Harnessing teachers' perspectives

Recognizing mathematically highly able pupils and orchestrating teaching for them in a diverse ability classroom

Elisabet Mellroth

Faculty of arts and social sciences

Pedagogical Work

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To Carl and Emma

To Jenny, Elisabet, Simon, John, Alva, and Filip you have motivated me and made me feel that my work is important.

Abstract

There is a lack of research that explores teachers' perspectives on teaching highly able pupils (HAPs) in the diverse ability classroom. Instead, pupils' perspectives are common, such studies often concluding that teachers need more professional development or that they fail to support HAPs. Consequently, previous research does not support teachers in how to implement teaching that includes HAPs. This thesis aims to harness teachers' perspectives of pedagogical possibilities that enable learning opportunities for mathematically highly able pupils (MHAPs). By comparing pupils' (n=264) relative achievement on two different tests, traditional and non-traditional, the first part of this thesis explores teachers' possibilities to recognize MHAPs. The results show that pupils who achieved highly on the non-traditional test, but not on the other, have higher problem-solving competence compared to pupils who achieved highly on the traditional test instead. The second part presents a teacher-initiated investigation of pupils' perceptions of challenging mathematical tasks. The findings indicate that the developed tool can help teachers find suitable tasks for MHAPs. The final part uses positioning theory to analyse teachers' (N=17) discussions and probe their perceptions on orchestrating teaching MHAPs. The findings show that the teachers have knowledge of how to recognize and support MHAPs. Specifically, the teachers express possibilities with challenging tasks and differentiated education to meet MHAPs' learning needs. Furthermore, the teachers perceived fewer rights than duties to orchestrate teaching for MHAPs', for example, to continuously asses them. The studies in this thesis support teachers to orchestrate teaching including MHAPs in the diverse ability classroom. Both practice and research can be guided by what the teachers in these studies perceived as possible to do to orchestrate such teaching. Thereby support is given to teachers, from teachers, through research.

Key-words: High ability, Gifted education, Professional development, Mathematics, Differentiated education, Inclusive education, Teachers, Positioning theory

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My dearest Sigge, I am sorry for all the time this work has taken. Thank you for always being there to cuddle with. Now – let´s go and do some agility!

TABLE OF CONTENT

Figures

Tables

List of papers

Text 1. Mellroth, E. (2014). *High achiever! Always a high achiever?: A comparison of student achievements on mathematical tests with different aims and goals*. Licentiate thesis, Karlstad, Sweden: Karlstad University.

Text 2. Mellroth, E. (2017). The suitability of rich learning tasks from a pupil perspective. In T. Dooley, & G. Guedet (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (pp. 1162-1169). Dublin, Ireland: Dublin City University and European Society for Research in Mathematics Education.

Text 3a. Mellroth, E., van Bommel, J., & Liljekvist, Y. (in press). Elementary teachers on orchestrating teaching for mathematically highly able pupils. Manuscript accepted for publication *The Mathematics Enthusiast*.

All authors have made substantial contributions to the conception and design of the study, as well as analysis and interpretation of data. However, as being main author, I have collected and analysed all data and had the main responsibility throughout the process.

Text 3b. Mellroth, E. (2018b). *Teachers' views on teaching highly able pupils in diverse ability classroom*. Manuscript submitted for publication.

1 Introduction

Most Highly Able Pupils (HAPs) are taught in classrooms that include a diverse range of abilities among learners (Shayshon, Gal, Tesler, & Ko, 2014), and it is reasonable to assume that most teachers teach in such diverse ability classrooms. However, empirical studies on gifted education seldom considers the context of the diverse ability classroom (Szabo, 2017), which may explain why teachers in such settings have difficulties in implementing existing research on the education of HAPs into their classrooms (Dodillet, 2017; Leikin & Stanger, 2011; Skolinspektionen, 2016). Furthermore, research on gifted education often takes a pupil's perspective, addressing pupils' needs (e.g., Rogers, 2007) and how they benefit from some teaching strategies (e.g., Johnsen, Haensly, Ryser, & Ford, 2002; Rogers, 2007). Very often such studies on gifted education conclude that teachers need more professional development on educating HAPs (e.g., Leikin & Stanger, 2011; Shayshon et al., 2014), but it is rare to include a teacher perspective in studies on gifted education.

If we take the diverse ability classroom as a starting point - what do teachers have to say about theories on high ability, about mathematical tasks, about HAPs and about their own power to orchestrate educational theories in their teaching? This thesis aims to address some of these questions. Teachers spend a lot of time teaching and have extensive experience of meeting the needs of a diversity of pupils. Therefore, acknowledging the perspectives and experiences such teachers can bring to research may result in practice and theory of educating HAPs becoming closer. In turn it might mean that both develop in parallel so that more teachers get support from research to orchestrate effective teaching to HAPs in diverse classrooms.

The overarching problem addressed in this thesis is that teachers are not supported in teaching mathematically highly able pupils (MHAPs), neither by structures and strategies for detecting them, nor how to implement knowledge from existing research when teaching HAPs in diverse ability classrooms. This thesis acknowledges that MHAPs are

highly able with a special ability in mathematics; when pupils with high ability in general are addressed, the abbreviation HAPs will be used.

It is known that most HAPs, amongst other things, learn at a faster speed than average pupils (e.g., Rogers, 2007). In general, HAPs also have special learning needs. For instance, if teaching progresses too slowly they might have difficulty in grasping the context (Rogers, 2007). HAPs prefer to get information presented in a complex and abstract way instead of a step-by-step progression (Rogers, 2007). MHAPs usually benefit from working with challenging problems (Benölken, 2015; Nolte & Pamperien, 2017). They quickly generalize mathematics and easily grasp the mathematical structure in problems (e.g., Krutetskii, 1976; Sheffield, 2003). Such challenging problems are preferably engaging, include deep mathematics and are open-ended (e.g., Benölken, 2015; Sheffield, 2003). In a review of mathematics education for MHAPs, Szabo (2017) showed that some pedagogical strategies are recommended when teaching MHAPs in the diverse classroom[1], including differentiating teaching through working speed, flexible grouping, modification of tasks or the use of open-ended tasks. Further, Szabo (2017) showed through his review that when MHAPs' learning needs not are met, they achieve far below their potential. He also showed that when teaching is adapted to meet their needs, they can achieve in relation to their potential.

In Sweden, it is clearly expressed that teachers' professional tasks include supporting HAPs' learning needs. The Swedish Education Act (SFS 2010:800) gives all pupils the legal right to get guidance for developing their knowledge as far as possible. However, HAPs are often hindered in their learning development, in Sweden as well as in other countries.

> Too many students sit in uniformly taught classrooms and wait while the teachers re-teach content to the whole group that a portion of the class

[1] Szabo uses heterogeneous classroom.

> mastered long ago. Too many students feel disconnected from learning…
> (Tomlinson & Imbeau, 2011, p. 5)

Swedish parents and pupils testify that it is common that pupils must wait for the rest of the class to complete a given task before they can continue. Some teachers tell them not to read further than a specific page in a book and not to continue working in the mathematics book, since the class is supposed to be on a specific page. Similar stories are also described in the literature (e.g., E. Pettersson, 2011).

These testimonies and stories are supported by the findings in a report from the Swedish Schools Inspectorate (Skolinspektionen, 2016). The report is based on an investigation of Swedish schools that are considered to be at risk of not providing pupils with the education to which they have legal right. The investigation focuses on the 25 percent of schools that are regarded to be at the highest risk. Pupils, teachers and principals in these schools were asked about the teaching and learning environment. Based on this, the Swedish Schools Inspectorate concludes:

- More than one third of pupils state that they are given too few challenging tasks in schools.
- It is common that teaching is adapted to meet the needs of pupils with learning difficulties or a perceived average level.
- Pupils who want to reach further in knowledge are not always being challenged enough.
- Pupils describe losing interest in school partly due to lack of challenges.
- Some teachers report lacking the opportunities, methods and material to support pupils further than passing level.

The focus in Swedish education has been on helping every pupil reach a passing level; little effort, if any, has been given to help HAPs[2] to excel in subjects (Persson, 2014). However, ways to recognize HAPs and

[2] Persson (2014) uses 'students with capability to excel'.

what they need to optimize their learning has been known for decades (e.g., Rogers, 2007). Many studies conclude that to be able to identify and support MHAPs' learning, teachers need professional development on how to meet their learning needs (e.g., Leikin & Stanger, 2011; Shayshon et al., 2014). It is, however, rare that research studies on gifted education have a teacher perspective, in the meaning of bringing in what teachers perceive on teaching MHAPs in the diverse classroom. To be able to operationalize theories on education of HAPs in schools, the teachers' perceptions provide relevant information regarding what those who teach find possible or difficult to implement.

The thesis is divided in three parts. Each part contributes to a greater understanding of teachers' perspectives of teaching HAPs, and each has a different focus: test results, pupils' views and teachers' views. Part 1 probes how mathematical competencies can be used to explain differences in achievement between groups of pupils on non-traditional tests. The non-traditional test aims to inspire joy and engagement in mathematics and is constructed to contain challenging tasks. Pupils' relative achievements on non-traditional national tests in mathematics were compared to the relative achievement on the traditional national test in mathematics for the same group of pupils (n=264). The comparison resulted in deeper analysis of mathematical competencies activated in groups of pupils who achieved high on the non-traditional test but not on the traditional and vice versa.

Part 2 focuses on pupils' perceptions of tasks specifically developed to support MHAPs. This part of the thesis is based on a report of an 18 month long professional development programme on education of MHAPs. Participants were mathematics teachers (N=7), teaching from grades 4-9 (ages 10-16). An investigation was developed by the participating teachers in collaboration with the facilitator. In literature on educating MHAPs it is written that tasks supporting learning needs of MHAPs should be engaging, challenging and joyful (e.g., Benölken, 2015; Sheffield, 2003). Therefore, the teachers wanted to probe how MHAPs and other pupils perceived tasks developed to support MHAPs.

Finally, teachers' perceptions are addressed in Part 3. The context of the studies for this part of the thesis is a three-year long professional development programme on gifted education specifically for mathematics teachers (n=15), teaching from grades 1-9 (ages 6-16). In the studies, teachers' views on operationalizing theories of gifted education in diverse ability classroom while teaching mathematics are explored.

1.1 Aim

The overarching aim is to capture the perspectives of teachers in order to highlight pedagogical possibilities in the diverse ability classroom that enable learning opportunities also for MHAPs.

More precisely the thesis seeks to investigate what teachers perceive regarding their own ability and power to teach MHAPs in a diverse ability classroom and, in the context of a professional development programme, probe their perceptions of the potential of theories on educating HAPs to enhance their own practice.

1.2 The aim and the three parts

To fulfil the aim, the thesis is guided by two overarching research questions, these are:

How can non-traditional tests and challenging mathematical tasks help teachers recognize and support MHAPs?

How can the experience of those who teach in diverse ability classrooms contribute to the research; what can their perceptions on orchestrating teaching that includes MHAPs contribute to theories on educating highly able pupils?

To respond to these questions the thesis is divided in three parts. The three parts are presented in the following sub-sections.

1.2.1 Part 1 – Text 1

Part 1 of the thesis is based on a study, using both quantitative and qualitative methods, presented in a licentiate thesis (Mellroth, 2014). In this study, relative achievement on Swedish national tests

performed in grade 3, year 2009, and 6, year 2012, was compared. In addition, the results of the national tests in grade 6 were also compared to relative achievement on *The Mathematical Kangaroo*[3], performed in grade 7, year 2013. Results from the same group of pupils were used in all three tests. To reveal possible explanations for differences in achievement, the tasks in The Mathematical Kangaroo were analysed, using a framework by Lithner et al. (2010), with respect to the possibilities they gave to show mathematical competencies.

In the study in Part 1 pupils' results on different tests were investigated. The national tests represented traditional tests used in educational settings, while The Mathematical Kangaroo represented a non-traditional test. Focus was on pupils who achieved highly on one of the respective tests, traditional or non-traditional, but not on the other.

In the results there are pupils that achieved highly on The Mathematical Kangaroo and lower on the national test, or vice versa. To explain this, the opportunities to display mathematical competencies in The Mathematical Kangaroo tasks were analysed. Through Part 1 of the thesis, it is examined how pupils with high mathematical abilities can be recognized using non-traditional tests in mathematics, although they do not necessarily show high mathematical ability through traditional tests in mathematics. Part 1 of the thesis therefore implicitly addresses the problem with identifying MHAPs through high achievement on traditional tests.

1.2.2 Part 2 – Text 2

The second part of the thesis is based on a report of a study, performed in the context of a professional development programme with seven participating mathematics teachers. The programme was running from November 2014 to June 2016; the programme and the study are described in Mellroth et al. (2016). The study evolved by, and was

[3] The Mathematical Kangaroo is an international competition aiming to stimulate interest and curiosity in mathematics (NCM, 2013) and to offer mathematical challenges (Wettbewerbsbedingungen, 2013).

performed in collaboration with, the participating teachers and is theorized in Text 2 (Mellroth, 2017). The study presented in Text 2 investigated a developed tool capturing pupils' perception of working with challenging mathematical tasks, chosen from Sheffield (2003). One of the two tasks is also elaborated on in Mellroth & Thyberg (2018). The tool was developed to help teachers identify mathematical tasks that MHAPs, and other pupils, perceive as stimulating. The tool was developed in collaboration between the participating teachers and the facilitator, i.e. the author of Text 2 and this thesis. In agreement, to make it practical to implement during regular teaching, the tool was made simple and aimed to take very little extra teaching time to implement. It consisted of a pre- and post-evaluation of pupils' perceptions. Because former research has shown that MHAPs are seldom stimulated and challenged in diverse classrooms (e.g., Leikin & Stanger, 2011; E. Pettersson, 2011), MHAPs perceptions were distinguished from others. Although the study focuses on the pupils' perspective, the teacher perspective on orchestrating teaching MHAPs in diverse classrooms is implicitly addressed, since the tool is developed with, and for teachers.

1.2.3 Part 3 – Text 3a and 3b

Part 3 of the thesis is based on two qualitative studies, both performed in the context of a professional development programme for teachers on educating MHAPs. Fifteen mathematics teachers participated and the programme was running from August 2015 to June 2017. Each study is presented in a separate paper (see, Mellroth, van Bommel, & Liljekvist, in press; Mellroth, 2018b).

The two studies probed what teachers expressed as possibilities and obstacles (Mellroth et al., in press), respectively rights and duties (Mellroth, 2018b) regarding orchestrating teaching MHAPs in diverse classrooms.

In this professional development programme the teachers read literature on teaching HAPs (e.g., Tomlinson, 2001; UNSW, 2004) and specifically MHAPs (e.g., Benölken, 2015; Krutetskii, 1976; Sheffield,

2003). Several discussions and workshops were performed in the programme, aiming to help the teachers reflect upon the literature and to connect it to the Swedish culture as well as their own practices. The study for Text 3a (Mellroth et al., in press) probed the discussion of a group of five elementary teachers during five workshops distributed over seven months. During the workshops they analysed and developed challenging mathematical tasks based on the literature used in the programme. The study for Text 3b (Mellroth, 2018b) probed the discussions of the teachers who participated in the sixth meeting in the professional development programme, when most of the literature was read. The teachers were divided in three groups based on teaching grades. The discussions were guided to encourage them to discuss the literature used in the programme in relation to their regular practice. The two studies explicitly address the teacher perspective on orchestrating teaching MHAPs in diverse classrooms.

1.3 Outline of the thesis

As the studies performed for this thesis are related to Swedish school traditions, chapter two gives an overview of some parts of the Swedish school law and curriculum that are of relevance for this thesis.

Chapter three gives a literature review on the research context for the studies in this thesis.

Chapter four describes theoretical considerations that have framed the research process.

The three parts are separated and described individually regarding their methods in chapter five, ethical considerations in chapter six, their analytical processes in chapter seven and their findings in chapter eight.

In chapter nine the findings of all studies are discussed and connected to the aim and the research questions. This chapter also includes reflections on methods used in the three different parts and the trustworthiness of this thesis is discussed at the end of chapter nine.

Finally, implications for practice and research are given in chapter ten.

2 The Swedish context

The Swedish Education Act (SFS 2010:800) states that education should be equal for all, meaning, for example, that it should be adapted to each pupil's needs. The steering document (Swedish National Agency for Education, 2011) clarifies that since differences among pupils must be considered, teaching can never be designed in the same way for all pupils. Therefore, these documents require that teachers orchestrate teaching to meet all pupils' learning needs and most often in the context of diverse ability classrooms.

In Sweden there is no differentiation based on high cognitive ability when pupils are placed in schools and classes. Since 99% of Swedish pupils attend compulsory school (estimated from Riksrevisionen, 2018; Skolverket, 2018a; Skolverket 2018b), there is a broad diversity of learning needs the teachers must address within a classroom. Further, according to the Swedish Education Act (SFS 2010:800) all pupils have the right for support and stimulation that will develop their knowledge. It is highlighted that pupils who reach the educational goals with ease should be given additional support and stimulation to further develop their knowledge.

The principal of a Swedish school has the autonomy to decide to have pull-out groups (SFS 2010:800), and it is common that pupils with learning disabilities are being taught in such groups with a special needs teacher (Nilhom & Göransson, 2016). Therefore, the Swedish school tradition is not unfamiliar with pull-out groups, which can be an argument for using pull-out groups also for mathematically highly able pupils (MHAPs) in Sweden. It is, however, rare with such pull-out groups and most MHAPs are educated in diverse ability classrooms.

Swedish teachers are therefore supposed to orchestrate teaching to accomplish the complex task of including all pupils in teaching. However even if they aim for it and know that it is their professional task to do so, they may not always be prepared for it, and they may not always get the required organisational support. Particularly, they are

not prepared to meet the needs of MHAPs in diverse ability classrooms (Mattsson & Bengmark, 2011).

3 Literature review

To orchestrate teaching to meet all pupils' learning needs, in the context of the diverse ability classroom, is not an easy task. Göransson and Nilholm (2014) even argue that it might be radical to believe that all pupils' learning needs, independent of their differences and without special needs resources, can be included in learning within the diverse classroom. This is, however, what most teachers are expected to do, especially in diverse ability classrooms. It is reasonable to assume that teachers would benefit from knowledge of how to differentiate teaching to be able to accomplish such teaching.

This thesis focuses on teachers' perspectives on how to provide teaching in the diverse classroom that includes MHAPs in learning. To orchestrate teaching in such ways, it is beneficial that teachers have knowledge of these pupils' specific learning needs (e.g., Vialle & Rogers, 2012). In the Swedish context, however, it is rare that teachers have completed professional training on educating MHAPs (Mattsson & Bengmark, 2011).

This chapter includes a literature review comprising identification of MHAPs, learning needs of MHAPs, teaching strategies to meet pupils' diverse learning needs, and professional development on teaching strategies meeting a diversity of learning needs - focusing on MHAPs.

3.1 Identifying MHAPs

High ability is not always seen as domain-specific; in this thesis MHAPs are considered to be highly able with a specific ability in mathematics. Therefore, literature addressing high ability in general is of relevance when discussing identification of MHAPs.

In this thesis it is assumed that the reason to identify MHAPs is to be able to offer teaching that enables learning, which is the same reason given for identification of other pupils, for example pupils with dyslexia. Identification of MHAPs can lead to some pupils getting support through special groups, special schools or through

differentiated education within the diverse classroom. But how are MHAPs identified?

In a review of mathematical high ability, it was found that in approximately 80% of empirical studies on MHAPs, IQ-measurements are used as a tool of identification (Szabo, 2017). However, in recent decades, IQ as a measurement tool for the identification of HAPs has been criticized (e.g., Nolte, 2012; Renzulli, 2005; Sternberg, 2017; Szabo, 2017; Ziegler, Ziegler & Stoeger, 2012). Most of the modern theories (e.g., Renzulli, 2005; Sternberg, 2017; Ziegler, 2005) do not use IQ measurements as the only source of identification. For example, Nolte (2012) showed that high ability in mathematics is not necessarily correlated to high results in intelligence tests. And Nolte and Pamperien (2017) argue that even if IQ is one precondition for high achievement in mathematics, an identification process of MHAPs should include challenging mathematical problems. Sternberg (2017) goes even further and claims that IQ as primary basis for identification of highly able individuals has outplayed its role. He argues that it is better to put a larger focus on individuals' creative, practical, wisdom-based and ethical skills in the identification process of highly able individuals. Therefore, in practice, the tools used to identify MHAPs are not coherent with modern theories of high ability.

A conclusion is that various types of assessment give teachers broad information about pupils' abilities and learning needs, and thereby provide more opportunities to identify MHAPs.

3.1.1 Assessment

Teachers are supposed to use assessment *for* learning (A. Pettersson, 2004); for example, continuous assessment can help the teacher to understand and respond to pupils' learning needs. Often when teachers are asked to assess pupils' knowledge they connect this to traditional tests. But when teachers are asked how they know if their pupils have learned something, they refer to teaching activities in the classroom (Dorr-Bremme & Herman, 1986). Both pupils and teachers sometimes focus on achieving high on the Swedish national tests, instead of

focusing on learning (A. Pettersson, 2007). It is, however, possible to use assessment to promote each pupil's strengths and to find ways to help the pupil develop those parts s/he can improve. To help teachers recognize all pupils' strengths and weaknesses, they can observe and document pupils' knowledge in several ways (Jönsson & Svingby, 2008).

> Put simply, when there are many ways to be successful, many more students are successful. Students are aware of the different practices that are valued and they feel successful because they are able to excel at some of them (Boaler, 2006, p. 42).

To be able to identify MHAPs, teachers need to have knowledge of high ability (Mohokare & Mhlolo, 2017). Pre-assessment can be used to grasp what the pupils already know and lead the teacher to extend the curriculum and include challenging problems for MHAPs (MacLeod, 2004; Mohokare & Mhlolo, 2017). It is naturally beneficial to measure abilities said to characterize MHAPs when assessment aims to recognize those pupils. For example, in problem solving they are in general able to grasp the formal structure of the posed problem, quickly generalize, be flexible and remember mathematical relationships (e.g., Fuchs & Käpnick, 2009; Krutetskii, 1976; Sheffield, 2003). Further, they may show an intense curiosity, and they can show great interest in *why* and *how* things are right or wrong (Mohokare & Mhlolo, 2017; Sheffield, 2003). But they may also be bored and act rebellious for having to work below their level (Mohokare & Mhlolo, 2017), which is to be taken into consideration both when constructing assessments and when evaluating pupils' results.

3.1.2 Achievement

High achievement is not always synonymous with high ability. As mentioned in the previous section, the construction of the task influences pupils' achievement – if the task is below the pupils' ability, s/he may not give the task a chance (Mohokare & Mhlolo, 2017). As indicated by Boaler (2006), teachers need to offer various forms of assessment to enable pupils to show their strengths and abilities. Pupils' achievement may also vary over both shorter and longer periods

of time, which is to be considered if assessment has the purpose to be used *for* learning.

In educational settings a pupil's subject grade is often influenced by her or his achievement on different tests. Pupils' achievement on similar tasks was probed in grades 3, 6 and 9 in a large quantitative study (A. Pettersson, 1990). Pettersson found that pupils with low achievement in grades 3 and 6 also displayed low achievement in grade 9, and those with high achievement in grade 3 and 6 also displayed high achievement in grade 9. How teaching is adapted to meet pupils' learning needs as well as pupils' individual prerequisites are essential aspects for achievement. Pettersson also found that the pupils with poor results had difficulties in understanding teachers' explanations, and they wanted more help than they were given. The high achieving pupils were not discussed in the study.

In contrast to A. Pettersson (1990), Häggblom (2000) showed, by following pupils' relative achievement from the age of 6 to the age of 15, that low achievers at the age of 6 can become high achievers at the age of 15 and vice versa. In addition, fewer than 20% of the pupils belonged to the same achievement group throughout the period of investigation. Häggblom's conclusion was therefore that the achievement of a pupil at the beginning of her or his time in school (age 6) says very little about her or his achievement at the end of compulsory school (age 15). Therefore, continuous assessment is suggested for teachers to be able to offer challenges and support to pupils, to constantly respond to their present readiness, interest and knowledge, and thereby to use assessment as a part of teaching to meet pupils' learning needs.

3.1.3 Assessment as an on-going process

This thesis addresses the settings in the diverse ability classroom. To orchestrate teaching in such a context it is beneficial for teachers to have knowledge of and ability to recognize HAPs' and other pupils' learning needs (Le Fevre, Timperley, & Ell, 2016; Tomlinson, 2001, 2016). Through continuously assessing pupils, teachers can differentiate instructions and offer challenges that develop knowledge

for all pupils (Tomlinson, 2001, 2016). It is, however, not necessary that a teacher identifies, with certainty, HAPs before adapting teaching towards those pupils' learning needs. Differentiated education is a pedagogical strategy possible to use for teaching in diverse ability classrooms; it is employed to respond to differences in pupils' learning needs, those with high ability, those with learning difficulties, those with another mother tongue than the national language, etc. (Tomlinson et al., 2003). The teacher is expected to address a diversity of pupils' learning needs when implementing differentiated education (Tomlinson et al., 2003).

To be responsive and adapt teaching towards pupils' learning needs, a broad and on-going assessment of pupils' current knowledge, their learning needs, their motivation and interest has shown its value (Corno, 2008). Knowledge of pupils' different learning needs helps teachers to adapt teaching to those who need complexity and to those who need simplicity. Which pupil needs what, complexity or simplicity, may change from time to time, since pupils' interest and achievement can change over the short and long term (see e.g., Corno, 2008; Häggblom, 2000).

Although this thesis addresses the diverse ability classroom, it focuses on teaching MHAPs, those who for example learn quicker and need more complexity in mathematics than other pupils.

3.2 MHAPs' learning needs

In this section the learning needs of MHAPs will be discussed. Since this thesis acknowledges that MHAPs are highly able with a special ability or interest in mathematics, learning needs of HAPs, in general, will also be discussed. The learning needs for most HAPs are assumed to be learning needs also for most MHAPs.

3.2.1 Acceleration and quick learners

HAPs are in general quick learners and their learning is hindered when teaching goes slowly (e.g., Rogers, 2007); most of them do not need repetition in their learning (e.g., Persson, 2010). They do not need as

much time as other pupils to work on and solve complex mathematical tasks (Nolte & Pamperien, 2017). They also need to move ahead in some form when they have passed the set goals in the curriculum (e.g., Gronostaj, Werner, Bochow, & Vock, 2016; Rogers, 2007). Due to their speed of learning, acceleration through grade skipping or subject acceleration may be a suitable way to meet HAPs' learning needs. There are studies that support acceleration as suitable for HAPs, for example Colangelo and Assouline (2009) argue that acceleration results in growth of academic achievement. Further, they state that it has been shown that acceleration either leads to positive impact, or no noticeable change, on social consequences like wellbeing, self-esteem and attitudes towards school. However, Sheffield (2015) argues that, if acceleration in mathematics aims to increase the number of individuals working in mathematics-intensive occupations, acceleration is counterproductive. Since 1990 the number of US pupils who have accelerated in mathematics has exploded, but only a minority of these pupils continued to study mathematics in tertiary education. Sheffield argues that those pupils get an inadequate foundation in mathematics required in, for example, a STEM career. It might therefore be suitable to reflect on the aims of acceleration for each pupil, as some might benefit from it while others will not.

3.2.2 Complexity and enrichments

HAPs often prefer learning situations that are abstract and complex rather than learning situations that build up knowledge in a step-by-step procedure (e.g., Rogers, 2007). According to the literature, HAPs sometimes need to be placed in groups where they can think and learn with like-minded peers (e.g., Nolte & Pamperien, 2017; Rogers, 2007; Vogl & Preckel, 2014). Sometimes they also need to be given opportunities to work individually (e.g., Rogers, 2007; Wolfensberger, 2012). Both Rogers and Wolfensberger found that working individually is beneficial for HAPs as it gives them the opportunity to go into depth in their area of interest without being forced to wait for their age peers. Working with enrichment tasks, sometimes with like-minded peers and sometimes individually, may be a good way to meet the needs of

abstract and complex learning situations. Enrichment in mathematics can be achieved, for example, through external activities such as offering mathematics clubs (see e.g., Benölken, 2015, 2017; Nolte & Pamperien, 2017). Enrichment in mathematics can also be done through differentiated instructions within the diverse ability classroom (Tomlinson, 2001, 2016) and by implementing challenging tasks for all pupils (Nolte & Pampetiren, 2017; Sheffield, 2003).

3.2.3 Challenges

To address their academic progress, HAPs benefit from daily challenges (e.g., Park, Lubinski, & Benbow, 2013; Rogers, 2007). Rogers writes that, due to the fact that teachers often lack time, training in gifted education and training in providing differentiation, they may have difficulties in catering for such challenges. Therefore, she suggests that those challenges may be better placed outside the diverse[4] classroom. In contrast, Nolte and Pamperien (2017) indicate that both teachers and pupils need to be familiar working with challenging mathematical tasks that usually need more time compared to common textbook tasks. How teachers respond to pupils' questions also has an effect on how well the task becomes a learning opportunity. For example, as Nolte and Pamperien mention, rather than confirming pupils' considerations, teachers can ask them to think over their own considerations. Eventually the pupils get used to the method and eventually become more able to prove and reason on their own solutions. The approach is similar to how Sheffield (2003) describes teachers' support of MHAPs, through questions and also similarly to the 'scaffolding and fading' process described by Mason and Johnston-Wilder (2006).

To provide learning opportunities through mathematical tasks, and meet the learning needs of MHAPs, Benölken (2015) and Fuchs and

[4] Rogers writes 'regular classroom', here considered to be the same as diverse classroom.

Käpnick (2009) suggest that such tasks should be challenging, engaging and joyful.

3.2.4 Mathematical tasks for HAPs

Offering challenging, non-traditional mathematical tasks can give teachers the possibility to recognize pupils who normally do not show their mathematical ability (Nolte & Pamperien, 2017). For example, it is possible to observe pupils' ability to generalize and to grasp the mathematical structure in a task, which is among the criteria for mathematical high ability (Kruteksii, 1976; Sheffield, 2003). Suitable tasks for MHAPs are often open-ended tasks, meaning that the task can either be developed further, or that it can be transformed into a new task by changing some conditions (Benölken, 2015; Nolte & Pamperien, 2017; Sheffield, 2003). Usually HAPs learn faster than others and they can cope with more complexity (Nolte & Pamperien, 2017; Rogers, 2007). Therefore, instruction for a task can be given in a shorter time and MHAPs can start working on a complex level earlier than others (Nolte & Pamperien, 2017).

It is, however, not enough to offer challenging tasks to support MHAPs. To be able to support them, teachers also need to choose and/or change tasks to become challenging; moreover it is beneficial if the teacher can solve such tasks (Hoth et al., 2017; Leikin, 2011). Therefore, including challenging tasks and discussions of their mathematical content and didactical possibilities in professional development for teachers develops teachers' competence to support MAHPs (Nolte & Pamperien, 2017). Further, tasks developed with the aim of challenging MHAPs may benefit all pupils (Nolte & Pamperien, 2017; Taflin, 2007).

3.2.5 Challenging tasks for all pupils

Challenging tasks, like problem solving where the solution process is unknown for the pupil (Schoenfeld, 1992), have proven to be valuable for all pupils' learning (Sheffield, 2003; Taflin, 2007), especially when pupils are guided to use their creative thinking (Jonsson, Norquist, Liljekvist, & Lithner, 2014). To be challenging for all, such tasks should in addition to open-ends also have easy entry points (Nolte &

Pamperien, 2017; Sheffield, 2003; Taflin, 2007). They offer possibilities for success on different levels, which allows pupils to engage in the solution process (Nolte & Pamperien, 2017; Sheffield, 2003). There are, however, differences in how most MHAPs work with such tasks compared to other pupils.

Nolte and Pamperien (2017) noticed that, compared to pupils in diverse ability classrooms, MHAPs in homogeneous groups needed less time to understand the task, work with the task, reach generalizations and discuss the solutions. MHAPs in homogenous groups also used deeper mathematical reasoning compared to pupils in diverse classrooms. But Nolte and Pamperien also showed that pupils in the diverse classroom used and practised on mathematical abilities such as generalization. Therefore, although MHAPs also have to work on challenges with like-minded peers (Nolte & Pamperien, 2017), it is reasonable to assume that carefully developed challenging tasks can benefit all pupils in the diverse ability classroom. But how the task will enable learning opportunities is also affected by how it is implemented. Therefore, teachers' implementation of tasks in the classroom is discussed in section 3.2.6.

3.2.6 Teachers' instructions and the task

To offer well-designed mathematical tasks is a central tenet of practice for teachers in mathematics, but the teacher's professional task covers much more (Franke, Kazemi, & Battey, 2007). For example, to support the development of pupils' creative ability, teachers can encourage and give pupils opportunities to choose their own solution process (Mhlolo, 2017; Sheffield, 2003). And to develop the problem-solving ability, it is also of relevance that it is accepted and encouraged to experience failure and to do detours (Kießwetter, 1999; Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012). When pupils only are given easily solved tasks, it can be assumed that they expect all tasks can be quickly solved and if they cannot solve them quickly they might think they are unsolvable (Nolte & Pamperien, 2017). How the teacher meets the pupils in the process is also significant in teaching, for example Nolte and Pamperien argue that, to support pupils' independent thinking

process, teachers can ask why they believe their considerations are correct, rather than confirming them.

Krutetskii (1976) wrote that a pupil must be involved in a mathematical activity, preferably problem solving, if teachers are going to be able to observe her or his mathematical high ability. But when implementing complex and challenging tasks in the diverse classroom it is likely that a task that is a problem for one pupil is not necessarily a problem for another (Schoenfeld, 1992). Therefore, such tasks should offer complexity on different levels, offering challenges to all (Benölken, 2015; Sheffield, 2003). As MHAPs usually can learn and take instruction faster than other learners this may mean that those pupils have to be patient regarding instruction in a diverse classroom. But, the teacher can encourage and allow them to move faster through the different levels of complexity or can allow them to start working on a higher, more complex level from the start. In his review, Szabo (2017) found, for example, that to allow MHAPs to work at a faster speed than others, teachers are recommended to use differentiated instructions.

Encouraging engagement is also mentioned as a criterion for mathematical tasks suitable for MHAPs (Benölken, 2015; Nolte & Pamperien, 2017), but this is of relevance for all pupils (Sheffield, 2003; Taflin, 2007). The mathematical tasks offered by Nolte and Pamperien (2017) are more complex than tasks usually used in school. To handle the complexity of the tasks, Nolte and Pamperien (2017) propose that instructions and worksheets must be more structured than is common for textbook tasks. Therefore, it is reasonable to assume that challenging mathematical tasks are suitable for all pupils in the diverse classroom, but teachers must differentiate their instructions to encourage MHAPs to reach the more complex levels. Szabo (2017) found that MHAPs appreciate that the teacher expects them to work hard, acknowledges them and invites them to structure their critical thinking. But are there any risks involved with not meeting the learning needs of MHAPs in teaching?

3.3 Not meeting learning needs

There are several researchers writing about the consequences of HAPs' learning needs not being met (e.g., Hoth et al., 2017), which include getting bored easily and the risk of giving up learning in school (e.g., Lassig, 2003; Rubenstein, Siegle, Reis, McCoach, & Burton, 2012). The results of Leikin and Stanger (2011) and Mhlolo (2017) indicate that teachers in a diverse[5] ability classroom do not give MHAPs enough support for their learning needs. According to Leikin and Stanger, those pupils are getting opportunities to demonstrate their present knowledge but are not getting support to extend their knowledge in mathematics. MHAPs do not differ from other pupils concerning the development of their knowledge, in that they also require learning support (Mhlolo, 2017; Nolte and Pamperien, 2017).

There are many possible reasons as to why MHAPs' learning needs are not met in a satisfactory way in diverse ability classrooms. One reason can be that many teachers require more professional learning to help them to identify these pupils, as mentioned in many studies (e.g., Leikin & Stanger, 2011; Mhlolo, 2017; Shayshon et al., 2014). Another reason can be that many teachers are not sufficiently prepared to address the huge diversity of learning needs that exists among pupils, especially not the HAPs (see e.g., Hoth et al., 2017). Teachers are therefore key factors for changing the teaching of HAPs (e.g., Tirri, 2017), and how they can orchestrate teaching to include HAPs as well as other pupils in learning is discussed in the next section.

3.4 Teaching strategies to meet diverse learning needs

It is a difficult and complex task to meet the diversity of abilities among pupils in a classroom. Different school traditions try to meet this difficulty with different pedagogical solutions and strategies. Some traditions have 'gifted classes', some have 'pull-out groups', while for

[5] Leikin and Stanger (2011) use 'mixed-ability classroom', which is considered to be the same as diverse classroom in this thesis.

others the idea of separating pupils due to their general, or subject-specific, ability is something unfamiliar. However, no two individuals are totally alike. Therefore, there will always be a diversity of pupils' abilities in a classroom, and for this reason teachers need knowledge of how to orchestrate teaching for a broad diversity of abilities.

3.4.1 Differentiation, inclusion and teaching

Differentiation, as described by Tomlinson (e.g., 2001, 2016), was developed in the US, which has an educational system very different to the Swedish system. Inclusive education as a pedagogical approach is relatively common within the Swedish context (see e.g., Hermansson, 2012; Skolverket, 2016). But, according to Nilholm and Göransson (2016), there is no political decision in favour of inclusive education in Sweden. In addition, there is no common agreement on how to understand inclusion and inclusive education in Swedish schools (Kotte, 2017; Nilholm & Göransson, 2016). Differentiation is, in this thesis, seen as a teaching strategy that enables inclusive education, in the sense that teaching is orchestrated to meet all pupils' learning needs, MHAPs as well as others. When addressing differentiated education, it is common to explicitly mention pupils with high ability and learning difficulties (e.g., Tomlinson 2001, 2005; Tomlinson et al., 2003). In comparison, texts on inclusive education often explicitly mention pupils with learning difficulties (e.g., Göransson & Nilhom, 2014; Florian, 2014), but it is rare that the learning needs of HAPs are mentioned.

According to Tomlinson (2001, 2016), to meet the learning needs of all pupils, teaching has to be differentiated in terms of content, process, product and learning environment. Gaitas and Martins (2017) found in their study that elementary teachers found it most difficult to perform differentiated instructions on content. The second most difficult task reported was assessment, which Gaitas and Martins (2017) believe is connected to teachers' sense of competence in adjusting instructions that help pupils move forward from their current knowledge and skills. To differentiate content in mathematics can mean to differentiate mathematical tasks given to the pupils.

There are some aspects to consider when implementing differentiation in the diverse ability classroom: a safe learning environment, quality curriculum, acknowledging differences, continuous assessment, and challenges with meeting various learning needs.

3.4.1.1 Safe learning environment

A safe learning environment is key to pupils' success (e.g., Tomlinson et al., 2003; Tomlinson, 2016). Through the learning environment teachers can help pupils to understand that they are welcomed and valued as unique individuals. In safe learning environments, teachers will have confidence in pupils' learning capacity and explicitly demonstrate that s/he will support them in learning. In such environments both failures and success are encouraged, and teachers and pupils are aware that both are required in learning. In such safe environments all pupils are given access to develop their knowledge (Corno, 2008; Tomlinson, 2001, 2016).

3.4.1.2 Quality curriculum

'Curriculum' in this thesis includes the combination of steering documents, the teachers' planning, and how teaching is implemented in the classroom. Organisation of the curriculum with clear instructions helps pupils be aware of what matters most in a topic (Tomlinson, 2005, 2016). Through this, pupils who struggle are helped to stay focused and do not drown in a lot of extra information, and HAPs are helped to know when they can go into more complexity rather than spending time on repeating tasks or simple work. When the complexity in mathematical tasks is increased, the structure of the implementation also has to increase. Nolte and Pamperien (2017) have noticed this in support programs for MHAPs.

To be able to implement a quality curriculum, teachers can benefit from having deep conceptual knowledge of teaching, developed from research theories and evidence in teaching (Le Fevre et al., 2016). It is also an advantage if they can use and verbalize research-based knowledge to understand and work effectively in novel situations (Le Fevre et al., 2016). It is also relevant that teachers have content

knowledge, for example in mathematics, to orchestrate teaching effectively (e.g., Hoth et al., 2017; Sullivan 2008). However, An, Kulm and Wu (2004) stated in their study that knowledge of teaching, and pupils' thinking, are the most critical aspects.

3.4.1.3 Acknowledge differences

According to Franke et al. (2007), teachers develop relationships with pupils which are not only built upon pupils' mathematical understanding but also on the pupils' past experiences and background. Franke et al. found that teachers who build such relationships make a difference for their pupils' success. In other words, teachers have to possess extensive knowledge, in pedagogics and about their pupils – their knowledge level as well as their interests, hopes, dreams and fears (Parsons & Vaughn, 2016). Teachers' knowledge of pupils' thinking is seen as an especially critical aspect of teaching (An et al., 2004). This means that to be able to teach MHAPs, teachers are required to have knowledge of how to first recognize them and second on how to guide them further in learning.

Teachers who acknowledge and are responsive to pupils' differences, can create a flexible and dynamic centre for teaching that expands to include more pupils in learning over time (Corno, 2008). Such teachers naturally use differentiation in teaching (Tomlinson, 2016) to teach pupils in diverse ability classrooms, those who need additional instructions and lots of repetition, and those who learn quickly and are several years ahead of their classmates in knowledge. Further, such teachers understand that all pupils bring value to the classroom and must be recognized and supported in their knowledge development (Tomlinson, 2016; Tomlinson et al., 2003).

3.4.1.4 Continuous assessment

Teaching is about continuously supporting pupils' learning, and it specifies pupils' learning trajectories (Kazemi & Franke, 2004). To be able to respond to pupils' learning needs, teachers must adapt their teaching towards pupils' readiness, interest and learning profiles. Therefore, *assessment is an on-going process* in successful teaching –

aiming to help each pupil to develop further from their present knowledge level (Bourke, Mentis, & Todd, 2011; Corno, 2008; Tomlinson, 2005, 2016).

The teacher orchestrates the content, the pupils and the instructions in relation to each other. This means that the teacher makes decisions *on the spot* to support both individuals and the class (Franke et al., 2007). Therefore, it demands flexibility and capacity to modify practices to address specific pupil needs, and also the ability to verbalize underlying principles for one's skills (Hatano & Inagaki, 1986). Teachers' ability to be flexible is also addressed in the knowledge quartet (Rowland, Huckstep, & Thwaites, 2003), where one of the dimensions used to describe teaching situations is *contingency*. This dimension concerns classroom situations that are almost impossible to plan for. It addresses teachers' ability to 'think on one's feet', i.e. to respond to pupils' ideas and be adaptive and deviate from her/his agenda when appropriate.

3.4.1.5 Challenges with meeting learning needs

It is challenging for teachers to develop their teaching towards greater flexibility[6] (Le Fevre et al., 2016) or to teach for mathematical high ability[7] (Franke et al., 2007); the teacher must re-evaluate many traditional mind-sets and skills associated with teaching. It demands engagement, ongoing learning and development of innovative solutions to improve pupils' outcome (Bransford et al., 2009). To develop in this way, teachers are helped by considering teaching holistically and by being able to judge its effect within a larger frame (Le Fevre et al., 2016). According to Le Fevre et al. (2016), it facilitates teachers to develop in this way when they see themselves as experts[8].

Research indicates that teachers are aware that teaching ought to be differentiated and that they try to find ways in their practice to meet

[6] Word used in Le Fevre et al. (2016) is adaptive teaching.

[7] They use proficiency

[8] They write adaptive experts.

the needs of each pupil (Shayshon et al., 2014). However, teachers' awareness does not mean that they know what is written in the literature on how, why and who the teaching should be differentiated for (Mhlolo, 2017; Shayshon et al., 2014).

3.5 Professional development to develop teacher competence

As pointed out by Corno (2008), researchers need a better understanding of what teachers actually do, at the same time as teachers need to know more about theories of practice. To be able to reach this double aim, it is assumed in this work that research on practice acts in close cooperation, which is strongly supported by, for example, suggested methods in design research (Plomp & Nieveen, 2010). A natural way to increase teachers' theoretical knowledge and develop their teacher competence is through professional development programmes, and research on such is common for different reasons. The studies in Parts 2 and 3 are connected to professional development programmes for teachers, therefore different aspects of such programmes are discussed in the following sections.

3.5.1 Professional development programmes

There are four key elements in designing a professional development programme: the programme itself, the participating teachers, the facilitator, and the context (Borko, 2004). Studies in Parts 2 and 3 of this thesis have one professional development programme for each part, participants are teachers in mathematics, the facilitator is the same for both, and both programmes have a similar context. Research on professional development can be organized into three hierarchic phases, each build on the previous. Phase 1) Research on a specific professional development programme at a single place. Phase 2) Research on a specific programme with more than one facilitator at more than one place. Phase 3) Increased complexity through implementing multiple programmes at multiple places (Borko, 2004). The studies for this thesis focus on relationship between the programme and the participating teachers, therefore they fall under phase 1 in Borkos' (2004) description.

3.5.2 Learning community in professional development

Naturally a programme always has an aim and a goal, which are guided by a certain view on education and learning that is meant to be transferred to the participants, i.e. the participants develop a *shared vision* (Sowder, 2007). Based on the evidence that knowledge is created and re-created in social situations, a professional development programme ought to offer social situations. While teachers learn new content, and develop their teaching competence, social situations such as discussions and workshops are useful to give them the opportunity to develop and clarify their individual values and beliefs (Sowder, 2007). In a professional development programme which aims to develop teachers' competence, it is critical that the teachers actively engage in creating knowledge, not just passively consume knowledge (Le Fevre et al., 2016). On a group level, phase 1 research has shown that teacher learning and instructional improvement are fostered by a strong professional learning community (Borke, 2004). Studies on, for example, QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning), where strong professional learning communities evolved, increased the teachers' use of challenging tasks in teaching (Borko, 2004; Silver, 1995).

3.5.3 Changes in teacher competence

Phase 1 research has shown that intensive professional development can increase teacher knowledge and change their teaching practice (Borko, 2004). For example, Chamberlin and Chamberlin (2010) showed that when pre-service teachers, during a professional development programme, implemented problem-solving tasks among MHAPs, the teachers improved their competence in teaching MHAPs. Among other things, the teachers were afterwards able to recognize the need to differentiate instructions for MHAPs.

Borko (2004) highlights three characteristics that can change for individual teachers in professional development programmes: subject matter knowledge for teaching, understanding student thinking, and instructional practices. Those three characteristics are mirrored in this study through, respectively, mathematical and didactical knowledge of

challenging tasks, understanding HAPs thinking, and differentiated education. According to Sowder (2007), research has provided evidence that it is efficient to focus on pupils' thinking and learning in professional development programmes. She refers to research showing that teachers provide better mathematics education when they understand pupils' reasoning and adapt their teaching to reflect their knowledge of the pupils' thinking, compared to when they focus on mathematical content.

3.5.4 Theoretical knowledge through professional development

Teachers are helped by deep conceptual understanding to be able to solve existing problems and to find innovative solutions to new problems (Le Fevre et al., 2016). Teachers who can use their deep understanding to act flexible, in an efficient and purposeful way in novel situations, can also verbalize the theoretical principle on which her or his acts are grounded, and through that address specific needs and conditions (Le Fevre et al., 2016). Although stable and well-established practices offer opportunities to act according to routine without constantly questioning whether the act is efficient, to develop and improve teacher competence must be an ongoing process (Le Fevre et al., 2016). To develop teachers' understanding of pupils' thinking and learning is therefore proposed to be both an aim and a strategy in professional development programmes (Sowder, 2007). The opportunity to discuss and hypothesize teaching together with colleagues allows greater insight into pupils' learning needs (Sowder, 2007). Collaboration is a core feature of professional development programmes that aims to improve teachers' competence (e.g., Desimone, 2009; Parsons, Ankrum, & Morewood, 2016).

3.5.5 Long-term or short-term professional development

To be successful and effective, Desimone (2009) argues for long-term professional development programmes. Although short-term professional development programmes can be criticized as inefficient, such programmes can under certain circumstances directly influence a teacher to change her or his teaching. Liljedahl (2010) showed that if a

teacher is, or becomes, aware of shortcomings in her or his teaching, and if the professional development programme can offer constructive ideas or examples of how to improve, then the teacher can directly change her or his teaching. Phase 1 research on professional development programmes often has participating teachers who are 'motivated volunteers' and therefore also are more motivated to try new ideas (Borko, 2004). It is reasonable to assume that such teachers also are more open to recognize shortcomings in their own teaching and to adapting and trying new ideas.

3.5.6 Professional development on teaching MHAPs

There seems to be a consensus among researchers that teachers must be given more professional development on gifted educational provision (e.g., Shayshon et al., 2014). There is also empirical evidence that professional development increases teachers' skills in supporting MHAPs (e.g., Chamberlin & Chamberlin, 2010). In such professional development, discussions of mathematical and didactical aspects of challenging mathematical tasks are included (Chamberlin & Chamberlin, 2010; Hoth et al., 2017; Nadjafikhah et al., 2012). Hoth et al. (2017) found that teachers with strong professional knowledge, i.e., regarding both mathematical content and pedagogics, were able to identify and support MHAPs.

However, Hoth et al (2017) also found that future and early career teachers who have difficulties in logical reasoning and in understanding mathematical structures also have difficulties in identifying and supporting MHAPs. They argue that teachers of MHAPs must have deep mathematical knowledge themselves, but they also argue that teachers can develop their professional knowledge to help them better identify and support MHAPs. When greater emphasis is placed on mathematical and didactical aspects in professional development, it increases teachers' understanding of the diversity among pupils' abilities, in particular their understanding of MHAPs (Hoth et al., 2017). Further, for similar reasons Nadjafikhah et al. (2012) suggest that teacher education should give deeper attention to

designing and implementing educational environments that promote creativity.

Teachers who continuously choose to develop can, by including challenging tasks in their professional development, be supported in developing both their mathematical and pedagogical skills (Singer, Sheffield, Freiman, & Brandl, 2016). Such professional development does not necessarily have to directly address high ability, improving teacher skills can still benefit the learning needs of MHAPs (Chamberlin & Chamberlin, 2010). To improve and develop teachers' understanding and teaching skills, it is also recommended that teachers collaborate and discuss MHAPs and their learning processes (Singer et al., 2016; Van Tassel-Baska & Hubbard, 2016). Therefore, Van Tassel-Baska and Hubbard (2016) suggest professional development, on how to identify and support HAPs, to be an ongoing process during the school year. For continuous development, they highlight collaboration and communication between teachers at the same school and across schools. Further, they suggest close cooperation with a university that has a focus on gifted education, and that can assist in the steps of implementing and developing strategies to meet the HAPs' learning needs.

3.5.7 Professional development and the researcher as facilitator

When someone facilitates and in parallel does research on a professional development programme, that person can be called a researcher-facilitator (see e.g., Olin, Karlberg-Granlund & Furu, 2016). This double role can bring research closer to practice, but the researcher needs to reflect on how her or his double role may influence the findings. The researcher-facilitator can intertwine with the teachers in different ways, for example s/he can facilitate and support the teachers to perform research in their own classrooms; s/he can participate in the programme and help to keep focus on the topic; and s/he can participate as consultants and contribute with her/his skills (Platteel, Hulshof, Ponte, van Driel, & Verloop, 2010).

A criticism is that it becomes difficult to withhold the analytical distance when being too close to the practice; on the other hand some say it is impossible to create understanding of a practice if you are not a member of it (Hoel, 2000). It is therefore helpful for the research-facilitator to clearly distinguish what her/his obligation is in the professional development programme as a researcher and as a facilitator.

Through their findings Olin et al. (2016) show how the double role of the researcher-facilitator can be separated into the role of researcher and the role of facilitator connected to parts of the professional development programme. For example, they highlight that there is no, or very limited, time for analysing and writing within the frame of professional development programmes, which therefore is clearly connected to the research. While it is the facilitator's task to, for example, contribute with organizing competence, to listen to and include teachers' (the participants') knowledge and enhance reflection. To systematically document and evaluate and present theoretical perspectives is another part the researcher is responsible for, according to Olin et al. To achieve learning through professional development it is important that the researcher-facilitator enhance the possibility for teachers to be actively engaged (Olin et al., 2016).

3.5.8 Collaboration in professional development

Usually professional development for teachers has an aim to develop practice which means that teachers often have to change something in their practice. When a facilitator and/or researcher asks them to implement something, for example a new teaching method or a mathematical task, they may be reluctant (Randi & Corno, 1997). Working in collaboration with teachers can, on the contrary, lead to them changing and adjusting their practice as a continuous process (Corno, 2008). According to Ponte et al. (2003), teachers' active involvement and professional knowledge is also required when analysing questions related to teaching practices.

To create an open-minded environment built on trust, where inputs from participants and facilitators have equal worth, is crucial to encourage collaboration (Olin et al., 2016). In such an environment researchers and facilitators learn from teachers, and teachers from the facilitator and researcher, each part bringing knowledge from different perspectives where none is more worthwhile than any other. It is the facilitator and researcher's work to capture the different perspectives from all participants and to create more knowledge from this for practice and for research (Olin et al., 2016).

A successful collaboration that avoids conflicts is preferably performed in an environment that nurtures development in practice and to create this, researchers, facilitators and teachers ought to clarify responsibilities in their collaboration (Platteel et al., 2010; Ponte et al., 2003). Otherwise misunderstandings and confusions, affecting the development process negatively, can easily happen, as described by Platteel et al. (2010) during their action research project. However, all participants also need to be flexible and ready to adjust the given responsibility depending on what is asked for during the collaborative process. For example, a researcher that at first may have an observation role can be asked by the participating teachers for greater involvement than is customary in research. Later on in the programme, this can lead to teachers successively developing their skills to examine and evaluate, and thereby develop their own practice (Platteel et al., 2010).

The review by Robutti et al. (2016) describes a successful collaboration between researchers, facilitators and teachers as a transformation process, where the early activities are often led by researcher-facilitators who share their knowledge with teachers. Gradually when both teachers and researcher-facilitators engage in activities and learn from experiences and reflections, this leads to teachers developing their own critical voice. They can use this to communicate with other teachers as well as with facilitators and researchers, for example on what they find to be possible or impossible in school settings. In addition, Robutti et al. (2016) also identified a barrier to successful collaboration as teachers lacking a sense of belonging – in other words

if the teachers did not feel that they belonged to, or perceive possibilities to influence, the professional development programme.

4 Theoretical considerations

In this chapter, theoretical considerations guiding this thesis are presented and discussed in terms of how they frame, and why they are suitable, for the performed studies. This thesis is a work in the educational field, which addresses teachers' professional task of orchestrating teaching that includes all pupils in learning, focusing on those who are highly able. An assumption for this work is that there is a broad diversity in abilities among pupils and that some of those pupils have the ability to learn at a faster rate and understand more abstract content compared to most other pupils. This means that they have learning needs that often go unmet in a diverse ability classroom, since most often teaching addresses the average (Tomlinson et al., 2003). Those pupils are called HAPs in this thesis and MHAPs if they specifically have these learning needs in mathematics. It is also an assumption for this thesis that one way to meet the broad diversity in pupils' ability is through differentiated education. The purpose of this chapter is to clarify theoretical considerations on high ability and differentiated education that frame the studies in this thesis. In addition, the last section of this chapter discusses how positioning theory has been used to capture the teachers' perspective, which belongs to the overarching aim of this thesis.

4.1 High ability

There is no universal definition of high ability[9] (see e.g., Mattsson, 2013; E. Pettersson, 2011). Several theories of high ability exist; one of the earliest was given by Terman in the 1920s (Terman & Oden, 1947) and one of the latest is by Sternberg (2017).

[9] High ability is used as synonymous to giftedness in this thesis.

4.1.1 Three paradigms of high ability

The field of gifted education is complex due to the lack of common theories, but also due to the role cultural beliefs play in approaches to both identification and provision in different countries and contexts (Dai & Chen, 2013). A person can be seen as highly able in one culture, but not in another. Therefore, it is necessary to discuss different views of high ability and gifted education in relation to the cultural context, such as the Swedish school system. Through reviewing definitions and models of high ability, Dai and Chen (2013) suggests that we should look upon what kinds of answers we get from the 'paradigm questions' *What? Why? Who?* and *How?*

- *What is high ability?*
- *Why should education bother about high ability?*
- *Who is highly able?*
- *How should education support HAPs?*

Those questions are essential to consider when theorizing a practice like education. By doing so, Dai and Chen divide views of high ability into three paradigms: *The Gifted Child Paradigm*, *The Talent Developing Paradigm* and *The Differentiation Paradigm* (Dai & Chen, 2013). Their standpoint is that high ability is a social construct that is created to serve practical ends. Their view is also adapted in this thesis since teaching is seen as a practical profession and since educational systems are influenced by the society they act in, for example by politics, culture and traditions. According to Dai and Chen, these paradigms give different answers to the four paradigm questions.

In Table 1 the three paradigms' view on the proposed four paradigm questions *What? Why? Who?* and *How?* (Dai & Chen, 2013) are briefly summarized.

Table 1 Summary of the three paradigms' view on the four paradigm questions What? Why? Who? and How? Table adapted from Dai and Chen (2013).

	Gifted Child Paradigm	Talent Development Paradigm	Differentiation Paradigm
What is high ability?	A static attribute, it is context-free and exceptionality is assumed.	Shapeable, it is about aptitudes for a particular domain and exceptionality is not assumed.	Emerges from pupils' need of differentiation and it is context-dependent.
Why bother?	To serve the highly able pupils with a goal to develop leader qualities.	To support development of domain-specific excellence and innovation.	To respond to manifested individual needs within schooling.
Who is highly able?	Pupils are classified through superior mental qualities, mainly measured by IQ tests.	Select or place pupils based on their aptitudes for a particular domain; IQ level is not sufficient as measurement.	Continuous assessment/diagnosis reveal strengths and needs for educational purpose in an educational context.
How should education support?	Programmes designed uniquely for highly able pupils and pull-out groups are supposed to fulfil the purpose.	Enrichment activities, learning connected to real life and mentoring are models to serve the highly able pupils.	Curriculum, instructions and other interventions in school should be adapted to the learning pace of the highly able pupil.

When reviewing theories of high ability, it is possible to start by analysing *who* the highly able are, according to the theory in question. The answer to this question often gives a hint as to *what* the theory is describing high ability to be, and sometimes also *why* and *how* those pupils should be recognized. For example, if high ability is something within the individual, as in *The Gifted Child Paradigm* - or if it is a developmental ability, a behaviour, as in *The Talent Development Paradigm* - or if it is as described in *The Differentiation Paradigm*, a

consequence of external factors like education, that can influence individuals to show their highly able behaviour.

4.1.1.1 What is high ability?

In *The Gifted Child Paradigm* high ability is viewed as something static. In contrast, *The Talent Development Paradigm* and *The Differentiation Paradigm* view high ability as something developable. According to Terman's description from the 1920s, high ability is something within the individual where the potential, although not necessarily expressed, can be identified by high IQ (Terman & Oden, 1947). By contrast, according to Sternberg's (2017) description, it is something the individual expresses and is not necessarily possible to identify through standardized tests such as IQ measurements. However, high ability can also be interpreted as an internal ability, where lack of positive intrapersonal and environmental catalysts can both favour and inhibit achievement (Gagné, 2005). Other models, though, view high ability as something that only can be seen in an individual's outstanding performance (Renzulli, 2005).

This thesis has the view that the way education is planned and implemented affects pupils' well-being as well as their further development, academically but also socially. It is acknowledging that pupils are different, and that some have higher ability than others in some things and sometimes in most things. Of minor interest for this work is whether this ability is developed or inborn. It is also acknowledged in this thesis that pupils with high ability have special learning needs. In addition, when teaching meets these needs, it helps them develop their knowledge, but it also helps them feel acknowledged.

4.1.1.2 Why should education bother about high ability?

Both Terman (Terman & Oden, 1947) and Sternberg (2017) display similarities regarding *Why* education should bother about HAPs. Terman used the IQ-measurement with the aim of identifying HAPs so that society could help them develop into future leaders. Sternberg (2017) also argues that it is important to identify individuals who can

become future leaders and who can solve the big problems of tomorrow. But Sternberg clearly states that IQ measurements are no longer appropriate to identify them.

> Today the greatest problems facing the world—those that we need gifted individuals to address—are not ones that IQ can directly address. (Sternberg, 2017, p. 4).

Sternberg (2017) claims that today there are a lot of 'high-IQ people' working on problems like global warming and air pollution, but they only reach modest progress. He means that to be able to solve the great problems the world is facing, highly able individuals are needed, who must be passionate problem-solvers rather than striving to enhance their own prestige. Further he means that to find them, a broad range of skills must be searched for, and that education is responsible for finding and developing the HAPs. Interpretations of the models of Gagné (2005) and Renzulli (2005) also suggest that education should support HAPs to develop as high achievers and/or innovative leaders and problem-solvers for the future, although they are not as specific as Sternberg.

Another reason why education should bother about HAPs is given by the view that the purpose for education to be differentiated is to match all pupils' strengths, interests and learning styles (Tomlinson et al., 2003), meaning that education should bother about all pupils. This view embraces possibilities and challenges within the diverse ability classroom.

The standpoint in this thesis in relation to the question "*Why* should education bother about HAPs?", is that education should bother about all pupils. No one should be excluded; all pupils have the right to be supported further in knowledge. As discussed in chapters 1 and 2, the learning needs of HAPs have, however, not been well met in the diverse ability classroom. Meeting their learning needs may lead to future innovators and great leaders, as mentioned for example by Sternberg (2017), but firstly it responds to an education that aims to meet all pupils' learning needs. This standpoint is the starting point for the further reasoning in this thesis.

4.1.1.3 Who is highly able?

To identify HAPs, Sternberg (2017) suggests competencies such as creative, practical, and wisdom-based and ethical skills to be of equal or more interest in the identification process compared to using IQ measurements. He also means that those competencies are developmental, and not are suitable to measure with standardized tests. Through the *Three-Ring Concept of Giftedness* by Renzulli, a clear stance is taken against the use of IQ measurements when identifying HAPs (Renzulli, 2005). He agreed that high IQ predicts high school achievement, but he argues that what he calls *creative-productive* competency will have a larger influence when predicting who might be able to come up with the most innovative ideas in future. In comparison, according to *The Differentiated Model of Giftedness and Talent* (DMGT), HAPs are identified through IQ (IQ>120) (Gagné, 2005). In this model a broad range of abilities includes 10% of a population and covers several domains.

In collaboration with school psychologists, in school health teams, Swedish teachers can sometimes, but rarely, get knowledge of pupils' results on cognitive measurements. Supported by the view of Renzulli (2005) and Sternberg (2017) that IQ measurements and other standardized tests are not enough to identify HAPs, this thesis is of the opinion that a broad assessment process is desired to identify HAPs. In this broad process, teachers' continuous assessment of pupils provides information on the pupils. The standpoint is that teachers with deep knowledge of pupils' different learning needs can notice HAPs. In some cases, those teachers require a broader assessment to fully understand the pupil. If so, those teachers can work together with school psychologists, parents and the pupil to create understanding of that specific pupil's learning needs, with the aim of orchestrating teaching that enables learning for that pupil.

4.1.1.4 How should education support them?

The Talent Development Paradigm and *The Differentiation Paradigm* share a common aim in supporting HAPs, which is to increase pupils' achievement and support them to develop extraordinary abilities.

There is, however, also a difference; the first paradigm focuses on identifying individuals and supporting them through specially designed pathways, for example gifted classes. In contrast the second focuses on continuously offering a rich and challenging learning environment for all pupils and proceeding to challenge those pupils who indicate that they require additional stimulation.

According to the DMGT model (Gagné, 2005), both intrapersonal and environmental catalysts can make it possible for HAPs to develop as far as possible. Intrapersonal catalysts can, for example, be health and personality, while provisions through education are an environmental catalyst. According to this model, education ought to be planned and implemented to support and challenge HAPs' internal and external learning needs. In comparison, Renzulli (2005) addresses external factors, for example to offer enrichments that make it possible for pupils to develop. In practice it means that, according to both theories (Gagné and Renzulli), pupils identified as highly able should be offered opportunities in education that enable them to develop in learning.

Finally, as expressed in *The Differentiation Paradigm* (see Dai & Chen, 2013), focus can also be placed on differentiating education in the diverse ability classroom, with the aim of supporting all pupils to develop in learning, without first identifying HAPs. To enable this, pupils' present knowledge can be assessed and thereafter teaching adapted to meet pupils' learning needs (e.g., Matthews & Foster, 2006). The view of how to support HAPs in education given by the differentiation paradigm is coherent with the Swedish context and school tradition.

4.1.2 Mathematically HAPs

Theories describing high ability in mathematics[10] often address mathematical abilities as an identification factor (e.g., Fuchs & Käpnick, 2009; Krutetskii, 1976; Sheffield, 2003; Pitta-Pantazi, 2017). Pitta Pantazi, Christou, Kontoyianni, and Kattou (2011) divide the abilities into five sets: spatial, quantitative, qualitative, causal and verbal ability. In these theories, mathematical abilities are more often described as abilities supporting problem solving such as the ability to quickly generalize, grasp the formal structure, be flexible in mathematical activities, or remember mathematical relationships (e.g., Fuchs & Käpnick, 2009; Krutetskii, 1976; Sheffield, 2003).

Research has provided knowledge of HAPs' learning needs (e.g., Gronostaj et al., 2016; Persson, 2010; Rogers, 2007; Vogl & Preckel, 2014) and risks when those needs are not met (e.g., Lassig, 2003). Therefore, addressing the development process of MHAPs' abilities (e.g., Fuchs and Käpnick, 2009; Sheffield, 2003) and not only the characteristics of mathematical high ability (e.g., Krutetskii, 1976; Pitta Pantazi et al., 2011) is of particular interest for the aim of this thesis.

Due to the aim of highlighting pedagogical possibilities that enable learning opportunities also for MHAPs, the identification process of MHAPs takes a smaller part. Greater focus is given to the question of how to provide teaching that develops those pupils' mathematical knowledge. As clarified in section 4.1.1, this thesis view on high ability is close to *The Differentiation Paradigm*, which means that differences in pupils' ability are acknowledged. Thereby follows the view that teaching ought to be adapted to meet this broad range of pupils, not primarily identifying who is highly able or not. Thereby, differentiated education as pedagogical method is of interest and will be discussed in the next section.

[10] In this thesis, 'highly able' is used; Fuchs and Käpnick use 'mathematical talent'; Krutetskii (1976) uses 'highly capable', Pitta-Pantazi (2017) uses 'mathematical giftedness'; Sheffield (2003) uses 'mathematically promising'.

4.2 Differentiated education

Differentiated education is a pedagogical strategy that is compatible with the Swedish context and Swedish Education Act (SFS 2010:800), especially through the focus on success for each pupil through responding to their differences (Tomlinson, 2016). It is also a strategy with the intention to include all pupils in learning, the highly able as well as all others. In this section the principles of differentiation (Tomlinson, 2016) are first presented. Thereafter, descriptions are given to how those principles are considered for, and thereby frame, the teaching strategies this thesis relates to when including MHAPs in learning within diverse ability classrooms.

Addressing the diverse ability classroom, equity through inclusion becomes applicable, which means that each pupil is provided opportunities to be successful in developing knowledge (Franke et al., 2007) - independent of their readiness or learning profiles (Florian, 2014; Le Fevre et al., 2016; Nilhom & Göransson, 2016; Tomlinson et al., 2003). To create inclusion, pupils with high ability as well as those with learning disabilities must be provided with learning opportunities that develop their knowledge. As Tomlinson (e.g., 2016) writes, those learning opportunities cannot be designed in the same way for pupils with different learning needs.

4.2.1 Principles of differentiation

The principles of *differentiation* (Tomlinson, 2016), here used as a shortened form of *differentiated education*, and how to work towards them, are shown in Figure 1. In this study, the *diverse*[11] *ability classroom* is the physical location where teaching is conducted, *differentiated instructions* are the instructions teachers plan for, and perform, in collaboration with their pupils.

[11] Tomlinson (2001, 2016) writes 'differentiated classroom'.

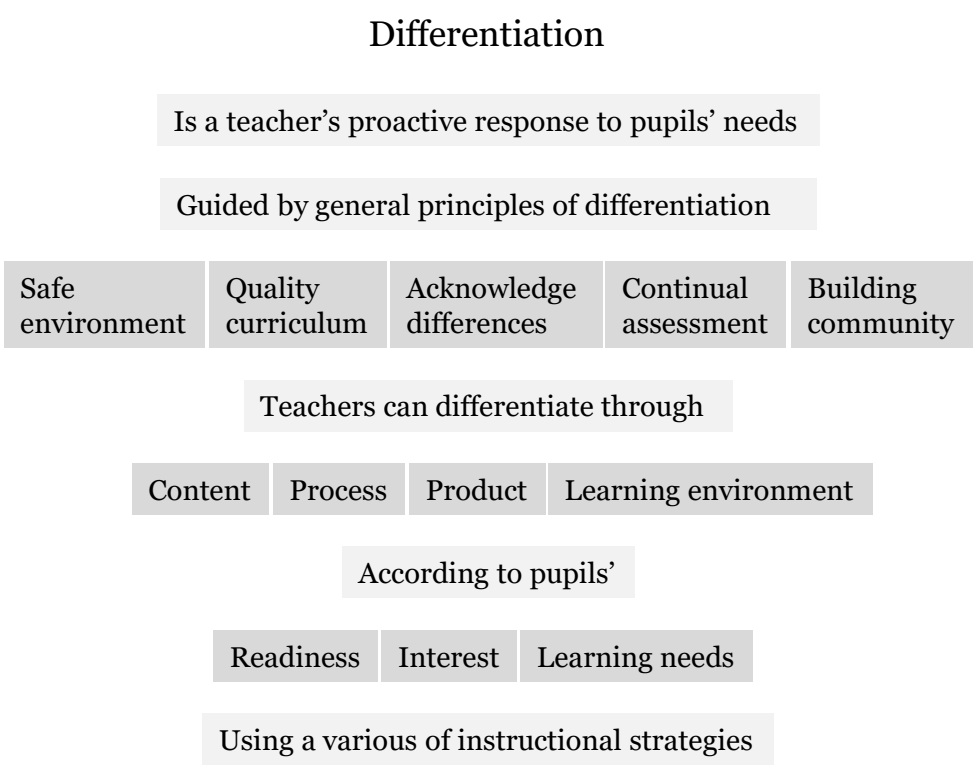

Figure 1 The principles of differentiation, adapted from Tomlinson (2016, p. 20).

The use of differentiated instruction does not mean that teachers plan a separate lesson for each pupil. According to Tomlinson et al. (2003), differentiated instruction is proactive. The teacher assumes that there are different learners with different needs in the classroom, and therefore plans a variety of ways for the pupils to reach learning. The teacher then still must fine-tune the instructions for individuals within the teaching situation.

> Perhaps a good way to begin an exploration of differentiated teaching is to look at the classroom through the eyes of two broad categories of students—those who are advanced and those who struggle. (Tomlinson, 2001, p.10)

Teaching is assumed to fit all pupils when teachers differentiate instructions regarding the components *content*, *process*, *product* and *learning environment* (Gaitas & Martins, 2017; Tomlinson, 2001, 2016), according to pupils' differences regarding *readiness*, *interest* and *learning needs* (Tomlinson, 2005). Bourke et al. (2011) also add

assessment[12] as a component of instructional strategies to differentiate. Tomlinson (2016), however, argues that instruction and assessment are inseparable, which also is assumed in this thesis, and therefore continuous assessment is seen as an on-going process that is intertwined in all instructions organized by the teacher.

4.2.1.1 Differentiate content through teaching

Content is here connected to the knowledge and skills pupils learn (Bourke et al., 2011; Tomlinson, 2005, 2016), and are provided through teaching, for example by mathematical tasks. Many HAPs benefit from abstract and complex content (Rogers, 2007; Tomlinson, 2001), while there are other pupils who need content with a clear step-by-step explanation. As shown in the review of Tomlinson et al. (2003), many studies have confirmed that teaching is likely to fail when teachers use tasks with the same content for all pupils in a class, due to the mismatch between the task and the knowledge level of many pupils.

4.2.1.2 Differentiate process through teaching

Process refers to how the content is taught (Gaitas &Martin., 2017; Tomlinson, 2005, 2016), and how the pupils are encouraged to work with the content. Many HAPs benefit when teaching gives them opportunities to engage in higher order thinking, such as to analyse, evaluate and create (see Krathwohl, 2002), and involves tasks which are open-ended and allow freedom of choice (e.g., Tomlinson, 2001, 2016). But HAPs as well as all other pupils may also differ in how they process and work with the taught content. For example, as Krutetskii (1976) found, some MHAPs are analytical in their mind and prefer working with algebraic representations while some are geometric and prefer working with geometric representations. To differentiate teaching in process, it requires teaching to be flexible, and to allow and encourage pupils to work with tasks in different ways.

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[12] They write 'high-quality assessment'.

4.2.1.3 Differentiate products through teaching

Product refers to pupils' presentation of what they learned (Bourke et al., 2011; Tomlinson; 2005, 2016). HAPs benefit from having deadlines and a real audience, extended or accelerated outcomes, i.e. outside and above the requested curricula (e.g., Tomlinson, 2001, 2016). It can also be assumed that pupils' work is continuously and/or formally assessed in teaching. Following the principles of differentiation, it becomes natural that pupils in diverse ability classrooms produce work of varying quality. To be able to help each pupil develop and achieve as highly as possible, teachers can adapt assessment situations to pupils' differences.

4.2.1.4 Differentiate the learning environment through teaching

Learning environment refers to an environment that supports pupils' learning, essentially through providing a safe environment where all pupils are recognized and valued as they are (Corno, 2008; Tomlinson, 2001, 2016). In the diverse ability classroom this can, for example, lead to flexible grouping (Pham, 2012), which meets the need for HAPs to sometimes be placed in groups with likeminded peers (e.g., Rogers, 2007; Vogl & Preckel, 2014), and at other times can lead to encourage pupils to work individually (e.g., Rogers, 2007; Wolfensberger, 2012). To create a differentiated learning environment, attitudes towards differences need to be addressed. For example, teachers must consciously work with creating understanding and acceptance of pupils' differences and thereby different learning needs (Tomlinson, 2016; Tomlinson et al., 2003).

4.3 The meaning in communication and positioning theory

To capture the teacher perspective included in the overarching aim, teachers' perceptions on how to orchestrate teaching that includes HAPs, are of interest. By using positioning theory (Harré, 2012, Harré & van Langenhove, 1999; Harré & Moghaddam, 2003) as an analytical framework it is possible to probe the meaning in communication and thereby the communicators' perceptions. In Part 3, teachers' discussions are probed to capture their perceptions of their own ability

and power to orchestrate teaching of HAPs in the diverse ability classroom.

A communication, such as a discussion, is always framed by and performed in a context that has an influence on what will or will not be said. The probed discussions in Part 3 are performed during a professional development programme by in-service teachers. This means that there are several 'actors' that can influence the teachers' discussions, for example the pupils, the school organisation to which the teachers belong, the facilitator of the programme, and the research community that influences the facilitator. Figure 2 shows how some of these 'actors' are related to each other and how they can thereby, implicitly or explicitly, influence teachers' discussions of pupils' learning. Rights and duties as defined in positioning theory (Harré, 2012) as guiding human behaviour, i.e. what someone *will* or *will not* do to someone or something. What someone does or doesn't do also includes what someone says or does not say. Figure 2 also shows how pupils, teachers, the school organisation, facilitators and research are connected to each other through rights and duties.

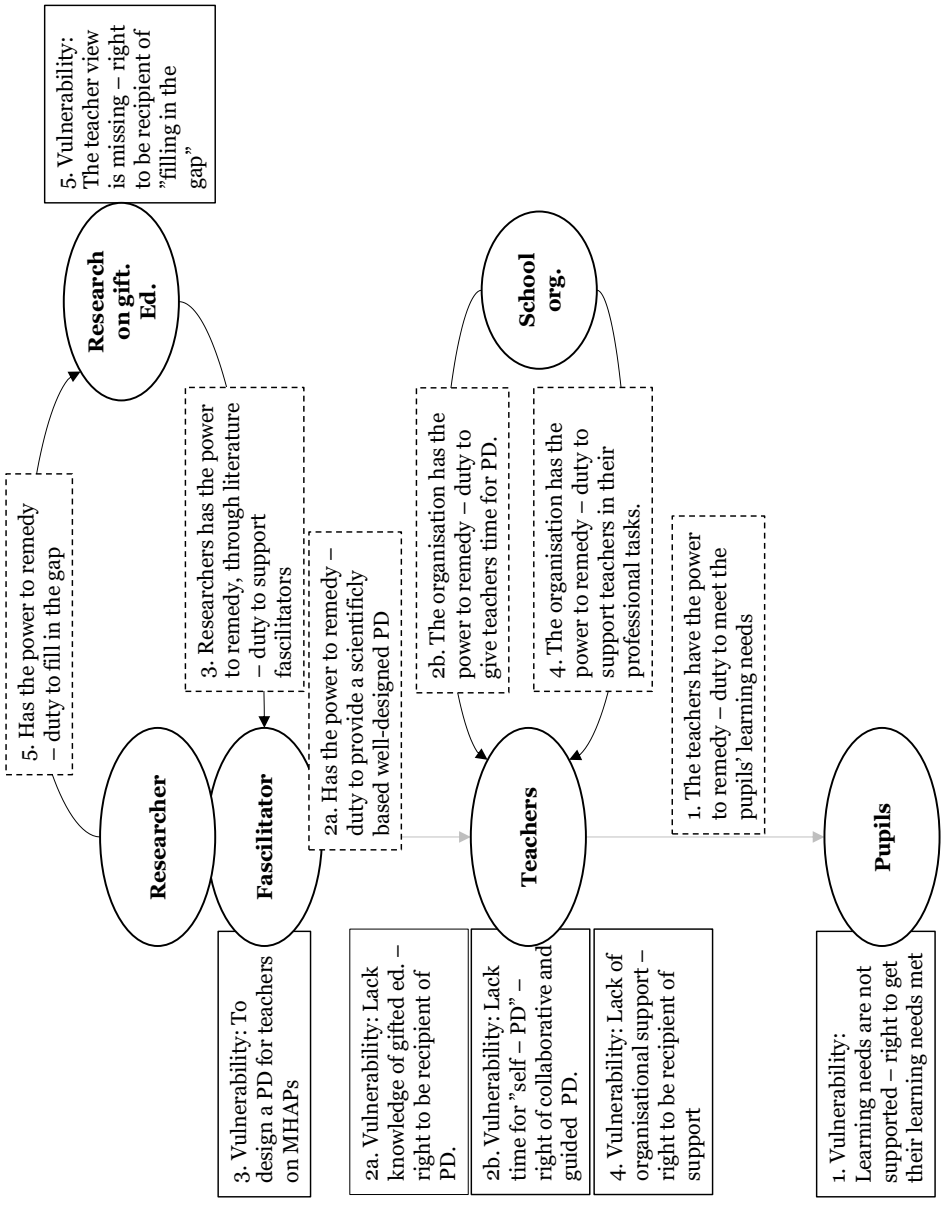

Figure 2 An illustration of connected rights and duties that can affect teachers' teaching. PD is used as an abbreviation for professional development.

4.3.1 Positioning vs position

In Positioning theory, human behaviour, for example speaking, can be seen to be guided by the following three perspectives.

A. What you can do.
B. What you do.
C. What you are permitted to do.

The three perspectives are connected to each other, B is a function of A and C. The perspectives A and C are discursive, while the interaction between them is cognitive (Harré, 2012; UNU-CRIS, 2015).

In this thesis it is interpreted that Davies and Harré (1990), with 'positioning', refer to the cognitive process between all three perspectives A, B and C, which, specified for the studies in Part 3, become:

a) What a teacher can say.
b) What a teacher says.
c) What a teacher is permitted to say.

The meaning of a position in this thesis follows the description by Harré (2012);

> Taken abstractly, a position is a cluster of beliefs with respect to the rights and duties of the members of a group of people to act in certain ways. (Harré, 2012, p. 196).

These clusters of beliefs may be implicit or explicit. Through analysing teachers' discussions in the context of the professional development programme on education of MHAPs, teachers' clusters of beliefs are captured through their given or taken positions.

The positioning theory is built up by the triad *position – speech act – storyline,* where all parts are included when the theory is used as tool to capture the meaning of human actions and describe how meaning and understanding are organized (Harré & van Langenhove, 1999; Harré & Moghaddam, 2003).

4.3.2 Speech acts and storylines

Herbel-Eisenmann et al. write: "*speech acts* are the meaning that those words/actions have for participants" (2015, p. 187). What people may say or may not say is based on their perceived rights and duties (Harré, Moghaddam, Pilkerton Cairnie, Rothbart, & Sabat, 2009). A speech act can be interpreted from an excerpt of what is said during teachers' discussions. It can be a section of a discussion or a part of a sentence. Interpretations of speech acts can also involve gestures and body language (Herbel-Eisenmann, Wagner, Johnson, Suh, & Figueras, 2015), although it is not done in this study. The context in which the speech acts are made has an influence on the interpretation of them. The context to which a speech act belongs is determined by the associated storyline.

Storylines are mutually implicitly or explicitly agreed context for the conversation (Harré & Moghaddam, 2003). The use of names on storylines can indicate what is expected in a speech act (Herbel-Eisenmann et al., 2015). To explore what teachers express on education of MHAPs in the diverse classroom, predefined storylines can be used, i.e. the context of the discussion is predefined.

4.3.3 Position as rights and duties

A position is a complex cluster of personal attributes, for example rights and duties (Harré, 2012; Harré & van Langenhove, 1999). Within positioning theory, rights and duties are seen to guide what you are permitted to do (UNU-CRIS, 2015).

Rights are connected to vulnerabilities for individuals as well as for systems.

Duties are connected to someone's or something's power to remedy someone's or something's vulnerability.

The interpretation of positioning theory in this thesis leads to the following definitions.

- Someone has a **right** to something when there is a vulnerability which permits the person to be helped.

    - *Vulnerability + permission -> right*
- Someone has a **duty** for someone or something when that person is permitted to use her or his power to remedy a vulnerability.
    - *Power + permission -> duty*

What eventually will happen in the interaction between someone with a right and someone with a duty depends on beliefs, perceptions and communication. If the one with power acts or not, depends on how the one with power perceives her or his duties and on how well the vulnerability has been presented. The one with power must understand that something is missing and understand her or his power (duty) to assign or resist (Harré, 2012). In addition, conscious choices must be made both by the one with a right and by the one with a duty; it is a dynamic social and cognitive process that establishes the positions, the cognitive interaction between two, A and C, of the three guiding perspectives, see section 4.3.1. The dynamics in this process is called positioning – during this process rights and duties are assigned, ascribed, resisted or rejected (Harré, 2012). During this process, power relations have an effect on dynamics. A sigh or a glance from the teacher might stop a pupil using her/his right to ask for help. For example, the principal might have given the teachers clear guidelines to focus on low-achieving pupils, which might prevent the teacher from focusing on HAPs.

Figure 2 shows how Part 3 of this thesis is placed in a larger picture and how rights and duties are connected to actors with power to influence teachers' discussions of pupils' learning. However, the overarching aim of this thesis is to bring the teacher perspective into research.

For the studies in Part 3 the larger picture is narrowed to focus on the teachers, described by Figure 3. Thereby, the findings will give an interesting insight into teachers' possible power to influence HAPs' learning in diverse ability classrooms.

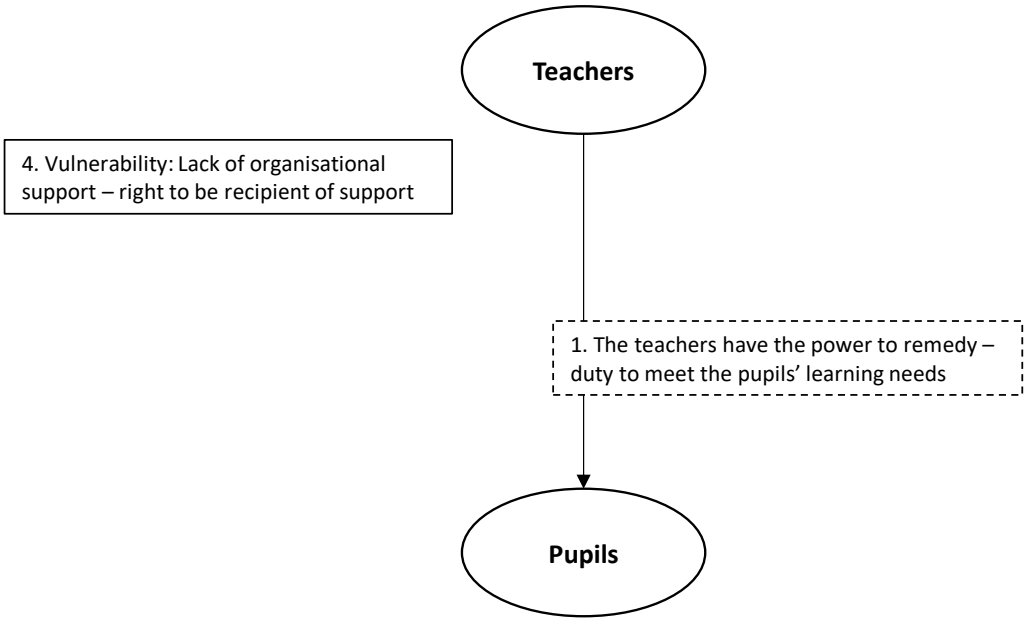

Figure 3 The focus of this study as a subsection of Figure 2.

4.3.4 Interactive and reflexive positioning

A position can be taken or given consciously or unconsciously, respectively explicitly or implicitly. The cognitive dynamic process, the positioning, can be described through a character or a metaphor (see e.g., Markström & Simonsson, 2011; Mosvold & Bjuland, 2016; Vanassche & Kelchtermans, 2014). Positioning goes both ways: when one positions oneself, other participants are also positioned (Herbel-Eisenmann et al., 2015), which leads to a right being connected to a duty. Positions are not permanent, they are dynamic and flexible, and are not freely chosen since they depend on other participants and the position taken by them (Evans, Morgan, & Tsatsaroni, 2006). What is logically possible for someone to do is implicitly limited by the position (right or duty) that person has in a certain context (Anderson, 2009; Harré & Moghaddam, 2003). Davies and Harré (1990) write the following:

> Positioning, as we will use it is the discursive process whereby selves are located in conversations as observably and subjectively coherent participants in jointly produced story lines. There can be interactive positioning in which what one person says positions another. And there can

> be reflexive positioning in which one positions oneself. (Davies & Harré, 1990, p. 49).

Positioning can happen in at least two ways that can be detected from a speech act:

- Interactive positioning is when someone positions someone else (Davies & Harré, 1990; Mosvold & Bjuland, 2016), for example a pupil or another teacher.
- Reflexive positioning is when someone positions himself/herself (Davies & Harré, 1990; Mosvold & Bjuland, 2016).

The interest in this study lies in how the teachers express themselves, and in line with Harré and Moghaddam (2003), it is interpreted that the context influences how and what they express; meaning that what the teacher expresses is authorized, for example, by the context of the professional development programme. From this it is possible to discuss how teachers are positioning themselves, as well as others, regarding orchestrating teaching MHAPs in diverse classrooms.

5 Method

In this chapter, methods and resulting data available for analysis are presented for each part of the thesis. Since the two involved professional development programmes can be assumed to have influence on the findings, the last section in this chapter discusses the construction of these programmes.

5.1 Part 1 – Text1

Part 1 of this thesis consists of a research study presented in a monograph (Mellroth, 2014). It is divided into two sub-parts. The first sub-part examines movements in relative achievement on individual level between mathematical tests. Movements in relative achievement over a four-year period (2009 – 2013) on two curriculum-bounded Swedish national tests, and one non-curriculum bounded, The Mathematical Kangaroo, were analysed. More specifically, the first subpart investigates the following:

> Relative achievement on the national test in grade 3 (2009) is compared with relative achievement on the national test in grade 6 (2012), n=568,
>
> Relative achievement on the national test in grade 6 (2012) is compared with relative achievement on the mathematical kangaroo in grade 7 (2013), n=264. (Mellroth, 2014, p. 43)

Test results of national tests were, at the time of the study, public data and therefore collected from the municipality office. To employ results of The Mathematical Kangaroo, the regular teachers implemented the test in their classrooms, with pupils and guardians signing an informed consent form, which explains the reduction in participant numbers from 568 to 264.

The second sub-part aims to explore how mathematical competencies (Lithner et al., 2010) can explain how some pupils are high achievers in one test but not in another.

Data for the studies, all of which were conducted in the same municipality, in Part 1 were:

- Test results from pupils (N=654) on national tests in mathematics, performed in grade 3 (2009).
- Test results from pupils (N=611) performed in grade 6 (2012).
- Test results from pupils (N=264) on The Mathematical Kangaroo performed in grade 7 (2013).
- The national test for year 6 in 2012, although classified data.
- The Mathematical Kangaroo test for grade 7 in 2013.

5.1.1 Method – Part 1

Pupils' test results on national tests in mathematics from grade 3 (2009, N=654) and grade 6 (2012, N=611) were collected from for the same group of pupils in a municipality. At the time of the study in Part 1, such data were public in Sweden, except for pupils with secret identity. To compare the same group of pupils' relative achievement on the national tests with a non-traditional test, The Mathematical Kangaroo, was chosen to be implemented in grade 7 (2013). In Sweden teachers to pupils in grades 3, 6, and 9 are very occupied with national tests in many subjects, including mathematics. To be able to use a non-traditional test without interfering too much with regular education it was chosen to implement it in grade 7. All 8 public schools in a municipality were asked to participate and all chose to do so, resulting in most of the pupils in grade 7 (2013) taking part. Of those, 264 pupils and guardians agreed on letting the results on The Mathematical Kangaroo be used as data for the study in part 1; of those 247 pupils had results on all three tests involved in the study.

In practice the teachers implemented The Mathematical Kangaroo with their pupils. All pupils did the test on approximately the same date and under the same conditions, for example with the same time limit. All tests were corrected by the researcher, including those who chose not to participate in the study. After all data for the study were collected, the teachers were given the corrected tests back and could hand them on to their pupils.

The pupils' test results were ranked and their relative achievement on the three tests was compared, as described in section 7.1. In addition,

as described more in the same section, the tasks on The Mathematical Kangaroo were analysed in terms of what opportunities they gave pupils to activate mathematical competencies (Lithner et al., 2010).

5.2 Part 2 – Text 2

The study for Part 2 was drawn from the report of a professional development programme on identifying and supporting mathematically highly able pupils (see Mellroth et al., 2016). The author of Text 2 was the facilitator of the programme as well as co-author of the report (see Mellroth, 2017). During the professional development programme, several investigations were performed in collaboration between the participating teachers and the facilitator. In Text 2 one of those is theorized and presented. To give the reader understanding of Text 2, it is here presented how the investigation was performed during the professional development programme.

5.2.1 Method – Part 2

Seven teachers, teaching grades 4-9, participated in the professional development programme. The specific investigation, presented in Text 2, was raised by questions of those teachers. As part of the programme, teachers read literature which highlights that mathematical tasks that meet MHAPs' learning needs are challenging, engaging and joyful (e.g., Benölken, 2015; Fuchs & Käpnick, 2009). The teachers meant that only pupils themselves can judge what they perceive, and therefore the teachers, in collaboration with the facilitator, developed a tool to measure pupils' perceptions of tasks.

The teachers and the facilitator, in collaboration, agreed on how to implement two rich learning tasks (chosen from Sheffield, 2003) in each of the teachers' diverse classrooms. To examine pupils' perceptions of the task in the beginning and at the end of the problem-solving process, the teachers implemented a pre- and a post-evaluation. During the professional development programme, teachers identified some pupils as MHAPs. In the results of the pre- and post-evaluation, teachers presented those pupils' evaluations separately.

The two tasks were implemented in two interventions, and teachers collected pupils' pre- and post-evaluation responses after each intervention. In the first intervention, i.e., the first task, 139 pupils (32 identified as MHAPs) responded to the evaluation. For the second intervention and the second task, 106 pupils (20 identified as MHAPs) responded to the evaluation. Ninety-five pupils (20 identified as MHAPs) did both the first and the second task. Each teacher summarized the results of the pre- and post-evaluation for her or his pupils and anonymised the data. The anonymised results collected by all teachers were thereafter summarized by the facilitator, who made a preliminary descriptive analysis. The facilitator discussed the preliminary results with the teachers, thereafter a final analysis was done. In the report (Mellroth et al., 2016), produced in collaboration between the teachers and the facilitator, the investigation and its results are presented.

In Text 2, the investigation and its results are viewed from a theoretical perspective. The same investigation and its results are viewed from a descriptive and practical perspective in the report (Mellroth et al., 2016). The report is written in Swedish, making it unavailable for many researchers in the field. Part 2 therefore theorized the investigation presented in the report, in order to place it in the field of research on education of MHAPs.

The investigation and its results are of interest in relation to the overarching aim of this thesis. To enable learning opportunities for MHAPs in the diverse classroom, mathematical tasks have been shown to be important (e.g., Nolte & Pamperien, 2017). For teachers to be able to judge and develop competence to choose mathematical tasks that works for all pupils and challenge MHAPs, s/he could benefit of a simple tool to judge the suitability of a task. In addition, the investigation is of interest for the aim of this thesis since it was asked for and developed by teachers. Having a teacher perspective on opportunities to include MHAPs in learning addresses the aim of this thesis.

5.3 Part 3 – Texts 3a and 3b

The two studies for Part 3, have used similar methods regarding data collection, preparation of data for analysis, and analysis. They are therefore presented together in this section; when differences occur, each study is discussed separately. The two studies are both based on small group discussions conducted in the context of the more extensive professional development programme.

5.3.1 Method – Part 3

In Part 3 the teacher perspective is captured through teachers' discussions in the context of a professional development programme. Positioning theory is used to probe the meaning of the teachers' communication. The teachers' clusters of beliefs, with respect to their perceived rights and duties, thus guide what they say or do not say. How those teachers became participants in the programme and thereby the studies are here presented.

5.3.1.1 Participants

In the selection process for participation in the professional development programme, all mathematics teachers working in public compulsory schools, in one municipality, were invited to apply. The announcement of the programme was distributed from the department of education to all principals through a digital platform. The programme was also announced through email to teachers involved in a network for mathematics teachers in the municipality. In addition, two other teachers, from another municipality that was running a project on high ability, were invited to join the programme. In the announcement of the programme, clear information was given that research would be conducted during the implementation.

As a result, 24 teachers applied to participate; their principals had signed their application in order to show their approval and support. During the first semester some teachers dropped out due to illness or high work load, resulting in 17 teachers participating in the professional development programme during data collection for the two underlying studies for Text 3a and 3b. Another two dropped out

during the last half year of the programme due to illness and change of work, therefore 15 teachers completed the whole professional development programme.

The study for Text 3a is based on five meetings held from September 2015 to April 2016. Patterns are probed by analysing reflexive and interactive positionings related to what teachers express as possibilities and obstacles regarding orchestrating teaching to support MHAPs. Predefined storylines, *Differentiation*, *Mathematical problems* and *Mathematical abilities*, were used in order to focus on what the teachers expressed about education of MHAPs in diverse ability classrooms. The participants were five out of the 17 teachers. Those teachers taught grades 1-3 (age 6-9) at the same school. The principal at this school was the only principal who actively announced the professional development programme among the staff and encouraged interested teachers to apply.

The study for Text 3b focuses on a discussion session during the sixth meeting in the programme in February 2016. In order to examine how the teachers connect theories of teaching MHAPs with their professional tasks as teachers, their expressed rights and duties were probed. The participants were 10 of the 17 teachers; in addition, there were two student teachers who had participating teachers as mentors. The seven teachers who were absent were either ill, or on holiday. Three discussion groups were arranged based on the grades the teachers taught, resulting in one group with four teachers teaching grades 1-3, one group with three teachers teaching grades 4-6 and one student teacher, and one group of three teachers teaching grades 7-9 and one student teacher.

5.3.1.2 Data collection of teachers' discussions

All meetings during this professional development programme were video-recorded, with the aim of recording the oral communication between teachers – no attention was given to visual quality; for example, sometimes someone or some was "out of picture". In total, more than 90 hours video film became potential data to study. The

facilitator of the programme, i.e., the author of this thesis, watched/listened to all video recordings after each meeting in the programme. In addition, supervision meetings were held with senior researchers before the next meeting in the programme, but after the facilitator had listened to the recordings.

To strive towards the aim of highlighting, from the perspective of teachers, possibilities with challenging tasks for MHAPs in the diverse classroom, the study for Text 3a follows teachers' discussions when analysing and developing such tasks. The selected group of teachers were chosen due to their common background, for example working in the same school. Thereby, the collective participation which is a core criterion in effective professional development (Desimone, 2009) was strengthened.

To limit the amount of transcription to a reasonable amount, the author used NVivo. The author listened to the recordings (9 hours in total) from the workshops at least once. Thereafter the author listened to them again, but now using NVivo to mark sections of extra interest in relation to the aim of Text 3a. Those sections (decreased to 4 hours and 42 minutes) were thereafter transcribed verbatim by the author and used as data for the study presented in Text 3a.

The study for Text 3b seeks to examine, from the perspective of teachers, how to orchestrate teaching in diverse ability classrooms that enables learning for MHAPs. The more detailed aim of this study was to probe how teachers perceived their professional task in the context of a professional development programme on educating HAPs. To achieve the goal, a guided discussion session was implemented when most of the theoretical parts in this professional development programme were covered (see Mellroth, 2018a). This specific session was therefore distinguished from other sessions in the programme since it was specifically planned to guide teachers to reflect on their current practice, their teacher professional task and theories on education of HAPs. Each of the three groups of teachers had discussions in parallel in separate rooms.

Full verbatim transcripts of each of the three groups' 90-minute discussion sessions were produced by an external company[13]. The author reviewed all transcripts by reading them while listening to the teachers' communication in the recordings. As data the author mainly used the transcripts, but the video recordings were also used as data in the analysis.

For both studies in Part 3, mainly transcripts of teachers' discussions were used as data, although video recordings were available data and were sometimes used to, for example, judge the tone of a teacher's voice. For each study, 3a and 3b, the discussions selected as data were carefully chosen to be coherent within each study, but also to give a broader picture towards the overarching aim of the thesis.

5.4 Professional development programmes in Part 2 and 3

Professional development programmes has either indirectly, for Part 2, or directly, for Part 3, been used as a method to produce data for studies in this thesis. The studies for Part 3 are conducted on a professional development programme that was designed to enable research. It can therefore be expected that the programme has influenced the teachers' discussions. To some extent the other professional development programme connected to Part 2 most likely influenced the study presented in Text 2. Therefore, it is necessary to account for the design of the two programmes so that the reader can understand its impact on the findings. There are several similarities between the two professional development programmes, therefore they are presented together; when there are differences they are distinguished from each other.

Both professional development programmes used as a base for the data collection in Part 2 and 3 of this thesis aimed to improve teacher competence in meeting the learning needs of MHAPs in the diverse ability classroom. It was a goal of these programmes to improve

[13] A company specialising in transcription, translation and language checking.

teachers' knowledge of HAPs' learning needs, specifically MHAPs, and through this process to also respond to the teachers' needs in relation to teaching HAPs in the diverse ability classroom. The teachers in these programmes were encouraged to connect their present knowledge and experience with the new knowledge acquired through the professional development programmes, aiming to help them make connections and create deep knowledge. Further to creating deep knowledge and engagement the programmes mostly built on collaboration between the teachers, and for Part 2 also between the teachers and the facilitator. In the professional development programme used for Part 3, experts in the field were invited to meet the teachers both through Skype meetings and in real life. Through such meetings the quality and the validity of the professional development programme were validated and supported. Some of the key authors within the specific field of literature became more real in this way, and the teachers were given the opportunity to discuss and pose questions to these experts. Both professional development programmes were grounded in research theory and research-based literature. Therefore, the design of the programmes in many ways fulfils the characteristics of professional learning environments that support teachers to become what Le Fevre et al. (2016) call adaptive experts.

This thesis does not aim to study the effectiveness of the professional development programmes, however they were designed to follow criteria of effective professional development (Desimone, 2009; Garet, Porter, Desimone, Birman, & Yoon, 2001). This means that in the design extra attention was given to the following identified core features of effective professional development: *content focus*, *coherence*, *active learning*, *collective participation*, and *duration* (Desimone, 2009). Since the programmes are assumed to have influenced the participating teachers' learning and perceptions, they are here presented in relation to core features of effective professional development.

5.4.1.1 Content focus and coherence

Both professional development programmes had a content focus on education of HAPs, pedagogical strategies to include HAPs in learning in the diverse ability classroom, and mathematical and didactical aspects of challenging tasks. Through the programmes the teachers acquired theories about education of HAPs in general (e.g., Gagné, 2005; Nissen, 2014; Sheffield, 2003; UNSW, 2004) and specifically of MHAPs with an extra focus on challenging mathematical tasks (e.g., Benölken, 2015; Krutetskii, 1976; Sheffield, 2003). In Part 3, differentiated education (Tomlinson, 2001; 2016) was emphasized as a pedagogical strategy to meet the diversity of pupils' learning needs, especially the HAPs. In both programmes the teachers had to reflect on their present way of teaching.

To create coherence, the present knowledge of the teachers was highlighted to the teachers as a strength to use when interpreting the theories acquired by the professional development programmes. In the discussions and, for Part 3, workshops, at each meeting, the teachers were encouraged to involve their present teacher experience and relate this to theories implemented through the programmes. For example, in workshops on challenging tasks, they were encouraged to connect their perceptions of how MHAPs would work on such tasks to the theories of Krutetskii (1976) (see Mellroth, 2018a). Thereby they were given opportunities to actively engage and collaboratively create new knowledge on how the theories could be implemented in their classrooms. The design of the programme made it possible to detect whether the teachers verbalized theoretical principles. That is, whether their perceptions were based on characteristics of adaptive teaching (Le Fevre et al., 2016), which in turn would be beneficial for teachers engaged in differentiated education.

5.4.1.2 Active learning and Collective participation

Active learning is the opposite to passive learning where one might just sit and listen (Desimone, 2009). In Part 2, seminars were given during the first 2 of 9 meetings in the professional development programme, thereafter focus shifted to discussions on developing and validating

teaching practice. In Part 3, seminars were given in the six first meetings of the professional development programme, but great focus was given on workshops and discussions, especially in the second half of the programme (see Mellroth, 2018a). In both programmes, it was clarified for the teachers that they together participated in creating new knowledge and connected the theories used in the programmes to the Swedish school context. In this way opportunities were given to create a *shared view* (Sowder, 2007) of how they can recognize MHAPs. In the workshops they created parts of lessons and they developed challenging tasks through implementing the theories presented in literature used in the programme. Thereby, passive learning did exist in both professional development programmes but the focus was on opportunities for active learning.

Collective participation means that participants have some sort of common background, for example, teaching at the same grades (Desimone, 2009). In the professional development programme connected to Part 2, the seven participating teachers worked together as a team throughout the whole programme. They taught pupils at different ages and only two worked at the same school. Therefore most of them did not have a common background. In the programme connected to Part 3 the participating teachers sometimes worked all together, but they were often divided in smaller groups, especially during workshops. Those groups were basically constructed depending on which grade the teacher taught in, resulting in four groups: one group of teachers teaching grades 7-9, one group teaching grades 4-6, and two groups teaching grades 1-3; in one of these groups, all teachers worked at the same school. Another purpose of these group discussions was to create a social situation where the teachers together created and negotiated knowledge, i.e. they were given opportunities to develop a *shared vision* (Sowder, 2007).

5.4.1.3 Duration

One criterion of efficient professional development programmes is their duration. Desimone (2009) found that they ought to be spread over at least one semester and include more than 20 hours of contact

time. The programme connected to Part 2 had nine two-hour meetings distributed over eight months. Thereafter three follow-up meetings were implemented, distributed over the following eleven months. The programme connected to Part 3 was spread over four semesters and the 13 meetings resulted in at least 80 hours of contact, therefore the criteria of duration are well fulfilled for both professional development programmes.

5.4.1.4 Researcher and facilitator roles

In Part 3 the researcher and the facilitator of the professional development is the same person, i.e. the author of this thesis. In this section differences in responsibility of the two roles are clarified and distinguished. The process of creating qualitative academic work, such as written articles, conference posters and papers, is mainly of interest for the researcher and the research community and was therefore a responsibility for the researcher, in a similar way to Olin et al. (2016). This means that the researcher role was responsible for processes like reading research articles, collecting data, analysing data and writing. In contrast the facilitator role was responsible for organizing the programme and meeting the teachers' expectations. This meant, for example, to choose and select among scientific literature and to design the programme according to criteria proven to be successful for professional development (e.g., Desimone, 2009). There was no time for analysing and writing included in the agreements for the programme. In Table 2, differences in responsibility between the researcher and the facilitator role are presented in more detail.

Table 2 Responsibilities of the researcher and the facilitator roles in Part 3.

Researcher	Facilitator
Researching as main activity	Facilitating as main activity
Participating to examine teachers' perceptions on teaching MAHPs.	Contribute with organisation and knowledge of how to find relevant scientific information. Create an open-minded environment, highlighting each participant's experience as equally worthwhile.
Evaluate and adjust content and work models, reassuring reflection and a critical perspective. For Text 3b, in collaboration with a senior researcher, construct questions for discussions.	Bringing in experts as speakers and being an extended arm between the teachers and principals when needed.
Continuously evaluate the programme with senior researchers to withhold the aim as well as the researcher role.	Continuously evaluate the programme to meet the needs of the teachers.
Contribute with theoretical perspectives both on content and structure, to deepen the discussions.	Upholding established roles, such as not mentioning names of pupils and remembering the importance of each teacher's participation.
Responsible for communicating the process, ethics and preliminary outcomes with research colleagues at the university.	Responsible for communicating the organisation and the structure of the programme with the teachers and their principals.
Scientifically analyse and write about the investigations performed during the programme. Thereby the teacher perspective of teaching MHAPs is disseminated to the research community.	Through written reports, conferences and social media disseminate the programme and its outcome to practice. Thereby the programme as such and the teachers' developed material is disseminated to practice.

6 Ethical considerations

All studies included in this thesis have followed the ethical guidelines given by the Swedish Research Council (Vetenskapsrådet, 2017), for example regarding providing information on the study to participants, participants' approval, storage of data (i.e., video recordings), and how the studies have been communicated (e.g., anonymisation of individuals). In relation to each part of this thesis, ethical discussions, during seminars with colleagues at the university and with collaborating teachers, have been strongly prioritized. Questions of whether and how the studies could affect someone psychologically or physically have been raised, and the conclusions have been, for all studies, that no individuals were placed at risk.

6.1.1 Ethical considerations - Part 1

At the time of the study for Part 1 results on national tests from pupils attending public schools were public data. Therefore, these data were easily accessible. The municipality, which stored data, was informed about the ethical considerations and that the study had been reviewed by the ethical committee at the university. All data was anonymised before starting the analysis process.

To be able to include the results of The Mathematical Kangaroo an informed consent form was given to both pupils and guardians. In the study only results from those with signed forms are included.

6.1.2 Ethical considerations - Part 2

Data for Text 2 consisted of an already published report of a professional development programme (Mellroth et al., 2016). The study presented in Text 2 is based on an investigation performed during the professional development programme, and ethical issues were addressed in the following way.

All teachers involved in the professional development programme for Part 2 (Mellroth et al., 2016) were informed that the programme, and the results of investigations performed in the programme, could be communicated both in popular science papers as well as in scientific

papers. The participants are anonymised in all texts, but since they were seven teachers and all are co-authors of the report of the programme (Mellroth et al., 2016), it is easy to identify them as one of seven. All teachers agreed to participate, and the ethical aspects were thoroughly discussed in the programme, especially since the teachers themselves collected data from their pupils. With regard to the indirect participation of the teachers' pupils, the pupils and their guardians were informed of the professional development programme and the forthcoming presentations of results, through both written and oral information given by the teachers. Guardians and pupils were also invited to contact the facilitator for more information, and to pose questions. It was only the teacher for each class who knew who the pupils were, since anonymisation was carried out before sharing the results with other participants in the professional development programme. In the discussions and the analysis process in the programme, the pupils were represented only by numbers. The ethical discussion is lacking in Text 2, but in the report of the programme (Mellroth et al., 2016) ethics is well discussed, and therefore the ethical issues regarding this study are available for all. All seven teachers in the professional development programme are acknowledged in Text 2 due to their indirect contribution.

6.1.3 Ethical considerations - Part 3

At the announcement of the professional development programme for Part 3, information was given on the parallel planned research; see appendix 1. The participating teachers applied for participation in the professional development program through a signed form which their principal also signed; see appendix 2. Together with information about the research also the way of collecting data through video recordings was announced; see appendix 3. All participating teachers signed an informed consent form; see appendix 4. In the form, information was given that ending participation in the research would not affect participation in the programme. This information was orally repeated several times during the programme, and none ended their participation. In all written texts, popular science as well as academic

texts, all participants are anonymised. The professional development programme has got national attention in popular science texts and in national conferences, and some of the participating teachers have even been interviewed. Therefore, it is easy to understand that a teacher mentioned in texts related to the programme is one of 15 teachers. There were 17 participating teachers during data collection for the studies for Texts 3a and 3b, but only 15 teachers who completed both two years of the programme.

7 Analytical procedure

Throughout the three parts, this thesis strives to meet the overarching aim and research questions. Part 1 is based on a mix of quantitative and qualitative studies, meeting the aim to probe the use of non-traditional tests in mathematics to help teachers recognize MHAPs. Part 1 therefore addresses the question: How can non-traditional tests and challenging mathematical tasks help teachers recognize and support MHAPs? The question indirectly also addresses teachers' competence to meet these pupils' learning needs, i.e., to enable learning opportunities for them.

Parts 2 and 3 are, on the other hand, based mainly on qualitative studies, meeting the aim of exploring teachers' perspectives regarding orchestrating teaching of HAPs in the diverse ability classroom. There are differences also between Parts 2 and 3. The study for Part 2 is based on an investigation developed by teachers with an aim to answer questions posed by them. The studies for Part 3 have been performed during a longer period, using mainly transcripts of teachers' discussions as data. The aim with these studies was to probe teachers' perceptions regarding their own ability and power to orchestrate teaching HAPs. This directly addresses the overarching aim to capture the perspectives of teachers in order to highlight pedagogical possibilities in the diverse ability classroom that enable learning opportunities also for MHAPs.

To be able to fulfil the respective aims, the three parts of the thesis have used different analytical frameworks, which will be presented in the following sections.

7.1 Part 1

To follow movements in relative achievement on the national tests (grades 3 and 6) to The Mathematical Kangaroo (grade 7) and to identify groups of pupils that were of interest for this study, several statistical procedures were performed using R-statistics (R core team, 2013) as a tool (see Mellroth, 2014).

To analyse opportunities provided by tasks to activate mathematical competencies, an analysis guide was developed (Mellroth, 2014). The guide aimed to analyse what mathematical competency a pupil would be *required* to activate, to produce a *reasonable* solution. Each task on the national test for grade 6 (performed in 2012) and on The Mathematical Kangaroo (performed in 2013) was analysed using the developed guide. Competency profiles of each test were produced as a result of using Mathematical competencies: A research framework (MCRF) (Lithner et al., 2010) as an analytical framework. The framework was chosen since it was developed for the Swedish context and had been used to analyse Swedish national tests. MCRF also has an aim to analyse opportunities to learn mathematics in tasks, which was the purpose with the analysis of the tasks in the study of Part 1.

For each of the six mathematical competencies in the MCRF – applying procedures, reasoning, communication, representation, connection and problem-solving (Lithner et al., 2010) – each task was given either the classification '0' or '1', according to the following conditions:

- If it is not necessary to activate, for this task, the competency is given the classification '0'. (Mellroth, 2014, p. 63)
- If it is necessary to activate the competency, for this task, the competency is given the classification '1'. (Mellroth, 2014, p. 63)

This way of analysing the tasks provided the possibility to produce a competency profile both for each test and also for each pupil. The aim was to reveal which competencies, and in which proportion, pupils in the identified groups activated, in relation to how many competencies the specific pupil activated in total.

The analysis resulted in a competency matrix for each task, see Figure 4.

Task 1	class
App	1
Rea	0
Com	1
Rep	0
Con	1
Pro	0

Figure 4 Competency matrix showing competency classification for task no. 1. (Mellroth, 2014, p. 65).

Thereafter, competency profiles could be produced for the full tests as well as for each pupil, if their results on each task on the test were known. An explanation adapted from Mellroth (2014, p. 66-67) is quoted here to show how those competency profiles are produced. 'Pupils' is used in this text instead of 'students', and Figure and Table numbers are changed to fit the text in this thesis.

Example from Mellroth (2014)

A test has 10 tasks and the competencies are distributed among the tasks in the example test shown in Figure 5. To solve all 10 tasks, it is necessary to activate the competencies as summarized in column 2 in Table 3. Relative values for each competency are shown in column 3. In this example, there is a specific pupil that has failed in tasks 2, 7 and 8. The pupil is therefore seen to have activated the reasoning competency two times fewer, the communication competency one time fewer, the representation competency two times fewer and the problem solving competency two times fewer than possible, summarized in columns 4 and 5 in Table 3. Data from column 3 result in the competency profile for the test; data from column 5 result in the competency profile for the specific pupil, Figure 6.

Task 1	Class
App	1
Rea	0
Com	1
Rep	0
Con	0
Pro	0

Task 2	Class
App	0
Rea	0
Com	0
Rep	1
Con	0
Pro	0

Task 3	Class
App	1
Rea	1
Com	1
Rep	0
Con	0
Pro	0

Task 4	Class
App	0
Rea	0
Com	0
Rep	0
Con	0
Pro	1

Task 5	Class
App	1
Rea	0
Com	1
Rep	1
Con	0
Pro	0

Task 6	Class
App	1
Rea	0
Com	0
Rep	0
Con	1
Pro	0

Task 7	Class
App	0
Rea	1
Com	1
Rep	0
Con	0
Pro	1

Task 8	Class
App	0
Rea	1
Com	0
Rep	1
Con	0
Pro	1

Task 9	Class
App	1
Rea	0
Com	1
Rep	0
Con	1
Pro	0

Task 10	Class
App	1
Rea	0
Com	0
Rep	0
Con	0
Pro	0

Figure 5 Competency distribution in each of the 10 tasks on the test in the given example. Adapted from Mellroth (2014, p.67).

Table 3 Example of data used to produce a competency profile for one test and one specific pupil. Adapted from Mellroth (2014, p.67).

Competency	No. of times activated *Maximum*	Relative values *Maximum*	No. of times activated *By the student*	Relative values *By the student*
Applying procedures (App)	6	0.27	6	0.40
Reasoning (Rea)	3	0.14	1	0.07
Communication (Com)	5	0.23	4	0.27
Representation (Rep)	3	0.14	1	0.07
Connection (Con)	2	0.09	2	0.13
Problem solving (Pro)	3	0.14	1	0.07
Total amount of competencies	*22*	*1.01*	*15*	*1.01*

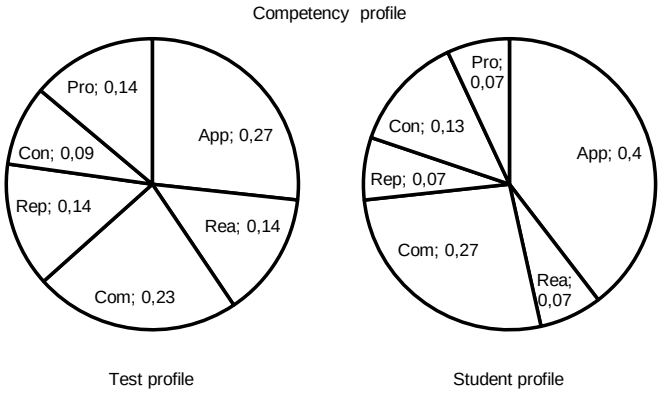

Figure 6 Example of a competency profile for one test and one pupil. Adapted from Mellroth (2014, p.68).

The competency profiles are not correlated to achievement; the profile of a pupil with full marks on a test can look exactly or almost the same for a pupil with lower marks. The profiles make it possible to compare opportunities given by the test to activate mathematical competencies with how frequently pupils did activate the competencies, both on an individual and on a group level.

On the pupil level, individual competency profiles were also produced, which in turn allowed two groups of pupils identified in sub-part one to be further analysed, i.e. those who were high achievers on one of the tests, 2012 or 2013, but not in the other. The second sub-part involved five steps:

> Competency analysis of the two tests involved on the task level,
>
> Producing competency profiles for the tests and individual competency profiles based on results in each task in the mathematical kangaroo,
>
> Comparison of competency profiles in the mathematical kangaroo for the identified groups,
>
> Investigating differences between the two identified groups in activated competencies by means of the mathematical kangaroo,
>
> Investigating tasks in the mathematical kangaroo that are of special interest. (Mellroth, 2014, p. 43).

The qualitative analysis procedure used both on tests and on pupils' achievement meets the aim of highlighting pedagogical possibilities that can be used to recognize MHAPs. For example, the results of used analyse procedure reveals what mathematical competencies a test gives

pupils possibility to activate, and to what extent. The analysis, on the pupil level, reveals strength and weaknesses in mathematical competencies for that specific pupil, regardless of whether or not s/he is a MHAP. In relation to the aim of this thesis the results of analysis therefore guide teachers to offer opportunities for MHAPs to show their ability. The quantitative analysis aims to investigate whether there is a group of pupils who have high mathematical abilities although they do not achieve on traditional tests. Altogether, the analysis procedure in Part 1 meets the aim to highlight pedagogical possibilities to recognize MHAPs through assessment in the diverse ability classroom.

7.2 Part 2

Text 2 is produced through theorizing an investigation performed during a professional development programme and used the report (Mellroth et al., 2016) as data. However, to be able to interpret the findings the analysing procedure performed within the professional development programme is here presented.

The performed investigation was teacher-initiated, in that the teachers wanted to investigate what their pupils perceived about two tasks that were said, by problem posers, to be engaging and challenging. After the professional development programme was completed, the author of Text 2 reviewed the process used in the investigation, theorized the background, the analysis and the results. During the programme the facilitator was responsible for the analysis process, although the teachers' comments were carefully considered. In Text 2 the investigation is discussed in more depth than in the report of the programme (Mellroth et al., 2016), and put in relation to previous research and thereby theorized.

To measure pupils' perceptions, the seven participating teachers and the facilitator, in collaboration, developed a tool to measure pupils' perceptions of mathematical tasks. The tool aimed to be easy to use, and to not change or interfere with regular teaching too much.

A pre-evaluation was created, see Figure 7. The pupils did the pre-evaluation directly after the task had been presented, but before starting to work with it.

Pre-evaluation
Your teacher has presented a mathematical problem. *Which emoji best matches your feeling about this problem?* 

Figure 7 Pre-evaluation of tasks, adapted from Mellroth et al. (2016, p. 19).

To measure if pupils felt differently about the task after working with it, a post-evaluation was also created, see Figure 8. The pupils did the post-evaluation after all the work with the task was completed, for example classroom discussions and presentations of solutions. The emojis were changed into words after a discussion with a researcher in educational psychology. This researcher indicated that when using the same sort of pictures in a pre- and post-evaluation it is a risk that the respondent gives the same answer in the post-evaluation as in the pre-evaluation, due to habit.

Post-evaluation
What words best describes how you felt about the task while working with it?
Very interesting
Interesting
Neither or
Uninteresting
Very uninteresting

Figure 8 Post-evaluation of tasks, adapted from Mellroth et al. (2016, p.19).

To summarise the results, descriptive analysis using statistical methods was performed on pupils' pre- and post-evaluations. Aiming to examine MHAPs' perceptions and compare them to other pupils, data from the evaluations were divided into two groups; One group of MHAPs and one group of the others. The responses from the pre- and the post-evaluation were transformed into numbers from 1 to 5, where 1 corresponded to the most positive perception and 5 the least. In the analysis, the change of pupils' perception from the pre-evaluation to the post-evaluation had three possible outcomes: more negative afterwards, more positive afterwards, and unchanged afterwards. The changes were not statistically verified.

To be able to verify if mathematical tasks fulfil what the problem poser claims they do, the teachers, in the professional development programme, found it of importance to investigate pupils' perception of the tasks. The report of the programme was written in Swedish and addressed practice. One way to disseminate the results, of what teachers perceived as important, to the research community is to theorize the investigation and its results and produce a scientific text. Thereby the process of transforming the teacher-initiated investigation to a scientific text, meet the second research question of this thesis: How can the experience of those who teach in diverse ability classroom contribute to research?

7.3 Part 3

The aim to highlight pedagogical possibilities to include MHAPs in learning within the diverse ability classroom is strived to be achieved through the two studies in Part 3. In addition, the teachers' perceptions are probed through their expressed possibilities, obstacles, rights and duties regarding teaching MHAPs. In those studies, perceptions are assumed to be built upon clusters of beliefs that can be captured through analysing communication. Therefore, positioning theory becomes suitable as an analytical framework. The focus of the two studies lies in interpreting the meaning in acts of communication expressed by teachers through speech acts, taking or giving different positions connected to different storylines.

Some differences exist in the analysis procedures of the two studies. In the study for Text 3a predefined storylines were used, in Text 3b the storylines are a result of the analysis. Further, Text 3a analyses elementary teachers' perceptions on possibilities and obstacles of mathematical tasks to orchestrate teaching of MHAPs, while Text 3b analyses teachers' expressed rights and duties to orchestrate teaching MHAPs in the diverse ability classroom.

7.3.1 Study for Text 3a

In line with Mosvold and Bjuland (2016), personal pronouns are used to identify whether positioning is reflexive or interactive. Through analysing teachers' reflexive positioning their perceptions on their own teaching of HAPs in the diverse classroom can be discussed. Analysing when the teachers use the interactive positioning, teachers' perceptions of others (i.e., persons not participating in the programme) or other factors (i.e., mathematical tasks) influencing their teaching of HAPs can be discussed.

7.3.1.1 Analysing interactive and reflexive positioning

To identify interactive and reflexive positioning, subject words and personal pronouns were used. The used words could not, however, be seen in isolation; sometimes the whole episode of communication had to be analysed to judge whether the word used was interactive or reflexive. For example, the word '*you*' was often used to mean '*me*' or '*we'* when the teachers talked about themselves in the third person.

Example of interactive positioning

T1B …then they are not thinking outside the box, but some do, and some come up with completely own solutions

(from transcriptions analysed in the study for Text 3a (Mellroth et al., in press))

In the utterance the teacher is referring to pupils and uses the personal pronouns 'they' and 'some'. The teacher is interactively positioning the pupils, the speech act is interpreted as if she acknowledges differences in pupils' ways of thinking and she also shows awareness of that some pupils might have extra abilities. The context of the whole episode

indicates that she is referring to HAPs. Further the context indicates that the teacher is discussing mathematical tasks suitable for MHAPs.

Example of reflexive positioning

T1B — then it is, this is how they have done here, so maybe we should, then you should explain. Then they have done the first challenge.

(from transcriptions analysed in the study for Text 3a (Mellroth et al., in press))

Although the teacher in this utterance addresses pupils' work, the context of the communication for the speech act shows that the teacher discusses how they (the teachers in the professional development) should reconstruct the formulation of a mathematical task to create more challenges for HAPs. The focus when interpreting this speech act is therefore on "maybe we should", which results in the analyse of reflexive positioning.

The analysis procedure for Text 3a meets the aim to, from a teacher perspective, highlight pedagogical possibilities, and obstacles, to meet the learning needs of MHAPs in the diverse ability classroom.

7.3.2 Study for Text 3b

To explain how rights and duties have been analysed in Part 3 of this thesis, some examples are given using transcripts from the performed studies, i.e., from discussions between teachers. To be able to analyse a discussion with positioning theory a holistic approach must be taken; therefore, a description of what the teachers are discussing is included. For the transcripts, the following abbreviations are used: Excerpt number X (NoX), Teacher Z, in group Y (TYZ).

Examples of analysed rights

In the speech acts interpreted from excerpts No 8 and 9, two teachers express some rights, underlined in the transcript.

No8:T3C — Because I was for example completely broken when a colleague of us had been to a seminar in high ability and it was so very good ehh: and then the three of us are sitting there who have so much knowledge in the same.

N09:T3D No but we should ask the principal to get two hours in June or something, where everyone should listen to us…

(from transcription in Mellroth (2018b))

Teacher T3C talks about a colleague, who did not participate in the long-term professional development programme on high ability that she and two of her other colleagues did. She expresses frustration when this colleague had been on a short external seminar on high ability and talked about this in an internal school meeting. She was frustrated since the principle did not acknowledge her and her two colleagues' knowledge of high ability, although the principle knew the three of them participated in the extensive professional development.

For the excerpt, N08:T3C, the speech act is interpreted to reveal a vulnerability at the school organisation. The vulnerability is lack of knowledge of teaching HAPs among her colleagues. The principle could have asked the three teachers to, for example, give them permission to arrange collegial meetings. The teacher feels that the principle should acknowledge and use her and her two colleagues' knowledge of educating HAPs to enhance their other colleagues' knowledge. That is, she feels they have the **right** to be acknowledged for their knowledge of high ability.

Teacher T3D adds to the discussion that they, the teachers who participate in the professional development programme, should ask for time for a collegial meeting where they can disseminate their knowledge of high ability. There is a vulnerability – time is lacking for such a collegial meeting. But the teachers are permitted to ask their principle for time to implement such meetings. Therefore, they have the **right** to ask for time to disseminate their knowledge.

Examples of analysed duties

In excerpts 6, 7 and 8 two teachers express some duties, dotted underlined in the transcript.

N06:T1D But if you have two perhaps, one a simple problem-solving and one could be geometrics, yeah something, then you do it like some sort of test. And if then someone shows huge interest we have some sort of guideline. (T1C

	agrees) And you can do some sort of read understanding diagnosis in grade two and remember to take this too
N07:T1A	Yeah it would be this one then, where was it, now then not unusual, no. Understand and uses abstract symbols very early could perhaps fit in there. But otherwise it's the rich vocabulary and things like that.
N08:T1B	That´s what we pick up meanwhile (T1A agree).

(from transcription in Mellroth (2018b))

Teacher T1D talks about implementing problem-solving tasks connected to different areas of mathematics, geometric and something else. She compares the implementation of the tasks with a test, in the sense of that the teacher can observe which pupils show extra interest and motivation to perform the task, that is to continuously assess the pupils' ability in mathematics. The teachers' continuous assessment of pupils aims to guide the teacher as to the suitable mathematical level on tasks: Is it an appropriate level to stimulate and support learning for the pupil? The speech act is interpreted as if the teacher has the power, and is permitted, in her teacher role, to choose and implement mathematical problems and through doing so continuously assess her pupils' mathematical ability. That is, she has the **duty** to continuously assess her pupils.

Teacher T1D also suggests adding diagnosis of mathematical high ability in already used diagnosis for reading in grade 2. She means it would broaden the diagnosis material and, by doing so, disseminate knowledge of HAPs' learning needs to colleagues, since the diagnosis procedure is well known and used by many teachers. It is interpreted that the teacher T1D feels she has the power to improve diagnostic material and that she at least is permitted to suggest improvements. Thereby she has the **duty** to disseminate knowledge of HAPs, in this case through diagnosis material for all pupils.

After the speech act of teacher T1D, T1A performs a speech act, interpreted from excerpt N07:T1A, without any analysed rights and duties. In this speech act teacher T1A connects to criteria of MHAPs. In the last speech act presented here by excerpt N08:T1B, teacher T1B

connects to what T1A just said. It is interpreted that T1B means that they, as teachers, have the power to observe and assess their pupils' learning needs. They are also permitted to do so, since they are teachers, and therefore they have the **duty** to continuously assess their pupils, in this case to assess them to judge whether they are HAPs.

Through analysing teachers' discussions, in the context of the professional development programme on education of MHAPs, this study probes what teachers perceive regarding their rights and duties on orchestrating teaching for MHAPs. This means that the study for Text 3b meets the aim of the thesis to highlight, from a teacher perspective, what can be seen as teachers' rights and duties to be able to orchestrate teaching that includes MHAPs.

8 Summary of findings

In the three sections of this chapter the findings of each of the three parts are summarized. In each section it is clarified how the findings of the respective part are interpreted in relation to the overarching aim, either directly and through the two research questions.

8.1 Findings - Part 1

The findings of Part 1 show that there is a group of pupils who achieve high on a non-traditional test in mathematics, The Mathematical Kangaroo, although they do not achieve high on traditional tests, the national tests. In the study for Text 1, pupils' relative achievements on the national tests in mathematics in grade 3 (2009), grade 6 (2012) and on The Mathematical Kangaroo in grade 7 (2013) were compared. The results show that a group of pupils achieve highly on the non-traditional test, despite not achieving highly on the traditional test in grade 6.

Two groups of pupils were identified for deeper analysis: Group 1 (n = 25) are pupils who achieved highly on The Mathematical Kangaroo (top 20%) but who were among the bottom 80% in the national test in grade 6. Group 2 (n = 23) are pupils who achieved highly in the national test in grade 6 (top 20%) but who were among the bottom 80% in The Mathematical Kangaroo. A deeper analysis of these groups aimed to explain some differences in achievement.

To compare relative strength and weakness in mathematical competencies, as defined by MCRF (Lithner et al., 2010), between these two groups of pupils, the analysis focused on pupils' relative activation of mathematical competencies on The Mathematical Kangaroo. This means that the findings revealed differences in how mathematical competencies were activated between the two groups of pupils.

The findings show that both groups activate the applying procedure competence more than other competencies. The relative comparison means that if one competency is activated more, some others must bear

the cost. For both groups it was the reasoning competence and the problem-solving competence that paid. When comparing the two groups of pupils with each other the findings show that Group 2 activate the connection competency more than Group 1, while Group 1 activate the problem-solving competency more than Group 2.

Interpretation of these findings in relation to the aim of this thesis indicates that the use of non-traditional tests in assessment can allow teachers to recognize pupils with higher problem-solving competence than most other pupils. The findings do not show whether those pupils are MHAPs. On the other hand, research studies and literature on high ability in mathematics often pinpoint the importance of using problem-solving, both to give MHAPs possibility to show their ability (Krutetskii, 1976; Nolte & Pamperien, 2017) and to support them in learning (Benölken, 2015; Sheffield, 2003). Therefore, the findings in Part 1 verify that teachers need to use a broad variety of assessment processes to give all pupils the chance to show their abilities. In addition, when using problem-solving in assessment, teachers may discover MHAPs who are not able to achieve on traditional tests.

The findings of Part 1 provide answers to the first research question. It is shown that non-traditional tests using challenging tasks, with an aim to awaken joy and engagement in mathematics, can help teachers recognize pupils with high problem-solving competence, possibly MHAPs, that do not become visible through traditional tests.

8.2 Findings - Part 2

Part 2 focuses on a teacher-initiated investigation performed during a professional development programme, in collaboration with the teachers and the facilitator. The process and the results are descriptively presented in a report of the professional development (see Mellroth et al, 2016). The report is addressed to practice, aiming to disseminate experiences from the programme. Text 2 places the investigation and its results in a theoretical perspective and addresses the research community. The results from the investigation come from

a descriptive analysis of pupils' pre- and post-evaluation of two implemented tasks.

Results of the pre- and post-evaluation of the first implemented task, Task 1, are shown in Figure 9; an explanation of how to interpret the graphs is given after the Figure.

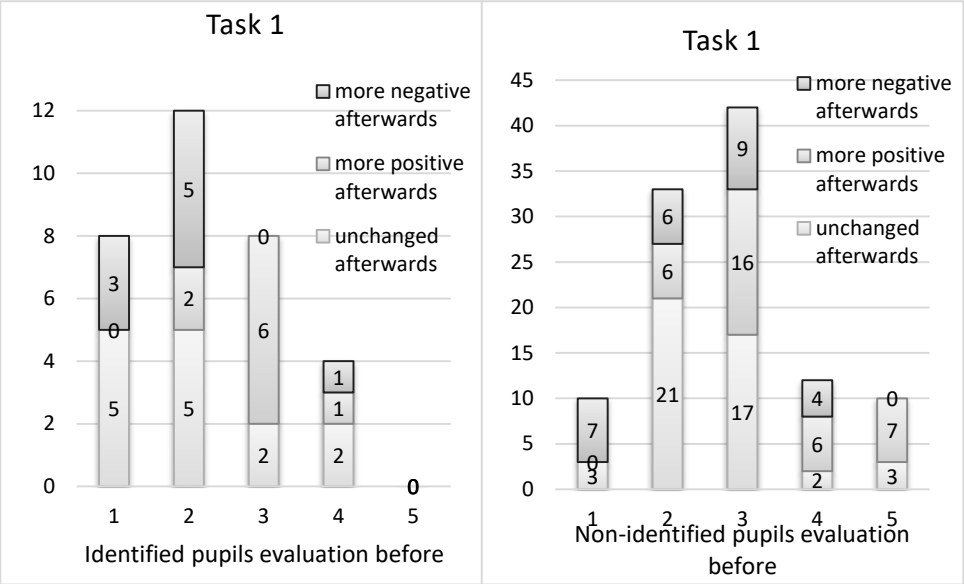

Figure 9 Summary of pupils pre- and post-evaluation of Task 1, adapted from Mellroth et al. (2016, p. 19).

Both the emojis on the pre-evaluation, Figure 7, and the words on the post-evaluation, Figure 8, were given numbers in the summarizing analysis. The number 1 was given to the most positive judgements and the number 5 was given to the most negative judgement. The left-hand graph in Figure 9 shows data from pupils identified by teachers as mathematically highly able, while the right-hand graph shows data from the other pupils; those are called non-identified. The bar numbered 3 in the right-hand graph shows that 42 non-identified pupils chose the middle emoji before they started to work on Task 1. Furthermore, the same bar shows that, of those 42 pupils, 9 gave the task a more negative judgement, 16 gave it a more positive judgement and 17 still gave it the middle judgement after they completed the task.

The results for Task 2 are shown in a similar way in Figure 10.

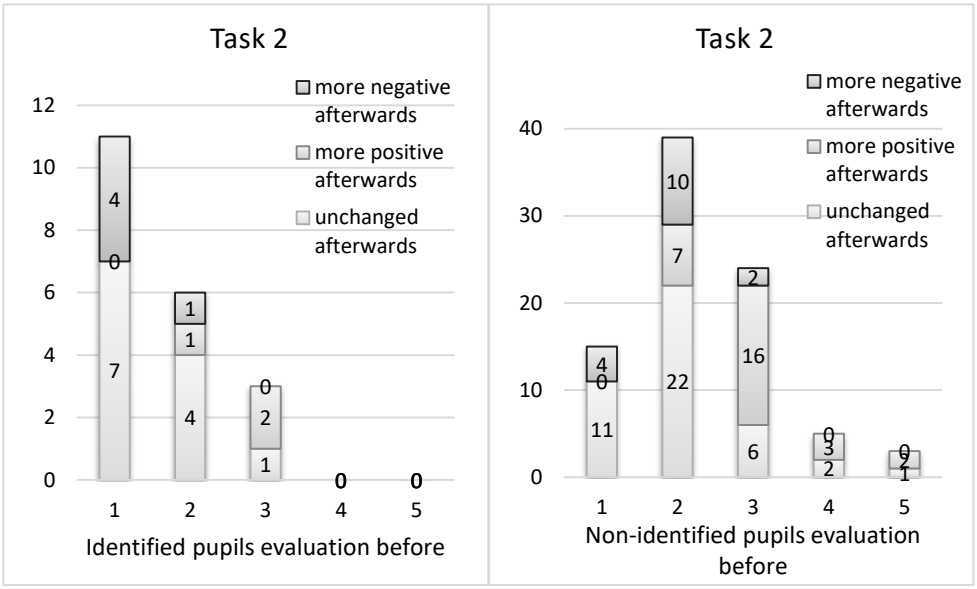

Figure 10 Summary of pupils pre- and post-evaluation of Task 2, adapted from Mellroth et al. (2016, p. 19).

From the summary of results, percentage changes were calculated to compare the pupils' perceptions of the tasks, as well as how identified MHAPs perceived the tasks in comparison to those who were not identified. However, no statistical verification was calculated.

The results presented in Figure 9 and Figure 10 show that MHAPs[14] perceived challenging tasks[15] more positively compared to other pupils. Moreover, they show that the other pupils became more positively disposed to such tasks while working with them. It is obvious that the evaluation tool has both a ceiling and a floor effect due to its construction; a pupil cannot be more positive in the post-evaluation if s/he chose the most positive emoji in the pre-evaluation. Despite this, the tool, measuring pupils' perceptions of mathematical tasks, has indicated its possibility to help teachers identifying mathematical tasks that are stimulating for MHAPs as well as others. Previous research has shown that teachers in diverse classrooms rarely provide MHAPs with

[14] 'Mathematically promising' is the term used in Mellroth (2017).

[15] 'Rich learning tasks' is the term used in Mellroth (2017).

learning opportunities that develop their knowledge (Leikin & Stanger, 2011).

The results show that MHAPs perceive the two implemented tasks more positively than other pupils, which is especially noticeable in Task 2 – see the concentration of bars to the left in the graphs for the identified pupils in Figure 9 and Figure 10. Those findings can also be interpreted as meaning that tasks developed to challenge MHAPs, together with the developed tool, can be used as an indication for teachers to recognize MHAPs. This means that the teachers, through observing, for example with the developed tool, which pupils become extra engaged and positive when implementing challenging tasks, are shown that those pupils can be highly able in mathematics.

The results also show that the tool can be used to give the teacher knowledge of how different tasks are perceived by pupils. When comparing the results of Task 1 and Task 2 in Figure 9 and Figure 10, it is clear that Task 2 is perceived more positively than Task 1 by MHAPs but also by the other pupils. This means that there is a possibility for teachers, by using the tool, to distinguish between tasks and find those that are more suitable than other tasks to challenge MHAPs and others. An implication of those findings is that if teachers regularly evaluate the suitability of tasks in this way, s/he probably builds up a competence in how to recognize such tasks. When teachers implement such tasks in the diverse ability classroom, it is likely that MHAPs are given learning opportunities.

This part of the thesis verifies that pupils, especially the MHAPs, perceived rich learning tasks (Sheffield, 2003) as stimulating. In addition, the tool allowed the teachers to see differences between tasks regarding pupils' perceptions. The findings therefore indicate that the tool fulfils its stated aim, although they are not statistically verified.

The findings can also provide justification for using the tool to orchestrate teaching that invites pupils to have an influence on their education. For example, a teacher can, at the end of a lesson, present two tasks and let the pupils fill in the pre-evaluation form. The results

will give the teacher knowledge of which task the pupils prefer and choose that task to work with during the next lesson.

8.3 Findings – Part 3

In order to harness a teacher perspective, two qualitative studies investigate how teaching can be orchestrated to include MHAPs in learning in the diverse ability classroom. To capture teachers' perceptions from discussions in the context of a professional development programme on educating MHAPs, positioning theory has framed the analysis. The findings reveal what the teachers perceive about orchestrating teaching of MHAPs in the diverse ability classroom.

8.3.1 Text 3a

The findings of the study presented in Text 3a show that the teachers perceive challenging tasks and differentiated education as providing the possibility to orchestrate teaching of MHAPs in the diverse classroom. The discussion about tasks was mostly connected to interactive positionings. For example, they mention tasks that allow different solution strategies and allow pupils to reach different levels in results as possibilities to include MHAPs in teaching. The discussion on teaching through differentiated education was mainly connected to reflexive positionings. In the storyline *Differentiation*, using reflexive positioning, the teachers addressed the importance of their role as a teacher, to support both MHAPs as well as others. The findings also show that the elementary teachers perceive their own mathematical knowledge as an obstacle, both to finding suitable tasks, and to providing enough support to MHAPs. These obstacles were mainly connected to reflexive positionings. The teachers did, however, express collaboration between teachers as a possible way to overcome this obstacle.

Through their discussions the teachers show deep knowledge of the literature involved in the professional development programme. Specifically, they show deep knowledge of the construction of challenging mathematical tasks. For such tasks, the teachers

highlighted the easy entry and the open end to be of extra importance for teaching in diverse ability classrooms. The easy entry was perceived as being of extra importance for the pupils with learning disability, and the open end as of extra importance for MHAPs. The participating teachers were all teaching in diverse classrooms. Despite the focus on high ability in the professional development programme, they did not forget their professional task to orchestrate teaching for all pupils, which is seen through all their discussions.

The teachers expressed that time to support pupils with learning difficulties is an obstacle to orchestrating teaching for HAPs. But they considered differentiated education (e.g., Tomlinson, 2016) as a possible pedagogical strategy to also include the HAPs. This is shown through their discussions when analysing and developing challenging tasks. Through these discussions they show understanding of how to differentiate instructions regarding content, which is one of the components to differentiate instructions (see e.g., Tomlinson, 2016).

The teachers express more possibilities than obstacles to orchestrating teaching that includes HAPs in learning in the diverse ability classroom. This is valid in relation to both reflexive positioning as well as interactive positioning, with one exception. When reflexive positioning is analysed within the storyline *Mathematical abilities*, the number of expressed possibilities (n=14) is almost the same as the number of expressed obstacles (n=15). Interactive positioning occurs almost twice as much as reflexive positioning.

8.3.2 Text 3b

The findings of the study for Text 3b show that the participating teachers have knowledge of HAPs regarding how to recognize them (e.g., Idsøe, 2014; MacLeod, 2005), their learning needs (e.g., Rogers, 2007), and how to support them in learning (e.g., Sheffield, 2003). Their knowledge is shown both through the teachers' expressed duties to disseminate knowledge of HAPs and through their expressed right to be acknowledged for their knowledge of HAPs. The findings also show that these teachers understand the principles of differentiated

education (e.g., Tomlinson, 2016). This is shown in the teachers' expressed right to assess pupils in several ways and contexts, and through their expressed duties to continuously assess pupils and to meet pupils' learning needs. Through their discussions they connect their arguments to theories of differentiated education and gifted education. These teachers therefore show competence in orchestrating differentiated education in the diverse ability classroom aiming to include HAPs as well as other pupils in learning.

Many studies end with a conclusion that teachers need more professional development on gifted education (e.g., Shayshon et al., 2014) or that teachers in diverse classrooms not are able to fully support HAPs in learning (e.g., Leikin & Stanger, 2011). In contrast, this study shows what teachers participating in professional development on high ability perceive about orchestrating teaching that includes HAPs in learning in the diverse ability classroom.

Through the use of positioning theory in the analytical framework, the teachers' expressed rights and duties were revealed. The teachers expressed far more duties than rights. The duty to continuously assess pupils is the most expressed duty, especially among the two groups of teachers teaching lower grades, i.e. grades 1-3 and grades 4-6. By 'continuous assessment' teachers mean that they have a duty to "keep their eyes open" and notice how their pupils react to and work on tasks. They combine their duty to continuously assess their pupils with their knowledge of MHAPs, connected to, for example, Krutetskii (1976) and Sheffield (2003). The third group of teachers, teaching grades 7-9, express the duty to acquire knowledge in almost the same amount as the duty to continuously assess pupils.

Further, the analysis of the storylines shows that the teachers mainly address HAPs, that is, high ability in general. MHAPs are specifically addressed in 20 of the 156 analysed storylines. For example, in their discussions the teachers argue that teachers must be aware of which criteria of high ability to look for, to be able to recognize HAPs. This means that the teachers perceive it as their duty to have knowledge of HAPs. The findings can also be interpreted as meaning that, if teachers

perceive they lack knowledge of HAPs, it is their responsibility to make sure they acquire the knowledge. The dominance of storylines addressing HAPs can be interpreted as if the teachers consider it important to have general knowledge of high ability, i.e. that they need this knowledge to be able to understand and support MHAPs.

Not many rights are expressed by the teachers, but they express their right to have access to easily used tools, for example to identify highly able pupils. The teachers at higher grades, grades 7-9, also express the right to get time, from their principals, to disseminate their knowledge as well as their right to be acknowledged, by their principals, for their expertise.

8.3.3 Part 3 connected to the aim

The findings of the two studies for Part 3 reveal teachers' possibilities and obstacles as well as rights and duties in orchestrating teaching HAPs in diverse ability classrooms.

Through capturing the teacher perspective on what would be possible to implement in diverse ability classroom, and their duty thereof, to enable learning for MHAPs, the two studies in Part 3 provide answers to the second research question. In addition, the obstacles that the teachers perceive provide additional breadth to the answer to research question two. Thereby the overarching aim is addressed through highlighting what teachers perceive as pedagogical possibilities to orchestrate teaching for MHAPs in the diverse ability classroom.

9 Discussion

In this chapter, reflections on the findings and on methods used in the study are given. The reflections are presented in relation to the overarching aim to capture the perspectives of teachers in order to highlight pedagogical possibilities in the diverse ability classroom that enable learning opportunities also for MHAPs.

9.1 Reflections on findings

The findings of this thesis give teachers support in teaching MHAPs in the diverse classroom. From a teacher perspective the findings show how teachers can identify MHAPs through assessment. But more importantly, the findings show possibilities, perceived by teachers, to include MHAPs in teaching in the diverse ability classroom. Previous research most often highlights that teachers need more knowledge and more professional development (Mhlolo, 2017; Shayshon et al., 2014) and how they fail in including MHAPs in teaching (Leikin & Stanger, 2011; Mhlolo, 2017). This thesis shows what teachers perceive as possible to do in teaching to include MHAPs, and their perceptions of such possibilities are in line with what previous research has described to be the learning needs of MHAPs.

This thesis shows that traditional tests are not enough to be able to identify MHAPs. Benölken (2015) argues that mathematical tasks suitable for MHAPs should be encouraging, and according to Krutetskii (1976), in order to be able to recognize MHAPs, they must be involved in a mathematical activity, preferably problem solving. Parts 1 and 2 show that such tasks also work as reliable instruments to recognize MHAPs and to assess pupils' strengths and weaknesses in mathematical competencies. Tests focusing on problem solving, like The Mathematical Kangaroo, can help teachers notice pupils with high mathematical ability even if they achieve low scores on traditional tests. The findings verify that employing a range of assessment tools gives pupils the possibility to show their abilities as well as their learning needs, which turns assessment into 'assessment *for* learning' (Bourke et al, 2011; A. Pettersson, 2004).

Pupils' achievement on non-traditional tests can also help teachers be aware of some pupils' learning needs that otherwise would be unrecognized, for example those who need challenging tasks in their learning. This means that teachers can use such tasks both in teaching and in assessments to include MHAPs in teaching. The conclusions are verified by the literature on mathematical tasks for HAPs (e.g., Benölken, 2015, Krutetskii, 1976, Sheffield, 2003). In addition, such tasks should offer complexity on different levels, since what is a mathematical problem for one pupil might not be a problem for another (Schoenfeld, 1992).

As Hoth et al. (2017) argue, teachers need deep mathematical knowledge to be able to support MHAPs. Furthermore, they also argue that challenging tasks should be included in professional development programmes to improve teachers' knowledge. In this study the teachers have not primarily worked with developing their own mathematical knowledge. The elementary teachers did express their own deficits in mathematical knowledge as an obstacle to supporting MHAPs, since it is connected to working with mathematically challenging tasks. Thus, the teachers in this study are aware of their shortcomings but express that collaboration between teachers can be a solution to overcome them. This means that if they do not have the desired subject knowledge themselves, they can collaborate with another teacher, with deeper subject knowledge, within the organisation.

The teachers' in this study express duties, related to knowledge of HAPs, that are connected to their perceived power to remedy a vulnerability related to lack of knowledge within the organisation. This means that the teachers in this study feel they have the knowledge, and competence, to disseminate knowledge of HAPs among their colleagues. According to positioning theory, the organisation has a duty, that is, the power, to use the teachers' competence. Whether the organisation will harness this competence or not, is another question. Figure 11, based on Figure 2, displays the connections between the rights (vulnerabilities) and duties (power) discussed in this paragraph.

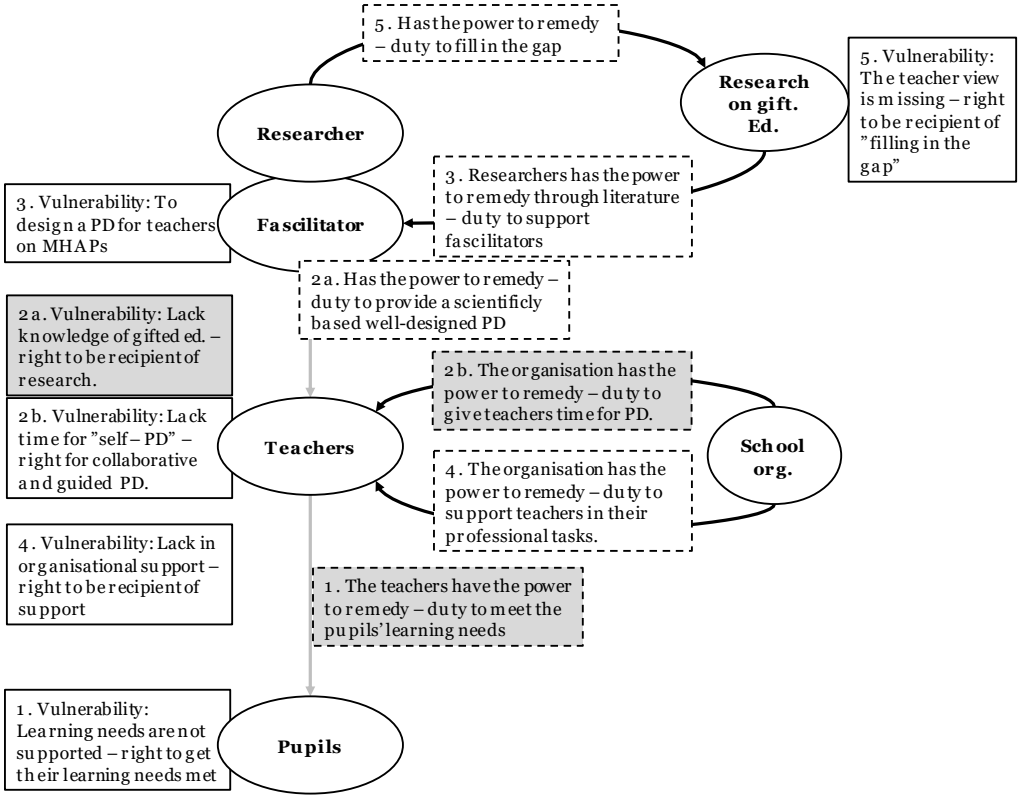

Figure 11 Marked boxes are related to the right connected to a vulnerability in the organisation, and duties, that is, teachers' power to disseminate knowledge and the organisations' power to remedy lack of knowledge.

Further, this thesis shows that, when teachers collaboratively participate in professional development on teaching MHAPs, they express possibilities to orchestrate teaching that includes MHAPs in learning. For example, they express differentiation of content, teachers' knowledge of high ability and collaboration between colleagues as possibilities. The expressed duties connected to teachers' professional tasks are in this study connected to the teachers' perceived power to remedy vulnerabilities. Firstly, the vulnerability among HAPs who are not taught according to their learning needs. Secondly, their own perceived vulnerability due to lack of organisational support to perform some of their professional tasks, see Figure 12, based on Figure 2. As examples on how to remedy the vulnerability related to the organisation, they suggest differentiated education and collaboration with colleagues such as the school health team and colleagues in other grades.

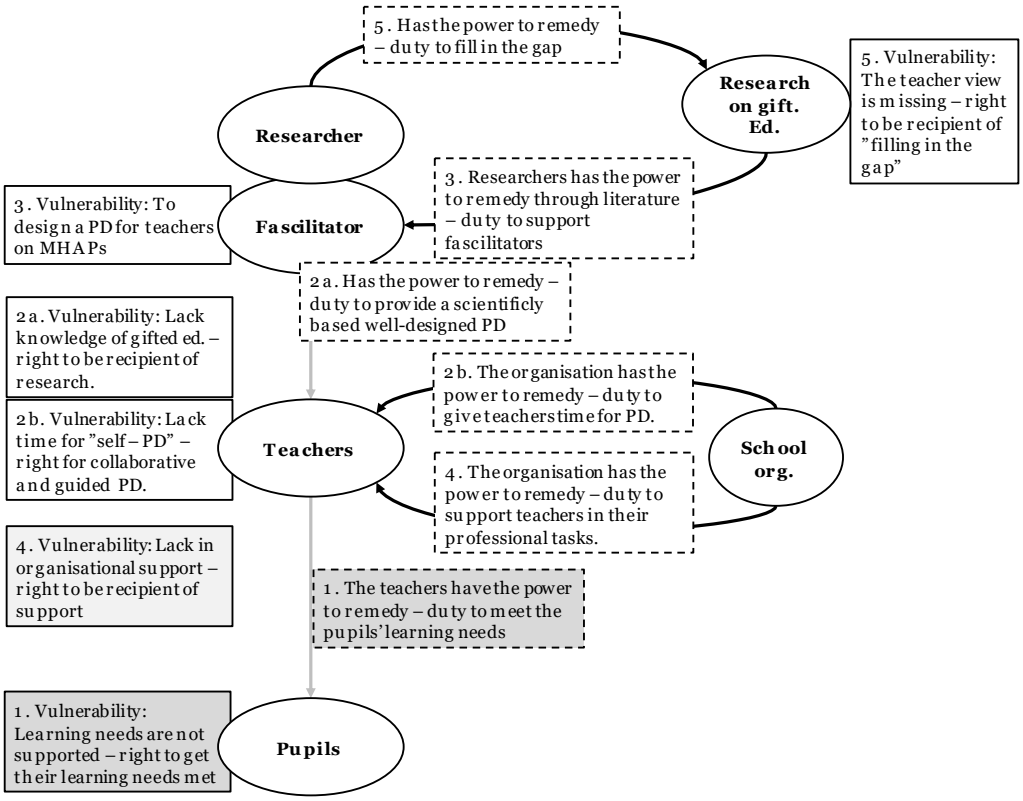

Figure 12 Marked boxes are related to rights, by a vulnerability in supporting HAPs and by a vulnerability in the organisation to support teachers, and a duty, that is, teachers' power to meet HAPs' learning needs.

The findings also show that teachers perceive more duties than rights when discussing their teaching. This means, according to positioning theory, that they feel they have the power to remedy some vulnerabilities related to teaching HAPs. Those duties are often placed in relation to their responsibility to continuously assess pupils, also focusing on MHAPs. Thus, the teachers show consciousness and understanding of how to identify MHAPs and, more importantly, of how to support them in their learning, which is a contradiction to previous research that claims that teachers do not have that knowledge (e.g., Leikin & Stanger, 2011).

The teachers' perceptions of how to recognize and support MHAPs are coherent with present research (e.g., Le Fevre et al., 2016; Krutetskii, 1976; Nolte & Pamperien, 2017; Szabo, 2017). These perceptions are shown through the teachers' discussions when they combine their duty to continuously assess pupils with their knowledge of MHAPs. Further,

the teachers' expressed possibilities and duties are coherent with what often is identified as important, for example task construction and pedagogical strategies, when meeting HAPs' learning needs (see e.g., Krutetskii, 1976; Rogers, 2007; Sheffield, 2003; Tomlinsson, 2001, 2016). Leikin and Stanger (2011) and Mhlolo (2017) argue that teachers in the diverse ability classroom do not know how to meet the learning needs of MHAPs. However, it is shown through the teachers' discussions, in this study, that they can verbalize their professional task to include MHAPs in learning. For example, they gave clear examples on how to adapt their teaching to include MHAPs in learning.

Teachers' perceptions of possibilities and obstacles in teaching HAPs in diverse classrooms are in this thesis mainly revealed through analysis of reflexive positionings. Vanassche and Kelchtermans (2014) demonstrated reflexive positioning to be a crucial factor in understanding practices. Their findings therefore support the findings in this thesis. In addition, this thesis shows that analysis of interactive positioning can be used to reveal teachers' perceptions of used material, for example mathematical tasks. This means that analysis of reflexive and interactive positioning respectively can reveal teachers' perceptions on pedagogical strategies as well as on teaching material.

The findings of this thesis show that when teachers participate in a professional development programme, designed according to existing research on educating HAPs (e.g., Krutetskii, 1976; Rogers, 2007; Sheffield, 2003; Tomlinsson, 2001, 2016), and implemented to follow core criteria in effective professional development (e.g., Desimone, 2009), they can connect theories with their own practice. The teachers in the studies for Parts 2 and 3 have chosen to participate in the professional development programmes; it is therefore assumed that they are open-minded and willing to broaden their thinking regarding HAPs. Supported by Liljedahls' (2014) findings of how such teachers act in practice, it is likely that these teachers also will implement their knowledge of recognizing and supporting HAPs' learning needs in their teaching.

Franke et al. (2007) argue that, to meet specific challenges in teaching, teachers and pupils, among others, can work collaboratively to build the trust required to share knowledge and ideas. It may be an overwhelming task for a teacher to build knowledge of each pupil's mathematical understanding and their past experiences, background, hopes, fears and dreams, which have been proven to have influence on pupils' success (Franke et al. 2007; Parsons & Vaughn, 2016). The findings in this thesis imply that those teachers perceive that teaching involves not only the classroom teacher, but all teachers in the organisation. Collaboration, as well as being a core feature in professional development (e.g., Desimone, 2009; Parsons et al., 2016), may also be a core feature among teachers in teaching. The findings of the thesis also indicate that everyday teaching can be seen and used as professional development.

From previous research and the findings of this thesis it can be concluded that, for teachers to orchestrate teaching for all pupils, MHAPs as well as others, collaboration among teachers is a core feature. This collaboration should be based on knowledge of differences in pupils' learning needs and on knowledge of activities, such as mathematical tasks, that can support or slow down pupils' learning.

9.2 Reflections on methods used

Throughout the three parts of the thesis, two research questions have guided the work. In the previous chapters (chapters 8 and 9) the studies have been addressed in relation to the aim and the research questions. Each part of the thesis has used a different method and contributed to one of the two questions. Part 1 has predominantly contributed to answering the first research question, whereas Part 2 and 3 have mostly contributed to the second. Although each part of the study has contributed to fulfilling the aim of the thesis, the contribution of Part 1 and 3 are both deeper and broader in comparison with Part 2. In this chapter, reflections on methods used and how they have contributed to fulfilling the aim of this thesis are made. The first section reflects on the mix of quantitative and qualitative methods used for

Part 1. The second section reflects on the production of a research paper from a report of a professional development programme and on the descriptive analysis. Finally, the third section reflects on the use of positioning theory in the analysis for Part 3.

9.2.1 Part 1 – A quantitative and a qualitative approach

The first step in the study for Part 1 was to investigate whether there were groups of pupils who achieved highly on one test but not on another, comparing traditional tests with non-traditional. To be able to do so, data for a larger group of pupils were needed. Since pupils' results on national tests at the time of the study were public, it was possible to collect data from a complete year group of pupils in one municipality. It is well known in Sweden that national tests take a lot of time and energy for both teachers and pupils to implement. In order not to interfere too much with that test procedure, and to keep the time between tests as limited as possible, The Mathematical Kangaroo was implemented in grade 7 (2013), for those pupils who did the national tests in grade 6 (2012). To include the pupils' results on The Mathematical Kangaroo, both guardians and pupils were required to sign an informed consent form, which reduced the amount of data analysed.

To compare pupils' achievement on different tests, relative achievement instead of actual achievement was used. This means that each pupils' results were given a ranking number on each test and it was these ranking numbers that were used in the comparison. This was done due to the differences in the tests' aims and constructions and since the aim of the comparison was to investigate groups of pupils who achieved high on one test and low on another, which the comparison through ranking positions fulfilled. It would have been possible to do a similar comparison using actual achievement, but since relative achievement was of interest, ranking became more appropriate to use.

To examine how Group 1 and Group 2 (see section 8.1 for definition of groups) differed in mathematical competency, a qualitative analysis was carried out on what possibilities the tests gave the pupils to activate

the competencies. For this, a guide to analysing mathematical competencies necessary to activate to solve a task was developed. The guide was based on an unpublished analysis document used in Boesen et al. (2014). Through this guide it was analysed what is *required and reasonable* to expect from a pupil in grade 7. The validity of the analysis was given by the 15-year experience of teaching by the author, and by discussing the guide with active teachers and senior researchers in mathematics education. Each task on the national test for grade 6 (2012) and The Mathematical Kangaroo (2017) was analysed using the developed guide. There was a minor difference between how the problem-solving competency was analysed. The reason for not being able to do it the same way was secrecy rules for the national test in grade 6 (2012).

Since the analysis of the task indicated what mathematical competencies a pupil needs to activate to solve the task, and since task results on The Mathematical Kangaroo were either fail or full points it was possible to produce a competency profile for each pupil on The Mathematical Kangaroo. Specifically, it was possible to compare competency profiles from Group 1 with Group 2. It would have been of great interest to do the same comparison on these groups on the national test in grade 6 (2012), but pupils' results on task level for the national test were not available data. To reach such data, the study could have been performed on results of the national test in grade 6 from year 2013 and from The Mathematical Kangaroo for the same group of pupils in the same year. This would, however, place another workload on top of an already high workload for grade 6 teachers. Other ways to explore differences in achievement between Group 1 and 2 would have been to analyse pupils' solution processes, written or oral. In addition, interviewing pupils would have been of interest to find out more reasons for the differences.

Nevertheless, even if there may be more explanations to search for, the method used in Part 1 does reveal reasonable explanations, related to pupils' mathematical competencies, of why some achieve highly on a non-traditional test but not on a traditional test. This clearly addresses

the first research question regarding how non-traditional tests can help teachers recognize pupils with high mathematical ability, where some may be MHAPs.

9.2.2 Part 2 – From a report to a research paper

The method used to produce a scientific paper from a report of a professional development programme is unusual but meets the overarching aim of this thesis to include the teacher perspective in research. The professional development programme was built on a method called the 'research circle', which includes a strong collaboration between teachers and a research-trained leader (Persson, 2009). There are different opinions about whether such a programme is considered as research or not (Persson, 2009); in this case the main aim was to develop teacher competence and not to produce research, although the participating teachers were aware that the results could be communicated in both popular science and scientific papers. Olin et al. (2016) have, by contrast, directly combined research with the implementation of research circles. For Part 2 in this thesis, the report of the professional development programme was used as data for Text 2 as a method to highlight a teacher perspective on orchestrating teaching of MHAPs in a diverse ability classroom.

The teachers put a lot of time and energy into the whole programme and the process of writing the report, which was written in Swedish addressing other teachers. To theorize and write a scientific paper on a part of the report was one way to disseminate the results of a teacher-initiated investigation. Text 2 therefore fulfils the aim to make the teachers' perspectives available for the national and international research community.

The chosen way of representing the results of the descriptive analysis, shown in Figure 9 and Figure 10 in section 8.2, saves space, which is important in a conference paper with a limited number of side pages. In addition, deeper analysis was not possible, since the report (Mellroth et al., 2016) was used as data for Text 2. It would have been ethically incorrect to use the original data collected during the

professional development programme. However, a more illustrative way to present the results would have been separate tables on pre-evaluations and others for post-evaluations. What can be shown using the available data in Mellroth et al. (2016) are the most positive and the most negative possible outcome of the post-evaluation respectively, see Figure 13.

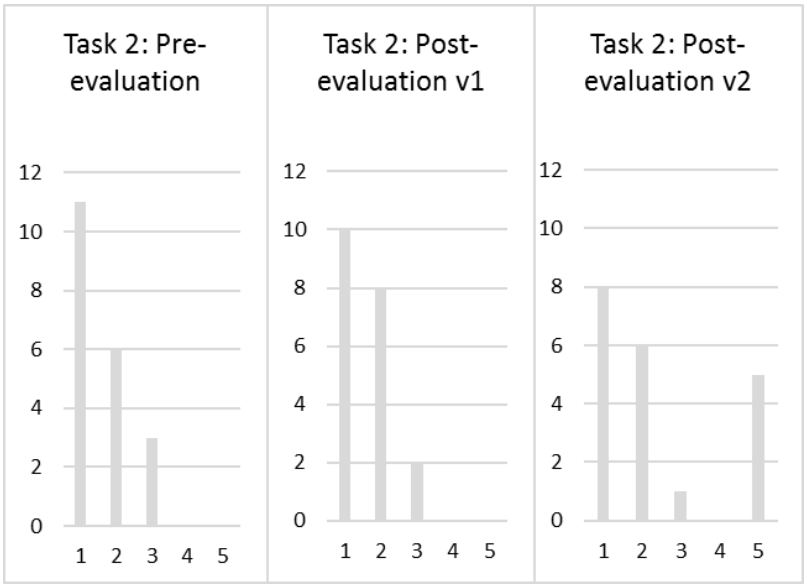

Figure 13 MHAPs' evaluation of Task 2. To the left their pre-evaluation is shown, post-evaluation v1 (in the middle) shows the most positive possible outcome; v2 (to the right) shows the most negative possible outcome.

With the most positive post-evaluation it means that it has been interpreted as if all pupils who became more positive after the implementation of Task 2 chose the most positive evaluation (no. 1), and that those who became more negative only chose one step lower in the post-evaluation than they did in their pre-evaluation, for instance changing from no. 2 to no. 3. In similar way the most negative post-evaluation means that the data is interpreted as if all pupils who became more negative after the implementation all chose the most negative choice (no. 5), and those who became more positive only changed one step higher in the post-evaluation compared to the pre-evaluation.

When addressing a teacher perspective and valuing collaboration between practice and research, the teacher-initiated study in Part 2 is

of interest and it can be further developed and fine-tuned with a direct aim to include both research and professional development. One possible way to do so is to use the methodology of action research, as for example Olin et al. (2016) did during their research circles.

9.2.3 Part 3 – Positioning theory

Both studies for Part 3 have a qualitative approach and were performed in one municipality in a city of medium size in Sweden. Different results may be found in larger cities as well as in smaller. Since the teachers volunteered to participate in the professional development programme it is reasonable to assume that the participating teachers (n=17) not are representative of all teachers in the municipality (approximately 600 in total). Nevertheless, in revealing that these perceptions do exist among teachers, the study contributes with a teacher perspective to research.

Positioning theory is a discourse analysis used in Part 3 to capture teachers' perceptions in their discussions. Discourse analysis focuses on how things are said, to provide key understanding of humans' social functioning (Robson, 2011), and is therefore a suitable choice for analytical framework.

Discourse analysis does not view 'what is said' in isolation, but states that the utterance has, at least, a prepositional content (what has been said earlier) and a performance content (what is being said) (Cohen, Manion, & Morrison, 2007). Based on these considerations, any kind of discourse analysis could have been used as analytical framework for the studies in Part 3.

However, positioning theory specifically aims to explore the meaning in communications (Harré, 2012, Harré & van Langenhove, 1999; Harré & Moghaddam, 2003). How participants are positioning themselves and others is revealed through the focus on the triad: *position – speech act – storyline* (Harré, 2012). This means that teachers' perceptions of their own and others' (for example colleagues') possibilities to orchestrate teaching including MHAPs can be revealed. In addition, the theory makes it possible to place the teachers'

communications in a larger context that partly reveals factors that may have influence on how the teachers can act, see Figure 2 in section 4.3. Another possibility positioning theory offers is that it systematically guides the analyser. This is helpful especially for withholding an analytical distance, which is of extra importance when the facilitator is the same person as the researcher, as well as when the researcher mostly works alone.

Other ways to capture the teachers' perceptions on orchestrating teaching for MHAPs in the diverse ability classroom could have been to conduct interviews with the teachers, individually and/or in groups, although such a choice would have placed an interviewer among the teachers. In the two studies for Part 3 the teachers are talking freely in groups without any external observer. Although they were video recorded it is reasonable to assume that the recordings had minor influence on their communication as the teachers had become accustomed to video recording during the professional development programme.

It is likely that there are other possible analytical frameworks that could have been used for the studies in Part 3 and yielded the same results. The use of positioning theory has contributed to fulfilling the aim of the studies. When trying to interpret discussions of others it is unlikely that anyone can do that completely objectively, but using the theory as an analytical frame made it possible to maintain structure and look at the data with an analytical mind.

9.3 Trustworthiness

The studies and the texts involved in this thesis were produced mainly by one person, which naturally increases the risk of giving a biased view of the findings. To avoid this, and to keep the process open for constructive criticism, the author has systematically and continually opened the work up for academic review. This was done throughout the whole research process, starting in February 2012 and ending in October 2018. Revisions were performed through regular academic seminars, with other doctoral students and senior researchers, where

chosen methods, theoretical considerations, analytical processes and findings were discussed and constructively criticised. The author also discussed the research process and presented parts of the work at several research conferences. In addition, she actively asked experts, professors and senior researchers in the field for feedback on her work and process. Through this systematic work and openness towards the research community the trustworthiness of this thesis is strengthened.

The context from which data are produced can also have an effect on the trustworthiness of the studies. This must be especially reflected on for Part 3 where the professional development programme and the studies were planned for in parallel. The programme was designed to fulfil an effective professional development programme (e.g., Desimone, 2009), which means that the programme is likely to have increased teacher competence. The studies do not, however, aim to measure the effectiveness of the programme, but rather what the teachers express.

Two senior researchers were consulted at frequent intervals regarding the research studies. The author of this thesis elaborated and discussed ideas with these researchers throughout the whole process of the two studies in Part 3, from planning and data collection to analysis. Those two senior researchers are co-authors to Text 3a and thus made a substantial contribution to the high academic quality of the study and the text. In addition, the analytical process of Text 3b was presented and discussed during a seminar with other doctoral students and senior researchers at Karlstad University. To secure the quality in the study for Text 3b, for example not to ask leading questions, the sixth meeting in the programme was planned in collaboration with one of the two senior researchers. Text 3b has been reviewed and given constructive criticism through the author's participation in courses for doctoral students. The process of producing Texts 3a and 3b has therefore followed principles to secure high academic quality, which strengthens the validity of the two studies as well as of the whole thesis.

10 Implications

This thesis addresses the problem that previous research does not support teachers in how to orchestrate teaching that includes MHAPs in learning.

With regard to practice, through this thesis teachers are given support regarding both how to recognize and how to support MHAPs in the diverse ability classroom. The findings imply that teachers' use of a variety of assessments increases MHAPs' possibilities to show their abilities. Using non-traditional tests can help teachers be aware of pupils who do not achieve on traditional tests. Furthermore, the findings of this thesis show practice what teachers themselves perceive as possible in teaching to meet the learning needs of MHAPs. What is expressed from teachers may be easier to adapt compared to a top-down approach. In addition, the expressed obstacles can be used as the basis of discussions about attitudes and the importance of subject knowledge.

For research, this thesis implies that having a teacher perspective can contribute with knowledge of what teachers perceive as possible to implement in teaching, and of whether their perceptions are in line with existing research on pupils' learning needs. Such an approach also captures teachers' expertise, which can contribute to deeper theoretical knowledge of how teaching can be orchestrated to include MHAPs in learning.

From the findings of this thesis, research can take a step forward, in collaboration with teachers, to develop and investigate pedagogical strategies that broaden education to include more pupils in learning. Specifically, for MHAPs, developmental studies in collaboration with teachers skilled in gifted education can contribute knowledge to successful education that includes those pupils in learning.

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Appendices

1. Announcement of the professional development programme for Part 3.

2. Application form for participation in the professional development programme for Part 3.

3. Information about research for Part 3.

4. Informed consent form for Part 3.

Appendix 1

[redacted]

[redacted] 2015-04-08
Elisabet Mellroth
Projektledare
elisabet.mellroth@[redacted]

[redacted]

Inbjudan till lärare att delta i ett skolutvecklingsprojekt

Augusti 2015 till och med maj 2017

Sammanfattning

Under de två kommande läsåren genomförs i [redacted] ett i Sverige unikt skolutvecklingsprojekt. Projektet har fokus på de särskilt begåvade barnens rätt att utmanas i en skola som är till för alla. I projektet kommer de deltagande lärarna att fortbildas om särskilt begåvade elever och de behov dessa har i undervisningssituationer. Tillsammans kommer projektdeltagarna med sina nyvunna kunskaper utveckla och testa undervisningsinnehåll och undervisningsmetoder i ämnet matematik med fokus på de särskilt begåvade eleverna. Projektets resultat förväntas gynna alla elever i klassrummet, inte bara de elever som står i fokus. [redacted] finansierar projektet. Med detta brev bjuder vi in matematiklärare på grundskolor och förskolor att delta i projektet.

Syfte

Syftet med detta projekt är att i den svenska kontexten och dess skolsystem med det heterogena klassrummet på vetenskaplig grund utveckla innehåll och undervisningssituationer i matematik med fokus på elever särskilt begåvade i matematik. Det vill säga; utveckla, pröva och utvärdera undervisning inom ämnet matematik som är speciellt anpassat för särskilt begåvade elever i matematik. I huvudsak handlar det om att skapa, utvärdera och definiera uppgifts/problemtyper, samt att utveckla undervisningsmetoder som fungerar i en svensk klassrumsmiljö. Fokus är på de särskilt begåvade, men resultatet förväntas gynna alla elever.

Målet är att öka kunskapen hos lärare om dessa elevers behov, styrkor och svårigheter, samt deras rättigheter. Ett mer detaljerat mål är att definiera vilket typ av innehåll som lämpar sig för de särskilt begåvade eleverna, både med avseende på matematiska förmågor och specifikt matematikinnehåll. Projektet avser också att identifiera några typer av undervisningsmetoder som är speciellt lämpliga för dessa elever i det vanliga klassrummet. Projektets resultat är tänkt att fungera som en kunskapsbas dels i den egna regionen men även nationellt.

[redacted]

Appendix 1

Vad ingår

Lärarens deltagande i projektet löper över två år med start i augusti 2015 och slut i maj/juni 2017.

Del 1: Första läsåret, 2015 – 2016, kommer i huvudsak bestå av fortbildning för de deltagande lärarna där ämnet är undervisning av särskilt begåvade elever både allmänt och specifikt i ämnet matematik. Fortbildningen lyfter olika aspekter kring undervisning av dessa elever. Utöver det kommer vi tillsammans välja och utveckla undervisningsmaterial i matematik lämpligt för dessa elever.

Del 2: Andra läsåret, 2016 – 2017, kommer handla om att i undervisningen implementera delar av det undervisningsmaterial som utvecklats under det första läsåret. Implementeringen kommer att analyseras och utvecklas; det kollegiala samtalet spelar en stor roll i detta arbete.

Arbetsinsats

Del 1: Under läsåret, 2015 – 2016, kommer cirka 80 timmar att krävas för respektive deltagare i projektet. I dagsläget är 8 heldagar planerade, 4 på hösten och 4 på våren. Viss förberedelsetid i form av bland annat inläsning av material ingår.

Del 2: Under läsåret, 2016-2017, ska materialet som utvecklats under del 1 implementeras i lärarnas ordinarie undervisning. Utöver detta tillkommer träffar för analys, reflektion och utveckling och för detta behöver de deltagande avsätta cirka 40 timmar.

Ekonomisk ersättning

Ekonomisk ersättning kommer att utgå för deltagande i projektet och det finns olika sätt att använda dessa medel, t.ex.

- projektet bekostar en vikarie,
- projektet ersätter den deltagande läraren med en timersättning utöver dennes ordinarie lön.

Material

Projektet bekostar material för deltagarna, mycket material kommer vara på engelska. Diskussioner kommer dock vara på svenska.

Möjlighet till erfarenhetsutbyte genom projektet

Inom projektet finns medel för att dela erfarenheter med skolor i Europa som har en fungerande verksamhet för särskilt begåvade elever. I praktiken innebär detta att några av deltagarna kan följa med på studiebesök t.ex. till Holland, Tyskland eller England. Troligtvis kommer dessa studiebesök att förläggas under vecka 44 eller på studiedagarna i juni när eleverna i Sverige gått på sommarlov. Erfarenhetsutbytet ligger utanför den beskrivna arbetsinsatsen. Möjlighet till extra ersättning eller vikarie finns inte för detta, tanken är att erfarenhetsutbytet ska ersätta studiedagar på hemmaplan. Projektet bekostar resa, boende och kostnader för respektive studiebesök.

Forskning

Karlstads universitet kommer att bedriva en forskningsstudie knuten till skolutvecklingsprojektet. Detta innebär att lärarna som deltar i projektet också kommer delta i en forskningsstudie. Deltagandet i forskningsstudien är anonymt, d.v.s. ingen identitet kommer att avslöjas i något sammanhang där forskningen presenteras. Allt

Appendix 1

deltagande i en forskningsstudie är frivilligt, vilket innebär att man när som helst kan avsluta deltagandet i forskningsstudien. [redacted] stödjer forskningsstudien och önskar att de deltagande lärarna är positiva till deltagande i forskningsstudien vid starten, det vill säga när man ansöker om att få vara med i skolutvecklingsprojektet. Väljer man att senare att avsluta deltagandet i forskningsstudien har man naturligtvis kvar sin plats i skolutvecklingsprojektet.

Anmälan för deltagande

Projektet strävar efter att 10-20 lärare inom grundskolan ska delta i projektet. Deltagaren ska vara undervisande lärare i matematik. Lärare för de lägre årskurserna kommer att prioriteras om det blir fler sökande än platser, men lärare för samtliga årskurser F – 9 är varmt välkomna. Vi kommer att sträva efter att de deltagande lärarna skall representera flertalet av skolorna i kommunen.

Rektor bör stödja och uppmuntra lärarens deltagande i projektet.

Anmälan till projektet gör man genom att fylla i anmälningsblanketten, scanna in den och maila till Elisabet Mellroth, elisabet.mellroth@[redacted] senast 28 maj 2015.

Fortsättning efter projektets slut

Detta är en unik satsning i Sverige, det är första gången som ett skolutvecklingsprojekt under en längre tid genomförs gällande skolpraktiken för särskilt begåvade elever. Efter projektets avslut kommer ett intyg delas ut som bekräftar deltagandet. I projektet räknar vi med att deltagarna kommer bli ambassadörer i att sprida kunskap kring undervisning av särskilt begåvade elever inte bara på sina egna skolor, utan även inom och utom kommunen. Det är mycket troligt att deltagarna kommer bli engagerade både regionalt och nationellt i önskan om att få ta del av deras erfarenhet och kunskap. De deltagande lärarna kommer att uppmuntras att dela sina erfarenheter på matematikbiennalen 2018.

Finansiering

Projektet finansieras av [redacted].

Elisabet Mellroth
Projektledare

[redacted]
Chef [redacted]

Appendix 2

[redacted]

[redacted] 2015-04-08
Projektledare: Elisabet Mellroth,
elisabet.mellroth@[redacted]

Anmälan till projektet:
[redacted]

Deltagande lärare:
Skola:
Kontaktuppgifter – deltagande lärare Email: Telefon:
Undervisar matematik i årskurs/årskurser under läsåret 2015-2016: Är behörig att undervisa matematik i årskurs:
Underskrifter Datum Datum Deltagande lärare Rektor Namnförtydligande Namnförtydligande
Scanna in dokumentet och maila till elisabet.mellroth@[redacted] senast 28 maj 2015. Alternativt skicka till: Elisabet Mellroth, [redacted]

[redacted]

Appendix 3

Förfrågan till lärare som deltar i [redacted] skolutvecklingsprojekt [redacted] 2015 –2017

Inledning

Forskningsstudien som denna förfrågan om deltagande syftar på utförs av Elisabet Mellroth, doktorand vid Karlstads universitet, institutionen för pedagogiskt arbete. I forskargruppen kring denna forskning ingår förutom Elisabet Mellroth hennes handledare f.n. Prof. Hector Pérez Pietro, fil. Dr. Jorryt van Bommel och fil. Dr. Yvonne Liljekvist.

Bakgrund och syfte

Denna studie studerar en fortbildning som ingår i ett skolutvecklingsprojekt gällande särskilt begåvade elever med fokus på matematik, forskningen kommer att fokusera på de deltagande lärarna, här kallade deltagarna. Syftet med forskningen är tvådelat, dels syftar forskningen till att utveckla och utvärdera själva fortbildningen, dels syftar forskningen till att undersöka hur attityder och kunskap gentemot särskild begåvning förändras hos deltagarna under genomförandet. Fortbildningen är den första i Sverige som görs inom området särskilt begåvade elever med avseende på både djup och tidsperiod (2 år), vilket motiverar att forskning görs på fortbildningen enligt det nämnda syftet. Studien avser att som empiri använda videoupptagning av de gemensamma träffarna i fortbildningen, samt material som deltagarna både använder och producerar genom olika medier (skriftligt, online, etc). Intervjuer kan även komma att ingå.

Ett mål med studien är att bidra med kunskap om viktiga lär och lärande strategier vid en fortbildning för aktiva lärare gällande matematiskt särskilt begåvade elever. Ytterligare ett mål är att bidra med kunskap om kritiska aspekter som kan uppstå när lärare genom matematiska uppgifter försöker utmana och stimulera särskilt begåvade elever i det heterogena klassrummet. Ett tredje mål är att undersöka hur lärares attityder och kunskaper kring särskilt begåvade elever och undervisningen av dessa förändras genom fortbildningen.

Hur går studien till?

Vid samtliga träffar videofilmas alla möten i syfte att kunna studera diskussionerna för att påverka innehållet/form på fortbildningen. Även enkäter förekommer för att kunna anpassa fortbildningen efter behov. Diskussionerna samt enkäterna kommer även att användas i forskningssyfte, vilket bland annat innebär att resultatet kopplas till vetenskapliga teorier. Vidare kommer deltagarna att producera olika typer av material, till exempel anteckningar vid diskussioner, analyser av uppgifter, konstruktion av uppgifter och liknande. Deltagarna kommer att göra olika typer av mätningar, till exempel analysera och dokumentera hur utvalda uppgifter fungerar i helklass. I dokumentationen kommer deltagarna eventuellt särskilja identifierade särskilt begåvade elever från övriga elever, information om individer kommer aldrig efterfrågas. Enbart undervisande lärare ska veta respektive elevs möjligheter

och svårigheter. Deltagarnas dokumentationer kring de olika mätningarna kommer att involveras i datainsamling för forskningssyftet. Deltagarna ombeds att utesluta uppgifter som kan identifiera någon elev, om sådant förekommer ansvarar forskargruppen för att alla identifikationsmöjligheter raderas. Materialet hanteras och analyseras av forskargruppen på Karlstads universitet.

Hantering av data och sekretess

Alla personuppgifter kommer vid analysarbetet att anonymiseras av forskargruppen. Ingen obehörig kommer att kunna ta del av uppgifter där någon individ kan identifieras, t.ex. videofilmerna. Allt material kommer att förvaras så att enbart forskargruppen har tillgång till dem.

Studien förväntas vara avslutad under hösten 2018. Enligt Personuppgiftslagen (1998:204) har deltagarna rätt att en gång per kalenderår få besked om vilka uppgifter som finns lagrade om sig.

Hur får jag information om studiens resultat?

Studiens resultat är planerat att publiceras i en doktorsavhandling under hösten 2018, studien kommer även att kommuniceras i form av artiklar i vetenskapliga tidskrifter, eventuellt även i populärvetenskapliga tidskrifter.

Frivillighet

Allt deltagande i studien är frivilligt och deltagaren har rätt att när som helst och utan särskild anledning välja att avbryta deltagandet i studien. Deltagarna kan också utan särskild anledning avstå från att delta i delar av studien. Kontakta i sådana fall någon i forskargruppen.

Ansvariga

Karlstads universitet är forskningshuvudman och personuppgiftsansvarig. Insamling och bearbetning av alla data genomförs av doktoranden Elisabet Mellroth. Undrar ni över något är ni välkomna att kontakta oss.

Hälsningar

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Appendix 4

Efter underskrift, lämna tillbaka till Elisabet Mellroth.

Samtyckesformulär för deltagande i forskningsprojektet gällande forskning på skolutvecklingsprojektet [redacted]

Jag bekräftar härmed att jag tagit del av informationen om deltagande i en forskningsstudie där dokumentation från skolutvecklingsprojektet [redacted] kommer att användas.

Information gällande dig kommer att behandlas helt anonymt, det ska inte vara möjligt att spåra skriven text till dig som individ.

☐ **JA.**

Jag samtycker till att Karlstads universitet behandlar personuppgifter om mig i enlighet med det ovanstående.

☐ **NEJ.**

Jag samtycker **inte** till att Karlstads universitet behandlar personuppgifter om mig i enlighet med det ovanstående.

Underskrift

Ort och datum

Namnförtydligande

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Harnessing teachers' perspectives

This thesis harnesses teachers' perspectives of pedagogical possibilities that enable learning opportunities for mathematically highly able pupils (MHAPs). Firstly, teachers' possibilities to recognize MHAPs through assessment are explored. The results show that pupils with high problem-solving competence can achieve highly on a non-traditional test, whereas they do not on traditional tests. Secondly, a teacher-initiated investigation of pupils' perceptions of challenging mathematical tasks is presented. The findings indicate that a developed tool can help teachers find suitable tasks for MHAPs. Finally, teachers' (N=17) discussions are analysed and their perceptions on orchestrating teaching MHAPs are probed. The findings show that the teachers have knowledge of how to recognize and support MHAPs. Specifically, the teachers express possibilities with challenging tasks and differentiated education. Furthermore, the teachers perceived they have knowledge and competence to orchestrate teaching for MHAPs. The findings show that their knowledge is consistent with previous research on teaching highly able pupils (HAPs) to meet their learning needs. This thesis gives support, on orchestrating teaching that includes HAPs in learning, to teachers, from teachers, through research.

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