Design of Concrete Pavements –
Design Criteria for Plain and Lean Concrete

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Licentiate Thesis
Preface

This licentiate thesis was carried out at the Swedish Cement and Concrete Research Institute (CBI) and at the school of Architecture and Built Environment at the Royal Institute of Technology (KTH), at the Division of Structural Design and Bridges.

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Abstract

New road infrastructure projects are important and constitute of large investments that have to serve the society for a long time. The investments have to be durable at the lowest life cycle cost and the pavements have to sustain loads from increasing traffic intensity and heavy traffic loads. In Sweden less than 1‰ of the national road network consists of concrete pavements. In parts of Europe and in the U.S., on the other hand, concrete roads are used to a large extent for highways as well as rural roads. To encourage the competition between different road materials in Sweden, the tools for designing robust concrete pavements have to be brought forward. In order to emphasize plain concrete pavements as an alternative in road construction, the design must also be competitive.

The current Swedish design method for concrete pavements is straightforward but offers no flexibility when designing roads with, for instance higher traffic loads. The method calculates concrete thicknesses on the conservative side since only a limited number of parameters are treated. Modern methods that take into account many more parameters in the design are being developed internationally. For a new Swedish design method, these parameters have to be established for actual conditions in Sweden. Also, the design has to be flexible and meet the demands from contractors and clients for a wider use.

The aim of this project is to develop a new design method for plain concrete pavements that is more flexible than today. A new design method is also intended for the Swedish Road Administrations’ (SRA) computer based public design guide, PMS Objekt.

Information for a new design method has been assembled mainly by investigating two newly developed design methods, VENCON2.0 in the Netherlands, and the Mechanistic-Empirical Pavement Design Guide (MEPDG) in the USA. Comparative calculations between the Swedish design method and the MEPDG are presented. The comparison is made on the level of input parameters and highlights the advantageous aspects of a semi-mechanistic design procedure where the functional properties of a concrete pavement are calculated incrementally over the design period.

Plain and lean concrete, separately, but also the in composite beams, have been studied in flexural fatigue testing. The results show that Tepfers’ fatigue criterion is valid for both plain and lean concrete when subjected to flexural fatigue loading. The results also show that the fatigue strength of composite beams of plain and lean concrete is mainly dependent on the strength of the lean concrete but that Tepfers’ fatigue criterion is applicable. The bond between plain in lean concrete is found to be strong and fatigue resistant, making the composite section able to accommodate higher stresses. The bond nevertheless contributes to the risk for reflection cracks in the plain concrete wear layer and a recommendation to focus on stresses in the bottom of the lean concrete is formulated. Also, well distributed expansion joints in the lean concrete are necessary.

A new project for measuring temperature gradients for use in concrete pavement design is also presented. This is done with means of concrete prisms placed in the pavement and are done in order to establish actual temperature gradients for various locations in Sweden. Also, the nonlinear gradients that act in the pavement as well as the negative temperature gradients will be analysed for the use in the design.
Finally, the thesis outlines a new design method for Swedish conditions. The method is possible to develop gradually and is based on FE-analysis for fast computations. In the design, stresses from traffic and temperature loads are calculated simultaneously in a number of critical locations in the concrete slab. The method will also make it possible to alter design features as slab lengths and widths, with various connections between the slabs.
Sammanfattning


Detta forskningsprojekt har behandlat de första stegen i arbetet för utvecklingen av en ny dimensioneringsmetod för betongvägar i Sverige. Den nya dimensioneringsmetoden ska vara mer flexibel för att kunna användas för dimensionering av olika former av betongvägar. Metoden ska också kunna implementeras i Vägverkets dimensioneringsprogram, PMS Objekt, som utgör det program som används för statliga vägar.


I dimensioneringsmetoden för vägar ingår utmattningskriterier för olika lager som en viktig del. Dagens kriterier för betongvägar behöver ses över och av den anledningen har utmattningsprovning av balkar av oarmerad betong och cementbundet grus (CG) utförts. Det har kunnat fastställas att Tepfers utmattningskriterium verkligen är tillämpbart för böjdraghållfasthet. För CG har provning visat att ett spänningskriterium kan användas med något reducerat utmattningshållfasthet i jämförelse med Tepfers kriterium. För samverkansbalkar av betong och CG har det även fastställts att vidhäftningen är mycket god och att Tepfers utmattningskriterium är applicerbart även här. Den goda vidhäftningen ger dock upphov till en risk för reflektionssprickor men den ökade bärförmågan bör ändå kunna utnyttjas om fokus läggs på spänningsnivån i underkant CG istället för underkant betong. Tät expansionsfogar i CG-lagret är också nödvändiga.

En annan viktig del av betongvägsdimensioneringen omfattar realistiska temperaturgradienter för olika delar av landet och därför presenteras en ny metod för att mäta temperaturgradienter i betong. Metoden innefattar mätningar av temperatur i betongkuber med inget mätutrustning som placerats ut invid flera vägar i Sverige. Tillsammans med tidigare utförda mätningar kommer nya data att sammanställas för att användas i en ny dimensioneringsmetod. Mätningarna kommer också att analyseras med anseende på negativa gradienter för att kunna beakta laster som påverkar betongens överkant, laster som orsakar sprickor som genereras ovanifrån s.k. top-down cracking.

Ett förslag till en ny dimensioneringsmetod för oarmerade betongvägar presenteras. Denna metod är möjlig att utveckla successivt och behandlar både trafik- och temperaturlaster.
samtidigt i ett FE-program. Metoden möjliggör en flexibel dimensionering där olika specifika egenskaper ska kunna varieras beroende på förutsättningar och krav som definieras under dimensioneringsperioden.
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Chapter 1

Introduction

1.1 General

Concrete pavement design has over the years become a more important part for the promoting of concrete roads. A high investing cost has to be motivated, and the benefits of a pavement with less maintenance over a longer design life have to be proved already before construction. Efforts to avoid premature performance failing of concrete roads are, at a larger degree, considered than for other pavement alternatives since rehabilitation techniques are expensive. A modern design methodology has to take into account all sorts of environmental conditions as well as future estimations on, for example traffic growth or environmental changes. The optimisation of materials in the pavement system, demands for long-term fatigue resistance at the lowest cost and ecologically sound choices must be considered. The understanding of the behaviour of a concrete road is vital for the design and the performance prediction.

The current Swedish design method was developed in the 1990’s in a trial to modernise the concrete pavement design, and create a chance for competition between road paving alternatives in the country, Petersson (1996). The design methodology is used for highway pavements, industrial pavements, and public pavements, accessible in Petersson (1996), SRA (2005), Silfwerbrand (1995), AB Byggtjänst (2002), and Farhang (2004a). It is a simple method in the way both traffic and temperature stresses are dealt with and provides a conservative design with just a limited number of calculation steps It also simplifies both the material and loading inputs, making it restrained for possible optimisation. Every simplification also comes with a safety margin that can be quantified.

The development of a new design procedure consists of the quantifications of different unknown aspects that are important for the performance of a concrete pavement. Advances in computer modelling are tempting for a number of applications, and offer the possibility to include many more, even new parameters in the design. Design methods with modern computer power can be done in a whole new way and the main advantage is that even though many more aspects are being considered, the calculations can be made very fast.

The Ph.D.-project “Design of Concrete Pavements” was started in 2004 at the Royal Institute of Technology and the Swedish Cement & Concrete Research Institute. The project aims at modernising the design of concrete pavements by assembling knowledge in Sweden and abroad, to create the basis for a new computerised design method that can be integrated in the Swedish Road Administration’s (SRA) public design tool, PMS Objekt (SRA, 2005). A new, computerised design method has to be calibrated for Swedish conditions, where different design aspects have to be investigated more profoundly. The design criteria for plain concrete, actual temperatures in the field, and real traffic data are some of the main issues that have to
be examined. In this thesis, a proposal for a new design method is presented. The thesis also presents three investigations on fatigue criteria as well as the description of an on-going project dealing with field measurements on temperature gradients in concrete.

1.2 Background

Concrete roads were probably first constructed in the USA in the beginning of the twentieth century, and spread to Europe in the twenties, Williams (1986). In Sweden three main concrete pavement epochs can be distinguished; in the twenties, the fifties, and the seventies with the construction of plain, unjointed concrete pavements. These roads were constructed in the south of Sweden, and apart from the highway E6 in Vellinge from 1978, only parts of evidence of these roads remains. In the nineties, on the other hand, a new effort to develop concrete pavements as an alternative to asphalt roads took place. A new design method was developed by Ö. Petersson, Petersson (1996), and the construction of four new highways was undertaken during a ten year period; a short part of a highway connecting E4 and the international airport at Arlanda in 1990, E6 Heberg – Långås at Falkenberg in 1993, E6 Fastarp – Heberg at Falkenberg in 1996, and E20 Eskilstuna – Arphus in 1999. The latest contribution is E4 Uppsala – Mehedeby, completed in 2006. Today, the total length of concrete roads in Sweden is 87 km. The concrete roads constitute less than 1 ‰, and are extremely underrepresented in comparison to the USA and several countries in Europe. For example, Germany has 28 % of its road network consisting of concrete pavements, that is almost as much as the total length of the primary road network in Sweden, European Commission (1999).

1.3 Types of Concrete Pavements

There are in principle three different kinds of concrete road designs; plain jointed concrete pavements (PJCP), continuously reinforced concrete pavements (CRCP), and jointed reinforced concrete pavements (JRCP), see Figure 1.1. The concrete roads in Sweden are PJCP’s and the typical Swedish pavement system consists of a wear layer of concrete, a bound base of asphalt (100 mm) or lean concrete (150 mm), two subbase courses of compacted unbound materials, SRA (1994, 2005), see Figure 1.2. The subgrade consists of the virgin material (varying from rock to clay and silt), or manmade materials, e.g. fillings.
Four types of joints exist for concrete pavements. Transversal joints are cut at approximately 5 m distances to reduce the tensile stresses that arise from temperature expansion. These joints are dowelled to guarantee a good load transfer between adjacent slabs. Longitudinal joints are cut to limit the slab width for roads with more than one lane. These joints are not equally loaded by traffic and therefore only connecting bars are placed here. Expansion or isolation joints are used between the concrete pavement and bridges or flexible pavements. Construction joints are used between concreting pauses, Löfsjögård (2003).
Figure 1.3. Slip-form paver at the construction of the PJCP at E4 Uppsala – Mehedeby in June 2006.

A concrete road is constructed with a slip-form paver which is a set of machines that places the concrete in one or two layers continuously. The concrete is vibrated, the dowels are put into place, and the surface is treated by the machines automatically. In the construction of E4 Uppsala – Mehedeby 2006, the slip-form paver had an average production speed of 1 m/min, i.e. 110 m³/h, see Figure 1.3.

1.4 General Design Procedure

The general design procedure for concrete pavements consists of the calculation of the number of load repetitions that a pavement can resist before failure. The methodology is iterative where a predefined pavement structure is calculated and evaluated compared to the loading conditions applied. If the pavement system is found not to meet the loading condition, the calculations are done all over again with increasing thicknesses or different choices of materials in the included layers.

1.5 Aim and Scope

Future highway projects in different regions in Sweden will have to be constructed to deal with the increasing traffic intensities from transportation. Safe, environmental friendly and effective roads have to be built in such a way that the society at large can take advantage of these big investments in infrastructure. Providing the tools for increased competition within the road construction market has for long been considered essential for maintaining a good level of competence and keeping the costs low.
The current design method in Sweden is not flexible and is also too simple when optimising the design. In contrast to modern design methods it cannot take into account a more varied climate, higher traffic loads or improved materials. The safety margins are not fully investigated and the method is badly adopted for a computerised tool. A modern design method ought to serve as a tool for designer and contractors to predict the future performance of a road to be constructed.

The primary aim of this thesis is to present an overview of modern design methods that have been developed over the last 10 – 15 years. The thesis presents the first step towards a new design method for concrete roads in Sweden by proposing a basic model that can be enhanced in the future. In the development of a new design procedure, many aspects of the design have to be examined and the thesis also presents three investigations on the fatigue of plain and lean concrete as well as measurements of temperature gradients in the field. In Sweden, CRCP or JRCP are less prioritised for highway construction at this time and therefore this thesis only deals with PJCP design.

1.6 Outline of Thesis

In Chapter 1, an introduction and the background of plain concrete pavement design in Sweden is presented.

In Chapter 2, an overview on different international methods is presented and compared to the current Swedish design.

In Chapter 3 and 4, experimental studies on fatigue are explained and measurements on temperature gradients in Sweden are presented.

Chapter 5 provides a new design concept for possible implementation in Sweden.

Conclusions and further work are presented in Chapters 6 and 7.

This thesis includes the following appended papers, which will be referred to with their numbers in the text.


Paper 2, “Flexural Fatigue of Plain Concrete Beams” by Johan Söderqvist and Johan Silfwerbrand, is submitted to the International Journal of Pavement Engineering. The paper is based on flexural fatigue tests made on plain concrete beams at the Swedish Cement and Concrete Research Institute. The objective of the paper is to investigate the fatigue criterion that is used in Sweden and verify if the criterion is valid for tensile stresses undergoing fatigue loading.
Paper 3, “Design Criteria for Lean Concrete”, by Johan Söderqvist and Johan Silfwerbrand, was presented at the 6th International DUT-workshop on Fundamental Modelling of Design and Performance of Concrete Pavements, Old-Tournhout, Belgium, September 15 – 16, 2006. The paper comprises an overview of design criteria for lean concrete used as a bound base layer under the pavement. A literature survey is presented as well as results from fatigue testing of beams.

Paper 4, “Flexural Fatigue of Composite Beams of Plain and Lean Concrete”, by Johan Söderqvist and Johan Silfwerbrand, submitted to the International Journal of Road Materials and Pavement Design. The objective of Paper 4 is to analyse how the crack development and the bond in composite beams of plain and lean concrete is influenced when subjected to fatigue loading.

Johan Söderqvist has drawn up the proposals for methodology, independently performed the trials, worked out the analysis and conclusions and written the papers. The co-author has contributed with the choice of subject and his view on methodology, analysis, conclusions, and text.
Chapter 2

National and International Design Methods

2.1 General

The design of concrete pavement consists of the calculations of the number of load applications that a specific pavement system can sustain before failure, taking into account the changes in climate, traffic, and material conditions, and summarising these effects during a set design period. In the design, the prediction of failure relies on data from field measurements, mechanistic, empirical, or statistical analysis. Material deterioration can be described directly by analysing the result of instant or fatigue loading, but multiple loading combinations in varied conditions have, to this day, been simplified in empirical methods. Nowadays, mechanistic design has, with proper assumptions and fundamental material knowledge, proved to be a reliable method for modelling the performance of a concrete pavement. Semi-mechanistic design methods, with material models that are calibrated using field data, are still the most successful methods because they relate computer models to actual performance in reality. These methods are possible to further develop thanks to the monitoring of roads in the field, roads that have been in service for decades. These methods combine the safety levels that come from an empirical approach together with powerful computational capabilities that can be used to explain the various phenomena that affect the pavement system during years of service.

In many countries, years of research have been put into the development of new design methods for concrete pavements. Computer based design methods with a high level of sophistication are introduced for highway agencies and designers with the aim of facilitating, not only for use in the designing of more durable pavements, but also by constituting as an economical validation tool for the optimisation of materials and calculating the effectiveness of a certain type of construction in relation to another. Many methods are based on the experience from road projects that have been in service for some time, making the new methods dependent on data from empirical models.

Among the concrete pavement design methods available, two methods have been investigated in detail, the U.S. and the Dutch one. The selection is based on two things; (i) modernity and (ii) availability. These two methods are both new, in the international research front, computer based, and available.

In this Chapter, the international design methods are presented and compared to the current design method used in Sweden. The current Swedish design as well as the first international method, the MEPDG from USA, have been investigated thoroughly in Paper 1, and are
therefore only presented briefly. The second method, from the Netherlands, VENCON2.0, is described more in depth but still only outlined in this thesis. The way of calculating stresses as well as the method’s specific aims of design is of interest when developing a new design method for Sweden.

2.2 Current Design Method in Sweden

2.2.1 Traffic Loads

Stresses due to traffic loads are present in the interaction between the wheel of a vehicle and the road surface. The pressure from the wheel develops stresses in the different material layers in the road structure and, it is foremost the stresses in the concrete slab, and the strains in the subbase that are of interest in the design. In Swedish design a 100 kN equivalent standard axle load (ESAL) is used. Every heavy truck corresponds to 1.3 passages of an ESAL. A heavy truck has a weight exceeding 3.5 metric tons.

Today, stresses due to traffic are calculated with an elastic multi-layer program, GIPI, van Cauwelaert (1986). In this program, it is possible to define five layers to be calculated, each of them given properties on thickness, module of elasticity, and Poisson’s ratio. Between the concrete and the bound base no bond is assumed. This is a conservative assumption. Contrarily, for industrial pavements, where a higher crack risk is accepted, bond is usually assumed, Silfwerbrand (1995, 2001).

2.2.2 Temperature Loads

Thermal loads arise in the concrete slab when its natural tendency to expand or contract is prevented. It is the deadweight of the concrete slab that prevents the slab to curl and in that way stresses are generated. A positive temperature gradient corresponds to an expansion on the top layer with tensile stresses as a consequence in the bottom layer. Curling stresses are directly dependent on the temperature gradients.

Stresses due to thermal loads are calculated with Eisenmann’s beam equation, Eisenmann (1979). The critical length $L_{cr}$ is the decisive factor that determines the geometry of the slab. The stresses at the bottom of a simply supported concrete beam is calculated according to the following equations:

$$
\sigma_{temp} = 1.2 \cdot \frac{\alpha \cdot \Delta t \cdot h \cdot E}{2 \cdot (1 - \nu)} \left( \frac{L}{L_{cr}} \right)^2 \quad \text{if } L \leq L_{cr} \tag{2.1}
$$

$$
\sigma_{temp} = 1.2 \cdot \frac{\alpha \cdot \Delta t \cdot h \cdot E}{2 \cdot (1 - \nu)} \quad \text{if } L > L_{cr} \tag{2.2}
$$

$$
L_{cr} = \sqrt{\frac{4 \alpha \Delta t E}{5 \cdot (1 - \nu) \cdot \gamma h}} \tag{2.3}
$$

where, $\alpha$ is the coefficient of thermal expansion of concrete, $\Delta t$ is the temperature gradient, $\nu$ is the Poisson’s ratio, $E$ is the modulus of elasticity, $h$ is the thickness of the concrete pavement, and $\gamma$ is the dead load per unit length.
The climate is characterised by temperature and sunshine and is described by the magnitude and length of the thermal gradients. In Sweden, an extreme gradient of 60°C/m has been chosen for 5% of the year, and 40°C/m for 20%. During the rest of the year, a thermal gradient of 0°C/m is assumed. These values, originated from field studies in Germany, were modified for Swedish conditions by Ö. Petersson, Petersson (1996), for the new Swedish specifications of the 1990’s.

2.2.3 Fatigue and Damage Accumulation

Thermal and traffic loads act simultaneously with different magnitudes, hour by hour, on the concrete pavement. The loads produce stresses and eventually, after sufficiently many repetitions, cause fatigue damage. The number of allowable load applications for each thickness under each part of the year is calculated with Tepfers’ fatigue equation, Tepfers (1978, 1979a, and 1979b)

\[
\frac{\sigma_{\text{max}}}{f_{c,0}} = 1 - 0.0685 \cdot \left(1 - \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}\right) \cdot \log N.
\]  

The number of allowable load applications is also calculated with respect to the deformation in the subgrade according to:

\[
N = \frac{8.06 \cdot 10^{-8}}{\varepsilon_{z}^{2}}
\]  

where, \(N\) is the number of load applications, \(\sigma_{\text{min}}\) is the minimum stress, i.e. temperature stress, \(\sigma_{\text{max}}\) is the maximum stress, i.e., the traffic and temperature stress superimposed, \(f_{c,0}\) is the concrete flexural strength, and \(\varepsilon_{z}\) is the vertical compressive strain on top of the subgrade. The stresses are only calculated in the bottom of the slab.

The accumulated fatigue damage that a certain pavement can resist is calculated with Miner-Palmgren’s damage hypothesis for flexural stresses in the concrete pavement, horizontal tensile strains in the base course or the vertical compressive strain of the subgrade. The accumulated damage is computed by summing fatigue damage incurred during each of six seasons of the year due to both traffic and thermal loads.

Miner-Palmgren’s damage hypothesis has the following form:

\[
\sum_{i=1}^{n} \frac{\alpha_{i} \cdot N_{i}}{N_{\text{allow}}} \leq 1
\]  

where, \(\alpha_{i}\) is the percentage of time in which damage \(i\) occurs, \(N_{i}\) is the total number of loads corresponding to damage \(i\), and \(N_{\text{allow}}\) is the number of allowable loads corresponding to damage \(i\).
2.3 Design Method in the USA

The USA have had an old tradition of constructing roads with cement stabilised materials and concrete and it is to no one’s surprise that the new design methods developed are very sophisticated. Design methods for highways and airfields have been developed locally in many states but larger project have been conducted by the Transportation Research Board (TRB), the U.S. Military Corps of Engineers, and the Federal Airport Association (FAA). The design method that is being launched for the public is the Mechanistic-Empirical Pavement Design Guide (MEPDG), Transportation Research Board (2004). The software can be downloaded from the internet and is currently being evaluated before it becomes the public tool for designing. Since the program can be used for both rigid and flexible pavements, it is a quite complicated program and will need a few years of investigation before all material models and algorithms’ in the system are fully functional for the design. The program has not been released in new versions because the risk of having people working with old versions is considered as a problem. The software is prepared for SI-units but has not yet been released because the lack of sponsorship. A transformation to SI-units come with a new, full investigation to check errors in the program. The MEPDG has been presented at several international conferences and information on the methodology used in the design is found mainly in Transportation Research Board (2004) but also in Darter (2001) and Darter (2004), among others.

The software calculates the degree of degradation of a concrete road by estimating the deterioration of different functional properties as cracking (percent slabs cracked), joint faulting (inches), and smoothness (IRI in inches/mile). The results are based on calibrated models for fatigue development and are presented incrementally over the design life. Three different levels of accuracy can be considered and the results are given with a percentage of probability. The design considers both bottom-up and top-down cracking, i.e. stresses that are generated in both the bottom and the top side of the concrete pavement. Further in-depth analysis of the design procedure and a comparison to the current Swedish design is found in Paper 1.

2.4 Design Method in the Netherlands

In the Netherlands, motorways have been designed with a software, VENCON 1.0, since the middle of the 1990’s. The demands for a more user friendly software have nevertheless emerged during the last ten years and therefore a complete upgrade of this program was undertaken. The new software, VENCON 2.0, CROW (2004), was developed and became available in 2005. The software is described in detail in van Leest (2006) and Houben (2006).

VENCON 2.0 is destined for plain concrete pavements and continuously reinforced concrete pavements and has a very flexible interface. For instance, the different geometrical properties, as the number or the width of the road lanes, can easily be changed. The design is based on fatigue strength calculations that are made in certain locations in the concrete slab; the longitudinal free edge, the longitudinal joints, and the transverse joint in the centre of the wheel load.

Mainly two concrete grades are used in road construction in the Netherlands, C28/35 and C35/45. The mean tensile strength ($f_{t,\text{mean}}$) for loading of short duration is calculated using a safety factor of 1.2 according to
\[ f_{\text{fm,mean}} = 1.3[1.05 + 0.05(f_{\text{ch}} + 8)]/1.2 \] (2.7)

where \( f_{\text{ch}} \) is the characteristic cube compressive strength at 28 days (European standard, NEN-EN 206).

The flexural strength is also defined as a function of thickness, \( h \) (in mm), of the concrete where

\[ f_{\text{f,mean,h}} = \left[ (1600 - h)/1000 \right] f_{\text{f,mean}} \] (2.8)

Traffic loading is calculated for the total number of axles per axle group. The frequency and average load for each axle group have been assembled from axle load measurements and can be used in the design. Different kinds of tyres are also included in the design, where single, double, wide base, and extra wide based tyres can be considered. The extra wide base tyre is for instance not yet allowed but have been included for future needs. The traffic measurements have also uncovered the number of overloaded axles that traffic the roads and the highest average wheel load is therefore 105 kN (corresponds to an axle load of 200 – 220 kN). The traffic stresses are calculated with means of the “new” Westergaard equation, Equation 2.9, for a circular tyre contact area, developed in Ioannides et al. (1987), at the bottom of the slab at the three above mentioned locations. The calculation also includes the load transfer (W) that is 60% for construction joints and 80% for contraction joints in a PJCP. For comparison, the LTE is 90% in transverse cracks in CRCP and 20% in longitudinal free edges on an unbound base.

For traffic stresses, the “new” Westergaard’s equation has the form

\[ \sigma_{\text{fl}} = \frac{3(1+\nu)P_{\text{cal}}}{\pi(3+\nu)h^2} \left[ \ln \left( \frac{E_hh^3}{100ka^4} \right) + 1.84 - \frac{4}{3} \frac{1-\nu}{2} + 1.18(1+2\nu)\frac{a}{l} \right] \] (2.9)

\[ l = \frac{4E_hk^2}{12(1-\nu)k} \] (2.10)

\[ P_{\text{cal}} = 1 - \frac{W}{200} \] (2.11)

where,

- \( \sigma_{\text{fl}} \) = flexural tensile stress (N/mm\(^2\))
- \( P_{\text{cal}} \) = wheel load, accounting for the load transfer (N)
- \( a \) = equivalent radius of circular contact area (mm)
- \( E_h \) = modulus of elasticity (N/mm\(^2\))
- \( \nu \) = Poisson’s ratio (-)
- \( h \) = thickness (mm)
- \( k \) = modulus of substructure reaction (N/mm3)
- \( W \) = load transfer that is dependent on the type of joint (%)
Temperature stresses are calculated along the edges of the slab. Default temperature gradients and frequencies are used that include seven positive temperature gradient classes from 0 to 60°C/m. These temperature gradients have been established from temperature measurements in concrete pavements during the years 2000 and 2001. The calculations of temperature induced stresses are done with respect to the magnitude of the temperature gradient. If the temperature gradient is small, the deadweight causes the beam (the calculation consider a beam along the edge of the slab) to remain fully supported, Equation (2.12). In case of a great temperature gradient the beam will only be supported over certain length $C$ at the ends because the curling upward is greater than the effect of the deadweight, Equation (2.13) and (2.14) for length and width, respectively. The governing equations for calculating temperature stresses are

$$\sigma_T = \frac{h \Delta t}{2} \cdot \alpha E$$  \hspace{1cm} (2.12)

$$\sigma_{T, L} = 1.8 \cdot 10^{-5} \cdot \left( \frac{L - \frac{2}{3} C}{h} \right)^2$$ \hspace{1cm} (2.13)

$$\sigma_{T, W} = 1.8 \cdot 10^{-5} \cdot \left( \frac{W - \frac{2}{3} C}{h} \right)^2$$ \hspace{1cm} (2.14)

$$C = 4.5 \cdot \frac{h}{k \Delta t} \quad \text{if} \quad C << 1$$ \hspace{1cm} (2.15)

where,

- $\sigma_T$ = flexural tensile stress due to temperature gradient (N/mm$^2$)
- $h$ = thickness of concrete (mm)
- $\Delta t$ = temperature gradient (C/mm)
- $k$ = modulus of substructure reaction (N/mm$^3$)
- $\alpha$ = coefficient of linear expansion (usually $10^{-5}$ (°C)$^{-1}$)
- $E$ = modulus of elasticity (N/mm$^2$)
- $C$ = supporting length (mm) in width or length of the slab
- $L$ = length of the concrete slab (mm)
- $W$ = width of the concrete slab (mm)

In the design, the substructure is characterised by the modulus of substructure reaction, $k$, that is calculated by using known values on the modulus of subgrade reaction $k_o$ for different subgrades listed in the Netherlands. This $k$-value represents the whole substructure beneath the concrete wear layer.

The fatigue relationship in the design is applied in all the three critical locations and accumulated for traffic and temperature stresses with Palmgren-Miner’s damage hypothesis, Equation (2.6).

The fatigue criterion has the following form:
where, $N_i$ is the number of load applications for a specific loading condition $i$, $\sigma_{\text{min}}$ is the minimum stress, $\sigma_{\text{max}}$ is the maximum stress, and $f_{\beta}$ is the concrete flexural strength. The fatigue equation is only valid when the maximum stress lies in the region of 50 to 83.3 % of the maximum flexural strength. Below this region, stresses are considered harmless. Above this region, stresses are considered to be too damaging for the pavement. The design only considers bottom-up cracking, i.e. stresses in the bottom of the concrete pavement.

Additional checks, which include the check for robustness and the check for traffic ability at opening of the pavement, are done as well.

### 2.5 Comparison of the Design Methods

The Dutch design method has many similarities with the current Swedish design method. The equations that have been chosen, for calculating both traffic and temperature loads are newly developed equations that originate from the same equations used in Sweden. An elastic multi-layer program is, however, used for calculating traffic stresses in Sweden today. The way of considering traffic loads in Dutch design, with different axle load frequencies for different types of roads is more sophisticated, but differs in principle only from the Swedish method in the number of loads, the different load magnitudes, and various tyres that are included in the design. The high axle loads of 200 – 220 kN that is included in the design comes from new traffic measurement in the Netherlands. Measurements conducted in Sweden have also indicated that trucks that traffic the national roads are overloaded and should be considered in the design, SRA (2003).

The temperature gradient distribution applied in the design is also more close to reality in comparison to the somewhat coarse values used in Sweden. A different fatigue equation is developed for the design which is more conservative than in Sweden, see Figure (2.1). It is also restricted for loads exceeding 83 % of the flexural strength and no damage is accounted for if the load is below 50 %. The main difference is nevertheless the advantageous features that enable the possibility to alter the design. The Dutch design method makes it possible, to a certain extent, to alter the widths and the lengths of the concrete slabs, choosing different conditions to connect the slabs according to the type of road that is designed. The software provides a concrete thickness with any desired information on the outcome from the calculations.
Figure 2.1. Difference between the Swedish criterion, Equation 2.4, and the fatigue criterion used in the Netherlands, Equation 2.15. The figure shows a S-N-diagram where the number of load applications, logN, for a given relative stress, $\frac{\sigma_{\text{min}}}{f_{c,fb}}$, and $R=\frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}$ is presented.

The MEPDG is also very flexible, in the same way as the Dutch method, but shifted towards a more mechanistic design. The design aims at predicting the functional properties as joint faulting, slab transverse cracking, and smoothness (IRI) based on the site conditions. The deterioration of the pavement is calculated incrementally over the design period, taking into account the various conditions that may change hourly.

Similar for all methods is the use of Miner-Palmgrens’ damage hypothesis. The hypothesis has been investigated by Tepfers et al. (1977) among others, and is up to this day the best method for summarising the effect of impact from more than one loading condition on concrete fatigue. It still remains a very approximate method that is used in lack of a more mechanistic alternative.

Critical locations that are considered in the investigated methods are presented in Figure 2.2. Table 2.1 shows a comparison between the different design methods presented in this Chapter. The aim of this study has been to highlight some of the features that are considered in the different methods, compared to the current Swedish design method.
Figure 2.2. Critical locations for bottom-up and top-down cracking in a PJCP considered in different design methods; Sweden (1), the Netherlands (2, 3, and 4), and the USA (4 and 5).
### Table 2.1. Comparison of design parameters for PJCP considered in the current design method in Sweden, VENCON 2.0 in the Netherlands, and the MEPDG in the USA

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Sweden</th>
<th>The Netherlands</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subgrade moisture/temperature</td>
<td>No</td>
<td>No</td>
<td>Monthly variation</td>
</tr>
<tr>
<td>Concrete strength over time</td>
<td>No*</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Material stiffness</td>
<td>Seasonal variation</td>
<td>No</td>
<td>Monthly variation</td>
</tr>
<tr>
<td><strong>Temperature loads</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear temperature gradient</td>
<td>3 gradients</td>
<td>7 gradients</td>
<td>Daily variations</td>
</tr>
<tr>
<td>Nonlinear temperature gradient</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Traffic loads</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axle loads</td>
<td>100 kN ESAL</td>
<td>10 axle groups</td>
<td>13 axle groups</td>
</tr>
<tr>
<td>Maximum axle load (axle type)</td>
<td>100 kN**</td>
<td>200-220 kN</td>
<td>100 kN / 450 kN</td>
</tr>
<tr>
<td>Type of axles considered</td>
<td>Single</td>
<td>Single</td>
<td>Single to Quad</td>
</tr>
<tr>
<td>Axle load distribution</td>
<td>No*</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Traffic wandering</td>
<td>No*</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Axle spacing</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Axle width</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Tyre pressure</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Dual tyre spacing</td>
<td>No*</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Pavement properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint spacing</td>
<td>Fixed*</td>
<td>Variable</td>
<td>Variable</td>
</tr>
<tr>
<td>Lane width</td>
<td>Fixed*</td>
<td>Variable</td>
<td>Variable</td>
</tr>
<tr>
<td>LTE - base layer</td>
<td>}100%</td>
<td>}60 or 80 %</td>
<td>Variable</td>
</tr>
<tr>
<td>LTE - dowels</td>
<td>}100%</td>
<td>}60 or 80 %</td>
<td>Variable</td>
</tr>
<tr>
<td>LTE - aggregate interlock</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shoulder type</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Traffic estimations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AADTT</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Monthly adjustment factors</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Vehicle class distribution</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Hourly truck traffic distribution</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Traffic growth factor</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Types of distresses considered</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of critical points in slab</td>
<td>1</td>
<td>3</td>
<td>All locations</td>
</tr>
<tr>
<td>Bottom-up cracking</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Top-down cracking</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Joint faulting</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>IRI</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Interface - wear layer/road base</td>
<td>No bond*</td>
<td>No bond(?)</td>
<td>Variable</td>
</tr>
</tbody>
</table>

**Note:** * – Considered in the Swedish design of industrial pavements, Silfwerbrand (2001), **– any load between 100 and 900 kN in the Swedish design of industrial pavements, Yes – considered, No – not considered, Variable – can be altered, Fixed – fixed to one value in the design.
Chapter 3

Experimental Studies on Fatigue

3.1 General

The fatigue criterion is the far most important factor in the design of concrete pavements. The criterion is used to predict failure and is normally investigated through material testing of small specimens (cubes, cylinders, or beams) in laboratories or full scale field test sections. Small scale testing is more convenient but include various estimations on how the actual behaviour in the field changes. Full scale testing, as full-size slabs, is rare because it needs large testing facilities. Full-scale testing on airfield pavements have been conducted since 60 years in the U.S. and has been the main data source for the development of slab fatigue criteria, Roesler (2004) and Smith and Roesler (2003). These tests offer real traffic loads, environmental conditions, traffic wander, and material properties to be included in the fatigue curve. Pavement testing, when accelerated, does not take into account all climatic variations and soil conditions, or the beneficial factors like increasing concrete strength.

There is a vast number of different fatigue criteria available in the USA and Europe and a comparison between different methods are presented in Figure 3.1. In Sweden, Tepfers’ fatigue criterion, Equation (2.4), has been chosen for concrete pavement design, Petersson (1996) and SRA (2005). The equation includes the effect of minimum to maximum stress ratio ($R$) and is not extremely conservative in comparison to other fatigue criteria (Figure 3.1). The equation is developed out of compressive and splitting tests and since the fatigue strength was equal in these two tests, it was concluded that the equation could be used for bending stresses as well.
Figure 3.1. The diagram shows fatigue curves for different fatigue criteria. Darter, Foxworthy, and NCHRP 1-26 are developed in the USA, Smith (2003). VENCON 2.0 comes from the Netherlands, CROW (2004) and Tepfers from Sweden, Equation 2.4, Section 2.2.2. Only VENCON 2.0 and Tepfers consider the stress ratio \(R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}\).

The experimental studies conducted in this thesis are presented in Papers 2 - 4. The tests have been done in order to verify the design criteria and are made on plain concrete, lean concrete and composite beams of plain and lean concrete. The static system chosen for the tests was retrieved from the preceding Swedish standard SS 13 72 12, that prescribes four point flexural tests (Nowadays, SS-EN 12390-5-2000 is practice in Sweden. Here, three or four point flexural tests are prescribed).

3.2 Methodology

Plain and lean concrete beams that measured 800 mm in length, 150 mm in width, and 100 mm in height were manufactured for the fatigue tests. The plain concrete beams were cast in steel forms at the laboratory. The lean concrete beams were manufactured by compaction of the material on a surface of 70 m² with a thickness equivalent to the beams’ height. The beams were extracted by sawing.

The fatigue tests were conducted at the Swedish Cement and Concrete Research Institute (CBI) in Stockholm with the Material Test System (MTS) 810 machine, Figure 3.2. The tests included static testing of at least three specimens to verify the static flexural strength and then applying the selected load levels in the subsequent flexural fatigue tests using a mean value from these tests \(f_{c,0}\) in Equation (2.4)). In the fatigue flexural tests, the loading was started at a low frequency (0.5 Hz) and increased rapidly to the maximum frequency (2.05 Hz) after a few cycles. This method was adopted to avoid any sudden loading, or concentrated loads, that could arise from the supports when applying the first load.
The loading caused a rough sampling of deflection and loads and the fatigue frequency was thus limited to 2.05 Hz. Deflection data were collected through external sampling. The sampling rate was 40 Hz. The applied loads are expected to be accurate within 5%, but the deviation is negligible in the context of fatigue loading where the standard deviation of the static flexural strength is higher (up to 15%). On the other hand, if the maximum load is greater than 80% of the maximum static strength, the loading accuracy and the true flexural strength is crucial for the results. Fatigue testing is often limited to stress levels below 80% of the static strength. This problem is discussed in Paper 1 but also in Tepfers (1978), among others.

### 3.3 Results

The fatigue test results showed that Tepfers’ fatigue criterion is applicable on the flexural fatigue of plain concrete, but also on the flexural fatigue of lean concrete. For lean concrete there exists a strain criterion today, developed by Örbom (1981). This criterion is very strict compared to international criteria, see Paper 3. Since both plain and lean concrete are cementitious materials it was of great interest to verify if a stress criterion could be applied instead. This was proved with somewhat lower fatigue strength. The composite beams of plain and lean concrete showed that the bond was extremely resistant and that repeated loading resulted in all-pervading cracks. The bond has a strengthening function in the pavement system and is likely to be advantageous if well distributed expansion joints are cut in the lean concrete layer.
Chapter 4

Temperature Measurements

4.1 General

Data of accurate temperature variations is a key factor for the design of concrete pavements. The temperature gradient in a concrete slab causes curling which generates stresses when the slab is restrained. The slab can be restrained by adjoined slabs but the dead weight of a slab also counteracts the natural movements. Thermal expansion on the upper side of a slab makes the slab rise over the centre, but the dead load acts in the opposite direction, generating tensile stresses in the bottom of the slab. Extreme temperature gradients, measured in for instance the USA or Germany, have been shown to reach 90°C/m over a 200 mm thick slab, Petersson (1996). Gradient of this magnitude can generate stresses near the ultimate tensile stress limit of the material. The thermal conductivity of the material is low and daily temperature variations have a pronounced effect on the concrete pavement. The temperature effects on concrete pavements also act at the same time as heavy traffic is present, i.e. rush hour traffic. This leads to superimposed stresses.

The temperature in the concrete is dependent on the air temperature, the solar radiation, precipitation, and the wind conditions. The specific conditions also change with the surrounding environmental characteristics, like shadows, hills or even the height over ocean.

In Sweden the current design only considers linear temperature gradients (presented in Section 2.2.2). These gradients herein from measurements in Germany, but have been altered to fit a less extreme Swedish climate by a limited number of field measurements, Petersson (1996). New temperature measurements in concrete slabs in various parts of Sweden are needed in a new design method to enable a more optimised design.

4.2 Methodology

Temperature measurements have been conducted within this project since 2005 with means of concrete prisms with the dimensions 400×400 mm and the height of 250 mm. Temperature measuring equipment has been installed inside the cubes at the time of casting and the prisms are dug down in the asphalt road pavement so that only the top surface is exposed to the varying environmental conditions, Figure 4.1. Four sensors measure the temperature at the depths of 25 mm, 75 mm, 170 mm, and 200 mm, respectively. The four points over the cross section will make it possible to investigate, not only the linear temperature gradient, but also the duration and shape of the nonlinear temperature gradients.
Figure 4.1. Installation of a concrete prism for temperature measurements in Sweden. Cables from temperature sensors are also put in the asphalt layer and connected to a data logger beside the road. The logger is of the type ConReg 706, used for concrete maturity monitoring in casting, CMT (2005).

The field measurements with these cubes are being conducted on three sites in Sweden; 300 km north of Stockholm in the region of Enånger, 60 km south of Stockholm in the neighbourhood of Strängnäs, and just south of Malmö in the south of Sweden (see Figure 4.2). These locations have been chosen because of the traffic volumes at these places, making these regions the most probable sites for concrete pavement construction in Sweden. Even though the Swedish west coast is heavily trafficked, no measurements are conducted there in this particular project. Here, measurements have already been done at the time for, and during ten years after the construction of highway E4 Falkenberg by VTI, Wiman (2002). Other measurements have also been conducted by Ö. Petersson in 1991, Petersson (1996), and A. Farhang in 1997 - 2000, Farhang (2000), in various locations, see Figure 4.2.
Figur 4.2. Locations for temperature measurements in Sweden. VTI, Petersson, and Farhang are measurements in actual concrete pavements. Söderqvist represents measurements in concrete prisms that are put into the existing asphalt pavement (two sites) and in existing concrete pavement (one site).

4.3 Results

The results from the measurements will give information on the frequency and magnitude of temperature gradients in different parts of Sweden. Measurements conducted by means of four sensors through the cross-section of a concrete prism will also make it possible to analyse the nonlinear temperature gradient. These gradients can reveal if stresses act in the bottom or top of the pavement and how important this effect is for the stress level in the design. Figure 4.3 illustrates the magnitude of a positive linear temperature gradient of 40 °C/m, but also a negative linear temperature gradient of 25 °C/m. A more accurate estimation of temperature gradients measured in the field that include positive, negative, and nonlinear gradients, will make the design method more reliable and at the same time, possibly less conservative.
Figure 4.3. Temperature varying over depth in a concrete prism in Enånger, late August, 2006. A positive linear temperature gradient has its peak in the afternoon and the negative temperature gradient has its peak in very early morning.
Chapter 5

New Design Method for Swedish Conditions

5.1 General

In the new proposal for the design of concrete pavements in Sweden, finite element analysis (FE-analysis) is suggested to be utilised for the calculation of stresses in different locations in the concrete slab. The design criteria for stresses in the concrete slab, the bound road base, and the subbase that are included in the current design can be used in the new methodology. Also, temperature loads with more accurate distributions can be included in the design. The new methodology can be applied on specific projects, and later on developed to account for true traffic loads as well as nonlinear temperature gradients.

5.2 New Method for Calculating Loads

For the calculation of traffic loads in the concrete slab, the layers under the concrete slab can be modelled as a Wrinkler foundation, Figure 5.1. The foundation stiffness is hereby characterised by a $k$-value, that is a proportionality constant between the vertical pressure and the deflection of the slab (N/mm$^2$). Values for the $k$-value exist for different materials but it is more accurate to predict this value out of geometry and Falling Weight Deflectometer (FWD) testing on the modulus of elasticity. The $k$-value is dependent on the modulus of elasticity of the subbase but also the thickness of the concrete slab as well as the ratio in modulus of elasticity between the concrete slab and the subbase.

![Figure 5.1. Theoretical model of a Wrinkler foundation where the stress in the foundation is proportional to the deflection of the slab in each point, from Petersson (1996).](image)
In the proposed method, the calculation of stresses and strains in the concrete slab are suggested to be done using a FE-program, for instance ISLAB2000, ARA (2004). To do this, the software is dependent on the \( k \)-value. To estimate the \( k \)-value in the subbase FWD data can be used, where the load and deflection is registered, or with known values on the modulus of elasticity of the road base materials.

The \( k \)-value can also be calculated in a separate elastic multi-layer program such as GIPI or BISAR 3.0 (1998), every time a new design is considered. The material data inputs in the program consist of the modulus of elasticity, the poisson’s ratio, and the thickness of the material layers. In the program, a FWD load is simulated and the \( k \)-value calculated with the data from the program, i.e. the load and corresponding deflection under the concrete slab. This value can be calculated for different seasons, with different temperatures and different moisture contents in the subbase. The \( k \)-value is then used in the FE-program to calculate the stresses in the concrete slab with varying load magnitudes and load locations, see Figure 5.2.

The use of a program like BISAR 3.0 will bring the design of concrete pavements more close to the design of flexible pavements since the subbase is modelled in the same way. The different design procedures will hereby be more comparable and this is advantageous for designers and clients when evaluating various pavement alternatives.

![Figure 5.2](image-url)

*Figure 5.2. An elastic multi-layer program is used to calculate the \( k \)-value using data on load and deflection from the actual pavement system (1). The \( k \)-value is used in the equivalent model in the FE-program, where stresses and strains are calculated in the concrete layer, using model (2) or the concrete and the bound road base, using model (3).*
The above mentioned procedure is basically the procedure that is used in the newly developed design method in the MEPDG. ISLAB2000 is a 2.5-dimensional FE-program, specially constructed for calculating stresses and strains in concrete slabs very fast. The program is simple in a way that a number of adjacent slabs and pavement shoulders can be modelled with specific interaction properties. One of the most helpful advantages is the possibility to calculate both temperature and traffic stresses at the same time. Nonlinear temperature gradients are also possible to apply.

In the new FE-program the calculation of stresses in the concrete slab can be utilised to calculate the fatigue from a 100 kN standard axle load. The load can be placed in the centre of the slab or on the edge of the slab, i.e. where the loads generate the highest stresses. The highest stresses in the bottom of a slab is generated by a load placed in the centre or the edge of the slab. In the top of the slab, the highest stresses are instead generated by a load placed near the corner or the edge. For certain situations, the temperature has a crucial effect on the total stress distribution. At a negative temperature gradient, the edges will rise, and if any load on the edges is present, the total stress will rise as well. In a later stage, the method can be enhanced by adding new parameters for different axle loads. These data have to be assembled before it can be included in the design.

To gain control over different extreme situations at least four different load locations should be considered for fatigue analysis; two load locations on top and two load locations in the bottom of the concrete slab. This approach would complete the current design method by considering the combination of loads near the edge of the slab when a negative temperature gradient is present.

The temperature is very important when designing concrete pavements. New temperature measurements in Sweden indicate that the temperature gradients are more complex and can be more accurately estimated compared to the current gradients. For instance, research data have shown that a negative temperature gradient is present during a short period of time (Farhang, 2004b). This is not accounted for today but will be included in the new design method. The use of linear temperature gradients is also a simplification of reality.

5.3 Fatigue Criteria and Damage Accumulation

A fatigue criterion that considers two load levels, as Tepfers’ fatigue criterion, or the Dutch correspondent equation, is valuable in a design that accounts for both temperature and traffic loads. A new fatigue criterion can be developed but for the time being, it is suggested that Tepfers’ criterion remains in Swedish design.

Damage is influenced by the loading frequency, the time at resting, and the magnitude of the loading, Hsu (1981). Calculating the accumulated damage is still generally done by means of Miner-Palmgren’s damage hypothesis in pavement design, Equation (2.6). The method is criticised by many because its linear illustration of damage simplifies the influence of damage from multiple loads. Any new or better method has, however, yet not been presented. The damage hypothesis works as an engineering tool more than a refined tool for accumulating different loads. In modern design methods, the damage hypothesis is used in combination with a calibrated damage model where the damage is varied statistically, as in the MEPDG.
5.4 Implementation of a New Swedish Design Method

The general idea of the proposed design method is presented in Figure 5.3. The development of a new method consists of the gathering of parameters for traffic and temperature loads that are unique in Sweden, Figure 5.3 (A and B). Different material properties and design criteria for new failure modes have to be investigated, Figure 5.3 (C and D) and new calculation methods have to be analysed in comparison to existing ones to guarantee the reliability of the method. The more practical implementation of a computer based, interactive program for the design is much dependent on the amount of parameters that are required in the design, i.e. the level of sophistication. The method proposed is meant to be developed gradually so that it replaces the current method without any too dramatically changes for constructors or clients. Also, the level of simplicity is a key factor to reach a wider use among future concrete pavement designers.
Figure 5.3. Proposed procedure for the design of PJCP in Sweden.
Chapter 6

Conclusions

6.1 Concrete Pavement Design

The design of plain concrete pavements involves a large number of parameters. The current Swedish design method was developed 15 years ago. The method is simple and limited since many parameters are neglected and therefore the safety margins applied are important. In different design methods found, new information on traffic and temperature distributions are incorporated into the design. These methods are more accurate because it is possible to quantify the safety margins. These methods are also computerised to a large extent, making them faster and more flexible.

This thesis presents a basis of a new design method that could be easily implemented in Sweden. The method can be developed gradually and is based on FE-analysis for fast calculation. In the process of validating the new method it is recommended that it is compared to some of the international design methods available and the current Swedish design method.

6.2 Flexural Fatigue Criteria

The laboratory tests conducted in this project have aimed at investigating the fatigue criterion in Swedish design. The tests involved beams of plain concrete, lean concrete, and composite beams of plain and lean concrete.

Tepfers’ fatigue criterion was originally developed out of compression and splitting tests and the test results for the plain concrete beams showed that the criterion is valid also for flexural fatigue.

The fatigue of lean concrete is, in Swedish pavement design, calculated using a strain criterion that is very strict compared to international standards. Since lean concrete is a cementitious material, the tests were done in order to analyse the possibility to apply a stress criterion instead, as done for plain concrete. The tests showed that a stress criterion is applicable with a small reduction in fatigue strength. The reduction compared to Tepfers’ fatigue equation can be illustrated by a reduction constant \( C \), that reduces the impact of amplitude between the load levels. A stress criterion for lean concrete would be less strict and consequently make lean concrete more competitive than today.
The composite beams were investigated to verify the cracking behaviour and the fatigue strength properties of the bond. In testing, all-pervading cracks were difficult to avoid. The cracking is explained by (1) the full bond between the materials that allowed stresses to pass through the interface, and (2) the concentrated stresses that are assembled at the crack opening, caused by the decreasing cross-section. The phenomenon of entire cracking of the cross-section is referred to as reflection cracks, and challenges the question on whether the bond really is desirable or not, i.e. strength versus risk of cracking. The bond might be detrimental because it ruins the whole section when the capacity of the lean concrete is surpassed and the cracking of the concrete wear layer is consequently difficult to control. However, if the design focuses on the fatigue and cracking in the bottom of the lean concrete instead of the bottom of the plain concrete, a much stiffer pavement can be obtained and stresses further down in the substructure are limited. Of course, cracking due to temperature expansion and shrinkage in the lean concrete has to be limited by cutting expansion joints in the material at short distances.
Chapter 7

Further Research

In order to establish a modern design method for plain concrete pavements in Sweden, research in various areas within concrete pavement design is required. Material testing for ascertaining the materials degradation parameters and how temperature or moisture affects the different materials properties is important. These are issues that have to be addressed in the development of durable and cost effective concrete pavements. Future research includes:

- Extended laboratory testing and/or road pavement testing regarding the fatigue criteria for plain and lean concrete to incorporate the most accurate and best fitting design criteria in Swedish design.

- Further research on the benefits or disadvantages with a bond between plain and lean concrete.

- Traffic monitoring to improve traffic estimations in the design. Measurements on traffic have to contain load axle weights, frequencies, and type of vehicles or tyre. Measurements have been conducted in Sweden but have to be extended, and assembled.

- Continuous temperature measurements for the establishment of real temperature gradients for various parts of Sweden.

- Further investigations for the development of a new method to replace Miner-Palmgrens’ damage hypothesis. The solution is regarded to be found in fracture mechanics.

- Further development of the proposed Swedish design method and verifications by performing comparison studies with international design methods, the old Swedish method, and long-term performance studies on real concrete roads in Sweden.
Chapter 8

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Paper 1
Design of Concrete Pavements: A Comparison between Swedish and U.S. Methods

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Johan Silfwerbrand², Member ISCP

Abstract

The Swedish regulations for roads have now been converted into a computerised design guide that will provide engineers with the necessary, up to date, tool for the design of roads, no matter the surfacing material. The design of concrete roads is, however, based on old tables that offer little change in the concrete thickness when altering parameters like climate, traffic, and material properties. In an ongoing project a new computer-based design method for concrete roads is being developed. The aim of the project is to facilitate the design method and establish a method that treats the concrete pavement with the same ease, accuracy and safety level as current asphalt pavement design methods. In the Ph. D. project an inventory of various design methods is conducted. The project has a focus on investigating and comparing the mechanistic-based design procedure available in the United States to the current Swedish aspects of design.

Introduction

In Sweden, less than one percent of the national roads are paved with concrete (European Commission 1999). The concrete roads of the 1960’s were constructed with long slab lengths on unbound road bases and had severe joint faulting problems. The unbound granular base under the slab eroded and cracks appeared as the loading capacity decreased. New attempts in the 1970’s resulted in a few roads that have been performing well but have now reached a critical point where they are in need of rehabilitation. The Swedish National Road Administration (SNRA) has during the last 15 years regained interest in concrete roads because of the growing problems with rutting on heavily trafficked asphalt roads and the need for competition between different road surfacing materials (Löfsjögård 2003).

The current design method for concrete roads in Sweden herein from the investigation conducted by Petersson (Petersson 1990) in the 1990’s. A thorough analysis of different design methods for concrete roads was performed and several methods for calculating traffic and temperature loads were considered. This work resulted in a new design method, later implemented in the Swedish regulations for roads. The methodology was evaluated with the design of two highways in the south of Sweden.

Modern concrete roads that have been constructed since 1990 are plain jointed concrete pavements (PJCP) on a cement bound or asphalt bound road base.

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and the results from the follow-ups show a high reliability and an excellent long-term performance (Löfsjögård 2003). The Swedish model for the design of concrete roads is based on Westergaard’s, by Eisenmann, improved equations. It is a design method used in several countries in Europe.

Nowadays, new computerised tools are available that can handle many more parameters in the design and make calculations instantly. Overcoming the difficulties of how to estimate traffic, variations in climate, and different material types, it would be possible to optimise and improve the accuracy in pavement design with these tools.

The Swedish regulations for roads have been converted to a computerised design guide that will provide engineers with the necessary, up to date, tool for the design of roads, no matter the surfacing material. This change in design methodology has, however, left the design of concrete pavements behind. The old tables for calculating pavement thicknesses are still used in new design, probably resulting in less optimised design.

The thicker the pavement is the less attractive it becomes and, therefore, a new more flexible design method is needed in Sweden, a method that can account for a greater variation of inputs and have a high level of reliability.

The new mechanistic-empirical design procedure, the Design Guide, that has been developed in the U.S. during the last ten years is based on mechanistic concepts but calibrated to observed data (Darter 2004, Transportation Research Board 2004). This design concept can be applied to both flexible and rigid pavements and the procedure is based on damage calculation accumulated incrementally over the entire analysis period. The mechanistic-based design procedure enables the functional properties, i.e., cracking, joint faulting and smoothness, to be evaluated throughout the pavement’s designated lifetime. This is necessary when estimating the total cost of a highway construction project and makes it possible to compare different surfacing materials. The design method is more attractive both for the designers and the contractors, as both economical and practical issues become possible to analyse.

Comparing this procedure to the Swedish aspect of design many questions arise on weather it would be possible to apply some of its features on projects in Sweden. It is of course fundamental that the design procedure is validated for the actual construction site and the actual materials as well as to the type of traffic and climate that the pavement will endure. In the U.S., it has been feasible to validate the design process to a huge bank of collected data and is, therefore, in some way optimised for these specific conditions. Taking this data to a new environment and using other materials in layers beneath the concrete, the parameters for particular parts of the design can be put faulty if not examined well. It is also courageous to replace design methods that differ too much in inputs, but also in redefining design criteria. The specifications for the design are always validated to test sections and reference projects. Specifications for all sorts of structures are always improved to new materials and new construction methods but follow the same principles.

The objective of this paper is to describe the differences between the new mechanistic-empirical design procedure for plain joint concrete pavements developed in the U.S., and the current Swedish design method. The paper will highlight the Swedish aspects of design and analyse the corresponding methodology in the Design Guide. The Design Guide software has been available for evaluation through the Transportation Research Board’s homepage and a limited investigation of the program has been done. The aim of this work is to investigate how new tools
and new modelling techniques could be used in Sweden and how these methods are developed.

A New Design Criteria

The new design procedure for plain joint concrete pavements in the U.S. is based on mechanistic concepts. In the development of the procedure many new parameters for the design are taken into account. The intention is to predict the functional properties as joint faulting, slab transverse cracking, and smoothness (IRI) based on the site conditions. These performance parameters are considered the most important in pavement design since they directly affect the riding comfort for the people travelling on the roads. Modern methods with high capacity for calculations and reliability prediction based on observed data have made it possible to consider further parameters in the design. It is nevertheless difficult to understand exactly which parameters to investigate and how these particular parameters change over time in ever changing conditions.

The Swedish design method

For pavement design, the procedure in Sweden is similar to the methods used in many European countries (Silfwerbrand 2001). The procedure consists of the determination of the acceptable number of load applications for a selected pavement type. Taking seasonal effects into account, the Miner-Palmgren’s hypothesis is used to determine the required pavement structure for each of the traffic loading and subgrade bearing capacity combinations. In the design, the diurnal combinations of thermal loads and traffic loads are critical for determining the bearing capacity of the road. For this it is necessary to calculate tensile stresses in the concrete pavement and the road base but also the vertical compressive strain on top of the subgrade. Tensile stresses in the road base are, however, often neglected for a cement bound base since it is considered cracked under the concrete pavement. The procedure is simple in a way that stresses are calculated assuming linear elastic properties of the materials. The likelihood of the method to lead to unsuitable thicknesses of the pavement is imminent since specific conditions are difficult to take into account, i.e. the true properties of the materials. It is believed that in most cases the method results in too thick pavements, taking into account the safety margins needed and the observed performance of roads constructed in Sweden.

Traffic load calculations in Swedish method. In traffic estimation the heavy trucks are represented by vehicles with a total load exceeding 3.5 metric tons and a wheelbase exceeding 3.3 meters. The load from these vehicles is substituted by an equivalent single axle load (ESAL) of 100 kN. The heavy trucks are given a factor of 0.9 to 2.0 ESAL’s depending on the type of pavement. Estimating the traffic, and consequently the number of load applications that the concrete slab has to withstand, it is also important to take into account the traffic wandering over the lanes, the design period, and the growth factor.

Stresses due to traffic were first calculated with Westergaard’s equations, improved by Eisenmann (Eisenmann 1979). These equations are used to calculate the stresses in the centre, on the edge, and in the corner of the concrete slab. Today, stresses due to traffic are calculated with an elastic multi-layer program, GIPI (van Cauwelaert 1986). In this program, it is possible to define five layers to be
calculated, each of them given properties on thickness, module of elasticity, and Poisson’s ratio. Between the concrete and the bound base no bond is assumed. This is not always the case (contrarily, bond is assumed in industrial pavements) and a higher flexural stress is hereby calculated in the concrete (Silfwerbrand 2001). The design is because of this rather conservative providing additional safety to the design. Calculations have to be made for each season of the year to capture the differences in strength of the materials and the depth of frozen material with the changing temperatures. The linear elastic layer program is infinite in the horizontal plane and stresses on the edges are therefore not possible to obtain. The case of loads near the edge is either avoided by markings on the road or by thickening the edge according to a distress twice as high as the computed stresses for internal loading. The last option is more common for industrial pavements. This practice is adopted from results from analysis with the two-dimensional finite element program ILLISLAB that indicated a 100% increase of stresses when approaching the edge of the slab (Silfwerbrand 1995). The load efficiency factor between doweled joints for concrete slabs on bound bases is considered to be 100 percent.

Traffic load calculations in U.S. Design Guide. The U.S. design Guide considers a broad variety of axle loads and axle configurations (Transportation Research Board 2004). The number of heavy trucks is calculated in the same manner as in Sweden but instead of one single axle, several different types of wheel loads can be modelled simultaneously. Mechanistically, this is interesting because the impact of more than one wheel on the concrete slab generates other types of distresses and damages on the pavement, e.g., top down cracking. The possibility to change anything from axle configuration to traffic distribution hour by hour is developed in a way that the traffic can be modelled with such accuracy that the main error lies within the traffic estimations. Table 1 shows the inputs for the Swedish design in comparison to the U.S. Design Guide.

Three main types of traffic related distresses for plain jointed concrete pavements are observed for analysis with the Design Guide; stresses that develop top-down cracking, bottom-up cracking, and joint faulting damage. The loads generating these distresses are identified at three critical locations on the concrete slab where the impact of the traffic loads is the most important (Transportation Research Board 2004, Darter 2001). For bottom-up cracking the critical point is situated at mid span on the edge of the slab. The same location is critical for top-down cracking but is produced by two axles acting simultaneously, developing tensile stresses on top of the slab between the axles. Faulting occurs when the loads are concentrated, and causing deflections on either side of a joint. The critical point regarding faulting is the corner of the slab.

For transverse joints, the total load transfer efficiency (LTE) includes the contribution of three major mechanisms of transfer; by concrete aggregates, by joint dowels, and by the base course (Transportation Research Board 2004).

Calibrated mechanistic based models have been developed to predict faulting, cracking, and IRI. The faulting is determined incrementally and considers the effect of previous maximum faulting, current faulting level, and differential energy. IRI depends on the initial smoothness, the change in distress, i.e., cracking and faulting, and age, subgrade type, and climate. The model predicts smoothness incrementally over the design life (Darter 2004, Transportation Research Board 2004).
Stresses due to both traffic and thermal loads are calculated with a finite element program, ISLAB2000. The procedure is explained further in the section describing thermal loads calculations.

**Table 1.** Comparison of design inputs for traffic calculations. The Swedish method against U.S. Design Guide

<table>
<thead>
<tr>
<th>Traffic Estimation Parameters</th>
<th>Sweden</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>AADTT</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Monthly adjustment factors</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Vehicle class distribution</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Hourly truck traffic distribution</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>AADTT distribution by vehicle class</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Traffic growth factor</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Axle load distribution factor</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Wheel location</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Traffic wandering</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Lane width</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Axle spacing</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Axle width</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Tyre spacing</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Tyre pressure</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Dual tyre spacing</td>
<td>-</td>
<td>X</td>
</tr>
</tbody>
</table>

**Discussion.** The U.S. Design Guide is, compared to the Swedish design method, more sophisticated considering the amount of inputs that are taken into account, see Table 1. The Swedish method does not take more than one single axle into consideration in comparison to the different axle spectra that defines traffic loads in the Design Guide. Introducing multiples axles into the design enables to consider the effect of tensile stresses on top of the concrete slab.

The reliability in traffic estimation also has a significant effect on the traffic calculations, and the Swedish design method is sensitive to the number of ESAL’s per heavy vehicle, e.g., increasing the factor from 1 to 2 equals a theoretical decrease in pavement design life from 20 to 10 years (SNRA 2003).

**Thermal load calculations with Swedish method.** Stresses due to thermal loads are calculated with Eisenmann’s beam equation (Eisenmann 1979). The critical length \(L_c\) is the decisive factor that determines the geometry of the slab. The stresses at the bottom of a simply supported concrete beam is calculated according to the following equations:

\[
\sigma_{\text{temp}} = 1.2 \cdot \frac{\alpha \cdot \Delta T \cdot h \cdot E}{2 \cdot (1 - \nu)} \left( \frac{L}{L_c} \right)^2 \quad \text{if } L \leq L_c \quad (1)
\]

\[
\sigma_{\text{temp}} = 1.2 \cdot \frac{\alpha \cdot \Delta T \cdot h \cdot E}{2 \cdot (1 - \nu)} \quad \text{if } L > L_c \quad (2)
\]
where, $\alpha$ is the coefficient of thermal expansion of concrete, $\Delta t$ is the temperature gradient, $\nu$ is the Poisson’s ratio, $E$ is the modulus of elasticity, $h$ is the thickness of the beam, and $\gamma$ is the dead load per unit length.

The climate is characterised by temperature and sunshine and is described by the magnitude and length of the thermal gradients. In Sweden an extreme gradient of 60°C/m has been chosen for 5 percent of the year, and 40°C/m for 20 percent. During the rest of the year, a thermal gradient of 0°C/m is assumed. These values, originated from field studies in Germany, were modified for Swedish conditions by Petersson (Petersson 1990) for the new Swedish specifications of the 1990’s.

**Thermal load calculations with the U.S. Design Guide.** In the Design Guide, the thermal loads in the concrete slab are calculated with a two-dimensional finite element program, ISLAB2000. The temperature variations in the Design Guide are treated by computerised means where the daily variations are assembled through weather station data, obtained from the National Climatic Data Centre database. (Transportation Research Board 2004)

The climatic information is given by providing the pavement location (longitude and latitude) and the elevation. It is recommended that the data be chosen from several weather stations to compensate for missing data in any one of the weather stations. The hourly inputs contain air temperature, precipitation, wind speed, percentage sunshine, and ambient relative humidity. Inputs provided from site such as seasonal or constant water table depth give the climate model sufficient information to monthly recalculate material properties over the entire design period.

The temperatures directly affect the concrete slab properties with transient hourly negative and positive nonlinear temperature differences between top and bottom, caused by solar radiation and computed using the Enhanced Integrated Climate Module (EICM). The temperature and other climate properties also have an indirect affect on the slab by changing the subgrade properties, i.e., subgrade strength and stiffness.

The analysis requires several input parameters described in Table 2. The structural model for stress computations only considers the concrete slab and the base course. For all layers beneath these two layers a dynamic k-value (psi/in) is assigned through calculations with a linear elastic layer program. In this program, a Falling Weight Deflectometer (FWD) load is simulated. The computed deflection that is produced by the FWD is then used to backcalculate the appropriate k-value. The k-value is recalculated for every month to reflect changes in material properties due to climate changes and is used directly for computations of stresses and deflections. (Transportation Research Board 2004)

To enable rapid solutions neural networks (NN) have been developed based on results from ISLAB2000. This approach is needed to deal with the large numbers of calculations that are required (Transportation Research Board 2004).

The thermal loads are calculated from a temperature profile of 11 point through the slab. The thermal gradients are generated from input data on temperatures in these points, creating nonlinear gradients.
Table 2. Parameters considered for calculation of distresses in the concrete slab. Comparison between the Swedish method and the U.S. Design Guide.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sweden</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Coefficient of thermal expansion</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Unit weight</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Subgrade properties</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Loads</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonlinear temperature gradient</td>
<td>-</td>
<td>Hourly input</td>
</tr>
<tr>
<td>Linear temperature gradient</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>Axle type, weight, and position</td>
<td>Single axle</td>
<td>Several combinations</td>
</tr>
<tr>
<td>Design features</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interface conditions</td>
<td>Bond/no bond</td>
<td>X</td>
</tr>
<tr>
<td>Joint spacing</td>
<td>X</td>
<td>Variable</td>
</tr>
<tr>
<td>Lane width</td>
<td>X</td>
<td>Variable</td>
</tr>
<tr>
<td>LTE - base layer</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>LTE - dowels</td>
<td>}100%</td>
<td>X</td>
</tr>
<tr>
<td>LTE - aggregate interlock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shoulder type</td>
<td>-</td>
<td>X</td>
</tr>
</tbody>
</table>

**Discussion.** Eisenmann’s equation, used in the Swedish method, is based on the assumption that the thermal gradient is linear and the chosen gradients are all positive, i.e., during daytime conditions. Negative gradients are not considered in the design since they are less significant and are counterbalanced by the traffic loads. The Design Guide, on the other hand, automatically computes hourly variations of both positive and negative nonlinear gradients.

Farhang investigated how the stresses are calculated and compared Eisenmann’s beam equation to results obtained by ISLAB2000 (Farhang 2004). The results showed that Eisenmann’s method produces almost twice as high stresses assuming linear, positive gradients. Farhang also showed that by introducing a nonlinear positive temperature gradient (based on measurements) the tensile stresses are reduced further.

**Damage from stresses and fatigue**

The concrete slab is, as mentioned above, affected by both thermal and traffic loads. These loads act simultaneously with different magnitudes, hour by hour. The loads produce stresses in the concrete and eventually, after sufficiently many repetitions, cause fatigue damage.

**Fatigue damage analysis for concrete in Sweden.** The number of allowable load applications for each thickness under each part of the year is calculated with Tepfers fatigue equation (Tepfers 1979)

\[
\frac{\sigma_{\text{max}}}{f_{\alpha}} = 1 - 0.0685 \cdot \left(1 - \frac{\sigma_{\text{max}}}{\sigma_{\text{max}}^*}\right) \log N.
\]
The number of allowable load applications are also calculated with respect to the deformation in the subgrade according to:

\[ N = \frac{8.06 \times 10^{4}}{e_{z}^{2}} \]  

where, \( N \) is the number of load applications, \( \sigma_{\text{min}} \) is the minimum stress, i.e. temperature stress, \( \sigma_{\text{max}} \) is the maximum stress, i.e., the traffic and thermal stress superimposed, \( f_{a} \) is the concrete flexural strength, and \( e_{z} \) is the vertical compressive strain on top of the subgrade. The stresses are only calculated in the bottom of the slab.

The accumulated fatigue damage that a certain pavement can resist is calculated with Miners-Palmgren’s damage hypothesis for flexural stresses in the concrete pavement, horizontal tensile strains in the base course or the vertical compressive strain of the subgrade. The stresses are only calculated in the bottom of the slab.

The accumulated damage is calculated with the following equation:

\[ \sum_{i=1}^{n} \alpha_{i} \cdot N_{i} \leq 1 \]  

where, \( \alpha_{i} \) is the percentage of time in which damage \( i \) occurs, \( N_{i} \) is the total number of loads corresponding to damage \( i \), and \( N_{\text{allow}} \) is the number of allowable loads corresponding to damage \( i \).

**Damage analysis in the Design Guide.** The Design Guide contains prediction models for calculating joint faulting, slab transverse cracking, and IRI. The models are calibrated with data from JPCP sections in the field.

The maximum bending stresses and bending strength are used to compute the number of allowable axle load applications and aggregate interlock wear due to each wheel load for each time increment using the fatigue relationship (Darter 2004, Transportation Research Board 2004):

\[ \log\left(\frac{N_{i,j,k,m,r}}{C_{i,j,k,m,r}}\right) = C_{1} \cdot \left(\frac{MR_{i}}{\sigma_{i,j,k,m,r}}\right)^{C_{2}} - 1 \]  

where, \( MR_{i} \) is the modulus of rupture at age \( i \) for concrete, \( N_{i,j,k,m,r} \) is the allowable number of loads applications at specified condition, \( \sigma_{i,j,k,m,r} \) is the applied stress at specified condition, and \( C_{1} \) and \( C_{2} \) are calibration constants.

The fatigue damage, for all traffic load increments over the design period is calculated in the same way as in Sweden with Miner-Palmgren’s hypothesis. The fatigue is calculated both on the top and bottom of the slab and the conversion to physical pavement distress, i.e., bottom-up and top-down transverse cracking, is related to observed cracking in the field through calibration.
Comparing Swedish and U.S. Design through a Case Study

In a trial to analyse the Design Guide, a section of a Swedish road is evaluated with the program. The particular section is a highway north of Stockholm, Sweden, that will be constructed as a plain jointed concrete pavement in 2006.

The following calculations with the Design Guide are not exact because no laboratory testing for this purpose have been done prior to this investigation. Many of the needed parameters are not directly considered in Swedish design and calculations are due to these reasons made on a level 3 basis; with a minimum of inputs. Materials parameters are, for instance, chosen from recommended values in the Guide. Traffic distribution by vehicle class and axle load values are also taken directly from the Guide.

The objective is to evaluate the impact of some of the parameters in the design of new concrete pavements. The inputs are changed one by one and the effect in performance at the end of the design life is evaluated. Different improvements may affect performance on faulting but have less effect on the cracking ability. It is beneficial to know what parameters are the most significant in the design and know under which circumstances they are valid, i.e., different climate or traffic conditions.

It is also essential to scrutinize the most important parameters and develop new criteria to be able to judge performance in future Swedish design.

General Information. The highway, E4 Uppsala, is situated north of Stockholm. A 23 km long section will consist of a plain joint concrete pavement with 2 lanes in each direction. Traffic is estimated to 12 500 AADT from which 17% are heavy vehicles, i.e., vehicles exceeding 3,5 metric tons. Construction starts in July 2006 and the highway will open to traffic in October 2007.

Material Data. The pavement (Figure 1) consists of a wear layer of concrete, a bound base of asphalt, two subbase courses of compacted unbound materials, and a subbase of crushed gravel. The subgrade consists of a typical Swedish, low strength and silty, material.

![Pavement system according to the Swedish specifications.](image)

Figure 1. Pavement system according to the Swedish specifications.
Climate conditions. For each climate zone, the number of accumulated negative degrees per day is given. The site is situated in climate zone 2 in Sweden and the climate properties are given for six seasons with a corresponding number of days, see Table 4. For climate zone 2, 300-600 accumulated negative degrees Celsius per day is estimated. This means, for instance, that 300 negative degrees per day equals 60 days with an average day temperature of –5°C. The maximal frost depth is 2.0 meters.

Traffic Data. The Average Annual Daily Traffic (AADT) for one lane, in one direction, is estimated to 12 500 vehicles of which 17 % are heavy trucks. Traffic is also estimated to increase annually with a growth factor of 2.6%. This value is not really reliable considering a design life of 40 years but should be accounted for over the first 15-20 years.

Design with the Swedish method

Material data. The pavements system is from start chosen from a standard design. Properties, e.g., elastic modulus and Poisson’s ratio for the different layers and different seasons, are retrieved from the Swedish specifications (SNRA 2004), see Table 3.

 Traditionally, concrete pavements for highways are constructed with concrete with a flexural strength of 6 MPa and an elastic modulus of 36 000 MPa.

Traffic loads. For the design of this road section a factor of 2.45 standard axles per heavy vehicle and a growth factor over 40 years of 2.6 % is estimated. The total number of estimated ESAL’s are shown in Table 3.

Table 3. Traffic calculation for E4 Uppsala

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AADT</td>
<td>12500</td>
</tr>
<tr>
<td>Percent of trucks, A</td>
<td>17 %</td>
</tr>
<tr>
<td>Number of axles per truck, B</td>
<td>2.45</td>
</tr>
<tr>
<td>Traffic growth, n</td>
<td>2.6 % (compound)</td>
</tr>
<tr>
<td>Design life, years</td>
<td>40</td>
</tr>
<tr>
<td>Number of ESAL’s, N&lt;sub&gt;dy&lt;/sub&gt;</td>
<td>134 million</td>
</tr>
</tbody>
</table>

The traffic stresses are calculated for a 100 kN standard axle with a tyre pressure of 0.8 MPa. The stresses induced by this axle are computed with an elastic multi-layer program and no bond between the concrete and the road base is assumed. The load transfer efficiency is considered to be 100 percent in dowelled joints between two slabs on bound bases. The calculations are based on a slab with 5 m joint distance.

Thermal loads. The pavement is affected by a positive thermal gradient of 40°C/m under 20 % of the year and 60°C/m under 5 % (SNRA 2004). These gradients take place under the summer period. Thermal stresses are calculated with Eisenmann’s beam model, Eq. 1-3.
Table 4. Material and climate data (SNRA 2004). (Note: 200 mm = 7.9 in, 36 GPa = 5.2 Mpsi)

<table>
<thead>
<tr>
<th>Material</th>
<th>Climate period</th>
<th>Winter Freeze-thaw</th>
<th>Spring Thaw</th>
<th>Spring</th>
<th>Summer</th>
<th>Autumn</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>80 days</td>
<td>10 days</td>
<td>31 days</td>
<td>15 days</td>
<td>76 days</td>
</tr>
<tr>
<td>Thickness [mm]</td>
<td>Modulus of elasticity [MPa]</td>
<td>Poisson's ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete</td>
<td>200</td>
<td>36 000</td>
<td>36 000</td>
<td>36 000</td>
<td>36 000</td>
<td>36 000</td>
</tr>
<tr>
<td>Asphalt base</td>
<td>100</td>
<td>11 500</td>
<td>10 000</td>
<td>9 000</td>
<td>8 500</td>
<td>2 500</td>
</tr>
<tr>
<td>Unbound base</td>
<td>80</td>
<td>1 000</td>
<td>1 500</td>
<td>3 00</td>
<td>450</td>
<td>450</td>
</tr>
<tr>
<td>Unbound subbase</td>
<td>220</td>
<td>450</td>
<td>450</td>
<td>450</td>
<td>450</td>
<td>450</td>
</tr>
<tr>
<td>Crushed rock</td>
<td>850</td>
<td>1 000</td>
<td>1 000</td>
<td>70</td>
<td>85</td>
<td>100</td>
</tr>
<tr>
<td>Subgrade</td>
<td>1 000</td>
<td>1 000</td>
<td>1 000</td>
<td>10</td>
<td>20</td>
<td>45</td>
</tr>
</tbody>
</table>

Results. The limitation for the pavement system, is the vertical compressive strain of the subgrade, see Table 5. The requirement for traffic class 7, i.e., traffic exceeding $19\cdot10^6$ standard axles, is nevertheless fulfilled. The thickness of the pavement structure is satisfactory against frost heave.

Table 5. Number of permissible axle loads for the pavement system calculated with the Swedish method. The subgrade is the weakest component in the system.

<table>
<thead>
<tr>
<th>Pavement layer</th>
<th>Allowable ESAL’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>$2.65\cdot10^8$</td>
</tr>
<tr>
<td>Asphalt base</td>
<td>$7.93\cdot10^8$</td>
</tr>
<tr>
<td>Subgrade</td>
<td>$2.28\cdot10^8$</td>
</tr>
</tbody>
</table>

Evaluation of the Design Guide

Materials. The materials for the structure are chosen to correspond to the actual material properties in the Swedish specifications. The Design Guide is most accurate after affecting thorough analysis of the materials in the laboratory. Because of this, the input values for the unbound materials are, without exception, chosen from the default values provided by the Guide. The common rule for the choice of elastic modulus, or corresponding strength properties, is that the deeper under the slab the material is situated, the lower the modulus. No stress-dependent properties are accounted for and the climate model is restrained from altering the modulus over the months, i.e., specified modulus values are given for each month.

Traffic. The Design Guide inputs for traffic is the number of AADT and the number of heavy vehicles. These parameters are computed in the same way as in the Swedish method. The inputs concerning the traffic are then chosen through the help of default values recommended in the guide as showed in Table 6.
Table 6. Inputs for traffic calculation in the Design Guide.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operational Speed</td>
<td>60 mph (100 km/h)</td>
</tr>
<tr>
<td>Volume Adjustment factors</td>
<td>Default, 100% for all vehicle classes</td>
</tr>
<tr>
<td>Vehicle class distribution</td>
<td>TTC 8 &quot;High percentage of single-trailer truck with some single-unit trucks.&quot;</td>
</tr>
<tr>
<td>Hourly truck traffic distribution</td>
<td>Default, maximum of 6% per hour around noon</td>
</tr>
<tr>
<td>Traffic growth</td>
<td>2.6% (compound)</td>
</tr>
<tr>
<td>Axle load distribution factors</td>
<td>Default, level 3</td>
</tr>
<tr>
<td>Mean wheel location</td>
<td>18 in (457 mm)</td>
</tr>
<tr>
<td>Traffic wander std deviation</td>
<td>10</td>
</tr>
<tr>
<td>Number of axles per Truck</td>
<td>Default for single-, tandem-, tridem-, and quad axles</td>
</tr>
<tr>
<td>Axle Configuration</td>
<td>Default</td>
</tr>
</tbody>
</table>

**Climate.** The New York climate was chosen to represent the climate of this particular design because it is well defined and does not bring too extreme temperature variations to the calculations.

**Design features – Structure.** The dowels, the slab size, and the joint spacing are the constructive features that have been altered in the design, see Table 7. Case 4 corresponds to Swedish practice regarding these features.

Table 7. Five different design features analysed with the Design Guide. (Note: 5.0 m = 16.4 in. 25 mm = 1 in.)

<table>
<thead>
<tr>
<th></th>
<th>Case 0</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint spacing</td>
<td>5.0 m</td>
<td>4.6 m</td>
<td>4.6 m</td>
<td>4.6 m</td>
<td>5.0 m</td>
</tr>
<tr>
<td>Dowel diameter</td>
<td>25 mm</td>
<td>25 mm</td>
<td>38 mm</td>
<td>38 mm</td>
<td>25 mm</td>
</tr>
<tr>
<td>Slab width</td>
<td>3.7 m</td>
<td>3.7 m</td>
<td>3.7 m</td>
<td>4.3 m</td>
<td>4.3 m</td>
</tr>
</tbody>
</table>

**Results.** The results show that the design features, e.g., joint design and edge support, have significant importance in crack development and damage accumulation under the design period.

The most effective parameters to avoid premature cracking, faulting and rapid damage accumulation is done by the use of widened slabs, decreased joint spacing, increased dowel diameter, increased thickness of the concrete slab, and higher strength of the concrete. The faulting is also affected by the capability of erosion of the base and how the slab shoulder is tied to the adjoining pavement. (Darter 2004, Transportation Research Board 2004)

The first model, Case 0, with the same parameters for joint spacing and dowel diameter as in Swedish practise shows the most damage. The slab width is, however, the same as the lane width, contrarily to Swedish practice where the slab is more or less square with a maximum width of 4.5 m. Because of the fact that the properties are not correct regarding materials we only consider this model as the basic model, the model from where we evaluate the following ones.
The model in Case 1 has shorter joint spacing. The percentage of cracking is decreased and illustrated by decreased bottom-up and top-down cracking. The LTE is about the same as for Case 0.

In Case 2, the joint spacing is shorter, the dowel diameter is enlarged, and the slab width is increased. In this case, the LTE is not affected at all over the entire design period, see Figure 2.

Shortening the joint spacing, increasing dowel diameter, and widening the slab yields the best result in comparison to the basic model. This is done in Case 3. Bottom-up cracking is significantly decreased.

In Case 4, the basic model is simply modified by a widened slab. The result shows on a significant improvement in cracking development. Faulting is, however, considerably increased after 20 years as shown in Figure 6 and the LTE is, consequently reduced, see Figure 2.

The main results from this limited study show that faulting is significantly depending on the dowel size, and that both top-down and bottom-up cracking can be controlled by slab size. The bottom-up cracking is more dependent of the width of the slab. IRI and LTE can be related to cracking and faulting and are, consequently, dependent of the same parameters, see Figure 6 and Figure 7.

**Figure 2.** Load transfer efficiency during design life for cases 0-4. Cases 2 and 3 show the beneficial influence of enlarged dowel diameter.
Figure 3. Bottom-up cracking over the design life for cases 0-4. Case 3 and case 4 show the efficiency of widened slabs to avoid bottom-up cracking.

Figure 4. Top-down cracking over the design life for Cases 0-4. Cases 3 and 4 show the efficiency of widened slabs in the development of top-down cracking. Case 1 and 2 have shorter joint spacing compared to Case 0.
Figure 5. Cases 0-4. Predicted slabs cracked over the design life, considering the effect from both bottom-up and top-down cracking. The damage is predicted on a 50% reliability level.

Figure 6. Faulting development over the design life for cases 0-4. The most effective parameters are dowel diameter and joint spacing.
Figure 7. IRI growth for cases 0-4. Similarly to faulting, the IRI is controlled by dowel diameter and joint spacing.

Discussion and Conclusions

The design of concrete pavements in Sweden is based on specifications established in the 1990’s. The procedure offers little change when altering parameters like climate, traffic, and material properties. In a first step towards the development of a new and more flexible method, an inventory of the U.S. Design Guide has been conducted. This paper focuses on the differences in traffic and thermal load calculations, as well as design criteria and damage analysis. Apart from the direct comparison of defined parts of the design procedure, a quantitative study of a highway project in Sweden has been performed with both methods.

The Swedish method considers distresses in the concrete slab, the bound base course, and the subgrade. The study shows, for instance, that the subgrade is decisive for the design. In the U.S. Design Guide the distresses in the pavement itself, i.e., cracking, smoothness, and joint faulting, are decisive in the design.

The Swedish method accounts for bottom-up cracking and calculations with the Design Guide show that bottom-up cracking is critical in the design. Nonetheless, both top-down cracking and the load transfer efficiency contributes to the overall performance. The Design Guide is flexible in such a way that, by changing parameters for joint design, it can demonstrate how cracking or faulting can be decreased.

A level of damage of approximate 50%, can in the Swedish method, in this particular case study, be formulated in the subgrade after 40 years, i.e., by dividing the total number of ESAL’s with the allowable number ESAL’s calculated for the subgrade. The U.S. Design Guide reveals instead, on a 50% reliability level, a predicted 5% damage in the concrete slab (Figure 5).

The level of distress used as design criteria in the U.S. Design Guide is to a certain extent dependent on the type of road that is designed. The designer...
establishes the allowable limit of cracking and expresses the criteria in terms of percentage of cracked slabs, percentage of LTE, or inches of faulting.

The calculations performed with the Design Guide are not accurate for the reason that the many input parameters, especially for materials in the subbase and subgrade, have not been evaluated regarding U.S. standards. Most inputs are taken directly from the recommended values provided by the Guide, i.e., level 3 model with the least possible inputs. The aim is, however, to model the Swedish pavement system and investigate in how to interpret the results.

The criteria for road construction in Sweden are different in that manner that it is difficult to put a number on the effective, accumulated damage. In the method, the safety margins are achieved by underestimating mechanistic properties.

The performance criteria must, nevertheless, be measurable in terms of relating percentages of distress to established criteria in preceding methods. Cracking is for instance, in Sweden, related to the flexural strength of concrete beams. The safety lies within the assumption that a slab is more resistant to stresses than a beam due to bi-axial load bearing, and that the stresses are calculated for the characteristic strength of the material, i.e., 28-days flexural strength (Silfwerbrand 1995, SNRA 2004).

The performance criteria in the Design Guide are developed with the intention that designers can decide what level of performance is acceptable for the actual design. The design can be weighed to the cost, i.e., using existent or available materials, chose simpler joint design features etc.. The modelling example considered in this paper could, for instance, be further improved by varying materials in the pavement system or choosing a stronger base course. The way of proceeding to achieve the best possible pavement depends much on the resources available and the engineering experience. The U.S. design procedure is different to methods applied in Sweden. Attempting to adopt a similar method requires new procedures to determine specific environmental effects, integrating new material characterisation methods, and assembling data for regional calibration of damage models.

Calibration data for a mechanistic based design, similar to the data used for the U.S. Design Guide, is for Swedish conditions very limited since less than 1% of the national roads consist of concrete pavements but many of the necessary tools to establish a new methodology in Sweden are already at hand. Climate data are, as an example, collected continuously for weather predictions and could be retrieved and applied in the same way as in the U.S. Design Guide.

New methods to estimate traffic are also being developed in Sweden. Here, information on actual loads and actual axle configurations that are trafficking the roads in Sweden are being assembled. Traffic is divided in traffic classes in the same manner as in the Design Guide and these results will be applicable in future design (SNRA 2002). Material properties in Sweden originate from FWD tests conducted by the Swedish National Road and Transport Research Institute. Typical values for elastic modulus for different materials have been established in accordance with the SNRA. New methods to estimate nonlinear material behaviour will have to be utilised to get necessary information for pavement design if a more optimised method is chosen.

Further Research

This Ph.D. project is a part in the development of a new design method for concrete roads in Sweden.
The project will involve further investigation in international methods for design of concrete pavements and especially a deeper understanding of the methodology applied in the U.S. Design Guide. Further analysis of the Design Guide will include a profound examination of the material characterisation methods, an analysis of the calibration procedure, and an evaluation of the procedure in Swedish conditions.

Furthermore, laboratory tests on concrete and lean concrete will be conducted in order to verify and analyse the existing fatigue criteria in Sweden, and parameters for climate affects will also be analysed within this project.

Together with the Swedish National Road and Transport Research Institute and the Swedish National Road Administration a new computer-based design method for concrete pavements will be chosen, modified for Swedish conditions, and implemented in a computerised design guide.

**Acknowledgment**

The financial support provided by Cementa AB, the Royal Institute of Technology, and the Swedish Agency for Innovation Systems is gratefully acknowledged.
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Silfwerbrand, J. (2001). “Swedish design of industrial pavements”. 7th Int. Conf. on Concrete Pavements, ISCP, Orlando, USA, Sept. 9-13, 791-806


Flexural Fatigue of Plain Concrete Beams

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Swedish Cement & Concrete Research Institute, Stockholm, Sweden

Abstract

This paper investigates the applicability of Tepfers’ fatigue equation to flexural strength of plain concrete. The flexural fatigue is an important property in pavement design and given that Tepfers’ fatigue equation, originally, was developed out of compression and splitting tests, the relationship to flexural tests has not been fully investigated. In this study twelve plain concrete beams have been subjected to flexural loading and the results have, with good agreement, been compared to other research results. Palmgren-Miner’s partial damage hypothesis has also been analysed with flexural tests with sound deviation compared to theory.

1 Introduction

The fatigue strength is an important property, which has to be taken into account in the design of various structures requiring long fatigue life. Although the major part of research on fatigue of plain concrete is devoted to compressive stresses, it is also recognized that tensile fatigue in bending is an important factor in the design of certain types of structures, such as roads and airfields. In Swedish pavement design, Tepfers’ fatigue equation is commonly used. The fatigue principle is based on compressive and splitting tests and, since the material can be described with the same equation for both compressive and tensile stresses, the same equation has also been used, tentatively, for tensile stresses in bending, i.e. flexural stresses.

This paper investigates fatigue strength of plain concrete beams subjected to flexural loading. First, twelve plain concrete beams were tested to determine the fatigue strength and, second, the results were compared with other tests performed on concrete beams, prisms, cylinders, and columns over the years. Tepfers’ fatigue equation is analysed and modified by the introduction of an additional constant in the formula. The Palmgren-Miner partial damage hypothesis is examined as well.

1.1 Previous Research

The fatigue strength of concrete is normally defined as a fraction of the static strength that the material can support for a given number of stress cycles, N. The most common way of illustrating this is by Wöhler curves, also called S-N-curves, see Figure 1. The logarithm of the number of load cycles to failure, N, at a specific maximum stress level, σ_{max}, is plotted in the diagram. For each curve the relationship between the minimum and the maximum stresses is held constant.
The principal form of the current fatigue formula has its origin in the work conducted by Aas-Jacobsen in 1970 (Aas-Jacobsen, 1970). Aas-Jacobsen performed a series of tests on prisms, prestressed beams and eccentrically loaded columns, and formulated

$$\frac{\sigma_{\text{max}}}{f_{\text{cfl}}} = 1 - \beta \cdot \left( 1 - \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \right) \log N$$

where,

- $N$ = the number of load applications
- $\sigma_{\text{min}}$ = the minimum stress
- $\sigma_{\text{max}}$ = the maximum stress
- $f_{\text{cfl}}$ = the concrete strength
- $\beta$ = a material constant
- $R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}$

The equation considers a linear relationship between the minimum and maximum stress ratio making the equation independent of the concrete strength. Aas-Jacobsen’s tests showed that failure will not occur due to fatigue if the load is less than 60 % of the static stress limit, that repeated loading has little effect on the ultimate load if the
specimen does not fail due to fatigue, and that the deflection at 1 million load cycles is 40 - 100 % larger than the deflection after the first load cycle. In Aas-Jacobsen’s investigation the material constant $\beta = 0.064$.

Tepfers (Tepfers & Kutti, 1979, Tepfers, 1979, and Tepfers, 1978) examined the fatigue of concrete with compressive and splitting tests on concrete cubes. The tests were also compared with other research results from compressive tests on ordinary and lightweight concrete (data are presented in (Tepfers, 1978)). Tepfers concluded that Aas-Jacobsen’s fatigue equation (Equation (1)) is valid for plain concrete but suggested $\beta = 0.0685$.

Plain concrete in flexure has been recognized as an important factor in the design of roads and airfields and therefore, and as the methods to test concrete in flexure are growing more accurate, this has been done in many countries. This type of testing is associated with inexactness because small imperfections in the specimens can cause large discrepancies (Tepfers & Kutti, 1979). Also, fatigue testing is time-consuming, especially in the high-cycle region, i.e. more than 1 000 load cycles.

Over the years, a number of studies have been conducted to investigate the fatigue of plain concrete. These studies have different approaches and the most significant ones are listed in this section.

Hsu (Hsu, 1981) has tested beams to investigate the three-variable relationship of $f$, $N$ and $R$ with the aim of developing a four-variable relationship $f$, $N$, $T$, and $R$, where $T$ is the period of repetitive loads. Hsu suggests two different fatigue equations, one for the low-cycle region, and one for the high-cycle region. For the high-cycle region Equation (2) is suggested.

$$\frac{\sigma_{\text{max}}}{f_{cc}} = 1 - 0.0662 \left(1 - 0.556 \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}\right) \log N - 0.0294 \log T \quad (2)$$

In comparison to Tepfers’ fatigue equation the $R$-value is decreased (by a factor of 0.556) and the loading time, $T$, is introduced. $\beta$ is still very near what Tepfers’ suggested. The resulting $R$-value reduces the impact of the amplitude, especially noticeable in case of high $R$-values.

Also, completely empirical models have been and are being developed to predict the fatigue life of concrete structures. These models are specifically developed from long-term testing on real structures as airfields and roads and are often formed as

$$\frac{\sigma_{\text{max}}}{MR} = AN - B$$

where $A$ and $B$ are experimental coefficients and $MR$ is the modulus of rupture, i.e. $f_{at}$.

Shi has investigated flexural fatigue strength with 78 plain concrete beams in 1994 (Shi et al., 1993). Shi uses a mathematical approach to modify the power formula with statistical tools to account for the stress ratio in Equation 2.

Oh (Oh, 1986) uses statistical tools, i.e. a Weibull distribution, to more convincingly describe the fatigue behavior of concrete as this kind of testing exhibits larger scattering than static tests. This approach makes it possible to calculate a probability of failure for a given level of maximum load.
Other research has aimed at investigating different conditions as material properties, time related aspect on loading, and stress reversals.

1.2 Fatigue Equation

In Sweden, Tepfers’ fatigue equation (i.e. Equation (1) with $\beta = 0.0685$) is commonly used in the design of concrete roads, airfields and industrial pavements where flexural stresses are dominant (Silfwerbrand, 1995, SRA, 2005). The compilation of various test results conducted by Tepfers showed that the fatigue equation, with satisfactory results, could be used for both compression and tension (i.e. splitting), and therefore, the same equation has been proposed and used for flexural stresses. This hypothesis is generally accepted but has not been fully investigated in Sweden. Tepfers also concluded that the deviation between measured and calculated results is significant when testing specimens with a stress ratio $R > 0.75$. Improvements in the development of measuring equipment and more complex arrangements in testing are considered to be useful in new testing to overcome these problems.

2 Laboratory Tests

2.1 Test Specimens

The concrete mixture is presented in Table 1. Test specimens were cast in four batches, each of which included eight $800 \times 150 \times 100$ mm beams, three 150 mm high cylinders with diameter 100 mm, and six $150 \times 150 \times 150$ mm cubes. The specimens were cured for four days in water and then transferred to conditioning at 20°C with a relative humidity of 50 % for 60 days. This was done to avoid uneven moisture content during testing.

<table>
<thead>
<tr>
<th>Table 1. Concrete mixture.</th>
<th>Quantity [kg/m^3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swedish Concrete Grade K60</td>
<td></td>
</tr>
<tr>
<td>CEM I, Std Degerhamn</td>
<td>375</td>
</tr>
<tr>
<td>Aggregate 0-8 mm, Underås</td>
<td>737</td>
</tr>
<tr>
<td>Aggregate 8-16 mm, Underås</td>
<td>1018</td>
</tr>
<tr>
<td>Water</td>
<td>149.4</td>
</tr>
<tr>
<td>Silica</td>
<td>26.3</td>
</tr>
<tr>
<td>Plasticiser, 92M</td>
<td>8.6</td>
</tr>
<tr>
<td>Air Entraining Agent, L14</td>
<td>300</td>
</tr>
<tr>
<td>w/c</td>
<td>0.40</td>
</tr>
<tr>
<td>w/b</td>
<td>0.37</td>
</tr>
</tbody>
</table>
2.2 Test Procedure

The flexural strength was tested in a Material Test System (MTS) 810 machine (Figure 2). For each batch three beams were used to determine the mean flexural strength. The beams were simply supported with a span of 700 mm, and tested in four-point flexural loading according to Swedish standard SS 13 72 12. A total of twelve tests were made on three different values of $\sigma_{\text{min}}$, corresponding to 5, 20 and 40 percent of the maximum static load. On each level four different values of $\sigma_{\text{max}}$ were tested. In addition, cylinders and cubes were cast to determine the elastic modulus and the compressive strength. The fatigue tests were conducted in load control with a sinus waveform at 0.05 Hz in the beginning and 2.05 Hz after 100 cycles. Two Linear Variable Differential Transformers (LVDT) were also mounted on every beam to measure the midspan deflection.

![Figure 2. Flexural fatigue testing with a MTS 810 machine. LVDT's are placed on both sides of the concrete beam, as shown.](image)

<table>
<thead>
<tr>
<th>$f_{\text{ct}}$</th>
<th>Type</th>
<th>$\sigma_{\text{min}}/f_{\text{ct}}$</th>
<th>$\sigma_{\text{max}}/f_{\text{ct}}$</th>
<th>$R = \sigma_{\text{min}}/\sigma_{\text{max}}$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Static load</td>
<td>-</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.05</td>
<td>0.6</td>
<td>0.083</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.05</td>
<td>0.7</td>
<td>0.071</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.05</td>
<td>0.8</td>
<td>0.063</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.05</td>
<td>0.9</td>
<td>0.056</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>Static load</td>
<td>-</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.2</td>
<td>0.6</td>
<td>0.333</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.2</td>
<td>0.7</td>
<td>0.286</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.2</td>
<td>0.8</td>
<td>0.250</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.2</td>
<td>0.9</td>
<td>0.222</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>Static load</td>
<td>-</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.4</td>
<td>0.6</td>
<td>0.667</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.4</td>
<td>0.7</td>
<td>0.571</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.4</td>
<td>0.8</td>
<td>0.500</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Fatigue</td>
<td>0.4</td>
<td>0.9</td>
<td>0.444</td>
<td>1</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21</td>
</tr>
</tbody>
</table>
3 Tests Results

3.1 Static Tests

Cubes were cast to determine the compressive strength of the concrete during the test period. The compressive strength was tested at 28 days, and at the start and end of the fatigue test period. The cubes were stored similarly to the concrete beams at 50% relative humidity and 20°C.

For each batch the flexural strength of the concrete was tested statically with three beams. The mean values from these test series were used to find the appropriate load level in the subsequent fatigue tests.

The modulus of elasticity, $E_c$, was determined at the start of the tests with 150 mm cylinders with radius 100 mm. The cylinders were stored in water until testing according to Swedish standard.

Also, the modulus of elasticity in bending, $E_b$, was determined from static tests on three beams using an equation, developed for four-point flexural loading, with the form

$$E_b = \frac{131(F_2 - F_1)l^3}{686(\delta_2 - \delta_1)bh^3}$$

where,

- $l$ = length
- $b$ = width
- $h$ = height
- $F_1$ = 1/4 $F_{cr}$
- $F_2$ = 3/4 $F_{cr}$
- $F_{cr}$ = failure load
- $\delta_{1,2}$ = midspan deflections corresponding to $F_1$ and $F_2$

All static material strengths are shown in Table 3.

3.2 Fatigue Tests

The fatigue tests were carried out according to the method described in previous sections. Results are presented in Table 4, Figure 3, and Figure 4. Specimens that exceeded 2 million loading cycles do not appear here. These tests were instead used to verify the partial damage accumulation and were subjected to one, or even two, higher load levels.

Since the fatigue tests were limited to ten beams and the discrepancies were substantial, the relationship between the expected and the measured number of loads can be used to visualize the fatigue strength (Figure 3). Before testing each beam was checked for voids and other irregularities and even though no voids appeared on the surface, some were found inside the beams after failure. It is difficult to predict the impact of these imperfections. By including research results from other tests it is possible to put the test results into perspective. Test results from compression and splitting tests are put together in Figure 5.
### Table 3. Static material strength values (mean values out of three tests).

<table>
<thead>
<tr>
<th>Batch</th>
<th>Age</th>
<th>( f_{cc} ) (Cubes)</th>
<th>( E_c ) (Cylinders)</th>
<th>( E_{fl} ) (Beams)</th>
<th>( f_{cfl} ) (Beams)</th>
</tr>
</thead>
<tbody>
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<td>86</td>
<td>33300</td>
<td>-</td>
<td>-</td>
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<tr>
<td>3</td>
<td>161</td>
<td>79</td>
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</tr>
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<td>84</td>
<td>-</td>
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</tr>
<tr>
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<td>231</td>
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<td>19500</td>
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<td>86</td>
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<td>294</td>
<td>86</td>
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<td>425</td>
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<td>21000</td>
<td>5.81</td>
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### Table 4. Summary of flexural fatigue tests

<table>
<thead>
<tr>
<th>Id</th>
<th>( \sigma_{max}/f_{cfl} )</th>
<th>( \sigma_{max}/f_{cfl} )</th>
<th>( R )</th>
<th>\log N_c</th>
<th>\log N_m</th>
<th>( \beta ) (C = 1)</th>
<th>( \beta ) (C = 0.7556)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
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<tr>
<td>139B</td>
<td>0.90</td>
<td>0.20</td>
<td>0.222</td>
<td>1.88</td>
<td>2.43</td>
<td>0.0530</td>
<td>0.0495</td>
</tr>
<tr>
<td>149E</td>
<td>0.90</td>
<td>0.40</td>
<td>0.444</td>
<td>2.63</td>
<td>2.41</td>
<td>0.0745</td>
<td>0.0623</td>
</tr>
<tr>
<td>139E</td>
<td>0.80</td>
<td>0.05</td>
<td>0.063</td>
<td>3.11</td>
<td>2.34</td>
<td>0.0912</td>
<td>0.0897</td>
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<tr>
<td>149H</td>
<td>0.80</td>
<td>0.05</td>
<td>0.063</td>
<td>3.11</td>
<td>4.41</td>
<td>0.0484</td>
<td>0.0476</td>
</tr>
<tr>
<td>139C</td>
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<td>0.20</td>
<td>0.250</td>
<td>3.89</td>
<td>3.82</td>
<td>0.0697</td>
<td>0.0645</td>
</tr>
<tr>
<td>149G</td>
<td>0.90</td>
<td>0.60</td>
<td>0.667</td>
<td>4.38</td>
<td>2.60</td>
<td>0.1155</td>
<td>0.0776</td>
</tr>
<tr>
<td>149F</td>
<td>0.90</td>
<td>0.60</td>
<td>0.667</td>
<td>4.38</td>
<td>2.27</td>
<td>0.1322</td>
<td>0.0888</td>
</tr>
<tr>
<td>992e</td>
<td>0.70</td>
<td>0.05</td>
<td>0.071</td>
<td>4.72</td>
<td>4.06</td>
<td>0.0797</td>
<td>0.0782</td>
</tr>
<tr>
<td>149D</td>
<td>0.80</td>
<td>0.40</td>
<td>0.500</td>
<td>5.84</td>
<td>5.42</td>
<td>0.0738</td>
<td>0.0593</td>
</tr>
<tr>
<td>139A</td>
<td>0.70</td>
<td>0.20</td>
<td>0.286</td>
<td>6.13</td>
<td>5.67</td>
<td>0.0740</td>
<td>0.0674</td>
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</table>

<table>
<thead>
<tr>
<th>Mean</th>
<th>0.0812</th>
<th>0.0685</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>0.0259</td>
<td>0.0148</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0007</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

**Note.** \( N_c \) is the calculated number, and \( N_m \) is the measured number of load repetitions. The 7th column show \( \beta \) values for \( N_m \) when \( C = 1 \) in Equation (3). The values of the 8th column show \( \beta \) values for \( N_m \) when \( C \) is selected to to fit \( \beta_{average} = 0.0685 \).
Figure 3. Correlation between the calculated number of load applications, $\log N_c$, and the measured number of load applications, $\log N_m$, according to Equation (1).

Figure 4. Damage as a percentage of the calculated number of load applications, $N_m/N_c$, and the load level that each beam is subjected to. 100 % is the fatigue limit according to Equation (1) with $C = 1$. 


damages / load level

\begin{align*}
\log N_m / \log N_c, C=1 &\quad \text{smin/ fcfl} \\
\text{smax/ fcfl} &\quad \text{mean value, C=1}
\end{align*}
9

3.3 Modified Fatigue Equation

In this study, the tests are slightly weaker than predicted with Equation (1), with a mean damage of 92%. The series with the higher total load can be distinguished with a somewhat lower fatigue capacity, see Figure 5. This could indicate that, with an increasing minimum load, the discrepancies would increase. This could be explained by the type of testing which is influenced by cracks and irregularities in the specimens. The supports are also inelastic and could lock ordinary movements and produce additional shear. To reduce the influence of the amplitude in Tepfers’ fatigue equation, a factor, $C$, is introduced into Equation 1 according to:

$$\frac{\sigma_{\text{max}}}{f_{\text{cl}}} = 1 - 0.0685 \cdot \left(1 - C \cdot \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \right) \cdot \log N$$  \hspace{1cm} (3)

where $C = 0.7556$ fits the test results in this particular study. These results are presented in Table 4 and Figure 6.

Performing the same procedure on previous results from both compression, splitting and flexural tests found in literature, the constant $C = 0.8035$, see Figure 7.

Figure 5. Correlation between the calculated number of load applications, $\log N_c$, and the measured number of load applications, $\log N_m$. Results from previous research consisting of both compression, splitting and flexural tests (data obtained from (Shi et al., 1993, and Tepfers, 1978)).
Figure 6. Damage as a percentage of the calculated number of load applications, $N_m/N_c$, and the load level that each beam is subjected to. 100% is the fatigue limit according to Equation (1) with $C = 0.7556$.

Figure 7. Correlation between the calculated number of load applications, $logN_c$, and the measured number of load applications, $logN_m$. Results from previous research consisting of both compression, splitting and flexural tests (also presented in Figure 4). The results are recalculated using Equation (3) with $C = 0.8035$. 

$$y = 0.8016x$$

$$y = 1.0539x$$
3.4 Measured Deflections

The deflections are measured during testing with two LVDT’s. In previous research it has been suggested that there is a linear relation between the range of deflection and the logarithm of the number of load applications. The measurements in this particular study indicate that the deflection indeed grows in a linear way and that the deflection increases just before failure. The deflection during testing is presented for each beam in Figure 8.

4 Miner-Palmgren’s Partial Damage Hypothesis

The accumulated fatigue damage can be calculated with Miners-Palmgren’s partial damage hypothesis. This formula takes into account the damage from each specific loading circumstance and accumulates the amount of fractional damage to predict the total damage. In pavement design, the accumulated damage is computed by summing fatigue damage incurred during each season of the year due to both traffic and thermal loads.

The basic shape of Miners-Palmgren’s damage hypothesis is

$$\sum_{i=1}^{n} \frac{a_i \cdot N_i}{N_{i,allow}} \leq 1$$
where, \( q_i \) is the percentage of time in which damage \( i \) occurs, \( N_i \) is the total number of loads corresponding to damage \( i \), and \( N_{i,\text{allow}} \) is the number of allowable loads corresponding to damage \( i \).

Two beams were used to verify Palmgren-Miner’s damage hypothesis. If a beam has reached over 2 million load repetitions without failing, the maximum load has been increased and the accumulated number of load repetitions is counted until failure. The partial sum according to

\[ \sum_{i=1}^{n} \frac{N_i}{N_{i,\text{allow}}} \]  

and,

\[ \sum_{i=1}^{n} \log \frac{N_i}{N_{i,\text{allow}}} \]  

are calculated and analysed. The results show that the partial sum of load repetitions reaches 167 and 141 %, respectively, and that the calculated sum for the logarithm of repetitive loads are higher (see Table 5). The accumulated damage is decreased when using Tepfers’ modified fatigue equation with \( C = 0.7556 \). The accumulated damage is shown in Figure 9.
Table 5. Number of load applications in each load level and the accumulated damage for each beam.

<table>
<thead>
<tr>
<th>Id</th>
<th>$\sigma_{\text{min}}/f_{\text{cfl}}$</th>
<th>$\sigma_{\text{max}}/f_{\text{cfl}}$</th>
<th>$N_m$</th>
<th>$N_c$ (C = 1)</th>
<th>$N_c$ (C = 0.7556)</th>
</tr>
</thead>
<tbody>
<tr>
<td>139D</td>
<td>0.2</td>
<td>0.7</td>
<td>2.462 $10^6$</td>
<td>5.742 $10^6$</td>
<td>3.911 $10^6$</td>
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<tr>
<td></td>
<td>0.2</td>
<td>0.8</td>
<td>3.808 $10^3$</td>
<td>1.353 $10^6$</td>
<td>1.302 $10^6$</td>
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<tr>
<td></td>
<td>0.2</td>
<td>0.9</td>
<td>1.080 $10^4$</td>
<td>7.815 $10^3$</td>
<td>1.420 $10^5$</td>
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<tr>
<td>992f</td>
<td>0.05</td>
<td>0.6</td>
<td>2.289 $10^6$</td>
<td>2.346 $10^6$</td>
<td>2.696 $10^6$</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.7</td>
<td>2.250 $10^4$</td>
<td>5.205 $10^4$</td>
<td>1.746 $10^6$</td>
</tr>
</tbody>
</table>

$$3 \sum_{i=1}^{N_{\text{i,allow}}} \frac{N_i}{N_m} = 1.67, 0.07$$

$$2 \sum_{i=1}^{N_{\text{i,allow}}} \frac{N_i}{N_c} = 1.41, 0.02$$

<table>
<thead>
<tr>
<th>Id</th>
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<th>$\sigma_{\text{max}}/f_{\text{cfl}}$</th>
<th>$N_m$</th>
<th>$N_c$ (C = 1)</th>
<th>$N_c$ (C = 0.7556)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.7</td>
<td>6.39</td>
<td>8.76</td>
<td>11.59</td>
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<td>4.03</td>
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<tr>
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<td>0.6</td>
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<td>6.37</td>
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</tr>
<tr>
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<td>0.7</td>
<td>4.35</td>
<td>4.72</td>
<td>6.24</td>
</tr>
</tbody>
</table>

$$3 \sum_{i=1}^{N_{\text{i,allow}}} \frac{\log N_i}{\log N_m} = 2.68, 2.02$$

$$2 \sum_{i=1}^{N_{\text{i,allow}}} \frac{\log N_i}{\log N_c} = 1.92, 1.45$$
Figure 9. Damage accumulation, calculated with Miners-Palmgren’s damage hypothesis with logarithmic values (Table 5). Comparison between Tepfers’ fatigue equation, Equation (1), and Tepfers’ modified equation, Equation (3). The fatigue limit is 100%.

5 Discussion

This investigation must be incorporated in the context of other research. In flexural testing, the static flexural strength, $f_{cl}$, is determined on separately tested beams and is therefore related to inexactness to some extent. However, this problem exists for all sorts of fatigue testing and a mean value is used instead. By using a minimum of three beams to determine the static strength, reliability is increased. Also, every beam was manually manufactured and small imperfections in each specimen have influence on the results. Deviations in $f_{cl}$ occur and the fatigue is very sensitive to this. A 5% deviation between assumed and true $f_{cl}$ is a fact for these tests and, according to Tepfers, this has an increasing effect, especially on higher $R$-values, i.e. $R > 0.8$. Tepfers also considered the long-term strength of concrete and concluded that $\sigma_{\text{max}}/f_{cl}$ exceeding the long-term strength of concrete is sensitive to the loading frequency because of creeping effects. For these reasons, no tests were performed for $R$-values in this region.

By introducing a constant, $C < 1$, in Tepfers’ fatigue equation, the influence of the amplitude is decreased. This approach can be compared to Equation (2), formulated by Hsu (Equation (2)). In this present study, $C = 0.7556$ correlates with the actual tests performed, and $C = 0.8035$ correlates with previous tests from both compression, splitting, and flexural tests found in literature. A new constant related to the $R$-value could indicate steeper curves in the Wöhler diagram (Figure 1) due to strength reducing factors from the set-up of the tests. Since the $R$-values in this study
varies from 0.071 to 0.667, it has not been possible to examine the possibility that the curves should be other than linear.

Measured deflections confirm the findings by Aas-Jacobsen that the deformations during fatigue loading increase with 40 – 100% from the first measured deflection after one load cycle (Aas-Jacobsen, 1970).

Palmgren-Miner’s partial damage hypothesis is generally used to accumulate the influence of multiple loading. Even though discrepancies of more than a factor of ten have been generally accepted, the two examples above show a 140 – 160% deviation. This indicates satisfactory results, in agreement with previously performed tests, and in particular (Tepfers et al., 1977).

6 Conclusions

Tepfers’ fatigue equation is, as a general rule, used in concrete pavement design in Sweden. In the design, the flexural strength is dominant. Since this equation originally was developed from compression and splitting tests, it is of great interest to investigate how well the equation is suited for flexural fatigue. The following conclusions may be drawn:

- The flexural fatigue can be described with Tepfers’ fatigue equation and is compatible with results from a number of previously performed compressive and splitting tests.
- The flexural fatigue can also be described with Tepfers’ fatigue equation by the introduction of a constant to decrease the impact of the R-value. In this study it was found that $C = 0.756$ can be fitted to the test results. For previous tests $C = 0.804$ is matched with the same procedure.
- For a calculated number of load applications exceeding $10^8$, the measured number of load applications is generally lower than predicted for both compressive and splitting fatigue tests made in the past. This can directly be derived from Figure 3.
- It is difficult to determine the damage if a specimen has been subjected to a small number of loads compared to the number of maximum load applications that the specimen can withstand. The Palmgren-Miner partial damage hypothesis is through this investigation still applicable to the fatigue of plain concrete, especially in the view of previous research in this area.
- Repeated loading increases the deflection at midspan of a beam and the deflection grows increasingly just before failure.

Plain concrete in flexure has so far been described with Tepfers’ fatigue equation by means of the assumption that the material can be described with the same equation for both compressive and tensile stresses, and consequently, also for flexural stresses. On the basis of this present study, this assumption is not contradicted, but rather confirmed. However, a larger scale investigation may reveal differences that were not possible to point out here.
References


Hsu, T. C., Fatigue of Plain Concrete, ACI Journal, Proceedings V. 78, No.4, July-August, 1981.


Paper 3
ABSTRACT: In Sweden, the fatigue of lean concrete has, up to today, been described with a strain criterion. By comparing different design criteria found in the literature, the Swedish design criterion becomes very rigorous, especially for high numbers of load repetitions. The lean concrete is due to this reason thick in the Swedish pavement system and, therefore, not as favourable as an asphalt base. In this paper, an international comparison of design criteria for lean concrete is done. Flexural tests on lean concrete beams, conducted in this study, show the unmotivated rigorousness of the Swedish fatigue criterion, and a new criterion based on tensile stresses is suggested.

KEY WORDS: Pavement, road base, lean concrete, flexural strength, fatigue.

1. INTRODUCTION

Swedish concrete pavement systems containing lean concrete tend to be thick. One example is the previous Swedish pavement design guide (SRA, 1994) where 150 mm of lean concrete is considered to be equivalent to a 100 mm asphalt stabilised base. In the current design guide (SRA, 2005), the design tables are replaced by a computer program. Its solution shows, however, the same result. For a person not familiar with Swedish pavement design, this fact might be surprising. Lean concrete is always (for all seasons) stiffer than asphalt and provides, thus, a better base in both asphalt and concrete pavement systems. The assumed superiority of the asphalt base is based on three reasons: (i) the Swedish design criterion for lean concrete is more rigorous than the corresponding one for asphalt, (ii) the asphalt base is more suitable for construction traffic than the lean concrete base, and (iii) several Swedish pavement engineers have the opinion that pavement systems containing asphalt base perform better than those containing lean concrete. The authors’ opinion is that the third reason lacks scientific support and that the used Swedish design criterion for lean concrete is too rigorous. This will be shown both in an international comparison and by analysing new fatigue test data.

2. STRAIN CRITERIA FOR LEAN CONCRETE

The Swedish design of lean concrete was originally developed by Björn Örbom (Örbom, 1981). It was based on field test results in Pennsylvania and adjusted to Swedish conditions by including measuring results from test roads of the Swedish National Road and Transport Research Institute (VTI). The design criterion for the Pennsylvania tests may be defined by the following expressions:

\[ N = 4.092 \cdot 10^{-6} \cdot \varepsilon_y^2 \cdot 2.597 ; 10000 \leq N \leq 3.5 \cdot 10^7 \]  

(1a)
where \( N \) is the allowable number of load repetitions and \( \varepsilon_y \) is the horizontal tensile strain in the lowest fibre of the lean concrete layer. The quantity is non-dimensional.

The Swedish design criterion was originally only shown as a curve in a diagram. VTI has, however, later developed an equation that is fitted to the diagram curve. It has the following expression:

\[
N = \frac{1.06 \cdot 10^{-10}}{\varepsilon_y^{3.86}}
\]

or

\[
\varepsilon_y = \frac{2.61 \cdot 10^{-3}}{N^{0.259}}
\]

The relationship (2a) can be found in both the previous and the current version of the Swedish pavement design guide, \( VÄG \ 94 \) (SRA, 1994) and \( ATB \ VÄG \ 2005 \) (SRA, 2005), respectively.

The Finnish researcher Józef Judycki (Judycki, 1991) has investigated how cementitious materials are characterized and, among other things, rewritten a number of fatigue criteria. The Belgian researchers De Henau & Verstraeten (Henau & Verstraeten, 1971) proposed the following equation:

\[
\varepsilon_y = \varepsilon_{y1} \cdot N^{-0.025} ; \ 10^4 \leq N \leq 10^8
\]

or rewritten

\[
N = \left( \frac{\varepsilon_y}{\varepsilon_{y1}} \right)^{-40}
\]

The American highway engineers Pretorius & Monismith (Pretorius & Monismith, 1972) proposed a curve that can be recalculated to the following equations:

\[
N = 1.288 \cdot 10^9 \cdot 10^{-57800 \cdot \varepsilon_y}
\]

\[
\varepsilon_y = 1.576 \cdot 10^{-4} - 1.73 \cdot 10^{-5} \cdot \log N
\]

In South Africa, Otte (Otte, 1978) proposed the following alternative relationships:

\[
\varepsilon_y = \varepsilon_{y1} \cdot (1 - 0.11 \cdot \log N)
\]
In the Netherlands, Ros et al. (Ros et al., 1982) proposed a curve that can be expressed with the following equation:

\[
N = 10^{\left(1 - \frac{\varepsilon_y}{\varepsilon_{y1}}\right)/0.11}
\]  

(6b)

\[
\varepsilon_y = \varepsilon_{y1} \cdot N^{-0.079}
\]

(7a)

\[
N = \left(\frac{\varepsilon_y}{\varepsilon_{y1}}\right)^{-12.66}
\]

(7b)

In the Netherlands, Ros et al. (Ros et al., 1982) proposed a curve that can be expressed with the following equation:

\[
N = \left(2.0 \cdot 10^{-4} / \varepsilon_y\right)^{13.5}
\]

(10a)

\[
\varepsilon_y = 2.0 \cdot 10^{-4} \cdot N^{-0.074}
\]

(10b)

Figures Nos. 1-5 show comparisons between the various fatigue criteria. The first three curves show how the allowable strain declines with increasing number of load repetitions. In Figures Nos. 2 and 3, also four Swedish field test results have been included. All figures contain a horizontal axis drawn logarithmically. Figure 2 has a linear vertical axis showing the strain in \(\mu\)strain whereas Figure 3 has a logarithmic vertical axis. We see that the results are well gathered close to the Swedish design criterion, especially in the log-log diagram (Figure 3). This is the diagram Örbom used when he established the Swedish design criterion. If he had used a diagram with linear vertical axis, he might instead have defined a curve closer to the curve proposed by Ros et al. In this sense, the Swedish design criterion for lean concrete is a coincidence.

Figure 4 shows how allowable strain declines in relation to allowable strain at a single load. Since a couple of the equations are not valid for small number of load repetitions \((N < 10000)\), Figure 5 shows how allowable strain declines in relation to allowable strain at \(N = 10000\). As shown in Figure 1, the Swedish design criterion gives large allowable strain for small numbers of load repetitions, i.e., for \(N < 1\) million. For higher numbers, it is more conservative than the criterion proposed by Ros et al. If we study how allowable strain declines in relation to allowable strain at single load instead, we observe that the Swedish criterion as its origin in Pennsylvania lies substantially below the other criteria. The fatigue is, consequently, assumed to occur much more dramatically.
Figure 1: Allowable horizontal tensile strain as a function of number of load repetitions.

Figure 2: Allowable horizontal tensile strain as a function of number of load repetitions. Figure 1 has been supplemented with measuring data provided from Swedish National Road and Transport Research Institute (VTI).
Figure 3: Allowable horizontal tensile strain as a function of number of load repetitions. Please, note that the figure shows a log-log diagram.

Figure 4: Allowable horizontal tensile strain in relation to allowable strain at single load.
3. STRESS CRITERIA FOR LEAN CONCRETE

Some criteria do not deal with tensile strain but tensile stress. These two groups of criteria are not that simple to compare. According to the theory of elasticity, stresses might be recalculated as strains. The following relationship is valid:

\[ \varepsilon_y = \frac{1}{E} \left( \sigma_y - \nu \left( \sigma_x + \sigma_z \right) \right) \]  \hspace{1cm} (11)

where, \( \sigma_x, \sigma_y, \) and \( \sigma_z \) are stresses, \( E \) is the modulus of elasticity, and \( \nu \) is Poisson’s ratio. In the bottom fibre of the lean concrete layer, the traffic stresses in two perpendicular directions \( \sigma_x \) and \( \sigma_y \) are approximately equal while the vertical stress \( \sigma_z \) might be neglected. Equation (11) might be rewritten as follows:

\[ \varepsilon_y = \frac{1}{E} \left( 1 - \nu \right) \sigma_y \]  \hspace{1cm} (12)

or

\[ \sigma_y = \frac{E}{1 - \nu} \cdot \varepsilon_y \]  \hspace{1cm} (13)

In Swedish concrete pavement design, Tepfers’ fatigue criterion (Tepfers, 1979) is used:

\[ \frac{\sigma_{\text{max}}}{f_{\text{ct}}} = 1 - 0.0685 \left( \frac{1 - \sigma_{\text{min}}}{\sigma_{\text{max}}} \right) \cdot \log N \]  \hspace{1cm} (14)

where, \( \sigma_{\text{min}} \) and \( \sigma_{\text{max}} \) are the minimum and maximum stress in the fatigue cycle, respectively, and \( f_{\text{ct}} \) is the flexural tensile strength of concrete. Usually \( \sigma_{\text{min}} \) is the thermal stress and \( \sigma_{\text{max}} \) is...
is the sum of the thermal and traffic stresses (Söderqvist & Silfwerbrand, 2005). If we neglect the thermal stress, Equation 14 may be simplified as follows:

\[
\frac{\sigma_y}{f_{ct}} = 1 - 0.0685 \cdot \log N
\]  
(15a)

or

\[
N = 10^{(1 - \sigma_y / f_{ct})/0.0685}
\]  
(15b)

If we assume that lean concrete has flexural tensile strength \( f_{ct} = 2.0 \) MPa, modulus of elasticity \( E = 17 \) 000 MPa, and Poisson's ratio \( \nu = 0.25 \), Equations (12) and (13) give us the following expression:

\[
\varepsilon_y = 8.82 \cdot 10^{-5} \cdot (1 - 0.0685 \cdot \log N)
\]  
(16)

At a concrete block pavement workshop organized in August 2000 on Lidingö island in Sweden, Shackel, Wellner and Huurman presented design criteria for lean concrete or other cement treated materials. Wellner (Wellner, 2000) assumes that the flexural tensile strength of a certain material equals 1/6 of its compressive strength and that the modulus of elasticity is 15 000 MPa if the material is uncracked and 5000 MPa if it is cracked. He uses the following criterion:

\[
\log N = 11.872 - 12.12 \cdot \frac{\sigma_y}{f_{ct}}
\]  
(17a)

which may be rewritten as follows:

\[
\sigma_y = f_{ct} \cdot (0.979 - 0.0825 \cdot \log N)
\]  
(17b)

If we for the sake of comparison reuse \( f_{ct} = 2.0 \) MPa, \( E = 17 \) 000 MPa and \( \nu = 0.25 \), we arrive at the following equations:

\[
\varepsilon_y = 8.82 \cdot 10^{-5} \cdot \frac{84 - 5 \cdot \log N}{100}
\]  
(19)
for Wellner and Huurman’s criteria, respectively.

Figure 6 shows a comparison between the Swedish concrete design criterion and Wellner and Huurman’s criteria. They are all also compared with the Swedish design criterion for lean concrete. As shown, the traditional Swedish design criterion for lean concrete is the most favourable for low numbers of load repetitions, i.e., for \( N < 3 \) million. For large numbers, it is the opposite. Figure 7 shows that the Swedish lean concrete criterion implies a more rapid fatigue than any of the others. Wellner’s criterion is the mildest and Huurman’s the most rigorous.

\[
\varepsilon_y = 8.82 \cdot 10^{-5} \cdot (0.979 - 0.0825 \cdot \log N) \quad (20)
\]

Figure 6: Allowable horizontal tensile strain as a function of the number of load repetitions according to the Swedish (or, e.g., Tepfers’) concrete criterion, Wellner’s criterion, and Huurman’s criterion after recalculation from stresses to strains. The Swedish lean concrete criterion according to VÄG 94 is shown for comparison.

Figure 7: Allowable horizontal tensile strain in relation to allowable strain at single load according to the Swedish (or, e.g., Tepfers’) concrete criterion, Wellner’s criterion, and Huurman’s criterion after recalculation from stresses to strains. The Swedish lean concrete criterion according to VÄG 94 is shown for comparison.
The laboratory tests conducted in this study are done in order to (i) evaluate the design criterion in Sweden and (ii) analyse a stress related approach on the fatigue of lean concrete. For plain concrete, Tepfers’ fatigue equation, Equation 14, is widely used, and the fatigue of lean concrete is compared to this equation through this study. The equation considers two stress levels and has been chosen in this study because of the similarities in material properties between plain and lean concrete. By introducing a correlation factor in this equation, a new criterion for lean concrete fatigue is suggested.

4.1 Test specimens

The lean concrete mixture chosen for the fatigue tests is retrieved from the Swedish design guide and fabricated in a concrete plant in Täby, Stockholm, Sweden. The plant was situated at a transportation distance of 30 min from the production site in Upplands-Väsby, north of Stockholm. Since the project took place in November 2004, with an air temperature of approximately 5 – 10 °C, the concrete was transported in open trucks. Data for the concrete mixture are presented in Table 1. The production site was prepared with a 70 m² base of compacted gravel. The lean concrete was tipped on the surface and compacted 3 – 4 times using a CA15 roller to a target thickness of 150 mm, see Figure 8. The level of compaction and the relative moisture content were monitored with a nuclear density gauge to guarantee a 97 % compaction level. After seven days, 13 beams were cut out of the surface. The specimens were stored in water for 12 –15 months before testing.

Table 1: Typical Swedish lean concrete mixture used in this study.

<table>
<thead>
<tr>
<th>Swedish Lean Concrete</th>
<th>Quantity (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEM I, Std Degerhamn, Swedish cement for civ. eng. struct.</td>
<td>110</td>
</tr>
<tr>
<td>Aggregate 0-8 mm</td>
<td>1576</td>
</tr>
<tr>
<td>Aggregate 8-16 mm</td>
<td>674</td>
</tr>
<tr>
<td>Water</td>
<td>115</td>
</tr>
<tr>
<td>w/c</td>
<td>1.05</td>
</tr>
<tr>
<td>Density</td>
<td>2476</td>
</tr>
</tbody>
</table>
4.2 Test procedure

The flexural strength and fatigue were tested in a Material Test System (MTS) 810 machine. At first, three beams were tested to retrieve the static flexural strength. The static strength properties were then used to pinpoint the load levels in the following fatigue tests, taking into account each beam’s individual cross-section. The beams were simply supported with a span of 700 mm, and tested in four point flexural loading according to Swedish standard SS 13 72 12, see Figures 9 and 10.
A total of 10 fatigue tests were made on two different values of $V_{\text{min}}$, corresponding to approximately 5 and 20 percent of the maximum static strength, $f_{\text{c}}$. On each level different values of $V_{\text{max}}$ were tested while the ratio $R = V_{\text{min}}/V_{\text{max}}$ was held constant on two levels. The test program is presented in Table 2. In addition, cylinders and cubes were tested to determine the modulus of elasticity, $E_{\text{u}}$, and the compressive strength, $f_{\text{c}}$. The fatigue tests were conducted in load control with a sinus waveform at 0.05 Hz in the beginning and 2.05 Hz from 100 cycles to failure. The beams were also covered with a plastic folio to keep the moisture level constant throughout the test. Two Linear Variable Differential Transformers (LVDT) were mounted on every beam to measure the midspan deflection.

Table 2: Program for fatigue testing of lean concrete.

<table>
<thead>
<tr>
<th>Id</th>
<th>$V_{\text{max}}/f_{\text{c}}$ (%)</th>
<th>$V_{\text{min}}/f_{\text{c}}$ (%)</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>62</td>
<td>5.2</td>
<td>0.0833</td>
</tr>
<tr>
<td>2</td>
<td>64</td>
<td>5.3</td>
<td>0.0833</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
<td>5.8</td>
<td>0.0833</td>
</tr>
<tr>
<td>4</td>
<td>75</td>
<td>6.3</td>
<td>0.0833</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td>6.7</td>
<td>0.0833</td>
</tr>
<tr>
<td>6</td>
<td>70</td>
<td>20.0</td>
<td>0.2857</td>
</tr>
<tr>
<td>7</td>
<td>75</td>
<td>21.4</td>
<td>0.2857</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
<td>22.9</td>
<td>0.2857</td>
</tr>
<tr>
<td>9</td>
<td>83</td>
<td>23.5</td>
<td>0.2857</td>
</tr>
<tr>
<td>10</td>
<td>85</td>
<td>24.3</td>
<td>0.2857</td>
</tr>
</tbody>
</table>

4.3 Static test results

Three 150 mm cubes were cast and tested in compression after 28 days according to Swedish practice. These cubes were compacted with a vibrating plate in three layers to guarantee a 97 % compaction level.
Cylinders drilled from the lean concrete beams were used to determine the compressive strength, \( f_{cc} \), at the end of the fatigue test period. The modulus of elasticity, \( E_c \), was also determined with cylinders cut out of the original lean concrete surface. All cylinders were stored at 100% relative humidity and 20°C as the lean concrete beams. The results are shown in Table 3.

Before the fatigue testing begun the flexural strength, \( f_{\text{cf}} \), of the lean concrete was tested statically with three beams, see Table 3. The mean value from this test series was used to find the appropriate load level in the subsequent fatigue tests. The static tests were also used to find the elastic modulus in flexure, \( E_f \).

The modulus of elasticity is different for cylinders and beams. Cylinders are often used for establishing this value but in the design, when the flexural strength determines the material’s capacity it is relevant to calculate the actual modulus of elasticity in flexure, \( E_f \). The static tests provided \( E_f = 16,000 \) MPa, approximately the same as suggested by Huurman and Wellner, see Section 3. The reason to the difference in \( E \) between cylinders and beams has not been investigated in this study. It might be a result of micro cracking, not visible, but possibly present during flexural loading.

Table 3 - Static material strengths. The late age is due to delayed start of testing.

<table>
<thead>
<tr>
<th>Type of strength</th>
<th>Type (mm)</th>
<th>Strength (MPa)</th>
<th>Age at test (days)</th>
<th>No. of tests (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{cc} ), 28d</td>
<td>Cubes (150×150)</td>
<td>16.3</td>
<td>28</td>
<td>3</td>
</tr>
<tr>
<td>( f_{cc} ), 400d</td>
<td>Cylinders (Ø100×100)</td>
<td>13.7</td>
<td>400</td>
<td>6</td>
</tr>
<tr>
<td>( f_{\text{cf}} ), 440d</td>
<td>Beams (800×150×100)</td>
<td>2.1</td>
<td>440</td>
<td>3</td>
</tr>
<tr>
<td>( E_c ), 500d</td>
<td>Cylinders (Ø100×200)</td>
<td>28,000</td>
<td>500</td>
<td>6</td>
</tr>
<tr>
<td>( E_f ), 440d</td>
<td>Beams (800×150×100)</td>
<td>16,000</td>
<td>440</td>
<td>3</td>
</tr>
</tbody>
</table>

Differences between compressive strengths are probably due to the fact that the 28-days compressive strength comes from the fabricated cubes that were compacted manually. The 400-day compressive strength comes from cylinders from the lean concrete surface, where a roller was used for the compaction of the material.

The flexural strength is in agreement with other research results, in the region of 1/3 to 1/2 of the flexural strength of plain concrete. In a previous study, plain concrete beam had a flexural strength of approximately 5.5 MPa (Söderqvist & Silfwerbrand, 2006).

4.4 Fatigue test results

The fatigue tests were carried out according to the method described in the previous sections. The specimens were carefully measured before testing to be able to apply the most accurate stresses with regard to the height and base of each individual specimen. Two different values on \( R = \sigma_{\text{min}}/\sigma_{\text{max}} \) corresponding to 0.2857 and 0.0833 were considered. Before each fatigue test, the number of loads, referred to as \( \log N_c \), that the current beam could withstand was calculated according to Equation 14. This number was then evaluated compared to the number of loads that the current beam could take before failure, \( \log N_m \).

The strain criterion is translated to a stress criterion and evaluated compared to results achieved in this study in Section 4.6. In Equation 14, Tepfers came to the conclusion that \( \beta = 0.0685 \) was valid for plain concrete (Tepfers, 1979). The evaluation of the fatigue tests in this study is mainly done by investigating the constant \( \beta \), but also by introducing a new constant, \( C \), in a modified fatigue equation, see Section 4.7. Equation 14 is presented in a S-N-diagram in Figure 11 and basic results from the fatigue tests are shown in Table 4.
In Figure 12, the results are also presented in relation to the predicted number of loads for each specimen according to Equation 14.

Table 4: Summary of flexural fatigue tests with results on fatigue life and different values on \( \beta \) with the correlation factor \( C \).

<table>
<thead>
<tr>
<th>Id</th>
<th>( \sigma_{	ext{max}}/f_0 )</th>
<th>( \sigma_{	ext{min}}/f_0 )</th>
<th>( R )</th>
<th>log( N _c )</th>
<th>log( N _m )</th>
<th>( \beta ), ( C = 1 )</th>
<th>( \beta ), ( C = 0.28 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>cg9</td>
<td>0.62</td>
<td>0.05</td>
<td>0.0833</td>
<td>6.05</td>
<td>6.04</td>
<td>0.0686</td>
<td>0.0644</td>
</tr>
<tr>
<td>cg10</td>
<td>0.64</td>
<td>0.05</td>
<td>0.0833</td>
<td>5.73</td>
<td>5.12</td>
<td>0.0767</td>
<td>0.0720</td>
</tr>
<tr>
<td>cg6</td>
<td>0.70</td>
<td>0.06</td>
<td>0.0833</td>
<td>4.78</td>
<td>4.42</td>
<td>0.0741</td>
<td>0.0696</td>
</tr>
<tr>
<td>cg7</td>
<td>0.75</td>
<td>0.06</td>
<td>0.0833</td>
<td>3.98</td>
<td>3.96</td>
<td>0.0689</td>
<td>0.0646</td>
</tr>
<tr>
<td>cg8</td>
<td>0.70</td>
<td>0.20</td>
<td>0.2857</td>
<td>6.13</td>
<td>3.49(^{+1})</td>
<td>0.1264</td>
<td>0.0935</td>
</tr>
<tr>
<td>cg12</td>
<td>0.70</td>
<td>0.20</td>
<td>0.2857</td>
<td>6.13</td>
<td>4.58(^{+2})</td>
<td>0.0918</td>
<td>0.0713</td>
</tr>
<tr>
<td>cg5</td>
<td>0.75</td>
<td>0.21</td>
<td>0.2857</td>
<td>5.11</td>
<td>4.00(^{+1})</td>
<td>0.0875</td>
<td>0.0679</td>
</tr>
<tr>
<td>cg13</td>
<td>0.75</td>
<td>0.21</td>
<td>0.2857</td>
<td>5.11</td>
<td>4.30(^{+2})</td>
<td>0.0813</td>
<td>0.0631</td>
</tr>
<tr>
<td>cg4</td>
<td>0.80</td>
<td>0.23</td>
<td>0.2857</td>
<td>4.09</td>
<td>3.25</td>
<td>0.0862</td>
<td>0.0670</td>
</tr>
<tr>
<td>cg11</td>
<td>0.85</td>
<td>0.24</td>
<td>0.2857</td>
<td>3.07</td>
<td>3.18</td>
<td>0.0660</td>
<td>0.0512</td>
</tr>
</tbody>
</table>

N: 10 10  
Mean: 0.0821 0.0685  
Standard Deviation: 0.0160 0.0106  
Variance: 0.0003 0.0001

Figure 11: Tepfers' fatigue equation, Equation 14. The number of allowable load applications in relation to the relative stress and different ratios on \( R \).
4.5 Deflections due to repeated loading

The deflections were measured with two LVDT's mounted on each side of the beams. The deflections were recorded at the beginning of the test and then every 50 to 10,000 cycles depending on the predicted number of loads to failure. In some rare cases the maximum deflection before failure was captured. All beams exceeded a 30% increase in deflections compared to the deflection in the first loading cycle. Aas-Jacobsen found a 40–100% increase in deflection for plain concrete (Aas-Jacobsen, 1970), and this was confirmed in the preceding study on plain concrete beams (Söderqvist & Silfwerbrand, 2006). The final deflection for lean concrete could reach a 180% increase. Deflections are presented in Figure 13.
Figure 13: Deflections for beams subjected to fatigue loading. Since the deflections were recorded manually, the final deflection is not necessarily the maximum deflection before failure.

4.6 The Swedish strain criterion

The tensile stresses in the road base are often neglected for a cement bound base since it is considered cracked under the concrete pavement. In Sweden, the use of a strain criterion has been chosen instead. This criteria is discussed in Section 2, i.e. Equation (2a) and (2b).

From Table 3, values on the modulus of elasticity, \( E_b \), and the flexural strength, \( f_{0b} \), is used together with Equation (13) and a Poisson’s ratio \( \nu = 0.25 \) to rewrite the strain criterion to a stress criterion. The produced equation has the following form:

\[
\sigma_y = 55.68 \cdot N^{-0.259}
\]  

(21)

The equation is plotted in Figure 17 together with results from the fatigue tests.

The obtained results from testing are spreading over a range of 10,000 to 1 million load applications but are clearly more resistible than predicted by the strain criterion. It can hereby, once again, be shown how strict the Swedish criterion is, not only in comparison to other international criteria, as discussed in previous sections, but also to new test data.
4.7 Tepfers’ modified fatigue equation

Equation 14 is, as a general rule, used in pavement design in Sweden for plain concrete. In the design, the flexural strength is dominant for both plain and lean concrete. In a previous study, plain concrete beams were tested in order to verify the fatigue criteria in flexure (Söderqvist & Silfwerbrand, 2006). In this study the same test system was used and the design criterion was in agreement with the flexural tests results. Since Tepfers’ fatigue criterion is consistent with ordinary concrete, it is not too unrealistic to think that it could be used for lean concrete, implying the material is uncracked.

In this study, the number of tests is limited but there is a tendency of coherence between the test results. All tests are slightly weaker than predicted with Equation 14, but the series with the higher minimum load can be distinguished with a somewhat lower fatigue capacity, see Figure 11. This could indicate that, with an increasing minimum load, the discrepancies would increase. This hypothesis can be shown by introducing a correlation factor, $C$, to Equation 14 according to:

$$\frac{\sigma_{\text{max}}}{f_{\text{ct}}} = 1 - 0.0685 \left( 1 - C \cdot \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \right) \cdot \log N$$

where $C = 0.28$ fits the test results in this particular study. These results are presented in Table 4 and Figure Nos. 15, 16 and 17.
Figure 15: Tests results and the correlation to Tepfers’ fatigue equation, Equation 14.

Figure 16: Tests results and the correlation to Tepfers’ modified fatigue equation, Equation (21), with $C=0.28$. 
Figure 17: Damage as a percentage of the calculated number of load applications considering a constant $C = 0.28$ in Equation 22. The mean value is in this case 102%.

5. DISCUSSION

The strain criterion in Sweden is developed out of field test result from Pennsylvania, USA. A discussion weather this criterion is fair, especially in relation to international criteria, has been presented. The strain criterion has also been translated into a stress criterion with actual material properties, to consider tensile stresses. In comparison to Tepfer’s fatigue criterion and new fatigue test data, the strain criterion is found to be discriminatory and this study may even raise a few question marks on whether this criterion really is not too strict.

Tepfer’s fatigue equation is generally applied to plain concrete in the Swedish design. The flexural fatigue of lean concrete, estimated through flexural tests in this study, is found to be in agreement with Tepfer’s fatigue equation, possibly with some modifications. The results are also compatible with results from previously performed fatigue tests on plain concrete. A larger scale investigation may, however, reveal the exact shape of a new approach that was not possible to obtain here. Since the fatigue of plain concrete is based on compression and splitting tests, it would also be valuable to perform these types of tests on lean concrete to more directly compare fatigue properties for these two materials.

6. ACKNOWLEDGEMENTS

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Paper 4
Flexural Fatigue of Composite Beams of Plain and Lean Concrete

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ABSTRACT

Lean concrete is often used in concrete or asphalt pavement design because it supplies additional strength and stiffness to the pavement system at a low cost. In concrete pavement design the lean concrete is mostly used for heavily trafficked roads, or industrial pavements where very high loads are present. The lean concrete contributes with strength and stiffness, and makes it possible to design pavements without dramatically increasing the overlaying concrete thickness. In this study, composite beams of plain and lean concrete have been manufactured and subjected to flexural fatigue testing. First, the flexural static strength was determined with four beams. Eight beams where subjected to flexural fatigue loading. The results are analysed using Tepfers’ fatigue equation, a fatigue criterion used for plain concrete. The study was conducted in order to analyse how the composite beam behaviour, the crack development, and bond between the materials were affected by cyclic loading.

INTRODUCTION

Lean concrete is generally described as a compacted, hydraulically bound material with a relative low cement and water content. It is widely used as a stabilised road base for highways as well as airports and industrial areas. The most common applications are roads with high traffic intensity or industrial pavements trafficked with very high loads. As an alternative to an asphalt road base, the lean concrete is much stiffer during all seasons of the year. It is of course affected by the temperature changes but with a minimal effect on strength. The lower content of cement in lean concrete makes it economically and environmentally advantageous to use it under the concrete pavement.

Lean concrete is used to distribute stresses and to minimise the overlaying concrete thickness. It has a strengthening function in the overall pavement system by contributing with additional strength in the material itself, but also by attaching to the concrete overlay. The bond is mechanistically desirable to reduce stresses but is thought to possibly be time dependent and diminish some time after construction. One disadvantage with bonding is the hypothesis that cracks that develop in the lean concrete can propagate through the overlaying pavement (reflection cracks).

In this study the interaction between plain and lean concrete is investigated with composite beams subjected to flexural stresses. The bonding between plain and lean concrete is investigated with static as well as fatigue flexural loading and pull-off tests. In the fatigue tests, the bonding is analysed by monitoring the deflection but also by visual inspections. The
main objective is to analyse the nature of the crack development in the composite beams and investigate how well Tepfers’ fatigue criterion is suited for the prediction of fatigue life. The load levels in the fatigue testing are chosen in relation to the material strength to capture the crack development in each material.

**Flexural Fatigue of Plain and Lean Concrete**

The fatigue strength of plain concrete is defined as the fraction of the static stress that the material can support for a given number of stress cycles, \( N \). A fatigue criterion that accounts for the variation in loading magnitude, originally developed by Aas-Jacobsen (Aas-Jacobsen, 1970) is:

\[
\frac{\sigma_{\text{min}}}{f_{c,\text{fl}}} = 1 - \beta \left( 1 - \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \right) \log N
\]

where,

- \( N \) = the number of load applications
- \( \sigma_{\text{min}} \) = the minimum stress
- \( \sigma_{\text{max}} \) = the maximum stress
- \( f_{c,\text{fl}} \) = the concrete flexural strength
- \( \beta \) = a material constant
- \( R \) = \( \sigma_{\text{min}} / \sigma_{\text{max}} \)

A common way of illustrating the fatigue strength is in a Wöhler diagram, with so called S-N-curves. In the Wöhler diagram, the logarithm of the number of load cycles to failure, \( \log N \), at a specific maximum stress level, is plotted. For each curve the relationship between the minimum and the maximum stresses, \( R \), is held constant, see Figure 1.
In Equation (1), the $\beta$-value is a material constant that is determined experimentally. Tepfers (Tepfers & Kutti, 1979, Tepfers, 1979, and Tepfers, 1978) examined the fatigue of concrete with compressive and splitting tests. The tests were also compared with other research results from compressive tests on ordinary and lightweight concrete (data are presented in (Tepfers, 1978)). Tepfers concluded that Aas-Jacobsen’s fatigue equation is valid for plain concrete and suggested $\beta = 0.0685$.

The fatigue of lean concrete, on the other hand, is often described with a strain criterion. In two preceding studies, lean concrete and plain concrete beams have been tested in bending and compared to Tepfers’ fatigue criterion (Söderqvist & Silfwerbrand, 2005 and Söderqvist & Silfwerbrand, 2006). The hypothesis is that both lean and plain concrete are cementitious materials and that Tepfers’ fatigue equation ought to be used for both. The tests were done in order to examine the hypothesis. The results show that the fatigue of lean concrete can be described with a stress criterion and therefore the fatigue life for a composite beam is chosen to be analysed with Tepfers’ fatigue equation in this study.

**BENDING STRESSES IN A COMPOSITE BEAM**

Fatigue testing requires the capability to calculate stresses in a specimen to conduct more effective testing. To reach a certain load level in a specific specimen, the stresses have to be calculated in advance with regard to the effective height, width, and material strength and stiffness of each layer. For a two layered composite beam with plain concrete on top and lean concrete in the bottom, and assuming full bonding, the stress distribution can be calculated by transforming the beam into one equivalent section, a *T-beam*, in one material, see Figure 2. The stresses can then be obtained by the simple theory of elasticity, using equation
\[ \sigma_{\text{PCC}} = \frac{M \cdot y_{\text{max}}}{I_{\text{PCC(equiv)}}} \]  

(2)

for the plain concrete layer and the relationship

\[ \sigma_{\text{LC}} = \frac{E_{\text{PCC}}}{E_{\text{LC}}} \sigma_{\text{PCC}} \]  

(3)

for the stresses in the lean concrete, where the stresses, \( \sigma \), the modulus of elasticity, \( E \), and the moment of inertia \( I \), are denoted with PCC for the plain concrete and LC for the lean concrete. \( I_{\text{PCC(equiv)}} \) is the moment of inertia of the equivalent section, the T-beam.

**Figure 2.** Cross-section of a beam. The composite section with two layers of different materials, on the left, is transformed into an equivalent section of one material with a reduced width, a T-beam, on the right.

Attention has to be made to the height relation between the two layers since the modulus of elasticity of both materials are fairly equal. If the height of both plain and lean concrete is the same, see Figure 3a, the failure load for the lean concrete layer will result in a load exceeding the failure load for the plain concrete when it has to carry the entire load. In this case, a brittle fracture through the whole section will occur. The stress level in the interface between the materials is also small since the neutral axis lies near the middle of the beam, and the interface is therefore unstressed or nearly unstressed. In this study, this has been tested with four beams.

In four beams, the thickness of the lean concrete is reduced by sawing to approximately half the plain concrete thickness, see Figure 3b. This has been done in order to try to reach the fatigue limit for the lean concrete while keeping the load level lower than the failure load for the plain concrete. By doing this, the bonding between the materials should be affected by the fatigue loading. In this case it would be interesting to see whether a crack that develop due to the repeated loading travels along or through the interface between the two materials.
Figure 3. Schematic stress distribution in a composite beam with a higher modulus of elasticity in top layer. For a and b, full bond generates lower relative stresses in bottom of top layer. In c, less or no bond generates higher tensile stresses in both layers. On the right, the resultant stress distribution after that the bottom layer (lean concrete) has failed is shown.

The benefits of a composite pavement system with plain and lean concrete can be illustrated by making an example with the calculation method mentioned above. Taking lean concrete and plain concrete with a flexural strength of 2.60 MPa and 5.50 MPa, respectively, and assuming full bonding, a 200 mm plain concrete layer with 150 mm lean concrete can take a 55 % higher load compared to a 200 mm plain concrete layer. Decreasing the thickness of the plain concrete layer to 180 mm with a remained thickness of 150 mm of lean concrete, the load can be increased with 38 %, see Table 1.
Table 1. Example of changing load capacity of different composite pavement systems. Flexural strength of lean and plain concrete is 2.60 MPa and 5.50 MPa, respectively.

<table>
<thead>
<tr>
<th>PCC (mm)</th>
<th>LC (mm)</th>
<th>Increased load capacity compared to reference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>150</td>
<td>55.0</td>
</tr>
<tr>
<td>180</td>
<td>150</td>
<td>38.0</td>
</tr>
<tr>
<td>150</td>
<td>150</td>
<td>14.0</td>
</tr>
<tr>
<td>200</td>
<td>100</td>
<td>16.0</td>
</tr>
<tr>
<td>180</td>
<td>100</td>
<td>0.3</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>-50.0</td>
</tr>
</tbody>
</table>

**SCOPE OF THE LABORATORY TESTS**

The laboratory tests are conducted in order to analyse the interaction between layers in a composite beams of plain and lean concrete subjected to fatigue loading. The fatigue strength is evaluated with Equation 1. The equation considers two stress levels and has been chosen in this study because of the similarities in material properties between plain and lean concrete. The objective of the tests is to investigate the fatigue strength of a composite beam but also to study the bond between the materials. The crack development is of great interest and here, the question is if the crack will propagate through both the lean and plain concrete, i.e. a so called reflection crack, or if the crack travels between the two materials, horizontally.

**TEST SPECIMENS**

Twelve composite beams with plain concrete on top and lean concrete in the bottom were used in this study. The beam dimensions (L × B × H) were approximately 800 × 150 × 150 mm (six beams) and 800 × 150 × 120 mm (six beams). All beams had from the start equal height of plain and lean concrete. Four beams were later cut to have a lean concrete layer that was approximately half the thickness of the plain concrete. Beam dimensions are presented in Table 2.
Table 2. Dimension of the cross-section of the tested beams, and the type of testing the beams were subjected to.

<table>
<thead>
<tr>
<th></th>
<th>$H_{LC}$ (mm)</th>
<th>$H_{PCC}$ (mm)</th>
<th>$B$ (mm)</th>
<th>Type of testing</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balk 1-1</td>
<td>67</td>
<td>71</td>
<td>156</td>
<td>Static</td>
<td></td>
</tr>
<tr>
<td>Balk 1-2</td>
<td>64</td>
<td>67</td>
<td>160</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>Balk 2-5</td>
<td>91</td>
<td>90</td>
<td>156</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>Balk 2-2</td>
<td>96</td>
<td>95</td>
<td>155</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>Balk 1-3</td>
<td>93</td>
<td>86</td>
<td>157</td>
<td>Fatigue</td>
<td></td>
</tr>
<tr>
<td>Balk 1-4</td>
<td>95</td>
<td>85</td>
<td>149</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>Balk 1-5</td>
<td>96</td>
<td>81</td>
<td>151</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>Balk 1-6</td>
<td>90</td>
<td>88</td>
<td>150</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>Balk 1-1</td>
<td>67</td>
<td>35</td>
<td>145</td>
<td>&quot;</td>
<td>Specimen failed at first loading</td>
</tr>
<tr>
<td>Balk 2-3</td>
<td>65</td>
<td>40</td>
<td>153</td>
<td>Subjected to three load levels, static test at the end</td>
<td></td>
</tr>
<tr>
<td>Balk 2-4</td>
<td>64</td>
<td>36</td>
<td>153</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>Balk 2-6</td>
<td>62</td>
<td>41</td>
<td>158</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

The lean concrete mixture was retrieved from the Swedish design guide (SRA, 2005) and fabricated in a concrete plant. Mixtures are presented in Table 4. The production site was prepared with a 70 m² base of compacted gravel. The lean concrete was tipped on the surface and compacted 3 – 4 times using a CA15 roller in two equal layers with the total thickness of 150 mm (Figure 4). The level of compaction and the relative moisture content were monitored with a nuclear density gauge to guarantee a 97 % compaction level. After seven days, twelve beams were cut out of the surface. The specimens were stored in water for 18 months before casting plain concrete on the beams. After casting the specimens were cured in water until testing. The surface of the lean concrete was only washed with water before applying the plain concrete.

Figure 4. Preparation of lean concrete layer. The material was compacted to a 97 % compaction level.
Table 3. Activities over time for fatigue testing of composite beams

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
<th>Age (Days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production of lean concrete on site</td>
<td>November 2004</td>
<td>1</td>
</tr>
<tr>
<td>Extraction of beams from surface, storage in 100 % RH</td>
<td>November 2004</td>
<td>14</td>
</tr>
<tr>
<td>Casting of plain concrete on lean concrete beams</td>
<td>June 2006</td>
<td>550</td>
</tr>
<tr>
<td>Static tests of composite beams</td>
<td>June – July 2006</td>
<td>600</td>
</tr>
<tr>
<td>Fatigue testing of composite beams</td>
<td>Sept – Oct 2006</td>
<td>630</td>
</tr>
</tbody>
</table>

Table 4. Typical Swedish plain and lean concrete mixtures used in this study.

<table>
<thead>
<tr>
<th>Swedish Lean Concrete</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEM I, Std Degerhamn*</td>
<td>110 kg/m³</td>
</tr>
<tr>
<td>Aggregate 0-8 mm</td>
<td>1576</td>
</tr>
<tr>
<td>Aggregate 8-16 mm</td>
<td>674</td>
</tr>
<tr>
<td>Water</td>
<td>115</td>
</tr>
<tr>
<td>w/c</td>
<td>1.05</td>
</tr>
<tr>
<td>Density</td>
<td>2476 kg/m³</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Swedish Concrete Grade K60</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEM I, Std Degerhamn*</td>
<td>375 kg/m³</td>
</tr>
<tr>
<td>Aggregate 0-8 mm, Underås</td>
<td>737</td>
</tr>
<tr>
<td>Aggregate 8-16 mm, Underås</td>
<td>1018</td>
</tr>
<tr>
<td>Water</td>
<td>149.4</td>
</tr>
<tr>
<td>Silica</td>
<td>26.3</td>
</tr>
<tr>
<td>Plasticiser, 92M</td>
<td>8.6</td>
</tr>
<tr>
<td>Air Entraining Agent, L14</td>
<td>0.3</td>
</tr>
<tr>
<td>w/c</td>
<td>0.40</td>
</tr>
<tr>
<td>w/b</td>
<td>0.37</td>
</tr>
<tr>
<td>Density</td>
<td>2400 kg/m³</td>
</tr>
</tbody>
</table>

* Swedish cement for civil engineering structures

TEST PROCEDURE

The flexural strength and fatigue were tested in a Material Test System (MTS) 810 machine (Figure 5). At first, four beams were tested to retrieve the static flexural strength. Here, both Linear Variable Differential Transformers (LVDT) and strain gauges were used to monitor the tests. The static strength properties were then used to pinpoint the load levels in the following fatigue tests, taking into account each beam’s individual cross-section. The beams were simply supported with a span of 700 mm, and tested in four point flexural loading according to Swedish standard SS 13 72 12, see Figure 6.
Figure 5. Material Test System 810 machine at the CBI was used for the fatigue testing of composite beams in this study.

Figure 6. Static system for flexural and fatigue tests.

**STATIC TEST RESULTS**

The static tests in this study are additional to tests made in an earlier study (Söderqvist & Silfwerbrand, 2006), where the material from the same lean concrete surface was tested. For the plain concrete, the same mixture was used as in an earlier study where plain concrete beams were subjected to fatigue loading (Söderqvist & Silfwerbrand, 2005).

The compressive strength of the lean concrete as well as the plain concrete was determined by means of cubes and cylinders. For the lean concrete, cubes were cast and tested in compression after 28 days according to Swedish practice. These cubes were compacted with a vibrating plate in three layers to guarantee a 97% compaction level. Plain concrete
cubes were cast at the same time as the overlaying concrete was cast on the lean concrete beams, and tested at 28 days.

The bond strength between plain and lean concrete is determined by pull-off tests made on cylinders drilled out of tested beams.

Four beams with equal height on both plain and lean concrete were tested statically to determine the static flexural strength. Strain gauges were mounted on both sides of the boundaries between the lean and plain concrete to monitor the failure load for the two materials respectively.

The modulus of elasticity in bending for both plain and lean concrete was also determined with the measured deflection through static tests, on specimens tested in an earlier study (Söderqvist & Silfwerbrand, 2005 and Söderqvist & Silfwerbrand, 2006) with the following equation:

\[
E_n = \frac{131(F_1 - F_2)l^3}{686(\delta_2 - \delta_1)bhl}
\]

where,

\[
\begin{align*}
    l & = \text{length} \\
    b & = \text{width} \\
    h & = \text{height} \\
    F_1 & = \frac{1}{4} F_{cr} \\
    F_2 & = \frac{3}{4} F_{cr} \\
    F_{cr} & = \text{failure load} \\
    \delta_{1,2} & = \text{midspan deflections corresponding to } F_1 \text{ and } F_2
\end{align*}
\]

The static modulus of elasticity is also determined with Ø100 x 200 mm cylinders for both plain and lean concrete. Results from the static tests are presented in Tables Nos. 5 - 8.

**Table 5.** Static material strengths for lean concrete. The late age is due to delayed start of testing. The test is made on lean concrete beams in an earlier study. The matieriel is nevertheless from the same batch and production site.

<table>
<thead>
<tr>
<th>Type of strength</th>
<th>Type strength</th>
<th>Strength and Stiffness</th>
<th>Age at testing (days)</th>
<th>No. of tests (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cubes (height x width x length)</td>
<td>cubes (150 x 150 x 150)</td>
<td>16.3</td>
<td>28</td>
<td>3</td>
</tr>
<tr>
<td>cylinders (diameter x heigth)</td>
<td>Cylinders (Ø100 x 100)</td>
<td>13.7</td>
<td>400</td>
<td>6</td>
</tr>
<tr>
<td>beams (length x width x height)</td>
<td>Beams (800 x 150 x 100)</td>
<td>2.1</td>
<td>440</td>
<td>3</td>
</tr>
<tr>
<td>$f_{cc} 28d$</td>
<td>Cylinders (Ø100 x 200)</td>
<td>28 000</td>
<td>500</td>
<td>6</td>
</tr>
<tr>
<td>$f_{cc} 400d$</td>
<td>Beams (800 x 150 x 100)</td>
<td>16 000</td>
<td>440</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 6. Static material strengths for plain concrete.

<table>
<thead>
<tr>
<th>Type of strength</th>
<th>Type (mm)</th>
<th>Strength and stiffness (MPa)</th>
<th>Age at test (days)</th>
<th>No. of tests (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_{cc\ 28d})</td>
<td>Cubes (150×150)</td>
<td>86.3</td>
<td>28</td>
<td>3</td>
</tr>
<tr>
<td>(f_{cc\ 146d})</td>
<td>Cylinders (Ø100×100)</td>
<td>97.5</td>
<td>146</td>
<td>5</td>
</tr>
<tr>
<td>(f_{fl\ 200d})</td>
<td>Beams (800×150×100)</td>
<td>5.5*</td>
<td>200</td>
<td>3</td>
</tr>
<tr>
<td>(E_{c\ 200d})</td>
<td>Cylinders (Ø100×200)</td>
<td>35 000*</td>
<td>200</td>
<td>6</td>
</tr>
<tr>
<td>(E_{fl\ 200d})</td>
<td>Beams (800×150×100)</td>
<td>21 000*</td>
<td>200</td>
<td>3</td>
</tr>
</tbody>
</table>

*Tests made on different beams in an earlier study, where the same mixture was utilised.

Table 7. Pull-off tests on cylinders from tested beams. All failures occurred in the lean concrete.

<table>
<thead>
<tr>
<th>Type of strength</th>
<th>Type* (mm)</th>
<th>Strength (MPa)</th>
<th>Age at test (days)</th>
<th>No. of tests (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry Specimens</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>balk 1-1</td>
<td>Cylinders (Ø72×100)</td>
<td>0.87</td>
<td>450</td>
<td>3</td>
</tr>
<tr>
<td>balk 1-2</td>
<td>&quot;</td>
<td>0.81</td>
<td>&quot;</td>
<td>3</td>
</tr>
<tr>
<td>balk 2-5</td>
<td>&quot;</td>
<td>0.24</td>
<td>&quot;</td>
<td>2</td>
</tr>
<tr>
<td>Wet Specimens</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>balk 2-5</td>
<td>&quot;</td>
<td>0.16</td>
<td>&quot;</td>
<td>1</td>
</tr>
<tr>
<td>balk 2-6</td>
<td>&quot;</td>
<td>0.52</td>
<td>&quot;</td>
<td>3</td>
</tr>
<tr>
<td>balk 2-3</td>
<td>&quot;</td>
<td>0.56</td>
<td>&quot;</td>
<td>2</td>
</tr>
<tr>
<td>balk 2-4</td>
<td>&quot;</td>
<td>0.71</td>
<td>&quot;</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 8. Static flexural testing of composite beams.

<table>
<thead>
<tr>
<th>Id</th>
<th>(F_{max}) (kN)</th>
<th>(f_{c, fc}),LC (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balk 1-1</td>
<td>15.00</td>
<td>2.84</td>
</tr>
<tr>
<td>Balk 1-2</td>
<td>12.58</td>
<td>2.57</td>
</tr>
<tr>
<td>Balk 2-5</td>
<td>24.19</td>
<td>2.66</td>
</tr>
<tr>
<td>Balk 2-2</td>
<td>25.20</td>
<td>2.50</td>
</tr>
<tr>
<td>Mean value:</td>
<td>2.64</td>
<td></td>
</tr>
<tr>
<td>Standard deviation:</td>
<td>0.14</td>
<td></td>
</tr>
</tbody>
</table>

| Balk 2-3* | 9.25           | 3.06                   |

* Beam was first subjected to 2 million loading cycles before conducting the static test

**FATIGUE TEST RESULTS**

A total of seven beams were tested in fatigue loading according to the tests procedure described in the previous section (one beam cracked at first loading due to mistreatment). First, four beams with equal height of plain and lean concrete were tested. All these beams showed an excellent bond between the two materials and failed in one single vertical crack, in a brittle behaviour. There were no crack development and as soon as the lean concrete failed, the plain concrete also failed due to the high load, see section *Bending Stresses in a Composite Beam*. In the next test series, the lean concrete layer was cut horizontally to
approximately half the thickness of the plain concrete. This procedure is adopted so that a failure load of the lean concrete would not exceed the failure load of the plain concrete, making it possible to analyse the bonding effect and a possible crack development in the lean concrete layer in the test specimens.

The specimens were carefully measured before testing to apply a load as close as possible to the desired load level with regard to the height and width of each individual specimen. A constant value of \( R = \sigma_{\text{min}} / \sigma_{\text{max}} \) corresponding to 0.22 was considered. Before each fatigue test, the number of loads, referred to as \( \log N \), that the current beam could withstand in the bottom of both the plain and lean concrete layer was calculated according to Tepfers’ fatigue equation and the calculation method described in the section Bending Stresses in a Composite Beam. This number was then evaluated compared to the number of loads that the current beam could take before failure, \( \log N_{\text{m}} \), for each of the two layers. The beams were covered with a plastic folio to keep the moisture level constant throughout the test. Two LVDT’s were mounted on every beam to measure the midspan deflection.

**Evaluation of Fatigue Test Results**

Each beam was measured after testing, and the dimensions of the crack cross-section of the beams were used to calculate a more accurate relative stress level. The load level that each beam was subjected to is calculated using the relation between the presumed maximum load, i.e. flexural static strength, and the applied load. Since the static test results had a large scatter on the static flexural strength, a 14 % deviation (i.e. the standard deviation from the static tests, see Table 8) from the mean value is considered in the evaluation of the fatigue tests. This means that if the flexural strength is varied, consequentially, the load levels and the number of predicted load applications will vary. For high load levels, when the maximum load generates stresses near the static stress limit, small changes in the static strength have a large impact in the number of predicted load applications. Since no crack development was observed during testing, calculations on fatigue strength is based only on the flexural strength of the lean concrete. Results are presented in Table 9 and Figure 7.

<table>
<thead>
<tr>
<th>Id</th>
<th>( H_{\text{PCC}}/H_{\text{LC}} )</th>
<th>( \sigma_{\text{mean}}/f_{\text{cfl}} )</th>
<th>( R )</th>
<th>( \log N )</th>
<th>( \log N_{\text{m}} )</th>
<th>( \log N_{\text{m}}/\log N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balk 1-3</td>
<td>1.08</td>
<td>0.81</td>
<td>0.61</td>
<td>0.22</td>
<td>3.61</td>
<td>7.33</td>
</tr>
<tr>
<td>Balk 1-4</td>
<td>1.12</td>
<td>0.76</td>
<td>0.57</td>
<td>0.22</td>
<td>4.58</td>
<td>8.06</td>
</tr>
<tr>
<td>Balk 1-5</td>
<td>1.19</td>
<td>0.69</td>
<td>0.52</td>
<td>0.22</td>
<td>5.78</td>
<td>8.96</td>
</tr>
<tr>
<td>Balk 1-6</td>
<td>1.03</td>
<td>0.69</td>
<td>0.52</td>
<td>0.22</td>
<td>5.84</td>
<td>9.00</td>
</tr>
<tr>
<td>Balk 2-4</td>
<td>1.75</td>
<td>0.88</td>
<td>0.66</td>
<td>0.22</td>
<td>2.33</td>
<td>6.36</td>
</tr>
<tr>
<td>Balk 2-6</td>
<td>1.91</td>
<td>0.98</td>
<td>0.74</td>
<td>0.22</td>
<td>0.46</td>
<td>4.94</td>
</tr>
</tbody>
</table>

**Note:** Columns 3, 6 and 9 show values when calculating with a static flexural strength that is decreased with 14 % and columns 4, 7, and 10 correspond to calculations done with a static flexural strength increased with 14 % from the mean value.
Figure 7. Fatigue test results. The diagram shows the damage \((\log N_m/\log N_c)\) and maximum load \((\sigma_{\text{max}}/f_{cfl})\). The columns represent the damage of each beam in relation to the predicted number of load applications. 100 % is the fatigue limit according to Equation 1. The markings represent the relative maximum load that each beam was subjected to. Specimens Balk 2-4 and Balk 2-6 are extremely persistent and therefore cut in this diagram, see values in Table 9.

The beams with a lean concrete layer of approximately half the thickness of the plain concrete were superior in fatigue loading. One possible explanation is the prestress in the specimens that is generated from the autogenous shrinkage of the plain concrete. This effect will be more pronounced in a thin lean concrete layer with less ability to counteract the prestressing.

One beam was also subjected to several load levels and the accumulated damage was calculated, see Table 10 and Figure 8. The beam did not fail due to the fatigue loading and was therefore tested statically at the end, see Table 10.

Table 10. Accumulated damage for three load levels. Results are evaluated using a 14 % variation of the calculated mean static flexural strength \(f_{cfl,LC} = 2.64 \text{ MPa}\).

<table>
<thead>
<tr>
<th>Id</th>
<th>(\sigma_{\text{max}}/f_{cfl})</th>
<th>(R)</th>
<th>(\log N_c)</th>
<th>(\log N_m)</th>
<th>(\Sigma \log N_m/\log N_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balk 2-3, 1\text{st}</td>
<td>0.54</td>
<td>0.66</td>
<td>0.22</td>
<td>6.42</td>
<td>8.62</td>
</tr>
<tr>
<td>Balk 2-3, 2\text{nd}</td>
<td>0.57</td>
<td>0.70</td>
<td>0.22</td>
<td>5.65</td>
<td>7.99</td>
</tr>
<tr>
<td>Balk 2-3, 3\text{rd}</td>
<td>0.61</td>
<td>0.74</td>
<td>0.22</td>
<td>4.88</td>
<td>7.36</td>
</tr>
</tbody>
</table>

Note: Columns 2, 5, and 8 show values when calculating with a static flexural strength that is decreased with 14 % and columns 3, 6, and 9 correspond to calculations done with a static flexural strength increased with 14 % from the mean value.
Figure 8. Beam subjected to three different load levels. Diagram showing the accumulated damage calculated with Miner-Palmgren’s damage hypothesis. In reality, the beam withstood all three load levels and finally failed under a static load of $F = 9.25$ kN equal to a flexural strength of $f_{c,l,C} = 3.06$ MPa (Table 10). The two lines correspond to two different values on the assumed static flexural strength ($f_{c,l}=2.38$ MPa and $f_{c,l}=2.90$ MPa).

DEFLECTIONS DUE TO REPEATED LOADING

The deflections were measured with two LVDT’s mounted on each side of the beams. The deflection was also used to determine if cracking occurred in the lean concrete layer. The deflections were recorded at the beginning of the test and then every 50 to 10 000 cycles depending on the predicted number of loads to failure. The maximum deflection before failure is difficult to capture since it is impossible to predict the exact time of failure. Deflections are presented in Figure 9. The deflection grows linearly versus the log$N$, with an accelerated growth just before failure.
DISCUSSION

In concrete pavement design, a lean concrete road base is used to reduce the stresses in the concrete overlay and the vertical strain in the subgrade, making it possible to design pavements for high traffic loads without increasing the concrete pavement thickness dramatically.

The tests show a very good bond between the materials. All cracks were inspected and pull-off tests were conducted. The ratio between the material strength and the modulus of elasticity is low and as the tensile stress reaches the stress strength limit of the material, a crack goes through the whole section. In these tests, peak stresses in the bottom of the lean concrete in the composite beam were investigated. The crack development did not stop in the interface between the plain and lean concrete, instead the whole beam cracked as soon as the stresses reached the fatigue limit in the lean concrete. Since the bonding was extremely good for all beams, the cracking of the beams can be explained by the natural behaviour of a crack development; the crack develops at the smallest cross-section, in the most stressed region, and opens perpendicularly to the stress direction. The crack develops because there is a concentration of stresses at the end of the crack, see Figure Nos. 10 - 11. Stresses and strains have to be taken care of by a smaller and smaller cross-section, and these stresses are multiplied instantly at the end of the crack. The bond between the lean and plain concrete allows strains and stresses to pass through the interface, and a stress concentration at the interface is therefore possible.
The tests also show that the deflections grow linearly in a logarithmic diagram. This also has been observed when testing plain and lean concrete separately. The deflection did not show any signs of intermediate cracking either.

**Figure 10.** Finite element model (FE-model) showing the stress concentration over the crack of a partly cracked beam with plain concrete on top and lean concrete in the bottom. Full bond is considered. The model constitutes of shell elements and is modelled in the finite element analysis (FEA) software *LUSAS Bridge* (LUSAS, 2006).

**Figure 11.** Stress distribution over the cross-section at the crack calculated with Equation (2), and the FEA software, *LUSAS Bridge*. Stresses are concentrated in the bottom of the plain concrete layer above the crack in the lean concrete. The stresses (8 MPa) exceed the strength limit (5.5 MPa) for the plain concrete, which results in reflection cracks. The bond is set to 100 % in the FE-model.
CONCLUSIONS

In the flexural fatigue tests, the first four beams had the same height of both plain and lean concrete. These four beams showed consistent fatigue properties; full bond and no crack development were observed. In the tests where the height of the lean concrete was approximately half of the plain concrete, the fatigue was superior compared to Tepfers’ fatigue criterion. These beams were tested in a trial to achieve partial cracking in the specimen. The load levels were chosen in a way that, even though the lean concrete would fail, the plain concrete would be sufficiently thick to carry the applied load. However, all beams cracked entirely and no partial cracking was observed. The cracking is explained by (1) the full bond between the materials that allowed stresses to pass through the interface, and (2) the concentrated stresses that are assembled at the crack opening, caused by the decreasing cross-section.

The phenomenon of entire cracking of the cross-section is referred to as reflection cracks, and challenges the question on whether the bond really is desirable or not, i.e. strength versus risk of cracking. The conclusion is that bond may be detrimental because it ruins the whole section and is difficult to control.

The higher flexural fatigue strength of the beams with a relatively thinner lean concrete layer is explained by a possible prestress from the shrinkage of the plain concrete on top. It is likely that a thinner lean concrete layer is more influenced by this prestress.

The deflection increased steadily throughout every test, and as the specimen come near the fatigue limit the deflections grow more rapidly. The increasing deflections work as a forewarning that the beam is about to crack. In this study, the average deflections increased with 100 % or more from the first deflection, compared to the deflections of observed for plain concrete beams (Söderqvist & Silfwerbrand, 2005), that just reached 60 – 100 %.

The conclusions from the study can be summarised as follows:

1. The bond between the plain and lean concrete is remarkably good still after 1 million loading cycles. The bond has been obtained without any additional treatment made on the surfaces of the specimens.
2. The static flexural tests did not indicate any bond loss. Neither did the flexural fatigue tests.
3. The static tests did not show any partial cracking in the lean concrete, the crack at the failure load always went through the entire cross-section.
4. The deflection increases more steadily versus the logarithm of loading cycles for the composite beams than for plain concrete beams.
5. The fatigue strength of composite beams of plain and lean concrete was equal or better than predicted with Tepfers’ fatigue criterion, Equation (1).
6. The bond is may be detrimental because, even though it brings more strength to the structure, overloading or fatigue loading causes all-pervading cracks.

In concrete pavement design the bond can be beneficial because a more stiff construction reduces stresses and strains in the subgrade. To account for a higher loading capacity, a bond could be considered but it would in that case be more important to focus on the stresses in the bottom of the lean concrete instead of the plain concrete. Furthermore, the thermal cracking of the lean concrete has to be avoided by sawing at short distances.
References


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