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Exoplaneter: En Interaktiv Visualisering av Data och Upptäcktsmetoder

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Exoplanets: Interactive Visualization of Data and Discovery Method

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Abstract

This report provides a description of the development and implementation of an Exoplanet visualization within the interactive astro-visualization software OpenSpace. Orbital Data from The Exoplanet Orbit Database were used to render the planetary systems around stars known to have exoplanets in orbit. The uncertainties of the data in the database were incorporated into the design of the visualization. A feature that visualizes the discovery method of exoplanets was also implemented.

Acknowledgments

I would like to thank the people at Linköping University and New York University who have been involved in making it possible for me to travel to New York in order to work on my thesis. Also, a thanks to everyone at OpenSpace for creating the opportunity. A big thank you to Alexander Bock for guiding and aiding me over the past months and also to Emil Axelsson for the supervision and Marie Rådbo for the astro guidance.

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1 Introduction

Since the discovery of 51 Pegasi b, the first exoplanet orbiting a star like our sun, was discovered in 1995 [11] [10] the number of known exoplanets has grown rapidly [20]. There are, to this day, more than 3,700 confirmed planets in our universe. The information about these planets comes from a range of different institutions and new instruments are being sent up into space to find even more exoplanets. It is a rising topic that will be of interest to visualize to get a spatial and temporal context of the planets and their planetary systems.

There are visualizing systems that focus on showing data that have been gathered about the universe. Two of them are Uniview [13] and OpenSpace [5]. While Uniview is a commercial software, OpenSpace is open source and it is for this software that the project explained in this report is developed.

1.1 OpenSpace

OpenSpace is an open source interactive data visualization software designed to visualize the entire known universe [5]. It targets immersive environments and it aspires to help astro-research while it is also a tool for science communication for the general public [21].

The project is a collaboration between Linköping University, the American Museum of Natural History (AMNH), NASA Goddard's Community Coordinated Modeling Center, New York University's Tandon School of Engineering and University of Utah's Scientific Computing and Imaging Institute. The development of the software has been ongoing since 2014.

1.2 Motivation

Outside of the Exoplanet Community the knowledge of exoplanets is limited. At best the general public knows that an exoplanet is a planet that orbits a star outside of our solar system. How many there actually are, how they are found and what information can be gathered about them are questions not many can answer. Although they are far away and their exact appearance is unknown it still lies in our interest to visualize what is known about exoplanets so that these questions can be answered.

1.3 Aim

The aim of the project is to create a 3D-vizualization of each individual exoplanet system in the context of the universe of OpenSpace. That is, to place out all found exoplanets in their position in space. The design of the exoplanet system should be so that the general shape and size of the planets' orbit around their star is clear while still conveying that there is an uncertainty to the measured data. These planets' orbits should be visualized even in the cases where some data might be missing and have to be filled in with default values.

There is also an aim to use the visualized exoplanet system to incorporate an element that explains the methods through which the exoplanets are discovered.

Being able to look at every planet that has ever been found in a 3D environment demands some thought into how to handle the rendered objects in the scene graph. The aim is that a user will never encounter any problems due to there being too many objects in the scene graph. The implementation should be so that the scene graph nodes will be added and removed without the user having to think about it.

1.4 Research questions

To achieve the aim described above, answers to the following questions will be sought throughout the work of this thesis:

1. How can real data be clearly distinguished when mixed with default values in a visualization?
2. How can the uncertainties of the data be visualized?
3. What methods can be used for adapting the scene graph as the viewer focuses on an exoplanet?

2 Related Work

Exoplanets have been a subject to ”artist’s illustration” where an exoplanet have been visualized with far more detail than is actually known. For an individual that is new to the subject Exoplanets it could lead to a false impression, when in fact only a handful of exoplanets have been captured in an image and even in those images the appearance of the surface is impossible to see. The figure 2.1 is an artist’s illustraion of the exoplanet 51 Eridani b and the figure 2.2 is a real image of the same planet.

Figure 2.1: An artist’s illustration of 51 Eridani b, Source: Danielle Futselaar / Franck Marchis / SETI Institute

There is another 3D visualization tool for exploring exoplanets, NASA’s ”Eyes on Exoplanets” [16]. It is a scientifically accurate software that lets the user fly to any exoplanet of their choosing of over a 1,000 exoplanets.

OpenSpace as a software differs from Eyes on Exoplanets in that it has many other features such as planetary surface rendering [4] and space weather visualization [6] and is developed to be used in domes while Eyes on Exoplanets only has the exoplanet discovery feature. As a whole OpenSpace is therefore a more complete astro-visualization software. The exoplanet visualization as it is implements in this project complements the Eyes on Exoplanets in the way that it has a discovery method feature that aims to explain the methods through which exoplanets are discovered. The design of the exoplanets and their orbits differs from NASA’s software since, in this project, a texture for the planets have been left out to avoid giving

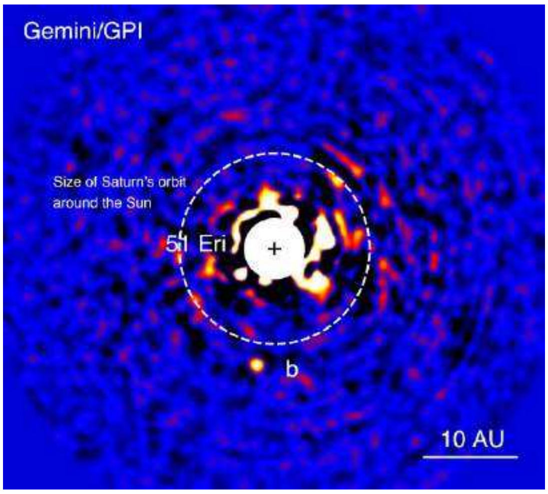

Figure 2.2: A real image of 51 Eridani b, Source: J. Rameau, University of Montreal / C. Marois, Herzberg Institute of Astrophysics

faulty impressions while Eyes on Exoplanets have used a version of ”artist’s illustrations” for the texture of their planets. Also, in this project there is a focus on visualizing the uncertainty of the data that is measured for the planets.

3 Theory

This chapter will help to provide the reader with the information needed to understand the choices made in this thesis. It will cover a brief introduction to some of the necessary knowledge of exoplanets and their orbit properties.

3.1 Exoplanets

Extrasolar planets, or exoplanets, are planets that are orbiting stars other than the Sun [9]. As of today, more than 3,700 planets outside of our solar system have been found [10]. Their mass ranges from twice the mass of our moon to 30 times the mass of Jupiter and the orbit time ranges from a few hours to thousands of years.

In April 2018 the space mission TESS, *The Transiting Exoplanet Survey Satellite*, was launched. The satellite will survey 200,000 of the brightest stars closest to the Sun. By using the Transit discovery method it is expected that TESS will find thousands of planet candidates that scientists can study further [12].

3.2 Discovery Methods

To this day there are six methods for studying the characteristics of an exoplanet. The methods are called Transit Photometry, Radial Velocity, Imaging, Microlensing, Timing and Astrometry. However, the majority of exoplanets are discovered through either Transit Photometry or Radial Velocity and the other methods are used for verifying.

Radial Velocity

Exoplanets can be found by observing their gravitational effect on the parent star as they orbit around it. This method is called *Radial Velocity*, (RV). As the planet orbits the star its gravity pulls on the star, causing it to wobble slightly. When the star is looked at from afar these small wobbles can be detected by splitting the star's light into a spectrum [9]. When the light leaves the star and moves through the outer edges of the star some of the light moves through hotter and some through colder regions of the star's surface. This means that some wave lengths will be absorbed, creating dark lines in the color spectrum [14]. When the star is moving away from the observation point the light waves are stretched and the dark lines in the spectrum move towards the red end of the spectrum and when the star is moving towards the observation point the light waves are compressed and the dark lines in the spectrum are shifted to the blue end [9], as displayed in figure 3.1 and 3.2.

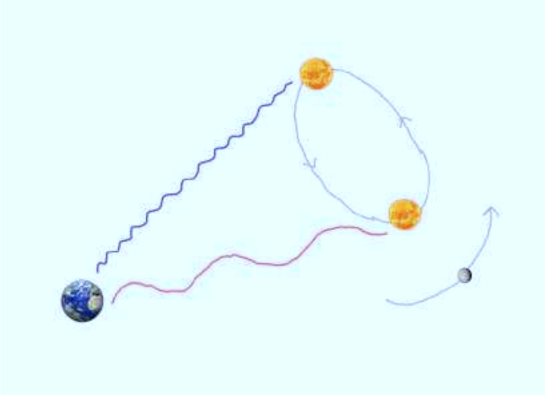

Figure 3.1: The light waves are stretched and compressed

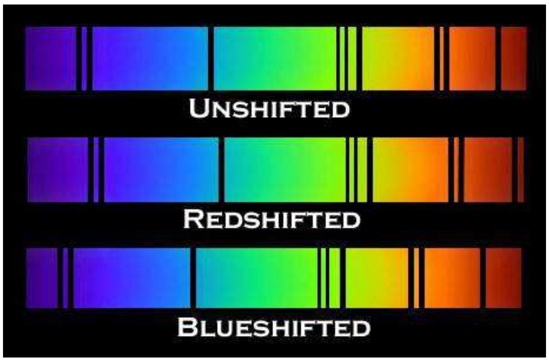

Figure 3.2: Shifting of the color spectrum, Source: NASA, ”Tracking Matter Around a Black Hole”

When these shifts occur at a fixed interval, it indicates that there is a body orbiting the star, which could possibly be one or more exoplanets. By analyzing these spectral shifts astronomers can deduce the gravitational influence of the exoplanet. When scientists have used RV they have only been able to give an estimate of the minimum mass of the exoplanets [17].

Transit Photometry

To detect exoplanets by the transit method, the light intensity of a star is measured. When a planet passes between the star and the observation point on Earth the light intensity decreases, see figure 3.3. The passage of the exoplanet is called a transit. If these decreases of light occur at regular intervals and last for a fixed length of time it is likely that a planet is orbiting the star.

A small planet orbiting a large star will cause a small decrease of light and a large planet orbiting a small star will cause a more noticeable drop in light intensity. Since scientist have a way of calculating the size of a star, they can estimate the size of the exoplanet from the amount of light it blocks.

In March of 2009 the Kepler Mission was launched and it has so far found thousands of planetary candidates using transit photometry [18].

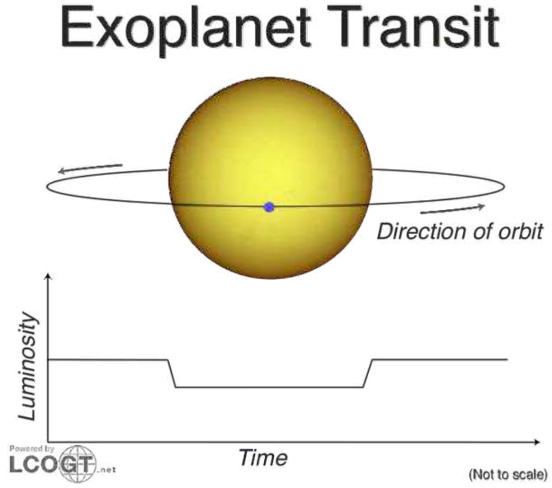

Figure 3.3: Light intensity curve during transit, Source: Las Cumbres Observatory, ”Transit Method”

When using the transit method the direction of the orbit is known and the inclination can be measured.

3.3 The Exoplanet Orbit Database

The data that is used for creating the visualization comes from The Exoplanet Orbit Database [1], (EOD). They describe their database as follows:

> ”We present a database of well determined orbital parameters of exoplanets, and their host stars’ properties.” [20]

In the database they have included all robustly detected planets that appear in peer-reviewed literature. They have a definition of ”robust” that requires that the period of the planet be certain to 15% (usually corresponding to seeing at least one or two complete orbits) [20]. Otherwise the creators of the database have applied their own judgement as to whether a planet’s detection and orbit are secure enough to be included in the database.

The data is provided in a *Comma Separated Values file* (CSV-file) where each line contains the data for one exoplanet. If a data value is missing there are simply no characters between the commas. The names of all the fields found in the Exoplanet Orbit Database are listed in appendix .1.

The only data used that is not found in the EOD is the star position. That information comes from NASA Exoplanet Archive (CalTech/NASA) and is prepared by AMNH/Hayden Planetarium for the original exoplanet visualization implementation in OpenSpace.

3.4 Orbit Parameter

The orbit parameters characterize the shape and size of the planet orbit. There are seven so-called Keplerian Elements that define an orbit [2]. For this thesis an eighth parameter, the period, has been used to animate the planet’s movement around the star. Each of the parameters are described in this section as to how they would characterize an orbit *outside* of the Solar System. The inclination, the longitude of ascending node and the argument of periapsis are measured from a plane of reference and a reference direction. The reference

---

[1] www.exoplanets.org

plane is the plane perpendicular to the line of sight from the Earth to the star which the planet orbits and the reference direction is a vector that lies in the reference plane. Exactly how the references are defined is explained in the following section Reference Frame 3.5

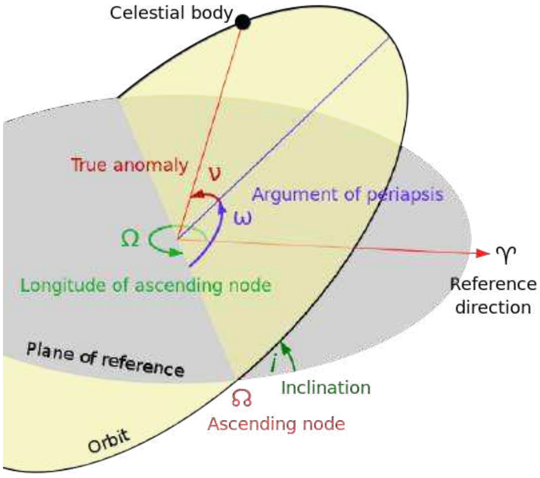

Figure 3.4: Orbital Elements, Source: Wikipedia, "Orbital Elements"

Longitude of Ascending Node

Longitude of ascending node, Ω, is the angle that orients the ascending node. The definition of the ascending node in orbits outside the Solar System is the point on the orbit where the planet passes through the reference plane moving away from the observer [1]. The angle is measured from the reference direction in the reference plane, see figure 3.4. However, this information is unknown for exoplanets, except in rare cases when the planet has been studied by direct imaging.

Inclination

The orbit lies in a plane called the orbit plane. The inclination, i, is the angle which the orbit plane is vertically tilted with respect to the reference plane, measured at the ascending node. The inclination is always a positive number between 0° and 180°.

Argument of Periapsis

The argument of periapsis, ω, defines the orientation of the orbit ellipse in the orbit plane. The periapsis is the point on the orbit that is closest to the star. The angle is measured from the ascending node to the periapsis in the orbit plane.

Eccentricity

The eccentricity, e, decides the shape of the ellipse. Its value needs to be in the range $0 \leq e < 1$. A value of zero will give a perfect circular orbit and a high value will give an elongated ellipse, see figure 3.5. A value of 1 or higher will be a parabola and then it is no longer an orbit.

Semi-Major Axis

The semi-major axis, a, is what determines the size of the orbit. It is half of the longest diameter of the ellipse. If the orbit is circular, then the semi-major axis is the radius. The semi-major axis can also be defined as the mean of the periapsis and the apoapsis distance,

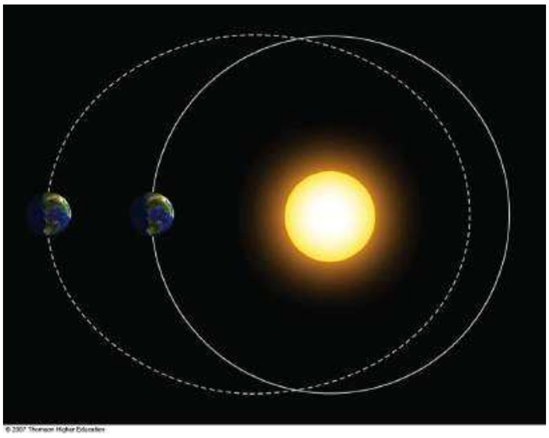

Figure 3.5: An elliptic orbit vs. a circular orbit, Source: Thomson Higher Education

l_{peri} and l_{apo}. The periapsis distance is the distance between the mass center and the closest point on the orbit and the apoapsis distance is the distance between the mass center and the furthest point on the orbit, see figure 3.6.

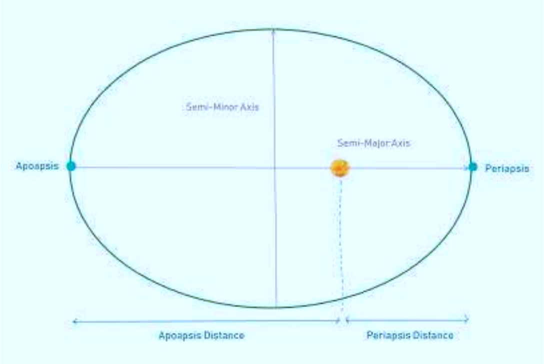

Figure 3.6: Ellipse Distances

The periapsis and the apoapsis distances can both be calculated from the semi-major axis and the eccentricity, see equation 3.1 and 3.2 [7].

$$l_{apo} = a(1 + e) \tag{3.1}$$

$$l_{peri} = a(1 - e) \tag{3.2}$$

Epoch

The epoch is the time frame which the orbital arguments were recorded.

True Anomaly

The true anomaly, ν, is where on the orbit the planet is located at the epoch. It is measured from the argument of periapsis in the orbital plane.

Period

The period is the time it takes for the planet to travel around the star one full lap. It is not a part of the Keplerian Elements since it does not change the appearance of the orbit. However, the period determines at what speed the planet will move around the star when animated.

3.5 Reference Frame

There are different coordinate systems in use in astronomy to express a body's position in space. There is the *Celestial* coordinate system that describes a position in space relative to Earth, this reference frame is also called *Geocentric*.

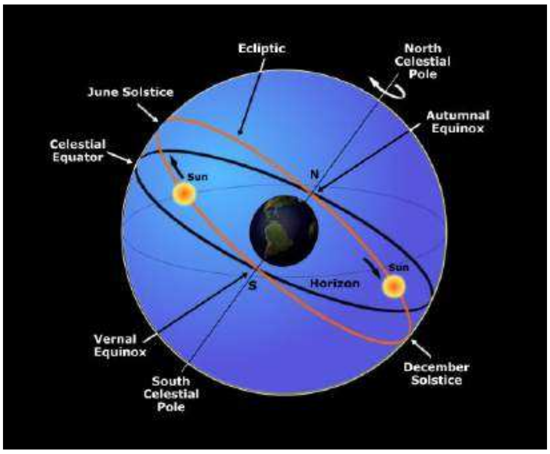

Figure 3.7: Celestial Sphere, Source: The Bluebird, "Celestial spheres: The Bluebird Guide to Solstices and Equinoxes"

For an orbit outside the solar system another reference frame is used. All the Keplerian Elements that describe an exoplanet's position is in reference to a reference plane and a reference direction. The tangent to the celestial sphere at the position of the star of interest is called the *Sky Plane* and is used as the reference plane. The normal to the Sky Plane is hence the vector that can be drawn from the star to the observation point. The reference direction from which the longitude of ascending node is measured is the perpendicular projection of the direction from the observer to the *North Celestial Pole* [19].

The North Celestial Pole is one of the reference points which make up the Celestial coordinate system. The vector from the Earth's center to the Celestial North is perpendicular to the Earth's equatorial plane, which is the reference plane for orbits around the Earth. The other reference point is the *First Point of Aries* (vernal equinox). The vector that is drawn from the Earth's center to the First Point of Aries hence lies in the reference plane [2]. The celestial sphere, the north celectial pole and The First Point of Aries is marked in figure 3.7.

To get the vector of the line between the observation point and the North Celectial Pole, $\vec{u}$, projected onto the Sky Plane, that has the normal $\vec{n}$, equation 3.3 is used [15].

$$proj_{Plane}(\vec{u}) = \vec{u} - \frac{\vec{u} \cdot \vec{n}}{\| \vec{n} \|^2}\vec{n} \tag{3.3}$$

4 Method

This chapter describes the implementation of the exoplanet system built up from the Exoplanet Orbit Database and the implementation of the discovery method visualization. The first part of this chapter will describe how the data from the database is processed. The second part will describe in detail the design of the planetary system. The third part will describe the implementation of the discovery methods visualization. The last part of the chapter will describe a Solar System reference feature.

4.1 Processing of Data

The data as it comes from the database is one large file that contains much more information then is needed for the visualization of the exoplanet and its orbit. To adapt the data to fit this project the necessary information is picked out and structured in a separate program which will enable a faster experience and adaptation of the scene graph. The separate program can be included in the start up process of OpenSpace.

The data is to begin with stored in a Comma Separated Value file (CSV-file) that is manually downloaded from exoplanets.org. After the data is parsed two new files are created. A binary file (bin-file), containing all the information that is needed for the visualization for each planet, and a Look-Up Table (LUT-file) that contains the planet names and the start bit where to find the data for the exoplanet in the bin-file, see figure 4.1

The program goes through the CSV-file line by line, each line containing all the data of one exoplanet, and stores the selection of the data that is needed in a struct, see listing 4.1. After each line iteration the Exoplanet struct is written into the bin-file. The name of the exoplanet is written into the LUT-file together with the location of the first byte in the bin-file where the structure was written into.

The separate LUT-file is created so that the retrieving of a certain planet's information is much faster than if the name would have been written into the struct and the larger bin-file would be looped to find a particular planet.

Listing 4.1: The Exoplanet struct

```
struct Exoplanet {
  float A;
  double AUPPER;
```

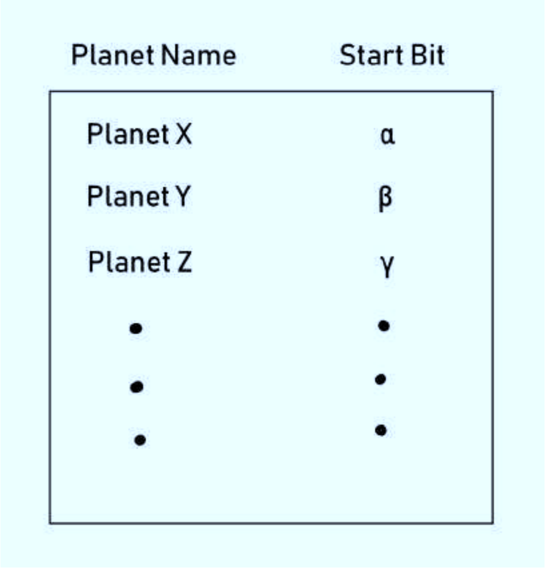

Figure 4.1: LUT structure

```
    double ALOWER;
    double UA;
    float BIGOM;
    float BIGOMUPPER;
    float BIGOMLOWER;
    float UBIGOM;
    int BINARY;
    float BMV;
    float ECC;
    float ECCUPPER;
    float ECCLOWER;
    float UECC;
    float I;
    float IUPPER;
    float ILOWER;
    float UI;
    int NCOMP;
    float OM;
    float OMUPPER;
    float OMLOWER;
    float UOM;
    double PER;
    float PERUPPER;
    float PERLOWER;
    float UPER;
    double R;
    double RUPPER;
    double RLOWER;
    double UR;
    float RSTAR;
    float RSTARUPPER;
    float RSTARLOWER;
```

```
    float URSTAR;
    double TT;
    float TTUPPER;
    float TTLOWER;
    float UTT;
    float POSITIONX;
    float POSITIONY;
    float POSITIONZ;
}
```

When the data of an exoplanet system is to be retrieved in OpenSpace, the LUT-file is iterated to find all the exoplanets that is orbiting the star. Since the exoplanet name is the star name plus a letter only the star name has to be known to find all the orbiting planets.

When an exoplanet is found in the LUT-file, the corresponding data in the bin-file is read into an Exoplanet struct and the struct is pushed to a vector. The name of the exoplanet is saved to another vector. After the LUT-file is iterated all the exoplanets orbiting the star are stored in the data vector and in the same order that the names of the exoplanets are stored in the name-vector.

Missing Data

All the orbit parameters, described in the Theory chapter 3.4, are some times not found for a particular exoplanet. In fact, some of the parameters are missing for almost all exoplanets in the database. In order to render an orbit all the parameters need to have a value, so in the cases where the data is missing a default value is put in its place.

This is true for all orbit parameters except for the semi-major axis and period. If one of them are missing the planet and its orbit is not visualized at all.

For the longitude of ascending node, the point on the orbit where the planet passes through the Sky Plane, moving away from the observer, the default value is 180°. If there is no inclination, then where the ascending node is placed has no meaning since it will never pass through the Sky Plane. When the value is set to 180°, and the orbit has an inclination, the ascending node is placed in the opposite direction from the reference direction. That way the planet will move away from the observer at the ascending node. The longitude of ascending node can neither be calculated with the Transit Method nor the RV Method, therefore the data is missing for almost all of the exoplanets. The data can only be calculated using Direct Imaging.

The inclination is missing for the majority of exoplanets in the database. However, for the ones that are registered, the values are around 90°. If the inclination is exactly 90° it means that the orbit is in line with the viewing direction from the Earth. The planet would never have been detected using RV or Transit if the orbit plane was at a small angle from the reference plane. Therefore, if the inclination is missing it will be filled in with 90°. They have after all been found with one of the discovery methods so a small inclination can be ruled out.

The argument of periapsis rotates the orbit so that the periapsis, the point on the orbit that is closest to the mass center, is positioned. If the eccentricity is zero, then the argument of periapsis does not matter. If there is an eccentricity to the orbit the argument of periapsis is almost never missing, and in the few cases it is the default value is 0°. In reality it could be any value between 0° and 360° and no value is more likely than the other.

The default value for the eccentricity is also zero. The epoch is set to 2009-05-19T07:11:34.080 and the True Anomaly is not a part of the database since its only used for describing the location of the planet at a certain time and OpenSpace is time varying.

The idea behind the chosen default values is to get the shape of the orbit that is as close as possible to the real one.

Data Uncertainty

A limit of uncertainty is given to data in the database. If the semi-major axis is given as A = 0.078 the uncertainty is given as ± 0.0013. However, in some cases the uncertainty limits will make the value go out of range. The allowed value of eccentricity is between 0 and 1. The exoplanet HD1461 b has the eccentricity value 0.14 and an uncertainty of ±0.19, so the lower limit will be -0.05 which is outside of the allowed value range. In these cases, the value is truncated to the nearest allowed value, which in this case is 0.

Star Name Alternatives

In the original implementation of exoplanet visualization in OpenSpace the names that are used comes from the NASA Exoplanet Archive and does not always match the names in the Exoplanet Orbit Database. To be able to use both star name alternatives a conversion table have been constructed for this thesis.

A list of names that differ between the two sources is found in table 4.1. Names marked with an asterisk have been found to be the same star according to the SIMBAD Astronomical Database [1]. This makes it possible to give either one of the two name alternatives of a star in order to add the visualization for that exoplanet system.

4.2 The Design of the Planetary System

The design of the exoplanet system is basically one or more spheres, the exoplanets, orbiting another sphere, the star. In the trace of the exoplanet a line is drawn that represents the orbit path. The sun is given a texture that is an alteration of the sun's texture. If the radius of the star is not given, the sphere is not visualized at all.

The exoplanet or exoplanets orbit the star in a Kepler orbit and the orbit is shown by an orbit trail. The visualization of the semi-major axis uncertainty is represented by a disc instead of a line. The inner edge of the disc is the lower value of the semi-major axis and the outer edge of the disc is the upper value of the semi-major axis.

The exoplanet itself is also represented by a sphere. If the radius is unknown, the position of the exoplanets is still known by the orbit line, although it is not represented by a sphere.

For the complete system to be visualized, three criteria have to be fulfilled. The star's location, the planets semi-major axis and period needs to be known.

Star Color

To color the texture of the star an RGB value is calculated from the star's B-V color index. The B-V index (blue magnitude minus visual magnitude) is a numerical expression that determines the color of an object. It is known that the B-V index is a number between -0.4 and 2.0 [3], where -0.4 is the bluest for the hottest stars and 2.0 is the reddest for the coolest stars.

In a texture file there is listed 256 RGB values from blue to red and all the possible colors a star can have in between the two as the star's temperature decreases. The B-V index is mapped to a number between 0 and 255 to get the right color using the equation in 4.1 and rounding the value to the closest integer.

$$\frac{B - V + 0.4}{2.0 + 0.4} * 255; \tag{4.1}$$

If a B-V index is not given for the star of a planet in the database, the value is estimated from the effective temperature which is more frequently given than the B-V index. A number of temperatures between 1,195K and 113,017K and their corresponding B-V index are given in a table [8], see table 4.2. To get the B-V index of any effective temperature the two closest

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[1] http://simbad.harvard.edu/simbad

Exoplanet Orbit Database	NASA Exoplanet Archive
HD 1237	GJ 3021 *
MOA-2009-BLG-387L	MOA 2009-BLG-387L
HD 126614 A	HD 126614 *
epsilon Ret	HD 27442 *
PH-1	PH1
gamma Leo A	gam 1 Leo *
OGLE-2007-BLG-368L	OGLE 2007-BLG-368L
alpha Ari	alf Ari
mu Ara	HD 160691 *
OGLE-05-169L	OGLE 2005-BLG-169L
tau Gru	HD 216435 *
iota Hor	HR 810 *
OGLE-05-071L	OGLE 2005-BLG-71L
OGLE235-MOA53	OGLE 2003-BLG-235L *
MOA-2008-BLG-310L	MOA 2008-BLG-310L
KIC 11442793	KOI-351 *
OGLE-2006-BLG-109L	OGLE 2006-BLG-109L
HD 137388	HD 137388 A
kappa CrB	kap CrB
XO-2	XO-2 N *
epsilon Tau	eps Tau
epsilon Eri	eps Eri
Kepler-448	KOI-12 *
omega Ser	ome Ser
MOA-2010-BLG-477L	MOA 2010-BLG-477L
GJ 176	HD 285968 *
HIP 2247	BD-17 63 *
MOA-2009-BLG-266L	MOA 2009-BLG-266L
Kepler-89	KOI-94 *
iota Dra	HIP 75458 *
MOA-2007-BLG-400L	MOA 2007-BLG-400L
upsilon And	ups And
OGLE-2011-BLG-0251	OGLE 2011-BLG-251L
OGLE-05-390L	OGLE 2005-BLG-390L
Kepler-420	KOI-1257 *
beta pic	bet pic
gamma cep	gam cep
MOA-2007-BLG-192L	MOA 2007-BLG-192L
MOA-2009-BLG-319L	MOA 2009-BLG-319L
omicron CrB	omi CrB
beta Gem	HD 62509 *
epsilon CrB	eps CrB
omicron UMa	omi UMa
HD 142022	HD 142022 A

Table 4.1: Alternative star names.

given temperatures are found and the index is interpolated linearly from the two corresponding B-V values.

T_{eff}	B-V index
1195	2.00
1675	1.95
2150	1.90
2579	1.85
2942	1.80
3234	1.75
3463	1.70
3640	1.65
3779	1.60
3892	1.55
3989	1.50
4076	1.45
4159	1.40
4241	1.35
4322	1.30
4405	1.25
4489	1.20
4576	1.15
4664	1.10
4755	1.05
4849	1.00
4948	0.95
5052	0.90
5164	0.85
5286	0.80
5418	0.75
5563	0.70
5722	0.65
5895	0.60
6082	0.55
6285	0.50
6500	0.45
6728	0.40
6967	0.35
7218	0.30
7483	0.25
7767	0.20
8084	0.15
8455	0.10
8917	0.05
9531	-0.00
10395	-0.05
11677	-0.10
13674	-0.15
16954	-0.20
22695	-0.25
33605	-0.30
56701	-0.35
113017	-0.40

Table 4.2: Efftective temperature and B-V index correlation

The Orbit Disc

For the Orbit Disc a new type of geometric object is created. It is a disc that lies on the orbit line and visualizes the uncertainty of the semi-major axis of the planets orbit. The inner rim of the disc is where the orbit line would be if the semi-major axis were at its lowest value and the outer rim of the disc is where it would be if the value were at its highest value. The semi-major axis can hence take on any value that makes the orbit line lie in the width of the disc.

The geometric object is made up of a square plane that lies in the orbit plane. The plane has its center at the mass center of the planetary system and the point furthest away from the mass center that would have to be colored is the apoapsis when the semi-major axis value is at its highest. The size of the plane is twice the apoapsis distance, to make sure the plane covers that point, see figure 4.2.

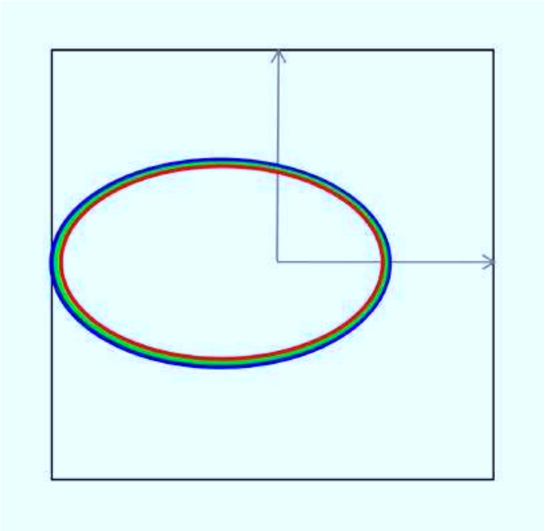


Figure 4.2: The Orbit Disc plane

The plane is then colored in the fragment shader to get the right shape and size of the disc. A point on the plane is subject to coloring if it lies inside the outer ellipse and outside the inner ellipse. Equation 4.2 has to be satisfied for a point to be inside an ellipse.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} \leq 1 \tag{4.2}$$

Where the ellipse center is at (h, k), the semi-major axis is a and the semi-minor axis is b. Equation 4.3 shows the relationship between the semi-major axis and the semi-minor axis.

$$b = a\sqrt{1-e^2} \tag{4.3}$$

Where e is the eccentricity.

The center of the ellipse is displaced from the center of the plane by a distance c along the x-axis, the distance can be calculated using equation 4.4.

$$c = \sqrt{a^2 - b^2} \tag{4.4}$$

All the values a, b, c, h and k need to be scaled to the size of the plane.

To color the disc one color at the outer rim, another color in the middle and a third color at the inner rim, the texture in figure 4.3 is used.

The calculations to see where along the texture to sample from are made by first normalizing the vector from the middle of the plane to the point (s, t). The normalized vector is then used

Figure 4.3: The Orbit Disc texture

to find where along its direction it intersects the outer ellipse by solving the following equation to get a scale factor. How the equation was derived can be seen in appendix .2.

$$scaleFactor = -\frac{b^2xc}{(bx)^2+(ay)^2} + \sqrt{(\frac{b^2xc}{(bx)^2+(ay)^2})^2 - \frac{(bc)^2-(ab)^2}{(bx)^2+(ay)^2}} \tag{4.5}$$

The normalized direction vector is multiplied by the scale factor to get the point where along the vector's direction it intersects the outer ellipse. The point where the direction vector intersects the inner ellipse is found by multiplying the outer point by the apoapsis distance of the inner ellipse divided by the apoapsis distance of the outer ellipse.

By dividing the distance between (s, t) and the outer point with the distance between the outer point and the inner point a sample factor is calculated and a the color from the texture at the sample is set to the point.

Scene Graph Node Position and Rotation

At the top of the scene graph for the exoplanet system visualization is its base node. The base node is added to the Solar System Barycenter node and a static translation is applied to it to place the exoplanet system correctly in space.

A rotation is also added to the base node to get the local coordinate system, where the z-axis is the normal to the sky plane and points from the star position to the Earth, the x-axis is the celestial north projected onto the sky plane and the y-axis completes the right hand coordinate system. To rotate one coordinate system to another, the matrix in 4.6 where *alpha* is the desired x-axis, *beta* is the desired y-axis, which is the cross product of *gamma* and *alpha*, and *gamma* is the desired z-axis. The matrix is added as a rotation to the base node.

$$\begin{bmatrix} alpha.x & alpha.y & alpha.z \\ beta.x & beta.y & beta.z \\ gamma.x & gamma.y & gamma.z \end{bmatrix} \tag{4.6}$$

The base node now represents the exoplanet system barycenter. A geometric node, representing the star, is attached to the base node without any further transformations. Another geometric node, representing the planet, is attached to the base node. The planet node has a transformation applied to it that will move it around the star in its orbit, specified by the orbital parameters. In the translation the orbit plane and the ascending node are orientated, and the planet is moved along the orbit line with time. To position the orbit around the star, three rotations are performed. The first rotation is around the z-axis of the local coordinate system to place the location of the ascending node. The rotation is hence made in the reference plane. The second rotation is made around the x-axis, now aligned with the ascending node, to get the correct inclination. The last rotation is made around the new z-axis, which is the normal to the orbit plane, to place the periapsis.

A node for the orbit trail is added to the base node as well. The same transformation node, as for the planet, is applied to the orbit trail.

The last node is the new geometric object, the Orbit Disc, that visualizes the uncertainty of the orbit. The planet moves in its orbit and that orbit will lie in a plane. In order to get the orbit disc to lie in the orbit plane the same three rotations that orientates the orbit plane and the ascending node for the planet and orbit line is added to the Orbit Disc node. The matrix that performs the rotation is calculated as in listing 4.2.

Listing 4.2: Rotation matrix for orbit plane

```
dmat4 orbitPlaneOrientation =
  rotate(ascendingNodeRadians, referencePlaneNormal) *
  rotate(inclinationRadians, ascendingNodeAxis) *
  rotate(argumentOfPeriapsisRadians, orbitPlaneNormal);
```

4.3 Discovery Method Visualization

In order to use OpenSpace as a tool for explaining the methods used for the discovery of exoplanets two visualization modes can be activated, one for each of the methods transit and radial velocity. When one of the modes is turned on some of the scene graph nodes change and an explaining graph is added.

Transit Method

A transit is the dip in light intensity that can be detected when a planet passes between the observation point and its star. The transit is seen in the visualization for this discovery method while also seeing an animation of the dip in light intensity in a graph as the planet transits.

The camera is moved so that the planetary system is seen ”edge on”, see figure 4.4. This is how the system would be seen if it could be viewed from the Earth.

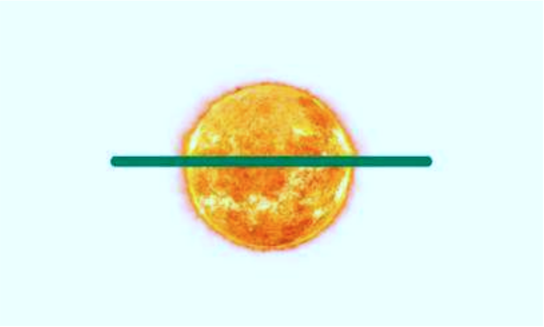


Figure 4.4: A planetary system seen edge on

The camera is hence placed a distance away from the star along the vector going from the star to the Earth, looking back at the star. The camera position is calculated by adding the displacement along the vector to the star position.

To make the transiting of the exoplanet and the decrease in light intensity more obvious, both the star and the planet are scaled to make them larger. When the nodes for the two objects are created, a scale transformation is added. At the time the nodes are created, they have the scale factor 1.0 and when the transit visualization mode is enabled, a new scale factor is set to the scale transformation. The value of the scale factor is calculated so that the radius of the star is half the length of the periapsis distance, as in equation 4.7.

$$scaleFactor = \frac{0.5 * periapsisDistance}{starRadius} \tag{4.7}$$

To make it clear that it is the decrease of the star’s light intensity that is observed in the transit method, a graph of the light curve is added to the screen space, see figure 4.5. The graph is animated so that an indicator of the current light intensity corresponds to the planet position in relation to the observation point.

The graph is made up of two images, the first of which is the graph itself, with the axes and the shape of the curve. The second image is the marker that is animated to move along the curve as the exoplanet moves.

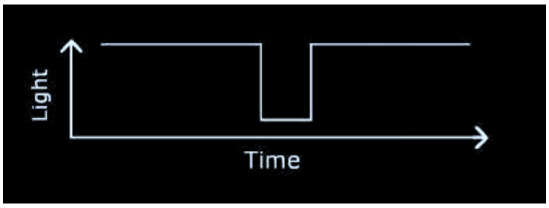


Figure 4.5: The light intensity over time

As long as the planet is not in transit the marker indicates full light intensity. When the planet enters the transit, the indicator moves down to the trough of the curve, indicating a lower light intensity. In order to know when the planet is in a position that appears to be in front of the star, the vector that can be drawn from the star to the planet is projected onto the sky plane and the length of the projected vector is compared with the scaled star radius. Due to perspective errors, the planet will not look like it is in front of the star if the comparison of the projected vector is to the full radius. To accommodate for the perspective error only a part of the star radius, 75% of it, is used in the comparisons in the calculations. The fraction that is used was chosen by decreasing the compared radius bit by bit and seeing where it looked like the planet entered and exited the transit. So if the length of the projected vector is longer than 75% of the scaled radius then the planet will look like it is in front of the star.

The indicator moves horizontally along the curve as the planet orbits the star. In order to calculate where on the orbit the planet is located at the current time two angles are calculated. One, α, is between the planet and the ascending node, the other, β, is between the planet and the observation point, see figure 4.6. The horizontal position of the marker moves between -1 and 1 and its exact position is determined by where on the orbit the planet is located. If the planet is in section R of the orbit as in figure 4.7, meaning that the angle between the planet and ascending node is less than 90°, then the marker position is calculated as in equation 4.8.

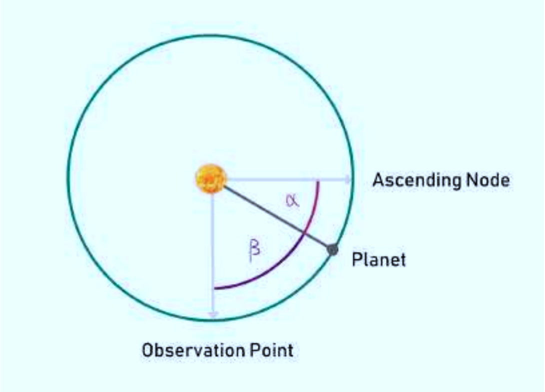


Figure 4.6: Angles to calculate planet position

$$position = \frac{\beta}{90} \tag{4.8}$$

If the planet is in section L of the orbit as in figure 4.7, the angle between the planet and ascending node is more than 90° and the marker position is calculated as in equation 4.9.

$$position = -\frac{\beta}{90} \tag{4.9}$$

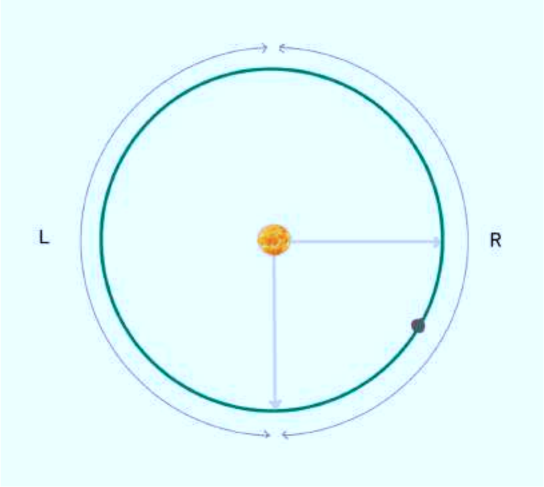


Figure 4.7: Sections of the orbit

Radial Velocity Method

In the Radial Velocity method the color spectra of the star's light changes as the gravitational pull that the exoplanet inflicts on the star moves it slightly toward and away from the observation point. The movement of the star is animated in the visualization for this discovery method and a graph is added, showing the change of the color spectra as the star is moving.

In order to see that movement, the camera is placed so that the system is seen "face on" when the RV method visualization is enabled. This means that the whole orbit line is seen as a ring or ellipse around the star, see figure 4.8.

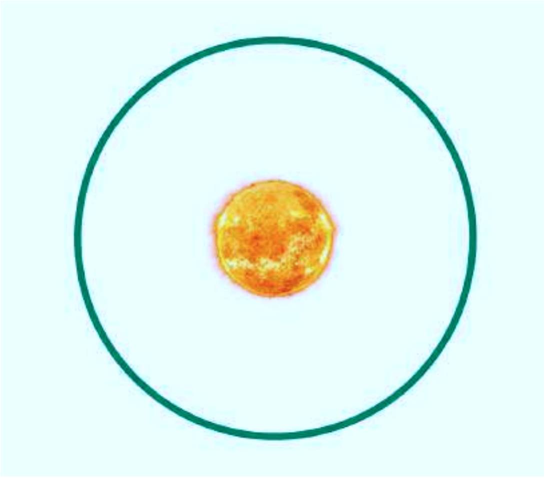


Figure 4.8: A planetary system seen face on

The vector which the camera is placed along is the cross product of the vector going from the star to the observation point and the vector going from the star to the ascending node. It is placed at a distance of four times the semi-major away from the star along that vector, looking back at the star.

For the purpose of education, the visualization only makes sense if there is only one exoplanet orbiting the star, even if the system has multiple exoplanets. With the help of the planet name vector that was created at initiation, the visibility of all but one planet is set to false. The visibility of the corresponding orbits and uncertainty discs is also hidden.

The star and the visible planet are scaled to make them larger. The scaling is done the same way as in the Transit Method visualization, by changing the value of the scale transformation applied to the nodes. The scale factor is calculated so that the radius of the star will be 20% of the periapsis distance, see equation 4.10.

$$scaleFactor = \frac{0.2 * periapsisDistance}{starRadius} \tag{4.10}$$

If there is no sphere for the exoplanet, only the orbit line and the disc, due to the fact that there is no record of the exoplanets radius, a sphere will be added to the scene as long as the RV method visualization is enabled.

An exaggerated motion of the star being pulled by the planet's gravitation is created by applying a Kepler Translation to the star. The parameters of the Kepler Translation are mostly the same as for the planets, except for the semi-major axis, which is much smaller than for the planets, and the mean anomaly which has the value of 180°. This will place the star in an orbit like the one that the planet is in only smaller and displaced by 180° from the planet.

A graph made up of two images is added to the screen space. The underlying image is of the color spectrum and the overlying picture is a set of black lines representing the gaps in the spectrum. The lines are then moved right and left to visualize the color shifts. The lines are moved right from the center when the planet is moving away from the observation point and they move to the left when the planet moves towards the observation point. The horizontal position of the lines moves between -1 and 1 and its exact position is determined by where on the orbit the planet is located. If the planet is in section FR of the orbit as in figure 4.9, where the angle between the planet and ascending node is less than 90° and the angle between the planet and the observation point is also less than 90°, then the position is calculated as in equation 4.11.

$$position = -\frac{\beta}{90} \tag{4.11}$$

If the planet is in section BR of the orbit in figure 4.9, where the angle between the planet and ascending node is less than 90° and the angle between the planet and the observation point is more than 90°, then the position is calculated as in equation 4.12.

$$position = -\frac{180 - \beta}{90} \tag{4.12}$$

If the planet is in section BL of the orbit in figure 4.9, where the angle between the planet and ascending node is more than 90° and the angle between the planet and the observation point is also more than 90°, then the position is calculated as in equation 4.13.

$$position = \frac{180 - \beta}{90} \tag{4.13}$$

If the planet is in section FL of the orbit in figure 4.9, where the angle between the planet and ascending node is more than 90° and the angle between the planet and the observation point is less than 90°, then the position is calculated as in equation 4.14.

$$position = \frac{\beta}{90} \tag{4.14}$$

4.4 Solar System Reference Visualization

When a planetary system is in focus it is hard to grasp how large or small the elements are. To get a better comprehension of the size a feature can be turned on to see the size of our Sun and the Earth as well as the Earth's semi-major axis. The sizes of the Sun and the Earth are shown with a white circle that is added at the same position as the systems star and

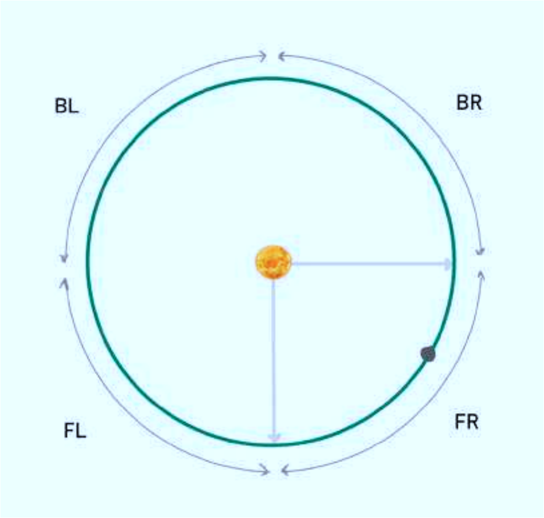


Figure 4.9: Sections of the orbit

exoplanets. The semi-major axis is also represented with a white circle that is placed in the same plane as the exoplanets orbit, centered around the star.

5 Results

In the original Exoplanet vizualisation in OpenSpace it was possible to see where in space the exoplanets were located, see figure 5.1 and 5.2.

With the visualization of exoplanets that has been implemented in this thesis, the majority of the shown exoplanets of the original visualization can be looked at up close. All planetary systems are added in the same way regardless of the scale or number of planets in orbit.

The Exoplanet Orbit Database contains information about 3011 planets, not including Kepler Candidates, while the NASA Exoplanet Archive has the location of about 2517 planets so at least a few hundred planets will not be visualized because the position is unknown. Also, the star names in the two files do not always match. Of the 2517 planets in the position-file, 397 do not have a matching name in the Exoplanet Orbit Database, however, 44 of those has been found to have a different name which can be found in the Exoplanet Orbit Database. That leaves at least 300 of 2517 planets that will not be visualized just by the names not matching between the two files.

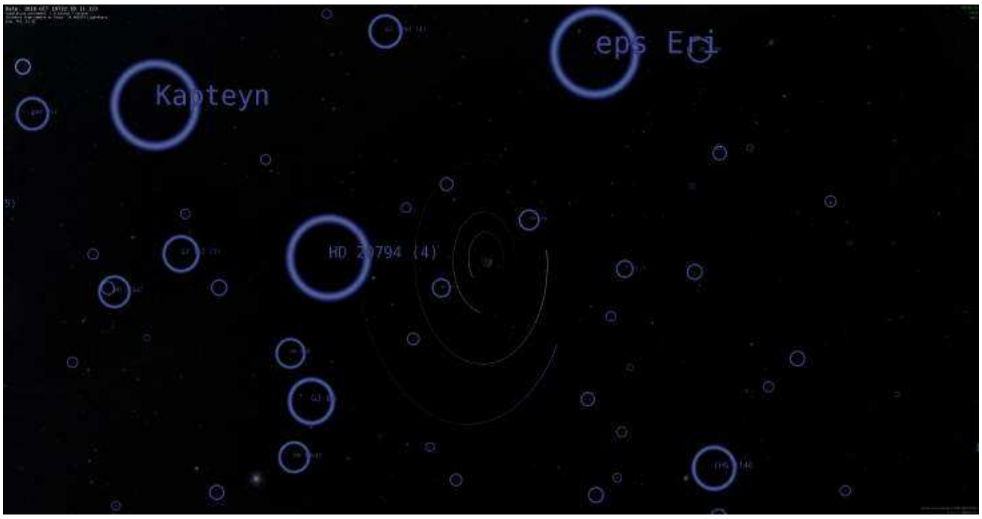


Figure 5.1: Exoplanets closest to the Solar System

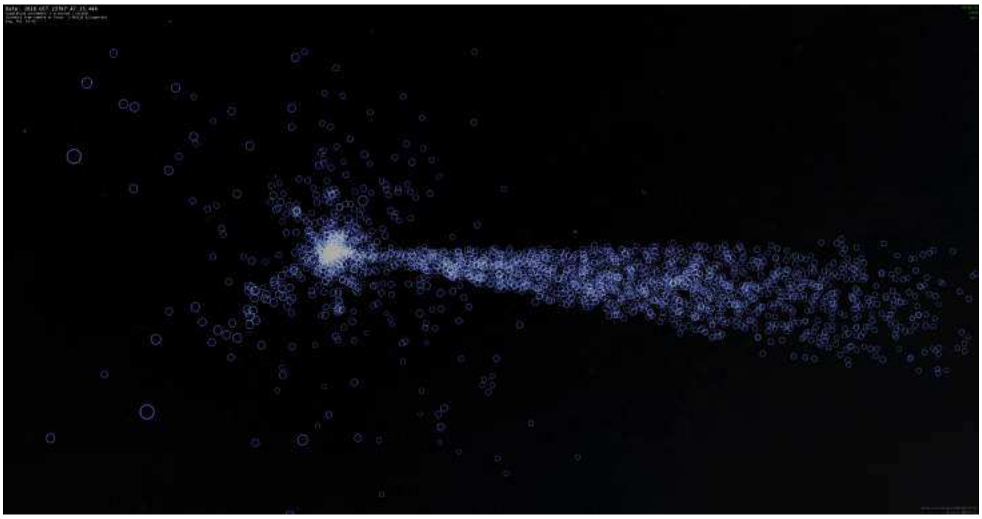


Figure 5.2: The spread of exoplanets in the universe

Figure 5.3: The planetary system around the star KOI-351

The star KOI-351 has 7 planets in orbit, see figure 5.3, and the star XO-2 N only has one, see figure 5.4.

The exoplanet HD 20782 b lies in an orbit with high eccentricity, $e = 0.970$, seen in figure 5.5.

The color of the star was determined by the B-V index of the star. If the B-V index was not given it was instead estimated using the effective temperature of the star. In figure 5.6 the star Kepler-427 is shown, the star had a B-V value of 0.71 and an effective temperature of 5800 Kelvin, it is hence colored by its B-V index. In figure 5.7 the star Kepler-528 is shown. The star had no recorded B-V index and an effective temperature of 5807 Kelvin, the color of the star is hence calculated from the temperature. The color of the two stars is very similar.

The known coldest star is TRAPPIST-1 seen in figure 5.8 and the known hottest star is Fomalhaut seen in figure 5.9.

A view from when the Transit Method Visualization is enabled is seen in figure 5.10.

A view from when the RV Method Visualization is enabled is seen in figure 5.11.

The Solar System Reference visualization is seen as white circles. One is centered around the star in focus and marks the semi-major axis of earth, the second is also centered around

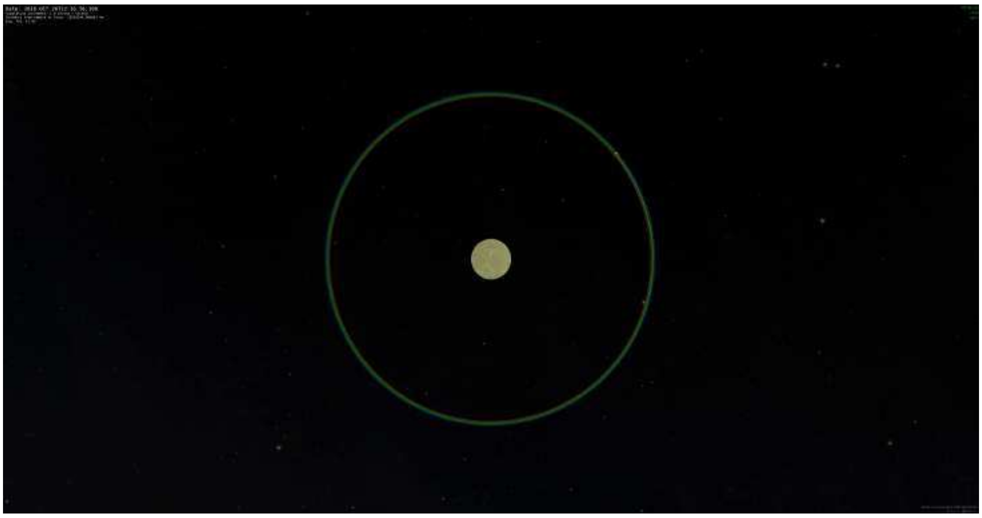


Figure 5.4: The planetary system around the star XO-2 N

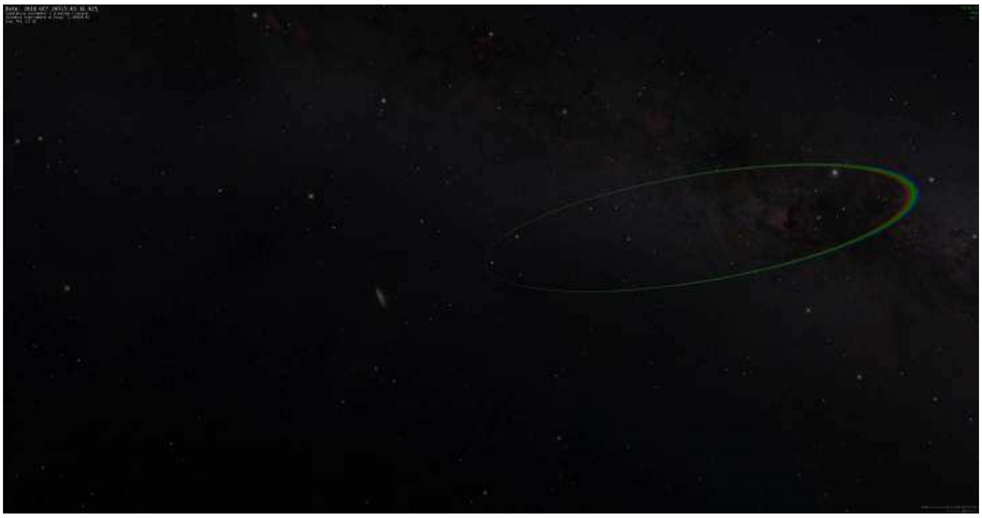


Figure 5.5: HD 20782 b has an orbit with a high eccentricity

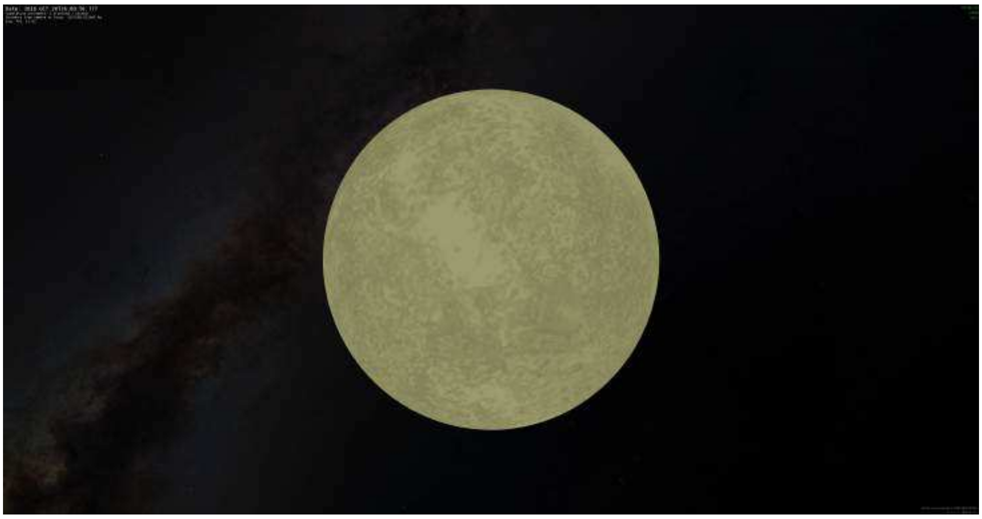


Figure 5.6: The color of Kepler-427 calculated from the B-V index

Figure 5.7: The color of Kepler-528 is calculated from the effective temperature

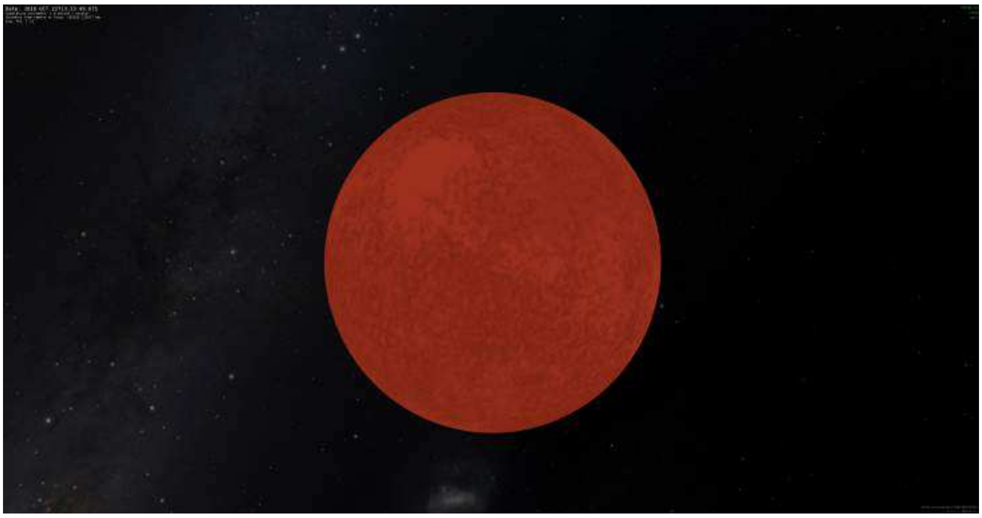


Figure 5.8: The coldest star TRAPPIST-1

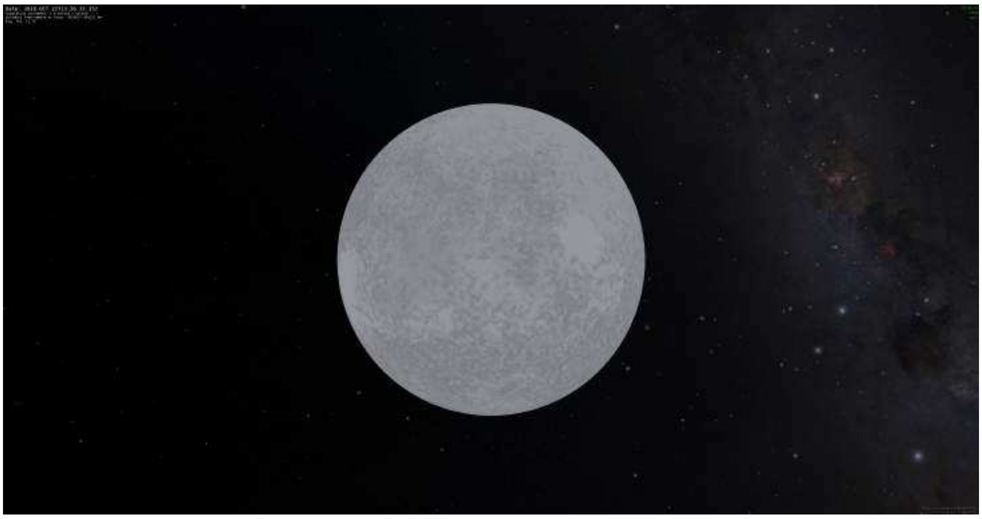


Figure 5.9: The warmest star Fomalhaut

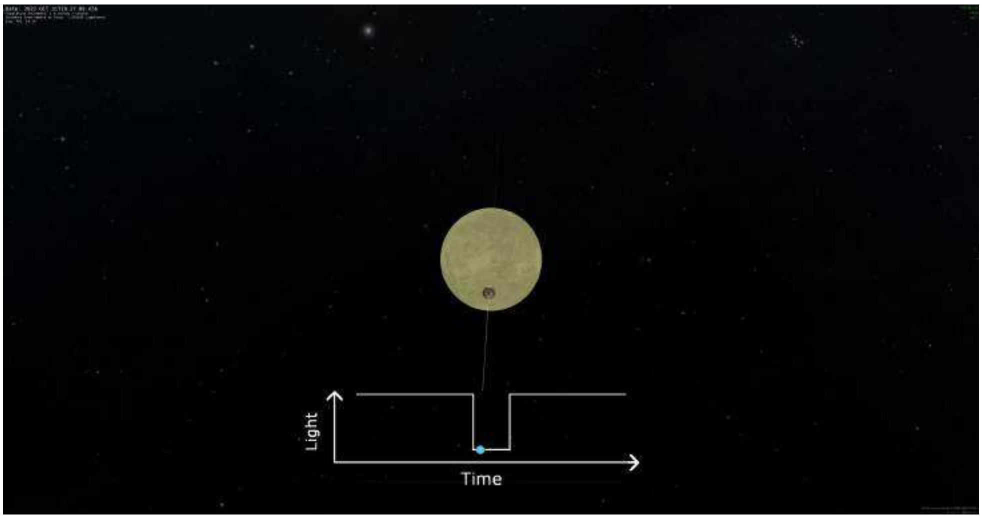


Figure 5.10: The result of the Transit Method Visualization

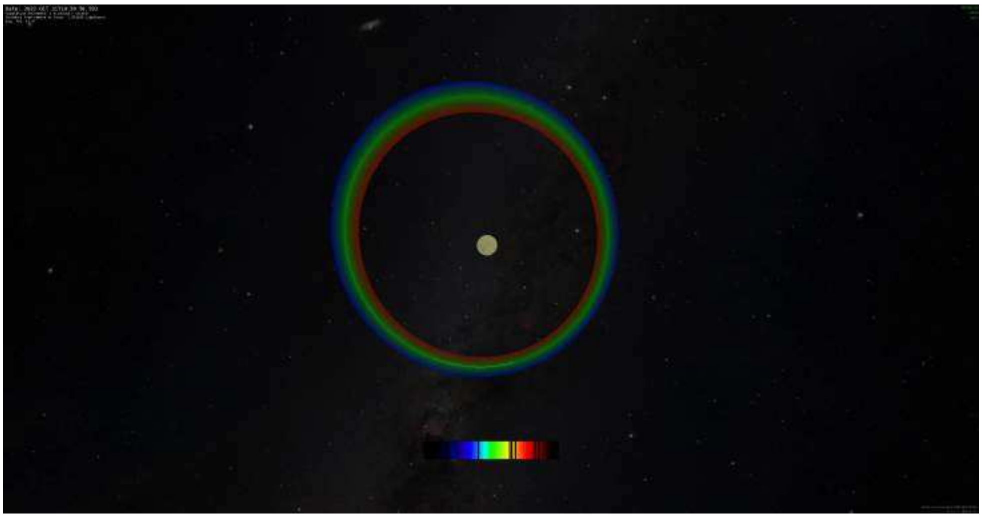


Figure 5.11: The result of the RV Method Visualization

the star and marks the size of the Sun and the third one is centered around all the planets in the system and mark the size of the Earth. In image 5.12 the semi-major axis reference is shown and in image 5.13 the sun size reference is shown. The planetary system the references have been added to is Kepler-69.

To add a planetary system, the name of the star is given in the GUI. In the same way a system can be removed from the visualization. Technically that leaves the user the possibility to add multiple planetary systems and fill the scene graph with object without removing them.

Figure 5.12: The semi-major axis of Earth shown around the star Kepler-69

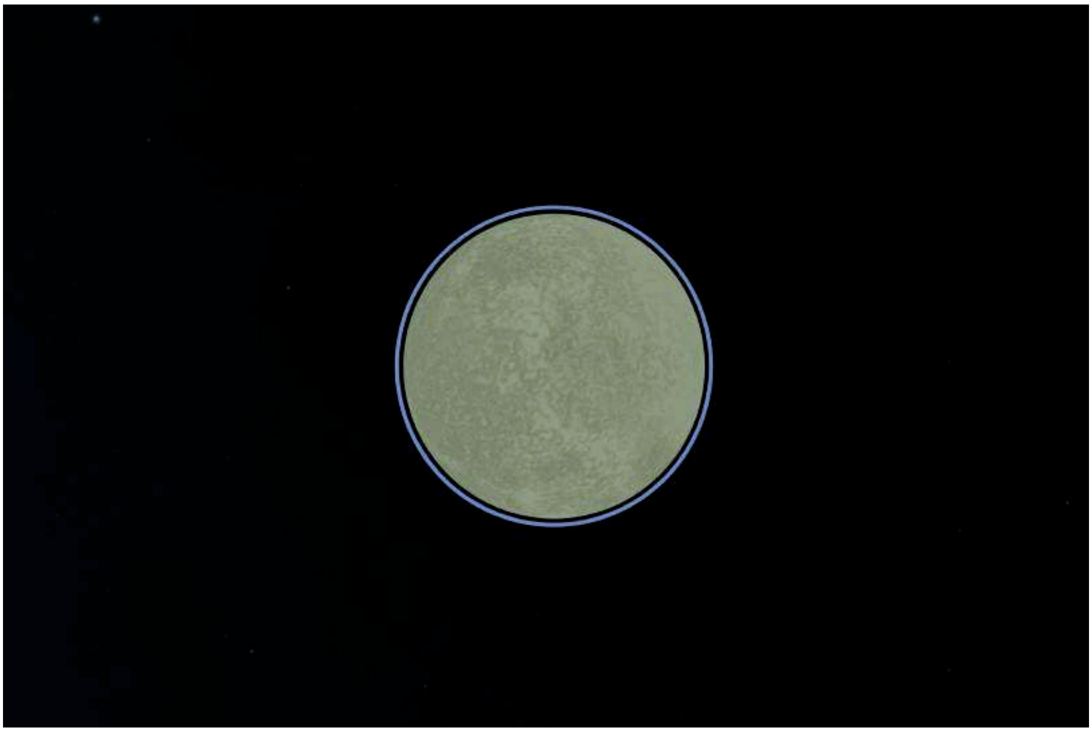


Figure 5.13: The size of the Sun shown around the star Kepler-69

6 Discussion

In this chapter the choices made in the method and the results that have come from them are evaluated. It also contains a section on future work.

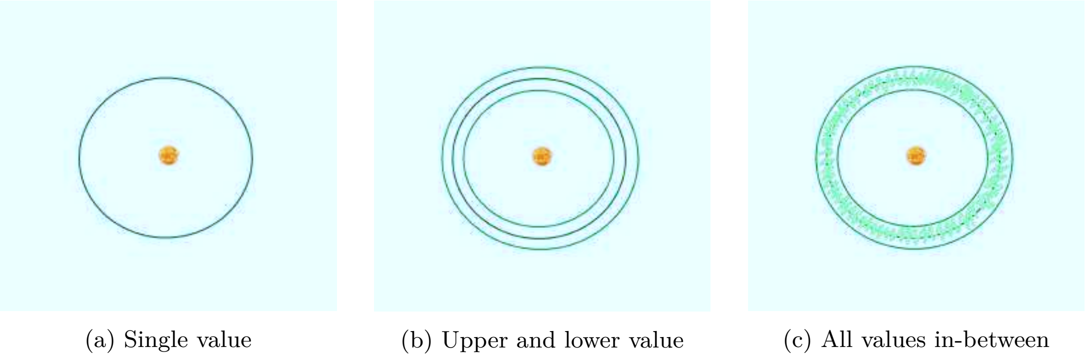
6.1 Results

The aim of the orbit visualization was to have so that the general shape and size was clear while still conveying that there is an uncertainty to the data. In the visualization, while the general shape and size is clear, it is not obvious that the appearance is an estimation of what the planetary system might look like. The only parameters that is marked with an uncertainty visualization is the semi-major axis, while in fact the other factors to the orbit's shape and orientation are not necessarily known to an exact value. The fact that only one orbit parameter is visualized with its uncertainty could, falsely, give the impression that all other parameters are know to be an exact value. Otherwise, it would be logical if their uncertainty too were visualized.

All the data that comes from the database has an uncertainty recorded. If the parameters of the orbit all had an uncertainty visualization it would send a clear message that the value is taken from the database. If the parameter then did not have an uncertainty visualized it would indicate that the value of it is default. For example, an orbit with an inclination of 90°, coming from the database, would also have an uncertainty of +0/-2.8 visualized and an orbit with an inclination of 90°, coming from the default value, would not have the uncertainty visualization.

The way the default values are chosen is so that they will give the most likely apperance of the orbit, which in the case for the inclination would be 90°. In doing so, it makes it harder to see if an orbit is constructed from real data or default values just by the apperance of it, unless the apperance diviates significantly from what it would look like with the defult value. For example, the default value would make the orbit circular, so if it is clearly an ellipse orbit it is a value from the database. However, what default values are set is just known by the developers of the code.

Adding all the planets' scene graph nodes from the start would mean thousands of object in the scene graph which simply would not be possible. Instead, a planetary system is added to the scene graph by giving the star name in the GUI. It was discussed if the scene graph nodes could be added in a more adaptive way. For example, if only the exoplanets within a



(a) Single value (b) Upper and lower value (c) All values in-between

Figure 6.1: Uncertainty disc texture

specific distance range from the camera could be added and removed as the camera moved. This would mean that the focus could only be set to the closest stars and it would not be possible to move the camera to a star far away since they are not a part of the scene graph and the camera focus can only be set to object in the scene graph. So, an active choice has to be made regarding which star to look at, either by turning on the original exoplanet visualization and choosing a name that is seen there or by knowing a name by heart.

6.2 Method

In the following text some parts of the application and their issues and decisions regarding them are discussed.

Orbital Parameters

The parameter Longitude of Ascending Node, Ω, should really be called the Nodal Point, accordingly to some literature on orbits outside the Solar System, and be defined as the angle of the nodal point that lies between 0° and 180° where the orbit intersects the reference plane. Furthermore, the inclination should be given in the span 0° to ±90° and the sign of the value should determine the motion of the planet. The value should be positive if the planet is moving away from the observer at the Nodal Point and negative if the planet is moving towards the observer at the Nodal Point. Although the angle of the node cannot be calculated with either Transit or RV, the indetermination of the sign can be eliminated through RV.

Disc Texture

The idea behind the texture that was created for the disc is to show that span of the possible values while at the same seeing the difference in the shape that the higher or the lower value would create.

If the value of the parameter would be given as an exact value a single line would mark the path of the planet, as in figure 6.1a. However, the value is given with an upper and lower uncertainty value. If these values were to be drawn out it would give the two orbits in figure 6.1b. The parameter could also take on any value in-between the two new lines and a disc, as in figure 6.1c, would represent the area in which the planet might orbit.

If the disc where to have one single color it might suggest to the user that the path of the planet could follow the orbit in figure 6.2. As long as all the parameters stays constant the orbit would not be able to take this shape. The disc was given another coloring where the orbit line created by the highest possible value was drawn in one color and the lowest possible value was drawn in another color and all orbit lines in-between the two was drawn in another color, creating the final disc texture as in figure 6.3. Then, if a planet would line in the red section on one side of the star it would also lie in the red section on the other side of the star.

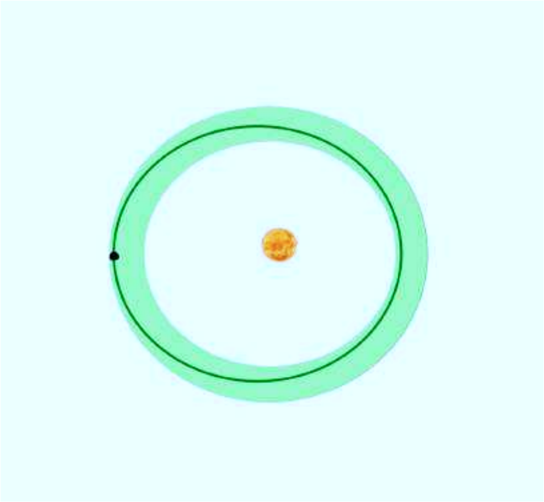


Figure 6.2: Not a possible orbit

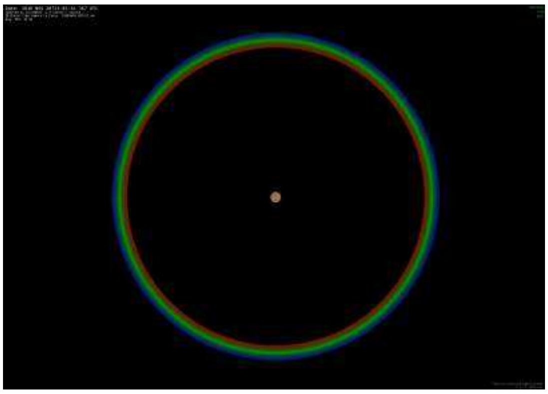


Figure 6.3: The uncertainty disc with three colors

Perspective Error

For the transit method visualization, it was necessary to know when the planet appeared to be in front of the star. Due to perspective errors, the calculations of when the planet is in front of the star did not match when it actually looks like it is in front of the star. The perspective error is illustrated in figure 6.4. The part of the orbit when the planet is in transit compared to the part of the orbit when the planet looks like it is in transit is visible in figure 6.5.

So the point on the orbit where the planet actually moves in and out of the transiting part will look like it is outside the transit part since everything is stretched out.

Discovery Methods Visualization

The size of the star when the discovery method visualization is activated was chosen so that it was nearly as large as the orbit. The interesting part of the transit method is the transiting part.

When the star is large relative to the orbit, the orbiting body comes nearer the star and the time during which the body is in transit becomes longer. The periapsis distance is the

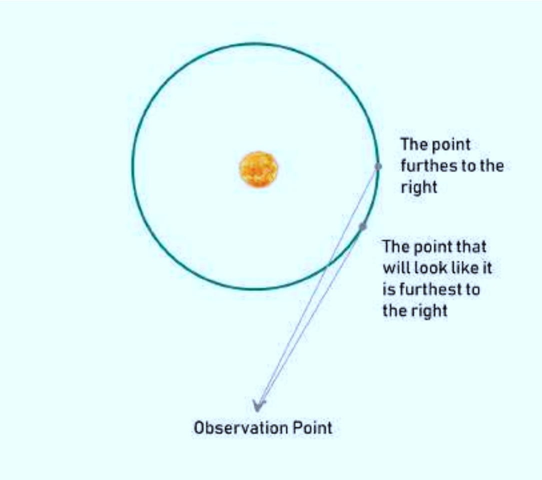


Figure 6.4: The perspective error

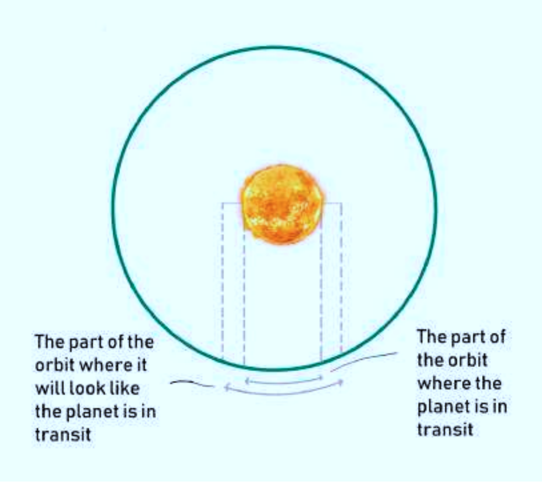


Figure 6.5: The transit and the visible transit parts

shortest possible distance the planet can be from the star, so the star can never be larger than the planet orbit, as it could be if the size is calculated using the semi-major axis or the apoapsis distance.

The graphs for the discovery method visualizations are both standardized and do not correspond to any real value from the planetary systems or change between the planetary systems.

The transit graph curve is simplified to easier fit the indicator to follow the curve. A realistic graph would gradually go down and up in light intensity, since only a part of the planet is blocking the light when it enters and exits the transit.

Actually calculating how much of the star's area is blocked by the planet and adapting the drop in light intensity from it would improve on the experience. Then it would also be possible to have all the planets in the system visible at the same time, see figure 6.6.

6.3 Future Work

In this thesis only the visualization for the semi-major axis uncertainty was implemented. However, all orbit parameters could be visualized using the same technique that was used for the semi-major axis. That is to draw out the orbit line that would come from the highest value

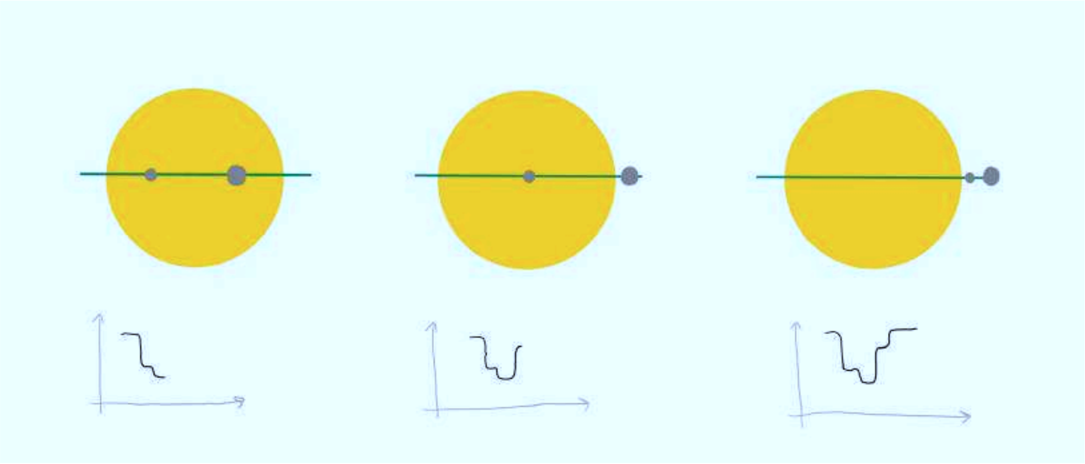


Figure 6.6: A transit graph with multiple planets

in one color and drawing out the orbit line created from the lowest value in another color and fill the area in-between the two lines with a color interpolation. How that would look with the eccentricity uncertainty is seen in figure 6.7.

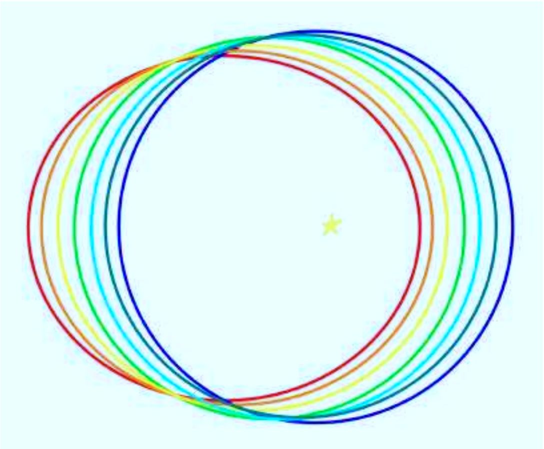


Figure 6.7: A sketch of how the eccentricity uncertainty would look

Using this technique to visualize the uncertainty of the values limits the software to only show one uncertainty at the time. If all uncertainty visualization were to be implemented a GUI to control which parameter that is show would have to be added.

An extension to the GUI where the user could add and remove planetary systems from the scene graph could also be made. All the planetary system could be added to a searchable list where the user, with a tickbox, could choose which planetary systems they wanted to look at. Then a maximum number of ticked planetary systems could be set so that the scene graph would never get large enough to create problems for the user. This method for adapting the scene graph is not as user friendly as is liked since the user actively have to remove objects from the scene graph by deciding what planetary system to remove.

7 Conclusion

From the result and experimentation with the visualization of exoplanets a conclusion can be drawn that real data is not always obvious from default values just by the appearance of the orbit. If the orbit deviates from a perfect circle or has an inclination that is obvious not in line with the view from the Earth they are clear indications that there is real data changing the appearance.

Another conclusion that can be drawn is that the uncertainty of the semi-major axis of a planet can be visualized with a disc that shows the span of possible values.

A method that can be used for adapting the scene graph is by only allowing a set number of planetary systems to be attached to the scene graph at the same time. It would prevent the program from crashing due to too many objects on the scene graph, which one of the aims. It would, however, not fulfill the aim that the user do not have to think about actively removing objects from the scene graph.

Bibliography

[1] Robert Grant Aitken. *The Binary Stars*. Dover Publications, Inc., 1964.

[2] AMSAT. *Keplerian Elements Tutorial*. 2018. URL: `https://www.amsat.org/keplerian-elements-tutorial/` (visited on 08/29/2018).

[3] Collins Dictionary of Astronomy. *effective temperature*. 2006. URL: `https://encyclopedia2.thefreedictionary.com/effective+temperature` (visited on 09/25/2018).

[4] Karl Bladin, Emil Axelsson, Erik Broberg, Carter Emmart, Patric Ljung, Alexander Bock, and Anders Ynnerman. "Globe Browsing: Contextualized Spatio-Temporal Planetary Surface Visualization". In: *IEEE Transactions on Visualization and Computer Graphics* 21.1 (Jan. 2018), pp. 802–811. ISSN: 1077-2626. DOI: `10.1109/TVCG.2017.2743958`.

[5] Alexander Bock, Emil Axelsson, Karl Bladin, Jonathas Costa, Gene Payne, Matthew Territo, Joakim Kilby, Masha Kuznetsova, Carter Emmart, and Anders Ynnerman. "OpenSpace: An open-source astrovisualization framework". In: *The Journal of Open Source Software* 2.15 (July 2017). DOI: `10.21105/joss.00281`. URL: `https://doi.org/10.21105/joss.00281`.

[6] Alexander Bock, Emil Axelsson, Carter Emmart, Masha Kuznetsova, Charles Hansen, and Anders Ynnerman. "OpenSpace: Changing the Narrative of Public Dissemination in Astronomical Visualization from What to How". In: *IEEE Computer Graphics and Applications* 38.3 (2018), pp. 44–57. ISSN: 0272-1716. DOI: `10.1109/MCG.2018.032421653`.

[7] Robert A. Braeunig. *Orbital Mechanics*. Accessed: 2018-08-09. URL: `www.braeunig.us/space/orbmech.htm`.

[8] Mitchell Charity. *Star color - details*. URL: `http://www.vendian.org/mncharity/dir3/starcolor/details.html` (visited on 09/04/2018).

[9] Heather Couper and Nigel Henbest. *SPACE Encyclopedia*. Dorling Kindersley, 1999.

[10] Exoplanet Exploration Program (NASA's Astrophysics Division). *10 Things: Exoplanets 101*. Mar. 2018. URL: `https://exoplanets.nasa.gov/news/1490/10-things-exoplanets-101/` (visited on 08/28/2018).

[11] Exoplanet Exploration Program (NASA's Astrophysics Division). *Exoplanets 101*. URL: `https://exoplanets.nasa.gov/the-search-for-life/exoplanets-101/` (visited on 08/22/2018).

[12] Rob Garner. *About TESS*. June 2018. URL: `https://www.nasa.gov/content/about-tess` (visited on 08/28/2018).

[13] Staffan Klashed, Per Hemingsson, Carter Emmart, Matthew Cooper, and Anders Ynnerman. "Uniview - Visualizing the Universe". In: *Eurographics 2010 - Areas Papers*. EG 2010 - Short papers. Norrköping, Sweden: Eurographics Association, 2010.

[14] Peter Linde. *Jakten på Liv i Universum*. Karavan, 2013.

[15] Mapelsoft. *Projection of a Vector onto a Plane*. URL: `https://www.maplesoft.com/support/help/maple/view.aspx?path=MathApps%2FProjectionOfVectorOntoPlane` (visited on 09/05/2018).

[16] California Institute of Technology NASA's Jet Propulsion Laboratory. *Eyes on Exoplanets*. URL: `https://eyes.nasa.gov/eyes-on-exoplanets.html` (visited on 11/15/2018).

[17] The Planetary Society. *Radial Velocity*. 2018. URL: `http://www.planetary.org/explore/space-topics/exoplanets/radial-velocity.html` (visited on 08/17/2018).

[18] The Planetary Society. *Transit Photometry*. 2018. URL: `http://www.planetary.org/explore/space-topics/exoplanets/transit-photometry.html` (visited on 08/24/2018).

[19] Dr. J. B. Tatum. *Celestial Mechanics*. Sept. 2018. URL: `http://astrowww.phys.uvic.ca/~tatum/celmechs.html` (visited on 09/09/2018).

[20] J. T. Wright, O. Fakhouri, G. W. Marcy, E. Han, Y. Feng, J. A. Johnson, A. W. Howard, D. A. Fischer, J. A. Valenti, J. Anderson, and N. Piskunov. "The Exoplanet Orbit Database". In: 123 (Apr. 2011), p. 412. DOI: `10.1086/659427`. arXiv: `1012.5676 [astro-ph.SR]`.

[21] Anders Ynnerman, Jonas Lowgren, and Lena Tibell. "Exploranation: A New Science Communication Paradigm." In: *IEEE COMPUTER GRAPHICS AND APPLICATIONS* 38.3 (n.d.), pp. 13–20. ISSN: 02721716. URL: `https://login.e.bibl.liu.se/login?url=https://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,uid&db=edswsc&AN=000434296900003&lang=sv&site=eds-live&scope=site`.

.1 Database Fields

Fields ending with UPPER or LOWER represent the upper or lower uncertainty in the parameter listed before them. Fields beginning with U represent uncertainties in the parameter listed before them, the value recorded is half of the span between the upper and lower limits. Note that the lower and upper uncertainties are not always symmetric.

Table .1: Fields of The Exoplanet Orbit Database

Field	**Data Type**	**Meaning**
NAME	String	Name of planet
OTHERNAME	String	Other commonly used star name
COMP	String	Component name of planet (“b”, “c”, etc.)
HD	Long Integer	Henry Draper number of star
HR	Integer	Bright Star Catalog number of star
HIPP	Long Integer	Hipparcos catalog number of star
SAO	Long Integer	SAO catalog number of star
GL	Float	GJ or Gliese catalog number of star
BINARY	Boolean	Star known to be binary?
RA	Double	J2000 Right ascension in decimal hours, Epoch 2000
DEC	Double	J2000 Declination in decimal degrees, Epoch 2000
RA STRING	String	J2000 Right ascension as a sexigesimal string, Epoch 2000
DEC STRING	String	J2000 Declination as a sexigesimal string, Epoch 2000
KEPID	Integer	The unique Kepler star identifier
KOI	Float	KOI object number
KDE	Boolean	If true, then this planet appears in the Kepler archive
EOD	Boolean	If true, then this planet has been vetted by the exoplanets.org team
MICROLENSING	Boolean	If true, then this planet was detected via microlensing
IMAGING	Boolean	If true, then this planet was detected via imaging
TIMING	Boolean	If true, then this planet was detected via timing
ASTROMETRY	Boolean	If true, then this planet was detected via astrometric motion
BMV	Float	$B - V$ color
V	Float	V magnitude
J	Float	J magnitude
H	Float	H magnitude
KS	Float	KS magnitude
DIST	Float	Distance to planetary system based on parallax in parsecs
DISTUPPER	Float	Upper uncertainty of distance to planetary system
DISTLOWER	Float	Lower uncertainty of distance to planetary system

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Table .1 – *Continued from previous page*

Field	**Data Type**	**Meaning**
UDIST	Float	Uncertainty of distance to planetary system
PAR	Float	Parallax in mas
PARUPPER	Float	Upper uncertainty of parallax
PARLOWER	Float	Lower uncertainty of parallax
UPAR	Float	Uncertainty of parallax
VSINI	Float	Projected equatorial rotational velocity of star
VSINIUPPER	Float	Upper uncertainty of projected equatorial rotational velocity of star
VSINILOWER	Float	Lower uncertainty of projected equatorial rotational velocity of star
UVSINI	Float	Uncertainty of projected equatorial rotational velocity of star
GAMMA	Float	Systemic radial velocity in km/s
GAMMAUPPER	Float	Upper uncertainty of systemic radial velocity
GAMMALOWER	Float	Lower uncertainty of systemic radial velocity
UGAMMA	Float	Uncertainty of systemic radial velocity
PER	Double	Orbital period in days
PERUPPER	Float	Upper uncertainty of period
PERLOWER	Float	Lower uncertainty of period
UPER	Float	Uncertainty of period
ECC	Float	Orbital eccentricity
ECCUPPER	Float	Upper uncertainty of orbital eccentricity
ECCLOWER	Float	Lower uncertainty of orbital eccentricity
UECC	Float	Uncertainty of orbital eccentricity
FREEZE ECC	Boolean	Eccentricity frozen in fit?
OM	Float	Argument of periastron in degrees
OMUPPER	Float	Upper uncertainty of argument of periastron
OMLOWER	Float	Lower uncertainty of argument of periastron
UOM	Float	Uncertainty of argument of periastron
K	Float	Semi-amplitude of stellar reflex motion in m/s
KUPPER	Float	Upper uncertainty of semi-amplitude of stellar reflex motion
KLOWER	Float	Lower uncertainty of semi-amplitude of stellar reflex motion
UK	Float	Uncertainty of semi-amplitude of stellar reflex motion
T0	Double	Epoch of periastron in HJD-2440000
T0UPPER	Float	Upper uncertainty of epoch of periastron
T0LOWER	Float	Lower uncertainty of epoch of periastron
UT0	Float	Uncertainty of epoch of periastron
DVDT	Float	Magnitude of linear trend in m/s/day
DVDTUPPER	Float	Upper uncertainty of magnitude of linear trend
DVDTLOWER	Float	Lower uncertainty of magnitude of linear trend
UDVDT	Float	Uncertainty of magnitude of linear trend
I	Float	Orbital inclination in degrees (for transiting systems only)
IUPPER	Float	Upper uncertainty of orbital inclination
ILOWER	Float	Lower uncertainty of orbital inclination

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Table .1 – *Continued from previous page*

Field	**Data Type**	**Meaning**
UI	Float	Uncertainty of orbital inclination
MSINI	Float	Minimum mass (as calculated from the mass function) in MJup
MSINIUPPER	Float	Upper uncertainty of minimum mass
MSINILOWER	Float	Lower uncertainty of minimum mass
UMSINI	Float	Uncertainty of minimum mass
A	Float	Orbital semi-major axis in AU
AUPPER	Float	Upper uncertainty of orbital semi-major axis
ALOWER	Float	Lower uncertainty of orbital semi-major axis
UA	Float	Uncertainty of orbital semi-major axis
MASS	Float	Mass of planet in Jupiter mass
MASSUPPER	Float	Upper uncertainty of planet mass
MASSLOWER	Float	Lower uncertainty of planet mass
UMASS	Float	Uncertainty of planet mass
SEP	Float	Separation between star and planet in Astronomical Units
SEPUPPER	Float	Upper uncertainty of separation between star and planet
SEPLOWER	Float	Lower uncertainty of separation between star and planet
USEP	Float	Uncertainty of separation between star and planet
TREND	Boolean	Linear trend in fit?
LAMBDA	Float	Projected spin-orbit misalignment
LAMBDAUPPER	Float	Upper uncertainty of projected spin-orbit misalignment
LAMBDALOWER	Float	Lower uncertainty of projected spin-orbit misalignment
ULAMBDA	Float	Uncertainty of projected spin-orbit misalignment
BIGOM	Float	Longitude of ascending node in degrees
BIGOMUPPER	Float	Upper uncertainty of longitude of ascending node
BIGOMLOWER	Float	Lower uncertainty of longitude of ascending node
UBIGOM	Float	Uncertainty of longitude of ascending node
DEPTH	Float	Transit depth $(R_p/R)^2$
DEPTHUPPER	Float	Upper uncertainty of transit depth
DEPTHLOWER	Float	Lower uncertainty of transit depth
UDEPTH	Float	Uncertainty of transit depth
T14	Float	Time of transit from first to fourth contact in days
T14UPPER	Float	Upper uncertainty of time of transit from first to fourth contact
T14LOWER	Float	Lower uncertainty of time of transit from first to fourth contact
UT14	Float	Uncertainty of time of transit from first to fourth contact
TT	Double	Epoch of transit center in HJD-2440000
TTUPPER	Float	Upper uncertainty of epoch of transit center

Continued on next page

Table .1 – *Continued from previous page*

Field	**Data Type**	**Meaning**
TTLOWER	Float	Lower uncertainty of epoch of transit center
UTT	Float	Uncertainty of epoch of transit center
R	Float	Radius of the planet in Jupiter radii
RUPPER	Float	Upper uncertainty of radius of the planet
RLOWER	Float	Lower uncertainty of radius of the planet
UR	Float	Uncertainty of radius of the planet
AR	Float	The ratio of the semi-major axis of a planet to the stellar radius (a/R)
ARUPPER	Float	Upper uncertainty of the ratio
ALOWER	Float	Lower uncertainty of the ratio
UAR	Float	Uncertainty of the ratio
B	Float	Impact parameter of transit
BUPPER	Float	Upper uncertainty of impact parameter of transit
BLOWER	Float	Lower uncertainty of impact parameter of transit
UB	Float	Uncertainty of impact parameter of transit
DENSITY	Float	Density of planet in g/cc^3
DENSITYUPPER	Float	Upper uncertainty of density of planet
DENSITYLOWER	Float	Lower uncertainty of density of planet
UDENSITY	Float	Uncertainty of density of planet
GRAVITY	Float	Planetary surface gravity [$log_{10}(cm/s^2)$]
GRAVITYUPPER	Float	Upper uncertainty of planetary surface gravity
GRAVITYLOWER	Float	Lower uncertainty of planetary surface gravity
UGRAVITY	Float	Uncertainty of planetary surface gravity
TRANSIT	Boolean	Is the planet known to transit?
DR	Float	Distance during transit in stellar radii
DRUPPER	Float	Upper uncertainty of distance during transit
DRLOWER	Float	Lower uncertainty of distance during transit
UDR	Float	Uncertainty of distance during transit
RR	Float	Planet/star radius ratio
RRUPPER	Float	Upper uncertainty of planet/star radius ratio
RRLOWER	Float	Lower uncertainty of planet/star radius ratio
URR	Float	Uncertainty of planet/star radius ratio
SE	Boolean	indicates that no secondary eclipse has been detected; indicates at least one secondary eclipse has been detected in the wavelengths listed below
SEDEPTHJ	Float	The secondary eclipse depth measured in the near infrared J band centered at 1.25 micron
SEDEPTHJUPPER	Float	Upper uncertainty of the secondary eclipse depth measured in the near infrared J band centered at 1.25 micron
SEDEPTHJLOWER	Float	Lower uncertainty of the secondary eclipse depth measured in the near infrared J band centered at 1.25 micron
USEDEPTHJ	Float	Uncertainty of the secondary eclipse depth measured in the near infrared J band centered at 1.25 micron

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Table .1 – *Continued from previous page*

Field	**Data Type**	**Meaning**
SEDEPTHH	Float	The secondary eclipse depth measured in the near infrared H band centered at 1.65 micron
SEDEPTHHUPPER	Float	Upper uncertainty of the secondary eclipse depth measured in the near infrared H band centered at 1.65 micron
SEDEPTHHLOWER	Float	Lower uncertainty of the secondary eclipse depth measured in the near infrared H band centered at 1.65 micron
USEDEPTHH	Float	Uncertainty of the secondary eclipse depth measured in the near infrared H band centered at 1.65 micron
SEDEPTHKS	Float	The secondary eclipse depth measured in the near infrared Ks band centered at 2.15 micron
SEDEPTHKSUPPER	Float	Upper uncertainty of the secondary eclipse depth measured in the near infrared Ks band centered at 2.15 micron
SEDEPTHKSLOWER	Float	Lower uncertainty of the secondary eclipse depth measured in the near infrared Ks band centered at 2.15 micron
USEDEPTHKS	Float	Uncertainty of the secondary eclipse depth measured in the near infrared Ks band centered at 2.15 micron
SEDEPTHKP	Double	The secondary eclipse depth measured in the Kepler photometry band in the optical from 400 to 865 nm
SEDEPTHKPUPPER	Double	Upper uncertainty of the secondary eclipse depth measured in the Kepler photometry band in the optical from 400 to 865 nm
SEDEPTHKPLOWER	Double	Lower uncertainty of the secondary eclipse depth measured in the Kepler photometry band in the optical from 400 to 865 nm
USEDEPTHKP	Double	Uncertainty of the secondary eclipse depth measured in the Kepler photometry band in the optical from 400 to 865 nm
SEDEPTH36	Float	The secondary eclipse depth measured with Spitzer in its IRAC1 band centered at 3.6 micron
SEDEPTH36UPPER	Float	Upper uncertainty of the secondary eclipse depth measured with Spitzer in its IRAC1 band centered at 3.6 micron
SEDEPTH36LOWER	Float	Lower uncertainty of the secondary eclipse depth measured with Spitzer in its IRAC1 band centered at 3.6 micron
USEDEPTH36	Float	Uncertainty of the secondary eclipse depth measured with Spitzer in its IRAC1 band centered at 3.6 micron
SEDEPTH45	Float	The secondary eclipse depth measured by Spitzer in its IRAC2 band centered at 4.5 micron

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Table .1 – *Continued from previous page*

Field	**Data Type**	**Meaning**
SEDEPTH45UPPER	Float	Upper uncertainty of the secondary eclipse depth measured by Spitzer in its IRAC2 band centered at 4.5 micron
SEDEPTH45LOWER	Float	Lower uncertainty of the secondary eclipse depth measured by Spitzer in its IRAC2 band centered at 4.5 micron
USEDEPTH45	Float	Lower uncertainty of the secondary eclipse depth measured by Spitzer in its IRAC2 band centered at 4.5 micron
SEDEPTH58	Float	The secondary eclipse depth measured by Spitzer in its IRAC3 band centered at 5.8 micron
SEDEPTH58UPPER	Float	Upper uncertainty of the secondary eclipse depth measured by Spitzer in its IRAC3 band centered at 5.8 micron
SEDEPTH58LOWER	Float	Lower uncertainty of the secondary eclipse depth measured by Spitzer in its IRAC3 band centered at 5.8 micron
USEDEPTH58	Float	Uncertainty of the secondary eclipse depth measured by Spitzer in its IRAC3 band centered at 5.8 micron
SEDEPTH80	Float	The secondary eclipse depth measured by Spitzer in its IRAC4 band centered at 8.0 micron
SEDEPTH80UPPER	Float	Upper uncertainty of the secondary eclipse depth measured by Spitzer in its IRAC4 band centered at 8.0 micron
SEDEPTH80LOWER	Float	Lower uncertainty of the secondary eclipse depth measured by Spitzer in its IRAC4 band centered at 8.0 micron
USEDEPTH80	Float	Uncertainty of the secondary eclipse depth measured by Spitzer in its IRAC4 band centered at 8.0 micron
RMS	Float	Root-mean-square residuals to orbital RV fit
CHI2	Float	χ^2_ν to orbital RV fit
NOBS	Integer	Number of observations used in fit
NCOMP	Integer	Number of planetary companions known
MULT	Boolean	Multiple planets in system?
PLANETDISCMETH	String	Method of discovery for the planet. Has value “RV” or “Transit”
STARDISCMETH	String	Method of discovery of first planet in system. Has value “RV” or “Transit”
DATE	Integer	Year of publication date
FIRSTREF	String	First peer-reviewed publication of planetary orbit
ORBREF	String	Peer-reviewed origin or orbital parameters
STAR	String	Name of host star
MSTAR	Float	Mass of host star
MSTARUPPER	Float	Upper uncertainty of mass of host star
MSTARLOWER	Float	Lower uncertainty of mass of host star

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Table .1 – *Continued from previous page*

Field	**Data Type**	**Meaning**
UMSTAR	Float	Uncertainty of mass of host star
RSTAR	Float	Estimated radius of the star in solar radii
RSTARUPPER	Float	Upper uncertainty of estimated star radius
RSTARLOWER	Float	Lower uncertainty of estimated star radius
URSTAR	Float	Uncertainty of estimated star radius
TEFF	Float	Effective temperature of host star
TEFFUPPER	Float	Upper uncertainty of effective temperature of host star
TEFFLOWER	Float	Lower uncertainty of effective temperature of host star
UTEFF	Float	Uncertainty of effective temperature of host star
RHOSTAR	Float	Density of star as measured from transit photometry and radial velocity information in g/cm^3
RHOSTARUPPER	Float	Upper uncertainty of star density
RHOSTARLOWER	Float	Lower uncertainty of star density
URHOSTAR	Float	Uncertainty of star density
LOGG	Float	Spectroscopic log g (surface gravity) of host star
LOGGUPPER	Float	Upper uncertainty of spectroscopic log g (surface gravity) of host star
LOGGLOWER	Float	Lower uncertainty of spectroscopic log g (surface gravity) of host star
ULOGG	Float	Uncertainty of spectroscopic log g (surface gravity) of host star
FE	Float	Iron abundance (or metallicity) of star.
FEUPPER	Float	Upper uncertainty of iron abundance (or metallicity) of star
FELOWER	Float	Lower uncertainty of iron abundance (or metallicity) of star
UFE	Float	Uncertainty of iron abundance (or metallicity) of star
SHK	Float	Mount Wilson Ca ii H & K S-value
RHK	Float	Chromospheric activity of star as R_{HK}
KP	Float	Kepler bandpass magnitude
JSNAME	String	Name of host star used in the Extrasolar Planet Encyclopedia
ETDNAME	String	Name of host star used in the Exoplanet Transit Database
SIMBADNAME	String	Valid SIMBAD name of host star
EANAME	String	Name of host star used in the Exoplanet Archive Database
SET	unknown	unknow

.2 Scale Factor

A point lies on the edge of an ellipse if

$$\frac{(s+c)^2}{a^2} + \frac{t^2}{b^2} = 1 \tag{1}$$

The coordinates for a point on the disc is also known and a vector between the middle of the plane, $(0, 0)$, and the point can be drawn. That vector normalized is (x, y). If the direction of the vector is followed it will intersect the ellipse. That intersection point will be found if the following equation is solved:

$$\frac{(xs + c)^2}{a^2} + \frac{ys^2}{b^2} = 1 \tag{2}$$

$$b^2(xs + c)^2 + a^2(ys)^2 = (ab)^2 \tag{3}$$

$$b^2((xs)^2 + 2xsc + c^2)a^2(ys)^2 = (ab)^2 \tag{4}$$

$$(bxs)^2 + 2b^2xsc + (bc)^2 + (ays)^2 = (ab)^2 \tag{5}$$

$$s^2((bx)^2 + (ay)^2) + s(2b^2xc) + ((bc)^2 - (ab)^2) = 0 \tag{6}$$

$$s^2 + s\frac{2b^2xc}{(bx)^2 + (ay)^2} + \frac{(bc)^2 - (ab)^2}{(bx)^2 + (ay)^2} = 0 \tag{7}$$

$$s = -\frac{b^2xc}{(bx)^2 + (ay)^2} + \sqrt{(\frac{b^2xc}{(bx)^2 + (ay)^2})^2 - \frac{(bc)^2 - (ab)^2}{(bx)^2 + (ay)^2}} \tag{8}$$