Mathematics achievement of early and newly immigrated students in different topics of mathematics

Jöran Petersson

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Abstract

This thesis aims to explore the mathematics achievement of second language immigrants in compulsory school as they continue their schooling in Sweden. Specifically, the thesis aims to generate more knowledge about different sub-categories of second language students, namely newly arrived immigrants, early arrived immigrants and other second language students in compulsory school. The data in this thesis consists of students' responses to test items and thus mainly contains mathematical symbols, essentially numbers in different representations, written by the students.

Doing so, this thesis problematizes the concept of second language students in mathematics in two aspects. One aspect is to assess the first and second language students' achievement in different mathematical content domains, instead of only assessing the total achievement. Another aspect is to see the second language students as different sub-categories of second language students.

Papers I and II of this thesis found that the achievement difference between first and second language students is not homogeneous. Instead the achievement difference between first and second language students is larger for concepts that are rare in mathematics textbooks. Moreover, the achievement difference between first and second language students varies with the content domain. Another way to say this is that first and second language students have different achievement profiles.

Papers III and IV of this thesis explored how sub-categories of second language students achieved on mathematics test items. Mathematics achievement studies on second language students often classify the second language students into a single category of students. Methodologically this imposes a concept of viewing second language students as homogeneous in proficiency in the language of instruction. This view is challenged in this thesis by dividing the second language students into newly arrived immigrants, early arrived immigrants and other second language students. These three sub-categories have different proficiency in Swedish language due to how long they have lived in Sweden. Papers III and IV found that these student categories both had different test achievement and, related to this, also used mathematical concept representations differently. In particular, the newly and early arrived immigrants seemed to experience on average different challenges during mathematics testing. The newly arrived students seemed more challenged with terminology but less with the mathematical content while the opposite seemed to hold for the early arrived students. An implication for teaching is that particularly early arrived second language children seem to be in urgent need of support in mathematical concept building from first day of schooling in the new country.

Keywords: *achievement profile, grade 9, mathematics achievement, mathematics education, second language learners, statistics literacy, student as-sessment.*

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To all devoted to
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List of papers

This PhD thesis consist a set of papers

Papers in part 1 of the thesis

I Petersson, J. (2012). Rare mathematics - a needle eye for teachers of second language learners. In G. H. Gunnarsdóttir, F. Greinsdóttir, G. Pálsdóttir, M. Hannula, M. Hannula-Sormunnen, E. Jablonka, U. T. Jankvist, A. Ryve, P. Valero, K. Waege (Eds.), *Proceedings of Norma 11, the 6th Nordic conference on mathematics education*. (pp. 483-492). Reykjavik: University of Iceland Press. < http://urn.kb.se/resolve?urn=urn:nbn:se:su:diva-116441 >

II Petersson, J. (2017; In press). First and second language students' achievement in mathematical content knowledge areas. *Nomad, 22*(2), XX-XX

Papers in part 2 of the thesis

III Petersson, J. Norén, E. (2017). To halve a fraction: An issue for second language learners. *Education Inquiry*, (Online first) doi: 10.1080/20004508.2016.1275187 < http://dx.doi.org/10.1080/20004508.2016.1275187 >

IV Petersson, J. (Manuscript). A multi-cultural perspective on students' concepts and achievements in stochastics education.

Contents

Abbreviations

2L	Second language learners
1L	First language learners

Preface

> Trött på alla som kommer med ord, ord men inget språk
> for jag till den snötäckta ön.
> Det vilda har inga ord.
> De oskrivna sidorna breder ut sig åt alla håll!
> Jag stöter på spåren av rådjursklövar i snön.
> Språk men inga ord.
>
> ("Från Mars -79", a poem by Tomas Tranströmer)

The cited poem was written by the 2011 Nobel Prize winner in Literature Tomas Tranströmer (Tranströmer, 1997, p. 161). In this poem, Tranströmer says that he is tired of those talking much without saying anything and for this reason went to the country side where he saw tracks of animals in the snow. The last row says "Language, but no words". One interpretation of this poem is that some expressions are not said or written in words, but despite this speak clearly. The same holds for numbers. They appear not only as digits. Numbers can be given implicitly when represented by coordinate points in a graph, as bars or sectors in a diagram or as algebraic expressions. Even when numbers appear explicitly in digits, they have different meanings if they appear as indices or powers. There may be symbols added to them such as decimal point, root of, minus sign, brackets and different division symbols for fractions. Some of these cases are illustrated on the book cover of this thesis. These added symbols have a thorough impact on how numbers are interpreted in mathematical activities both as enacted by novice students and as norms agreed among professional mathematicians. There simply are more to numbers than only digits can tell.

This thesis aims to explore the mathematics achievement of second language immigrants' in compulsory school as they continue their schooling in Sweden. Specifically, the thesis aims to generate more knowledge about different sub-categories of second language students, namely newly arrived immigrants, early arrived immigrants and other second language students in compulsory school. The data in this thesis consists of students' responses to test items and thus mainly contains mathematical symbols, essentially numbers in different representations, written by the students. The students' responses have been assessed by mathematics teachers including the author. In these responses, the teachers did not see many words, but a great deal of the students' mathematical 'language'. Moreover – the students' proficiency in the natural language was reflected in an intricate way in their fluency in the mathematical language; particularly for newly and early immigrated students.

Some personal anecdotal experience of teaching mathematics to immigrant second language students is the following. For a while I taught in adult education, mainly distance education, where students showed up to ask for personal instruction or for taking a final exam. One man from the former French colonies in North Africa gave very neat and spotlessly complete solutions to algebra problems that would have been intricate to my previous Swedish upper secondary school students. A woman from Russia showed very good knowledge of functions, being advanced mathematics in the Swedish curriculum, but gave a poor impression in percentage calculation, which in the Swedish curriculum is elementary mathematics. A man from Iran did not answer any test item containing the word "interpret", but gave correct and complete responses to all other test items. He knew the meaning of the word "interpret" in non-mathematical context, but not in mathematics. I started to realise that being an immigrant second language speaker in Swedish is not only about being a second language speaker. These three students, of course, brought knowledge from following previous mathematics education, but from following some mathematics syllabus that in some aspects is different from the Swedish mathematics syllabus. There were traces of a language not expressed in words alone, but also in how mathematical expressions appeared on their sheets. In the present PhD project, I have had the opportunity to explore this further and in detail.

Introduction

> "When an alien lives with you in your land, do not ill-treat him. The alien living with you must be treated as one of your native born. Love him as yourself, …" (The Bible, Leviticus 19:33-34).

In Sweden, there is presently a lively debate, both colloquial and academic, on immigrants' equity in access to school (e.g. Bunar, 2010). This can be illustrated with the number of book title search hits for the Swedish word "Nyanlända", (meaning "Newly arrived" in English). In the Swedish royal library database, the word "Nyanlända" until about 2010 gave at most a few hits per year, while there has recently been a large increase in the number of hits, as illustrated in Figure 1.

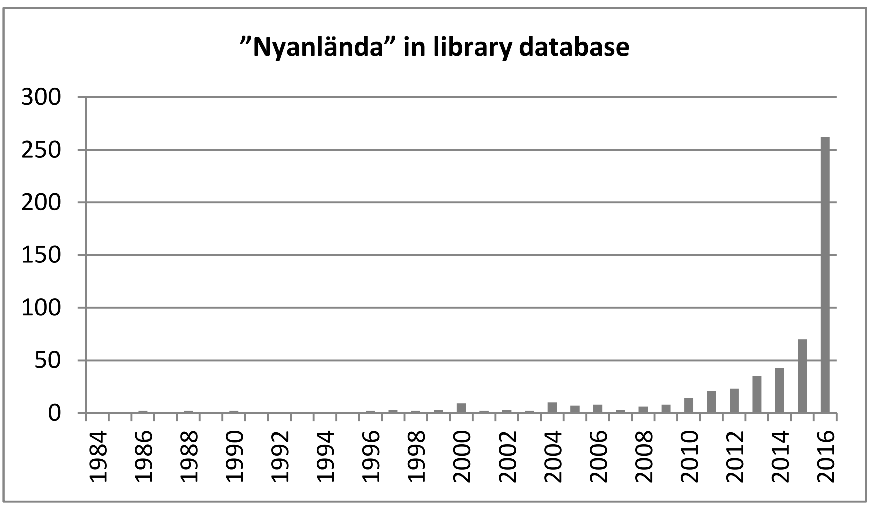


Figure 1. Occurrences per year of the word "Nyanlända" in the library database < www.libris.kb.se >.

More ambitious than equity in access to education; the debate is on immigrants' access to equity in school success. This is because one of the tasks of the Swedish school is to be compensatory, that is, to compensate for systemic discrimination. For example, Skolverket (2016) recently published a web-course giving further training for teachers' work in mapping newly arrived second language immigrants' knowledge in school subjects. This issue of equity among natives and immigrants is a history-old issue, as indicated by the Bible quotation given above.

The present study suggests seeing newly arrived second language immigrant students as bringing a mathematical competence flavoured by their previous schooling as likely first language students outside Sweden. In so doing, the present study problematizes categorising immigrants mainly as second language students being potential under-achievers. Moreover, the present study suggests that early arrived immigrants may on average have face major challenge in, as beginning second language students, learning the fundamental mathematical ideas in early school years on which success in the later school years (and thus access to further education) builds.

The target groups in the present study are early and newly arrived immigrants, where especially the latter can be seen as bi-curricular students, since they may have experiences from both the Swedish and some other curriculum. This induces a comparative perspective and a multilingual perspective to the research background. Since the methods for categorising the students, for collecting data and data itself were different in, on the one hand papers I and II and, on the other hand, papers III and IV, this thesis was written in two parts.

In part one, comprising papers I and II, the students were categorised into first and second language students as reported in the test data. In papers I and II were made secondary analyses of data of student achievement from national tests in the ninth schoolyear.

In part two, comprising papers III and IV, the students were categorised into first language students, and three sub-categories of second language students, namely newly arrived, early arrived and other second language students as reported by the students in a survey. As method for data collection students' responses to written tests were used. Test items were designed to be similar to those analysed in papers I and II. A test was given to students in the ninth schoolyear. Selected students were invited to comment on their responses in order to validate the interpretation of their concept use, if the written response was unclear or difficult to interpret.

A mathematical idea can be made 'public' as a drawn or written representation and thus made available for interpretation by the participants in the classroom being the teacher, another student or the student himself/herself (Bakker & Hoffmann, 2005; Morgan, 2001). There are several different analytic frameworks for classifying different forms of mathematical communication. Paper I used the enacted and endorsed rules of Sfard (2008). Papers III and IV used a framework by Prediger and Wessel (2011) since this includes the three dimensions of representation forms (Duval, 2006), of multilingualism and of formal/informal language. However, the dimension of formal/informal language was exchanged for Sfard's (2008) enacted and endorsed rules in order to assess mathematical quality.

Previous research has focused on the perspectives of socio-economic background and bilingualism. The present study adds a third perspective, namely that of distinguishing between newly arrived and early arrived immigrants in school. Newly arrived students near the end of compulsory school in most cases have been first language students following some other curriculum for a long time. This may bring, into their new mathematics classroom a knowledge profile flavoured by their previous schooling. They may have a good knowledge in school mathematics, but are beginners in Swedish, the language of instruction. In contrast, early arrived immigrants have had most of their schooling in their second language, which may have been an obstacle to them in developing a rich body of mathematical knowledge (Cuevas, 1984). The present study generated four papers, of which one is a published conference papers, two are journal articles in press and one is a manuscript.

> "… each of us hears them in his own native language …" (The Bible, Acts 2:8).

Organisation of the thesis

The organisation of the present study is in two parts for reasons mentioned above. Each part consists of a research background, a research question, a chapter on methodology and analytic framework followed by paper summary. Part 1 compares first and second language students' achievement via secondary data from national tests and leads to papers I and II. Part 2 compares first and second language students' achievement through primary data from a test designed by the author and leads to papers III and IV. A concluding discussion will draw the two parts together. Eventually, after part 2 are given a summary in Swedish, appendices with test instruments, letter of consent, the survey and the full papers I–IV.

Part 1

Comparative education studies

In educational studies, a common research design is to compare different student categories with each other. For example, first and second language students might be compared with respect to achievement, attitudes etc. Egidius (2006) defined comparative education as the study of similarities and differences between school systems in different countries and cultural groups, and discussing reasons for it. This definition allows for both school culture and cultural groups of students. Egidius defined school culture as traditions and attitudes that characterise the school's practices. Together, these definitions of comparative education and of school culture open a wide research field in mathematics education. For example, the discussion document of the 13th ICMI study (Leung, Graf, Lopez-Real, 2006) raised a lot of issues in comparative studies. School cultures consist of governmental documents, textbooks and other teaching materials. With this follows aspects on how gender, giftedness, multilingualism and disadvantaged students are treated in the class. Even assessment itself is not culture-neutral since there are different assessment traditions such as multiple-choice test items versus open response test items. For example, TIMSS has a large proportion of multiple-choice test items while these are few in Swedish national tests in mathematics, which instead have open response test items (Mullis, Martin, Foy, & Arora, 2012; PRIM-gruppen, 2009). Moreover, there is teacher education. There are teachers' and students' attitudes, beliefs and roles in teaching and learning. To capture and analyse these various aspects of mathematics education requires different data such as documents of governmental and textbook material, videos of classroom processes, surveys of attitudes and beliefs and assessment instruments for achievement.

Since comparative education comprises a large area of research questions, methods and study objects, the present review on comparative studies has a focus on achievement and is structured in the two aspects of school systems and cultural groups of students covered by Egidius' (2006) definition. The description of comparative research on school systems follows Travers and Weinzveig (1999), who in SIMS identified the three levels of the attained curriculum; intended curriculum; and the implemented curriculum. The description of comparative research into cultural groups of students includes their socioeconomic background and migrant students.

- At the level of the attained curriculum, a comparative study asks questions about learning outcome, attitudes to using mathematics in daily life, work and choice of continued education. Typical data are from surveys and achievement tests designed for measuring some specific aspect(s) of mathematical knowledge.
- At the level of the intended curriculum, there are questions about how and why curricula are different in different school systems. The data could be documents that regulate the school systems such as curricular documents.
- At the level of the implemented curriculum practices in the classroom are studied. It works with what beliefs and attitudes of teachers and students that are connected to the classroom practices. Typical data are from surveys, interviews and classroom observations.
- Besides comparing different countries with each other, the definition of comparative studies includes comparing how an educational system could work out for different cultural subgroups within a country or educational system. One example of cultural subgroups could be the socioeconomic strata of the population.
- Another example of a cultural group could be immigrants, which in their new country may become second language learners and how transition into the mainstream education is organised for these students.

The attained curriculum – the student level

In a world with migrants, the achievement outcome in large-scale international studies can be used for other purposes than comparing the achievement of one country with another. One example is to compare average achievements of immigrant students from countries where students on average achieved differently. Giannelli and Rapallini (2016) did this and found that, in the new country, immigrants from high achieving countries achieved higher than immigrants from low achieving countries.

When reading, for example, the TIMSS-results, the results display several properties of the achievement for different countries. One property is the total achievement. Students in some countries on average achieve differently from average students in other countries as seen in Figure 2. Another property is that countries may achieve similarly in total, but have achieved differently in different mathematical content areas. Figure 2 shows a diagram, where the absolute achievement difference between England and Russia and between Sweden and Serbia may look small, but where the achievement per mathematical content area is differently distributed.

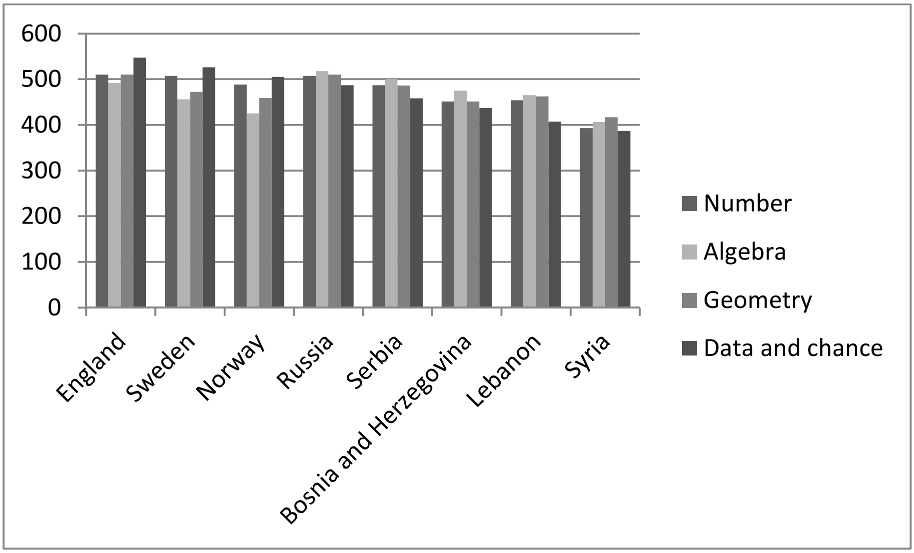


Figure 2. TIMSS 8th grade 2007 achievement of some countries (Mullis, Martin & Foy, 2008).

Thus, it seems too naïve to compare only the total achievement. There is a need of other ways for comparing average student achievement in different countries. This need is partly motivated by migration. For example, in Sweden a large proportion of the immigrants are from the Balkans and the Middle East. In Figure 2, the average achievement profile for students in England, Norway and Sweden shows a shape with a valley for algebra and geometry, and this is more pronounced for Norway and Sweden. Students in the countries of Russia, Serbia, Bosnia and Herzegovina, Lebanon and Syria show an achievement peak for algebra and geometry. This peak is less pronounced for Russia and more pronounced for Lebanon. The achievement profile allows us to compare not only total achievement, but also mathematical content areas of relative strength and weakness for students in different countries.

Two large frameworks used for international comparative studies in mathematics education are those of PISA and TIMSS. PISA aims to evaluate students' readiness for work and further study as post compulsory-school citizens, with the OECD perspective of a being citizens in a democratic market economy in an industrial country. TIMSS instead aims to evaluate school from an inside-school perspective. These two different aims lead to the design of different mathematical frameworks, and as a consequence, to design of different test instruments. When comparing the two frameworks of PISA and TIMSS, the functional model of Rico (2006), also described in Sáenz (2009), is useful. Rico's (2006) functional model gives three aspects of mathematical learning, namely school mathematical content, cognitive processes in mathematics and problem context situated in some cultural experience. While PISA and TIMSS are similar in mathematical content, they differ in the aspects of mathematical processes and problem contexts. The mathematical content domains in PISA and TIMSS are similar. For PISA, they are ‘change and relationships’; ‘space and shape’; ‘quantity’; and ‘uncertainty and data’. For TIMSS 8th grade they are ‘algebra’; ‘geometry’; ‘number’, and ‘data and chance’. However, the cognitive domains in PISA and TIMSS are different. For PISA, a basis is the question “What is important for citizens to know and be able to do in situations that involve mathematics?” (OECD, 2013, p. 24). In order to answer this question, PISA defined mathematical literacy as follows.

> Mathematical literacy is an individual’s capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (OECD, 2013, p25)

A model for assessing mathematical literacy in PISA is to give the student a problem formulated in some context, which can be modelled mathematically. The student then enters a solving process, which constitutes a modelling cycle. In the solving process, the student formulates the problem mathematically, employs mathematics to solve it and interprets the solution back into the context of the problem formulation. TIMSS aims to measure the students’ knowing, applying, reasoning in mathematics. The cognitive domain of knowing, involves recalling terminology and definitions; recognising mathematical objects as equivalent (for example ½ = 0.5); classifying objects by some common property; and it also involves procedures for computing and measuring. The application domain involves solving problems in contexts of pure or applied mathematics with an emphasis on familiar and routine problems. It includes determining and implementing strategies for problem solving, and representing mathematical ideas in figures, diagrams or relations that model the mathematics in the situation. The reasoning domain in TIMSS involves solving multi-step systematic problems in unfamiliar mathematical or applied contexts; and it includes justifying and generalising conclusions.

The aim of PISA to evaluate readiness for work and further study leads to a framework and test item design that does not need to take any consideration different curricula in different countries and school systems. TIMSS, in contrast, aiming to evaluate school from an inside-school perspective, needs to take each participating country’s curriculum into consideration. This gives PISA and TIMSS different relations to the participating countries’ curricula as illustrated in Figure 3.

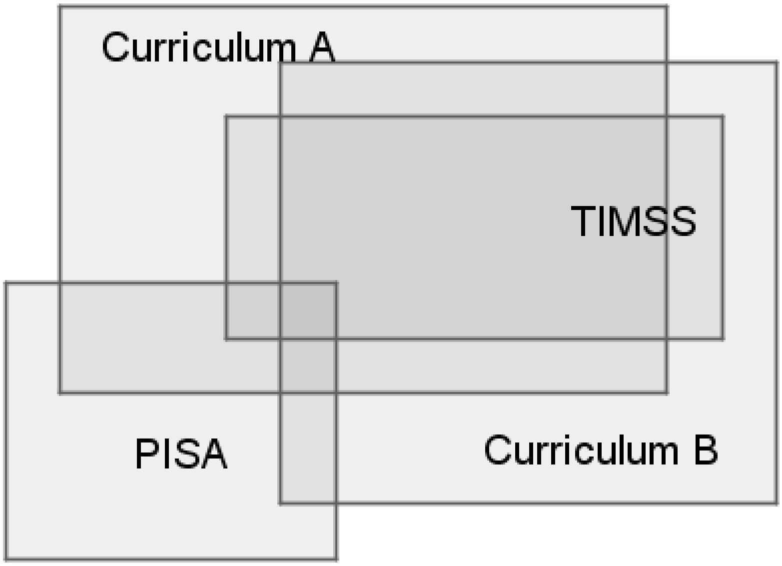


Figure 3. TIMSS's and PISA's relation to different curricula.

Figure 3 illustrates that TIMSS aims to examine the intersection of different curricula. For most participating countries TIMSS achieves this aim well (Lange, 2007). To take the example of Sweden, Skolverket (2006) found the Swedish mathematics syllabus and TIMSS to be essentially compatible. The test instrument for PISA is instead designed independently of participating countries' curricula. Since the present study aims to explore migrants' continued mathematics schooling in compulsory school in their new country, the curriculum focus of TIMSS is more relevant than the PISA focus, being readiness as citizens for work and further study.

One issue in international comparative education studies and a possible explanation of different achievement is that different school systems may have different policies for age at school start and for retention and/or faster promotion due to the student's school achievement, while others have promotion guided only by age of the student. The PISA studies sample by age; 15-year-old students which, for the reasons mentioned above, may be in different school years (OECD, 2013). Although TIMSS tries to cope with this by sampling in specific grades, this does not fully resolve the challenges of sampling. Kitchen (2000) used four countries for illustrating that the dispersions in age in TIMSS student samples are different for different countries. In for example TIMSS, the students are sampled from grades 4 and 8 (in TIMSS denoted populations 1 and 2). This still does not mean that all children in the sample have the same length of school experience, and there are several reasons for this (Mullis et al., 2012). In different school systems, primary school starts at different ages though usually between ages 5–7 and in some school systems children start their first school year in the month or the term of birth instead of the year of birth. This means that children in 4th grade may have different ages in different school systems. TIMSS has a policy of not testing very young children. Thus TIMSS selects students at a grade where the student cohort has an average age at least 9.5 years old for the 4th grade sample and 13.5 years old for the 8th grade sample. A consequence of this is that school systems where children start early may be tested in a later grade.

In secondary school, there are other challenges when it comes to identifying student populations, namely that students follow different programmes with different mathematics content. TIMSS resolved this by having one test ‘population 3 general’ on mathematical literary and another test ‘population 3 advanced’ on advanced mathematics. One outcome of this was a scattered image of countries, which achieved well in both mathematical literacy and advanced mathematics, countries that achieved well in one but not the other and countries that achieved poorly in both mathematical literacy and advanced mathematics (McKnight & Valverde, 1999).

The intended curriculum – the curricular level

In FIMS (the first TIMSS 1964), the differences between curricula of different participating countries were ignored when comparing the achievement outcome. This was later criticised (Freudenthal, 1975). Travers and Weinzveig (1999) described how this critique was dealt with by discriminating between curriculums as expressed in governmental documents (the intended curriculum), curriculum as expressed in the classroom (the implemented curriculum), and learning outcome (the attained curriculum). SIMS and the following TIMSS studies aimed to test mathematics that is to be found in most curricula of the participating countries (e.g. Beaton, Mullis, Martin, Gonzales, Kelly & Smith, 1996). Despite this, differences in the emphasis of different curricula are reflected in students’ achievements (the attained curriculum) in different mathematical content areas as illustrated in Figure 2 (p. 25). For example, in TIMSS 8th grade 2011, most of the participating countries in North and Western Europe in ‘data and chance’ achieved above their own overall average score with statistical significance. In contrast to this, most of the participating countries in Eastern Europe and the Middle East in ‘data and chance’ achieved below their own overall average score with statistical significance (Mullis, et al., 2012). There may be several reasons for the different observed achievement profiles between students from culturally different geographic regions; that in a specific mathematical content area they achieve significantly above or below their own average achievement (cf. Figure 2 p. 25). For example, Wu and Zhang (2006) reported that some countries do not include probability, since it is linked to gambling, which is forbidden in, for example, Islam, and that the Chinese curriculum did not include probability and statistics until 1999. As a comparison, Sweden included descriptive statistics but not probability already in its first national mathematics syllabus in 1962 (Skolöverstyrelsen, 1962). Probability was included in school years 7-9 in the second Swedish national mathematics syllabus in 1969 (Skolöverstyrelsen, 1969) and presently this is taught already in school years 1-3 (Skolverket, 2011a). Other examples are Son and Senk (2010), who studied how fractions are presented in (the intended) curricula and found differences between Korea and USA. Bessot and Comiti (2006) traced differences in teaching of vectors and graphs in France and Vietnam back to different curricular developments. Moreover, very few countries (e.g. China and France) have kept deductive geometry in grades 6–9 (Wu & Zhang, 2006).

Though Wu and Zhang (2006) call for curricula reforms through collaboration between countries, they also warn against uncritical adoption of other countries' curriculum formulations and textbooks, since views on teaching and learning are deeply rooted in the local culture (see e.g. Li & Ginsburg, 2006). Curricular reforms induced by changes in society and comparison with other countries must thus be adapted to the local educational conditions and values. A curriculum may also, at least temporarily, act as a goal document rather than a description of the status. For example, Van de Walle (2004) stated that the reform curriculum in the USA is still to be realised in a large majority of schools. The 13th ICMI study discussion document mentions that some mathematics educators express concern about impoverishment through mainstreaming of the rich variety in the world's different cultures of mathematics education, but also justify comparative studies with other arguments such as the following (Leung, Graf, Lopez-Real, 2006): Why there are differences in school systems and whether a certain educational system is competitive and effective in some sense. Mathematics education is in constant change due, for example, to changes in society and technology, and this suggests studying what we can learn from effects of these changes.

In the case of Sweden, Emanuelsson and Sahlström (2006) described the overarching ideology of the Swedish intended curriculum (Lpo94 at the time, now replaced by Lgr11) as 'one school for all', and that heterogeneous classes should be managed by individualisation. In other words, the Swedish school system can be characterised by small organisational differentiation but major pedagogical differentiation. Lindblad (2006) described the Swedish curriculum (Lpo94) in Bernstein's terms as weakly framed with respect to teaching (see Bernstein, 2000).

The implemented curriculum – the classrooms level

Since textbooks are a part of the classroom practice, this leads to the relation between textbooks and how the curriculum is implemented in the classroom.

A note is that, in Sweden, the local school decides what teaching material to use. There is no governmental sanction of textbooks. Instead the market economy, being the publishers and the teachers (customers) decide what counts as good teaching material. Internationally, there are differences between different geographic regions. For example, Park and Leung (2006) stated that “textbooks articulate what should be taught in the curriculum” (p. 237), and Wood (2006) reported that textbook dominates instruction at all levels in the USA although the textbooks may be oriented towards either traditional or reform pedagogy. Also in Sweden, textbooks to a large extent influence the classroom activities by both teacher and students (Johansson, 2006). The following are a list of examples of research projects on comparing textbooks in different countries. Fan (1999) found differences in how arithmetic is presented in textbooks from the USA and China. Yeap, Ferrucci and Carter (2006) found differences in arithmetical problems in textbooks in Singapore and America. Charalambous, Delaney, Hsu, and Mesa (2010) found differences in how fractions are presented in textbooks from Cyprus, Ireland, and Taiwan. Alajmi (2012) found differences in how fractions are presented in textbooks from USA, Japan and Kuwait. Cao, Seah and Bishop (2006) compared Australian and Chinese textbooks. There seem to be few studies on comparing the mathematics classrooms in the Middle East, from which a large proportion of Swedish immigrants come, with classrooms in western and northern Europe. However, the case of ‘East and West’ has been explored in detail. Park and Leung (2006) contrasted textbooks in the Far East (China, Japan, Korea) and the West (England and the United States) and found that textbooks in the Far East are better at delivering mathematical ideas, while textbooks in the West are better in motivating students to learn.

All the listed differences are not just a question of formulations in the textbooks. Li and Ginsburg (2006) found that formulations in curriculum and textbooks match societal norms. This means that a change in one calls for a change in the other. Zheng (2006) exemplified this with the idea of ‘problem solving’, which was imported from abroad into Chinese mathematics education. However, ‘problem solving’ was adapted into exercising strategies for problem solving, and thus strengthened the Chinese teaching tradition instead of changing it. This example illustrates the earlier mentioned phenomenon of views on teaching as deeply rooted in the local culture (Li & Ginsburg, 2006). A second example is from Wu and Zhang (2006). They stated that “examinations drive education” (Wu & Zhang, 2006, p. 185) and that examination is an important aspect of the implemented curriculum in particular in the Far East, where a mind-set is ‘to teach what will be examined’. This slogan is also reflected in the students’ choice of mathematics courses. According to Wu and Zhang, the large majority of students in China choose the course without calculus, since calculus is not a part of the entrance examinations. Moreover, calculators have previously been forbidden in examinations in China, so calculators were rarely used in education despite the fact that the written (intended) curriculum at that time encouraged this.

Moreover, international comparative studies have carried out surveys of students and principals. The surveys have given background information about the school, about the students' attitudes, beliefs and emotions as regards mathematics. From TIMSS surveys, Leung (2006) analysed variables such as students' and teachers' attitudes to mathematics, teaching style and teacher competence, and found that these variables cannot explain the differences in achievement between countries in Europe and North America compared with counties in the Far East. Leung concluded that there must be something other than these variables explaining the differences, and suggested the Confucian culture and values that are associated with it in the view on education. This means that it is not only the choice or emphasis on different mathematical content in textbooks or examination practices that differs between countries. It is also that the teaching style differs between countries (Andrews, 2009; Cogan & Schmidt, 1999; Ma, 1999). In the (first) TIMSS video study, classroom activities were video recorded in Germany, Japan and USA. The recordings showed differences between the countries in the implemented curriculum (Kawanaka, Stigler & Hiebert, 1999; Stigler & Hiebert, 1999). Schmidt et al. (1996) described typical patterns of pedagogical processes as 'characteristic pedagogical flow', and found these to be country-specific.

Another study was the LPS – Learner's Perspective Study (Clarke, Keitel & Shimizu, 2006a). While the TIMSS video study recorded single lessons by many teachers, the LPS instead recorded several lessons by each (of a smaller number of) teachers and with several participating countries. While the TIMSS video study essentially focused on the teachers, the LPS also recorded the students in the classroom with the aim of studying the teacher-student interaction in different countries. One outcome of the LPS was that countries that achieve similarly in international comparisons may have very different classroom characteristics. This suggests that there is not a single best way to teach. Rather "these untranslatable practices have a power as catalysts for discussion and reflection on the practice of our classrooms and the values that underlie them" (Clarke, Keitel & Shimizu, 2006b, p. 11). In the LPS, Emanuelsson and Sahlström (2006) carried out a study that followed up how the ideology of individualisation in the curriculum is put into practice in classroom interaction. They compared the classroom interaction with respect to two contrasting classes in Sweden. One class, denoted SW1, had a poor mathematics achievement, and parents were dominated by the working class or the unemployed. The second class, denoted SW2, had good achievements, and parents were dominated by the professional middle class. The outcome of the study was that class SW1 invested more time in the citizenship aspect of the mathematics curriculum, while the SW2 class spent more time on the subject content aspect of the mathematics curriculum. Such differences between subpopulations inside a country highlight the socio-economic background as one important aspect in shaping the mathematics classroom culture and leads to the next section.

Social groups in comparative education studies

When it comes to cultural groups of students, the social capital in the spirit of Bourdieu has shown to be important in explaining student achievement. Although social capital correlates with individual student achievement, it is important to bear in mind that there is no determinism, but an increased probability, in the correlation between social capital and achievement, (Noyes, 2008). The social capital is often operationalised as socio-economic status, which Secada (1992, p 626) defined as "…some combination of familial income, education and employment". However, in the same article, Secada complained, 25 years ago, about the lack of research on disadvantaged students. "Also, what became increasingly clear as I reviewed this topic was its marginal status relative to mainstream mathematics education research" (Secada, 1992, p 654). The next edition of the NCTM Handbook wrote on the same topic that "…over the past 10 years studies in this area have gained prominence…" (Diversity in Mathematics Education Centre for Learning and Teaching, 2007) and gave two reasons for this development. One reason was the 'social turn', which paid attention to the role of culture in learning in educational research; for example, situated cognition, cultural historic activity theory. A second reason was the development of advanced quantitative models such as hierarchical linear modelling that, in large scale studies, can model complex relationships of social structures, which are difficult to study with qualitative methods.

The role of social capital has also been observed at a classroom level. Zevenbergen (2001), like Emanuelsson and Sahlström (2006), compared the classroom interaction in two classrooms, where one classroom had students with high socio-economic background and the other classroom had students with low socio-economic background. Zevenbergen collected data in a yearlong ethno-methodological study from classrooms in two socially different areas. In her data, she chose to analyse triadic dialogues, which consists of the three turn-takings teacher-student-teacher, as shown in Table 1 (see e.g. Wells, 1993). There were, of course, both longer and shorter dialogues in her data but, although triadic dialogues do not constitute a high-level dialogue, there was an interesting pattern in the results. In the working-class school, the triadic dialogues were often on declarative knowledge and in the middle-class school more often on constructing knowledge, as illustrated in Table 1.

Zevenbergen's conclusion was that linguistic habitus can be used as symbolic capital and as such be exchanged into academic success. Thus linguistic habitus is one variable that in a socio-economically segregated school might work on the classroom level.

Table 1. Zevenbergen's triadic dialogues.

Turn taker	Low SES	High SES
Teacher initiates	What is the name of this geometrical figure?	What similar and different properties have these geometrical figures got?
Student(s) respond(s)	Student's response in one word is possible.	Student's response in full sentences is expected.
Teacher follows up	"Correct" or "Wrong".	"Correct" or "Wrong".

Migrants and multilingualism in comparative education studies

Besides socio-economic categories, cultural groups have also been categorised in terms of migration status and first or second language speakers. In particular, not all second language students may master linguistic habitus at a high level in their second language, though Clarkson (1992) gave counter-examples to this. Moreover, migration status and being a second language student may sometimes go hand in hand with having a lower socio-economic background (Hansson & Gustafsson, 2011; Hansson, 2012). In education statistics, both migration status and language status are used for categorising people. These two concepts are related, but they are not equivalent, as illustrated in Figure 4.

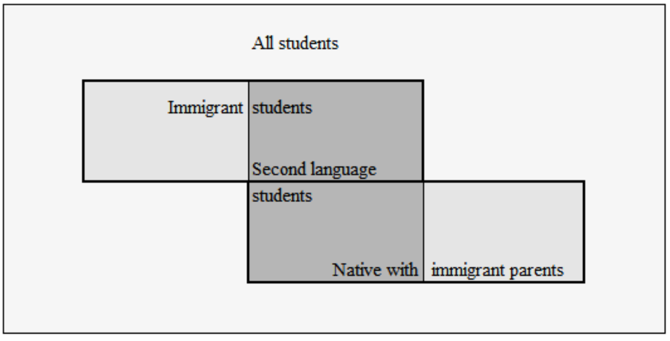


Figure 4. The relation between language status and immigrant status illustrated with a Venn-diagram.

In countries where skilled immigration is encouraged, the differences in PISA 2003 were smaller, while in many European countries, the achievement differences were larger (OECD, 2006). On a more detailed level, the achievement gap between native and immigrant students was smaller for immigrants coming from countries that on average achieved well in PISA 2012 when compared with immigrants from countries that on average achieved less well (Giannelli & Rapallini, 2016). Now migration itself may not always be a problem for a student. Consider, for example, a Swedish-speaking student that migrates from the Swedish-speaking part of Finland to Sweden. This student will transit into a new school system but much will still be similar, such as language, culture and probably the student's social capital. Similar migration situations are possible in parts of Europe where, for example, one language is spoken in several countries, as is the case with French, German and some other languages. There are also students who are children of immigrants that have found a new job in some other (or the same) country with another language. These students become second language students, but they are likely to have only a short, if any, interruption in their schooling due to the migration, and it is likely that there is time for planning the school transition. Moreover, they are likely to keep their socio-economic background, since their parents probably stay within the same profession.

The case for immigrant students that are refugees might be different (Lindqvist, 1991). Some may have traumatic experiences from the country of origin or may presently experience economic stress due to their parents being unemployed and this may affect their school achievement (Hjern & Angel, 2000; Rousseau, Drapeau & Corin, 1996). They often have a period of interruption in schooling during the migration process. This interruption may be in addition to the language being new for the students; and that the school system might be different. Moreover, the students' parents might have been established in the employment market and having an appropriate education in the country of emigration, but may now find themselves in a different situation in the country of immigration.

The case for the parents of the immigrant students might be different. While an immigrant from within the EU might choose to move after they get a new job, immigrants from outside the EU first move and then start the process of finding a job. Socialstyrelsen (2016) describes that it may take some time for those having a degree in education or medical care to learn sufficient Swedish; get their education validated and in some cases complemented with further education; they also need a complementary education in the Swedish system of school and medical care respectively. The parents of immigrant students may for these reasons be outside the employment market due to not knowing the language, not having relevant education or due to discrimination in some staff recruitment process.

Therefore, immigrant students may have changed from high to low socioeconomic status due to migration, at least for a period of establishment in the new situation. The situation of immigrants and second language students has been highlighted in large-scale studies such as PISA and TIMSS. For example, OECD (2006) in PISA 2003 observed statistically significant achievement differences between native students and students with a migrant background. A study similar to that of Emanuelsson and Sahlström (2006) is Hansson (2010; 2012). Hansson used advanced quantitative modelling of classroom culture using Swedish TIMSS 8th grade 2003 data with a focus on multilingual classrooms. Her research questions focus on how the teachers' responsibility for students' learning is put into practice in terms of whole class instruction, individual student working and mathematical content. The result was a positive correlation between student achievement and whole class instruction. There was also a correlation between more individual student working and more students having an immigrant background or low socioeconomic status. This means that these students got less teacher support, though they could be expected to be in more need of teacher support. This can be interpreted as a segregation effect between different mathematics classrooms in Sweden. The possibility of segregation in classroom activities is enhanced by a residential segregation. Skolverket (2004, p. 45) found that, for schools with a high proportion of students who are immigrants or who are native, but with both parents who are immigrants, it followed that the parents, of both native speakers and immigrant students, have a shorter education on average. Moreover, with the shorter education of these parents followed, on average, lower school achievements of the students. Since most schools enrol students from the local neighbourhood, this can be interpreted as a residential segregation effect. Another potential school segregation effect is that schools with a high proportion of these students also have a slightly lower ratio of qualified teachers and slightly higher ratio of turnover of employed teachers. However, Skolverket (2004, p. 57) could not differentiate between such potential teacher effects at school level and the larger residential effect. Skolverket (2004) based the results on a regression model for socioeconomic status and school achievement. The school achievement was the dependent variable (output variable) and was measured as the average leaving grade in school year 9, being the last year of Swedish compulsory school. Independent variables (input variables) were parents' education, income and employment rate and number of guardians in the students' home. Independent variables were also the four following student categories.

- Students with Swedish background; defined as students born in Sweden and having at least one parent born in Sweden.
- Students with immigrant background; defined as students born in Sweden and having two parents born abroad.
- Students that immigrated before the school start.

- Students that immigrated after the school start.

The distinction between students that immigrated before and after the school start allows the identification of specific properties in school performance among these two student categories. When the socio-economic background was accounted for, this regression model could explain the differences in school achievement between students with a Swedish background and students with an immigrant background and between students with a Swedish background and students that immigrated before the school start. In contrast, this model of socio-economic background could not explain the differences between students with a Swedish background and students who had immigrated after the school start. Böhlmark (2008) could confirm that the later the students had immigrated, the lower were their leaving grades on average. However, he found this effect less pronounced for mathematics. To control for social background, the study by Böhlmark was done as a sibling study, where the school achievement of the older and younger siblings (both immigrants) were compared with age at immigration. Most of the immigrants in Böhlmark's study were also second language students. To give a more detailed picture, Böhlmark also used geographical continent of emigration as background information and found that students that have immigrated from outside Europe and North America achieved less well than other immigrants. There is also a geographical component in Xenofontos (2015). He interviewed teachers, who mentioned that students who had emigrated from different parts of the world, also achieved differently. The experience of these teachers is in line with the results of Giannelli and Rapallini (2016) mentioned earlier. Elmeroth (1997) found a similar result for students with both parents born abroad. She used parents' migration status as a background variable and compared students having both parents born abroad with other students. She found that those with parents born inside Europe were similar to other students in terms of attitude to and achievement in school. In contrast, students with parents born outside Europe had a more positive attitude, but achieved less well. Elmeroth added a socioeconomic component. For students with parents born inside Europe, the parents were often skilled, industrially or academically, while for students with parents born outside Europe, the parents more often had a shorter education.

Pásztor's (2008) target group was second-generation Turkish immigrants in Belgium, Austria, Germany and Switzerland. A second-generation immigrant is in PISA defined as a person, both of whose parents are immigrants. The dependent variable was mathematics achievements in PISA 2003. Pásztor compared three successively incremental regression models.

- Model (1) only used migration background as an independent variable.

- Model (2) added socioeconomic status and gender as independent variables to model (1).
- Model (3) added school variables as independent variables to model (2).

The school variables were attendance in kindergarten, repeated school years and tracking (vocational schools). Pásztor found that, of the three models, model (3) gave the best fit, and thus concluded that the three variables of migration, socioeconomic background and school all play a role in mathematics achievement. Pásztor concluded that socioeconomic variables are not enough to explain the differences in mathematics achievement; school variables are also important. However, one possible inferential weakness is that Pázstor only mentioned the R-square analysis and not the adjusted R-square analysis. For the (non-adjusted) R-square analysis, the model fit will increase irrespective of the relevance of adding further explanatory variables.

Ufer, Reiss and Mehringer (2013) collected linguistic, socioeconomic and cognitive data as background variables from German students and their parents. From a parent questionnaire, the students were categorised as monolingual and bilingual (including multilingual). The students took tests at the end of both first and second school years in German language proficiency, in general cognitive proficiency and in mathematics proficiency. In their regression model, mathematics achievement was the dependent variable. Independent variables were general cognitive proficiency, socioeconomic status and language proficiency. Between the student categories there were only small differences in mathematics achievements for algorithmic tasks, but large differences for conceptually demanding tasks. In the test in the first school year, socioeconomic status had some explanatory power, but not in the second school year. Ufer et al. (2013) interpreted the result as if the language proficiency early in school had an influence on the mathematics acquisition. They also saw a danger that the algorithmic proficiency of all students can create an illusion of competence for the teachers, especially for students that are weak in the language of instruction. The results from the statistical modelling of Ufer et al. (2013) are in line with a modelling in terms of linguistic habitus by Zevenbergen (2001). The research referred above (Böhlmark, 2008; Hansson, 2010, 2012; Hansson & Gustafsson, 2011; Skolverket, 2004; Ufer et al. 2013; Zevenbergen, 2001) suggests that, besides social capital, one key factor in school success in mathematics is access to linguistic capital.

Summary and a working hypothesis

The present study aims to generate more knowledge about newly arrived immigrant students compared with early arrived immigrant students and first language students in the mathematics classroom. Comparative studies point out that one country may have teaching traditions and goals that are different compared with those of some other country. Given the research referred to above, comparative studies are interesting for this thesis in at least three aspects. One aspect is that immigrant students may bring different experiences of mathematics education when it comes to both knowledge (Giannelli & Rapallini, 2016) and attitudes. For example, as noted in Mullis et al. (2008), students in different countries may on average have different achievement profiles as illustrated in Figure 2 (p. 25). A second aspect is that students, who move between countries, might become second language students. A third aspect is that, irrespective of whether these migrating students are children of skilled or non-skilled immigrants, their socio-economic status might be changed. For a newly arrived immigrant, the three aspects of changing teaching and learning traditions, changing school language and changing socio-economic status might interact in a way that, on average, have consequences for their possibilities to learn effectively in the mathematics classroom.

From the research referred earlier we can sum up that differences between the two categories of first language and second language students respectively have been observed in both testing and in the students' background. However, it seems that much less has been explored in mathematics education when it comes to the area of that immigrated students may have followed a curriculum different from that of the new country. In the previous research, comparative research has found the following factors to vary between countries: curricula, mathematics classrooms and total achievement as well as content-specific achievement. There have been studies on immigrant students' total achievement in school mathematics. In those studies, with few exceptions, such as Clarkson (1992), the immigrants have on average achieved less well than the native students. The common explaining variables for the lower achievement are those of having a lower socio-economic status and of being a second language learner. One observation is that studies discussing experiences of different curriculum as one part of explaining the achievement differences were not found though a few added a geographical component (Böhlmark, 2008; Elmeroth, 1997; Xenofontos, 2015).

One tentative research question that emerges out of this is to ask about differences in mathematics achievement profile (see Figure 2, p. 25) between students with experiences from following only the Swedish curriculum and students with shorter or longer schooling following a curriculum, specifically a mathematics syllabus, different from that of Sweden. A working hypothesis could be that immigrant students with longer experiences of being taught within some curriculum different from that of Sweden may have an achievement profile different from that of students that have only followed the Swedish curriculum.

A definite research question

The research question below stems from the observation in Figure 2 (p. 25) that students in different countries on average may have different achievement profiles in mathematical content areas. In Sweden, a large proportion of the immigrants are from outside Europe and many of these are second language students in Swedish schools (see Figure 4, p. 33). Some of these countries have achievement profiles with a different shape from that of students in Sweden as illustrated in Figure 2 (p. 25). Now the research questions for part one in this thesis are on achievement profiles:

> Given that first and second language students in Sweden on average achieve differently, what do these average differences look like at test item level? What do the achievement profiles at a mathematics content area level look like for first and second language students?

The research question on achievement profiles can be said to be driven by the data format in the sense that it is closely connected to achievement in written tests as described earlier in the comparative research. This research question guided papers I and II in the present study.

Methodology, part 1

Introduction

Several mathematical jokes are not only funny. They also highlight important epistemological properties of mathematical and other sciences. One example is the following from < http://www.sgoc.de/math.html >.

> A biologist, a statistician and a mathematician are on a photo-safari in Africa. They drive out onto the savannah in their jeep, stop and scout the horizon with their binoculars.
>
> The biologist: "Look! There's a herd of zebras! And there, in the middle: A white zebra! It's fantastic! There are white zebras! We'll be famous!"
>
> The statistician: "It's not significant. We only know there's one white zebra."
>
> The mathematician: "Actually, we know only there exists one zebra which is white on one side."

This mathematical joke occurs in several versions. In another version, they see a green sheep in the Scottish countryside. This joke highlights the differences in how to infer knowledge within different ontologies. In the qualitative research tradition, the ontology for making convincing arguments is logical reasoning that connects empirical results with the conclusion. In the quantitative research tradition, the convincing arguments are instead probabilistic reasoning that connects empirical results with the conclusion. When we observe some phenomena by interview, (classroom) observation, enquiry or assessment tests, we can infer that the phenomenon exists. For example, consider the following test questions:

> How many minutes is 0.75 hour?
> What is half of 1/5?

Suppose student A solves both the problems using decimal representations 0.75x60 and 0.2/2 and student B solves both problems using fraction representations (60/4)x3 and (1/2)(1/5). Similarly to the mathematician in the joke above, we can infer the existence of representation change from fractional to decimal by student A and from decimal to fractional by student B. But under which conditions can we adopt the biologist’s role and infer that student A prefers decimal representation and student B prefers fraction representation from only two observations of test problem solutions? Let us assume that student A is a newly arrived immigrant and student B is native speaking student. Under which conditions can we adopt the statistician's role and infer that decimal representation is more common in one student category than another? Such and other methodological issues will be discussed in this chapter.

Rationale for the method

The method in papers I and II was the following. Data collected were students' background categorised as being first or second language student and students' achievement on test items in Swedish national tests in mathematics for school year 9. These data were secondary data originally collected by the PRIM-group at Stockholm University. The PRIM-group is a research group with focus on assessment of knowledge in mathematics. The PRIM-group has been given the commission for devising, constructing, implementing and evaluating the national tests in mathematics for compulsory school. The national tests in mathematics in Sweden are taken at a day specified by the National Agency for Education, who also specifies the means that the test taker is allowed to use, such as calculator and formulae sheet. During the tests the students cannot ask questions, with the exception that second language students can get help from the teacher with translating words or explaining difficult words without revealing the mathematical content of the test item. The tests are taken in the local school of the students and are supervised and assessed by the local teachers. In papers I and II, the achievement of first and second language students was compared using statistical methods. The following sections discuss practical, theoretical and ethical matters on collecting information about the students' background and achievement; choice of test instrument; defining achievement profile; and statistical tools for comparing achievement.

Categories for the students' background

In the present study, the students were categorised into the two disjointed groups of first and second language students. Norén and Björklund Boistrup (2013) argued for the necessity of researchers to categorise students in order to learn about mathematics classroom practices. They suggested to use categories that already exist in public and official documents, but also to see the categories as non-permanent and context dependent. The latter is important not least for the fact that a beginner second language student may in time become a highly proficient user of the second language. Given the research question on achievement profiles, a student's language status should reflect the student's school situation. In Swedish schools, all students are assigned to follow one of the courses 'Swedish' and 'Swedish as a second language'. The decision is made by language experts and regulated by the governing school act (Grundskoleförordning, 1994; Skolförordning, 2011). These laws give the following two criteria for being assigned to 'Swedish as a second language' course. One criterion is that the student has a mother tongue other than Swedish, which may include both immigrants and Swedish native children of immigrants and national minority language groups. The other criterion is that the student is considered to need support in Swedish language development. Both these criteria must be fulfilled for being assigned in order for the student to follow the course 'Swedish as a second language'.

Ethical aspects on collecting secondary data

What ethical aspects are there when collecting information about students' backgrounds? Just as collecting data via observations or interviews is to collect data from people, so is collecting data via a survey or test or database. That is an ethical matter. The Swedish law on ethics for research on people §10 (SFS 2003:460) is based on the ethical standards by the Swedish Research Council and requires the researcher to consider alternative research designs interfering less in the informant's integrity. In papers I and II, second-hand data was used. The data were originally collected in a random sample made by the PRIM-group for evaluation purpose. From the Swedish National Agency for Education (being the Swedish governmental organisation for education), the PRIM-group at Stockholm University has been given the commission for constructing, devising and implementing and evaluating national tests in mathematics for compulsory school. The data used in paper II contains no information other than each student's achievement per test item and whether the student in school is a first or second language student. In paper I, a sample of teacher-assessed test was inspected. This sample was also collected by the PRIM-group. Else the students are anonymous and the PRIM-group has an ethically based integrity-defending policy for collecting, storing and re-distributing data about individuals. One ethical argument is that tests, particularly through secondary data, reduce the intervention in the classroom. This is in line with the policy of using research designs interfering less in the informant's integrity (SFS 2003:460).

Choosing data format

What data format to use for observing achievement profiles? There are several reasons for using written tests for measuring achievement profiles. With the formulation of the research question on achievement profiles follows that the students' achievement is in focus. Tests are common tools for measuring individual achievement, for example, in national tests and in international comparative studies such as TIMSS and PISA. In papers I and II, the choice was to use test results from the compulsory national test in mathematics as a secondary data source. To use secondary data saves time and effort in the design work, since the test items in the national test have been piloted by the PRIM-group. It also saves time and effort in collecting and assessing the primary data. These and other aspects to consider when choosing test results as data, are discussed in the following.

When using tests, there are (at least) three aspects of validity that should be considered (e.g. Dager Wilson, 2007). These are the following. The design of the test instrument reflects the research questions. The students are familiar with the test. The equity with respect to test item language and test item context is considered for students taking the test. These aspects are discussed below.

Research questions and test design

We start by discussing the first aspect of validity, which is how the design of the test instrument reflects the research questions. The achievement profiles in Figure 2 (p. 25) were based on results from the written test in TIMSS (Mullis et al., 2012). To give an answer to the research question on achievement profiles comparable with the original TIMSS result, one way is to use a test comparable to those of TIMSS as research method. Now, the Swedish national test in mathematics in school year 9 is compulsory and has three written parts B1, B2 and C. Of these three, part B1 covers the widest range of mathematical content knowledge areas and in this aspect better than part B2 and C of the national test reflects the research question on achievement profiles. A more practical argument for using a test, specifically a national test, is that observing students' activities during lessons could give detailed information but is time consuming. The same holds true for clinical interviews (Hurst, 2008; Goldin, 1997). One limitation in using secondary data from national tests is that the original test item formulations are often under secrecy.

For the research question on achievement profiles, access was needed to the formulations of the test item. Thus, the sample of tests was restricted to those that were made public. The secrecy for the national tests lasts some period of time, and is governed by the law on official and secret acts (SFS 2009:400). Similar conditions apply to PISA and TIMSS. There are at least two reasons for this (Downing & Haladyna, 2006). One reason for the secrecy is for comparative purposes. It makes it possible to evaluate changes over time within the same school system or after changes within the school system, by comparing the achievement of two student cohorts. Another reason is the following. It takes time and effort to construct the test instruments, especially non-standard problems, to explore them in different student groups, and to evaluate and probably modify the test instruments before they are ready for use. For these reasons, the test problems have a large economic value and are re-used a few times before they are considered as being “consumed” (Lundahl, 2009).

Students’ familiarity with the test

A second aspect of validity is the students’ familiarity with the test instrument. Some aspects of this area are the required format for the responses and the mathematical content of the test items. There are also test conditions such as time length and use of calculators.

As concluded earlier, part B1 in the national test is the part that fits best with the research question on achievement profile. In part B1, the format of the responses is typically a numerical value and in few cases an algebraic expression or a point in a coordinate system and the students do not need to show any calculations or give any justifications for their responses. Since much of school mathematics ends with a numerical value or an algebraic expression, this format as such is familiar for all students. Also the mathematical content should be familiar since part B1 in the national test covers a subset of content included in the mathematics syllabus. The test time for part B1 and B2 together is 80 minutes of and the students are recommended to spend no more than 30 minutes on part B1. In the aspect of test time, part B1 is not different from an ordinary classroom test. Moreover, calculator is not allowed on part B1 and the students should be familiar with this since the mathematics syllabus states as a goal that the students can do calculations both with and without calculators.

A more psychological aspect is that from the students’ perspective, the national test might be seen as high stake tests, since the national test is the largest mathematics test that the students take. Dager Wilson (2007) defined high stake tests as tests whose results are used for making decisions that have serious consequences for the student, teacher, school or school system. For the Swedish context, this wide definition allows any classroom test (including the national tests) to be a high stake test for the individual student, since classroom tests constitutes one important tool for assessment and thus for determining grades for individual students. On an intermediate level, the national tests have a role as high stake tests for the teacher and the local school as an evaluative tool, while international tests such as TIMSS and PISA might be high stake tests for comparing different school system on a national level, but not for the (individual) student or school. Since there are at least psychological reasons for the students to see the national tests as high stake tests, schools typically prepare the students to make them familiar with what the test is like in terms of test conditions and also spend some time on reviewing the mathematics content.

Equity for students taking the test

The third validity aspect is on equity for different student categories taking the test. Dager Wilson (2007) wrote that differences in achievement between two student categories can reflect actual differences, but could also reflect construct irrelevant differences. The latter can be caused by issues in language or in particular problem contexts (Campbell, Adams & Davis, 2007). As mentioned above, the Swedish national test in mathematics in school year 9 has three written parts B1, B2 and C. Of these written parts, part B1 has the lowest correlation between the students' mathematics achievement and the students' reading ability (Skolverket, 2009; Skolverket, 2010). Moreover, part B1 when compared with other the parts of the national test typically has direct questions and few real life application contexts. Together, these aspects of the second language students' reading and understanding problem texts and possibly not so familiar problem contexts made part B1 of the national tests in mathematics suitable as a test instrument for papers I and II. In addition, the students may ask teachers about, for example, translating words, but the teachers do not answer mathematical questions.

Defining achievement profile

The research question on achievement profiles asks for comparing achievement profiles for different student categories. One subsequent question is: How to define a measure of achievement profile? Olsen (2006) used as mathematics achievement profile, the test item achievement minus the total average achievement. TIMSS uses the same definition, but applied to mathematical content areas (Mullis, et al., 2012). Another suggestion, inspired by Figure 2 (p. 25), is to define an achievement profile building on the shape of the achievement distribution per mathematical content areas. There are several ways to model a quantitative definition of achievement profile. It could be a difference or a quotient of achievement between two specified mathematical content areas or groups of content areas as suggested in the bullet points below.

- (Algebra achievement) – (Number achievement)
- (Algebra achievement) / (Number achievement)
- (Group of content areas) – (Other group of content areas).

With achievement numerically defined as a proportion (percentage) of correct responses, a draw-back of a quotient between achievements, in two content areas, is the risk of a zero, or near-zero, in the denominator. For this mathematical reason, the difference is more robust, since it will always return a number in the interval [-1, 1] where the sign gives the direction. Moreover, it is preferable to compare one single content area with one other for the following reason. For the case of Sweden in Figure 1, suppose the content areas are aggregated as (algebra + data and chance) and (number + geometry). The differences between these two groups would be 982 – 979 = 3. This difference is tiny despite that the difference between, for example, algebra and number is much larger than that. To sum up, in paper II, the first bullet point above was used for comparing achievement profiles.

Comparison and statistical inference source

When comparing for example achievement, one common tool is inferential statistics. Here the achievements of the first and second language students were normal approximated. Now the two student categories can be compared using a hypothesis test. An outline for a hypothesis test is the following. Formulate the null hypothesis H0 as ‘there is no difference between the two student categories’. The alternative hypothesis H1 is that there is a difference between the two student categories. For a normally distributed variable, the test is to measure the size of the difference between the achievements of the two student categories. If this size is sufficiently large, then the alternative hypothesis holds, i.e. the difference is real and not just due to random variation. If not, then we cannot rule out the null hypothesis, i.e. it is possible that there is no difference. A complication in our case is the need to correct for repeated hypothesis testing. Suppose a hypothesis test gives the outcome that the null hypothesis is rejected at the chosen significance level 0.05. What this means is that, if there was in fact no difference except for random variation, then the risk of the test erroneously concluding that there is a difference, is 5%. If this process is repeated, there is an increased risk of concluding there is a difference when there is no difference. To compensate for this increased risk, there is a need of a correction of the significance level. A correction which is simple to derive is the Šidák correction (Šidák, 1967). For the 18 tests made in paper I and significance value 0.05, the corrected significance value, using the Šidák correction is $1-(1-0.05)^{(1/18)} \approx 0.0028$. However, a condition for using the Šidák correction is that that the 18 test items are considered as independent, which may not be the case. A correction not requiring independence is the Bonferroni correction. This correction is more difficult to derive, but easy to apply and is $0.05/18 \approx 0.0028$ in this case, which when rounded off is the same as the Šidák correction. For this corrected significance value, the critical value for the normal distribution is u=2.77 and the null hypothesis could be rejected if the test variable has a value larger than this.

For paper II, the achievement profiles were compared using a t-test. With the definition of achievement profile in the first bullet point above, the first research question, used in paper II, can be stated as a statistical hypothesis testing as follows. Hypothesis; the achievement profile (number achievement minus algebra achievement) is the same for first language students as for second language students. If this hypothesis is correct, the difference between the achievement profiles for first and second language students should be near zero. Stated this way, the t-test in statistics can test the hypothesis as described in paper II.

Summary of papers I & II

Paper I

Paper I compared first and second language students with respect to achievement on test items given in the national test in mathematics 2009. In brief, the result was that for typical arithmetic test items; the achievement differences were small between the two student categories of first and second language students, while the achievement differences were large for test items that, either as concept or as context, occur rarely in mathematics textbooks.

The data analysed in paper I consisted of results from the mandatory national test in school year 9, given 2009. The data was collected by the PRIM-group as a random national sample for evaluation of the national test. The data consisted of one batch of copies of teacher assessed student responses to the tests and another an achievement data batch of test results with credits per test item and students background as first or second language student.

The achievement data batch of credits per test item gave information on for which tasks the achievement differences between first and second language students were small or large. Since the achievement per student category is a sum of ones and zeros, a normal approximation of the achievement was made for the comparison of the two student categories. As there were achievements on 18 test items to compare, Bonferroni correction for multiple significance tests was used.

The test items were categorised after their mathematical conceptual content and after test item context using the functional model of Rico (2006, see also Sáenz, 2009). One outcome was that, for arithmetic test items in an intra-mathematical context (as opposed to an applied – extra-mathematical context), there were only small differences in achievement between first and second language students. It was notable that the two test items for which the first language student achieved on average the lowest, the second language students achieved on average the same for one and slightly better for the other. Among test items with moderate differences were those on equations and proportionality. A main result was that test items with large and statistically significant differences between first and second language students, had in common that they rarely occur in textbooks or non-school mathematics. Examples of such test items are determining the median, working with decimal hours and using the numerically given scale '1:X' on a map for calculating a distance. The case of subtracting negative numbers is special in the sense that this mathematical content in textbooks may occur as an 'honours course section' or even a separate 'honours course textbook', which not all students follow. This, together with mathematical concepts that occur only rarely in textbooks, may create a learning environment in which not all students have access to all the mathematical content. It may be that immigrants in school have a greater risk of not having access to these learning opportunities, for example, due to immigrating after the topic was taught. It may also be that that second language students have immigrated shortly before the topic was taught, when they were still novices in the language, and for that reason could not successfully participate in the teaching of some specific rare topic. This would give teachers of immigrant second language students a special responsibility to make sure that these students are given access to learning also the rarely occurring content in the mathematics teaching.

The data batch of teacher-assessed student responses made it possible to obtain information on what kind of errors the students had made. For this purpose, Sfard's (2008) idea as used of endorsed rules such as when-routines and how-routines. One example was a test item on determining how many minutes is 0.75h (test item number 4 in part B1 of the 2009 national test in mathematics). Of the incorrect responses, some were incomplete such as '15 minutes'. Other incorrect responses were to make a decimal conversion '75 minutes' or '7.5 minutes' indicating that the responder viewed the task as similar to converting 0.75 metres into centimetres. Using Sfard's (2008) terminology, it seems as if these students had a when-routine of 'when there is a unit conversion, use the how-routine of decimal conversion'. With this interpretation, the erroneous response of decimal conversion can be linked to a mathematical error and not to a linguistic misinterpretation of the test item formulation. Another example was to determine the largest temperature difference in a table of positive and negative temperatures (test item number 2 in part B1 of the 2009 national test in mathematics). Some responses corresponded to subtracting the absolute values of the most extreme temperatures or subtracting the absolute values of the temperatures with largest absolute value. These responses, showing a how-routine of ignoring the sign of negative numbers also showed that the problem-solving student had linguistically interpreted the test item correctly but had made mathematical mistakes that resulted in an incorrect response. In other words, it was possible to exclude the linguistic formulation of the test item as a source of the incorrect response and instead identify the source as a lack of a specific mathematical knowledge. Success on this test item correlated with success on test items on equations in the following way. First and second language students who responded correctly to this test item achieved similarly on the test items on equations. However, the proportion of first language students who responded correctly to these test items was higher than the proportion of second language students.

Since the achievement differences were particularly large for concepts that rarely occur in textbooks, this suggests that not only language of the second language students plays a role, but also access to mathematics teaching. For example, the second language students may have had on average less access to participation in lessons on these infrequent topics. As mentioned above, reasons for this could be that they immigrated after the period, when the rare topic had been taught or if they had immigrated shortly before when they were still new beginner in the language and for that reason could not successfully participate in the teaching on some specific rare topic. Moreover, the uneven distribution of the achievements suggested that first and second language students might have different achievement profiles. In that sense, paper I was a catalyst for paper II.

Paper II

Both paper I and paper II used data from the Swedish national tests in mathematics, though in different ways. While paper I found that the differences in achievement of first and second language students were unevenly distributed over the test items, paper II took this one step further. Paper II set out to explore whether Swedish first and second language students have different achievement profiles measured as shape of the achievement distribution in different mathematical content areas. Paper II found that this was the case and also suggested that one possible explanation for this is that newly arrived immigrants might bring knowledge and experiences from being taught in some curriculum different than the Swedish one. The arguments in paper II essentially are based on Swedish immigration statistics and achievement profiles on the school year 9 national test in mathematics, where the test items have been classified according to the mathematical content areas of TIMSS 8^{th} grade.

Using data from Statistics Sweden (2016), paper II showed that a majority of those who had immigrated in school years 7–9 are from the Middle East and Eastern Europe (including OSS). Several countries in these two regions show a mathematical content achievement profile in TIMSS different from Sweden's. TIMSS 8th grade studies measure mathematics knowledge in the four mathematical content areas of Number, Data and chance, Algebra, and Geometry (Mullis et al., 2008; Mullis et al., 2012). Different countries may achieve differently in these four mathematical content areas as is illustrated in Table 2. The last row in Table 2 illustrates the idea of different achievement profiles for different countries. In TIMSS 2011, for example, Russia achieved better than Sweden in both numbers and algebra, and Sweden achieved better than Turkey in both numbers and algebra. Thus, these three countries achieve on three different levels. However, the differences calculated in the last row of Table 2 can be used as a measure of the knowledge profile of the country. For example, Swedish students compared with themselves, showed a relative strength in the mathematical content knowledge area of number, while Russian and Turkish students showed a relative strength in algebra. This means that comparing the achievement profiles means comparing the shape of the distribution and not the absolute achievement level.

Table 2 TIMSS 2011 achievements of Sweden, Russia and Turkey (Mullis et al., 2012)

Country	Sweden	Russia	Turkey
Overall score	484	539	452
Number	504	534	435
Algebra	459	556	455
Difference Algebra – Number	-45	22	20

The method in paper II was to compare the difference calculated in the last row of Table 2 as a measure of the knowledge profile for Swedish first and second language students. As data students' results on released test items of the Swedish national test in mathematics 2007–2009 were used. In order to make the outcome comparable with the achievement profiles of countries in the Middle East and Eastern Europe these test items had been classified according to the TIMSS' framework of mathematical content knowledge areas. The comparison was made for the mathematical content knowledge areas of number versus algebra and data and chance versus algebra, since the test items in geometry were too few for a comparison. In absolute achievement, both first and second language students achieved less well in algebra than in number and in data and chance, but when the achievement profiles were compared, the second language students showed a smaller achievement difference between both algebra and number and algebra and data and chance than did the first language students. In other words, this means that the achievement profiles had a valley in algebra for both student categories, but the valley was deeper for first language students than for second language students. To explain the outcome, it was suggested that newly arrived immigrants have contributed to this. An argument for this is that a large proportion of the newly arrived immigrants come from the Middle East and Eastern Europe, for which TIMSS shows good results in algebra. A suggestion for further research is to differentiate between different mathematical content areas when comparing mathematics achievement of second language students and to differentiate between second language students who have immigrated in late school years and those who have not.

Part 2

Multilingualism

This chapter on multilingualism starts with a definition of multilingual mathematics classrooms and problematizes the concept of ‘first and second language’ as being dependent on the setting. This is followed by a brief overview of the following different theoretical models for describing different aspects of multilingualism. It ends with a summary and a suggested research question.

- Cummins’ (2008) threshold hypothesis modelled multilingualism as different levels of proficiency in both first (L1) and second (L2) language respectively.
- Drawing on Halliday’s functional linguistics, Pimm (1987) described mathematical communication as a topic-specific linguistic register.
- Seeing the communication in the mathematics classroom through a discourse perspective allows viewing the student as a participant (and not only an acquisitionist) making use of both linguistic and non-linguistic resources for a purposeful communication (Moschkovich, 2002).
- A student who is not sufficiently proficient in the language of learning and teaching may experience an increased cognitive load due to being a second language student (Campbell et al., 2007).
- Drawing on Bakhtin, there are unavoidable tensions in the multilingual mathematics classroom between linguistic homogeneity and diversity (Barwell, 2014).
- There have been different attitudes to and policies for multilingualism such as subtractive, additive and dynamic multilingualism (Garcia, 2011).
- Multilingualism occurs at different levels of the educational systems, such as curricular, school and classroom levels (e.g. Setati, 2005).

When it comes to the setting of European multilingual mathematics classrooms with respect to migration, Meyer, Prediger, César and Norén (2016) described there being ‘still much to do in research’.

Multilingualism – what is the problem?

Could you manage the following mathematical tasks given in North Sámi in Text 1?

Text 1, a mathematical task in North Sámi:
Rehkenaste cuovvovaš differánssaid!
93-38=
56-39=
65-29=

Text 2, a mathematical definition in North Sámi:
Primalohku lea lunddolaš lohku mii lea stuorit go 1
ja man sáhttá juohkit dušše ovttain dehe iežainan

Although you may not know North Sámi, you probably could guess the meaning of the third word in the first example and guess what the task is. If you could not, you could probably look at the mathematical symbols and guess what the task is, since the mathematical symbol language is essentially the same in countries sharing the same alphabet and probably even in, for example, Japan, where the Latin form of the Hindu-Arabic numerals is also used. Then, what is the problem with having a mathematics class in a second language? If you already know the mathematical concepts needed for solving the mathematical task, and can guess what the task formulation asks for, there is maybe not much of a problem. Did you understand Text 2 above? It gives the definition of a prime number in North Sámi and illustrates what the problem is for those who do not yet know for example the concept of subtraction or prime number in Texts 1 and 2 above; they need the concept explained in a language that they understand. This is a challenge even in a classroom where teacher and students share the same first language; and perhaps more so when not all participants in the classroom have one language in common that all are sufficiently familiar with for the purpose of the mathematics classroom communication (Schleppegrell, 2007).

What is multilingualism?

Multilingualism can be defined for individuals (e.g. Council of Europe, 2001; Garcia, 2011). But what makes the *mathematics classroom* multilingual? Barwell gave the following definition of the multilingual mathematics classroom:

> That is, mathematics classrooms are considered to be multilingual if two or more languages are used overtly in the conduct of classroom business. And mathematics classrooms are *also* considered to be multilingual if students *could* use two or more languages to do mathematics, even if this does not actually occur,... (Barwell, 2009a, p. 2) (Italics in original).

The first part of the definition is unproblematic, since a context in which at least two different languages are overtly used, is multilingual. The second part would include most Swedish classrooms, since English, being (the only) compulsory foreign language in the Swedish curriculum could be, but is only occasionally if at all, used in mathematics lessons. In that respect, Barwell's definition is too wide and needs a reformulation. There is a need for a definition that excludes the example of a possible but not actual use of English in Swedish mathematics classrooms, but includes the situations described by Lager (2006). Lager interviewed second language students, who said that, during a written test, they translated, though not overtly, the test items into their first language. Then they solved it and then translated it back into English and gave a response in the language of instruction, which was their second language. A possible reformulation of the second part of the definition above is to require that at least one individual is a second language student in the language of instruction, irrespective of the individual using their own first language or not.

Barwell et al. (2016) problematized the concept of being first or second language speaker. At home, a student may be a first language speaker. In a classroom, a student may be a second language student in the language of instruction and during breaks they may switch between being first language speaker or second language speaker in a set of multiple non-ordered languages. One example of this is Xenofontos (2015), who described a school where some students were first language speakers at home. In school, they were second language students, but the second language for the immigrant students varied with the context. In the mathematics classroom, their second language was standard Greek; during the breaks, it was instead a Greek-Cypriot dialect; and during the Turkish lessons Turkish was their second language. This may also hold true in formerly colonised countries where, say, French constitutes the lingua franca in the (mathematics) classroom, Arabic constitutes the lingua franca in the playground and the Berber language constitutes the first language at home. The situation in Sweden might be similar, with Swedish in school and some non-Swedish language(s) at home and possibly in the school playground. The classroom setting is multilingual also for deaf students, having sign language *and* some other written language(s) (Garcia & Sylvan, 2011; Healy, Ramos, Fernandes & Piexoto, 2016; Molander, Halldén & Lindahl, 2007). For example, in Sweden, deaf students are trilingual with sign-language plus written Swedish and written English. For the purpose of understanding multilingual classrooms, a set of theoretical models has been developed.

Multilingual proficiency modelled as a threshold

Cuevas (1984) and Parszyk (1999) noted that to learn mathematics requires skills in the language of instruction that second language students may not yet have become proficient in. Moreover, achievement studies such as TIMSS have noted that, in many countries, second language students achieve less well than first language students (Mullis et al., 2008; Mullis et al., 2012). TIMSS defined ‘second language student’ as when the student reports speaking the language of test at home never or seldom. Analogously, a student, who reports speaking the language of test at home often or always, is considered as a first language student. Australia is one exception to this pattern, with second language students on average performing better in TIMSS-studies than native Australian. In particular, Clarkson (1992) found bilingual students to not be a unidimensional group. Those who were competent in both of their two languages performed better than monolingual students despite the latter having better educational resources.

This phenomenon might be described by Cummins’ *threshold hypothesis* as a theoretical model (Clarkson, 2007). Cummins (2008) distinguished between BICS (Basic Interpersonal Communicative Skills) and CALP (Cognitive Academic Linguistic Proficiency skills). Cummins described the domains for BICS communication as cognitively undemanding and contextual with instant feedback via face-to-face communication including gestures, emotional expressions supporting meaning, and often spoken. In contrast, the CALP communication is defined as context-reduced (or de-contextual), cognitively demanding and often written school subject language. The border between the domains of BICS and CALP is diffuse and what communicative act that counts as one or the other may sometimes not be definitive. However, Cummins’ threshold hypothesis states that a bilingual student who is proficient in both languages on a CALP level benefits from it in school while one who has a low proficiency level in both languages experiences negative cognitive effects in school. In educational systems, Cummins’ threshold hypothesis has had the impact of creating educational resources that supports development of proficiency in the student’s first language.

A distinction similar to that of Cummins has been made by Bialystok in terms of balanced and unbalanced bilinguals (Bialystok, 1999, 2001; Bialystok & Majumder, 1998). A balanced bilingual has equal proficiency in both languages. Bialystok gave monolingual, balanced bilingual and unbalanced bilingual children non-verbal tasks designed to test selective attention. The selective ability consists in focusing relevant information where the tasks contain misleading distractors among the relevant information. Bialystok found that the balanced bilingual children performed better on these non-verbal tasks than the monolingual children. However, the advantage in ability of selective attention seems to disappear as children grow into young adults and their monolingual peers catch up in cognitive development (Bialystok, Martin & Viswanathan, 2005).

Multilingualism and mathematical registers

To move from BICS to CALP is much more than extending a vocabulary. Though Pimm (1987) does not see mathematics as a language, he showed that linguistic tools could still be successfully applied to mathematical communication. One of these linguistic tools is linguistic registers. A main idea in linguistic registers is that the same word may have different meanings in mathematics and some other subject-specific or everyday contextual communication. There are etymological reasons for this. Besides transliterating (importing without translation) or translating mathematical terms from other languages, one way of creating new mathematical words is to use metaphors. Consider the mathematical word *derivative*. The etymology for this word is de-river with the original meaning of off-flow (cf. the Greek saying 'panta *rei*' and Latin *rivum* = brook). In mathematics, this metaphorical meaning was used for expressing that a derivative flows off (derives) from the original function. Another example is the term 'right angle'. The etymology of the word *rectangle* is a *vertical/upright angle* (cf. the word Homo *erectus*), where the word upright is a metaphorical picture of what the angle looks like. But the Latin word *recte* also means *right* (opposite to wrong) and in English with meanings such as right (wrong), right (left) and right angle. Pimm gave an example of a child who invented the term left-angled as a mirror image of right-angled. Pimm denoted this case *semantic contamination*, with which he meant some erroneous concept usage due to other meaning(s) than the mathematical. It is noteworthy that, even within mathematics, several meanings are possible, which Pimm illustrated with the following question. What is the difference between 16 and 7? Pimm had observed different correct, but maybe unexpected, answers from children in different mathematical contexts. The difference could be that 16 is multi-digit and 7 is not; and that 16 is even and 7 is odd; and that 7 is a prime number and 16 is not; and that 16 is a square and 7 is not. So, even within the mathematical register, meaning making is a central activity when participating in mathematical communication.

Lager (2006) gave examples of how semantic contamination may cause added difficulties during a test situation for Spanish-speaking students who, in his study, were English as second language students. The setting in Lager's study was a written test followed up by interviews. Lager described the test situation as a double challenge, since the outcome of the test and the interviews following showed that the work of the students may have included a series of translations. First into everyday Spanish, then into Spanish mathematics register and after solving the problem translation back into everyday English and eventually writing the answer in formal English mathematical register. Besides vocabulary challenges, Lager found a number of semantic confusions of words in the mathematics register of the students.

As discussed earlier, having access to a mathematical terminology in the student's own first language could change how classroom mathematics is discussed and perceived by the students (Meaney, Fairhall & Trinick, 2008). Moreover, moving between registers and at the same time between languages could be a source of misconceptions in bilingual mathematics classrooms through semantic contamination, as illustrated by Lager (2006). This might be the case in mathematics classrooms without a well-developed mathematical terminology. For example, some indigenous peoples did not until recently have a culture of academically advanced mathematics, and may not have a rich mathematical terminology. Though Chichewa is one of the official languages in Malawi, Kazima (2007, p. 173) reported as late as 2007 that "…there is no official mathematics register in Chichewa." It holds for many African languages that the mathematics register is not well developed (Setati & Adler, 2000). As a comparison, the Swedish academy since 1874 publishes SAOL, a word list of standardised Swedish language, including many mathematical words, with spelling, declension etc. (Svenska akademien, 2015). Moreover there is a dictionary of standardised Swedish mathematics terminology in school (Kiselman & Mouwitz, 2008).

Kazima (2007) reported that Malawi students had difficulties in translating between English and Chichewa, since some mathematical terms in the English mathematics register did not match translations into the everyday meaning (BICS) of corresponding words in Chichewa. As an example, the word 'impossible' has a mathematical meaning in probability that is different from its everyday meaning in Chichewa, which is 'unlikely'. Thus, moving between registers, and at the same time between languages, could be a source of misconceptions in bilingual mathematics classrooms as in Kazima (2007) but also a resource as in Lesser and Winsor (2009) and Planas (2014).

As Pimm (1987) exemplified, mathematics communication can be expressed in a wide range of resources for representation. Duval (2006) systematized these into four registers for representations of which natural language (written and spoken) is only one; see Figure 8 (p. 87). The other three are symbolic computations (numerical and algebraic), iconic (drawing and geometrical figures) and diagrams (statistical diagrams and function graphs). In order to develop a theoretical model for describing communication in mathematics classrooms with language diversity, Prediger and Wessel (2011; see also Prediger, Clarkson & Bose, 2016) merged the three theoretical models of BICS and CALP by Cummins (2008); of representation registers by Duval (2006) and of mathematics registers by Pimm (1987). The resulting model is illustrated in Figure 5.

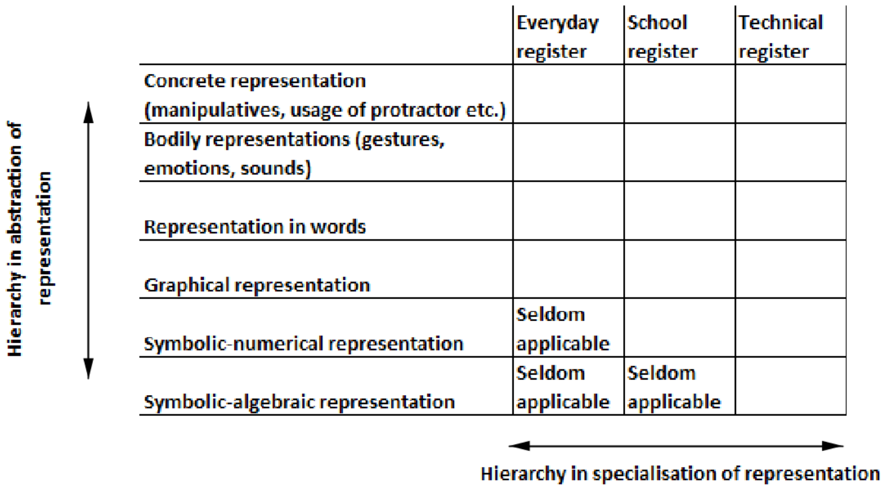


Figure 5. Representation registers with hierarchical dimensions (Prediger & Wessel, 2011; Prediger et al., 2016).

The boxes in Figure 5 are to be filled with representations of some chosen mathematical concept in different registers and representations in first and second languages of the classroom. The model in Figure 5 has three dimensions, where the dimension of first and second language has to be imagined by the reader. The other dimensions are; one dimension describing the symbolic abstraction in the mode of representation in (different modes of translanguaging) and one dimension describing the specialisation in the registers of representation. Concerning the hierarchy in abstraction level of the representation in Figure 5, Prediger et al. (2016) noted that its degree of abstractness depends on the mathematical topic. The everyday register corresponds to Cummins' BICS while CALP are split in school register and technical register. The technical register is characterised by being mainly intra-mathematical and unambiguous. Though the core of each of the three registers is clear, the borders between them are sometimes permeable and fluid. The school register is correlated with the socioeconomic background of the learners, and Prediger et al. (2016, p. 201) mention that the school register is sometimes viewed as a *condition for learning* rather than a *goal for learning*. This has consequences for the teaching design. Prediger et al. suggest that students, if they find the (school register L2) task text difficult, start by translating it into the everyday register in language L1/L2 of their choice and check their rewording with some peer student. The students go on with re-representing the text (in school/everyday register in L1 or L2) into a graphical representation and check whether the drawn representation matches the original and new text. Prediger et al. suggest this teaching model be further explored in empirical research. For teaching mathematics in the multilingual classroom, the model in Figure 5 is interesting, since it includes the non-linguistic representations discussed by Duval (2006). Specifically, a second language student with previous mathematics education is likely to quickly learn to master the non-linguistic representations. For example, several graphic representations such as diagrams, graphs and geometric shapes are recognisable irrespective the language background. Most languages use the decimal position system for number representation and only the ten digits and the decimal point may sometimes look different between two languages. The case is similar for algebraic expressions, since the only symbols used are numbers, arithmetic operations and some symbol for the unknown. When it comes to representation in words, the situation is more complex. According to Cummins' (1979) interdependence hypothesis, the CALP is transferrable between languages. Though Cummins' interdependence hypothesis works with learning languages, it has some bearing on mathematics communication as well, which the following example illustrates. Suppose a second language student can explain some mathematical concept in the student's first, but not second language. This student can use the concept with proficiency as long as it is expressed in non-linguistic registers. Moreover, for this student, the representation in words is similar to learning a technical term. Suppose another second language student does not know the same mathematical concept. This student needs to learn both the technical term and the concept via the school register.

Multilingualism seen through a discourse perspective

Moschkovich (2002) discussed three perspectives on multilingual mathematics communication. One perspective focuses on second language students' work with acquiring mathematical vocabulary and syntax in the language of instruction. An example of research in this perspective is Monaghan (2009), who carried out corpora studies focusing on specific words used in school mathematics. A second perspective uses functional linguistics, by Halliday, to focus on students' semantic meaning making of mathematical communication as discussed above. Moschkovich noted the possibility viewing these two perspectives as deficiency models or limitations for second language mathematics learners, since they focus on students' lack in vocabulary and struggle with multiple meanings in different linguistic registers such as mathematics and everyday registers. One should note that this may also hold for first language students (Petersson, 2014). Moschkovich (2002) suggested a third and pragmatic perspective as participating in mathematical discourses. The shift, from the first perspective of vocabulary acquisition to the second perspective on linguistic registers and the third perspective on participating in a discourse in research caused a change in curriculum from individual and silent classrooms to classrooms emphasising communication and interaction. Moschkovich described this shift as presenting new challenges and opportunities for multilingual students.

Moschkovich's (2002) perspective on students as participants in mathematical discourses is based on Gee's (2012) concept of a discourse, which has two elements. The first element is a pair of *sender* and *receiver*. The receiver is some socially meaningful group and the sender wants to be identified as a member of, or communicator with, the receiver. The second element in Gee's definition is a *message*, which the receiver perceives as having a socially accepted content of thoughts, acts or values, etc. In Gee's discourse, the mode of communication (verbal/non-verbal, see Figure 5, p. 61) is subordinate to the message. According to Moschkovich, Gee's definition of discourse allows a wide range of resources for communication such as material, social and linguistic resources for participating in the mathematical classroom (see e.g. Langer-Osuna, Moschkovich, Norén, Powell & Vazquez, 2016). Material resources could be tools such as rulers and protractors drawing mathematical figures or using calculators. Social resources could be gestures and facial/emotional expressions. Such material and social resources would hardly be regarded as being linguistic or semantic resources in functional linguistics. Moschkovich (2002) underlined that the perspective of participating in a discourse makes the second language student not a subject with (linguistic) deficiency, such as being semi-lingual, in the language of instruction but as a participant using multiple linguistic resources such as translanguaging. The concept of translanguaging Garcia and Sylvan (2011) is vaguely defined as how teachers and students communicate in multilingual classrooms making use of several communicative resources. This definition allows also non-linguistic communication, for example, gestures and the use of material manipulatives. Canagarajah (2011) described translanguaging as building, negotiating and symbiotically using a repertoire of languages for general communicative purposes. Translanguaging includes code-switching, which is when using more than one language in a conversation. One Swedish example of a situation where translanguaging showed to be a resource in the mathematics classroom is from Norén and Andersson (2016). In their study, a group of four Arabic-Swedish bilingual students were stuck in a discussion on how to interpret a task in statistics. The teacher, who knew Arabic, intervened by translating the task into Arabic, which enabled the students to start working on solving the task.

To sum up, the discourse participant perspective is more overt in emphasizing the non-verbal aspects of communication than is functional linguistics. This enables viewing the multilingual student as making use of several resources for competently communicating mathematically (Langer-Osuna et al., 2016). When Moschkovich (2002) wrote her article, she described the research using this third perspective as in its beginning.

Multilingualism as an increased cognitive load

It is clear that proficiency in at least the language of instruction is crucial for success in mathematics learning. The challenge for second language students has also been modelled as an increased demand on cognitive load while processing information. The cognitive load theory applies to general information processing (Paas, Renkl & Sweller, 2003). Campbell et al. (2007) used it for modelling *the problem space* in relation to solving mathematical problems. The problem space consists of four dimensions as illustrated in Figure 6.

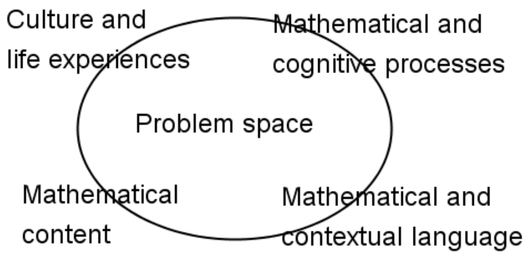


Figure 6. The problem space (Campbell et al., 2007).

The model in Figure 6 is similar to, and completes, the functional model of Rico (2006) and Sáenz (2009) with the fourth dimension of language. Moreover Campbell et al. (2007) give explicit attention to the case when some students may not be familiar with the culture context of the mathematical task. When solving a mathematical problem, for example during a test, the solver uses several cognitive resources while working with the problem. Each of these contributes to the cognitive load needed when working with the problem. Campbell et al. (2007) illustrated the dimensions of the problem space with the test problem "The Laundry Problem", restated below:

> Sandy's family does its laundry at a coin-operated laundromat. It costs $1.25 per load to use the washing machines and 25¢ per load to use the dryers for 10 minutes. Sandy's family has 5 loads of laundry to do and each load will need to be in a dryer for 30 minutes. Which expression will give Sandy's family the total cost of doing these loads of laundry?
> A. ($1.25 + $0.25) × 3 × 5
> B. [$1.25 + (3 × $0.25)] × 5
> C. [(3 × $1.25) + $0.25] × 5
> D. 3 × ($1.25 + $0.25) × 5

When solving this problem, the solver, of course, needs to know the mathematical content of arithmetic and to parse the calculations. With a test problem situated in a specified real life context, there is also the dimension of culture (e.g. Barwell, 2002). Has the student had personal experience of using a laundromat, or do they do the washing at home? Has the student even culturally experienced a laundromat indirectly via for example a TV-programme? There is the language in the problem text. What happens if the student uses one of the contextual wordings ‘wash up’ or ‘do the washing’ instead of ‘doing laundry’? There is also the mathematical language where the problem text contains costs and times and how these are related, which the solver needs to penetrate in order to find a correct solution (see for example Barwell, 2009b; Gerofsky, 1996; Lager, 2006). For the second language student, both the dimensions of culture and language may add to the cognitive load needed for solving the problem. Giving tests in the first language of the students may avoid the contribution to the cognitive load from the mathematical and contextual language, but probably not the contribution from cultural and life experiences. There are studies in which bilingual Spanish-English students achieved better on Spanish versions than on English versions of essentially the same test (Holland 1960; Meeker and Meeker 1973; Mycue 1968). A similar result holds for bilingual Arabic-Swedish students (Norén, 2010; Norén & Andersson, 2016).

Linguistic tensions in multilingual classrooms

Bakhtin (1981) suggested a model for a tension in language between standardization and diversity. The standardization is often driven by some group, which in some aspect(s) is in a majority in the society while the diversity is often advocated by some minority group. The reasons for, in some way, being in a majority or minority could vary. The reasons could be political, demographic, linguistic (including languages, dialects and sociolects); economic or cultural (see e.g. Duranti, 1997). Barwell (2014; 2016) described different aspects of how these tensions might work in multilingual classrooms in mathematics. These tensions might work in parallel as a pair of centripetal and centrifugal forces. For example, Barwell (2014) noted three of the following four tensions working in parallel in one classroom.

One tension is between the students’ oral and written explanations. In their oral explanations, the students made use of a wider spectrum of communicative resources, such as translanguaging, than when giving explanations in written English (Barwell, 2014; 2016). Also, Pimm (1987) and Setati and Adler (2000) have noted that students experience written (and formal) mathematical language as being more difficult than oral mathematical explanations.

A second tension is between the word problem context and the student’s own experiences. The word problems in Barwell (2002; 2014) and Halai (2009) were in English (being the second language of the students) and, just as in Campbell et al. (2007), both the non-shared cultural experiences of the mathematical problem and the complex contextual language of the mathematical problem were sources of difficulty for the students despite the calculations being straightforward.

A third tension is between one language being the language of learning and teaching and one or several other languages used outside school. Barwell (2014) described how this occurred in a mathematics classroom in Canada, where the students were Cree-speakers and the teacher was English and a non-Cree speaker. In this situation, Cree was often used between the students while English was used in whole classroom communication and often in written mathematical explanations. Similar situations may occur in many school settings over the world where the teacher and the students do not share a common first language.

A fourth tension is between some high-status language and other languages. The best known case is probably the situation in South-Africa, where proficiency in English is connected with opportunities for social capital such as access to further education and improved employment opportunities, while many other languages were associated with apartheid and withhold from increased social capital (Setati, 2008). A less politically charged situation was described by Parszyk (1999) in a Swedish preschool and primary school, where several parents emphasized the need for 'Swedish only' in school despite the access to teachers that were bilingual in Swedish and the mother tongue of the children.

Subtractive, additive, recursive and dynamic multilingualism

In school, there have been different views on developing a new language. Lambert (1974) and Wong Fillmore (1991) described how immigrants in the USA due to, for example, social pressure did not further develop their native language(s), but instead might replace their native language(s) with only English. This phenomenon Lambert denoted subtractive bilingualism, and it has monolingualism in the language of learning and instruction as the norm. In French Canada, Lambert described how children instead actively developed both their French and English language proficiency into what he denoted additive bilingualism.

Besides subtractive and additive bilingualism, Garcia (2011) described recursive and dynamic bilingualism. Recursive bilingualism is when specific parts of a language are developed for a particular function or purpose. One example of recursive bilingualism is the development of a mathematics linguistic register in the Maori language (Barton, Fairhall & Trinick, 1998).

While subtractive, additive and recursive bilingualism usually denotes one language used at a time, dynamic bilingualism denotes the ability to use several modes of language including bilingualism adjusting to the present communicative situation. While Garcia (2011) described dynamic multilingualism as using (only) linguistic resources, it differs from translanguaging, which also allows non-linguistic communicative resources, illustrated in Figure 5 and discussed above (see e.g. Garcia and Sylvan, 2011; Langer-Osuna et al., 2016; Moschkovich, 2002). Similar to dynamic bilingualism is plurilingualism, which is the idea that a person can use several languages at different abilities and for different purposes such as education, commerce, public or private life (Council of Europe, 2001; Garcia, 2011).

Use of dynamic multilingualism in mathematics classrooms has been observed in several studies. Clarkson (2009) has observed that multilingual students in Australia used their first language (L1) for informal reasoning about the mathematical tasks and then, when giving the formal solution, switched to the language of instruction, which was the students' second language (L2). Setati (2005) and Setati and Adler (2000) noted that both teachers and students in South Africa more often used English for discussing mathematical procedures, while the first languages were more often used for discussing mathematical concepts. This result is similar to that of Ufer et al. (2013), in which the bilingual students were proficient in algorithmic test tasks, but not in conceptually demanding test tasks, when compared to monolingual students. In Catalonia (Spain), Planas and Setati (2009) observed that South American immigrant bilingual students (having Spanish as L1) used Catalan to become familiar with the mathematical tasks and the (Catalan) mathematical vocabulary, while they used Spanish for reasoning and solving the mathematical tasks. With a dynamic use of multilingualism, the idea that code switching is confusing and a bad habit can be challenged (Barwell, 2009c).

Multilingualism at curricular, school and classroom level

From the discussion above, for example in the tensions in language between standardization and diversity, we see that multilingualism is to be dealt with at several levels such as: at curricular level, at the local school organisation and in the classroom activities. Globally, the reasons for the existence of multilingual mathematical classrooms may vary. There might be aboriginal or immigrant peoples with minority language(s) or there might be some dominant minority language due to colonialism. In all cases, the curricular policy has consequences for how multilingualism may be dealt with in the local school and in the classrooms.

At the curricular level, there might be official national language policies that are implemented in the educational system. In South Africa, there are eleven official languages and, though English is smaller than several other

South African languages, there are strong centripetal forces that work for English as the language of learning and teaching. Examinations arranged by the educational authorities are given in English, and English is needed for studying at higher levels (Adler, 1998; Setati, 2008). A third centripetal force is that English may also serve as a lingua franca in larger towns in South Africa, where English may be the only language shared by all students in the mathematics classroom. Though some smaller towns may be linguistically more homogeneous, knowledge of English is a socioeconomic argument for enhanced mobility and employment opportunities in other areas within the country where that specific language is in minority.

In New Zeeland, Barton et al. (1998) developed both curriculum and mathematics terminology in the Maori language instead of using English for the mathematical terms. One driving force in that project was that, if an indigenous language is not usable in most aspects of life in the contemporary society, that indigenous language might become fossilized. On the other hand, a question in their project was whether there is a value in developing a Maori mathematics terminology. Anyway, Maori students who go on studying higher mathematics are likely to do so in English. In that case, it might be useful already to be familiar with English mathematics terminology. A counterargument is that mathematics terminology is not only a question of changing vocabulary. There was also a change and development in the practice of ethno-mathematics in mathematics classrooms and a change in how mathematical activities were discussed and perceived by the students (Meaney, Fairhall and Trinick, 2008). Lindberg (2009) reminded us that Sweden as late as 2009 introduced a law for the protection of the Swedish language. The recent introduction of this law can be interpreted as if there was earlier no need of such a law, but that the Swedish language recently, and in some areas, has been pushed by other languages in, for example, terminology in science, commerce and technology. Presently, Sweden has five official minority languages; Finnish, Meänkieli (spoken mainly near the border between Sweden and Finland), Romani-Chib, Sámi and Yiddish (SFS 2009:724). Moreover, the national language policy has resulted in a separate Sámi curriculum (Jannok Nutti, 2010; Skolverket, 2011b).

At school level, the implementation of the curriculum may be affected by local opportunities and limitations. Nilsson and Axelsson (2013) studied how the integration process is implemented in different Swedish schools. They found that the access to further education for teachers of second language and the organisation of the preparatory classes in the Swedish language varied between schools. In the smaller schools, the preparatory classes were widely mixed in age, and in larger schools the age interval was narrower. Garcia and Sylvan (2011) demonstrated a programme of organizing teaching for newly arrived immigrants in the USA, and found it successful when compared with other schools with a similar mix of students. At the school level, Svensson, Meaney and Norén (2014) presented the Swedish students' view of access to support with homework. Compared with first language students, the second language students viewed themselves as underprivileged in the sense that their parents were less able to give them support in homework when compared to their 'Swedish' classmates, irrespectively whether this was the case or not. Thus, the second language students saw a possible lack of equity in the case of support with homework.

Research literature from different parts of the world describes locally different multilingual landscapes. In some geographical regions, the multilingual landscape is dominated by essentially bilingual classrooms, while in other regions, the situation is more multilingual (Langer-Osuna et al. 2016). In the southern USA, it is common to see essentially Spanish-English bilingual classrooms (Lesser, Wagler, Esquinca & Guadalupe Valenzuela, 2013; Lesser & Winsor, 2009; Dominguez, López-Leiva, Khisty, 2014). In New Zeeland, Barton et al. (1998) have focused on the Maori-English bilingual situation by developing a mathematics register and teaching materials in the local language. In a bilingual landscape, the school could sometimes handle the situation by employing a teacher who is bilingual in the two languages of the classroom. This is often the case, for example, in Quebec (Lambert, 1974), in Malta (Farrugia, 2009) and in parts of South Africa (Setati, 2005; Setati & Adler, 2000). In other situations, this could not be arranged for practical reasons of the language competency of the teacher employed (Barwell, 2014). In other geographical regions, the multilingual landscape is more complex, with several simultaneous languages in the same classroom (Garcia & Sylvan, 2011; Meyer et al., 2016) and this changes the conditions for how to arrange the mathematics classroom activities. For example, a student might be the only speaker of their first language and the teacher seldom masters more than one of the languages other than the language of instruction. The language situations described here can be illustrated by the three simplified models in Figure 7.

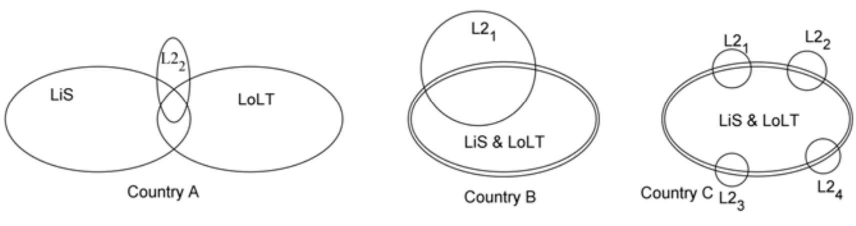


Figure 7. Languages in society and classroom and L2 constellations.

Country A in Figure 7 illustrates the situation in, for example, South Africa where the 'language of learning and teaching' (LoLT) does not coincide with the 'language in society' (LiS) used for other activities. So, essentially all students are second language learners in school and there might be other second language groups sharing neither LoLT nor LiS. In countries B and C, LiS and LoLT coincide. Country B illustrates the situation in parts of the USA where a large proportion of the second language students in the classroom share Spanish as first language. Country C illustrates the situation in many European countries where the second language students in the classroom do not share the same first languages. In countries A and B in Figure 7, it is sometimes possible that a bilingual teacher can serve the students in both the major languages used in the classroom, while this is not possible with the multilingual situation in country C.

At classroom level, a crucial point is the teachers' knowledge of how the multilingual mathematics classroom communication can be arranged. Xenofontos (2015) found that the teachers may sometimes have a naïve view of multilingualism such as if the students knew the language (the BICS and not the CALP), the teaching would be easy. This view is in contrast to the views discussed earlier of multilingualism as layered in everyday and academic register, as an increased cognitive load which is not only linguistic but also culturally flavoured, and through a discourse perspective. However, the communication could also be used in a way that fits the students' competencies in the two languages, as is the case with dynamic use of multilingualism in mathematics classrooms as discussed earlier. There are further suggestions on how the individual teacher can orchestrate the multilingual classroom irrespective the opportunities and limitations at the school level. Moschkovich (2009) described the work of a teacher who views multiple meanings not as obstacles but as resources. The teacher provided support by clarifying and comparing students' work and took time in discussing what was similar and different in how she interpreted the students' work. In doing this, the teacher supported the discussion rather than evaluating the students' work. Chval and Khisty (2009) described a classroom where the students improved their mathematical writing and academic language via writing mathematically. During the writing process, the students encountered the mathematical and academic registers in contexts that supported meaning making and concept development, at the same time as the students could discuss mathematics. In other words, this classroom supported the development of dynamic multilingualism.

Summary and a tentative research question

Research on multilingual mathematics classrooms has pointed out a range of theoretical models that both identify challenges and offer opportunities for the multilingual mathematics classroom. Some of them have been used in research on Swedish mathematics classrooms. Parszyk (1999) have observed linguistic tension between the language of instruction and the students' mother tongues. Norén and Andersson (2016) observed students and a teacher using translanguaging, which made it possible for the students to productively participate in the mathematical activity. In Germany, Prediger et al. (2016) proposed a teaching design, which they suggested be further explored in empirical research. Their design builds on encouraging the students to move between several linguistic registers (everyday, school and technical register) and representational registers (linguistic, graphic, numeric, algebraic registers) as they work with a mathematical task.

Much of the research reviewed here and in part 1 of this thesis has discussed the conditions for learning such as proficiency in the language of instruction, cultural unfamiliarity with contexts of problems in textbooks and tests, and socio-economic background such as different educational levels of the parents. However, it seems that much less has been explored in mathematics education when it comes to the area of heterogeneity at the proficiency level of second language students. When students have been categorised after migration status, there were studies with a division in sub-categories of immigrant students. For example, OECD (2006) categorised students into being native, native with two immigrant parents, or immigrants. Skolverket (2004) made a similar categorisation with the addition of immigrants categorised as immigrated before or after age of school start. However, one observation in the previous research is that, when students instead were classified in language status, research on second language students in the mathematics classroom, classifies these students as ‘a single category’ of second language students. This imposes a concept of methodologically viewing second language students as homogeneous. Putting second language students in a single category is done despite results from, for example, Clarkson (1992), Cummins (2008) and Prediger et al. (2016) who saw proficiency in everyday-language and school-language as different levels of proficiency. Clarkson (1992) exemplified with empirical mathematics achievement results, though the latter two studies presented theoretical models and did not give empirical results. In the previous research, only Böhlmark (2008), was found to empirically connect school achievement with time after immigration, which is related to having gained opportunities for learning the new language of instruction. Moreover, paper II in this thesis suggested that newly arrived immigrants may contribute to the observed differences in achievement profiles between first and second language students. Thus, there seems to be a need for more research on sub-divisions of immigrants with respectively shorter and longer experiences of the new language of instruction.

One tentative research question that emerged out of this is to ask about differences between second language students who immigrated recently or longer ago, but presently participate in the same mathematics classroom. This question could be about achievement or about possibilities and challenges to participate in the mathematics classroom. Inspired by the model in Figure 5 (p. 61) by Prediger et al. (2016), a working hypothesis can be formulated. A working hypothesis could be that students, who immigrated early in their schooling, may master a purposeful school register in neither their first nor their second language in the mathematics classroom. For students who have had a long experience of being first language students in their country of emigration and are newly immigrated, the situation may instead be that they master the school register and technical register in their first language though not yet in their second language.

A definite research question

The research question below stems from the model in Figure 5. A student who immigrated in, say, the last year of Swedish compulsory school will be a beginner in Swedish. If this student has had a regular schooling in the country of emigration, the student may master all the registers and representation in the student's first language. Moreover, the student may have followed a mathematics syllabus different from that of Sweden, and this may have given them a mathematical body of knowledge with a possibly different emphasis than the Swedish mathematics syllabus as suggested in paper II in this thesis. Drawing on, for example, Cummins' (1979) interdependence hypothesis, this student may quickly learn to master the numerical and algebraic representations in the second language, though the second language school register may take much longer time to develop. In a similar manner, a student that immigrated during first year of compulsory school is likely to not yet master the school register in the student's first language. This student will, according to Cummins (2008), need several years to develop proficiency in the second language sufficient for successful learning. That is, this student may master the school register in neither first nor second language. This leads to a research question on representing and using mathematical concepts in mathematical communication for part 2 of this thesis:

> Students in the same classroom may have different lengths of stay in Swedish schooling. First language students in most cases have all their schooling following the Swedish curriculum. While some second language students have had all or most of their schooling in Sweden, some may also have their major experiences from following some other curriculum, in which they may have been first language students. When second language students with different duration of experience of Swedish school are compared with each other and with first language students, how do these student categories achieve on tests and represent and use mathematical concepts?

This research question on representations and concept usage in mathematical communication can be said to be driven by the theoretical model in Figure 5. This research question guided papers III and IV in the present study. This research question was driven also by the results in paper II in which first and second language students were reported to have different achievement profiles and by the results in paper I, in which first and second language students were reported to have large achievement differences on specific test items.

Methodology, part 2

Rationale for the method

The following methods were used for data collection for the research question on students' achieving on, representing and using mathematical concepts. A method for sampling students was designed (Norén, Petersson, Sträng, & Svensson, 2015). A test was designed and given to the students in order to collect information about their achievement and their representation of different mathematical concepts. In some cases, the students' test response to some individual test item was unclear when it came to classifying them with respect to concept use. In these cases, interviews were used to complete the information retrieved from the students' test responses. A survey was used for classifying the students according to their background as first or second language students and in which school year they have immigrated. The following sections discuss practical, theoretical and ethical matters on collecting information about the students' background; sampling students; choice of test instrument; interviewing students; and analytical framework.

Categories for the students' background

When it comes to background information, the research question asks about two things. One is on the students as being first or second language students with different duration of experience of the Swedish language in the mathematics classroom. The other is the students' achievement on test items and how they chose to represent mathematical concepts. The research question requires background information about both language status and migration status.

For the migration status, several definitions are used in school. Skolverket (2007) defined *nyanlända elever* (newly arrived immigrant students) as those who start primary or secondary education within 3 years after immigration and does not have Swedish as a mother tongue. A more restrictive definition is *sent anlända elever* (lately arrived immigrant students). This definition includes those who immigrate during school years 6–12 and have a mother tongue other than Swedish. One school act (SFS 2013:69, §2) defines a newly arrived immigrant an immigrated student arriving in the first four semesters during school years 6–9 of. According to the first of these definitions, a student that immigrated in school year 5 will be a newly arrived student from grade 5 to grade 8, while the same student will not be a newly arrived immigrant according to the two other definitions. In a research review, Bunar (2010) concluded that, for administrative reasons, a student in primary and secondary education is regarded as a newly arrived immigrant for two years. During these years, the student may follow a preparatory course in Swedish. This course is preparatory and is not the same as the compulsory school course 'Swedish as a second language'. Depending on the individual student's progression in the Swedish language, the student enters ordinary classes as soon as possible. The transition into ordinary classes could be earlier in some subjects, such as physical education, and later in some other language intensive subjects. The content in the mathematics syllabus for compulsory school (Skolverket, 2011a) could also be a ground for differentiating between newly and early arrived immigrants. In later school years, more formal mathematics is introduced together with more advanced mathematical concepts such as algebraic expressions, arithmetic with negative numbers etc. that in many mathematics textbooks appear en masse in school year 8.

Table 3. Definition of student categories in school year 9

Newly2L: Newly arrived second language students	Students who in Swedish school follow the course 'Swedish as a second language' and due to immigration have entered the Swedish school system during school years 8–9.
Early2L: Early arrived language students	Students who in Swedish school follow the course 'Swedish as a second language' and due to immigration have entered the Swedish school system after school start but not later than 7th grade.
Other2L: Other second language students	Students who in Swedish school follow the course 'Swedish as a second language' and are neither newly arrived nor early arrived immigrant; they may have immigrated before school start age or have not immigrated at all.
Swe1L: First language students	Students who in Swedish school follow the course 'Swedish (as a first) language'.

Cummins (2008) gave empirical and linguistic base for distinguishing between students having conversational proficiency (BICS) or academic proficiency (CALP) respectively in the new language. Cummins gave an approximate time span of reaching conversational proficiency in about two years and academic proficiency in five to seven years. Since the time span of two years has support from empirical research in applied linguistics and is coherent with the Swedish administrative routines, the definitions in Table 3 were made for the present study. Moreover, sampling students in grade 9, being the last year of compulsory school, will give a mix of students with the longest possible span of experience from only the Swedish curriculum to mostly some other curriculum. By the definition, all students in the three first categories in Table 3 follow the course "Swedish as a second language". The category "Other second language students" in Table 3 was constructed as an intermediate group distinguished from, on the one hand, first language students and, on the other hand, second language students who are newly arrived immigrants and early arrived immigrants as given in Table 3.

Properties of the student subpopulations

Before discussing how to sample students, there is a need to know something about the properties of the populations in the student categories in Table 3. We start with estimating the population of the newly arrived immigrants. Since the Swedish compulsory school uses age as a basis for promotion to the next school year, the number of students in the last year of compulsory school is, in practice, the same as the number of sixteen-year-old persons. Thus, a rough estimation of the population of newly arrived immigrants can be made as follows. According to Statistics Sweden (2016), there are about 100,000 persons that in 2015 were 16 years old, and about 15,000 of these were born abroad, though are not necessarily second language speakers. So, of the students aged 16, being the most common age for finishing compulsory school, on average about 1,000 persons have immigrated each year. This rough estimation gives that in the last year of compulsory school, the population of newly arrived immigrants (only by immigrant status, not by language status) are about 2,000 individuals, which is about 2% of the 16-year-old students. A more exact estimate of the population size can be made as follows. In 2015, the number of 16-year-old individuals born abroad was 15 074, while in 2013 the number of 14-year-old individuals born abroad was 11,533 (Statistics Sweden, 2016). The difference between these two figures is 3,541, which is a net flow of immigrants and can be used as an estimate of the number of individuals that in 2015 are 16-years old and have immigrated during the last two years. In fact, the net flow gives a lower estimate, since some people may have emigrated from Sweden (or died though mortality is low in this age group). For example, on country level, the number of individuals born in Denmark during this period was seven persons fewer, which indicates a net migration back to Denmark during these two years. Statistics

Sweden does not show statistics on a combined status of being an immigrant and status as second language student, but Table 4 suggests that a majority of at least newly arrived immigrants are second language learners in Swedish schools. The reason for this is that from the other Nordic countries a large proportion of the students immigrated before school start age, and the languages Danish and Norwegian, though not Finnish, are similar to Swedish. A note is that some migrants between Finland and Sweden might move between Swedish-speaking communities.

Table 4. 16 years old immigrants 2015 (Statistics Sweden, 2016).

Region of birth	Total number	Of which newly arrived	Of which early arrived	Of which before school start
Africa	6,983	30%	40%	30%
Asia	3,166	32%	55%	14%
Other countries except Nordic	4,184	10%	38%	52%
Nordic countries except Sweden	741	4%	9%	87%

Seen as a sample for educational statistics purpose, immigrants are in many aspects a heterogeneous group. Immigrants are a diverse group with respect to age of immigration, as illustrated in Table 4. In the present study, some students immigrated during school year 9 and others immigrated during school year 1. Moreover, the age at immigration varies with geographic region. Immigrants from Africa and Asia are in a majority among the newly arrived immigrants, while those in Table 4 from ‘other and Nordic countries’ immigrated early or before school start. Another way to say this is that immigrants from, say, Europe are likely to be among those who immigrated before school start, while immigrants from, say, the Middle East are likely to have immigrated at school age. There is also heterogeneity in schooling time. Some immigrants might have as long schooling as their new classmates, while others might have had a long absence from school for different reasons (Skolverket, 2007). There is heterogeneity in language background. While some immigrants’ mother tongues use the Latin alphabet, and have at least a few words similar with Swedish, others have neither. There is heterogeneity in the adults’ educational level as shown in Table 5. Table 5 exemplifies that immigrants even from neighbouring countries such as Afghanistan-Iran and Kenya-Somalia have quite different educational distributions. As a comparison, Table 5 shows the education distribution of those born abroad and born inside Sweden. The educational level of many immigrants is (as yet) unknown to the Swedish authorities, as seen in Table 5.

Table 5. Educational level and country of birth, ages 16–74 and years of immigration 1987-2015 (Statistics Sweden, 2016).

Population category	Pre-secondary	Secondary	Tertiary	Unknown
Born in Afghanistan (N=14,273)	40%	15%	15%	30%
Born in Iran (N=55,215)	19%	35%	32%	14%
Born in Kenya (N=5,296)	32%	22%	25%	21%
Born in Somalia (N=31,197)	53%	19%	8%	20%
Born in Sweden (N=5,758,597)	17%	47%	36%	1%
Born abroad (N=1,425,082)	23%	34%	35%	8%

Sampling from the subpopulations

Given the student categories in Table 3, what strategy, for example probability or non- probability samples, should be used when designing a student sample? Heesch, Storaker and Lie (2000) used the probability sample data in TIMSS 1997 for a study of Norwegian immigrants. In her study, the number of immigrants was small and in some cases she could not achieve statistically significant results, despite interesting patterns in the data. Thus, there is a need for the sample to be large enough in each of the student categories in Table 3. Such issues will be discussed here.

A probabilistic sample

With a random sample, there is a risk of segregation effects. Earlier studies have found that the parents' education, with statistical significance, correlates with the school achievements (Skolverket, 2004; Zevenbergen, 2001). It is also known that socio-economic background correlates with language status and immigration status (Hansson, 2012; Hansson & Gustafsson, 2011). With a random sample model, where the educational background is not included, this segregation effect is pronounced since, instead of comparing students with different language status, it might be students with different socio-economic background that are compared. In paper II, this was dealt with by constructing a measure for the achievement profile (see Figure 2). A measure of achievement profile building on the shape of the achievement distribution per mathematical content areas compares a group of student with themselves. In that sense, the measure is likely to be invariant to differences in the social background.

A purposive sample

We can estimate how large the size of a random sample needs to be if all four student categories were to be represented. If we suppose that a minimum sample of newly arrived immigrants is 30 students, then we must sample about 30/3541 ≈ 0.8% of the whole subpopulation of newly arrived immigrants estimated above and in Table 4. This corresponds to at least 800 randomly chosen individuals of the whole population of 16-year-old students. This sample size could be achieved in, for example, the evaluations sample used for the national tests, but is difficult to administer when it comes to distributing, collecting and assessing the outcome. For example, the secondary data sample in paper I was 740 first language students and 120 second language students, and in paper II, there were 2,253 first language students and 248 second language students.

One option is to use purposive sampling to oversample the student categories in focus (Cohen & Manion, 1994). Now most students are enrolled in local schools and, due to residential segregation, there is a tendency to socio-economic segregation in some schools (Hansson, 2010, 2012; Skolverket, 2004). To sample all and entire classes in schools with a high proportion of first and second language students and immigrants gives an opportunity to decrease the sample size and possibly to avoid some of the socio-economic differences. The sample, used in papers III and IV and given in Table 6 is an example of this. The absent (non-participating) students are discussed at the end of the section 'Data collection'.

Table 6. Participating students' achievement in Swedish language and mathematics.

Student category	Participating students	Absent students	Swedish grade ≥ passed	Mathematics achievement
Newly2L	23	5	52%	49%
Early2L	67	7	78%	43%
Other2L	56	7	86%	48%
All2L	146	19	77%	46%
Swe1L	113	20	97%	56%

However, there is a risk of losing external validity in the sense that the purposive sample might clearly have properties other than a random sample (Kruuse, 1998). To control for this, the student sample in Table 6, was compared with a random evaluation sample of students from all over Sweden (Norén, Petersson, Sträng, & Svensson, 2015). The evaluation sample was collected by the Swedish National Agency for Education and is a part of the annual evaluation of the national test. The author received this evaluation sample from Pettersson (personal communication, February 14, 2013). The evaluation sample only categorises students as first or second language students. The second language students in the evaluation sample achieved 46% (Skolverket, 2012), which is the same as all second language students in the present study taken together in Table 6. The first language students in the evaluation sample achieved 60%, which is slightly better than in the purposive sample in Table 6. A likely explanation for this is that the first language students in the purposive sample might have had a lower socio-economic background (Hansson, 2010, 2012; Skolverket, 2004). Since the differences between the purposive and evaluation samples are small, it is likely that the purposive sample has similar achievement properties as a random sample on a national level and that the result can be generalised on a national level.

Ethical aspects on categorising the students' background

As stated in the research background on comparative studies, students' school achievement correlates with their parents' educational level. Although students' school achievements correlate with their parents' average socio-economic status, there are arguments against collecting the parents' educational level or employment status. One ethical argument is that survey questions should follow the ethical guidelines and be designed to avoid intruding the integrity by involving the conditions of other family members than the student. A statistical inference argument is that the parents' educational level is not a strong predictor of the individual students' achievements. Ufer, Reiss and Mehringer (2013) in a regression model on German school data found that language ability took over the significant explanatory role of mathematics achievements from the socio-economic level already in school year 2. There is also a validity argument against using a survey for collecting parents' educational background. In the PISA survey, questions on the parents' educational titles will confuse the students. Wester (Anita Wester, Stockholms stad, utbildningsförvaltningen, personal communication) confirms that students, with both Swedish and other mother tongues, have difficulties in understanding these questions and often overestimated their parents' education. Thus, there is a risk in acquiring incorrect data if asking for the background of the students' parents.

One alternative is to collect information about which country the student emigrated from. The reason for this is that, as stated in the research background, Böhlmark (2008) found that students, who emigrated from different geographic regions, achieve differently. Moreover, Table 4 shows a spread in which geographic region, the students emigrated from. There is also the option of asking the students which language is their mother tongue. Asking about country or region of emigration and about mother tongue might lead to ethical and legal dilemmas, since they are closely related to ethnic background, which is a sensitive piece of information according to the law on personal acts (SFS 1998:204). The law on ethics for research on people (SFS 2003:460) refers to the law on personal acts for this case. The exception to the law on ethics for research on people is that the informant is at least 15 years old, informed and that there is documented consent.

But what is 'ethnic background'? The law on personal acts (SFS 1998:204) does not define this. According to the sociologists Björklund and Hannerz (1983), one definition of an ethnic group is that the group fulfils at least the following four characteristics:

Self-reproducing.
Self-identifying.
Common origin.
Distinctive feature.

With this definition, for example, the Swedish nobility might constitute an ethnic group. This definition is typically used by the researcher on the research subjects, and Björklund and Hannerz suggest an often used alternative definition where the ethnic group is instead defined by the research subjects themselves. In this sense, the research subjects construct a border of those belonging or not to some certain ethnic group. The two definitions above could be characterized as oriented towards essential versus constructed definition. In the first definition, an outside observer puts some constructed frame on the research subject. In the second definition, the research subjects choose themselves to fit some certain pattern of characteristics to be counted as a member.

Using the first definition, we can ask the following question. Do individuals born in a specific country belong to a specific ethnicity? The Swedish National Encyclopaedia (*Nationalencyklopedin, 2013, Somalia*) states that Somalia is mainly uniform in language, religion and culture. This is not the case for people born in Iraq, since Iraq consists of several groups fulfilling the four characteristics given earlier by Björklund and Hannerz (1983) with respect to religion and language. The religious map of Iraq since ancient times contains separate groups as, for example, Sunni, Shia, Catholic and Orthodox, and the language map is divided into the major languages Arabic and Kurdish (*Nationalencyklopedin, 2013, Irak*). An outcome is that 'born in

Somalia' with high probability indicates ethnicity, while being 'born in Iraq' only partly does. The categories of 'being immigrant' and 'being a second language student' do not indicate ethnicity at all. The two cases of individuals born in Somalia and Iraq exemplify that there are no simple answers to the question of ethnicity. The answer could be that the country of birth implies ethnicity with high probability for at least some cases. On the other hand, the answer could be that it does not with certainty imply belonging to an ethnic group. For the definition of the student categories in Table 3 on language status and first school year in Swedish school, there is no problem with respect to Björklund and Hannerz's first concept of ethnic group. But, given the information in Table 4 and Figure 2, it is interesting to have more fine-grained information about the students' school background by differentiating between students having school experience from countries in different geographic regions. For this reason, a supplementary question in the survey in Appendix IV was to ask about the country of previous schooling and the mother tongue. The survey in Appendix IV was given together with a letter of informed consent in Appendix III along with the ethical standards in the law on ethics for research on people (SFS 2003:460). According to this law, the students were given the option to not participating in the survey. The country of previous schooling is less connected to ethnicity than country of birth, and for some immigrants it may not be the same as country of birth. In Swedish school, students fulfilling some conditions of using a mother tongue other than the Swedish language can take a course in their mother tongue. The grade of this course is recorded in the document of leaving grade from public school. This document is public in Sweden. This means that, for some students, the information about mother tongue is publicly available through the leaving grade. This makes it ethically less problematic to ask a question about a student's mother tongue.

When it comes to reporting achievement results, the researcher should be cautious about reporting them in terms of ethnicity. Leder (2015) gave a case on indigenous people in Australia. In the Australian national assessment programme, indigenous students had been reported as having low achievements. However, Leder discussed how a careful interpretation of the data showed that social background could explain much of the differences. For indigenous students living in urban areas with the same access to school resources as average non-indigenous people, the achievement difference was small. However, for indigenous students living in remote areas, with less access to school resources, the achievement difference with average non-indigenous people was large. So, comparison of student categories must be made with care.

Data collection

Designing the test

Along with the research question follows that in papers III and IV the focus is on the students' achievement, representation and usage of mathematical concepts. For test item secrecy, the data available from the national test is only the points that each student gets for each test item. So, in order to answer the research question, a separate test had to be constructed as a test for which the collected data could be the students' full and written responses. When the test was designed, the same aspects of validity described in part 1 were considered as when choosing test instrument for papers I and II.

The idea of representing mathematical concepts is closely related to written or drawn communication (Dörfler, 2006; Pimm, 1987; Prediger et al., 2016). Moreover, a test is a common and time-efficient tool for collecting students' individual written mathematical communication. This holds true especially if we want to collect data from several students in order to study how frequent some specific representation of some concept is among different student categories, and if we want to do this for several concepts in different mathematical areas.

Following Cummins' (1979) interdependence hypothesis and Prediger et al.'s (2016) model on representational registers, a student who is a beginner in the Swedish language but experienced in school mathematics is likely still to be able to process a test item and give a response relying on essentially using graphical, numerical and algebraic representation forms. So, when designing the test, it was taken into account that some of the test takers are beginners in the Swedish language. One means of doing this was to reduce the culture experiences and contextual language in the problem space in Figure 6 (Campbell et al., 2007). This means that the test items were chosen to essentially have an intra-mathematical context and word formulation, which in most cases was simple instructions like 'calculate', 'solve', 'how much is' and 'what is'. This would reduce the element of testing language proficiency while aiming to test mathematics proficiency, which would otherwise lower the validity with respect to equity for different student categories taking the test.

The tests in Appendices I and II for papers III and IV were designed to be similar to part B1 in the national test used for school year 9 with respect to length, problem formulation and mathematical content. This would benefit the validity aspect since the students get an opportunity to become familiar with and prepare for the conditions of the national test. In order to measure knowledge in mathematics rather than language proficiency, the linguistic complexity in the test item formulations was kept low. Since the test and test conditions were similar to those of the national test, the students could be expected to be more motivated to take the test as seriously as an ordinary teacher test. This might not be the case if the test had been administered as a researcher's test and designed as a test being less relevant for the students' ordinary classroom work.

As seen in Tables 7a, 7b and 7c, the test items were designed by modifying or in some cases re-using test items from previous national tests. This design process made use of test items which have already been carefully tested and evaluated when the national test items were originally designed. In the modified test items, the numbers and coefficients have been exchanged. Since the time allowed for the test was limited to one lesson, it had to be short, and for this reason geometry and probability was not included.

Table 7a. Test items on algebra in the designed test.

Test item formulation	Source formulation
Lös ekvationen 2x + 3 = 11. [Solve the equation.].	2009ÄP9B1#7; Lös ekvationen 17=3x+5.
4x + 5y = 11. Vad är 12x + 15y? […What is…?]	2009ÄP9B1#15; Hur mycket är 4x+6y om 2x+3y=12?

Table 7b. Test items on statistics in the designed test.

Test item formulation	Source formulation
Vad är medianen för temperaturen? [Find the median temperature.] (A table was given).	2008ÄP9B1#14; Under en vecka i mars läste Markus av följande temperaturer kl 13.00. Beräkna medeltemperaturen. (A table was given).
Philip går i klass 9B och väger 70 kg. Hur lång är han? [Philip in class 9B weighs 70 kg. How tall is he?] (A scatter plot was given).	2009ÄP9B1#10a; Philip går i klass 9B och väger 65 kg. Hur lång är han? (A scatter plot was given).
Vilken är medianlängden i klassen? [What is the median height in the class?] (A scatter plot was given).	2009ÄP9B1#10b; Identical (A scatter plot was given).
Se diagrammet! Ungefär hur många procent mindre nederbörd föll i Örebro än i Karlstad under juli månad? [See the diagram. About how many percent less precipitation fell in Örebro than in Karlstad during July?]	Not similar, but on reading graphs: 2008ÄP9C#6; Hur manga elever deltog I undersökningen? [How many students participated in the survey?] (A bar diagram was given).

Table 7c. Test items on numbers in the designed test.

Test item formulation	Source formulation
Beräkna 12 – 23 + 9. [Calculate…]	2010ÄP9B1#3; Beräkna 15 – 28 + 5
Vilket tal ska stå i rutan så att likheten stämmer? 1,365 – ___ = 1,305. [Which number fits the box?]	2007ÄP9B1#3; Skriv ett tal i rutan så att likheten stämmer. 1,795 – _____ = 1,705
Beräkna 6,32 – 3,44. [Calculate…]	2006ÄP9B1#2; Beräkna 15,3 - 8,25
Hur många grader skiljer det mellan de dagar där temperaturskillnaden är störst? [How many degrees is the difference between days with the largest temperature difference?] (A table was given).	2009ÄP9B1#2; Hur många grader skiljer det mellan de städer där temperaturskillnaden är störst? (A table was given).
Hur många minuter är 0,75 h? [How many minutes is 0.75h?]	2009ÄP9B1#4; Identical
Vad är hälften av 1/5? [What is half of… ?]	2006ÄP9B1#7; Vad är hälften av 1 1/2 ?
Figuren består av rektanglar och trianglar. Hur stor del av figuren är grå? [The figure consists of rectangles and triangles. How large a part is grey?] (A figure was given).	2007ÄP9B1#8b; Identical
Vad är 2/3 av 60? [What is…?]	2006ÄP9B1#7; Vad är hälften av 1 1/2 ?
Skriv talet 1 430 i grundpotensform. [Write in scientific notation.]	2007Ä9B1#11; I Sverige köper vi 120 miljoner tulpaner under vårvintern. Skriv antalet i grundpotensform
Beräkna 3^2+2^3. [Calculate…]	None, though powers in scientific form has occurred (e.g. test item 3).
Beräkna $\sqrt{9+16}$ [Calculate…]	2007ÄP9B1#14; Identical
a = 2 och b = 4. Beräkna a(b + 2) + b. [Calculate…]	2008ÄP9B1#17; a=4 och b=-3. Bestäm värdet av a(a + 1) + b

The algebra test items chosen are the two given in Table 7a. One is an elementary equation and the other is a more advanced test item. In the latter, one solution is to use proportionality. The quotient between the coefficients in the two linear expressions is 3, and multiplying this with the known right-hand side gives the value of the other linear expression. Another way is to assign a value to one unknown, for example y=1, then solve the equation for the other unknown and substitute the two unknowns into the linear expression. Strictly speaking, this solution presupposes the statement that you would get the same answer no matter what value is assigned to y, i.e. that the problem has a well-defined solution. This statement is true precisely because the two linear expressions are proportional.

In paper I (Petersson, 2012), the median was one test items that showed large achievement differences between first and second language students. For this reason, two test items on the median were included as seen in Table 7b. The data format was varied in order to explore what role the representation of the data plays for the given student responses. In one test item, the data was a univariate table of positive and negative integers. In the other test item, the data was a bivariate diagram with coordinates given by the axes.

Typically, the mathematical content area of number dominates part B1 in the national test (see paper II, Petersson, in press). The same holds true for the designed test. An emphasis was put on test items containing subtraction and proportionality, since paper I (Petersson, 2012) found that among these kinds of test items there were both similarities and large differences in achievement between first and second language students. For example, the test item of calculating 12–23+9 in Table 7c contains a ‘detached minus sign’, which is known to be difficult and often misinterpreted as calculating 12–(23+9) instead (Herscovics & Linchevski, 1994). Test items on powers were included, since such test items had showed variations in achievement differences between first and second language students on previous national tests. One test item with brackets and variables was included containing elements of both number and algebra.

The test consisted of 17 essentially single step mathematical problems. Calculators were not allowed and time allowed for the test was about 40 minutes. The test was distributed by the teachers as an ordinary classroom test. The students may ask teachers about translating words, but just as during national tests, the teachers do not answer mathematical questions. The researcher collected them from the teachers and assessed the students’ responses and gave the individual test results as feedback to the teachers. The students took the tests (Appendices I and II) between a few weeks and a month before the national test. This gave an opportunity to offer the test, administered through their teachers, as an exercise test preparing for the national test.

Analytical framework for assessing the students’ responses

While the part B1 in the Swedish mathematics national test asks for short answers, mainly a number, the test designed for the research question in part two of this thesis asks for fuller responses that can be assessed with respect to representation and use of mathematical concepts. Consider, for example, the following two versions of a test item:

Determine half of a fifth as a fraction!
Determine half of a fifth!

If the response format to the two versions above of a test item is a multiple choice, the assessment is essentially a singleton assessment as correct or incorrect. When the response format is instead open, such as short answer or extended full response, a response might contain various kinds of representation forms. For example, the short answer “0.1” to the first test item above is partially correct, since the value of the number is the same as one tenth, but is not given in fraction representation. In this example, the assessment is not only incorrect or correct, but also has an intermediate level of partially correct. In the second version of the test item formulation above, the student may solve the problem by representing it using only fractions or convert it to decimal or percentage or graphically via the iconic register representation in Figure 8. Specifically for proportionality problems, Vergnaud (1983) showed that there are several alternative ways to model and solve them. Double coding systems acknowledging variations in expressing the response, has been used on, for example, TIMSS (Agnell, Kjaernsli & Lie, 2000). The conclusion of this example is that an analytic framework for assessment of students’ responses needs a dimension allowing various forms of representation. One such framework is the one in Figure 8 by Duval (2006).

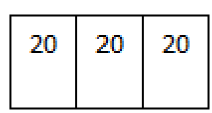
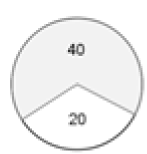
Representation registers	Examples
Natural language - oral or written.	“Two thirds of sixty”
Written with mathematical symbols	(60/3)x2 = … (60x2)/3 = … 0.67x60 = …
Iconic – drawing.	
Diagrams and graphs.	

Figure 8. A spectrum of representation registers (Duval, 2006) with examples.

The framework in Figure 8 illustrates the complexity of representation forms in mathematical communication and the cognitive requirements needed to access mathematical objects. The strength of Duval's model in Figure 8 is that it can handle the complexity of representation registers in mathematical communication (Hoffmann, 2006). Duval (2006) underlined that transitions within and between the registers in Figure 8 constitute a threshold for learning mathematics. As such, the transitions in the student's realisations are a potentially rich source of information for assessment made by the interpreter (Dörfler, 2006; Sfard, 2008). However, it must be kept in mind that both the student's realisation and the assessor's interpretation are situated in the classroom mathematics context. The model in Figure 8 by Duval was developed for the purpose of describing drawn and written mathematical communication. Duval suggested that the spectrum of representations used in the student's written response can be one source for assessing mathematical quality. During the solution process, the student must be able to make mathematically meaningful interpretations of the task, stated in some source representation(s). The student may make transitions within some representation, for example numerical calculations. And the student may make transitions between some representations, for example, between geometric and algebraic representations. Eventually, the student gives a response in some target register(s), possibly chosen by the student.

Duval also emphasised the plurality in perceiving a visual object; there is a discrepancy between looking at visual figures as an imitation of the object or as representing relations "the mathematical way they are expected to be looked at" (Duval, 2006, p. 116). This expresses not only a choice of representation registers, but also a way to assess mathematical quality. This can be used as a way to assess the quality of a student's written response, namely to measure how close the realisation is to a mathematical object. This can be described in terms of *enacted rule*, *endorsed rule* and following a *norm* for using mathematical objects (Sfard, 2008). Here, the enacted rule is the student's use and conceptualisation as expressed in some realisation. The teacher's assessment of the student's response follows an endorsed rule for how to interpret a realisation in a school mathematics context. If a strict mathematical response is expected, the assessing teacher may even follow a norm, which, in the case of mathematics, is an interpretation agreed among professional mathematicians.

With the purpose of capturing the translingual mathematics classroom communication, Prediger and Wessel (2011) and Prediger et al. (2016) extended Duval's model with the two dimensions of language diversity and specialisation in the registers of representation, as was illustrated in Figure 5 (p. 61). In a test situation, the communication is more restricted. Written communication is a standard format and material resources such as rulers and protractors might be used, but bodily representations are likely excluded. Moreover, it is not clear how the dimension of specialisation in the registers of representation applies to the situation of assessing test responses, since these are essentially assessed in degree of mathematical quality and to a less extent in specialisation of registers. So, though the dimension of registers of representation is relevant as a framework for assessment, the dimension of specialisation needs to be reworked.

The analytical framework discussed in part 1 gave some suggestions for assessing mathematical qualities in a student's response to a test item. Duval (2006) expressed the idea of distinguishing between the common, often iconic, way of looking at figures and the mathematical way of looking at figures. But, there are not only strict mathematical ways of looking at various representations. Sfard (2008) distinguished between enacted realisations that follow either a mathematical norm or an endorsed rule or neither. A similar idea is expressed in the overarching goals in the Swedish curriculum, namely that the student "can use mathematical reasoning for further studies and in everyday life" (Skolverket, 2011a, p. 15). This gives a societal context for interpreting what the norm and the endorsed rules are, namely that using mathematics *for everyday life* is close to Sfard's *endorsed rule*, while using mathematics *for further study* is close to Sfard's *norm*.

	Mathematically not appropriate	Mathematical for everyday life – endorsed	Mathematical for further study – norm	
	Erroneous	Relevant or similar but not	Correct but incomplete	Correct and complete
Graphical representation	Erroneous figure			Correct figure
Percentage representation	12.5%	10	20%	10%
Decimal representation	0.125		0.2	0.1
Fraction representation	$\frac{1}{2}$; $\frac{5}{2}$		$\frac{2}{10}$	$\frac{1}{10}$

Dimension of semiotic representation

Dimension of mathematical quality

Figure 9. A framework for assessing student responses, applied to responses to the test item 'What is half of $\frac{1}{5}$?'

In the framework in Figure 9, the horizontal dimension assesses the mathematical quality and the vertical dimension assesses the mode of representation. The framework allows several graphical, numerical and algebraic representations, and these may work in parallel, as illustrated in Figure 10.

Dimension of semiotic representation

	Mathematically not appropriate	Mathematical for everyday life – endorsed	Mathematical for further study – norm	
	Erroneous	Relevant or similar but not correct	Correct but incomplete	Correct and complete
Word representation		Mean or midrange of data		
Graphical representation	Median of axis marks	Median of vertical coordinates	Median of horizontal coordinates, but single data point missed.	Median of horizontal coordinates
Numerical representation		Error in calculation (when applicable)		

Dimension of mathematical quality

Figure 10. A framework for assessing student responses, applied to responses to the test item of determining the median from data in a bivariate scatter plot.

Aggregation of responses

All the students' responses to the test items were compiled into a database. In this database, the responses were aggregated into larger response categories using the framework illustrated in Figures 9 and 10. For example, responses to the test item in Table 7c on determining half of a fifth were aggregated into responses containing only correct fraction representation, partially correct responses 'two tenths' containing only correct fraction representation but not correct response, responses converting the fraction into the decimal number representation 0.2 followed by halving it, partially correct decimal representation responses such as 'stop at 0.2' and incorrect decimal representation responses such as 0.25 etc. There was percentage representation of 'half of 20% = 10%' etc. So in the aggregation there was the dimension of mathematical quality such as correct, partially correct and mathematically inappropriate. There was the dimension of representation form such as graphic, numeric and algebraic. Within, for example, numeric representation, there were numbers as fractions, decimals or percentages. Some responses were difficult to interpret and for this reason were left unclassified.

Interviews

For the aspect of representing and using mathematical concepts of the research question, the test items in Appendices I and II were used. This presupposes that the students' responses to the test items are detailed enough to make a valid interpretation. Now, an individual student's response in a written test may sometimes lack detailed information about representation of the concept in question. When this is the case, task-based interviews can be added in order to gain a valid interpretation and a more complete picture of the individual student's knowledge and familiarity in using some mathematical concept or representation form (Hurst, 2008). In the present study, some students for some test items gave only an answer or gave an unclear solution as written response. Some of these students were invited to be interviewed with the purpose of gaining more detailed information about which concept was underlying the representation form they had used. They were invited to comment on their solutions to the test problems. Interviews were used in paper IV (but not in paper III) since some responses were difficult to classify in the framework illustrated in Figure 10 based on only the written response.

According to the teachers of the students in this thesis, the individual students' participation in the ordinary classroom activities is important for the quality of the learning. Moreover, the teachers' wish was to keep the interruption of the teaching to a minimum during the interviews. This is an issue of research ethics of consent of participation. For this reason, the moments when students were interviewed were chosen to be essentially during breaks and during individual classroom work. With this research design, the researcher could keep to a minimum the intervention and interruption in the classroom and teaching moments. Moreover, from the students' perspective, the students did not have to miss any teaching, since most of them were focused on gaining good study results during their last semester in compulsory school.

According to the law on ethics for research on people (SFS 2003:460), they were interviewed after having giving their documented consent to being interviewed. A few of the invited students chose to not participate. The interviews were held within a few weeks or a month after the test. The interviews were electronically recorded. A few students preferred to participate with the microphone turned off, and in these cases, the interviewer took written notes instead. During the interviews, each student was first shown the test item in the test and asked to solve it. If the student's response during the interview was different from the one they gave during the test, the student was shown the test response and was asked to comment on how they think they solved the test item during the test. The interviews for each problem lasted from a few seconds to a few minutes. The length of the interviews is similar to those in Bakker and Hoffmann (2005, p 343) who conducted mini-interviews lasting from 20 seconds to 4 minutes. Another example is Herscovics and Linchevski (1994), who observiewed (interviewed and observed) 22 students, each during 2x45 minutes, solving 50 algebra problems, giving on average 2 minutes per problem. In their study, the students had no previous teaching in algebra, and in this sense the problems were of a mathematically unknown character and thus challenging. This is in contrast to the present study, in which the students before the interviews have both been taught all the mathematics that occurred in the test items and then have given written responses to all test items during a test.

The interviews in paper IV in the present study were inspired by task-based interviews. Task-based interviews are a special kind of clinical interview. In a task-based interview, the interviewee interacts with a purposefully designed task (Maher & Sigley, 2014). The intent is to gain information about the interviewee's representations and use of mathematical concepts (Goldin, 1997). For example, Hurst (2008) recommended task-based interviews as useful for helping students to recognise embedded mathematical concepts.

Kvale (1997) and Cohen and Manion (1994) see the interview as a professional inquiry and as an asymmetric dialogue. The asymmetry consists in the fact that the researcher has the power to intrude the interviewee with questions on the topic of the interview. Davis (1984) wrote that task-based interviews are difficult in the sense that the interviewer's question might lead the interviewee to reflect the researcher's ideas rather than their own. This is related to the asymmetry in power and status (Troyna & Carrington, 1989). The researcher has the roles of researcher and adult and from the individual student's perspective, possibly also as having the role of a teacher. Moreover, the interviewer might be a first language speaker, as in the present study, while the informant maybe has Swedish as second language. There is also the question of loyalty. This holds true, for example, when the researcher interviews the students, who may ask 'will you tell this to my teacher?', thus assuming the researcher to be loyal to the teacher instead of independent. Troyna and Carrington (1989) shared their experiences from research on educational science and gave a set of ethical advice to researchers in this area. The researcher must not have loyalty to some certain immigrant group or any certain group in school such as students, teachers or principal, but instead must have loyalty in fundamental principles of social justice and equity and participatory democracy. As a researcher, I followed this piece of advice.

In line with Kvale (1997), who gave several possible aspects of an interview situation, the following aspects were considered. The interviewer should keep the interviews descriptive and specific in the sense that the aim is to acquire detailed descriptions of the solution process used by the interviewee for some selected mathematical problem. The interviewer was sensitive to changes in showed reasoning and mathematical representations. The interviewer should encourage the interviewee to see the interview as a positive experience in terms of new insights in the mathematical knowledge of the interviewee. See also Cohen and Manion (1994).

Kvale (1997) suggested a battery of introductory questions, such as "Can you tell me about…!", "Describe…!", "Do you remember…?". In this study, these suggestions were often used as introductory questions. Another recommendation that was followed was to wait with direct questions till the end of the interview on a selected problem and close the whole interview by asking if the informant has any questions. Kvale gave examples of 'moving-on actions' intended that were used for the middle of the interview. Such moving on actions are nodding, humming, being quiet to give the interviewee time for reflection and asking "could you explain in more detail…?" and similar actions.

Original transcripts of interviews were written in Swedish. The oral language genre in the interviews was transcribed into written genre but still close to the original oral language. Kvale (1997) gave arguments for doing so: The oral interviews contained grammar errors, repetitions and slips of the tongue without mathematical meaning. To include them would not add to the mathematical message of the interview, but would make parts of the transcripts laborious to follow. Pauses shorter than 2 seconds were marked with dots (…) and pauses longer than 2 seconds were marked with their length, for example as (3 s pause). The interview transcripts and solutions of the students were labelled with tags. For example "6E12" means school 6, class E student 12.

Each interview could cover several different problems. The researcher constructed a database containing which interviewee who contributed with interesting aspects to which test item. This database was used in order to give structure to the interview data and to help identify common patterns in representations and concept use. Translation into English was done when the papers were written (Meaney, 2013).

Ethical aspects on data collection

When the data generation activities were planned, some ethical questions arose. The students would in fact get less ordinary lesson time due to interference of the lessons with data collection. Would the students regard this situation as if the researcher wins and the student loses? Would the students productively participate under such conditions? Could the data generation be designed as a win-win situation? The Aristotelian virtue ethics is contextual and emphasises integrity and cooperation with the praxis (Kvale, 1997). In the spirit of virtue ethics, the data generation through test was designed as a pre-national test that gave the students opportunity to feedback from their teachers about their strong and weak parts in mathematical knowledge. Moreover, the interviews gave the students an opportunity to have guided reflection on selected mathematical topics.

Non-participating students

The absent students in table 6 are those who took the national test but did not take the test designed in tables 7a, 7b and 7c and are thus absent in papers III and IV. In order to control for them, their results in the national test were analysed in the following way. Using the framework in Figure 10, their responses to the three national test items on data and chance explored in paper IV were classified into the categories given by the columns of table 8 as correct response, no response and other incorrect responses.

Table 8. Present and absent (in brackets) students' achievement on three national test items on data and chance.

Student category	Present (& Absent) students	Correct response	No response	Other incorrect response
Newly2L	23 (5)	52% (20%)	10% (40%)	38% (40%)
Early2L	67 (7)	44% (43%)	11% (0%)	44% (57%)
Other2L	56 (7)	52% (19%)	8% (19%)	40% (62%)
Swe1L	113 (20)	62% (65%)	7% (7%)	31% (28%)

Table 8 shows that the main differences between present and absent students are the following. Among Newly2L and Other2L the absent students had a smaller proportion of correct responses than the present students. Moreover, the absent Newly2L had a larger proportion than the present Newly2L of 'No responses'.

Summary of papers III & IV

Paper III

Paper III follows up the suggestion for further research in paper II, namely to partition second language students into sub-groups with a different duration of experience of the Swedish language. Paper III distinguishes between second language students who are newly arrived immigrants, that are early immigrants and other second language students that have immigrated before school start or not immigrated at all as defined in Table 3. The theoretical ground for doing so is Cummins' (2008) distinction between BICS and CALP. In paper III, the average Swedish language proficiency of the three student categories was different and in coherence with Cummins' distinction between BICS and CALP.

This task raises several methodological challenges. One challenge is that a mathematics test designed for newly arrived immigrants needs to have test item formulations that are linguistically simple and culturally familiar in order to test mathematics knowledge rather than to test knowledge in the second language or in the culture (Campbell et al., 2007). A second challenge is that each test item should be mathematically narrow enough to fit into one specific mathematical content area, as defined by, for example, TIMSS (Mullis et al., 2012). A third challenge is to collect a sample. Since newly arrived immigrants are few in numbers, a random sample of entire classes would require an unmanageably large sample. On the other hand, a purposive sample from schools with a large proportion of immigrant second language students may have properties that are different from a random sample. Examples of this are socio-economic background including parents' education and residential segregation (Hansson, 2012; Hansson & Gustafsson, 2011). The residential segregation may in turn lead to school segregation, since schools with a large proportion of second language students on average have a slightly lower ratio of qualified teachers and a slightly higher ratio of turnover of employed teachers (Skolverket, 2004). There is also a positive correlation between having a larger proportion of individual school work and schools having high proportions of students with an immigrant background or low socio-economic status (Hansson, 2012). In order to manage this, paper III suggested using results from the compulsory national test for comparing the purposive sample with the national random sample used for evaluating the national test.

With this outlined method, paper III presents the students' responses to essentially one specific test item, namely to determine "What is the half of 1/5 ?" The student categories were defined as in Table 3, and the students' responses were categorised with support of the model in Figure 9 inspired by Prediger et al. (2016). One outcome of paper III is that the newly arrived immigrants achieved a little bit better than early arrived immigrants, but what was more interesting was the different incorrect responses that some of the newly and early arrived immigrants made. Among the newly arrived immigrants, a common incorrect response was to answer '0.2' indicating an incomplete response. This response was most common among the newly immigrated students and less common the longer the students' experience of the Swedish language. Among the students, who gave this response, almost everybody correctly solved another test item, asking them to determine two thirds of sixty, numerically formulated as 2/3 of 60. Since these students could solve the numerically formulated test item and did set 1/5 = 0.2, but did not 'divide 0.2 by 2', these together were interpreted as if these students did not know the word 'half'. In the given test item, the word 'half' ('halv') in Swedish has an irregular declension 'hälften'. A category of mathematically inappropriate responses was 'incorrect halving', consisting of responses denominator/2, numerator/2, 0.5/2.5 and similar. This category of responses was more common among other second language students than among newly arrived immigrants and was interpreted as a difficulty in correctly applying 'half of' to a fraction. Moreover, newly arrived students showed a larger proportion of correct fraction responses than did early arrived students. The outcome of paper III was that newly and early arrived immigrants should be seen as two separate groups having different challenges when being taught and tested respectively on, for example, fractions in their second language Swedish. This occurred despite the problems in the present study not being word problems. Paper III was co-authored by Petersson and Norén. In the preparation of paper III, Petersson contributed with the research question, design of the study, collecting and analysing data. In the writing of paper III, Norén contributed with parts of the literature background and Petersson contributed with mathematical background, research question, method, results discussion, conclusion and parts of the literature study. Petersson and Norén together revised the paper.

Paper IV

With the method outlined in paper III, paper IV presents the students' achievements and responses to test items on statistical literacy such as extracting a single data point from diagrams, determine measures of location and using probability. Just as in paper III, the student categories in paper IV were those defined as in Table 3 based on Cummins' (2008) distinction between BICS and CALP and reflected in the students' language proficiency as showed in their leaving grades in Swedish language.

Three of the test items in Table 7b are on extracting a single data point from diagrams, two were on reading a bar diagram and one on reading a bivariate diagram. There were small achievement differences between all four student categories. However, there were some interesting patterns in the errors they made. In the bar diagram, the students should extract the precipitation from the month of July. In Swedish, July is spelled 'juli'. This is similar to 'juni', which is the Swedish word for June. Two newly immigrant students consistently confused these two months while no other second language student did so. In Figure 10, inspired by Prediger et al. (2016), this error is situated in the representation as a word. The task on reading from a bivariate scatter diagram was to extract the height of a person weighting 70 kg. For this test item, a majority (N=210 of 259) of the students gave an exact response. There were eighteen students, (zero newly arrived immigrants and six in each of the other three student categories) who gave a response that corresponds to 'the height of a person that weighs *about* 70 kg'. In a mathematics test, it could be expected to give an exact response unless an approximate response is explicitly asked for. In that sense, the approximate response should be interpreted as belonging to the box in Figure 10 of numerical representation and Mathematics for everyday life register instead of numerical representation and Mathematics for further study. From the students' ordinary national test, one test item was included in the data. This test item is not in Table 7b, since it is still under secrecy, but is on interpreting a frequency table. This test item is related to determining a weighted arithmetic mean, which earlier research has found to be more difficult than an average of data without weights (Pollatsek, Lima & Well, 1981). For this test item, one result in paper IV is that the newly arrived immigrants had a higher proportion of correct responses and a lower proportion of no response (left blanks) when compared with the other two categories of second language students.

Two test items in Table 7b were on determining a measure of location, which on earlier national tests has been found to be more difficult for second language student than for first language students. These were: Determine the median of a list of signed integers; and determine the median of data in a bivariate diagram. One contrast in the students' responses to these two test items was that the newly arrived immigrants compared with the other two categories of second language students had a higher proportion of correct responses, but also a higher proportion of no response (left blanks). This result could be interpreted in the light of paper I in the present study, where it was reported that the 'median' is a concept for which second language students have achieved low, and the suggested explanation was that the median concept seems to get little space in the mathematics teaching. Together, paper I and these results in paper IV suggest that a reasonable working hypothesis is that newly immigrant students may have less access to teaching on ‘the median’. The infrequent occurrences of the median concept suggest classifying it in box in Figure 10 as a word representation and Mathematics for further study.

From the students’ ordinary national test, two test items was included in the data. These were on probability. There were minor differences between early and newly arrived immigrants, and both categories performed less well than the first language students. However, among the newly arrived students there were notable proportions of responses, which were interpreted as if the students had not fully understood what the test item asked for.

Discussion

Along with the aim of exploring the second language students' mathematics achievement in Swedish compulsory school, this thesis asked research questions on comparing how first and second language students achieve in and respond to mathematics test items. Doing so, this thesis focused on the learning outcome of second language students as seen through tests and did not focus on the communicative acts performed in the learning context of the mathematics classroom, in which the first and second languages students participated. The first part of this thesis divided the students into first and second language students, which on average correspond to students having different proficiency in Swedish language. The second part of this thesis made a more fine grained division of the second language students into the following three sub-categories: newly arrived and early arrived students and other second language students. These three groups of second language students, together with the first language students, on average correspond to students having different proficiency in Swedish language. In one end there were those having only BICS (defined by Cummins, 2008 as Basic Interpersonal Communicative Skills) in Swedish and in the other end there were those having CALP in Swedish (Cognitive Academic Linguistic Proficiency skills, Cummins, 2008).

An outcome of the present study is the following: Newly and early arrived immigrants appeared as two separate groups with respect to educational history and language proficiency. In tests, the newly arrived students seemed more challenged by terminology but less so by the mathematical content while the opposite seemed to hold true for the early arrived students, despite the problems in the present study not being word problems. The main scientific contributions of paper I was to discover that mathematical concepts that occur rarely in teaching and textbooks are the same concepts for which second language students achieve significantly below the level of first language students. The main scientific contributions of paper II was to discover that Swedish first and second language students have different achievement profiles in mathematics as seen through their results on the Swedish national test in mathematics. With this also followed the development of a new measure of achievement profile as achievement in one content area minus achievement in another achievement content area, with content areas defined as in TIMSS (Mullis et al., 2012). The main scientific contributions of papers III and IV were to separate between newly and early arrived students, having Swedish as second language, and conclude that these two student categories seem to face different challenges when tested in different mathematical content areas. This indirectly implies that these two student categories also face different challenges as they participate in the mathematics classroom activities.

Discussion of paper I

Part 1 of this thesis asked about achievement profiles on a test item level and on a mathematical content area level. Paper I compared first and second language students' national test achievements on a test item level, and paper II did the same on a mathematical content area level. Though differences in total achievements between first and second language student categories were expected, one interesting result was that these differences were non-uniformly distributed.

In detail, paper I found that the differences for arithmetic test items were smaller for typical standard problems and larger when the test item contained a concept, which only occasionally occurs in the Swedish mathematics textbooks and teaching. Examples of such test items, explored in paper I, are the median, decimal hours, scale and arithmetic with negative numbers. Paper I suggested that some second language students may not have had an equal opportunity when compared with first language students to participate in the rare lessons on these concepts. Paper I suggested two reasons for this result.

The first reason suggested is that the mathematical term 'the median', occurring rarely, could be more unknown among second language students. The second reason suggested is that some of the second language students are newly arrived immigrants that have started to participate in the ordinary mathematics lessons after the concept has been covered in the mathematics textbooks and the teaching, and thus may have missed the instruction on the concept of median. In the research literature, models for explaining similar situations are found in Campbell et al. (2007), Lager (2006) and Ufer et al. (2013). One explaining model is the test item language. It could be the test item formulation that is complicated (Campbell et al., 2007). As suggested in paper I, it could be specific mathematical terms, such as 'the median', being unfamiliar (Lager, 2006). It may also be that some of the immigrated second language students have not met the median concept in their countries of previous schooling, but this was not discussed in paper I.

Another explaining model is if the mathematical content in a test item is conceptually demanding (Campbell et al., 2007; Ufer et al., 2013). For example, the erroneous responses to the two test items on arithmetic with negative numbers and with decimal hours showed that most students had correctly understood the language in the test item formulations as a difference of temperatures and as a conversion between hours and minutes. The errors were typically mathematical and not linguistic. This shows that it was the mathematical content that was conceptually demanding for second language students, while first language students achieved well on these test items.

A subsequent question, discussed in paper I, is why the concept of the arithmetic with negative numbers is significantly more difficult for second language students than for first language students. The suggestion in paper I is access to teaching on rare concepts. Second language students may, to a larger extent, follow a 'basic course' in mathematics instead of an 'advanced course' in the teaching in the same classroom. For example, Emanuelsson and Sahlström (2006) stated that the Swedish curriculum suggests heterogeneous classes to be managed by individualisation. Another aspect of this is that on average high achieving classes spent more time on the mathematical content aspect of the mathematics curriculum, and on average low achieving classes spent more time on of the citizenship aspect of the mathematics curriculum. Now, being a second language student correlates with lower achievement and residential segregation (Hansson, 2012; Hansson & Gustafsson, 2011). Moreover, Hansson (2012) and Hansson and Gustafsson (2011) found that the organisation of the teaching is different in high achieving and low achieving classes. To conclude, it seems that first and second language students on average have different access to teaching on the same mathematical concepts.

Discussion of paper II

While paper I explored achievement profiles of first and second language student on test item level, paper II did this on a mathematical content area level. This needs a definition of 'achievement profile', and paper II used the one defined in part 1 of this thesis. Now, some second language students are immigrants that have experienced teaching following some other curriculum than the Swedish one for a shorter or longer time. These students might flavour the shape of the achievement profile of the second language students. In order to compare and contrast Swedish first and second language students' achievement profiles with other countries, the TIMSS definition of mathematical content areas was used (Mullis et al., 2008; Mullis et al., 2012).

One condition for the difference in achievement profile between first and second language students to be detectable is that there are sufficient second language students with an achievement profile that is clearly different from the first language students. Now, paper II found the knowledge profiles of first and second languages students to be detectable and significantly different. The second language students in paper II showed a smaller gap in achievement differences between the mathematical content areas algebra and number than did the first language students.

For the following reason, this result is surprising if we see it only from the perspective of the students as being second language students or only through the perspective of the students as having an on average lower socio-economic background. In the study by Ufer et al. (2013), the second language students achieved similarly to first language students on items testing algorithmic skills but less well than first language students on cognitively more demanding test items. If we assume that test items, in national tests, for which all the four student categories on average achieve poor results, are cognitively demanding, we could expect a larger gap between first and second language students for test items with a low proportion of correct answers and a smaller gap for test items with a high proportion of correct answers. Contrary to this expectation, paper II showed no such trend. Moreover, the second language students achieved similarly to first language students on some of the algebra test items with a low proportion of correct responses.

To explain this result, paper II suggested the following hypothesis; that among the second language students, some are newly arrived immigrants with another achievement profile due to experiences from following some non-Swedish curriculum. Support for this hypothesis was found in Petersson (2013b) for the case of algebra achievement. For the case of fraction, the hypothesis in paper II was explored and confirmed as described in paper III.

Discussion of paper III

Paper III set out to explore the hypothesis in paper II of separating between newly and early arrived second language students. Specifically, paper III explored students' responses to the written test item of determining half of a fifth using the analytical framework in Figure 9 (p. 89). This analytical framework allows categorising a student's response in both the dimensions of representation and of mathematical quality.

The result was that the different student categories in Table 3 (p. 75) had significantly different proportions of response formats in two dimensions given in Figure 9 (p. 89). One dimension is the representations, essentially numerical, as fraction, decimal, percentage. The other dimension is mathematical quality as mathematically inappropriate response, mathematically correct but incomplete response and correct and complete response. While newly arrived immigrants less often made mathematical errors, but lacked sufficient vocabulary to understand the task in the test item of halving a fifth, the early arrived immigrants instead more often gave a response corresponding to a mathematically erroneous use of the tested concept. In short, Paper III showed that among the responses that were not mathematically correct and complete, there was on average a difference in mathematical quality between newly and early immigrants, as illustrated in Figure 9.

The result in paper III distinguishing between newly and early arrived and other second language students makes the view of second language students more complex. Despite, for example, Cummins' (2008) threshold hypothesis of BICS and CALP, it seems to be standard to report students in multilingual classrooms as being either first or second language students. Methodologically, this imposes a view of second language students as one homogeneous group. TIMSS allows a more fine-grained description, since the students participating in TIMSS report themselves as using the language of test at home never, seldom, often or always (Mullis et al., 2008; Mullis et al., 2012). However, TIMSS aggregate these responses into the two categories of first and second language students. Moreover, Böhlmark (2008) found that the closer to the last year of compulsory school that the students immigrate to Sweden, the lower are their average leaving grades.

The newly arrived students that participated in the study for paper III achieved better results than the early immigrants, despite a weaker knowledge of Swedish language and immigrating closer to the last year of compulsory school. This shows that there are more factors in describing the students' complex situation in the mathematics classroom than the students' *present-day* situation with respect to language status (e.g. BICS and CALP by Cummins, 2008) and socio-economic status (e.g. Hansson, 2010, 2012; Zevenbergen, 2001). The individual student's *educational history* seems to be important as well (Bishop, 1991; Cummins, 1979; Giannelli & Rapallini, 2016: Li & Ginsburg, 2006). Moreover, a good knowledge of fractions is considered as a gatekeeper for success in algebra learning (Post, Behr & Lesh, 1988). This effect was noted as higher achievements in both fractions (paper III) and algebra (Petersson, 2013b) for newly arrived students than for early arrived students. The role of the students' educational history was also illustrated in Petersson (2013a), where the newly arrived second language students achieved better than all the other student categories including first language students on correctly calculating $12 - 23 + 9$. This example contains a 'detached minus sign', which is known to be difficult (Herscovics & Linchevski, 1994).

Discussion of paper IV

Similarly to paper III, paper IV set out to explore the hypothesis in paper II of separating between newly and early arrived second language students for the content area of data and chance. The student categories were the same as in paper III and defined in Table 3. The analytical framework was the same as in paper III, but adapted to the test items on statistics and probability as illustrated in Figure 10.

Paper IV showed no large achievement difference between newly and early arrived students, though there were some differences in the way the students responded when their responses were categorised as illustrated in Figure 10. Some of these differences were related to the use of language. One example is that, of the second language students, only two and both newly arrived, consistently confused the words for the months ‘juni’ and ‘juli’ (in English June and July). Using the analytic model in Figure 10, a reasonable interpretation of this kind of error is as a mathematically correct treatment given confusion in word representation. With this interpretation, the error occurred in the representation as word in the models in Figures 8 (Prediger et al., 2016) and 12 and might be connected with knowing language at the level of BICS (Cummins, 2008).

A second example is that newly arrived immigrants often left blank on test items on the median, but still achieved slightly higher than early arrived students. The reason for this is unclear, but two possible interpretations are the following. It could be that some of these students did not know the word ‘median’ just as some students in paper III did not know the word ‘hälften’ (English ‘half’), and it could also be that some students do not know the concept of median, likely for reasons discussed in paper I. However, that newly arrived students on average scored higher than early arrived and other second language students may suggest that some have encountered this concept earlier. It may possibly be an example of transferring the term for this concept from their CALP in their first language to their second language used in school, as suggested in Cummins’ (1979) interdependence hypothesis.

Though Pimm (1987) did not see mathematics as a language, he successfully applied linguistic concepts on mathematical communication. In a formal mathematical context, a number is typically interpreted as exactly that number, while in an informal context; the same number could correspond to an approximate value of that number. Both the formal and informal interpretations were present in the test item giving a bivariate diagram and asking to find the height of a person with the weight of 70 kg. Among all student categories except newly arrived immigrants, there were a handful of responses showing a concept use of ‘about 70 kg’ when reading the diagram in the test item. This result suggests that the informal interpretation might be more common among student categories with a longer experience of the Swedish mathematics curriculum.

The small difference between newly and early arrived students in achievement in the mathematical content area of data and chance can be interpreted as follows. Olsen (2006) identified a Nordic achievement profile. In TIMSS 8th grade, for example, the Nordic countries have an achievement profile with a peak for the mathematical content area of data and chance as illustrated in Figure 2 (p. 25) (Mullis et al., 2008; Mullis et al., 2012). In contrast, several of the countries in Eastern Europe and the Middle East have a valley for this content area. Now, early arrived second language students may experience added challenges in learning mathematics (Campbell et al., 2007). At the same time, those who are newly arrived may bring knowledge from having followed some curriculum with less emphasis on data and chance (Bishop, 1991; Giannelli & Rapallini, 2016). Together, this may contribute to both newly arrived, early arrived and other second language students achieving poor results in this content area. To this should be added that the reasons for the poor results of newly and early arrived students seemed to not be the same. The newly arrived students had a larger proportion of 'no response' than other student categories on the two test items on the median and on one of the test items on chance. Moreover, on the other test item on chance, the newly arrived students had a large proportion of responses, which was interpreted as if they did not fully understand the task of the test item. For the same test items, the early arrived students had larger proportions of other mathematically inappropriate responses. Together this suggests that the early arrived students had understood the task in the test item but have vague concepts about the median and chance, possibly due to having been taught these concepts in their second language in which they had not reached CALP proficiency. The large proportion of 'no response' among the newly arrived students could be interpreted as if several of them either did not understand the language in the test items or had not received instruction about the median concept. An analysis of the non-participating students in table 8 shows similar results. This suggests that the results for paper IV would be similar if the non-participating students also would have participated in the study.

Three perspectives on a student's background

The research background in the present study showed that there is a large body of research on second language students from the perspectives of socio-economic background in part 1 of this thesis and of multilingualism in part 2 of this thesis. A third perspective, which this thesis showed to be important, is to acknowledge the immigrant's educational history by following up what previous experiences and knowledge, the immigrated student, in Sweden typically a second language student, brings into the mathematics classroom. These three perspectives are briefly reviewed below.

One aspect of describing students' school success is their socio-economic background. In order to improve the accuracy of the model Pásztor (2008) added variables to this aspect describing the school situation. A critique against using students' socio-economic background as a variable for describing school success is that it has low explanatory power. Ufer et al. (2013) found that, when compared with general school achievement and language proficiency, the socio-economic background lost its explanatory power already in the second school year. One way to reformulate this result is to say that a child's school success is better modelled by the child's own educational history than the child's parents' educational level. The result by Ufer et al. (2013) supports the conclusion that previous school achievement has a higher predictive value than, for example, the parents' education (in a similar way to that a weather forecast using one-day-old data gives a better predication than when using several days old data). Another way is to compare the activities in socio-economically different mathematics classrooms, which Emanuelsson and Sahlström (2006), Hansson (2010, 2012) and Zevenbergen (2001) did. They saw segregation in teaching styles in classrooms with different socio-economic status. This means that the individual students' socio-economic background may come in implicitly into the mathematics classroom on a group level.

A second aspect of the student's school success is that of proficiency in the language of instruction, for example in terms of BICS and CALP (Cummins, 2008). Due to being second language students, these students face added challenges in the learning context (Campbell et al., 2007; Cuevas, 1984; Lager, 2006; Parszyk, 1999). The outcome of these added challenges play out in test outcome (Mullis et al., 2008; Mullis et al., 2012; Skolverket, 2004, 2012; Ufer et al., 2013). With the background of being a second language student, unfamiliarity with the cultural context in tasks and test items may also follow, adding to the learning challenges (Campbell et al., 2007). However, in classrooms where there are several second language students having the same first language, there may be opportunities for various ways of using multilingualism. For example, the students may use both their first and second languages in the classroom but for different purposes (Garcia, 2011; Moschkovich, 2002; Planas & Setati, 2009; Setati, 2005; Setati & Adler, 2000). The students may use their first language to discuss conceptual matters and their second language to discuss procedural matters. Moreover, being a second language student is not a uniform or fixed state. The second language students may be more or less proficient in the language of instruction (Clarkson, 1992, 2007, 2009), and they develop proficiency in the second language from BICS to CALP over time (Cummins, 2008).

A third perspective is the individual student's educational history. A second language student who has immigrated in later school years brings an accumulated body of knowledge into the new mathematics classroom (Bishop, 1991; Giannelli & Rapallini, 2016). This third perspective complements the two perspectives of socio-economic background and multilingualism, since a migration may sometimes cause a student from being a first language student with one socio-economic background to become a second language student with some other socio-economic background. Thus, modelling school success of immigrant students in school age using the new and present linguistic and socio-economic background may ignore the educational history of the student as a likely first language student. Moreover, the migration may sometimes lead to a change from one curriculum and teaching tradition to another, since these may vary between countries and regions (Li &

Ginsburg, 2006). Ignoring differences in curriculum and classroom experiences between countries, when studying immigrants, methodologically assumes curricula in different countries to be uniform. This is not the case (Bessot & Comiti, 2006; Fan, 1999; Li & Ginsburg, 2006; Son & Senk, 2010; Wu & Zhang, 2006). In acknowledging the experiences and the knowledge, which an immigrated second language student brings into the mathematics classroom, it follows that Cummins' (1979) interdependence hypothesis may be applicable: The CALP is transferrable between languages. The second language student, who has immigrated in late school years, may, for some test items, only need to learn the terminology in the second language. In such cases, they may have an advantage over classmates, who have immigrated earlier in school and thus having to learn both the terminology and the concept in the second language. So again, modelling school success of immigrant students of school age using only the new and present situation may ignore the educational history of the student. Moreover, when people communicate mathematical ideas, they use more than only natural language. They use several representational registers exemplified in figure 5 (p. 61). For example, the subtraction tasks in text 1 given in North Sámi (p. 56) were probably understandable through the numerical register despite the text, given through the linguistic register, was not understandable for most readers. This illustrates that the language dimension of BICS and CALP is obtuse as a single tool for assessing proficiency in mathematical communication.

Language as data or as a background variable

The research carried out in the present thesis and its four papers has focused on the students' responses to and achievement in mathematics tests. Moreover, the test item formulations in part two in this thesis were chosen to be linguistically uncomplicated with the purpose of reducing the language proficiency needed to understand the test items. The students were categorised after a combination of language status and migration status. The categories for language status were as first and second language students. The categories for migration status were newly arrived second language students, early arrived second language students and other second language students defined as in table 3. Doing so, this thesis interpreted the results, that is, what they brought out from it in terms of test results, as related to what the students had brought into their present mathematics classroom from previous experiences – their educational history. The mathematics learning classroom itself was methodologically considered to be a black box since no classroom observations were made. Moreover, natural language communication seen as first or second language is not a part of the data, but comes in indirectly for categorising the students. This may raise the question of where is the second language in this thesis and I try to answer this as follows.

We can compare the research methods used in this thesis with those commonly used for research on multilingual mathematics classrooms. When doing research on multilingual mathematics classrooms, it is common to use direct observations of the communicative acts in the learning environment such as classroom observations and interviews with students (e.g. Canagarajah, 2011; Clarkson, 2009; Garcia, 2011; Moschkovich, 2002; Planas & Setati, 2009). There have also been studies focusing the role of language in test item formulations when taking a test in the second language (e.g. Barwell, 2009b; Campbell et al., 2007; Gerofsky, 2006; Heesch et al., 2000; Lager, 2006; Norén & Andersson, 2016). In such studies, the (written) language that is the second language is clearly a part of the data collected and plays the main role of when interpreting the results.

Despite the invisibility of the second language in data in the present study, the interpretation of the results makes claims about the opportunities to learn mathematics for both newly and early arrived second language immigrant students. To summarize, we can categorise research on multilingual mathematics classroom into type I or type II depending on whether they use or do not use language itself as data. Type I research on multilingual mathematics classroom analyses the second language itself in the communicative acts when learning and taking tests in mathematics. Type II research on multilingual mathematics classroom analyse what has happened before entering or after exiting the mathematics classroom. Besides papers I–IV in this thesis, the interview study by Svensson, Meaney and Norén (2014) can also be classified as type II research on multilingual mathematics classroom, since it studied second language students' opinions about support with homework. Other common type II research on multilingual mathematics classrooms are comparative studies on both lesson structure (Hansson, 2012, Hansson & Gustafsson, 2011) and test achievement (Elmeroth, 1997; Giannelli & Rapallini, 2016; Ufer et al., 2013).

Implications for teaching and for further research

This thesis, from theory and empirical data, exemplified that specifically newly and early second language students experience different challenges in the mathematics tests. The short timed test used in papers III and IV gave a snapshot of the situation of Newly2L and Early2L. As exemplified in paper III, newly arrived second language students may bring useful knowledge from previous schooling, probably in their first language. Thus, they may make use of transferring their mathematical CALP from their first to their second language, though the terminology may challenge them. Second language students that arrive in early school years will have to build their mathematical foundation in their second language, which they in the beginning master at a BICS level. Thus, they are obstructed in mathematics learning, due to both unfamiliar terminology and learning new concepts in a second language. Just as second language students in Ufer et al. (2013) underachieved on cognitively demanding tasks, the early arrived students and the other second language students achieved less well than the newly arrived students on the test items discussed in paper III.

Under this assumption, particularly early arrived second language children seem to be in urgent need of support in mathematical concept building from the first day of schooling in the new country. This gives the outcome of the present study a political dimension, as discussed in Romberg (1992). Such a support may help children that immigrate in early school years to overcome everyday conceptualisations of mathematical ideas and to develop school conceptualisations. To explore how this would be efficiently done in the daily practices in the multilingual mathematics classroom needs more research, for example using observational data and perhaps intervention studies. Empirical data in the present study was students' written responses to test items. The data in this thesis, being diagrammatic, fit well into the drawn and written representations in the model in Figure 5 (p. 61) by Prediger and Wessel (2011). If a study on the same topic as in the present study instead used data from observations of the mathematics classroom, all representations in the model in Figure 5 would have to be included.

An explicit suggestion for further research on teaching mathematics in multilingual classrooms would be to explore how teaching using figurative arithmetic and algebra could support a clearer visual representation of mathematics and at the same time be less dependent on natural language and numerical-algorithmic representations as main representations. For example, a geometric model in multi-digit multiplication gives a visually clear and mathematically precise and transparent illustration of both the distributive law and the commutative law, while these laws might be difficult to explain and much less transparent within the representations of words and numbers for some vertical standard algorithm. For this suggestion for further research, one assumption is that engaging multiple forms of representations may promote conceptual proficiency.

An explicit suggestion for further research on assessing mathematics in multilingual classrooms would be the following: To explore how using both the dimensions of mathematical quality and representation form in Figures 9 and 10 may help understanding students' responses to test items as being formally or informally mathematical or being mathematically inappropriate or if another representation form could better support the individual student's mathematical work. This kind of interpretation may support the teacher in helping individual students to learn the concept and also as a catalyst for reflecting on alternatives for improving the teacher's own practice in the multilingual mathematics classroom (Clarke, Keitel & Shimizu, 2006b). The citation in the section of earlier research can be repeated: There is ‘still much to do in research’ when it comes to multilingualism (Meyer, Prediger, César & Norén, 2016).

Svensk sammanfattning

Introduktion

I Sverige finns det för närvarande en livlig debatt, både inom och utanför akademin, om invandrarelevers jämlikhet avseende tillgång till skolan (t. ex. Bunar, 2010). Mer ambitiöst än så handlar debatten om jämlikhet mellan förstaspråkare och andraspråkare (det vill säga elever som följer grundskolans kurser svenska respektive svenska som andraspråk) avseende skolresultat. Denna debatt är viktig eftersom en av uppgifterna för den svenska skolan är att vara kompensatorisk, det vill säga att stödja missgynnade elevgrupper. Som en del i denna debatt syftar denna avhandling till att utforska invandrade andraspråkares studieresultat i matematik i deras fortsatta skolgång i svensk grundskola. Särskilt syftar denna avhandling på att belysa undergrupper av andraspråkare, såsom nyanlända, tidigt anlända och övriga andraspråkare.

Denna avhandling är uppdelad i två delar med liknande disposition: En forskningsgenomgång, som mynnar ut i en forskningsfråga följt av metodavsnitt och sammanfattning av delens artiklar. Del 1 är en jämförande studie om förstaspråkares och andraspråkars provresultat och mynnar ut i artikel I och II. Även del 2 är en jämförande studie, men med fokus på flerspråkighet, där nyanlända immigranters och tidigt anlända immigranters provresultat jämförs och mynnar ut i artikel III och IV. Efter detta kommer avhandlingens diskussionsavsnitt.

Denna avhandling ger två huvudresultat. I del 1 är det att förstaspråkare och andraspråkare visar olika kunskapsprofiler på prov i matematik. I del 2 är huvudresultatet att elevers kunskaper i matematik verkar variera beroende på om eleverna har invandrat tidigt eller sent till grundskolan. Det visar sig vara två grupper med olika behov och förutsättningar. Trots att de senare är nybörjare i sitt nuvarande undervisningsspråk, så kan de i en del avseenden dra fördel av sin tidigare skolgång som matematikelever på sitt förstaspråk samt att de kan ha följt en läroplan, som betonar matematiska kunskapsområden annorlunda än den svenska läroplanen. Omvänt har andraspråkare, som har invandrat tidigt i sin skolgång, bättre kunskaper i det nya undervisningsspråket, men har också nackdelen av att ha fått lära sig grunderna i matematik på sitt andraspråk under större delen av sin skolgång. Genom att skilja mellan andraspråkare som invandrade tidigt respektive sent i grundskolan, konstaterar denna avhandling att det inte är nog att ta hänsyn till socioekonomisk bakgrund och kompetens i undervisningsspråket. Ett nödvändigt tredje perspektiv blir att ta med den matematiska kompetens som särskilt nyinvandrade elever kan ha med sig från tidigare skolgång på sitt första språk.

Forskningsbakgrund om jämförande studier

Egidius (2006) definierar jämförande (komparativa) studier som studiet av likheter och skillnader mellan skolsystem och kulturgrupper, där kulturgrupper avser traditioner och attityder som karakteriserar skolans praktik. Viktigt för särskilt artikel II i denna avhandling är att elever från olika länder visar olika resultatprofiler på matematikprov, se figur 2. I exempelvis TIMSS (Mullis, Martin & Foy, 2008) visar svenska elever en lägre lösningsproportion i algebra än i taluppfattning och statistik/sannolikhetslära. En resultatprofil på matematikprov beskriver formen på fördelningen av lösningsproportionerna för olika matematiska kunskapsområden och kan definieras exempelvis som en differens av lösningsproportionen i två olika matematiska kunskapsområden. En styrka hos resultatprofilerprofiler på matematikprov är att de inte jämför absoluta resultat hos en elevgrupp utan jämför hur olika elevgruppers provprestation fördelar sig över olika matematiska kunskapsområden. Det torde göra resultatprofiler på matematikprov mindre känsliga för exempelvis socioekonomiska skillnader. Just elevernas socioekonomiska bakgrund är en faktor som korrelerar med elevernas skolprestation om än med stor individuell variation. En mekanism bakom dessa skillnader är att klassrum som domineras av elever med lägre socioekonomisk bakgrund kan ha en annan form på interaktionen i klassrummet än klasser som domineras av elever med högre socioekonomisk status (Emanuelsson & Sahlström, 2006; Hansson 2010, 2012; Zevenbergen, 2001). Exempelvis kan innehållet i turtagningen 'lärare initierar – elever svarar – lärare återkopplar' se olika ut i dessa olika klassrum liksom även fördelningen av genomgångar och enskilt elevarbete.

Jämförande studier har också gjorts på förstaspråkare och andraspråkare i samma skolsystem. Dels kan dessa elevgrupper i genomsnitt ha olika socioekonomisk bakgrund (Elmeroth, 1997; Hansson, 2010, 2012; Skolverket, 2004). Dels finns det en geografisk komponent där elever invandrade från olika länder i genomsnitt presterar olika (Böhlmark, 2008; Elmeroth, 1997; Giannelli & Rapallini, 2016; Xenofontos, 2015). Dels finns det enålderskomponent där elever i genomsnitt får lägre avgångsbetyg ju senare i skolåldern de har invandrat (Böhlmark, 2008). Det finns även en skolkomponent där skolor med stor andel andraspråkare i genomsnitt har något högre personalomsättning och något lägre andel behöriga lärare (Skolverket, 2004). Ufer, Reiss och Mehringer (2013) fann att även djupet i kunskapen skilde sig mellan förstaspråkselever och andraspråkselever. I deras studie lyckades dessa två elevgrupper ungefär lika bra på rutinuppgifter medan det fanns statistiskt signifikanta skillnader för begreppsmässigt mer krävande provuppgifter.

Forskningsbakgrund om flerspråkighet

Att vara andraspråkare är inget entydigt begrepp. TIMSS låter elever besvara enkäter om hur ofta eleverna använder undervisningsspråket. Elever som anger att de hemma använder undervisningsspråket alltid eller ofta kategoriseras som förstaspråkare medan elever som anger att de sällan eller aldrig använder undervisningsspråket hemma kategoriseras som andraspråkare. Att ha svenska som andraspråk i svenska skolan regleras av skollagen (Grundskoleförordning, 1994; Skolförordning, 2011). Dessutom kan en elev som är andraspråkare i skolan vara förstaspråkare i andra sammanhang såsom hemma eller under skolgårdens raster (Barwell m.fl., 2016; Xenofontos, 2015). Det finns ett spektrum av attityder till hur man hanterar flerspråkighet i klassrummet. Exempel på dessa är om skolsystemet uppmuntrar till flerspråkighet i matematikundervisningen och hur flerspråkighet praktiskt kan hanteras i det enskilda klassrummet.

För att förstå situationen för andraspråkare i matematikundervisningen, har ett antal teoretiska modeller utvecklats. Några av dessa beskrivs här kort. En modell är att inte se andraspråklighet som något antingen eller, utan istället se det som olika grader av färdighet i såväl förstaspråket som andraspråket (Clarkson, 1992; Cummins, 2008). Denna modell understryker att det är en stor skillnad på att ha svaga kunskaper i både förstaspråket och andraspråket respektive att ha goda kunskaper i både förstaspråket och andraspråket. En annan modell är språkliga register, vilket Pimm (1987) med framgång tillämpade på matematisk kommunikation. En huvudtanke i språkliga register är att samma ord kan ha olika betydelse i matematik och vissa andra ämnesspecifika eller vardagliga sammanhang. Ett exempel är ordet volym, som beroende på sammanhanget kan betyda exempelvis ljudstyrka, geometrisk volym samt volym som mängd varor (försäljningsvolym). En kraftig utvidgning av att se matematisk kommunikation som en språklig aktivitet är att också tillåta andra uttrycksformer såsom konkreta material (exempelvis miniräknare och gradskivor) och handlingar liksom symboler (exempelvis figurer, numeriska och algebraiska). De teoretiska modellerna i figurerna 5 och 8 utvecklades för att beskriva just detta och modellen i figur 5 passar även för att beskriva flerspråkig matematikundervisning (Prediger m.fl., 2016). Alla elever kan behöva växla mellan flera språkliga register och matematiska representationsformer, men andraspråkseleverna kan också behöva växla mellan förstaspråk och undervisningsspråk under lektionsarbete och prov. Detta innebär ett merarbete för andraspråkarna (se exempelvis Barwell, 2009b; Gerofsky, 2006, Lager, 2006; Norén & Andersson, 2016). Det extra merarbetet, som andraspråkselever möter kan modelleras som en ökad kognitiv belastning (Paas m.fl., 2003). Just för matematikundervisningen sker den ökade belastningen när andraspråkare försöker förstå en uppgiftstext och när invandrare försöker förstå uppgiftens kulturella kontext, vilket illustreras i figur 6 (Campbell m.fl., 2007). Att betrakta den matematiska kommunikationen genom inte bara naturligt språk som uttrycksformer utan även genom andra uttrycksformer (figur 8 samt Duval, 2006; Pimm, 1987), gör det möjligt att förskjuta tyngdpunkten från att betona andraspråkares brister i undervisningsspråket till att se dem som medverkande aktiva deltagare i en matematisk diskurs (Moschkovich, 2002).

Forskningsfrågor

Givet forskningsbakgrunden formulerades följande två forskningsfrågor:

> En forskningsfråga i avhandlingens del 1 är om resultatprofiler på matematikprov: I genomsnitt presterar förstaspråkare och andraspråkare olika på nationella prov. Hur ser dessa genomsnittliga skillnader ut på enskilda provuppgifter? Hur ser prestationsprofilerna på matematikprov ut för förstaspråkare och andraspråkare?
>
> En forskningsfråga i avhandlingens del 2 är om representationer och register i matematisk kommunikation: Elever i samma klassrum kan ha olika lång erfarenhet av svensk skolgång. Förstaspråkselever har i de flesta fall hela sin skolgång enligt svensk läroplan. Vissa andraspråkselever kan också ha kortare eller längre skolgång enligt någon annan läroplan och då kanske som förstaspråkselever. När andraspråkare med olika lång erfarenhet av undervisning i svensk skola jämförs med varandra och med förstaspråkare, hur presterar de på matematikprov och hur representerar och använder dessa olika elevgrupper matematiska begrepp?

Analytiskt ramverk

För båda forskningsfrågorna är det rimligt att använda skriftlig kommunikation; mer specifikt skriftliga prov. Ett ramverk som passar detta är ett som omfattar flera matematiska uttrycksformer (Bakker & Hoffmann, 2005). Semiotik enligt Peirce består av tre delar, nämligen begreppet självt, en realisation av begreppet samt en tolkare av realisationen. I en provsituation är realisationen elevens skriftliga svar. Tolkningen av elevens provsvar kan vara en rik källa till information vid bedömning och det gäller inte minst när eleven har använt flera olika register (Duval, 2006; Dörfler, 2006; Sfard, 2008).

Sammanfattning av studiens publikationer

Publikation I

Den första artikeln syftade till att undersöka om förstaspråkare och andraspråkare i genomsnitt har annorlunda lösningsproportion på enskilda provuppgifter och möjliga orsaker till detta. Data i den första artikeln bestod av resultaten från 2009 års nationella prov i matematik för årskurs 9. Data samlades in av PRIM-gruppen som ett riksomfattande slumpmässigt urval för utvärdering av det nationella provet. Den första datamängden bestod av kopior av lärarbedömda elevlösningar. Den andra datamängden bestod av provresultat med poäng per provuppgift. Eleverna delades in i kategorierna förstaspråkare och andraspråkare.

Den första datamängden med elevlösningar gjorde det möjligt att få information om vilka typer av fel, som eleverna gjorde. Ett exempel är provuppgiften att bestämma hur många minuter 0,75 h är (provuppgift nummer 4 i del B1 i nationella provet i matematik 2009). Av de felaktiga svaren var några ofullständiga, såsom ’15 minuter’. Andra felaktiga svar var '75 minuter’ eller ’7,5 minuter’, vilket indikerar att eleven såg provuppgiften som ungefär som att konvertera mellan meter och centimeter. Ett annat exempel var att bestämma den största temperaturskillnaden i en tabell med positiva och negativa temperaturer (provuppgift nummer 2 i del B1 i nationella provet i matematik 2009). Från svaren drogs slutsatsen att en del elever hade gett ett felaktigt svar som motsvarar att subtrahera de absoluta värdena av de mest extrema temperaturerna eller subtrahera de absoluta värdena av temperaturerna med största absolutvärdet. Denna typ av analys av de enskilda elevlösningarna visade att det i flera fall är möjligt att dra slutsatsen att eleven hade tolkat provuppgiften språkligt korrekt, men hade gjort matematiska misstag som resulterade i ett felaktigt svar. Med andra ord, var det möjligt att utesluta den språkliga uppgiftsformuleringen som en källa till felaktiga svar och istället identifiera källan som en brist på en specifik matematisk kunskap.

Den andra datamängden var poäng per provuppgift. Den gav information om för vilka provuppgifter, som skillnaderna i lösningsproportioner mellan förstaspråkare och andraspråkare var små respektive stora. Ett resultat var att för aritmetiska provuppgifter i en intramatematisk uppgiftsformulering (i motsats till en extramatematisk uppgiftsformulering), fanns det bara små skillnader i prestation mellan förstaspråkare och andraspråkare. Bland provuppgifter med måttliga skillnader i lösningsproportion var ekvationer och proportionalitetsproblem. Ett huvudresultat var att provuppgifter med stora och statistiskt signifikanta skillnader mellan förstaspråkare och andraspråkare, hade det gemensamt att de förekommer på endast enstaka ställen i läroböcker och sällan utanför skolmatematiken. Exempel på sådana provuppgifter är att bestämma medianen, att arbeta med decimala timmar och att använda en numeriskt given skala '1: X' på en karta för att beräkna avstånd. Fallet med att subtrahera negativa tal är speciellt i den meningen att detta matematiska innehåll inte säkert finns med i det som i läroböckerna ofta kallas grundkurs, men istället finns i läroböckernas fördjupningsavsnitt. Ibland finns grundkursen och fördjupningsavsnitten i olika läroböcker, där respektive elev använder en av dessa två böcker. Att en del elever följer grundkursen, kan ibland skapa en lärsituation där dessa elever inte alltid får tillgång till det matematiska innehåll, som förekommer sällan i grundkursen och kanske främst i fördjupningsavsnittet. Det kan vara så att andraspråkande invandrare i skolan har en större risk att inte få tillgång till dessa lärsituationer, till exempel på grund av invandring efter att en serie lektioner om just det aktuella avsnittet har avslutats. Detta ger lärare till andraspråkare ett särskilt ansvar att se till att dessa elever får tillgång till den matematik som vid endast få tillfällen förekommer i matematikundervisningen.

Publikation II

Den första artikeln kom att fungera som katalysator till den andra artikeln eftersom ett resultat från artikel I var att skillnaderna i lösningsproportioner mellan förstaspråkare och andraspråkare var ojämnt fördelade över de olika provuppgifterna. Liksom artikel I, använde artikel II data från svenska nationella prov i matematik för årskurs 9, men på ett annat sätt.

Forskningsfrågan i artikel II är om vilka resultatprofiler på matematikprov som förstaspråkare och andraspråkare visar på matematikprov, mätt som fördelningen av lösningsproportioner i olika matematiska innehållsområden (jämför figur 2). De matematiska innehållsområdena definierades på samma sätt som i TIMSS (Mullis m.fl., 2012). Artikel II fann att förstaspråkare och andraspråkare visar olika resultatprofiler på matematikprov. Artikel II föreslog att en möjlig förklaring till detta är att nyanlända andraspråkare kan ha med sig kunskap och erfarenheter från att ha blivit undervisat enligt någon läroplan, som är annorlunda än den svenska läroplanen, exempelvis genom att betona ett matematiskt kunskapsområde mer och ett annat kunskapsområde mindre jämfört med svensk kursplan i matematik.

Med hjälp av data från SCB (Statistics Sweden, 2016), visade artikel II att en majoritet av de elever som invandrade under skolåren 7-9 är från Mellanöstern och Östeuropa (inklusive OSS). Flera länder i dessa två regioner visar resultatprofiler på matematikprov, vilka skiljer sig från den svenska. TIMSS studier mäter matematik kunskaper inom fyra matematiska innehållsområden. Dessa är taluppfattning; data och slump; algebra; samt geometri (Mullis m.fl., 2008; Mullis m.fl., 2012). Olika länder kan ha olika lösningsproportioner för olika matematiska kunskapsområden, vilket visas i tabell 2. Den sista raden i tabell 2 illustrerar idén med resultatprofiler på matematikprov för olika länder. I TIMSS 2011 hade exempelvis Ryssland bättre resultat än Sverige i både taluppfattning och algebra och i sin tur hade Sverige bättre resultat än Turkiet i både taluppfattning och algebra. Alltså visade dessa tre länder prestationer på tre olika nivåer. Emellertid kan de skillnader som beräknades i den sista raden i tabell 2 användas som ett mått på länders resultatprofiler på matematikprov. Till exempel, när svenska elever jämförs med sig själva, så visade de en relativ styrka i det matematiska innehållsområdet tal medan ryska och turkiska elever omvänt visade en relativ styrka i algebra.

Metoden i artikel II var att jämföra förstaspråkares och andraspråkares resultatprofiler på matematikprov, där resultatprofilen beräknades som på sista raden i tabellen 2. Som data användes elevernas resultat på frisläppta nationella prov i matematik för årskurs 9 från åren 2007-2009. Dessa består av ett riksomfattande slumpvis urval av elever. Provuppgifterna klassificerades enligt TIMSS ramverk för matematiska innehållsområden så att elevernas resultatprofiler på matematikprov kunde jämföras med resultatprofiler på matematikprov för olika länder i Mellanöstern och Östeuropa. Jämförelsen gjordes för resultatprofiler på taluppfattning minus algebra samt data och slump minus algebra eftersom det var alltför få provuppgifter i geometri för en jämförelse.

De forskningsetiska aspekterna på datainsamlingen i artiklarna I och II är tämligen okomplicerad. Insamlade data är sekundärdata från PRIM-gruppen och innehåller ingen annan information än varje deltagande elevs poäng på respektive provuppgift samt om eleven är förstaspråkare eller andraspråkare. PRIM-gruppen arkiverar och lämnar ut data enligt forskningsetiska principer. Eftersom data är sekundärdata behövs forskaren inte heller störa elevernas ordinarie lektionsarbete eftersom data redan har samlats in via den ordinarie skolverksamheten i form av de obligatoriska nationella proven. Däremot finns en potentiell nackdel med ett slumpmässigt urval och det är att det finns en risk för segregationseffekter. Tidigare studier har visat att föräldrarnas utbildning korrelerar signifikant med skolprestationer (Skolverket, 2004; Zevenbergen, 2001). Det är också känt att socioekonomisk bakgrund korrelerar med språkstatus och invandringsstatus (Hansson, 2012, Hanson & Gustafsson, 2011). Med ett slumpmässigt urval, där föräldrarnas utbildningsbakgrund inte ingår, kan en jämförelse av förstaspråkares och andraspråkares provresultat istället bli en jämförelse av olika socioekonomiska gruppers provresultat. Detta problem hanterades genom att i artikel II jämföra elevernas resultatprofiler på matematikprov definierade enligt ovan (se figur 2). Att jämföra resultatprofiler på matematikprov innebär att jämföra resultatets fördelning över olika kunskapsområden istället för att jämföra absoluta resultat.

Publikation III

Med tanke på resultaten i artikel II, diskuterar delar av papper III metoder för att studera resultatprofiler på matematikprov för nyanlända andraspråkare. Detta ger upphov till flera metodologiska utmaningar. En utmaning är att ett matematikprov avsett för nyanlända andraspråkare måste ha provuppgiftsformuleringar, som är språkligt enkla så att de testar kunskap i matematik snarare än kunskap i andraspråket. En annan utmaning är att varje provuppgift bör vara matematiskt tillräckligt smal för att passa in i ett visst matematiskt innehållsområde. En tredje utmaning är att välja ett stickprov. Eftersom nyanlända andraspråkare är få till antalet, skulle ett slumpmässigt urval av hela klasser kräva ett ohanterligt stort stickprov. Å andra sidan kan ett riktat urval från skolor med en stor andel andraspråkare, ha egenskaper som skiljer sig från ett slumpmässigt urval. Exempel på detta är den socioekonomiska bakgrunden inklusive boendesegregation och föräldrarnas utbildning. Bostadssegregation kan i sin tur leda till skolsegregering eftersom skolor med en stor andel andraspråkare i genomsnitt har en något lägre andel av behöriga lärare och en något högre andel av personalomsättning bland lärarna. Det finns också en positiv korrelation mellan att ha en större andel av enskilt arbete i klassrummet och skolor som har en hög andel elever med invandrarbakgrund eller låg socioekonomisk status. För att hantera denna utmaning, föreslog artikel III att som jämförelse använda resultaten från de obligatoriska nationella proven för det riktade urvalet respektive PRIM-gruppens slumpmässiga stickprov.

Artikel III presenterar elevernas svar på i huvudsak ett specifikt problem, nämligen att bestämma "Vad är hälften av 1/5?" I artikel III definierades elevkategorierna som i tabell 3. Ett resultat i artikel III är att de nyanlända andraspråkarna hade något högre lösningsproportion än de andraspråkare, som har invandrat tidigt i skolåren. Mer intressant var olika felaktiga svar, som några av de nyanlända andraspråkarna och tidigt invandrade andraspråkarna gav. Bland de nyanlända var det vanligt att svara "0,2", vilket motsvarar en ofullständig lösning. Den relativa frekvensen av detta svar var vanligast bland de nyanlända eleverna och mindre vanligt ju längre erfarenhet eleverna hade av det svenska språket. Bland de elever, som svarade 0,2 hade nästan alla korrekt löst en annan uppgift om att bestämma två tredjedelar av sextio. Detta tolkades som om dessa elever inte förstod ordet "halv", som på svenska har en oregelbunden böjning "hälften".

En annan kategori av felaktiga svar var "felaktig halvering". I denna kategori var svaren nämnare/2, täljare/2, 0,5/2,5 och liknande svar. Svar i denna kategori tolkades som okunskap i att korrekt kunna halvera en bråkdel. Denna svarskategori var vanlig bland särskilt bland de andraspråkare som inte var nyanlända. Utfallet av artikel III var att nyanlända och tidigt invandrade andraspråkare bör ses som två separata grupper av andraspråkare med olika utmaningar när de på sitt andraspråk undervisas och testas i olika matematiska områden. Detta gäller trots att provuppgifterna i den aktuella studien inte är lästal. Artikel III författades av Petersson och Norén. Till artikel III bidrog Petersson med matematisk bakgrund, forskningsfråga, metodutformningen av studien, insamling, analys och sammanställning av data samt diskussion och slutsats. Petersson och Norén skrev litteraturgenomgången tillsammans.

Publikation IV

Med den metod som anges i artikel III, presenterar artikel IV elevernas lösningsproportioner och svar på provuppgifter i statistik och sannolikhetslära. Provuppgifterna var att avläsa en enskild datapunkt från ett diagram, att bestämma lägesmått och att använda sannolikhet. Precis som i artikel III kategoriserades eleverna i artikel IV som i tabell 3.

I tabell 7b är tre provuppgifterna om att avläsa enskilda datapunkter från diagram. Två är om att avläsa ett stapeldiagram och en om att avläsa ett bivariat diagram. Skillnaderna i lösningsproportioner var små mellan samtliga fyra elevkategorier för dessa tre provuppgifter. Men det fanns några intressanta mönster i de fel, som eleverna hade gjort. I stapeldiagrammet skulle eleverna avläsa nederbörden för juli månad. De svenska orden för månaderna juni och juli liknar varandra. Två nyanlända andraspråkare förväxlade dessa två månader med varandra medan ingen annan andraspråkselev gjorde det felet. I analysmodellen i figur 10, kan detta fel placeras i representationsformen ord.

Uppgiften i tabell 7b om att avläsa ett bivariat punktdiagram var att avläsa längden på en person som väger 70 kg. En majoritet (N = 210 av 259) av eleverna gav ett exakt och korrekt svar på denna provuppgift. Däremot gav arton elever, (inga nyanlända och sex elever i var och en av de övriga tre elevkategorierna) ett svar som motsvarar "längden av en person som väger ungefär 70 kg”. På ett prov i matematik kan eleverna förväntas ge ett exakt svar såvida inte ett ungefärligt svar uttryckligen efterfrågas. I det avseendet bör det ungefärliga svaret tolkas som i figur 10 hemmahörande i rutan för ord som representationsform och matematik för vardagslivet i stället för ord som representationsform och matematik för fortsatta studier.

Som en provuppgift om att tolka data togs med en provuppgift som ingick i elevernas ordinarie nationella prov. Denna provuppgift är sekretessbelagd, men beskrivs som att tolka en frekvenstabell. Denna provuppgift har likheter med att bestämma medelvärdet av viktade data, vilket i tidigare forskning har visat sig vara svårare än att bestämma medelvärde ur oviktade data (Pollatsek, Lima & Well, 1981). Ett resultat i artikel IV är att nyanlända hade högre lösningsproportion och lägre andel av ’ej svarat’ på denna provuppgift jämfört med övriga andraspråkare.

I tabell 7b är två provuppgifter om att bestämma ett lägesmått. På tidigare nationella prov har detta visat sig vara svårare för andraspråkare än för förstaspråkare. Dessa provuppgifter var; att bestämma medianen av en lista av positiva och negativa heltal samt att bestämma medianen av data i ett bivariat diagram. På dessa provuppgifter hade de nyanlända eleverna hade visserligen något högre lösningsproportion än övriga andraspråkare, men också högre andel ’ej svarat’ än övriga andraspråkare. Detta resultat kan tolkas i ljuset av artikel I, där det rapporterades att begreppet median, är ett begrepp som andraspråkare har låg lösningsproportion i, med den föreslagna förklaringen att begreppet median förefaller vara sällsynt i betydelsen att det får lite utrymme i matematikundervisningen. Utifrån resultaten i artikel I och IV är en rimlig slutsats att nyinvandrade elever kan ha färre möjligheter till att kunna delta i undervisning om begreppet median. I figur 10 skulle dessa provuppgifter om median kunna klassificeras i rutan ord som representationsform och matematik för fortsatta studier.

Som provuppgifter om sannolikhet togs med två provuppgifter, som ingick i elevernas ordinarie nationella prov. För dessa provuppgifter fanns det smärre skillnader mellan nyanlända och tidigt anlända andraspråkare vad det gäller lösningsgrad. Båda dessa elevkategorier hade lägre lösningsproportion än förstaspråkare. Däremot hade de tidigt anlända eleverna högre andel av matematiskt irrelevanta svar medan de nyanlända hade högre andel av ’ej svarat’ och av att svara på ett sätt som kan tolkas som att de inte fullt ut har förstått vad provuppgiften frågar om.

Diskussion

Forskningsbakgrunden i denna avhandling visade att det finns en stor mängd forskning om andraspråkare sett från de båda perspektiven socioekonomisk bakgrund och flerspråkighet. Ett tredje perspektiv är att ta i beaktande erfarenheter och kunskaper som invandrade elever har med sig från tidigare skolgång i matematik. Dessa tre perspektiv överblickas kort nedan.

En aspekt för att beskriva elevernas skolresultat är deras socioekonomiska bakgrund. Till denna aspekt lade Pásztor (2008) variabler som även beskriver skolgången och fick förbättrad noggrannhet i sin modell. En kritik mot att använda elevernas socioekonomiska bakgrund som variabel för att beskriva skolresultat är dock att den har lågt förklaringsvärde. Ufer m.fl. (2013) fann att den socioekonomiska bakgrunden förlorade sitt förklaringsvärde redan i andra skolåret jämfört med bakgrundsvariablerna allmänna skolresultat och språkkunskaper. Deras resultat innebär att elevernas tidigare skolresultat har ett högre prediktivt värde än exempelvis deras föräldrars utbildning. Ett annat sätt att uttrycka detta resultat är att en väderprognos med hjälp av en dag gamla uppgifter ger en bättre väderprognos än vid användning av flera dagar gamla data. Ett annat sätt är att jämföra lektionsverksamheten i socioekonomiskt olika klassrum, vilket Hansson (2010, 2012) och Zevenbergen (2001) gjorde. De såg segregation i undervisningens utformning i klassrum med i genomsnitt olika socioekonomisk status. Detta innebär att de enskilda elevernas socioekonomiska bakgrund indirekt kan komma in i matematikundervisningen på klassrumsnivå.

En annan aspekt av elevens skolresultat är kunskaper i undervisningsspråket. Andraspråkare kan vara mer eller mindre skickliga i undervisningsspråket (Clarkson, 2009, Cummins, 2008). Andraspråkare kan i matematikundervisningen uppleva en högre kognitiv belastning eftersom de är elever i inte bara matematikämnet utan även undervisningsspråket och eventuellt också genom övningsuppgifternas kulturella sammanhang (Campbell m.fl., 2007). En modell för detta illustreras i figur 6. Dessutom kan möjligheterna till och sätten för att använda andraspråket variera mellan olika klassrum (Garcia, 2011; Moschkovich, 2002; Setati, 2005). En migration kan ibland leda till att en elev, som har varit en förstaspråkare med en viss socioekonomisk bakgrund i det nya landets klassrum blir en andraspråkare med någon annan socioekonomisk bakgrund. Dessutom kan migrationen ibland leda till ett byte från en läroplan och undervisningstradition till en annan läroplan och undervisningstradition, eftersom dessa kan variera mellan länder och regioner (Li & Ginsburg, 2006). Att modellera skolresultat hos andraspråkselever, som har invandrat i skolåldern, med den nya språkliga och socioekonomiska situationen kan ibland ignorera den tidigare situationen för eleven som en trolig förstaspråkare. Dessutom ignorerar den som modell även den under skoltiden invandrade elevens matematiska kunskap som en kumulativ kunskaps som överförs till den nya matematikundervisningen (Bishop, 1991; Giannelli & Rapallini, 2016). Detta leder till den tredje aspekten, som utforskas i denna avhandling. Att ignorera skillnader mellan läroplaner och klassrum i olika länder när man studerar invandrare förutsätter indirekt att skillnader mellan olika länders läroplaner är försumbara. Detta är inte fallet (Bessot & Comiti, 2006; Fan, 1999; Li & Ginsburg, 2006; Son & Senk, 2010; Wu & Zhang, 2006).

I linje med denna tredje aspekt, jämförde artikel II i denna studie de resultatprofiler på matematikprov som förstaspråkare och andraspråkare visar på de svenska nationella proven i matematik i sista skolåret i svenska grundskolan. Den definition, som används för resultatprofiler på matematikprov var lösningsproportion i ett matematiskt innehållsområde minus lösningsproportion i ett annat matematiskt innehållsområde. En förutsättning för att kunna påvisa skillnader i resultatprofiler på matematikprov hos förstaspråkare och andraspråkare är att de länder, som immigranterna kommer ifrån, har resultatprofiler på matematikprov som skiljer sig från Sveriges. Detta är fallet eftersom de flesta andra språkstudenter har sin bakgrund i Mellanöstern och Östeuropa och för flera av dessa länder visar TIMSS resultatprofiler på matematikprov, som skiljer sig från den hos svenska elever. Se figur 2 (Mullis m.fl., 2008; Mullis m.fl., 2012). Exempelvis presterade svenska elever i TIMSS sämre i algebra än i taluppfattning medan vissa länder i figur 2 inte presterade olika i algebra och taluppfattning eller rentav bättre i algebra än i taluppfattning. Data i artikel II var det slumpmässiga urvalet av resultat från nationella prov 2007-2009 insamlat av PRIM-gruppen för utvärdering av det nationella provet. Artikel II fann att skillnaderna i resultatprofiler på matematikprov hos förstaspråkare och andraspråkare var detekterbara och skilde sig åt med statistisk signifikans. För att göra resultaten i artikel II jämförbara med de resultatprofiler på matematikprov, som framgår i TIMSS rapporter (Mullis m.fl., 2012), klassificerades de provuppgifter, som ingick i artikel II, enligt TIMSS ramverk. Andraspråkarna i artikel II visade en mindre differens mellan prestationer i de matematiska innehållsområden algebra och taluppfattning än vad förstaspråkarna gjorde. Detta resultat är förvånande om vi endast tar hänsyn till elevernas språkbakgrund och i genomsnitt lägre socioekonomiska bakgrund. Exempelvis fann Ufer m.fl. (2013) att andraspråkare hade ungefär samma lösningsproportioner som förstaspråkare på provuppgifter som prövar algoritmiska färdigheter, men sämre än förstaspråkare på kognitivt mer krävande provuppgifter. Om vi antar att provuppgifter med låg lösningsproportion i nationella prov är kognitivt krävande, så borde vi kunna förvänta oss en större differens mellan de två elevgrupperna på denna typ av uppgifter och ett mindre gap mellan förstaspråkare och andraspråkare för provuppgifter med en hög andel korrekta svar. Artikel II fann i stället ett motsatt resultat. På några av provuppgifterna i algebra med låg andel korrekta svar, hade dessutom andraspråkarna samma lösningsproportion som förstaspråkarna. För att förklara detta resultat, föreslog artikel II följande hypotes; att bland andraspråkare är vissa elever nyanlända andraspråkare med annan resultatprofil på matematikprov än förstaspråkare på grund av sina erfarenheter från matematikundervisning enligt någon annan läroplan. Denna hypotes kunde delvis bekräftas för algebra i Petersson (2013b). Artikel III utforskade denna hypotes för fallet bråkräkning genom provuppgiften att bestämma hälften av en femtedel.

Artikel III visade att nyanlända och andraspråkare som invandrare i tidiga skolår, kan ge olika typer av felaktiga svar, se figur 9. När provsvaren från nyanlända immigranter jämfördes med övriga andraspråkare, så visade sig två skillnader. Dels saknade de nyanlända immigranterna oftare språkkunskap nog att förstå provuppgiften än övriga andraspråkare. Dels gjorde de matematiska felaktigheter mer sällan än övriga andraspråkare. Lösningsproportionerna för respektive elevkategori för provuppgiften i artikel III följde ett mönster liknande det för algebra i Petersson (2013b). Detta är i linje med att se kunskaper och resonemang i proportionalitet och bråkräkning som en tröskel för goda kunskaper i algebra (Post, Behr & Lesh, 1988).

Artikel IV visade inga stora skillnader i lösningsproportion mellan nyanlända och tidigt anlända andraspråkare, även om det fanns några skillnader i hur eleverna hade svarat eller inte svarat, vilket illustreras i figur 10. Exempelvis förekom i alla elevkategorier utom nyanlända andraspråkare svar, som motsvarar begreppet ’ungefär 70 kg’ när de avläste ett diagram i provuppgiften. Dessutom var det vanligare att nyanlända andraspråkare gav blankt svar på provuppgifter om median jämfört med andra elevkategorier. En i genomsnitt lägre lösningsproportion bland andraspråkare än bland förstaspråkare var förväntat (Böhlmark, 2008; Hansson, 2012). Givet resultatet i artikel II, var det också väntat att detta gäller särskilt för det matematiska innehållsområdet data och slump och att detta även borde gälla nyanlända andraspråkare. Detta eftersom elever med svensk skolgång enligt TIMSS har högre lösningsproportion i data och slump än många länder, från vilka elever invandrar till Sverige (Giannelli & Rapallini, 2016; Mullis m.fl., 2008). Dock, jämfört med de tidigt anlända, hade de nyanlända en lägre andel av matematiskt irrelevanta provsvar och högre andel av provsvar som visar att de språkligt inte förstått vad provuppgiften frågar om. Ett sådant exempel kan vara att inte känna till ordet ”median”.

Som en alternativ förklaring till hypotesen i artikel II, diskuterade artikel I möjligheten till att delta i undervisning om begrepp som förekommer sällsynt i läroböcker eller huvudsakligen i läroböckernas fördjupningsavsnitt. Exempelvis kan en elev ha invandrat efter att undervisningen om ett sällan förekommande begrepp är avklarad för det skolåret. Alternativt kan en andraspråkare, som invandrat i tidiga skolår och kanske finner matematiken svår, välja att hoppa över fördjupningsavsnitten i matematikläroboken. Följden av detta är att de därför kanske inte möter en del av de begrepp, som tas upp i de fördjupade avsnitten. Denna förklaring bekräftades i Petersson (2013a) för provuppgiften att beräkna 12–23+9. Denna typ av numeriska uttryck är känd för att vara svår för eleverna och Herscovics och Linchevski (1994) kallar det för ett uppdelande minustecken då det är vanligt att eleverna delar upp uttrycket vid minustecknet och därmed tolkar det som 12-(23+9). På provuppgiften om uppdelande minustecken hade nyanlända andraspråkare högre lösningsproportion än förstaspråkare, som i sin tur hade högre lösningsproportion än andraspråkare, som hade invandrat i tidiga skolår (Petersson, 2013a). Elevernas felaktiga svar visade att de var obekanta med den här typen av subtraktioner och ofta gav svar som motsvarar ett uppdelande minustecken.

Trots att de nyanlända andraspråkarna, som deltog i denna studie, hade lägre kunskaper i svenska, hade de högre lösningsproportion på provuppgifterna i matematik än de tidigt anlända andraspråkarna. Detta visar att det finns fler faktorer för att beskriva elevernas situation i matematikundervisningen än elevernas nuvarande kunskaper i undervisningsspråket och socioekonomiska bakgrund. Den enskilda elevens utbildningshistoria verkar också vara viktigt (Bishop, 1991; Giannelli & Rapallini, 2016: Li & Ginsburg, 2006). De utbildningshistoriska mekanismer som ligger bakom de mönster, som provresultaten visar för nyanlända och andra elevkategorier, beskrivs i denna avhandling, nämligen de som föreslås i artikel I och II och utforskas i artikel III och IV. För de andraspråkare, som invandrade i tidiga skolår verkar det som att resultaten i denna avhandling liknar dem i tidigare forskning om andraspråkare: Deras begränsade språkkunskaper utgör ett hinder i matematiklärandet liknande det i exempelvis Ufer m.fl. (2013). Att lägga grunderna i matematik på sitt andraspråk förefaller kräva mycket av dessa elever. Under dessa omständigheter förefaller andraspråkare, som har invandrat i tidiga skolår, vara i behov av särskilt stöd i sitt matematiklärande redan från första dagen i skolan i det nya landet. Detta ger resultatet av denna avhandling en politisk dimension (se exempelvis Romberg, 1992). Ett sådant stöd ska inriktas mot att ge god förståelse av grundläggande matematiska begrepp och avhjälpa vanliga missbegrepp. För att undersöka hur detta effektivt skulle kunna göras i skolans dagliga praxis i flerspråkig matematikundervisning krävs mer forskning, till exempel med hjälp av observationsdata och kanske interventionsstudier. Empiriska data i den aktuella studien var elevernas skriftliga svar på provuppgifter. Eftersom dessa data var diagrammatiska, passar de väl in i modellen i figur 5 av Prediger och Wessel (2011). Om en studie på samma tema som i denna studie istället skulle använda data från observationer av matematikundervisningen, skulle modellen i figur 8 behöva använda även representationsformer såsom material och gestaltande representationer. Citatet i avsnittet om tidigare forskning kan upprepas: Det är "fortfarande mycket att göra i forskning" när det gäller flerspråkighet (Meyer, Prediger, César & Norén, 2016).

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Appendix I – test A

Prov i matematik, åk9
Inga hjälpmedel på dessa matematikuppgifter!
Skriv namn, klass och svar (och vid behov stödanteckningar) på separat papper!

---

1. Beräkna 12 – 23 + 9

2. Hur många minuter är 0,75 h?

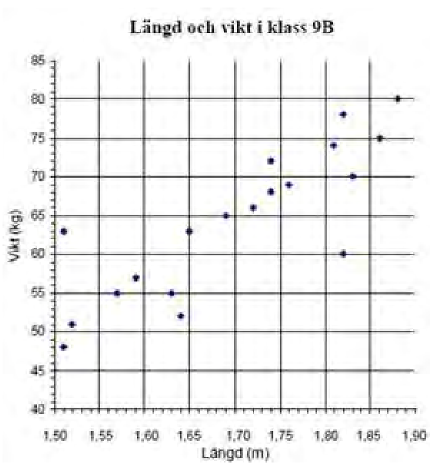
3. Skriv talet 1 430 i grundpotensform.

4. Vilket tal ska stå i rutan så att likheten stämmer?

$$1{,}365 - \square = 1{,}305$$

5. Vad är hälften av 1/5.

6. Diagrammet visar längd och vikt i klass 9B.



a) Philip går i klass 9B och väger 70 kg. Hur lång är han?
b) Vilken är medianlängden i klassen?

7. Beräkna 6,32 – 3,44

8. Figuren består av rektanglar och trianglar. Hur stor del av figuren är grå?

9. Beräkna 32 + 23.

10. Lös ekvationen 2x + 3 = 11

11. Vad är 2/3 av 60?

12. I tabellen anges temperaturen i °C för fem dagar i mars.

Dag	Temp.
Måndag	-9
Tisdag	6
Onsdag	0
Torsdag	7
Fredag	-3

a) Hur många grader skiljer det mellan de dagar där temperaturskillnaden är störst?
b) Vad är medianen för temperaturen?

13. Beräkna $\sqrt{9+16}$.

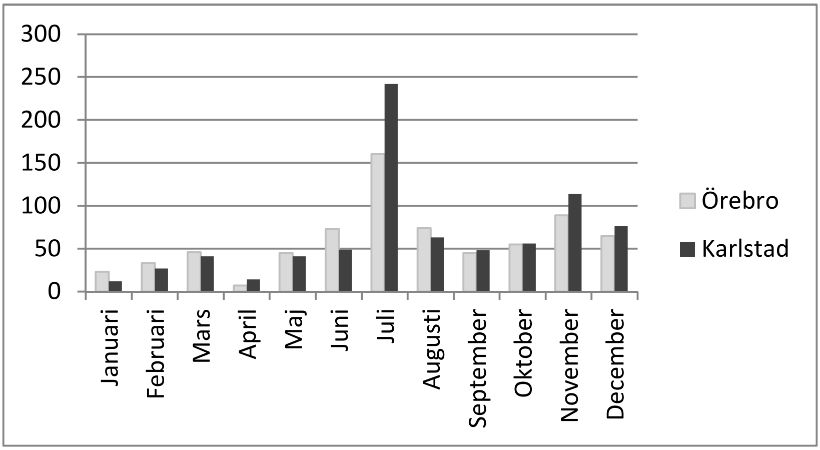
14. 4x + 5y = 11. Vad är 12x + 15y?

15. a = 2 och b = 4. Beräkna a(b + 2) + b

Appendix II – test B

Prov i matematik, åk9

1a. Se diagrammet! Hur mycket nederbörd föll det i juli månad i Örebro?
1b. Hur mycket nederbörd föll det i juli månad i Karlstad?
1c. Ungefär hur många procent mindre nederbörd föll i Örebro än i Karlstad under juli månad?



Appendix III – letter of consent

Till elever i åk 9

Jag heter Jöran Petersson och forskar i matematikdidaktik vid Stockholms Universitet. I ett forskningsprojekt undersöker jag hur åk9-elever som har gått i svensk skola olika länge eller har flerspråkig bakgrund löser uppgifter inom några matematiska områden.

Varför behövs din tillåtelse?

Lagen om etisk prövning (SFS 2003:460) säger att forskaren måste be om tillstånd från de personer som ska delta i ett forskningsprojekt. Om deltagarna är äldre än 15 år får de själva ge tillstånd när frågorna inte handlar om det som personuppgiftslagen kallar känsliga uppgifter. Lagen säger också att deltagarna ska informeras om studiens syfte och att de även har rätt att avbryta sitt deltagande.

Insamlad information kommer endast att användas för forskningsändamål. Inga namn på personer, och skolor kommer att redovisas. Studiens resultat kommer att publiceras i vetenskapliga artiklar och i en doktorsavhandling och presenteras på seminarier och konferenser. Insamlade data kommer att hanteras och lagras enligt arkivlagen (SFS 1990:782) och personuppgiftslagen (SFS 1998:204).

Tillåtelse

Du ger din tillåtelse genom att ringa in JA på frågorna nedan.
Du ger inte din tillåtelse genom att ringa in NEJ på frågorna nedan.

- Intervjuaren frågar hur eleven löser uppgifter i matematik. JA / NEJ
- Jag tillåter att intervjuerna får spelas in som ljudfil. JA / NEJ

Elevens namn: ______________________________

Elevens skola och klass: ______________________________

Dagens datum (År-Månad-Dag): ______________________________

Underskrift (elev): ______________________________

Appendix IV – survey

Namn ____

Frågor om din skolbakgrund

Jag är ☐ pojke ☐ flicka

Från vilket skolår har du gått i svensk skola?

☐ från Åk 1 ☐ från Åk 2 ☐ från Åk 3 ☐ från Åk 4 ☐ från Åk 5
☐ från Åk 6 ☐ från Åk 7 ☐ från Åk 8 ☐ från Åk 9

I vilka länder har du gått i skola? (Du kan kryssa eller skriva flera alternativ.)

☐ Sverige
☐ Andra länder (vilka, skriv gärna åk)

____

____

____

Vilken kurs i svenska läser du? (kryssa)

☐ Svenska ☐ Svenska som andraspråk

Går du på lektioner i modersmål? (kryssa)

☐ Nej ☐ Ja

Har du studiehandledning på ditt modersmål?

☐ Nej ☐ Ja

Du kan kryssa/skriva flera alternativ: Vilka språk talar du vardagligen med?

Din mamma ☐ Svenska ☐ Andra språk (vilka)

____

Din pappa ☐ Svenska ☐ Andra språk (vilka)

____

Dina syskon ☐ Svenska ☐ Andra språk (vilka)

____

Dina skolkamrater ☐ Svenska ☐ Andra språk (vilka)

____

Övriga kompisar ☐ Svenska ☐ Andra språk (vilka)

____