

The Black Litterman Asset Allocation Model

An empirical comparison of approaches for estimating the subjective view vector and implications for risk-return characteristics

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Master of Science Thesis in Economics
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Abstract

Background

In the early 90's, Black and Litterman extended the pioneering work of Markowitz by developing a model combining qualitative and quantitative research in a delicate optimization process. It allows for a subjective view parameter in a quantitative model and with absent views, the investor will have no reason to deviate from the market equilibrium portfolio. As one can imagine, the investors' views incorporated in the Black-Litterman model is crucial and is the unique advantage or problem of the model, depending on the user's ability to properly forecast expected return. However, it has yet to be covered thoroughly in the academic literature how different approaches for estimating subjective views actually yield a more attractive risk-return profile.

Purpose

In this study we intend to use the Black-Litterman model with subjective views generated from analysts' forecasts and a statistical valuation multiple in order to compare and analyze how portfolios differentiate regarding asset allocation and risk-return characteristics.

Methodology

Two different valuation approaches are compared and analyzed in the Black-Litterman Asset Allocation Model by running historical simulations on risk adjusted performance. To generate elements for the subjective view vector we use analysts' forecasts and a statistical valuation multiple approach from a fixed effect panel regression. The empirical study has a Swedish perspective with simulations based on data from the OMXS30, with a analyzed period stretching from March of 2008 to March 2018.

Conclusions

Even though analysts' forecasts proved to be the most accurate approach estimating the direction of the stock price and outright return for all given time horizons, the statistical counterpart was the superior when applied in a risk adjusted context in the Black-Litterman model. This holds true for the larger portion of occasions when modifying key input variables such as transaction costs, risk aversion, certainty level and time horizon. Our empirical findings show that the Black-Litterman model is suitable for investment managers committing to the CAPM approach to estimate expected return in the long turn, but who still is managing an *alphad* driven portfolio in the short term, capitalizing on mispricing.

Key Words

Analysts' Forecasts, Statistical Valuation Multiple, Equilibrium Portfolio, Portfolio Optimization.

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1

Introduction

With Markowitz (1952) presenting his work on portfolio theory, he introduced a brilliant quantification of the two basic objectives of investing: maximizing return whilst minimizing risk. Before the pioneering work of Markowitz, the benefits from diversification was yet to be fully discovered apart from the most intuitive advantage of not putting all eggs in one basket. The key message of the mean-variance formulation of the portfolio optimization problem was that an accurately chosen combination of assets could sum up to a portfolio, maximizing expected return and minimizing the volatility, than any of the assets could do separately.

Even though the Markowitz' mean-variance framework establishes a foundation for modern portfolio theory, practical attempts to actually implement it into the portfolio optimization turned out to be more difficult than first could be expected. Mostly because of the complicatedness in finding appropriate estimates for expected return and covariance between individual assets. The implication that follows is that the mean-variance framework composes highly concentrated portfolios only incorporating a minority of the assets and being heavily sensitive to changes in input. (Michaud, 1989)

The aforementioned concerns implied a rather exhausting challenge even for professional investment managers, including the fixed income research function of Goldman Sachs. The unit was at the time working with global portfolios consisting of correlated bonds and currencies in different markets all around the globe (Litterman and Group, 2004). The optimal portfolio weights for these assets when embedded in an portfolio proved to be highly sensitive to even the smallest modification in expected returns. Not surprisingly, this made the portfolio optimization process all but easy.

It was Fischer Black who first expressed the notion to include the International CAPM equilibrium as a reference point, inspiring discussions of how to make appropriate investment decisions in the optimization process (Litterman and Group, 2004). The model has come to be called Black-Litterman Asset Allocation Model and except incorporating the international CAPM equilibrium as a point of departure, it mathematically combines qualitative and quantitative research in an optimization model. Apart from the reference point, the investor has the ability to specify subjective views both in absolute and relative terms without forcing a complete set of expected returns (Black and Litterman, 1991). The equilibrium portfolio, in combination with the subjective view vector, can yield an optimal portfolio considering both the implied market expectations and the private opinions of the particular investor. Due to the intuitive portfolio composition and less extreme weights, the model has been widely appreciated in practice and is up until this day implemented by fund managers (Avanza, 2018).

The delicacy of the Black Litterman Asset Allocation Model is the capability to, in accordance with views on only parts of the expected return vector, automatically adjust the entire vector. In combination with using the global CAPM equilibrium as a starting point, the model suggests less extreme portfolio weights and is less sensitive to input variations relative to predecessors. As one can imagine, the investors' views incorporated in the Black-Litterman model is crucial and is the unique advantage or problem of the model, depending on the user's ability to properly forecast expected return. It allows for a subjective view factor in a quantitative model and with absent views, no active portfolio management is possible and the investor will have no reason to deviate from the market equilibrium portfolio. (Cheung et al., 2009)

Numerous methods have been reported to determine a stock's fair price and there is no unity among practitioners on what approach is leading to the most accurate results. Discounted Cash Flow Valuation (Henceforth DCF) and Peer Valuation are two widely used approaches by professional analysts to determine a company's fair share price. Clement (1999) implies in his study that professional analysts' forecasts exhibit systematic differences in forecast accuracy and predictability depending on the analyst's experience, number of firms covered and the access to resources. On the other hand, Butler and Lang (1991) argue in a paper that individual analysts show little or no evidence of any systematic differentiation in forecast ability over time. Even though various studies can agree upon the disagreement of how successful analysts are in forecasting future stock prices, a paper by Becker and Gürtler (2008) concludes that implementing analysts' forecasts in the investor's subjective views vector outperforms all other Black-Litterman strategies regarding risk-adjusted return.

When valuing a firm's true enterprise value by the means of Peer Valuation, one should focus on comparing fundamental variables such as growth, risk and cash flows (Damodaran, 2002). From the given circumstance, statistical mod-

elling can offer an alternative for certain investors to estimate expected return by running regressions (Damodaran, 2002). It can assist investors to more accurately estimate statistical valuation multiples even in the absence of valid comparison and do still stack up well against traditional approaches (Mckinsey&Company, 2012). Contrarily, a study by Fried and Givoly (1982) provides results indicating that analysts' forecasts are generally more accurate than econometric forecast models to predict future earnings whilst the statistical counterpart is shown to be more successful in capturing the correlation between unexpected earnings and movements in share prices.

However, it has yet to be covered thoroughly in the academic literature how different approaches for estimating subjective views actually yield a more attractive risk-return profile rather than only committing to the CAPM equilibrium portfolio. It is therefore of further interest to analyze how different valuation methods render the portfolio performance with focus on risk-adjusted return rather than outright return. By incorporating analysts' forecasts in the subjective view vector and compare it to results from statistical valuation multiples, one can imagine it to be of significance for professional investment managers to establish which of the two valuation approaches is the more appropriate to implement.

1.1 Purpose

In this study we intend to use the Black-Litterman model with subjective views generated with analysts' forecasts and a statistical valuation multiple in order to compare and analyze how portfolios differentiate regarding asset allocation and risk-return characteristics.

1.2 Hypothesizing Questions

- For Black-Litterman portfolios with subjective views generated from analysts' forecasts or a statistical valuation multiple, what are the differences in risk-return characteristics?
- How does the two valuation approaches alter the asset allocation in the Black-Litterman model?
- How does decisions faced by investors in terms of key input variables in the model alter the portfolio performances?
- What practical considerations for investors are important to take into account before committing to one approach rather than the other?
- How does analysts' forecasts perform in a risk adjusted context compared to previous research?

1.3 Delimitations

We are restricted to use shares included in the OMXS30 index since they are the most frequently traded and largest companies on the Stockholm Stock Exchange and covered by professionals why this restriction is of importance for the execution of this study. The data set in this study stretches over a ten year period between March 13, 2006 to March 13, 2018. The investable universe of our simulations is restricted to the OMXS30 (OMX Stockholm 30 Index), a market weighted price index consisting of the 30 most actively traded stocks on the Stockholm Stock Exchange (Nasdaq, 2016). The index is rebalanced semi-annually why the index composition changes over time. Since March 1, 2008, the following seven companies have joined and left OMXS30:

Table 1.1: Joiners and Leavers in Index Constituents

Date	Joiner	Leaver
2017-06-12	Essity	Lundin Petroleum
2017-01-02	Autoliv	Nokia
2014-07-01	Kinnevik	Scania
2009-07-01	Getinge	Eniro
2009-07-01	Modern Times Group	Vostok Gas
2008-12-10	Lundin Petroleum	Autoliv

Since the companies Essity, Autoliv, FPC, Kinnevik, Getinge and MTG have joined OMXS30 during the time period, those are excluded while conducting the historical simulations. This study will therefore be conducted on a 24-asset universe.

1.4 Contribution

By applying valuation methods stemming from outright return focused academia in a quantitative equilibrium model, we aim to distinguish results in a risk-adjusted context to add another focus that has up until this day not been investigated as comprehensively. With regards to the Black-Litterman model, a large portion of previous research have focused to a large extent on the mathematical underpinnings of the model, why we intend to add on research regarding the contribution of the subjective view vector for the overall portfolio and its risk-return characteristics.

2

The Black-Litterman Model and Subjective Views

2.1 Capital Asset Pricing Model

The starting point of the Black-Litterman model is the expected returns from the market equilibrium. The Capital Asset Pricing model (CAPM) is a market equilibrium model, assuming expected returns of all assets will converge towards an equilibrium in such manner that if all investors hold the same belief, demand will perfectly meet supply (He and Litterman, 2002).

In the mid 1960s, Sharpe, Lintner and Treynor were among the first to establish the foundation of the CAPM. The model's theoretical starting point is that in a competitive landscape, the expected return of an asset is a function of its covariance with the overall market portfolio and the expected return and variance of the market (Brealey et al., 2010). The CAPM is not restricted to a smaller asset universe, but could theoretically model any asset where the appropriate market portfolio consists of all such assets, and is therefore practically close to unobservable in reality. In many cases though, economists utilize the model in a smaller exclusive context where the investable universe is limited to publicly traded securities on global financial markets. In this scenario, the theoretical market portfolio is often proxied with a broad market index (Bodie et al., 2011). According to Bodie et al. the CAPM is resting upon a set of assumptions¹ To validate the

¹The assumptions behind the model can be summarised in seven bullets: 1) The market consists of many investors and they are all price takers. 2) All investors have the same investment horizon. 3) No taxes or transaction costs. 4) All investors can invest in the same risk-free rate. 5) All investors are rational with the same risk-return preferences. 6) the market is characterized by information symmetry and returns are normally distributed. 7) The market portfolio consists of all publicly traded assets.

model where the algebraic representation is presented as follows:

$$E(R_i) - R_f = \beta_i (E(R_m) - R_f) \quad (2.1)$$

$$\beta_i = \frac{\text{Cov}(R_i; R_m)}{\text{Var}(R_m)}$$

where,

$E(R_i)$ is the expected total return on the i :th asset.

R_f is the risk-free rate of interest.

$E(R_m)$ is the expected total return on the market portfolio.

β_i is in what extent the i :th asset move together expected market return.

In short, the CAPM models the basic linear relationship between risk and return where investors get rewarded for taking more systematic, in other words the non-diversifiable risk. This follows from that an asset's β_i is the only risk that the investor cannot eliminate through diversification. The risk premium in terms of the compensation an investor requires for bearing risk will therefore be a function of the risk aversion of an investor so that:

$$E(R_m) - R_f = \frac{1}{\beta_m} \sigma_m^2 \quad (2.2)$$

where,

β_m is a parameter for the average risk aversion in the market.

σ_m^2 is the market capitalization portfolio variance of expected returns.

The CAPM states that the expected risk premium from any asset is a product of β_i and the market risk premium. Therefore any investment should lie on the security market line (SML) that creates the linear relationship between risk and return. If an investment falls out of the SML and appears more attractive for investors (in terms of the ratio between systematic risk and expected risk), the equilibrium price for this asset will rise due to supply and demand, until considered equally attractive as other securities offered on the market. (Brealey et al., 2010)

2.2 Black Litterman Asset Allocation Model

In the coming section, a step-by-step overview of the B-L model will be presented. Since the model was first introduced in the 1990s, various interpretations of the original model have been developed. Many of which have disagreed what variables to actually include in the model and on the basis of theoretical approach on how to derive variables of the original model. The scope of the following presentation is to a large extent limited to the original model and how it was first described by Black and Litterman (1991) and later presented in greater detail by He and Litterman (2002).

2.2.1 Reversed Optimization

The first step in the B-L model is to derive the reference point in terms of the implied equilibrium expected return vector. This neutral market portfolio will act as the center of gravitation for the weight vector that will converge to the equilibrium (Walters, 2011). The essence with the approach is that an investor without any private views on assets will automatically invest in the market capitalization weighted portfolio. The algebraic representation of the implied excess return vector is in accordance with equation 2.3:

$$= W_{mkt} \quad (2.3)$$

where,

μ is a (N x 1) column vector of implied excess equilibrium return, N equals number of assets.

γ is a scalar representing the risk aversion coefficient of the average investor in the market.

Σ is the (N x N) covariance matrix of excess returns.

W_{mkt} is a (N x 1) column vector of market capitalization weights.

To expand on the risk aversion coefficient; it can be derived mathematically from equation 2.3 by multiplying both sides with W_{mkt} , substituting the vectors with scalar terms and finally by dividing both sides with $\frac{\sigma_m^2}{m}$, we obtain:

$$= \frac{E(R_m) - R_f}{\frac{\sigma_m^2}{m}} \quad (2.4)$$

where,

$E(R_m)$ is the total expected return on the market capitalization portfolio.

R_f is the risk-free interest rate.

$\frac{\sigma_m^2}{m}$ is the market capitalization portfolio variance of expected returns.

Equation 2.4 is commonly known as the Sharpe ratio and is the risk adjusted return, i.e. the reward an investor requires for bearing the risk of the market portfolio. In the Black-Litterman model the implied equilibrium expected return is closely linked to the CAPM equilibrium. A relationship that can be observed by multiplying both sides of equation 2.4 with the variance of the market (Bodie et al., 2011):

$$E(R_m) - R_f = \frac{\sigma_m^2}{m} \quad (2.5)$$

The delicacy of this derivation is that equation 2.5 is identical to equation 2.2 and proves that the market risk premium is a product of the average market risk aversion and the market variance. Put differently, it can be derived directly from the CAPM market equilibrium.

2.3 The Black-Litterman Master Formula

The Prior Distribution

A central attribute of the B-L model is the assumption that expected returns are not observable fixed values, but should rather be regarded as stochastic variables around a normally distributed population mean. Under these circumstances, expected returns have to be modelled with respect of a probability distribution (He and Litterman, 2002). On the contrary, actual returns are regarded as observable random variables and are derived from historical data. An essential piece in the B-L puzzle is how to separate the above-mentioned variables so that the actual return vector is normally distributed around a mean vector with a covariance matrix. The algebraic representation can be presented as:

$$\begin{aligned} r & \sim N(\mu; \Sigma); \text{ where,} \\ \mu & = (\mu + \tau^{-1}v); \\ \Sigma & = N(0; \tau^{-1}\Sigma + \Sigma_0) \end{aligned} \quad (2.6)$$

For the less statistical experienced reader this entails that the prior view of expected returns is a stochastic variable distributed around a normally distributed mean and creating a covariance matrix, where τ is a scalar representing the level of uncertainty one can have over the prior views.

The View Distribution

The private views of the investor form the conditional distribution and is expressed in the model with three components, P , Q and Σ . P is $[k \times n]$ matrix expressing the asset weights within each view and whether the investor has views in absolute or relative forms. Q is a $[k \times 1]$ vector containing the returns for each view, i.e. the expected return the investor estimates a certain asset to yield (an absolute view) or the expected difference in return between assets (a relative view). Mathematically, for a relative view the sum of all weights will equal 0, whilst for an absolute view the sum of the weights will be 1 (Walters, 2011). This can be expressed algebraically as:

$$P = \begin{pmatrix} P_{1,1} & \dots & P_{1,k} \\ \vdots & \ddots & \vdots \\ P_{n,1} & \dots & P_{n,k} \end{pmatrix} \quad Q = \begin{pmatrix} Q_1 \\ \vdots \\ Q_n \end{pmatrix} + \begin{pmatrix} 1 \\ \vdots \\ n \end{pmatrix} \quad (2.7)$$

For the third component of the view distribution, the model requires the investor to express and specify how certain one is on the subjective views, specified in the matrices P and Q . Different studies estimate the uncertainty matrix by different means often taking form as a diagonal covariance matrix. In this study we intend to follow the example of Becker and Görtler (2008), arguing in their study that an uncertainty matrix estimated in accordance with a diagonal covariance matrix will generate elements larger than if expressing a confidence level from

0% to 100%. For the algebraic representation of taking form as a diagonal covariance matrix, refer to Appendix A.

Rather than using the covariance matrix as certainty measure, Becker and Gürtler (2008) propose a different implementation, defining the certainty matrix as following:

$$= \begin{matrix} 2 & & & & 3 \\ \begin{matrix} 1;1 \\ \vdots \\ 0 \end{matrix} & \begin{matrix} \ddots \\ \vdots \\ \vdots \end{matrix} & \begin{matrix} \ddots \\ \vdots \\ \vdots \end{matrix} & \begin{matrix} 0 \\ \vdots \\ \vdots \end{matrix} & \begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} \\ & & & & \end{matrix} \quad (2.8)$$

By implementing certainty in this manner, the user can express how sure one is of his or her private views in the interval of 0% to 100%.

The Posterior Distribution

The mechanism to express views combined with the CAPM equilibrium prior distribution is merged in the Bayesian framework² through an algebraic expression known as the Black-Litterman Master Formula. Merging the prior distribution with the view distribution give rise to expected returns of the posterior distribution (denoted $\bar{\mu}$), distributed as:

$$N(\bar{\mu}; M^{-1}) \quad (2.9)$$

where the mean of the distribution is given by:

$$\bar{\mu} = [(\mu)^{-1} + P^0^{-1}P]^{-1}[(\mu)^{-1} + P^0^{-1}Q] \quad (2.10)$$

And the covariance matrix is given by:

$$\bar{M}^{-1} = [(\mu)^{-1} + P^0^{-1}P]^{-1} \quad (2.11)$$

The essence of the calculation of $\bar{\mu}$ is that uncertainty regarding the prior distribution (denoted μ) and uncertainty regarding the view distribution (denoted P) are appropriately taken into account. The mechanism shapes an inverse relationship between the investor's certainty of one's own views and its implication for the mean of the posterior distribution. The essence entails that if the uncertainty regarding the view distribution increases, $\bar{\mu}$ will gravitate to the mean of the prior distribution, the equilibrium weight vector w and away from the mean of the view distribution Q. On the contrary, if the uncertainty regarding the prior decreases, the mean of the posterior distribution will converge to the mean of the view distribution and diverge from the prior.

As a final note, in order to provide a comprehensive summary of the derivation of the final return vector we choose to present the illustration of Idzorek (2007) in figure 2.1 below:

²The original version of the B-L model is of a Bayesian nature on the basis of the utilization of CAPM to constitute the prior distribution that is later revised using the view distribution derived from the posterior distribution. For a more detailed explanation, refer to Appendix B.

Figure 2.1: Graphic summary of the Black-Litterman Master Formula

2.4 Derivation of the P/E Multiple

Numerous methods have been reported on how to determine a stock's fair price and there is no unity on what approach leads to the best results. Two widely practiced methods are the Discounted Cash Flow Valuation and the Peer Valuation. A DCF considers the company's intrinsic value regarding future cash flows and the Peer Valuation approach values a company relative to companies with similar characteristics. The intuition behind the Peer Valuation is that companies exhibiting similar financial attributes should trade at similar multiples and by comparing peers, a fair share price can be determined (Damodaran, 2002).

Anderson and Kostmann (2007) compare the estimation validity and precision of multiples by running OLS-regressions to determine coefficients for each fundamental driver behind the multiples. The study tested prediction power of what the authors described as widely used multiples and resulted in evidence of disparity in proper multiple predictability. They found that the Price to Earnings (P/E) makes the most accurate multiple estimating a future share price.

In order to derive the fundamental drivers behind the P/E, one can start by revising one of the more simpler valuation methods, known as Gordon's Equation and is based solely on a company's dividends determining the value of a firm as:

$$P_0 = \frac{DIV_1}{R_e - g} \quad (2.12)$$

where,

DIV_1 is expected dividend for next year.

R_e is cost of equity.

g is expected long term growth in dividends.

Expected dividend can also be expressed by multiplying the expected earnings per share (EPS) with the payout ratio.

$$DIV_1 = PayoutRatio \cdot EPS \quad (2.13)$$

Following the fact that the P/E-ratio is calculated on current numbers, we can express upcoming profits as:

$$EPS = EPS \cdot (1 + g) \quad (2.14)$$

By combining equation 2.13 and 2.14 we can reformulate Gordon's Equation to express the value of a share as:

$$P_0 = \frac{PayoutRatio \cdot EPS \cdot (1 + g)}{R_e - g} \quad (2.15)$$

Finally, by dividing both sides with EPS we obtain the P/E-ratio expressed with cash flow variables on a firm with stable growth.

Even though Gordon's Equation is a wild simplification of the reality, the derivation represents what P/E-ratio a firm should trade at considering fundamental characteristics. The derivation of fundamental drivers further declares that a firm with a higher P/E-ratio is not per definition more expensive and is not necessarily overvalued compared to peers but could rather be because of a higher expected growth, higher cash flows or a higher risk, *ceteris paribus*.

As previously discussed, the P/E-ratio can be expressed with expectations on future earnings per share which is known as the Forward P/E and can be algebraically presented as:

$$\frac{P_0}{EPS} = Forward \frac{P}{E} = \frac{PayoutRatio}{R_e - g} \quad (2.16)$$

P/E is the most commonly used multiple and is defined, according to Souza and Deng (2001), as the market capitalization divided by net income and is only considering the equity side of a company. From the fact that the multiple is based on a company's earnings, it might differ from year to year depending on what

accounting principles are applied. Souzzo and Deng (2001) mean that the best way to handle the problem is by using the adjusted earnings per share, reflecting the earnings before extraordinary items and goodwill amortization that restrain comparability between firms. Souzzo and Deng further argues that when the P/E multiple attains a negative value, in other words when a firm generating a loss per share, the multiple cannot be analyzed.

A practical concern for the investor is what type of earnings to use when calculating the P/E ratio. Damodaran (2002) presents three different types of P/E multiples calculated as:

- Current P/E - By using the earnings from the latest annual report.
- Trailing P/E - By using the earnings from the last four quarterly reports.
- Forward P/E - By using the expected earnings in analyst forecasts.

To promote comparability between the statistical valuation multiple and analysts' forecasts, this study will be based on forward P/E so that it along with the analysts' forecasts approach, in a similar fashion take into account expectations on future EPS-growth. This leaves us with three key fundamental drivers behind expected return as derived in detail described in equation 2.16.

2.5 Prior Research

Since the B-L model is an extension from the original work of Markowitz' optimization model, several studies have interpreted the work aiming to quantify and explain the dynamics and mechanisms of the model. Even if the mathematical underpinnings have been thoroughly covered in the academic literature, the scope of studies focusing on the subjective view vector is limited and has not been studied as extensively. On the other hand, the precision and accuracy of analysts' forecasts is a polarized topic among researchers. Due to the limitation of previous work focusing on the subjective view vector, the upcoming chapter will present the prior research resembling the purpose of this particular study.

2.5.1 Econometric Approaches

Beyond the quantification of the model, subjective views generated by econometric models have been incorporated in the B-L model in order to estimate elements for the subjective view vector. Andregård and Pezoa (2016) compare two models in order to reduce the subjectivity of the B-L model by using a GARCH (1.1) to estimate views on expected return and pays regards to the time dependent variance. They conclude that variances from GARCH (1.1) combined with the implementation of a statistical model in order to diminish the subjective component, will give the best outcome of the B-L model. The findings open up for further research to estimate subjective views generated from other econometric models, where we find inspiration for our study in the work of Anderson and Kostmann

(2007) using an OLS-regression based on financial theory, expressed in equation 2.16.

Geyer and Lucivjanská (2016) propose an extension on how to estimate views based on a predictive regression. By incorporating a predictive regression in the Bayesian framework of the B-L model, both the views and uncertainty level is generated for the investor. The authors argue that the extended setup liberates the investor from the task to form confidence levels for each view, since it is implicitly taken into account in the predictive model by putting an informative prior on R-squared. Compared to previous setups, this allows the investor to neglect the uncertainty matrix which otherwise can be challenging to interpret. From applying the predictive regression on seven international financial markets, the study concludes that with views generated from the predictive regression and uncertainty levels endogenously determined from R-squared, other strategies are outperformed. The results point out the same conclusions as the study of Andregård and Pezoa (2016) motivating further investigation in this study to compare a statistical approach with a qualitative method.

Francis and Philbrick (1993) further test an econometric model by comparing analysts' forecasts accuracy to univariate time-series models based on historical earnings data in order to measure differences in the predictability. The result shows that analysts is having a timing and informational advantage due to the fact that analysts release their forecasts after public announcement dates. Other studies point in the same direction where Brown et al. (1987) implies that analysts use more information available on the market why it is of significance to test the claimed advantage in a risk adjusted context.

2.5.2 Analysts' Forecasts and Recommendations

In an article, Treynor and Black (1973) present seven stylized facts on their view of how security analysis can improve portfolio selection. The authors argued that any portfolio can be thought upon as having both a highly diversified part, a riskless part and an active part that in general is exposed to both specific risk and market risk. To optimally select the active portfolio, the allocation is only dependent on appraisal risk and appraisal premiums and not at all on systematic risk or the market premium. The appraisal ratio, in other words how much the optimal portfolio will deviate from the market portfolio, is solely dependent on the quality of the security analysis and how efficiently the active portfolio is balanced. Treynor and Black conclude that the potential contribution to the portfolio performance made from analysts, depends only on how well forecasts correlate with actual returns and not at all on the magnitude of the returns.

Instead of using an econometric method, Becker and Gürtler (2008) aim to further develop the subjective views of the B-L model by implementing a qualitative method to quantify subjective views. In their work, professional analysts' forecasts form the second Bayesian layer. They argue that the issue arising from

using analysts outright target prices and recommendations is that security analyses are not available to the extent required for the overall investor. Instead, Becker and Gürtler present an implementation of analysts' dividend forecasts to determine private views of expected returns. To quantify the uncertainty an investor holds with views, Becker and Gürtler observe the number of analysts covering the asset to determine a confidence level. They argue that previous research indicate that a higher number of analysts increase the confidence level. The study concludes that by using dividend forecasts for generating subjective views outperform other strategies and while bearing the stylized facts of Treynor and Black (1973) in mind, the study implies that dividend forecasts seems to correlate with actual returns. For a more detailed explanation on how the study of Becker and Gürtler implements of dividend forecasts, refer to Appendix C.

Rather than limit the estimations to dividend forecasts, He et al. (2013) empirically examine investment value from implementing analyst recommendations on Australian stocks in the B-L model. The study summarizes that stocks with positive recommendations on average outperform the benchmark index while stocks with unfavourable recommendations contrarily underperform. In accordance with the study of Becker and Gürtler (2008), the results from using analysts' forecasts indicate that the investment strategy outperforms the benchmark both in terms of outright return and risk-return measures. The authors further acknowledge that since the study is carried out with a daily rebalancing policy no ex post abnormal returns are achieved after transaction costs. He et al. (2013) admit that a good place to start in order to avoid transaction costs is to rebalance less frequently, but they conclude that the portfolio performance decreases when stretching the rebalance period, probably due to the fact that recommendations are path dependent in the short term. Put differently, consensus recommendations are in short term self-fulfilling and can be another explanation why forecasts correlate with actual returns in shorter term as described in Treynor and Black (1973).

It is not the first time the topic of analysts' recommendations has been researched. In a study by Liu et al. (1990) the results show that stock prices and trading volumes move in line with the released recommendation over a three days period. Womack (1996) summarizes similar results that stock prices move in the same direction as the analyst forecasts, indicating that analysts forecasts can in short-term be self-fulfilling. Having recommendations being self-fulfilling on a shorter basis, the question to be asked is what stock characteristics attract analysts.

Previous research show that analysts are in general optimistic about stocks with positive historical price momentum, high historical growth and high trading volume (Jegadeesh et al., 2004). The study concludes that to follow these recommendations to naively can turn out to be costly for the investor as the level of consensus recommendation only adds value among stocks exhibiting favourable fundamental values. The study finds that recommendations are adversely corre-

lated with contrarian indicators and positively correlated with momentum indicators. Put differently, the results show that analysts are rather inadequate in predicting turnarounds but are overoptimistic about stocks with a strong historical record, a topic well-covered by behavioural finance literature (Kahneman, 2011). In his book, Kahneman argues that the greatest challenge for the investor is how to control for one's biases that leads to less rational decisions. Easterwood and Nutt (1999) further investigate whether analysts systematically overreact to new information and find that analysts not only are overoptimistic about positive information but also underreact to negative information. In a study by Campbell and Sharpe (2009), the bias has been shown to appear in analysis releases where forecasts are available by professional analysts and being anchored toward realized values presented in recent months. These results are consistent with research implying an over optimism bias among consensus and could impact the results in this study.

Another reason for analysts to be overoptimistic is to create transactions and to keep a good relationship with the firms covered. A study from Arand and Kerl (2012) used a unique data set of reported conflicts of interest to identify an association and relationship between over-optimism in recommendations and conflicts of interest. The results provide indication that stocks for which conflicts of interest have been reported also perform lower risk-adjusted returns compared to the unconflicted counterparts.

3

Methodology and Data

In this chapter, we present the methods and procedures that have been carried out to obtain the empirical findings. To gain a comprehensive understanding of in what order methodology has been executed, we wish to describe the procedure in a systematic manner. Each step of the process will thereafter be presented accordingly.

1. Develop valuation approaches and evaluate with respect to outright return to gain an understanding of predictability power.
2. Set up an investment process with historical simulations in order to evaluate risk-return characteristics.
3. Construct two different view portfolios and build a benchmark reflecting the equilibrium portfolio.
4. Conduct sensitivity analysis, modifying values for critical input variables.

To gain a holistic overview over the optimization process, the procedure is illustrated graphically in figure 3.1:

Figure 3.1: Optimization Scheme

3.1 The Authors' Expectations and Hypothesis

Worth mentioning before carrying out the procedures to gather the empirical findings, is that the authors of this study had on beforehand expectations of analysts' forecasts to be a superior method to generate views compared to the statistical counterpart. The authors hypothesized that analysts would perform better in both outright return and also in terms of risk adjusted return which would be in line with previous research favouring analysts' forecasts as an approach in the Black-Litterman model.

3.2 Approaches to Generate Subjective Views

3.2.1 Statistical Valuation Multiple

Essentially, when conducting a relative valuation between companies, one should focus on comparing fundamental variables such as growth, risk and cash flows (Damodaran, 2002). This grants us the opportunity to apply statistical methods on a complete investment universe to account for differences in fundamental drivers that contribute value of a company. We can now understand the reason why we dedicated so much effort to isolate the fundamental variables behind the P/E multiple. To proxy expected growth, risk and cash flows, we use analysts' forecasts on earnings per share growth, beta of each stock and payout ratio as reported in annual reports. By gathering information during our holding period 2008 – 2018 for each company respectively, we create a panel data set.

As far as the authors are concerned, most work on statistical valuation multiples are based on simple OLS-regression (Damodaran, 2002), however, research suggests that a panel data set can empower the econometric analysis (Hsiao, 2007). Hsiao implies that by using panel data, the increased degrees of freedom and the greater capacity to capture complexity in the data improves the estimators in the model.

Forward P/E

From running fixed effect regressions on our panel data set we obtain coefficients for each fundamental driver behind the forward P/E. The coefficients sum up to a regression equation reflecting what P/E a company should be trading at based on fundamental drivers. Below we present an example of how the regression equation can be arrayed:

$$\text{Forward } \frac{P}{E}_i = \beta_0 + \beta_1 \cdot \text{PayoutRatio}_i + \beta_2 \cdot \text{EPSGrowth}_i + \beta_3 \cdot \text{Beta}_i + \epsilon_i \quad (3.1)$$

By assigning true current values for the independent variables for every single asset, we compare the regression P/E ratio with the currently traded forward P/E. The deviation in percentage results in a positive or negative expected return and can be illustrated as:

Table 3.1: Numerical Illustration

Forward P/E	12,0
Regression P/E	16,0
Over-/Undervalued	33,3%

3.2.2 Analysts' Forecasts

Analysts' forecasts consist of professionals' interpretation of specific stocks' estimated price performance. Since the ability to forecast differ between analysts and target prices distinctly vary – the analysts' forecasts parameter in this study will consist of the consensus mean of all analysts' target prices for a specific stock. The analysts' forecasts are used in the Black-Litterman model as a calculated expected return from the current stock price compared to the consensus forecasted target price. The expected return is calculated as:

$$E(R_i) = \frac{TP_{i,t} - CP_{i,t}}{CP_{i,t}} \quad (3.2)$$

where,

$E(R_i)$ is the expected return for the i :th asset in period t .

$TP_{i,t}$ is the consensus target price for the i :th asset in period t .

$CP_{i,t}$ is the current trading stock price of the i :th asset in period t .

3.3 Evaluating Outright Return

Before comparing risk-return characteristics between subjective views from analysts' forecasts or the statistical multiple approach, the predictability of the two approaches will first be evaluated in an outright return context. As there is no unity of analysts' ability to properly forecast expected return, it is of significance to study analysts' forecast accuracy. The predictability of the statistical valuation multiple has proven to be accurate in a previous study Anderson and Kostmann (2007) and in order to make a comparison, we evaluate predictability by estimating the movement of the expected returns on a three-, six-, nine- and twelve month basis. The results will therefore represent the average prediction accuracy of the direction of stock movements.

3.4 Evaluation of Risk-Return Characteristics

The scope of this study is to uncover how the B-L model behaves while performed with subjective views from either analysts' forecasts or a statistical valuation multiple approach, with real market data over an extended period of time. The empirical research will consist of historical simulations to compare portfolios with two outright return valuation approaches to analyze results in a risk adjusted context.

3.4.1 The Investment Process

Starting on the first trading day of March 2008 (denoted t_0), the covariance matrix of returns (Σ) is calculated on weekly rates of returns of the past 504 trading days - starting from the observation on day t_{-504} up until t_0 . The motive behind initiating the investment process in March is mainly driven by the circumstance that a majority of financial information is released and available for the market during this period through annual reports.

By estimating the covariance matrix and calculating the market capitalization weight for each asset in t_0 , we compute the implied equilibrium excess return vector μ . To generate subjective views for assets about which the investor holds private views, both analysts' forecasts and the statistical multiple approach are incorporated in the model. By doing so, we estimate elements for the subjective view vector, Q .

Through the master formula of Black-Litterman we combine the prior and view distribution and derive the posterior vector of expected returns and insert it into the optimization process. The output weight vector returned from the optimizer is the decision basis for buying shares in accordance and at current prices in t_0 where the initial value of the portfolio in this study is 100 MSEK.

3.4.2 Historical Simulations

The comparison of the B-L model with either subjective views from analysts' forecasts or statistical valuation multiple is based on simulated performance. This particular study intends to use a total of three weighting schemes to construct three different portfolios that are compared in terms of risk-return performance. Apart from evaluating the B-L portfolios based on analysts' forecasts and the corresponding portfolio based on statistical valuation multiple, a third portfolio reflecting the equilibrium portfolio from the 24-asset universe will be compared acting as an benchmark.

Building the View portfolios

Through the framework from the Black-Litterman model and Subjective views section, a covariance matrix and an implied excess return vector is derived. Independent of method used to generate subjective views, the elements to be inserted in the vector is first adjusted with the reigning risk-free rate of return. By assigning values for all key input variables, the Black-Litterman Master Formula calculates a posterior expected return vector. In this study, the optimization process of the posterior expected return vector and the covariance matrix is handled iteratively. The objective is set to yield the maximized expected risk adjusted return, but is subject to two constraints. Firstly, the weight vector is constraint by not allowing for negative values since taking short positions is considered to be unlikely for a vast majority of investors. Secondly, under the assumption of investors to be fully invested, the weight vector is constrained to sum up to unity. Including constraints to the optimization process may result in a sub-optimal weight vector compared to its unconstrained counterpart.

Building the Benchmark Portfolio

To construct a benchmark against which the two view portfolios can be measured and evaluated, the 24-asset universe picked from the OMXS30 is used, reflecting the equilibrium portfolio. As Black and Litterman (1991) argue, for an investor who holds no subjective view about future performances of certain securities, there is little or no reason to deviate from the market capitalization weight vector. This benchmark portfolio can be considered a passive alternative that only requires the investor to rebalance the portfolio in accordance with the development of market capitalization of the securities included in the portfolio. It is therefore of significance to analyze whether applying views to the equilibrium portfolio can add value for the investor.

3.5 Sensitivity Analysis

To maintain the results at a high level of relevancy for a broader range of investors, the simulations are conducted setting the input variables to various values. The input variables reflect decisions that an investor typically has to deal with based on some sort of judgement and rationale. We perform sensitivity analysis on our results by first defining a default level for each variable and then modifying the value for one variable at the time, holding all other variables constant. This allows us to closely observe and analyze how each individual variable affects the characteristics of the portfolios. The default settings are all but randomly chosen, but are rather selected to imitate the settings of He and Litterman (2002) and Black and Litterman (1991).

3.5.1 Investment Horizon (τ)

A bilateral dilemma for the investor is how frequent one should rebalance the portfolio. The benefit to rebalance more often is that the investor is given the opportunity to consider new information in the market. On the other hand, a more frequent rebalancing involves a higher amount of transactions and may generate an increase in operating and financial transaction costs. To reflect the trade-off between considering new information and increased transaction costs for different investors, the simulations are conducted using various values for τ . Since most analysts' forecasts have a time horizon between six to twelve months and the regressions have proved to be most accurate between three to six months, the default value for τ is set to six months corresponding to 126 trading days (Bonini et al., 2010).

3.5.2 Transaction Costs (c)

In reality, institutional as well as private investors face different transactions costs why we test for alterations in portfolio performance at different levels of c . Even though the variable in itself does not have any influence on the ex-ante optimization process, it has a direct effect on ex-post returns and performance why it is of relevance to evaluate and assess the robustness of the results for each portfolio. When setting the default level of transaction costs, we follow the example of Black and Litterman (1991) and set c to 0%. Even though 0% real transaction costs may be a rarity in reality for the overall investor, it still allows for consistent comparability between the portfolio simulations as well as in the study of Black and Litterman.

3.5.3 Risk Aversion (γ)

To conduct a sensitivity analysis on how investors' optimism can render into alterations in the portfolios' characteristics, numerous values are assigned to the risk aversion parameter γ . With a starting point in the article of He and Litterman (2002), we follow the example and set the default level of the risk aversion parameter equal to 2.5.

3.5.4 Certainty level ()

The level of certainty translates into the model by the means of the diagonal omega matrix where the investor has to determine the degree of certainty associated with a certain view. Even though various studies have presented different methods to estimate the omega matrix we intend to follow the example of Idzorek (2007) who determines the certainty factor as a percentage from 0% to 100%. This entails that if the investor does not express any confidence about private future predictions (0%), the model fully trusts the market equilibrium. Vice versa, if the investor expresses full confidence about the future (100%), the model fully trust the investors' views. The confidence factor () in terms of a percentage 0-100 is incorporated in our model in the same way as Idzorek presented in his work. The default setting for the confidence level is in accordance with the predictability of the valuation approaches. As the methods on average predicts the direction of a movement 58% of the occasions, the default level of the certainty level variable is set to 58%.

3.6 Data

3.6.1 Black Litterman Model

The data set in this study consists of daily closing prices of a twelve year period between March 13, 2006 to March 13, 2018. To improve the comparison of our B-L optimal portfolios, we optimize a portfolio of equities included in an index. The investment simulations are conducted using individual stocks rather than any broad index. We have restricted the investment universe to the Swedish equity market as simplifies when accounting for structural changes in the composition of such an index. On the other hand, our results may be more sensitive to the portfolio weights compared to a portfolio only consisting of broad indices, since individual stocks tend to swing more in terms of volatility and return.

The investable universe of our simulations is the OMXS30 (OMX Stockholm 30 Index), a market weighted price index consisting of the 30 most actively traded stocks on the Stockholm Stock Exchange Nasdaq (2016). The index is rebalanced semi-annually why the index composition changes over time. Since March 1, 2008, the following seven companies have joined and left OMXS30:

Table 3.2: Joiners and Leavers in Index Constituents

Date	Joiner	Leaver
2017-06-12	Essity	Lundin Petroleum
2017-01-02	Autoliv	Nokia
2014-07-01	Kinnevik	Scania
2009-07-01	Getinge	Eniro
2009-07-01	Modern Times Group	Vostok Gas
2008-12-10	Lundin Petroleum	Autoliv

Since the companies Essity, Autoliv, FPC, Kinnevik, Getinge and MTG have supervised the OMXS30 during the time period, those are excluded while conducting the historical simulations. The data of closing prices and consensus estimates have been sourced from Thomson Reuters Eikon

3.6.2 Analysts' Forecasts

The coverage of companies included in OMXS30 are all but equally distributed, with the lowest number of 8 analysts to the highest number of 34 analysts presenting their target prices. To find the expected return for each asset, daily data of analysts' forecasts are collected during the period March 2008 to March 2018 and sourced from Thomson Eikon Reuters

3.6.3 Statistical Valuation Multiple

To proxy the independent variables in the regression model, we use payout ratio from annual reports, betacalculated in relation to OMXS30 on a 24 month basis and analysts' estimates on EPS-growth. Data are sourced from Thomson Eikon Reuters and Datastream 5.0 but due to insufficient data on dividend payout ratio, the dividend payout was gathered from each annual report for the analyzed period. To avoid results that lack realism due to limited liquidity and cover from analysts, the sample is restricted to OMXS30 stocks only.

In order to reflect excess returns, all calculations are adjusted with the risk-free rate. Our decision to use a Swedish 10 year government bond is based on the fact that we strongly argue that a longer duration is a better proxy for a risk-free asset rather than choosing a government bond with shorter duration due to lower reinvestment risk and default risk. We further argue that this is in line with the investment horizon of this study with historical simulations stretching over a ten year holding period. In addition, this study has a Swedish perspective meaning an implicit assumption that our investor is exposed to risk located in Sweden why we use the risk-free rate data sourced from Sverige Riksbank

3.6.4 Econometric Analysis

The purpose of this study is not to immerse itself in econometrics but is rather supported and backed by financial theory. But to achieve consistent and unbiased estimators we have conducted a brief econometric analysis. For the more statistical interested reader, descriptive statistics can be found in Appendix D.

Although a panel data set should increase reliability compared to an ordinary cross-sectional data set, it does not come entirely without challenges (Hsiao, 2007). To attain consistent and unbiased estimators one should control for stationarity and autocorrelation. Within the scope of this particular study, multicollinearity will not constitute any major issue. In a data set, it is preferred to have variables completely independent of one another so that the variables effect

on the dependent variable can be isolated. Still, in our case, the problem can be considered to be of less importance since our aim is to make future predictions and it is not of any value how much of the model can be explained from the variables individually. As a reminder, the inclusion of the variables has a strong connection to financial theory as illustrated in section 2.5.

To ensure credible estimators of the model, we test for stationarity in a Fisher Phillips Perron unit root test, from which we present the results below:

Table 3.3: *, **, *** indicates statistical significance at 10%, 5% and 1%

Variable name	Forward P/E	Payout Ratio	EPS Growth	Beta
Fisher PP Level	0.3218	-2.4187**	-4.3035***	0.5972

To properly account for autocorrelation in our data set, we perform our panel regression with a fixed effect. By using fixed effect model (FEM) we implicitly assume that individual observations may bias or have an impact on the predictor or the outcome variable. To solve for this, each entity in the cross-section is given a unique and individual intercept to adjust for heterogeneity and time independent effects (Bartels, 2009). We find the economic argument behind using FEM due to the fact that the data set consists of stocks from various sectors. Since different sectors can exhibit different means in terms of P/E, we find reason to believe that the intercept can be different for different sectors.

3.7 Method Criticism

When using a statistical approach to model a solution, one must always consider the characteristics of the data set to ensure the predictability and credibility. A recurring comment from the opposition of econometric studies is the presence of multicollinearity. In a data set it is preferred to have variables completely independent of one another so that the variables effect on the dependent variable can be isolated. In the case of our study, it should be stated that risk and growth are frequently correlated. The problem arising is that the constant in our model can be misleading and it is difficult to determine the effect of risk and growth respectively. Still, the problem is of less importance in our study since our aim is to make future predictions and it is not of significant value how much of the model can be explained from the variables individually.

In our regression analysis of the panel data set, the model is estimated with the assumption of linear relationships between variables. Damodaran (2002) argues that this is not always the case for the variables driving cash flows in a firm. The effects on our study are vague but it is possible we would have obtained different results from a regression method based on nonlinear relationships. Further, a majority of previous research has implemented a GARCH (1.1) with the advantage of taking into consideration the time-varying volatility of financial assets, an attribute that our method does not wear. This could imply that even if our panel

regression rests upon financial theory it might be the case that other methods are in practice more appropriate for this kind of purpose.

Another widely discussed dilemma is that financial markets can be characterized by noise. This phenomenon can take form as deviations from the classic financial theories such as Efficient Market Hypothesis, indicating that movements in stock prices are not always logical and based on fundamental values. This involves consequences so that statistical significant relationships can be hard to obtain and the validity of fundamental drivers behind the value of a firm can be questioned.

Something often debated when conducting financial modelling is that the results can never be better than the data used. "Garbage in garbage out" is an expression that casts light on the fact that if the input data in a study includes inaccuracies and deficiencies, the results will also be misleading no matter how good a model is. The data used in this study is solely of a secondary form and according to Rienecker and Jørgensen (2014) it is of uttermost relevance to use a critical selection when using secondary sources. The data was mainly collected from Thomson Eikon Reuters financial data base and Datastream 5.0 To minimize the risk of deficiencies, all data gathered was closely examined and questioned for validity. In this manner, parts of the data gathered could be determined to exhibit inaccuracies where the data needed instead was gathered from respective annual report. On the other hand, since we have delimited this study to the Swedish stock market and a smaller investment universe, one can argue that garbage can be the best data alternative we have.

In some cases, firms utilize different accounting principles and follow different fiscal years. Accounting differences can have an impact for the trustworthiness of this study. To minimize the actual impact on our study we have consistently used numbers from the day after a firm releases the annual report. Not surprisingly, whilst using analysts' forecasts to model subjective views, a few considerations has to be properly taken into account. Firstly, analysts present their reports and forecasts for specific stocks during different periods and at different dates, more commonly in conjunction with quarterly reports. In order not to include information in our modelling that was not yet available at the market, we have used March as a starting point constituting the first trading day for our investment period. Since the majority of the Large Cap companies in Sweden present their annual report in the end of February demonstrating current figures. By March a strong majority of all companies included in our investment universe have released financial information. It is plausible the results might be different if the rebalancing periods started on different dates or performed during other horizons.

Since analysis departments and individual analysts differ in their ability to predict target prices, it might cause an under-/over performance when using the analysts' mean in terms of consensus. This is not accurately taken into account

while using the mean of all analysts' forecasts implicating that results might differ if we are selective in the use of analysts' forecasts. This leads to the question whether all analysts' should have the same weight in analyst mean estimates. Further, the number of analysts' covering a specific stock changes over time why some of the stocks might have had mean estimates consisting of only a few analysts' predictions while back testing. This decreases the reliability in the mean estimates if it only consist of a few number of analysts.

4

Results and Analysis

The range of our empirical findings stems from the performance of 3360 simulations of how two subjective view portfolios generated by two different subjective view approaches have developed under the holding period between 2008 and 2018. In the following section, we will summarize of how the two subjective view strategies would have performed in the Black-Litterman model. We will also present figures for an equilibrium portfolio (henceforth EQ-portfolio) reflecting the overall investment universe and acting as a benchmark.

4.1 Outright Return Performance

Before advancing to risk-adjusted return one should first determine differences in capability and accuracy to make prediction of outright expected return. Table 4.1 illustrates how accurate respective valuation approach is with respect to expected outcome and actual outcome. Put differently, how accurate each approach is in predicting the direction of stock movements. As the results suggest, analysts' forecasts are more accurate in predicting the direction of the expected return in any given period. Moreover, the results indicate that both approaches are most accurate on a six months basis where analysts on average accurately predict the right direction of the stock movement in 60% of the occasions. With the exception of fundamental valuation multiple on a twelve months basis, the min- and max figures in table 4.1 indicates that the variation in predictability increases when stretching the time horizon.

Table 4.1: Summary Statistics Outright Return

Forecast Accuracy Outright Return	Period (p)	3	6	9	12
Average Accuracy	Fundamental Valuation Multiple	48,63%	52,40%	51,49%	52,17%
	Analysts' Forecasts	54,06%	60,00%	53,21%	53,75%
Min	Fundamental Valuation Multiple	29,27%	24,39%	21,95%	29,27%
	Analysts' Forecasts	45,00%	30,00%	30,77%	20,00%
Max	Fundamental Valuation Multiple	65,85%	68,29%	73,17%	78,05%
	Analysts' Forecasts	65,00%	70,00%	76,92%	90,00%

We find a plausible explanation behind the results to be the fact that analysts' recommendations can be self-fulfilling since the price follows the direction of the recommendation on a short-term basis (Womack, 1996). This would explain why analysts are more accurate on three and six months basis compared to longer periods. What can not be explained from the fact that analysts are favoured short-term by their own momentum is why six months within the scope of this study is superior to the shorter time span of three months. A study by Liu et al. (1990) further indicates that recommendations create their own momentum on a short-term, which in our case imply that the shorter time horizon, the better accuracy of analysts' forecasts. Put differently, the analysts' ability to accurately predict stock movements can not alone be explained by the short-term momentum but with a shorter rebalancing policy we argue that the effect would have been more significant in line with Liu et al.

Another aspect of the results is that the analyzed period is characterized by a strong stock market performance. Overoptimism in forecasts and recommendations will under a booming market be favourable even if the forecasts in themselves are wrong, why it is hard to justify only predicting the right direction in 60% of the occasions. It is not difficult to understand why the flaws of overoptimism is hard to detect in a booming market. Under more volatile circumstances though, the results could prove to be different as Jegadeesh et al. (2004) show that analysts are overoptimistic about momentum indicators and reluctant to contrarian indicators, which mean that analysts fail to predict turnarounds in the market. The findings of Easterwood and Nutt (1999) support the argument that analysts could be worse off in a volatile market since the study implies that analysts underreact to negative information and overreact to positive information. On the other hand we argue that the statistical approach would still be better of compared to analysts in a more volatile market due to the more systematic attitude. To contradict the aforementioned research, a study by Brown et al. (1987) indicate that analysts are able to interpret more information on the market, which is more in line with our empirical findings.

As we have now unraveled what method is more accurate regarding predicting directions of future performances it is of high significance to progress to the Black-Litterman Model implementation to investigate what happens when looking at risk adjusted return.

4.2 Optimal Asset Allocation

While setting key input variables to the respective default level ¹, table 4.2 illustrates results from the optimized portfolio with subjective views from analysts' forecasts (henceforth B-L Analysts), and the optimized portfolio with subjective views from a statistical multiple approach (henceforth B-L Regression). The figures illustrate how the two portfolios tend to have rather different average asset weights and the statistical valuation method gives rise to higher volatility in individual asset weights over time. Logically, different asset weights will be the key determinant of differences in risk return characteristics where we hypothesize based on the results that the B-L Regression will generate a higher risk adjusted return compared to using analysts' forecasts. First and foremost because a higher asset weight volatility indicates that the valuation method is more adaptable and flexible for any rebalancing point in time and induce a better portfolio composition between the rebalancing periods. This contradicts previous findings of Brown et al. (1987) that we believe are disregarding the analysts' bias.

Table 4.2: Weight Distribution for Different Portfolios

Asset	B-L Regression				B-L Analysts				EQ-Portfolio			
	Average	Min	Max	Weight Volatility	Average	Min	Max	Weight Volatility	Average	Min	Max	Weight Volatility
SSAB	0,1%	0,0%	0,9%	0,2%	6,4%	0,0%	20,2%	5,8%	1,4%	0,4%	3,8%	1,0%
Electrolux	2,6%	0,0%	9,7%	3,3%	1,3%	0,0%	5,0%	1,6%	1,8%	1,3%	2,4%	0,3%
AstraZeneca	7,1%	0,0%	26,8%	8,0%	1,2%	0,0%	5,6%	1,5%	1,5%	0,8%	2,4%	0,4%
SKF	1,5%	0,0%	8,5%	2,2%	1,0%	0,0%	2,9%	1,1%	1,6%	0,0%	2,8%	0,8%
Tele2	5,8%	0,0%	19,9%	6,7%	6,1%	0,0%	19,2%	6,1%	1,5%	0,9%	2,4%	0,5%
Boliden	4,2%	0,0%	15,0%	4,8%	0,4%	0,0%	2,8%	0,8%	1,0%	0,0%	2,0%	0,5%
Volvo	6,4%	0,0%	24,7%	6,7%	5,5%	0,0%	8,6%	2,1%	5,9%	0,5%	7,6%	1,6%
Investor	1,5%	0,0%	10,8%	2,8%	6,4%	0,3%	14,0%	3,1%	4,6%	0,2%	7,0%	1,8%
SHB	3,3%	0,0%	8,0%	2,8%	3,3%	0,0%	6,7%	1,9%	5,7%	4,2%	7,5%	0,7%
Sandvik	0,7%	0,0%	4,4%	1,4%	2,3%	0,0%	5,2%	1,8%	4,1%	2,6%	6,1%	1,1%
Swedish Match	0,7%	0,0%	7,7%	1,8%	2,0%	0,0%	5,1%	1,4%	1,3%	0,4%	2,1%	0,4%
ABB	0,4%	0,0%	3,5%	0,9%	3,9%	0,0%	6,5%	1,8%	3,3%	2,2%	5,0%	1,0%
Securitas	1,3%	0,0%	6,9%	2,0%	0,6%	0,0%	3,9%	1,0%	1,2%	0,7%	2,3%	0,4%
Atlas Copco A	3,3%	0,0%	20,5%	5,6%	5,1%	0,0%	9,1%	2,8%	6,8%	4,3%	8,9%	1,4%
Ericsson	9,6%	0,0%	26,1%	6,9%	8,1%	2,8%	16,1%	2,9%	8,2%	4,5%	15,2%	2,4%
Nordea	10,3%	0,0%	22,9%	6,7%	4,2%	0,0%	9,1%	2,9%	5,3%	2,4%	8,5%	1,3%
Nordea	17,5%	2,0%	34,2%	8,9%	12,6%	6,8%	25,7%	4,2%	11,5%	9,9%	14,2%	1,1%
Telia	9,5%	0,0%	27,6%	8,3%	6,7%	3,9%	10,7%	1,7%	6,6%	3,7%	9,6%	1,9%
Alfa Laval	0,2%	0,0%	2,6%	0,6%	1,1%	0,0%	5,3%	1,4%	1,5%	0,0%	2,8%	0,8%
Skanska	0,2%	0,0%	2,2%	0,6%	1,5%	0,0%	4,5%	1,4%	1,7%	1,0%	2,4%	0,4%
Assa Abloy	0,4%	0,0%	4,5%	1,0%	1,7%	0,0%	7,5%	2,5%	2,2%	0,0%	5,2%	2,1%
H&M	1,6%	0,0%	9,3%	2,3%	10,7%	5,7%	19,4%	3,6%	11,3%	7,2%	15,4%	1,9%
Atlas Copco B	4,3%	0,0%	17,3%	5,0%	4,7%	0,0%	7,9%	2,6%	6,0%	0,8%	7,9%	1,6%
SEB	7,5%	0,0%	29,2%	7,5%	3,4%	0,0%	7,1%	2,5%	4,0%	0,1%	5,7%	2,0%

One can expect that implementing two view strategies result in a more concentrated allocation relative to the EQ-portfolio. As can be seen in figure 4.1 the portfolio optimization excludes more or less every asset at some point in time allowing assets to form 0% of the portfolio. This is not very surprising since if the valuation methods give rise to a negative view of an asset, the intuitive move for any investor is not to include the asset in the portfolio. Not surprisingly, the predictability of the valuation strategy is of uttermost importance as the exclusion of assets reduces the diversification effect introducing idiosyncratic risk in the portfolio. In line with Treynor and Black (1973) this would suggest the actively managed part of the portfolio to increase, which should be in the interest of the investor if one chooses to implement subjective views in the portfolio. Firm-specific

¹ As a reminder the default settings are set to: $c = 0$, $f = 180$ days, $\alpha = 2,5$, $\beta = 0,56$

risk is a bilateral matter in the B-L model as it both allows to generate alpha compared to the EQ-portfolio and also attaches great responsibility on the investor to be skilled in predicting expected return.

Figure 4.1: Average number of assets included

Portfolio managers have praised the allocation dynamics of the B-L model since it gravitates towards the market capitalization weights suggesting a less extreme portfolio composition. This is in line with the investment philosophy advocating a diversified portfolio. A crucial part when constructing a portfolio is not to allow the asset allocation to interfere with the investor's judgement. The intuitive portfolio allocation of both the B-L analysts and B-L regression, not to include an asset when the valuation methods suggest a negative expected return is reflected in figure 4.1.

4.3 Risk-Return Characteristics

With key input variables set to their respective default level, table 4.3 illustrates both return and risk figures for each portfolio.

Table 4.3: $c = 0\%$, $f = 180$ days, $\lambda = 2,5$, $\sigma = 0,56$

Summary Statistics	B-L Regression	B-L Analysts	EQ
Return Rate (CAGR)	6,01%	3,12%	4,81%
Volatility (annual)	23,99%	24,68%	24,09%
-Lowest	11,66%	10,57%	11,41%
-Highest	56,13%	60,37%	56,17%
Sharpe Ratio	0,2506	0,1266	0,1997
Tracking Error	0,43%	0,29%	-
Information Ratio	0,1902	-0,1201	-
Alpha Compared with EQ	0,0057	-0,0043	-
Beta compared with EQ	1,0760	1,0316	-
Residual Error	0,0298	0,0356	-
Correlation with EQ	96,01%	98,25%	-
Average Reallocation	36,9%	24,0%	5,8%
Highest Cumulative Loss	-45,17%	-49,45%	-40,99%
Highest Cumulative Gain	42,65%	45,83%	39,73%

c = Transaction Costs, f = Rebalancing Period,
 λ = Risk Aversion, σ = Uncertainty Level

As suggested, all figures are fairly different in terms of performance which is rather surprising since both portfolios have a correlation to the EQ-portfolio close to 100%. Over the investigated holding period the B-L Regression has generated a risk-adjusted return higher than both its analyst counterpart and the EQ-portfolio. The B-L Analysts has yielded a lower ex-post Sharpe ratio compared to the EQ-portfolio giving further evidence that a high confidence level (0,56) allows the portfolios to deviate from the market reference point in search for alpha and append high responsibility to the investor to be skilled in prognosticating subjective views. The results are ambiguous since the analysts' forecasts were superior in predicting the direction of stock movements in an outright return context. Taking the risk dimension into account indicates that implementing the optimistic analysts' forecasts into the optimization process rather comes with costs than to add value for the investor. Noteworthy, the beta of the B-L Analysts is lower compared to the statistical counterpart while the portfolio still has a higher volatility. This indicates in accordance with Brealey et al. (2010) that analysts' forecasts incorporate firm-specific risk in the portfolio which the investor does not get compensated for, judging from the Sharpe-ratio. Bearing the previous in mind, we argue that analysts' forecasts do not consider a general equilibrium approach and generate forecasts deviating from the SML. The results in table 4.3 contradict the findings of Becker and Gurtler (2008) and could be ex-

plained by the practical implementation of analysts' forecasts. Rather than using expected dividends, this study incorporates actual target prices that we argue is subject to a higher risk of containing overoptimism and/or anchoring.

On the other hand, in terms of tracking error ² the B-L Analysts seems to track the EQ-portfolio benchmark more closely that can be an attractive attribute for many portfolio managers ruling under restrictions to track a benchmark. The tracking error has a direct link to the average rebalancing of the portfolios, where less frequent trading indicates the final weight vector to gravitate towards the underlying prior weight vector.

Figure 4.2: Graphic Representation of Portfolio Performance

From the graphic illustration of portfolio performances one can see how all three portfolios react in the same manner during different time periods. One plausible reason for the similarities is the fact that the market capitalization portfolio is used as a basis for all portfolios in the B-L model (Litterman and Group, 2004). Even though all portfolios are derived fundamentally from the market capitalization weights, it is remarkable to see the differences demonstrated in the portfolio performance. From the given circumstance, we can determine that the graphic deviations is a direct product of the implemented subjective view approaches. This further signifies the impact of subjective views in the optimization process.

²This study defines tracking error as a measurement of risk associated with the actively managed part of the portfolio. It is a measure of how closely the portfolio follows the benchmark.

4.4 Sensitivity Analysis

4.4.1 Investment Horizon

To identify and analyze how the investment horizon and the investor's inclination to rebalance the portfolio may determine the outcome, we set the rebalancing policy to different values as table 4.4 suggests.

Table 4.4: $c=0\%$, $\lambda=2,5$, $\sigma=0,56$

Summary Statistics	Rebalancing Policy (f)	3	6	9	12
Return Rate (CAGR)	B-L Regression	7,23%	6,01%	4,87%	3,94%
	B-L Analysts	1,99%	3,12%	2,75%	1,56%
	EQ	1,35%	4,81%	4,97%	4,88%
Volatility	B-L Regression	24,21%	23,99%	23,73%	24,10%
	B-L Analysts	25,13%	24,68%	23,69%	23,79%
	EQ	24,44%	24,09%	23,92%	24,18%
Sharpe Ratio	B-L Regression	0,2988	0,2506	0,2051	0,1635
	B-L Analysts	0,0790	0,1266	0,1162	0,0654
	EQ	0,0552	0,1997	0,2076	0,2018
Tracking Error	B-L Regression	0,54%	0,43%	0,41%	0,12%
	B-L Analysts	0,52%	0,29%	0,35%	0,37%
	EQ	-	-	-	-
Reallocation	B-L Regression	26,27%	36,86%	39,02%	47,44%
	B-L Analysts	36,40%	24,03%	28,07%	30,38%
	EQ	11,58%	5,80%	9,52%	9,41%

c = Transaction Costs, f = Rebalancing Period,
 λ = Risk Aversion, σ = Uncertainty Level

As illustrated above, extending the investment horizon tends to result in mixed reactions regarding the sharpe ratio for the particular portfolios. Extending the rebalancing policy seems to have negligible effect on volatility and thus the results from varying rebalancing frequency are in essence captured by the return figures.

In terms of outright return, the optimal investment horizon seems to be inhomogeneous for the three portfolios where the shortest rebalancing policy seems to favour the view portfolios, with the B-L Regression in front. That a shorter time horizon seems to favour all portfolios can be attributable to a momentum effect where the view portfolios are given the possibility to incorporate new positive information on a booming market, giving rise to momentum. According to Brown et al. (1987) this should benefit the analysts since the findings of the study imply that they are better at absorbing new information. This is in line with previous research pointing out that analysts' forecasts are favoured by momentum

since their predictions rest upon historical outcomes (Jegadeesh et al., 2004). The same relationship seems to hold true for the statistical valuation multiple during our analyzed period. The optimal rebalancing policy for the EQ-portfolio is 9 months and of a longer character compared to the view portfolios. This supports the general validity that for investors who have no time nor energy to rebalance investments more frequently, the EQ-portfolio makes a more attractive alternative. Regardless of how much energy and time employed by the investor, a shorter rebalancing of the portfolios will make the allocation account for the more recent market data. Even though the B-L Regression on a short term basis generates the highest observed Sharpe ratio, the real-world investor should bear in mind that more frequent rebalancing also troubles with increasing transaction costs.

4.4.2 Transaction Costs

Because real-world investors have to deal with different transaction costs, it is not obvious what rebalancing frequency is optimal for all investors promoting a sensitivity analysis on different levels of transaction costs even if it has no ex ante effect on the optimization process.

Even though transaction costs can take on radically different shapes for a fund manager and an overall investor, within the scope of this study we will for simplicity only consider a financial cost in form of a brokerage fee.

Average reallocation can act as a broad indicator for volatility in portfolio composition over the holding period as it represents how certain asset holdings change in average at every rebalancing point in time. As expected, table 4.5 illustrates how inserting views in the portfolio optimization is associated with larger fluctuations in reallocation relative to the EQ-portfolio. As one can imagine, the portfolio with the highest average reallocation also is the most sensitive to increased transaction costs. As the figures suggest, at a transaction cost level of 2,5% the B-L Regression no longer produces the ex post highest Sharpe ratio due to the EQ-portfolio not reallocating assets as pendulously. What this together with the reallocation figures suggest, views in the optimization process give rise to larger swings in the portfolio weights and does not surprisingly impose costs to the investor due to increased amount of transactions. Once again this reflects the importance of a successful model to accurately estimate returns from where we can determine that using analysts' forecasts only impose costs for the overall investor relative the EQ-portfolio since it cannot compensate for the transaction costs. The break-even brokerage fee between the B-L Regression and the EQ-portfolio can be estimated to approximately 2,31%. If this is considered as a realistic or abnormally high brokerage fee will probably depend on if the investor is private or institutional and on what stock market the investor operates.

Table 4.5: $f=180$ days, $\gamma=2,5$, $\sigma=0,56$

Summary Statistics	Transaction Costs (c)	0,0	1,0	2,5	5,0
Return Rate (CAGR)	B-L Regression	6,01%	5,31%	4,27%	2,56%
	B-L Analysts	3,12%	2,68%	2,01%	0,92%
	EQ	4,81%	4,70%	4,53%	4,24%
Volatility	B-L Regression	23,99%	24,01%	24,06%	24,21%
	B-L Analysts	24,68%	24,69%	24,72%	24,79%
	EQ	24,09%	24,10%	24,11%	24,12%
Sharpe Ratio	B-L Regression	0,2506	0,2212	0,1775	0,1059
	B-L Analysts	0,1266	0,1084	0,0814	0,0372
	EQ	0,1997	0,1949	0,1877	0,1759
Tracking Error	B-L Regression	0,54%	0,43%	0,43%	0,45%
	B-L Analysts	0,52%	0,29%	0,29%	0,31%
	EQ	-	-	-	-
Reallocation	B-L Regression	36,86%	36,86%	36,86%	36,86%
	B-L Analysts	24,03%	24,03%	24,03%	24,03%
	EQ	5,80%	5,80%	5,80%	5,80%

c = Transaction Costs, f = Rebalancing Period,
 γ = Risk Aversion, σ = Uncertainty Level

4.4.3 Risk Aversion

A quite critical decision for the investor is how to deal with the future risk aversion of the average investor in the market, i.e. how to best estimate the future risk adjusted return on the market. To illustrate the impact of such a decision, table 4.6 demonstrates figures on portfolio performance while altering the value of the risk aversion.

Table 4.6: $c=0\%$, $f=180$ days, $\sigma=0,56$

Summary Statistics	Risk Aversion (γ)	1,5	2,5	3,5	4,5
Return Rate (CAGR)	B-L Regression	6,20%	6,01%	5,62%	5,55%
	B-L Analysts	2,37%	3,12%	4,02%	4,21%
	EQ	4,81%	4,81%	4,81%	4,81%
Volatility	B-L Regression	24,95%	23,99%	24,32%	24,28%
	B-L Analysts	25,07%	24,68%	24,13%	24,22%
	EQ	24,09%	24,09%	24,09%	24,09%
Sharpe Ratio	B-L Regression	0,2485	0,2506	0,2309	0,2287
	B-L Analysts	0,0944	0,1266	0,1665	0,1738
	EQ	0,1997	0,1997	0,1997	0,1997
Tracking Error	B-L Regression	0,51%	0,43%	0,29%	0,25%
	B-L Analysts	0,40%	0,29%	0,24%	0,20%
	EQ	-	-	-	-
Reallocation	B-L Regression	39,49%	36,86%	27,65%	23,68%
	B-L Analysts	34,08%	24,03%	22,37%	15,83%
	EQ	5,80%	5,80%	5,80%	5,80%

c = Transaction Costs, f = Rebalancing Period,
 γ = Risk Aversion, σ = Uncertainty Level

As can be observed, γ does not have any measurable impact on the allocation decision for the EQ-portfolio as other figures remain constant when the risk aversion is set to different values. This should not come as a surprise since a higher value will increase all elements proportionately in the implied excess equilibrium return vector, ceteris paribus

The same does not go for the B-L Regression and B-L Analysts where alterations in the risk aversion give rise to seemingly mixed results as the return rate and volatility change in both directions, even if only in a quite modest degree. This might not appear all that intuitive and as far as the authors are concerned, there is no universal economic principle signaling that the optimistic expectations of one single investor would have any significant impact on the market portfolio. The explanation to the not so intuitive mechanism lies in the mathematical underpinnings of the expected return vector. The subjective views are in absolute terms independent from the values assigned the market optimism parameter. This entails that when γ takes on higher values, as long as the investor does not hold private views in exact proportion to the implied market equilibrium returns, the final expected return vector will be adjusted when fed into the optimizer. Put differently, when assigning higher values for the risk aversion parameter the subjective views will have less influence on the final vector of expected returns that is ultimately optimized on. What the investor should gen-

erally keep in mind is that the output from the optimization is a product of not only the covariance matrix but also the proportions in the final expected return vector. This might explain why we in table 4.6 can observe a consistent increase in tracking errors when decreasing λ since the final weights converge to the prior distribution.

The Sharpe ratio for the B-L Regression has without exception increased when the risk aversion parameter is lowered, fuelling the argument that the view strategy undoubtedly has added value. In contrast, the B-L Analyst has conversely generated a lower Sharpe ratio when lowering the risk aversion parameter, making it difficult to determine whether attaching views have consistently added value in the optimization process. What can be established is that once again our empirical findings suggest that the validity of the estimation method for generating subjective views seems to be of uttermost importance. The logical explanation for the discovered differences in Sharpe ratio for the two view portfolios is that analysts' forecasts have in many cases been bad in estimating expected return why the portfolio has been favoured when increasing λ , reducing the influence of the analysts.

4.4.4 Confidence Level

To gain an enhanced understanding of to what extent the valuation methods generating views have actually added value to the investment, table 4.7 illustrates the portfolio performance when simulating different values for λ .

Logically, alterations in λ have no impact on the allocation in the EQ-portfolio, why the performance remains constant. The risk adjusted return of the B-L Regression portfolio truly attests to that the view generating process has added value as it increases as we assign more confidence to the views held. The same relationship between risk adjusted return and confidence level does not hold true when employing the analysts' forecasts in the optimization process, as the Sharpe ratio is consistently lowered as we increase our beliefs. The results are not very dazzling given the empirical findings from alterations in the risk aversion parameter as it points at the same mechanism. Also, it should be mentioned that we can not find any radical shift in the B-L Regression in regards to volatility when altering the confidence level whilst a more ample change can be witnessed in the B-L Analysts.

As uncertainty of subjective views increases we can clearly observe in the results that performance converges towards the EQ-portfolio, a direct effect from the dynamic mechanism of the model making the posterior weight vector closing in on the prior weight vector. In line with what could be expected, the tracking error increases significantly for both view portfolios when we attach a stronger confidence for the elements in the view vector, making the portfolios diverge from the equilibrium weight vector. Not surprisingly, given the skew towards the subjective view vector, the reallocation for any rebalancing point in time in-

Table 4.7: $c = 0\%$, $f = 180$ days, $\gamma = 2,5$

Summary Statistics	Confidence Level ()	100%	56%	40%	20%
Return Rate (CAGR)	B-L Regression	6,35%	6,01%	5,71%	5,25%
	B-L Analysts	2,06%	3,12%	4,14%	4,23%
	EQ	4,81%	4,81%	4,81%	4,81%
Volatility	B-L Regression	25,30%	23,99%	24,35%	24,11%
	B-L Analysts	26,15%	24,68%	24,49%	24,37%
	EQ	24,09%	24,09%	24,09%	24,09%
Sharpe Ratio	B-L Regression	0,2509	0,2506	0,2347	0,2176
	B-L Analysts	0,0786	0,1266	0,1692	0,1735
	EQ	0,1997	0,1997	0,1997	0,1997
Tracking Error	B-L Regression	0,65%	0,43%	0,17%	0,13%
	B-L Analysts	0,67%	0,29%	0,20%	0,15%
	EQ	-	-	-	-
Reallocation	B-L Regression	42,46%	36,86%	22,32%	13,34%
	B-L Analysts	52,61%	24,03%	21,91%	17,32%
	EQ	5,80%	5,80%	5,80%	5,80%

c = Transaction Costs, f = Rebalancing Period,
 γ = Risk Aversion, σ = Uncertainty Level

creases, which can be a further explanation why the Sharpe ratio is so radically affected.

5

A Comparison to Previous Research

Analysts' forecasts in the B-L model have in previous research appeared to be a superior strategy to generate subjective views, but our results shed new light on the topic. Within the scope of this particular study we provide some evidence that in a risk-adjusted context, analysts' forecasts in the B-L model underperforms other portfolios in all analyzed aspects in the sensitivity analysis.

One can imagine that the contradiction can arise from the fact that different studies execute different methods. For instance, He et al. (2013) rebalance the portfolio on a daily basis and conclude that the performance decreases when stretching the rebalancing policy. On the other hand, as the authors argue themselves, the recommendations can short-term be self-fulfilling as previous studies indicate and do in fact not add any investment value (Jegadeesh et al., 2004). This is in line with the results of this study where the analysts' forecasts are superior in outright return (where they are favoured by their own momentum short-term) but underperform in terms of risk-adjusted return in all situations compared to other analyzed portfolios.

Further, the B-L model demands the investor to be very specific with one's private views and it is not enough to only express a notion of if the asset is over- or undervalued. Similarly, our results imply that analysts' ability to properly forecast the direction of stock movements is not enough to specify the elements in the subjective view vector. Instead, the model requires a very specific value of expected return and lower deviation from actual outcomes will favour the weight allocation. Accordingly, overoptimism among analysts is a disadvantage in the model as it might give rise to larger deviations from actual outcome even with the ability to properly forecast directions of stock movements.

The results can also differentiate due to the characteristics of the analyzed period. The analyzed period of this particular study can be described as initiating the holding period with a recession followed by a strong and increasing stock market. As previous research suggest analysts tend to base their recommendations on historical price momentum and as one can imagine, a strong business cycle should then imply overoptimistic recommendations and misleading forecasts. It is plausible, that under a more volatile and decreasing investment period the analysts' forecasts would be inferior in outright return also. As the study of Jegadeesh et al. (2004) argue, analysts' forecasts are negatively correlated with contrarian indicators why in a more volatile market, analysts are worse of forecasting the future.

Becker and Gürtler (2008) incorporate analysts' dividend forecasts as subjective views and argue that the setup is superior to other strategies regarding risk-adjusted return. The method differentiates from how this particular study intends to use forecasts since it in a systematic manner implements dividend expectations in a mathematical setup. Compared to using outright analyst recommendations, we argue that by systematically using dividend forecasts in a mathematical valuation formula, the method is much more alike the statistical valuation multiple rather than the analysts' forecasts in this study. As Jegadeesh et al. (2004) put it, naive adherence to analysts' recommendations can be costly and by using dividend forecasts in a similar fashion as the statistical valuation multiple one can argue that the forecasts are filtered from biases. For instance, we argue that a target price and recommendation is much more likely to be subject to conflict of interest and over-optimism rather than a dividend forecast.

Previous studies have not neglected the possibility to generate views with statistical models and have used various econometric approaches. In this study, the statistical valuation multiple derived from a panel regression indicates that even with a simple econometric approach one can add investment value in the optimization process. Andregård and Pezoa (2016) are among others who argues that a GARCH (1.1) has strong econometric ground to be a superior model to properly estimate expected return, since it properly accounts for heteroskedasticity of the variances. Even if our model can be viewed upon as a much more simpler econometric approach, we argue that the model in this context has a stronger economic stronghold since it is directly derived from financial theory. This is the main advantage for using this type of simple approach and since both advanced econometric approaches and our simple statistical model have turned out to be successful, it is of interest to further investigate and compare the quantitative methods in terms of risk-adjusted characteristics.

Our results are in line with the previous research suggesting that a statistical approach outperforms other strategies. A plausible explanation why statistical methods are superior can be that a systematic quantitative approach filters the outcome from possible biases, since it reduces the qualitative and subjective factor. The essence of all investing is to buy cheap and sell expensive and can sound

rather trivial and as a simple task. But as Kahneman (2011) puts it, the greatest challenge for the investor is how to control for one's biases that leads to less rational decisions. We argue that by implementing a systematic quantitative approach, one do automatically control for potential biases and the outcome of the model will be ltered from the human factor.

6

Practical Implications for the Real World Investor

Our empirical findings do undoubtedly uncover significant differences in portfolio performance and characteristics, depending on how the investor operates to generate subjective views, even if in theory the views are extracted from the same fundamental inputs. In the upcoming chapter we aim to unveil what the B-L model and implemented views implicate for the investor in practice.

6.1 Committing to the Equilibrium Approach

Before we start discussing when and if one should implement a valuation approach to apply subjective views, we have to unravel whether or not the Black-Litterman framework can add value to the investment. If the market is in equilibrium and as efficient as the theory behind CAPM indicates, the investor has in practice no reason to deviate from the market capitalization weight vector. To illustrate how deviations from the equilibrium portfolio may impact the investment, a relative performance of a constant allocation is presented in table 6.1:

Table 6.1: $f=180$ days, $\lambda=2,5$, $\sigma=0,56$

Summary Statistics	Transaction Costs (c)	0,0	1,0	2,5	5,0
Return Rate (CAGR)	EQ	4,88%	4,70%	4,53%	4,24%
	Constant	4,39%	4,39%	4,39%	4,39%
Volatility	EQ	24,18%	24,10%	24,11%	24,12%
	Constant	24,06%	24,06%	24,06%	24,06%
Sharpe Ratio	EQ	0,2018	0,1949	0,1877	0,1759
	Constant	0,1827	0,1827	0,1827	0,1827
Tracking Error	EQ	-	-	-	-
	Constant	0,1137%	0,1152%	0,1183%	0,1252%

c = Transaction Costs, f = Rebalancing Period,
 λ = Risk Aversion, σ = Uncertainty Level

The CAPM theory is supported by the figures as they suggest that the market capitalization portfolio generates a higher risk adjusted return compared to its constant allocation counterpart. What should not be neglected is that the benefit from holding a static allocated portfolio is the avoidance of transaction costs why it is of significance to simulate a break-even transaction cost level. Over the holding period the figures indicate that the static portfolio yields an ex post Sharpe ratio equal to the EQ-portfolio when transaction costs is set to approximately 1,85%.

6.2 Valuation Approach in Accordance with Investment Philosophy

What we have aimed to research is partly what aspects are the most important ones to consider, before committing to either analysts' forecasts or a statistical valuation multiple approach. Needless to say, the approach generating the most accurate estimations on expected return on average, does not necessarily correspond to the approach generating the highest ex-post risk adjusted return. The explanation from this not so intuitive phenomenon can be deduced to how accuracy on outright return has been looked upon. We have considered to what extent the approaches have correctly estimated the direction of the expected return. Correspondingly, our empirical findings suggest analysts' forecasts are on average more correct estimating whether the stock price should go up or down, for any given time horizon. This implies for the real-world investor that having a notion if an asset will go up or down is not enough for the B-L framework, but instead the expressed view must be very specific. A reason why analysts fail to impress in the equilibrium model may be that forecasts and recommendations from analysts lack an equilibrium mindset. This could further explain why forecasts and recommendations are argued to contain overoptimism, a bias that does not go hand in hand with an equilibrium mindset.

Table 6.2: intuitionism in weight distribution

Asset	Statistical Fundamental Multiple		Analysts' Forecasts	
	Average Weight	Average Expected Return	Average Weight	Average Expected Return
SSAB	0,2%	0,0%	6,7%	43,9%
Electrolux	2,5%	11,8%	1,3%	6,5%
AstraZenica	7,3%	20,6%	1,2%	6,3%
SKF	1,6%	8,8%	0,9%	7,1%
Tele2	5,9%	22,2%	6,3%	31,8%
Boliden	4,4%	36,9%	0,3%	2,5%
Volvo	6,7%	14,4%	5,5%	10,5%
Investor	1,4%	7,0%	6,4%	15,3%
SHB	3,4%	2,5%	3,2%	0,6%
Sandvik	0,8%	1,5%	2,2%	5,3%
Swedish Match	0,8%	-11,6%	2,0%	3,8%
ABB	0,4%	-12,4%	3,9%	16,8%
Securitas	1,4%	10,1%	0,5%	2,7%
Atlas Copco A	3,3%	-10,4%	4,9%	8,5%
Ericsson	9,7%	10,0%	8,3%	11,7%
Swedbank	10,0%	7,7%	4,1%	8,1%
Nordea	17,5%	17,8%	12,7%	14,4%
Telia	8,9%	16,6%	6,8%	7,1%
Alfa Laval	0,2%	-17,0%	1,1%	6,3%
Skanska	0,2%	-10,7%	1,5%	6,0%
Assa Abloy	0,3%	-18,7%	1,6%	4,0%
H&M	1,5%	-31,3%	10,7%	4,5%
Atlas Copco B	4,0%	5,6%	4,5%	8,5%
SEB	7,5%	13,5%	3,3%	7,8%

As part of the explanation, we believe the intuitive mechanism of subjective view vector in the B-L model has influenced the results. A significant difference between the two valuation approaches is how optimistic the values for expected return are. What we can conclude from table 6.2 is how statistical fundamental multiples in our study seems to be more pessimistic and yield more negative expected returns on average. As discussed before, practitioners have praised the intuitive dynamics of the model not including an asset when an element attains a negative value in the subjective view vector. As the statistical valuation multiple approach proved to be more pessimistic and expecting more negative returns, we can observe from table 6.2 that these assets are underrepresented in the average portfolio composition. As previously discussed, this is not precisely in line with ordinary diversification theory, but it allows for missing out on the clear losers. For instance, SSAB has between the holding period of 2008 and 2018 only performed a poor, negative return of approximately -72%. Needless to say, excluding this particular asset will have a significant outcome on the risk adjusted characteristics. To generalize, the two valuation approaches seems to vary in what assets they consider attractive in an outright return context which is a key determinant in the end, behind the differences in risk adjusted characteristics.

Our empirical findings show that the Black-Litterman model is suitable for investment managers committing to the CAPM approach, estimating expected return in the long term but still managing an alpha driven portfolio in the short term, capitalizing on mispricing. This can be the case for mutual funds ruling under the requirement of a certain tracking error but still charging clients for

some sort of active management. This is also in line with how Treynor and Black (1973) categorize a portfolio with both a passive and active part. By applying subjective views in the expected return vector, the portfolio weights will deviate from the benchmark weights. By simulating historical performance, the manager can get a sense of the tracking error to the benchmark. For managers evaluated on the basis of performance compared to a certain benchmark, the B-L model is an attractive alternative as it resembles the performance of a benchmark.

Noteworthy, the adequacy and appropriateness of the valuation approaches further depends on the category of portfolio managers and the investment philosophy. As fund managers rarely operate under no restrictions, we still see the usefulness of view generating models exhibiting a closer tracking error and therefore follows the benchmark portfolio more closely. For funds ruling under strict order to closely track a benchmark but still seek to add qualitative research to a quantitative model using analysts' forecasts can be an alternative if the rebalancing policy is on a short term basis. Our empirical findings support previous research indicating that analysts' create their own momentum and recommendations to be self-fulling. With that said, an even shorter rebalancing period than three months could be beneficial for the B-L Analysts and could be subject for further research. Using consensus estimates is not the only alternative to implement analysts' forecasts but investment managers can rather use the in-house knowledge or individual analysts' experience within a certain field or sector. For managers under more latitude we still argue that a more systematic, statistical valuation approach is more beneficial in a quantitative equilibrium model as the portfolio performance of this study illustrates. On the other hand, for portfolios designed to create value by deviating from a benchmark, employing the B-L model incorporating private views is not the most consistent methodology with the investment philosophy.

As a final note, we do not consider investors limited to the CAPM equilibrium approach to see the usefulness of the B-L framework. The prior vector of expected returns (μ) is not constrained to a equilibrium approach but the elements can rather be derived from any benchmark the manager deems fit. the optimization process can then be carried out in similar manner so that the optimization weights gravitate towards the preferred index.

7

Conclusion

For an investor with subjective views of future performances, the key differences in risk-return characteristics between the analyzed approaches can be summarized in three key pillars. First, analysts' forecasts proved to be the best estimator of stock prices future movements but is inferior as an approach to generate risk adjusted return compared to its statistical counterpart. Second, for any given modification of key variables, the statistical valuation multiple approach in the B-L model gives rise to a higher tracking error, making it deviate more from the equilibrium weight vector compared to analysts' forecasts. Third, the statistical approach exhibits a higher reallocation on average, making it more aggressive and reactive to the latest market information.

Paying regards to the actual alterations in asset allocation depending on what valuation approach is implemented in the B-L framework, our empirical findings indicate that both approaches yield a more concentrated allocation relative to the EQ-portfolio. This is regarded as a pure result from the intuitive dynamics of the model, excluding an asset in the portfolio when the corresponding element in the expected return vector attains a negative value. As the statistical method generates more pessimistic expected returns compared to its analyst counterpart it automatically renders a more extreme weighting scheme, reducing the diversification effect and allowing for idiosyncratic risk in the portfolio. The statistical valuation multiple approach renders higher volatility in individual asset weights over time, a plausible explanation why the B-L Regression generates higher risk adjusted return since it is an indication of adaptability and flexibility in new information.

Sensitivity analysis on the most critical investment decisions reveals what input variables impact the portfolios the most. We can conclude that stretching

the rebalancing policy affects both view portfolios in a negative fashion, while the equilibrium portfolio generally benefits from a longer time horizon. This supports the argument that for an investor with less time and energy, the market portfolio is an attractive alternative. Applying subjective views is unambiguously correlated with increased average reallocation why transaction costs have a fairly significant impact on the portfolios.

Our empirical findings show that the Black-Litterman model is suitable for investment managers committing to the CAPM approach to estimate expected return in the long term, but who still is managing an alpha driven portfolio in the short term, capitalizing on mispricing. For the portfolio manager yet to commit to an approach to generate subjective views in the model, we recommend two considerations. First, for managers operating under strict restrictions to track a benchmark we argue analysts' forecasts can be an attractive method on a short term rebalancing policy. For managers under more latitude, we still argue that a more systematic statistical valuation approach is more beneficial in a quantitative model as the portfolio performance of this study illustrates.

Our empirical findings contradict previous research promoting analysts to be a superior and attractive method to generate subjective views. This study sheds new light on the topic and that when using more realistic and real world inputs, analysts fail to impress in a risk adjusted context. Previous studies have rebalanced portfolios much more frequently favouring analysts with their own momentum while not taking into account the transaction costs aspect. Previous research does not use outright analyst recommendations but rather dividend forecasts in a systematic valuation model, why we argue it more corresponds to the statistical valuation multiple approach of this study. As a final note, we argue that our findings imply that analysts' biases do not go hand in hand with an equilibrium model due to over-optimism in estimates.

8

Further Research

We have provided some evidence of the investment value added from using a statistical approach in the optimization process. Our simple econometric approach supported by financial theory outperformed other strategies and it is of significance to further investigate the differences between a more advanced econometric approach and the model used in this particular study and evaluate validity between financial theory and econometric theory.

Analysts' forecasts have in this study failed to impress but it is still up for debate whether the results can be different if conducted with shorter rebalancing policies in the B-L model. Since previous research indicate that analysts are favoured by their own recommendations on a short term basis it is interesting to conduct a similar study on a weekly or monthly rebalancing basis.

An advantage with the Black-Litterman framework is that the investment universe is not limited to specific stocks but a portfolio can rather be optimized on different asset classes and broader indices. This would include a broader investment universe allowing more assets in the portfolio composition. By using the same approach as this study, it would be of importance to evaluate if the same results holds when views are generated on broader indices or asset classes.

Bibliography

- Anderson, C. and Kostmann, O. (2007), 'Relativvärdering – en studie av multiplars förmåga att generera överavkastning'. Cited on pages 10, 12, and 20.
- Andregård, V. and Pezoa, C. (2016), Black-littermans allokeringsmodell : En empirisk studie av prognosvariansen och dess betydelse för portföljprestationen, Master's thesis, Linköping University, Department of Management and Engineering. Cited on pages 12, 13, and 42.
- Arand, D. and Kerl, A. G. (2012), 'Sell-side analyst research and reported conflicts of interest', *European Financial Management* 21(1), 20–51. Cited on page 15.
- Avanza (2018), 'Avanza auto white paper'.
URL: <https://www.avanza.se/avanza-auto.htm> Cited on page 2.
- Bartels, B. (2009), 'Beyond fixed versus random effects: A framework for improving substantive and statistical analysis of panel, time-series cross-sectional, and multilevel data'. Cited on page 25.
- Becker, F. and Gürtler, M. (2008), 'Quantitative forecast model for the application of the black-litterman approach', (IF27V2). Cited on pages 2, 8, 9, 13, 14, 33, and 42.
- Black, F. and Litterman, R. (1991), 'Global asset allocation with equities, bonds, and currencies', *Fixed Income Research* 2(15-28), 218. Cited on pages 2, 6, 21, and 22.
- Bodie, Z., Kane, A. and Marcus, A. (2011), *Investments and Portfolio Management*, McGraw-Hill/Irwin series in finance, insurance and real estate, McGraw-Hill/Irwin. Cited on pages 5 and 7.
- Bonini, S., Zanetti, L., Bianchini, R. and Salvi, A. (2010), 'Target price accuracy in equity research', *Journal of Business Finance Accounting* 37, 1177–1217. Cited on page 22.
- Brealey, R., Myers, S. and Allen, F. (2010), *Principles of Corporate Finance*, McGraw-Hill Education. Cited on pages 5, 6, and 33.

- Brown, L., Richardson, G. and Schwager, S. (1987), 'An information interpretation of financial analyst superiority in forecasting earnings', *Journal of Accounting Research* 25(1), 49–67. Cited on pages 13, 30, 31, and 35.
- Butler, K. and Lang, L. (1991), 'The forecast accuracy of individual analysts - evidence of systematic optimism and pessimism', *Journal of Accounting Research* 29(1), 150–156. Cited on page 2.
- Campbell, S. D. and Sharpe, S. (2009), 'Anchoring bias in consensus forecasts and its effect on market prices', *Journal of Financial and Quantitative Analysis* 44(02), 369–390. Cited on page 15.
- Cheung, W. H. W., Bronner, G., Feiereisen, R., Hofmeyr, A., Lai, H. W., Luo, W., Manwani, A., Meucci, A., Sharma, D. and Vandermark, S. (2009), *The black-litterman model explained*. Cited on page 2.
- Clement, M. B. (1999), 'Analyst forecast accuracy: Do ability, resources, and portfolio complexity matter?', *Journal of Accounting and Economics* 27(3), 285 – 303. Cited on page 2.
- Damodaran, A. (2002), *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset*, Wiley finance series, Wiley. Cited on pages 2, 3, 10, 12, 18, 19, and 25.
- Easterwood, J. C. and Nutt, S. R. (1999), 'Inefficiency in analysts' earnings forecasts: Systematic misreaction or systematic optimism?', *Journal of Finance* 54(5), 1777–1797. Cited on pages 15 and 30.
- Francis, J. and Philbrick, D. (1993), 'Analysts decisions as products of a multitask environment', *Journal of Accounting Research* 31(2), 216–230. Cited on page 13.
- Fried, D. and Givoly, D. (1982), 'Financial analysts' forecasts of earnings: A better surrogate for market expectations', *Journal of Accounting and Economics* 4(2), 85–107. Cited on page 3.
- Geyer, A. and Lucivjanská, K. (2016), 'The black–litterman approach and views from predictive regressions: Theory and implementation', *The Journal of Portfolio Management* 42(4), 38–48. Cited on page 13.
- Gordon, M. J. (1959), 'Dividends, earnings, and stock prices', *The Review of Economics and Statistics* 41(2), 99–105. Cited on page 63.
- He, G. and Litterman, R. (2002), 'The intuition behind black-litterman model portfolios'. Cited on pages 5, 6, 8, and 22.
- He, P. W., Grant, A. and Fabre, J. (2013), 'Economic value of analyst recommendations in australia: an application of the black–litterman asset allocation model', *Accounting and Finance* 53(2), 441–470. Cited on pages 14 and 41.

- Hsiao, C. (2007), 'Panel data analysis—advantages and challenges', *TEST* 16(1), 1–22. Cited on pages 19 and 24.
- Idzorek, T. (2007), *A step-by-step guide to the Black-Litterman model*, Elsevier, pp. 17–38. Cited on pages 9 and 23.
- Jegadeesh, N., Kim, J., Krische, S. D. and Lee, C. (2004), 'Analyzing the analysts: When do recommendations add value?', *Journal of Finance* 59(3), 1083–1124. Cited on pages 14, 30, 36, 41, and 42.
- Kahneman, D. (2011), *Thinking, fast and slow*, Farrar, Straus and Giroux, New York. Cited on pages 15 and 43.
- Litterman, B. and Group, Q. (2004), *Modern Investment Management: An Equilibrium Approach*, Wiley Finance, Wiley. Cited on pages 1, 2, and 34.
- Liu, P., Smith, S. D. and Syed, A. A. (1990), 'Stock price reactions to the wall street journal's securities recommendations', *Journal of Financial and Quantitative Analysis* 25(03), 399–410. Cited on pages 14 and 30.
- Markowitz, H. (1952), 'Portfolio selection', *Journal of Finance* 7(1), 77–91. Cited on page 1.
- Mckinsey&Company (2012), 'Not enough comps for valuation? try statistical modeling'.
URL: <https://www.mckinsey.com/business-functions/strategy-and-corporate-nance/our-insights/not-enough-comps-for-valuation-try-statistical-modeling>
Cited on page 3.
- Michaud, R. O. (1989), 'The markowitz optimization enigma: Is 'optimized' optimal?', *Financial Analysts Journal*. Cited on page 1.
- Nasdaq (2016), 'Rules for the construction and maintenance of the omx stockholm 30 index', 1.6. Cited on pages 4 and 23.
- Rienecker, L. and Jörgensen, P. (2014) *Att skriva en bra uppsats*, Liber. Cited on page 26.
- Souzzo, Peter. Cooper, S. S. G. and Deng, Z. (2001), 'Valuation multiples: A primer', *Global Equity Research*. Cited on pages 11 and 12.
- Treynor, J. L. and Black, F. (1973), 'How to use security analysis to improve portfolio selection', *The Journal of Business* 46(1), 66–86. Cited on pages 13, 14, 31, and 48.
- Walters, J. (2011), 'The black-litterman model in detail'. Cited on pages 7, 8, 59, and 61.
- Williams, J. (1938), *The Theory of Investment Value*, Harvard University Press. Cited on page 63.
- Womack, K. (1996), 'Do brokerage analysts' recommendations have investment value?', *Journal of Finance* 51(1), 137–67. Cited on pages 14 and 30.

Appendix

A

Walters' Illustration of Estimating

In various studies, Σ takes form as a diagonal covariance matrix which is expressed algebraically as:

$$P = Q + \Sigma^{(V)} \quad (A.1)$$

where, $\Sigma^{(V)}$ is an unobservable vector of error terms, with normal distribution so that:

$$\Sigma^{(V)} \sim N(0; \Sigma) \quad (A.2)$$

What the expression above entails is that Σ represents the expected variances of the elements in vector Q, algebraically expressed as:

$$\Sigma = \begin{pmatrix} \sigma_1^2 & & \\ & \ddots & \\ & & \sigma_k^2 \end{pmatrix} \quad (A.3)$$

Walters (2011) illustrates in his study a method to estimate and calculate Σ in accordance with the following expression:

$$\Sigma = \text{diag}(P^{-1} \Sigma^{(V)} P) \quad (A.4)$$

This method generates an uncertainty matrix corresponding to the covariance matrix of the included securities in the portfolio.

B

Bayes' Theorem

The original version of the B-L model is of a Bayesian nature on the basis of the utilization of CAPM to constitute the Prior Distribution that is later revised using the View Distribution derived from the Posterior Distribution. The ultimate contribution of Bayes' findings is how one should relate to having a prior view and how to incorporate additional inputs into the decision making, describing the very foundation of the B-L model. The formula is expressed as Walters (2011):

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \quad (B.1)$$

The left-hand side of the equation describes the conditional probability of A, given B and is also referred to as the Posterior Distribution. When solving for the posterior distribution we utilize the condition distribution of B given A, also known as Sampling Distribution and the probability of A, also referred to as the prior distribution. The probability of B serves as a normalizing constant (Walters, 2011).

C

Becker & Gürtlers use of analysts' forecasts

In their work, the subjective view vector is based on the dividend discount model developed by Williams (1938) and Gordon (1959) from where they derive an expected return. This is calculated by the following formula:

$$EK_i^{(t)} = \sum_{t=1}^T \frac{\hat{D}_i^{(t+1)}}{(1 + r_i^{(t)})} + \frac{\hat{D}_i^{(t+T)} \cdot (1 + g_i^{(t)})}{(1 + r_i^{(t)}) \cdot (1 + r_i^{(t)})^T} \quad (C.1)$$

where,

$r_i^{(t)}$ is the expected stock return of company i is calculated at a given point in time t .

$EK_i^{(t)}$ is the market value of equity.

T is phase one where detailed estimations of the dividends D_i of company i are available.

$g_i^{(t)}$ is phase two, the remaining time a constant growth rate of dividends is assumed.

The following financial data are provided every month t concerning the expected dividends, mean $\hat{D}_i^{(t+1)}$, standard deviation $\sigma_{D_i}^{(t+1)}$, highest and lowest estimation $D_{i;hi}^{(t+1)}$ and $D_{i;lo}^{(t+1)}$, number of analysts' dividend forecasts $D_i^{(t+1)}$.

Becker & Gürtler assume a constant dividend payout ratio by application of these growth rates because dividend and earnings growth are equal in this case.

D

Descriptive Statistics for the Statistical Valuation Multiple

Table D.1: Descriptive Statistics

	Forward P/E	Estimated gEPS	Beta	Payout ratio
Mean	13,3724	6,02%	0,9934	57,63%
Standard Error	0,3477	1,29%	0,0173	2,39%
Median	13,8101	6,54%	0,9566	59,31%
Minimum	4,7895	-50,52%	0,7112	0,00%
Maximum	19,1395	24,74%	1,4378	99,03%
Kurtosis	0,0201	5,8940	0,9116	0,9461
Skewness	-0,5097	-1,8696	0,7317	-0,6643
Observations	85	85	85	85

Table D.2: Correlation Matrix

	Forward P/E	Estimated gEPS	Beta	Payout ratio
Forward P/E	1			
Estimated gEPS	0,40305	1		
Beta	-0,00713	0,003687	1	
Payout ratio	0,631968	0,512118	-0,02322	1

E

Yearly Portfolio Performance

Figure E.1: Yearly Risk Return Characteristics