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Designing Urban Road Congestion Charging Systems

-Models and Heuristic Solution Approaches

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Abstract

The question of how to design a congestion pricing scheme is difficult to answer and involves a number of complex decisions. This thesis is devoted to the quantitative parts of designing a congestion pricing scheme with link tolls in an urban car traffic network. The problem involves finding the number of tolled links, the link toll locations and their corresponding toll level. The road users are modeled in a static framework, with elastic travel demand.

Assuming the toll locations to be fixed, we recognize a level setting problem as to find toll levels which maximize the social surplus. A heuristic procedure based on sensitivity analysis is developed to solve this optimization problem. In the numerical examples the heuristic is shown to converge towards the optimum for cases when all links are tollable, and when only some links are tollable.

We formulate a combined toll location and level setting problem as to find both toll locations and toll levels which maximize the net social surplus, which is the social surplus minus the cost of collecting the tolls. The collection cost is assumed to be given for each possible toll location, and to be independent of toll level and traffic flow. We develop a new heuristic method which is based on repeated solutions of an approximation to the combined toll location and level setting problem. Also, a known heuristic method for locating a fixed number of toll facilities is extended, to find the optimal number of facilities to locate. Both heuristics are evaluated on two small networks, where our approximation procedure shows the best results.

Our approximation procedure is also employed on the Sioux Falls network. The result is compared with different judgmental closed cordon structures, and the solution suggested by our method clearly improves the net social surplus more than any of the judgmental cordons.

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Joakim Ekström

Contents

1 Introduction

In every day life we experience the inconvenience of congestion in various situations. Whether we are standing in line at a grocery store, waiting in a telephone queue, or driving to work during the rush hour, we may experience congestion. In the example of the grocery store, congestion can of course, easily, be alleviated by hiring more staff and adding additional cashiers. As customers, we however, understand that a shop cannot increase capacity to completely eliminate the time waiting in queue, and still have a profitable business. This argument can be extended to road traffic were we cannot expect the road capacity to be high enough for the traffic to flow freely, even during the rush hours. There is an important difference between the example of congestion at the grocery store and road traffic congestion. In a grocery store the capacity can to some extent be adjusted, by having more personnel during the rush hours. Road capacity is on the other hand fixed, and to increase capacity require major investments.

Since the car was introduced in the beginning of the 20th century there has been an ever increasing demand for road infrastructure, and whenever a road is congested the solution has been to increase the capacity. Increasing capacity will however lead to increased demand, and this relationship between capacity and demand is well accepted. Still, the generic solution to alleviate congestion is even today to expand the road network with even more capacity.

Besides the relationship between capacity and demand, new road infrastructure is expensive and road pricing is often mentioned as one alternative tool to address the problem of congestion. The objective of road pricing is often unclear, and is not always to reduce congestion, but to finance new road infrastructure. When the objective of road pricing is to reduce congestion, the pricing scheme (a combination of toll locations and toll levels) is referred to as congestion pricing. The general idea behind congestion pricing is to let the road users pay for the congestion they force on the other road users, and this can be extended to incorporate other negative effects a road user will have on the surroundings, such as emission of pollutants, noise and accidents.

Traffic researchers and planners have acknowledged congestion pricing as an important part of the traffic system for a long time. The very few implemented congestion pricing systems suggests that this has not been properly communicated to the politicians and the public. The first operational congestion pricing scheme was Singapore in 1975 and in the recent years systems have been introduced in London, 2003, and in Stockholm, 2006.

The transportation system is complex and to design a congestion pricing scheme which give the desired effects is not a trivial task. Planning tools can be used to evaluate the effects a congestion pricing scheme will have on the traffic in a city. In a large city the alternative designs of a congestion pricing scheme are immense and to find a best design is both difficult and time consuming. Transport economists evaluate the efficiency of a pricing scheme by social welfare measures, and these measures can be used as objectives in an optimization framework. By formulating the problem of finding efficient congestion pricing schemes in an optimization framework, tools and theories from the field of optimization theory is used. This has primarily been done for the case when the toll locations are considered as fixed, and will be referred to as the level setting problem. In the level setting problem the cost of collecting the tolls is disregarded, but in practice there are setup and operational costs for the toll collection system. Introducing these costs in the optimization framework will allow us to not only maximize the social welfare, but the social welfare minus the cost of collecting the tolls. This problem will be referred to as the combined toll location and level setting problem, and is about were to locate the toll collecting facilities, as well as finding out the toll levels to charge the road users at each such facility.

So far, the main literature on optimal congestion pricing has focused on the level setting problem. The question of where to locate the toll facilities have so far mainly been addressed in methods for finding efficient closed cordons (Sumalee, 2005). Verhoef (2002a) suggest a method for locating a given number of toll facilities, without any restriction on the structure of the pricing scheme. To find an optimal solution is considered as difficult for both the level setting and combined toll location and level setting problem.

1.1 Objective and contributions

This thesis focuses on the quantitative aspects of congestion pricing schemes. The main objective is to develop models and methods for finding locations for toll facilities and the corresponding toll levels, to charge the road users, within a traffic network.

This thesis contributes to the field of congestion pricing analysis in the following way.

The thesis

- presents an exact mathematical formulation of the combined toll location and level setting problem, where the setup and operational cost are explicitly considered in the objective. The problem is formulated with both general elastic car demand and a multinomial logit model for the modal choice between car and public transportation.

- proposes a sensitivity analysis based method for solving the level setting problem. The method is an extension to a procedure for finding optimal capacity improvements in a traffic network (Josefsson, 2003).

- presents two heuristic procedures for solving the combined toll location and level setting problem. The first heuristic is an extension of a previously published heuristic (Verhoef, 2002a) to find optimal toll locations, given the number of tolls to locate. The second heuristic employs a continuous approximation of the combined toll location and level setting problem, which allows it to be solved with the proposed sensitivity analysis based method.

- presents numerical results to demonstrate how the proposed methods can be used to find efficient congestion pricing schemes. These numerical results also contribute to the discussion on efficient cordon structures.

Parts of the work presented in this thesis have been published in Ekström et al. (2008a) and Ekström et al. (2008b).

1.2 Method and delimitations

In this thesis a static traffic modeling framework is adopted for describing the change in travel times, traffic flows and demands, generated by a congestion pricing scheme. Social welfare measures are used to measure the efficiency of a pricing scheme but no considerations are made for equity or acceptability issues. By combining the static traffic modeling framework and the social welfare measures, mathematical models for finding efficient pricing schemes can be formulated. This modeling framework for optimal congestion pricing is well established and is considered to give valuable insight to how a congestion pricing scheme can be efficiently designed, despite any shortcoming the static transportations models may have. In a congestion pricing schemes, we assume the tolls to be collected from the road users at specific locations, i.e. link tolls or cordon tolls, and area or distance based tolls are not considered.

A solution method to an optimization problem is considered as heuristic if it is not possible to theoretically guarantee convergence towards the global optimum. Today there are no known optimization methods which can solve the congestion pricing problems presented in this thesis, and we have to rely on heuristic procedures to find as good solutions as possible.

The static modeling framework will give a simplified description of the transportation system, but as Verhoef (1999) points out, the analytical relationships in static models are appealing when searching for optimal congestion pricing fees. The static framework rely on the assumptions that the traffic conditions are stable over time, that the travelers have perfect information about the traffic conditions, and that the congestion on a road segment do not spill over to the surrounding network. Despite the somewhat unrealistic assumptions, the static modeling framework have prevailed for many years, and have proven to give valuable insight to the transportations systems that have been studied. If the transportation system is heavily congested, and the demand for traffic can not be accommodated by the traffic network, the static modeling framework is known to produce less reliable traffic flows and travel times.

1.3 Outline

Chapter 2 gives an introduction to traffic modeling, social welfare measures and optimal congestion pricing. Chapter 3 presents the mathematical formulations of the level setting problem and the combined toll location and level setting problem. The models are presented with both general elastic car demand and a multinomial logit model for modal choice. A sensitivity analysis based method for solving the level setting problem is proposed in Chapter 4, together with numerical examples for two networks. In Chapter 5, two different heuristic procedures for solving the combined problems are presented. Chapter 6 concludes the thesis and suggests further research directions.

2 Background

The field of congestion pricing is multidisciplinary and catches the attention of researchers from the fields of traffic engineering, transport economy and optimization theory. The theoretical background can be traced back to the work of Pigou (1920) and Knight (1924) followed by Beckmann et al. (1956), Marchand (1968) and Vickrey (1969). Even though researchers in the middle of the 20th century recognized congestion pricing as a tool to reduce congestion on roads, increasing capacity has prevailed as the generic solution. Congestion pricing is however implemented in several cities today (e.g. Singapore, London, and Stockholm). Road pricing in general is more common, but not with the aim to alleviate congestion, but to finance infrastructure.

It is well recognized among traffic researchers that increased capacity cannot be viewed as a sustainable solution of the problem of congestion. Lindsey and Verhoef (2000) mention three important reasons to why increased capacity will not, in the long term, solve the problem of congestion; in most large cities there is a lack of available land to be used for building new roads; constructing new roads to improve capacity is very expensive; and the latent demand will diminish the effect of congestion when capacity is increased.

When the demand for car traffic exploded in the middle of the 20th century there was a need to understand the increasingly complex transportation systems. Both engineers and economists addressed this problem, and the field of transportation modeling appeared. These models were primarily used to address the problem of how the existing road infrastructure could be extended to handle the ever increasing demand for car traffic. Today traffic models are one of the most important tools for traffic engineers.

Focusing at the quantitative aspects of congestion pricing, efficiency measure from the field of transport economy together with transportation modeling can give valuable insight into the performance of a pricing scheme. How to design a most efficient scheme is however not obvious, and involves where to collect the tolls, i.e. where to locate the toll facilities, and what toll levels to charge at each such facility. If only the quantitative aspects of congestion pricing schemes are considered, the problem can be formulated in a mathematical programming framework. These problems are often non-convex and therefore difficult to solve even for small networks. To solve these problems for large scale networks are even more difficult, and different heuristic methods have been suggested.

The remainder of this chapter will give an introduction to traffic modeling, social welfare measures and optimization of congestion pricing schemes.

2.1 Modeling transportation systems

The transportation systems in mid-size and larger cities are complex systems of road infrastructure and public transportation, and include the people using it. The basis of a transportation system is the travelers which need to move between different locations within a city. Ortúzar and Willumsen (1990) point out that the demand for transport is derived, not an end itself. With the exception of sight-seeing, this means that the trip is not the purpose, but what attract a person to make the trip, e.g. work, shopping, leisure and healthcare. Obviously the trip makers are better of with fast and low cost trips, but also the society in general is better of if the road users are productive, rather than spending time queuing or waiting.

To evaluate the effects of a congestion pricing scheme, tools from the field of transportation modeling are used. These models have been developed during the last half of the 20th century, and are still developing. When transportation modeling was introduced it was not to evaluate congestion pricing, but to decide how the traffic network could be enhanced to improve the quality of service. To model all the components of the transportation system is often unnecessary and when discussing road tolls the focus will be on car traffic models.

A road traffic model, as any other type of model, captures some aspects of the system which is being studied, but not all. In the context of traffic modeling, this means that depending on what type of problem is being studied different aspects of the traffic networks are described more or less detailed by the model. One of the key aspects of a traffic model used to model the road users' response to a congestion pricing scheme, is how the congestion is modeled, and how congestion affects route choices and traffic demand. It is important to understand that congestion appear on a road segment (link) but also affect the road users' travel choices (e.g. route choice and modal choice).

The engineering approach to congestion recognizes that when the density on a road segment is increased, the interactions among the vehicles (or drivers) will result in increased travel times, and sometimes in reduced flows. This phenomena is visualized in the fundamental diagram of traffic (Pipes, 1967), or the average cost curve which is usually preferred by economists (e.g. Verhoef, 1999). The average cost curve gives the travel time (cost) with respect to the traffic flow. Unfortunately the average cost curve is backward bending, and for a certain flow, there are two different travel times. One for stable conditions, when the traffic is functioning normally below the capacity of the road facility, and one for unstable conditions, when the traffic conditions are highly irregular, and the demand is higher than capacity, sometimes referred to as hyper congestion.

Traffic modeling approaches can be divided into static and dynamic modeling frameworks. Depending on temporal and spatial resolutions the modeling approaches can be further grouped into microscopic, mesoscopic and macroscopic models. Dynamic models recognize the time dynamic nature of traffic, with ever changing traffic conditions, and are well equipped to model both the spatial and temporal distribution of congestion. In dynamic models, congestion can be modeled in detail by microscopic approaches, in which each single vehicle is described in detail, with position, speed, acceleration, and driver behavior. When the density of a road segment increase, the interactions among the vehicles will result in lower speeds and either density or mean travel cost can be used as a measure of the level of congestion. In macroscopic approaches the traffic conditions are described by aggregated measures (e.g. flow, density and speed). Mesoscopic models can be placed between microscopic and macroscopic models, and usually model single vehicles or group of vehicles but without describing the interactions between them.

The other main modeling approach is static, in which the traffic conditions are assumed to be stable over a longer time period, and congestion is modeled by a travel time function of traffic flow, corresponding to the stable part of the average cost curve. Static models are by nature macroscopic, and describe the traffic conditions in term of flow, demand and average travel cost.

In general, dynamic models rely on simulation and static models on analytical relationships. Whichever modeling framework is used, modeling a traffic network incorporates route choice and demand models. One important difference is that within a dynamic modeling framework, departure time choices can be included (Wie and Tobin, 1998) in the demand model. A description of the static modeling framework, which will be adopted for the analysis of congestion pricing, will now follow.

2.1.1 The traffic network

The traffic network is modeled by a set of links $\mathcal{A}$ and a set of origin destination (OD) pairs $\mathcal{I}$. For each link $a \in \mathcal{A}$ there is a travel cost function $c_a(v_a)$ of flows v_a. The link travel cost functions are assumed to be continuous and smooth.

For each OD pair $i \in \mathcal{I}$ there is a set of routes Π_i, each route $p \in \Pi_i$ with flow f_p. The flow, v_a, on link a is given by

$$v_a = \sum_{i \in \mathcal{I}} \sum_{p \in \Pi_i} f_p \delta_p^a, \tag{2.1}$$

where δ_p^a takes the value of 1 if route p traverses link a, and 0 otherwise. Note that there can be an infinite number of elements in Π_i if the network contains cycles, and this imply that one link can be traversed several times by one route. This is however not a problem, since we only will consider routes for which the travel cost is equal to the minimum travel cost of any route in the same OD pair, and there is a finite set of such routes. The travel cost function $c_a(v_a)$ can include components of both travel time and monetary costs, and the travel time is weighted relative to the monetary cost by the value of time (VOT). When $c_a(v_a)$ include other components than monetary costs it is denoted as generalized travel cost (Williams, 1977). In reality the VOT is perceived differently by individual travelers, but for the travel cost functions used in this model a mean value across the population is used. By grouping the users into different groups of socioeconomic characteristics (Dafermos, 1973), or by assuming that the continuous distribution of VOT is known across the population (Dial, 1996, 1997), more advanced models, compared to the ones presented here, can be used.

2.1.2 Traffic equilibria

The route choice model we will adopt, assumes that within an OD pair, road users choose a route with minimal cost in the traffic network, and no user can reduce their travel cost by changing route. This is referred to as Wardrop's user equilibrium or Wardrop's first principle, and the behavior is said to be user optimal (Wardrop, 1952). If the road users would instead choose routes so that the total travel cost in the traffic network was minimized, this solution is said to be system optimal (compared to user optimal), and this is referred to as Wardrop's second principle. In practice a system optimal behavior can not be assumed, but in a congestion pricing context there are some interesting parallels as will be discussed later on.

In the standard formulation of the user equilibrium, the car demand is assumed to be fixed, i.e. there is a given number of trips between each origin and destination. A more realistic assumption is that the demand will depend on the travel costs and this is modeled by variable demand, or sometimes referred to as elastic demand (Sheffi, 1984). In the elastic demand model we will adopt, it is assumed that an individual only makes a car trip if this is beneficial, i.e. the individual surplus associated with the car trip is larger than the surplus related to any other alternative (i.e. transit trips, slow mode trips or no trip at all). The relationship between travel cost and demand is expressed by the inverse travel demand function, which for OD pair $i \in \mathcal{I}$ is given by $D_i^{-1}(q_i)$. The inverse travel demand function is assumed to be a continuous and smooth function of travel demand q_i in OD pair $i \in \mathcal{I}$.

The travel cost functions are assumed to be separable and increasing and the inverse demand functions to be separable and decreasing. Under these assumptions, the user equilibrium problem has a unique link flow, v, and OD demand, q, solution (Patriksson, 1994) and the necessary and sufficient Wardropian conditions for the user equilibrium with elastic demand can be formulated as

$$\begin{aligned} &\sum_{a\in\mathcal{A}} c_a(v_a)\delta_p^a = \pi_i \Leftrightarrow f_p \geq 0, \quad \forall i \in \mathcal{I}, \quad \forall p \in \Pi_i \\ &\sum_{a\in\mathcal{A}} c_a(v_a)\delta_p^a \geq \pi_i \Leftrightarrow f_p = 0, \quad \forall i \in \mathcal{I}, \quad \forall p \in \Pi_i \\ &D_i^{-1}(q_i) = \pi_i, \qquad\qquad \forall i \in \mathcal{I} \end{aligned} \tag{2.2}$$

where the travel cost in OD pair $i \in \mathcal{I}$, along route $p \in \Pi_i$ is given by $\sum_{a\in\mathcal{A}} c_a(v_a)\delta_p^a$. The user equilibrium conditions state that no user in OD pair i will travel on a route with a travel cost higher than the minimum travel cost. In OD pair i the minimum travel cost, π_i, will equal the cost of traveling, given by $D_i^{-1}(q_i)$. The routes with minimum travel cost in each OD pair are referred to as equilibrium routes.

An equilibrium solution is obtained by solving the user equilibrium problem with elastic demand (Sheffi, 1984):

$$\begin{aligned} \min_{q,v} \; G(q,v) = \quad & \sum_{a\in\mathcal{A}} \int_0^{v_a} c_a(x)dx - \sum_{i\in\mathcal{I}} \int_0^{q_i} D_i^{-1}(w)dw \\ s.t \quad & \sum_{p\in\Pi_i} f_p = q_i, \quad \forall\, i \in \mathcal{I} \\ & f_p \geq 0, \quad \forall\, i \in \mathcal{I},\; p \in \Pi_i \\ & q_i \geq 0, \quad \forall\, i \in \mathcal{I} \\ & v_a = \sum_{i\in\mathcal{I}} \sum_{p\in\Pi_i} f_p\delta_p^a, \quad \forall\, a \in \mathcal{A}. \end{aligned} \tag{2.3}$$

The solution to this problem are link flow, route flow and demand vectors, v, f and q, corresponding to a user optimal behavior by the road users. Note that the route flows f are not unique, i.e. there can be many different route flows satisfying the same link flows and OD demands solving (2.3).

Different methods have been applied to solve the fixed demand user equilibrium problem, and one of the first and still commonly used methods is the Frank-Wolfe method (Frank and Wolfe, 1956). Another method were proposed by Larsson and Patriksson (1992) and is called the Disaggregated Simplicial Decomposition (DSD) method. The DSD method has re-optimization abilities, which can be useful if several user equilibria with similar but not equal travel cost functions have to be solved. For a comprehensive review of different available methods see Patriksson (1994). To solve the user equilibrium problem with elastic demand (2.3), the excess demand formulation can be applied, reformulating (2.3) into a fixed demand problem (Sheffi, 1984).

When the travel costs are changed and demand is decreased or increased, as specified by the elastic demand function, there is no information on how the rest of the transportation system is affected. An increased travel cost for the car users, will most likely not only reduce the car demand, but also increase demand for public transportation and slow mode trips, and decrease the number of total trips. To have a demand model which can specify how the rest of the transportation system is affected is therefore of great interest. A common modeling framework in these situations are discrete choice models (Williams, 1977). The modeling framework can incorporate travel decision on different hierarchical levels, with modal choice, destination choice and trip generation. The discrete choice model presented here only conserns the modal choice, and the total travel demand in each OD pair is assumed to be fixed.

In discrete choice models, the travel cost is usually replaced by the utility of traveling. The utility of traveling in OD pair $i \in \mathcal{I}$ with travel mode n is $V_i^n = U_i^n + \epsilon_i^n$. Where U_i^n is the average utility of alternative n, equal for all users of alternative n in OD pair $i \in \mathcal{I}$, and ϵ_i^n is the random variation in utility, which is not known, and differ over the population. The average utility U_i^n can incorporate travel cost, travel time and other measures of quality of services, and the different factors are weighted to form the average utility. If the only component in the utility function is the travel cost, the average utility is the negative of the travel cost.

The multinomial logit model (MNL) (McFadden, 1970) is a discrete choice model in which ϵ_i^n is assumed to be i.i.d. Gumble distributed with location parameter 0 and scale parameter α. The probability P_i^n of a random individual choosing travel mode n in OD pair i can be expressed as (Williams, 1977)

$$P_i^n = \frac{e^{\alpha U_i^n}}{\sum_{k\in\mathcal{N}} e^{\alpha U_i^k}}, \tag{2.4}$$

where $\mathcal{N}$ is the set of alternative travel modes.

Consider the choice between car and public transportation, in OD pair $i \in \mathcal{I}$, among the travelers with access to car. Let T_i be the total demand for this group of users, and assume that the measurable utility is equal to the travel cost for each mode. The car demand, q_i, can then be expressed as

$$q_i = T_i \frac{e^{\alpha(-\pi_i)}}{e^{\alpha(-\pi_i)} + e^{\alpha(-k_i)}}, \tag{2.5}$$

where π_i and k_i are the cost of traveling by car and public transportation respectively. If it is assumed that the public transportation cost is fixed, inverting (2.5) give the inverse demand function

$$D_i^{-1} = \pi_i = k_i + \frac{1}{\alpha} \ln\left(\frac{T_i}{q_i} - 1\right), \tag{2.6}$$

which is defined for $0 < q_i < T_i$, and within this interval the function is decreasing.

The combined user equilibrium and modal split problem can now be formulated as

$$\begin{aligned}
\min_{q,v}\ G(q,v) = \quad & \sum_{a\in\mathcal{A}} \int_0^{v_a} c_a(x)dx \qquad (2.7)\\
& -\sum_{i\in\mathcal{I}} \int_0^{q_i} \left(k_i + \frac{1}{\alpha}\ln\left(\frac{T_i}{q_i} - 1\right)\right) dw \\
s.t \quad & \sum_{p\in\Pi_i} f_p = q_i, \quad \forall\ i \in \mathcal{I} \\
& f_p \geq 0, \quad \forall\ i \in \mathcal{I},\ p \in \Pi_i \\
& q_i \geq 0, \quad \forall\ i \in \mathcal{I} \\
& v_a = \sum_{i\in\mathcal{I}} \sum_{p\in\Pi_i} f_p \delta_p^a, \quad \forall\ a \in \mathcal{A}.
\end{aligned}$$

with public transportation demand $H_i = T_i - q_i$, for each OD pair $i \in \mathcal{I}$. The MNL model do not need to be restricted to the choice between car and public transportation, as presented here, The model can be extended with travel cost functions for the public transportation system, and with additional travel modes and hierarchical decisions of when and where to travel (Ortúzar and Willumsen, 1990).

An important difference between the general elastic demand function and the MNL model, is that in the MNL model the demand in each OD pair is fixed and given by the parameter T_i. The car demand and public transportation demand is then given by the MNL model.

Since the inverse demand function (2.6) is not defined for zero flow, the excess demand reformulation cannot be applied (Sheffi, 1984). Instead the partial linearization method in Evans (1976) will be adopted, to solve the combined user equilibrium and modal split problem. In Evans algorithm the problem is solved by repeatedly solving fixed demand equilibrium problems, and updating the demand in between.

2.2 Congestion pricing

In this section quantitative measures for evaluating congestion pricing are presented.

Road users make their travel decisions based on the perceived travel cost (private cost), in contrast to the full cost, which include the congestion (delay) a road user impose on the fellow users. The difference between the full and private cost is often referred to as the congestion externality. By internalizing the congestion externality, i.e. to let the road users experience the full cost of their travel decision, the efficiency of the traffic system can be increased (Newbery, 1990). This is the general idea of congestion pricing and can be traced back to the work of Pigou (1920) and Knight (1924), and will be further discussed in the context of optimal congestion pricing.

Congestion pricing schemes can be divided into cordon schemes and area based schemes. In a cordon based congestion pricing scheme, the road users pay a toll when passing certain points in the traffic network. With the technology available today, a reasonable assumption is that tolls can be collected without any impact on the travel time, i.e. no users need to reduce their speed in order to pay the toll. From a traffic modeling perspective, there is no need of knowing the exact location of the toll facility, only on what links, in the traffic network, tolls will be collected. The toll locations in a cordon based congestion pricing system do not necessarily form a closed cordon, i.e. a set of links whose removal would make the network unconnected. Common cordon structures are single and multiple closed cordons, and screen lines and spurs (May et al., 2002).

In an area based congestion pricing scheme, the road users pay a fee to be allowed to drive within an area. This type of scheme requires that all moving vehicles within the area are monitored. Area pricing schemes are further discussed and compared to closed cordon pricing in Maruyama and Sumalee (2007).

2.2.1 Evaluating congestion pricing schemes

To estimate the effects of a congestion pricing scheme, a common microeconomic approach is employed. Welfare is measured by the social surplus, sometimes, in the literature, referred to as social welfare, net benefits or total cost. The social surplus is formulated as the difference between total benefits and total costs (Sumalee, 2004; Verhoef, 2002b; Yang and Zhang, 2003; Yin and Lawphongpanich, 2008) or as the sum of the consumer surplus and operator benefits (toll revenues) (Bellei et al., 2002; de Palma and Lindsey, 2006).

A congestion pricing scheme which give a positive change in the social surplus will leave the users better of as a group, but individual users may be worse of. This assumes that the collected tolls are redistributed to the road users, e.g. by investment in road infrastructure (Small, 1992), and ideally the collected tolls can be used to compensate the road users who are worse off, after the pricing scheme is introduced. This is further discussed by Eliasson (1998) and Eliasson and Mattsson (2006).

The static traffic modeling framework presented earlier will be adopted to compute traffic flows, demands and travel costs, which are important input to the evaluation of a congestion pricing scheme. The travel cost function, $\hat{c}_a(v_a)$ including the link toll τ_a for a link a in $\mathcal{A}$ is

$$\hat{c}_a(v_a) = c_a(v_a) + \tau_a,$$

and $\hat{\pi}_i$ is the minimum OD travel cost, including any tolls, in OD pair $i \in \mathcal{I}$.

The user benefit, UB, is determined, according to the Marshallian measure (Verhoef, 2002b), by the integral

$$UB = \sum_{i \in \mathcal{I}} \int_0^{q_i} D_i^{-1}(w)dw.$$

The user costs, UC, is the total travel cost in the network, and is given both in term of link costs and OD costs:

$$UC = \sum_{a \in \mathcal{A}} \hat{c}_a(v_a)v_a = \sum_{i \in \mathcal{I}} \hat{\pi}_i q_i.$$

The net user benefit, denoted consumer surplus, is the user benefits minus user costs

$$CS = UB - UC.$$

If the demand is determined according to the MNL mode choice model (2.4), the change in consumer surplus, for OD pair i, ΔCS_i, can be computed as (Williams, 1977)

$$\Delta CS_i = \frac{1}{\alpha} T_i \ln \frac{\sum_{k \in \mathcal{N}} e^{-\alpha \hat{t}_i^k}}{\sum_{k \in \mathcal{N}} e^{-\alpha \hat{z}_i^k}}. \tag{2.8}$$

where $\hat{t}_i^k$ and $\hat{z}_i^k$ is the travel cost in OD pair i using mode k in the toll and no-toll scenario respectively. Williams (1977) denotes this measure as the change in consumer benefit, while Small (2006) refer to it as the change in consumer surplus. Note that the common formulation of this measure (Small, 2006; Small and Rosen, 1981) require (2.8) to be divided by the marginal utility of income to transform the utility measure into a monetary value. This is however not necessary when the travel costs are measured in a monetary unit.

The travel cost, excluding the tolls, is sometimes referred to as the social cost (Yang and Zhang, 2003), or the total cost (Verhoef, 2002b). The social cost, SC, is given both in term of link costs and OD costs:

$$SC = \sum_{a \in \mathcal{A}} c_a(v_a)v_a = \sum_{i \in \mathcal{I}} \pi_i q_i.$$

The total toll revenue, R is computed as

$$R = \sum_{a \in \mathcal{A}} \tau_a v_a,$$

and is denoted the operator surplus in Sumalee (2005). Note that UC can be expressed as

$$UC = SC + R. \tag{2.9}$$

The social surplus, SS is expressed as (Sumalee, 2004; Verhoef, 2002b; Yang and Zhang, 2003; Yin and Lawphongpanich, 2008)

$$SS = UB - SC$$

or the equivalent formulation (Bellei et al., 2002; de Palma and Lindsey, 2006)

$$SS = CS + R.$$

Since (2.8) gives the change in consumer surplus, we express the change in social surplus rather than the social surplus itself. The change in social surplus is

$$\Delta SS = \Delta UB - \Delta SC \tag{2.10}$$

or the equivalent formulation

$$\Delta SS = \Delta CS + \Delta R. \tag{2.11}$$

For the inverse demand formulation of the user equilibrium problem, the change in social surplus is

$$\begin{aligned} \Delta SS = \quad & \left(\sum_{i \in \mathcal{I}} \int_0^{q_i} D_i^{-1}(w) dw - \sum_{i \in \mathcal{I}} \int_0^{q_i^0} D_i^{-1}(w) dw \right) \\ & - \left(\sum_{i \in \mathcal{I}} \pi_i q_i - \sum_{i \in \mathcal{I}} \pi_i^0 q_i^0 \right), \end{aligned} \tag{2.12}$$

where the zero index imply the no-toll scenario.

The equivalent formulation for the MNL mode choice model is

$$\Delta SS = \frac{1}{\alpha} \sum_{i \in I} T_i \ln \frac{\sum_{k \in \mathcal{N}} e^{-\alpha \hat{t}_i^k}}{\sum_{k \in \mathcal{N}} e^{-\alpha \hat{z}_i^k}} + \sum_{a \in \mathcal{A}} \tau_a v_a. \tag{2.13}$$

In de Palma and Lindsey (2006) the change in social surplus is expressed as

$$\Delta SS = \Delta CS + (1 + \text{MCPF})R - \Delta C_{EXT} \tag{2.14}$$

where MCPF is the marginal cost of public funds, and C_{EXT} is externalities other than congestion, such as emissions of pollutants, traffic noise and accidents. Expression (2.13) is a special case of (2.14) where MCPF and C_{EXT} is assumed to be zero.

To set up and operate a congestion pricing scheme is costly, and it is therefore of great interest to not only measure the social surplus, but the net benefits. The net benefit is sometimes denoted as net social surplus (Santos et al., 2001) or gross total benefit (Sumalee, 2004) and is the difference between the social surplus and the cost of collecting the tolls. The cost of collecting the tolls will be denoted as operator cost, OC, and the change in net social surplus,NSS, can be formulated as

$$\Delta NSS = \Delta SS - OC. \tag{2.15}$$

2.3 Optimal congestion pricing schemes

Congestion pricing problems can be viewed as a special case of a network design problem and formulated as a bi-level programming problem (Migdalas, 1995). In the classic network design problem, the question is how to add road capacity (e.g. Leblanc, 1975). In congestion pricing problems, the question is instead on what links in the traffic network to locate toll facilities and how much to charge at each such facility. For the network design problem in general, and the congestion pricing problem in particular, the bi-level formulation gives a convenient interpretation. At the upper-level, the regulator (road authority) decide on how the congestion pricing scheme is designed, trying to maximize either the social surplus or the net social surplus. At the lower level the road users are responding to the pricing scheme in order to minimize their own, individual, travel cost. The regulator can then take appropriate counter actions, and this can continue until the regulator can find no better design of the pricing scheme. The bi-level formulation has among others, been adopted by Yang (1996); Yang and Bell (1997); Zhang and Yang (2004) and is further discussed by Clegg et al. (2001)

in the context of a more general network design problem. The bi-level program can be formulated as a mathematical program with equilibrium constraints (MPEC), by applying the complementarity constraints (2.2) (Lawphongpanich and Hearn, 2004; Sumalee, 2004; Verhoef, 2002b).

2.3.1 First-best pricing

The social surplus can be maximized by letting the road users pay for their external effects (Beckmann et al., 1956). This pricing principle is usually referred to as marginal social cost pricing (MSCP). If we consider a road segment $a \in \mathcal{A}$, with link flow v_a and travel cost function $c_a(v_a)$, the optimal toll is

$$\tau_a = \frac{\partial c_a(v_a)}{\partial v_a} v_a. \tag{2.16}$$

The optimal toll is equal to the marginal change of travel cost, if the flow would increase, multiplied by the current flow, and this is the increase in travel cost that the users currently traveling on the link would experience if the flow was increased.

It can easily be shown that MSCP will result in a system optimal demand and flow pattern (Sheffi, 1984). The MSCP solution requires tolls on every link with a positive flow but is not the only toll pattern which result in system optimal flow. If the demand is elastic, all toll patterns which produce system optimal flow, will toll the road users the same amount (Yin and Lawphongpanich, 2008). When the demand is fixed there can however be pricing schemes which result in system optimal flow with different total toll revenues (Hearn and Ramana, 1998).

Yildirim and Hearn (2005) utilizes alternative objective functions to find alternative toll patterns, which give a system optimal flow, and also minimize the number of toll facilities or minimize the maximum toll in the network. If the operator cost is considered, finding a first-best pricing scheme which minimizes the number of toll facilities will give the first-best pricing scheme with highest net social surplus. The change in social surplus compared to the no-toll scenario may, however, be negative. If the change in net social surplus is positive, this is a lower bound on the improvement in net social surplus, which can be achieved by a congestion pricing scheme.

Marginal cost pricing is not limited to single mode networks (i.e. auto or transit networks). The same principles are valid in multimodal networks (Hamdouch et al., 2007).

2.3.2 Second-best pricing

In a second-best pricing scheme, the collection of tolls is restricted. The restriction can be to

- only allow a limited number of predetermined toll levels
- require all toll levels to be equal
- require the toll levels to be within a fixed interval
- only consider a subset of links as tollable
- require that the toll locations form a closed cordon

In practice, several of these restrictions will be combined. An analogue formulation to restriction of tollable links is that there is a limit on the toll levels, set to zero, for the links which are not tollable.

The improvement of the social surplus by a first-best solution is an upper bound on the improvement that can be achieved by any second-best pricing scheme. A second-best pricing scheme which yields the system optimal flow and demand pattern is by definition a first-best pricing scheme.

There are several reasons to why second-best pricing schemes are necessary. In practice there will most certainly be restrictions on what links that can be tolled, either out of practical or political considerations, and it might not be possible to find a first-best pricing scheme which complies to these restrictions. Also, if there is a cost associated with having a toll facility, a first-best pricing scheme does not need to maximize the net social surplus, only the social surplus.

The level setting problem

In the level setting problem the set of tollable links is given, and the question is what toll level to choose for each toll facility. The objective to maximize is the change in social surplus, (2.12) or (2.13). For the inverse demand user equilibrium formulation Verhoef (2002b), employ the MPEC formulation to formulate the level setting problem as:

$$\begin{aligned}
\max_{\tau} \quad & \sum_{i\in\mathcal{I}} \int_0^{q_i} D_i^{-1}(w)dw - \sum_{a\in\mathcal{A}} c_a(v_a)v_a \\
s.t. \quad & f_p \left[\sum_{a\in\mathcal{A}} \left(c_a(v_a) + \tau_a\lambda_a\right)\delta_p^a - D_i^{-1}(q_i) \right] = 0, \quad \forall\ p \in \Pi_i,\ i \in \mathcal{I} \\
& \sum_{p\in\Pi_i} f_p = q_i, \quad \forall i \in \mathcal{I} \\
& q_i \geq 0, \quad \forall i \in \mathcal{I} \\
& f_p \geq 0, \quad \forall i \in \mathcal{I}, \quad \forall p \in \Pi_i \\
& v_a = \sum_{i\in\mathcal{I}} \sum_{p\in\Pi_i} f_p\delta_p^a, \quad \forall\ a \in \mathcal{A} \\
& \tau_a \geq 0, \quad \forall a \in \mathcal{A},
\end{aligned} \tag{2.17}$$

where λ_a is equal to 1 if link a is tollable and 0 otherwise. Since the social surplus in the no-toll scenario is constant (compare to 2.12) it will not affect the optimization, and is therefore not part of the objective. This optimization problem may be non-convex and therefore it can be difficult to find a global optimal solution.

Assuming that the set of equilibrium routes will not change when tolls are introduced, the complementarity constraint

$$f_p \left[\sum_{a\in\mathcal{A}} \left(c_a(v_a) + \tau_a\lambda_a\right)\delta_p^a - D_i^{-1}(q_i) \right] = 0, \quad \forall p \in \Pi_i, \quad \forall i \in \mathcal{I}$$

can be written as

$$\sum_{a\in\mathcal{A}} \left(c_a(v_a) + \tau_a\lambda_a\right)\delta_p^a - D_i^{-1}(q_i) = 0, \quad \forall p \in \Pi_i^*, \quad \forall i \in \mathcal{I} \tag{2.18}$$

where Π_i^* is the set of equilibrium routes with travel cost equal to the minimum travel cost. By rewriting problem (2.17) for only the equilibrium routes and applying Lagrangian relaxation on this set of constraints, Verhoef (2002b) derive the optimal tolls analytically. For a small network the optimal tolls can be expressed analytically similar to the MSCP tolls.

Consider the two link network in Figure 2.1, with link travel cost functions $c_1(v_1)$ and $c_2(v_2)$. The demand in the only OD pair is $q_{12} = v_1 + v_2$, and the inverse demand function is $D_{12}^{-1}(q)$.

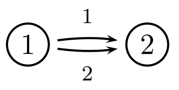

Figure 2.1: A two link network

If only link 1 is tollable, an optimal second-best toll level, τ_1, can be expressed as

$$\tau_1 = \frac{dc_1(v_1)}{dv_1} v_1 + \frac{\frac{dD^{-1}(q)}{dq}}{\frac{dc_2(v_2)}{dv_2} - \frac{dD^{-1}(q)}{dq}} \frac{dc_2(v_2)}{dv_2} v_2. \tag{2.19}$$

The analytical expression for the optimal second-best toll can be regarded as an extension of the MSCP toll, $\tau_1 = \frac{dc_1(v_1)}{dv_1} v_1$, when only on of the two links are tollable. If the inverse demand function is decreasing and the travel cost functions increasing, (2.19) shows that the optimal toll on link 1 is lower than the MSCP toll would have been, and also depends on the marginal cost on link 2.

To derive the optimal second-best toll for a small network gives some insight, and second-best pricing in the two link network is further discussed by Marchand (1968), Verhoef et al. (1996) and Liu and McDonald (1999). To formulate a closed form expression for a large network, as for the first-best pricing scheme, is not practical. Verhoef (2002b) and Lawphongpanich and Hearn (2004) further explore the analytical expression that can be derived from Lagrangian relaxation (Verhoef, 2002b) and the KKT-conditions (Lawphongpanich and Hearn, 2004), and suggests different methods to solve the level setting problem.

The method suggested by Verhoef (2002b) relies on the linear system of equations that can be derived from the Lagrangian relaxation of the complementarity constraints in (2.17), for the case when the set of routes are restricted to the equilibrium routes. The method can be briefly outlined as

1. Start with a feasible toll vector $\bar{\tau}$.

2. Solve the user equilibrium problem with elastic demand (2.3). The solution yields the link flows, v^*, and demands, q^*, and the set of equilibrium routes Π_i^*, $\forall i \in \mathcal{I}$.

3. Given the equilibrium solution (v^*, q^*), solve the linear system of equations, given by the Lagrangian relaxation of the complementarity constraints in (2.17), for the restricted set of routes Π_i^*, $\forall i \in \mathcal{I}$. This yields the Lagrangian multipliers and an updated toll vector $\bar{\tau}$.

4. If the stopping criteria is not fulfilled, continue with Step 2.

Verhoef (2002b) stop the algorithm when the toll levels do not change between successive iterations, but give no proof of convergence. Verhoef also argues that for a large network the number of iterations must be limited, since a user equilibrium problem has to be solved in each iteration, which can be computationally burdensome. This method is further explored by Shepherd et al. (2001) under the name CORDON which recognize some problems related to the accuracy of the method for solving the user equilibrium problem. Yildirim (2001) points out that besides the method being rather complex there is no guarantee of the existence of the multipliers computed in Step 3. However, for the combined toll location and level setting problem the Lagrangian multipliers, which are computed in Step 3, can be used for finding suitable links to toll (Verhoef, 2002a).

The combined toll location and level setting problem

In the level setting problem the toll locations were considered to be fixed. To let the locations be variable, results in a problem in which not only the toll levels need to be decided, but also the toll locations. In the level setting problem the operator cost was disregarded and the objective to maximize was the social surplus. When the toll locations are not predetermined, the operator cost will affect how many tolls to locate. If there is no cost of locating toll facilities and all links are considered as tollable, the obvious solution is to collect MSCP tolls at every link in the traffic network. This is however not the case in reality, and the total operator cost will most certainly be higher for each toll that is added.

Two different approaches can be distinguished on how to model the problem of both locating the toll facilities and setting the toll levels. The first approach can be viewed as a direct extension of the level setting problem, where the same objective of maximizing the social surplus is used and the number of toll facilities to locate is predetermined, but not their locations. This approach does not guarantee a net benefit, since the objective to maximize is still the social surplus, not the net benefit. Verhoef (2002a) address this problem and suggests a methodology, which is further discussed by Shepherd et al. (2001) and Shepherd and Sumalee (2004). The suggested method is based on an approximation of the welfare gain for each tollable link, and is further presented in Chapter 5. Verhoef (2002a) shows that it is difficult to get an accurate prediction of the welfare gain for links which are used by several routes in the same OD pair. Shepherd et al. (2001) however demonstrates the method on a mid-size network, and Shepherd and Sumalee (2004) employs the method in a genetic algorithm (GA) framework.

Sumalee (2004) introduces the cost of collecting a toll on link a as C_a. The total operator costs can then be calculated as

$$OC = \sum_{a \in \mathcal{A}} \lambda_a C_a \tag{2.20}$$

where λ_a is 1 if link a is tolled, and 0 otherwise.

When evaluating a pricing scheme by the net social surplus (2.15) and letting the operator cost be computed as (2.20), Verhoef et al. (1996) acknowledge that the two link network in Figure 2.1 may not have an optimal solution in which only one route is tolled for any positive collection costs, $C_1 > 0$ and $C_2 > 0$. There is no reason to assume that this cannot be the case for larger networks. To restrict the number of toll facilities by a budget constraint, rather than using the net social surplus as objective, may yield solutions which give lower net benefit, or even a negative net benefit. From this argument follows the second approach to model the combined toll location and level setting problem, by maximizing the net social surplus. This formulation is adopted by Sumalee (2004).

When it comes to cordon structures, closed cordons are appealing from a practical perspective, and several of the implemented pricing schemes (e.g. Stockholm and Singapore) apply closed cordon structures. One of the main obstacles when it comes to optimizing the efficiency of closed cordon pricing schemes is how to find the closed cordons. Sumalee (2004) and Zhang and Yang (2004) suggest different techniques based on graph theory to address this problem, and the cordon design is then manipulated by a GA.

3 Modeling congestion pricing

In this chapter two different congestion pricing problems are formulated in a bi-level framework. The level setting problem, in which the toll levels are variable, but not the toll locations, and the combined toll location and level setting problem in which both toll locations and toll levels are variable. The users of the transportation network are modeled in a static framework, with user optimal route choices. Travel demand is modeled by the inverse demand formulation in (2.3), or by the combined user equilibrium and modal choice formulation in (2.7). The user equilibrium problem with elastic demand given by the inverse demand function can incorporate modal choice by using the inverse demand formulation of the MNL modal choice model (2.6). The framework can be extended to incorporate additional modes, hierarchical travel decisions and modeling of congestion in the public transportation network.

Applying the ideas of Pigovian taxation, also known as marginal social cost pricing (MSCP), to the road traffic system, optimal toll levels which maximize the social welfare can easily be computed by (2.16). MSCP require all links with positive flow and link travel cost larger than the free flow travel cost, to be tollable, Yildirim and Hearn (2005), however, shows that the same level of social surplus can be reached with tolls on fewer links. Also, any pricing scheme which achieves the same improvement in social surplus as MSCP tolls, is denoted as a first-best pricing solution or, synonymously a system optimal solution. All first-best pricing schemes will result in a system optimal flow and corresponding demand pattern. In practice there is likely to be practical and political restrictions for which links that are tollable and it might therefore not be possible to find any feasible first-best pricing scheme. A congestion pricing schemes which maximizes the social surplus under such restrictions, and does not give a system optimal flow and demand pattern, is denoted as a second-best pricing scheme.

3.1 A bi-level formulation

Bi-level models in a general network design context are discussed by Migdalas (1995). The general congestion pricing problem is illustrated in Figure 3.1. At the upper-level the road authority tries to maximize the benefit from the congestion pricing scheme. In our case the benefit is either the social surplus or the net social surplus. The toll levels, τ, have to be feasible, i.e. positive toll levels are only allowed for tollable links, and are chosen by the road authority. On the lower-level the users make their travel decisions to maximize their own utilities, given τ, which result in demands, q, traffic flows, v, and travel costs π and c for the OD pairs and links respectively. The road authority has to anticipate this change in behavior, and adjust the toll levels accordingly.

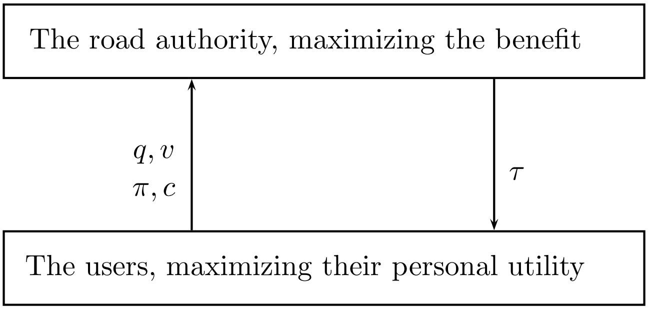

Figure 3.1: The general congestion pricing bi-level model

Let $\mathcal{T}$ be the set of tollable links. A set of feasible toll variables can then be formulated as

$$\mathcal{X} = \{\tau | \tau_a \geq 0 \ \forall a \in \mathcal{T},\ \tau_a = 0 \ \forall a \in \mathcal{A} \setminus \mathcal{T}\} \tag{3.1}$$

where $\mathcal{A} \setminus \mathcal{T}$ is the set of links which are not tollable.

The general congestion pricing problem can be stated as the bi-level optimization problem

$$\begin{aligned} \max_{\tau} \quad & F(q(\tau), v(\tau), \tau) & (3.2) \\ s.t. \quad & \tau \in \mathcal{X} & \\ & \{q(\tau), v(\tau)\} = \{\arg \min_{(q,v) \in \mathcal{R}} G(q, v, \tau)\} & (3.3) \end{aligned}$$

where $G(q, v, \tau)$ corresponds to the objective in either (2.3) or (2.7), and $\mathcal{R}$ is the set of patterns (q, v) describing the travel demand and link flow patterns which are feasible in the user equilibrium problem. The upper-level objective $F(q(\tau), v(\tau), \tau)$ is either to maximize the social surplus, corresponding to the level setting problem, or the net social surplus, for the combined toll location and level setting problem. The Bi-level problem in general is non-convex and therefore hard to solve for a global optimum. We observe that the bi-level formulation can be expressed as a mathematical problem with equilibrium constraints (MPEC) by replacing (3.3) with the corresponding complementarity constraints (2.2).

3.2 The level setting problem

In the level setting problem the cost of collecting the tolls is usually not part of the objective, and the objective to maximize solely expresses the social surplus. Since the toll locations are predetermined, the cost for collecting the tolls is constant, and may be subtracted from the objective to get the net social surplus.

We use the definition of the social surplus from Chapter 2 and formulate the upper-level objective as

$$\begin{aligned} \max_{\tau \in \mathcal{X}} F(q(\tau), v(\tau), \tau) = & \sum_{i \in \mathcal{I}} \int_0^{q_i(\tau)} D_i^{-1}(w) dw \\ & - \sum_{a \in \mathcal{A}} c_a(v_a(\tau)) v_a(\tau), \end{aligned} \tag{3.4}$$

when the elastic demand is given by the inverse demand formulation.

Adding the toll levels to the link cost functions in (2.3), the lower-level problem for computing $v(\tau)$ and $q(\tau)$ becomes:

$$\begin{aligned}
\min_{q,v} \; G(q,v,\tau) = \; & \sum_{a\in\mathcal{A}} \int_0^{v_a} (c_a(w)+\tau_a)dw \\
& -\sum_{i\in\mathcal{I}} \int_0^{q_i} D_i^{-1}(w)dw \\
s.t \quad & \sum_{p\in\Pi_i} f_p = q_i, \quad \forall\, i \in \mathcal{I} \\
& f_p \geq 0, \quad \forall\, p \in \Pi_i,\; i \in \mathcal{I} \\
& q_i \geq 0, \quad \forall\, i \in \mathcal{I} \\
& v_a = \sum_{i\in\mathcal{I}} \sum_{p\in\Pi_i} f_p \delta_p^a, \quad \forall\, a \in \mathcal{A}.
\end{aligned} \tag{3.5}$$

An upper-level objective function for the case of the MNL modal choice model between car and public transportation, can be formulated as

$$\begin{aligned}
\max_{\tau\in\mathcal{X}} F(v(\tau),\hat{\pi}(\tau),\tau) = \; & \frac{1}{\alpha}\sum_{i\in I} T_i \ln \frac{e^{-\alpha\hat{\pi}_i(\tau)} + e^{-\alpha k_i}}{e^{-\alpha\pi_i^0} + e^{-\alpha k_i^0}} \\
& + \sum_{a\in\mathcal{A}} v_a(\tau)\tau_a.
\end{aligned} \tag{3.6}$$

The lower-level problem, corresponding to this upper-level objective, is the combined user equilibrium and modal choice problem (2.7) which give $v(\tau)$, and from which the minimum OD travel costs including the tolls, $\hat{\pi}(\tau)$, can easily be extracted.

Note that for (3.4) and (3.6) to be comparable, the social surplus in the no-toll scenario has to be deducted from the objective in (3.4).

The first-best pricing problem, when all links are tollable, is a special case of the second-best level setting problem with $\mathcal{T} = \mathcal{A}$. The first-best problem can be formulated at a single level, either by the lower-level objective, with τ_a replaced by $\frac{\partial c_a(v_a)}{\partial v_a} v_a$, or by the upper-level objective, with the lower-level constraints.

The system optimal solution, ΔSS^{SO}, is an upper bound on the possible improvement in social surplus for any combination of toll locations in the level-setting problem.

3.3 The combined location and level setting problem

In this chapter we have so far disregarded the cost of collecting the tolls. In practice, there is an economy of scale in a congestion pricing system. The cost per toll facility is most likely to be lower in a system with several facilities, than in a system with just a few. Economy of scale will however be disregarded in favor for a simpler model. The model we will apply is similar to the one presented by Sumalee (2004). The cost for locating a toll facility is assumed to be link specific in order to capture any special link characteristics and independent of toll level and link flow.

In the combined toll location and level setting problem we wish to maximize the net social surplus (2.15) by finding optimal toll locations and corresponding toll levels. Each tolled link a, will add C_a to the operator cost. The combined toll location and level setting problem is

$$\begin{aligned} \max_{\tau \in \mathcal{X}} F(q(\tau), v(\tau), \tau) = & \sum_{i \in \mathcal{I}} \int_0^{q_i} D_i^{-1}(w) dw \\ & - \sum_{a \in \mathcal{A}} c_a(v_a(\tau)) v_a(\tau) - \sum_{a \in \mathcal{A}} C_a \text{sign}(\tau_a), \end{aligned} \tag{3.7}$$

for the inverse demand formulation of the elastic demand. The link flows, $v(\tau)$, and demands, $q(\tau)$, are given by the solution to (3.5). We define

$$\text{sign: } \Re \rightarrow \{-1, 0, 1\}, \text{ where } \text{sign}(x) = \begin{cases} 1, \text{ if } & x > 0 \\ 0, \text{ if } & x = 0 \\ -1, \text{ if } & x < 0. \end{cases}$$

When the MNL mode choice model (2.7) is used to model the elastic car demand, the upper-level objective can be formulated as

$$\begin{aligned} \max_{\tau \in \mathcal{X}} F(v(\tau), \hat{\pi}(\tau), \tau) = & \sum_{i \in I} T_i \frac{1}{\alpha} \ln \frac{e^{-\alpha \hat{\pi}_i} + e^{-\alpha k_i}}{e^{-\alpha \pi_i^0} + e^{-\alpha k_i^0}} \\ & + \sum_{a \in \mathcal{A}} v_a(\tau_a) \tau_a - \sum_{a \in \mathcal{A}} C_a \text{sign}(\tau_a), \end{aligned} \tag{3.8}$$

with $v(\tau)$ and $\hat{\pi}(\tau)$ given by the solution to the lower-level combined user equilibrium and modal choice problem. The last sum in (3.8) and

(3.8) is the operator cost, which will be positive for every link where a toll is collected. Note that there can still be constraints on the set of tollable links $\mathcal{X}$, e.g. due to practical or political considerations.

The first-best problem corresponding the combined toll location and level setting problem is the minimum toll booth formulation in Yildirim and Hearn (2005). In their formulation the cost of collecting the tolls is assumed to be equal on all links and a first-best solution with a minimum number of tolled links is sought for. The first-best minimum toll booth solution may however yield a negative change in the net social surplus.

An upper bound on the net social surplus, $\overline{\Delta NSS}$, is

$$\max \left\{ \left(\Delta SS^{SO} - \min_{a \in \mathcal{A}} C_a \right), 0 \right\},$$

and any feasible solution to the combined toll location and level setting problem is a lower bound, $\underline{\Delta NSS}$.

4 Solving the level setting problem

In this chapter a sensitivity analysis based approach for solving the level setting problem is presented.

Yang (1997) presents a sensitivity analysis approach for solving network design and congestion pricing problems which is based on the work by Tobin and Friesz (1988). The sensitivity analysis in Tobin and Friesz (1988) is however restricted by network structure, and may not return a value which can be interpreted as a gradient (Patriksson and Rockafellar, 2003). We will adopt a sensitivity analysis approach based on the work by Patriksson and Rockafellar (2003), which is further discussed in Josefsson (2003), to estimate directional derivatives and use them in a bi-level optimization heuristic.

The sensitivity analysis based approach presented in this chapter is heuristic in the sense that there will be cases when the directional derivatives are not ascent directions. Even if we always find ascent directions the sensitivity analysis based approach can only find a local optimum which may not correspond to the best solution.

Consider the bi-level formulation of the level setting problem (3.4) or (3.6). If it would be possible to compute the gradient ∇F, an ascent method could be used in the search of a local optimum. Applying the rule of chain to the upper-level objective it would be sufficient to have the Jacobians $\frac{\partial v}{\partial \tau}$, and $\frac{\partial q}{\partial \tau}$ or $\frac{\partial \hat{\pi}}{\partial \tau}$, depending on the demand model, for each tollable link $a \in \mathcal{T}$, to compute ∇F. This is not the case but we will compute directional derivatives and use the as if they were these Jacobians.

4.1 Sensitivity analysis of the elastic demand user equilibrium problem

Consider the user equilibrium problem with elastic demand given by the inverse demand formulation (3.5). Given a toll vector $\bar{\tau}$ this problem is

$$\begin{aligned}
\min_{q,v} G(q,v,\bar{\tau}) = \quad & \sum_{a\in\mathcal{A}} \int_0^{v_a} (c_a(x) + \bar{\tau}_a)dx - \sum_{i\in\mathcal{I}} \int_0^{q_i} D_i^{-1}(w)dw \quad (4.1) \\
s.t. \quad & \sum_{p\in\Pi_i} f_p = q_i, \quad \forall\, i \in \mathcal{I} \\
& f_p \geq 0, \quad i \in \mathcal{I}, \forall\, p \in \Pi_i \\
& q_i \geq 0, \quad \forall\, i \in \mathcal{I} \\
& v_a = \sum_{i\in\mathcal{I}} \sum_{p\in\Pi_i} f_p \delta_p^a, \quad \forall\, a \in \mathcal{A},
\end{aligned}$$

with optimal solution (v^*, q^*). By performing sensitivity analysis on the current link flows and demands (v^*, q^*) in a direction, τ', a directional derivative of the link and demand flows with respect to changes in toll levels can be approximated.

Patriksson and Rockafellar (2003) formulate the sensitivity analysis problem for the traffic assignment problem with elastic demand given by the inverse demand function, as a variational inequality. Josefsson (2003) further explores this problem, and formulates it as a mathematical program for the fixed demand case. We will extend the formulation in Josefsson (2003) to elastic demand, and formulate the sensitivity analysis problem for a change in the toll vector $\bar{\tau}$, in the direction τ'. Note that the directional derivative denoted $\nabla_{\tau'} v(\tau)$ is a vector of link flow perturbations, v', in the direction τ', and $\nabla_{\tau'} q(\tau)$ is the corresponding demand perturbation, q'. We are interested in the direction where $\tau'_b = 1$ if $b = a$, $\tau'_b = 0$ if $b \neq a$. Patriksson and Rockafellar (2003) shows that by linearization of the link cost and inverse demand functions, around $(v^*, q^*, \bar{\tau})$ in the direction τ', v' and q' can be regarded as directional derivatives in the direction τ'.

Following Josefsson (2003) we formulate the sensitivity analysis problem very similar to the user equilibrium problem, but for a restricted set of routes. Let Π_i^1 be the set of routes given by the solution to problem (4.1) with positive route flows, Π_i^2 the set of routes with zero flows, but still routes with minimum cost, and Π_i^3 the set of non-equilibrium routes. There is no restriction of the route flow perturbations for the routes in Π_i^1. For the routes in Π_i^2, the route flow perturbation must be non-negative, and finally the routes in Π_i^3 is restricted to zero.

Similar to the link cost functions in the user equilibrium problem we will follow Patriksson and Rockafellar (2003) to formulate the link flow perturbation cost function as

$$c_a'(v_a', \tau_a') = \bar{\tau}_a \tau_a' + \frac{\partial \hat{c}_a(v_a^*, \bar{\tau}_a)}{\partial v_a} v_a'$$

and the inverse demand perturbation function

$$D_i^{-1'}(q_i^*) = \frac{\partial D_i^{-1}(q_i^*)}{\partial q_i} q_i'.$$

The relationship between link and route flow perturbations can be expressed as

$$v_a' = \sum_{i \in \mathcal{I}} \sum_{p \in \Pi_i} f_p' \delta_p^a.$$

Now, we can formulate the sensitivity analysis problem as

$$\begin{aligned}
\min_{q', v'} Q(q', v', \tau') = \quad & \sum_{a \in \mathcal{A}} \int_0^{v_a'} \left(\bar{\tau}_a \tau_a' + \frac{\partial \hat{c}_a(v_a^*, \bar{\tau}_a)}{\partial v_a} x \right) dx \qquad (4.2) \\
& - \sum_{i \in \mathcal{I}} \int_0^{q_i'} \frac{\partial D_i^{-1}(q_i^*)}{\partial q_i} w dw \\
s.t. \quad & \sum_{p \in \Pi_i} f_p' = q_i', \quad \forall i \in \mathcal{I} \\
& v_a' = \sum_{i \in I} \sum_{p \in \Pi_i} \delta_p^a f_p', \quad \forall a \in \mathcal{A} \\
& f_p' \text{ free}, \quad \forall p \in \Pi_i^1, \quad i \in \mathcal{I} \\
& f_p' \geq 0, \quad \forall p \in \Pi_i^2, \quad i \in \mathcal{I} \\
& f_p' = 0, \quad \forall p \in \Pi_i^3, \quad i \in \mathcal{I}.
\end{aligned}$$

A slightly modified version of the Disaggregated Simplicial Decomposition method (DSD), presented in Josefsson (2003), can be used to solve this problem. If the DSD method is also used for solving the user equilibrium problem, the set of routes, Π_i^1, Π_i^2 and Π_i^3 can easily be extracted.

4.2 Sensitivity analysis of the combined user equilibrium and modal choice problem

Consider the combined user equilibrium and modal choice problem

$$\begin{aligned}
\min_{q,v} \; G(q,v,\bar{\tau}) = \; & \sum_{a\in\mathcal{A}} \int_0^{v_a} (c_a(x)+\bar{\tau})dx \\
& -\sum_{i\in\mathcal{I}} \int_0^{q_i} \left(k_i + \frac{1}{\alpha}\ln\left(\frac{T_i}{w_i}-1\right)\right) dw \\
s.t \quad & \sum_{p\in\Pi_i} f_p = q_i, \quad \forall\, i\in\mathcal{I} \\
& f_p \geq 0, \quad \forall\, p\in\Pi_i,\; i\in\mathcal{I} \\
& q_i \geq 0, \quad \forall\, i\in\mathcal{I} \\
& v_a = \sum_{i\in\mathcal{I}}\sum_{p\in\Pi_i} f_p\delta_p^a, \quad \forall\, a\in\mathcal{A}
\end{aligned} \tag{4.3}$$

with optimal link flows v^* and demands q^*, for a given toll vector $\bar{\tau}$.

Note that the MNL modal choice model has the inverse demand function (2.6). Following the sensitivity analysis of the a user equilibrium with elastic demand given by the inverse demand function, the corresponding MNL modal choice formulation is

$$\begin{aligned}
\min_{q',v'} Q(q',v',\tau') = \; & \sum_{a\in\mathcal{A}} \left(\bar{\tau}_a\tau'_a v'_a + \frac{1}{2}\frac{\partial\hat{c}_a(v_a^*,\bar{\tau}_a)}{\partial v_a}(v'_a)^2\right) \\
& +\frac{1}{2}\sum_{i\in\mathcal{I}} \frac{T_i}{\alpha q_i^*(T_i-q_i^*)}(q'_i)^2 \\
s.t. \quad & \sum_{p\in\Pi_i} f'_p = q'_i, \quad \forall i\in\mathcal{I} \\
& \sum_{i\in I}\sum_{p\in\Pi_i} \delta_p^a f'_p = v'_a, \quad \forall a\in\mathcal{A}
\end{aligned} \tag{4.4}$$

$$f'_p \; free, \quad \forall p \in \Pi_i^1, \quad i \in \mathcal{I}$$

$$f'_p \geq 0, \quad \forall p \in \Pi_i^2, \quad i \in \mathcal{I}$$

$$f'_p = 0, \quad \forall p \in \Pi_i^3, \quad i \in \mathcal{I},$$

which can be solved by the same type of partial linearization method (Evans, 1976) as the combined user equilibrium and modal choice problem, discussed in Section 2.1.2.

4.3 Sensitivity analysis based algorithms

Now we turn to the problem of finding a local optimum to the bi-level problem (3.4). Setting every element in τ' to zero except for one link b and solve the sensitivity analysis problem (4.2) for this direction will generate the directional derivatives $\nabla_{\tau'} v(\tau)$, $\nabla_{\tau'} q(\tau)$ and $\nabla_{\tau'} \hat{\pi}(\tau)$. These directional derivative will be used as if they were the derivative $\frac{\partial v}{\partial \tau_b}$, $\frac{\partial q}{\partial \tau_b}$ and $\frac{\partial \hat{\pi}}{\partial \tau_b}$. Applying these derivatives to (3.4) and (3.6), the derivative $\frac{\partial F(q^*, v^*, \bar{\tau})}{\partial \tau_b}$, can be computed as

$$\sum_{i \in \mathcal{I}} D_i^{-1}(q_i^*) \frac{\partial q_i}{\partial \tau_b} - \sum_{a \in \mathcal{A}} \left(c_a(v_a^*) + \frac{\partial c_a(v_a^*)}{\partial v_a} v_a^* \right) \frac{\partial v_a}{\partial \tau_b} \tag{4.5}$$

or for MNL version (3.6)

$$\sum_{i \in \mathcal{I}} \frac{-T_i e^{-\alpha \hat{\pi}_i^*}}{e^{-\alpha \hat{\pi}_i^*} + e^{-\alpha k_i}} \frac{\partial \hat{\pi}_i}{\partial \tau_b} - \sum_{a \in \mathcal{A}} \bar{\tau}_a \frac{\partial v_a}{\partial \tau_b} + v_b^*. \tag{4.6}$$

Note that there is no guarantee that these derivatives are ascent directions, since they realy are directional derivatives which may not correspond to the derivatives if F is not differentiable. Verhoef (2002b) discuss this problem, and give examples of when (3.4) is not differentiable.

In the objective of (3.6) the variables are link flows, v, and OD travel costs, $\hat{\pi}$, and the OD travel cost perturbation in OD pair $i \in \mathcal{I}$ can be computed as

$$\hat{\pi}'_i = \min_{p \in \Pi_i^1 \cup \Pi_i^2} \sum_{a \in \mathcal{A}} \delta_p^a \left(\bar{\tau}_a \tau'_a + \frac{\partial \hat{c}_a(v_a^*, \bar{\tau}_a)}{\partial v_a} v'_a \right)$$

and used in (4.6)

Two different versions of sensitivity analysis based algorithms will be presented. The algorithms are iterative methods and in each iteration the sensitivity analysis problem, (4.2) or (4.4), needs to be solved for each tollable link.

The first algorithm (**A1**) follows the algorithm suggested by Josefsson (2003) for a network design problem. The algorithm can for the congestion pricing problem be outlined as

0. *Initiate.* Set each element in the toll vector τ^0 equal to zero, $\tau_a^0 := 0, \; \forall a \in \mathcal{A}$, and iteration counter $n := 1$. Solve (4.1) or (4.3), with $\bar{\tau} = \tau^0$ and compute the social surplus SS^0. Let $\underline{\Delta SS} := 0$ be the solution corresponding to the toll vector $\underline{\tau} := \tau^0$.

1. *Sensitivity analysis.* Solve (4.2) or (4.4), for each link $a \in \mathcal{T}$ and compute ∇F by either (4.5) or (4.6).

2. *Projection.* Project each element in ∇F onto the fesible space, which give ∇F_{proj}.

3. *Search direction.* Compute a search direction $d^n = \frac{\nabla F_{proj}}{||\nabla F_{proj}||}$

4. *Maximum step length.* Calculate a maximum step length,

$$t_{max} = \max_t t \quad s.t \; \tau_a^n + t d_a^n \geq 0, \forall a \in \mathcal{A}.$$

5. *Line search.* Find an appropriate step length, $t \leq t_{max}$, by an inexact line search according to the procedure described in 4.3.1. The step is $t^n = \max\{t, \frac{1}{n}\}$.

6. *Update.* $\tau^{n+1} := \tau^n + t^n d^n$, and set $n := n + 1$.

7. *Solve UE.* Solve the user equilibrium problem for $\bar{\tau} = \tau^n$ and compute the social surplus SS^n, and change in social suplus $\Delta SS^n = SS^n - SS^0$. If $\Delta SS^n > \underline{\Delta SS}$, set $\underline{\Delta SS} = \Delta SS^n$ and $\underline{\tau} := \tau^n$.

8. *Check termination criteria.* If the termination criterion is not fulfilled continue with Step 1.

In the projection in Step 3 of **A1**, each element in ∇F, $\frac{\partial F}{\partial \tau_a}$ is projected onto the feasible space of τ_a. The projection is

$$\left[\frac{\partial F}{\partial \tau_a}\right]_{proj} = \begin{cases} \frac{\partial F}{\partial \tau_a} & \text{if } \tau_a > 0 \\ \max\left\{0, \frac{\partial F}{\partial \tau_a}\right\} & \text{if } \tau_a = 0. \end{cases}$$

In **A1** there is a risk that the maximum step length will reduce the improvement that can be achieved in each iteration. This is cured in the second algorithm. Instead of computing a maximum step length the toll vector is projected onto the feasible space. In the second version of the algorithm (**A2**), Step 2 and 4 are replaced by

2. *Projection.* No projection is needed. Set $\left[\frac{\partial F}{\partial \tau_a}\right]_{proj} := \frac{\partial F}{\partial \tau_a}$.

4. *Maximum step length.* Set $t_{max} := \infty$.

4.3.1 Line search, A1

Given the gradient ∇F a step length is computed by an inexact Armijo line search (Armijo, 1966). For a step size t to be accepted, in a maximization problem, the inequality

$$F(\tau^n + td^n) - F(\tau^n) \geq \sigma t \nabla F(\tau^n) d^n \tag{4.7}$$

must hold, and $t \leq t_{max}$. $0 < \sigma < 1$ is the Armijo factor, and the step size t must not only give a positive improvement of the objective, but a sufficiently large improvement. In the line search it is checked if the Armijo rule is fulfilled. If not, the step size is reduced by β, until (4.7) holds, and if the initial step size is accepted, the step length is increased by the same factor β as long as $F(\tau^n + td^n)$ is increased, and $t \leq t_{max}$. Note that evaluating $F(\tau^n + td^n)$ requires a user equilibrium problem to be solved.

4.3.2 Line search, A2

A similar inexact line search is used, as for **A1**. There is however one important difference. The evaluated toll vector $\tau^n + td^n$ is projected onto the feasible space, given by the non-negative constraints, and the projection is

$$[\tau_a^n + td_a^n]_+ = \max\{\tau_a^n + td_a^n, 0\}, \quad \forall a \in \mathcal{A}.$$

4.3.3 Termination criteria

In both algorithms the termination criteria can be related to the norm of the direction, $||d^n|| \leq \epsilon$. This will have the drawback that if the optimal solution is a point where the function is not differentiable, the method may never terminate. Therefore the termination criterion is extended with a specification of maximum number of iterations.

4.3.4 Speed ups

For each tollable link a sensitivity analysis problem has to be solved in each iteration. For some of the tollable links, the toll levels may stay at zero and to reduce the computational burden these tolls can be set to zero for a number of consecutive iterations.

If the DSD method is used when solving the user equilibrium problem, the re-optimization ability can be explored. By keeping the set of equilibrium routes between each iteration, and only update the travel cost functions, i.e. the toll levels, the time required by the DSD method to solve the user equilibrium problem will be reduced. Since the user equilibrium problem needs to be solved several times during the line search, this can greatly improve the overall computational performance of the algorithm.

4.4 Numerical results

The sensitivity analysis approach has been evaluated on two different networks. The Nine node network, for which Yildirim (2001) has presented both a first-best (FB) and second-best (SB) solutions, and a version of the fixed demand Sioux Falls network (Leblanc, 1975), which has been extended with a modal choice model. The two algorithms **A1** and **A2** have been implemented in MATLAB.

4.4.1 The Nine node network

The Nine node network has 18 links with travel cost functions on the form $c_a(v_a) = T_a(1 + 0.15(v_a/K_a))$, where T_a is the free flow travel cost, and K_a the link capacity (Yildirim, 2001). The inverse demand function is on the form $D_i^{-1}(q_i) = \psi_i - \varsigma_i q_i$, and parameters for both the travel cost and inverse demand functions are presented in Table 4.1. In Table 4.1 the user and system optimal flows, costs and demands, and the marginal social cost pricing (MSCP) tolls, are also presented. Yildirim (2001) presents both a first-best and a second-best solution, for this network. The first-best solution gives $\Delta SS = 116.43$, and for the second-best solution Yildirim allow nine links to be tolled, marked with a star in Figure 4.1, which give $\Delta SS = 85.17$.

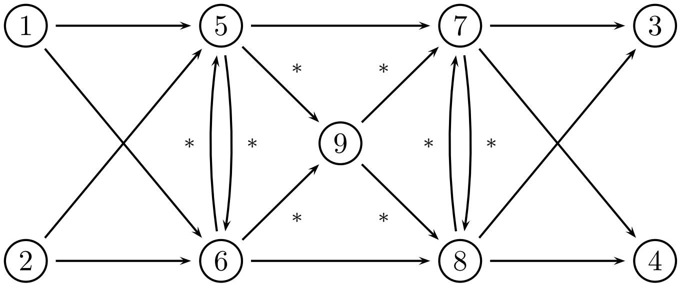

Figure 4.1: The Nine node network. Tollable links in the second-best scenario are marked with *

A1 and **A2** have been applied to the Nine node network. The increase in social surplus, ΔSS, after each iterations is plotted in Figure 4.2 for both the first-best and second-best example. The increases in social surplus are compared to the figures given in Yildirim (2001). In Table 4.2, the deviation from the optimal solution, ΔSS^*, in Yildirim (2001) is presented after 10, 50 and 500 iterations. The final toll levels are presented in Table 4.3, and are the results after 500 iterations. Note that in the second-best example only 4 out of the 9 tollable links are tolled.

There are very small differences between the toll levels computed by **A1** and **A2**. For both the first-best and second-best example, the methods converge towards the known optimal objective value.

Table 4.1: Paramters $(T_a, K_a, \psi_i, \varsigma_i)$ togheter with the user optimal and the system optimal solutions for the Nine node network

	UO		*SO*		
Link (T_a, K_a)	*Flow*	*Cost*	*Flow*	*Cost*	*MSCP tolls*
1 - 5 $(6, 11)$	12.06	6.99	9.46	6.78	0.78
1 - 6 $(7, 9)$	0	7.00	0	7.00	0
2 - 5 $(2, 2)$	49.61	9.45	30.4	6.56	4.56
2 - 6 $(8, 35)$	0	8.00	15.50	8.54	0.54
5 - 6 $(5, 20)$	0	5.00	0	5.00	0
5 - 7 $(2, 5)$	61.67	5.71	39.86	4.40	2.40
5 - 9 $(3, 44)$	0	3.00	0	3.00	0
6 - 5 $(10, 6)$	0	10.00	0	10.00	0
6 - 8 $(9, 48)$	0	9.00	15.50	9.44	0.44
6 - 9 $(7, 29)$	0	7.00	0	7.00	0
7 - 3 $(2, 34)$	23.88	2.22	21.5	2.19	0.19
7 - 4 $(7, 9)$	11.06	8.30	13.01	8.52	1.52
7 - 8 $(1, 49)$	26.74	1.09	5.35	1.02	0.02
8 - 3 $(4, 13)$	0	4.00	0	4.00	0
8 - 4 $(4, 5)$	26.74	7.21	20.84	6.51	2.51
8 - 7 $(2, 47)$	0	2.00	0	2.00	0
9 - 7 $(5, 42)$	0	5.00	13.7	5.25	0.25
9 - 8 $(5, 5)$	0	5.00	0	5.00	0
OD-pair (ψ_i, ς_i)					
1-3 $(20, 2)$	2.55		1.64		
1-4 $(40, 2)$	9.51		7.81		
2-3 $(60, 2)$	21.32		19.86		
2-4 $(80, 2)$	61.28		26.03		

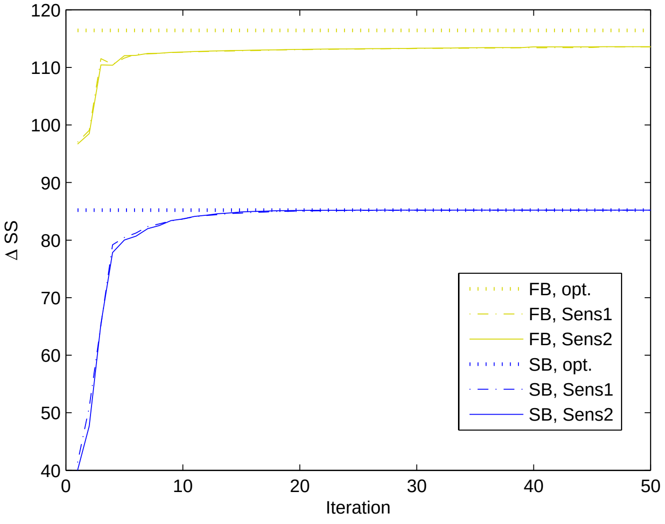

Figure 4.2: ΔSS for iteration 1 to 50, for A1 and A2, on the first-best and the second-best example

Table 4.2: $\frac{\Delta SS^* - \Delta SS}{\Delta SS^*}$, in %, after 10, 50 and 500 iterations, for the first-best and second-best example

	It. 10	*It. 50*	*It. 500*
FB, A1	3.26	2.44	< 0.001
FB, A2	3.24	2.43	< 0.001
SB, A1	1.79	0.006	0.005
SB, A2	2.00	0.006	0.005

Table 4.3: Toll levels for the first-best and the second-best example after 500 iterations

Link	FB A1	FB A2	SB A1	SB A2
1 - 5	0.16	0.15	0	0
1 - 6	0	0	0	0
2 - 5	3.9	3.89	0	0
2 - 6	0.38	0.4	0	0
5 - 6	0	0	0	0
5 - 7	2.85	2.88	0	0
5 - 9	0.26	0.25	1.11	1.11
6 - 5	0	0	0	0
6 - 8	0.38	0.4	0	0
6 - 9	0	0	0	0
7 - 3	0.32	0.34	0	0
7 - 4	1.6	1.55	3.73	3.73
7 - 8	0	0.02	4.57	4.57
8 - 3	0	0	0	0
8 - 4	2.6	2.53	0	0
8 - 7	0	0	0	0
9 - 7	0	0	0	0
9 - 8	0	0.07	1.11	1.11

4.4.2 The Sioux Falls network

The Sioux Falls network was fist presented by Leblanc (1975) and has 24 nodes, each node constituting both an origin and destination. There are 528 OD pairs and 76 links, and the network can be considered as a mid-size network. The car demand is fixed and is summarized to 360 600 vehicles per day.

The version of the Sioux Falls network which will be used for numerical experiment is extended with an MNL mode choice model, between car and public transportation, for the travelers with access to car. Even if this network resembles the actual network of Sioux Falls, we consider it as purely fictitious and only use it to demonstrate and evaluate the sensitivity analysis based approach.

By assuming that the travel costs for public transportation will not change when tolls are introduced, and assuming that the modal split in the no-toll scenario is known and correspond to the MNL model with travel costs π^0 and k^0, the pivot point version of the MNL model (Kumar, 1980) can be used. With the pivot point version of the MNL model, the public transportation costs do not need to be known, only the market share of either mode in the no-toll scenario. In Appendix A a simplified formulation of the combined modal split and user equilibrium problem and the corresponding social welfare measure is derived for the pivot point version of the MNL model. Note that the sensitivity analysis problem (4.4) does not include the public transportation cost in the objective, and will not change when the pivot point version of the MNL model is used. The directional derivatives (4.6) do however need to be derived for the pivot point version of the MNL model, and this is also done in Appendix A.

The Sioux Falls network with travel cost functions on the form $c_a(v_a) = T_a\left(1+\left(\frac{v_a}{K_a}\right)^4\right)$ is presented in Appendix B, with parameters for the travel cost functions and modal split model. For each OD pair the input parameters are car demand in the no-toll scenario A_i, the total travel demand T_i, and the car travel costs in the no-toll scenario π_i^0. In its original form the Sioux Falls network describe the traffic during 24 hour, we will however only consider the morning rush our and assumes that the traffic is a tenth of the daily traffic during this hour. The total car demand in the no toll scenario is therefore 36 060, and the link capacities are changed accordingly. The demand for public transportation in the no-toll scenario is generate for each OD pair, by adding a random number between -5 and 100 to the car demand. The total demand for public transportation in the no-toll scenario is 62 566 which give the total demand is 97 626, which is fixed and will not change when tolls are introduced.

The link flows and costs for the no-toll scenario are presented in Appendix B. The dispersion parameter in the mode choice model is estimated to 0.05 and give a mean elasticity per OD pair, for car travel with respect to a change in car travel cost of -1.0 . Travel costs are given in minutes, compare to the original version which present travel costs in 0.01 hours, and the value of time is assumed to be 1 SEK[1] per minute.

[1] 1 SEK $\approx$ 13 cent

The MSCP solution can easily be computed by solving the combined user equilibrium and modal choice problem with travel cost functions

$$\hat{c}_a(v_a) = c_a(v_a) + \frac{\partial c_a(v_a)}{\partial v_a} v_a,$$

which yield the first-best link flows, v^{MSCP}, and demand, q^{MSCP}. The MSCP tolls, τ^{MSCP} can then be computed as

$$\tau_a^{MSCP} = \frac{\partial c_a(v_a)}{\partial v_a} v_a.$$

The resulting change in social surplus $\Delta SS^{MSCP} = 83\ 828$.

For the Nine node network both the algorithms **A1** and **A2** were applied, with small differences in the final solution and for the Sioux Falls network we therefore only apply algorithm **A2**. Four different scenarios are evaluated for the Sioux Falls network, the first-best scenario and three different judgmental cordon structures. The judgmental cordons are named J1, J2 in Figure 4.3, and the third cordon structure J3 is the combination of J1 and J2.

In Figure 4.4 the change in social surplus, ΔSS, is plotted during the 100 first iterations, for the four scenarios respectively. After about 50 iterations the change in the objective value is small for all four scenarios. For the first-best scenario we can compare the objective function value, with the known optimum solution with MSCP tolls. After 350 iterations the solution deviate 0.59% from the optimal solution. The change in social surplus compared to the solution given by the MSCP tolls is presented in Table 4.4, and the final toll levels for the three second-best scenarios are presented in Table 4.5. It is not surprising that the combination of J1 and J2 into J3 produce a higher increase in social surplus compared to either J1 or J2 separately. If J2 is compared to J3 the benefit from the 10 additional tolls in J1 is only 13 670, not even half of the benefit produced by J1 alone.

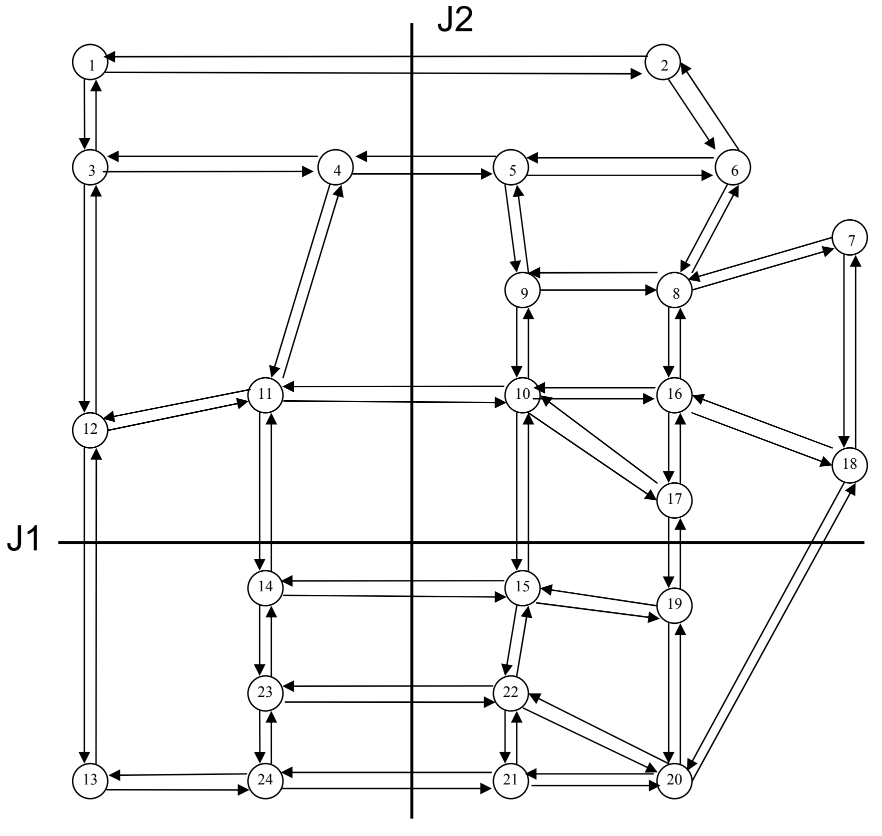

Figure 4.3: Judgmental cordons in the Sioux Falls network

Table 4.4: The change in social surplus, ΔSS compared to the MSCP solution, ΔSS^{MSCP}, as $\Delta SS/\Delta SS^{MSCP}$

Scenario	ΔSS	$\Delta SS/\Delta SS^{MSCP}$
J1	33 968	0.41
J2	41 880	0.50
J3	55 541	0.67
MSCP	83 828	–

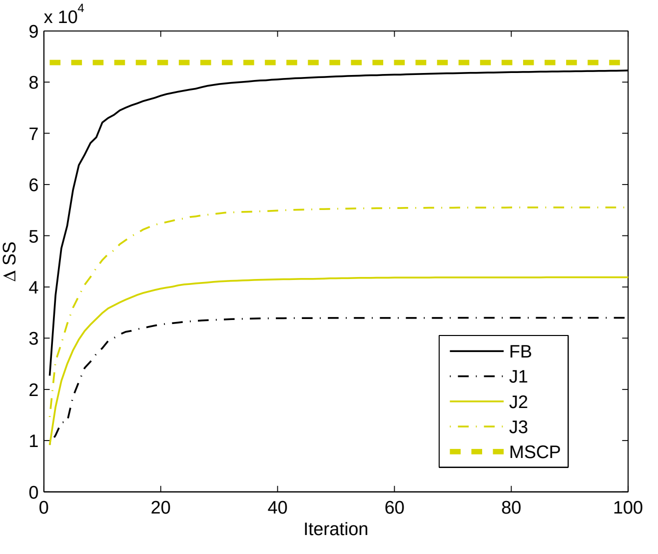

Figure 4.4: ΔSS for iteration 1 to 100, for the first-best, and the judgmental cordons, within the Sioux Falls network, compared to the known optimal MSCP solution

Table 4.5: Toll levels for the three judgmental cordons, the first-best scenario and the MSCP tolls

Link	*J1*	*J2*	*J3*	*FB*	*MSCP*
1 - 2	–	14.3	9.3	0.2	0.0
1 - 3	–	–	–	0	0.0
2 - 1	–	14.4	9.4	0.2	0.0
2 - 6	–	–	–	4.5	2.2
3 - 1	–	–	–	0	0.0
3 - 4	–	–	–	0.7	0.7
3 - 12	–	–	–	0.1	0.1
4 - 3	–	–	–	0.7	0.8
4 - 5	–	16.0	10.9	1.9	0.5
4 - 11	–	–	–	2.7	2.1
5 - 4	–	16.1	11.0	1.9	0.5
5 - 6	–	–	–	4.4	3.4
5 - 9	–	–	–	5.1	6.4
6 - 2	–	–	–	4.5	2.3
6 - 5	–	–	–	4.4	3.5
6 - 8	–	–	–	11.2	14.3
7 - 8	–	–	–	3.9	4.0
7 - 18	–	–	–	1.4	0.1
8 - 6	–	–	–	11.4	14.5
8 - 7	–	–	–	3.8	4.0
8 - 9	–	–	–	6.4	7.2
8 - 16	–	–	–	7.7	6.1
9 - 5	–	–	–	5.2	6.4
9 - 8	–	–	–	6.2	7.0
9 - 10	–	–	–	4.1	3.5
10 - 9	–	–	–	4.1	3.6
10 - 11	–	18.7	13.4	8.1	7.9
10 - 15	16.5	–	11.0	8.2	8.7
10 - 16	–	–	–	15.2	17.3
10 - 17	–	–	–	8.9	10.0
11 - 4	–	–	–	2.7	2.2
11 - 10	–	18.5	13.2	8.0	7.8
11 - 12	–	–	–	7.3	6.9

Continued on Next Page...

Table 4.5 – Continued

Link	*J1*	*J2*	*J3*	*FB*	*MSCP*
11 - 14	17.4	–	12.0	9.1	9.8
12 - 3	–	–	–	0.1	0.1
12 - 11	–	–	–	7.3	6.8
12 - 13	13.7	–	9.3	0.6	0.1
13 - 12	13.9	–	9.4	0.6	0.1
13 - 24	–	–	–	10.8	12.3
14 - 11	17.6	–	12.0	9.2	9.7
14 - 15	–	18.5	13.2	8.0	7.7
14 - 23	–	–	–	4.6	4.8
15 - 10	16.6	–	11.1	8.3	8.8
15 - 14	–	18.7	13.4	8.1	7.9
15 - 19	–	–	–	1.7	1.5
15 - 22	–	–	–	5.1	5.3
16 - 8	–	–	–	7.7	6.1
16 - 10	–	–	–	15.3	17.5
16 - 17	–	–	–	6.9	7.3
16 - 18	–	–	–	0.9	0.5
17 - 10	–	–	–	8.9	10.1
17 - 16	–	–	–	7.0	7.2
17 - 19	16.0	–	11.8	4.6	4.5
18 - 7	–	–	–	1.4	0.1
18 - 16	–	–	–	0.9	0.5
18 - 20	13.5	–	9.1	0.9	0.6
19 - 15	–	–	–	1.7	1.5
19 - 17	16.1	–	11.8	4.5	4.5
19 - 20	–	–	–	6.2	7.2
20 - 18	13.6	–	9.1	0.9	0.6
20 - 19	–	–	–	6.3	7.3
20 - 21	–	–	–	3.5	3.0
20 - 22	–	–	–	2.9	4.3
21 - 20	–	–	–	3.4	2.9
21 - 22	–	–	–	3.4	2.4
21 - 24	–	19.4	14.4	8.6	8.5
22 - 15	–	–	–	5.1	5.2

Continued on Next Page…

Table 4.5 – Continued

Link	*J1*	*J2*	*J3*	*FB*	*MSCP*
22 - 20	–	–	–	2.9	4.3
22 - 21	–	–	–	3.4	2.4
22 - 23	–	19.2	13.9	8.6	8.4
23 - 14	–	–	–	4.6	4.8
23 - 22	–	19.0	13.7	8.4	8.2
23 - 24	–	–	–	2.9	2.0
24 - 13	–	–	–	10.8	12.3
24 - 21	–	19.3	14.2	8.4	8.3
24 - 23	–	–	–	2.8	2.0

Now, let us modify the second scenario to not include tolls on links (10-11) and (11-10), and denote the new cordon as J4. The cordon is no longer closed, and allows traveling between the two zones that were formerly closed by the cordon, without paying a toll. The first step in **A2** is to set the toll levels to zero, and the solution corresponding to this initial solution is denoted as 4a. If we instead use the optimal toll levels for J2 as initial solutions, and only set (10-11) and (11-10) to zero we get a different solution, denoted 4b. It is well known that the level setting problem may have multiple local optima, and by changing the initial solution, a local optima which yield a negative change in the social surplus of $-6\ 057$, is found. The convergence is however slow, and after 200 iterations the objection function value is still increasing. If the directional derivatives are not ascent directions in every iteration, the convergence will be slow, and this is actually the case here.

The second-best toll levels from solution 4a and 4b for J4 are presented in Table 4.7. The best change in social surplus for J4, given by the solution 4a, is 8 781, which can be compared to 41 880 for J2. It is obvious that the closed cordon structure is much more appealing. Comparing the toll levels for J2 with solution 4a for cordon J4, the toll levels in solution 4a are much lower. In Table 4.6 the change in social surplus is broken up into the change in consumer surplus and toll revenues. Note that the change in consumer surplus and toll revenue are proportional between cordon J2 and solution 4a for J4, with approximately a 80% reduction of both the loss in consumer surplus and the total toll revenues. Comparing

J2 and 4b for J4, there is a 38% reduction of the loss in consumer surplus and a 54% reduction in the total toll revenues. The reason for the negative change in social surplus is that the toll revenues are more reduced than the loss in consumer surplus.

Table 4.6: The change in social surplus for the solution to J2, and solution 4a and 4b for J4, broken up into consumer surplus (CS) and toll revenues (R)

Cordon	ΔCS	R	ΔSS
J2	−151 625	193 505	41 880
J4(a)	−34 746	43 527	8 781
J4(b)	−94 723	88 666	−6 057

Table 4.7: Toll levels for the tolled links in solution 4a and 4b for J4

Link	*4a*	*4b*
1 - 2	2.6	13.1
2 - 1	2.6	13.2
4 - 5	4.4	7.4
5 - 4	4.3	7.4
14 - 15	5.0	13.5
15 - 14	5.0	13.6
21 - 24	6.0	18.5
22 - 23	5.5	17.7
23 - 22	5.5	17.5
24 - 21	6.0	18.3

Two other initial solutions have been used for both cordon J2 and J4, and are presented in Table 4.8. A uniform random distribution generating number between 0 and 20 have been used to generate the alternative initial solutions. The final solution for J2 does not change when the new initial solutions are applied. For J4 the final solutions however differ, and the toll levels and objective function value after zero and 200 iterations respectively are presented in Table 4.8, together with the initial objection function value. The convergence is also slow which suggest that the directional derivatives are not ascent directions in every iteration. Besides reducing the efficiency of the congestion pricing scheme significantly, the open cordon structure also seems to give rise to several local optima.

Table 4.8: Alternative initial solutions (Init), corresponding final solutions (Final), and the change in social surplus for each toll vector

	Alt 1		*Alt 2*	
Link	*Init*	*Final*	*Init*	*Final*
1 - 2	16.6	8.2	15.9	9.4
2 - 1	10.1	8.0	19.1	10.5
4 - 5	14.2	5.5	10.5	3.7
5 - 4	8.6	5.2	17.6	5.0
14 - 15	3.9	7.8	5.4	4.5
15 - 14	13.6	7.8	5.0	6.3
21 - 24	6.1	8.7	17.5	7.6
22 - 23	10.8	8.3	14.7	7.0
23 - 22	3.0	8.3	2.7	4.8
24 - 21	14.0	8.8	0.2	5.2
ΔSS	$-17\ 627$	6 085	$-26\ 086$	6 224

5 Heuristic approaches for the combined toll location and level setting problem

A straight forward approach to the combined toll location and level setting problem would be to solve the level setting problem for all combination of links, and for each combination compute the net social surplus. The number of ways to combine the links will however even for a network of moderate size be vast. Note that the level setting problem is non-convex for a general network, and there is no guarantee that the final solution to the level setting problem is the global optimal one. The genetic algorithms presented in Sumalee (2004, 2005) and in Zhang and Yang (2004) only evaluate a subset of the possible combinations, where the subset is combinations of toll locations that form closed cordons. Hopefully the optimal solution lies within this subset, but to restrict the combination of toll locations have some drawbacks. There might be scenarios when it would be beneficial to combine different kind of cordon structures, e.g. a closed cordon around the city center and a single toll on a ring road circulating the city.

In this chapter, two different heuristic approaches for the combined toll location and level setting problem are presented. The first approach (the incremental approach) is a direct extension of the method suggested by Verhoef (2002a) for solving the problem of locating a fixed number of toll facilities and finding optimal toll levels. This approach is only considered for the inverse demand formulation (3.4). The second approach (the approximation approach) can be viewed as an extension of the sensitivity analysis based method, presented in Section 4.3. The approximation approach is evaluated for both the inverse demand formulation (3.4), and the multinomial logit (MNL) modal choice version (3.6).

Numerical results are presented for both methods. To evaluate the performance of a method for solving the combined problem is however difficult. The possible number of combinations of toll facilities is large even for a small network, which makes it hard to compute an optimal solution by exhaustive search, and even for a fixed location, the level setting problem is non-convex.

5.1 The incremental approach

To solve the problem of maximizing the social surplus for a given number of toll facilities, i.e. to find the optimal locations and optimal toll levels, Verhoef (2002a) suggests a heuristic that exploits a ranking of possible locations by a location index. This location index, computable for each link, is an approximation of the possible gain in the objective value, if a toll is located on that specific link. The location index can be computed for a given user equilibrium and these indices can then be used to specify which link or links to toll. Given the fixed set of links to toll, second-best toll levels can be computed. The procedure of using the location indices to find the optimal locations and toll levels has been described thoroughly by Verhoef (2002a) and Shepherd et al. (2001). In this section the method will only be briefly described.

The location index for link a is an approximation of the improvement in social surplus if a toll facility is located on link a and is computed as

$$I_a = \frac{1}{2} S_a L_a,$$

where S_a is the predicted optimal toll from the first iteration in CORDON (see Section 2.3.2) and L_a is the marginal effect on the social surplus of adding a toll on link a.

To locate n tolls, three different strategies are suggested by Verhoef (2002a). The first one is to compute I_a once for each tollable link based on the no-toll equilibrium, and chose the n links with highest score. In the second strategy, one toll is located at a time. Before an additional toll is added, I_a is recomputed. This strategy requires a level setting problem to be solved after each toll is located. For the third strategy, I is computed for all possible combinations of tollable links, based on the no-toll scenario. Note that in this strategy, I is the index or rather estimated welfare gain of simultaneously adding several tolls. Strategy one requires the least computational time, but in (Verhoef, 2002a) the results presented for this strategy are not very good. The third strategy is very computational burdensome, since the number of combinations are vast for a larger network. Strategy two is maybe the most appealing one, since (some) link interactions are taken into account without the necessity of evaluating all possible combinations, the drawback is that in each iteration a level setting problem has to be solved, which is not necessary in strategy 1 and 3.

The incremental approach is an extension of strategy two which will be modified to certify that each additional added toll facility will produce an improvement of the net social surplus, i.e. the increase in social surplus must be at least as large as the collection cost. If there is no improvement, we terminate the algorithm. In each iteration we add the link which at the moment seems to produce the highest improvement of the net social surplus, i.e. the link with the highest value of the location index I_a.

The computation of S_a and L_a in strategy 2 requires linear systems of equations to be solved, given an equilibrium solution. The linear systems of equations is derived from the Lagrangian relaxation of the complementarity constraints for the equilibrium routes (2.18) in problem (2.17) and this is described in detail by Verhoef (2002a).

To compute S_a the system of linear equations is

$$\sum_{b\in\mathcal{A}} \delta_p^b \left(\lambda_b S_b - \sum_{i'\in\mathcal{I}} \sum_{p'\in\Pi^*_{i'}} \delta_{p'}^b f_{p'}^* \frac{\partial c_b(v_b^*)}{\partial v_b} \right) \tag{5.1}$$

$$+ \sum_{i'\in\mathcal{I}} \sum_{p'\in\Pi^*_{i'}} \gamma_{p'} \left(\sum_{b\in\mathcal{A}} \delta_p^b \delta_{p'}^b \frac{\partial c_b(v_b^*)}{\partial v_b} \right)$$

$$- \sum_{p'\in\Pi^*_i} \gamma_{p'} \frac{\partial D_i^{-1}(q_i^*)}{\partial q_i} = 0, \quad \forall i \in \mathcal{I}, p \in \Pi_i^*$$

$$\sum_{i\in\mathcal{I}} \sum_{p'\in\Pi^*_i} \delta_p^a \gamma_p = 0, \tag{5.2}$$

with unknowns γ_p for each equilibrium route, and S_b for each link $b \in \mathcal{A}$. The constant λ specifies for which link a, S_a is being computed, with $\lambda_b = 1$, if $b = a$, and 0 otherwise. The set Π_i^* is the equilibrium routes in OD pair $i \in \mathcal{I}$ corresponding to the link flows, v^*, route flows f^*, and demand, q^*. The relationship between links and routes are given by δ_p^a, which is 1 if route p traverses link a, and 0 otherwise. For each equilibrium route, γ_p is the Lagrangian multiplier corresponding to the complementarity constraint. There will be one equation for each route with a positive flow, and one equation for the considered toll, furthermore, there is one variable for each such route and one for the toll. If we let N_r be the number of all such routes, the linear system of equations will be of dimension $(N_r + 1) \times (N_r + 1)$. To compute L_a, the linear system of equations is made up of (5.1), with S_b replaced by the current toll levels τ_a, and $\lambda_a = 1$ for all tollable links. This linear system of equations will have dimension $(N_r) \times (N_r)$, and L_a is computed as

$$L_a = \sum_{i \in \mathcal{I}} \sum_{p \in \Pi_i^*} \delta_p^a \gamma_p.$$

The need to find all routes which will have a positive flow in the equilibrium solution requires the user equilibrium problem to be solved with a high accuracy.

The incremental approach can be stated as:

0. Let all candidate links a (tollable links) belong to the set $\mathcal{C}$, and let the set of selected locations $\mathcal{T} := \emptyset$. Set $\tau_a := 0, \forall a \in \mathcal{A}$. Solve the lower level user equilibrium problem and let NSS^0 be the corresponding net social surplus, i.e. the objective of (3.8). Set the iteration counter $n := 1$.

1. Compute the location indices, $I_a \ \forall \ a \in \mathcal{C}$ and find link a^* with the highest index value I_a. Update the set $\mathcal{T} := \mathcal{T} \cup \{a^*\}$.

2. Solve the level setting problem (3.4) and compute the net social surplus (3.8), NSS^n. If $NSS^n \geq NSS^{n-1}$, set $n := n + 1$ and return to 1. Otherwise set $\mathcal{T} := \mathcal{T} \backslash \{a^*\}$, solve the level setting problem (3.4) and terminate the algorithm.

In Verhoef (2002a) the level setting problem (3.4) is solved with the CORDON method (Shepherd and Sumalee, 2004; Shepherd et al., 2001; Verhoef, 2002b). We will however solve the level setting problem with the sensitivity analysis based method presented earlier.

5.2 The approximation approach

As will be shown in the numerical experiments, one disadvantage of the incremental approach is its inability to estimate the effects of tolling parallel routes. Further, the method requires the user equilibrium problem to be solved with high accuracy and relies on matrix operations to solve a linear system of equations of dimension $(N_r + 1) \times (N_r + 1)$, which may lead to difficulties for large networks. An alternative approach is therefore suggested, with the aim to better be able to predict welfare gains when tolling parallel routes.

The approach is based on repeated solutions of an approximation of problem (3.8) or (3.8) depending on which demand model is used. The approximation is made by replacing $\text{sign}(\tau_a)$ with a continuous function of τ_a. The function is chosen as

$$\gamma(k, \tau_a) = \frac{2}{1 + e^{-k\tau_a}} - 1, \tag{5.3}$$

but other functional forms may be considered. Note that for any $\tau_a \geq 0$, $\gamma(k, \tau_a)$ tends to $\text{sign}(\tau_a)$ when k tends to infinity. Moreover, $\gamma(k, \tau_a) = 0$ at $k = 0$. This is illustrated in Figure 5.1.

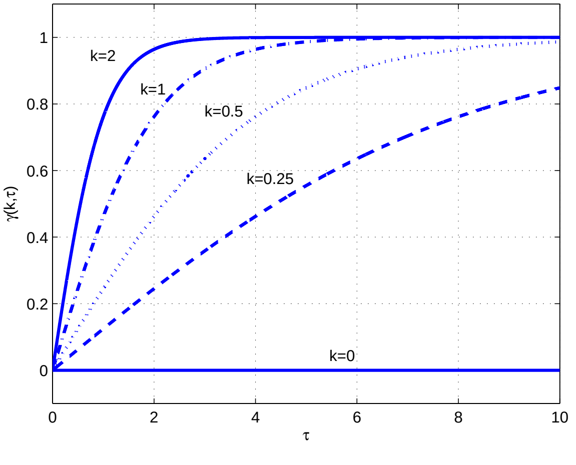

Figure 5.1: The function $\gamma(k, \tau)$ for different values on k

Replacing sign(τ_a) in problem (3.8) by $\gamma(k, \tau_a)$ gives

$$\max_{\tau \in \mathcal{X}} F(\tau) = \sum_{i \in \mathcal{I}} \int_0^{q_i(\tau)} D_i^{-1}(w)dw - \sum_{a \in \mathcal{A}} c_a(v_a(\tau))v_a(\tau) - \sum_{a \in \mathcal{A}} C_a \gamma(k, \tau_a) \quad (5.4)$$

and for the MNL modal choice version

$$\max_{\tau \in \mathcal{X}} F(\tau) = \frac{1}{\alpha} \sum_{i \in I} T_i \ln \frac{e^{-\alpha \hat{\pi}_i(\tau)} + e^{-\alpha k_i}}{e^{-\alpha \pi_i^0} + e^{-\alpha k_i^0}} + \sum_{a \in \mathcal{A}} v_a(\tau)\tau_a - \sum_{a \in \mathcal{A}} C_a \gamma(k, \tau_a) \quad (5.5)$$

where $\mathcal{X}$ describe the set of tollable links (3.1).

In (3.8) and (3.8) the collection cost for link a is given by the discrete function $C_a$sign(τ_a). When replacing this discrete function with a continuous one, the same type of sensitivity analysis based method as was presented previously can be used to solve (5.4) and (5.5). The only change in **A1** and **A2** is that there will be a contribution from the collection cost to the derivatives. This contribution will, for link a, be equal to $-C_a \frac{\partial \gamma(k, \tau_a)}{\partial \tau_a}$, and is added to (4.5) and (4.6).

The approximation approach is an iterative solution procedure. In each iteration of the method, (5.4) or (5.5) is solved and the solution provides a number of locations with a τ-level above zero. Increasing the value of k gives a better approximation of the actual collection cost and in the procedure, the parameter k is increased by a pre-specified amount in each iteration.

In each iteration of the method, the current solution is improved until a (local) optimum is reached. Thus we let the final solution to (5.4) or (5.5) in one iteration be the initial solution in the next iteration.

The approximation approach can be stated as

0. Start with a feasible solution τ^0 and set $k := 0$. Set iteration counter $n := 1$.

1. Find a new solution by solving the problem (5.4) or (5.5) depending on the choice of demand model, with initial solution τ^{n-1}. Every link a with a positive τ-level, indicates a toll location. If $\gamma(k, \tau_a) > \theta$ (θ close to 1), for every link a with a positive toll, continue with 3.

2. Increase k by ϵ, set $n := n + 1$ and return to 1.

3. Set $k := \infty$, in practice a large enough value for $\gamma(k, \tau_a) \approx 1$, for every link a with $\tau_a > 0$, and solve problem (5.4) or (5.5), starting at τ^n.

Note that in each iteration, τ^n is a feasible toll vector in (3.8) and (3.8), and can thus be used to compute a lower bound, $\underline{\Delta NSS}$, on the optimal solution.

After the first iteration, the solution will correspond to a system optimal solution, but most likely not the MSCP solution. When k is increased, each toll location will be associated with a collection cost between 0 and C_a. If the collection cost for a toll facility is higher than the improvement in social surplus produced by that toll it may be dropped (τ_a is set to zero). A location which has been dropped may later enter ($\tau_a > 0$) at any time. For $k > 0$, τ_a will not be the same as the optimal toll level on link a, since there is a relation between the collection cost and τ in (5.3). When k is raised and $\gamma(k, \tau_a)$ tends to $\text{sign}(\tau_a)$, τ will eventually reach the second-best toll level solution (Figure 5.1).

In the last iteration k will be infinitely large, and this correspond to solving a level setting problem for a fixed location given by the positive τ-levels. In practice we do not require k to be infinitely large but just large enough to satisfy the termination criterion. The termination criterion is related to how large proportion of the actual collection cost is included in the objective function value.

5.2.1 Two examples

To illustrate the ideas behind the approximation approach, two examples will be presented.

First, consider the simple network of two nodes and one link in Figure 5.2, with travel cost function $c(v) = 2.5 + 0.01v$ and the inverse demand function $D^{-1}(q) = 25 - 0.05q$. The first-best toll, $\tau = 3.2$, increases social surplus by 100 and for a collection cost below 100, the optimal solution is to locate a toll on the link. In Figure 5.4, the objective in (5.4) is plotted with respect to the τ-value for $C = 90$ and $C = 120$. The objective function value is plotted for several iterations with the approximation approach, and each iteration is represented by different values on k. For each iteration, k is incremented and the optimal τ-value is driven towards either the real toll level, indicating that a toll on this link will be beneficial ($C = 90$), or to zero, indicating that the toll will give no benefit to the net social surplus ($C = 120$).

Next, the network is expanded with one more node and two links (Figure 5.3). The inverse demand function for the only OD-pair (1,3) is $D_{13}^{-1}(q_{13}) = 25 - 0.05q_{13}$ and the travel cost functions are, $c_1(v_1) = 0.5 + 0.01v_1$, $c_2(v_2) = 0.02v_2$ and $c_3(v_3) = 2$. Assuming the possibility to toll every link, the MSCP tolls $\tau_1 = 2.30$, $\tau_2 = 2.55$ and $\tau_3 = 0$ improve social surplus by 58.2 and reduce the total demand from 400 to 358. Out of this improvement, 56.1 is due to change in demand and 2.1 is due to change in route choices. If the collection cost is larger than 2.1, it is advantageous to replace the two tolls with one single toll on link 3. The optimal τ-values after each iteration (with $\epsilon = 5$) are presented in Table 5.1 for $C = 1.5$ and $C = 2.5$. The τ-levels give for the first iteration with $k = 0$ the same level of social surplus as the MSCP solution previously stated. For $C = 1.5$, two links have non-zero τ-values at the final iteration, which implies that at this collection cost, it is beneficial to both reduce the demand and reroute the road users. If the collection cost is increased to 2.5 the benefit from adding a second toll is lower than the collection cost. Therefore all road users are only charged the toll on link 3, and thus, the route choice is not affected.

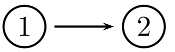

Figure 5.2: A single link network

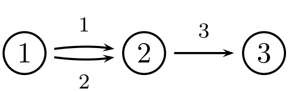

Figure 5.3: A three node network

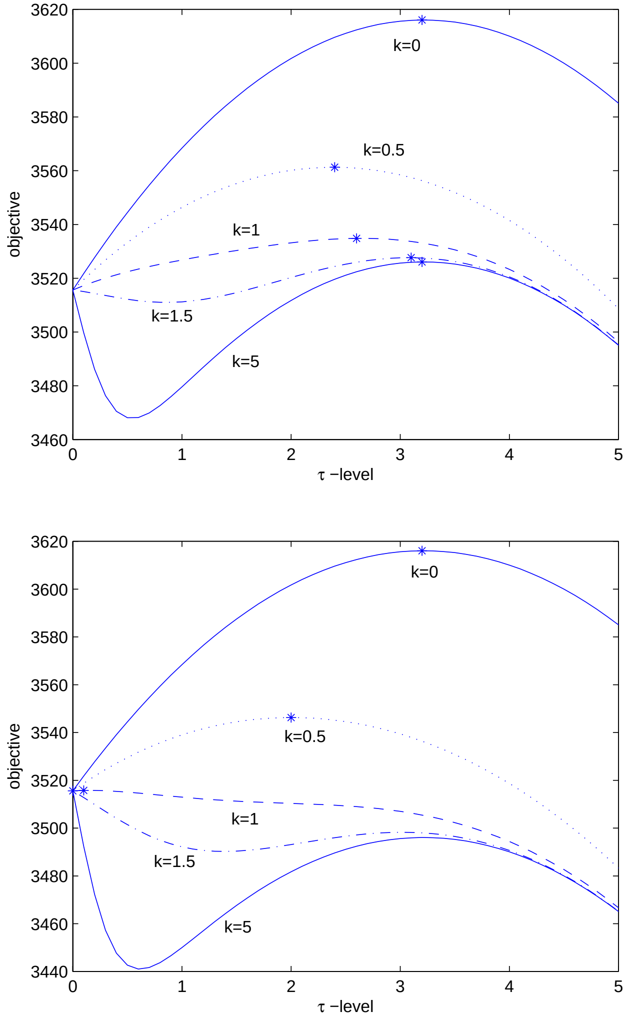

Figure 5.4: Plot of the objective of problem (5.4) for the single link network with $C = 90$ (upper) and $C = 120$ (lower), maximum for each k is marked with *

Table 5.1: τ-levels at each iteration in the approximation approach and final net social surplus for the three node network.

	$C = 1.5$			$C = 2.5$		
k	*Link 1*	*Link 2*	*Link 3*	*Link 1*	*Link 2*	*Link 3*
0	0.68	0.93	1.62	0.68	0.93	1.62
5	0	0.21	2.32	0	0.17	2.33
10	0	0.20	2.32	0	0.11	2.35
15	0	0.23	2.31	0	0	2.39
20	0	0.24	2.31	0	0	2.39
25	0	0.25	2.30	0	0	2.39
30	0	0.25	2.30	0	0	2.39
Final net social surplus	3200.0			3198.4		

5.3 Numerical results

In this section, numerical results are presented for three different networks. Both the incremental and approximation approach are evaluated on two small networks, with linear elastic demand functions. The approximation approach is also evaluated on a version of the Sioux Falls network with modal choice between car and public transportation. The two small networks are primarily used to evaluate the performance of the two heuristic solution procedures. The Sioux Falls network allows for further analysis on optimal cordon structures, and the results are compared to the judgmental cordons presented in Chapter 4.

The two methods have been implemented in MATLAB. Computing the location indices in the incremental approach requires the user equilibrium problem to be solved with a high accuracy which is not necessary when using the approximation approach. Both the incremental and approximation approach require numerous user equilibrium problems to be solved and by using a solving the user equilibrium problem with less accuracy, the overall running time with the approximation approach can be significantly reduced.

5.3.1 The Four node network

The incremental approach and the approximation approach have been evaluated on the Four node network (Figure 5.5) with link travel cost functions on the form $t_a(v_a) = \alpha_a + \beta_a v_a$. The network has two OD pairs (1-2 and 1-4), with linear inverse demand functions $D^{-1}(q_i) = \psi_i - \varsigma_i q_i$, and parameter values are presented in Table 5.2. It is possible to reach system optimal flows with tolls on four links, either 1,2,3,4 or 1,2,4,5. The user optimal (UO) and system optimal (SO) flows and costs are given in Table 5.2, together with the MSCP tolls. The collection cost is assumed to be equal for all links, i.e. $C_a = C$, for all $a \in \mathcal{A}$.

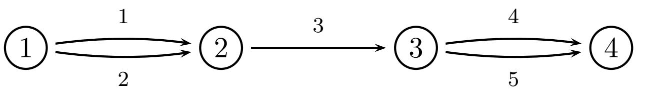

Figure 5.5: The Four node network

Table 5.2: UO and SO solutions for the Four node network

	UO		*SO*		
Link (α_a, β_a)	*Flow*	*Cost*	*Flow*	*Cost*	*MSCP tolls*
1 $(2.5, 0.002)$	538	3.58	510	3.52	1.02
2 $(2.5, 0.0007)$	1537	3.58	1459	3.52	1.02
3 $(2.5, 0.001)$	1004	3.50	946	3.45	0.95
4 $(1.5, 0.002)$	631	2.76	431	2.36	0.86
5 $(2.5, 0.0007)$	373	2.76	515	2.86	0.36
OD-pair (ψ_i, ς_i)					
$1-2$ $(25, 0.02)$	1071		1023		
$1-4$ $(50, 0.04)$	1004		946		
Social Surplus		31633.7		31827.5	

Note that if $C = 0$, the optimal solution will be system optimal but for the case when $C > 0$ the collection cost will be considered and it might therefore be optimal to locate fewer tolls than what is required for system optimum. The benchmark optimal location has been computed by solving the level setting problem for each possible combination of locations. The level setting problem is solved with the sensitivity analysis based method presented in Chapter 4. Note that for a second-best solution there might, however, be cases when the local optimal solution does not correspond to the global one.

In Table 5.3, the solutions obtained from using the incremental approach and the approximation approach are presented and compared to the benchmark solution. The solutions are computed for different values on the collection cost, C. The approximation method fails to find the optimal set of locations for $C = 70$ and $C = 110$. In the incremental approach, the links will always be chosen in the order 3, 4, 2. With a collection cost of $C = 70$, $C = 80$ and $C = 100$ the optimal solution, however, is to toll only link 4, a solution which will not be found by the incremental approach. If computing the location index one by one for links 1 and 2, these links will show very low index values. With a collection cost of $C = 10$, these two links would however have a positive effect, a solution which is identified by the approximation approach but not by the incremental approach.

The reason the approximation method fails to identify the optimal toll location on two occasions is explained by the fact that there might be cases when one gets stuck in the ”wrong” local maxima. Consider the case when $C = 110$. After some iterations, all τ-values are zero except for link 4, and the objective can be plotted with respect to τ_4 (Figure 5.6) for some of the iterations, represented by the different k-values. For k between 3 and 6, the objective is concave, and the global maximum at $\tau_4 = 0.33$ for $k = 6$ is the starting point of the maximum search at iteration 7. Since the objective increases in the neighborhood of 0.33 for $k = 7$, the search results in the local maximum $\tau_4 = 0.42$ although the highest value is obtained at $\tau_4 = 0$. Observe that Figure 5.6 only shows some of the iterations ($\epsilon = 0.25$). Different values on ϵ have been tested and in this example, a smaller size of ϵ seems to have no effect on the results but in general the size of ϵ will affect the final result.

Table 5.3: Resulting tolls and improvement of the net social surplus when using the incremental approach and the approximation

Collection cost	*Method*	*Link 1*	*2*	*3*	*4*	*5*	ΔNSS
10	Inc.	0	0	2.33	0.50	0	147.8
	Approx.	1.02	1.02	1.31	0.50	0	153.8
	Bench.	1.02	1.02	1.31	0.50	0	153.8
20	Inc.	0	0	2.33	0.50	0	127.8
	Approx.	0	0	2.33	0.50	0	127.8
	Bench.	0	0	2.33	0.50	0	127.8
60	Inc.	0	0	2.33	0.50	0	47.8
	Approx.	0	0	2.33	0.50	0	47.8
	Bench.	0	0	2.33	0.50	0	47.8
70	Inc.	0	0	2.33	0.50	0	27.8
	Approx.	0	0	2.33	0.50	0	27.8
	Bench.	0	0	0	0.52	0	30.5
80	Inc.	0	0	0	0	0	0
	Approx.	0	0	0	0.52	0	20.5
	Bench.	0	0	0	0.52	0	20.5
100	Inc.	0	0	0	0	0	0
	Approx.	0	0	0	0.52	0	0.5
	Bench.	0	0	0	0.52	0	0.5
110	Inc.	0	0	0	0	0	0
	Approx.	0	0	0	0.52	0	−9.5
	Bench.	0	0	0	0	0	0
115	Inc.	0	0	0	0	0	0
	Approx.	0	0	0	0	0	0
	Bench.	0	0	0	0	0	0

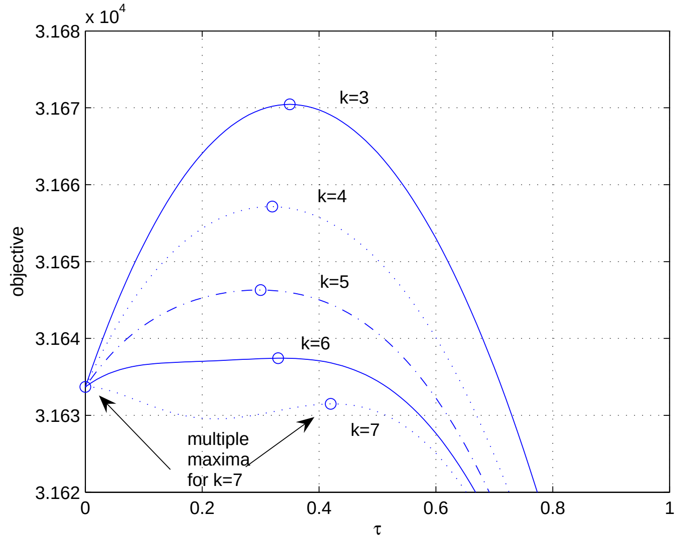

Figure 5.6: Plot of the objective for problem (5.4) with respect to τ_4 at different iterations in the approximation approach, for $C = 110$, local maxima for each k are marked with a circle

5.3.2 The Nine node network

The Nine node network layout is presented in Figure 5.7, and is identical to the network presented in Chapter 4. For the network used here, the travel cost functions are, however, non-linear and on the form $t_a(v_a) = T_a(1 + 0.15(v_a/K_a)^4)$, where T_a is the free flow travel cost and K_a the capacity of the link (Yildirim and Hearn, 2005). As before, the inverse demand functions are on the form $D_i^{-1}(q_i) = \psi_i - \varsigma_i q_i$ and parameter values for the travel cost and demand functions are given in Table 5.4.

Yildirim and Hearn (2005) present different first-best pricing solutions for this network and manage to find an optimal solution with tolls on only 5 links. This information can be used to compute a lower bound ($\underline{\Delta NSS}$) of the optimal solution to (3.8), to which the solutions produced by the incremental and approximation approaches can be compared. User optimal and system optimal solutions, together with the MSCP tolls, are presented in Table 5.4.

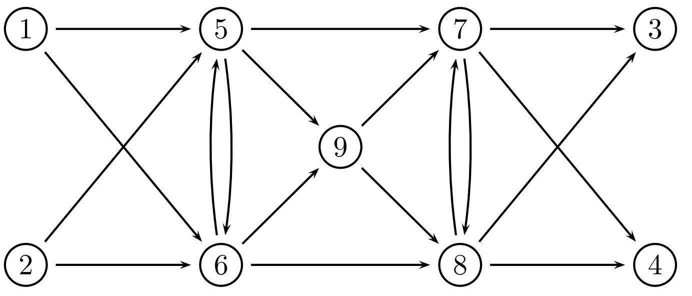

Figure 5.7: The Nine node network layout

Results are presented in Table 5.5 for both the incremental approach and approximation approach, for three different collection costs. For $C = 1$ and $C = 2$, the approximation approach performs better than the incremental approach and for $C = 3$ both methods give the same result. In this network, it is very beneficial to toll link 6 and it is therefore reasonable that both methods identify this link as part of the optimal set of tolls for all three collection costs. Besides the toll on link 6, the solutions obtained from the two methods have no other toll facilities in common.

Disregard link 6 for a moment and focus on links 3 and 9, and 2 and 11. To toll only link 3 would give a benefit of 0.2 and to toll only link 9 would give a benefit of 0.02. These results are compared with the benefit of tolling only links 2 and 11, which would give a benefit of 1.6 and 1.9 respectively. It therefore seems reasonable that besides tolling link 6, the benefit would be higher if links 2 and 11 were also tolled, compared to adding tolls on links 3 and 9. Tolling links 2, 6 and 11 is indeed the solution obtained by the incremental approach. If these two combinations are evaluated together with a toll on link 6, the result is somewhat different. The benefit from adding tolls on links 3, 6 and 9 is 143.0 and the benefit from adding tolls on links 2, 6 and 11 is 141.9. To see how the simultaneous tolling of links 3 and 9 is affected by a toll on link 6, the benefit from tolling links 3, 6 and 9 is compared to the benefit from tolling links 3 and 9. The benefit of tolling links 3 and 9 is 3.8 and the benefit of tolling only link 6 is 138.4. Since the benefit of adding tolls on links 3, 6 and 9 is 143.0, it is concluded that the benefit is further improved when all three links are tolled simultaneously. That the approximation approach succeeds in identifying links which will produce high benefit when tolled simultaneously despite the fact that the benefits are low (for links 3 and 9) when tolled separately, is an important result.

In the Nine node network, there are differences between the magnitude of the toll levels, and one possible problem with the approximation approach is that the same value on k is used for all links. Notice that as k tends to infinity, $\gamma(k, \tau_a)$ tends to 1 and $\frac{\partial}{\partial \tau_a}\gamma(k, \tau_a)$ tends to 0, for every link a with $\tau_a > 0$. The effect of this will be that despite the fact that a location with a high τ-level will have a larger contribution to the collection cost, the contribution to the derivative of the net social surplus measure with respect to τ_a will be smaller for the location with the larger τ-value (see Figure 5.1). This indicates that it is necessary to chose the incremental scheme for k carefully.

In Table 5.6, we give the preliminary computation time for the two methods. The incremental approach requires a more accurate solution of each user equilibrium problem but it is apparent that the approximation approach still requires more computation time.

In each iteration of the approximation approach, the approximated problem (5.4) or (5.5), depending on what demand model is used, has to be solved. When **A1** were applied to the Nine node network in Chapter 4, there where very small differences compared to using **A2**. When the approximation is applied and $-\sum_{a \in \mathcal{A}} C_a \gamma(k, \tau_a)$ is added to the objective, some τ-levels will be driven towards zero. When any τ_a is close to zero the step length used in **A1** will be restricted, and the convergence rate will be reduced. This was noted when using the approximation approach on the two small networks, and there is no reason to assume that this will not be the case for larger network. Therefore **A2** was developed, and will be used in the next example.

Table 5.4: Link parameters (T_a, K_a), and OD parameters (ψ_i, ς_i), together with the user optimal and the system optimal solutions for the Nine node non-linear network

	UO		*SO*		
Link $(T_a,\ K_a)$	*Flow*	*Cost*	*Flow*	*Cost*	*MSCP tolls*
1 $(5, 12)$	0	5.0	0	5.0	0
2 $(6, 18)$	10.9	6.1	9.7	6.1	0.3
3 $(3, 35)$	34.5	3.4	31.7	3.3	1.2
4 $(9, 35)$	15.4	9.1	16.0	9.1	0.2
5 $(9, 20)$	0	9.0	0	9.0	0
6 $(2, 11)$	26.4	12.0	18.0	4.1	8.6
7 $(8, 26)$	8.0	8.0	13.7	8.1	0.4
8 $(4, 11)$	0	4.0	0	4.0	0
9 $(6, 33)$	26.3	6.4	25.7	6.3	1.3
10 $(7, 32)$	0	7.0	0	7.0	0
11 $(3, 25)$	20.8	3.2	19.5	3.2	0.7
12 $(6, 24)$	13.8	6.1	12.2	6.1	0.2
13 $(2, 19)$	0	2.0	0	2.0	0
14 $(8, 39)$	0	8.0	0	8.0	0
15 $(6, 43)$	26.1	6.1	25.7	6.1	0.5
16 $(4, 36)$	0.2	4.0	0	4.0	0
17 $(4, 26)$	8.0	4.0	13.7	4.0	0.2
18 $(8, 30)$	0	8.0	0	8.0	0
OD-pair $(\psi_i,\ \varsigma_i)$					
1-3 $(20, 2)$	2.55		1.64		
1-4 $(40, 2)$	9.51		7.81		
2-3 $(60, 2)$	21.32		19.86		
2-4 $(80, 2)$	61.28		26.03		
SS		1396.3		1539.3	
ΔSS		0		143.0	

Table 5.5: Results for different collection costs in the Nine node network

	C = 1		*C = 2*		*C = 3*	
Link	*Inc.*	*Approx.*	*Inc.*	*Approx.*	*Inc.*	*Approx.*
1	0	0	0	0	0	0
2	2.2	0	0	0	0	0
3	0	2.1	0	2.1	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	8.0	8.0	8.0	8.0	8.0	8.0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	2.1	0	2.1	0	0
10	0	0	0	0	0	0
11	2.5	0	2.5	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0
ΔNSS	138.9	140.0	136.6	137.0	135.4	135.4
$\Delta \underline{NSS}$	138.0		133.0		128.0	

Table 5.6: Computation time in CPU-seconds for the two different methods

Approximation			Incremental		
$C = 1$	$C = 2$	$C = 3$	$C = 1$	$C = 2$	$C = 3$
288	241	205	210	160	118

5.3.3 The Sioux Falls network

The same version of the Sioux Falls network as in Chapter 4 will be used for numerical experiments, for details on this network see Chapter 4, and Appendix B. In Chapter 4, three different judgmental cordon structures were evaluated for this network, and in this section we compare these judgmental cordons with the result when applying the approximation approach. All links are considered as tollable and the cost of collecting a toll in the Sioux Falls network is assumed to be equal for all links, and is set to 1 500 SEK[1] per hour. Travel costs are given in minutes, and the value of time is assumed to be 1 SEK per minute.

A2 has been used to solve (5.5) with a fixed number of iterations (40), to limit the computational time, and in each iteration of the approximation approach θ was set to 0.99.

In Table 5.7, the value of k, the number of tolled links and the lower bound of the net social surplus are presented for each iteration and in Figure 5.8 and 5.9 the final solution is presented. The road users are tolled when crossing the lines in Figure 5.8, and the tolls divide the network into the four zones (Table 5.8) are part of closed cordons. Besides the toll facilities in the closed cordons there are three additional toll facilities (links 5-6, 6-5 and 17-19). The second-best toll levels are given in Figure 5.9. The total flow on the tolled links is reduced by around 20%, and the total car demand (all OD pairs) is reduced by around 10%. The resulting net social surplus is 32 419 and this result can be compared to the three judgmental cordons (Table 5.9). The solution suggested by the approximation approach produce an improvement in the net social surplus which is 36% higher compared to J2, which is the judgmental cordon with the highest improvement of the net social surplus.

In the solution obtained with the approximation approach, the toll levels range between 2.8 SEK and 21 SEK (Figure 5.9) and the social surplus (i.e. excluding the collection costs) will increase to 72 919, which can be compared to the MSCP solution for which social surplus increase with 83 828. The increase in social surplus for the second-best solution obtained by the approximation approach, with 27 toll facilities, is only 13.0% lower than the first-best solution whit 76 tolled links, but with

[1] 1 SEK $\approx$ 13 cent

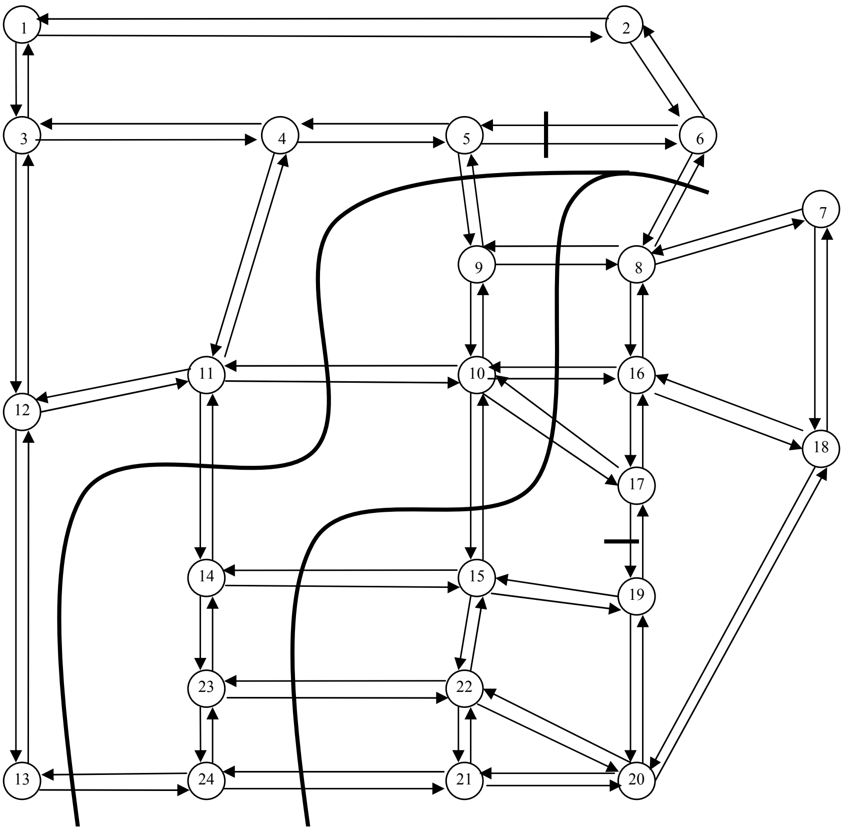

Figure 5.8: Final closed cordon structure

49 fewer toll facilities.

When studying the result, a reasonable question is if both the closed cordons in Figure 5.8 are really necessary. Comparing with the results for the judgmental cordons we remember that the benefit from applying the combined judgmental cordon, did not improve very much from only using the single one with best improvement in social surplus. Turning to the cordon structure suggested by the approximation approach, could a better result be obtained with only one closed cordon, and the toll levels re-optimized? Table 5.8 present the four zones which are separated by the closed cordons in Figure 5.8. If the toll facilities, separating zone 3 and 4, from zone 1, are removed, and the toll levels are re-optimized (denoted as Approx1), the net social surplus is reduced to 12 832 . The

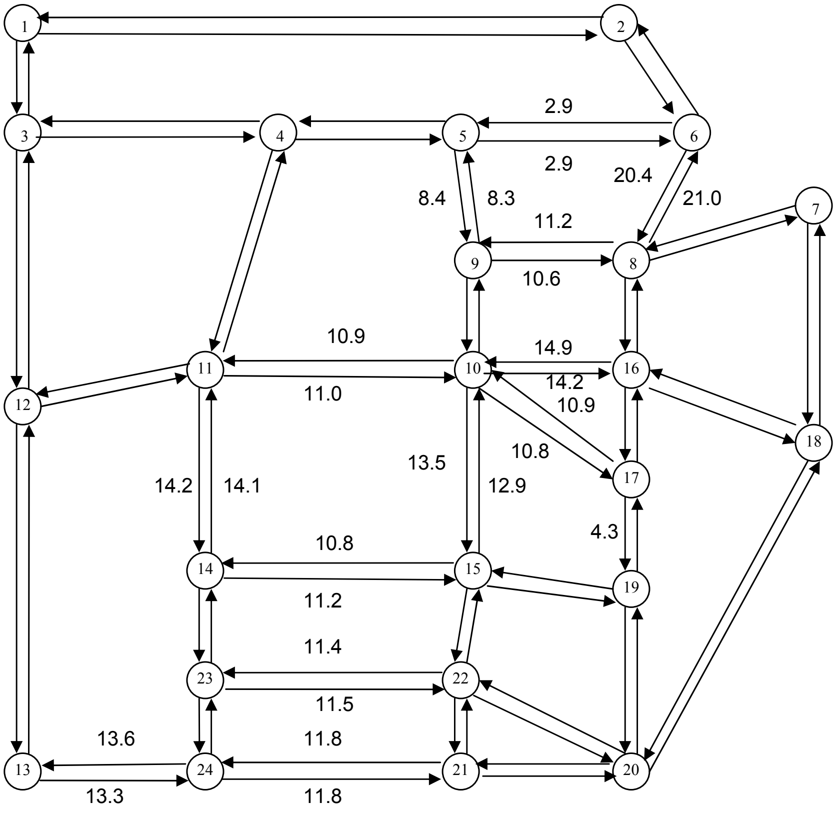

Figure 5.9: Final toll levels

same is done for the tolls separating zone 3 and 4, from zone 2 (denoted as Approx2), which give a net social surplus of 20 977.

In Table 5.9 all six cordons structures are compared, for the change in net social surplus, social surplus and the number of toll facilities. If net social surplus from the two single judgmental cordons are added together (42 848) and compared with the sum of the net social surplus from (Approx1) and (Approx2) (33 809) the judgmental cordons seems to be the one which would produce the most benefit. When the closed cordons are combined the approximation approach presents a far better solution. The increase in net social surplus for the approximation approach is also clearly better than any of the two cordons (Approx1) and (Approx2) separately.

Table 5.7: Number of tolled links and lower bound ($\Delta \underline{NSS}$), after each iteration

Iteration	k	Number of tolled links	$\Delta \underline{NSS}$
1	0	76	−33 579
2	0.05	72	−26 651
3	0.10	68	−20 829
4	0.15	67	−20 702
5	0.20	50	3 511
6	0.25	48	6 275
7	0.30	48	6 022
8	0.40	44	12 022
9	0.50	42	13 393
10	0.60	40	15 631
11	0.70	34	22 642
12	0.80	29	29 916
13	0.90	27	32 330
14	1.00	27	32 367
15	1.25	27	32 402
16	∞	27	32 419

As a last comparison we remove the three toll locations which are not part of any closed cordon (links 5-6, 6-5 and 17-19). Re-optimizing the toll levels for the remaining toll locations give a net social surplus of 33 043, which is an additional increase of 624. In the solution from the approximation approach the toll levels on these three links are considerably smaller, compared to the toll levels on the other tolled links. The reason that the approximation approach chose to toll these three links is most likely connected to the problem of the method getting stuck in the "wrong" local maxima, as was discussed for the Four node network. Still, the net social surplus is only increased by 1.9%, when the three toll locations are removed.

Table 5.8: The four zones separated by the solution given by the approximation approach

Zone	*Nodes*
1	1 to 6 and 11 to 13
2	7, 8 and 15 to 22
3	9 and 10
4	14, 23 and 24

Table 5.9: Net social surplus, social surplus and number of tolled links with six different cordon structures

Scenario	ΔNSS	ΔSS	*Number of tolled links*
J1	18 968	33 968	10
J2	23 880	41 880	12
J3	22 541	55 541	22
Approx1	12 832	41 332	19
Approx2	20 977	40 477	13
Result from the approximation approach	32 419	72 919	27

6 Conclusions and further research

In this thesis, heuristic solution procedure have been developed for solving two different, but related, congestion pricing problems.

For the level setting problem a sensitivity analysis based approach has been applied, which, for a small network, has shown to converge towards the known optimal solution for both the first-best case and a second-best example. For the Sioux Falls network, three judgmental cordon structures are evaluated with the same method. The toll levels suggested by the method greatly improve the social surplus for all three cordon structures.

Two heuristic approaches have been presented, for finding the optimal number of toll facilities, their locations and individual toll levels, with the objective to maximize the net social surplus. The incremental approach has a known problem with estimating the improvement in social surplus from tolling parallel routes. This is discussed by Shepherd et al. (2001), Verhoef (2002a) and Shepherd and Sumalee (2004), and this has become clear in the numerical results for both the Four node and the Nine node network. Even when there is no problem with parallel routes, the incremental approach may fail to identify the correct set of tolls.

The approximation approach has been shown to be able to find the optimal set of tolls and their corresponding levels for several different collection costs in the Four node network. An important result is that the approximation approach is able to identify sets of links which will produce a high increase of the net social surplus when tolled simultaneously. Compared to the incremental approach, the approximation approach may therefore be more suitable for finding appropriate toll locations in large road networks.

Applying the approximation approach to the Sioux Falls network, the method finds an attractive cordon structure which yields a high increase in the net social surplus. If the collection cost is not included, the increase in social surplus is only 13.0% from the benefit of first-best tolling, and with tolls located on considerable fewer links.

In the Sioux Falls network, the approximation approach also suggests additional toll locations besides the one that are part of the closed cordons. The result would however further improve if these additional toll locations were not chosen. A similar problem appears when applying the approximation approach on the Four node network. First of all this implies that one should be careful with solutions that contain toll locations which are not included in closed cordons, and further analyze, the need for these additional toll locations. Secondly the magnitude of the toll levels on these three additional links are much lower compared to the rest of the toll levels, and it is possible that this difference in magnitude present a potential problem for the method. If there is a difference in magnitude between different toll levels, the incremental scheme of the parameter k may have a larger impact on the result. Further experiments need to be conducted to investigate the importance of the incremental scheme of the parameter k. The possibility to use differentiated schemes for tolls with different magnitudes and different functional form of $\gamma(k, \tau_a)$ are other topics which it would be interesting to explore.

For further research, efficient cordon structures are an interesting area. In the numerical examples it has been shown that the closed cordon structure greatly improves the social surplus compared to an open cordon, and this suggests that closed cordon structures are efficient structures. When applying an open cordon structure the sensitivity analysis method is more sensitive to the starting solution, and this imply that besides giving large improvement of the social surplus, closed cordons have fewer local optima, and result in a more smooth landscape of the objective function, in the level setting problem. To further investigate the properties of efficient cordon structure is an interesting area for further research.

Choosing the numbers of toll locations by only considering the net social surplus, i.e. an improvement in social surplus larger than the setup and operational costs, might have practical disadvantages. The pricing scheme which gives the best improvement in net social surplus might still be too expensive. To combine the model with a budget restriction can therefore be an interesting area of further research. Also, there may be additional restrictions on the toll levels, e.g. a finite number of allowed toll levels, or that the same toll level is required for all toll locations within the same closed cordon. Methods for finding efficient pricing schemes given these additional requirements are necessary to address the problem of finding congestion pricing schemes which can be implemented in practice.

In cordon based congestion pricing systems there is often a transaction cost associated with each collected toll. The size of this cost will depend on the technique used to collect the toll and an interesting extension of the model would be to let part of the collection cost depend of the link flow. Other possible extensions to the model in order to make it more useful in practice, would be to apply more advanced demand models and/or multiple user groups, with different value of time or even charged with different tolls. Also, it would be interesting to extend the congestion pricing models to incorporate other transportation externalities, such as emissions of pollutants and noise, and measures of equity or acceptability.

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A Pivot point modal choice

In this appendix the pivot point version (Kumar, 1980) of the multinomial logit (MNL) model is presented for mode choice between car and public transportation. The public transportation costs are assumed to be fixed and the market shares for each mode of transportation in the no-toll scenario are known.

The traffic network is given by the set of links $\mathcal{A}$ and the set of origin destination (OD) pairs $\mathcal{I}$. For each link $a \in \mathcal{A}$ there is a travel cost function $c_a(v_a)$ of flows v_a.

For each OD pair $i \in \mathcal{I}$ there is a set of routes Π_i, each route $p \in \Pi_i$ with flow f_p. The minimum travel cost in OD pair i, including any tolls is $\hat{\pi}_i$. The flow, v_a, on link a is given by

$$v_a = \sum_{i \in \mathcal{I}} \sum_{p \in \Pi_i} f_p \delta_p^a$$

where δ_p^a takes the value of 1 if route p traverses link a, and 0 otherwise

Given that the public transportation costs, k_i, are fixed, the pivot point version of the MNL mode choice model gives the car demand in OD pair $i \in \mathcal{I}$ as

$$q_i = T_i \frac{P_i^{car}}{P_i^{car} + P_i^{pt} e^{\alpha(\hat{\pi}_i - \pi_i^0)}},$$

where P_i^{car} and P_i^{pt} are the market shares, in the no-toll scenario for car and public transportation respectively and T_i the total number of travelers with access to car in OD pair $i \in \mathcal{I}$. The total demand is fixed and π_i^0 is the car travel cost in OD pair $i \in \mathcal{I}$ in the no-toll scenario. Note that the car demand (or public transportation demand, $T_i - q_i$) is computed without any information of the public transportation travel costs, instead the market share of each mode in the no-toll scenario are required to be known. The market shares are, however, assumed to correspond to the choice probabilities in the no-toll scenario given by the MNL model with travel costs π^0 and k^0.

The car demand can also be expressed as

$$q_i = T_i \frac{A_i}{A_i + K_i e^{\alpha(\hat{\pi}_i - \pi_i^0)}}$$

where A_i and K_i are the car and public transportation demand respectively in the no-toll scenario that correspond with equilibrium OD travel cost π_i^0. Note that $T_i = A_i + K_i$, and two out of these three parameters needs to be given as input together with π_i^0 for each OD pair $i \in \mathcal{I}$.

The inverse travel demand function is

$$\hat{\pi}_i = D_i^{-1}(q_i) = \pi_i^0 + \frac{1}{\alpha}\left[\ln \frac{A_i}{K_i} + \ln\left(\frac{T_i}{q_i} - 1\right)\right],$$

and the pivot point version of the combined user equilibrium and modal choice problem can be formulated as

$$\begin{aligned} \min_{q,v}\ G(q,v) = \quad & \sum_{a \in \mathcal{A}} \int_0^{v_a} c_a(w)dw \\ & - \sum_{i \in \mathcal{I}} \left[\pi_i^0 q_i + \frac{q_i}{\alpha} \ln \frac{A_i(T_i - q_i)}{K_i q_i} + \frac{T_i}{\alpha} \ln \frac{T_i}{T_i - q_i}\right] \\ s.t \quad & \sum_{p \in \Pi_i} f_p = q_i, \quad \forall\ i \in \mathcal{I} \\ & f_p \geq 0, \quad \forall\ p \in \Pi_i,\ i \in \mathcal{I} \\ & q_i \geq 0, \quad \forall\ i \in \mathcal{I} \\ & v_a = \sum_{i \in \mathcal{I}} \sum_{p \in \Pi_i} f_p \delta_p^a, \quad \forall\ a \in \mathcal{A}. \end{aligned} \tag{A.1}$$

For the MNL modal choice between car and public transportation, the change in consumer surplus in OD pair $i \in \mathcal{I}$, ΔCS_i is (Williams, 1977)

$$\Delta CS_i = \frac{T_i}{\alpha} \ln \frac{e^{-\alpha\hat{\pi}_i} + e^{-\alpha k_i}}{e^{-\alpha\pi_i^0} + e^{-\alpha k_i^0}}, \tag{A.2}$$

where k_i^0 and k_i are the costs of traveling by public transportation before and after tolls are introduced respectively, in OD pair $i \in \mathcal{I}$. When the travel cost for public transportation is fixed, i.e. $k_i = k_i^0$, the probability of choosing public transportation in OD pair i is

$$\frac{K_i}{T_i} = \frac{e^{-\alpha k_i}}{e^{-\alpha \pi_i^0} + e^{-\alpha k_i^0}}. \tag{A.3}$$

Also, note that

$$\frac{e^{-\alpha \hat{\pi}_i}}{e^{-\alpha \pi_i^0} + e^{-\alpha k_i^0}} = \frac{e^{-\alpha \hat{\pi}_i}}{e^{-\alpha \pi_i^0}} \frac{e^{-\alpha \pi_i^0}}{e^{-\alpha \pi_i^0} + e^{-\alpha k_i^0}} = \frac{A_i}{K_i} e^{\alpha(\pi_i^0 - \hat{\pi}_i)}. \tag{A.4}$$

Applying (A.3) and (A.4) to (A.2), the change in consumer surplus for OD pair $i \in \mathcal{I}$ can be formulated as

$$\Delta CS_i = \frac{T_i}{\alpha} \ln \left[\frac{A_i}{T_i} e^{\alpha(\pi_i^0 - \hat{\pi}_i)} + \frac{K_i}{T_i} \right].$$

The change in social surplus, ΔSS, is

$$\Delta SS = \frac{1}{\alpha} \sum_{i \in \mathcal{I}} T_i \ln \left[\frac{A_i}{T_i} e^{\alpha(\pi_i^0 - \hat{\pi}_i)} + \frac{K_i}{T_i} \right] + \sum_{a \in A} v_a \tau_a.$$

The level setting problem for the pivot point version of the combined user equilibrium and modal choice problem can be formulated as

$$\begin{aligned} \max_{\tau \in \mathcal{X}} F(v(\tau), \hat{\pi}(\tau), \tau) = \quad & \frac{1}{\alpha} \sum_{i \in \mathcal{I}} T_i \ln \left[\frac{A_i}{T_i} e^{\alpha(\pi_i^0 - \hat{\pi}_i(\tau))} + \frac{K_i}{T_i} \right] \\ & + \sum_{a \in \mathcal{A}} v_a(\tau) \tau_a \end{aligned} \tag{A.5}$$

where $v(\tau)$ is given by the solution to the problem (A.1), from which the minimum OD travel costs $\hat{\pi}(\tau)$ can easily be extracted. The set $\mathcal{X}$ is

$$X = \{\tau | \tau_a \geq 0 \ \forall a \in \mathcal{T}, \ \tau_a = 0 \ \forall a \in \mathcal{A} \setminus \mathcal{T}\}$$

where $\mathcal{T}$ and $\mathcal{A} \setminus \mathcal{T}$, the set of tollable and not tollable links respectively.

For a given solution to (A.1), with link flows v^* and OD travel costs $\hat{\pi}^*$, corresponding to the tolls $\bar{\tau}$, the directional derivatives can be computed by the sensitivity analysis model, presented in Chapter 4. These directional derivatives are used as if they constituted the Jacobians $\frac{\partial v}{\partial \tau}$

and $\frac{\partial \hat{\pi}}{\partial \tau}$, to compute $\nabla F(\tau)$. Each element in $\nabla F(\tau)$ is the derivative of F with respect to a change in the toll level on link b, $\frac{\partial F}{\partial \tau_b}$ and can be computed as:

$$\sum_{i \in \mathcal{I}} \frac{T_i A_i e^{\alpha(\pi_i^0 - \hat{\pi}_i^*)}}{A_i e^{\alpha(\pi_i^0 - \hat{\pi}_i^*)} + K_i} \frac{\partial \hat{\pi}_i}{\partial \tau_b} - \sum_{a \in \mathcal{A}} \bar{\tau}_a \frac{\partial v_a}{\partial \tau_b} + v_b^*.$$

B The Sioux Falls network

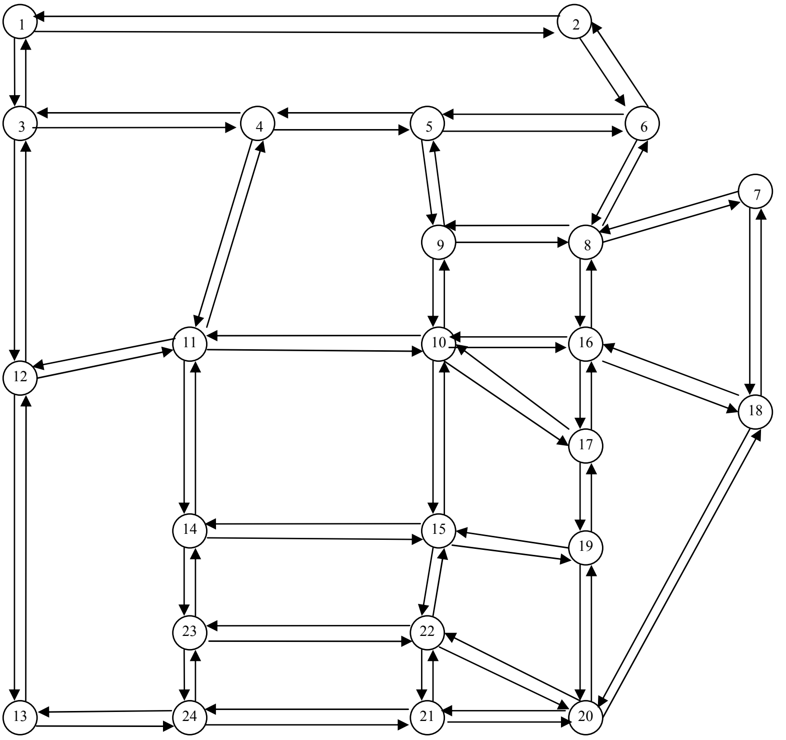

Figure B.1: The Sioux Falls network model

Table B.1: Parameters for the link cost functions to be used with the Sioux Falls network model, and link flows, v_a, and costs $c_a(v_a)$ in the no-toll scenario

Link	T_a	K_a	v_a	$c_a(v_a)$
1 - 2	3.6	4161.8	449.5	3.60
1 - 3	2.4	3760.6	811.9	2.41
2 - 1	3.6	4161.8	451.9	3.60
2 - 6	3	796.7	596.7	3.94
3 - 1	2.4	3760.6	809.5	2.41
3 - 4	2.4	2749.4	1400.6	2.56
3 - 12	2.4	3760.6	1002.2	2.41
4 - 3	2.4	2749.4	1403.1	2.56
4 - 5	1.2	2857.4	1800.6	1.39
4 - 11	3.6	788.8	520.0	4.28
5 - 4	1.2	2857.4	1803.1	1.39
5 - 6	2.4	795.1	879.8	6.00
5 - 9	3	1606.9	1578.1	5.79
6 - 2	3	796.7	599.2	3.96
6 - 5	2.4	795.1	880.6	6.01
6 - 8	1.2	787.1	1249.3	8.81
7 - 8	1.8	1260.1	1210.2	3.33
7 - 18	1.2	3760.6	1579.4	1.24
8 - 6	1.2	787.1	1252.6	8.89
8 - 7	1.8	1260.1	1204.1	3.30
8 - 9	6	811.5	688.3	9.10
8 - 16	3	810.8	838.9	6.44
9 - 5	3	1606.9	1579.7	5.80
9 - 8	6	811.5	683.7	9.02
9 - 10	1.8	2236.1	2174.4	3.41
10 - 9	1.8	2236.1	2181.4	3.43
10 - 11	3	1606.9	1772.7	7.44
10 - 15	3.6	2171.2	2312.6	8.23
10 - 16	2.4	780.1	1104.7	12.05
10 - 17	4.8	802.4	810.0	9.78
11 - 4	3.6	788.8	530.0	4.33
11 - 10	3	1606.9	1760.4	7.32
11 - 12	3.6	788.8	836.5	8.15
11 - 14	2.4	783.6	977.6	8.21
12 - 3	2.4	3760.6	997.4	2.41
12 - 11	3.6	788.8	840.5	8.24
12 - 13	1.8	4161.8	1228.8	1.81

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Table B.1 – Continued

Link	T_a	K_a	v_a	$c_a(v_a)$
13 - 12	1.8	4161.8	1237.9	1.81
13 - 24	2.4	818.1	1112.1	10.60
14 - 11	2.4	783.6	981.4	8.31
14 - 15	3	823.9	903.6	7.34
14 - 23	2.4	791.3	840.0	5.45
15 - 10	3.6	2171.2	2319.2	8.29
15 - 14	3	823.9	908.0	7.42
15 - 19	1.8	2340.3	1908.3	2.60
15 - 22	1.8	1542.5	1841.0	5.45
16 - 8	3	810.8	840.7	6.47
16 - 10	2.4	780.1	1107.3	12.14
16 - 17	1.2	840.4	1169.5	5.70
16 - 18	1.8	3162.3	1527.8	1.90
17 - 10	4.8	802.4	810.0	9.78
17 - 16	1.2	840.4	1168.4	5.68
17 - 19	1.2	775.1	995.3	4.46
18 - 7	1.2	3760.6	1585.5	1.24
18 - 16	1.8	3162.3	1533.3	1.90
18 - 20	2.4	3760.6	1897.7	2.56
19 - 15	1.8	2340.3	1911.7	2.60
19 - 17	1.2	775.1	994.2	4.45
19 - 20	2.4	803.8	868.8	5.68
20 - 18	2.4	3760.6	1899.3	2.56
20 - 19	2.4	803.8	871.1	5.71
20 - 21	3.6	813.1	630.2	4.90
20 - 22	3	815.6	700.0	4.63
21 - 20	3.6	813.1	624.0	4.85
21 - 22	1.2	840.4	862.0	2.53
21 - 24	1.8	785.0	1030.9	7.15
22 - 15	1.8	1542.5	1838.6	5.43
22 - 20	3	815.6	700.0	4.63
22 - 21	1.2	840.4	860.7	2.52
22 - 23	2.4	803.4	966.2	7.42
23 - 14	2.4	791.3	839.5	5.44
23 - 22	2.4	803.4	962.6	7.35
23 - 24	1.2	816.0	790.3	2.26
24 - 13	2.4	818.1	1111.2	10.57
24 - 21	1.8	785.0	1026.0	7.05
24 - 23	1.2	816.0	786.2	2.23

Table B.2: Parameters for the pivot point version of the MNL model, to be used with the Sioux Falls network model

O	D	A_i	T_i	π_i^0	O	D	A_i	T_i	π_i^0
1	2	10	114.8	3.60	12	24	50	119.6	12.41
1	3	10	39.3	2.41	13	1	50	152.6	6.63
1	4	50	158.7	4.97	13	2	30	152.8	10.23
1	5	20	86.0	6.36	13	3	10	50.2	4.23
1	6	30	148.6	7.54	13	4	60	183.8	6.79
1	7	50	175.0	19.66	13	5	20	76.1	8.18
1	8	80	202.9	16.36	13	6	20	100.9	14.18
1	9	50	96.9	12.15	13	7	40	148.4	26.29
1	10	130	341.2	15.56	13	8	60	156.7	22.99
1	11	50	141.7	9.25	13	9	60	158.4	13.97
1	12	20	99.6	4.82	13	10	190	443.8	17.38
1	13	50	178.2	6.63	13	11	100	282.9	10.06
1	14	30	151.8	17.46	13	12	130	294.0	1.81
1	15	50	172.5	23.79	13	14	60	159.7	18.27
1	16	50	113.5	22.80	13	15	70	197.4	25.61
1	17	40	117.6	25.34	13	16	60	174.4	26.95
1	18	10	113.2	20.90	13	17	50	170.2	27.16
1	19	30	151.3	26.39	13	18	10	68.7	25.05
1	20	30	98.1	23.45	13	19	30	136.5	28.21
1	21	10	108.8	24.28	13	20	60	166.4	22.50
1	22	40	81.1	26.81	13	21	60	134.5	17.65
1	23	30	92.1	19.46	13	22	130	328.6	20.18
1	24	10	100.4	17.23	13	23	80	258.2	12.83
2	1	10	16.0	3.60	13	24	80	239.7	10.60
2	3	10	29.6	6.01	14	1	30	128.9	17.61
2	4	20	56.3	8.57	14	2	10	65.9	21.21
2	5	10	35.9	9.96	14	3	10	27.0	15.20
2	6	40	138.4	3.94	14	4	50	164.8	12.64
2	7	20	63.6	16.06	14	5	10	53.4	14.03
2	8	40	95.9	12.76	14	6	10	29.7	20.03
2	9	20	36.6	15.75	14	7	20	94.5	19.41
2	10	60	193.4	19.16	14	8	40	161.4	22.74
2	11	20	81.7	12.85	14	9	60	185.8	19.06
2	12	10	112.8	8.42	14	10	210	519.9	15.63
2	13	30	103.9	10.23	14	11	160	416.0	8.31
2	14	10	59.0	21.06	14	12	70	141.2	16.46

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Table B.2 – Continued

O	D	A_i	T_i	π_i^0	O	D	A_i	T_i	π_i^0
2	15	10	103.9	27.39	14	13	60	152.8	18.27
2	16	40	130.1	19.20	14	15	130	312.6	7.34
2	17	20	56.3	24.90	14	16	70	162.5	20.07
2	19	10	85.6	25.56	14	17	70	197.7	14.38
2	20	10	103.0	19.85	14	18	10	20.2	18.17
2	22	10	17.1	24.48	14	19	30	115.0	9.94
3	1	10	86.5	2.41	14	20	50	168.6	15.61
3	2	10	54.8	6.01	14	21	40	176.0	14.75
3	4	20	122.3	2.56	14	22	120	313.8	12.79
3	5	10	67.8	3.95	14	23	110	292.7	5.45
3	6	30	129.5	9.95	14	24	40	120.3	7.70
3	7	10	60.0	22.07	15	1	50	161.6	23.88
3	8	20	67.0	18.76	15	2	10	99.3	27.48
3	9	10	34.9	9.74	15	3	10	23.8	21.47
3	10	30	75.3	13.15	15	4	50	194.3	18.91
3	11	30	126.6	6.84	15	5	20	131.2	17.52
3	12	20	66.8	2.41	15	6	20	98.2	23.52
3	13	10	71.9	4.23	15	7	50	121.6	12.07
3	14	10	30.8	15.06	15	8	60	206.7	15.40
3	15	10	88.3	21.38	15	9	100	248.9	11.72
3	16	20	74.7	25.20	15	10	400	871.9	8.29
3	17	10	105.3	22.94	15	11	140	319.3	15.73
3	22	10	104.6	24.40	15	12	70	235.9	23.88
3	23	10	77.3	17.06	15	13	70	142.6	25.70
4	1	50	147.1	4.97	15	14	130	313.1	7.42
4	2	20	129.5	8.57	15	16	120	265.7	12.73
4	3	20	121.3	2.56	15	17	150	385.1	7.04
4	5	50	162.7	1.39	15	18	20	70.3	10.83
4	6	40	160.9	7.39	15	19	80	226.4	2.60
4	7	40	144.3	19.50	15	20	110	220.6	8.27
4	8	70	170.9	16.20	15	21	80	192.4	7.97
4	9	70	165.4	7.18	15	22	260	567.3	5.45
4	10	120	270.8	10.59	15	23	100	240.6	12.87
4	11	140	331.1	4.28	15	24	40	134.1	15.13
4	12	60	191.3	4.97	16	1	50	159.7	22.92
4	13	60	147.5	6.79	16	2	40	86.9	19.32
4	14	50	183.0	12.49	16	3	20	129.3	25.33
4	15	50	154.6	18.82	16	4	80	234.2	22.76
4	16	80	193.9	22.64	16	5	50	178.1	21.37

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Table B.2 – Continued

O	D	A_i	T_i	π_i^0	O	D	A_i	T_i	π_i^0
4	17	50	168.8	20.37	16	6	90	260.6	15.36
4	18	10	72.4	20.74	16	7	140	345.4	3.14
4	19	20	81.7	21.42	16	8	220	456.1	6.47
4	20	30	127.9	23.30	16	9	140	303.7	15.57
4	21	20	100.2	24.44	16	10	440	940.8	12.14
4	22	40	158.5	24.28	16	11	140	331.4	19.59
4	23	50	195.5	17.94	16	12	70	141.2	27.74
4	24	20	89.9	17.39	16	13	60	124.3	27.08
5	1	20	127.4	6.36	16	14	70	163.5	20.19
5	2	10	33.2	9.96	16	15	120	278.0	12.76
5	3	10	117.9	3.95	16	17	280	604.8	5.70
5	4	50	123.5	1.39	16	18	50	190.4	1.90
5	6	20	61.5	6.00	16	19	130	317.6	10.16
5	7	20	127.0	18.11	16	20	160	349.5	4.45
5	8	50	172.4	14.81	16	21	60	165.2	9.35
5	9	80	169.3	5.79	16	22	120	297.7	9.08
5	10	100	196.2	9.20	16	23	50	112.0	16.50
5	11	50	188.9	5.67	16	24	30	142.1	16.51
5	12	20	55.9	6.37	17	1	40	175.4	25.38
5	13	20	66.4	8.18	17	2	20	97.5	25.01
5	14	10	84.5	13.88	17	3	10	18.0	22.97
5	15	20	64.9	17.43	17	4	50	180.3	20.41
5	16	50	144.3	21.25	17	5	20	99.1	19.02
5	17	20	41.8	18.99	17	6	50	168.7	21.05
5	19	10	118.8	20.03	17	7	100	204.7	8.82
5	20	10	76.2	21.91	17	8	140	319.6	12.15
5	21	10	59.5	25.41	17	9	90	214.4	13.22
5	22	20	89.1	22.89	17	10	390	792.4	9.78
5	23	10	50.1	19.33	17	11	100	282.5	17.23
6	1	30	100.5	7.56	17	12	60	203.1	25.38
6	2	40	98.7	3.96	17	13	50	142.4	27.20
6	3	30	115.9	9.97	17	14	70	235.4	14.49
6	4	40	154.8	7.40	17	15	150	310.5	7.06
6	5	20	90.6	6.01	17	16	280	646.3	5.68
6	7	40	142.3	12.12	17	18	60	195.8	7.58
6	8	80	177.0	8.81	17	19	170	381.6	4.46
6	9	40	114.9	11.80	17	20	170	400.2	10.14
6	10	80	237.2	15.21	17	21	60	214.9	15.04
6	11	40	146.5	11.68	17	22	170	402.2	12.52

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Table B.2 – Continued

O	D	A_i	T_i	π_i^0	O	D	A_i	T_i	π_i^0
6	12	20	83.4	12.38	17	23	60	141.0	19.94
6	13	20	94.6	14.19	17	24	30	92.0	22.19
6	14	10	98.4	19.90	18	1	10	34.7	21.02
6	15	20	41.2	23.45	18	4	10	66.5	20.87
6	16	90	238.3	15.25	18	6	10	58.0	13.46
6	17	50	100.3	20.95	18	7	20	83.7	1.24
6	18	10	58.6	13.35	18	8	30	119.1	4.57
6	19	20	67.0	21.62	18	9	20	42.5	13.67
6	20	30	146.8	15.91	18	10	70	168.0	14.04
6	21	10	16.6	20.81	18	11	20	98.9	21.48
6	22	20	115.6	20.54	18	12	20	53.4	25.84
6	23	10	116.9	25.35	18	13	10	80.2	25.18
6	24	10	119.0	24.79	18	14	10	40.8	18.29
7	1	50	177.8	19.79	18	15	20	96.7	10.87
7	2	20	81.1	16.19	18	16	50	148.1	1.90
7	3	10	67.3	22.19	18	17	60	163.8	7.60
7	4	40	97.5	19.63	18	19	30	111.8	8.26
7	5	20	102.6	18.24	18	20	40	173.9	2.56
7	6	40	108.6	12.23	18	21	10	50.9	7.46
7	8	100	295.8	3.33	18	22	30	97.2	7.18
7	9	60	191.3	12.44	18	23	10	47.3	14.60
7	10	190	418.3	15.28	19	1	30	98.2	26.48
7	11	50	173.2	22.72	19	2	10	45.0	25.65
7	12	70	163.1	24.60	19	4	20	76.4	21.51
7	13	40	121.2	26.42	19	5	10	67.8	20.12
7	14	20	133.0	19.53	19	6	20	110.8	21.70
7	15	50	166.7	12.10	19	7	40	107.2	9.47
7	16	140	297.3	3.14	19	8	70	146.8	12.80
7	17	100	283.1	8.84	19	9	40	121.5	14.32
7	18	20	101.0	1.24	19	10	180	404.0	10.89
7	19	40	89.0	9.50	19	11	40	76.5	18.33
7	20	50	116.7	3.79	19	12	30	124.7	26.49
7	21	20	98.8	8.69	19	13	30	131.0	28.30
7	22	50	161.1	8.42	19	14	30	84.6	10.03
7	23	20	73.9	15.84	19	15	80	182.5	2.60
7	24	10	75.4	15.85	19	16	130	329.4	10.13
8	1	80	202.4	16.45	19	17	170	417.3	4.45
8	2	40	79.6	12.85	19	18	30	158.5	8.23
8	3	20	37.9	18.86	19	20	120	284.7	5.68

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Table B.2 – Continued

O	D	A_i	T_i	π_i^0	O	D	A_i	T_i	π_i^0
8	4	70	167.8	16.30	19	21	40	169.8	10.57
8	5	50	96.4	14.91	19	22	120	282.4	8.05
8	6	80	195.3	8.89	19	23	30	139.5	15.47
8	7	100	266.7	3.30	19	24	10	102.0	17.73
8	9	80	164.7	9.10	20	1	30	72.5	23.58
8	10	160	318.7	12.51	20	2	10	56.4	19.98
8	11	80	219.3	19.96	20	4	30	109.7	23.42
8	12	60	178.9	21.27	20	5	10	90.4	22.03
8	13	60	116.7	23.09	20	6	30	114.8	16.02
8	14	40	76.7	22.83	20	7	50	143.4	3.79
8	15	60	135.0	15.40	20	8	90	221.8	7.13
8	16	220	496.6	6.44	20	9	60	124.2	16.23
8	17	140	281.0	12.14	20	10	250	541.6	16.60
8	18	30	93.6	4.54	20	11	60	153.5	24.04
8	19	70	201.3	12.80	20	12	50	126.8	24.44
8	20	90	250.4	7.09	20	13	60	204.4	22.62
8	21	40	147.7	11.99	20	14	50	174.7	15.74
8	22	50	103.8	11.72	20	15	110	314.7	8.31
8	23	30	102.7	19.14	20	16	160	373.6	4.46
8	24	20	81.4	19.15	20	17	170	336.5	10.16
9	1	50	132.1	12.16	20	18	40	137.6	2.56
9	2	20	51.1	15.76	20	19	120	320.7	5.71
9	3	10	85.9	9.76	20	21	120	337.6	4.90
9	4	70	208.4	7.19	20	22	240	498.3	4.63
9	5	80	231.4	5.80	20	23	70	208.9	12.05
9	6	40	125.2	11.80	20	24	40	129.8	12.05
9	7	60	173.3	12.32	21	1	10	113.0	24.36
9	8	80	167.7	9.02	21	4	20	109.9	24.51
9	10	280	602.3	3.41	21	5	10	38.9	25.48
9	11	140	350.2	10.85	21	6	10	62.2	20.87
9	12	60	208.7	12.17	21	7	20	53.1	8.64
9	13	60	143.7	13.98	21	8	40	176.7	11.97
9	14	60	141.8	19.07	21	9	30	92.4	19.68
9	15	90	265.9	11.64	21	10	120	240.1	16.25
9	16	140	299.4	15.46	21	11	40	154.3	23.13
9	17	90	259.5	13.19	21	12	30	149.0	19.54
9	18	20	130.4	13.56	21	13	60	145.0	17.72
9	19	40	99.3	14.24	21	14	40	101.4	14.83
9	20	60	140.1	16.12	21	15	80	252.9	7.96

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Table B.2 – Continued

O	D	A_i	T_i	π_i^0	O	D	A_i	T_i	π_i^0
9	21	30	60.2	19.62	21	16	60	128.8	9.30
9	22	70	143.2	17.10	21	17	60	213.8	15.01
9	23	50	162.3	24.52	21	18	10	88.7	7.41
9	24	20	55.0	24.58	21	19	40	164.0	10.56
10	1	130	343.6	15.59	21	20	120	257.0	4.85
10	2	60	133.3	19.19	21	22	180	402.8	2.53
10	3	30	72.9	13.19	21	23	70	143.5	9.39
10	4	120	339.4	10.62	21	24	50	184.4	7.15
10	5	100	241.2	9.23	22	1	40	134.0	26.88
10	6	80	190.7	15.23	22	2	10	48.5	24.61
10	7	190	408.0	15.19	22	3	10	54.4	24.47
10	8	160	353.3	12.45	22	4	40	166.1	24.34
10	9	280	596.3	3.43	22	5	20	74.1	22.95
10	11	400	857.1	7.44	22	6	20	42.7	20.65
10	12	200	407.6	15.60	22	7	50	116.0	8.42
10	13	190	379.0	17.41	22	8	50	100.2	11.75
10	14	210	463.2	15.66	22	9	70	194.5	17.15
10	15	400	886.3	8.23	22	10	260	527.8	13.72
10	16	440	973.1	12.05	22	11	110	269.8	21.16
10	17	390	802.8	9.78	22	12	70	147.3	22.06
10	18	70	151.8	13.95	22	13	130	335.8	20.25
10	19	180	446.6	10.83	22	14	120	274.4	12.86
10	20	250	520.0	16.50	22	15	260	601.5	5.43
10	21	120	302.8	16.21	22	16	120	239.9	9.08
10	22	260	616.5	13.69	22	17	170	397.8	12.48
10	23	180	424.8	21.11	22	18	30	154.7	7.18
10	24	80	246.4	23.36	22	19	120	265.3	8.03
11	1	50	96.0	9.30	22	20	240	568.3	4.63
11	2	20	49.4	12.90	22	21	180	365.7	2.52
11	3	30	141.0	6.90	22	23	210	421.9	7.42
11	4	150	340.2	4.33	22	24	110	239.6	9.68
11	5	50	188.5	5.72	23	1	30	153.0	19.46
11	6	40	152.2	11.72	23	3	10	21.6	17.05
11	7	50	167.2	22.51	23	4	50	122.7	18.08
11	8	80	191.3	19.78	23	5	10	120.0	19.47
11	9	140	292.4	10.75	23	6	10	37.3	25.47
11	10	390	791.3	7.32	23	7	20	87.3	15.77
11	12	140	295.1	8.15	23	8	30	85.5	19.10
11	13	100	239.4	9.97	23	9	50	165.6	24.50

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Table B.2 – Continued

O	D	A_i	T_i	π_i^0	O	D	A_i	T_i	π_i^0
11	14	160	404.9	8.21	23	10	180	455.6	21.07
11	15	140	326.5	15.56	23	11	130	335.5	13.75
11	16	140	360.7	19.37	23	12	70	204.9	14.64
11	17	100	243.4	17.11	23	13	80	168.7	12.83
11	18	10	63.0	21.27	23	14	110	225.0	5.44
11	19	40	122.3	18.15	23	15	100	196.6	12.78
11	20	60	158.3	23.83	23	16	50	125.3	16.43
11	21	40	169.7	22.97	23	17	60	200.8	19.82
11	22	110	215.6	21.01	23	18	10	118.5	14.53
11	23	130	286.2	13.66	23	19	30	56.8	15.38
11	24	60	120.2	15.92	23	20	70	221.0	11.97
12	1	20	107.8	4.82	23	21	70	200.2	9.31
12	2	10	83.3	8.42	23	22	210	473.8	7.35
12	3	20	138.2	2.41	23	24	70	160.6	2.26
12	4	60	173.0	4.97	24	1	10	101.3	17.20
12	5	20	77.0	6.36	24	4	20	62.6	17.36
12	6	20	55.9	12.36	24	6	10	94.1	24.75
12	7	70	200.6	24.48	24	7	10	84.3	15.69
12	8	60	192.0	21.18	24	8	20	57.5	19.03
12	9	60	154.5	12.15	24	9	20	98.2	24.54
12	10	200	396.0	15.56	24	10	80	218.5	23.30
12	11	140	319.1	8.24	24	11	60	184.2	15.98
12	13	130	334.1	1.81	24	12	50	114.3	12.38
12	14	70	218.4	16.46	24	13	70	201.8	10.57
12	15	70	231.6	23.80	24	14	40	92.9	7.67
12	16	70	223.7	27.61	24	15	40	131.7	15.01
12	17	60	153.6	25.35	24	16	30	120.5	16.36
12	18	20	100.2	25.71	24	17	30	127.0	22.06
12	19	30	131.8	26.39	24	19	10	86.1	17.61
12	20	40	95.4	24.31	24	20	40	167.1	11.90
12	21	30	150.0	19.46	24	21	50	96.4	7.05
12	22	70	194.8	21.99	24	22	110	247.6	9.58
12	23	70	201.3	14.64	24	23	70	216.8	2.23