

The Ecology of Mary's Mathematics Teaching

Tracing Co-determination within School Mathematics Practices

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Abstract

Teachers' mathematics teaching has been studied in many different ways. Such studies not often include more contexts than the teacher's teaching practice. An assumption in this thesis is that in order to create a deeper understanding of mathematics teachers' teaching we also need to study the contexts around mathematics teachers, and in relation to each other. Together such contexts create an environment for teachers' teaching. The determination of how mathematics is taught is not decided in any of the contexts alone. Rather, all contexts participate in the determination of how mathematics is taught and teachers need to negotiate how different contexts privilege both mathematics and mathematics education. In this study, I have studied one teacher's, Mary's, teaching practice as well as three contexts from her close environment, the teacher group she participated in, the textbooks she used, and the national curriculum she was bound to follow. To study how mathematics and mathematics teaching was privileged in the four studied contexts became a way to trace how the contexts participate in the determination, in short, their co-determination of how mathematics is taught.

With an aim to deepen the understanding of how the environment of a teacher's teaching enables and constrains mathematics teaching, the four contexts were studied in relation to each other in different ways, in four studies. First, the context of Mary's mathematics teaching was studied in relation to the teacher group in how the justifications of Mary's mathematics teaching was constituted in relation to a teacher group discussion. Second, Mary's teaching of problem-solving was studied in relation to how problem-solving was privileged in both mathematics textbook and national curriculum. Third, praxeology was explored as an analytical tool to understand how mathematics was privileged in teaching practice in relation to the privileging of mathematics in textbooks. Fourth, all four contexts were studied to trace arguments and principles for teaching rational numbers and how these enable and constrain the teaching of rational numbers.

To address these different contexts, ATD as described by Chevallard was adopted. In ATD, the environment of contexts with influence of teachers' practices, is described as an ecology with levels that co-determine each other. The studied contexts represented some of these levels of co-determination. The privileging of mathematics and mathematics teaching was studied from a varied data material. Data from Mary's teaching practice was transcripts of classroom observations and interviews. Data from the teacher group was transcripts of teacher meetings. Data from the textbook context was the textbooks and teacher guides Mary used. Data from the context of the national curriculum was the mathematics syllabus accompanied with clarifying and explanatory comments.

The analyses revealed a strong resemblance of the mathematical communication between the different contexts. They all emphasised similar approaches to problem-solving, aspects of rational numbers, mathematical values, or explanations of angles. Mary, however, anchored her arguments for mathematics teaching in partially different theoretical principles than those privileged in the ecology. Theoretical principles were not explicitly communicated in any context. They were inferred from the communication. An implication generated by these findings is the importance for teachers to engage in the principles behind the privileging expressed in contexts they need to negotiate. These principles need to be discussed and challenged. Another implication is the relevance of allowing for teachers to engage in research literature, and to have influences from other sources than their immediate contexts. The thesis also point to the need to study textbooks and national curriculum, not in terms of how they are enacted by teachers, but what they privilege. By doing so teachers practices may be understood in the sense of what teachers have to negotiate, where the consequence is a deeper understanding of constraints and affordances for teachers' teaching practices.

Keywords: *Mathematics teaching, Mathematics teachers, ATD, Co-determination, Praxeology.*

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To Josef, Ida and Rasmus

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Anna Pansell, Åre October 2018

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1 Introduction

This is a study of the influence of the environment on a Swedish mathematics teacher's teaching in grade five. This environment includes: the group of teachers she collaborated with, the mathematics textbooks she used and the national curriculum supposed to guide her mathematics teaching. In the following sections, I will introduce the study from different angles, including a presentation of the teacher, Mary and the Swedish context as well as the rationale for the study.

1.1 Understanding Teachers' Teaching

If we want to really understand teachers' teaching, and if we want mathematics education to develop, an assumption in this thesis is that we need to study not only what is taking place in mathematics classrooms, but also study practices in other contexts, such as curriculum and textbooks, and how they relate to each other. These contexts may be studied from a variety of angles, for example, how they are organised or how they sequence content matter. To really understand what influences mathematics teaching we specifically need, as I see it, to study how the contexts are interrelated to each other, focusing on how mathematics and mathematics teaching were privileged. In doing this, it becomes possible to see how mathematics teaching is constituted.

The title of this thesis is "The ecology of Mary's teaching; Tracing co-determination in school mathematics practices." Some of the words in this title may need some clarification. Ecology is usually found in Biology and it translates in Merriam Webster as "a branch of science concerned with the interrelationship of organisms and their environments"¹. This explanation is not far from how ecology is seen in this study. A teacher's teaching is surrounded by other contexts where different teaching practices are privileged. Together, these contexts form an environment for teachers' teaching. The contexts included in this study are; Mary's teaching, the teacher group, the textbook, and the national curriculum. There are relationships between these contexts that constitute each other. I have used an anthropological theory of didactics, ATD (Chevallard, 2006), which will be described in the following sections, mainly

¹https://www.merriam-webster.com/dictionary/ecology?utm_campaign=sd&utm_medium=serp&utm_source=jsonld

in the theoretical considerations. ATD describes the ecology as a collection of levels which co-determine mathematics teaching in schools. The contexts described above can be placed within this system of levels, where the levels are described to co-determine each other. I would like to emphasise the “co”, in co-determine. One level is not determining another while all levels determine what the teachers do. The levels are more to be understood as being interrelated to each other, where the determination may go both ways and where not all levels participate in the co-determination at all times. The concepts presented here will be used, and further explained, throughout the thesis.

How the contexts privilege mathematics and mathematics teaching, can contribute with an explanation about what enables and constrains mathematics teaching. If we want to develop mathematics education. I view this as essential to take into account. In the following pages I will develop what I see as the rationale for this study, but first, let me start from the very beginning, with my point of departure.

1.2 The Starting Point

The most exciting lessons I have taught, as a primary school teacher, were the ones where I explored mathematics together with my students. Not when we practiced routine exercises, but when we were engaged in reasoning about mathematical properties. This could be a discussion about the difference between area and perimeter in a whole class, or discussing how the counting sequence is constituted with a single student. To be a part of the students' explorations and to be able to show them a world of mathematics where there is joy in solving a problem has been my pleasure. This sounds like I had these conversations all the time. That was not the case. I have had so many lessons I am not proud of, and sometimes I have felt bound by constraints outside of my control. With the experience of mathematics teaching as complex, difficult, fantastic and exciting, I have searched for ways to understand more about mathematics teaching. This led me to mathematics teacher education.

In my work as a teacher educator, I have had the privilege to visit schools to help them with projects. In such a project, I had one conversation that will always stay with me. A teacher wanted my advice on something she had just taught. I suggested that she could turn the lesson upside down and begin with the problem and discuss the properties she wanted the students to know about, during the problem-solving process, instead of beginning with the properties. She looked at me for a while, sighed, and told me how that was her way of teaching science. She asked herself out loud why she had never tried this in mathematics. This conversation taught me that mathematics (as probably any school subject) comes with a tradition that could restrict teachers to a limited set of practices. My wish to understand more of what mathematics teaching is

in the everyday experience of teachers and what mechanisms shape their teaching led me to apply for a PhD scholarship.

I was free to choose what to study in my PhD project, and I took a question one of my colleagues often posed as my point of departure. I thought about this question often, in my own teaching practice. He asked, “On what do you base your decisions, when you choose to teach the way you do?” Therefore, with an interest in the grounds of teachers’ mathematics teaching as well as in how mathematics teaching can be affected by other contexts, I designed my study as a single case study. One teacher’s teaching was studied in depth, as well as how this teacher’s teaching connected to other contexts where mathematics teaching was privileged in different ways.

During a few months in 2011 and 2013, I followed a primary school mathematics teacher whom I have called Mary (pseudonym). I studied her teaching but as I was interested in how it was affected by its surroundings, I have also studied how mathematics teaching was privileged in the contexts closest to Mary: the teacher group she collaborated with, the textbooks she used, and the national curriculum. I have been driven by a curiosity to understand the ecology of Mary’s teaching, and how the contexts within the ecology privilege mathematics and mathematics teaching and how this may contribute to possibilities and constraints to Mary’s mathematics teaching.

1.3 Introducing Mary

At the time of the study, Mary was a mathematics teacher qualified to teach mathematics and science in school years 1-7. She had completed her teacher education around the turn of the millennium and had worked as a teacher, mainly in grade 4 and 5, since then. Mary worked in a public school with a good reputation in the area. Mary was an appreciated teacher at her school, often getting positive feedback from parents, colleagues and students. Mary’s school had also achieved awards thanks to their students’ results. Mary was appointed to “lead teacher” in mathematics, a special form of position a municipality could bestow on teachers they wanted to promote. As a lead teacher, she was asked to lead her colleagues in collegial learning in mathematics. The teachers at Mary’s school were organised by school year, with year 4 teachers in one team and year 5 teachers in another team, about 4-5 teachers in each. At the time of the study, Mary worked within the team teaching year 5. Occasionally, she met the teachers teaching mathematics in the same school year as her. In these meetings, the teachers discussed and coordinated their mathematics teaching.

During the time I spent in Mary’s classroom, she appeared to be a very engaged mathematics teacher. She came “to life” when she talked about mathematics and how much fun it was to teach it. Mary drew on both her education (both in-service and pre-service) and experience, when she commented that

she had come to a point in her career when she knew how she wanted to teach mathematics. The teaching experiments and mistakes she had made at the beginning of her career had helped to ground her way of teaching. When asked, during interviews, what was important to her as a mathematics teacher she always claimed conceptual understanding and problem-solving. The former was visible in Mary's lessons where concepts were discussed and explained in almost every lesson (see Article 3, of this thesis). She described it as important to present solutions in a clear manner. Another thing Mary often highlighted in her teaching was the importance to "think outside the box". She often praised solutions or statements showing something unexpected or different from the mainstream.

1.4 The Swedish Context

This study was based in Sweden in one teacher's practice. Swedish teachers most often work in public schools even if some do work in private schools. The difference between them is not their access to funding, all schools are funded in the same way by the government. The difference is in how and by whom the schools are managed. Public schools are managed by the municipalities, and private schools are managed by private companies or cooperatives. A school with public funding, public or private, cannot ask for any kind of payment from their students. The Swedish school was, at the time for the study, organised in "lågstadium" (ages 6-9), "mellanstadium" (ages 10-12), "högstadium" (ages 13-15), and "gymnasium" (ages 16-18). From age 6 to 15, school is compulsory. Normally the "lågstadium" and "mellanstadium" are grouped together in one school, which was the case in Mary's school. In the following pages, I present the conditions under which a mathematics teacher in Sweden worked at the time of the study, as well as a brief historical overview of the school subject of mathematics in Sweden. The purpose for doing this has been to offer a background for the Swedish educational system which produced both curriculum and textbooks and within which Mary and the other teachers worked.

1.4.1 Working Conditions for Swedish Teachers

In 2015, a fulltime teacher in Sweden worked 45 hours every week compared to normal fulltime employment of 40 hours. Instead, teachers had a paid holiday during all school holidays. A teacher was expected to spend 35 of these hours in school, and she could decide for herself where to spend the remaining 10 hours. There was no official number of hours a teacher had to teach, but an analysis of time management for teachers showed that about 60% of a teacher's time was spent on teaching. The same analysis also showed that 10% was spent on planning and assessing and the rest of the time was used for

administration and teacher conferences (Skolverket, 2015)². In Mary's case I would describe these numbers to be accurate, except for the 10% of preparation time. My guess is that Mary spent those hours as well as several hours of her own time unpaid on lesson preparation.

1.4.2 Mathematics Teaching in Swedish Classrooms

Studies show that, at the time of the study, more than half of the lesson-time in Swedish mathematics classrooms was usually spent on individual work or small group activities. At the same time, a third of the lesson-time was spent on working with exercises from the textbook of which the major part concerned practicing procedures (Bergqvist et al., 2010; Skolinspektionen, 2009).

There was no governmental expectation with respect to which textbook teachers should use. Teachers were free to choose textbooks, at least together with their colleagues, although schools could specify a book for the whole school. In a quality report about mathematics teaching in Sweden, the grounds for choosing a specific textbook was often found to be connected to how the mathematical content was treated. In this, procedure management was privileged (Skolinspektionen, 2009). Another ground for choosing a specific book was, in a study connected to the quality report, found to be whether it was easy for students to work on by themselves. Teachers were, in the same study, described as being guided by textbooks in their teaching, relying on them to interpret the national curriculum (Bergqvist et al., 2010).

National tests were, at the time of the study, taken by all students in grade 3, 6 and 9. Even though the tests were supposed to be solely a part of the teachers' assessment, they became high-stake tests posing stress on both students and teachers (Sjöberg, Silfver, & Bagger, 2015).

1.4.3 Curricular Development in Sweden

The historical context a teacher takes part in is possibly one of many factors that may influence teachers' teaching (Herbel-Eisenmann, Lubienski, & Id-Deen, 2006). Therefore, I present an overview of the curricular development in Sweden so that the activities described in the thesis can be placed in a historical and curricular context. The findings of this thesis can then be seen in the light of their historical context, and the possible historical influence may be discussed. I begin with the first years of public schooling and how mathematics was seen and included in the first national curricula until the current curricula, both in relation to teachers' freedom to choose, but also how mathematics is included. When I presented the mathematics that was included in the curriculum, I have focused on what was included for grade five.

² The present study was carried out a few years prior to 2015 but these conditions still applied for the years before Skolverket's (2015) report was written.

Public education of some kind has been offered in Sweden since the beginning of the 1800s. The express goal of this education was to teach children morality, and mathematics was one means of accomplishing this. In mathematics, the aim became to teach the students how to calculate, textbooks were named “Arithmetics” and the goal was to calculate quickly and correctly, not to think, only to do (Lundin, 2008). At the end of the 1800s, a national curriculum became available, formed as a course plan with brief recommendations on teaching methods, still with the emphasis on arithmetic skills (e.g. Svensk författningssamling, 1878). Such recommendations were also issued in 1889 and 1900. There were small changes between them. These two course plans had the exact same guideline about written calculations. For mental calculations in 1889 the students should be able to calculate in the number-area 1-30 in the first three grades, but in 1900 they should be able to do mental calculations in the number area 1-50 (Svensk författningssamling, 1889; Svensk författningssamling, 1900). In 1919, the curriculum changed name to teaching plan. In this curriculum geometry had been added to calculations. The description of what to teach in calculations was reformulated but the content was still the same, only without a restriction to a specific number area (Svensk författningssamling, 1919). The 1955 curriculum, was more extensive with some descriptions of how to teach the content matter. Calculations changed name to “Mathematics”, the content still consisted of calculations and geometry. The content to teach the first three grades still consisted of calculations, written and mental, now in the number area 1-10000. The guidelines described how and when to teach this (Skolöverstyrelsen, 1955)

The first modern Swedish national curriculum was published 1962,. Now, the curriculum became more detailed, describing the “content, planning and teaching methods and eventually how, in what order and to what extent each mathematical topic ought to be treated” (Prytz, 2013, p. 312). Mathematics still mostly consisted of calculations and geometry but everyday problems were also added as content in the mathematics curriculum (Skolöverstyrelsen, 1962). This detailed curriculum reflected a centralisation of the school system.

The subsequent national curriculum was published in 1969 (Skolöverstyrelsen, 1969), with a course plan and a commentary material on mathematics, which still consisted of calculations, geometry and everyday problems. With this curriculum, the New Math (Phillips, 2015) was introduced in Sweden, only to disappear a few years later. The curriculum, again, described “the mathematical content, planning and teaching methods and in what order the content ought to be treated” (Prytz, 2013, p. 312).

The curriculum of 1980 was to some extent different; the structure remained with a shorter course plan than before but with substantial commentary material. Mathematics was divided into the main content categories, which was more diverse than the previous curricula. These categories were, arithmetic, real numbers, percentage, measurements and units, geometry, algebra and functions, statistics and probability (Skolöverstyrelsen, 1980). This

can be seen as the beginning of the process of decentralisation, where teachers were expected to choose how to order the content or how to teach, unlike before when this was described in the curriculum (Prytz, 2013), a process that continued with the national curriculum published in 1994 (Skolverket, 1999). This curriculum only described what the students should know and be able to do in school year 5 and 9, alongside justifications and purpose for the subject as well as a section on assessment. Further recommendations were found in the commentary material that was to be seen as a support for teachers, but not a demand (Prytz, 2013).

The 1994 curriculum introduced a competence based curriculum, which is both described and evaluated by Bergqvist et al. (2010), linking to a more international trend, this curriculum took direct influence from NCTM's standards (NCTM, 1991), which was discussed, in terms of how to incorporate the standards in the Swedish curriculum (Ahlberg, Emanuelsson, Johansson, & Runesson, 1989). The competencies were both used as a description of the characteristics of mathematics as well as the goals for mathematics education (Bergqvist et al., 2010; NCTM, 1991). The 1994 curriculum did not only describe what content knowledge the students were supposed to learn (number sense, arithmetics, geometry, measurements, and statistics), but also what competencies they were supposed to develop (e.g. mathematical reasoning, problem-solving, conceptual understanding, and mathematical modelling) (Skolverket, 1999). Instead of a detailed description of what mathematics content to teach, teachers now had a more unspecified description of what content to cover, as well as something new, the competencies, to incorporate into their mathematics teaching. Interpreting these competency related goals was described as difficult for teachers by Bergqvist et al. (2010).

The current national curriculum, which was launched just prior to the first data collection, was published in 2011 (Skolverket, 2011a). Still in the process of decentralisation, this curriculum points out what the purpose, content and assessment of mathematics should be, and very little about teaching methods (Prytz, 2013). Teachers were obliged to carry out national tests, where the goals of the curriculum were assessed. These tests have had some constraining effect on teachers (e.g. Bagger, 2015). With an aim of clarifying the goals for teachers, the goals for all subjects were written in the same way, with the same use of words. The result was a similar wording for different school subjects, such as mathematics, home economic and history, which in itself was in conflict with the theoretical grounds for the curriculum (Jahnke, 2014). The curriculum's division of mathematics into core content (what teachers should teach) and abilities (competencies), was discussed by Dahl (2014) to result in students' struggling to know what is expected of them. This core content was basically a list of the minimum amount of content matter teachers had to cover in their mathematics teaching. In mathematics it was divided into: understanding and use of numbers, algebra, geometry, probability and statistics, relationships and change, and problem-solving. The assessment criteria, which were

based on the competencies, were called knowledge demands, describing what abilities the students needed to have by the end of grade 3 and 6 for primary school. These assessment criteria were based on five abilities, similar to the competencies from the previous curriculum: problem-solving, conceptual understanding, methods, mathematical reasoning, and communication (Skolverket, 2011a).

Primary school teachers were, according to the 2011 curriculum, free to choose how to teach mathematical content and in what order, but in this freedom there were limitations regulating them. The overall communication guiding Swedish teachers' teaching was altered, due to the introduction of the 2011 curriculum. From assessment criteria there was a change to knowledge demands, and from goals to core content. This change of wording was combined with an introduction of grades from grade 6 (to compare with grade 8 in the previous curriculum). The scope for teachers to choose how to teach what and when was limited by these and other measures. At the same time school mathematics in Sweden has developed into a variety of content matter, compared to the privileging of arithmetics from the first years of public schooling in Sweden. As a result, teachers are battling new demands, a limited freedom and control, and a curricular history with a very limited scope of mathematics. In this study, it has been important to relate the findings of how mathematics was communicated in the ecology of Mary's mathematics teaching to its curricular history. This will be addressed in Section 7. It has also been important to understand the decentralisation, demands and control of the curriculum as influencing contexts for Mary's teaching, which will be addressed in the following section.

1.5 Rationale for the Study

As described above, the Swedish school has been in a process of decentralisation since the late eighties (Popkewitz, 1996; Skott, 2004). This decentralisation can be seen in the national curricula from 1980 (Skolöverstyrelsen, 1980), 1994 (Skolverket, 1999), and 2011 (Skolverket, 2011a). Within a decentralised system, teachers can be described as autonomous. Skott (2004) even argues that decentralisation forces autonomy on teachers, which implies that teachers are expected to serve as a central link between school mathematics and the mathematics classroom. Autonomous teachers are also expected to make informed decisions on the formation of their teaching. Teachers' teaching is, though, framed by an institutional context where conflicting demands may work against this autonomy (Skott, 2004). The Swedish school system is still not entirely decentralised, and how free each teacher is to choose methods and materials may differ between municipalities. If teaching is meant to be formed by teachers, it is important to study the circumstances for both teachers and their teaching. Teachers will never be without institutional constraints. In

Sweden, teachers are bound to follow the national curriculum, which may both enable and constrain mathematics teaching.

Teacher professionalisation has been described as a parallel process to de-centralisation (Carlgren, 2004; Lundgren, 2006a). In Sweden, teachers receive a licence to teach grade-specific school subjects and school years, as a part of the professionalisation process. Professionalisation may lead to a development of professional concepts and codes (Abbott, 1988) with the consequence that descriptions of teaching as a profession become more specialised as research about teachers and their practice develops (Lundgren, 2006b). Professional teachers are described as having the right to form their teaching (Hargreaves, 2000; Skott, 2004) as well as be expected to be collegial, taking part in a professional collaboration (Hargreaves, 2000; Klette, 2002; Stengel, 2010). Moreover, while increased professionalisation may offer teachers the status of professionals, this status conveys demands for more specialised qualifications. A professional teacher is as a consequence of increased professionalisation expected to justify the practices the teachers adopt in their classrooms (Carlgren, 2004). The combination of such controlled autonomy and the professionalisation may result in teachers justifying teaching practices they did not have the freedom to choose.

Restrictions to teachers' autonomy as well as the opportunities for teachers to be professional may come in different shapes. When teachers are controlled and subjected to detailed measurements and detailed curricula, they become de-professionalised and the practice becomes reproduced from expert to novice (Hargreaves, 2000; Klette, 2002). During the years of this study, there has been an increased control of teachers in Sweden. One example was when the agency School Inspectorate, was founded in 2008, at the same time as the Agency of School Development was closed. School development was replaced with school control. These two reforms together have been described as setting the teachers free without enabling them to govern and regulate themselves (Wermke & Forsberg, 2017). Boistrup (2010), criticise how the teachers can be blamed for educational failures. She draws from a report from the School Inspectorate (Skolinspektionen, 2009) which claimed that a reasonable assumption, for teachers' problems to decipher the national curriculum, was that they had not tried to develop and interpret the different parts of the syllabus. This is a clear deficit explanation of teachers, in the sense that the teachers need to be fixed. In a study of the construction of teachers in international policy documents, Montecino and Valero (2017) conclude that mathematics teachers are portrayed as incomplete with constant deficits to overcome due to requirements from society and market. To remedy the deficit, permanent training of teachers is offered as a solution but, at the same time, it constitutes a way of maintaining control of this never-ending process for teachers. As Montecino and Valero (2017) say, the teacher is constructed as "a man in debt" (p. 150).

An example of this type of portrayal in the context of this study, is an in-service programme announced during the data collection period, to remedy failing results in international assessments, named “The lift of mathematics” [Matematiklyftet]³. The very name suggests that teachers need to be lifted up by someone else. Gutiérrez (2013) criticises the assumption that the (lack of) teacher knowledge is the direct answer to problems of students’ learning. She claims that teachers need tools, to complement their mathematical knowledge, a political knowledge, to negotiate a world of high stakes testing and standardisation so they can resist mandates, which are not in the best interest of their students. Another example, from the Swedish context, is from the public debate, where whole class teaching was often mentioned as a privileged teaching method, often by the Minister of Education. In the Swedish wording, whole class teaching is “katederundervisning”. Kateder is in Sweden, the teacher’s desk⁴. Katederundervisning refers to the teacher’s place in the classroom, the teacher teaches from his desk. This term has been criticised with the argument that the term is too unspecific and simplified (Eriksson, 2011). This privileged teaching method may be difficult for a teacher to relate to since it is unspecific. Where to teach in the classroom does not say anything about how to teach and there is no rationale for this privileged practice. The privileging of such a method in public debate may work as a restriction for teacher’s practice. It is simply not clear what to draw from this unspecific privileged practice. This suggestion from the Minister of Education and the in-service programme suggesting that teacher needs to be lifted up are examples of what contributed to deficit explanations of Swedish teachers.

Swedish teachers have recently been described as being under the pressure of a PISA shock that contributed to the government’s increased control of Swedish schools (Wermke & Forsberg, 2017). Frequent PISA discussions in the media have created what Thavenius (2014) labels media-panic. He advocated a discussion of the results and what is actually measured in PISA, a discussion rarely found in the public debate. To improve Swedish education in general and mathematics education in particular, the government has increased national assessment in schools over the past ten years with national assessments in more subjects and age groups than before. Controlling teachers’ performance with large scale assessments and inspectorates has been described as working against teacher autonomy with consequences for both teaching and teachers’ professional identity (Ball, 2003). Even though the decentralisation of the Swedish school system may suggest that teachers are autonomous, there is a governmental control restricting teachers’ autonomy.

³ <https://larportalen.skolverket.se/#/moduler/1-matematik/alla/alla>

⁴ The word kateder comes from the priest’s chair, one example is Cathedra Petri in St. Peter’s Church in Rome

An institutional perspective questions teachers' autonomy claiming it to be an illusion. Instead, the assumption that institutional constraints participate in the determination of what practice becomes possible is adopted (Winsløw, 2012). In order to understand institutional constraints, mathematics education has been studied with the anthropological theory of didactics (Chevallard, 2006). Chevallard (2002b) describes educational situations as located within an institutional ecology, where different contexts together enable or constrain the development of specific practices (Chevallard, 2002b). In this institutional perspective, teachers' practice is described in relation to institutional and pedagogical constraints, found in different contexts of the educational system (Chevallard, 2002a; Chevallard, 2002b; Winsløw, 2012)⁵. A teacher who comes to work at a new school might find decisions already made, for example, the choice of what textbook to use (Chevallard, 2002a). Resources determining what teaching become possible may be textbooks, syllabus, conversations with other teachers and online materials (Winsløw, 2012). Some of these resources are examined in this study as contexts, in relation to a teacher's mathematics teaching.

There are other ways to study teachers' teaching in relation to an ecology. In a study drawing on Bernstein (2000), Gellert, Espinosa and Barbé (2005) study how performativity imposed from the political and pedagogical context affect teacher identity. In a study drawing on Foucault (1993), Montecino and Valero (2017) discuss the governing of teachers through international policy and assessment. In a study drawing on Engeström (1998), Jaworski and Potari (2009) study both micro and macro aspects of teaching and learning. They claim that an understanding of macro factors brings a depth, which the micro factors alone cannot achieve. These studies show how other theories can be used to address the ecologies of teachers' teaching. What ADT did for this study was to offer both constructs to address the ecology as well as constructs to address how the mathematics content was communicated within them, in terms of both mathematical and didactic practices.

To study an ecology is to study the environment of teachers' teaching. An ecology consists of levels of co-determination, which have wanted and unwanted impact on the teaching practice. Such ecologies are most often studied in parts, from the perspective of two contexts originating in one of these levels. Valero (2004) argues for finding ways to knit micro contexts (classroom) together "with the multiple layers of contexts in which that micro context is inserted" (p. 17). It is here, in the intersection between classroom studies and studies of educational policy, I want to position this study. I describe an ecology for a teacher's teaching including at least three other contexts, from different levels, which could be said to co-determine this teaching. A broad description of such an ecology is a contribution to the understanding of how the micro and macro contexts of teaching work together in co-determining how

⁵ ATD, praxeologies and ecology will be further elaborated in the theory section.

mathematics teachers' practices are formed, and how the ecology as a whole enables and constrains mathematics teaching.

1.6 Unit of Analysis

The unit of analysis in this thesis is a mathematics teacher's teaching as part of an ecology of co-determining levels. The data comes from both Mary's classroom practice as well as from her collegial context, the textbooks that she used, and the national curriculum that she follows.

The decision of what data to analyse has partly been made based on my interest in the environment of Mary's teaching. The first study, where a teacher meeting was analysed was a way to begin to know Mary in relation to the ecology. From Mary's discussions with the other teachers it was possible to see both her justifications but also those of the other teachers which gave an understanding about the teacher group as a part of Mary's environment. In the second study I chose to study problem-solving because Mary often emphasised the importance of problem solving, both towards her students and in interviews when she talked about her mathematics teaching. She also dedicated about a third of her classroom time to problem-solving activities. Studying problem-solving teaching in different contexts was a way to understand a teaching practice she privileged herself, in relation to her textbooks and discuss this in relation to the national curriculum. In the third study, I chose to study Mary's teaching in relation to the textbooks she used, because she often discussed textbooks, and also since the second study indicated how textbooks worked as a constraint to her teaching. I chose to focus the fourth study on the teaching of one mathematical concept, as inferred from data from all the contexts studied: Mary, the teacher group, the textbook, and the national curriculum. I chose to focus on a concept because Mary often emphasised the importance of conceptual understanding when we discussed her teaching in the interviews. My choice to focus on the teaching of one concept, delimited the possible data into a reasonable amount to be studied. My choice to study Mary's teaching in relation to all contexts, was based on the previous studies where one or more context had been shown to both constrain and enable Mary's teaching and I wanted to investigate all contexts as a whole.

1.7 Aim and Research Questions

The overarching aim of this thesis was to deepen the understanding of how an ecology of a teacher's teaching enables and constrains mathematics teaching. The ecology in this study includes Mary's teaching, and three other contexts: the teachers she collaborated with, the textbooks she used, and the national curriculum. These contexts were studied in terms of how mathematics and mathematics teaching are communicated in them separately, and in the ecology as a whole. The study has been carried out in parts where, in the first study, the interactions between the teacher's teaching and the teacher group have been studied (Article 1). In the second study, the teacher's teaching, textbooks and national curriculum were studied (Article 2). In the third study, the teacher's teaching and the textbook were studied (Article 3). Finally, in the fourth study, all four contexts were studied (Article 4). The studies that include different parts of the ecology are, in the findings (Section 5) and discussion (Section 7) here in the preamble, all brought into the understanding of how the ecology enables and constrains the teaching.

The main questions asked in the four articles lead to the understanding of the ecology for Mary's mathematics teaching:

- How are the justifications for Mary's mathematics teaching constituted, in relation to a teacher group discussion? (Article 1)
- How is Mary's teaching of problem-solving constituted in relation to the curricular materials available to her? (Article 2)
- How can praxeology be used to study a mathematics teacher's teaching practice, and the teaching practice as inferred from the textbook as interrelated? (Article 3)
- How are some theoretical principles for teaching rational numbers expressed in the ecology of a teacher's teaching, and how may the way these principles are expressed enable and constrain teachers' teaching of rational numbers? (Article 4)

1.8 Outline of the Thesis

This is a compilation thesis, which means that it consists of this preamble, one conference paper and three journal articles, hereinafter all called articles. The conference paper and two articles are published, and the fourth article is a manuscript. Each article answers one research question.

The four articles are:

- I. Justifications for mathematics teaching: A case study of a mathematics teacher in collegial collaboration
- II. The teaching of mathematical problem-solving in Swedish classrooms: a case study of one grade five teacher's practice
- III. Mathematics Teachers' Teaching Practices in Relation to Textbooks: Exploring Praxeologies
- IV. Principles and Arguments for the Teaching of Rational Numbers in Different Contexts.

After this introductory section, in Section 2 I describe the research field in relation to teaching in different contexts, such as teachers' teaching, curriculum and textbooks. In Section 3, I present the theoretical frameworks used in the studies together with the theoretical considerations on which this research has been based. In Section 4, I present the methodology of the thesis and I provide an account of my operationalisations of the theories and address methods for data collection and analysis. I also discuss ethical issues and the quality of the study. In Section 5, I present the findings by summarising the four articles together and reflecting on the fourth research question. In Section 6, I draw conclusions based on the findings. In Section 7, I discuss the findings and the conclusions in relation to the aim of the study, the literature review, and the rationale of the study presented in this introduction section. Section 8 is a summary of the whole thesis, in Swedish.

2 Mathematics Teachers' Teaching: A Literature Review

During the nine years I have been working with different articles about Mary's teaching as part of an ecology I have conducted various searches for research addressing different aspects of mathematics teachers and their teaching. The literature review can, consequently, be described as cumulative. New research papers were added to my database over the years. Sometimes I have made new searches, which have been more or less systematic. All research I have found relevant to the thesis has been added along the way. When I have searched systematically I have always included [teach*] in addition to other words relevant to the situation. One example was the search for research about textbooks in relation to mathematics teachers. I searched for [teach* AND textbook* AND mathematic*] in both EBSCO and Google Scholar, I also widened the search to include research about textbooks in any subject [teach* AND textbook*]. Similar searches were made for different fields. To search for research papers that may have escaped these searches I have also searched through certain research journals for papers studying teachers' teaching in general, in mathematics and in relation to one or more of the contexts studied in this thesis —teachers' teaching, teacher group, textbook and national curriculum. I have searched through Educational Studies in Mathematics, Research in Mathematics Education, Journal for Research in Mathematics Education, ZDM, Nordic Studies in Mathematics Education, Mathematics Education Research Journal, and Journal of Curriculum Studies. The past nine years have resulted in a database with over 1100 peer reviewed research papers.

The following literature serves as a background to this study with research of teachers' teaching as well as research connected to the different contexts of this study. In this review the contexts of teacher group, textbook and curriculum will also be defined. The review outlines the research that I have been reading during this study, for inspiration and to be challenged by.

2.1 Mathematics Teachers' Teaching

For teachers to be seen as part of an ecology requires research methods and perspectives which will acknowledge that they are situated within an institutional system. All studies of mathematics teachers cannot have the same interest and mathematics teachers have also been successfully studied with more individual perspectives. An example of this is research describing teachers' decision-making as an individual process to be a strong determinant for mathematics teaching, where teachers' experience and beliefs contribute to the decision-making process (Bishop & Whitfield, 1972; Bishop, 1976; Borko, Roberts, & Shavelson, 2008; Paterson, Thomas, & Taylor, 2011; Schoenfeld, 2011a; Schoenfeld, 2011b; Shavelson & Stern, 1981). Other studies describe how teachers' individual knowledge about subject matter, pedagogy or student knowledge, their Pedagogical Content Knowledge (PCK), is a main reason for determining what can be taught in the classroom (Shulman, 1987; Taylan & da Ponte, 2016; Van den Hurk, Houtveen, & Van de Grift, 2017). Others study teachers' knowledge about subject matter as one way to understand teachers better (e.g. Rowland, Huckstep & Thwaites, 2005). Skott, van Zoest and Gellert (2013) discuss theoretical frameworks in mathematics education research and conclude that studies with constructivist frameworks view teaching as a product of the teacher's pre-existing knowledge and beliefs. Such individual factors are sometimes related to social factors, such as classroom management (Schoenfeld, 2011b), social interaction with others (Steffe & Thompson, 2000), how mathematical concepts are communicated or visualised (Pettersson, Stadler & Tambour, 2013), or factors outside the scope of the teacher that may restrict the teacher's opportunity to use her pedagogical content knowledge (Schoenfeld, 2011b; Steffe & Thompson, 2000; Van den Hurk et al., 2017). These studies describe important issues for teachers and teaching. However, this study, where the teacher is viewed as embedded in different contexts of an ecology, required a social perspective. A social perspective invites us to shift the focus from the teacher alone, and to include the environment and the regulating effects within it (Lerman, 2000a). From a social perspective, teaching is then seen as a product of teachers' participation in different social practices (Skott et al., 2013).

When widening the perspective to include a political dimension of mathematics education, there is a body of research describing a socio-political theme in mathematics education research. Such studies aim to uncover how different practices privilege and exclude individuals. Education here is not only understood in different social arenas, but also transformed into more socially just practices rethinking truths about mathematics: who is good in mathematics, and what makes a quality teacher (Gutiérrez, 2013). Valero (2004) argues that studies labelled as socio-cultural sometimes lose the multi-contextual and political nature of cognition when they settle for a declaration of students and

teachers being social beings, without including the broader context of the study. Instead, she asks for studies discussing mathematics education as a political and social act, which needs to be understood in relation to the many contexts it is a part of. Such studies could be answering questions about what it is “that makes particular kinds of school mathematics education practices develop in ways that are valued as the ‘right’ way of teaching” (p. 16) mathematics. This quote implies that rather than understanding each context separately, understanding them together as intertwined is a more powerful reading.

This study is an attempt to understand educational contexts as embedded in each other, which is why Mary’s teaching was studied as one context of the ecology. Her teaching was represented in both her lessons and in how she discussed her teaching activities in interviews. Mary’s teaching, enacted in lessons and expressed in interviews, was studied in relation to the contexts of the same ecology: The teacher group represented by four teachers’ discussions in teacher meetings; the textbooks represented by the textbook and the teacher guide; and the national curriculum represented by the syllabus and other texts where the syllabus is explained. An ecology of a teacher’s teaching includes more contexts than these. To incorporate some of these contexts I continue this literature review with research about teaching as part of a culture and an educational system.

2.2 Teaching as Part of a Culture

Teachers’ practices are part of many contexts. One of them is the culture they practice within. Teachers’ practices are then viewed to reflect values and objectives of their school system (see e.g. Andrews, 2010), and they have been described as aligning with the culture they operate within (Andrews, 2007). Mathematics and mathematics teaching develop differently, within different cultures (Andrews, 2010; Bishop, 1991; d’Ambrosio, 1985). A country’s history influences the curricula that may be formed. A strong catholic tradition is described as creating a different environment for a curriculum than a protestant tradition (Andrews, 2010). Different cultures value mathematics differently, both what mathematics is and who can participate in mathematics (Bishop, 1991).

From a Swedish perspective, mathematics education has deep roots in public schooling (see Section 1.4.3 in this thesis). The education of the masses, not only the rich children, has been described by e.g. Lundin (2008) as conveying a goal of disciplining the children into obedient citizens. For teachers to manage many students in a classroom, mathematics became de-mathematised and mechanised, where doing arithmetic (quick and correct) became the focus (Lundin, 2008). Later, this attempt on public schooling has developed into a school system grounded in the labour movement and in a social security system where all children should be offered equal opportunities for schooling.

Do it quick and do it right, has recently been described by Boistrup (2017) as one assessment discourse in Swedish classrooms. Do it quick and do it right was not the only assessment discourse described, but it can be seen as a trace of the quick and correct mathematics teaching from the first years of public schooling in Sweden. Since then, Sweden has had about fourteen different national curricula in which calculations have had a prominent role in all of them, as described in Section 1.4.3. It was only after 1980 that problem-solving and other mathematical competencies became privileged in the curriculum (Prytz, 2013). The Swedish culture of public schooling, which from the beginning was a means to discipline the future working class, has generated an emphasis on calculations (Lundin, 2008) which is still emphasised in the national curriculum (Skolverket, 2011a).

Mary's teaching is situated in the Swedish culture of public schooling with a curriculum that has historically privileged arithmetics as mathematics. The Swedish culture will not be specifically included in the analyses, but it will be discussed in relation to the findings, in Section 7.

2.3 Teaching as Part of an Educational System

Teaching has been addressed as part of an educational system where curriculum, state and school are different contexts or institutions. How this system is constituted has been described to have effects on the freedom of teachers to form their teaching. One example of this is Barbé et al. (2005) who describe restrictions from the society, the mathematical community, the educational system, school, and classroom. These restrictions were found to constrain teachers from fully exploring mathematics, instead solely moving on to the next task, and instead of evaluating the value of what had just been explored. Skott (2004) describes how teachers are expected to meet the expected classroom practices and learning outcomes formulated outside the classroom. In doing so, teachers participate in many contexts of the educational system. These contexts may express conflicting grounds for mathematics teaching, which teachers need to negotiate.

A number of studies address how performativity is imposed from the society by means of increased control of teachers signalling privileged teaching practices. Ball (2003) stresses performativity in the sense of regulation, forcing teachers to organise themselves as a response to evaluations, and that it not only changes what practice is privileged, it also changes what it means to be a teacher. Gellert et al. (2013) also describe how educational policy within a culture of performativity may disturb the identity formation of teachers, and also create a fragile base for professional development. Montecino and Valero (2017) describe how teachers today should be both products and sales agents of policy. As products of policy, teachers are described as being governed to

be effective quality teachers and controlled by continuous training and standardised tests. Boistrup (2017), describes how time for teacher-driven professional development may help teachers to resist control and competing demands and develop their practice. In-service teacher training may be a way to control teachers, but on the other hand, if driven by teachers, it may provide a way to resist control. In a study by Povey, Adams and Everly (2017), a teacher is struggling to resist performativity agenda, engaging in past curricula as a means to see current educational policy as a product of its history, but also as possible to change. The authors argue that teachers need to navigate the system and resist mandates which are not in their students' best interest.

An educational system includes many different contexts. These contexts have been studied as an ecology of classroom practices and how different contexts of this ecology co-determine possible teaching practices (Artigue & Winsløw, 2010; Chevallard, 2002b). Through such analyses, it may be possible to identify conditions and constraints for the construction of content in school settings (Achiam & Marandino, 2014). Another way to address such an ecology has been as a construction of a pedagogic discourse, as described by Bernstein (2000). Studies where the authors draw on Bernstein sometimes describe tensions between and within discourses and practices, such as how contradictory demands from different discourses may narrow the view of teachers (Tsatsaroni, Ravanis, & Falaga, 2003), or how the construction of subject knowledge or competencies in the classroom is framed (Liu & Hong, 2009). Such tensions and contradictory demands require analysis of the discourses “within which teachers make their decisions and deploy their actions” (Tsatsaroni et al., 2003, p. 400).

Before academic mathematical knowledge is written in a curriculum of any kind, it has to be transformed to something teachable, into school mathematics (Bernstein, 2000; Chevallard & Bosch, 2014; Dowling, 2014). One way to address this transformation has been didactic transposition (Chevallard, 1989; Chevallard & Bosch, 2014). Didactic transposition has been studied in terms of how knowledge transforms through institutions from academic discipline to the classroom under the influence of institutional and societal conditions and constraints (Achiam & Marandino, 2014). Didactic transposition has also been studied in terms of how a mathematical concept has been reinterpreted in one context of the educational system, such as how the textbooks' presentation of irrational numbers focussed on what to do with them, leaving the mathematical need for irrational numbers unclear (González-Martín, Giraldo, & Souto, 2013). Institutional contexts may also be studied in relation to each other, for example, studying what is taken into account in the formation of a museum exhibit where both society and academics co-determined what reinterpretations would become possible in a museum exhibit (Achiam & Marandino, 2014). Another aspect emphasised in the literature is that every choice of expression (such as images or gestures) in relation to an exercise

recreates the content so it will be relevant to the situation (Kress & Sidiropoulou, 2010). In this study I have used didactic transposition to discuss the change mathematics undergoes through the contexts.

In terms of didactic transposition, transformations of mathematics content, have been studied most often where two contexts are related to each other. One example is textbooks and teacher practice as two contexts influencing each other, where the studied teacher is described as following the mathematical presentation of the textbooks rather than the definition from the discipline of mathematics (González-Martín, 2015). In an American study, textbooks and teaching were compared, showing significant differences between the teachers' teaching and how the same topic was presented in the textbooks (Freeman & Porter, 1989). Theoretical contexts underpinning educational policy have also been studied in how they in different ways are linked to mathematics teaching practices (e.g. Jaworski & Gellert 2003).

On one hand Swedish teachers are expected to be autonomous in how to achieve the goals of the curriculum. On the other hand, they are under influence from several surrounding contexts. This is why it is important to include how mathematics was privileged in the curriculum, and other contexts, in order to understand how the privileging of mathematics and mathematics teaching influence teachers' practice.

2.4 Teaching in Relation to Different Contexts

In this thesis, teaching is understood as a micro-context within a macro-context, similar to how Valero (2004) describes "multiple layers of contexts in which the micro context is inserted" (Valero, 2004, p. 17). Here, macro-context is the ecology and the micro-context is called the context of the teacher's teaching in line with Chevallard's (2002b) use of these concepts. In the following pages, I describe research concerning the contexts that were analysed in the four articles (teacher group, textbook and national curriculum).

2.4.1 Teaching in Relation to Colleagues

Teacher collaboration has been described as involving negotiations of roles and responsibilities. Teacher collaboration also becomes necessary for teachers as the teaching profession becomes more complex (Hargreaves, 2000). Collaboration between teachers has been described as having a positive impact on both teaching and student achievement (Honingh & Hooge, 2014; Moller, Stearns, Mickelson, Bottia, & Banerjee, 2014; Vescio, Ross, & Adams, 2008). Research of teacher collaboration often studies teacher collaboration in professional development projects where the collaboration is part of the project (Chong & Kong, 2012; Gresalfi & Cobb, 2011; McNicholl, 2013; Riveros, 2012). There are some studies of teacher collaboration that naturally

occur in schools, that would take place whether there was a researcher there or not (e.g. Coburn, Mata, & Choi, 2013; Doppenberg, den Brok, & Bakx, 2013; Eddy Spicer, 2011; MacPherson, 2010). Consequently, we know more about how teacher collaboration works within the context of a project, and less about what happens in teacher collaboration in everyday school life, as is investigated in the project of this thesis.

Teacher collaboration has been described as one way for teachers to reduce achievement gaps, when they engage in mathematics teaching together, both in collaborative planning, but also in defining goals and professional development needs (Moller, Mickelson, Stearns, Banerjee, & Bottia, 2013). Teachers' opportunities to participate in teacher communities have been described as having an impact on their opportunities to form an identity as mathematics teachers. On the other hand, the lack of such communities, as in an example about a substitute teacher, may enable a teacher to hold on to a privileged teaching practice and resist unwanted practices (Palmér, 2010). A teacher may be engaged in real-life communities but also in others.

A teacher's opportunity to engage in mathematics depends on what is privileged by the colleagues, if other issues are privileged above mathematics in collaboration it will restrict the opportunity for the participating teachers also outside the collaboration (Skott, 2013). Such restrictions may also come from policy and how teachers' social networks are affected by their organisational context, attempts to influence teachers through their collaborative networks (Coburn et al., 2013).

In Sweden, research on teacher collaboration is often presented as a product of professional development projects, such as learning studies (e.g. Holmqvist, Gustavsson, & Wernberg, 2007; Kullberg, Runesson, & Mårtensson, 2014). Other examples of teacher-researcher collaboration studies are action research projects (Boistrup, 2017; Gade, 2012). There are few studies where the natural collaboration of mathematics teachers is studied. One example is Palmér's (2012) description of a novice teacher in her first year of teaching. She expressed a privileged teaching practice which was inconsistent with how she taught. Studying how the teacher colleagues and this teacher collaborated, there was more consistency with how the novice teacher taught (Palmér, 2012). This suggests that the privileged practices in the teacher groups a teacher collaborates with influences what this teacher teaches. This is problematic if teacher collaboration is also a necessity for teachers to navigate a complex profession, as Hargreaves (2000) describes it. There is a need for studies of teacher collaboration, what the privileged practices regarding specific mathematics content matter are and how this influence the teachers' classroom practices.

2.4.2 Teaching in Relation to Textbooks

Mathematics teachers' teaching has both internationally (Barr, 1988; Sosniak & Stodolsky, 1993) and in Sweden (Bergqvist et al., 2010; Boesen et al., 2014; Englund, 1999; Skolverket, 2003; SOU 2004:97, 2004)⁶ been shown to be influenced by the mathematics textbook. Teachers have been described as basing their teaching on the textbook's disposition and content (Barr, 1988). Others describe how teachers relate their teaching to the textbook but as one of many resources (Sullivan, Clarke, Clarke, Farrell, & Gerrard, 2013).

Research has described mathematics textbooks as being a central resource for, and even as steering, mathematics teaching, mostly in terms of what to teach and in what order (Barr, 1988; Haggarty & Pepin, 2002; Lepik, Grevholm, & Viholainen, 2015). Textbooks have been described as being a translation of policy into practice (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002) or as being the educational institutions' information about what mathematics to teach and how to do so (Barbé et al., 2005). The influence on mathematics teaching from textbooks has both been described as indirect, when the textbook facilitates teaching for teachers (Pepin & Haggerty (sic), 2003), or a direct influence when teachers use textbooks exclusively without additional materials (Sosniak & Stodolsky, 1993). However, the outcome of this influence does not seem to be teachers' mirroring their textbooks, meaning that two teachers using the same textbook do not share the exact same teaching practice (Son & Kim, 2015). Teachers are also described as choosing additional content or teaching with a different structure than the textbook (Sosniak & Stodolsky, 1993) or taking more factors, for example assessment data, into account preparing mathematics teaching (Sullivan et al., 2013).

Mathematics textbooks have been described as one context, where both mathematical and didactic knowledge is organised. An example of this is Gonzalez-Martin, Giraldo and Souto (2013) who describe how exercises and procedures were privileged above explanations in Brazilian textbooks. Jablonka, Ashjari and Bergsten (2016) describe the explanations and instructions in the textbook as a didactic layer in textbooks which can be described as mathematics texts. They show how different didactic layers succeed differently in communicating the specificity of mathematics.

In a Swedish context, textbooks have been described as dominant in mathematics (Johansson, 2006) both as a main source for teachers (Boistrup, 2015; Jablonka & Johansson, 2010; Skolinspektionen, 2009; Skolverket, 2003) and what students spend their time on, working with exercises from the textbook, during mathematics lessons, for about 70% of the time on average (Bergqvist et al., 2010). Textbooks in Sweden are not regulated or controlled. The government stipulates national criteria for learning outcomes (Skolverket, 2011a), leaving the teachers with the choice of what textbook to use and how. Going

⁶ SOU: Statens [the government's] Offentliga [public] Utredningar [investigations].

deeper into the Swedish context, Lundin (2008) describes how mathematics textbooks were originally arithmetic workbooks where there were many exercises easy enough for students to manage by themselves due to the practical problems of large classroom teaching. The role of textbooks was described by Lundin, to keep the students busy without much need of teacher assistance.

Research on how textbooks affect mathematics teaching or mathematics teachers has been done in many ways, as described above. How teachers use textbooks and teacher guides in their day-to-day practice has been studied, but most often with interviews (e.g. Ahl, Gunnarsdóttir, Koljonen, & Pálsdóttir, 2015). Freeman and Porter (1989) is one example of a study of teachers and textbooks where both interviews and classroom observations are used showing that teachers do not always privilege what the textbooks emphasised.

Mathematics textbooks privilege mathematics in different ways. The textbooks are one context, meaning that they may influence what and how a teacher teaches as well as that they are influenced in turn by other contexts in the ecology.

2.4.3 Teaching in Relation to Policy Documents

How teachers engage in policy documents have been described as depending on various factors. One example is how important contents are described as being lost in teaching without clear curricular guidelines (van Loon, Driessen, Teunissen, & Scheele, 2014). Boesen et al. (2014) also conclude that complex reforms need to be implemented very clearly in the curricular guidelines if they are to have the intended impact on classroom practice. However, this does require that teachers follow the policy guidelines, and that the curriculum always privileges the best possible practice. As described earlier, policy might impose performativity on teachers (e.g. Ball, 2003) which, Gutierrez (2013) claims, leaves teachers little room to reflect on how the students do in relation to this policy. She describes how teachers need to negotiate the system for the best interest of their students. A clearly written curriculum may lead teachers to follow the guidelines, but it may also make it easier for teachers to see what the guidelines impose and uncover whether the guidelines will benefit their students or not. Herbel-Eisenmann (2007) argue instead for curriculum developers to consider what ideological goals should underlie curriculum materials.

If an explicit goal of curriculum authors is to make mathematics available to all students, yet some of their language choices support the mystique of mathematics, consideration needs to be given to the forms of mathematical discourse embodied in the materials (Herbel-Eisenmann, 2007).

To have a clear communication in a curriculum is not enough, following Herbel-Eisenmann, it is also important how the text reflects the underlying goals that make the text available to teachers (and others).

Teachers have been described as navigating the governmental influence through the national curriculum, which sometimes inhibits their internal deliberations when they take the curriculum and other aspects of the teaching into account when deciding what actions to take (Ryan & Bourke, 2013). If teachers do not examine these deliberations, their professional actions will serve to maintain the structural and cultural forms, and they will not challenge these. It is important to preview how teachers process their circumstances, such as curricular demands. In doing this, it would be possible to see what teachers consider to be effective or engaging practices in their context and why, regardless of what the national curriculum proposes.

Andrews (2016b) describes how a country's curriculum is deeply rooted in culture. Current social factors have been shown to have impact on how mathematics is construed in the curriculum, such as international large-scale assessment, for example. Such tests have been described as having influence on educational policy. Frejd and Bergsten (2016) describe how policy makers refer to PISA results to prompt curricular change without any concern for how mathematical knowledge is construed in PISA and not by mathematics as a discipline. Conflicting grounds in policy making have also been described by Jahnke (2014) who described conflicting views during the process of forming the current national curriculum in Sweden. The current policy in Sweden, expressed in the national curriculum, is consequently only the best negotiation of the conflicting grounds and views, expressed in the clearest way possible at a specific moment in time. One cannot assert that this curriculum has all the answers for how mathematics teaching should be carried out; it is only one of many contexts. As such, policy and national curriculum need to be studied in relation to other contexts.

2.5 The Contribution of this Thesis

Above I have described parts of the research field of mathematics education, where mathematics teachers' teaching is addressed. These studies often address one or two contexts, such as classroom practice and mathematics textbooks. When it comes to other contexts, such as the curriculum or policy, they are often mentioned as possible influences, but not included in the study. I have adopted a strategy where it was possible to go into depth with the connections between the contexts. The four articles together explore a variety of such connections within the ecology, developing an account of how an ecology functions for how mathematics and mathematics teaching are privileged and how this may influence teachers' practices.

3 Theoretical Considerations

In this study, I have made many theoretical decisions. Big, overarching decisions, which I have described as overall theoretical considerations and smaller choices of what theories and analytical tools to use. In the following pages, I will give an account of these choices, but also explain both theories and analytical frameworks. My explanations of what theoretical tools I have used and why will also reveal my theoretical positioning. This will make the grounds for my interpretations presented in the findings (see Section 5), as well the grounds for my final discussion (see Section 7), more transparent. Two overall decisions were made. First, the decision of where to position the study theoretically, then what theoretical and analytical frameworks to use. These choices have been made during the course of the study, as is described in the section below.

3.1 Overall Theoretical Considerations

Two themes, relevant to this study have been described in mathematics education research, a social (Lerman, 2000b), and a socio-political (Gutiérrez, 2013). The social theme marks a shift from seeking reasons and responsibility for learning inside individuals into seeing learning mathematics as increasing participation in mathematical practices, where specific forms of knowledge as well as specific forms of participation are produced (Lerman, 2000b). It then becomes of interest to study the social settings and spaces, where learning activities happen, and how they are structured. In the socio-political theme, knowledge, power and identity are seen to be interwoven and constituted within social discourses. It is not enough just to “*understand* mathematics education in all of its social forms but to *transform* mathematics education in ways that privilege more socially just practices” (Gutiérrez, 2013, p. 40). Taking on a social perspective, the unit of analysis is social practices (Lerman, 2000b). In a socio-political perspective, the analysis goes beyond only recognising the context of the study. There is an assumption in social and socio-political perspectives that the macro-sociological context influences the micro-contexts where mathematics teaching and learning take place. In a social perspective, the unit of analysis is concluded to be: the person acting in social practice (Lerman, 2010b). The unit of analysis in a socio-political perspective

is, instead, the different contexts both micro and macro, which makes it possible to trace the connections between the contexts (Valero, 2010). In the introduction (Section 1.5), I concluded that purely individual perspectives were not sufficient to answer my research questions. To only study what a teacher knows, does not tell the whole story about the teacher's mathematics teaching practice. I included social structures, otherwise substantial parts of the story about Mary's teaching would be lost. Mary, herself often discussed how she felt restricted by other contexts both within and outside the school. Instead of only placing the study within a social perspective, and being content to see Mary within her social setting, I have analysed Mary's teaching, as well as three other contexts of the ecology, and I wanted to reveal questions on social and political issues in the Swedish educational practices of mathematics, such as how the contexts enable and constrain mathematics teaching, which is described to be socio-political (Valero, 2004). I would argue that this study is situated at the border between a social and a socio-political perspective. The reason for this is that I study Mary within her social practices, but I also include other contexts with the aim of tracing privileged practices between them. In this, I also reveal how other contexts participate in determining what teachers do in order to achieve a deeper understanding of teachers' teaching practices as related to other contexts.

This theoretical position requires a theory or an analytical framework where practices in different interwoven contexts are studied. There are several theories that could have been used. Foucault (2008) describes a *dispositive* where social dispositions, present in curriculum and textbooks, can be analysed to reveal circumstances under which a teacher constructs her teaching. With a dispositive, it becomes possible to relate activities to specific situations, but also to see the influences on these activities (Raffnsøe, Gudmand-Høyer, & Thaning, 2014). The dispositive functions as a map, a network where certain processes are operative. Boistrup (2017) operationalised this, in a dispositive where regulatory decisions and teacher discourses constitute a map explaining structures affecting assessment in mathematics classrooms.

In Bernstein's (2000) theory of pedagogic discourse, the pedagogic device is described as a mechanism through which pedagogic discourse is established and maintained. The device is run by three sets of rules, distributive, recontextualising and evaluative. Lerman (2000b) describes how Bernstein's theory is often used to reveal inequalities in education and how they are socially reproduced. Bernstein (2000) has some theoretical constructs that could enable the analysis of a teacher's teaching in relation to an ecology. One example is recontextualisation, which describes how specialised knowledge, such as formal mathematics or research in mathematics education, is transformed for the purpose of teaching students (Parker, 2009). This could have helped me to see structures in Mary's teaching and to go beyond Mary and see the social and institutional contexts through her teaching. They did, however, not offer me

specific tools for the analysis of the teaching of specific mathematical contents as well as how this teaching could be seen as part of a system of contexts.

The anthropological theory of didactics (ATD) (Chevallard, 2006; Chevallard & Bosch, 2014) with didactic transposition (Chevallard & Bosch, 2014) that describes, similar to Bernstein's (2000) recontextualisation, how mathematics is adapted through different institutions to be teachable in a mathematics classroom. These institutions, in this study called contexts, can also be described to co-determine each other (Chevallard, 2002b), which, in this study, offered me a language to describe the relationship between Mary's teaching and the other contexts. In ATD praxeologies, describing knowledge as a know-how (praxis) and know-why (logos) can be used to describe both mathematical and didactic knowledge in different contexts (Barbé et al., 2005). For the purpose of this study, this was a tool, specific enough to study how mathematical practices were privileged in the mathematics classroom, as well as in other contexts. ATD offered me a set of theoretical tools to analyse and describe Mary's teaching as part of an ecology, also called ecology in ATD (Chevallard, 2002b).

To study how mathematics and mathematics teaching are communicated in different contexts, I used a theory providing tools to specifically study communication. Studying a teacher and her communication, it was essential to be able to view more than spoken communication as communication. In multi-modal social semiotics, semiotic resources are seen as situated within social practices (Kress, 2009; Selander & Kress, 2010) and different semiotic resources are simultaneously used to communicate (Jewitt, 2009). Within a multimodal perspective, it is also possible to view different functions of communication with different meta-functions described by Halliday (2004). In the following pages, both ATD and social semiotics will be explained in more detail.

3.2 Anthropological Theory of Didactics

To understand mathematics teaching as part of an ecology, I aligned with the anthropological theory of didactics (ATD) (Chevallard, 2006; Chevallard & Bosch, 2014). ATD has been used to study didactic knowledge located in different social practices, where someone intends to teach someone something (Chevallard & Sensevy, 2014). Within ATD, there are theoretical constructs for the analysis of knowledge and practice which have been used in this study in order to understand the contexts studied: Mary's mathematics teaching, the teacher group, the textbooks and the national curriculum. In the following pages I will describe the theoretical framework and how it relates to this study.

3.2.1 Didactic Transposition

Chevallard (2007) describes the point of didactic transposition theory to be the consideration of knowledge to be changed and adapted to new institutions. As a definition, Chevallard and Bosch (2014) describe didactic transposition as “the transformations an object or a body of knowledge undergoes from the moment it is produced, put into use, selected, and designed to be taught until it is actually taught” (p. 170). Lundin (2008) describes how school mathematics was de-mathematised due to the need of public schooling for the commoners, which led to an emphasis on arithmetics, which is far from mathematics as a discipline. The transposition of both mathematical and didactic knowledge, where the knowledge is appropriated from the scholar discipline to become teachable and learnable in the classroom, is described as a fundamental didactic phenomenon (Achiam & Marandino, 2014; Chevallard & Bosch, 2014). The difference is that mathematical knowledge originates from the discipline of mathematics and didactic knowledge originates in a number of disciplines where ideas, concepts and principles explain different aspects of mathematics teaching. Transposition of didactic knowledge therefore becomes very complex to study.

Transposition conveys an interrelationship between the contexts of an ecology. If knowledge is transformed it is between these contexts this transformation takes place. In this study this interrelationship is a basic assumption, on which I discuss of how the contexts studied participate in the co-determination of what becomes taught in Mary’s classroom. From the scholarly institutions, such as universities, the knowledge about mathematical concepts is changed as it is re-described and redefined in other institutions, in this study the Agency of Education or a textbook publisher. In these new descriptions of how to teach mathematical concepts, new elements are also integrated from a variety of social practices and in the end these descriptions of how to teach mathematical concepts and why are adapted to be teachable in a specific classroom (Chevallard & Bosch, 2014).

The actors of the transpositive work between the society, the scholar institutions, and the classroom are referred to as the noosphere. The noosphere consists of those who think about teaching, in this study the contexts of the national curriculum and the textbook. The noosphere serves as a midway between the teaching institutions and society (Chevallard, 2007), and negotiates the demands from the society and at the same time upholds the illusion of school knowledge being authentic to scholar knowledge. The knowledge taught in schools has to appear as genuine, and the process of transposition itself becomes invisible. This is why it is important to understand on what grounds the knowledge to be taught is constructed and to be aware of didactic transposition and the illusion that follows (Chevallard & Bosch, 2014).

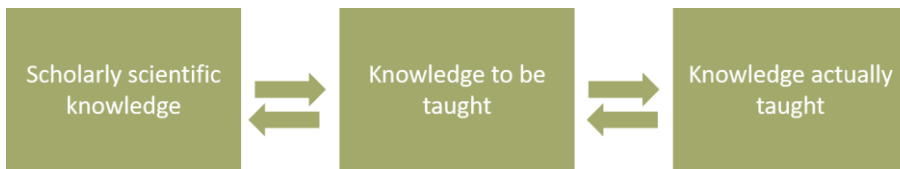


Figure 1, Didactic transposition (Chevallard & Bosch, 2014)

Didactic transposition (see Figure 1) is described as a tool to see beyond this illusion and to not only see what organisation of knowledge is present, but also what could have been. To accomplish this, researchers need to be freed from the contexts studied and be aware of their own perspective. This can be expressed in a reference epistemological model REM, from which the methodological proposal for analysis is described. In this model, possible tasks, techniques, technologies and theories are described (Barbé et al., 2005).

A REM about rational numbers should encompass analysis and answers from the different perspectives to questions such as: What are rational numbers? How to describe them in terms of praxeologies /.../? What is their connection to other praxeologies, for instance, other number sets? (Østergaard, 2013, p. 9)

This model is used to make the analysis explicit, to distance the researcher from the data material and to enhance a detailed and objective analysis (Wijayanti & Winslow, 2017). In this study, different frameworks have been used instead of a REM, for example three approaches to problem-solving (Shroeder & Lester, 1989) in Article 2, and sub-constructs of rational numbers (e.g. Charalambous & Pitta-Pantazi, 2007) and mathematical values (Bishop, 1991) in Article 4.

In the transposition process, teachers are said to be involved in internal transposition, drawing from the noosphere, which describes the knowledge to be taught. The noosphere is engaged in external transposition drawing from the scholarly scientific knowledge (e.g. Barbé et al., 2005; Østergaard, 2015). The illustration of the didactic transposition implies a linear process. This is a simplification. The noosphere is not just one organisation, which is evident in this study. A number of practices outside of school participate in the transposition process, with different objectives and views about the knowledge actually taught, driven by other agendas, for example ideological (Chevallard & Bosch, 2014). The contexts included in this study are only some examples of many possible contexts from the noosphere, they are nevertheless some of the contexts close to Mary. She has read the texts and she participates in the discussions in the teacher meetings.

3.2.2 Praxeology

Knowledge is, in ATD, described in terms of praxeology (Winsløw, 2012), where knowledge is organised in terms of know-how (praxis) and know-why (logos). In an educational situation there are two praxeologies at play, a school mathematical, describing what mathematics to do and why, and a didactic, describing what mathematics teaching to do and why (Barbé et al., 2005). In this study I have studied both mathematical and didactic knowledge. Focusing on how these two types of knowledge were expressed in the different contexts enabled me to trace privileged practices and how they related to each other. In the following pages I elaborate on what forms these two kinds of praxeology can take.

Mathematical Praxeology

A praxeology is how ATD describes knowledge, a practice (know how) and a discourse on practice (know why) together (Chevallard & Sensevy, 2014). A specific mathematical praxeology creates a structure of mathematical knowledge for a specific use (Jablonka & Bergsten, 2010) of a mathematical content. In the case of this study, this could concern angles or volume in grade five (see Article 3). This structure is built on two main components, praxis (know how) and logos (know why). Praxis is described as including tasks that are solved with certain techniques.

In a mathematical praxeology, the task is a type of mathematical task, for example, how to solve an equation, or how to calculate the sum of two fractions. For each task, there is one or more techniques for how to solve this task. To explain why the techniques apply, there is a logos, a technology, and to put the technology into a wider context of meanings there is a theory. The theory explains, justifies or produces technology, establishing a deeper level of justification of the practice (Barbé et al., 2005; Bosch & Gascón, 2006; Chevallard, 1998). Theory consists of the underlying principles of a given skill, or set of ideas setting out principles or laws of known/observed phenomena (Chevallard et al., 2015). In mathematics, theory consists of definitions and axioms (Østergaard, 2013). Praxis and logos are described as interrelated, affecting each other; praxis entails logos and logos backs up praxis explaining and justifying the actions (Chevallard, 2006).

Mathematical praxeologies were focused on in Article 3 where the mathematical knowledge expressed in Mary's teaching and in a mathematics textbook was analysed. In Table 1, I show a mathematical praxeology as an example of how the task, technique, technology and theory might be seen in a primary school, when the mathematical task is to measure a distance.

Table 1. Example of a mathematical praxeology

| <i>Praxis</i> | |
|---|--|
| <i>Task</i> | <i>Technique</i> |
| To measure a distance | Measure a distance with a ruler. |
| <i>Logos</i> | |
| <i>Technology</i> | <i>Theory</i> |
| We need to measure with standardised methods, and units, in order to compare our results. | The definition of a distance and standardised agreements of units (the SI system). |

This is only one example of many possibilities. A mathematical praxeology is socially situated. Consequently, a praxeology concerning, for example, linear algebra is not the same in upper secondary as it is at university (Winsløw, Barquero, De Vleeschouwer, & Hardy, 2014). Bosch and Gascón (2006) show how the institutional and social environment (ecology) can constrain a praxeology and one problem is to know from which context what restriction comes. In this study, where a teacher’s practice together with several contexts was studied, an assumption was that these contexts co-determine how a specific mathematical praxeology is constituted in a specific situation.

Didactic Praxeology

The *didactic praxeology* can be seen as the praxeology of those (teachers) who engage others in a specific mathematical praxeology. This is the praxeology that has been studied in Article 4, it was also used to reinterpret the findings of Articles 1-3. In a didactic praxeology, the teacher’s didactic task is to teach a specific mathematical praxeology (Barbé et al., 2005; Gellert et al., 2013), in relation to this study this could be to teach how to measure an angle, where the knowledge about “how to measure an angle” represents the mathematical knowledge. The didactic techniques refer to what is done to face the task at hand, to teach a mathematical praxeology (e.g. Gellert et al., 2013), in the case of this study this could be: visualise a fraction by dividing an orange. Technology provides backing for the technique used with explanations and arguments (Østergaard, 2013), an example of such an argument from this study could be that understanding rational numbers in relation to the number line is essential for the development of number sense, which is why teachers should engage their students in activities where rational numbers are ordered by size. Didactic theory constitutes a deeper level of justification of practice, the underlying ideas and principles which could both found and generate technologies, techniques and tasks (Chevallard, 2006), in relation to this study this could be principles directly connected to either mathematics or learning. One example is a principle that mathematics is a problem-solving activity where mathematical ideas may be discussed (see the revisiting of Article 2, in Section 5.2.2).

In Table 2, I show a didactic praxeology as an example of how the didactic task, technique, technology and theory might be seen in primary school, when the didactic task is to teach the meaning of the equal sign.

Table 2. Example of a didactic praxeology

| <i>Praxis, the know-how for teachers</i> | |
|--|---|
| <i>Task</i> | <i>Technique</i> |
| How to teach the meaning of the equal sign. | Use a balance scale as a metaphor for the equal sign in a problem-solving activity. |
| <i>Logos, the know-why for teachers</i> | |
| <i>Technology – rationale for techniques</i> | <i>Theory – overall rationale</i> |
| The equality of the two sides of the equal sign is an important idea to understand, which is why a scale is excellent so show the meaning of the equal sign. | Mathematics essentially consists of ideas that may be expressed with symbols. |

In a didactic praxeology, the techniques, technology and theory may differ even if the task is the same. The didactic task is to teach a mathematical praxeology. If this mathematical praxeology was inferred to have a praxis but little, the mathematics teaching would be providing the students with tasks and telling them how to solve them with no explanations. If this praxeology, on the other hand, included logos but little praxis, the mathematics teaching would consist of the teacher explaining and defining mathematical properties with little exercises for the students. The didactic technique consists of the teaching methods for teaching the mathematical praxeology. The technology consists of the arguments for this technique, and the theory consists of the principles in which the technology is anchored. The praxeologies presented in this thesis, were construed by me, and my co-authors. I do not claim to have found the right or true praxeology. I have only interpreted the situation I have observed.

3.2.3 Levels of Co-determination

How mathematics content is taught in a classroom is, in ATD, assumed to be affected by hierarchically ordered levels that co-determine what is taught and how (Chevallard, 2002; Winsløw et al., 2014). These *levels of co-determination* (see Figure 2) together form an ecology of classroom practice (Chevallard, 2002), with conditions that influence (co-determine) a teacher's teaching. These levels of co-determination go from the student activities in the classroom all the way to the civilisation.

| | |
|-----------------|--|
| 9. Civilisation | Swedish or western culture |
| 8. Society | State – the national curriculum |
| 7. School | Teaching institution |
| 6. Pedagogy | Teaching principles – textbook and teacher group |
| 5. Discipline | Mathematics |
| 4. Domain | Rational numbers |
| 3. Sector | Theoretical principles for the mathematics teaching activities |
| 2. Theme | Arguments for the mathematics teaching activities |
| 1. Subject | Mathematics teaching activities |

Figure 2. Levels of didactic co-determination in relation to the components of the ecology in this study (inspired by, Chevallard, 2002b; Rasmussen & Winsløw, 2013)

Chevallard (2002b) describes this as the ecology of a praxeology. It holds the possibilities and constraints for this praxeology that each level imposes at a given moment in time (see also, Rasmussen & Winsløw, 2013). This implies that the reasons for the praxeology taught in a classroom could reside in a level far from this classroom, where the teachers do not participate. In this study, I have studied four contexts from some of these levels of co-determination to trace how these contexts co-determine what is taught in Mary's classroom. As I wrote in the introduction (Section 1.1), I want to emphasise the co, in co-determine. Mary and the contexts all participate in the determination of what is taught. In this study, I have showed some traces of how this co-determination was performed. How I have studied the contexts as parts of the ecology will be further elaborated in Section 4.

3.3 Multimodal Social Semiotics

A basic assumption in *multimodal social semiotics* is that semiotic resources are socially situated. A culture offers semiotic resources (Kress, 2009). A semiotic resource is, in this thesis, the actions, materials and artefacts used to communicate (Jewitt, 2009). In this thesis, a multimodal approach was necessary since the data is both pictures (e.g. on the white board or in textbooks), actions (e.g. gestures) and many other resources used for meaning-making. Too much information would be lost if only written and spoken communication were to be included. In this thesis, I make a general assumption that communication is multimodal and to see the communicated meaning I explored a diversity of modalities of the communication. I view different modalities (including gestures and pictures) as text throughout the thesis.

Halliday's (2004) three meta-functions are used to describe different meanings in a text as three different versions of reality: textual, ideational and interpersonal (Björkqvall, 2009). The *textual* meta-function describes the text as a whole and the opportunities for participants in a communication to perceive the meaning of the text (Björkqvall, 2009). This can be seen through the roles different communicative resources (e.g. pictures, gestures, speech) play in communication. If the layout is changed in a written text, the whole meaning of the text could change (Kress & Van Leeuwen, 2006). The composition of a text becomes important in terms of what is salient, for example. Saliency can be afforded by different resources such as size, font or colour (Björkqvall, 2009), but also the volume, rhythm or pitch of spoken communication can bring saliency to an element of communication (Kress & Van Leeuwen, 2006). This creates differences between the different parts in a composition by catching the reader's attention (Van Leeuwen, 2005). The semiotic principle is that the more saliency an element has in a text, the more important it becomes to the meaning potential.

The *ideational* meta-function is used to describe the content of communication (Herbel-Eisenmann & Otten, 2011; Selander & Kress, 2010). This could be manifested by what is focused or addressed (see article 1) or in how people or things are represented (Jewitt, 2009).

The *interpersonal* meta-function can be used to describe social relationships between the participants in an interaction, for example between teachers talking to each other in a teacher group or between a teacher and the text in a textbook (Kress & Van Leeuwen, 2006; Morgan, 2006). The communication establishes and preserves relationships between members of a group through how social relationships are expressed, for example expressions of power (Kress, Jewitt, Ogborn, & Tsatsarelis, 2001).

In relation to ATD, a multimodal semiotic approach enabled me, inspired by the ideational meta-function, to focus on what mathematics was communicated and, inspired by the textual meta-function, how this communication was

expressed. This was used as a framework in Article 1, but the inspiration remained in the following articles. A multimodal approach to the communication also helped me to include more aspects of communication than just spoken and written words.

3.4 The use of Theories in the Articles

These theories have been used to different extents in the different articles. Different theoretical tools have been included in different articles. In the first article, the need for a multimodal approach emerged which led me to use multimodal tools for the analysis. In the work with Article 2, I continued with a multimodal approach with analytical tools borrowed from the literature review. The findings in Article 2 led me to explore the anthropological theory of didactics and praxeologies; mathematical praxeologies in Article 3; and didactic praxeologies in Article 4. In this preamble, I also revisit the first three articles as didactic praxeologies, not to generate new findings, but to describe the findings of all four articles in the same way. In Table 3, I have summarised how the theories have been used in the four articles. This is only to display where these tools have been used, not to relate the theoretical tools to each other.

Table 3. Theoretical tools used in the four articles

| | Article 1 | Article 2 | Article 3 | Article 4 |
|---------------------------|-----------|-----------|-----------|-----------|
| Multimodal analysis | X | | | |
| Multimodal approach | X | X | X | X |
| Mathematical praxeologies | | | X | X |
| Didactic praxeologies | | | | X |

4 Methodology

In this methodology for a study situated on the border between a social and socio-political paradigm, adopting an anthropological theory of didactics, I give an account of how this theory has been used in this study. The nature of the study as well as the specific methods used, is also described in the following sections.

4.1 ATD in Relation to the Case of Mary

Following the theoretical considerations presented in the previous section, I will now give an account of how I have used these theories in this study. Having positioned the study at the border of a social and socio-political perspective, I see individuals as social beings, in Lerman's (2000a) words, in practice. This implies a view that knowledge is not located within the individual only affected by the social context, but that "Knowledge has to be understood relationally, between people and settings" (p. 13). In this, I embrace how Valero (2004) describes mathematics education as embedded in many layers that value different ways of teaching. In this study, I have chosen to see this as ecologies of a teacher's teaching in relation to contexts from the levels of co-determination. The layers described by Valero (1999) as social contexts do coincide in a sense with Chevallard's (2002b) levels of co-determination, but with a more political interest in the ideology that governs mathematics education. However fascinating I find this, I wanted to study such contexts but also the communication of mathematics in them, in detail. This was enabled by the combination of contexts from the levels of co-determination and praxeology.

The research interest in this study is in the contexts close to the classroom. Later, in the final discussion (Section 7) I will discuss them in relation to societal contexts further from the classroom. In illustrations of the levels of co-determination (e.g. Winsløw et al., 2014), the levels of co-determination are placed in a hierarchy separated from each other. I recognise that this was one way of showing a very complex system in one illustration, I would like to show an alternative way of drawing this illustration, showing how I see these levels as inscribed in each other, see Figure 3.

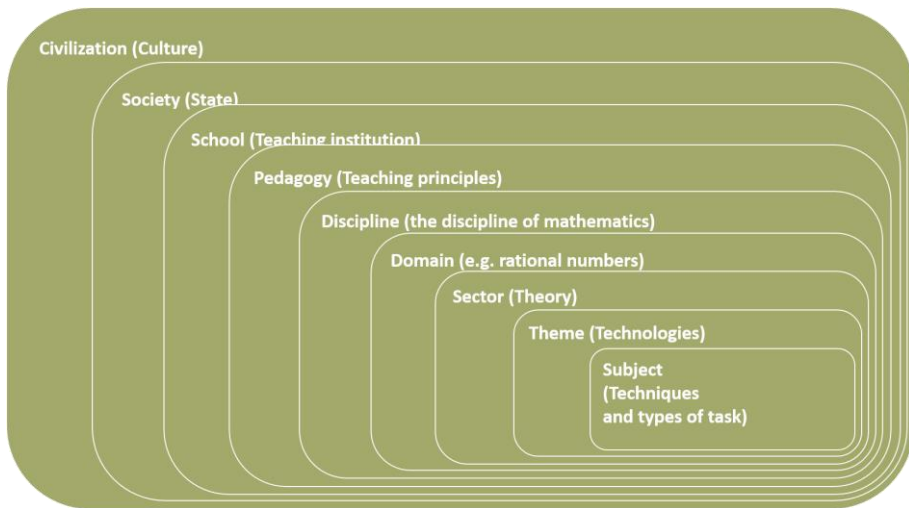


Figure 3. Levels of co-determination as inscribed in each other.

This illustration of inscribed levels is meant to show how the levels are rather surrounded by each other than ordered in a hierarchy. Following Chevallard (2002b), each praxeology is co-determined by these levels. As described in Section 1.6, the unit of analysis in this study is the ecology of a teacher's teaching. This implies that there has to be a praxeology at the centre (or bottom) of the ecology, in this case this is the didactic or mathematical praxeology of Mary's teaching. Analytically there have been many praxeologies in this study. Each type of task, in each context may have its own. It is when praxeologies are collected into regional organisations, which consist of praxeologies unified by the same theory (Winsløw, 2011), and sorted into the levels of co-determination, that it is possible to see what the whole environment of Mary's teaching looks like. In doing that, mathematical praxeologies have been seen as the task in didactic praxeologies and it is the didactic praxeologies that constitute the ecology.

Mary's teaching is one context, which includes the levels subject, theme and sector, which according to Chevallard (2002b) constitutes a praxeology. The levels of domain and discipline were in this study seen as the levels where mathematics was defined and explained. These levels were included as resources that may be drawn from in the technology and theory in both mathematical and didactic praxeologies. The analytical frameworks in Articles 2 and 4 are examples of how these levels were included. I see the pedagogy level in this study as being represented by contexts where teaching principles were expressed, which was the teacher group and textbooks. The level called school represents the teaching institution, in this study Mary's school. This level was not studied systematically. The level called state was represented by the context of the national curriculum since it constituted the official requirements for mathematics teaching in Swedish schools, issued by the state. The level called

civilisation was not analysed but the findings will be discussed in relation to the Swedish culture, in Section 7.

In this study, I focused specific mathematics content matters in the different studies: automated multiplication skills, problem-solving, angles and rational numbers. Each level that was represented by a context, which was included in the analysis, conveyed descriptions of these content matters and of how to teach them. This implies that there were praxeologies of the content matter under scrutiny on each level. A picture of the levels of co-determination adjusted to this study, and the contexts studied, would then be centred around a praxeology of the teacher's teaching, inscribed in praxeologies from the levels of pedagogy and society. These contexts also express praxeologies that are part of the co-determination, see Figure 4.

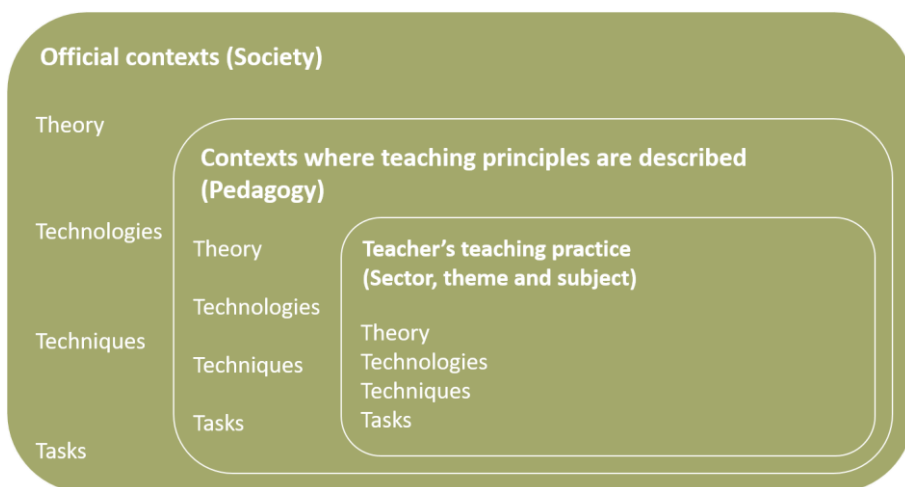


Figure 4. Levels of co-determination seen from the perspective of this study.

This system of how mathematical or didactic knowledge is expressed in different contexts in the levels of co-determination is a picture of how I see the ecology of a mathematics teacher's teaching. Focussing on mathematical and didactic praxeologies of the same content matter gave me something to trace and by extension a way to trace the co-determination between them. The ecologies, described in the findings are my interpretations of the praxeologies in the different contexts. I have interpreted praxeologies from the communication about the content matter in each level. In these interpretations, I have seen communication as multimodal and I have included modes other than just spoken and written language in the analyses. I do not claim to describe the full depth of knowledge from any of these contexts, only what was communicated in these situations. I do not see the construed ecologies as fixed or true, only as an interpretation of how they expressed knowledge of mathematics and mathematics teaching.

4.2 Case Study

Case studies concern events and phenomena in their full actuality, as they occur in everyday life (Mitchell, 1984). This study is a single instrumental case study (Stake, 1995), through which I aimed to understand more about the ecology of Mary's mathematics teaching. Distinct from an intrinsic case study that aims to understand the case itself, an instrumental case study serves as a means to illuminate the research question (Stake, 1995). For example, when a case illuminates the experiences of a mentoring team, it can contribute to illustrate potential viability of team mentoring (Wasburn, 2007). Or a case of two teachers, where their teaching gave insights on instructional patterns (Bullough Jr., 2015). Instrumental case studies are sometimes used to test a theory or a hypothesis in practice (Rule & John, 2015). That was not what I did in this study. This study is described as an instrumental case study because the purpose was to go beyond the case (c.f. Stake, 1995). In the case of Mary, I have sought to understand her teaching as part of her ecology. The interest was not in how Mary herself chose how to teach. The interest was in Mary's teaching as part of the system within which she worked, which would make the study an instrumental case study.

Mary was also construed as a 'telling' case (Andrews, 2016a; Mitchell, 1984) which serves "to make previously obscure theoretical relationships suddenly apparent" (Mitchell, 1984, p. 239). Both instrumental and telling case studies aim at gaining insights, but a telling case also intends to reveal theoretical relationships. A telling case could describe the role of policy in a community of enquiry being visualised (Dixon & Green, 2009). Studying the damage from an explosion of a bag filter could give an insight into modelling the consequences of an explosion (Marmo, Piccinini, & Danzi, 2015). A telling case could also use a local example such as the globalisation of a company in one specific country to suggest propositions for cross cultural research (Holden, 2001). This study is a telling case since the study of Mary's teaching as part of the ecology gives insights into how the environment for mathematics teaching in Sweden can be described, and how didactic co-determination can be traced in different contexts of the ecology.

Mary was an experienced teacher with a valid teacher education for the grades she taught. Mary was also especially engaged in mathematics, which she regarded as her favourite subject to teach. Being such a well-qualified, experienced and engaged teacher, Mary became my only case since I saw an opportunity to describe an ecology based on her teaching. Including more teachers than Mary would have drawn my attention from the relationship between a teaching and the system around it. A single case study also gave me the opportunity to do a longitudinal study, which in itself could justify a single case study if it is carried out at two or more points in time or if the case is representative (Yin, 2014). Studying a single case not only facilitates a thorough analysis of the teacher within her social setting (Hammersley & Gomm,

2009) it also permits a depth unlikely with multiple cases (Donmoyer, 2009; Gerring, 2006). In this single case study, I offer a thick description (c.f. Geertz, 1994) of the ecology of one teacher's mathematics teaching.

4.3 Data Collection

In order to understand an ecology of a mathematics teacher's teaching, the data needs to reflect different contexts on different levels of this ecology. Studying an ecology, the contexts was regarded as a part of the study, not only recognised but also analysed. In the following, I will give an account of what contexts were included and why as well as how they relate to Mary's teaching. The theories described in the previous section offer a rich variety of opportunities to analyse this data. I will also give an account of how I have used these theories.

4.3.1 Data sources

To understand an ecology of a mathematics teachers' teaching I needed to find a teacher who wanted to have me there. My requirement for the participating teacher was a teacher with a mathematics teacher education, not a general teacher education. I wanted a teacher with at least ten years of experience of teaching mathematics in the grades 4-6 and who currently taught grade 4 or 5. The reason for not wanting a teacher teaching grade six was that there were national tests in grade six. I wanted a teacher who expressed an engagement for teaching mathematics and who was well regarded at her school, by both parents and colleagues. I realised that it was not an ordinary teacher I was looking for and I searched within my network as a teacher and teacher educator in Sweden. Mary met all my requirements and she was happy to participate in my study. The context of Mary's mathematics teaching is represented by her mathematics lessons and her communication about her mathematics teaching in interviews.

The other contexts studied were studied based on what contexts were important for Mary in her mathematics teaching. One of these was the teacher group of mathematics teachers. In the first cycle of this study, these teachers met regularly discussing mathematics teaching in grade five, exactly the teaching practice I was studying. I asked the teachers for permission to observe their meetings and include them as data material in the study. They agreed and I included the teacher group as one context, which is represented by the teachers' discussions about mathematics teaching.

Mary and the teacher group all used a regular mathematics textbook which was accompanied with a teacher guide. Mary also used a number of additional textbooks for different subject matters. She often discussed these books in particular as well as textbooks in general, which was one reason for including

them as a context in the study. Another reason for including textbooks was how research points to the influence of textbooks on teachers' mathematics teaching, as described in Section 2.4.2. All textbooks Mary used, and when possible, teacher guides as well as other textbook-like documents Mary used in her teaching were included in the context of mathematics textbooks.

In Sweden, teachers are required to follow a national curriculum, which includes a syllabus for mathematics and which is accompanied by commentary material where the syllabus is explained. Mary, as well as the teacher group, often commented on the Swedish national curriculum, which had just been launched. The national curriculum, especially the mathematics syllabus and its commentary material, was included in this study as the context of the national curriculum.

4.3.2 Data Collection Process

Collecting data from the four different contexts in this study could be carried out for several years, which is why there were some elements of delimitations. One of them was time. I could only spend a limited amount of time in Mary's classroom, out of respect for both Mary and her students as observation may affect practice. Another was content, which in its turn solved the time issue. I participated in Mary's teaching, and in other activities concerning mathematics teaching, on a number of occasions over several months while she taught a specific content matter. The time was limited to the time Mary spent on this content matter. This was done in two cycles with about two-and-a-half years in between (due to reasons unrelated to the study), see Figure 5 and the following description.



Figure 5. Timeline for data collection

At the beginning of the first data collection cycle, I participated in Mary's lessons and the teacher meetings where mathematics teaching was discussed. I conducted one interview in the middle of the cycle, only to talk about the plan for the coming weeks. When the classroom data collection was complete, I conducted an interview where Mary and I discussed the teaching of the content matter she had just taught. During the time I participated in Mary's teaching, I also gathered all documents she and her students used in connection with the subject matter covered in the lessons.

Just before data collection cycle 2, I participated in some teacher meetings in order to understand the conditions of Mary's teaching. During this time, Mary and I also planned data collection cycle 2. After data collection cycle 2, I returned to Mary with detailed synopses (see Appendix 1) of a number of lessons. Over two days, Mary read and annotated these synopses. By the end of each day, I conducted interviews where Mary had the opportunity to further discuss the lessons she had just read and commented. For a more detailed timeline of the whole data collection, see Table 4.

Table 4. Detailed timeline for data collection

| Activity | Participants and content |
|---|--|
| Data collection cycle 1; 3 October – 12 December 2011 | |
| 5 teacher meetings | Mathematics teachers in grade five |
| 4 lessons | Mary, teaching fractions and decimal numbers |
| 2 interviews | Mary, about her teaching |
| Before data collection cycle 2; 11 November – 7 December 2013 | |
| 4 teacher meeting | Teachers in grade five, not specifically about mathematics |
| Data collection cycle 2; 30 January – 22 May 2014 | |
| 4 lessons | Mary, teaching angles |
| 4 lessons | Mary, teaching problem-solving |
| 6 lessons | Mary, teaching statistics |
| 2 lessons | Mary, teaching volume |
| 2 teacher meetings | Mathematics teachers in grade five |
| 1 teacher meeting | All teachers at the school about auscultations in each other's classroom |
| 2 interviews | Mary, about her teaching |

| | |
|---|--|
| Additional data collection after data collection cycle 2; 12 December 2014 – 5 March 2015 | |
| 2 days of data checking | Mary reads and comments synopses of lessons |
| 2 interviews | Interview with Mary after her reading the synopses |
| 1 lecture | I participated in a lecture Mary described as an inspiration for her problem-solving teaching. |

The choice of content matter (as described above) and time for my participation in Mary’s teaching resulted in what contexts to collect data from. In the first cycle of data collection, Mary and her colleagues met every other week to discuss mathematics teaching in grade five, where the data collection took place. The reason for these teacher meetings was a local project where the teachers wanted to explore a close collaboration about mathematics teaching. In the second cycle, Mary met her mathematics teacher colleagues on fewer occasions. In relation to this study and the four contexts studied, I here draw a refined picture of the ecology of Mary’s mathematics teaching, as seen in this study, see figure 6.

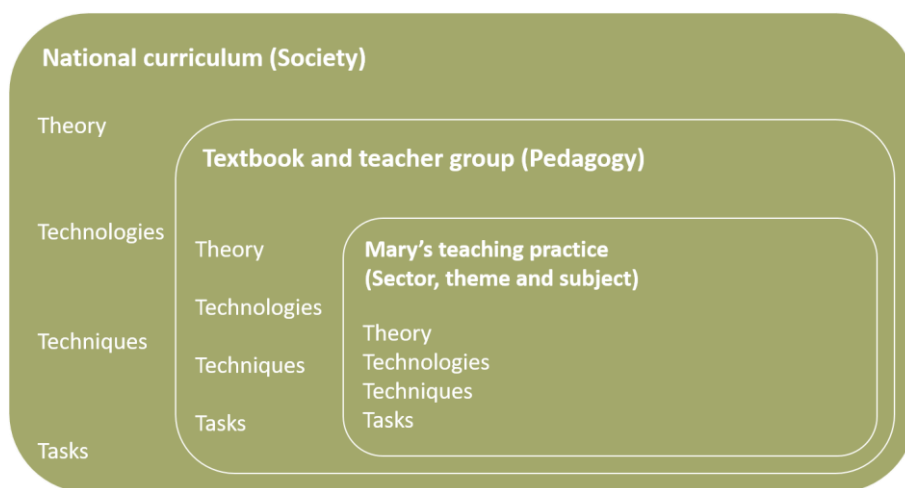


Figure 6. The levels of the ecology of Mary’s teaching, in relation to the contexts studied in this study.

Figure 6 displays how the level of society is represented by the context of the national curriculum, the level pedagogy is represented by two contexts, the textbook and the teacher group, and the context of Mary’s teaching is the levels sector, theme and subject where the didactic praxeology of the classroom practice is described.

This data collection gave rise to a rich data set. In this data set, there were various kinds of data, such as transcripts, synopses, documents and books. In

Table 5, I have summarised the full data set, which will be further described and elaborated on in the following sections.

Table 5. Summary of the data set

| Data | Number of occasions | Pages | Minutes |
|--|---------------------|----------------|---------|
| Transcripts from audio recordings of teacher meetings | 6 | 50 | 180 |
| Transcripts and synopses of audio recordings of interviews | 5 | 44 | 570 |
| Transcripts and synopses of audio, and video recordings of lessons | 18 | 91 | 1075 |
| Documents | | | |
| Textbook | | 174 | |
| Teacher guide | | 22 | |
| Teacher literature | | 290 | |
| National curriculum | | 52 | |
| Documents produced by Mary and her colleagues | | 30 | |
| | | $\Sigma = 568$ | |

The data used in Article 1 was one transcript from a teacher meeting from the first data collection cycle. I used transcripts from four teacher meetings as background material, but I based the article on the analysis of one transcript from one of the meetings.

The data used in Article 2 consisted of transcripts from four lessons on angles and four lessons on problem-solving from the second data collection cycle. I also used the regular textbook Mary, and her colleagues, used in their mathematics teaching, as well as an additional textbook Mary used only for her teaching on angles. Mary used another additional textbook, in her problem-solving teaching, which I also included in the analysis of the textbook as well as the additional documents Mary used in her problem-solving teaching. The national curriculum was not analysed *per se*, but I related to the writings about problem-solving in the national curriculum in the discussion.

The data used in Article 3, consisted of transcripts from the lessons about angles, volume and fractions and the textbook used in these lessons. In this study, Mary's teaching was only studied in relation to the context of the textbook.

In Article 4, I included data from all four contexts connected to Mary's teaching of fractions. This included transcripts of the fraction lessons, transcripts from the teacher meetings where the teachers discussed the teaching of fractions, the mathematics textbook used in these lessons, and the national

curriculum. The data not included in an article was used as background knowledge about Mary and her teaching.

4.3.3 Methods for Data Collection

The different contexts required different data collection methods. In the following section, I will give an account of the different data collection methods and in the context in which they were used.

Lesson Observations

I observed Mary's mathematics teaching by attending her lessons. I also participated in these lessons. I cannot argue to have been a participant observer to a full extent with all skills described by e.g. Musante and DeWalt (2010), but I did participate in the lessons. I did help out in the classroom, for example when students asked for help, and I tried not to disturb at the same time. I did not sit still just observing. I captured the lessons with a video camera, with audio recordings and by collecting the documents used in the lesson. I placed one single camera at the back of the classroom making sure that the picture frame captured all the students as well as Mary. I only used one camera because I did not want to disturb Mary and her students more than necessary. It took some time for them to adjust to the one camera so I concluded that adding another camera could have had an unnecessary constraining effect and I wanted Mary to be as free as possible to work, as she wanted to, even if my camera and I were there. This camera also worked well to capture the sound from the back of the classroom. When necessary, I manoeuvred the camera so it followed Mary as she moved around the classroom. A microphone, worn by Mary, also captured the sound from wherever she stood in the classroom. These audio recordings captured the episodes when Mary stopped to talk to individual students, which the video camera did not capture. This microphone was a small device, which I could attach to her clothes so it was out of her way. As a reserve recording I used Notability⁷ to record the sound on a tablet, where I was at the same time and I could make simple field notes on the tablet. This gave me an extra recording of the conversation, as a backup.

Apart from collecting data, my role during the lesson was to be an extra adult in the classroom, helping students when required. When Mary talked to the class, I stayed at the back listening. I tried to make Mary as comfortable as possible to have me there. Mary was instructed to teach as normal as possible. I realise that the video camera and the fact that she knew that I was going to analyse her teaching could have influenced her to teach differently than if I was not there. I clarified to Mary and her colleagues that I was not there to assess or analyse her teaching in itself. However, as a telling, instrumental case, Mary still worked well as a mirror of a mathematics teaching within the

⁷ <https://itunes.apple.com/se/app/notability/id360593530?mt=8>

Swedish school system. I also carried out the data collection of lessons and teacher meetings before the longer interviews so Mary could teach without my interview questions interfering with her plans.

Observations of Teacher Meetings

I audiotaped the teacher meetings during the first cycle. It was enough to capture the sound of the four teachers' discussions and I did not have any problem following who said what when I transcribed the meetings. I acted as an observer during these meetings. I wanted to hear their discussions, not to participate in them. In order to pass as unnoticed as possible, I chose to audiotape the meetings as the only data collection method. During the second cycle, as I was now familiar to the teachers, they allowed me to use a video camera. I placed this camera beside the table where the teachers sat, fixed in one position. Otherwise, I captured the teacher meetings in the same way as in the first cycle, with an audio recording, as an observer.

Interviews

I undertook various interviews with Mary. Two long interviews were semi-structured (Kvale & Brinkmann, 2014). There was a plan for the interview as well as flexibility for unplanned questions. Planning the interviews, I was inspired by stimulated recall (Haglund, 2003). The interviews were organised around video clips from the videotaped lessons, events we could discuss. Mary always asked to start the discussion. I wanted the discussion to be about what she was interested in and what she saw in these video clips. We planned the interview to take a whole day so we did not have any time pressure. There was enough time to see and discuss the clips and we could take a pause whenever necessary. The interviews were audio-recorded. I could easily follow who said what, since the only participants were Mary and I.

During these whole-day interviews, I chose video clips showing lesson activities from the whole data collection cycle. We looked at the introduction of each activity until the students started to work. I sequenced the video clips chronically and I stopped when there was a question about something new or when Mary began to explain something new. It could be a clip when Mary partitioned a circle into different parts discussing how to name them, followed by a similar exercise when a rectangle was partitioned, which would have been a new video clip. I structured these interviews as a discussion between Mary and myself, even though I wanted the data to consist of her comments on her teaching. I posed questions when I wanted her to develop or explain something. We watched the video clips chronologically so she also could comment on the structure of the teaching activities. In addition to video clips I also asked Mary to write, in bullet points, what had been important to her, when she taught this content matter.

I conducted short interviews in the middle of each cycle. In the unstructured interviews (Kvale & Brinkmann, 2014) there was no agenda from me. They

consisted of conversations about what was to come in the teaching sequence. These interviews offered an opportunity for Mary to comment on her teaching in a more structured way than all the small conversations we have had over a cup of coffee during these months of data collection.

I conducted a third kind of interview in connection with Mary's reading of the synopses. As described before, Mary spent two days annotating detailed synopses of lessons (Appendix 1). Working with the synopses, Mary received three tasks from me to give her different foci in annotating the synopses. The first task was to memory-check the synopses, to see if she remembered the lessons as I had described them. In this task she was also invited to comment in whatever way she wanted. The synopses included images from the lesson and the assignments, the students had worked on. The second task was for Mary to comment on where she thought she had made a choice in the lesson. Finally, the third task was for Mary to comment on what affordances and constraints on her teaching she could infer from these synopses.

At the end of each day after Mary had been working with the synopses of lessons we sat down for unstructured (Kvale & Brinkmann, 2014) interviews. I did not plan any questions before these interviews, we based our discussions on Mary's reading of the synopses. I asked Mary to comment on the synopses, and the lessons the synopses described. I asked follow up questions, asking Mary to develop an answer or clarify. Consequently, Mary structured the interviews on the spot.

I chose to take an unobtrusive role in the interviews. I only structured the stimulated recall interviews. During the longer interviews, I was careful with my questions. I preferred to prompt Mary with examples from her teaching, to let her talk about her teaching, as she wanted to. I did not want to influence Mary with questions, as I was genuinely interested to hear what she wanted to discuss about her teaching.

Documents

In addition to interviews, meetings and lessons, I collected documents connected to Mary's mathematics teaching. Documents collected from the classrooms were, for example, textbooks, assignments, tests and posters posted on the wall. Other documents were texts Mary related to in her teaching, in teacher meetings or in the interviews.

The national curriculum consists of a syllabus where the demands of knowledge outcome mandated from the government are written (Skolverket, 2011a), but also commentary material, issued by the National Agency of Education (Skolverket, 2011b), which claims to elaborate on the concise wordings of the syllabus. In this thesis, both of these are seen as part of the same context, named the national curriculum. I added the commentary material to the national curriculum, even if Mary had not read it. The national curriculum in Sweden was very short and the comments became a way to get access to the communication about mathematics in this context. It was in the comments

that I could find justifications for what teachers were expected to teach. Even if Mary had not read them, she had listened to the Agency of Education, when they introduced the new curriculum. The same agency, and often the same people who introduce a curriculum, are the ones who authored the commentary material, exemplifying the goals of the new national curriculum.

4.4 Methods for Data Processing

I processed the data in different stages, as the process evolved and as the need arose. One important part of the analytical process, for the researcher, is to become acquainted with the data (Braun & Clarke, 2006; Lapadat & Lindsay, 1999) which I have done in different ways. In this section, I describe how I have processed the data prior to the analytical process.

4.4.1 Transcripts

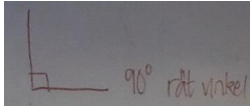
I transcribed the teacher meetings and interviews verbatim and I added non-verbal communication when necessary. I chose to transcribe the teacher meetings as a linear discussion. When the teachers' comments overlapped, I could still treat them as if they came one after another. This worked well since the teachers caused few instances of overlapping speech. In the transcripts, I sometimes included the tone of Mary's voice due to all the little "hm..." or "yes" or "aah..." Mary uttered during the conversation. These could mean very different things and the tone and volume of Mary's voice became one way of distinguishing different meanings. I developed a simple multimodal transcript where this was included. Most of her "hmm" and "yes" were uttered with a low tone and with a weak voice which was marked as [l,w (for low tone, weak voice)]. Sometimes she raised the volume and tone which was marked as [h,s (for high tone, strong voice)].

I transcribed the lessons both verbatim and with a multimodal approach (Zhao, Djonov, & van Leeuwen, 2014), where gestures and pictures were also included as communication besides the spoken communication. Verbal communication only became unsatisfactory for the analysis. The teacher showed angles both on the whiteboard, and with her hands. She enacted a rotation, which was difficult to show in a drawn picture on a whiteboard, but important for the analysis of her teaching. I would have missed parts of the message during the lessons if I had excluded these drawings and actions in the transcripts.

I included different forms of communication in the transcripts (Kress, 2015). To capture complexity in communication, the transcript expressed more than one mode (Boistrup & Selander, 2009; West, 2009; Zhao et al., 2014). Different multimodal transcripts often present different modes in col-

umns with one column for each mode. In this study, I have included the necessary modes in order to give an account of the communication about the concepts. The spoken language has one column, actions one and drawings one. Actions consisted of all the gestures Mary made when she described or explained the current content matter. Drawings consisted of the pictures drawn on the whiteboard in order to show how the properties of a concept were represented in the picture (see Table 6 for an example).

Table 6. Example of a multimodal excerpt

| Time | Speech | Actions | Whiteboard |
|----------|---|---|--|
| 00:02:18 | Sebastian: A right angle, like this /.../ | Mary shows a right angle between two fingers (mirroring a gesture Sebastian made, however not on camera). | |
| 00:02:26 | Mary: You drew, in the air, that a 90 degree angle looks like this. | Mary draws a right angle, she also writes 90 and right angle. |  |

I made these multimodal transcripts when the analytical process required inclusion of more communication than spoken communication.

4.4.2 Synopses

Some of the lessons have been summarised into synopses (Appendix 1). These synopses included detailed descriptions of what happened during the lessons, including images of what Mary wrote on the white board. I expanded these synopses during the process. Synopses of the lessons I engaged in a lot, analysing, are more detailed. They include transcripts of interactions that concerned the focus of study (problem-solving), descriptions of gestures, images from the classroom and the tasks the students engaged in (see Article 2). The synopses came out of the process, since transcripts of the lesson became unsatisfactory; there was a need for a description including much more information of what resources Mary used. Transcripts (in this case synopses) should contain the information the researcher needs (Braun & Clarke, 2006) which was the very reason for these synopses.

To summarise a lesson, rather than write verbatim transcripts, entails a choice of what to include and exclude. In these cases, I chose to include all

activities concerning the content of the lesson, leaving all conversation about other things, such as classroom management. What I wrote in detail developed during the analytical process. The parts I did not develop in the synopses were when Mary disciplined the class or when another teacher disturbed the lesson. All activities related to mathematics teaching are represented in the synopses, in words or in images.

4.5 Methods for Data Analysis

This project moved in different directions during the years I visited Mary, analysing and writing. These movements entailed me using different theoretical approaches as I moved together with the study. The data material and my interactions with Mary prompted all the twists and turns I made. In this section, I will describe my analytical process and give an account of the methods I have used for data analysis, when and why.

4.5.1 Analytical Process

The work with Article 1 was the starting point for my analytical process. With the aim of understanding Mary's teaching as embedded in other contexts I began an open coding process with the transcripts of the teacher meetings, similar to what Braun and Clarke (2006) propose as an inductive thematic analysis. In the first tentative analysis, it became clear to me that the spoken word alone was not enough to understand the meaning of the communication. The tone of voice was one way to distinguish different meanings. If Mary spoke with engagement, this inferred engagement could change the meaning of the communication. Another insight from the tentative findings implied that the content of the discussion, for example the mathematic content and the roles of relationships and artefacts were emerging themes. This led me to adopt a multimodal approach and meta-functions of communication (Van Leeuwen, 2005) as an analytical approach (see article 1).

The point of departure in Article 2 was to study Mary's problem-solving teaching. Again I began with an inductive thematic analysis (Braun & Clarke, 2006), which resulted in emerging themes that led me to include a framework from the literature to analyse, now with a deductive thematic analysis, Mary's problem-solving teaching in terms of teaching for, about and through problem-solving (drawing on Schroeder & Lester, 1989). The multimodal approach was still there, even if I did not use a multimodal analytical tool. I analysed synopses where Mary's communication, as well as pictures and photos, were included. This way I could analyse how the teaching of problem-solving was privileged in Mary's teaching as well as in the textbooks (see Article 2).

In Article 3, I analysed the multimodal communication in Mary’s teaching. I chose her lessons about angles and inferred them as mathematical praxeologies (Barbé et al., 2005), how the knowledge about this content matter was organised in Mary’s classroom. I construed mathematical praxeologies from the communication in the classroom practice as well as from the textbooks’, used in the classroom. Finally, I discussed how the mathematical knowledge was organised in Mary’s teaching compared to how it was organised in the textbooks (see article 3).

In Article 4, I analysed the communication involved in teaching rational numbers in all four contexts. To begin with I did a deductive thematic analysis (Braun & Clarke, 2006), where I searched for traces of didactic technology and didactic theory. The themes of these traces led me to include a framework from the literature describing sub-constructs of rational numbers (e.g. Charalambous & Pitta-Pantazi, 2007) to analyse technology, and a framework of mathematical values (Bishop, 1991). With these frameworks I continued with a new phase of deductive analysis. The findings were expressed as the theoretical block of a didactic praxeology of the ecology as a whole (see Article 4). In Section 5, I have also revisited the findings of the other three articles to represent them as didactic praxeologies.

In Table 7, I show an overview of the methods used for both data processing and analysis. This is only to display where the different methods were used, not to relate the methods to each other.

Table 7. Methods used in the four articles

| | Article 1 | Article 2 | Article 3 | Article 4 |
|---------------------------------------|-----------|-----------|-----------|-----------|
| Multimodal analysis, meta-functions | X | | | |
| Multimodal transcripts | X | X | X | X |
| Thematic analysis, inductive | X | X | | |
| Thematic analysis, deductive | X | X | X | X |
| Praxeological analysis (mathematical) | | | X | X |
| Praxeological analysis (didactic) | | | | X |

4.5.2 Thematic Analysis

I have chosen to draw on Braun and Clarke (2006) carrying out thematic analysis in the studies. A thematic analysis can be inductive, only driven by data or deductive using a theoretical interest or a coding frame to map the data (Braun & Clarke, 2006). The analysis in both Articles 1 and 2 was both inductive and deductive. The first readings of data were inductive. I coded and commented on the data in the software Videograph⁸, in a timeline following the video. I will from now on refer to reading, listening, and watching as reading.

⁸ <http://www.dervideograph.de/enhtmStart.html>

In a thematic analysis, it is possible to search for explicit patterns, directly connected to what the participants talk about (semantic approach). Another way is to search for what is underneath (latent approach) the explicit language (Braun & Clarke, 2006). The analyses in Article 1-3 are mostly of the semantic kind. In Article 1, I did, however infer relational aspects of the communication. In Article 4 as well as in the revisiting sections in Section 5, there are also latent analyses.

In Articles 1 and 2, I first read and listened to the chosen data as I tried to find relevant potential codes. In the subsequent phase, I interpreted the initial coding, and tentative themes emerged. Depending on what themes I interpreted in the data, I turned to different analytical frameworks, to continue the coding process in relation to these frameworks. In Articles 3 and 4, I chose to conduct praxeological analyses. In these, I still coded the data material with codes borrowed from both the theory and categories from the literature, connected to the mathematical content matter. The process of thematic analyses was thus also used deductively in Articles 3 and 4. For Article 3, data was coded in Videograph with codes connected to *task*, *techniques* and *technology/theory*. For Article 4, data was coded in MaxQDA⁹ with codes connected to both praxeology and the taught content matter.

From the teacher meetings I inferred three themes. The first theme related to artefacts as part of the school system, such as multiplication tables, and the role they played in the teachers' discussion. The second theme was "what" Mary focused on in her justifications, and a third theme concerned interpersonal aspects such as how Mary related to her students in her justifications. In these themes I saw conceptual similarities with social semiotics (Halliday, 2004; Morgan, 2006; Van Leeuwen, 2005) which led me use the three meta-functions of communication described by Halliday (1977) and Kress (2009), see the section about multimodal analysis, see Section 4.5.3.

In the initial analysis of Mary's problem-solving teaching, I found tentative themes suggesting differences in how Mary used and described problem-solving. Sometimes problem-solving strategies were in focus and mathematical subject matter was focused on at other times, with problem-solving as a goal instead. These themes led me to the three approaches to problem-solving teaching described by Schroeder and Lester (1989). I used these descriptions as an analytical framework together with other categories of problem-solving teaching summarised by Andrews and Xenofontos (2014). Through these categories, I could analyse how Mary organised her teaching in terms of teaching for, about and through problem-solving. I also analysed the textbooks with the same categories, which enabled me to relate them to each other.

The analyses of Article 3 and 4 will be described in Section 4.5.4. They are described as praxeological analyses. The frameworks that were included in

⁹ <https://www.maxqda.com/>

Article 4 could have been describe as a deductive thematic analysis, in a praxeological analysis, these frameworks was instead used as a reference epistemological model, as described in Section 3.2.1.

4.5.3 Multimodal Analysis

I adopted a multimodal approach to the communication in the study right from the start. Since I realised that different kinds of tone in Mary's voice gave different kinds of meanings to the communication in the teacher meetings, I have seen communication in a wider perspective than spoken and written words, see the section about transcripts. More so, when I analysed how Mary explained mathematical content matter and I realised that Mary explained and defined mathematical properties with both gestures and spoken words.

The analysis of teacher meetings began with coding Mary's engagement. In this, the little "mmm" and "hm" became a challenge, but also important since it was in these sounds Mary often agreed or disagreed. Most of the time, these little sounds were interpreted as that Mary was listening, somewhat active. Sometimes I interpreted the same sound as agreement. When she spoke with a low tone and with a weak voice it interpreted just as an indicator that she was an active listener. When she raised the volume and tone, it was interpreted as engagement. It was, however, not enough to consider the tone and volume of Mary's voice. The circumstances for the utterance were also taken into account. When these small sounds appeared after Mary had proposed something or protested and someone else started to interpret what she just had said, even the silent low toned "mmms" were interpreted as engagement "Yes, you understood me right". These different ways of categorising Mary's engagement were used to find the situations where she was an active participant during the conversation. These interpretations were possible since I am a Swedish mathematics teacher myself and the type of communication was familiar to me.

Three functions in a text are described as representation, interaction and message (Halliday, 1977; Kress, 2009). These functions provided an opportunity to view the communication from different perspectives, in relation to the ecology. In social semiotics, I found tools to view content based and relational aspects as well as the role of artefacts in the same analysis (see Article 1).

Inspired by the interpersonal meta-function, I coded the data of the teacher meetings with a focus on Mary's actions. How Mary responded to what was said, who had agency in the conversation, who/what was the expert, what relationship Mary had to the textbook teacher group students or to problem-solving was identified as the interpersonal meta-function. Analysing the ideational meta-function of communication, I coded what mathematics, what

classroom activities or what materials Mary and the other teachers were talking about. Finally, the textual meta-function was only used to see what roles the artefacts, the timed test and the textbook, played in the communication.

4.5.4 Praxeological Analysis

I have construed praxeologies after analysing how knowledge was organised in terms of praxis and logos. The two different kinds of praxeologies, mathematical and didactic, construed in this study are analysed in different ways, even if there are significant similarities. In the following sections, I give an account of how the two kinds of praxeologies were analysed.

Mathematical Praxeologies

As described in the theory section, a mathematical praxeology consists of mathematical tasks, techniques, technologies and theories, as shown in Table 8.

Table 8. *Mathematical praxeology*

| <i>Praxis</i> | |
|--|---|
| <i>Task</i> | <i>Technique</i> |
| A type of task | How to solve the task |
| <i>Logos</i> | |
| <i>Technology</i> | <i>Theory</i> |
| The rationale for the technique, for example arguments for a method. | Principles and sets of ideas, for example definitions and axioms. |

My first step in analysing how mathematical knowledge was organised in the different contexts was to infer different kinds of tasks. I categorised all tasks of the lessons included in the study into different types according to mathematical content matter. Examples of types of tasks were 'how to draw an angle' or 'how to convert from one volume unit to another'.

The techniques I interpreted for each type of task were the communicated procedure(s) described to solve the types of tasks. The technology for each technique consisted of the communicated explanations that were described to justify the procedure(s) presented. If there were no procedures or explanations, I left these boxes in the praxeology blank.

Logos represents the rationale for the praxis divided in two parts. Technologies were inferred from the explanations of the techniques communicated in the different contexts. Theory was inferred from the more overall communication, which could also justify the technology. In this, I searched for communication similar to scholar mathematics, such as, when a concept was defined. Definitions, axioms and proofs are examples of how a deeper level of rationale

could be communicated in the scholarly institutions of mathematics. The difference between technology and theory lies in how the rationale is communicated and what it can be inferred to draw from. An argument for a method grounded in experience or in ideas about, for example, different ways to convert units is a technological argument. Theoretical rationale draws from principles, in this case of mathematics. In the case of units, these principles could be definitions of a unit, common agreements like the SI system or the definitions of different quantities. Theory draws more clearly on the domain and discipline of mathematics.

Didactic Praxeologies

A didactic praxeology is the didactic tasks, techniques, technologies and theory that describe the didactic knowledge of how to teach, in this study, mathematics. In a didactic praxeology, the task is how to teach a mathematical praxeology, as shown in Figure 7 where the mathematical praxeology is placed as the didactic task.

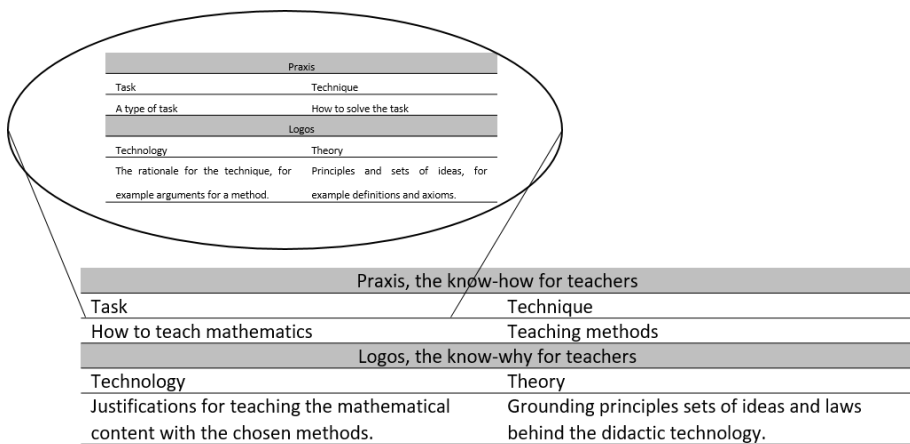


Figure 7. Didactic praxeology

Figure 7 shows that the didactic task is how to teach a specific mathematical praxeology, as described above. The didactic techniques were inferred from the communication about how mathematical knowledge could or should be taught. In Mary’s case, this was inferred from all classroom observations, Mary’s actions teaching mathematics as well as interviews in which Mary described privileged teaching methods.

The technologies and theories together form a language of the didactic knowledge reflected in the different contexts. Didactic technologies were inferred from the communication with a rationale for the techniques. In Article 4, I used a framework explaining different teaching activities of rational numbers (c.f. Charalambous & Pitta-Pantazi, 2007), as a reference technology.

This framework worked as a tool for the analysis of the didactic technology of teaching rational numbers. The reason for this choice of framework was that I had identified in data that the arguments for specific teaching practices of rational numbers often concerned properties of rational numbers. The didactic technology in full was inferred as the distribution of the categories from the framework. The way these categories were distributed together with the meaning of each construct, argued for a specific way of teaching rational numbers. In this preamble, I revisited the findings from article 1, 2 and 3, and inferred them as didactic technologies (see Section 5). This way, I inferred the arguments for different teaching activities from the different contexts as didactic technology.

Didactic theory was first inferred from the communication where I found a more general rationale for the mathematics teaching. This rationale was inferred as, for example, theoretical principles or ideas which anchored the mathematics teaching. There was little explicit theoretical rationale, which was why I included the framework of mathematical values (Bishop, 1991) in the analysis. The explicit theoretical rationale I did find were often more generally mathematical and not connected to a specific content matter. This was why the mathematical values worked well for the analysis of didactic theory since they described different ways to value mathematics. In Article 4, I inferred didactic theory as traces of the mathematical values. In the revisiting sections, Section 5, I instead, inferred different principles from the didactic technology and from the teaching activities. One such example was when the teacher group discussed how to use the mathematics textbook for high achieving students, arguing for the necessity of the students' working at their level of exercises, which were to be found in the subsequent textbook. Here I inferred a didactic theoretical principle to be that mathematics is ordered per difficulty. Since there were little theoretical principles included in the communication about mathematics teaching throughout the whole ecology, most theoretical rationales were inferred in similar ways, from statements and activities.

4.6 Trustworthiness of the Study

There are different descriptions of quality in research, written in relation to qualitative research (e.g. Adler & Lerman, 2003; Goodchild, 2011; Hermerén, 2017; Niss, 2010; Tracy, 2010). These studies describe a number of criteria of qualitative research mainly concerning the research design, research ethics and the report of research. One might argue, similar to Goodchild (2011) that these criteria are all about research ethics. It is not only how the participants are treated, it is also how the study was designed and reported to give a fair account of what they said or did. In the following pages I will discuss the

quality of my study from these three perspectives in relation to different criteria. I have grouped these criteria into three main categories under which I have collected criteria from the references above. The main categories are; research design, research report and research ethics.

4.6.1 Research Design

The research topic of my study was something I was interested in and wanted to study. To meet the criteria of the topic being worth studying it has to be beneficial for others (Hermerén, 2017; Tracy, 2010). Understanding the grounds for a mathematics teacher's teaching in relation to an ecology may reveal insights about what could guide, support or hinder teachers' teaching. This is important to understand if we want to develop mathematics education.

To achieve quality in the design of a study's reflexivity, transparency and critical reflection is an important criterion to meet. To have a critical reflection about what has been done and to expose weaknesses and limitations (Goodchild, 2011), but also to be transparent in describing the research process (Tracy, 2010). In this study, transparency has been moving in two directions. In relation to the reader, I have attempted to show the analytical processes as clearly as possible. The analyses have often been made in steps where theoretical tools have been added along the way, often prompted by what I have seen in the initial analyses and this has been important for me to describe. In relation to Mary, it has been important to keep her up to date with the study. Sometimes this has been more formal when Mary has read transcripts or synopses followed by interviews and sometimes it has been over a cup of coffee where I have described my current work, inviting her to comment on my interpretations and findings. Just before publishing the thesis I also talked Mary through all of the articles, findings and descriptions of her and her colleagues, and asked for her comments. The teacher group has not been able to do the same. I have met them informally at the school when I have been back to meet Mary and I have then described my research and what I have found, but not in a formal way. Now they are no longer at the same school and it is not possible to gather them together to talk them through the thesis as I have done with Mary. It has, however, been more important for me to include Mary, since it is about her teaching I am writing. My choices are indeed guided by my experience and knowledge. No researcher knows all possibilities and has all experiences. One limitation of the study may be the use of different analytical tools along the way. It may have been possible to refine the analytical tool built on praxeologies and levels of co-determination if I had used this from the beginning.

My experience as a mathematics teacher in a Swedish primary school has given me an advantage in knowing how it is to teach grade five in Sweden. A limitation regarding my experience is indeed that my knowledge about mathematics teaching and the Swedish school system may have cluttered my view.

I have tried to solve this with extensive detail in transcripts and in how I have reported the analyses in my four articles. I do claim to have had an insider's perspective, which is a quality criterion according to Adler and Lerman (2003), in respect of understanding the complexity of the studied practice. I am, however, not Mary and to keep in touch with her has been essential to discuss interpretations and descriptions. She has always agreed with my interpretations, if she would have contested an interpretation or a description I would have changed it, or addressed the discrepancy between us. This is also attached to a quality criterion, which Goodchild (2011) calls voice, saying that the research should reflect the participants and they should be able to recognise themselves in the texts. When I showed Mary the whole thesis, I asked her to read the introduction of her. When she had finished reading this description she commented with the words: "That's me!" She also described how she felt flattered that I had been able to see this much from her mathematics teaching. She became very interested in some theoretical constructs where she recognised herself, for example in Bishop's (1991) value openness. She claimed this value expressed her most important ground for how she taught. The description of mathematics being open to anyone was similar to Mary's emphasis of why mathematical communication is essential to teach. This tells me that I have been able to keep close to Mary without excessive interpretations. This also tells me that my choices of analytical tools have been suitable for studying Mary's teaching.

My reflexivity in this study lies mostly in how I have formed the study according to what I have seen in the data, but also in how I have organised my participation in Mary's everyday life at the school without disturbing her and especially without educating her. It has been important that she has been comfortable to have me there.

A trustworthy study is conducted with a research design where sufficient, appropriate and complex data is used, described as a rich rigour by Tracy (2010). In a study like the present, it is difficult to determine when there is sufficient data. There will always be another conversation to follow or another document to collect. However, for the time I spent in the school I collected all the data I could. This data was seen to be enough to understand both the teacher's teaching as well as the ecology. In a school context, with respect for students and teachers it is important to only collect the necessary data and intrude as little as possible. On the other hand, from a researcher's perspective, all situations are interesting and could offer new insights. The data collection in this study is a negotiation between the two. I have also had the opportunity to go back and collect additional data. Mary has been generous enough to participate in data checking and follow-up interviews. Videotaping the lessons allowed me to return to them to see fine differences in gestures and drawings which was important during the analysis. The interviews are the weak spot in the design of this study. As I was cautious to not influence Mary with questions, the whole-day interviews did not include in-depth conversations about

Mary's teaching. Mary did comment a lot but as she was not restricted to mathematics, she saw her own gestures, student behaviour and other things relevant to her, but not the study. I asked questions, but more as a response to what Mary discussed and less as well thought-through questions. The reason for this was that I did not want to influence Mary with my questions. The shorter interviews following Mary's readings of lesson synopses were much more focussed on mathematics teaching as she had time to read and reflect beforehand.

4.6.2 Research Report

Through the writing of the articles and this preamble, I have compiled a research report. One quality criterion for such reports is that they describe methods and results openly together with the grounds for the results (Goodchild, 2011; Hermerén, 2017) so they are robust to criticism (Niss, 2010). I have described how and why the methodology has changed during the process. To address this change, I have also revisited the first three articles in the findings to describe the results as didactic praxeologies, so they will be described the same way as in Article 4. This was a way to create a detailed report where I give an account of the findings in relation to my whole methodology.

I have attempted to write a clear report with a clear structure, which is regarded as a quality criterion (Niss, 2010). In this preamble, one important part of the structure has been to let the ecology permeate the whole text. Another consisted of the revisiting parts, described above. To achieve the best clarity and structure possible, I have asked colleagues with different backgrounds to read and comment on my texts. These readings have showed me what is clear and what needs to be developed. A limitation is that I write in my second language. Therefore, I have had all texts proof-read by native English speakers or professional proof-readers.

Throughout the work I have maintained two interests. One is the interest in Mary's teaching as part of an ecology. This interest has had consequences for the methodology. However, since I have maintained this interest there is also a common thread through the thesis, even though the methods have changed. Since the beginning I have also maintained the interest in how specific mathematical contents were communicated. Together with the interest in the ecology, this interest has resulted in different descriptions of how mathematics practice is privileged in the different contexts. In this, I have attempted to achieve coherence.

4.6.3 Research Ethics

To meet the criteria of research ethics it is essential to organise the study so that the participants are protected (Goodchild, 2011; Hermerén, 2017; Tracy, 2010). The first step to securing an ethical study was to ask the participants to

give an informed consent to participate in the study. Mary gave her consent (Appendix 2) twice. The first consent was to be videotaped and interviewed in the first data collection. When I came back to Mary I asked her for her consent to do the same thing, but as the only teacher in my thesis. For a teacher to be studied as one of many teachers is one thing, but to be the only teacher whose teaching is studied is another and this have to be articulated to the teacher. In this consent I informed her about the study, that she would be anonymous and that the data material would be stored securely and that participation in the project was voluntary and that she could leave the project at any time. This informed consent described the study in a general way. Exactly what to study and how it has evolved in relation to the data material as described in the methodology made it impossible for me to explain in detail what she accepted to participate in. I have solved this during the process by involving Mary as much as possible, as described above. I have also reminded her about the possibility to leave if she wants to.

The other participating teachers have also given their informed consent to audio and video recordings. For the students, the parents gave their informed consent for their children to participate in the classroom in videotaped lessons. The parents had a choice to allow me to be in the classroom, filming but not to include their child in the data material and some parents took that option. In those cases, when one of those children spoke which was audiotaped even if the child was not videotaped, I marked this as “unusable material” in the transcripts. In the letters of consent (Appendices 3 and 4) the participants were informed about the conditions of the study and their right to withdraw from the study at any time during the study. The letters of informed consent were inspired by Boistrup (2010).

I have taken measures, other than the informed consent, to ensure the privacy of the participants, which is another ethical criterion (Adler & Lerman, 2003; Hermerén, 2017). When teachers and students are present in excerpts they are labelled with pseudonyms and there is no information in the study that could reveal their actual identity. The data material has also been stored in a secured location at Stockholm University.

Finally I would like to say something about who this research is for, to articulate this has been regarded as a quality criterion (Adler & Lerman, 2003). It is connected to the ethics of the study. It would not be ethical to study one teacher and not give something back to her and to teachers in general. The contribution of this thesis lies in the description of the teaching in relation to the ecology. This thesis is written on behalf of teachers, and by extension, students who benefit when teachers are enabled to teach.

5 Findings

In the four articles, I used, at least partly, different analytical frameworks to investigate the ecology of Mary's teaching. As described in the analytical process, the choice of ATD and praxeologies was made in between the work with Articles 2 and 3. This section summarises each article. The first three articles are also revisited from the point of view of the ecology of didactic praxeologies. This allows me to present the findings in all four articles in a more consistent way.

5.1 Article 1: Justifications for Mathematics Teaching: A Case Study of a Mathematics Teacher in Collegial Collaboration

5.1.1 Summary of Article 1

In Article 1, Mary's teaching was studied in relation to the teacher group she collaborated with. The article describes how a mathematics teacher, Mary, justified her mathematics teaching in a teacher meeting. Mary's teaching was represented by Mary's statements, questions and comments during the meeting. Mary's communication, which focused on the teaching of mathematics to grade five students, was analysed in relation to what was communicated by the teachers as a group. When studying this communication within the teacher meeting, I asked the following question: "How are the justifications for Mary's mathematics teaching constituted, in relation to a teacher group discussion?" In this meeting, the teachers discussed how to test automated multiplication skills. Their descriptions of how they assessed automated recall skills were very similar, they used about 80-100 exercises in 5-8 minutes. The number of exercises and minutes varied between the different teachers' descriptions, but not the format of the test. Another topic during the meeting was the use of mathematics textbooks for later school years to find suitable exercises for advanced students.

After the initial analysis, three major themes were identified. One related to artefacts as part of the school system, for example multiplication tables and the role they played in the teachers' discussion. Another was "what" Mary

focused on in her justifications, and a third concerned interpersonal aspects such as how Mary related to her students in her justifications. These themes were inferred to be similar to the textual, ideational and interpersonal meta-functions of social semiotics (Evans, Morgan, & Tsatsaroni, 2006; Halliday, 2004; Van Leeuwen, 2005). In short, the textual meta-function concerns the role of artefacts. The ideational meta-function reflects the content of communication (Herbel-Eisenmann & Otten, 2011). The interpersonal meta-function describes the roles and relationships between people. The analysis was made using the meta-functions (Van Leeuwen, 2005), see also (Boistrup & Selander, 2009), as a framework. As examples of the textual mode of communication, the roles of the multiplication test and the textbooks were analysed. The content of communication revealed what Mary focused on, and addressed in her observable justifications. The interpersonal function of the communication was how Mary related to the students in her justifications.

Mary both resisted and supported the idea of the multiplication tests. The resistance could be described in relation to ideational aspects, for example automated recall as important knowledge. Mary's resistance could mainly be described as relational aspects of the communication. She expressed concerns about students' having anxiety attacks during multiplication tests. The support for the test was justified based on the importance of the students having automated recall skills so they can calculate more easily. A timed test does not give time to think about the calculations. This was used as a rationale for the tests, because automated multiplication skills were inferred as important for the ability to calculate.

Mary supported the use of textbooks for differentiating mathematics teaching. It was justified from an ideational aspect, the need for challenges for the "high achieving" students and from a relational aspect the idea that these students were considered to be able to decide for themselves when it was time to take on more advanced exercises. The teachers agreed on the need for more advanced exercises so these students were not held back.

Looking at both the textbook and the multiplication test, from a textual aspect, we could infer a tradition from the roles these artefacts had. The test had a role as a tradition keeper since it is a part of the school tradition. The textbook in its role in differentiating mathematics teaching had a very strong position, which was visible when the teacher group unanimously justified this idea.

5.1.2 Revisiting Article 1 as Didactic Praxeologies

The findings in Article 1 can be re-formulated in terms of didactic praxeologies (Chevallard, 2006, see also Section 3.2.2 in this thesis): one regarding the discussion of the assessment of automated multiplication skills, and the other regarding the discussion of textbooks as a means to differentiate mathematics teaching. I present the two praxeologies below, with a short description of how

the findings in Article 1 may be represented as didactic praxeologies. In a didactic praxeology, *task* describes the teachers' teaching task, *technique* describes how they did the task, *technology* describes the arguments for why the task should be carried out that way, and *theory* describes the principles that could be inferred to ground the technology and techniques. Finally, I will comment on what becomes visible in the praxeology compared to the initial analysis in Article 1.

The first didactic praxeology regards the didactic task that gave rise to the teachers' discussion 'to assess automated multiplication skills', see Table 9.

Table 9. Didactic praxeology, from a teacher meeting, of the assessment of automated multiplication skills.

| <i>Praxis, the know-how for teachers</i> | |
|--|---|
| <i>Task</i> | <i>Technique</i> |
| How to assess automated multiplication skills. | Use a timed test with 100-120 multiplication table exercises to solve within five minutes. |
| <i>Logos, the know-why for teachers</i> | |
| <i>Technology – rationale for techniques</i> | <i>Theory – overall rationale</i> |
| Atomised skills are assessed with a short time-frame because the students should not have to think about their solutions, they should know them by rote. Atomised number facts are a prerequisite for fluent calculation skills. | Number facts are foundational in mathematics. Skills are learned separately before they are used. |

The technique in the first didactic praxeology was inferred from the teachers' description of a five-minute test with 100-120 exercises. It was this technique the teachers mostly discussed and argued for and against. The technology was inferred from the conclusions the teacher group agreed on during this discussion. Timed tests were described as necessary when atomised skills were assessed, because it was the atomisation that was assessed and there should be no need for time to think. Atomised multiplication skills were also described as important for the ability to engage in calculations. These descriptions both argue for timed tests as the assessment method for atomised skills, which is why this was inferred as the technology.

In this discussion, there was no communication explicitly drawing on theoretical principles for mathematics teaching and learning. The arguments were based on experience and opinions of the function of the test. Theory as principles grounding this rationale had to be inferred from the teachers' discussion. In this case, I inferred two principles from the discussion. All emphasis on the importance of atomised number facts was inferred as a principle that number facts are foundational in mathematics. This can be compared to how

Sayers and Andrews (2015) describe number sense for grade one students to consist of eight components. The inferred principle in the teachers' discussion was inferred to be that number facts was one component, of a foundational number sense for grade five. The teachers' arguments about these skills being important for coming calculations was inferred as a principle of mathematics learning, that mathematical skills are learned separately before they are used.

If a didactic praxeology would have been inferred from Mary's arguments in the discussion, separated from the other teachers' communication, it would have been possible to identify another theoretical principle: Mary's resistance to the test was based on how the students felt while taking the test. This could be inferred as a principle about making mathematics teaching available to all students (also those who don't take well to timed tests). Other than that, the praxeology would have been similar to the praxeology of the other teachers, but with less exercises and longer time on the test.

The second didactic praxeology was constructed around the task that gave rise to the discussion about more advanced textbooks 'to differentiate mathematics teaching for "high achieving" students' (see table 10).

Table 10. Didactic praxeology, from a teacher meeting, of differentiating mathematics teaching for high achieving students.

| <i>Praxis, the know-how for teachers</i> | |
|--|---|
| <i>Task</i> | <i>Technique</i> |
| How to differentiate mathematics teaching for "high achieving" students. | Give the "high achieving" students the subsequent textbook, when they have showed that they can do the exercises in the current textbook. |
| <i>Logos, the know-why for teachers</i> | |
| <i>Technology – rationale for techniques</i> | <i>Theory – overall rationale</i> |
| "High achieving" students need more, and more difficult exercises so they are not held back. | Mathematics is ordered as per difficulty Students need to learn on their level. |

The technique in the second didactic praxeology was inferred from the teachers' discussion about the method of giving the 'more advanced' students the subsequent textbook when they have shown that they understand the content of the present school year. The technology was inferred from the agreements in the teachers' discussion where the teachers agreed on the importance of not holding the more advanced students back. From these arguments I also inferred two theoretical principles, one of mathematics and one of mathematics learning. The practice of using the next textbook for high achievers was inferred as a trace of a principle that mathematics is ordered as per difficulty where one difficulty grounds another. The same practice was also inferred as a trace of a principle that students learn mathematics best on their level. In this

case, a praxeology from Mary's communication in the meeting would have mirrored the praxeology in Table 10.

The construction of the findings from Article 1 as didactic praxeologies clarified what grounded these teachers' discussions of mathematics teaching. The arguments for, and principles of the discussed mathematics teaching conveyed a strong emphasis on mathematics as structured in separate skills of varying difficulty, which conveys that mathematics needs to be taught with the right level of difficulty.

Something else possible to infer from the two discussions was the strong position of calculations. When the teachers agreed on how to differentiate their mathematics teaching, they agreed that what would ground the need for a more advanced textbook was the student's ability to do the exercises in the present textbook. This implies an emphasis on working with exercises, most often calculations, instead of to further explore the ideas of the present book. When the teachers discussed multiplication skills, calculations were described as the reason of why these skills were important to learn. From this, I conclude calculations to be a privileged mathematical activity in these two discussions from the teacher meetings.

5.2 Article 2: The Teaching of Mathematical Problem-Solving in Swedish Classrooms: a Case Study of one Grade Five Teacher's Practice

5.2.1 Summary of Article 2

In Article 2, Mary's teaching was studied in relation to the textbooks that were used in her classroom and the findings were discussed in relation to the national curriculum to answer the following question: How is Mary's teaching of problem-solving constituted in relation to the curricular materials available to her? The article focused on Mary's teaching of problem-solving in relation to how problem-solving was described in the curricular texts she used in this teaching. Transcripts from Mary's mathematics lessons and the mathematics textbooks she used were analysed and the national curriculum was included in the introduction, but discussed in relation to Mary's teaching and the textbook.

Following curricular development in Sweden and abroad, problem-solving was described as having a central place in mathematics teaching during the last twenty years. From this literature, three approaches to problem-solving teaching (Schroeder & Lester, 1989) were chosen as an analytical framework. Teaching for problem-solving refers to teaching mathematical content

knowledge so problems may be solved. Teaching about problem-solving refers to teaching problem-solving strategies. Teaching through problem-solving refers to problem-solving as a method to problematise mathematical content knowledge so students may learn. Data from four problem-solving lessons and four content-based lessons on angles was described in detailed synopses. These synopses, as well as Mary's written comments of the synopses, together with transcripts from interviews and the textbooks Mary used in her problem-solving teaching, were used as data in the study.

The initial readings resulted in themes with respect to aspects like lesson structure. During the angle lessons, students received a lot of information about the topic before they worked with exercises, but in the problem-solving lessons, the information was almost only about the problem, not the mathematical topic at hand. Here the discussion about the mathematical content came at the end of the lesson when Mary and her students discussed the solutions whereas the angle-lessons did not end with a discussion. For the structure of the problem-solving process, Mary used two different schemes of a problem-solving process where one is a checklist of strategies that should be considered and carried out for each problem (Sternier, 2007), the other is a list of different strategies, similar to what Polya (1957) described. This structure of the problem-solving process, that on the one hand supported thoroughness as well as "any strategy goes" and on the other hand there were strategies to choose from in a list indicating that they (if not better) at least were more difficult down the list. Another theme was how problem-solving was treated in the textbook, two pages at the end of each chapter, compared to how Mary did it, one lesson each week.

In these themes, Mary emphasised the mathematics content and problem-solving strategies differently in different types of lessons and the textbook had yet another approach. This made me take interest in how problem-solving as teaching practices was constituted in Mary's teaching and in the textbook, in relation to the three approaches: teaching for, about and through problem-solving. The three approaches were adopted as an analytical framework, which was used to read and code the data.

Mary enacted all three problem-solving approaches, although she attended more to teaching about problem-solving than the other two. This could be seen both in the organisation of her lessons, and in the feedback the students received during lessons. However, the same emphasis could be found in the textbooks Mary used. This organisation and emphasis of problem-solving was described to be effectively encouraged by how the regular mathematics textbook separated problem-solving from the other content. The national curriculum was also described as privileging teaching *about* problem-solving over teaching *for*, or *through* problem-solving. It was at the intersection of Mary's teaching and institutional structures, it became clear how the weakly framed curriculum and other resources contributed to tensions in Mary's problem-solving teaching. Drawing on Schroeder and Lester (1989), who claimed that

all three approaches are necessary for students to gain deepen in-depth understanding of mathematics, we discuss Mary's uneven enactment of the three and how that could be related to the lack of specificity in the Swedish national curriculum, and the unregulated textbooks. The curricular resources used by Mary, provided full support for teaching *about* and *for* problem-solving, but little support for teaching *through* problem-solving. Due to the emphasis on teaching *about* problem-solving, Mary, and the textbooks privileged different problem-solving strategies. The communication privileged by the academic mathematics, such as generalisations, were rarely visible in either Mary's teaching or in the curricular texts.

5.2.2 Revisiting Article 2 as Didactic Praxeologies

The analysis from Article 2 gave rise to three different didactic praxeologies of problem-solving. In teaching for problem-solving, problem-solving is a part of the technology, the rationale for why mathematics concepts should be taught. In teaching about problem-solving, problem-solving and problem-solving strategies are the task, what to teach, and finally in teaching through problem-solving, problem-solving is the technique, the means of teaching, for example, a mathematical concept, see Tables 11, 12 and 13.

Table 11. Didactic praxeology, from Mary's communication inferred as teaching for problem-solving.

| <i>Praxis, the know-how for teachers</i> | |
|---|--|
| <i>Task</i> | <i>Technique</i> |
| How to teach basic mathematical content matter. | Begin each lesson with a discussion of the current content matter. Separate content based lessons from problem-solving lessons. |
| <i>Logos, the know-why for teachers</i> | |
| <i>Technology – rationale for techniques</i> | <i>Theory – overall rationale</i> |
| Basic mathematical content knowledge is a prerequisite for problem-solving. | Problem-solving is an application of mathematics. |

In this first didactic praxeology of problem-solving, the task was inferred to be how to teach basic mathematical content matter. The technique, in this praxeology, was inferred from how Mary organised her mathematics teaching differently in her problem-solving lessons compared to her regular mathematics lessons where she taught mathematics explaining and discussing the content matter before the students got to work, but also from the very fact that she separated problem-solving from the regular mathematics teaching. The technique for teaching this mathematics content matter, in relation to problem solving, was to discuss the content matter at the start of each lesson, and to

teach the content matter separate from the teaching of problem solving. This separation of problem-solving from the regular mathematics teaching was also privileged in the mathematics textbooks Mary used. The technology was inferred from how Mary discussed the role of problem solving, sometimes as the goal of mathematics teaching. She described how the students needed to know, for example, the four principles of calculations and other basic mathematical skills so that they could engage in problem-solving. There was also a support inferred from the curriculum in a rationale for problem-solving where the use of mathematics in problem-solving was emphasised. From this, a didactic technology states that mathematical content knowledge is a prerequisite for problem-solving, which would argue for the didactic techniques. A didactic theoretical principle was also inferred from the same communication, that problem-solving is an application of mathematics.

Table 12. Didactic praxeology, from Mary's communication inferred as teaching about problem-solving.

| <i>Praxis, the know-how for teachers</i> | |
|---|--|
| <i>Task</i> | <i>Technique</i> |
| How to teach mathematical problem-solving strategies. | Show the students a structure for a problem-solving process. Show, and discuss a rich variety of solutions. |
| <i>Logos, the know-why for teachers</i> | |
| <i>Technology – rationale for techniques</i> | <i>Theory – overall rationale</i> |
| Being a problem-solver is to be able to solve a problem using many different methods. | Mathematical work requires flexibility and creativity. |

The task in this praxeology was inferred to be ‘how to teach mathematical problem-solving strategies’. The techniques for this were inferred from Mary’s teaching in the problem-solving lessons. Mary used a structure for a problem-solving process which was inferred as one technique for teaching problem-solving strategies. At the end of each problem-solving lesson, where the students showed and discussed their solutions, Mary privileged a variety of solutions when she asked for different and unusual solutions. The discussion and privileging of a rich variety of solutions was also inferred as a technique. The technology was inferred from the emphasis of different solutions as a quality in problem-solving. From all this, and from Mary’s privileging of creative solutions when she praised the students who had thought outside the box, a theoretical principle about mathematical work was inferred. Flexibility and creativity were inferred as essential for mathematical work.

In the didactic praxeology of teaching through problem-solving (see Table 13), problem-solving was inferred as a technique for the teaching of mathematical content matter. The didactic task in this praxeology would be ‘how to teach mathematical content matter’. From Mary’s teaching, this was inferred from different occasions where she drew on mathematics, discussing with her students, when they were engaged in problem-solving. From this a didactic technology was inferred. If posing a problem is a relevant teaching technique for teaching mathematical content, there should be an argument that mathematical ideas can appear when problematised and by extension learned. From the use of problem-solving to teach mathematics by problematising mathematical ideas, a theoretical principle emerges about mathematics being ideas rather than procedures or facts.

Table 13. Didactic praxeology, from Mary’s communication inferred as teaching through problem-solving.

| <i>Praxis, the know-how for teachers</i> | |
|---|---|
| <i>Task</i> | <i>Technique</i> |
| How to teach mathematical content matter. | Pose a problem where this content is problematised. |
| <i>Logos, the know-why for teachers</i> | |
| <i>Technology – rationale for techniques</i> | <i>Theory – overall rationale</i> |
| In problem-solving properties of mathematical ideas can appear so they can be discussed and also learned. | Mathematics is a problem-solving activity where mathematical ideas are central. |

Teaching about problem-solving, was the dominant approach out of the three approaches (for, about and through) used for the analysis in the article in Mary’s teaching. Either this approach seems to be affected by these curricular materials, or Mary and the materials can be seen as parts of the same teaching culture. The curricular tradition where textbook use has been shown to have a negative correlation to problem-solving (Boesen et al., 2014), strongly supports the praxeologies of teaching for and about problem-solving. Even if Mary practised the teaching through problem-solving approach, the other two approaches were more visible in her teaching as well as in her communication about problem-solving teaching.

In this reformulation of Article 2 as didactic praxeologies it becomes clear that the three approaches to problem-solving are grounded with different theoretical principles as well as arguments for mathematics teaching. These principles do, however work together. Mathematical principles can be applied in a flexible and creative way. The study did focus on the distribution of these principles. In Mary’s teaching, the textbook as well as in the national curriculum, it was the flexibility and creativity and the application of mathematics that were privileged. The reformulation also showed how problem-solving has

different roles in the different praxeologies. This was detected, but not visible without a framework such as praxeology. In one lesson all three praxeologies are evident, which means that Mary balanced three meanings of problem-solving at the same time.

5.3 Article 3: Mathematics Teachers' Teaching Practices in Relation to Textbooks: Exploring Praxeologies

5.3.1 Summary of Article 3

In Article 3, Mary's teaching was studied in relation to the textbook that was used in her classroom. The article explored the affordances of adopting praxeology (Chevallard, 2006) as a framework, to study classroom practices, in relation to how similar practices were privileged in the textbook. The aim of the article was to explore how praxeology can be used to compare a mathematics teacher's practice, as observed in the classroom, with the practice, as inferred from the textbook. The teacher's practice and the textbooks' were studied as parts of the same social environment, which Lerman (2000b) describes as a study with a social perspective. A literature review of research on teaching shows that studies with a social perspective, such as sociological studies, often concern a macro-perspective of a teacher's ecology. An alternative to this is described as the "practice turn", where both the micro-perspective of the individual teacher and the macro-perspective of the ecology are studied (Cetina, Schatzki, & Von Savigny, 2005). The aim of the study was to study the micro-sociological layer to a classroom practice (the teacher's teaching and the teaching described in the textbook). More specifically, the particularities of mathematics instruction in a classroom in relation to instructions described in mathematics textbooks were studied, while adopting praxeology as a framework. In this, it was possible to relate how a mathematics teacher's praxeology, as observed in the classroom, corresponded with the praxeology, as inferred from the textbook.

Through the teaching of the primary school mathematics teacher, Mary, it was possible to infer how the knowledge about some content matters was organised in mathematical praxeologies. In a praxeological analysis of how the content matter, angles, was taught (in Mary's mathematics teaching) or described to be taught (in the textbook) two mathematical praxeologies were compared, see Table 14.

Table 14. Mathematical praxeologies of how to measure angles.

| <i>Textbook praxis</i> | | <i>Classroom praxis</i> | |
|---|---------------------------------------|---|---|
| <i>Task</i> | <i>Technique</i> | <i>Task</i> | <i>Technique</i> |
| How to measure an angle | Measuring an angle with a protractor. | How to measure an angle | Measuring an angle with a protractor, and decide what scale to use on the protractor through a comparison with a right angle. |
| <i>Textbook logos</i> | | <i>Classroom logos</i> | |
| <i>Technology/Theory</i> | | <i>Technology/Theory</i> | |
| An angle is defined as two rays meeting in a common point, but also by naming the parts of an angle. The unit degree is a measurement of a rotation. Acute and obtuse angles are described to be smaller and bigger than 90° respectively. | | An angle is defined by naming the parts of an angle. The unit degree is a measurement of a rotation. The scope of acute and obtuse angles is described. Acute angles are described to go from zero towards 90° and obtuse angles are described to go from 90° and up. | |

The *tasks* in both contexts were inferred from the exercises in the textbook. Mary used the textbook in her classroom practice, so the exercises were the same in both contexts. The *techniques* were very similar. Both contexts described how an angle is measured with a protractor. In both contexts there was a practical description of how to measure an angle. The textbook used an illustration to show how to position a protractor and Mary showed, with her actions, how it was used. Mary did, however, also show a technique where an imagined right angle was used to estimate if the angle to be measured was acute, right or obtuse, and which scale to read on the protractor. The *technology/theory* for this praxis also had similarities in the two contexts. In both, there was a technical definition of an angle based on the parts of the angle. In the definition of an angle, the two contexts also related to the acute, right and obtuse angles. The textbook stated that an acute angle is smaller than 90° and an obtuse angle is bigger than 90° . In Mary's explanation the differences between these types of angles were also described, but as measurements of a rotation, and she drew from the notion of limits when she described how the acute angle can be very close to 90° . This was inferred as a language drawing from scholar mathematical notions, and forms of expressions.

In this study, it was concluded that praxeologies worked well as a means to compare contexts and to gain access to the details of how a mathematical content was organised. In the methodology explored as described above, praxeologies with similar tasks were compared, in terms of the techniques and technologies/theories that were communicated. The analysis revealed what was communicated, but also how differences in how the mathematical content was organised in the two different contexts. In this comparison, we could also see that the use of a textbook does not, at least not completely, predetermine what is taught and how. Even if the textbook was used as the primary source for exercises, Mary did draw from more general and theoretical notions compared to the language in the textbook which consisted more of statements. The textbook may have constrained Mary's teaching due to the absent general and theoretical rationale for mathematical activities.

5.3.2 Revisiting Article 3 as Didactic Praxeologies

The analysis in article 3, gave rise to two mathematical praxeologies. To revisit this article as didactic praxeologies I have inferred the task to be how to teach the mathematical praxeologies described above, see Tables 15 and 16. From the detailed description of the analysis in the article, it is also possible to infer *techniques*, and *technology/theory*.

Table 15. Didactic praxeology for how to measure angles, from Mary's teaching

| <i>Praxis, the know-how for teachers</i> | | |
|---|--|--|
| <i>Tasks</i> | | <i>Techniques</i> |
| How to teach: | Define angles by naming the parts. | Demonstrate how to measure angles with a protractor. |
| How to measure an angle with a protractor, deciding what scale to read through a comparison with a right angle. | Describe the unit degrees as a measurement of a rotation. Describe the scope of acute and obtuse angles. Describe acute angles to go from zero towards 90 and obtuse angles to go from zero and up. | Name the different types of angles, acute, obtuse and right as well as the parts of an angle. Show how to compare the measured angle with an imagined right angle. Visualise a rotation with gestures. |

Technology – rationale for techniques

Theory – overall rationale

The students need to be familiarised with some angles so they can be used as references for rough estimations of angles, which argues for a presentation of acute, obtuse and right angles as types of angles.

Angles being a representation of a rotation argues for a dynamic presentation of angles.

Angles are mathematical objects that can be named and measured.

Angles represent a mathematical idea, rotation.

Mary showed how to use the protractor to measure angles in practice on the white board, she drew angles, both to show the parts of an angle, but also to show what right, acute and obtuse angles look like. She also showed the scope for acute and obtuse angles, creating an angle with her arms and she showed how the angle became bigger and smaller as she moved one arm. All these actions were inferred as didactic techniques in this praxeology. Mary told her students that she wanted them to have a method for comparing an angle they are supposed to measure, to a right angle, so they can decide for themselves what scale to use on the protractor. From this, an argument emerged for demonstrating the three types of angles so they can be used as a reference for further measurements. Mary's demonstration of the angle as a rotation was also inferred as this representation of a rotation demands a dynamic presentation of angles and not just a static showing specific angles. From these two theoretical principles about angles, it was inferred that angles are mathematical objects to name and measure but they also represent a mathematical idea.

Table 16. Didactic praxeology for how to measure angles, from the teaching described in the textbook

| <i>Praxis, the know-how for teachers</i> | |
|---|--|
| <i>Task</i> | <i>Technique</i> |
| <p>How to teach: How to measure an angle with a protractor.</p> <p>An angle is two rays with a common starting point. An angle has two angular legs meeting in an angle point and the angle is marked with a bow. The unit degrees is a measurement of a rotation. An acute angle is smaller than 90° and an obtuse angle is bigger than 90°.</p> | <p>Show the students how to place the protractor on the angle, in an illustration.</p> |
| <i>Logos, the know-why for teachers</i> | |
| <i>Technology – rationale for techniques</i> | <i>Theory – overall rationale</i> |
| <p>The students need to be familiarised with some angles so they can be used as references for rough estimations of angles, which argues for a presentation of acute, obtuse and right angles as types of angles.</p> | <p>Angles are mathematical objects that can be named and measured.</p> |

In the textbook, there was an illustration of a protractor positioned on an angle. The information to the students was that a protractor is used to measure angles. To show such an illustration to the students was inferred as a didactic *technique*. The illustrations were inferred as a visualisation of angles which in turn was inferred as the rationale for showing them, the *technology*. From these activities the same technology was inferred as from Mary's teaching. That the three types of angles need to be demonstrated so they can be used as references for further measuring. Similar to Mary's teaching, I inferred a theoretical principle from this. Angles are mathematical objects that can be named and used. The textbook did not stress the angle as a rotation. There was a brief remark but this was hidden in a description about how to measure angles which is why

I did not infer the theoretical principle of angles being a representation of a mathematical idea from the textbook's explanations of angles.

In Article 3, one finding was that Mary used a more specific language, closer to how angles are defined mathematically. This re-interpretation of the findings in Article 3, as two didactic praxeologies has shown what this difference may be grounded in. From Mary's teaching, two theoretical principles could be inferred. Both mathematics as objects and mathematics as ideas. From the textbook, only the former principle could be inferred. Depending on what principles that underlie teaching, the communication about the content matter should be different and in this case, Mary's students were invited to view angles as something more than objects that should be named and measured.

5.4 Article 4: Principles and Arguments for the Teaching of Rational Numbers in Different Contexts

5.4.1 Summary of Article 4

In Article 4, Mary's teaching was studied in relation to the three other contexts from the ecology, the teacher group, the textbook, and the national curriculum in order to answer the following question: "How are some theoretical principles for teaching rational numbers expressed in the ecology of a teacher's teaching, and how may the way these principles are expressed enable and constrain teachers' teaching of rational numbers?" The aim of the study was to investigate the grounds of the teaching of rational numbers in the different contexts of the ecology, and how these grounds can be described to facilitate specific mathematics teaching practices. In this case it was the teaching of rational numbers which was studied.

The ecology of Mary's teaching was in this case four contexts (Mary, teacher group, textbook, curriculum), and in these four contexts there was an expressed knowledge of how to teach rational numbers. This knowledge was interpreted as didactic praxeologies describing what didactic task (what to teach), what didactic technique (rationale for how to teach this specific task), what didactic technology (why this specific task is taught this way), and didactic theory (sets of ideas or principles that anchor mathematics teaching) (Chevallard, Bosch, & Kim, 2015; Chevallard & Bosch, 2014). In the article, it was the didactic technology and theory of the teaching of rational numbers that were studied.

To study a didactic theory, a framework was adopted where a set of principles was described in terms of six different mathematical values grouped as opposing pairs (Bishop, 1991), see Table 17.

Table 17. The opposing pairs of mathematical values, developed with inspiration from (Österling, Grundén, & Andersson, 2015)

| Dimension | Pairs of opposing values | |
|---|--|--|
| <i>Ideological values:</i> the ideology of mathematics | <i>Objectism</i> – concretising and applying ideas of mathematics | <i>Rationalism</i> – reasoning, argument and proofs |
| <i>Sentimental values:</i> What sensations mathematics can bring | <i>Control</i> - a sense of certainty and power through mastery of rules | <i>Progress</i> – the sense of having the courage to engage in mathematics even if it is unknown territory, e.g. trying alternative solutions. |
| <i>Sociological values:</i> who can do mathematics | <i>Mystery</i> – fascination and mystique of mathematical ideas and their origin | <i>Openness</i> – mathematics is democratically open for anyone to use and explain |

These values were used to describe the principles of mathematics that could be inferred to anchor the teaching. Having chosen these two frameworks, three research questions were posed. How are traces of the mathematical values distributed in the ecology? How are traces of the sub-constructs distributed in the ecology? How can the distribution of sub-constructs be understood in relation to the distribution of mathematical values?

To study a didactic technology systematically, a framework was adopted where rational numbers were described to include six sub-constructs, which in turn gave rationale for different teaching activities (Charalambous & Pitta-Pantazi, 2007).

- Part-whole – rational numbers as part-whole relationships
- Ratio – rational numbers as a proportion, a comparison of two numbers
- Operator – rational numbers as a transformer that shrinks or enlarges a number or a discrete object
- Quotient – rational numbers as the relationship between two quantities
- Measure – the size of rational numbers which can be placed on a number line
- Decimal – rational numbers as a generalisation of a decimal numeration to a fractional situation

The findings showed how theoretical principles in terms of mathematical values were privileged in the communication. Objectism, control, and openness, were privileged over: rationalism, progress, and mystery. The findings

also showed how technological principles in terms of sub-constructs of rational numbers, were distributed. Quotient and decimal were privileged, operator was underprivileged, while measure and part-whole were neither/nor. The general picture of how the mathematical values were privileged was very similar between the contexts. Using brief summaries of how each category was visible, Table 18 shows how the sub-constructs and mathematical values were privileged in the ecology as a whole.

Table 18. Didactic praxeology for how to teach rational numbers

| <i>Praxis, the know-how for teachers</i> | |
|--|---|
| How to teach rational numbers | <ul style="list-style-type: none"> - Explore equivalent fractions and decimal numbers, as translations. - Explore properties of decimal numbers, especially place value in relation to the positional system of numbers. - Let the students practice calculations with decimal numbers. - Visualise part-whole relationships as parts of an object. |
| <i>Logos, the know-why for teachers</i> | |
| <i>Technology</i> – rationale for techniques | <i>Theory</i> – overall rationale |
| <ul style="list-style-type: none"> - Rational numbers are expressed in different forms of expressions as in equivalent fractions and decimal numbers. To be able to translate between these forms of expression is to have a language for rational numbers (the quotient sub-construct). - Decimal numbers are a central aspect of rational numbers, which is why it is important to understand the properties of them (the decimal sub-construct). - Visualising part-whole relationships enables an understanding of fractions (the part-whole sub-construct). - One way of problematising fractions is to discuss an object partitioned into parts with different proportions to the whole (the ratio sub-construct). - One important aspect of knowing decimal numbers being able to identify | <ul style="list-style-type: none"> - Mathematical concepts needs to be visualised and concretised (objectism). - Mathematics represents predictable methods, facts and procedures which provides security to mathematical work (control). - Communication is central in mathematics, with talking about mathematical concepts as well as presenting and discussing solutions representing central activities (openness). - To engage in mathematics is to be able to use different strategies, to be creative and to think outside the box (progress). - Mathematical ideas need to be problematised and discussed (rationalism). - Mathematics can sometimes be to explore unknown territory or to search for the unknown (mystery). |

them on a number line (the measure sub-construct).

- One way to problematise fractions is to pose problems where fractions as parts of an amount are to be calculated (the operator sub-construct).
-

Table 18 presents only the most common activities, as didactic techniques, from which the arguments and theoretical principles arose. The technological arguments are shown as examples of how the sub-constructs were recognised, with the most common sub-construct first. The theoretical principles are shown as examples of how traces of the mathematical values were recognised, with the most common mathematical value first.

Placing these technologies and theories in relation to each other, it became possible to describe how the privileging of practices grounded on objectism, in which representations of rational numbers rather than the mathematical ideas behind them were privileged; for example, part-whole represented as partitioned objects or quotient as equivalent representations of rational numbers; teaching activities grounded on control, where routine facts and predictable methods were privileged, such as rote learning of equivalent fractions and decimal numbers; or learning place values in decimal numbers to enable calculations with decimals. Openness grounds teaching activities whenever communication is central; for example, expressing rational numbers as different forms of expression. The privileged mathematical values were seen to ground not only which sub-constructs were privileged, but also how they were taught.

The privileging of objectism was discussed in order to generate an emphasis of sub-constructs where visualisations were more common, for example part-whole. Privileging control may, instead, generate an emphasis on number facts like equivalent fractions and decimal numbers (in this study, quotient). Even though it may be perfectly reasonable in grade five for mathematical values not to be evenly distributed, we need to consider what is lost when, for example, rationalism and progress are not privileged. With more balance between the theoretical principles underpinning the communication of mathematics teaching at all contexts, students of all grades may be invited to participate in broader mathematical reasoning and exploration of mathematical ideas. The ecology of Mary's mathematics teaching is very homogenous, which was described as a strong environment to go against. If we want to change mathematics teaching it is not enough to target the teacher with training activities, it is the whole environment that needs to change.

6 Conclusions

I set out to study mathematics teaching as part of an ecology. Exactly how to do that has been developed along the way. During the project, I have made a number of analyses of different parts of this ecology. I have tried different theoretical approaches and different ways to understand how a teacher's environment, in terms of an ecology, works, what was privileged in it, and how this may affect what was actually taught to students. I have written one conference paper and three journal articles where this work has been described to answer four different research questions.

- How are the justifications for Mary's mathematics teaching constituted, in relation to a teacher group discussion? (Article 1)
- How is Mary's teaching of problem-solving constituted in relation to the curricular materials available to her? (Article 2)
- How can praxeology be used to study a mathematics teacher's teaching practice, and the teaching practice as inferred from the textbook as interrelated? (Article 3)
- How are some theoretical principles for teaching rational numbers expressed in the ecology of a teacher's teaching, and how may the way these principles are expressed enable and constrain teachers' teaching of rational numbers? (Article 4)

In the following pages I will describe conclusions I have drawn from this study. In the discussion (Section 7) I will discuss them in relation to the research filed as described both in the introduction (Section 1) and in the literature review (Section 2).

From the four articles and the revisiting of the three first, in terms of ATD, I draw some conclusions about the ecology of Mary's teaching and Mary's opportunities to work as a professional teacher within it. One conclusion is that the theoretical environment in the ecology of Mary's teaching does not include much explicit theory. All theoretical principles in the didactic praxeologies were inferred implicitly from the activities and communication. This could be a result of, on the one hand, Mary, her colleagues, and, on the other hand, the textbook and curriculum authors not knowing enough theory to express it. This, in my opinion, is unlikely. Instead, I conclude that the theoretical dimension of didactic knowledge in this ecology was not explicit enough, in

any of the contexts. Mathematics teaching is not explicitly discussed with arguments grounded in theoretical principles. Justifications are rather given in statements about the importance of content matter or procedures. Teaching activities are also justified with subsequent teaching in mind. Knowing equivalent representations of rational numbers was inferred to be important for subsequent calculation activities. Theoretical principles not being explicitly communicated prevented these principles from being challenged and discussed.

Another conclusion is that even if the ecology as a whole was focused on *praxis* rather than *logos*, Mary did, go beyond the privileged practices in the ecology as a whole. As shown in Article 3, she drew from angles as a representation of a mathematical idea and not only as a mathematical object, which was the case in the textbook. In Article 1, Mary based her arguments, partly on other principles compared to the principles inferred from the teacher group as a whole, when she, as the only teacher, resisted the timed test. In Article 2, problem-solving was found to be task, technique and theory where problem-solving as a task was privileged in all contexts. Mary privileged problem-solving as a means to teach mathematical ideas, which was not the case in the textbooks. In Article 4, as Mary's theoretical ground, she almost always privileged the mathematical values described as closer to mathematics more than the other contexts. She emphasised fractions as a mathematical idea more than the other contexts did. This indicates that Mary was not only engaged in internal transposition, drawing from the noosphere (textbook and curriculum authors). To some extent, she was engaged in external transposition drawing from other sources, including scholar mathematics. She often drew from the textbook context, but she also drew from other sources, sometimes the result was more depth than what was visible in the ecology. This is also an example of how a teacher is not subject to other contexts' determination. Mary participates actively in the determination of how she teaches. The privileging of timed tests in the teacher group influenced how Mary did, but not entirely. In other words there was a co-determination. Mary did in the end do multiplication tests, but they were adjusted in a way that met her express criteria. If Mary's mathematics teaching was the only studied context, the earlier discussion about the tests would have been lost. This discussion influenced her teaching, even if she did not entirely follow the privileged practice of the teacher group. Mary's privileging of mathematics and mathematics teaching practices were similar to the other contexts, but she clearly had a scope to draw from other sources or be creative.

There were many things enabling Mary's mathematics teaching, such as support from individual colleagues and lectures she was inspired by. One thing did come through from the whole study, as an enabling factor for Mary and her colleagues, that is the mathematics textbooks. To have tasks, already organised by content matter was described by the teachers as necessary, so they could manage their time. Mary expressed this, but it was also visible in the teacher group's discussions in Article 1. The fact that they used the textbook

as a means to differentiate their teaching and as the source for assessment material shows that the textbook was an important tool. For Mary, it was also obvious that the freedom not to be bound to the regular textbook, but also to be able to choose other materials, in the same manner, was important to her. In Article 4, Mary added exercises and methods from other sources. She could also adjust the material to the class and content she taught, for example, choosing a more advanced textbook for her teaching of angles as in Article 3. This freedom was inferred as an affordance of its own.

Finally, I draw some conclusions about what constraints to a teacher's teaching I can see in these four studies. The unspecified curriculum was pointed out as a restriction in Article 2. This is possibly a restriction, but with a more explicit theoretical backing, it might have been easier to interpret the curriculum. My conclusion is, as claimed above, that the lack of explicitly expressed theoretical principles for the teaching activities, in the whole ecology, is a restriction in itself. This is not helping teachers to go behind the simple statements about what is important. The curriculum offer little or simplified information about why different content matter is important. This also restricts teachers in interpreting the curriculum and, by extension, it prevents teachers from being professionals.

The privileging of calculations, both as an activity (Article 1), or a prerequisite for mathematical activities (Article 2), but also as an application of mathematical content (Article 4), could also be a restriction to a teacher's whole practice. If calculations are the objective of mathematics teaching, it prevents teachers like Mary, who expresses an interest in mathematical concepts and problem-solving, from teaching this in-depth. An example is the teaching of rational numbers that could be about exploring rational numbers in relation to the number line and proportional reasoning, but instead it became procedures to translate rational numbers into decimal numbers and make them possible to calculate. This was evident in the whole ecology.

The privileging of a simplified school mathematical language, where definitions are taken for granted, may also be a restriction. In Article 2, there was no communication in the textbooks or in the curriculum supporting generalisations or a privileging of effective or elegant solutions to a problem. Instead, a variety of solutions was privileged in all three contexts included in the study. Naturally, students need to be invited to participate in problem-solving and their solutions should be included and encouraged. However, if effective solutions are never discussed, the students could also be happy with a simple illustration, when they, instead, could learn more elegant solutions. This was also evident in Article 3, when definitions were more taken for granted in the textbook than in Mary's teaching. Communication where concepts were defined and derived was not privileged in either the teacher group, textbook, or in the national curriculum, at least not in the recommendations for school years 4-6.

Finally the homogenous ecology where similar concepts, practices, arguments and theoretical principles were privileged, were concluded to be a restriction in itself. Privileging the same praxis with little explicit logos it is only reasonable for teachers to stay within this ecology. There is little influence to challenge these practices. When they are challenged, as by Mary when she questioned the timed tests or when she asks for more problem-solving activities, she needs to go against her whole environment.

These are the conclusions from the four studies where I have traced didactic co-determination. There were more restrictions and more opportunities visible within the data material, but these are what was seen in the ecology as a whole. These conclusions as well as the findings will be discussed in relation to research fields and the Swedish context in the following section.

7 Discussion

In this study, I aimed to deepen the understanding of how an ecology of a mathematics teacher's teaching enables and constrains teaching practices. In this section I will discuss my findings with an emphasis on how mathematics and mathematics teaching was privileged in Mary's teaching and in the other contexts of the ecology. The discussion will begin with the big picture in discussing how teaching in relation to an ecology may be understood both from an international perspective as well as from the Swedish context.

7.1 Understanding Teachers' Teaching

This study represents an attempt at a systematic analysis of the micro-context of a mathematics teacher's teaching in relation to the macro-context represented by the three contexts: teacher group, textbook and curriculum. The first and third article each included two contexts, which is the most common in other studies. The second article included three contexts, while the fourth article included four contexts. With all four articles seen together, it became possible to describe how the micro-context of the mathematics classroom relates to the contexts of the macro-context seen as an ecology.

In all analyses made in this study, there was a strong resemblance between the contexts. They all emphasised similar approaches to problem-solving, sub-constructs of rational numbers, mathematical values, or explanations of angles. This can be seen as a confirmation of educational contexts being situated within a culture, which has been described from different perspectives (Andrews, 2010; Bishop, 1991; d'Ambrosio, 1985). One example is how calculations stands out as a privileged practice which has been described as deeply rooted in the history of mathematics education in general (d'Ambrosio, 1985) as well as in Sweden (Lundin, 2008). Studies of teachers' knowledge and their classroom practices will only show what was happening in the classroom. This may give a deep understanding of this context, which, for example, Rowland, Huckstep and Thwaites (2005) achieved when they developed a framework for classroom observations. We will, however, never know what constituted the teaching practice and the expressed knowledge possible at that time. If we want an understanding of what enables and constrains teachers' mathematics teaching practices, it is essential to understand how the co-determination of mathematics teaching works, with a focus on how the teaching of mathematics

is constituted. Teachers' teaching needs to be studied in relation to the levels of the ecology and how mathematics and mathematics teaching practice is privileged in different contexts. To educate teachers about the educational history could possibly be important but teachers are only one part of the culture. Restrictions to teachers' practices need to be discussed and debated in relation to the many contexts that teachers are part of.

Nations write curricula to define what should be taught in schools, to produce future citizens as ideal persons with a set of privileged practices and knowledge (see Cummings, 1999). Teachers are trusted with different degrees of freedom in different countries; from the strict control China has had (Xu, 2011) to the loose American school system which enables widely spread uses of curriculum, as described by Remillard (2005). This study was carried out in a system in between these two extremes. In Sweden, teachers are expected to be autonomous, but claimed to be in a difficult situation where this autonomy is more or less an illusion (Skott, 2004). This study contributes with insights into how the unspecified competence-based Swedish curriculum may enable as well as constrain mathematics teaching. One example from this study is the practice of assessing automated recall skills with timed tests. There was no advice in the national curriculum suggesting this method, only a goal about the students having a number sense (Skolverket, 2011a). The teachers were thus left to decide how to teach and how to assess the automated recall skills of multiplication. With the conflicting demands of students' well-being versus the importance of automated recall skills, and a strong tradition of how such skills were assessed, leaving the teachers with no alternative practices. This is a clear example of the autonomy being only an illusion. With no advice from the curriculum to guide the teachers, and within a strong tradition, it was not possible to choose an alternative practice. The teachers' collectively made the only possible choice. I am not convinced that the answer to this is to fill the national curriculum with advice. Another way is to give the teachers more space to engage with research, so they may have more sources of possible practices than their tradition and experience provide.

The context of the classroom is not situated only in a culture or in a curricular context. It is situated in both these contexts together with several others, together forming a macro-context. To study one of these contexts as an example of the macro-context in relation to the classroom has been done in previous research (cf. Haggarty & Pepin, 2002; Jablonka et al., 2016; Nie et al., 2013; Sullivan et al., 2013). Many studies conclude the necessity of taking macro-contexts into account (Gellert et al., 2013; Gutiérrez, 2013; Jaworski & Potari, 2009; Sullivan et al., 2013). This study takes on such a challenge and shows how several contexts can be studied with the same framework so that the teacher's teaching of mathematical content matter can be understood in relation to its macro-context. The contexts studied had a direct influence on Mary's teaching. Not only as something to relate to. These contexts were analysed in the same way as Mary's teaching, which conveyed opportunities to

say something about what it was that permeated the contexts and what may have been the common grounds.

Seeing each context as an expression of a praxeology, didactic or mathematical, made it possible for me to trace didactic co-determination within this ecology. Searching for, not only tasks and techniques, but also technologies and theories, made it possible to discuss the principles that may have grounded the teaching activities. This methodology of addressing multiple contexts may be a contribution to the international field of research in mathematics education. This study serves as an example of such a study and as such it is a contribution to the insights and conclusions I have been able to draw, studying a teacher's teaching in relation to an ecology with a specific interest in mathematics and didactics of mathematics.

7.2 A Swedish Perspective of Understanding Teachers' Teaching

This thesis is situated in the Swedish context. Thus, it contributes specifically to Swedish research field in mathematics education. Other studies, also conducted in a Swedish context, discuss institutional affordances and constraints from a Swedish perspective. Communication from different official contexts has been discussed as weaving its way into the formation of classroom practices as institutional traces (Boistrup, 2010) and governing elements (Boistrup, 2017), or as a voice of the institutionalised authority (Norén, 2010). Another way of addressing a teacher's environment in a Swedish context has been to see a teacher as a part of different communities of practice, showing how sensitive new teachers can be to the environment they begin to work within (Palmer, 2013). In relation to these studies, this study offers a way to actually trace the connections between the contexts, by studying what practices, in relation to specific content matter, were privileged in the mathematical and didactic praxeologies in the different contexts.

Other studies from a Swedish context describe Swedish mathematics teachers as dependent on their textbooks (Bergqvist et al., 2010; Boesen et al., 2014; Englund, 1999; Skolverket, 2003; SOU 2004:97, 2004). Mary, and her colleagues, did structure their mathematics teaching according to the textbook, but Mary used the regular book more as one of many resources, similar to how Sullivan et al. (2013) describe how textbooks are used. This was probably necessary, looking at the time Swedish teachers spend on planning and assessing (10%) (Skolverket, 2015). In this time, it is probably wiser to use exercises in a book, and use the ten percent to plan demonstrations and explanations. Mary clearly used her time to develop problem-solving lessons, and searching for problems to challenge and engage her students. This practice was not privileged in the textbook, but the textbook enabled it, since there

were prepared exercises. Mary saved time, using the pre-produced exercises in the textbook. There is, however, a risk that the possibility a textbook constitutes also becomes a restriction. In this study, the textbook (including the teacher guide) does not offer a clear theoretical rationale for the didactic activities. The mathematical rationale is also limited to simple explanations, where definitions were taken for granted. Without clearly communicated logos for teaching activities, teachers will not be encouraged to negotiate the theoretical grounds for their teaching. If procedures are privileged above explanations, in textbooks, as Gonzales et al. (2013) show, there is a risk that the textbook also restricts teachers from engaging in explanations, both when it comes to mathematical and didactic activities. The solution is, however, not that a mathematics teacher has to work without a textbook. The solution may rather be, as Mary does, to use textbooks when they are useful and at the same time reach for other sources and other ways to communicate.

Calculations were privileged in all contexts and in all the studies. How calculations were privileged differed between the studies. In the teacher meeting the privileged position of calculations caused the teachers to use an assessment method they had already described as contributing to students having anxiety attacks. In Article 4, calculations were privileged in all four contexts: communication about teaching rational numbers, as a result of their privileging of equivalent fractions and decimal numbers, the properties of decimal numbers as well as calculations with decimal numbers. This privileging of calculations was made at the expense of other notions of rational numbers that were left unexplored. Lundin (2008) shows how calculations have had a prominent position in Swedish mathematics education, right from the time when school became a public institution. He claimed that calculations became a way to discipline the students. This tradition runs deep within the Swedish school system. Which is still evident in how calculations are privileged in both national curriculum and textbooks, which are, according to Andrews (2016b) deeply rooted in this culture.

The case of Mary enabled me to describe how a teacher's practice is not just a product of a teacher's knowledge and effort, which is the focus of many studies of mathematics teachers (e.g. Goulding, Rowland & Barber, 2002; Shavelson & Stern, 1981; Taylan & da Ponte, 2016). Even if such studies give important insights, they do not include other contexts even if they often point to possible influence from them. An example of this is Taylan and da Ponte (2016) who discuss possible influences from other contexts (teacher education and research) but these contexts were not part of the study. The knowledge a teacher expresses or enacts in a specific study may or may not reflect the full potential of this teacher, and there is no way to know if it did. Mary knew more and was able to do more than what was seen in her classroom, which was evident in the interviews when she commented on what she could have done or when she drew from literature she did not actively use in her classroom. She was in different ways restricted from using her full potential and

the important question to pose is what constrained her to a limited set of practices. The findings of this study, together with other studies from a Swedish context, show why it can never be enough to target only teachers with professional development projects. Instead, educational policy and textbooks also need to be discussed, criticised and developed. If the ecology remains the same, it still conveys the same communication of mathematics teaching, even if teachers are taught a new teaching method. The theoretical grounds for mathematics teaching need to be discussed and debated in many arenas. Policy makers and other actors of the ecology, as well as teachers, need to negotiate the grounds for mathematics education within the culture. Professional development programmes such as “Matematiklyftet”, which was launched for all mathematics teachers in Sweden during the course of this study, may provide teachers, as individuals or groups, with arguments to resist what they see as harmful to their students, in other words, to be creative insubordinate (Gutierrez, 2016). On the other hand, this was, for a few years, an ongoing professional development imposed on teachers from authorities and such programmes may, according to Montecino and Valero (2017) also be a way to maintain control of teachers. To be under such professional development, which was launched as a remedy for failing results, also conserves deficit explanations of teachers where the teacher is described as inadequate. This is not productive. We need to see both deficits as well as assets within the school system as a whole, and discuss how teachers can be supported to negotiate their teaching on different grounds where they have rich opportunities to use the width and depth of their experience and knowledge

7.3 Didactic Theory in the Ecology

The didactic theory was not explicitly expressed in the contexts of this study, but it was possible to infer from communicated and enacted teaching activities. The inferred didactic theory was found to be central to the co-determination between the contexts. Teachers’ opportunities to form their teaching have been described to be located mostly in praxis, in what tasks to give and what methods to present (Winsløw, 2011). Teachers’ authority is then delimited from the outset (Barbé et al., 2005; Winsløw, 2011). Someone else has already decided what teachers should teach, and the mathematical structure has already been defined and described. Even if this is true, the teachers in this study still drew from notions possible to infer as theoretical principles. As Jaworski and Gellert (2003) describe, theory and practice are always interrelated, even if the way they are linked together may differ.

The contexts were very similar in how they expressed the grounds for their teaching, Mary did, however stretch the boundaries ever so little. When she differed from the ecology in her mathematical activities, it was possible to see how the principles differed as well. One example was in the discussion about

automated recall skills where she based her arguments on both the principle about making mathematics teaching available for all students, as well as the principles the teacher group drew from, which concerned number facts and skills learned separately. Another example was how Mary privileged angles not only as an object as the textbook did, but also as a representation of a mathematical idea. In Article 4, it was also possible to see how Mary drew from the notions of rational numbers and mathematical values in her own way, which was evident when she privileged part-whole more than the other contexts. She also privileged the mathematical values closer to how Bishop (1991) privileged them, compared to the other contexts. This is not to say that Bishop's privileging is the right way to value mathematics in a grade five classroom. What I do say is that Bishop privileges the values closer to the discipline of mathematics, which is what Mary also did, more than the other contexts. A consequence of this observation could be that the textbook and the national curriculum marked a limit for Mary. It may not have been essential for Mary to draw more from principles valuing mathematical reasoning or problematising more, since the curriculum did not. Then it becomes important to investigate what would enable teachers to explore theoretical and technological principles and constructs they would like to base their mathematics teaching on, rather than investigate what she already knows in the situation she is already in. Such studies do show how teachers get new perspectives in their formation of mathematics teaching (e.g. Jaworski & Potari, 2009).

This study had a specific interest in mathematics teaching and how mathematical content matter was privileged in the different contexts. This conveyed a limitation of the theoretical principles that were inferred from the data material. It was only principles connected to mathematics or didactics of mathematics that were included in the didactic praxeologies. Through these it was possible to see how different activities were connected to different principles, as in Articles 1 and 3, or different distribution of principles, as in Article 4. Article 2 shows how problem-solving could be theory, technique and task, where problem-solving as a task was emphasised more in the four contexts. The theoretical rationale was not explicitly expressed in the contexts studied, which is why it was inferred from the activities and communication. If teachers would argue for different teaching activities and the theoretical underpinnings of them, they might also be freer to use the teaching methods and activities they see as productive for their students, instead of using methods causing their students anxiety, as in the case of the timed test in the teacher meeting. A discussion where the theoretical grounds were addressed may have had a different solution, without the teachers having to choose between automated recall skills and student wellbeing.

The role of didactic theory is to generate and found technology, techniques and tasks (Chevallard et.al. 2015) which means that if the didactic theory would be a part of teachers' explicit communication, teachers would generate teaching activities within their own practices according to their conditions,

similar to how a teacher develops teaching activities when engaging in activity theory (Jaworski & Potari, 2009). This does, however take time, and time was precious for Mary and her colleagues.

The fact that the theoretical grounds were not explicitly expressed did not mean that a teacher (or textbooks/curriculum authors) did not know or were aware of didactic theories. The theoretical grounds do, however, become tacit in all contexts. Skott (2004) describes how the social contexts a teacher participates within can restrict the autonomy expected of teachers in a decentralised system. In this study, all contexts describe mathematics teaching without an explicit theoretical rationale, which in itself restricts a teacher working within this ecology, to communicate any theoretical rationale herself. The rationale remains a tacit knowledge. In the communication it can seem as if there are common agreements on what to do and how. A silent knowledge does not have to be expressed as long as everything works (Jahnke, 2014). Theoretical principles may not always need to be expressed, especially if everything works, but if we want to develop mathematics teaching we need to, at least sometimes, express our theoretical principles so they can be challenged and discussed. Teachers (and experts from different fields) wrote both the national curriculum and textbooks. These teachers are a part of the same educational system as Mary, as well as the contexts studied. This could imply that the system works without discussing a theoretical rationale, which was why no one expressed it. So, the curriculum was written without a clearly expressed theoretical rationale, which gave teachers the comfort of following the curriculum, sharing the same language about mathematics teaching and learning. As discussed in the summary of Article 1, if the ecology does not include an expressed theoretical rationale, there is a risk that teachers repeat what others have done, without questioning.

It is not realistic to demand teachers to be up-to-date in all research fields in connection to their teaching. This does not mean that teachers should not engage with research at all. The risk with conclusions, like the ones in this study, is that policy-makers will impose research on teachers as decrees. I align with Biesta (2010) who claims that research should rather be a part of teachers' negotiations than a formula to follow. It is not about making teachers obey research. It is to enable teachers to make decisions based on a broader ground than what is included in textbooks and curricula. One way to make research accessible for teachers' is to encourage researchers to write popular scientific texts for teachers, in Swedish (in this case). This is, however, not a privileged genre for researchers to write in (see Boistrup, 2014). Another way could be to include the theoretical principles and scientific grounds in texts like the commentary material to the national curriculum. If the explanations of the mathematics syllabus also included explicit theoretical principles, or references to research findings, it would be easier for teachers to relate to the

basic ideas behind the syllabus. By extension, this could make the tacit theoretical didactic knowledge more visible, and placed within a context of research.

7.4 Teachers as Professionals

Taking the contexts outside the described ecology of this study, for example public debate, into consideration, there are clearly restrictions to the opportunities to be professional within the ecology of Mary's teaching. There are activities in the Swedish society creating a possible restriction together with an ecology without a communicated theoretical rationale. There has been an increased control over both students and teachers, and PISA has been frequently discussed in the media (Thavenius, 2014). The public debate privileges vague practices such as "katederundervisning" (criticised by Eriksson, 2011) and governmental authorities describe teachers as under-educated, governed by textbooks and unknowing of the national curriculum (Skolinspektionen, 2009). All these activities in the public debate reflect phenomena that restrict teacher autonomy (Wermke & Forsberg, 2017), and by extension, lead to a de-professionalisation of teachers (Hargreaves, 2000) with consequences for both teaching and teachers (Ball, 2003). These deficit explanations of teachers have been described as blaming them for educational failures (Boistrup, 2010) or construct teachers as incomplete, which leads to permanent training of teachers in order to maintain control over them (Montecino & Valero, 2017). Looking at this study, the ecology of Mary's mathematics teaching can be described as homogenous. It was rather Mary, who reached beyond the ecology. It is clearly not reasonable to describe teachers as incomplete and unknowing when teachers teach as the curriculum prescribes, and do more.

Mary was the one who reached beyond the contexts close to her teaching. Mathematics as a collection of mathematical ideas could be inferred from Mary's teaching, compared to the textbook where mathematics was inferred as described objects. She also reached for something more than the ordinary when she privileged solutions "outside the box". Dahl (2014) describes how the opportunity to engage in vertical discourses of mathematics more in line with a scholar discourse rather than the vertical, every day discourse of school mathematics, could make mathematics more accessible for all students. This is what Mary did when she included definitions in her explanations. Even if subtle, Mary reached beyond the descriptions of the noosphere, indicating that she also drew from scholarly knowledge. It is possible that Mary's technology for these activities could be deepened, something Barbé et al. (2005) describe in their study as well. They suggest that constraints in the external transposition could be the reason for deficiencies in teacher's rationale for their teaching. If that is the case, it would be interesting to see what would happen if

teachers were given more opportunities to discuss and develop their theoretical rationale.

The ecology of Mary's teaching was very homogenous in how mathematics and mathematics teaching and learning were expressed. This can be interpreted as a strong culture. I have already described that increased control compromises a teacher's opportunities to be professional. If we add this strong culture, which Mary apparently had reasons to reach beyond, I conclude that the system around Mary also constrained Mary's opportunities to be professional, even if she to some extent reached beyond curriculum and textbook. A teacher autonomy, as in the Swedish national curriculum, does clearly not mean that teachers are autonomous, which Skott (2004) also conclude. A more varied ecology where teachers would have alternative practices to negotiate may be one way to challenge this homogenous ecology. Another way to do this is to give teachers the opportunity to read and discuss the very principles that may ground their teaching as well as research, problematising both the principles and what they may generate during teaching activities.

7.5 The Research Process

In this study, I have used different methods and theories to achieve an understanding of teaching as part of an ecology. I maintained an interest of how mathematics and mathematics teaching were communicated, and I maintained an interest in what enables and constrains a teacher to be professional within this ecology. The analytical frameworks I have included in this preamble are the ones used in the four articles. Multimodal social semiotics, by which I studied the communication in a teacher meeting from three different aspects (Kress, 2009; Van Leeuwen, 2005), have not been used in the other articles except from a multimodal approach to communication. Without this approach, I would never have seen what I saw. A teacher's communication is indeed multimodal. Mary drew pictures, and made gestures when explaining mathematical properties to her students. Without seeing this as communication I would have little to analyse from Mary's mathematics teaching. Through the multimodal approach I also first saw the privileging of mathematics as objects, when I realised that it was often objects that carried the meaning of fractions. Through the multimodal analyses in Article 1, more specifically the ideational analysis, I also became interested in what mathematics was communicated and how, not only in the mathematics lessons, but also in the teacher group and other contexts. I continued to study how problem-solving was communicated in two contexts. This can be described as an ideational analysis, even if it was not framed that way in Article 2.

In Article 2, I began with a deductive thematic analysis (Braun & Clarke, 2006) which led me to include concepts from the literature: teaching for, about and through problem-solving (Schroeder & Lester, 1989). With these concepts

as codes I carried out a second deductive thematic analysis. These analyses were conducted on both Mary's teaching and the textbook, and the findings showed that it was Mary who varied her problem-solving activities more than the textbook did. These findings made me search for a theoretical construct which allowed me to systematically study a teacher's teaching in relation to other contexts. The revisiting of the findings of Article 2 shows how a powerful tool as praxeologies (Chevallard, 2006) may reveal another angle to the same findings. Construing one didactic praxeology for each approach to problem-solving provides a very complex picture of problem-solving teaching. In a teacher's classroom there is no separation of the three praxeologies, but they are all there, which was evident in Mary's communication, both within and outside the classroom. This conveys that problem-solving is constituted as task, technique and technology, in the same classroom, which are all grounded in different perhaps conflicting theoretical principles. This would not have been possible to see without using praxeologies.

ATD and praxeologies (Chevallard & Bosch, 2014), were used to analyse how mathematics and mathematics teaching were privileged in different contexts of the ecology. This enabled a systematic analysis of the communication, and to interpret the tasks, techniques, technology and theory that could be inferred from the communication, which made it possible to not only say what was privileged in the contexts, but also to interpret some of the theoretical rationale behind this communication. Analysing how different contexts privilege mathematics and mathematics teaching in terms of praxeologies, I have developed a methodology for how to study the relationship between micro and macro contexts, which is a contribution of this thesis.

ATD is not a universal tool that solves all. It has been powerful in this study and it has been widely used in research but I did have some problems with the constructs. The levels of co-determination (Chevallard, 2002b) are ordered as a hierarchy. I have instead, showed them as inscribed in each other. I would like to argue that they are not necessarily ordered in a hierarchy, if that would be the case pedagogy would be above the academic discipline. Chevallard describes the levels as co-determining, which hints that the levels are not just determining from top to bottom. A teacher may have influence over the society and the national curriculum, but not the same way as the national curriculum has influence over teachers. I regard my picture of these levels as inscribed in each other as a contribution to the discussion of ATD and co-determination. I also ask for a continued discussion of how the co, in co-determining works. In this study it is possible to see influences from different contexts, and the connections between the contexts shows that it is not always top down or a linear process.

In ATD, a reference model (see e.g. Wijayanti & Winslow, 2017) is often used to make the praxeological analysis transparent. For a mathematical praxeology where technology and theory are strictly about the mathematics it is possible to achieve such a model. In a didactic praxeology the arguments

for a didactic technique may consider how the mathematical content is taught in relation to mathematical properties. The arguments may also come from more general aspects, they could be arguments about inclusion of students with special needs or arguments for using manipulatives. To make a reference model for all possible theoretical principles would be an impossible task. Other possible principles may be sociological, where theoretical principles of democratic competence are emphasised (e.g. Skovsmose, 1990). A third example of principles that could have been taken into account are principles about inclusion of students in mathematics education (e.g. Popkewitz and Lindblad, 2000). To focus on how mathematics and mathematics teaching were communicated entailed a focus on the arguments for how mathematics was taught, and principles of mathematics and mathematics teaching and learning. To study theoretical grounds other than the ones in direct relation to the mathematical communication was not possible within this study. On the other hand, to navigate the study around mathematics and mathematics teaching gave me an opportunity to explore this in four contexts.

Mary being a single case was also a limitation of the study. It is reasonable to ask if more teachers would have benefitted the study. More teachers would of course have given more information about mathematics teachers in the Swedish system. To have data from more teachers would also have taken me on a different journey, possibly to compare between teachers. In that case I would have missed the relationships between teaching and the contexts studied. Mary being a telling case, did of course not offer solid ground for generalisations. She did, however, offer rich insight to what possibilities and constraints a Swedish mathematics teacher experiences. Mary being a Swedish teacher, engaged, well-educated and well regarded, but still a very normal teacher, can show us how a teacher navigates a homogenous ecology when she wants to do more than what is privileged in the ecology.

7.6 Implications

In the light of my findings, it is possible to see some implications, for both teachers as well as the broad context of mathematics education. I will also discuss implications for further research.

Teachers are embedded in an environment where, sometimes, conflicting demands on what they do in their classrooms compete for their attention. In the light of the findings of this thesis, I can see opportunities for teachers to navigate these demands. The theoretical principles I have described were not explicitly communicated, but they were possible to infer from the communication about mathematics and mathematics teaching. If teachers had the opportunity to discuss possible principles and what they might generate in terms of teaching activities, these principles would also be part of the teachers' com-

munication. There is a possibility that teachers would privilege different principles if they were up for discussion. This can sometimes be discerned in Mary's communication, for example, when she discussed problem-solving. She privileged problem-solving as strategies and emphasised how the solutions were reported. In interviews she did, however, become very enthusiastic when she discussed how mathematics, different from routine and facts, could be explored through problem-solving. If Mary had had the opportunity to discuss different grounds for problem-solving, and what teaching activities they might generate, it is possible that she would change her problem-solving practice in another direction. I asked, in an interview, if it would be possible to introduce a new content matter with a problem. Mary immediately replied that she really wanted to explore that more. I would argue that it is not only about what teachers know about these theoretical grounds, but what teachers have the opportunity to discuss. In the teacher meetings, I have observed the teachers to be occupied with solutions for the immediate future, such as a coming test. In the light of the time these teachers had for preparations this might be very reasonable. Nevertheless, these teachers did ask for different kinds of discussions and I would argue that this is exactly what they need. They need a discussion where the theoretical principles are addressed and challenged.

Seeing the findings of this study in relation to a Swedish context it is important to relate those to how teachers and mathematics teaching are described by authorities in Sweden. We have the public debate where the Minister of Education suggested teaching methods such as "katederundervisning" as a remedy for failing results. This concept was seen as too unspecific for teachers to relate to (Eriksson, 2011), saying nothing about task, technique (other than the placement of the teacher), technology or theory. I would argue that Mary did a lot of "katederundervisning" (the Swedish term for whole class teaching), she also let her students do exercises in the textbook and she engaged them in problem-solving as well as exploring mathematics with them. Almost all these activities were student-active. It is impossible to say whether she fulfilled the requirements for the privileged practice of "katederundervisning". What we do know is how she in her teacher-driven, student-active mathematics teaching communicated mathematics with both depth and variation. One implication for mathematics education in Sweden would, in the light of this public debate, be to focus more on how mathematics is communicated in the mathematics classroom and why rather than where in the classroom this is done.

The Swedish School Inspectorate described teachers as inadequate, under-educated, governed by textbooks and unknowing of the national curriculum (Skolinspektionen, 2009). I would argue that Mary told a different story. She was only one teacher, but neither was she a rare kind. There are many teachers with similar education and experience who manage their classes well. In the case of Mary I argue that it is the national curriculum and the textbook that need to develop. For teachers, both the teacher group and Mary, it is not more

fidelity to the curriculum that is the solution. That would only be more of the same. An implication for Swedish mathematics education could be, in the light of this study, more scope to read research literature and to discuss the very principles their teaching is anchored in as well as the principles they would like to use as grounds for coming teaching. That way, teachers are included in the theoretical discussion of mathematics teaching.

For institutions of mathematics teaching education, an implication could be to give in-service courses where teachers may have the opportunity to more systematically read research about mathematics teaching and discuss what principles they would like to take from the research into their negotiations about their coming mathematics lessons similar to how Jaworski and Potari (2009) presented a teacher with a theoretical tool to promote both teacher reflection and development of teaching. Teacher education may benefit in different ways from this study. One is how I have used didactic praxeologies to describe privileged practices in different contexts. This could be used in teacher education to visualise how different practices are privileged in teacher education compared to the school where the students do their in-service practicum.

In this study I have described how the theoretical principles of mathematics teaching are not part of the teachers' communication. They can still be inferred from the communication. One way to continue to study this, would be together with teachers. To go into depth with what principles can be inferred from their practice, what principles they would want to ground their teaching in and how they could start with their preferred theoretical principles and explore what technology they convey and what teaching activities that may be generated from this. This could be done as action research where the teachers are co-researchers or as an intervention where teachers who are exposed to research principles become visible, similar to Povey, Adams and Everley's (2017) study where a teacher was exposed to articles about teachers and performativity, which made her negotiate her teaching in relation to this.

The Swedish national curriculum, and more specifically the mathematics part, which teachers are bound to follow, did not include explicit references either to research or to what principles it is grounded in. Inferring mathematical values, it is possible to see all of them, and there was a definition of mathematics, but it was not clear why different goals and activities were privileged. In further studies the theoretical grounds for the curriculum need to be studied. Both in direct relation to the writing of the curriculum as Jahnke (2014) did, but we also need to study the product, the texts and views that these texts take departure in. The theoretical grounds of a curriculum need, in a time where teachers are expected to be professional, to be explicit so they can be negotiated and contested.

My hope with this study is, that it contributes to a deeper understanding of the complexity of mathematics teaching for policy makers as well as decision makers of educational systems. I hope that having such an understanding will

lead to an inclusion of contexts from the ecologies of mathematics teachers in the discussion about what needs to be developed and changed in order to develop mathematics teaching practices. My hope is also that it inspires further mathematics education research in addressing teachers and teaching to include broader contexts of teaching than those of teachers and classroom practices.

I wish that every headmaster, who works with as engaged and educated a teacher as Mary, would give them rich opportunities to read and experience influences from different arenas, including research, and to negotiate different grounds for developing mathematics teaching. In this, I hope that mathematics teachers (and of course teachers in other subject areas) will have time, opportunities and tools to read and discuss theoretical principles for their mathematics teaching. As a consequence of this, I hope that they will develop a more critical approach towards national curricula, textbooks and public debate, and negotiate them in relation to their experience, their students' wellbeing and their theoretical grounds. Through this, we might save teachers' souls from performativity (see Ball, 2003) and might have an opportunity to transform mathematics teaching into a teaching practice which is well-grounded in different, explicitly expressed and open for further negotiations, theoretical principles.

8 Sammanfattning

Att förstå lärares undervisning är inte bara att förstå vad som händer i klassrummet, det är också att förstå hur andra kontexter påverkar vad som blir undervisat i klassrummet. Det kan vara kontexter som, andra lärare, läroböcker och läroplaner. Dessa skapar en miljö för lärares undervisning, en ekologi där olika kontexter samspelar med varandra.

Jag har följt en matematiklärare, som jag kallar Mary, i hennes matematikundervisning i årskurs fem. Marys matematikundervisning sker i samspel med många kontexter varav jag har studerat tre. Marys kollegor som möts regelbundet för att diskutera matematikundervisningen i årskurs fem är en av dessa kontexter. Andra är matematikboken och läroplanen. Tillsammans utgör dessa kontexter vad jag kallar undervisningsekologi. Mary är en engagerad, utbildad, erfaren och uppskattad matematiklärare. Mary arbetar som lärare i svensk skola vilket placerar henne i ett sammanhang där lärare ska vara fria att välja själva hur de ska undervisa, men där de är kontrollerade och beskrivs som otillräckliga av media och av skolmyndigheter. När man ska göra särskilda satsningar på lärare så kallas de för lyft, som om lärare behöver lyftas upp av någon annan.

Hur man ska undervisa matematik beskrivs i många olika sammanhang. Lärares undervisning är bara en av dem. Därför behöver vi studera hur matematik och matematikundervisning privilegieras i olika kontexter som samverkar till att avgöra vad matematikundervisning utgörs av. Vetenskapliga studier undersöker i stor utsträckning lärares undervisning som enda kontext. I dessa studier adresseras ofta lärares matematiska eller didaktiska kunskaper. Ibland studeras lärares undervisning i relation till en annan kontext, till exempel läroplanen. Även i sådana studier kan fokus vara på lärarens kunskap om läroplanen och inte på vilken bild av matematik eller matematikundervisning som läroplanen förmedlar eller hur lärare påverkas av den. Ibland fokuseras lärares undervisning eller kunskap men där forskarna pekar på möjligt inflytande från andra kontexter, som inte inkluderats i studien. Om vi inte studerar lärares undervisning i relation till omgivande kontexter så kommer vi inte att förstå vad som samverkar till att avgöra vad matematikundervisning består av, och varför. Om vi inte förstår detta så riskerar vi att rikta utbildningsinsatser mot lärare, men där det kanske var en annan kontext, eller alla omgivande kontexter, som behövde utvecklas eller förändras.

Med syftet att fördjupa förståelsen för vilka möjligheter och begränsningar som skapas i en matematiklärares undervisningsekologi har jag samlat data

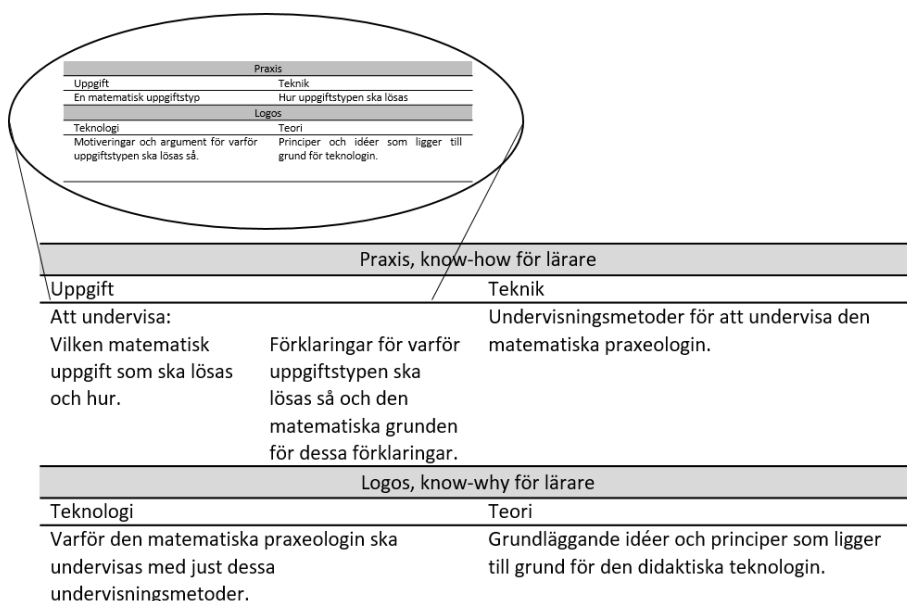
ifrån Marys undervisning (klassrumsobservationer och intervjuer), från lärargruppen som Mary ingår i (observationer av lärmöten), läroböckerna som Mary använder (elevböcker och lärarhandledningar), läroplanen (kursplanen i matematik och kommentarmaterialet).

För att kunna studera de fyra kontexterna i undervisningsekologin för Marys matematikundervisning har jag använt mig av ATD (antropologisk didaktisk teori). ATD beskriver didaktisk transponering. En sång som transponeras låter annorlunda men fortfarande som samma sång, den kan förändras lite så att förändringen blir nästan obefintlig eller mycket så att den nästan inte känns igen, som när man transponerar från dur till moll. Didaktisk transponering beskriver hur matematisk eller didaktisk kunskap omtolkas och omformuleras i olika institutioner så att den ska kunna undervisas i skolan. Kunskap beskrivs i ATD som en handling som består både av ett görande och ett vetande. Detta kallas tillsammans för praxeologi. En matematisk praxeologi beskriver matematisk kunskap i termer av uppgiftstyper, tekniker som används för att lösa uppgiftstypen, teknologi med argument och motiveringar för varför teknikerna ska användas till just dessa uppgiftstyper och teori som består av grundläggande idéer som teknologin grundar sig på, se Tabell 19.

Tabell 19. Matematisk praxeologi

| Praxis | |
|--|---|
| Uppgift | Teknik |
| En matematisk uppgiftstyp | Hur uppgiftstypen ska lösas |
| Logos | |
| Teknologi | Teori |
| Motiveringar och argument för varför uppgiftstypen ska lösas så. | Principer och idéer som ligger till grund för teknologin. |

I en didaktisk praxeologi, som beskriver undervisningskunskap, är uppgiften istället att undervisa en matematisk praxeologi. Då blir tekniken hur man ska göra för att göra matematisk kunskap tillgänglig för andra, alltså undervisningsmetoder. Teknologin är motiveringar och argument för varför dessa undervisningsmetoder ska användas för att göra denna matematiska praxeologi tillgänglig. Teorin består av teoretiska principer och idéer som ligger till grund för teknologin, se figur 8.



Figur 8. Didaktisk praxeologi

I Figur 8 visas hur den matematiska praxeologin tar plats som didaktisk uppgift i den didaktiska praxeologin. Både didaktiska och matematiska praxeologier antas i ATD vara påverkade av olika nivåer som samverkar till att avgöra vad matematikundervisning utgörs av, vilket brukar beskrivas som en hierarki av nivåer som i Figur 9.



Figur 9. Samverkande nivåer.

I Figur 9 har jag också visat hur dessa nivåer kan sammanföras med in studie och med ett svenskt sammanhang. De fyra kontexterna som jag har studerat kan ses som delar av dessa nivåer. I min studie ingår nivå 1, 2 och 3 som representeras av lärarens matematikundervisning. 6 som utgörs av två kontexter där undervisningsprinciper uttrycks, i lärobok och lärargrupp. Nivå 8 utgörs av läroplanen som staten har utfärdat. Jag har valt att beskriva dessa nivåer som inskrivna i varandra istället för hierarkiskt ordnade, se Figur 10.



Figur 10. Samverkande nivåer inskrivna i varandra, relaterade till de kontexter som studerats i denna studie.

I de fyra kontexterna privilegieras matematik och matematikundervisning på olika sätt, detta kan beskrivas i termer av praxeologier. Alltså har varje kontext ett uttryck för didaktisk kunskap, som kan studeras som en praxeologi. Det är dessa praxeologier som jag har studerat.

I den första artikeln studerades hur en lärare argumenterar för sin matematikundervisning i ett lärarmöte. Diskussionen handlade om att testa multiplikationstabellen på tid och om att använda matematikböcker för att tillgodose högpresterande elevers behov. Resultatet visar att Marys utgångspunkter skiljer sig något jämfört med lärargruppens. Mary argumenterar mer för eleverna som mår dåligt av tabelltestet jämfört med den övriga gruppen vilket kan tolkas som att hon utgår mer ifrån en teoretisk princip att alla elever har rätt att må bra när de lär sig matematik. Lärargruppen argumenterar emot baserat på en teoretisk princip om att tabellkunskaper är centrala matematiska färdigheter som kräver att bli bedömda med tidsbegränsade test, mer än vad Mary gör. De argumenterar alltså till synes utifrån samma utgångspunkter, men de privilegierar de två teoretiska principerna olika.

I den andra artikeln studerades Marys problemlösningundervisning i relation till tre sätt att närma sig problemlösning. Det fanns drag av alla tre sätt att närma sig problemlösningundervisning i både Marys undervisning och i

läroböckerna, men med en tydlig övervikt för två av dem. Om man tänker att alla dessa tre sätt att närma sig problemlösningsundervisning behövs så finns det en snedfördelning i Marys klassrum, samma snedfördelning finns men är ännu tydligare i läroböckerna, den kan också ses i läroplanen. Alltså finns inte hela förklaringen för hur problemlösningsundervisningen blir, hos Mary. Den finns också i läroboken och i läroplanen som båda är del av samma kultur som Mary.

I den tredje artikeln utforskades ett teoretiskt verktyg som beskriver kunskap som både ett görande och ett vetande för att beskriva vilken kunskap som privilegieras i olika kontexter, i det fallet matematikbok och i lärarens matematikundervisning. Resultaten indikerar att nyanserna, som framträder när man analyserar både vilket görande och vetande som kommuniceras i en undervisningskontext, gör att fler skillnader framträder. I det exempel som analyseras i artikeln kan man se hur Marys förklaringar av mätning av vinklar bottnar mer i vinkeln som en representation av en matematisk idé, rotationen, jämfört med matematikbokens förklaring som bottnade mer i vinkeln som ett matematiskt objekt som man sätter namn på och mäter.

I den fjärde artikeln studerades vilka teoretiska principer och argument för undervisning om rationella tal som kunde ses i de fyra kontexterna. Resultaten visar att teoretiska utgångspunkter inte kommuniceras explicit i någon av de fyra kontexterna. Bakomliggande teoretiska principer får tolkas från kommunikation och aktiviteter från de olika kontexterna som rör rationella tal. Resultaten visar också att de fyra kontexterna privilegierar de teoretiska principerna och argumenten på liknande sätt. Undervisningsekologin är homogen. Att undervisa på ett annat sätt än vad som privilegieras i undervisningsekologin innebär avsteg från samtliga omgivande kontexter. Lärare behöver alltså få engagera sig i kontexter som privilegierar olika teoretiska principer, till exempel vetenskaplig litteratur, för att få möjlighet att urskilja och diskutera hur andra kontexter påverkar matematikundervisningen.

Utifrån artiklarnas resultat dras några slutsatser om hur undervisningsekologin påverkar Marys matematikundervisning. I hela undervisningsekologin privilegieras görandet framför vetandet. Trots det så kan man se hur Mary ger uttryck för fler eller andra teoretiska principer än ekologin i allmänhet. Hon lyder inte de omgivande kontexterna, hon förhandlar de teoretiska principer som privilegieras där med andra faktorer, såsom elevernas välbefinnande. Läroboken har en mycket framträdande roll i ekologin, den används som uppgiftssamling men också för att utmana högrepresterande elever. Detta skulle kunna ses som en begränsning, men i denna studie blir det också tydligt att matematikböckerna också möjliggör Marys matematikundervisning. Matematikböckerna erbjuder både en struktur för skolmatematiken och uppgifter som eleverna kan arbeta med, vilket gör att Mary sparar tid och kan utveckla sin problemlösningsundervisning. Läroplanens otydliga sätt att privilegiera matematik och matematikundervisning blir en begränsning för Mary eftersom det inte blir tydligt vilka teoretiska principer som ligger till grund för de olika

målen. I hela ekologin privilegieras räkning som både aktivitet, förutsättning för matematisk aktivitet och som tillämpning av matematik. Denna ständiga privilegiering av räkning begränsar utforskandet av matematiska idéer, till exempel när rationella tal blir bråk som översätts till decimaltal så att de kan räknas med. Över lag i ekologin privilegierades en förenklad matematisk kommunikation. Till exempel när olika lösningar privilegierades framför effektiva eller generella lösningar, eller när definitioner inte kommuniceras utan tas för givet, som i matematikbokens förklaringar av vinklar.

Som en följd av dessa resultat och slutsatser anser jag att lärare behöver få tid och möjlighet att engagera sig i vetenskapliga och andra kontexter för att bredda sin ekologi och få tillgång till fler och andra teoretiska principer att förhandla med när de formar sin matematikundervisning. Genom att få tillgång till en större bredd och variation av teoretiska principer så kan lärare få större möjligheter att motstå influenser som till exempel när utbildningsministern föreslår katederundervisning som en lösning på försämrade resultat i internationella mätningar, eller när myndigheter beskriver lärare som otillräckliga och okunniga. Ett sätt att göra teoretiska principer tydliga för lärare är att inkludera referenser, vetenskapliga och andra, i läroplanen så att grunderna för de mål som står där blir tydliga.

Min förhoppning med denna studie är att den bidrar till att beslutsfattare i skolsystemet får en större förståelse för komplexiteten att undervisa matematik. Det behöver ta med lärares undervisningsekologi i diskussioner om vad som behöver utvecklas eller förändras för att matematikundervisningen ska utvecklas. Min förhoppning är också att studien inspirerar till vidare forskning som fokuserar lärare och undervisning som i högre grad studerar fler kontexter än läraren och klassrummet med fokus på den matematik som kommuniceras.

Jag önskar att varje rektor som har en Mary på sin skola, alltså en engagerad och utbildad lärare, ska ge henne möjlighet att fördjupa sig och hämta influenser på många olika håll för att utveckla sin undervisning. Min förhoppning är att varje matematiklärare ska få tid, möjlighet och verktyg att läsa och diskutera teoretiska principer för den undervisning de genomför, och som en konsekvens av det får större möjlighet att kritiskt granska både läromedel, läroplaner och allmän debatt.

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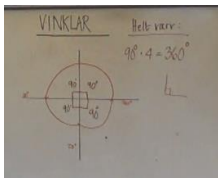
Lesson 1 angles

Mary starts the lesson with a good morning and a presentation of me and the camera. Mary says that they are starting up a new topic, angles. She comments that they have been working with this the year before so she wonders what they know about angles.

Tom gives an explanation about acute, right and obtuse angles showing how they look like by hands. Tony says that the degrees tells how open and wide the angle is. Catrin connects angles to geometrical forms and to rectangles and squares that have right angles and triangles that have acute angles. Mary wrinkles her forehead asking if they only has acute angles and Catrin corrects herself to triangles can have different angles. Mary says that this is one of the things they will look into this lesson. Mary repeats the utterance about squares and rectangles saying that this is right. She repeats the words right, obtuse and acute asking if they know anything more. Wilhelm who takes advanced mathematics says the Pythagorean Theorem, sine and cosine. Mary asks him if they can wait with this saying it is a bit too advanced.



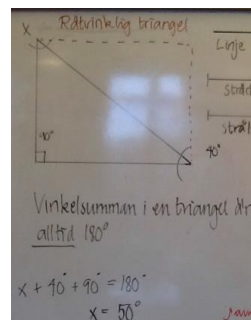
Mary asks the class to pair up with one pencil each. Mary asks them to make a right, an acute and an obtuse angle she moves around the classroom looking at the students' angles. She concludes with a: Good, you know this! Mary asks the students to put the pencils in a cross to make four right angles. She asks them what will happen if she moves one of the pencils. Tony gets to tell that it will be two acute and two obtuse angles; Mary asks him to come to the white board to show where these angles are. Then all the students get to try this at their benches. Mary holds her pencils up again so they show four right angles. She asks her students how many degrees they have together. The answer comes fast, 360 degrees.



Mary starts to draw at the whiteboard, showing how four right angles together create a whole lap and 360 degrees. Mary makes a connection to snowboard-tricks. It is winter time in Sweden and the students will soon go on a holiday when many of them will be in the slopes skiing downhill. Mary asks about two laps and one student says seven-twenty as they say 720 degrees in the snow board slope.

Mary goes from degrees to the protractor, holding one up asking the students what it is. Vilgot knows and Mary holds the small protractors up that the students will use by themselves later.

Mary asks the students why you should know how to measure angles. Tony suggests that it is used when you want to measure how much you rotated when jumping with a snowboard. Tim suggests that it is because walls in a house won't be skew. Mary agrees and adds that even if not everyone will be an architect it is good to be able to measure an angle in real life, she throws in the comment that “mathematics is very much in real life”.





**Stockholms
universitet**

2013-01-27

Dnr SU

Jag heter Anna Pansell och är doktorand i matematikämnets didaktik på Stockholms universitet. Detta innebär att jag studerar matematikundervisning och mitt arbete kommer att resultera i en doktorsavhandling.

Min studie handlar om matematiklärare och deras matematikundervisning. Detta innebär att jag följer matematiklärare i deras arbete med undervisningen även i arbetet utanför klassrummet. För att kunna studera matematikundervisningen är det nödvändigt att följa arbetet i klassrummet.

Jag önskar att få följa dig i de möten som rör matematikundervisningen och under några lektioner. Jag vill göra både ljud- och bildupptagningar. Jag vill också träffa dig för att prata om matematikundervisning intervjusituationer som jag också vill spela in.

Resultaten kommer att presenteras i en doktorsavhandling men också i andra texter där jag presenterar min forskning såsom vetenskapliga artiklar eller presentationer på konferenser. Inga bilder eller videosekvenser kommer att förekomma i någon text eller presentation. Alla deltagares namn kommer att avidentifieras. Uppgifter som möjliggör identifiering kommer att behandlas konfidentiellt och under tystnadsplikt i enlighet med personuppgiftslagen.

Jag lovar att studien kommer att genomföras i enlighet med Vetenskapsrådets forskningsetiska principer för humanistisk/samhällsvetenskaplig forskning. Alla originaldokument (filmer, ljudinspelningar, pappersdokument) och arbetskopior kommer att förvaras oåtkomliga för obehöriga. Medverkan i studien är frivillig och du kan när som helst under projektet avbryta din medverkan. Hör gärna av dig om du har några frågor!

Med vänlig hälsning

**Institutionen för matematikämnets och
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Medgivande

Detta medgivande avser tillstånd för Anna Pansell (och eventuella medhjälpare) att video- och ljuddokumentera undervisningssituationer och samtal om matematikundervisning där du deltar samt tillstånd att använda materialet för den ovan beskrivna forskningen.

Jag säger ja till medverkan i Anna Pansells forskningsstudie.

Datum: _____

Underskrift

Namnförtydligande



**Stockholms
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Dnr SU

Jag heter Anna Pansell och är doktorand i matematikämnets didaktik på Stockholms universitet. Detta innebär att jag studerar matematikundervisning och mitt arbete kommer att resultera i en doktorsavhandling.

Min studie handlar om matematiklärare och matematikundervisning. Detta innebär att jag följer matematiklärare i deras arbete med undervisningen och även i arbetet utanför klassrummet.

Jag önskar att få delta i lärarmöten. Samtalen vill jag göra både ljud- och bildupptagningar av.

Resultaten kommer att presenteras i en doktorsavhandling men också i andra texter där jag presenterar min forskning såsom vetenskapliga artiklar eller presentationer på konferenser. Inga bilder eller videosekvenser kommer att förekomma i någon text eller presentation. Alla deltagares namn kommer att avidentifieras. Uppgifter som möjliggör identifiering kommer att behandlas konfidentiellt och under tystnadsplikt i enlighet med personuppgiftslagen.

Jag lovar att studien kommer att genomföras i enlighet med Vetenskapsrådets forskningsetiska principer för humanistisk/samhällsvetenskaplig forskning. Alla originaldokument (filmer, ljudinspelningar, pappersdokument) och arbetskopior kommer att förvaras oåtkomliga för obehöriga. Medverkan i studien är frivillig och du kan när som helst under projektet avbryta din medverkan. Hör gärna av dig om du har några frågor!

Med vänlig hälsning

**Institutionen för matematikämnets och
naturvetenskapsämnenas didaktik**



Stockholms
universitet

Medgivande

Detta medgivande avser tillstånd för Anna Pansell (och eventuella medhjälpare) att ljuddokumentera samtal om matematikundervisning där du deltar samt tillstånd att använda materialet för den ovan beskrivna forskningen.

Jag säger ja till medverkan i Anna Pansells forskningsstudie.

Datum: _____

Underskrift

Namnförtydligande



**Stockholms
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2014-01-27

Medgivande för deltagande i forskningsstudie

Jag heter Anna Pansell och är doktorand i matematikämnets didaktik på Stockholms universitet. Detta innebär att jag studerar matematikundervisning och mitt arbete kommer att resultera i en doktorsavhandling.

Min studie handlar om matematiklärare och deras matematikundervisning. Detta innebär att jag följer matematiklärare i deras arbete med undervisningen även i arbetet utanför klassrummet. Men för att kunna studera matematikundervisningen är det nödvändigt att följa arbetet i klassrummet.

Jag har besökt klassen för att presentera mig och för att de ska få träffa mig. Jag kommer därefter att följa lärarens arbete genom att videofilma några lektioner samtidigt som jag spelar in den kommunikation som läraren deltar i. Fokus för studien ligger inte på enskilda elever utan på läraren.

Resultaten kommer att presenteras i en doktorsavhandling men också i andra texter där jag presenterar min forskning såsom vetenskapliga artiklar eller presentationer på konferenser. Inga bilder eller videosekvenser kommer att förekomma i någon text eller presentation. Eventuella deltagares namn kommer att avidentifieras. Uppgifter som möjliggör identifiering kommer att behandlas konfidentiellt och under tystnadsplikt i enlighet med personuppgiftslagen.

Jag lovar att studien kommer att genomföras i enlighet med Vetenskapsrådets forskningsetiska principer för humanistisk/samhällsvetenskaplig forskning. Alla originaldokument (filmer, ljudinspelningar, pappersdokument) och arbetskopior kommer att förvaras oåtkomliga för obehöriga. Medverkan i studien är frivillig och deltagarna kan när som helst under projektet avbryta sin medverkan. Hör gärna av er per mail eller telefon om ni har några frågor!

Med vänlig hälsning

**Institutionen för matematikämnets och
naturvetenskapsämnenas didaktik**



Stockholms
universitet

Medgivande

Detta medgivande avser tillstånd för Anna Pansell (och eventuella medhjälpare) att video- och ljuddokumentera undervisningssituationer där du/ditt barn deltar samt tillstånd att använda materialet för den ovan beskrivna forskningen.

Kryssa för ett av nedanstående alternativ och skriv under.

Elevens namn: _____

Vi (elev och målsman) säger ja till elevens medverkan i Anna Pansells forskningsstudie.

Vi (elev och målsman) säger nej till elevens medverkan i Anna Pansells forskningsstudie men vi tillåter videofilmning av undervisning där eleven är med i bakgrunden.

Vi (elev och målsman) säger nej till all medverkan i Anna Pansells forskningsstudie.

Datum: _____

Målsmans underskrift

Elevens underskrift
