to 0 because the weights given to the welfare of the $n-m$ people becomes arbitrarily close to 0.

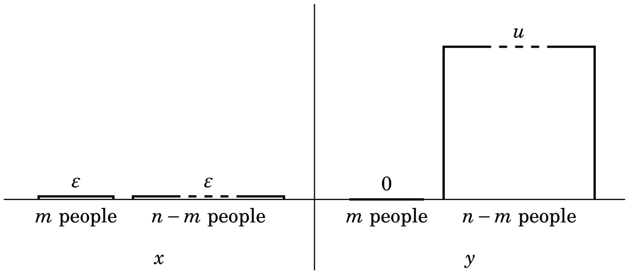

Figure 3.20 Illustration of proposition 3.5

Proof. Consider the inequation

(3.63) $v_q(x) > v_q(y)$.

First, consider (3.63)'s left hand side.

$$\begin{aligned} v_q(x) &= \sum_{i=1}^{n} q^{i-1}\varepsilon \\ &= \varepsilon \sum_{i=1}^{n} q^{i-1} \\ &= \varepsilon \sum_{i=0}^{n-1} q^{i}. \end{aligned}$$

If n tends to infinity, this tends towards a fixed positive number, i.e., by proposition 3.1,

$$\varepsilon \sum_{i=0}^{\infty} q^{i} = \frac{\varepsilon}{1-q} > 0.$$

Next, consider (3.63)'s right hand side.

$$\begin{aligned} v_q(y) &= \sum_{i=m+1}^{n} q^{i-1}u \\ &= u \sum_{i=m+1}^{n} q^{i-1} \\ &= u \sum_{i=m}^{n-1} q^{i} \\ &= u \cdot \left(\sum_{i=0}^{n-1} q^{i} - \sum_{i=0}^{m-1} q^{i} \right). \end{aligned}$$

And, if, in addition to n, m tends to infinity, the right hand side of the inequation tends to 0, i.e., by proposition 3.1,

$$u \cdot \left(\sum_{i=0}^{\infty} q^i - \sum_{i=0}^{\infty} q^i \right) = u \cdot \left(\frac{1}{1-q} - \frac{1}{1-q} \right) = 0.$$

Hence, for any given n, given m is large enough, the left hand side will be larger than the right hand side. □

3.C Derivation of the deontic from the axiological

Let $\succ$ be the betterness relation, i.e. $\succ \in O^2$. Next, it is shown how deontic conditions can be derived from axiological conditions by the

Undominanted Choice Connection: for all $X \subseteq O$,

$$C(X) = U(X, \succ) := \{x \in X : \text{for all } y \in X,\ y \not\succ x\}.$$

First, note that

(3.64) for all $x \in X \subseteq O$,

$$x \notin U(X, \succ) \iff \text{for some } y \in X,\ y \succ x.$$

The axiological substantive conditions imply their deontic counterpart by

Proposition 3.6. *The Undominanted Choice Connection implies, for all $x \in O$, $X \subseteq O$, and well-formed formula $\varphi(x, y)$,*

$$\begin{aligned} &\textit{for all } y \in O\left[\varphi(x,y) \implies y \succ x\right] \\ \implies &\left[\textit{for some } y \in X,\ \varphi(x,y) \implies x \notin U(X,\succ)\right]. \end{aligned}$$

The left and right hand side of the conditional is the formal structure of the substantive axiological and deontic substantive conditions, respectively.

Proof. Assume

(3.65) $\varphi(x, y) \implies y \succ x$;

(3.66) for some $y \in X$, $\varphi(x, y)$.

Therefore,

(3.67) for some $y \in X$, $y \succ x$.

Therefore, by (3.64),

(3.68) $x \notin U(X, \succ)$. □

Next, we turn to the structural conditions. For $x \in O$ and $k \in \mathbb{Z}_{>0}$, let $k * x$ denote the the *k-replication of* x, i.e. $k * x \in O$ with

$$u(k * x) = (\underbrace{u_1(x), \ldots, u_n(x), u_1(x), \ldots, u_n(x), \ldots, u_1(x), \ldots, u_n(x)}_{k\text{-times}}).$$

According to

Replication Invariance: for all $x, y \in O$, and $k \in Z_{>0}$,

$$x \succ y \implies k * x \succ k * y.$$

According to

Deontic Replication Invariance: for all $x, y \in O$,

$$\text{for all } X \subseteq O[y \in X \implies x \notin U(X, \succ)]$$
$$\implies \text{for all } k \in \mathbb{Z}_{>0}, \text{ and } X \subseteq O[k * y \in X \implies k * x \notin U(X, \succ)].$$

Proposition 3.7. *The Undominanted Choice Connection implies that if Replication Invariance holds, then Deontic Replication Invariance holds.*

Proof. Assume,

(3.69) for $x, y \in O$, for all $X \subseteq O[y \in X \implies x \notin U(X, \succ)]$.

Therefore, by ,

(3.70) $x \succ y$.

Therefore, by Replication Invariance,

(3.71) for all $k \in \mathbb{Z}_{>0}$, $k * y \succ k * x$.

Therefore, by ,

(3.72) for all $k \in \mathbb{Z}_{>0}$, and $X \subseteq O[k * y \in X \implies k * x \notin U(X, \succ)]$.

□

Proposition 3.8. *The Undominanted Choice Connection implies, for all* $x, y \in O$,

$$y \succ x \iff \textit{for all } X \subseteq O[y \in X \implies x \notin U(X, \succ)].$$

Proof. Let $x, y \in O$, and $X \subseteq O$. First, prove the left to right direction of the equivalence. Assume

(3.73) $y \succ x$ and $y \in X$.

Therefore, by the Undominanted Choice Connection,

(3.74) $x \notin U(X, \succ)$.

Next, prove the right to left direction via the contrapositive. Assume

(3.75) $y \not\succ x$.

Therefore, for $X = \{x, y\}$ and by the Undominanted Choice Connection,

(3.76) $y \in X$ and $x \in U(X, \succ)$. □

Finally, according to

No Prohibition Dilemmas: for all $X \subseteq O$, there exists $x \in X$ such that $x \in C(X)$.

Proposition 3.9. *If $X \subseteq O$ is finite and $\succ$ satisfies Acyclicity, then $U(X, \succ)$ satisfies No Prohibition Dilemmas.*

Proof. Assume that,

(3.77) $\succ$ satisfies Acyclicity.

Therefore, by proposition 2.1,

(3.78) $\succ$ satisfies Irreflexivity.

For contradiction, assume that, for some finite feasible set $X \subseteq O$, $U(X, \succ)$ does *not* satisfy No Prohibition Dilemmas, i.e.

(3.79) there exists a *finite* feasible set $X \subseteq O$ such that, for all $x \in X$, $x \notin U(X, \succ)$.

Therefore, by (3.64),

(3.80) for all $x \in X$, there is $y \in X$ such that $y \succ x$.

And, by the assumption that X is finite,

(3.81) there is a sequence $(x_1, x_2, x_3, \ldots, x_{k-1}, x_k) \in X^k$ with *maximal* length, i.e. with a maximal number of distinct elements, such that $x_1 \succ x_2$, $x_2 \succ x_3$, ... and $x_{k-1} \succ x_k$.

Therefore, by (3.80),

(3.82) there is a $y \in X$ such that $y \succ x_1$.

Therefore, by Irreflexivity,

(3.83) $x_1 \neq y$.

Therefore, by (3.81),

(3.84) there is $i \in \{2, \ldots, k\}$ such that $y = x_i$.

Therefore, by (3.81) and (3.82),

(3.85) $y \succ x_1$, $x_1 \succ x_2$, $x_2 \succ x_3$, ... and $x_{i-1} \succ x_i = y$.

Therefore,

(3.86) $\succ$ violates Acyclicity.

Therefore, by contradiction, $U(X, \succ)$ satisfies No Prohibition Dilemmas.[50] □

[50] Karl Nygren suggested this proof to me.

4

Absolute Comparability reconsidered and the lexitotal principles

4.1 Absolute Comparability reconsidered

As it turns out, a crucial background assumption for the impossibility results is Absolute Comparability. In virtue of this assumption utilities have two properties. Real numbers satisfy

Completeness: for any real numbers r and r', either r is at least as great as r' or r' is at least as great as r.

So, first, by Absolute Comparability, this means that, for *any* two welfare levels, the one is at least as great as the other or *vice versa*.

Completeness of welfare levels is a controversial assumption.[1] Instead, it is often claimed that there can be incommensurability, i.e. there are two welfare levels such that neither welfare level is higher than the other nor are they exactly equally as high.[2] To take one famous example of incommensurability, compare two careers, a successful musical career with a successful career as a lawyer.[3] These careers can be claimed to be incommensurable in welfare,, because they involve very different values; aesthetic value in the case of the musical career, and the value of protecting the legal rights of fellow citizens in the case of the legal career.[4] And similarly, it might be claimed that these different careers have an effect on the welfare of the person who takes up these careers that make the resulting welfare levels incommensurable.

However, it is hard to see how giving up completeness of welfare levels helps to solve the moderate trade-offs paradoxes. All that is needed for the paradoxes to arise is that in *some* cases where welfare levels and differences are interpersonally comparable. That such comparisons can be done in some cases seems hard to

[1] For discussion of Completeness see e.g. Chang (1997).

[2] See subsection 2.A.5 for a definition of incommensurability for moral betterness. The definition for the individual betterness relation is often defined analogously.

[3] The example is adapted from Raz (1986: 332).

[4] Hsieh (2016: sec 3.1).

deny—independent of the claim that this can *always* be done, i.e. Completeness. And, it seems that such comparisons can be done, at least in principle, in the examples of the sequences that I suggested—life shortening and additional pain.

Next, real numbers also satisfy

> **Archimedeaness:** for any real numbers r and r', if r' is positive, then there is some positive integer k such that k times r' is greater than r.

So, second, by Absolute Comparability, this means that a large enough number of small positive welfare differences add up to a sum total difference greater than any given (finitely) large welfare difference. That this is a crucial assumption can be seen from the proof of Result 3*. First, consider again the illustration of the First Moderate Trade-offs Paradox in figure 3.1 (on page 52). For the application of Minimal Finite Superiority between the first and the last alternative of the sequence, x_0 and x_m, it is necessary that the welfare difference for the best-off people can be arbitrary large, in the illustration 32 (= 81 − 49). Therefore, it is necessary that the level is lowered by that amount over the steps of the sequence from x_0 to x_m. Next, for the application of Minimal Infinite Superiority, it is necessary that in each step the difference in utility for the best-off people can be arbitrarily small, in the illustration .001. Therefore, over the sequence, m times an arbitrarily small difference in welfare for each of the best-off individuals must together amount to an arbitrarily large difference for each of them, in the illustration 32, i.e.

(4.1) there is a positive number m such that $m \cdot 0.001 \geq 32$.

The same holds for the Third Moderate Trade-offs Paradox Sequence and its first subsequence where Super Ultra Minimal Infinite Superiority is repeatedly applied.

As mentioned in section 3.2, Fleurbaey, Tungodden and Vallentyne claim to assume Absolute Comparability only "for the sake of argument". The assumption of Absolute Comparability rather than a *more* limited form of measurability and interpersonal comparability makes possible the formulation of more theories. However, there are still *less* limited forms of measurability and interpersonal comparability. Absolute Comparability rules out, what I call,

> **Infinite Welfare Superiority:** there are types of welfare components such that some amount of one component is better for a person than any amount of the other.[5]

So, contrary to the claim of the authors, Absolute Comparability, and more generally representation of welfare on a single real number scale, introduces also a *constraint* on the range of possible theories. As I will show next, this is especially problematic in the present context since some theories that employ Infinite Welfare Superiority can fulfil all the conditions of the impossibility results.

[5] Mill (1991: 138) calls this "superiority in quality".

4.2 The lexitotal principles solve the Third Moderate Trade-offs Paradox

Next, I will show that, once Absolute Comparability is dropped, the Weak Simple Total Principle solves the Third Moderate Trade-offs Paradox. To prove that all the structural conditions of Result 3* (and also Result 3) are satisfied, i.e. Acyclicity (and Replication Invariance), the Weak Simple Total Principle has to be strengthened from a sufficient to a necessary *and* sufficient condition for betterness. For the precise proof that this principle satisfies all the conditions of Result 3*, see section 4.A. The idea is based on work by Carlson (ms) and Thomas (2018) on impossibility results in population ethics by Arrhenius (2000).

4.2.1 The Weak Simple Lexitotal Principle

For simplicity, assume that welfare consists of only two kinds of welfare components, call them *higher* and *lower pleasures*, which are lexically ordered, i.e.

(4.2) any amount of higher pleasure is better than any amount of lower pleasure,[6]

both intra- and interpersonally, i.e. between a person's possible lives and between different people's possible lives. More precisely, assume

(4.3) welfare consist in two lexically ordered components, higher and lower pleasure only, each separately measurable on an absolute scale,[7]

i.e. amounts of welfare can be represented by a pair of real numbers (h,l) where h is the amount of higher and l the amount of lower pleasures, and

(4.4) amount of welfare (h,l) is *greater than* amount of welfare $\left(h',l'\right)$ if and only if (h,l) is *lexically superior* to $\left(h',l'\right)$,

i.e. if and only if (1) the higher pleasure h is greater than h', *or* (2) the higher pleasures h and h' are equal *and* the lower pleasure l is greater than l'.[8]

For example, consider table 4.1. Person 1's (total) amount of welfare is greater than person 2's (total) amount of welfare since person 1 has a greater amount of higher pleasure (3 is greater than 2), even though person 2 has a greater amount of lower pleasure (10 is greater than 1).

Finally, the Weak Simple Total Principle refers to the sum total of welfare. But this does not rule out the lexical superiority of higher pleasures, for it is natural to assume that

(4.5) the *sum total of welfare* in alternative x is the *component-wise sum* of pleasures in x,

[6] Mill (ibid.: ch. 2) suggests this distinction between higher and lower pleasures.
[7] Again, for example, a ratio scale would suffice, but I will stick to an absolute scale for simplicity.
[8] This is the special case of ordered pairs of the general definition of lexical superiority in 2.5 (on page 26).

Table 4.1 Example of lexically ordered higher and lower pleasure

	Pleasures	
Person	Higher	Lower
1	3	1
2	2	10
Sum	5	11

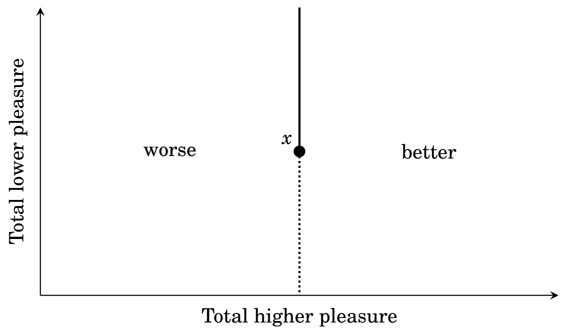

Figure 4.1 Better and worse alternatives according to the Weak Simple Lexitotal Principle

i.e. the sum of higher and lower pleasures calculated separately. In table 4.1, the sum of welfare of persons 1 and 2 with its two components is represented in the last row.

According to the

Weak Simple Lexitotal Principle: for any alternatives x and y, if the sum total of welfare in x is greater than in y, then x is better than y,[9]

i.e. if (1) the amount of higher pleasure of the sum of welfare in x is greater than in y, or (2) the amount of higher pleasure of the sums of welfare are equal in x and y and the amount of lower pleasure of the sum of welfare in x is greater than in y, then x is better than y.

In this framework, alternatives can be represented on the (two-dimensional) plane. Figure 4.1 shows an alternative x. Alternatives better than x are (1) strictly to the right of x, i.e. have greater total higher pleasure, or (2) on the edge strictly above x, i.e. have the same total higher pleasure and strictly greater lower pleasure. Alternatives worse than x are (1) strictly to the left of x, i.e. have lower total higher pleasure, or (2) on the dotted edge strictly below x, i.e. have the same total higher pleasure and strictly lower total lower pleasure.

[9] This is just the Weak Simple Total Principle from section 2.3. The different name is just to make explicit that it operates with lexically ordered welfare components.

4.2.2 The Weak Simple Lexitotal Principle solves the Third Moderate Trade-offs Paradox

The central idea to solve the paradox is to choose particular benefit sizes for Super Ultra Minimal Infinite Superiority. Assume that

(4.6) a welfare amount (h,l) is *positive* if and only if it is greater than $(0,0)$,

i.e. (1) the higher pleasure h is greater than 0, or (2) the higher pleasure h is 0 and the lower pleasure l is greater than 0;

(4.7) the *sufficiently large* positive benefit, b, is some positive amount of *higher* pleasure;

(4.8) the *sufficiently small* positive benefit, b', is some positive amount of *lower* pleasure *only*.

So, the sum of any number of sufficiently large benefits always contains some higher pleasure while the sum of any number of sufficiently small benefits never contains any higher pleasure. So, the former sum will always be greater than the latter sum. With this it can be proven that the Weak Simple Lexitotal Principle fulfils all conditions of Result 3* (and Result 3), of course while dropping Absolute Comparability. Here, I will just demonstrate this at the illustration of the Third Moderate Trade-offs Paradox from section 3.4. Actually, to prove that the structural conditions are satisfied, i.e. Acyclicity (and Replication Invariance), the Weak Simple Lexitotal Principle has to be strengthened from a sufficient to a necessary *and* sufficient condition for betterness. For the precise proof, see section 4.A.

First, consider the illustration of the Infinite Superiority Sequence, repeated in figure 4.2. Under the assumption (4.2) of lexicographic ordered welfare components, the Infinite Superiority Sequence cannot be represented by the box diagrams used earlier since this presupposes measurability and comparability of welfare on a single real number scale, as granted by Absolute Comparability. But I will start with a proxy representation to smoothen the transition. Assume that

(4.9) a positive welfare difference in *at least* an integer in utility is a positive welfare difference in *higher* pleasure;

(4.10) a positive welfare difference in *less than* an integer in utility is a positive welfare difference in *lower* pleasure *only*.

Here, by "utility" I refer to the previous representation of welfare by single real numbers as opposed to the current representation of welfare in terms of two real numbers.

So, starting at x_0, in each step there is a gain in higher pleasure by one of the m people and a loss in only lower pleasure by the m' people. More precisely, there is a gain of 9, hence by (4.9) of higher pleasures, for a worst-off, and there is a very slight loss below 1, hence by (4.10) of lower pleasure only, for each of the m' best-off. Therefore, according to the Weak Simple Lexitotal Principle, in each step the successor is better than its predecessor, independently of how large m' is. And this is also the implication according to Super Ultra Minimal Infinite Superiority.

However, according to the Weak Simple Lexitotal Principle, the combined losses in lower pleasure of the m' people never make up for a loss in higher pleasure. So, the end of the sequence, x_m, which involves a loss of 32, hence by (4.9) of higher pleasures, for each of the m' best-off cannot be reached in this way. This is where the appeal to (4.2), lexicographic ordering of higher and lower welfare, comes in, but would be ruled out by Absolute Comparability as pointed out in section 4.1.

Figure 4.3 represents the Infinite Superiority Sequence more accurately under the current lexically ordered welfare assumptions. The diagram shows the sum total of higher and lower pleasure of alternatives in the sequence. Starting from x_0 there is a gain in the sum total of higher pleasure and a loss in the sum total of lower pleasure in each step. In x_m there is a lower amount of higher pleasure than in x_0. But, by assumption, a loss in higher pleasure cannot be reached by added losses only in lower pleasure no matter how many. Hence, x_m which has a lower sum total of higher pleasure than x_0 cannot be better according to the Weak Simple Lexitotal Principle.

Second, the Weak Simple Lexitotal Principle fulfils Non-Anti Pigou-Dalton (see figures 4.4 and 4.5). This is because a loss by *one* person together with the equal size gain, i.e. same absolute amount of higher and lower pleasures, by each of *more than one* person increases the sum total of welfare. However, according to the Weak Simple Lexitotal Principle,

(4.11) a loss in *higher* pleasure can only be compensated by a greater gain in *higher* pleasure.

This is compatible with Non-Anti Pigou-Dalton because the condition applies only for the equal benefit trade-offs between better-off and worse-off people. But the end of the Non-Anti Pigou-Dalton Sequence, i.e. y'_l, where the better-off have lost more than 8, i.e. higher pleasure, but the worst-off have gained only 0.001, i.e. lower pleasure only, cannot be reached either. This is because, by (4.11), a loss in higher pleasure for one person has always to be compensated by a gain in higher pleasures for persons—even if they are worse off than the one.

In summary, repeated application of Super Ultra Minimal Infinite Superiority and Non-Anti Pigou-Dalton can only lead to a greater difference in higher pleasure for the worst-off than for the best-off. And, in particular, the end of the Third Moderate Trade-offs Paradox, $z_{m+m\cdot m''}$, as illustrated in figure 3.3 (on page 56), where, as compared to the beginning of the sequence, z_0, the m' people have a loss in higher pleasure while the other people have a gain in lower pleasure only cannot be reached in this way. So, Acyclicity is not violated and the inconsistency is resolved. And this solves the paradox in a way that is consistent with the intuitive judgements, (3.39) to (3.41).

Objection

It might be objected that the reduction in lower pleasure along the Infinite Superiority Sequence has to stop at some point because, according to Domain Richness as stated by Fleurbaey, Tungodden and Vallentyne, only "non-negative welfare levels" are permitted. But as the sequence continues to remove the same amount of lower pleasure from the best-off people it inevitably goes to *negative* lower pleasures.

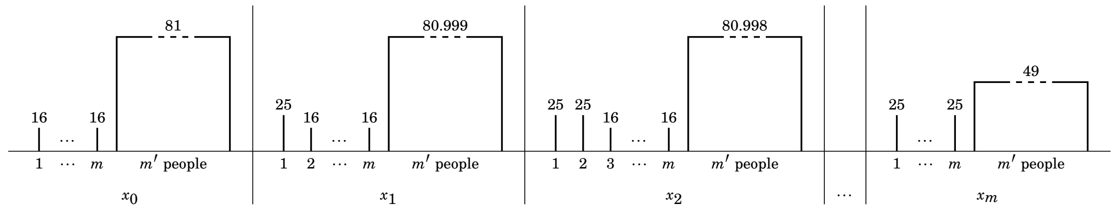

Figure 4.2 Illustration of the Infinite Superiority Sequence (repeated figure 3.5 on page 58)

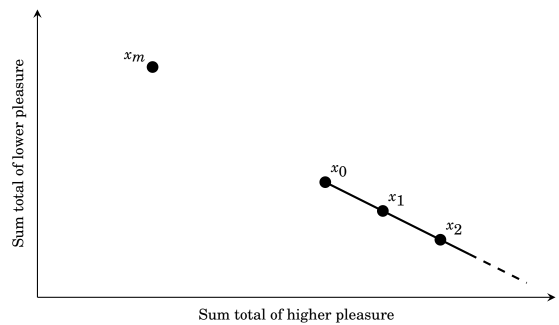

Figure 4.3 Illustration of the Infinite Superiority Sequence according to the Weak Simple Lexitotal Principle

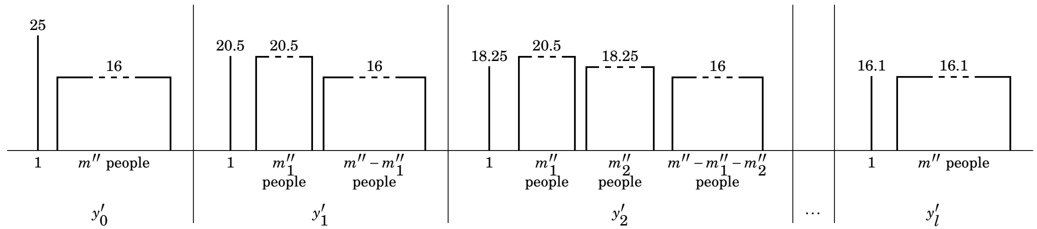

Figure 4.4 Illustration of the Non-Anti Pigou-Dalton Sequence (repeated figure 3.8 on page 62)

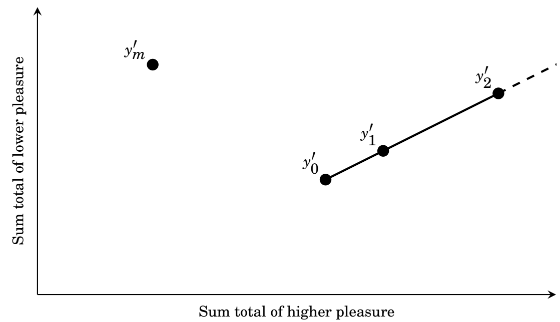

Figure 4.5 Illustration of the Non-Anti Pigou-Dalton Sequence according to the Weak Simple Lexitotal Principle

Table 4.2 The Higher and Lower Pleasure Pigou-Dalton Case

	x		y	
Person	h	l	h	l
1	10	1	10	0
2	0	1	0	2
Sum	10	2	10	2

This objection can be answered in two ways. According to Fleurbaey and Tungodden, "[t]he restriction to non-negative numbers is not important but it helps showing that our results are valid in the presence of a lower bound."[10] So, first, even without the restriction the result would still be important. It would still hold if there is in fact no lower boundary on welfare. And it is at least controversial whether such a lower boundary exists.

And, second, having *no* lower boundary on the *lower* welfare component *if* the higher pleasure is *positive*, is actually compatible with having a lower boundary on, and in particular non-negative, *overall* welfare levels. According to the two-component welfare theory, it is natural to assume that

(4.12) an amount of welfare is *non-negative* if and only if it is positive or equal to (0,0),

i.e., by (4.6), (1) the higher pleasure is positive, or (2) the higher pleasure is 0 and the lower pleasure non-negative.

Next note that, according to Super Ultra Minimal Infinite Superiority, the subtraction of lower pleasure must be possible only if the best-off are sufficiently well off. So, assume that they have *positive* higher pleasure. Therefore, by (4.12), their welfare level will be non-negative no matter how many lower pleasures are subtracted. Therefore, the solution of the Third Moderate Trade-offs Paradox provided by the Weak Simple Lexitotal Principle is compatible with a lower bound on welfare.

I will return to more substantive objections to the Weak Simple Lexitotal Principle in section 4.4.

4.3 Implications for other results

4.3.1 Counterexample to Result 3

The Weak Simple Lexitotal Principle, while it satisfies Non-Anti Pigou-Dalton, does not satisfy the stronger Pigou-Dalton. For example, suppose the two alternatives represented in table 4.2, i.e.

(4.13) in x, person 1 has 10 higher and 1 lower pleasure, and person 2 has no higher and 1 lower pleasure;

[10] Fleurbaey and Tungodden (2010: fn. 2).

(4.14) in y, person 1 has 10 higher and no lower pleasure, and person 2 has no higher and 2 lower pleasure.

So,

(4.15) person 1 is at a higher welfare level than person 2 (in both x and y);

(4.16) person 1 is better off by 1 lower pleasure in x compared to y;

(4.17) person 2 is better off by 1 lower pleasure in y compared to x.

Therefore, according to Pigou-Dalton,

(4.18) y is better than x.

In contrast, the Weak Simple Lexitotal Principle is *silent*, since the sum totals of welfare in x and y are equal. So, the Weak Simple Lexitotal Principle has to be extended in order to capture Pigou-Dalton.

Note that the Weak Simple Lexitotal Principle cannot be simply extended so that it satisfies Pigou-Dalton, by applying non-relational finite priority-weights to each of the welfare components, higher and lower pleasures, separately, analogue to the case of the Weak Simple Priority Total Principle. To the contrary, if priority-weights are applied separately, as just suggested, then, in the last example, person 1 will get priority over person 2 in getting a unit of lower pleasures since person 1 is worse off in y than person 2 in x with respect to the *lower* pleasure component. Hence this principle will imply that

(4.19) x is better than y.

But this *violates* the implication of Pigou-Dalton, (4.18).

However, the Weak Simple Lexitotal Principle can be extended in another way so that it satisfies Pigou-Dalton. Note that Pigou-Dalton applies only to alternatives where the sum total of welfare is equal. So, a simple extension of the Weak Simple Lexitotal Principle to satisfy Pigou-Dalton is to add Weak Leximin, which satisfies Pigou-Dalton, as a tie breaker in this case. More precisely, according to the

Weak Leximin Lexitotal Principle: for alternatives x and y, if

1. the sum total of welfare in x is greater than in y,

or

1. the sum total of welfare in x and y is equal, and the worst-off person in x is better off than the worst-off person in y, and in case of ties, the second worst-off in x is better off than the second worst-off in y, etc.,

then x is morally better than y.

Consider again the case from above in table 4.2. The sum totals of welfare in x and y are equal. Therefore, by the tie breaking condition, the Weak Leximin Lexitotal Principle, condition 1, the implication by Pigou-Dalton follows, i.e. (4.18) y is better than x, since person 1, the worse-off in x and y, is better off in y than in x.

Note that the Weak Leximin Lexitotal Principle implies the Weak Simple Lexitotal Principle. Hence, the other betterness rankings reached in subsection 4.2.2 still hold. So, the Weak Leximin Lexitotal Principle solves the Third Moderate Trade-offs Paradox as well. And, in particular, the Pigou-Dalton Sequence is handled in the same way as the Non-Anti Pigou-Dalton Sequence.

4.3.2 Counterexample to all other axiological impossibility results by Fleurbaey et al.

The Weak Leximin Lexitotal Principle (and the Weak Simple Lexitotal Principle) does not satisfy all of the substantive trade-off conditions of the other axiological results, e.g. Result 1*, since they use the stronger versions of infinite superiority, e.g. Minimal Infinite Superiority, which does not include a "sufficiently large benefit size" for the worst-off but holds for *any* benefit for the worst-off.

However, at least in principle, the types of pleasure can be extended to meet these conditions as well. Above, in section 4.2, a rather simple model with only two welfare components, higher and lower pleasure, was assumed. Instead, assume that pleasure consists of further welfare components again lexically ordered. More precisely, assume that there is an *infinite* number of welfare components, u^1, u^2, u^3, $\ldots$, and that,

(4.20) for all $i > j$, any amount of u^i is better than any amount of u^j.

For clarity, I will write the ordered components u^1, u^2, $u^3, \ldots$ as an ordered infinite list $\boldsymbol{u}$, i.e.

$$\boldsymbol{u} = \left(u^1, u^2, u^3, \ldots\right).$$

Next, we need to adapt *lexically superiority* in (4.4) accordingly. Assume the following replacement of (4.4),

(4.21) for amounts of welfare $\boldsymbol{u} = \left(u^1, u^2, u^3, \ldots\right)$ and $\boldsymbol{u}' = \left(u'^1, u'^2, u'^3, \ldots\right)$, u is greater than u' if and only if u is lexically superior to u'.[11]

Now, the restriction to a "sufficiently large benefit size" can be lifted as follows. First, take any *positive* welfare difference $\boldsymbol{b} = \left(b^1, b^2, b^3, \ldots\right)$, i.e. there is an integer i such that $b^i > 0$ and, for all $j < i$, $b^j = 0$. For example, $\boldsymbol{b} = (0, 4, 1, \ldots)$ is such a positive welfare difference. Second, construct the positive welfare difference $\boldsymbol{b}' = \left(b'^1, b'^2, b'^3, \ldots\right)$ accordingly with $b'^{i+1} = 1$ and, for all $j < i$, $b'^j = 0$.[12] Then, $\boldsymbol{b}$ is greater than any number of $\boldsymbol{b}'$ added to itself. For example, the positive welfare difference $\boldsymbol{b} = (0, 4, 1, \ldots)$ is greater than any number of welfare differences $\boldsymbol{b}' = (0, 0, 1, \ldots)$ added to itself because, for any real number r, $\boldsymbol{b}$ is lexically superior to $r \cdot \boldsymbol{b}' = (0, 0, r, \ldots)$.

[11] See the definition of lexical superiority in section 2.5.

[12] The existence of such a $\boldsymbol{b}'$ for every $\boldsymbol{b}$ presupposes an infinite number of welfare components.

4.3.3 Counterexample to the deontic impossibility results: the Deontic Simple Lexitotal Principle

As shown in section 4.A, the (non-weak) Simple Lexitotal Principle fulfils all the conditions of Result 3*. So, by the Undominanted Choice Connection, the deontic analogues of these conditions are fulfilled by the deontic analogue of Simple Lexitotal Principle,

> **Deontic Simple Lexitotal Principle:** a feasible alternative is permissible if and only if no feasible alternative has a greater sum total of welfare,

i.e., a feasible alternative is permissible if and only if, for all feasible alternatives y, y is *not* morally better than x according to the Weak Simple Lexitotal Principle, i.e. the sum total of y is *not* lexically superior to the sum total of x.

The Simple Lexitotal Principle implies all the conditions of Result 3*. And, as shown in section 3.5, each condition implies its deontic analogue via the Undominanted Choice Connection. Hence, the deontic analogue of the Simple Lexitotal Principle via the Undominanted Choice Connection, Deontic Simple Lexitotal Principle, implies all the deontic analogues. In particular, the Deontic Simple Lexitotal Principle also solves the Deontic Third Moderate Trade-offs Paradox.

4.3.4 Other results

Section 4.B shows that the Simple Lexitotal Principle fulfils a number of conditions of another important result by Blackorby, Bossert and Donaldson (2005) and that their justification for restricting their discussion to Absolute Comparability is similarly wanting as in the case of Fleurbaey, Tungodden and Vallentyne.

4.4 Objections

4.4.1 Infinite superiority for the best off, and the Non-Anti Pigou-Dalton Paradox

It might be objected that the Weak Simple Lexitotal Principle implies too strong infinite superiority. Consider a case where,

(4.22) in x, one *best-off* person benefits from an additional unit of higher pleasure;

(4.23) in y, *a very large number of worst-off* persons each benefit from an additional unit of lower pleasure.

According to the Weak Simple Lexitotal Principle,

(4.24) x is better than y.

More generally, the Weak Simple Lexitotal Principle violates the condition that

(4.25) there is a number of worse-off people such that benefiting these people slightly matters more than benefiting one better-off person significantly.

More precisely, it violates

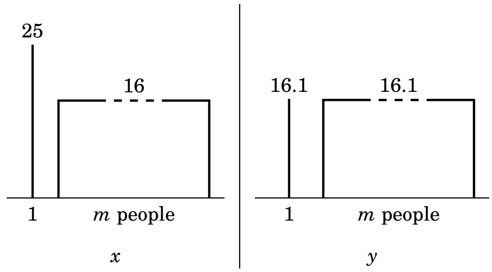

Figure 4.6 Illustration of Strong Infinite Superiority and Strong Infinite Superiority

Very Strong Finite Superiority: for any welfare levels w and w' such that w is lower than w', and positive welfare differences b and b', there exists a number of people m such that, for all alternatives x and y, if

1. at least m people are at or below w in x and better off by b in y compared to x;
2. one person is at or above w' in y and is better off by b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x.

A special case of Very Strong Finite Superiority is illustrated in figure 4.6. There are

(4.26) m people at level 16 in x and better off by 0.1 in y compared to x;

(4.27) one person at level 16.01 in y and better off by 9 in x compared to y.

Therefore, if the number m is large enough, according to Very Strong Finite Superiority,

(4.28) y is better than x.

And note that this holds

(4.29) independent of how small the gain for the m people is.

Both the Weak Simple Lexitotal Principle and the Weak Leximin Lexitotal Principle violate Very Strong Finite Superiority.

Are there independent considerations in favour of Very Strong Finite Superiority? First, given Absolute Comparability, the total sum of utility counts in favour of Very Strong Finite Superiority.[13] However, as shown above in subsection 4.2.2, this claim does not follow when Absolute Comparability is rejected.

[13] Another name for Very Strong Finite Superiority for a similar condition in the population ethics literature is *Inequality Aversion*, see Arrhenius (2000: 93). As Thomas (2018) points out "Inequality Aversion" seems to be a misnomer because it is not necessary to appeal to any idea of equality, not even priority for the worse-off, since the Weak Simple Total Principle implies it.

How intuitively plausible is Very Strong Finite Superiority? Consider again Scanlon's Jones and the Transmitter from chapter 1. Presumably, we are supposed to assume that Jones is worse-off than the viewers or their welfare levels are (roughly) the same, i.e. Jones being rescued is worse-off than the viewers not being able to watch the World Cup, or they are at roughly the same level. But one might go even further. Consider Temkin's, what I call,

Better-off Jones and the Transmitter:

> Now suppose we add some details to [Jones and the Transmitter]. [...] First, imagine that the match will continue for three hours, so that Jones will suffer three hours of extremely painful shocks unless we turn off the transmitter. Next, imagine that among the worst-off people watching the game are 100 who have a chronic illness that produces pain of the same intensity as Jones is suffering while being shocked. In particular, imagine that each minute of pain that Jones will endure will produce the same degree of suffering in him, and that that is the same as the degree of suffering felt by those with the chronic illness during each suffering felt by those with the chronic illness during each minute that they are in pain. Imagine further that the 100 with the chronic illness are such rabid fans that watching the game distracts them from their pain, effectively serving as a psychological anesthetic. Finally, imagine that one only needs to turn the transmitter off for two minutes to rescue Jones.[14]

Temkin claims, and I agree, that it still seems that it is better to help Jones. More generally, the principle behind this is, roughly, that

(4.30) it is better to consolidate benefits (and spread out burdens thinly) than to spread out benefits thinly (and consolidate burdens).[15]

More precisely, according to

Strong Infinite Superiority: for any welfare levels w and w', there are positive welfare differences b and b' such that b is greater than b' such that, for any number of people m', and alternatives x and y, if

1. one person is at or below w in x and better off by at least b in y compared to x;
2. m' people are at or above w' in y and better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x.

So, in contrast to the minimal infinite superiority conditions, Minimal Infinite Superiority and Super Ultra Minimal Infinite Superiority, infinite superiority holds even in cases where the many are worse off, i.e. it is sometimes better if one best-off benefits rather than *any* number of worst-off.

Consider again figure 4.6. Assume, for Strong Infinite Superiority,

(4.31) w and w' are 16 and 16.1, respectively;

(4.32) b and b' are 8.9 $(= 25 - 16.1)$ and .1, respectively.

Therefore,

[14] Temkin (2012: 71-2) and Temkin (2005: 220-1).
[15] This is roughly what Temkin (2012: 67-8) calls the Minimize Great Additional Burdens View.

(4.33) x is better than y.

This contradicts (4.28). And it is intuitive that 9 years more lifetime for one person is better than a couple of hours for any number of people *even if* they are worse off than this person would have been without the benefit. Both the Weak Simple Lexitotal Principle and the Weak Simple Lexitotal Principle satisfy Strong Infinite Superiority.

It might be objected that Strong Infinite Superiority, while it might seem acceptable at first, leads to paradoxes. In particular, consider the Non-Anti Pigou-Dalton Sequence (subsection 3.4.2). Note that the first and last alternatives in the sequence, y'_0 and y'_l, have the same welfare distribution as x and y in figure 4.6. Hence according to Strong Infinite Superiority, given the previous assumptions,

(4.34) y'_0 is better than y'_l.

Hence, we get another paradox, I call the

Non-Anti Pigou-Dalton Paradox: there is a number of people m'', such that

1. (3.22) y'_i is better than y'_{i-1}, for $i = 1, \ldots, l$;
2. (4.34) y'_0 is better than y'_l.

However, in reply it can be claimed that, similarly to the intuitive judgement in the Third Moderate Trade-offs Paradox (section 3.6), it is intuitive that

(4.35) successive alternatives are better than their predecessor at first, starting with y'_1 is better than y'_0;

(4.36) y'_0 is better than y'_l;

(4.37) there is a non-dominated alternative better than both y'_0 and y'_l.

And, as for the Third Moderate Trade-offs Paradox, the Weak Simple Lexitotal Principle captures these verdicts as seen in subsection 4.2.2.

Next, it might be objected that the Weak Simple Lexitotal Principle does not give sufficient priority to the worse off because it implies the even stronger

Extreme Strong Infinite Superiority: there are positive welfare differences b and b' such that, for any welfare levels w and w', number of people m', and outcomes x and y, if

1. one person is at w in x and better off by at least b in y compared to x;
2. m' people are at w' in y and better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x.

While Strong Infinite Superiority leaves open the possibility that the size of benefit, b', can depend on the welfare levels w and w', i.e. how well off and worse-off the people who stand to benefit are, Extreme Strong Infinite Superiority considers those levels irrelevant.

And Extreme Strong Infinite Superiority is implied by the Weak Simple Lexitotal Principle because higher pleasures are better than any number of extra units of lower pleasure independently of the welfare levels of people who receive those benefits. For example, one person might already have an enormous amount of higher pleasures, still each extra higher pleasure to this person is better than any number of lower pleasure for each of any number of people that have no pleasures whatsoever.

And, because the Weak Leximin Lexitotal Principle implies the Weak Simple Lexitotal Principle, the Weak Leximin Lexitotal Principle implies Extreme Strong Infinite Superiority, too. There may be other variants of the Weak Simple Lexitotal Principle that can avoid Extreme Strong Infinite Superiority. But I have to leave this for further research.

4.4.2 The scope of the lexitotal principles

Furthermore, it might be objected that the Weak Simple Lexitotal Principle seems not to solve all instances of the Third Moderate Trade-offs Paradox, since an appeal to different welfare components is implausible in some cases.

Consider again the original welfare numbers in the Third Moderate Trade-offs Paradox Sequence. Assume them to correspond to life years. .001 life years amount to a couple of hours.[16] It seems plausible to me that a gain in 9 years for one worse-off person outweighs a loss in a couple of hours for any number of better-off people. In order to explain this, it seems the Weak Simple Lexitotal Principle has to claim that 9 years is a loss in higher welfare components while a couple of hours is a loss in lower welfare components only. But surely, multiple losses of a couple of hours for each of the better-off people along the sequence amounts to 32 years for each. But if 9 years is a loss in higher welfare components, so is a loss in 32 years. So, it seems that losses in lower welfare components add up to losses in higher welfare components. But this what the lexitotal principles deny.

It might be replied that extending or shortening lives more and more might at some point actually add or remove certain higher goods from a life, like friendships, achievements, etc. So, this case would be unlike one where adding small differences in lower pleasure never adds up to a difference in higher pleasure, i.e. there is a small difference in lifetime which amounts to a large difference in welfare (though there might be no specific point at which this change occurs due to vagueness). So, this would not be an instance of the paradox.

Now, to stay away from this possible complication, consider only lives that do not have higher pleasures. Now ask: Can a loss of many years still make such a live so much worse that it outweighs the loss of a day for any number of other people?

First, it is important to consider the right kind of being. Most plausible are cases where the life of the being extends in an important way over time. For example, some fish might not be such beings. Arguably, some fish are beings living pretty much only in the "here and now" with almost no memories of the past and desires into the future. Hence, arguably, but this is highly controversial, such beings would lose, in the important sense, as much by being killed in a year or in two years because it

[16] Again nothing hinges on this choice of particular numbers.

does not have any interests extending further into the future.[17] (Obviously, such a being might still be capable of serious momentary suffering.)

So, let us take a rather normal being like most of us (though some non-human animals might qualify as well). Now, assume that this person does not take much interest in friendships, achievements, etc. This person enjoys the pleasure from good health and food, and nice sunsets, and that's pretty much it. We might think that such a life has not much good in it. Still this being is happy and looks with pleasure back on all her memories and forward to experiences in the future. Such a being loses out by having a day cut off her life. But no higher pleasure (over and above what is experienced at each day) seems to be lost at any cut.

Do we think that cutting such a life short by 40 years cannot be outweighed by avoiding cutting short similar lives of any number of such persons by a day? I am inclined to affirm this. But I admit that the intuition might be a bit less strong than in more usual cases.

Here is another case that might be more persuasive. It appeals to different durations of pain. Assume that everyone lives an equally normal life with the same ups and downs in higher and lower pleasures. Suppose these lives have to be extended in either of two possible ways. Either

(4.38) one person's life gets extended by horrible torture for a long period, say many years,

or

(4.39) many people's lives are extended by horrible torture but only for a short time, say a couple of hours.

I think that many people will be inclined to think that the latter is better than the former alternative no matter how many people are tortured each for a short enough time. And this can be captured by minimal infinite superiority.

However, it is not clear to me how this is possible to capture by an appeal to different kinds of welfare components because it seems that the lives contain the same welfare components. The only difference is the *amount* of torture rather than its *kind*.

It might be objected that there *are* lexically ordered welfare kinds involved:

(4.40) many years of horrible torture involve a higher kind of ill-fare than a short duration of horrible torture.

And, in general, it might be suggested that all similar cases can be explained by different kinds of welfare.

However, I do not think this reply is satisfactory. Remember that the original problem was to find a satisfactory answer to instances of the moderate trade-off paradoxes. But if it is claimed that many years of torture are ill-fare of a different kind, then *at some point* along the paradox sequences,

[17] Singer (1993: 95) discusses a similar desire satisfaction account to explain the special significance of the right to life.

(4.41) *one* additional day of torture (for the m' better-off people) is not (or at least not determinately) better than *many* years of torture for one worse-off.

This is because, the reply claims, all the days of torture for the better-off will at some point have added up to many years of torture and there are many people who each suffer these many years of torture. Hence, according to the Weak Simple Lexitotal Principle, these many years of torture to each of these many people will together have a greater disvalue than the many years of torture to the one. But intuitively, (4.41) is implausible.

In any case, it seems to me that (4.41) is a too general response. According to the Weak Simple Lexitotal Principle, the context of the paradox sequence is irrelevant to the evaluation of alternatives. *Given the other feasible alternatives in the sequence*, (4.41) is compatible with the plausible verdict that I suggested in section 3.6. But, according to Weak Simple Lexitotal Principle,

(4.42) there is a pair of successive alternatives from the sequence such that (4.41) holds *even if there are no other feasible alternatives*.

But this last claim contradicts minimal infinite superiority in a very simple case with only two alternatives. If there are any cases in which we should be able to trust intuitions concerning minimal infinite superiority it seems to be simple cases like this.

And I do not see how this problem can be circumvented by a more sophisticated version of the Weak Simple Lexitotal Principle. For example, introducing additional kinds of welfare will only have the effect that there are *more* successive pairs such that (4.41) holds which rather than lessen the problem makes it arguably even more severe.

So, tentatively, I suggest that an appeal to Infinite Welfare Superiority is insufficient to solve the paradox, at least in the whole range of cases. And that the assumption of Absolute Comparability is justified, at least for simplicity when discussing solutions to the just discussed instances of the moderate trade-offs paradoxes.

So, what other solutions to the paradoxes are possible? In the next chapter, I will discuss and reconsider the substantive moral conditions and argue that they might be over-generalizations.

4.A The Simple Lexitotal Principle fulfils the conditions of Result 3*

The Weak Simple Lexitotal Principle is based on lexically ordered welfare components. First, I introduce the underlying lexical superiority relation. For simplicity, I will focus on the simple two-component version, i.e., for all $\boldsymbol{v} = (v_1, v_2), \boldsymbol{v}' = \left(v_1', v_2'\right) \in \mathbb{R}^2$,

$$\boldsymbol{v} >_{\text{lex}} \boldsymbol{v}' \mathrel{\dot{\Longleftrightarrow}} v_1 > v_1' \vee \left(v_1 = v_1' \wedge v_2 > v_2'\right).$$

The multi-component version is a simple but tedious extension which I will skip here. First, note that

(4.43) for $\boldsymbol{v}, \boldsymbol{v}' \in \mathbb{R}^2$,

$$\boldsymbol{v} \not>_{\text{lex}} \boldsymbol{v}' \wedge \boldsymbol{v}' \not>_{\text{lex}} \boldsymbol{v} \iff \boldsymbol{v} = \boldsymbol{v}'.$$

Proof.

$$\begin{aligned}
& \boldsymbol{v} \not>_{\text{lex}} \boldsymbol{v}' \wedge \boldsymbol{v}' \not>_{\text{lex}} \boldsymbol{v} \\
\iff & v_1 \not> v_1' \wedge \left(v_1 \neq v_1' \vee v_2 \not> v_2'\right) \wedge v_1' \not> v_1 \wedge \left(v_1' \neq v_1 \vee v_2' \not> v_2\right) \\
\iff & \underbrace{v_1 \not> v_1' \wedge v_1' \not> v_1}_{\iff v_1 = v_1'} \\
& \wedge \Big(\underbrace{\cancel{\left(v_1 \neq v_1' \wedge v_1' \neq v_1\right)}}_{\lightning v_1 = v_1'} \vee \underbrace{\cancel{\left(v_1 \neq v_1' \wedge v_2' \not> v_2\right)}}_{\lightning v_1 = v_1'} \\
& \quad \vee \underbrace{\cancel{\left(v_2 \not> v_2' \wedge v_1' \neq v_1\right)}}_{\lightning v_1 = v_1'} \vee \underbrace{\left(v_2 \not> v_2' \wedge v_2' \not> v_2\right)}_{\iff v_2 = v_2'} \Big) \\
\iff & v_1 = v_1' \wedge v_2 = v_2' \\
\iff & \boldsymbol{v} = \boldsymbol{v}'
\end{aligned}$$

□

So, in particular,

$$\begin{aligned}
\boldsymbol{v} =_{\text{lex}} \boldsymbol{v}' &\iff \boldsymbol{v} = \boldsymbol{v}' \\
\boldsymbol{v} \geq_{\text{lex}} \boldsymbol{v}' &\iff \boldsymbol{v} = \boldsymbol{v}' \vee \boldsymbol{v} >_{\text{lex}} \boldsymbol{v}'
\end{aligned}$$

Next, it is shown that lexical superiority satisfies a number of structural conditions.

Proposition 4.1. *$>_{lex}$ is a strict complete pre-ordering,*

i.e. $>_{\text{lex}}$ satisfies Irreflexivity, Transitivity and Completeness.

Proof. Let $\boldsymbol{v} = (v_1, v_2), \boldsymbol{v}' = \left(v_1', v_2'\right), \boldsymbol{v}'' = \left(v_1'', v_2''\right) \in \mathbb{R}^2$. Irreflexivity: Assume, for contradiction, $\boldsymbol{v} >_{\text{lex}} \boldsymbol{v}$. Note that

$$\begin{aligned}
& \boldsymbol{v} >_{\text{lex}} \boldsymbol{v} \\
\iff & v_1 > v_1 \vee (v_1 = v_1 \wedge v_2 > v_2) \\
\implies & v_1 > v_1 \vee v_2 > v_2.
\end{aligned}$$

Therefore, by contradiction,

$$\boldsymbol{v} \not>_{\text{lex}} \boldsymbol{v}.$$

Transitivity:

$$\begin{aligned}
& \boldsymbol{v} >_{\text{lex}} \boldsymbol{v}' \wedge \boldsymbol{v}' >_{\text{lex}} \boldsymbol{v}'' \\
\iff & \left(v_1 > v_1' \vee \left(v_1 = v_1' \wedge v_2 > v_2'\right)\right) \\
& \wedge \left(v_1' > v_1'' \vee \left(v_1' = v_1'' \wedge v_2' > v_2''\right)\right)
\end{aligned}$$

$$
\begin{aligned}
&\iff \underbrace{\left(v_1 > v_1' \wedge v_1' > v_1''\right)}_{\implies v_1 > v_1''} \vee \underbrace{\left(v_1 > v_1' \wedge v_1' = v_1'' \wedge v_2' > v_2''\right)}_{\implies v_1 > v_1''} \\
&\quad \vee \underbrace{\left(v_1 = v_1' \wedge v_2 > v_2' \wedge v_1' > v_1''\right)}_{\implies v_1 > v_1''} \\
&\quad \vee \underbrace{\left(v_1 = v_1' \wedge v_2 > v_2' \wedge v_1' = v_1'' \wedge v_2' > v_2''\right)}_{\implies v_1 = v_1'' \wedge v_2 > v_2''} \\
&\implies v_1 > v_1'' \vee \left(v_1 = v_1'' \wedge v_2 > v_2''\right) \\
&\iff \boldsymbol{v} >_{\text{lex}} \boldsymbol{v}''
\end{aligned}
$$

Completeness: follows immediately from (4.43). □

Proposition 4.2. *$>_{lex}$ satisfies Asymmetry.*

Proof. Follows directly from propositions 2.2 and 4.1. Here is also a direct proof:

$$
\begin{aligned}
&\boldsymbol{v} >_{\text{lex}} \boldsymbol{v}' \\
&\iff v_1 > v_1' \vee \left(v_1 = v_1' \wedge v_2 > v_2'\right) \\
&\implies \underbrace{\left(v_1 > v_1' \vee v_1 = v_1'\right)}_{\implies (v_1' \geq v_1)} \wedge (\underbrace{v_1 > v_1'}_{\implies (v_1' \neq v_1)} \vee \underbrace{v_2 > v_2'}_{\implies (v_2' \not> v_2)}) \\
&\implies \neg\left(v_1' > v_1\right) \wedge \neg\left(v_1' = v_1 \wedge v_2' > v_2\right) \\
&\iff \neg\left(v_1' > v_1 \vee \left(v_1' = v_1 \wedge v_2' > v_2\right)\right) \\
&\iff \boldsymbol{v}' \not>_{\text{lex}} \boldsymbol{v}
\end{aligned}
$$

□

Proposition 4.3. *$>_{lex}$ satisfies Acyclicity.*

Proof. Follows directly from propositions 2.3 and 4.1 and 4.2. □

Define standard component-wise addition, the additively neutral element, the additively inverse element, and scalar multiplication on $\mathbb{R}^2$, i.e., for all $\boldsymbol{v} = (v_1, v_2), \boldsymbol{v}' = \left(v_1', v_2'\right) \in \mathbb{R}^2$, and $r \in \mathbb{R}$,

$$
\begin{aligned}
\boldsymbol{v} + \boldsymbol{v}' &\doteq \left(v_1 + v_1', v_2 + v_2'\right); \\
\boldsymbol{0} &\doteq (0,0); \\
-\boldsymbol{v} &\doteq (-v_1, -v_2); \\
r\boldsymbol{v} &\doteq (rv_1, rv_2).
\end{aligned}
$$

Note that

(4.44) for $\boldsymbol{v}, \boldsymbol{v}' \in \mathbb{R}^2$ with $\boldsymbol{v}' >_{\text{lex}} \boldsymbol{0}$,

$$\boldsymbol{v} + \boldsymbol{v}' >_{\text{lex}} \boldsymbol{v}.$$

Proof. Assume

(4.45) $\boldsymbol{v} + \boldsymbol{v}' = \left(v_1 + v_1', v_2 + v_2'\right);$

(4.46) $\boldsymbol{v}' >_{\text{lex}} \mathbf{0} \iff v'_1 > 0 \lor \left(v'_1 = 0 \land v'_2 > 0\right)$.

1. If $v'_1 > 0$, then $v_1 + v'_1 > v_1$.
2. If $v'_1 = 0$ and $v'_2 > 0$, then $v_1 + v'_1 = v_1$ and $v_2 + v'_2 > v_2$.

Therefore, $\boldsymbol{v} + \boldsymbol{v}' >_{\text{lex}} \boldsymbol{v}$. □

With the preliminaries above, we are in a position to precisely formulate the Simple Lexitotal Principle and proof that it satisfies the conditions of Result 3*. For $i \in N$, and $x \in O$, denote the higher and lower pleasure of i in x by $h_i(x) \in \mathbb{R}$ and $l_i(x) \in \mathbb{R}$, respectively. Represent the welfare of i in x by

$$\boldsymbol{u}_i(x) = (h_i(x), l_i(x_i)) \in \mathbb{R}^2.$$

For $x, y \in O$, and $i, j \in N$,

$$\boldsymbol{u}_i(x) \text{ is higher than } \boldsymbol{u}_j(y) :\iff \boldsymbol{u}_i(x) >_{\text{lex}} \boldsymbol{u}_j(y).$$

Denote the distribution of welfare in x by

$$\begin{aligned}\boldsymbol{u}(x) &= (\boldsymbol{u}_i(x))_{i\in N} = (u_1(x), \ldots, u_n(x)) \\ &= (h_i(x), l_i(x))_{i\in N} = ((h_1(x), l_1(x)), \ldots, (h_n(x), l_n(x))) \in \mathbb{R}^{2n}.\end{aligned}$$

According to the

Simple Lexitotal Principle: for all $x, y \in O$,

$$\boldsymbol{v}(x) = \sum_{i\in N} \boldsymbol{u}_i(x)$$

and

$$\boldsymbol{v}(x) >_{\text{lex}} \boldsymbol{v}(x) \iff x \succ y,$$

where, by the standard component-wise addition,

$$\begin{aligned}\sum_{i\in N} \boldsymbol{u}_i(x) &= \left(\sum_{i\in N} h_i(x), \sum_{i\in N} l_i(x)\right) \\ &= (h_1(x) + \ldots + h_n(x), l_1(x) + \ldots + l_n(x)) \in \mathbb{R}^2.\end{aligned}$$

First, the structural conditions are proved.

Proposition 4.4. *The Simple Lexitotal Principle satisfies all conditions of Result 3*.*

Proof. Follows from propositions 4.5, 4.6 and 4.9. □

Proposition 4.5. *The Simple Lexitotal Principle satisfies Acyclicity.*

Proof. Follows from proposition 4.3. □

Next, consider Replication Invariance. For $x \in O$, and $k \in \mathbb{Z}_{>0}$, denote the *k-replication* of x by $k * x$, i.e.

$$\boldsymbol{u}(k * x) \doteq (\underbrace{\boldsymbol{u}_1(x), \ldots, \boldsymbol{u}_n(x), \boldsymbol{u}_1(x), \ldots, \boldsymbol{u}_n(x), \ldots, \boldsymbol{u}_1(x), \ldots, \boldsymbol{u}_n(x)}_{k\text{-times}}).$$

Proposition 4.6. *The Simple Lexitotal Principle satisfies Replication Invariance,*

i.e., according to the Simple Lexitotal Principle, for all $x, y \in O$, if $x \succ y$, then $k * x \succ k * y$.

Proof. Assume,

(4.47) $x \succ y$.

Therefore, by the Simple Lexitotal Principle,

(4.48) $\sum_{i \in N} \boldsymbol{u}_i(x) >_{\text{lex}} \sum_{i \in N} \boldsymbol{u}_i(y)$.

By propositions 4.7 and 4.8,

$$\begin{aligned} \sum_{i \in N} \boldsymbol{u}_i(k * x) &= k \cdot \sum_{i \in N} \boldsymbol{u}_i(x) && \text{[by proposition 4.7]} \\ &>_{\text{lex}} k \cdot \sum_{i \in N} \boldsymbol{u}_i(y) && \text{[by (4.48) and proposition 4.8]} \\ &= \sum_{i \in N} \boldsymbol{u}_i(k * y). && \text{[by proposition 4.7]} \end{aligned}$$

Therefore, by the Simple Lexitotal Principle,

(4.49) $k * x \succ k * y$. □

Proposition 4.7. *For all $x \in O$, and $k \in \mathbb{R}$,*

$$\sum_{i \in N} \boldsymbol{u}_i(k * x) = k \cdot \sum_{i \in N} \boldsymbol{u}_i(x).$$

Proof. Let $x \in O$, and $k \in \mathbb{R}$. Then

$$\begin{aligned} \sum_{i \in N} \boldsymbol{u}_i(k * x) &= \left(k \cdot \sum_{i \in N} h_i(x), k \cdot \sum_{i \in N} l_i(x)\right) \\ &= k\left(\sum_{i \in N} h_i(x), \sum_{i \in N} l_i(x)\right) \\ &= k \cdot \sum \boldsymbol{u}_i(x). \end{aligned}$$ □

Proposition 4.8. *For all $\boldsymbol{v}, \boldsymbol{v}' \in \mathbb{R}^2$, and $r \in \mathbb{R}$,*

$$\boldsymbol{v} >_{lex} \boldsymbol{v}' \implies r\boldsymbol{v} >_{lex} r\boldsymbol{v}'.$$

Proof. Let $\boldsymbol{v} = (v_1, v_2), \boldsymbol{v}' = (v_1', v_2') \in \mathbb{R}^2$ with $\boldsymbol{v} >_{\text{lex}} \boldsymbol{v}'$, and $r \in \mathbb{R}$. There are two cases to consider:

1. If $v_1 > v_1'$, then $rv_1 > rv_1'$, then $r\boldsymbol{v} >_{\text{lex}} r\boldsymbol{v}$.
2. If $v_1 = v_1'$ and $v_2 > v_2'$, then $rv_1 = rv_1'$ and $rv_2 > rv_2'$, then $r\boldsymbol{v} >_{\text{lex}} r\boldsymbol{v}$.

□

Next, the substantive conditions are proved. Here is the analogue of the precise statement of the simple general form of many conditions, see figure 2.2. It can be formalized using a relation $B \subseteq O^2 \times (\mathbb{Z}_{>0} \times \mathbb{R}^2 \times \mathbb{R}^2)^2$ with

(4.50) $\boldsymbol{B}(x, y, \langle m, \boldsymbol{w}, \boldsymbol{b}\rangle, \langle m', \boldsymbol{w}', \boldsymbol{b}'\rangle) \iff$ for all $M, M' \subseteq N$,

B1. $|M| \geq m$, and, for all $i \in M$,

$$\boldsymbol{u}_i(x) \leq_{\text{lex}} \boldsymbol{w} \text{ and } \boldsymbol{u}_i(y) \geq_{\text{lex}} \boldsymbol{u}_i(x) + \boldsymbol{b};$$

B2. $|M'| \leq m'$, and, for all $i \in M'$,

$$\boldsymbol{u}_i(y) \geq_{\text{lex}} \boldsymbol{w}' \text{ and } \boldsymbol{u}_i(x) \leq_{\text{lex}} \boldsymbol{u}_i(y) + \boldsymbol{b}';$$

B3. for all $i \in N - (M \cup M')$,

$$\boldsymbol{u}_i(x) = \boldsymbol{u}_i(y).$$

Accordingly, the conditions can be re-stated concisely. For example, according to

Minimal Finite Superiority: for all $x \in O$, $\boldsymbol{w}, \boldsymbol{w}' \in \mathbb{R}^2$, there are $\boldsymbol{b}, \boldsymbol{b}' \in \mathbb{R}^2$ such that $\boldsymbol{b} >_{\text{lex}} \boldsymbol{b}'$ such that, for all $y \in O$,

$$\boldsymbol{B}(x, y, \langle 1, \boldsymbol{w}, \boldsymbol{b}\rangle, \langle 1, \boldsymbol{w}', \boldsymbol{b}'\rangle) \implies y \succ x.$$

Non-Anti Pigou-Dalton: for all $x, y \in O$, $\boldsymbol{b} \in \mathbb{R}^2$, and $\boldsymbol{w}, \boldsymbol{w}' \in \mathbb{R}^2$ such that $\boldsymbol{w} <_{\text{lex}} \boldsymbol{w}'$, there is $m \in \mathbb{Z}_{>0}$ such that

$$\boldsymbol{B}(x, y, \langle m, \boldsymbol{w}, \boldsymbol{b}\rangle, \langle 1, \boldsymbol{w}', \boldsymbol{b}\rangle) \implies y \succ x.$$

In addition there is

Extreme Strong Infinite Superiority: there are $\boldsymbol{b}, \boldsymbol{b}' \in \mathbb{R}^2$ such that, for all $\boldsymbol{w}, \boldsymbol{w}' \in \mathbb{R}^2$, $m' \in \mathbb{Z}_{>0}$, and $x, y \in O$,

$$\boldsymbol{B}(x, y, \langle 1, \boldsymbol{w}, \boldsymbol{b}\rangle, \langle m', \boldsymbol{w}', \boldsymbol{b}'\rangle) \wedge \boldsymbol{b} > \boldsymbol{b}' \implies y \succ x.$$

Proposition 4.9. *The Simple Lexitotal Principle satisfies Extreme Strong Infinite Superiority (and hence Super Ultra Minimal Infinite Superiority), Minimal Finite Superiority (and hence Ultra Minimal Finite Superiority), and Non-Anti Pigou-Dalton.*

Proof. Follows from 4.10. In particular, for Extreme Strong Infinite Superiority let $\boldsymbol{b} = (1,0)$ and $\boldsymbol{b}' = (0,1)$. □

Proposition 4.10. *For all $x, y \in O$, $m, m' \in \mathbb{Z}_{>0}$, $\boldsymbol{w}, \boldsymbol{w}' \in \mathbb{R}^2$, and $\boldsymbol{b}, \boldsymbol{b}' \in \mathbb{R}^2$, if*

1. $m \cdot \boldsymbol{b} >_{lex} m'\boldsymbol{b}'$;
2. $\boldsymbol{B}\big(x, y, \langle m, \boldsymbol{w}, \boldsymbol{b}\rangle, \langle m', \boldsymbol{w}', \boldsymbol{b}'\rangle\big)$,

then

$$\sum_{i \in N} \boldsymbol{u}_i(y) >_{lex} \sum_{i \in N} \boldsymbol{u}_i(x).$$

Proof. The proof is similar to the proof of proposition 2.8. Let $x, y \in O$, $m, m' \in \mathbb{Z}_{>0}$, $\boldsymbol{w}, \boldsymbol{w}' \in \mathbb{R}^2$, and $\boldsymbol{b}, \boldsymbol{b}' \in \mathbb{R}^2$. It is to be shown that

$$(4.51) \quad \sum_{i \in N} \boldsymbol{u}_i(y) >_{\text{lex}} \sum_{i \in N} \boldsymbol{u}_i(x)$$

where

$$\sum_{i \in N} \boldsymbol{u}_i = \sum_{i \in M} \boldsymbol{u}_i + \sum_{i \in M'} \boldsymbol{u}_i + \sum_{i \in N-(M \cup M')} \boldsymbol{u}_i.$$

By B3, for all $i \in N - (M \cup M)'$, $\boldsymbol{u}_i(x) = \boldsymbol{u}_i(y)$, hence this is equivalent to

$$(4.52) \quad \sum_{i \in M} (\boldsymbol{u}_i(y) - \boldsymbol{u}_i(x)) >_{\text{lex}} \sum_{i \in M'} (\boldsymbol{u}_i(x) - \boldsymbol{u}_i(y)).$$

Consider the left side of this inequation. By B1,

$$(4.53) \quad \text{for all } i \in M,\ \boldsymbol{u}_i(y) \geq_{\text{lex}} \boldsymbol{u}_i(x) + \boldsymbol{b}.$$

Therefore,

$$(4.54) \quad \sum_{i \in M} (\boldsymbol{u}_i(y) - \boldsymbol{u}_i(x)) \geq_{\text{lex}} m\boldsymbol{b}.$$

Symmetrically,

$$(4.55) \quad \sum_{i \in M'} (\boldsymbol{u}_i(x) - \boldsymbol{u}_i(y)) \leq_{\text{lex}} m'\boldsymbol{b}'.$$

Therefore, by $m \cdot \boldsymbol{b} > m' \cdot \boldsymbol{b}'$,

$$(4.56) \quad \sum_{i \in M} (\boldsymbol{u}_i(y) - \boldsymbol{u}_i(x)) > \sum_{i \in M'} (\boldsymbol{u}_i(x) - \boldsymbol{u}_i(y)).$$

This concludes the proof. □

4.B Characterization theorem by Blackorby et al.

It can be shown that other important results are similarly challenged by the Simple Lexitotal Principle. I will just give an example. According to Blackorby, Bossert and Donaldson's

> **Theorem 4.22:** a moral betterness relation satisfies Continuity, Strong Welfare Impartiality, Strong Pareto, Same-People Independence, and Replication Invariance if and only if it is generalized utilitarian,[18]

i.e. there exists a continuous and increasing $\phi \in \mathbb{R}^{\mathbb{R}}$ such that, for all $x, y \in O$,

$$x \succ y \iff \sum_{i \in N} \phi(u_i(x)) > \sum_{i \in N} \phi(u_i(y)).$$

This result is particularly interesting since it characterizes generalized utilitarianism, i.e. gives necessary and together sufficient conditions for when the principle holds. And since generalized utilitarianism rules out Super Ultra Minimal Infinite Superiority*, Theorem 4.22 implies as a corollary another impossibility result,

> **Result 4.22*:** no moral betterness relation satisfies Continuity, Strong Welfare Impartiality, Strong Pareto, Same-People Independence, and Replication Invariance and Super Ultra Minimal Infinite Superiority*.

However, it can be shown that an impossibility does not hold if Absolute Comparability and Continuity is dropped.

To justify Absolute Comparability, or Ratio Comparability, Blackorby, Bossert and Donaldson merely claim that *lesser* welfare information would constrain theories:

> To define most of the principles for social evaluation discussed in this book, we require more information about individual well-being than the ordinal information contained in a goodness relation.[19]

But, as argued in the case of Fleurbaey, Tungodden and Vallentyne in section 4.1, it seems a shortcoming to generally rule out accounts that require *more* welfare information beyond Absolute Comparability without argument.

To justify Continuity, Blackorby, Bossert and Donaldson claim that

> Continuity is an axiom that prevents a social-evaluation ordering from exhibiting "large" changes in the social ranking in response to "small" changes in individual utilities.[20]

This seems true as applied to some theories that violate the condition, e.g. Weak Leximin. However, for theories violating the condition due to lexically ordered welfare components, as the Weak Simple Lexitotal Principle, this claim seems false since for these theories a change in a higher welfare (no matter how small!) is "large" compared to a change in the lower welfare components.

Next, let us turn to each condition of Theorem 4.22 not already discussed above. According to

[18] Blackorby, Bossert and Donaldson (2005: 127).
[19] Blackorby, Bossert and Donaldson (2009: 29).
[20] Blackorby, Bossert and Donaldson (2005: 70).

Continuity: the relation $\succeq$ is continuous if and only if, for all $x \in O$, $\{y \in O : y \succeq x\}$ and $\{y \in O : x \succeq y\}$ are closed sets (relative to u),

i.e. for all $x \in O$, and $(x_i)_{i\in\mathbb{N}} \in O^{\mathbb{N}}$, if,

1. for all $i \in \mathbb{N}$,

$$x_i \succeq x;$$

2. there exists $x_0 \in O$ such that

$$\lim_{i\to\infty} \boldsymbol{u}(x_i) = \boldsymbol{u}(x_0),$$

then $x_0 \succeq x$.

Proposition 4.11. *The Weak Simple Lexitotal Principle does not satisfy Continuity.*

Proof. Let $x \in O$ such that $\boldsymbol{u}(x) = (0,1)$, and $(x_i)_{i\in N} \in O^{\mathbb{N}}$ such that, for all $i \in \mathbb{N}$, $\boldsymbol{u}(x_i) = \left(\frac{1}{i}, 0\right)$. Therefore, by the Weak Simple Lexitotal Principle,

1. for all $i \in \mathbb{N}$,

$$x_i \succ x;$$

2. there exists $x_0 \in O$ such that

$$\lim_{i\to\infty} \boldsymbol{u}(x_i) = \left(\lim_{i\to\infty} \frac{1}{i}, \lim_{i\to\infty} 0\right) = (0,0) = \boldsymbol{u}(x_0),$$

but, by the Weak Simple Lexitotal Principle, $x \succ x_0$. □

Let S_n be the set of all permutations of $\{1,\dots,n\}$, i.e. $S_n \subseteq \{1,\dots,n\}^{\{1,\dots,n\}}$ such that, for all $\sigma \in S_n$, σ is a one-to-one function (and, hence, correspondence). For $\boldsymbol{v}, \boldsymbol{v}' \in \mathbb{R}^n$, $\boldsymbol{v}$ is a permutation of $\boldsymbol{v}'$ if and only if, there exists $\sigma \in S_n$ such that

$$\boldsymbol{v}' = \boldsymbol{v}_\sigma := \left(v_{\sigma(1)},\dots,v_{\sigma(n)}\right).$$

According to

Strong Welfare Impartiality: for all $x, y \in O$, if $\boldsymbol{u}(y)$ is a permutation of $\boldsymbol{u}(x)$, then $x \sim y$.

Proposition 4.12. *The Simple Lexitotal Principle satisfies Strong Welfare Impartiality.*

Proof. The proposition follows from the commutativity of addition. Let $x, y \in O$, such that $\boldsymbol{u}(y)$ is a permutation of $\boldsymbol{u}(x)$, i.e., there exists $\sigma \in S_n$ such that, for all $i \in N$,

$$\begin{aligned}\boldsymbol{u}_i(y) &= (h_i(y), l_i(y)) \\ &= \left(h_{\sigma(i)}(x), l_{\sigma(i)}(x)\right)\end{aligned}$$

$$= \boldsymbol{u}_{\sigma(i)}(x).$$

Then,

$$\begin{aligned}\sum_{i\in N} \boldsymbol{u}_i(y) &= \sum_{i\in N} \boldsymbol{u}_{\sigma(i)}(x)\\ &= \left(\sum_{i\in N} h_{\sigma(i)}(x), \sum_{i\in N} l_{\sigma(i)}(x)\right)\\ &= \left(\sum_{i\in N} h_i(x), \sum_{i\in N} l_i(x)\right) && \text{[by commutativity of addition]}\\ &= \sum_{i\in N} \boldsymbol{u}_i(x).\end{aligned}$$

Therefore, by the Simple Lexitotal Principle,

$$x \sim y. \qquad \square$$

Next, "[s]uppose a social change affects only the utilities of the members of a population subgroup. Same-People Independence requires the social assessment of the change to be independent of the utility levels of people outside the subgroup."[21] More precisely, according to

Same-People Independence: for all $M \subseteq N$ such that $M \neq \emptyset$ and, for all $x, y, x', y' \in O$, if,

1. for all $i \in M$, $u_i(x) = u_i(y)$ and $u_i(x') = u_i(y')$;
2. for all $j \in N - M$, $u_j(x) = u_j(x')$ and $u_j(y) = u_j(y')$,

then

$$x \succeq y \iff x' \succeq y'.$$

So, the people in M are unaffected, i.e. their welfare level is the same, in the comparison between x and y, as well as in the comparison between x' and y'. Same-People Independence says that, hence, the ranking of x compared to y, and x' compared to y' is independent of their welfare levels.

Proposition 4.13. *The Simple Lexitotal Principle satisfies Same-People Independence.*

Proof. Let $M \subseteq N$ such that $M \neq \emptyset$, and $x, y, x', y' \in O$ such that Same-People Independence condition 1 and 2 are satisfied. By the Simple Lexitotal Principle,

$$x \succeq y \iff \sum_{i\in N} \boldsymbol{u}_i(x) \geq_{\text{lex}} \sum_{i\in N} \boldsymbol{u}_i(y).$$

[21] Blackorby, Bossert and Donaldson (2005: 115).

And

$$
\begin{aligned}
& \sum_{i\in N} \boldsymbol{u}_i(x) \geq_{\text{lex}} \sum_{i\in N} \boldsymbol{u}_i(y) & \\
\Longleftrightarrow \quad & \left(\sum_{i\in N} h_i(x), \sum_{i\in N} l_i(x)\right) \geq_{\text{lex}} \left(\sum_{i\in N} h_i(y), \sum_{i\in N} l_i(y)\right) & \\
\Longleftrightarrow \quad & \left(\sum_{i\in N-M} h_i(x), \sum_{i\in N-M} l_i(x)\right) \geq_{\text{lex}} \left(\sum_{i\in N-M} h_i(y), \sum_{i\in N-M} l_i(y)\right) & \text{[by 1]} \\
\Longleftrightarrow \quad & \left(\sum_{i\in N-M} h_i(x'), \sum_{i\in N-M} l_i(x')\right) \geq_{\text{lex}} \left(\sum_{i\in N-M} h_i(y'), \sum_{i\in N-M} l_i(y')\right) & \text{[by 2]} \\
\Longleftrightarrow \quad & \left(\sum_{i\in N} h_i(x'), \sum_{i\in N} l_i(x')\right) \geq_{\text{lex}} \left(\sum_{i\in N} h_i(y'), \sum_{i\in N} l_i(y')\right) & \text{[by 1]} \\
\Longleftrightarrow \quad & \sum_{i\in N} \boldsymbol{u}_i(x') \geq_{\text{lex}} \sum_{i\in N} \boldsymbol{u}_i(y') &
\end{aligned}
$$

Therefore, by the Simple Lexitotal Principle,

$$x \succeq y \Longleftrightarrow x' \succeq y'.$$ □

Strong Pareto: for all $x, y \in O$, if,

1. for all $i \in N$, the welfare level of i in x is at least as high as in y;
2. there exists $j \in N$ such that the welfare level of j in x is higher than in y,

then $x \succ y$.

Proposition 4.14. *The Simple Lexitotal Principle satisfies Strong Pareto.*

Proof. Let $x, y \in O$ such that 1 and 2, i.e.,

1. for all $i \in N$, $\boldsymbol{u}_i(x) \geq_{\text{lex}} \boldsymbol{u}_i(y)$;
2. there exists $i \in N$ such that $\boldsymbol{u}_i(x) >_{\text{lex}} \boldsymbol{u}_i(y)$.

Therefore,

(4.57) $\sum_{i\in N} h_i(x) > \sum_{i\in N} h_i(y)$ or
$[\sum_{i\in N} h_i(x) = \sum_{i\in N} h_i(y)$ and $\sum_{i\in N} l_i(x) > \sum_{i\in N} l_i(y)]$.

Therefore,

(4.58) $\sum_{i\in N} \boldsymbol{u}_i(x) >_{\text{lex}} \sum_{i\in N} \boldsymbol{u}(y).$

Therefore, by the Simple Lexitotal Principle, $x \succ y$. □

5

Trade-off conditions reconsidered and the minimax complaint principles

5.1 Non-aggregation and the minimax binary complaint principles

In section 2.5, I suggested that one way to capture minimal infinite superiority is to appeal to "non-aggregation" which commonly means in this context that, roughly,

(5.1) welfare of different individuals cannot be aggregated, or combined (e.g. additively) so that the aggregate can be morally weighed against the welfare of other individuals.

For example, Weak Maximin is satisfies both non-aggregation and Minimal Infinite Superiority. But it violates Minimal Finite Superiority. It does these things because the situation of the worst-off is seen as having infinite priority over the situation of the better-off. It also does not satisfactorily solve the moderate trade-off paradoxes since it judges the last alternative in the respective sequences best. But, intuitively, this is the worst alternative.

However, there are closely related principles that satisfy Minimal Infinite Superiority and Minimal Finite Superiority. One common way to understand Weak Maximin is to say that the worst-off has a greater *complaint* against losing than the better-off. It will be helpful for the further discussion to decompose this principle into two components. First, according to the Weak Minimize the Maximum Complaint Principle, for short

Weak Minimax Complaint: for any alternatives x and y, if the maximum complaint against x is smaller than that against y, then x is better than y.[1]

Here a *complaint* against an alternative means an *individual* person's complaint against that alternative. And, because we assume that other things are equal apart from effects on welfare, the complaint is based on the effect on the person's *welfare* in the alternative. Sometimes, instead of complaint against an alternative, *claim against* an alternative is used. I consider those two notions interchangeable

[1] See Brink (1993: 264) who follows Parfit's naming in a 1989 lecture. See also Tungodden (2003: 34).

here. The term is often used by contractualists but also more widely, e.g. by pluralists and consequentialists, in order to emphasize distributive or non-aggregative considerations.[2]

The second component is an interpretation of "complaint". There are different ways to interpret "complaint" that lead to different versions of Weak Minimax Complaint. On a simple interpretation, the

> **simple complaint:** for any alternative x and y, the maximum complaint against x is smaller than that against y if and only if the worst-off in x is better off than the worst-off in y.

The combination of the Weak Minimax Complaint and the simple complaint gives the Weak Simple Complaint Minimax Principle, or better known as

> **Weak Maximin:** for any alternatives x and y, if the worst-off in x is better off than the worst-off in y, then x is better than y.

Furthermore, there is a lexical version of Weak Minimax Complaint, *Weak Lexicographically Minimize the Maximum Complaint*, for short

> **Weak Lenimax Complaint:** for any alternatives x and y, if the maximum complaint against x is smaller than that against y, or, in cases of ties, second-largest complaint against x is smaller than that against y, etc., then x is better than y.

The combination of the Weak Lenimax Complaint and simple complaint gives the Weak Simple Complaint Lenimax Principle, or better known as

> **Weak Leximin:** for any alternatives x and y, if the worst-off person in x is better off than the worst-off person in y, and in case of ties, the second worst-off in x is better off than the second worst-off in y, etc., then x is better than y.

However, there are other interpretations of a "complaint" that lead to different specifications of the minimax principle which satisfy Minimal Infinite Superiority but do not violate Minimal Finite Superiority. Consider Minimal Infinite Superiority. Note that not only is the one person worse off, whom it is better to benefit rather than the many better-off, but this person would also *benefit more* than each of the better-off. So, instead of basing complaints on the welfare level, they could be based on welfare differences. Famously, in "Should the Numbers Count?", Taurek (1977) writes:

> My way of thinking about these trade-off situations consists, essentially, in seriously considering what will be lost or suffered by this one person if I do not prevent it, and in comparing the significance of that *for him* with what would be lost or suffered by anyone else if I do not prevent it. This reflects a refusal to take seriously in these situations any notion of the sum of two persons' separate losses. [...] The discomfort of each of a large number of individuals experiencing a minor headache does not add up to anyone's experiencing a migraine. In such a trade-off situation as this we are to compare your pain or your loss, not to our collective or total pain, whatever exactly that is supposed to be, but to what would be suffered or lost by any given single of us.[3]

[2] See e.g. Scanlon (1982: 123), Brink (1993: 261), Scanlon (1998: 230), Nagel (1995) and Adler (2012: ch 5). Temkin (1986: 102) uses the term "complaint with respect to equality".

[3] Taurek (1977: 307-8).

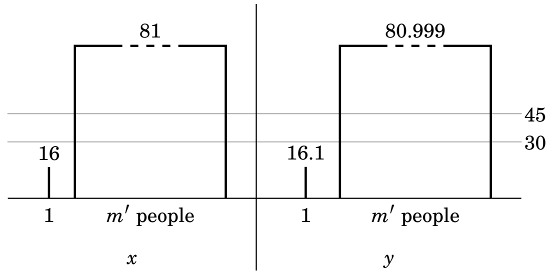

Figure 5.1 Illustration of Minimal Infinite Superiority (repeated figure 2.1 on page 18)

So, Taurek appeals to *pairwise* comparisons of individual persons' *losses*, i.e. welfare differences between two alternatives. More precisely, on the simplest version, the

> **simple binary complaint:** for any alternatives x and y, a person's complaint against alternative x (relative to alternative y) is the *shortfall* in this person's welfare in x from y.

And this combined with Weak Minimax Complaint gives

> **Weak Minimax Simple Binary Complaint:** for any alternatives x and y, if the maximum simple binary complaint against x is smaller than against y, then x is better than y,

i.e. if the maximum shortfall of some person in x from y is smaller than the maximum shortfall of any person in y from x, then x is better than y. This fulfils Minimal Infinite Superiority. To illustrate, consider figure 2.1 (on page 18 repeated in figure 5.1). Note that

(5.2) the shortfall in the one person's welfare in x from y is 9;

(5.3) the shortfall in any of the m' people's welfare in y from x is 0.001.

Therefore, by Weak Minimax Simple Binary Complaint, the implication of Minimal Infinite Superiority follows, i.e. (2.6) y is better than x, under the assumptions from before, (2.4) and (2.5) (on page 18).

But, in contrast to Weak Leximin, it also fulfils Minimal Finite Superiority. To illustrate, consider figure 2.5 (on page 25 repeated in figure 5.2). Note that

(5.4) the shortfall in the better-off person's welfare in x from y is 32;

(5.5) the shortfall in the worse-off person's welfare in y from x is 0.1.

Therefore, by Weak Minimax Simple Binary Complaint, the implication of Minimal Finite Superiority follows, i.e. (2.30) y is better than x, under the assumption from before, (2.29) (on page 25).

Like Weak Maximin, Weak Minimax Simple Binary Complaint implies a kind of infinite superiority but not the one captured by Infinite Priority. Rather it implies, what I call

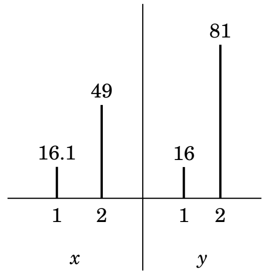

Figure 5.2 Illustration of Minimal Finite Superiority (repeated figure 2.5 on page 25)

Infinite Difference Superiority: for any alternatives x and y, numbers of people m and m', welfare levels w and w', and welfare differences b and b' such that b is greater than b', if

1. m people are at w in x and each is better off by b in y compared to x;
2. m' people are at w' in y and each is better off by b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

then y is better than x.

However, Weak Minimax Simple Binary Complaint does not satisfy Non-Anti Pigou-Dalton, and hence not Pigou-Dalton, because it is silent in cases where the benefits are equal. The lexicographic version implies Non-Anti Pigou-Dalton but not Pigou-Dalton.

To accommodate Pigou-Dalton, we could appeal to both losses, i.e. welfare differences, and how well-off persons are, i.e. welfare levels, in determining an individual's complaint in order to fulfil Pigou-Dalton as well. For example, in *Mortal Questions*, Nagel writes that

> The main point about a measure of urgency is that it is done by pair-wise comparison of the situations of individuals. The simplest method would be to count any improvement in the situation of someone worse off as more urgent than any improvement in the situation of someone better off; but this is not especially plausible. It is more reasonable to accord greater urgency to larger improvements somewhat higher in the scale than to very small improvements lower down.[4]

This idea is captured by the

priority binary complaint: for any alternatives x and y, person i's complaint against alternative x (relative to alternative y) is the shortfall in i's *non-relational finitely priority-weighted* welfare in x from the most i could get in either of the two alternatives.[5]

And according to the

[4] Nagel (1979) and see also Scanlon (1998: 123).

[5] Non-relational finite priority weight is defined in section 2.3.

> **Weak Minimax Priority Binary Complaint:** for any alternatives x and y, if the maximum priority binary complaint against x is smaller than that against y, then x is better than y.

Fleurbaey, Tungodden and Vallentyne show that this principle satisfies all *substantive* conditions of Result 3.[6] It also fulfils Ultra Minimal Finite Superiority*. However, as shown in chapter 3, it follows that Weak Minimax Priority Binary Complaint violates Acyclicity in the Third Moderate Trade-offs Paradox Sequence and, hence does not solve the Third Moderate Trade-offs Paradox. And, as shown in section 3.6, there is no simple deontic principle based on Weak Minimax Priority Binary Complaint that solves the Deontic Third Moderate Trade-offs Paradox. For example, according to

> **Deontic Weak Minimax Binary Complaint:** for any alternatives x and y, if the maximum binary complaint against x is smaller than that against y, and x and y are feasible, then y is impermissible.

Analogously to the axiological, this deontic principle satisfies all the *substantive* conditions of Result 5. It also fulfils the deontic analogue of Ultra Minimal Finite Superiority*. However, as shown in chapter 3, it follows that Deontic Weak Minimax Binary Complaint violates No Prohibition Dilemmas in the Third Moderate Trade-offs Paradox Sequence and, hence does not solve the Deontic Third Moderate Trade-offs Paradox.

5.2 Deontic Minimal Infinite Superiority reconsidered and Alpha rejected

Next, I will argue that minimal infinite superiority might be an over-generalization, i.e. the condition seems to be an inductive generalization from intuitions in a too limited set of cases and overlooks alternatives. I will discuss this in deontic terms for now and return to the axiological version in chapter 7. Considering deontic theories is interesting because they allow for more context dependence: what is a permissible action can depend on the set of feasible alternatives in a situation, for short the *feasible set*. Arguably, this is in contrast to axiological theories that assign final value to alternatives since final value is often seen as choice context-independent (e.g. value based on *intrinsic* features of alternatives).

If Absolute Comparability (as well as the other background conditions) is assumed, then one of the substantive deontic trade-off conditions has to be violated in order to solve the moderate trade-off paradoxes. These trade-off conditions were first introduced in cases with only two alternatives. Considering these cases, it does not seem that the implication of the these conditions should be rejected in cases with *only two* feasible alternatives. But are the implications of these conditions plausible independently of the alternatives in the feasible set, in particular in more complex cases like the Third Moderate Trade-offs Paradox (see section 3.3)?

[6] Fleurbaey, Tungodden and Vallentyne (2009: 275).

In section 3.6, I suggested that, intuitively, this is not the case. Instead there should be a permissible alternative *in this* sequence.[7] Below, I will discuss the more complex Third Moderate Trade-offs Paradox but for now consider again the First Moderate Trade-offs Paradox Sequence from section 3.3, repeated below in figure 5.3. Remember that the axiological judgement implied by Minimal Infinite Superiority is that (3.13) x_i is morally better than x_{i-1}, for all $i = 1, \ldots, m$. So, the deontic analogue of Minimal Infinite Superiority, call this Deontic Minimal Infinite Superiority, implies that

(5.6) x_{i-1} is impermissible if x_i is feasible, for all $i = 1, \ldots, m$.

But instead, it might be claimed that,

(5.7) for all $i = 1, \ldots, m$, x_{i-1} is impermissible if x_{i-1} and x_i are the only feasible alternatives, but, for some i, x_{i-1} is permissible if x_1 to x_m are the only feasible alternatives.

So, the claim is that Deontic Minimal Infinite Superiority is an over-generalization and instead we might more cautiously claim

Deontic Two-Option Minimal Infinite Superiority: for any welfare levels w and w' such that w is lower than w', and positive welfare difference b, there is a positive welfare difference b' such that b' is smaller than b such that, for any numbers of people m', and alternatives x and y, if

1. one person is at or below w in x and better off by at least b in y compared to x;
2. at least m' people are at or above w' in y and each is better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

and only x and y are feasible, then x is impermissible.

However, if (5.7) is correct, then we also have to reject the rational choice condition

Alpha: if an alternative is permissible in a given feasible set, then it is also permissible in any subset containing the alternative.[8]

This is equivalent to that

(5.8) if an alternative is impermissible relative to a given feasible set, then it is also impermissible relative to any superset of the feasible set.

And, in particular, if an alternative is impermissible relative to a pair of alternatives, then it is also impermissible relative to any superset (which includes the pair).

(5.7) violates Alpha because its former conjunct together with Alpha violates its latter conjunct, i.e. if, for all $i = 1, \ldots, m$, x_{i-1} is impermissible if x_{i-1} and x_i are the only feasible alternatives, then, by Alpha, for all $i = 1, \ldots, m$, x_{i-1} is impermissible if x_1 to x_m are feasible. And, more generally,

[7] This is compatible with accepting that there can be No Prohibition Dilemmas, see section 3.6.

[8] Fleurbaey, Tungodden and Vallentyne (2009: 283). This condition is suggested as a general condition for rational choice also known as "Chernoff", and Independence of Irrelevant Alternatives (not to be confused with Arrow (1951)'s condition of the same name), see Sen (1979: 17).

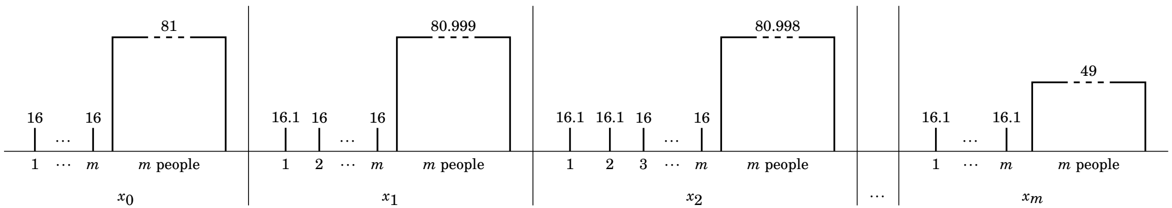

Figure 5.3 Illustration of the First Moderate Trade-offs Paradox (repeated figure 3.1 on page 52)

(5.9) Deontic Two-Option Minimal Infinite Superiority and Alpha imply Deontic Minimal Infinite Superiority.

Similar for any other substantive trade-off condition. For example, for the two-option variant of Deontic Super Ultra Minimal Infinite Superiority*, call it Deontic Two-Option Super Ultra Minimal Infinite Superiority*,

(5.10) Deontic Two-Option Super Ultra Minimal Infinite Superiority* and Alpha imply Deontic Super Ultra Minimal Infinite Superiority*.

So, Alpha allows us to derive impossibility

> **Result 8*:** no theory of moral permissibility satisfies No Prohibition Dilemmas, Alpha, Deontic Two-Option Non-Anti Pigou-Dalton, Deontic Two-Option Super Ultra Minimal Infinite Superiority*, and Deontic Two-Option Ultra Minimal Finite Superiority*.[9]

Is it, in principle, defensible that the permissibility of an alternative depends on the feasible set in the way (5.7) suggests and which contradicts Alpha? Consider the following case suggested by Sen (1993). Suppose at the dinner table you have a choice between some of the following alternatives:

> x**:** take nothing;
>
> y**:** take an apple;
>
> z**:** take another apple.

It might not show good manners, Sen suggests, to take the *last* available fruit. So, given your manners are rightly adjusted,

(5.11) if only x and y are feasible, only x is permissible;

(5.12) if only x, y, and z are feasible, y is permissible.[10]

And these judgements seem at least not obviously *inconsistent*. However, they violate Alpha. If it is impermissible to choose y if only x and y are feasible, then, by (5.8), it is also impermissible to choose y if in addition z is feasible.

It might be instructive to see that another, related rational choice condition often assumed to be mandatory is controversial. According to

> **Beta:** if two alternatives are permissible relative to a given feasible set, then the one is permissible if and only if the other is permissible in any superset of the given feasible set.

[9] Note also that by the contrapositive of (5.10) and the analogue for Ultra Minimal Finite Superiority*, Result 8* is implied by the deontic analogue to Result 3*. Fleurbaey, Tungodden and Vallentyne (2009: 283) state a similar but weaker Result 8, i.e. one with stronger conditions.

[10] Cf Sen (1993: 501).

Alpha together with Beta are equivalent to a number of proposed rationality conditions, most famously the axiom of revealed choice.[11] However, Beta has been rejected on the basis that it does not capture (a certain kind of) supererogation. Consider the following case from Parfit (1982). Suppose you the have a choice between some of the following alternatives:

x**:** do nothing;

y**:** at great cost to yourself, saving only one arm of a stranger;

z**:** at the same cost to yourself, saving both arms of a stranger.

Parfit suggests both that, if the cost to yourself is great enough,

(5.13) if only x and y are feasible, then both x and y are permissible;

(5.14) if only x, y, and z are feasible, then only x and z are permissible.[12]

This is because, arguably, it is beyond the call of duty to save a stranger's arms at great cost to yourself, and it is impermissible, *given* that one bears the cost and it is the *same* in either case, to only save one arm of a stranger if it is feasible to save both at the same cost. But if this is true, then Beta has to be rejected. This is because, according to Beta and (5.13), x is permissible if and only if y is permissible in any feasible set containing both. But this contradicts (5.14).

Another proposal to capture Sen's case is the fine-grained individuation approach. Instead of there being the same alternative, e.g. x, in the different choice situations, there are different alternatives.[13] For example, if the apple is the last available fruit, then the alternative is not just x but x-when-only-x-and-y-are-feasible; and if there is another apple, then the alternative is x-when-only-x-y-and-z-are-feasible. However, this approach has at least two problems. First, intuitively meaningful questions about alternatives such as "Is alternative x which is permissible in one choice context also permissible in another choice context?", turn out not to be meaningful on this approach.[14] Second, the approach trivializes the conditions. Arguably, one reason these conditions are important is that violating them makes practical decision making more complicated. I think the severest problems a rejection of Alpha faces concern practical rationality in more complex cases, including dynamic choice. The violation of Alpha highlights this.[15]

I am not committed to one or the other approach. In fact, I think it does not matter much for the focus of this thesis which approach we take. The problems and possible solutions can be translated from one into the other framework. For simplicity, let us just grant that Alpha is questionable and explore moral principles that violate it.

[11] See Sen (1971: 310-14). McClennen (1990: 23) calls the conjunction of Alpha and Beta *Context Free Choice*. Both Sen and McClennen reject it.

[12] Cf Parfit (1982: 131).

[13] For example, this is the general approach of Voorhoeve (2014: 79). See also Broome (1991: sec 5.3) and Voorhoeve (2013: 414-5) for the re-individuation to rescue the axiological condition of transitivity.

[14] Dietrich and List (2017: 427-8).

[15] Temkin (2012) provides a discussion of some of these challenges to "practical reasoning". See also Gustafsson (2014) for a discussion of dynamic choice and "anti-aggregation". However, a discussion of these important practical challenges is beyond the scope of this thesis.

5.3 The global complaint principles solve the moderate trade-offs paradoxes

I have suggested that the permissibility of alternatives cannot be derived from pairwise comparisons of alternatives. If this diagnosis is correct, then in turn it might be plausible to relativize a person's complaint not only to another single alternative but the whole feasible set. This is the idea of the global interpretation of "complaint",

> **priority global complaint:** for any feasible set, person i's complaint against alternative x (relative to the feasible set) is the shortfall in i's non-relational finitely priority-weighted welfare in x from the most i could get in the feasible set.

In other words,

(5.15) relative to a feasible set, a person's complaint against an alternative is equal to the person's greatest *priority binary complaint* against it when compared to each alternative in the the feasible set.[16]

And according to the *Deontic Lexicographically Minimize the Maximum Priority Global Complaint Principle*, for short

> **Deontic Lenimax Global Complaint:** for any alternatives x, x is permissible if and only if there is no feasible alternative with a smaller maximum priority global complaint, or, in cases of ties, a smaller second-largest priority global complaint, etc.

By (5.15), Deontic Lenimax Global Complaint is equivalent to Weak Minimax Priority Binary Complaint in feasible sets with only two alternatives. Hence it implies all the two-option versions of the substantive conditions of the moderate trade-off paradoxes.

Next, consider the Third Moderate Trade-offs Paradox Sequence. The *feasible set* consists of all alternatives in the sequence. For simplicity, assume that the *priority-weighting* of welfare mentioned in the priority global complaint is done by taking the square root of welfare levels. First, consider again the Infinite Superiority Sequence from subsection 3.4.1, repeated in figure 5.4. The corresponding global complaints are illustrated in figure 5.5. The most that each of the first m people can get in this set of alternatives is a welfare level of 25, or priority-weighted 5 ($= \sqrt{25}$). The most that each of the other m' people can get is 81, or priority-weighted 9 ($= \sqrt{81}$).

Consider first x_0 and x_1. In x_0, person 1 has a welfare level of 16, or priority-weighted 4 ($= \sqrt{16}$). So her global complaint against x_0 is 1 ($= 5-4$). In x_1, person 1 has no global complaint. The other $m-1$ worst-off people have the same global complaint of 1 against both x_0 and x_1. The m' best-off people have no complaint against x_0 and only a slight complaint in x_1 which is much smaller than 1. So, the greatest global complaint against x_1 is *lexically* greater than against x_0. Therefore, according to Deontic Lenimax Global Complaint,

[16] See section 5.B for a formal definition of the priority global complaint in terms of the priority binary complaint.

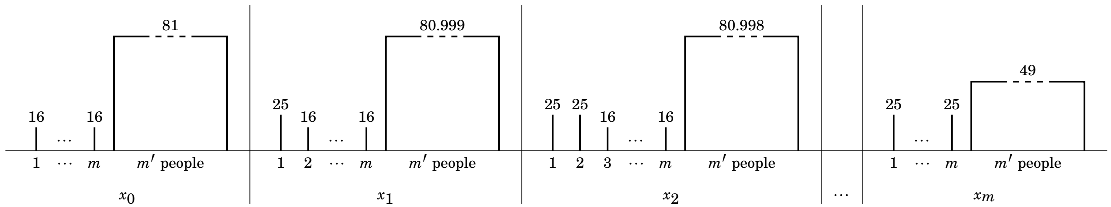

Figure 5.4 Illustration of the Infinite Superiority Sequence (repeated figure 3.5 on page 58)

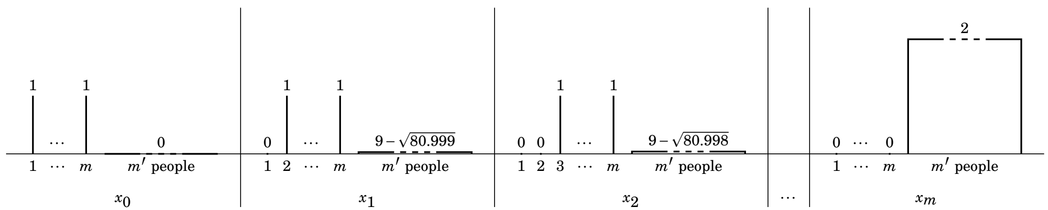

Figure 5.5 Illustration of global complaints in the Infinite Superiority Sequence

(5.16) x_0 is impermissible.

By similar reasoning, x_1 is impermissible which can be seen when contrasted with x_2.

However, the global complaint of each of the m' people increases steadily down the sequence. By the end of it, in x_m, the welfare levels of the m' people are at 49, or priority-weighted 7 ($= \sqrt{49}$). Hence their global complaint in x_m is 2 ($= 9 - 7$). So the greatest global complaint in x_m is clearly greater than the greatest global complaint in x_0, which is 1. Therefore, according to Deontic Lenimax Global Complaint,

(5.17) x_m is impermissible.

Furthermore, the global complaint of any of the first m people is never greater than 1 in any alternative. So, according to Deontic Lenimax Global Complaint there is an alternative in the sequence with a (slightly) higher global complaint than its preceding alternative. But then this preceding alternative is not rendered impermissible by any of its successors which will have a greater complaint. Therefore, according to Deontic Lenimax Global Complaint,

(5.18) for some i, x_i is permissible,

This is how Deontic Lenimax Global Complaint "stops" the sequence. And this is in line with the intuitive verdict suggested in section 3.6, (3.46) to (3.48), thereby solving the paradox. It seem plausible to me that the "best" alternative is somewhere in between the two extremes x_0 and x_m, and that the steady decrease for the best-off ends up being "too much" at some point.

In summary, Deontic Lenimax Global Complaint violates Alpha since it implies (5.7). However, this "flipping" of permissibility depending on the feasible set seems quite in line with intuitive verdict in this case. It does not seem true that at any particular point there is reasonable doubt that x_{i-1} is rendered impermissible by x_i when considering only these two alternatives. But when considering the Third Moderate Trade-offs Paradox Sequence as a whole it seems clear that it should "stop" at some point in between the extremes z_0 and $z_{m+m\cdot m''}$.

5.4 Counterexamples to the proof of Result 7

Now, I turn to a deontic *possibility* result by Fleurbaey, Tungodden and Vallentyne. They weaken the minimal infinite superiority conditions to the deontic two-option version. Exactly what a not *too* severely weakened Minimal Infinite Superiority condition should claim is not easy to establish. This is due to the possibly complex feasible set dependence of deontic theories once condition Alpha is rejected. This holds, in particular, for Deontic Lenimax Global Complaint. And, unfortunately, Fleurbaey, Tungodden and Vallentyne fail to find such a condition. That is why they severely limit the condition to apply only in cases with only *two options*.[17] According to their

[17] Fleurbaey, Tungodden and Vallentyne (2009: 282).

Result 7: There are theories of moral permissibility that satisfy No Prohibition Dilemmas, Deontic Strong Pareto, Deontic Pigou-Dalton, Deontic Replication Invariance, Deontic Two-Option Super Ultra Minimal Infinite Superiority, and Deontic Ultra Minimal Finite Superiority.[18]

The only condition not discussed before is

Deontic Strong Pareto: for any alternatives x and y, if everyone's welfare level in x is at least as high as in y and one person's welfare level in x is higher than in y and x and y are feasible, then y is impermissible.

This is the deontic analogue to the classic (axiological)

Strong Pareto: for any alternatives x and y, if everyone's welfare level in x is at least as high as in y and one person's welfare level in x is higher than in y, then x is better than y.

As a possibility proof of Result 7, they suggest Deontic Lenimax Global Complaint. However, they omit to prove in detail that it fulfils all the conditions of Result 7. And as it turns out there can be no such proof since it violates at least two of the conditions.

5.4.1 Counterexample 1: the Pigou-Dalton Sequence

Not only does Deontic Lenimax Global Complaint "stops" the Infinite Superiority Sequence and solve the moderate trade-off paradoxes, but it also "stops" the Pigou-Dalton Sequence, repeated in figure 5.6. While Deontic Lenimax Global Complaint includes a priority-weighting of welfare levels in order to satisfy priority for the worse-off, this is not enough to fulfil

Deontic Pigou-Dalton: for any alternatives x and y, positive welfare difference b, and persons i and j, if

1. i is at least as well off as j in y;
2. i is better off by b in x compared to y;
3. j is better off by b in y compared to x;
4. everyone else is at the same welfare level in x and y,

and x and y are feasible, then x is impermissible.

As for Minimal Infinite Superiority, the condition is satisfied by Deontic Lenimax Global Complaint in the two-option context. But there are contexts with more alternatives where the principle fails to satisfy the condition.

In the Pigou-Dalton Sequence (repeated in figure 5.6), the only alternative not rendered impermissible by Deontic Pigou-Dalton is the last alternative, $y_{m''}$. However, consider the priority global complaints, illustrated in figure 5.7. According to Deontic Lenimax Global Complaint, the priority-weighted loss of person 1 in $y_{m''}$

[18] Fleurbaey, Tungodden and Vallentyne (ibid.: 281).

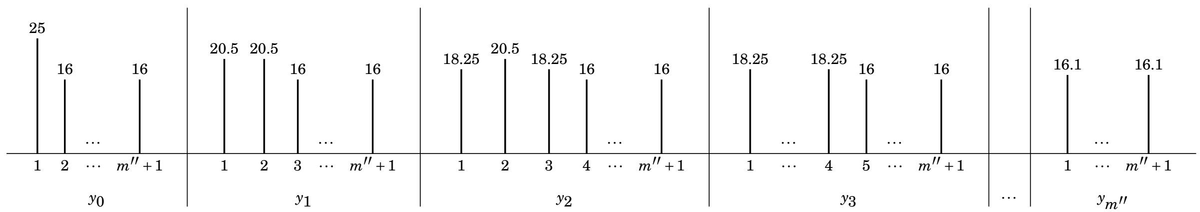

Figure 5.6 Illustration of the Pigou-Dalton Sequence (repeated figure 3.7 on page 62)

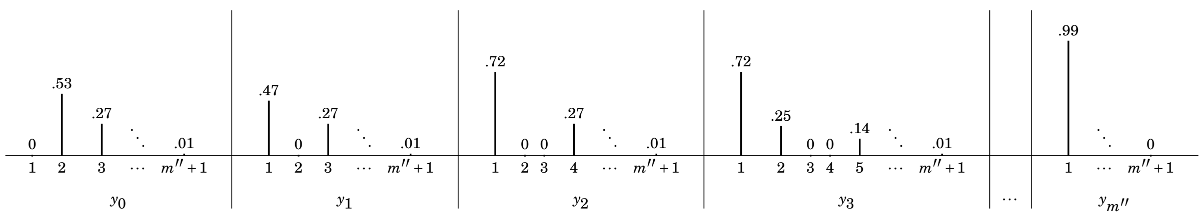

Figure 5.7 Illustration of global complaints in the Pigou-Dalton Sequence

compared to y_0 is close to 1 ($\sqrt{25}-\sqrt{16.1}$) and no one has a global complaint against y_1 as close to 1. The greatest complaint against y_1 is by person 2 of roughly 0.53 ($\sqrt{20.5}-\sqrt{16}$). Everyone else has an even smaller complaint since their increase along the sequence from level 16 is less. Therefore, according to Deontic Lenimax Global Complaint,

(5.19) $y_{m''}$ is impermissible

since it does not minimize the maximum global complaint. This establishes the claim that Deontic Lenimax Global Complaint violates Deontic Pigou-Dalton.

I suggested before, in subsection 4.4.1, that I find the implications of Deontic Lenimax Global Complaint in the PD-Sequence intuitive and for very similar reasons as in the Infinite Superiority Sequence discussed above. Hence a restriction of the Deontic Pigou-Dalton condition seems plausible. So, rather than rejecting Deontic Lenimax Global Complaint for not implying Deontic Pigou-Dalton, the condition should be weakened to its two-option version, for lack of a better alternative.

5.4.2 Counterexample 2: the Symmetric Minimal Finite Superiority Sequence

Now, I will argue that Deontic Lenimax Global Complaint is implausible after all because it does not capture a modest extension of the two-option version of Deontic Ultra Minimal Finite Superiority.

Remember, according to

Deontic Ultra Minimal Finite Superiority: there is a population size n' such that, for any alternative x, there are welfare differences b and b' such that b is larger than b' such that, for any alternative y, if

1. the population is larger than n';
2. everyone, except for one, is better off by at least b in y compared to x;
3. the one person is better off by at most b' in x compared to y,

and x and y are feasible, then x is impermissible.

Consider the case illustrated in figure 5.8, i.e., in x, everyone is at welfare level 16.1; in y, person 1 is at 16 and everyone else at 81. Note that this is a version of the illustration of Ultra Minimal Finite Superiority (figure 3.9 on page 64). Hence under the previous assumptions, (3.23) to (3.25), by Deontic Ultra Minimal Finite Superiority,

(5.20) x is impermissible.

The (priority-weighted) welfare levels and global complaints are represented in table 5.1. The welfare level of all n people in x is 16.1, or priority-weighted roughly 4.01 ($\sqrt{16.1}$). And the welfare level of person 1 in y_1 is 16, or priority-weighted 4, and of the other persons in y_1 81, or priority-weighted 9. So the shortfall from the most the individuals can get in the feasible set consisting of x and y_1 is roughly 0.01 ($= 4.01 - 4$) in y_1 for person 1, and 5 ($= 9 - 4$) for the others in x. Therefore, according to Deontic Lenimax Global Complaint,

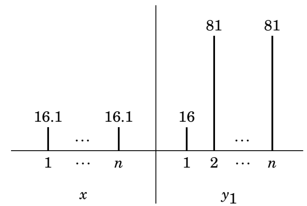

Figure 5.8 the Symmetric Ultra Minimal Finite Superiority Sequence (part)

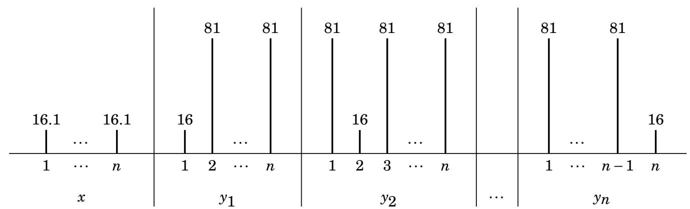

Figure 5.9 the Symmetric Ultra Minimal Finite Superiority Sequence

Table 5.1 The Symmetric Ultra Minimal Finite Superiority Sequence (part)

	Welfare		Priority-weighted		Global complaints	
Persons	x	y_1	x	y_1	x	y_1
1	16.1	16	4.01	4	–	0.01
2	16.1	81	4.01	9	4.99	–
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	16.1	81	4.01	9	4.99	–

Table 5.2 The Symmetric Ultra Minimal Finite Superiority Sequence

	Welfare				Priority-weighted				Global complaints			
Persons	x	y_1	$\cdots$	y_n	x	y_1	$\cdots$	y_n	x	y_1	$\cdots$	y_n
1	16.1	16		81	4.01	4		9	4.99	5		–
2	16.1	81		81	4.01	9		9	4.99	–		–
⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋱	⋮
$n-1$	16.1	81		81	4.01	9		9	4.99	–		–
n	16.1	81		16	4.01	9		4	4.99	–		5

(5.21) only y_1 is permissible

since it has the smallest maximal (global) complaint. This is consistent with (5.20).

Next, consider the

Symmetric Ultra Minimal Finite Superiority Sequence: illustrated in figure 5.9,

i.e., in x, everyone is at welfare level 16.1; in y_1, person 1 is at 16 and everyone else at 81; in y_2, person 2 is at 16 and everyone else at 81; ...; in y_n; person n is at 16 and everyone else at 81.

The (priority-weighted) welfare levels, and complaints are represented in table 5.2. The most that each can get in the feasible set containing x, y_1, ..., and y_n is 81, or priority-weighted 9. Therefore, the greatest global complaint in x is 4.99 ($= 9 - 4.01$) and in y_i is 5 ($= 9 - 4$), for all $i = 1, \ldots, n$. Therefore, by Deontic Lenimax Global Complaint,

(5.22) only x is permissible

since it has the smallest maximal (global) complaint.

Before, I suggested that other conditions could plausibly be replaced by weaker versions. Similarly, it might be claimed that Deontic Ultra Minimal Finite Superiority should be rejected and be replaced by a weaker version. And again, in general, I find a weakening of the condition plausible. As before, lacking a better alternative, this would leave us again with the two-option version:

Deontic Two-Option Minimal Finite Superiority: for any alternative x, there are positive welfare differences b and b' such that b is larger than b' such that, for any alternative y, if

1. one person is better off by at least b in y compared to x;
2. one person is better off by at most b' in x compared to y;
3. everyone else is at the same welfare level in x and y,

and only x and y are feasible, then x is impermissible.

However, in contrast to the implications of Deontic Lenimax Global Complaint in the other sequences, its implication in the Symmetric Ultra Minimal Finite Superiority Sequence is implausible: if x is impermissible and y_i is permissible in the set consisting of only x and y_i, for *all* $i = 1, \ldots, n$, as it is implied by the "two-option" version of Deontic Ultra Minimal Finite Superiority, then in a set consisting of only x and y_1 to y_n, x is still impermissible and, since y_1 to y_n are completely symmetric, y_1 to y_n are permissible. This argument does neither appeal to Deontic Ultra Minimal Finite Superiority nor to Alpha—both of which I am prepared to reject. So, while I am inclined to believe that Deontic Ultra Minimal Finite Superiority is an over-generalization and instead its two-option version holds, a modest extension of the two-option version to cover the Symmetric Ultra Minimal Finite Superiority Sequence seems to hold as well.[19]

[19] In subsection 7.2.1, I will make a suggestion for a general modest extension of the two-option version implied by the so called *Condorcet criteria* from voting theory.

It might be objected that Deontic Lenimax Global Complaint could be defended against my objection by an appeal to fairness. It might be claimed that there is some (additional) unfairness in alternative y_1 in the Symmetric Ultra Minimal Finite Superiority Sequence not present in the case with only two alternatives. In the Symmetric Ultra Minimal Finite Superiority Sequence, choosing y_1 respects the strong complaint of person 1 while not respecting the *equally* strong complaint of the other persons. By contrast, in the two-alternative case, by choosing y_1 this is not true since only person 1 has the strong complaint. This additional unfairness gives (additional) reasons against doing y_1 in the Symmetric Ultra Minimal Finite Superiority Sequence. The analogue applies for $y_2, \ldots,$ and y_n. And this is one reason why Alpha should be rejected.

This objection can be answered. I accept that Alpha should be rejected and I accept, at least for the sake of argument, that there is additional unfairness in doing y_1 in the Symmetric Ultra Minimal Finite Superiority Sequence. If this unfairness would always outweigh finite superiority considerations, then it might be that choosing y_1 would be impermissible in this case. But this is implausible. Respecting the strong complaint of the other individuals is possible only by choosing x. But this leaves everyone at the very low well-being level 16.1 and that respects everyone's claim only minimally.

Note that the result does not rely on any specific numbers chosen. It can be generalized and holds as long as the difference of the two lower (priority-weighted) levels, in the illustration 16 and 16.1 (priority-weighted 4 and 4.01, respectively), is small enough and hence the global complaints arbitrary similar enough, and the highest (priority-weighted) level, in the illustration 81 (priority-weighted 9), can be chosen arbitrarily high. Hence it implies more generally that Deontic Lenimax Global Complaint violates Ultra Minimal Finite Superiority in a very implausible way, analogously to what happens in the illustration.[20] Hence, I suggest that Deontic Lenimax Global Complaint should be rejected.

It might be objected that the counterexample can be circumvented by a variant of Deontic Lenimax Global Complaint that appeals, instead of the *minimization* of the maximum global complaint (or claim) *against* an alternative, to the *maximization* of the maximum global claims *to* an alternative, which are the symmetric analogue to global complaints, i.e. relative to the lowest level in the feasible set. While it is actually true that *this* counterexample can be circumvented in this way, it does not help much.

First, it is an odd feature that complaints (or claims) against an alternative are not symmetric to claims to an alternative given the narrow "other things equal" context of our discussion.[21] Second, as I show in section 5.B, there are analogous symmetric counterexamples to principles that maximize the maximum global claim to an

[20] See section 5.B for a formal proof.

[21] Arguably, there are asymmetries when a change in population is considered. Many people accept the

Asymmetry: causing people to exists with negative welfare is *im*permissible but *not* causing people to exist with positive welfare is *not* impermissible.

alternative where these principles violate Super Ultra Minimal Infinite Superiority, instead of Ultra Minimal Finite Superiority, and this seems counter-intuitive.

Finally, it might be claimed that the problem is the too weak finite superiority that the minimax (and lenimax) principles support. However, as I show in the next chapter, an appeal to the *sum* of global complaints violate Deontic Two-Option Minimal Infinite Superiority or are vulnerable to the same counterexamples.

5.A The minimax and maximax binary claim principles

5.A.1 In terms of complaints (or claims) against alternatives

Let $c_i(x)$ be the strength of i's complaint against x, for short i's *complaint (or claim) against* x. According to

Weak Minimax Complaint: for all $x, y \in O$,

$$\max_{i \in N} c_i(x) < \max_{i \in N} c_i(y) \implies x \succ y.$$

Next, we need to specify $c_i(x)$. There are two common ways to do so comparative to other alternatives. One way is the

priority binary complaint: there is a strictly increasing and strictly concave function ϕ such that, for all $x, y \in O$, $i \in N$, the *priority binary complaint* against alternative x relative to y is

$$c_i(x, y) := \max_{z \in \{x,y\}} \phi(u_i(z)) - \phi(u_i(x)) \in \mathbb{R}_{>0}.$$

The following identity makes the the proofs below clearer. Let $[]^+$ be the *positive part*, i.e., for all $r \in \mathbb{R}$,

$$[r]^+ := \max\{r, 0\} = \begin{cases} r & \text{if } r > 0 \\ 0 & \text{else} \end{cases}.$$

Proposition 5.1. *For all* $a, b \in \mathbb{R}$*,*

$$\max\{a, b\} - a = [b - a]^+.$$

Proof. By definition,

$$\max\{a, b\} - a = \begin{cases} b - a & \text{if } b > a, \\ 0 & \text{else.} \end{cases}$$

It might be claimed that this can be explained by an appeal to the asymmetry between claims against and claims to an alternative. While possible people's claims against an alternative matter, claims to an alternative don't. For an early discussion see e.g. Parfit (1984: sec. 134) and, in particular footnote 32.

$$= \begin{cases} b-a & \text{if } b-a>0, \\ 0 & \text{else.} \end{cases}$$
$$= \max\{b-a, 0\}$$
$$= [b-a]^+.$$ □

Corollary 5.1. *For all $x, y \in O$, and $i \in N$,*

$$c_i(x,y) = \left[\phi(u_i(y)) - \phi(u_i(x))\right]^+.$$

Proof. Follows directly from the definition of the priority binary complaint and proposition 5.1. □

According to

5.1: there exists a strictly increasing and strictly concave ϕ such that, for all $x, y \in O$,

$$\max_{i\in N} c_i(x,y) < \max_{i\in N} c_i(y,x) \implies x \succ y,$$

i.e.

$$\max_{i\in N}\left[\phi(u_i(y)) - \phi(u_i(x))\right]^+ < \max_{i\in N}\left[\phi(u_i(y)) - \phi(u_i(x))\right]^+ \implies x \succ y.$$

Proposition 5.2. *Weak Minimax Priority Binary Complaint satisfies Strong Pareto, Pigou-Dalton, Minimal Infinite Superiority, and Minimal Finite Superiority.*

Proof. Follows from propositions 5.3 and 5.4. □

Proposition 5.3. *Weak Minimax Priority Binary Complaint satisfies Strong Pareto.*

Proof. Assume

(5.23) for all $i \in N$, $u_i(x) \geq u_i(y)$;

(5.24) there exists $i \in N$ such that $u_i(x) > u_i(y)$.

Therefore, by ϕ is strictly increasing,

(5.25) for all $i \in N$, $\phi(u_i(x)) \geq \phi(u_i(y))$;

(5.26) for some $i \in N$, $\phi(u_i(x)) > \phi(u_i(y))$.

Therefore,

$$\begin{aligned} \max_{i\in N} c_i(x,y) &= \max_{i\in N}\left[\phi(u_i(y)) - \phi(u_i(x))\right]^+ \\ &= 0 \\ &< \max_{i\in N}\left[\phi(u_i(x)) - \phi(u_i(y))\right]^+ \\ &= \max_{i\in N} c_i(y,x). \end{aligned}$$

Therefore, by Weak Minimax Priority Binary Complaint, $x \succ y$. □

Proposition 5.4. *Weak Minimax Priority Binary Complaint satisfies Pigou-Dalton, Minimal Infinite Superiority, and Minimal Finite Superiority.*

Proof. Assume, for $x, y \in O$, $m, m' \in \mathbb{Z}_{>0}$, $w, w' \in \mathbb{R}$, and $b, b' \in \mathbb{R}_{>0}$,

(5.27) $B\big(x, y, \langle m, w, b\rangle, \langle m', w', b'\rangle\big)$.

Let

(5.28) M be the set of at least m people at or above w that benefit by at least b;

(5.29) M' be the set of at most m' people at or above w' that benefit by at most b'.

Therefore, by ϕ is (strictly) increasing and (weakly) concave,

$$c_i(x,y)\begin{cases} \geq \phi(w+b) - \phi(w) & \text{if } i \in M, \\ = 0 & \text{else;} \end{cases}$$

$$c_i(y,x)\begin{cases} \leq \phi\big(w'+b'\big) - \phi\big(w'\big) & \text{if } i \in M', \\ = 0 & \text{else.} \end{cases}$$

Therefore,

$$\max_{i \in N} c_i(x,y) \geq \phi(w+b) - \phi(w);$$

$$\max_{i \in N} c_i(y,x) \leq \phi\big(w'+b'\big) - \phi\big(w'\big).$$

Therefore,

$$\phi\big(w'+b'\big) - \phi\big(w'\big) < \phi(w+b) - \phi(w) \implies \max_{i \in N} c_i(y,x) < \max_{i \in N} c_i(x,y)$$

Therefore, by Weak Minimax Priority Binary Complaint,

$$\phi\big(w'+b'\big) - \phi\big(w'\big) < \phi(w+b) - \phi(w) \implies y \succ x$$

Therefore, the claim follows for Minimal Infinite Superiority and Minimal Finite Superiority since ϕ is strictly increasing and weakly concave, and for Pigou-Dalton since, additionally, ϕ is strictly concave. □

Next, the Lenimax Complaint is introduced. For $x \in O$, let

$$\boldsymbol{c}(x) := (c_1(x), \ldots, c_n(x)) \in (\mathbb{R}_{>0})^n,$$

i.e. $\boldsymbol{c}(x)$ is the ordered list of people's complaints against x. For $\boldsymbol{v} \in \mathbb{R}^n$, let

$$\boldsymbol{v}_{()} := \big(v_{(1)}, \ldots, v_{(n)}\big) \in \mathbb{R}^n$$

be a permutation of $\boldsymbol{v}$ with $v_{(i)} \geq v_{(i+1)}$ for $i = 1, \ldots, n-1$. In particular,

$$\boldsymbol{c}_{()}(x) = \big(c_{(1)}(x), \ldots, c_{(n)}(x)\big) \in (\mathbb{R}_{>0})^n$$

is an ordered list of people's complaints against x re-ordered from the highest to the lowest. According to *Lexicographically Minimize the Maximum Complaint*, for short

Lenimax Complaint: for all $x, y \in O$,

$$x \succ y \iff \boldsymbol{c}_{()}(x) <_{\text{lex}} \boldsymbol{c}_{()}(y),$$

i.e.

$$\begin{aligned} x \succ y \iff &\text{there exists } j \in N \text{ such that, for all } i < j, \\ &c_{(i)}(x) = c_{(i)}(y) \text{ and } c_{(j)}(x) > c_{(j)}(y). \end{aligned}$$

Next, according to

Lenimax Priority Binary Complaint: there exists a strictly increasing and strictly concave function ϕ such that, for all $x, y \in O$,

$$x \succ y \iff \boldsymbol{c}_{()}(x, y) <_{\text{lex}} \boldsymbol{c}_{()}(y, x).$$

Proposition 5.5. *Lenimax Priority Binary Complaint implies Weak Minimax Priority Binary Complaint.*

Proof. The claim follows from

$$\begin{aligned} \max_{i \in N} c_i(x, y) < \max_{i \in N} c_i(y, x) &\iff c_{(1)}(x, y) < c_{(1)}(y, x) \\ &\implies \boldsymbol{c}_{()}(x, y) <_{\text{lex}} \boldsymbol{c}_{()}(y, x). \end{aligned}$$

Then, by Lenimax Priority Binary Complaint,

$$\max_{i \in N} c_i(x, y) < \max_{i \in N} c_i(y, x) \implies x \succ y.$$ □

Proposition 5.6. *Lenimax Priority Binary Complaint satisfies Strong Pareto, Pigou-Dalton, Very Strong Finite Superiority, and Minimal Finite Superiority.*

Proof. Follows from propositions 5.5 and 5.2. □

Proposition 5.7. *Lenimax Priority Binary Complaint satisfies Replication Invariance.*

Proof. Assume that

(5.30) $x \succ y$.

Therefore, by Lenimax Priority Binary Complaint, $\boldsymbol{c}_{()}(x, y) <_{\text{lex}} \boldsymbol{c}_{()}(y, x)$, i.e.

(5.31) there exists $j \in \{1, \ldots, n\}$ such that, for all $i < j$,
$c_{(i)}(x, y) = c_{(i)}(y, x)$ and $c_{(j)}(x, y) < c_{(j)}(y, x)$,

i.e. the $j-1$ greatest binary complaints against x match those against y while the j-th greatest binary complaint against x is smaller than against y. Therefore, for any k-replication of these alternatives, the $k\cdot(j-1)$ greatest binary complaints match while the $(k\cdot(j-1)+1)$-th greatest binary complaint against x is smaller than against y. Therefore,

(5.32) for all $k \in \mathbb{Z}$,
there exists $j' := k\cdot(j-1)+1$ and for all $i < j'$,
$c_{(i)}(k*x, k*y) = c_{(i)}(k*y, k*x)$ and $c_{(j')}(k*x, k*y) < c_{(j')}(k*y, k*x)$.

Therefore,

(5.33) for all $k \in \mathbb{Z}$, $\boldsymbol{c}_{()}(k*x, k*y) <_{\text{lex}} \boldsymbol{c}_{()}(k*y, k*x)$.

Therefore, by Lenimax Priority Binary Complaint,

(5.34) for all $k \in \mathbb{Z}$, $k*x \succ k*y$. □

Proposition 5.8. *Lenimax Priority Binary Complaint violates Acyclicity.*

Proof. Follows from impossibility Result 3*, propositions 5.6 and 5.7. □

5.A.2 In terms of claims to alternatives

Instead of *complaints against*, we can appeal to *claims to* alternatives. Let $c_i^*(x)$ be the strength of i's claim to x, for short i's *claim to x*. According to

Maximax Claim: for all $x, y \in O$,

$$\max_{i\in N} c_i^*(x) > \max_{i\in N} c_i^*(y) \implies x \succ y.$$

Next, we need to specify $c_i^*(x)$. One way is the

priority binary claim: there is a strictly increasing and strictly concave function ϕ such that, for any $x, y \in O$, $i \in N$, the *priority binary claim to* alternative x relative to y is

$$c_i^*(x,y) := \phi(u_i(x)) - \min_{z\in\{x,y\}} \phi(u_i(z)).$$

Proposition 5.9. *For all $a, b \in \mathbb{R}$,*

$$a - \min\{a,b\} = [a-b]^+.$$

Proof. By definition,

$$\begin{aligned} a - \min\{a,b\} &= \begin{cases} 0 & \text{if } a \le b, \\ a-b & \text{else.} \end{cases} \\ &= \begin{cases} a-b & \text{if } a \ge b, \\ 0 & \text{else.} \end{cases} \end{aligned}$$

$$= \max\{a, b\} - b.$$

Therefore, the proposition follows immediately from proposition 5.1. □

Corollary 5.2. *For all $x, y \in O$, and $i \in N$,*

$$c_i^*(x,y) = \left[\phi(u_i(x)) - \phi(u_i(y))\right]^+.$$

Proof. Follows directly from the definition of priority binary claim and proposition 5.9. □

Proposition 5.10. *The priority binary claim and the priority binary complaint are symmetric to each other, i.e., for all $x, y \in O$, and $i \in N$,*

$$c_i^*(x,y) = c_i(y,x).$$

Proof. Follows from corollary 5.1 and corollary 5.2. □

According to

Weak Maximax Binary Priority Claim: there exists a strictly increasing and strictly concave ϕ such that, for all $x, y \in O$,

$$\max_{i \in N} c_i^*(x,y) > \max_{i \in N} c_i^*(y,x) \implies x \succ y.$$

Corollary 5.3. *Weak Maximax Binary Priority Claim and Weak Minimax Priority Binary Complaint are coextensive, i.e. they imply the same betterness relation.*

Proof. Follows immediately from the definitions of Weak Maximax Binary Priority Claim and Weak Minimax Priority Binary Complaint and proposition 5.10. □

Corollary 5.4. *Weak Maximax Binary Priority Claim satisfies Strong Pareto, Replication Invariance, Pigou-Dalton, Very Strong Finite Superiority, and Minimal Finite Superiority.*

Proof. Follows from 5.7 and propositions 5.3. □

Next, according to

Leximax Priority Binary Claim: there exists a strictly increasing and strictly concave ϕ such that, for all $x, y \in O$,

$$x \succ y \iff \boldsymbol{c}^*_{()}(x,y) >_{\text{lex}} \boldsymbol{c}^*_{()}(y,x).$$

Corollary 5.5. *Leximax Priority Binary Claim and Lenimax Priority Binary Complaint are coextensive, i.e. they imply the same betterness relation.*

Proof. Follows immediately from the definitions of Leximax Priority Binary Claim and Lenimax Priority Binary Complaint and proposition 5.10. □

Corollary 5.6. *Lenimax Priority Binary Complaint violates Acyclicity.*

Proof. Follows from propositions 5.8 and corollary 5.5. □

5.B The lenimax and leximax global claim principles

Next, the results of subsection 5.4.2 for the lenimax and leximax global claim principles are proven.

5.B.1 Deontic Lenimax Global Complaint

According to

> **Deontic Lenimax Complaint:** for all $x \in X \subseteq O$,
>
> $$x \in C(X) \iff \text{for all } y \in X,\ \boldsymbol{c}_{()}(x) \not>_{\text{lex}} \boldsymbol{c}_{()}(y),$$

i.e. $C(X) = U(X, >)$ according to Lenimax Complaint. Note that, by the definition of $>_{\text{lex}}$, for $\boldsymbol{v}, \boldsymbol{v}' \in \mathbb{R}^n$,

$$\boldsymbol{v} \not>_{\text{lex}} \boldsymbol{v}' \iff \boldsymbol{v} = \boldsymbol{v}' \text{ or } \boldsymbol{v} <_{\text{lex}} \boldsymbol{v}'.$$

Again, we need to specify $c_i(x)$. An alternative to the priority binary complaint is the *maximum priority binary complaint*, for short

> **priority global complaint:** for $x \in X \subseteq O$, $i \in N$, and strictly increasing and strictly concave ϕ, the *priority global complaint* against x (relative to X) is
>
> $$\begin{aligned} c_i(x, X) &:= \max_{z \in X} c_i(x, z) \\ &= \max_{z \in X} \left[\phi(u_i(z)) - \phi(u_i(x))\right]^+. \end{aligned}$$

According to

> **Deontic Lenimax Global Complaint:** there is a strictly increasing and strictly concave ϕ such that, for all $x \in X \subseteq O$,
>
> $$x \in C(X) \iff \text{for all } y \in X,\ \boldsymbol{c}_{()}(y, X) \not>_{\text{lex}} \boldsymbol{c}_{()}(x, X),$$

i.e. $C(X) = C(X, >)$ according to Lenimax Priority Binary Complaint.

Proposition 5.11. *Deontic Lenimax Global Complaint fulfils all the two-option versions of the substantive conditions of all deontic results, i.e. Deontic Two-Option Pigou-Dalton, Deontic Two-Option Minimal Infinite Superiority, and Deontic Two-Option Minimal Finite Superiority.*

Proof. The proposition follows from propositions 5.6, 5.12 and 3.6. □

Proposition 5.12. *The priority global complaint against an alternative relative to a pair of alternatives is equal to the priority binary complaint against that alternative relative to the other alternative, i.e.*

(5.35) $c_i(x, \{x, y\}) = c_i(x, y)$.

Proof. Follows immediately from the definitions of the priority binary complaint and the priority global complaint. □

Consider the (generalized version of)

Symmetric Ultra Minimal Finite Superiority Sequence: illustrated in figure 5.10 and table 5.3.

Proposition 5.13. *Deontic Lenimax Global Complaint violates Deontic Ultra Minimal Finite Superiority in the Symmetric Ultra Minimal Finite Superiority Sequence,*

i.e., for all $w, \varepsilon \in \mathbb{R}_{>0}$, $n \in \mathbb{Z}_{>0}$, and $X = \{x, y_1, \dots, y_n\} \subseteq O$ if

1. $w > \varepsilon > 0$;
2. for all $i \in N$, $u_i(x) = \varepsilon$;
3. for all $j = 1, \dots, n$, $u_i(y_j) = \begin{cases} 0 & \text{if } i = j, \\ w & \text{else.} \end{cases}$,

then, according to Deontic Lenimax Global Complaint,

$$x \in C(X).$$

Proof. For simplicity, assume that $\phi(0) = 0$. This can be assumed without loss of generality.[22] According to Deontic Ultra Minimal Finite Superiority, an alternative is impermissible if the population is large enough and some other feasible alternative

(5.36) gives everyone except one more benefits and the difference is large enough;

(5.37) gives one person less benefits but the difference is small enough.

In Symmetric Ultra Minimal Finite Superiority Sequence, for $i = 1, \dots, n$, in y_i compared to x,

(5.38) everyone except person 1 gets w rather than ε;

(5.39) person 1 gets 0 rather than ε.

Therefore, according to Deontic Ultra Minimal Finite Superiority for w large enough and ε small enough,

(5.40) x is impermissible.

[22] This is because, for all ϕ', there is a transformation $\phi(u) := \phi'(u) - \phi'(0)$ such that $\phi(0) = 0$ and

$$\begin{aligned} \phi'(u) > \phi'(u') &\iff \phi'(u) - \phi'(0) > \phi'(u') - \phi'(0) \\ &\iff \phi(u) > \phi(u), \end{aligned}$$

i.e. the ordering of welfare levels is invariant.

Table 5.3 The Symmetric Ultra Minimal Finite Superiority Sequence

	$u_i(\cdot)$				$\phi(u_i(\cdot))$				$c_i(\cdot, X)$			
i	x	y_1	$\cdots$	y_n	x	y_1	$\cdots$	y_n	x	y_1	$\cdots$	y_n
1	ε	0		w	$\phi(\varepsilon)$	0		$\phi(w)$	$\phi(w)-\phi(\varepsilon)$	$\phi(w)$		–
2	ε	w		w	$\phi(\varepsilon)$	$\phi(w)$		$\phi(w)$	$\phi(w)-\phi(\varepsilon)$	–		–
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$n-1$	ε	w		w	$\phi(\varepsilon)$	$\phi(w)$		$\phi(w)$	$\phi(w)-\phi(\varepsilon)$	–		–
n	ε	w		0	$\phi(\varepsilon)$	$\phi(w)$		0	$\phi(w)-\phi(\varepsilon)$	–		$\phi(w)$

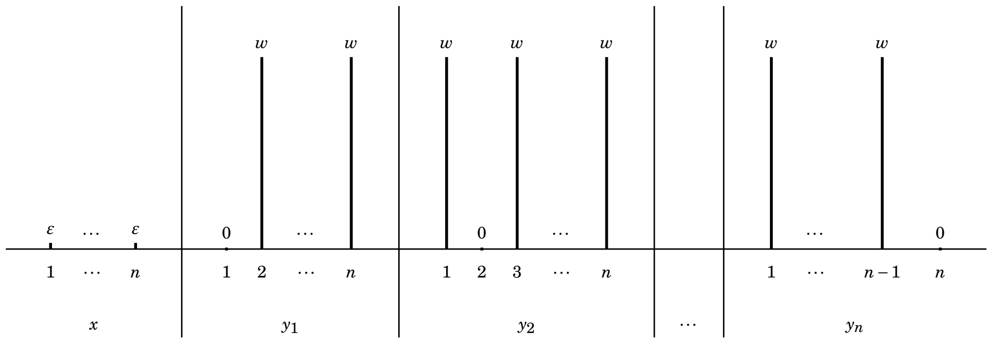

Figure 5.10 The Symmetric Ultra Minimal Finite Superiority Sequence

However,

(5.41) $c_i(x,X) < c_i(y_i,X)$

i.e. the global complaint in x is smaller than in y_i, for all $i = 1,\ldots,n$, because

$$\begin{aligned} c_i(x,X) &= \phi(w) - \phi(\varepsilon) \\ &< \phi(w) \\ &= \phi(w) - \phi(0) \\ &= c_i(y_i,X), \end{aligned}$$

Therefore, according to Deontic Lenimax Global Complaint,

(5.42) x is permissible.

But this contradicts (5.40). □

Note that (5.41) holds even if ϕ is weakly concave.

5.B.2 Deontic Leximax Global Claim

Next, it is shown that a symmetric case based on Deontic Super Ultra Minimal Infinite Superiority can be made against a theory that appeals to *claims to* rather than complaints against alternatives. Start with the definition of the

priority global claim: there is a strictly increasing and strictly concave ϕ such that, for all $x \in X \subseteq O$, $i \in N$, the (strength of the) *priority global claim* to x (relative to X) is

$$\begin{aligned} c_i^*(x,X) &:= \max_{y\in D} c_i^*(x,y) \\ &= \max_{y\in D} \left[\phi(u_i(x)) - \phi(u_i(y))\right]^+. \end{aligned}$$

According to the *Deontic Lexicographically Maximize the Maximum Priority Global Claim Principle*, for short

Deontic Leximax Global Claim: for all $x \in X \subseteq O$,

$$x \in C(X) \iff \text{for all } y \in X,\ \boldsymbol{c}^*_{()}(x,X) \not<_{\text{lex}} \boldsymbol{c}^*_{()}(y,X).$$

According to

Deontic Super Ultra Minimal Infinite Superiority: there are welfare levels w and w' such that w is lower than w', and positive welfare differences b and b' such that b' is smaller than b such that, for any alternatives x and y, if

1. the worst-off in x are at or below w in x;
2. one worst-off in x is at or below average and w in y and better off by at least b in y compared to x;

3. the best-off in x are the best-off in y and at or above w' in y and better off by at most b' in x compared to y;
4. everyone else is at the same welfare level in x and y,

and x is permissible, then y is impermissible.

Next, we proof the analogue to 5.13 for global claims to alternatives. Consider the

Symmetric Minimal Infinite Superiority Sequence: in table 5.3.

Proposition 5.14. *Deontic Leximax Global Claim violates Deontic Super Ultra Minimal Infinite Superiority in the 5.B.2.*

Proof. According to Deontic Super Ultra Minimal Infinite Superiority, for w and ε large and small enough, respectively,

(5.43) for all $i = 1, \dots, n$, y_i is *im*permissible.

However, consider the priority global claims to alternatives in table 5.3.

(5.44) For all $i = 1, \dots, n$, y_i has a greater maximum priority global claim than x.

Therefore, according to Deontic Leximax Global Claim,

(5.45) for all $w > \varepsilon > 0$, and $i = 1, \dots, n$, y_i is permissible.

But this contradicts (5.43) and shows the claim. □

Table 5.4 The Symmetric Minimal Infinite Superiority Sequence

	$u_i(\cdot)$				$\phi(u_i(\cdot))$				$c_i^*(\cdot, X)$			
i	x	y_1	$\cdots$	y_n	x	y_1	$\cdots$	y_n	x	y_1	$\cdots$	y_n
1	$w-\varepsilon$	0		w	$\phi(w-\varepsilon)$	0		$\phi(w)$	$\phi(w-\varepsilon)$	–		$\phi(w)$
2	$w-\varepsilon$	w		w	$\phi(w-\varepsilon)$	$\phi(w)$		$\phi(w)$	$\phi(w-\varepsilon)$	$\phi(w)$		$\phi(w)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$n-1$	$w-\varepsilon$	w		w	$\phi(w-\varepsilon)$	$\phi(w)$		$\phi(w)$	$\phi(w-\varepsilon)$	$\phi(w)$		$\phi(w)$
n	$w-\varepsilon$	w		0	$\phi(w-\varepsilon)$	$\phi(w)$		0	$\phi(w-\varepsilon)$	$\phi(w)$		–

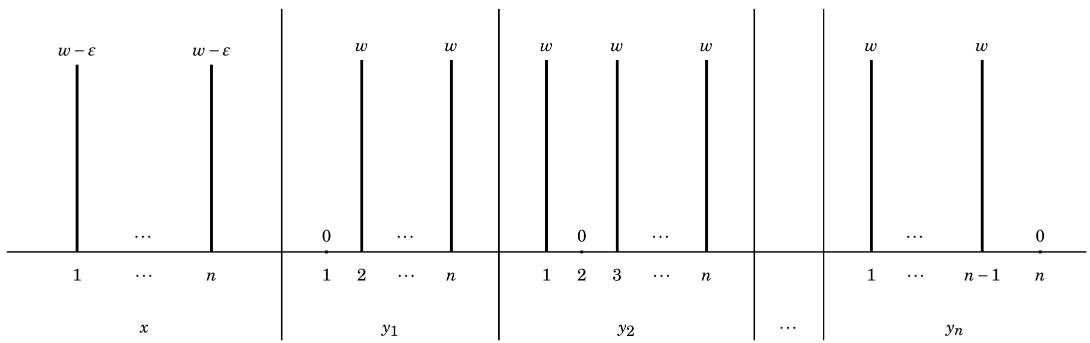

Figure 5.11 The Symmetric Minimal Infinite Superiority Sequence

6

The total claims principles

Many accept a version of strong finite superiority according to which "we ought to save a large number from being permanently bedridden rather than save one from death" and a version of minimal infinite superiority according to which "we ought to save one from death rather than a multitude from a very minor harm, no matter how large this multitude".[1] Voorhoeve (2014) claims that a principle I call the Deontic Relevant Claims Total Principle (DTRC) "satisfactorily explains" both judgements.[2] I argue that the explanation is unsatisfactory because his simplified statement of DTRC is severely incomplete, and straight-forward variations are inconsistent with either (a modest extension of) the version of minimal infinite superiority, or the basic requirement that at least one feasible alternative is not impermissible in the moderate trade-offs paradoxes. And, hence, DTRC does not solve the paradoxes.[3]

6.1 The Deontic Total Relevant Claims Principle (DTRC)

The simple total principles, e.g. Weak Simple Total Principle and Weak Simple Priority Total Principle, can be equivalently expressed in terms of claims and, hence, violate the minimal infinite superiority conditions.[4] But other principles have been suggested that appeal to the total sum of *complaints* in a more refined fashion.

Consider two central cases Voorhoeve discusses.[5] Suppose that, in

Case 1: you can either do

x**:** which fully cures one individual's terminal illness (thereby restoring him to good health for a normal life span), or

[1] Voorhoeve (2014: fn. 2).

[2] Voorhoeve (ibid.: 64).

[3] In a recent article John Halstead (2016) also argues against the account in Voorhoeve (2014). However, in contrast to Halstead, my arguments do not depend on any additional assumptions to succeed. It accepts *all* of the premises in Voorhoeve (ibid.) and shows how they are inconsistent with straightforward interpretations of DTRC.

[4] For a proof see section 6.A.

[5] Voorhoeve (ibid.: 64–5).

y**:** which fully cures a number, m, of other individuals from complete disability (they would otherwise be mentally unimpaired but permanently bedridden, which for them will entail a life just somewhat better than an early death).

According to Voorhoeve, many believe,

Deontic Strong Finite Superiority*: "you ought to save a large number of people from complete disability rather than one from premature death [...]."[6]

In Case 1, according to Deontic Strong Finite Superiority*, for m large enough, x is impermissible.

Next, suppose that, in

Case 2: you can either do

x**:** which fully cures one individual's terminal illness, or

y**:** which fully cures a number, m, of other individuals from an illness which would cause a very minor harm.

According to Voorhoeve, many believe a special version of Minimal Infinite Superiority,

Deontic Minimal Infinite Superiority*: "there is a harm small enough such that no number of such very minor harms to people who will in any case have good lives can outweigh curing one young person's terminal illness [...]."[7]

In Case 2, according to Deontic Minimal Infinite Superiority*, for *no* number of individuals m, x is impermissible.[8]

Case 1 and Case 2 are illustrated in table 6.1 and 6.2, respectively. The first column labels each individual with a number, $i = 1, \ldots, m+1$. The other columns represent the lifetime welfare levels resulting from alternatives x and y. For simplicity, the different welfare levels are normalized to numbers between 0 and 1. 1 represents full life in good health, and 0 represents premature death. Some very small positive number, ε, represents welfare of a life with complete disability, and $1-\varepsilon$ represents welfare of a life with (good health less) a very minor harm. (Both m and ε need not be the same number in different cases. For simplicity, I am using only those variables.)

Voorhoeve claims that, if both Deontic Strong Finite Superiority* and Deontic Minimal Infinite Superiority* are correct, a principle he calls *Aggregate Relevant Claims* (*ARC*) satisfactorily explains them.[9] According to Voorhoeve's *simplified*

6 Voorhoeve (2014: 65); but he does not explicitly label this condition.

7 Voorhoeve (ibid.: 65); but he does not explicitly label this condition.

8 Although it does not strictly follow from Voorhoeve's version, I believe he even wants to say that y is impermissible in this case because this is implied by his version of the Deontic Total Relevant Claims Principle below, and how the problem is discussed in the literature, see Voorhoeve (ibid.: fn. 2).

9 Voorhoeve (ibid.: 64).

Table 6.1 Welfare levels in Case 1

Person	x	y
1	1	0
2	ε	1
⋮	⋮	⋮
$m+1$	ε	1

Table 6.2 Welfare levels in Case 2

Person	x	y
1	1	0
2	$1-\varepsilon$	1
⋮	⋮	⋮
$m+1$	$1-\varepsilon$	1

version of ARC,[10] or what I call the *Deontic Total Relevant Claims Principle*, for short

DTRC:

1. Each individual whose well-being is at stake has a claim on you to be helped. (An individual for whom nothing is at stake does not have a claim.)
2. Individuals' claims *compete* just in case they cannot be jointly satisfied.
3. An individual's claim is *stronger*:
   a) the more her well-being would be increased by being aided; and
   b) the lower the level of well-being from which this increase would take place.
4. A claim is *relevant* if and only if it is sufficiently strong relative to the strongest competing claim.
5. You should choose an alternative that satisfies the greatest sum of strength-weighted, relevant claims.[11]

DTRC is a theory on how to base permissibility on the aggregate of *satisfied strength-weighted claims* to being aided, for short, *claims* to being aided.

Note that DTRC, condition 5, might be read in different ways. On one reading, it states a necessary and sufficient condition for permissibility. Alternatively, it states a necessary condition for permissibility and a sufficient condition for obligation in cases where *only one* alternative satisfies the greatest sum of relevant claims. Then in cases where *more than one* alternative maximizes the sum of relevant claims some tie breaking principle needs to be appealed to. My arguments are compatible with both readings.

Next, I shall apply DTRC to Case 1 and 2 in order to show how it is supposed to capture Deontic Strong Finite Superiority* and Deontic Minimal Infinite Superiority*, respectively.

First, note that by DTRC 4 and 5, only claims that are *sufficiently* strong relative to competing claims are aggregated. However, DTRC itself does not specify the meaning of "sufficiently strong". Voorhoeve suggests a moderate interpretation in order to capture both Deontic Strong Finite Superiority* and Deontic Minimal Infinite Superiority*. He assumes that

[10] Voorhoeve simply labels this version ARC. But this can be misleading since it is only a *simplified* version of a more general principle which he also refers to as ARC but does not state. I will return this simplification at the end of this section and section 6.2.

[11] Voorhoeve (ibid.: 65).

Table 6.3 Relevant claims in Case 1

Person	x	y
1	a	–
2	–	b
⋮	⋮	⋮
$m+1$	–	b
Sum	a	mb

Table 6.4 Relevant claims in Case 2

Person	x	y
1	a	–
2	–	–
⋮	⋮	⋮
$m+1$	–	–
Sum	a	0

(6.1) the claim to be saved from complete disability *is* sufficiently strong relative to the claim to be saved from premature death;

(6.2) the claim to be saved from illness which will cause a very minor harm is *not* sufficiently strong relative to the claim to be saved from premature death.[12]

In the following, I shall follow him in assuming (6.1) and (6.2).

The resulting *relevant* claims, according to DTRC, in Case 1 and 2 are illustrated in table 6.3 and 6.4, respectively. The different strengths of relevant claims are represented by positive numbers, a and b. A dash represents that the individual has no relevant claim. The sum of relevant claims is represented in the last row of each table.

In Case 1, the sum of relevant claims by choosing x is a and by choosing y is mb. Note that $mb > a$ if $m > a/b$. Therefore, by DTRC 5, there *is* a number of individuals m such that x is impermissible and y is permissible. This is consistent with Deontic Strong Finite Superiority*. In Case 2, the sum of relevant claims by choosing x is a and by choosing y is 0.[13] Therefore, by DTRC 5, for all m, x is permissible and y is impermissible. So there is *no* number of individuals m such that x is impermissible. This is consistent with Deontic Minimal Infinite Superiority*.

Importantly, Voorhoeve restricts the application of DTRC.

> [DTRC] takes this relatively simple form in the limited class of cases I consider. In these cases, you must decide whom to benefit by improving their well-being over what would otherwise happen due to natural causes. (No alternative involves making anyone worse off than they would have been but for your intervention.) To make it plausible that the well-being at stake is of concern, I focus on cases in which you can improve people's health-related well-being. I also assume that no one has claims based on desert, that no one is responsible for her level of well-being, and that no one has prior entitlements. In addition, my cases involve neither risk nor changes in the population.[14]

For the sake of argument, I will just accept these assumptions.[15]

[12] Voorhoeve (2014: 67).

[13] By convention, the value of an empty sum, i.e. a summation where the number of terms is zero, is 0. Alternatively, DTRC has to be appropriately extended to deliver the result that x is impermissible.

[14] Voorhoeve (ibid.: 66–7).

[15] However, there are additional restrictions not mentioned needed for DTRC to be plausible. One is that the population has a *finite* number of individuals, see e.g. Vallentyne (1993). Another is that the act is *not* part of a *sequence* of acts, see e.g. Gustafsson (2014). For the sake of argument, I will assume that both restrictions are fulfilled.

6.2 The Two-Level Restriction

However, a main problem with DTRC is that its application is severely restricted in yet another way rendering it severely incomplete as a theory of aggregation in many relevant cases:

> [...] the alternatives under consideration give an individual either the best or the worst feasible outcome for him. (I therefore will not consider cases in which some alternatives are of intermediate value for an individual.)[16]

So, for the applicability of DTRC, Voorhoeve assumes the

> **Two-Level Restriction:** consider only cases in which there are *no more than two different* feasible welfare levels per individual.

Surprisingly, Voorhoeve does not motivate this restriction. However, to see the severity of this restriction, consider a case involving that you can fully, or *almost* fully cure a young individual's *terminal illness*. So your alternatives result in her welfare levels 1, $1-\varepsilon$, and 0. For example, it may be possible by almost fully curing a large number of individuals' terminal illness, leaving each with an illness which will cause a very minor harm, to use the saved scarce resources to cure an additional individual's terminal illness. Surely, this seems like an ordinary and practically relevant case in which a theory of aggregation should give a verdict. But, by the Two-Level Restriction, DTRC is completely silent in such cases that involve an alternative that is of "intermediate value for some individual". Therefore, DTRC is severely incomplete.

In the following, I will consider such cases and explore straightforward extensions of DTRC that drop the Two-Level Restriction. Dropping the restriction makes the appeal to people's claims more complex. In cases that don't adhere to the restriction, for some individual there is not just one claim of a certain strength to be cured but different claims of different strengths. There are two straightforward ways to distinguish these different claims.

First, define a

> **global claim:** an individual's claim to an alternative is relative to her *lowest* welfare level over *all* feasible alternatives.

In the cases considered, consistent with Voorhoeve's assumptions, the lowest welfare level feasible for each individual is the one where the individual is not aided at all. So, global claims fix one interpretation of "being aided" in DTRC 1, and set the baseline for the strength of claims to the lowest level in DTRC 3. Accordingly, if you can either fully, *almost* fully, or not at all cure a young individual's terminal illness, there are two claims with different strengths to consider, one for each alternative where the individual's welfare level is higher than the lowest feasible: the claim to be fully cured, and the claim to be almost fully cured.

Second, define a

[16] Voorhoeve (2014: 66–7).

binary claim: an individual's claim to an alternative is relative to her lower welfare level over a pair of feasible alternatives.

The lower welfare level feasible for each individual in a pair of alternatives is the one where the individual is aided less. So, binary claims fix another interpretation of "being aided" in terms of being aided more, and set the baselines for the strength of claims to the lower level. Accordingly, if you can either fully, *almost* fully, or not at all cure a young individual's terminal illness, there are three claims with different strength to consider. One for each pair of alternatives where one welfare level is higher than the other: the claim to be fully cured in comparison to being not at all cured, the claim to be almost fully cured in comparison to being not at all cured, *and* the claim to being fully cured in comparison to be almost fully cured.

As a final restriction Voorhoeve assumes that "claims to an alternative are either all relevant or all irrelevant".[17] This seems like another severe restriction but, for the sake of argument, my cases will adhere to it.

6.3 Global DTRC

The first extension, arguably, sticks most closely to the formulation of DTRC. According to

Global DTRC: same as DTRC where claims are global claims,

i.e. an individual's claim to an alternative is relative to her *lowest* welfare level over *all* feasible alternatives.

Global DTRC gives the same verdict as DTRC in cases with only two feasible alternatives, in particular in Case 1 and 2.[18]

But Global DTRC fails to give a plausible verdicts in some relevant cases that do not fit the Two-Level Restriction because it violates (a extremely modest extension of) Deontic Minimal Infinite Superiority*. Return to the case from the previous section. Suppose that, in

Simple Extended Case 2: you can either do

x**:** which fully cures one young individual's terminal illness and *almost fully* cures some number, m, of other young individuals' terminal illness leaving them with an illness which will cause a very minor harm,

y**:** which *fully* cures those other m young individuals' terminal illness, or

z**:** which cures no one.

The resulting welfare levels are illustrated in table 6.5.

As the name suggests, Simple Extended Case 2 in a sense extends Case 2. The extension being the additional alternative, z, where all die prematurely (due to

[17] Voorhoeve (2014: 67).

[18] Global DTRC also gives the verdict Voorhoeve suggests in the other cases he discusses to which I return throughout the paper.

Table 6.5 Welfare levels in Simple Extended Case 2

Person	x	y	z
1	1	0	0
2	$1-\varepsilon$	1	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$m+1$	$1-\varepsilon$	1	0

Table 6.6 Relevant global claims in Simple Extended Case 2

Person	x	y	z
1	a	–	–
2	–	a	–
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$m+1$	–	a	–
Sum	a	ma	

"natural causes"). But *this* extension should not matter for the permissibility of x and y. It should not matter whether there is an additional alternative where *no one* is aided and all die. Therefore, it seems obvious that the verdict in this case should not differ from the verdict in Case 2, i.e. you should do x.

The resulting relevant claims according to Global DTRC are illustrated in table 6.6. Except for individual 1, three different welfare levels per individual are feasible. Therefore, by the Two-Level Restriction, DTRC does not apply to Simple Extended Case 2. Dropping the Two-Level Restriction, what are the claims at stake? According to global claim, each individual has a claim to be fully cured from terminal illness.

Next, we should ask, what are the strengths of these claims? In Simple Extended Case 2, for everyone the lowest level feasible is premature death. Therefore, by Global DTRC 3,

(6.3) the individuals' claims to be fully cured from terminal illness are the strongest competing claims, and

(6.4) the individuals' claims to be fully cured from terminal illness are equally strong.

Therefore, by Global DTRC 4,

(6.5) the individuals' claims to be cured from terminal illness *are* relevant.

If these are the only relevant claims, then the sum of relevant claims of x is a and of y is ma. So, for $m > 1$, only y satisfies the greatest sum of relevant claims. Therefore, by Global DTRC 5, there *is* a number of individuals m such that x is impermissible and y is permissible. But this violates Deontic Minimal Infinite Superiority*.

It might be objected that Deontic Minimal Infinite Superiority* is meant to apply only to cases with *no more than two* alternatives, as in Case 2. This would be another restriction which renders the theory severely incomplete. I will return to this objection in section 6.5. For the sake of argument, I accept that Deontic Minimal Infinite Superiority* does not apply to *all* cases. But, as argued above, it is reasonable that Deontic Minimal Infinite Superiority* should at least (be modestly extended to) give the same verdict in Simple Extended Case 2 as in Case 2.

Note that since y is permissibly chosen in Simple Extended Case 2 but not in Case 2, Global DTRC apparently violates Alpha, i.e. if an alternative is permissible in a given feasible set, then it is also permissible in any subset containing the alternative.[19]

Apparently, by Alpha, if y is permissible in Simple Extended Case 2, then y is permissible in Case 2. But, as shown above, according to Global DTRC y is permissible in Simple Extended Case 2 and y is *im*permissible in Case 2. Discussing another case, Voorhoeve argues that DTRC can be saved from violating Alpha by "fine-grained individuation of alternatives" that includes the choice context in the description of alternatives.[20] For the sake of argument, I will assume that Alpha should be rejected. But importantly, my objection above does not appeal to Alpha. It only appeals to Deontic Minimal Infinite Superiority*, or at least a modest extension of it.

Next, it might be objected that there are additional claims to consider. Since, for $i = 2, \ldots, m+1$, there are more than two welfare levels feasible for individual i, it might be claimed that individual i has an additional claim to be almost fully cured from the terminal illness, and,

(6.6) for $i = 2, \ldots, m+1$, individual i's claim to be almost fully cured from terminal illness *is* sufficiently strong relative to the strongest competing claim,

i.e. the claim to be fully cured from terminal illness.

But this is questionable since these individuals' claims seem not to *compete* in the relevant sense. According to Global DTRC 2, "[i]ndividuals' claims *compete* just in case they cannot be jointly satisfied". But it is questionable whether an individual's claim to be fully cured cannot be jointly satisfied with *her* claim to be almost fully cured.

But it might be claimed that Global DTRC 2 could be skipped or reformulated. For the sake of argument, assume (6.6). The resulting relevant claims are illustrated in table 6.7. By Global DTRC 4 and (6.6),

(6.7) for $i = 2, \ldots, m+1$, individual i's claim to be almost fully cured *is* relevant.

These relevant claim are represented by another positive number c. So the sum of relevant claims in x is $a + mc$. Note that

(6.8) $a > c$,

i.e. the claim to be fully cured is stronger than the claim to be almost fully cured.

Therefore, by (6.8) and elementary algebra,

[19] See section 5.2. Voorhoeve (2014: 65) calls the condition *Basic Contraction Consistency*.

[20] Voorhoeve (ibid.: 79), follows Broome (1991: ch. 5).

Table 6.7 Relevant global claims in Simple Extended Case 2

Person	x	y	z
1	a	–	–
2	c	a	–
⋮	⋮	⋮	⋮
$m+1$	c	a	–
Sum	$a+mc$	ma	

Table 6.8 Welfare levels in Symmetric Extended Case 2.

Person	x_1	y_1	x_2	y_2	⋯	x_{m+1}	y_{m+1}
1	1	0	$1-\varepsilon$	1		$1-\varepsilon$	1
2	$1-\varepsilon$	1	1	0		$1-\varepsilon$	1
3	$1-\varepsilon$	1	$1-\varepsilon$	1	⋱	$1-\varepsilon$	1
⋮	⋮	⋮	⋮	⋮		⋮	⋮
m	$1-\varepsilon$	1	$1-\varepsilon$	1		$1-\varepsilon$	1
$m+1$	$1-\varepsilon$	1	$1-\varepsilon$	1		1	0

(6.9) there is a positive integer m such that $ma > a + mc$,

i.e. the sum of relevant claims in y is greater than in x, if $m > \frac{a}{a-c}$.Therefore, there is a number of individuals m such that only y satisfies the greatest sum of relevant claims. Therefore, by Global DTRC 5, there *is* a number of individuals m such that x is impermissible and y is permissible. Again, this violates (the modest extension of) Deontic Minimal Infinite Superiority*, and Global DTRC fails to give a plausible verdict.

Another objection might be that, in Simple Extended Case 2, z is a morally *irrelevant alternative*. z can be disregarded since it is just obviously wrong to let everyone die. Some rule utilitarians, libertarians, and contract theorists appeal to exceptions for "moral catastrophes".[21] I find such claims *ad hoc*—especially in a theory about aggregation.

But even this objection can be granted. Consider

Symmetric Extended Case 2: illustrated in table 6.8 which contains only the resulting welfare levels from Case 2 in x and y and all permutations of those.

The relevant claims are illustrated in table 6.9. Again, for $i = 1, \ldots, m+1$, individual i has a terminal illness since i dies prematurely in y_i. Therefore, the relevant global claims are distributed as in Simple Extended Case 2 in x and y and permutations of those. Therefore, as in Simple Extended Case 2, there is a number of individuals m such that only y_1, y_2, ..., and y_{m+1} satisfy the greatest sum of relevant claims.

[21] See e.g. Hooker (2002), Nozick (1974: 30 fn) and Otsuka (2018: 200), and Otsuka (2018: 200). Otsuka even rules out some "manifestly unreasonable option" even if they do not seem to be moral catastrophes.

Table 6.9 Relevant global claims in Symmetric Extended Case 2.

Person	x_1	y_1	x_2	y_2	$\cdots$	x_{m+1}	y_{m+1}
1	a	–	c	a		c	a
2	c	a	a	–		c	a
3	c	a	c	a	$\ddots$	c	a
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
m	c	a	c	a		c	a
$m+1$	c	a	c	a		a	–
Sum	$a+mc$	ma	$a+mc$	ma	$\cdots$	$a+mc$	ma

Therefore, by Global DTRC 5, there *is* a number of individuals m such that x_1, x_2,..., and x_{m+1} are impermissible. Again, this violates (a modest extension of) Deontic Minimal Infinite Superiority*, and Global DTRC fails to give a plausible verdict. And to object here that some alternatives are morally irrelevant is question begging because the case involves only alternatives which are at the core of the aggregation problem posed by Case 2.

6.4 Binary DTRC

The problem with Simple Extended Case 2 and Symmetric Extended Case 2 is easily avoided by a *binary* interpretation of claims. However, in this case DTRC 5 has to be reformulated in order to give a final verdict in cases with more than two feasible alternatives. According to

> **Binary DTRC:** same as DTRC where claims are binary claim, i.e. an individual's claim to an alternative is relative to her lower welfare level over a pair of feasible alternatives, and except for 5 which is replaced by
>
> B5. You should choose an alternative that satisfies the greatest sum of strength-weighted, relevant claims in pairwise comparison with all other feasible alternatives.

Binary DTRC gives a verdict consistent with Deontic Strong Finite Superiority* and Deontic Minimal Infinite Superiority* in all cases so far considered.

In particular, consider again Simple Extended Case 2. The resulting relevant claims according to Binary DTRC are illustrated in table 6.10. First, consider the pairwise comparison between x and y as illustrated in table 6.10a. This is the same comparison as in Case 2. Therefore, the relevant claims are the same.

Next, consider the pairwise comparison between x and z as illustrated in table 6.10b. In z, no one's claim is satisfied. Note that the comparison between x and z highlights an insufficient formulation of DTRC 4. In this comparison there are *no competing* claims. Therefore, for every claim, it is *not true* that "it is sufficiently strong relative to the strongest competing claim". Hence, by DTRC 4 *no* claim is relevant. But clearly individual 1's claim in x to be fully cured from terminal illness should *be* relevant. So DTRC has to be further amended by replacing 4 with

Table 6.10 Relevant binary claims in Simple Extended Case 2.

(a) x compared to y

Person	x	y
1	a	–
2	–	–
⋮	⋮	⋮
$m+1$	–	–
Sum	a	0

(b) x compared to z

Person	x	z
1	a	–
2	c	–
⋮	⋮	⋮
$m+1$	c	–
Sum	$a+mc$	0

(c) y compared to z

Person	y	z
1	–	–
2	a	–
⋮	⋮	⋮
$m+1$	a	–
Sum	ma	0

4.* A claim is *relevant* if and only if it is sufficiently strong relative to the strongest competing claim *or there is no competing claim*.

Therefore, only x satisfies the greatest sum of relevant claims in pairwise comparison with all other feasible alternatives. Therefore, by Binary DTRC B5, x is the only permissible alternative. This is consistent with (the modest extension of) Deontic Minimal Infinite Superiority*.

However, Binary DTRC does not solve the moderate trade-off paradoxes and the Non-Anti Pigou-Dalton Paradox since it implies all the substantial conditions of these paradoxes. To see this note that Super Ultra Minimal Infinite Superiority, Ultra Minimal Finite Superiority*, and Pigou-Dalton are satisfied because (3a) and (3b) hold, i.e. both increasing the welfare difference and lowering the welfare level increase a person's claim.

Furthermore, Binary DTRC faces an additional paradox by Parfit (2003), and discussed by Voorhoeve.[22] Suppose that, in what I call the

Strong Finite Superiority Paradox Sequence: you can either do

x**:** which fully cures one individual's terminal illness,

y**:** which fully cures m_1 other individuals' moderate impairment, or

z**:** which fully cures m_2 other individuals' minor impairment.

The Strong Finite Superiority Paradox Sequence is illustrated in figure 6.1. Since this case is already discussed in the literature and my space is limited, I will skip the details. With Parfit, Voorhoeve assumes that, in the Strong Finite Superiority Paradox Sequence,

(6.10) if only x and y are feasible, then the m_1 individuals' claims *are* relevant,

(6.11) if only y and z are feasible, then the m_2 individuals' claims *are* relevant, and

(6.12) if only x and z are feasible, then the m_2 individuals' claims are *not* relevant.[23]

[22] Parfit (2003: 384); Voorhoeve (2014: sec. V and VI). Similar sequences are discussed by Temkin (2012).

[23] Voorhoeve (2014: 76–7).

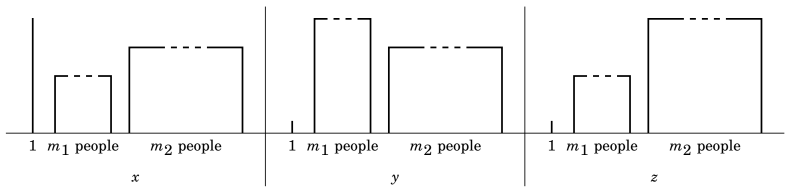

Figure 6.1 The Strong Finite Superiority Paradox Sequence

Therefore, by Binary DTRC B5, (6.10), (6.11) and (6.12), there are numbers of individuals m_1 and m_2 such that x, y and z are impermissible. Therefore, Binary DTRC violates No Prohibition Dilemmas, i.e. for any set of feasible alternatives, at least one feasible alternative is not impermissible.

There are some strong arguments for No Prohibition Dilemmas.[24] Furthermore, Voorhoeve claims that "[DTRC] always selects at least one alternative from a feasible set".[25] Note that the Strong Finite Superiority Paradox Sequence satisfies the Two-Level Restriction. Voorhoeve claims that in the Strong Finite Superiority Paradox Sequence, according to DTRC,

(6.13) only y is permissible.[26]

Voorhoeve's argument for why DTRC implies this verdict is that it renders the m_2 individuals' claims irrelevant when all three alternatives are feasible. As shown above, this claim is incompatible with Binary DTRC.

Furthermore, I suppose that those inclined to accept both Deontic Strong Finite Superiority* and Deontic Minimal Infinite Superiority* will intuitively share Voorhoeve's verdict that (6.13). Hence Binary DTRC does not give the intuitive verdict in the Strong Finite Superiority Paradox Sequence and does not solve what I call the *Strong Finite Superiority Paradox*. In general, the verdict of Binary DTRC does not violate Alpha and hence does not solve any of the paradoxes discussed in the previous chapters.

However, the intuitively correct verdict is implied by Global DTRC. But, as I have argued in the previous section, Global DTRC fails for other reasons.

6.5 On the possibility of DTRC

As discussed in section 3.5, in "On the Possibility of Nonaggregative Priority for the Worst Off" Fleurbaey, Tungodden and Vallentyne deliver several *impossibility*, or inconsistency, results concerning moral permissibility based on seemingly plausible conditions.[27] At the centre of their investigation are very weak conditions of minimal finite superiority and infinite superiority. The latter is a very weak minimal infinite superiority condition which focuses only on the *worst*- and the *best*-off individuals.

In face of these impossibility results it is important to see which condition DTRC denies in order to escape inconsistency. What makes Voorhoeve escape is the Two-Level Restriction for DTRC. In contrast, Fleurbaey, Tungodden and Vallentyne assume Domain Richness, i.e. for any logically possible welfare distribution with a finite number of individuals and non-negative welfare levels, there is an alternative that generates that distribution.[28] As noted by Fleurbaey, Tungodden and

[24] McConnell 2014: sec. 4; some responses in sec. 5.
[25] Voorhoeve (2014: 78).
[26] Voorhoeve (ibid.: 78).
[27] Fleurbaey, Tungodden and Vallentyne (2009).
[28] Fleurbaey, Tungodden and Vallentyne (ibid.: 260).

Vallentyne, their impossibility results hold even if the welfare levels have additionally an upper boundary.[29] But they appeal to cases that assume more than two welfare levels feasible per individual.[30] Hence, by the Two-Level Restriction, DTRC is completely silent on which condition it violates in the cases they discuss.

Not only do Fleurbaey, Tungodden and Vallentyne deliver impossibility but also *possibility* results. One possibility result, *Result 6*, is based on a minimal finite superiority condition that gives people below a certain threshold absolute priority.[31] However, this kind of minimal finite superiority is explicitly ruled out as implausible by Voorhoeve, because "an arbitrarily small gain for a person below the threshold can outweigh any gain above the threshold, no matter how large it is and no matter how many people would receive it."[32] Therefore, such theories are incompatible with DTRC.

The other possibility result, *Result 7*, contains only extremely minimal infinite superiority conditions that have similar implications as the Two-Level Restriction.[33] These two-option infinite superiority conditions apply merely in cases with *only two* feasible alternatives. In such cases there are *no more than two* welfare levels feasible per individual and hence the Two-Level Restriction is satisfied. However, as Fleurbaey, Tungodden and Vallentyne note, it is doubtful whether their result is significant.

> [T]he conditions imposed for Result 7 do not leave room for a significant form of nonaggregative priority. After all, the two-option priority conditions [. . .] are completely silent about priority for the worse off when there are more than two options.[34]

However, they speculate that there could be a variation of the possibility result with a less minimal infinite superiority condition. Unfortunately, they failed to find it.[35]

They also note, in *Result 8*, that the possibility disappears once Alpha is invoked.[36] So, Alpha appears to be a main obstacle to moral theories that capture both minimal finite and infinite superiority. As I argued in the previous sections, even granting the rejection of Alpha, DTRC faces severe problems.

6.6 Complaints

Fleurbaey, Tungodden and Vallentyne suggest a principle that seems promising to fulfil a less minimal infinite superiority condition, the Deontic Lenimax Global Complaint (see section 5.3):

> a feasible alternative [is] permissible if and only if no other feasible alternative has a smaller maximum [global] complaint, or, in cases of ties, no smaller second-largest [global] complaint, etc. [R]elative to

[29] Fleurbaey, Tungodden and Vallentyne (2009: 271).
[30] Fleurbaey, Tungodden and Vallentyne (ibid.: 273).
[31] (Fleurbaey, Tungodden and Vallentyne [ibid.: 277-9]).
[32] Voorhoeve (2014: fn. 6). Fleurbaey, Tungodden and Vallentyne (2009: 278-9) note that this absolute priority would be one possible way out of the impossibility results.
[33] Fleurbaey, Tungodden and Vallentyne (ibid.: 281).
[34] Fleurbaey, Tungodden and Vallentyne (ibid.: 281).
[35] Fleurbaey, Tungodden and Vallentyne (ibid.: 282).
[36] Fleurbaey, Tungodden and Vallentyne (ibid.: 283).

Table 6.11 Relevant global claims in Simple Extended Case 2

Person	x	y	z
1	–	a	a
2	–	–	a
⋮	⋮	⋮	⋮
$m+1$	–	–	a
Sum	0	a	ma

a given feasible set, a person's [global] complaint in a given option is equal to the shortfall in priority weighted benefits from the most that the individual can get *in that feasible set*.[37]

So while global claims relativize an individual's claim to the *lowest* welfare level feasible, symmetrically, global complaints relativize to the *highest* level feasible. However, the Deontic Lenimax Global Complaint fails to imply Deontic Strong Finite Superiority* in Case 1 since it focuses only on the *maximum* complaint.

But it might be claimed that Global DTRC, which implies Deontic Strong Finite Superiority* in Case 1, can be reformulated in terms of global complaints. In the following, I will use dashes to mark conditions and cases reformulated in terms of complaints. According to

Global DTRC': Global DTRC reformulated in terms of global complaints, in particular,

5'. You should choose an alternative that minimizes the sum of strength-weighted, relevant complaints.

Global DTRC' gives a verdict consistent with Deontic Minimal Infinite Superiority* in all two-alternative cases considered so far, since in these cases global complaints are symmetrical to global claims, i.e. if some individual has a claim in one alternative there is a corresponding complaint in the other. Interestingly, and in contrast to binary complaints, there is an asymmetry in cases with more than two alternatives. Global DTRC' is consistent with (the modest extensions of) Deontic Minimal Infinite Superiority* even in all other cases discussed so far. For example, in Simple Extended Case 2, it renders the m individual's complaints in x irrelevant since the difference between the welfare level $1-\varepsilon$ in x and the *highest* level, 1 in y, is so small. The corresponding global complaints are illustrated in table 6.11. Therefore, only x is permissible. This is consistent with (the suggested moderate extension of) Deontic Minimal Infinite Superiority*. The analogue holds for the Symmetric Extended Case 2.

However, Global DTRC' faces closely related severe problems. Consider

Case 2': illustrated in table 6.12,

and

[37] Fleurbaey, Tungodden and Vallentyne (ibid.: 281). Similar suggestions have been made by Nagel (1979: ch. 8); and Scanlon (1998: ch. 5).

Symmetric Extended Case 2': illustrated in table 6.13.

The resulting relevant complaints according to Global DTRC' in Case 2' and Symmetric Extended Case 2' are illustrated in table 6.14 and 6.15, respectively. The different complaints are represented by positive numbers, a and c.

Note that the relevant complaints in Case 2' and Symmetric Extended Case 2' are symmetric to the relevant claims in Case 2 and Symmetric Extended Case 2, illustrated in table 6.4 and 6.9 on page 148 and 154. Therefore, by Global DTRC', in Case 2' there is *no* number of individuals m such that y is permissible and x is impermissible, and in Symmetric Extended Case 2' there *is* a number of individuals m such that x_1, x_2,..., and x_{m+1} are impermissible. But the latter violates (a modest extension of) the version of minimal infinite superiority decisive in the former case. So Global DTRC' fails as well to give a plausible verdict.

It might be replied that the version of minimal infinite superiority implied by Global DTRC' in Case 2' is not the Deontic Minimal Infinite Superiority* condition as stated by Voorhoeve. This is because the condition "who will in any case have good lives" is not met. However, in my objection, I have not appealed to Deontic Minimal Infinite Superiority* but only to the version of minimal infinite superiority that is implied by Global DTRC' in Case 2'.[38] The argument was only that Case 2' and Symmetric Extended Case 2' should be judged in an exactly similar way.

It might be objected that Global DTRC' could be defended against my objection by an appeal to fairness. It might be claimed that there is some (additional) unfairness in Symmetric Extended Case 2' by doing x_1 not present in Case 2' by doing x. In Symmetric Extended Case 2', by doing x_1 one respects the strong complaint of individual 1 while not respecting the *equally* strong complaint of the other individuals. In Case 2', by doing x this is not true since only individual 1 has the strong complaint. This additional unfairness gives additional reasons against doing x_1 in Symmetric Extended Case 2'. The analogue applies for $x_2, \ldots,$ and x_{m+1}. And this is one reason why Alpha should be rejected.

This objection can be answered. For the sake of argument, I accepted that Alpha should be rejected and I accept that there is additional unfairness in doing x_1 in Symmetric Extended Case 2'. If this unfairness would always outweigh infinite superiority considerations, then it might be that choosing x_1 would be impermissible. But this is implausible. Respecting the strong complaint of the other individuals is possible only by choosing some y_i. But this leaves everyone at most at the very low welfare level ε and respects everyone's claim only minimally. The same applies for $x_2, \ldots,$ and x_{m+1}.

6.7 Conclusion

If correct, the arguments presented show that it is hard, if not impossible, to state a plausible version of DTRC even given only modest requirements on this theory. This is because, first, DTRC is completely silent when there are more than two

[38] Note that Voorhoeve's DTRC implies versions of minimal infinite superiority that don't meet the "who will in any case have good lives" condition as well, see (Voorhoeve [2014: 83–4]).

Table 6.12 Welfare levels in Case 2'

Person	x	y
1	1	0
2	0	ε
3	0	ε
$\vdots$	$\vdots$	$\vdots$
m	0	ε
$m+1$	0	ε

Table 6.13 Welfare levels in Symmetric Extended Case 2'

Person	x_1	y_1	x_2	y_2		x_{m+1}	y_{m+1}
1	1	0	0	ε		0	ε
2	0	ε	1	0		0	ε
3	0	ε	0	ε	$\ddots$	0	ε
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
m	0	ε	0	ε		0	ε
$m+1$	0	ε	0	ε		1	0

Table 6.14 Relevant global complaints in Case 2'

Person	x	y
1	–	a
2	–	–
3	–	–
$\vdots$	$\vdots$	$\vdots$
m	–	–
$m+1$	–	–
Sum	0	a

Table 6.15 Relevant global complaints in Symmetric Extended Case 2'

Person	x_1	y_1	x_2	y_2		x_{m+1}	y_{m+1}
1	–	a	a	c		a	c
2	a	c	–	a		a	c
3	a	c	a	c	$\ddots$	a	c
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
m	a	c	a	c		a	c
$m+1$	a	c	a	c		–	a
Sum	ma	$a+mc$	ma	$a+mc$		ma	$a+mc$

feasible welfare levels per individual, and, second, the three natural versions of DTRC (Global, Binary, and Hybrid) fail to capture (modest extensions of) Deontic Minimal Infinite Superiority* or violate No Prohibition Dilemmas in cases where this is intuitively implausible. And it is harder than sometimes expected since I accepted, for the sake of argument, the rejection of conditions that appeared to be main obstacles to a principle that captures versions of both minimal finite and infinite superiority. Finally, while there is an interesting asymmetry between claims and complaints, switching from claims to complaints is no viable option either.

As a final remark note that the arguments presented are quite general. For example, they are independent of DTRC 2, priority for the worse off in DTRC 3b, and the particular choice of welfare levels in Deontic Strong Finite Superiority* and Deontic Minimal Infinite Superiority*.

6.A The simple total claims principles

In this section, two versions, the binary and the global, of the simple total complaints and claims principles are stated, followed by the proof of their equivalence to the simple total (welfare) principles. This shows that they neither solve the moderate trade-off paradoxes. According to the (axiological)

Weak Simple Total Complaints: if the total sum of complaints against x is smaller than the total sum of complaints against y, then x is better than y, i.e., for all $x, y \in O$,

$$\sum_{i \in N} c_i(x) < \sum_{i \in N} c_i(y) \implies x \succ y.$$

Next, consider the generalization of the binary complaints with *weakly* concave priority-weight function ϕ, for short the

generalized binary complaint: for $x, y \in O$, $i \in N$, and strictly increasing and weakly concave ϕ, the *generalized binary complaint against* x relative to y is

$$c_i(x, y) := \max_{z \in \{x,y\}} \phi(u_i(z)) - \phi(u_i(x)).$$

According to

Weak Simple Total Generalized Binary Complaints: there exists a strictly increasing and weakly concave function ϕ such that, for all $x, y \in O$,

$$\sum_{i \in N} c_i(x, y) < \sum_{i \in N} c_i(y, x) \implies x \succ y.$$

Next, consider the deontic versions. According to

Deontic Simple Total Complaints: for all $x \in X \subseteq O$,

$$x \in C(X) \iff \text{for all } y \in X, \sum_{i \in N} c_i(x) \leq \sum_{i \in N} c_i(y).$$

According to

Deontic Simple Total Generalized Binary Complaints: there exists a strictly increasing and weakly concave function ϕ such that, for all $x \in X \subseteq O$,

$$x \in C(X) \iff \text{for all } y \in X, \sum_{i \in N} c_i(x,y) \leq \sum_{i \in N} c_i(y,x).$$

Next, consider the generalization of the global complaints.

generalized global complaint: for $x \in X \subseteq O$, $i \in N$, and strictly increasing and weakly concave ϕ, the *generalized global complaint against* x relative to X is

$$c_i(x,X) := \max_{z \in X} \phi(u_i(z)) - \phi(u_i(x)).$$

According to

Deontic Simple Total Generalized Global Complaints: there exists a strictly increasing and weakly concave function ϕ such that, for all $x \in X \subseteq O$,

$$x \in C(X) \iff \text{for all } y \in X, \sum_{i \in N} c_i(x,X) \leq \sum_{i \in N} c_i(y,X).$$

The equivalence between the simple total (welfare) principles and the simple total complaints principles shows

Proposition 6.1. *The sum total of complaints against x is smaller than against y if and only if the total sum of (priority-weighted) welfare in x is greater than in y, i.e., for all real-valued functions ϕ, and $x, y \in O$,*

$$\sum_{i \in N} c_i(x) \leq \sum_{i \in N} c_i(y) \iff \sum_{i \in N} \phi(u_i(x)) \geq \sum_{i \in N} \phi(u_i(y)).$$

Proof. Let $x, y \in X \subseteq O$. First, consider the global complaints.

$$\begin{aligned}
\sum_{i \in N} c_i(x) &= \sum_{i \in N} c_i(x,X) \\
&= \sum_{i \in N} \left(\max_{z \in X} \phi(u_i(z)) - \phi(u_i(x)) \right) \\
&= \sum_{i \in N} \max_{z \in X} \phi(u_i(z)) - \sum_{i \in N} \phi(u_i(x)).
\end{aligned}$$

Therefore,

$$\begin{aligned}
& \sum_{i \in N} c_i(x) \leq \sum_{i \in N} c_i(y) \\
\iff \quad & -\sum_{i \in N} \phi(u_i(x)) \leq -\sum_{i \in N} \phi(u_i(y)) \\
\iff \quad & \sum_{i \in N} \phi(u_i(x)) \geq \sum_{i \in N} \phi(u_i(y)).
\end{aligned}$$

This shows the claim for *global* complaints against. For *binary* complaints, it follows immediately from proposition 5.12, i.e. the equivalence between binary complaints and global complaints relative to feasible sets with two alternatives. □

The equivalence between the simple total (welfare) principles and the total *claims* principles is shown by

Corollary 6.1. *The sum total of claims to x is smaller than to y if and only if the total sum of (priority-weighted) welfare in x is greater than in y, i.e., for all $x, y \in O$,*

$$\sum_{i\in N} c_i^*(x) \geq \sum_{i\in N} c_i^*(y) \iff \sum_{i\in N} \phi(u_i(x)) \geq \sum_{i\in N} \phi(u_i(y)).$$

Proof. Analogous to the proof of proposition 6.1 by using

$$\sum_{i\in N} c_i^*(x) = \sum_{i\in N}\left(\phi(u_i(x)) - \min_{z\in X}\phi(u_i(z))\right).$$ □

Finally, we also have the symmetry between the simple total complaints and complaints principles.

Corollary 6.2. *The the sum total of (priority) claims to x is greater than to y if and only if the sum total of (priority) complaints against x is smaller than against y, i.e., for all $x, y \in O$,*

$$\sum_{i\in N} c_i^*(x) \geq \sum_{i\in N} c_i^*(y) \iff \sum_{i\in N} c_i(x) \leq \sum_{i\in N} c_i(y).$$

Proof. Follows from proposition 6.1 and corollary 6.1. □

6.B The geometric total claims principles

In this section, I argue that a principle, suggested by Carlson (2000), shares the problems of the other complaints and claims principles, i.e it either does not solve the moderate trade-offs paradoxes, or violates Ultra Minimal Finite Superiority in the Symmetric Ultra Minimal Finite Superiority Sequence or Super Ultra Minimal Infinite Superiority in the Symmetric Super Ultra Minimal Infinite Superiority Sequence.

Carlson states his theory in terms of the sum of weighted “harms”. However, he does not clearly define the concept of harms he is using. Carlson’s usage of harms presumably indicates that there are no priority-weights attached to the welfare levels, i.e. he refers to simple rather than priority complaints.[39] However, my following arguments do not presuppose that the priority-weight function is strictly concave hence they cover mere harms, or simple complaints, as well.

According to Carlson’s *Moderate Trade-offs Theory*,[40] what I call

[39] See section 5.1 for a definition of simple and priority complaints.
[40] Carlson (2000: 248–9).

Geometric Total Complaints Principle: there exists $0 < q < 1$ such that, for any alternative $x \in O$,

$$v(x) = v_q(x) := \sum_{i=1}^{n} q^{i-1} c_{(i)}(x).$$

Here v represents the value of alternatives.[41] As I argued before, there are different ways to specify complaints, i.e. $c_{(i)}(x)$. First, if they are specified as binary complaints, then Geometric Total Complaints Principle cannot solve the moderate trade-off paradoxes as the impossibility results in chapter 3 show. So, turn to the global complaints, i.e., there is a strictly increasing and weakly concave *continuous* function ϕ[42] such that, for $x \in X \subseteq O$, and $i \in N$,

$$c_i(x, X) = \max_{z \in X} \phi(u_i(z)) - \phi(u_i(x)).$$

Accordingly,

$$v_q(x, X) := \sum_{i=1}^{n} q^{i-1} c_{(i)}(x, X).$$

According to

Deontic Geometric Total Global Complaints Principle: there exists $0 < q < 1$ such that, for any $x \in X \subseteq O$,

$$x \in C(X) \iff \text{for all } y \in X,\ v_q(x, X) \leq v_q(y, X).$$

Proposition 6.2. *Deontic Geometric Total Global Complaints Principle violates Deontic Ultra Minimal Finite Superiority in the Symmetric Ultra Minimal Finite Superiority Sequence,*

i.e., for all $w, \varepsilon \in \mathbb{R}_{>0}$, $n \in \mathbb{Z}_{>0}$ such that ε is small enough and n is large enough, and $X = \{x, y_1, \ldots, y_n\} \subseteq O$ if

1. $w > \varepsilon > 0$;
2. for all $i \in N$,

$$u_i(x) = \varepsilon;$$

[41] For the definition of the value function see section 2.B.

[42] Continuity is a simplifying assumption, see footnote (44). A real-valued function $\phi \in \mathbb{R}^D$ is *continuous* if and only if, for all $c \in D$,

$$\lim_{x \to c} \phi(x) = \phi(c).$$

3. for all $j = 1, \dots, n$,

$$u_i(y_j) = \begin{cases} 0 & \text{if } i = j, \\ w & \text{else,} \end{cases}$$

then, according to Deontic Geometric Total Global Complaints Principle,

$$x \in C(X).$$

The Symmetric Ultra Minimal Finite Superiority Sequence is repeated in table 6.16.

Proof. For simplicity, assume again that $\phi(0) = 0$.[43] Assume 2 to 3. Therefore, see also table 6.16,

$$c_i(x, X) = \phi(w) - \phi(\varepsilon);$$

$$c_i(y_j, X) = \begin{cases} \phi(w) & \text{if } i = j, \\ 0 & \text{else.} \end{cases}$$

Therefore, by the limit of the geometric sequence (proposition 3.1) and $0 < q < 1$,

$$\begin{aligned} \lim_{n\to\infty} v_q(x, X) &= \lim_{n\to\infty} \sum_{i=1}^{n} q^{i-1}(\phi(w) - \phi(\varepsilon)) \\ &= (\phi(w) - \phi(\varepsilon)) \lim_{n\to\infty} \sum_{i=0}^{n-1} q^i \\ &= \frac{\phi(w) - \phi(\varepsilon)}{1-q}; \end{aligned}$$

$$\begin{aligned} \lim_{n\to\infty} v_q(y_j, X) &= q^0 \cdot \phi(w) + q^1 \cdot 0 + q^2 \cdot 0 + \dots \\ &= \phi(w). \end{aligned}$$

Therefore, by $1 - q > 0$,

$$\begin{aligned} & \lim_{n\to\infty} v_q(y_j, X) < \lim_{n\to\infty} v_q(x, X) \\ \iff \quad & \phi(w) < \frac{\phi(w) - \phi(\varepsilon)}{1-q} \\ \iff \quad & \phi(w) - q \cdot \phi(w) < \phi(w) - \phi(\varepsilon) \\ \iff \quad & q \cdot \phi(w) > \phi(\varepsilon). \end{aligned}$$

[43] See footnote (22).

Table 6.16 The Symmetric Ultra Minimal Finite Superiority Sequence (repeated table 5.3 on page 140)

	$u_i(\cdot)$				$\phi(u_i(\cdot))$				$c_i(\cdot,X)$			
Person	x	y_1	$\cdots$	y_n	x	y_1	$\cdots$	y_n	x	y_1	$\cdots$	y_n
1	ε	0		w	$\phi(\varepsilon)$	0		$\phi(w)$	$\phi(w)-\phi(\varepsilon)$	$\phi(w)$		–
2	ε	w		w	$\phi(\varepsilon)$	$\phi(w)$		$\phi(w)$	$\phi(w)-\phi(\varepsilon)$	–		–
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$n-1$	ε	w		w	$\phi(\varepsilon)$	$\phi(w)$		$\phi(w)$	$\phi(w)-\phi(\varepsilon)$	–		–
n	ε	w		0	$\phi(\varepsilon)$	$\phi(w)$		0	$\phi(w)-\phi(\varepsilon)$	–		$\phi(w)$

Table 6.17 The Symmetric Minimal Infinite Superiority Sequence (repeated table 5.4 on page 143)

	$u_i(\cdot)$				$\phi(u_i(\cdot))$				$c_i^*(\cdot,X)$			
Person	x	y_1	$\cdots$	y_n	x	y_1	$\cdots$	y_n	x	y_1	$\cdots$	y_n
1	$w-\varepsilon$	0		w	$\phi(w-\varepsilon)$	0		$\phi(w)$	$\phi(w-\varepsilon)$	–		$\phi(w)$
2	$w-\varepsilon$	w		w	$\phi(w-\varepsilon)$	$\phi(w)$		$\phi(w)$	$\phi(w-\varepsilon)$	$\phi(w)$		$\phi(w)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$n-1$	$w-\varepsilon$	w		w	$\phi(w-\varepsilon)$	$\phi(w)$		$\phi(w)$	$\phi(w-\varepsilon)$	$\phi(w)$		$\phi(w)$
n	$w-\varepsilon$	w		0	$\phi(w-\varepsilon)$	$\phi(w)$		0	$\phi(w-\varepsilon)$	$\phi(w)$		–

Now, by ϕ being continuous,

(6.14) $\lim_{\varepsilon\to 0}\phi(\varepsilon)=\phi(0)$.

Hence, by $\phi(0)=0$ and ϕ being strictly increasing,

(6.15) for any given (positive) q and w, there is an ε small enough such that this inequality holds.[44]

Therefore, for all $0<q<1$, for any n large enough, and ε small enough, for all $j=1,\ldots,n$,

$$v_q\left(y_j,X\right)<v_q(x,X).$$

This shows the claim. □

Similarly it can be shown that if, instead of global complains (or claims) against an alternative, global claims to an alternative are used, i.e., there is a strictly increasing and weakly concave continuous function ϕ such that, for $x\in X\subseteq O$, and $i\in N$,

$$c_i^*(x,X)=\phi(u_i(x))-\min_{z\in X}\phi(u_i(z))$$

and, accordingly,

$$v_q^*(x,X):=\sum_{i=1}^{n}q^{i-1}c_{(i)}^*(x,X),$$

this principle violates Deontic Super Ultra Minimal Infinite Superiority in the Symmetric Super Ultra Minimal Infinite Superiority Sequence (repeated in table 6.17). More precisely, according to this principle, the

Deontic Geometric Total Global Claims Principle: there exists $0<q<1$ such that, for any $x\in X\subseteq O$,

$$x\in C(X)\iff \text{for all } y\in X,\ v_q^*(x,X)\geq v_q^*(y,X).$$

Proposition 6.3. *Deontic Geometric Total Global Claims Principle violates Deontic Super Ultra Minimal Infinite Superiority in the Symmetric Super Ultra Minimal Infinite Superiority Sequence.*

Proof. Consider the limits of the values of the alternatives in the sequence:

$$\lim_{n\to\infty}v_q^*(x,X)=\sum_{i=1}^{\infty}q^{i-1}\phi(w-\varepsilon)=\frac{\phi(w-\varepsilon)}{1-q};$$

[44] Note that continuity of ϕ is not necessary for this result to hold. It is only needed that $\phi(\varepsilon)$ can be sufficiently small relative to the given q and $\phi(w)$, e.g when ϕ is unbound from above.

$$\begin{aligned}
\lim_{n\to\infty} v_q^*\left(y_j, X\right) &= \lim_{n\to\infty}\left(\sum_{i=1}^{n} q^{i-1}\phi(w) - q^{n-1}0\right)\\
&= \lim_{n\to\infty}\sum_{i=0}^{n-1} q^{i}\phi(w)\\
&= \frac{\phi(w)}{1-q}. && \text{[by proposition 3.1]}
\end{aligned}$$

Therefore, by $1-q>0$,

$$\begin{aligned}
& \lim_{n\to\infty} v_q^*(x,X) < \lim_{n\to\infty} v_q^*\left(y_j, X\right)\\
\iff \quad & \frac{\phi(w-\varepsilon)}{1-q} < \frac{\phi(w)}{1-q}\\
\iff \quad & \phi(w-\varepsilon) < \phi(w).
\end{aligned}$$

Therefore, for n large enough,

$$v_q^*(x,X) < v_q^*\left(y_j, X\right).$$

This shows the claim. □

7

The maximum claim Condorcet principles

7.1 Simplified paradoxes and sequences

This chapter continues the discussion of non-aggregative principles from chapter 5, more precisely, of *maximum claim principles*, i.e. principles that take the maximum claims against (or complaints) or claims to alternatives as the overriding reason. I will suggest new maximum claim principles that get around the shortcomings of those discussed earlier. As I have shown in chapter 5, going beyond maximum claim principles is not necessary to capture minimal finite superiority. However, for many people non-aggregative principles seem hard to accept. I will discuss possible alternatives to the maximum claim principles discussed below in the next chapter, section 8.3.

For simplicity, in the discussion in this chapter we will assume that

(7.1) concern for equal distribution and priority for the worse-off doesn't matter.

As I have shown in chapters 4 and 5, priority for the worse-off is not a necessary feature to capture minimal infinite superiority. It is only necessary to capture Pigou-Dalton.[1]

This assumption has two purposes here. First, we can simplify the moderate trade-offs paradoxes and the symmetric finite and infinite superiority sequences. Second, it will make the discussion clearer and make it easier to identify the crucial features of these new principles.

7.1.1 The Simple Moderate Trade-offs Sequence

Consider a simple version of the moderate trade-offs sequences, the

Simple Moderate Trade-offs Sequence: illustrated in figure 7.1 and table 7.1,

[1] However, the cases and principles can easily be adapted so that the welfare numbers are in terms of *priority-weighted* welfare instead of unweighted welfare.

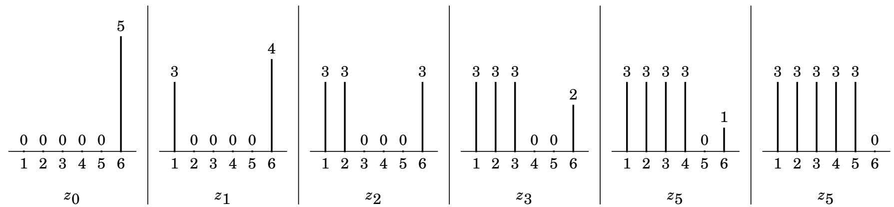

Figure 7.1 The Simple Moderate Trade-offs Sequence

Table 7.1 The Simple Moderate Trade-offs Sequence

	Welfare					
Persons	z_0	z_1	z_2	z_3	z_4	z_5
1	0	3	3	3	3	3
2	0	0	3	3	3	3
3	0	0	0	3	3	3
4	0	0	0	0	3	3
5	0	0	0	0	0	3
6	5	4	3	2	1	0

i.e., in z_0, persons 1 to 5 are at welfare level 0, and person 6 is at welfare level 5; in z_1, person 1 is at 3, persons 2 to 5 are at 0, and person 6 at 4; ...; in z_5, persons 1 to 5 are at 3, and person 6 is at 0.

The main differences to the First Moderate Trade-offs Paradox Sequence are as follows. First, instead of a large number of people, m, there is only one person, person 6, who is relatively slightly worse off in each step from z_0 to z_5. However, this difference does not matter for the evaluation according to the maximum claim principles. Second, this person is not always better off than the other people. However, this difference does not matter under the simplifying assumption (7.1).

The verdict analogous to the axiological verdict in the previously discussed moderate trade-offs paradoxes, see section 3.6 claims (3.39), (3.40), and (3.41), is that

(7.2) successive alternatives are better than their predecessors at first, starting with z_1 is better than z_0;

(7.3) z_0 is better than z_5;

(7.4) there is a undominated alternative better than both z_0 and z_5,

i.e. at least one of z_1 to z_4 is not worse than any other alternative and better than both z_0 and z_5.

And the verdict, analogous to the deontic verdict in these paradoxes (see again section 3.6), that (3.46), (3.47), and (3.48), is that

(7.5) z_0 is impermissible;

(7.6) z_5 is impermissible;

(7.7) there is a permissible alternative.

I recognize that (7.2) to (7.7) might not have much intuitive appeal. However, as I just argued, under the two assumptions from above, the focus on maximum claim principles and (7.1), the evaluations in the Simple Moderate Trade-offs Sequence are analogous to the evaluations in the First Moderate Trade-offs Paradox Sequence.

To see how the Simple Moderate Trade-offs Sequence leads to the analogue paradox for some of the maximum claim principles discussed in chapter 5, consider the following verdict arrived at by Weak Minimax Simple Binary Complaint (introduced in section 5.1). Consider the binary complaints in the Simple Moderate Trade-offs Sequence, illustrated in table 7.2. First,

(7.8) the greatest binary complaint against z_0 compared to z_1 has strength 3 and the greatest binary complaint against z_1 compared to z_0 has strength 1

stemming from person 1 and 6, respectively;

(7.9) no other person has any binary complaints against z_0 compared to z_1 or vice versa.

Therefore, according to Weak Minimax Simple Binary Complaint,

(7.10) z_1 is better than z_0.

Table 7.2 Binary complaints in the Simple Moderate Trade-offs Sequence

	Binary complaints							
Persons	z_0	z_1		z_4	z_5		z_0	z_5
1	3	0		0	0		3	0
2	0	0		0	0		3	0
3	0	0	...	0	0		3	0
4	0	0		0	0		3	0
5	0	0		3	0		3	0
6	0	1		0	1		0	5

Analogously, and more generally,

(7.11) the greatest binary complaint against z_i compared to z_{i+1} has strength 3, and the greatest binary complaint against z_{i+1} compared to z_i has strength 1, for $i = 0,1,2,3,4$

stemming from person $i+1$ and person 6, respectively;

(7.12) no other person has any binary complaints against z_i compared to z_{i+1} or vice versa, for $i = 0,1,2,3,4$.

Therefore, according to Weak Minimax Simple Binary Complaint,

(7.13) z_{i+1} is better than z_i, for $i = 0,1,2,3,4$.

However,

(7.14) the greatest binary complaint against z_0 compared to z_5 has strength 3, and the greatest binary complaint against z_5 compared to z_0 has strength 5

stemming from each of persons 1 to 5 and person 6, respectively. Therefore, according to Weak Minimax Simple Binary Complaint,

(7.15) z_0 is better than z_5.

But, by (7.13) and (7.15),

(7.16) there is a betterness cycle.

This contradicts the axiological verdict (7.4).

Similarly, as seen before, its simple deontic version, Deontic Weak Minimax Binary Complaint, implies that all alternatives, z_0 to z_5 are impermissible. This contradicts the deontic verdict (7.7).

As for the other moderate trade-offs paradoxes, Deontic Lenimax Global Complaint solves the deontic version of the paradox. To see this, consider the global complaints in the Simple Moderate Trade-offs Sequence, illustrated in table 7.3. So,

(7.17) z_3 lexicographically minimizes the maximum global complaint

since z_3 has a smaller maximal global complaint than z_4 and z_5, and has a *lexicographically* smaller maximal global complaint than z_0, z_1 and z_2. Therefore, according to Deontic Lenimax Global Complaint,

Table 7.3 Global complaints in the Simple Moderate Trade-offs Sequence

	Global complaints					
Persons	z_0	z_1	z_2	z_3	z_4	z_5
1	3	0	0	0	0	0
2	3	3	0	0	0	0
3	3	3	3	0	0	0
4	3	3	3	3	0	0
5	3	3	3	3	3	0
6	0	1	2	3	4	5
max	3	3	3	3	4	5

Table 7.4 Illustration of the Simple Symmetric Finite Superiority Sequence

	Welfare		
Persons	x	y_1	y_2
1	1	3	0
2	1	0	3

(7.18) z_3 is the only permissible alternative.

And this implies (7.5) to (7.7) solving the paradox. But Deontic Lenimax Global Complaint faces severe problems in other sequences, as we have seen before, and as I will show again in a simplified version in the next subsection.[2]

A problem with Weak Minimax Simple Binary Complaint is that, in contrast to Deontic Lenimax Global Complaint, it takes *too* seriously pairwise comparisons, deriving directly from each an overall moral verdict, i.e. that an alternative is morally better than another. As argued in chapter 5, instead the overall moral verdict should depend on the choice context.

7.1.2 The Simple Symmetric Finite Superiority Sequence and the Simple Symmetric Infinite Superiority Sequence

Next, consider a simple version of the Symmetric Ultra Minimal Finite Superiority Sequence, the

Simple Symmetric Finite Superiority Sequence: illustrated in table 7.4,

i.e., in x, person 1 and 2 are each at welfare level 1; in y_1, person 3 is at 3 and person 2 is at 0; in y_2, vice versa.

The main differences to the Symmetric Ultra Minimal Finite Superiority Sequence are as follows. First, instead of a large number of people, $n-1$, there is only one person, person 2, relatively slightly worse off in y_1 compared to x. However, this difference does not matter for the evaluation according to the maximum claim

[2] See also subsection 5.4.2.

Table 7.5 Global complaints in the Simple Symmetric Finite Superiority Sequence

	Global complaints		
Persons	x	y_1	y_2
1	2	0	3
2	2	3	0
max	2	3	3

principles. Second, the differences between the gain for the best-off and the loss for the worst-off person in y_1 compared to x are not as extreme as in the Symmetric Ultra Minimal Finite Superiority Sequence. However, this difference does not matter under the simplifying assumption (7.1). And analogously for y_2 compared x.

Analogous as in the Symmetric Ultra Minimal Finite Superiority Sequence, the verdict is that

(7.19) y_1 and y_2 are both better than x;

(7.20) only x is impermissible.

Again, I recognize that (7.19) and (7.20) might not have much intuitive appeal. However, again under the two initial assumptions from above, the evaluations in the Simple Symmetric Finite Superiority Sequence are analogous to the evaluations in the Symmetric Ultra Minimal Finite Superiority Sequence.

Some maximum claim principles imply the verdict. Weak Minimax Simple Binary Complaint implies (7.19) since

(7.21) the greatest binary complaint against x compared to y_1 and y_2 has strength 2

stemming from person 1 and 2, respectively;

(7.22) the greatest binary complaint against y_1 and y_2 compared to x has strength 1

stemming from person 2 and 1, respectively. (Furthermore, the greatest binary complaints against y_1 compared to y_2 and vice versa are equal.)

However, some maximum claim principles fail to imply the verdict. Consider the global complaints in the Simple Symmetric Finite Superiority Sequence, illustrated in table 7.5. According to Deontic Lenimax Global Complaint,

(7.23) x is permissible

since x minimizes the maximum global complaint. But this contradicts (7.20).

A problem with Deontic Lenimax Global Complaint seems to be that, in contrast to Weak Minimax Simple Binary Complaint, it does *not* take seriously enough pairwise comparisons. It does not take into account that the great complaint against y_1 by individual 2 stems from the comparison with another alternative, y_2, in which another person, individual 1, would incur just the same loss compared to y_1.

Remember that the analogous principle appealing to claims (to) an alternative implied the intuitive verdict in this case, see section 5.B. I suggested a symmetric variant of the Symmetric Ultra Minimal Finite Superiority Sequence, the Symmetric

Table 7.6 Illustration of the Simple Symmetric Infinite Superiority Sequence

	Welfare		
Persons	x	y_1	y_2
1	2	3	0
2	2	0	3

Table 7.7 Global claims in the Simple Symmetric Infinite Superiority Sequence

	Global claims		
Persons	x	y_1	y_2
1	1	0	3
2	1	3	0
max	1	3	3

Minimal Infinite Superiority Sequence which Deontic Lenimax Global Complaint solved but the analogue in terms of claims (to) didn't. So, it seems important that a plausible principle gets both cases right. Again, here is a simplified version of the sequence, the

Simple Symmetric Infinite Superiority Sequence: illustrated in table 7.6.

Analogous to the Symmetric Minimal Infinite Superiority Sequence, the verdict is that

(7.24) x is better than both y_1 and y_2;

(7.25) only x is permissible.

First, consider Weak Minimax Simple Binary Complaint.

(7.26) The greatest binary complaint against x compared to y_1 and y_2 has strength 1

stemming from person 1 and 2, respectively;

(7.27) the greatest binary complaint against y_1 and y_2 compared to x has strength 2

stemming from person 2 and 1, respectively. Therefore, according to Weak Minimax Simple Binary Complaint, the intuitive verdict (7.24) follows. (Furthermore, the greatest binary complaints against y_1 compared to y_2 and vice versa are equal.)

Next, consider the global complaints in the Simple Moderate Trade-offs Sequence, illustrated in table 7.7. Note that

(7.28) only x minimizes the maximum global complaint.

Therefore, according to Deontic Lenimax Global Complaint, the intuitive verdict (7.25) follows.

7.2 The deontic Condorcet methods

In voting theory structurally similar problems arise and many voting methods have been studied to solve these problems. This also shows that the problems discussed before are not limited to infinite and finite superiority conditions but appear in other contexts, e.g. majority voting.

One important subset of these methods are *Condorcet voting methods*.[3] Simplifying a bit, they are designed to select, based on majorities, the set of permissible candidates from the set of feasible candidates. More precisely, they are based on the following dominance relation in a pairwise choice:

(7.29) a candidate x dominates another y if and only if the majority of voters prefer x over y,

i.e. the number of voters who prefer x over y is greater than the number of voters who prefer y over x, for short $N(x,y) > N(y,x)$.

These methods can be generalized. The *deontic Condorcet methods* are designed to select the set of permissible alternatives from the set of feasible alternatives based on the following dominance relation in a pairwise choice:

(7.30) an alternative x dominates another y if and only if there is most reason to choose x over y in a pairwise choice.

Here we focus on maximum claim deontic Condorcet methods:

(7.31) there is most reason to choose x over y in a pairwise choice if and only if the maximum binary claim to x is greater than that to y,

i.e. the maximal shortfall in individual utility in x from y is greater than that in y from x. Note that I have used claims *to* an alternative rather than claims (or complaints) *against* an alternative. However, the dominance relations in terms of claims to and against are equivalent since claims to and against are symmetric in a pairwise choice.[4]

7.2.1 The Condorcet criteria

Now, we can state the defining criterion for a

deontic Condorcet method: a method is a *deontic Condorcet method* if and only if it satisfies the

Condorcet winner criterion: if a feasible alternative x dominates every other feasible alternative, then only x is permissible.

[3] See, e.g., Fishburn (1977) for an early overview, and Schulze (2011: sec. 5) for a more current overview of satisfied conditions of voting methods.

[4] See proposition 5.10.

Table 7.8 Pairwise claims (to) in the Simple Symmetric Infinite Superiority Sequence

	Claims to			
Persons	x	y_1	x	y_2
1	0	1	2	0
2	2	0	0	1
max	2	1	2	1

Table 7.9 Illustration of the Simple Symmetric Finite Superiority Sequence*

	Welfare		
Persons	x	y_1	y_2
1	1	3	0
2	1	0	2.5

So, according to Condorcet voting methods, if a majority of voters prefers a candidate over each other candidate, then it is the only permissible candidate. Analogously, according to maximum claim deontic Condorcet methods, if the maximum binary claim to an alternative is greater than that to each other alternative, then it is the only permissible alternative.

A maximum claim deontic Condorcet method implies the plausible verdict in the symmetric infinite superiority sequences. Consider the pairwise claims to alternatives in the Simple Symmetric Infinite Superiority Sequence, see table 7.8. x dominates both y_1 and y_2. Therefore, by the Condorcet winner criterion, (7.25), i.e. x is the only permissible alternative.

We have already seen a simple method, the Undominanted Choice Connection, that implies the Condorcet winner criterion. If there are only two candidates, it seems plausible that the dominance relation immediately implies the verdict, i.e. the set of permissible candidates. In this case the permissible candidates are plausibly the undominated candidates, i.e. the candidates that are not dominated by the other candidate. It seems tempting to generalize from this condition in cases with only two alternatives to that of any (finite) number of alternatives which gives the Undominanted Choice Connection, i.e. an alternative is permissible if and only if it is undominated. This principle implies the Condorcet winner criterion.

In contrast, Deontic Lenimax Global Complaint violates the Condorcet winner criterion, hence it is not a deontic Condorcet method. Consider a variant of the Simple Symmetric Finite Superiority Sequence, the

Simple Symmetric Finite Superiority Sequence*: illustrated in table 7.9.

According to Deontic Lenimax Global Complaint,

(7.32) only x is permissible,

see table 7.10. However, y_1 dominates x and y_2, see table 7.11. Hence, according to the Condorcet winner criterion,

Table 7.10 Global complaints in the Simple Symmetric Finite Superiority Sequence*

	Global complaints		
Persons	x	y_1	y_2
1	2	0	3
2	1.5	2.5	0
max	2	2.5	3

Table 7.11 Pairwise claims (to) in the Simple Symmetric Finite Superiority Sequence*

	Claims to			
Persons	x	y_1	y_1	y_2
1	0	2	3	0
2	1	0	0	2.5
max	1	2	3	2.5

(7.33) only y_1 is permissible.

There is a symmetric relative to the Condorcet winner criterion. According to the

> **Condorcet loser criterion:** if an alternative is dominated by every other alternative, then it is impermissible.

A maximum claim method that satisfies the Condorcet loser criterion implies the plausible verdict (7.20), i.e. that x is impermissible, in the symmetric finite superiority sequences. For example, consider the pairwise claims to alternatives in the Simple Symmetric Finite Superiority Sequence, see table 7.12. y_1 and y_2 both dominate x. Therefore, x is impermissible.

By definition, the Undominanted Choice Connection implies immediately the Condorcet loser criterion. Furthermore, neither y_1 dominates y_2 nor vice versa. Hence it implies the plausible verdict (7.20), i.e. only x is impermissible.

As seen, by (7.23), Deontic Lenimax Global Complaint violates the Condorcet loser criterion.

Remember that Fleurbaey, Tungodden and Vallentyne (2009: 282) suggested the "two-option" restriction of Minimal Infinite Superiority but left interesting strengthening for future research, see section 5.4. I also suggested that a number of other conditions need to be restricted, e.g. to "two-option" conditions. Now, we can

Table 7.12 Pairwise claims (to) in the Simple Symmetric Finite Superiority Sequence

	Claims to			
Persons	x	y_1	x	y_2
1	0	2	1	0
2	1	0	0	2
max	1	2	1	2

make some more progress on extending beyond the "two-option" conditions in two ways. First, the Condorcet winner criterion suggest an interesting extension:

(7.34) for any feasible alternative x, if each other feasible alternative is impermissible according to a "two-option" condition compared to x, then x is the only permissible alternative.

Let's call the extension of the Deontic Two-Option Minimal Infinite Superiority, the Deontic Condorcet Winner Minimal Infinite Superiority. As seen above, for maximum claim principles, the plausible implications by this condition are illustrated by Simple Symmetric Infinite Superiority Sequence.

Second, the Condorcet loser criterion suggest a an interesting extension of the two-option conditions:

(7.35) for any feasible alternative x, if x is impermissible according to a "two-option" condition compared to *each* other feasible alternative, then x is impermissible.

Let's call the extension of the Deontic Two-Option Minimal Finite Superiority, the Deontic Condorcet Loser Minimal Finite Superiority. As seen above, for maximum claim principles, the plausible implications by this condition are illustrated by Simple Symmetric Finite Superiority Sequence.

7.2.2 The Condorcet paradox

The other structurally similar problem that arises for Condorcet voting methods is that they can lead to a dominance cycle, a so-called majority cycle or *Condorcet paradox*. Consider the ranked preferences of voter 1, 2, and 3 in table 7.13. Table 7.14 shows the number of voters, N, who prefer a candidate over another. For example, the second row, first column shows the number of voters who prefer x over y, $N(x,y)$, i.e. 2; the first row, second column shows the number of voters who prefer y over x, $N(y,x)$, i.e. 1. First,

(7.36) two people, 1 and 2, out of three prefer x over y.

Hence, by (7.29), i.e. majority dominance,

(7.37) x dominates y,

illustrated by an arrow pointing from y to x in figure 7.2. Second,

(7.38) two people, 1 and 3, out of three prefer y over z.

Hence,

(7.39) y dominates z.

However, also

(7.40) two people, 2 and 3, out of three prefer z to x.

Hence

(7.41) z dominates x.

Table 7.13 Voter preferences in a Condorcet paradox

	Preference		
Voter	1.	2.	3.
1	x	y	z
2	z	x	y
3	y	z	x

Table 7.14 Cross table of number of voters, N, who prefer a candidate (row) over another (column)

N	x	y	z
x	–	2	1
y	1	–	2
z	2	1	–

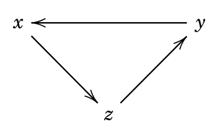

Figure 7.2 Majority dominance relation cycle between candidates

Table 7.15 Cross table of maximum binary claim, $c_{\max}$, to an alternatives (row) over another (column) in the Simple Moderate Trade-offs Sequence

$c_{\max}$	z_0	z_1	z_2	z_3	z_4	z_5
z_0	0	1	2	3	4	5
z_1	3	0	1	2	3	4
z_2	3	3	0	1	2	3
z_3	3	3	3	0	1	2
z_4	3	3	3	3	0	1
z_5	3	3	3	3	3	0

Therefore, by (7.37), (7.39) and (7.41),

(7.42) there is a majority dominance cycle,

illustrated in figure 7.2. Hence, we get an instance of the Condorcet paradox.

And, as I have shown in subsection 7.1.1, similar cycles can be created by the maximum claim dominance relation in the Simple Moderate Trade-offs Sequence. Table 7.15 shows the maximum binary claim to each alternative over each other alternative, analogue to table 7.14. Figure 7.3 illustrates the cycle of greater maximum binary claims, analogue to figure 7.2. For example, symmetric to (7.8),

(7.43) the maximum binary claim to z_1 over z_0, which is 3, is greater than the maximum binary claim to z_0 over z_1, which is 1,

as can be seen from the second row, fist column and the first row, second column position of table 7.15. Hence, analogue to (7.10),

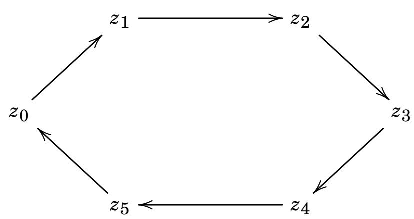

Figure 7.3 Cycle of greater maximum binary claims in the Simple Moderate Trade-offs Sequence

(7.44) z_1 dominates z_0

which is illustrated by an arrow pointing from z_0 to z_1 in figure 7.3. The other dominance relations in the cycle are established analogously.

Different Condorcet methods differ in how they deal with cyclic dominance. The Undominanted Choice Connection implies that *no* alternative is permissible when there is a dominance cycle. This seems implausible for majority dominance in the example of the Condorcet paradox. Instead, every candidate seems a permissible choice. And, as I have argued there, it does not solve the moderate trade-offs paradoxes for the maximum binary claim dominance.

In contrast, as shown in section 3.6, the Schwartz Choice Connection, according to which an alternative is permissible if and only if it belongs to the minimal undominated set, implies that *all* alternatives are permissible. This seems plausible in the example of the Condorcet paradox above. However, as I have argued, it does not solve the moderate trade-offs paradox.

It might be suggested that these two principles take only a part of the available dominance information from table 7.15 into account. Alternatively, consider the following principle. This principle is based on a relation called

covering: an alternative x *covers* another y if and only if x dominates y and, for all alternatives z, if z dominates x, then z dominates y.

According to the

Uncovered Set Principle: an alternative is permissible if and only if it is uncovered,

i.e. no alternative covers it. In other words, the permissible alternatives are the undominated alternatives of the covering relation. So, in order to check whether an alternative is permissible according to the Uncovered Set Principle, it is not sufficient to check whether it is dominated by another alternative. Hence, that there is a cycle among all alternatives is not sufficient to determine whether an alternative is permissible or not.[5]

Figure 7.4 shows *all* greater maximum binary claims in the Simple Moderate Trade-offs Sequence. However, again, *all* alternatives are permissible since *none* is

[5] It can be shown that the set of permissible alternatives according to the Uncovered Set Principle is a subset of that of the Schwartz Choice Connection, see Bordes, Le Breton and Salles (1992).

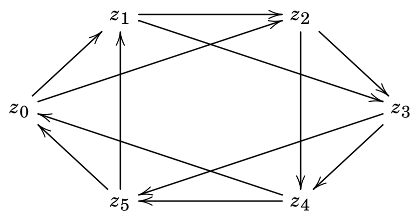

Figure 7.4 All greater maximum binary claims in the Simple Moderate Trade-offs Sequence

covered. To see this consider z_0:

(7.45) only z_1 and z_2 dominate z_0;

but

(7.46) there is an alternative, z_3, that dominates z_1 and z_2 but not z_0.

Hence

(7.47) z_0 is uncovered.

And, symmetrically, all alternatives are uncovered, see figure 7.4.

Different versions of the covering relation have been suggested that lead to different uncovered sets principles.[6] However, note that *any* principle *merely based* on the binary dominance relation between alternatives, i.e. most reason to choose one alternative over another, will not reach the intuitive verdict due to the symmetry of the dominance relation between alternatives in this case.

It might also seem that due to this symmetry, any principle that does not go beyond the dominance relation between alternatives will fail to reach the (supposedly) correct verdict in the Simple Moderate Trade-offs Sequence. But that is false. Carlson (1996) suggests the following rule:

> **R2:** Choose an alternative which is optimal among those alternatives which weakly dominate each earlier alternative.[7]

Importantly, R2 presupposes that there is a natural ordering of a sequence with a "natural starting point", so that "earlier" alternatives can be determined.[8] Consider again figure 7.4. Taking z_0 as this starting point,

(7.48) only z_2 and z_3 weakly dominate each earlier alternative;

(7.49) z_3 dominates z_2.

[6] See e.g. Miller (1980), Bordes (1983) and Duggan (2013).
[7] Carlson 1996: 149.
[8] See Carlson (ibid.: 155-6).

Therefore, by R2, (7.18), i.e. z_3 should be chosen. This solves the Deontic Third Moderate Trade-offs Paradox in accordance with Deontic Lenimax Global Complaint. However, below I provide a different solution which implies (7.18) but gives this result even without presupposing a natural starting point. That is an advantage over Carlson's approach because in many cases it is unclear what the natural ordering of the feasible alternatives is.

Carlson also presents three further rules. R1 and R3 are very similar to R2. R2 and R3 are refinements of R1 to deal with indifference. However, R1 and R3 can be disregarded for simplicity here since they also presuppose a natural starting point. R4 is merely based on the dominance relation and hence, as shown above, does not solve the Deontic Third Moderate Trade-offs Paradox and, hence, does not provide a general principle.

I suggest that to solve the moderate trade-offs paradox we need to consider not only the the dominance relation but also the relative *strength* of reasons. It might be objected that this is a disadvantage because it requires more information. However, this not a disadvantage in the current context. Consider table 7.15. The maximum binary claims provide not only the dominance relation but relative strength information. And since this information is available in the cases I discuss, it seems unproblematic to require it. Indeed, it seems that such information is necessary to reveal important asymmetries in the Simple Moderate Trade-offs Sequence.

7.3 Three variants of strength of reasons

For Condorcet voting methods, the strength of reason to choose an alternative x over y is its *voting score*, $s(x,y)$, based on the number of people who rank x over y and the number of people who rank y over x.[9]

Again, this idea can be generalized to the *strength of reason to choose* an alternative x over another *y in a pairwise choice*, for short $R(x,y)$, or, equivalently, the strength of reason *not* to choose y over x in a pairwise choice. On maximum claim methods, $R(x,y)$ is a function of the maximum binary claim to x compared to y and vice versa, i.e. the maximal shortfall in individual utility in x from y and vice versa. In particular, (7.31) still holds, i.e. there is most reason to choose x over y in a pairwise choice if and only if the maximum binary claim to x is greater than that to y.

There are three variants of the pairwise voting score to be distinguished that imply different verdicts. First, on the *opposing* variant, the voting score is the number of people who rank the one over the other, $N(x,y)$. Second, on the *winning* variant, the voting score is the number of people who rank the one over the other *if* this number is greater than the number of people who rank the other over the

[9] Note that this should not be confused with *cardinal voting methods* which allow *each voter* not only to (ordinal) rank each candidate but also to assign each candidate a score. Cardinal voting methods are sometimes called *utilitarian voting methods* since the cardinal scores are simply summed up. However, we have already seen that simple summation of cardinal welfare is insufficient, e.g. to solve the moderate trade-offs paradox. Furthermore, on the maximum claim methods discussed presently, an individual's claim (based on cardinal welfare) already is a cardinal score for each alternative for each individual. Hence, these methods already incorporate cardinal information at the individual level.

Table 7.16 Cross table of the voting score

(a) Opposing variant

s	x	y	z
x	–	2	1
y	1	–	2
z	2	1	–

(b) Winning variant

s	x	y	z
x	–	2	0
y	0	–	2
z	2	0	–

(c) Marginal variant

s	x	y	z
x	–	1	−1
y	−1	–	1
z	1	−1	–

Table 7.17 Strength of reasons in a pairwise choice in the Simple Symmetric Finite Superiority Sequence*

(a) Opposing reasons

R	x	y_1	y_2
x	–	1	1
y_1	2	–	3
y_2	1.5	2.5	–
max	2	2.5	3

(b) Winning reasons

R	x	y_1	y_2
x	–	0	0
y_1	2	–	3
y_2	1.5	0	–
max	2	0	3

(c) Marginal reasons

R	x	y_1	y_2
x	–	−1	−.5
y_1	1	–	.5
y_2	.5	−.5	–
max	1	0	.5

one, and 0 otherwise, i.e. $N(x,y)$ if $N(x,y) > N(y,x)$, and 0 otherwise. Third, on the *marginal* variant, the voting score is the difference between the number of people who rank one over the other and the number of people who rank other over the one, i.e. $N(x,y) - N(y,x)$. Table 7.16 shows the voting scores on each variant for the Condorcet paradox from above. First, Table 7.16a is the same as table 7.14. Second, in table 7.16b, for example, the second row, first column is 0 because the number of voters who prefer y over x, $N(y,x) = 1$, is smaller than the number of voters who prefer x over y, $N(x,y) = 2$. Finally, in table 7.16c, for example, the second row, first column is −1 because this is the number of voters who prefer y over x, $N(y,x) = 1$, minus the number of voters who prefer x over y, $N(y,x) = 2$.

Analogously, there are three variants of the strength of reasons based on the maximum binary claim to an alternative over another alternative which fulfil (7.31). Table 7.17 represents these three variants in the Simple Symmetric Finite Superiority Sequence*. On the

opposing reasons variant: the strength of reason to choose an alternative in a pairwise choice is the maximum binary claim to it in that choice,

(or, equivalently, the strength of reason to *not* choose an alternative in a pairwise choice is the maximum binary claim *against* it in the pairwise choice.) For example, therefore, on the opposing reasons variant,

(7.50) the strength of reason to choose x over y_1 in a pairwise choice is 1 and to choose y_1 over x is 2,

i.e. $R(x,y_1) = 1$ and $R(y_1,x) = 2$ since the maximum shortfall in individual utility in y_1 from x and in x from y_1 is the shortfall in utility of individual 2 (level 0 from level 1) and individual 1 (level 1 from level 3), respectively, see table 7.11. On the

winning reasons variant: the strength of reason to choose an alternative in a pairwise choice is the maximum binary claim to it in the pairwise choice if it is greater than the maximum binary claim against it in that choice; otherwise it is 0.

For example, therefore, on the winning reasons variant,

(7.51) the strength of reason to choose x over y_1 in a pairwise choice is 0 and to choose y_1 over x is 2, i.e.

$R(x, y_1) = 0$ (since $1 < 2$) and $R(y_1, x) = 2$ (since $2 > 1$).
On the

marginal reasons variant: the strength of reason to choose an alternative in a pairwise choice is the difference between the maximum binary claim to it in that choice and the maximum binary claim against it in that choice.

For example, therefore, on the marginal reasons variant,

(7.52) the strength of reason to choose x over y_1 in a pairwise choice is -1 and to choose y_1 over x is 1, i.e.

$R(x, y_1) = -1$ $(= 1 - 2)$ and $R(y_1, x) = 1$ $(= 2 - 1)$.
All these variants have may seem initially plausible. And on the face of it, it is not clear to me how to decide between them. Next, I am going to discuss different Condorcet principles based on the strength of reasons. I will apply each strength of reasons variant in order to see whether any can help solve the problems in the simple sequences discussed above.

7.4 Deontic Minimax SD Condorcet

Again, several different Condorcet voting methods have been constructed to generate a plausible verdict based *merely* on the pairwise voting score but which take into account pairwise comparisons with other alternatives in order to solve the Condorcet paradox. One such principle, in general terms, is

Deontic Minimax Condorcet: an alternative is permissible if and only if it minimizes the maximum strength of reasons to choose any alternative over it in a pairwise choice,

(or, equivalently, minimizes the maximum strength of reasons *not* to choose it over any alternative in a pairwise choice).

Consider again table 7.17. The last row in each table states the maximum number in each column, i.e. the maximum strength of reason to choose any alternative over the alternative in the head of the column. For example, on the opposing reasons variant, the maximum strength of reason to choose any alternative over x is 2.

According to Deontic Minimax Condorcet, on the winning and marginal reasons variant only y_1 is permissible but on the opposing reasons variant only x is permissible. Hence, the opposing reasons variant does not imply the plausible verdict and is not a deontic Condorcet method. Hence, this variant should be rejected.

Table 7.18 Strength of reasons in a pairwise choice in the Simple Symmetric Finite Superiority Sequence

(a) Opposing reasons

R	x	y_1	y_2
x	–	1	1
y_1	2	–	3
y_2	2	3	–
max	2	3	3

(b) Winning reasons

R	x	y_1	y_2
x	–	0	0
y_1	2	–	0
y_2	2	0	–
max	2	0	0

(c) Marginal reasons

R	x	y_1	y_2
x	–	−1	−1
y_1	1	–	0
y_2	1	0	–
max	1	0	0

Table 7.19 Strength of reasons in a pairwise choice in the Simple Symmetric Infinite Superiority Sequence

(a) Opposing reasons

R	x	y_1	y_2
x	–	2	2
y_1	1	–	3
y_2	1	3	–
max	1	3	3

(b) Winning reasons

R	x	y_1	y_2
x	–	2	2
y_1	0	–	0
y_2	0	0	–
max	0	2	2

(c) Marginal reasons

R	x	y_1	y_2
x	–	1	1
y_1	−1	–	0
y_2	−1	0	–
max	0	1	1

Note that the opposing reasons variant, in contrast to the two other variants, does not weigh up the claims in pairwise comparisons and reproduces the verdict of Deontic Lenimax Global Complaint which, as suggested above, does not take pairwise comparisons seriously enough. In fact, the opposing variant is *equivalent* to minimizing the maximum global complaint. According to the opposing reasons variant of Deontic Minimax Condorcet, an alternative x is permissible if and only if

(7.53) x minimizes the maximum over all alternatives of the maximum over all individuals' binary complaint,

which is equivalent to

(7.54) x minimizes the maximum over all individual's global complaint

since an individual's global complaint is the individual's maximum binary complaint over all alternatives.

7.4.1 The Simple Symmetric Finite Superiority Sequence and the Simple Symmetric Infinite Superiority Sequence

Next, consider the strength of reasons in a pairwise choice in the Simple Symmetric Finite Superiority Sequence, see table 7.18. Both the winning and marginal (but not the opposing) reasons variant of Deontic Minimax Condorcet imply the intuitive verdict (7.20) that x is (the only) impermissible alternative.

Next, consider the strength of reasons in a pairwise choice in the Simple Symmetric Infinite Superiority Sequence, see table 7.19. Both the winning and marginal (and the opposing) reasons variant of Deontic Minimax Condorcet imply the intuitive verdict (7.25) that x is the only permissible alternative.

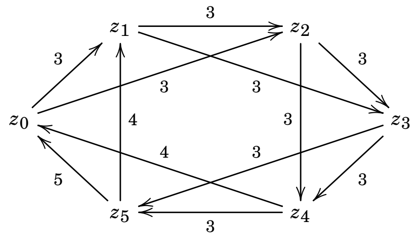

Figure 7.5 Strength of winning reasons in a pairwise choice in the Simple Moderate Trade-offs Sequence.

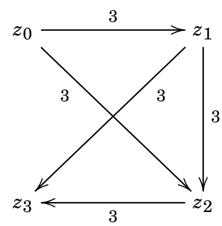

Figure 7.6 Strength of winning reasons in a pairwise choice in the Simple Moderate Trade-offs Sequence after once eliminating the alternatives not in the Deontic Minimax Condorcet set.

7.4.2 The Simple Moderate Trade-offs Sequence

Next, consider the strength of reasons in a pairwise choice in the Simple Moderate Trade-offs Sequence, see table 7.20. According to the winning (and opposing) reasons variant, only z_0 to z_3 are permissible, see also figure 7.5. But this violates the verdict (7.5), i.e. that z_0 is impermissible.

However, note that the winning reasons *among* z_0 to z_3 are not symmetric, represented in table 7.21b and figure 7.6:

(7.55) among z_0 to z_3, for z_0 to z_2 the maximum is 3 while for z_3 it is 0.

Hence, the following simple extension of Deontic Minimax Condorcet might be suggested: among tied alternatives iteratively use Deontic Minimax Condorcet to decide among them. More precisely, this new principle can be defined as follows. First, define the

Deontic Minimax Condorcet set: given a set of alternatives, the *Deontic Minimax Condorcet set* is the subset of all alternatives that minimize the maximum strength of reason to choose any alternative over it in a pairwise choice.

So, Deontic Minimax Condorcet claims that

(7.56) an alternative is permissible if and only if it is in the Deontic Minimax Condorcet set of the feasible set.

Next, according to, what I call, Deontic Minimax Sequential Dropping Condorcet, for short

Table 7.20 Strength of reasons in a pairwise choice in the Simple Moderate Trade-offs Sequence.

(a) Opposing reasons

R	z_0	z_1	z_2	z_3	z_4	z_5
z_0	–	1	2	3	4	5
z_1	3	–	1	2	3	4
z_2	3	3	–	1	2	3
z_3	3	3	3	–	1	2
z_4	3	3	3	3	–	1
z_5	3	3	3	3	3	–
max	3	3	3	3	4	5

(b) Winning reasons

R	z_0	z_1	z_2	z_3	z_4	z_5
z_0	–	0	0	0	4	5
z_1	3	–	0	0	0	4
z_2	3	3	–	0	0	0
z_3	0	3	3	–	0	0
z_4	0	0	3	3	–	0
z_5	0	0	0	3	3	–
max	3	3	3	3	4	5

(c) Marginal reasons

R	z_0	z_1	z_2	z_3	z_4	z_5
z_0	–	−2	−1	0	1	2
z_1	2	–	−2	−1	0	1
z_2	1	2	–	−2	−1	0
z_3	0	1	2	–	−2	−1
z_4	−1	0	1	2	–	−2
z_5	−2	−1	0	1	2	–
max	2	2	2	2	2	2

Table 7.21 Strength of reasons in a pairwise choice in the Simple Moderate Trade-offs Sequence after once eliminating the alternatives not in the Deontic Minimax Condorcet set.

(a) Opposing reasons

R	z_0	z_1	z_2	z_3
z_0	–	1	2	3
z_1	3	–	1	2
z_2	3	3	–	1
z_3	3	3	3	–
max	3	3	3	3

(b) Winning reasons

R	z_0	z_1	z_2	z_3
z_0	–	0	0	0
z_1	3	–	0	0
z_2	3	3	–	0
z_3	0	3	3	–
max	3	3	3	0

(c) Marginal reasons

R	z_0	z_1	z_2	z_3	z_4	z_5
z_0	–	−2	−1	0	1	2
z_1	2	–	−2	−1	0	1
z_2	1	2	–	−2	−1	0
z_3	0	1	2	–	−2	−1
z_4	−1	0	1	2	–	−2
z_5	−2	−1	0	1	2	–
max	2	2	2	2	2	2

Deontic Minimax SD Condorcet: an alternative is permissible if and only if it is in the smallest set reached by iteratively eliminating all alternatives not in the Deontic Minimax Condorcet set.

Table 7.21 shows the strength of reasons in a pairwise choice after once eliminating the alternatives that are not in the Deontic Minimax Condorcet set. On the winning reasons variant, according to this principle and by (7.55), (7.18), i.e. only z_3 is permissible, follows since it minimizes the maximum strength of reason to choose any alternative over it in a pairwise choice *among* z_0 to z_3. This is the intuitive result also reached by Deontic Lenimax Global Complaint. (On the opposing variant, according to Deontic Minimax SD Condorcet, z_0 to z_3 are the only permissible alternatives, again violating (7.5).)

In contrast, according to the marginal reasons variant *all* alternatives are permissible. Hence, on the marginal variant, both Deontic Minimax Condorcet and Deontic Minimax SD Condorcet violate the intuitive verdict in this case, i.e. (7.5). So, this variant of the principle should be rejected.

7.4.3 Objections

While, as seen above, the winning variant of Deontic Minimax SD Condorcet manages to give the intuitive verdicts in the sequences discussed, it has a couple of troublesome features.

First, Deontic Minimax SD Condorcet (and Deontic Minimax Condorcet) does not satisfy the Condorcet loser criterion. While it sometimes coincides with the Condorcet loser criterion, as seen in the Simple Symmetric Finite Superiority Sequence in subsection 7.4.1, this is not always the case. And in some cases, the violation seems clearly implausible. Consider the case illustrated in table 7.22. It seems plausible that

(7.57) w is impermissible

since everyone except for one is relatively much better off while one person is slightly worse off in x, y, and z than in w. And this verdict is implied by the Condorcet loser criterion since

(7.58) w is dominated by each other alternative

since the maximum claim to x, y, and z compared to w is 1.9 and the maximum claim to w compared to x, y, and z is only .1, see table 7.23. However,

(7.59) w minimizes the maximum strength of reasons to choose any alternative over it in a pairwise choice,

see table 7.23 and 7.24. Therefore, according to the winning and opposing reason variant of Deontic Minimax SD Condorcet (and Deontic Minimax Condorcet),

(7.60) w is the only permissible alternative.

Table 7.22 Welfare in the Symmetric Condorcet loser criterion Case

	Welfare			
Persons	x	y	z	w
1	2	0	1	.1
2	1	2	0	.1
3	0	1	2	.1

Table 7.23 Opposing reasons in the Symmetric Condorcet loser criterion Case

R	x	y	z	w
x	–	2	1	1.9
y	1	–	2	1.9
z	2	1	–	1.9
w	.1	.1	.1	–
max	2	2	2	1.9

Table 7.24 Winning reasons in the Symmetric Condorcet loser criterion Case

R	x	y	z	w
x	–	2	0	1.9
y	0	–	2	1.9
z	2	0	–	1.9
w	0	0	0	–
max	2	2	2	1.9

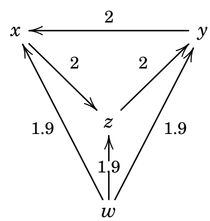

Figure 7.7 Winning reasons in the Symmetric Condorcet loser criterion Case

Table 7.25 Strength of opposing reasons in a pairwise choice in the Simple Symmetric Finite Superiority Sequence

R	x	y_1	y_2	max
x	–	1	1	1
y_1	2	–	3	3
y_2	2	3	–	3

Table 7.26 Strength of opposing reasons in a pairwise choice in the Simple Symmetric Infinite Superiority Sequence

R	x	y_1	y_2	max
x	–	2	2	2
y_1	1	–	3	3
y_2	1	3	–	3

But this violates (7.57).

Here is a second, familiar problem, see subsection 5.4.2. Remember that the maximum strength of reason *against* an alternative is calculated via the maximum in each column of the pairwise reason table, i.e. the strength of reason to choose any alternative over it in a pairwise choice has been calculated via the claims (or complaints) *against* alternatives. However, it might be objected that this seems an arbitrary choice. Why not appeal instead to

> **Deontic Maximax Condorcet:** an alternative is permissible if and only if it maximizes the maximum strength of reason to choose it over any alternative in a pairwise choice,

(or, equivalently, it maximizes the maximum strength of reason *not* to choose any alternative over it in a pairwise choice). Deontic Minimax Condorcet directs us to calculate the maximum over all other alternatives of the maximum binary claim *against* an alternative, i.e. the maximum in each *column*, and then determine which alternative *minimizes* these (dis)values. Instead, Deontic Maximax Condorcet directs us to calculate the maximum over all other alternatives of the claim *to* an alternative, i.e. the maximum in each *row*, and then determine which alternative *maximizes* these values.

And Deontic Maximax Condorcet implies different verdicts from Deontic Minimax Condorcet in some cases. For example, the opposing reasons variant is equivalent to the maximization of the maximum global claim (to), see also section 5.B. For example, consider the opposing reasons in tables 7.25 and 7.26.

It implies the asymmetric verdicts to this variant of Deontic Minimax Condorcet in the symmetric cases, i.e. in the Simple Symmetric Finite Superiority Sequence, it implies the intuitive verdict that y_1 and y_2 are the only permissible alternatives, while in the Simple Symmetric Infinite Superiority Sequence, it implies the same, but in this case counter-intuitive, verdict.

Next, consider the winning reasons in table 7.27.[10] The greatest claim to an alternative is to z_0 with strength 5. Hence, according to Deontic Maximax Condorcet, z_0 is the only permissible alternative. But this verdict differs from Deontic Minimax Condorcet, according to which z_0 to z_3 are permissible, and also from Deontic Minimax SD Condorcet, according to which z_3 is the only permissible alternative.

Finally, the marginal reasons variants of Deontic Minimax Condorcet and Deontic Maximax Condorcet are equivalent, but we have rejected them for other reasons before in subsection 7.4.2.

Because of these objections, it seems worth considering alternative principles.

7.5 Deontic Ranked Pairs SD Condorcet

The basic idea of the Ranked Pairs Condorcet method is to honour the strength of reasons to choose an alternative over another by *ranking* it in accordance with this strength.[11] More precisely, first, define the

> **Ranked Pairs-ranking:** given a set of alternatives, in order of the strength of reason to choose an alternative over another in a pairwise choice, iteratively rank all alternatives accordingly if they do not create ranking cycles with rankings of the alternatives with at least as great strength.

So, the procedure to generate the Ranked Pairs-ranking is this:

1. Determine all strengths of reason in pairwise choice.
2. Order these strengths of reason by their size.
3. Consider the greatest strength of reason and rank alternatives if they do not create a ranking cycle with other rankings of the greatest strength.
4. Consider the second greatest strength of reason and rank alternatives accordingly if they do not create a ranking cycle with other rankings of the greatest strength or the second greatest strength.
5. Work down through the strength of reason, from the greatest to the smallest, and rank alternatives accordingly if they do not create a ranking cycle with other rankings of at least as great a strength.

The Ranked Pairs-ranking will not contain cycles and hence, by Deontic Ranked Pairs Condorcet, there will be at least one undominated alternative. Second, according to

> **Deontic Ranked Pairs Condorcet:** an alternative is permissible if and only if it is undominated in the Ranked Pairs-ranking of the feasible set.

[10] The table also shows the opposing and marginal reasons for completeness.

[11] (Tideman [1987] and Zavist and Tideman [1989]). The Ranked Pairs Condorcet method is used since it is easy to explain and solves the problems discussed. But there are other promising methods, e.g. the *beatpath method* Schulze (2011). For example, Thomas (ms) discusses the beatpath methods as a candidate to solve problems in population ethics. Unfortunately, the discussion of the merits and problems of this and other methods has to await another occasion.

Table 7.27 Strength of reasons in a pairwise choice in the Simple Moderate Trade-offs Sequence.

(a) Opposing reasons

R	z_0	z_1	z_2	z_3	z_4	z_5	max
z_0	–	1	2	3	4	5	5
z_1	3	–	1	2	3	4	4
z_2	3	3	–	1	2	3	3
z_3	3	3	3	–	1	2	3
z_4	3	3	3	3	–	1	3
z_5	3	3	3	3	3	–	3

(b) Winning reasons

R	z_0	z_1	z_2	z_3	z_4	z_5	max
z_0	–	0	0	0	4	5	5
z_1	3	–	0	0	0	4	4
z_2	3	3	–	0	0	0	3
z_3	0	3	3	–	0	0	3
z_4	0	0	3	3	–	0	3
z_5	0	0	0	3	3	–	3

(c) Marginal reasons

R	z_0	z_1	z_2	z_3	z_4	z_5	max
z_0	–	−2	−1	0	1	2	2
z_1	2	–	−2	−1	0	1	2
z_2	1	2	–	−2	−1	0	2
z_3	0	1	2	–	−2	−1	2
z_4	−1	0	1	2	–	−2	2
z_5	−2	−1	0	1	2	–	2

Table 7.28 Winning reasons in the Simple Symmetric Finite Superiority Sequence*

R	x	y_1	y_2
x	–	0	0
y_1	2	–	3
y_2	1.5	0	–

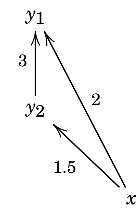

Figure 7.8 Ranked Pairs-ranking (with strength of winning reason to choose in pairwise comparison) in the Simple Symmetric Finite Superiority Sequence*

Again, the strength of reasons in a pairwise choice can be determined in terms of the opposing, winning, and marginal variant. For example, consider the opposing reasons variant in the Simple Symmetric Finite Superiority Sequence*, see table 7.17. Figure 7.8 shows correspondingly the Ranked Pairs-ranking with the corresponding strength of reason to choose in a pairwise comparison. The greatest strength of reason to choose an alternative over another in a pairwise choice is 3 with

(7.61) y_1 over y_2.

The second greatest with strength 2.5 is y_2 over y_1. However, this ranking would create a cycle together with the previously established ranking with greater strength, (7.61), i.e. y_2 over y_1 over y_2. Hence, this ranking is skipped. The third greatest with strength 2 is

(7.62) y_1 over x.

Finally, the fourth greatest with strength 1.5 is

(7.63) y_2 over x.

This completes the ranking. Therefore, Deontic Ranked Pairs Condorcet implies (7.33), i.e. only y_1 is permissible, which is the implication of the Condorcet winner criterion.

More generally, Deontic Ranked Pairs Condorcet implies the Condorcet winner criterion. An alternative x that dominates all other alternatives is not involved in any dominance cycles. Therefore, x will dominate all other alternatives without itself being dominated in the Ranked Pairs-ranking. Therefore, according to Deontic Ranked Pairs Condorcet, only x is permissible.

It also implies the Condorcet loser criterion. An alternative x that is dominated by all other alternatives is not involved in any dominance cycles. Therefore, x will also be dominated by all other alternatives in the Ranked Pairs-ranking. Therefore, x

is not undominated in the Ranked Pairs-ranking. Therefore, according to Deontic Ranked Pairs Condorcet, x is impermissible.

Deontic Ranked Pairs Condorcet avoids the asymmetry objection that the deontic minimax and maximax Condorcet methods faced. By basing permissibility on the ranking of alternatives, instead of claims (or complaints) against alternatives (or alternatively claims to an alternative), the symmetry between claims against and to alternatives is respected: an alternative x is ranked over another y with a certain strength according to the maximum binary claim to x relative to y, or equivalently, against y relative to x.

7.5.1 The Simple Symmetric Finite Superiority Sequence and the Simple Symmetric Infinite Superiority Sequence

That the Deontic Ranked Pairs Condorcet implies both the Condorcet winner criterion and Condorcet loser criterion shows that it gives the correct verdict in the Simple Symmetric Finite Superiority Sequence and the Simple Symmetric Infinite Superiority Sequence. However, it might be instructive to derive these verdicts directly in each case.

Next, consider the strength of reasons in a pairwise choice in the Simple Symmetric Finite Superiority Sequence, see table 7.18. On the opposing reasons variant, the greatest strength of reason to choose an alternative over another in a pairwise choice is 3 with y_1 over y_2 and y_2 over y_1. Hence, this creates a cycle and is skipped. The second greatest strength is 2 with

(7.64) y_1 and y_2 over x.

Finally, the third greatest strength is 1 with x over y_1 and y_2 which creates a cycle and are skipped. Therefore, Deontic Ranked Pairs Condorcet implies the intuitive verdict (7.20) that x is the only impermissible alternative.

On the winning reasons variant, the greatest strength with 2 is

(7.65) y_1 and y_2 over x.

Therefore, according to Deontic Ranked Pairs Condorcet, again (7.20) follows.

On the marginal reasons variant, the greatest strength with 1 is

(7.66) y_1 and y_2 over x.

Therefore, according to Deontic Ranked Pairs Condorcet, again (7.20) follows.

Symmetrically, all three variants imply the intuitive verdict (7.25) that x is the only permissible alternative in the Simple Symmetric Infinite Superiority Sequence.

7.5.2 The Simple Moderate Trade-offs Sequence

Next, consider the strength of reasons in a pairwise choice in the Simple Moderate Trade-offs Sequence, see table 7.29. The Ranked Pairs-ranking in the opposing reasons variant is illustrated in figure 7.9. On the opposing reasons variant, the strongest reason to choose an alternative over another in a pairwise choice is with strength 5

Table 7.29 Strength of opposing reasons in a pairwise choice in the Simple Moderate Trade-offs Sequence.

R	z_0	z_1	z_2	z_3	z_4	z_5
z_0	–	1	2	3	4	5
z_1	3	–	1	2	3	4
z_2	3	3	–	1	2	3
z_3	3	3	3	–	1	2
z_4	3	3	3	3	–	1
z_5	3	3	3	3	3	–

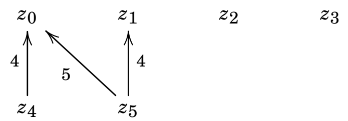

Figure 7.9 Opposing reasons variant of the Ranked Pairs-ranking (with strength of reason to choose in pairwise comparison) in the Simple Moderate Trade-offs Sequence

(7.67) z_0 over z_5.

The second greatest with strength 4 are

(7.68) z_0 over z_4;

(7.69) z_1 over z_5.

The third greatest with strength 3 includes z_1 over z_0, z_2 over z_1, z_3 over z_2 and z_4 over z_1 which creates a cycle with (7.68). The fourth greatest with strength 2 is z_0 over z_2 over z_4 and z_1 over z_3 over z_5 which creates a cycle with (7.68) and (7.67), respectively.

The smallest strength 1 includes z_0 over z_1 over z_2 over z_3 over z_4 over z_5 which creates again a cycle with (7.67). Therefore, according to Deontic Ranked Pairs Condorcet,

(7.70) z_0 to z_3 are the only permissible alternatives.

The Ranked Pairs-ranking on the winning reasons variant is illustrated in figure 7.10. On the winning reasons variant, the greatest strength of reason to choose an alternative over another in a pairwise choice is with strength 5

(7.71) z_0 over z_5.

The second greatest with strength 4 is

(7.72) z_0 over z_4;

(7.73) z_1 over z_5.

The third greatest with strength 3 includes z_1 over z_0, z_2 over z_1, z_3 over z_2 and z_4 over z_1 which creates a cycle with (7.72). Therefore, according to Deontic Ranked Pairs Condorcet,

Table 7.30 Strength of winning reasons in a pairwise choice in the Simple Moderate Trade-offs Sequence.

R	z_0	z_1	z_2	z_3	z_4	z_5
z_0	–	0	0	0	4	5
z_1	3	–	0	0	0	4
z_2	3	3	–	0	0	0
z_3	0	3	3	–	0	0
z_4	0	0	3	3	–	0
z_5	0	0	0	3	3	–

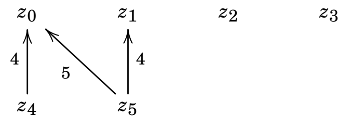

Figure 7.10 Winning reasons variant of the Ranked Pairs-ranking (with strength of reason to choose in pairwise comparison) in the Simple Moderate Trade-offs Sequence

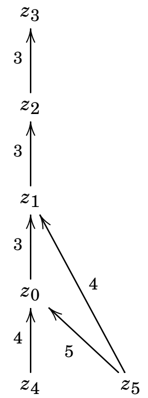

Figure 7.11 Winning reasons variant of the iterated Ranked Pairs-ranking (with strength of reason to choose in pairwise comparison) in the Simple Moderate Trade-offs Sequence

(7.74) z_0 to z_3 are the only permissible alternatives.

However, remember that the winning reasons *among* z_0 to z_3 are not symmetric, i.e. the winning reasons in the left upper square of table 7.20b, represented in table 7.21b: among z_0 to z_3, with greatest strength, strength 3

(7.75) z_1 over z_0, z_2 over z_1, and z_3 over z_2,

(and compatible with these rankings, z_2 over z_0, z_3 over z_1, and z_3 over z_0). Combining these rankings with those from the previous step gives the ranking shown in figure 7.11.

Hence, I suggest the following simple extension of Deontic Ranked Pairs Condorcet: among the undominated alternatives iteratively use Deontic Ranked Pairs Condorcet to decide among them. More precisely, this new principle can be defined as follows. First, define the

Deontic Ranked Pairs Condorcet set: the *Deontic Ranked Pairs Condorcet set* is the set of all undominated alternatives in the Ranked Pairs-ranking.

So, Deontic Ranked Pairs Condorcet claims that

(7.76) an alternative is permissible if and only if it is in the Deontic Ranked Pairs Condorcet set.

According to, what I call, Deontic Ranked Pairs Sequential Dropping Condorcet, for short

Deontic Ranked Pairs SD Condorcet: an alternative is permissible if and only if it is in the smallest set reached by iteratively eliminating all alternatives not in the Deontic Ranked Pairs Condorcet set.

Table 7.31 shows the strength of reasons in a pairwise choice after one has eliminated the alternatives not in the Deontic Ranked Pairs Condorcet set. On the winning reasons variant, according to this principle and by (7.75), only z_3 is permissible since it is the only undominated alternative in the Ranked Pairs-ranking *among* z_0 to z_3. This is the result also reached by Deontic Lenimax Global Complaint, (7.18).

Note that, on the opposing variant, according to Deontic Ranked Pairs SD Condorcet, z_0 to z_3 are permissible alternatives. Finally, on the marginal reasons variant, no alternatives are ranked over another since all possible rankings create cycles. Hence, these variants should be rejected.

7.6 A problem: Deontic Strong Pareto

However, there is serious doubt that Deontic Ranked Pairs SD Condorcet, as it stands, is a plausible principle. This is because it violates the following very weak efficiency condition: Deontic Strong Pareto. Both Deontic Minimax SD Condorcet and Deontic Ranked Pairs SD Condorcet violate Deontic Strong Pareto. Consider the

Strong Pareto Case: illustrated in table 7.32.

(7.77) y_1 is equally as good for everyone as x;

(7.78) y_1 is better for person 1 than x.

Therefore, according to Deontic Strong Pareto,

(7.79) x is impermissible.

However, consider Deontic Minimax SD Condorcet and the strength of reasons in table 7.33.

(7.80) x minimizes the maximum strength of reasons to choose any alternative over it in a pairwise choice.

Therefore, according to Deontic Minimax SD Condorcet,

(7.81) x is permissible.

Table 7.31 Strength of reasons in a pairwise choice in the Simple Moderate Trade-offs Sequence after once eliminating the alternatives not in the Deontic Ranked Pairs Condorcet set.

(a) Opposing reasons

R	z_0	z_1	z_2	z_3
z_0	–	1	2	3
z_1	3	–	1	2
z_2	3	3	–	1
z_3	3	3	3	–

(b) Winning reasons

R	z_0	z_1	z_2	z_3
z_0	–	0	0	0
z_1	3	–	0	0
z_2	3	3	–	0
z_3	0	3	3	–

(c) Marginal reasons

R	z_0	z_1	z_2	z_3	z_4	z_5
z_0	–	−2	−1	0	1	2
z_1	2	–	−2	−1	0	1
z_2	1	2	–	−2	−1	0
z_3	0	1	2	–	−2	−1
z_4	−1	0	1	2	–	−2
z_5	−2	−1	0	1	2	–

Table 7.32 Welfare in the Strong Pareto Case

	Welfare			
Person	x	y_1	y_2	y_3
1	0	2	1	0
2	0	0	2	1
3	1	1	0	2
4	2	2	1	0

Table 7.33 Winning reasons in the Strong Pareto Case

R	x	y_1	y_2	y_3
x	–	0	0	2
y_1	2	–	0	2
y_2	2	2	–	0
y_3	0	0	2	–
max	2	2	2	2

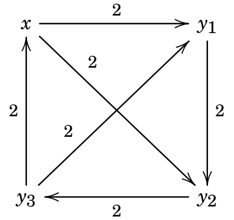

Figure 7.12 Winning reasons variant of the Ranked Pairs-ranking (with strength of reason to choose in pairwise comparison) in the Strong Pareto Case

Table 7.34 Winning reasons after eliminating alternatives according to Deontic Strong Pareto

R	y_1	y_2	y_3
y_1	–	0	2
y_2	2	–	0
y_3	0	2	–
max	2	2	2

This is inconsistent with (7.79). Hence Deontic Minimax SD Condorcet violates Deontic Strong Pareto.

Next, consider Deontic Ranked Pairs SD Condorcet and the strength of reasons in table 7.33. The greatest and only strength of reason to choose an alternative over another in a pairwise choice is 2 with

(7.82) x and y_1 over y_3, y_3 over y_2, y_2 over y_1, and y_2 and y_1 over x.

So, this creates a cycle, illustrated in figure 7.12. Therefore, according to Deontic Ranked Pairs SD Condorcet, again

(7.83) x is permissible.

And this is again inconsistent with (7.79). Hence Deontic Ranked Pairs SD Condorcet violates Deontic Strong Pareto. There may be several ways to extend these Condorcet principles to satisfy Deontic Strong Pareto. The simplest is to use a lexical method that first eliminates all alternatives that are impermissible according to Deontic Strong Pareto. According to

> **Deontic Ranked Pairs SD Condorcet*:** an alternative is permissible if and only if it is permissible according to Deontic Ranked Pairs SD Condorcet among the remaining alternatives of the feasible set after eliminating all alternatives that are impermissible according to Deontic Strong Pareto.

So, the procedure is as follows:

1. eliminate all alternatives impermissible according to Deontic Strong Pareto;
2. choose an alternative that is permissible according to Deontic Ranked Pairs SD Condorcet among the remaining alternatives.

Consider again the case in table 7.32. By Deontic Ranked Pairs SD Condorcet* 1 and (7.79), x is eliminated. There is no other alternative considered impermissible according to Deontic Strong Pareto. The strength of winning reason among the remaining alternatives is illustrated in table 7.34. The greatest and only strength of reason to choose an alternative over another in a pairwise choice is 2 with

(7.84) y_1 over y_3, y_3 over y_2, and y_2 over y_1.

Therefore, according to Deontic Ranked Pairs SD Condorcet* 2,

(7.85) only y_1 to y_3 are permissible.

Similarly, Deontic Minimax SD Condorcet could be modified to imply (7.85).

Table 7.35 Strength of opposing reasons in the Strong Pareto Case

R^*	x	y_1	y_2	y_3
x	–	(0,0,0,0)	(0,0,1,1)	(0,0,0,2)
y_1	(2,0,0,0)	–	(1,0,1,1)	(2,0,0,2)
y_2	(1,2,0,0)	(0,2,0,0)	–	(1,1,0,1)
y_3	(0,1,1,0)	(0,1,1,0)	(0,0,2,0)	–
max	2	2	2	

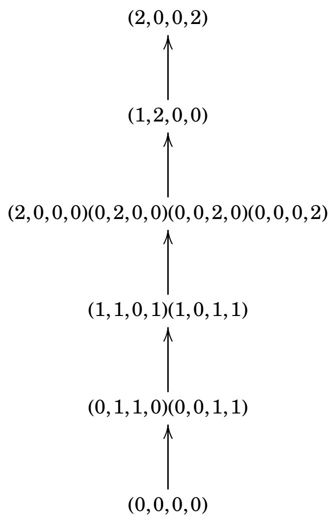

Figure 7.13 Lexical ranking of strength of reasons in the Strong Pareto Case

7.A Lexical Ranked Pairs Condorcet

Here is a sketch of a possible alternative to Deontic Ranked Pairs SD Condorcet*. The basic idea is that too much relevant information is lost when representing the strength of reason to choose in a pairwise choice by the *maximum* claim. Instead, the strength of reason can be the *distribution* of claims in pairwise choice, for short R^*. These complex strengths of reasons can still be ranked by their strength and used in the Deontic Ranked Pairs Condorcet.

Consider again the welfare distributions in table 7.32. No person has a binary claim to x compared to y or vice versa except for person 1 who has a binary claim of strength 2 to y compared to x. These claim distributions together with all other binary claim distributions are illustrated in table 7.35. In order to determine the Ranked Pairs-ranking, we need a ranking among these claim distributions. Note, however, that it is sufficient that a cardinal ranking, as provided by the maximum strength, is not necessary. One such ranking is a lexically one, as illustrated in table 7.13. The lexically greatest strength of (2,0,0,2) is

(7.86) y_1 over y_3.

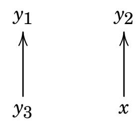

Figure 7.14 Ranked Pairs-ranking in the Strong Pareto Case

The lexically second greatest strength of $(1,2,0,0)$ is

(7.87) y_2 over x.

Next, the lexically equal strength of $(2,0,0,0)$, $(0,2,0,0)$, $(0,0,2,0)$, and $(0,0,0,2)$ lead to a cycle, y_1 over x, y_2 over y_1, y_3 over y_2, and x over y_3. All other also lead to cycles. Hence, the Ranked Pairs-ranking is as illustrated in Ranked Pairs-ranking in the Strong Pareto Case. Finally, according to Deontic Ranked Pairs Condorcet,

(7.88) y_1 and y_2 are the only permissible alternatives.

Note that this does *not* violate Deontic Strong Pareto because x is impermissible. However, it seems noteworthy that this is *not* because y_1 is ranked over x in the Ranked Pairs-ranking. So, the mechanism is more complicated. I still suspect that Deontic Strong Pareto fulfils Deontic Strong Pareto. But I am leaving the proof to future research.

Next, consider the Simple Moderate Trade-offs Sequence. I will not go through the details here and just show the results. The claim distributions are illustrated in table 7.36. The lexical ranking of these claim distributions is illustrated in table 7.15. Finally, the Ranked Pairs-ranking is illustrated in table 7.16. So, according to Deontic Ranked Pairs Condorcet,

(7.89) only z_3 is permissible.

This is the same verdict as reached by Deontic Lenimax Global Complaint and solves the paradox.

Table 7.36 Strength of opposing reasons in the Simple Moderate Trade-offs Sequence

R^*	z_0	z_1	z_2	z_3	z_4	z_5
z_0	–	(0,0,0,0,0,1)	(0,0,0,0,0,2)	(0,0,0,0,0,3)	(0,0,0,0,0,4)	(0,0,0,0,0,5)
z_1	(3,0,0,0,0,0)	–	(0,0,0,0,0,1)	(0,0,0,0,0,2)	(0,0,0,0,0,3)	(0,0,0,0,0,4)
z_2	(3,3,0,0,0,0)	(0,3,0,0,0,0)	–	(0,0,0,0,0,1)	(0,0,0,0,0,2)	(0,0,0,0,0,3)
z_3	(3,3,3,0,0,0)	(0,3,3,0,0,0)	(0,0,3,0,0,0)	–	(0,0,0,0,0,1)	(0,0,0,0,0,2)
z_4	(3,3,3,3,0,0)	(0,3,3,3,0,0)	(0,0,3,3,0,0)	(0,0,0,3,0,0)	–	(0,0,0,0,0,1)
z_5	(3,3,3,3,3,0)	(0,3,3,3,3,0)	(0,0,3,3,3,0)	(0,0,0,3,3,0)	(0,0,0,0,3,0)	–

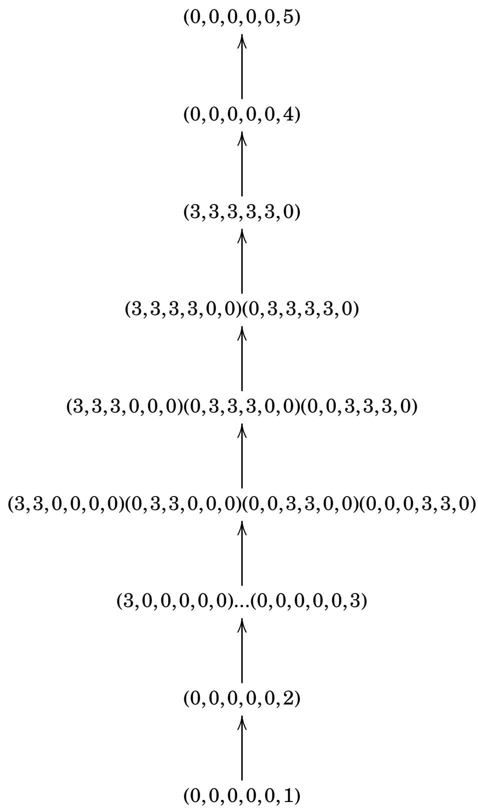

Figure 7.15 Lexical ranking of strength of reasons in the Simple Moderate Trade-offs Sequence

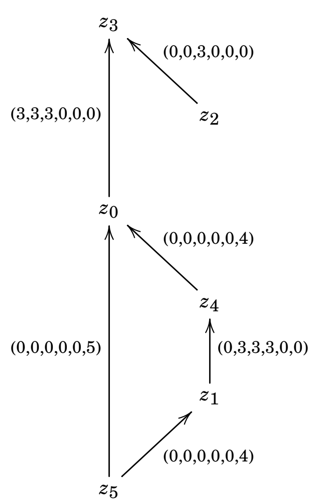

Figure 7.16 Ranked Pairs-ranking in the Simple Moderate Trade-offs Sequence

8

Further problems: axiology, impartiality, and non-aggregation

This chapter briefly discusses a number of possible objections to the principles suggested in the last chapter. It also discusses preliminary answers to them but whose details are left for future research.

8.1 Axiology reconsidered

This section discusses the extent to which the results about the deontic theories from chapter 7 can be extended to axiological theories.

The Condorcet methods discussed in the previous chapter were deontic (chapter 7). So, it might be objected that only solutions to the *deontic* moderate trade-offs paradoxes (section 3.5) have been presented. Hence, the axiological moderate trade-offs paradoxes (sections 3.3 and 3.4) remain unsolved. And it might be claimed that any reasonable moral theory should have an axiology. Hence, the possibility of a comprehensive moral theory that solves the moderate trade-offs paradoxes remains an open question.

One important feature in solving the deontic moderate trade-offs paradoxes was an appeal to choice context-dependence by a denial of Alpha (section 5.2), and, therefore, a denial of choice context-independence. In contrast to deontic theories, axiological theories that assign final value to alternatives are often claimed to be choice context-independent (e.g. because value is based on *intrinsic* features of alternatives).

However, this is controversial. Not everyone agrees that moral value is context-independent. It might be claimed that "x is morally better than y" is an incomplete expression as long as the choice-context has not been fixed. So, the claim is that betterness is not a binary but a *ternary* relation that holds between two alternatives and a feasible set, i.e. "x is better than y, given the feasible set X", or, for short, "x is better than y relative to X".[1] Or, on another way to understand this, instead of a unique binary betterness relation, there is a *family* of binary betterness relations,

[1] This approach is also suggested e.g. Handfield (2016: 6-7). See also Roberts (2014: 310) and Cusbert (2017) who interpret Temkin (2012: ch. 11) along these lines.

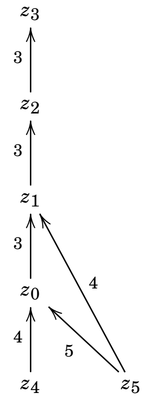

Figure 8.1 Winning reasons variant of the iterated Ranked Pairs-ranking (with strength of reason to choose in pairwise comparison) in the Simple Moderate Trade-offs Sequence (repeated figure 7.11 on page 199)

consisting of different binary betterness relations each relative to a different feasible set.

Next, it should be noted that some Condorcet methods provide not only a set of permissible alternatives but a (partial) ordering of the feasible alternatives, i.e. a ranking of them that is asymmetric and transitive. So, it might be claimed, this ordering *is* the betterness ranking relative to a feasible set. One such ranking is given by the (iterated) Ranked Pairs-ranking.[2] Figure 7.11 (repeated in figure 8.1) shows the iterated Ranked Pairs-ranking in the Simple Moderate Trade-offs Sequence. And it is easy to see that the iterated Ranked Pairs-ranking solves the axiological paradox, i.e. it fulfils that (7.2) successive alternatives are better than their predecessors at first, starting with z_1 is better than z_0, (7.3) z_0 is better than z_5, and (7.4) there is a undominated alternative better than both z_0 and z_5,

. Of course, as the deontic theories that solve the moderate trade-offs paradoxes violate Alpha, the axiological theories violate a condition, I call

Binary Independence: for any two feasible sets X and Y, and any alternatives x and y in both X and Y, x is at least as good as y relative to X if and only if x is at least as good as y relative to Y.

The Ranked Pairs-ranking is also both asymmetric and transitive. However, it is important here to distinguish between two different ways in which asymmetry and transitivity might be understood when value is relativized to feasible sets. One way to understand the conditions is this:

[2] In fact, it can be shown that the Ranked Pairs-ranking gives a *Condorcet ranking*, i.e. a ranking that ranks the Condorcet winner, if it exists, first, the Condorcet loser, if it exists, last, and the same for every subset of candidates in between, recursively. The beatpath method also provides a ranking of alternatives, see footnote (11).

Asymmetry: for any feasible set X, and alternatives x and y in X, if x is better than y relative to X, then y is *not* better than x relative to X.

Transitivity: for any feasible set X, and alternatives x, y and z in X, if x is better than y relative to X and y is better than z relative to X, then x is better than z relative to X.

Another way to understand the conditions involves rankings *across* feasible sets.

Super Asymmetry: for any feasible sets X and Y, and alternatives x and y in both X and Y, if x is better than y relative to X, then y is not better than x relative to Y.

Super Transitivity: for any feasible sets X, Y and Z, and alternatives x in both X and Z, y in both X and Y, and z in both Y and Z, if x is better than y relative to X and y is better than z relative to Y, then x is better than z relative to Z.

The first way to understand asymmetry and transitivity is the sense in which one can say that the Condorcet rankings satisfy these conditions. Arguably, it is Asymmetry and Transitivity that are most important for practical reasoning and, hence, morality. For example, it is plausible to think that

(8.1) one should do (or bring about) what is best among the feasible alternatives, or, at least, what is not worse than any feasible alternative.

Note that you need not be a committed to consequentialism, i.e. the view that what is permissible depends only on the value of the consequences, to accept this principle; arguably, all plausible moral principles agree with (8.1), given that other things are equal. Now, Asymmetry and Transitivity is sufficient to make sure that, in each situation, one can do what (8.1) demands. This is because Asymmetry and Transitivity together imply

Acyclicity: for any feasible set X, and for any alternatives x_1, x_2, …, and x_n in X, if x_1 is morally better than x_2 relative to X, x_2 is morally better than x_3 relative to X, … and x_{n-1} is morally better than x_n relative to X, then x_n is *not* morally better than x_1 relative to X.

And that is why, arguably, the conditions that involve rankings within a feasible set are the most important for practical reasoning.[3] However, this account faces also other problems of practical reasoning that are involved with a violation of Alpha.[4]

It might be objected that an axiology, as opposed to a deontic theory, does not necessarily presuppose choice among alternatives. However, it might be replied that when considering the value of (non-feasible) alternatives, we are still considering their value against *comparison classes* of other (non-feasible) alternatives. Those comparison classes will play the role of the feasible sets and are fixed by the context of conversation. For example, if we are considering what value a single alternative

[3] See, e.g. Handfield (2016) for a discussion.
[4] See footnote (15).

has, it might often seem natural to consider this alternative against an empty world. So, this alternative and the empty world provide the comparison class. The comparison with an empty world leads to the question of how good it is if people exist at a certain welfare level rather than if they do not exist. To take another example, we might consider whether one alternative is better than another. The obvious comparison class in this case consist in only these two alternatives.

However, arguably, there will be no axiological moderate trade-offs paradox if betterness is relative to comparison classes. In the moderate trade-offs sequences, it is reasonable that the comparison class would contain all the alternatives of the sequence. But, then as suggested above, the axiological paradox can be avoided.

Finally, one might not have to claim that *all* senses of betterness involve comparison classes. For example, Bykvist (2009) imagines worlds where there are no past, present or future agents or believers. Arguably, such worlds are not possibly evaluated by value concepts that involve comparison classes. However, there may well be different senses of betterness. And it might be claimed that the sense central to *practical* reasoning does involve comparison classes and that this is the important sense for moral theory. For example, Temkin (2012: sec. 1.5) appeals to, what he calls, a *reason-implying* sense of betterness. He says,

> Roughly, on this use, outcome A is better than outcome B, all things considered, if one would have more reason to prefer A to be realized than B, from an impartial perspective.[5]

8.2 Impartiality as permutation

In this section, I will briefly discuss the impartiality conditions claiming that outcomes with permuted welfare distributions are equally good, or both either permissible or impermissible. However, the maximum claim Condorcet principles, both deontic and axiological, violate these conditions. This seems to strongly undermine the plausibility of these theories. But I will suggest that there are reasonable alternatives that sufficiently capture impartiality.

Arguably, impartiality is a central condition of morality, for both axiological and deontic theories. For example, Harsanyi claims:

> Since Adam Smith, moral philosophers have often pointed out that the moral point of view is essentially the point of view of a *sympathetic* but *impartial* observer. It is the point of view of a person taking a positive sympathetic interest in the welfare of *each* participant but having no partial bias in favor of *any* participant.[6]

It is controversial whether impartiality is essentially the moral point of view. For instance, one might plausibly claim that one is morally permitted to be partial towards one's near and dear. But at least given the "other things equal" clause we are presupposing in this thesis, impartiality seems very hard to deny.

According to

Strong Welfare Impartiality: the value of an alternative is invariant under permutation of its welfare distribution,

[5] Temkin (2012: 13).
[6] Harsanyi (1977: 48-9).

Table 8.1 The Simple Permutation Case

Person	Welfare x	Welfare y
1	1	3
2	2	1
3	3	2

Table 8.2 Opposing reasons in the Simple Permutation Case

R	x	y
x	–	1
y	2	–
max	2	1

i.e. if an alternative has a permuted welfare distribution of another, then they are equally good,[7] and, according to its deontic analogue,

Deontic Strong Welfare Impartiality: the permissibility of an alternatives is invariant under permutation of its welfare distribution,

i.e. if a feasible alternative has a welfare distribution that is the permutation of another, then it is permissible if and only if the other is permissible. These conditions are usually justified by appeal to impartiality, i.e. that it does not matter "who is who" or which particular individual is affected by the choice.

However, all maximum claim principles discussed previously, including all the variants Deontic Minimax Condorcet and Deontic Ranked Pairs Condorcet discussed in the previous chapter, violate Deontic Strong Welfare Impartiality. To see this, consider the

Simple Permutation Case: illustrated in table 8.1.

Obviously, the welfare distribution of y is a permutation of the welfare distribution of x. Hence, according to Strong Welfare Impartiality, x is equally as good as y, and, according to Deontic Strong Welfare Impartiality, x is permissible if and only if y is permissible.[8]

However, consider the opposing reasons in the Simple Permutation Case, illustrated in table 8.2. The maximum claim to y is greater than that to x. So, according to the axiological maximum claim principles, y is better than x, and, according to the

[7] See, for example, Kavka (1979: 291), Kamm (1994: 85), Hirose (2001: 341), (Broome 2004: 135), (Adler 2012: 71), (Blackorby, Bossert and Donaldson 2013: 4). Other names for Strong Welfare Impartiality in the literature are "Anonymity", "Neutrality", and "Symmetry".

[8] More generally, it seems that any principle that satisfies Strong Infinite Superiority violates Strong Welfare Impartiality. I do not prove this here but as an instructive example see e.g. Voorhoeve (2014: sec. VIII).

Table 8.3 The Simple Permutation Case'

	Welfare	
Person	x'	y'
1	2	1
2	1	3
3	3	2

deontic maximum claim principles, y is permissible and x is impermissible. Hence, these principles violate Strong Welfare Impartiality.

However, arguably, a weaker condition sufficiently captures impartiality:

> **Deontic Weak Welfare Impartiality:** the permissibility of an alternative is invariant under permutation of the welfare profiles over feasible alternatives,

i.e. if the welfare distribution of *all* feasible alternatives is permuted *by the same permutation*, then their normative status does not change. Analogously, according to

> **Weak Welfare Impartiality:** the (relative) value of an alternative is invariant under permutation of the welfare profiles over feasible alternatives,

i.e. if the welfare distribution of *all* feasible alternatives is permuted by the same permutation, then they don't change their (relative) values. This is the classic *anonymity* condition from social choice theory,[9] and, arguably, all that impartiality asks for in the sense that it should not matter "who is who". Consider again table 8.1. Weak Welfare Impartiality does not imply that x is equally as good as y. Only that if the *complete rows* of welfare information are permuted, then the relative values of x and y are unchanged. For example, assume that

(8.2) y is better than x;

(8.3) person 1 gets what person 2 gets and vice versa, call these alternatives x' and y',

i.e., in x', person 1 gets 2 and person 2 gets 1, and, in y', person 1 gets 1 and person 2 gets 3, as illustrated in table 8.3. Then

(8.4) y' is better than x'.

And the maximum claim principles satisfy the weak welfare conditions because they do not single out any particular individual whose welfare counts for more than the welfare of anyone else. They say that claims matter but also no person's claim matters more.

This does not conclusively show that the maximum claim principles are impartial. For example, return to Harsanyi who also claims that an

[9] See, e.g. Roberts (1980: 447). This condition is the utility profile variant of Arrow (1951)'s classic Anonymity condition, see also Sen (1979: 72).

> [i]ndividual *i*'s choice among alternative social situations would certainly satisfy this requirement of impartiality and impersonality, if he simply *did not know in advance* what his own social position would be in each social situation [. . .]. More specifically this requirement would be satisfied if he thought that he would have an *equal probability* of being *put in the place* of anyone among the *n* individual members of society [. . .].[10]

As an example, consider again table 8.1. If a person would have had equal chances of ending up in any of the places of person 1, 2 and 3, then the person would have been indifferent between x and y because the alternatives would have the same probable outcomes: 1, 2, or 3 each with probability 1/3. However, Harsanyi's view of moral impartiality is controversial.[11] But I will have to leave further discussion of it for future research.

8.3 Beyond non-aggregation

Many people claim that maximum claim principles are implausible because they imply too minimal finite superiority conditions. For example, they violate Deontic Strong Finite Superiority*, i.e. you ought to save a large number of people from complete disability rather than one from premature death. I tend to be sceptical that this violation is as obviously wrong as it seems to many at first. However, an argument to that effect is beyond the scope of this thesis. Instead, I turn to alternative approaches.

It might be responded that the maximum claim principles should be modified, so that they only provide one moral factor and that a full theory about welfare trade-offs appeals also to other moral factors that imply stronger finite superiority conditions. However, on its own, this response is very unsatisfactory because it does not tell us how this theory will solve the problems that have been at the centre of the discussion in this thesis. Instead, in the following I briefly discuss variants of the principles suggested in the previous chapter that imply stronger finite superiority conditions.

The Condorcet methods discussed in the previous chapter can be used with strength of reasons in pairwise comparisons other than the maximum claim, so that they will satisfy a version of strong finite superiority. Remember that the Condorcet methods discussed in the last chapter assume only that the reason to choose an alternative in pairwise choices is ordinally comparable across different pairwise comparisons.

Now, consider the approaches discussed in chapter 6. Carlson (2000)'s principle, the Geometric Total Complaints Principle (see section 6.B), delivers values that can be ranked across pairwise choices. If the strength of reason in pairwise choices is determined by these values, the resulting Condorcet version of the principle will satisfy some stronger finite superiority conditions, as exemplified by Deontic Strong Finite Superiority*.

A problem with the global version of the Geometric Total Complaints Principle, as with other previously discussed global principles, is that it does not imply the plausible verdicts in both the symmetric minimal finite and infinite superiority

[10] Harsanyi (1977: 49-50).

[11] See e.g. Broome (1991: 55-57).

Table 8.4 Illustration of the Strong Finite Superiority Paradox Sequence

	Welfare		
Persons	x	y	z
1	1	0	0
m_1 people	.5	1	.5
m_2 people	.75	.75	1

sequences (section 6.B). However, the Deontic Ranked Pairs SD Condorcet version of the principle will avoid these problems since it satisfies both the Condorcet winner criterion as well as the Condorcet loser criterion (section 7.5).

It might be that these Condorcet variants of Geometric Total Complaints Principle face other problems but their exploration is beyond the scope of this thesis.

In contrast, Voorhoeve (2014)'s principle, DTRC (see section 6.1), does *not* deliver comparable strength of reasons to choose across pairwise choices. This is because which claims to alternatives are *relevant* depends on the alternative compared with. So, the sums of relevant claims which should be maximized according to DTRC are not comparable either.

For example, consider the Strong Finite Superiority Paradox Sequence (on page 155). Assume the welfare distributions are as in table 8.4. For simplicity, disregard priority-weighting for the worse-off or, alternatively, assume that the numbers represent already priority-weighted welfare levels. So, the strength of an individual's claims is just the difference between welfare levels in alternatives. Remember that, according to DTRC 4, "[a] claim is relevant if and only if it is sufficiently strong relative to the strongest competing claim". The original assumptions of the Strong Finite Superiority Paradox Sequence (6.10) to (6.12) mean that the only irrelevant claims are the m_2 individuals' claims when x and z are compared. Under these assumptions, table 8.5 shows the relevant binary claims to alternatives and their sums. Given that m_2 is sufficiently larger than m_1, the strength of reason to choose z over y, $.25m_2$, will be greater than the strength of reason to choose any other alternative over another in a pairwise choice. Table 8.6 shows the strength of opposing and winning reasons based on the sums of relevant claims.

First, consider Deontic Minimax Condorcet. On either strength of reasons variant, y has a greater maximum strength of reason *not* to choose it than any other alternative. So, according to this principle, y is impermissible. But this is inconsistent with the intuitive verdict suggested by Voorhoeve that (6.13) only y is permissible.

Next, consider Deontic Ranked Pairs Condorcet. By the strength of reasons on both the opposing and winning reasons variants,

(8.5) z is ranked over y.

So, according to this principle on both variants, y is impermissible. Again this is inconsistent with (6.13).

Maybe there are ways to resolve these problems. But their exploration is beyond the scope of this thesis, too.

Table 8.5 Relevant binary claims in the Strong Finite Superiority Paradox Sequence.

(a) x compared to y

Persons	x	y
1	1	–
m_1 people	–	.5
m_2 people	–	–
Sum	1	$.5m_1$

(b) x compared to z

Persons	x	z
1	1	–
m_1 people	–	–
m_2 people	–	–
Sum	1	0

(c) y compared to z

Persons	y	z
1	–	–
m_1 people	.5	–
m_2 people	–	.25
Sum	$.5m_1$	$.25m_2$

Table 8.6 Strength of reasons in a pairwise choice in the Strong Finite Superiority Paradox Sequence

(a) Opposing reasons

R	x	y	z
x	–	1	1
y	$.5m_1$	–	$.5m_1$
z	0	$.25m_2$	–
max	$.5m_1$	$.25m_2$	$.5m_1$

(b) Winning reasons

R	x	y	z
x	–	0	1
y	$.5m_1$	–	0
z	0	$.25m_2$	–
max	$.5m_1$	$.25m_2$	1

9

Conclusion

My overarching aim in this thesis was to make progress towards finding defensible moral principles that capture limited moral trade-offs between different people. These limited trade-offs can be expressed by two minimal conditions. First, according to *minimal infinite superiority*, significantly benefiting one person matters more than slightly benefiting each of any number of better-off people. Second, according to *minimal finite superiority*, significantly benefiting some number of people matters more than slightly benefiting one person.

Making progress towards finding such principles is no easy task especially since none of the simple classic principles satisfies these conditions. The simple total principles, according to which the maximal sum total of welfare has overriding moral importance, violate minimal infinite superiority. The maximin principles, according to which the claim of the worst-off has overriding moral importance, violate minimal finite superiority.

What follows is an overview of what I consider to be the main conclusions and contributions of this thesis.

The impossibility results proved in chapter 3 (in particular Result 3* and Result 5*) show that there is no moral principle, axiological or deontic, that satisfies minimal infinite superiority and minimal finite superiority together with another minimal trade-off condition, a weak version of Pigou-Dalton, minimal structural conditions, e.g. Acyclicity and No Prohibition Dilemmas, and minimal background assumptions, e.g. Absolute Comparability. The impossibility results mark out clear boundaries within which each moral principle has to operate: any moral principle will violate at least one of these conditions or assumptions. The proofs of these results involve sequences of alternatives, for short the *proof-sequences*. Applied to the proof-sequences, the seemingly plausible minimal trade-off conditions imply a violation of Acyclicity or No Prohibition Dilemmas, i.e. they imply the axiological verdict that every alternative is worse than another or the deontic verdict that every alternative is impermissible. The impossibility results are improvements over the previous impossibility results by Fleurbaey, Tungodden and Vallentyne (2009) because they are based on logically weaker trade-off assumptions and dispense with a structural assumption, namely Replication Invariance.

Furthermore, I argued that the proposed solutions to impossibility results suggest the rejection of the remaining minimal structural conditions, Acyclicity and No Prohibition Dilemmas. I argued that these “solutions” are unsuccessful because they fail to imply intuitive verdicts in the proof-sequences. In particular, the intuitive verdict is not one of a moral dilemma. In a formal part, I also showed how the deontic conditions can be derived from the axiological conditions which makes it easy to obtain one result from the other.

So, I turned to the remaining options of moral principles that reject either a background assumption or a trade-off condition. In chapters 4 to 6, I discussed several recently proposed moral principles that have been claimed to imply plausible verdicts in cases where minimal infinite and finite superiority plausibly holds. However, I showed that they fail to do so.

In chapter 4, I showed that proposed lexitotal principles, total principles based on lexicographically ordered welfare components, e.g. higher and lower pleasures as suggested by Mill (1863), challenge the impossibility results. Some of these principles can satisfy all the conditions of the results while only rejecting Absolute Comparability. This makes the lexitotal principles promising and because I also argued that the justification given by Fleurbaey, Tungodden and Vallentyne for accepting Absolute Comparability is unsatisfactory. However, while this is an impressive result in theory, I suggested that the principles do not imply intuitive verdicts to the comparison of alternatives in some instances of the proof-sequences.

In chapter 5, I suggested that the minimal limited trade-off conditions of the impossibility results are over-generalizations. By this I meant that the conditions seem to be an inductive generalization from intuitions in a too limited set of cases and that alternative conditions are overlooked. In particular, in the case of the deontic conditions, I suggested, in light of the intuitive verdict in the proof-sequences, that the trade-off conditions are too insensitive to the choice context, i.e. the set of feasible alternatives, for short *the feasible set*. This brought us to a revision of the deontic trade-offs conditions.

In chapters 5 and 6, I showed that several previously suggested deontic principles based on global claims give plausible verdicts in the proof-sequences. A global claim is an individual's claim based on comparisons of alternatives in the whole feasible set that are not in general reducible to a single pairwise comparison of alternatives.

In chapter 5, I discussed one of these principles, a global claim version of the well-known minimax complaint principle modelled along suggestions by Nagel (1979) and Scanlon (1982), according to which the maximum individual claim against an alternative has overriding moral importance. However, I also showed that this minimax complaint principle implies verdicts contrary to the limited trade-offs conditions in other sequences, for short the *symmetric sequences*, where, in contrast to the proof-sequences, variants of the conditions should cover these cases. In these symmetric sequences, one alternative is superior (or inferior) to every other alternative in simple cases that involve only two alternatives by the lights of the minimal infinite or finite superiority conditions.

In chapter 6, I showed that several previously suggested total claims principles share the fate of the minimax complaint principles. According to one of these total claims principles, the total relevant claims principle suggested by Voorhoeve (2014), the maximum total sum of strong enough claims has overriding moral importance. According to different total claims principle, the geometric total claims principle suggested by Carlson (2000), the maximum total sum of rank-weighted claims has overriding moral importance.

In chapter 7, I proposed new deontic principles based on Condorcet methods from voting theory and on rankings of alternatives based on maximum pairwise claims. Condorcet voting methods are designed to deal with cases structurally similar to the proof-sequences and the symmetric sequences. And I showed that the new principles imply the intuitive verdicts in both the proof-sequences and the symmetric sequences that the previously discussed principles failed to imply.

Finally, in chapter 8, I canvassed possible further problems for the new principles I suggested and how they could meet these problems. The first problem is that the new principles are deontic principles. So, axiological principles that imply intuitive verdicts in the proof-sequences are still lacking. However, this might not be very troubling because I suggested that some of the new principles also deliver a context-sensitive approach to axiology.

Second, the new principle violates an often-accepted impartiality condition. According to impartiality, in a slogan, it should not matter "who is who". The condition that the new principle violates says that if only an alternative's distribution of welfare-levels is permuted, then its moral evaluation should not change. However, I suggested that there is an alternative condition which sufficiently captures impartiality. This condition only says that if an alternative's distributions of welfare levels is permuted while all other alternatives' welfare distributions are permuted in the same way, then its moral evaluation (relative to the other alternatives) should not change.

Finally, as the minimax claim principles discussed earlier, the new principle is a non-aggregative principle. A non-aggregative principle implies that welfare of different individuals cannot be aggregated, or combined (e.g. additively) so that the aggregate can be morally weighed against the welfare of other individuals. However, non-aggregative principles are very controversial. I confessed that I am a bit less sceptical towards non-aggregative principles than many others but could not pursue further arguments in this direction in the remaining space. Instead, I suggested that the new principle might be modified to be based on the total sum of claims.

Swedish summary

Mitt övergripande mål i denna avhandling är att göra framsteg i sökandet efter försvarbara moralprinciper som fångar upp begränsade moraliska avvägningar mellan personer. Dessa begränsade moraliska avvägningar kan uttryckas med två minimala villkor. För det första, enligt minimal oändlig superioritet, så väger det tyngre att betydligt gynna en person än att lite gynna var och en av något antal personer som har det bättre. För det andra, enligt minimal begränsad superioritet, så väger det tyngre att betydligt gynna något antal personer än att lite gynna en person.

Att göra framsteg i sökandet efter sådana principer är ingen enkel uppgift, särskilt eftersom ingen av de enkla klassiska principerna uppfyller dessa villkor. De enkla totalprinciperna, enligt vilka den största summan av total välfärd har överordnad moralisk vikt, står i strid med minimal oändlig superioritet. Maximinprinciperna, enligt vilka anspråken hos de som har det sämst har överordnad moralisk vikt, står i strid med minimal ändlig superioritet.

Nedan följer en översikt över vad jag anser vara de viktigaste slutsatserna och bidragen i denna avhandling.

Omöjlighetsresultaten som bevisades i kapitel 3 (särskilt Result 3* och Result 5*) visar att det inte finns någon moralprincip, vare sig axiologisk eller deontisk, som uppfyller minimal oändlig superioritet och minimal ändlig superioritet tillsammans med ett annat minimalt avvägningsvillkor, en svag version av Pigou-Dalton, minimala strukturella villkor, t.ex. Acyclicitet och No Prohibition Dilemmas, samt minimala bakgrundsantaganden, t.ex. Absolute Comparability.

Omöjlighetsresultaten ger tydliga gränser som varje moralprincip måste verka inom. Varje moralprincip kommer att bryta mot minst ett av dessa villkor eller antaganden. Bevisen för dessa resultat involverar sekvenser av alternativ, vi kan kalla dem 'bevissekvenser'. Tillämpade på bevissekvenserna innebär de till synes rimliga minimala avvägningsvillkoren en avvisning av Acyclicity eller No Prohibition Dilemmas, dvs de implicerar det axiologiska utfallet att varje alternativ är värre än ett annat eller det deontiska utfallet att varje alternativ är otillåtet. Omöjlighetsresultaten är förbättringar jämfört med tidigare omöjlighetsresultat från Fleurbaey, Tungodden och Vallentyne (2009) eftersom de bygger på logiskt svagare avvägningsantaganden och undviker ett strukturellt antagande, nämligen Replication Invariance.

Vidare argumenterade jag att de föreslagna lösningarna på dessa omöjlighetsresultat tyder på att de återstående minimala strukturella villkoren, Acyclicity och No Prohibition Dilemma, ska avvisas. Jag argumenterade att dessa "lösningar" misslyckas eftersom de inte ger de intuitivt riktiga utfallen i bevissekvenserna. Framförallt är det intuitivt riktiga utfallet inte ett moraliskt dilemma. I en formell del visade jag också hur deontiska villkoren kan härledas från de axiologiska villkoren, vilket gör det enkelt att nå det ena resultatet från det andra.

Jag vände mig därför till de återstående alternativen för moralprinciper, vilka avvisar antingen ett bakgrundsantagande eller ett avvägningsvillkor. I kapitel 4 till 6 diskuterade jag flera nyligen föreslagna moralprinciper som påstås ge rimliga resultat i fall där minimal oändlig och ändlig superioritet rimligtvis föreligger. Jag visade dock att de inte ger rimliga utfall.

I kapitel 4 visade jag att föreslagna lexitotalprinciper, totalprinciper baserade på lexikografiskt ordnade välfärdskomponenter, utgör en utmaning för omöjlighetsresultaten. Vissa av dessa principer kan uppfylla alla villkor för resultaten och endast avvisa Absolute Comparability. Detta gör lexitotalprinciperna lovande. Jag argumenterade också att motiveringen som Fleurbaey, Tungodden och Vallentyne gav för att acceptera Absolute Comparability är otillfredsställande. Emedan detta är ett imponerande resultat i teorin föreslog jag att principerna inte ger intuitivt rimliga resultat i jämförelse av alternativ i vissa fall av bevissekvenserna.

I kapitel 5 föreslog jag att de minimala begränsade avvägningsvillkoren för omöjlighetsresultat är övergeneraliseringar. Med detta menade jag att villkoren tycks vara en induktiv generalisering från intuitioner i ett alltför begränsat antal fall och att alternativa villkor förbises. Framför allt när det gäller de deontiska villkoren föreslog jag, mot bakgrund av det intuitivt rimliga utfallet i bevissekvenserna, att avvägningsvillkoren är för okänsliga för valmöjligheterna, dvs mängden utförbara alternativ, 'den utförbara mängden'. Detta föranledde en översyn av villkoren för deontiska avvägningar.

I kapitel 5 och 6 visade jag att flera tidigare föreslagna deontiska principer baserade på globala anspråk ger rimliga utfall i bevissekvenserna. Ett globalt anspråk är en individs anspråk baserat på jämförelser av alternativ i hela mängden utförbara alternativ, vilka inte i allmänhet kan reduceras till parvisa jämförelse av alternativ.

I kapitel 5 diskuterade jag en av dessa principer, en global anspråksversion av den välkända minimax complaint principle modellerad utifrån förslag av Nagel (1979) och Scanlon (1982), enligt vilken det maximala individuella anspråket mot ett alternativ har en överordnad moralisk vikt. Dock visade jag också att denna minimax complaint principle ger resultat som strider mot de begränsade avvägningsvillkoren i andra sekvenser, låt oss kalla dem de symmetriska sekvenserna, där, i motsats till bevissekvenserna, varianter av villkoren borde täcka dessa fall. I dessa symmetriska sekvenser verkar ett alternativ överlägset (eller underlägset) alla andra alternativ i enkla fall som endast involverar två alternativ i ljuset av de minimala oändliga eller ändliga superioritetsvillkoren.

I kapitel 6 visade jag att flera tidigare föreslagna principer om totala anspråk delar ödet med minimax complaint principle. Enligt en av dessa principer om totala anspråk, the total relevant claims principle som föreslogs av Voorhoeve (2014), har den maximala totalsumman av tillräckligt starka anspråk överordnad moralisk vikt. Enligt different total claims principle, den geometriska principen om totala anspråk som föreslås av Carlson (2000), har den totala summan av rankningsvägda anspråk överordnad moralisk vikt.

I kapitel 7 föreslog jag nya deontiska principer baserade på Condorcet-metoder från röstningsteori och på rangordningar av alternativ baserade på maximala parvisa anspråk. Condorcet-metoder är utformade för att hantera fall som strukturellt liknar bevissekvenserna och de symmetriska sekvenserna. Jag visade att de nya principerna ger de intuitiva rimliga resultaten i både bevissekvenserna och de symmetriska sekvenserna där de tidigare diskuterade principerna inte gav detta resultat.

I kapitel 8 diskuterade jag möjliga ytterligare problem för de nya principerna som jag föreslagit samt hur principerna kunde möta dessa problem. Det första problemet är att de nya principerna är deontiska principer. Det saknas fortfarande axiologiska principer som ger intuitivt rimliga utfall i bevissekvenserna. Detta kanske inte är särskilt oroande eftersom jag föreslog att vissa av de nya principerna också ger en kontextkänslig hantering av axiologi.

För det andra bryter den nya principen med ett ofta accepterat villkor om opartiskhet (impartiality). Enligt opartiskhet ska det inte spela någon roll ”vem som är vem”. Villkoret som den nya principen bryter säger att om bara ett alternativs fördelning av välfärdsnivåer permuteras, så påverkas dess moraliska vikt inte. Jag föreslog emellertid att det finns ett alternativt villkor som tillräckligt väl ger uttryck för opartiskhet. Detta villkor säger bara att om ett alternativs fördelning av välfärdsnivåer är permuterad medan alla andra alternativs välfärdsfördelningar är permuterade på samma sätt, så påverkas dess moraliska vikt (i förhållande till de andra alternativen) inte.

Slutligen, som var fallet för minimax claim principles som diskuterats tidigare, är den nya principen en icke-aggregerande princip. En icke-aggregerande princip innebär att olika individers välfärd inte kan aggregeras eller kombineras (t.ex. summeras) så att aggregatet kan vägas moraliskt mot andra individers välmåga. Icke-aggregerade principer är mycket kontroversiella. Jag erkände att jag är lite mindre skeptisk gentemot icke-aggregerande principer än många andra, men jag hade inte utrymme att ytterligare argumentera i denna riktning. Istället föreslog jag att den nya principen skulle kunna modifieras så att den baseras på totalsumman av anspråk.

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