MASTER'S THESIS

Inhomogeneous Cosmology

Jesper Lindkvist

Master of Science in Engineering Technology
Engineering Physics

Luleå University of Technology
Department of Engineering Sciences and Mathematics

Inhomogeneous Cosmology

Jesper Lindkvist

ABSTRACT

Telescopes all around the World are daily collecting huge amounts of data from the sky. However, this information must be interpreted before we can say anything about the physics of the cosmos. Therefore it highly depends on the model one uses for the dynamics of the universe. The standard model of modern cosmology says that the universe is both homogeneous and isotropic at large scales. This is clearly not the case in the local neighborhood of our galaxy, and all the way out to superclusters of galaxies. The real universe, rather being inhomogeneous, will automatically have an effect that behaves in the same way as dark energy does. This may alleviate the need for an accelerating universe.

Preface

I would like to thank my examiner and supervisor Johan Hansson for sharing the idea of an inhomogeneous cosmology with me. The project has enlightened me to a new way of thinking. I know now that we do not know much at all. We will never fully understand that which we call the universe.

Jesper Lindkvist

Contents

CHAPTER 1

Introduction

1.1 What is cosmology?

Cosmology is simply the study of the universe as a whole; its origin, evolution, composition and destiny. On these big scales, all interactions besides gravity can be neglected. The dynamics is therefore ruled by Einstein's equations; ten coupled partial differential equations. The matter will determine the geometry that in turn will decide the dynamics for the matter. This goes on in a closed loop.

One can say that modern cosmology started when it was realized that the laws that apply here on Earth also govern the cosmos. However, not much could be done in a detailed way until the early twentieth century when Einstein developed his general theory of relativity. It describes space and time as a unity and at this time, scientists started thinking of the universe as "any" dynamical system. One great progress that has been made in the past century is the accuracy in telescopes. Now we can see objects at distances never imagined 100 years ago. Together with space-based telescopes and detectors, this has ushered in cosmology as a "falsifiable" exact science as late as during the last two - three decades.

1.2 Relativistic cosmology

Back in the 1920's, people believed that the universe should be contracting because of the attracting force of gravity. On the contrary, Hubble discovered that the universe was in fact expanding. More recent studies show that the universe even seems to be accelerating [1], [2]. This means that some part of the energy density of our universe must undertake a form quite different than the one we know of.

Einstein's equations,

$$R_{ab} - \frac{1}{2}g_{ab}R + g_{ab}\Lambda = \frac{8\pi G}{c^4}T_{ab} \tag{1.1}$$

are not analytically solvable if one does not make assumptions about the symmetry. One may argue that the universe might be homogeneous and isotropic on really large scales. This is what is called the cosmological principle and is the main building stone for relativistic cosmology. It is critical for the standard model of cosmology today that it holds. By assuming the cosmological principle, it is possible to reduce Einstein's equations to two coupled linear ordinary differential equations (Friedmann's equations) [3]. In the newtonian limit this reduces to Hubble's law and means that the recessional velocity of objects is proportional to the distance to them,

$$v = Hd, \tag{1.2}$$

where v is the recessional velocity, d the relative distance and H the Hubble parameter.

If a light-emitting object is receding from an observer, one observes a red-shifted light due to the doppler effect. The definition of redshift is

$$z = \frac{\Delta\lambda}{\lambda}, \tag{1.3}$$

where λ is the wavelength of the emitted light and $\Delta\lambda$ is the difference between the observed and emitted light. For relativistic doppler shift one can rewrite z in a way that only depends on the recessional velocity,

$$z = \sqrt{\frac{1 + v/c}{1 - v/c}} - 1, \tag{1.4}$$

or equivalently

$$v/c = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}. \tag{1.5}$$

For the slow receding limit one simply gets,

$$z = v/c. \tag{1.6}$$

In a homogeneous universe, one can only have three different geometries of spacetime; open, flat or closed. See figure 1.1. For a certain density, the universe will become flat. This is called the critical density and is defined as:

$$\rho_{crit} = \frac{3H_0^2}{8\pi G}, \tag{1.7}$$

One may now introduce a density parameter which is the real density relative to the critical one,

$$\Omega = \frac{\rho}{\rho_{crit}}. \tag{1.8}$$

For a flat universe $\Omega = 1$, for a closed $\Omega > 1$ and for an open $\Omega < 1$.

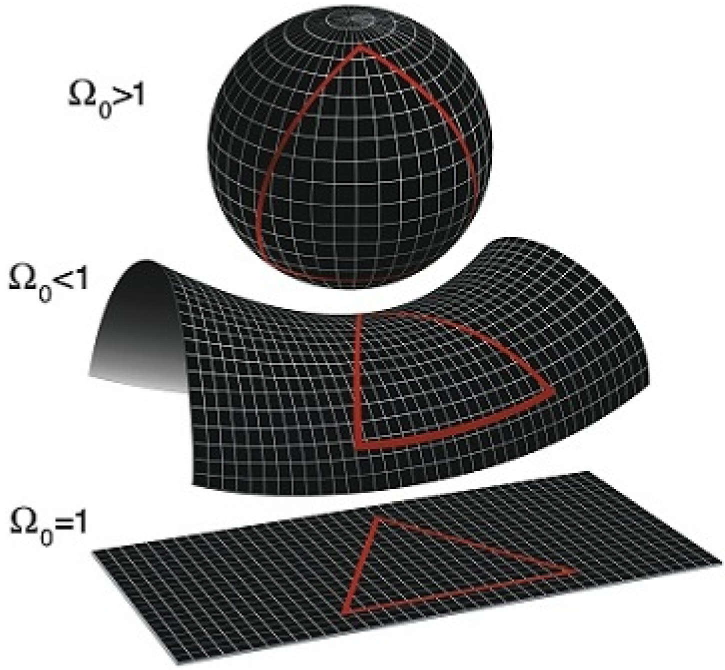

Figure 1.1: The three different curvatures of a homogeneous universe [4].

1.3 The equation of state

Using the ideal gas equation for non-relativistic matter:

$$p = \rho RT = \rho C^2 \tag{1.9}$$

where ρ is the mass density, R the gas constant, T the temperature and $C = \sqrt{RT}$ the thermal speed of the particles. Defining w such that $p = w\rho c^2$. Calculating w:

$$w = \frac{C^2}{c^2} \approx 0. \tag{1.10}$$

Using the Friedmann-Lemaître-Robertson-Walker (FLRW) metric for the universe, i.e. flat and homogeneous, will link the density and the cosmic scale factor, a,

$$\rho \propto a^{-3(1+w)}. \tag{1.11}$$

Non-relativistic matter will have $\rho \propto a^{-3} = V^{-1}$, as expected. Relativistic matter has $p = \frac{1}{3}\rho c^2$ and therefore $\rho \propto a^{-4}$. An energy density that does not change with the cosmic scale factor has $w = -1$. A negative value of w corresponds to a negative pressure.

1.4 Large scale structures

Based on redshift survey data, Geller and Huchra discovered the "Great Wall" in 1989. The wall is a big sheet of galaxies, being 500 million light-years long, 200 million light-

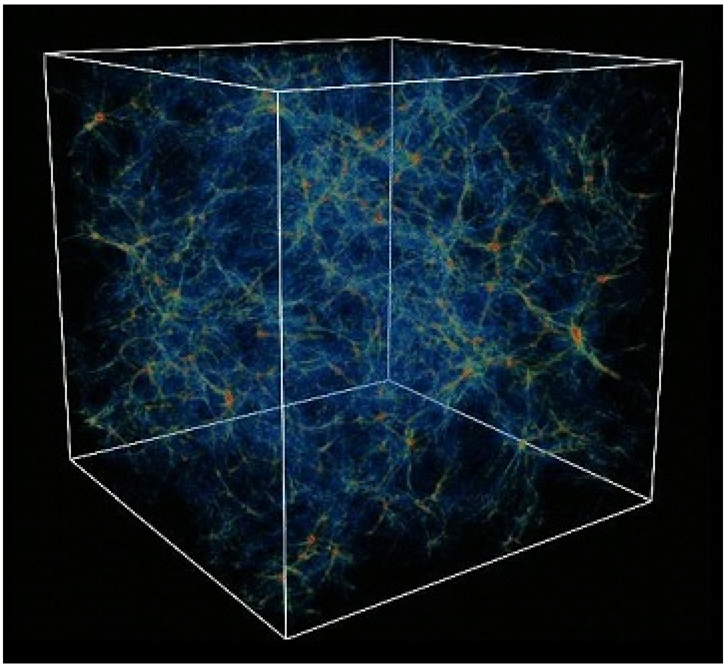

Figure 1.2: The filaments shown [6] are from a computer simulation.

years wide and only 15 million light-years thick [5].

In our universe, inhomogeneities seem to arise even on the biggest of scales. From our Earth in the Solar system, the Sun amongst all stars in the Milky Way, the Milky Way in clusters of galaxies, these clusters in clusters of clusters and so on. The biggest structures we know of now are the cosmic filaments (see figure 1.2) of super clusters and the big voids of vacuum between these clumps of matter.

The large structures give rise to the idea that the universe may not be homogeneous at all, which most modern cosmology is based on, i.e. the cosmological principle.

CHAPTER 2

Observations

2.1 Supernova Ia - Standard candle

The only thing that is measured by telescopes is the red-shifted light of the far away objects we are trying to investigate. To be able to translate this information to a distance, one needs to have an object that one knows the luminosity and spectrum of. A supernova Ia can be considered such an object and is often referred to as a ”standard candle” in the universe.

In a double-star system where the more massive star has already gone through its stellar cycle and ended up being a compact white dwarf, where the other will eventually become a red giant. The red giant will be very large compared to its small neighbor and will therefore have a very low density at its surface. The gas of the red giant will escape it and begin to go in an accretion disc around the white dwarf, see figure 2.1. If the accretion continues for long enough, the white dwarf will eventually approach the Chandrasekhar limit (around 1.4 Solar masses) and become unstable. It will then explode in a supernova and give off a characteristic light curve, since most supernovae of this kind are created in a similar way.

The light seen from one of these supernovae is so intense that it will outshine the whole galaxy it inhabits. If one searches the millions of galaxies all around us, the chance of one occurring is promising.

By using the light from these supernovae, several high redshift observations have been made [1], [2]. The results show that the universe seems to be accelerating.

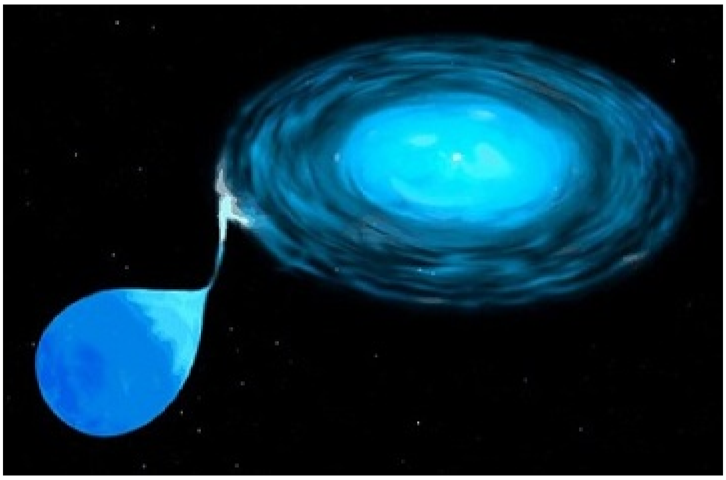

Figure 2.1: Shown [7] is an accretion disc of a gas giant's outer layers towards a white dwarf.

2.2 The cosmic microwave background

Looking out into the space between all the galaxies and stars, it seems to be pitch black. However, in the 1960's, Penzias and Wilson discovered that the whole universe seemed to have a background glow, similar in all directions. It peaked in the microwave region of the electromagnetic spectrum. A closer study of this radiation showed that it was a black body spectrum with a temperature of about 3K [8]. It is the most precise black body spectrum ever to be observed.

When the universe was young, it consisted of a hot compact plasma and radiation. As the universe expanded, it also grew colder. When it was cold enough, stable atoms could form. Now, these atoms could not absorb the thermal radiation that was all around them. Thus, the radiation could travel freely through space and as a consequence, the universe became transparent to light. The radiation that was released then is what the Big Bang theory explains as the Cosmic Microwave Background (CMB).

Considering the reaction $p + e^- \rightleftarrows H + \gamma$ for when it is mainly dominated by the right-going reaction, gives a reasonable value of a temperature at 3000K, where the photons can no longer ionize hydrogen. Since the CMB is 3K now, the wavelength has become about 1000 times longer and the universe 1000 times larger, since $\frac{\lambda_0}{\lambda} = \frac{R_0}{R}$, which also corresponds to a redshift of $z \sim 1000$.

In more recent days, the CMB has been observed with more powerful equipment, the WMAP, and it seems that the CMB has tiny fluctuations of the order of μK, see figure 2.2. This in turn shows that the universe is very close to flat [4]. This anisotropy seen in the CMB is responsible for the structure formation that followed, which in turn formed all the galaxies and clusters that we see today, i.e. all inhomogeneities.

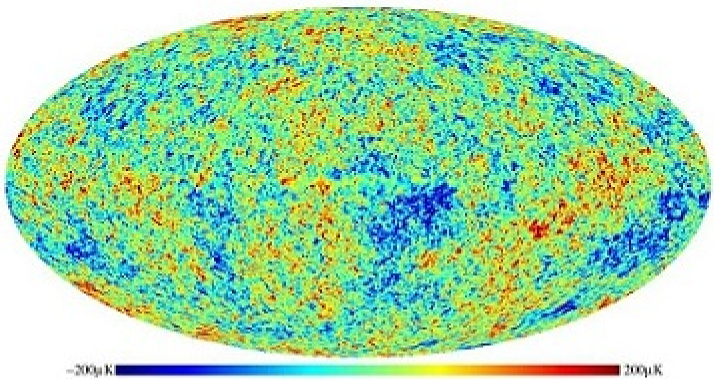

Figure 2.2: Shown [4] is the whole sky in the microwave regime. Different colors stand for different temperatures. Here, one can clearly see the small fluctuations in temperature.

CHAPTER 3

The "standard model" of cosmology today

The standard model of cosmology today is called the Lambda Cold Dark Matter (ΛCDM) model. Since the CMB shows that the universe is close to flat and the supernova Ia data shows an acceleration, the cosmological constant is reintroduced in Einstein's equations,

$$R_{ab} - \frac{1}{2}g_{ab}R + g_{ab}\Lambda = \frac{8\pi G}{c^4}T_{ab}. \tag{3.1}$$

This model is based on the cosmological principle and assumes flat space-time. Object can travel away from each other faster than the speed of light if they are spatially separated. This is still consistent with special relativity, which is only a local theory. This means that if distances between objects are cosmologically small ($z \sim 0$), then nothing can travel faster than the speed of light. Λ is the cosmological constant which is associated with dark energy. The energy density parameter of dark energy is given as

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}, \tag{3.2}$$

where $\rho_\Lambda = \frac{\Lambda}{8\pi}$. The cold dark matter is simply non-relativistic matter that cannot be seen with light detection. This is necessary to account for the gravitational effects seen in galactic rotational curves and gravitational lensing of clusters. The Big Bang theory is used to explain the early universe. The model has cosmic inflation to account for the "horizon problem" in the CMB, where two spots cannot have been in casual contact to form a thermal equilibrium.

The universe is flat, so the energy densities has to add up to 1. An approximate fit of these parameters is [4]:

$$\Omega_\Lambda = 0.726, \Omega_{DM} = 0.228, \Omega_{LM} = 0.046, \tag{3.3}$$

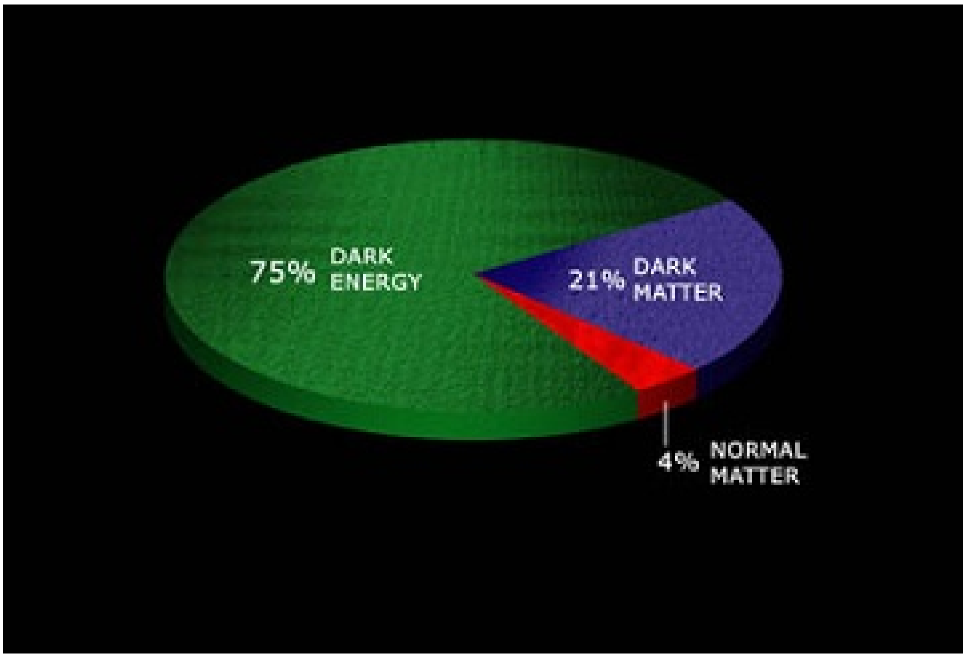

Figure 3.1: The energy density of the universe is illustrated as a pie [9].

where dark matter and light matter add up to become $\Omega_M = \Omega_{DM} + \Omega_{LM} = 0.274$. An illustrative representation can be seen in figure 3.1.

Allowing quintessence instead of a cosmological constant is essentially allowing Λ to vary in space and time. This means that the equation of state may differ from a value of $w = -1$ (constant energy density). All values with $w < 0$ give a negative pressure. A value of $w < -1/3$ will lead to an accelerating universe which is observed today. Letting $w < -1$ leads to the "Big Rip" where all galaxies, stars and eventually atoms are torn apart by the dark energy.

CHAPTER 4

A new model

4.1 Why inhomogeneous?

One assumes Friedmann-Lemaître-Robertson-Walker (FLRW) metric only because it is simple. Assuming a flat and homogeneous spacetime results in an analytical solution to Einstein's equations. The assumption makes the structure variable, $a(t)$, contain all the dynamics. In "real life" Einstein's equations give rise to ten coupled, nonlinear, partial differential equations. These are analytically unsolvable for a general case.

4.2 Theory

The goal is to create a model to measure how "inhomogeneous" the observable universe is. We considered a universe with Newtonian gravitation (gravitation is infinitely fast). Even though Newtonian cosmology is not completely mathematically consistent in an infinite universe, as it depends on the boundary conditions infinitely far away, we can dodge that difficulty by treating the Big Bang as a "normal" explosion in a pre-existing space with a finite number of particles corresponding to the observable universe in a general relativistic setting. The idea is to express the real acceleration in terms of the acceleration that would be felt if the universe was completely homogeneous, plus a correction term arising from the inhomogeneity.

Nomenclature:
d, the distance from an object to the observer.
v, the observed velocity of an object in the radial direction.
a, the observed acceleration of an object in the radial direction.

If one considers an Earth-bound observer looking out into the universe, she can measure these three quantities in one way or another.

Starting by defining an acceleration called the "homogeneous acceleration", denoted a_h. This is the acceleration that would be experienced by an object if the distribution would have been homogeneous and isotropic on all scales. Simply put, if a test particle is somewhere in a homogeneous sphere, the only net effect of gravitation is the mass within a smaller sphere with a radius equal to the distance from the test mass to the center. The acceleration of this test mass becomes

$$a_h = -\frac{Gm}{d^2}, \tag{4.1}$$

where m is the mass of the small sphere. This mass can be expressed by the volume ratio times the total mass of the sphere, M,

$$m = M\frac{d^3}{R^3}, \tag{4.2}$$

where R is the radius of the total sphere. The acceleration can be expressed as

$$a_h = -\frac{GMd}{R^3}. \tag{4.3}$$

If one knows the total mass, then one way of deciding R is to check the mean-distance of the observed supernovae. If one assumes that we can see all supernovae in a certain sphere, one may then take the mean-value of d and multiply with four-thirds to get the radius, as the geometric center lies at three-fourths of the radius in a cone-fragment of a sphere. The equation for the acceleration becomes

$$a_h = -\frac{27GMd}{64\langle d\rangle^3}, \tag{4.4}$$

where $\langle d\rangle$ is the observed mean-value of the distance, d.

If one knows the mean-density, ρ, the acceleration becomes

$$a_h = -\frac{4\pi\rho Gd}{3}. \tag{4.5}$$

Now to obtain a quantitative measure of inhomogeneity. If one starts with the acceleration, one may simply add and subtract the "homogeneous acceleration", a_h,

$$a = a_h + (a - a_h), \tag{4.6}$$

extract a factor $\frac{v^2}{d}$ from the bracket

$$a = a_h + \frac{v^2}{d}\left(\frac{ad}{v^2} - \frac{a_h d}{v^2}\right). \tag{4.7}$$

Introducing the Hubble parameter, $H = \frac{v}{d}$, one gets

$$a = a_h + dH^2 \left(\frac{ad}{v^2} - \frac{a_h d}{v^2} \right). \tag{4.8}$$

The terms inside the brackets are dimensionless. As can be seen, this result yields the same behavior as the standard model with a cosmological constant, where instead of the bracket one has the Ω_Λ term [10]. From now on the correction terms for the inhomogeneity will be denoted by

$$Q = \left(\frac{ad}{v^2} - \frac{a_h d}{v^2} \right). \tag{4.9}$$

4.3 An analytical inhomogeneous example

Let us consider an inhomogeneous universe consisting of two large bodies and one observer between them. The two bodies have an equal mass and therefore half the mass of the universe each. The calculations will be done using newtonian gravity. If the velocity and accelerations can be expressed in terms of the distance from the observer, d, the correction term for inhomogeneity, Q, can be calculated accordingly.

This is a two-body problem and the acceleration in the radial direction is

$$a = -\frac{GM}{8d^2}. \tag{4.10}$$

The gravitational potential, V, is defined according to

$$F = -\bigtriangledown_{2d} V, \tag{4.11}$$

where F is the force acting on one of the particles in its radial direction, which is given by

$$F = -\frac{GM^2}{16d^2}. \tag{4.12}$$

Solving for V gives

$$V = -\frac{GM^2}{8d}. \tag{4.13}$$

If the particles have escape velocity (corresponding to flat space-time), the virial theorem states that

$$2K + V = 0, \tag{4.14}$$

where K is the kinetic energy of one of the particles, $K = -\frac{Mv^2}{4}$. Solving for v^2 gives

$$v^2 = \frac{GM}{4d}. \tag{4.15}$$

The only factor not known in Q is the "homogeneous acceleration", a_h. The total mass of the universe is M so we use Eq. (4.4) for the acceleration,

$$a_h = -\frac{27GMd}{64\langle d\rangle^3}. \tag{4.16}$$

In our case $\langle d\rangle = d$ because both particles are at the same distance from the observer. So the "homogeneous acceleration" becomes

$$a_h = -\frac{27GM}{64d^2}. \tag{4.17}$$

Eq. (4.9 with 4.10, 4.15, 4.17) give the value for Q,

$$Q = \left(-\frac{1}{2} + \frac{27}{16}\right) = 1.1875. \tag{4.18}$$

The real acceleration will be

$$a = a_h + dH^2 1.1875. \tag{4.19}$$

This example shows that if one observes an inhomogeneous universe and still "pretends" that it is homogeneous, it will yield a correction term for the inhomogeneity that behaves in the same way as a cosmological constant would. This may then remove the need of dark energy with its mysterious negative pressure.

It is argued that in a FLRW-universe with a cosmological constant it is just a strange and completely unexplained "cosmic coincidence" that $\Omega_M \sim \Omega_\Lambda$ now [11]. Ω_M dominates in the early universe and Ω_Λ in the late. Since an appreciable amount of structure must form before intelligent life can evolve and observe the universe, it should then be natural to exist in a time when the apparent "acceleration" (really due to inhomogeneity) becomes observable.

4.4 Newtonian simulations with special relativity

I have done an N-body simulation with newtonian gravitation that takes into account the effects of special relativity. The objects are treated as particles with a small gas-like distribution. If they get really close to each other, the gravitational potential will not be infinite. The particles will therefore not collide but simply pass through each other without having an infinite acceleration. Just as expected for "test-particles" of cosmology; galaxies. They are distributed randomly in a sphere so that the large-scale density is almost uniform, but still inhomogeneous. The particles begin with escape velocity as initial condition, as this is equivalent to a flat universe in the general relativistic setting. The acceleration is then calculated and the next distance and velocity for all the particles are iterated using a finite difference method in time. Plotting the distance from the center of the sphere against the red-shift, and comparing to a few fully relativistic

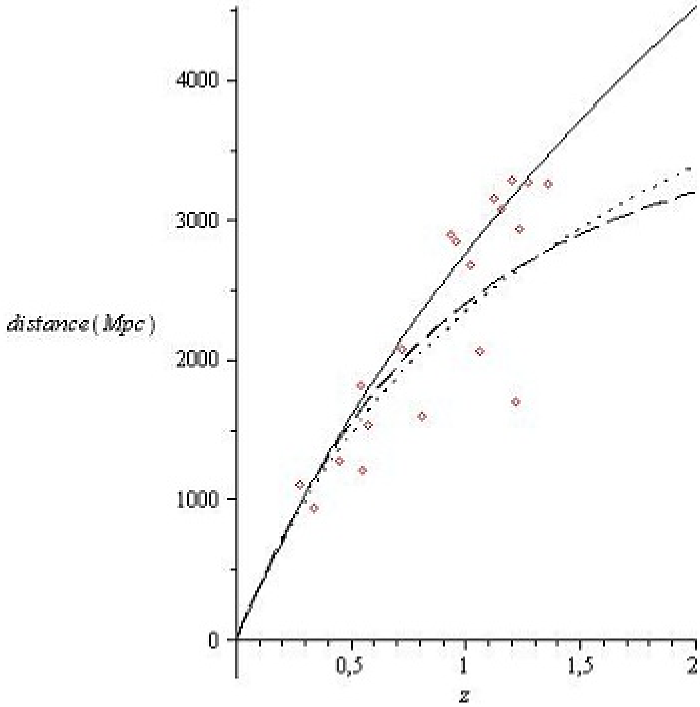

Figure 4.1: Shown [11] is the prediction of three different homogeneous models shown together with the simulated inhomogeneous "particles". The solid line is an open FLRW-model with $\Omega_M = 0.2$, $\Omega_\Lambda = 0$. The dotted line is a flat FLRW-model with $\Omega_M = 1$, $\Omega_\Lambda = 0$. The dashed line is the (flat) newtonian prediction with corrections for special relativity. An inhomogeneous matter distribution may thus be interpreted wrongly as to suggest that Ω_M is lower than it actually is, e.g. $\Omega_M = 0.2$ if interpreted through a homogeneous and isotropic "standard model" cosmology. The simulated particles correspond to $\Omega = \Omega_M = 1$. The assumption of homogeneity and isotropy mislead the physical interpretation if the real distribution is inhomogeneous, as is the case for our universe.

models, show some interesting results in figure 4.1. All models seem to have almost the same appearance up to $z \sim 1$ and the particles seem to line up better with the model where the energy density for matter is, $\Omega_M = 0.2$.

Chapter 5

Conclusions

The question we must ask is how the inhomogeneous distribution can be overlooked when one observes the large scale structures that we do. At the early era when the CMB was released, the universe can best be described as a homogeneous system with small perturbations. However, for the present universe this is not the case, and has not been so for a very long time. And because of the inhomogeneity, dark energy may not even be needed to explain the acceleration of the universe. In fact, the inhomogeneous model could remove the need for an accelerating universe all together.

CHAPTER 6

Discussion

Why do the majority of cosmologists continue to only consider Einstein's equations for a homogeneous matter distribution (with "small" perturbations) when that clearly is not what we observe? Modeling with a realistic inhomogeneous distribution should ideally be done by all cosmologists in this area. I understand why modelers want to use a homogeneous universe, as Einstein's equations become immensely simpler if a homogeneous distribution is assumed - the full Einstein equations (ten coupled non-linear PDEs) reduce to Friedmann's equations (two coupled linear ODEs) and all dynamics become encoded simply in the cosmological scale-factor, $a(t)$. Anyway, who said that nature was easy?

Appendix A

N-body simulation program code for Maple

```
restart: with(plots): with(plottools): with(RandomTools): with(Statistics):
with(linalg): Digits := 10: randomize():

np := 100:

ni := 1000:

ts := 10^(-3.0):

dmin := 10^(-3.0):

vmin := 2^(-5.0):

rad := 2^(1/3.0):

G := 1:

H := 75000:

M := 1:

c := 2.99792458*10^8:

m := M/np:
```

```
mu := (1/2)*m:

disR := Distribution(PDF = (t \rightarrow piecewise(0 <= t <= rad, 3*t^2/rad^3,
0)), CDF = (t \rightarrow piecewise(0 <= t <= rad, t^3/rad^3, rad < t, 1, 0)),
Mean = (3/4)*rad):

disTheta := Distribution(PDF = (t \rightarrow piecewise(0 <= t <= evalf(Pi),
(1/2)*sin(t), 0)), CDF = (t \rightarrow piecewise(0 <= t <= evalf(Pi),
1/2-(1/2)*cos(t), evalf(Pi) < t, 1, 0)), Mean = (1/2)*evalf(Pi));

dR := RandomVariable(disR):

dTheta := RandomVariable(disTheta):

for j from 0 to np do

r[j, 0] := Generate(distribution(dR)):

theta[j, 0] := Generate(distribution(dTheta)):

phi[j, 0] := Generate(distribution(Uniform(0, 2*evalf(Pi)))):

x[j, 0] := r[j, 0]*sin(theta[j, 0])*cos(phi[j, 0]):

y[j, 0] := r[j, 0]*sin(theta[j, 0])*sin(phi[j, 0]):

z[j, 0] := r[j, 0]*cos(theta[j, 0]):

d[j, 0] := vector([x[j, 0], y[j, 0], z[j, 0]]) end do:

for j from 0 to np do

v[j, 0] := 0:

for q from 0 to j-1 do

if (x[j, 0]-x[q, 0])^2+(y[j, 0]-y[q, 0])^2+(z[j, 0]-z[q, 0])^2 > dmin^2 then

v[j, 0] := v[j, 0]+1/((x[j, 0]-x[q, 0])^2+(y[j, 0]-y[q, 0])^2+(z[j, 0]-z[q,
0])^2)^.5:
```

```
else

v[j, 0] := v[j, 0]+3/(2*dmin)-((x[j, 0]-x[q, 0])^2+(y[j, 0]-y[q, 0])^2+(z[j,
0]-z[q, 0])^2)/(2*dmin^3):

end if:

end do:

for q from j+1 to np do

if (x[j, 0]-x[q, 0])^2+(y[j, 0]-y[q, 0])^2+(z[j, 0]-z[q, 0])^2 > dmin^2 then

v[j, 0] := v[j, 0]+1/((x[j, 0]-x[q, 0])^2+(y[j, 0]-y[q, 0])^2+(z[j, 0]-z[q,
0])^2)^.5:

else

v[j, 0] := v[j, 0]+3/(2*dmin)-((x[j, 0]-x[q, 0])^2+(y[j, 0]-y[q, 0])^2+(z[j,
0]-z[q, 0])^2)/(2*dmin^3):

end if:

end do:

v[j, 0] := (1-1/(1+v[j, 0]*G*mu)^2)^.5; Gamma[j, 0] := 1/(1-v[j, 0]^2)^.5:

end do:

for j from 0 to np do

xxx[j, 0] := 0:

yyy[j, 0] := 0:

zzz[j, 0] := 0:

for k from 0 to j-1 do

if (x[k, 0]-x[j, 0])^2+(y[k, 0]-y[j, 0])^2+(z[k, 0]-z[j, 0])^2 > dmin^2 then
```

```
xxx[j, 0] := xxx[j, 0]+(x[k, 0]-x[j, 0])/((x[k, 0]-x[j, 0])^2+(y[k, 0]-y[j,
0])^2+(z[k, 0]-z[j, 0])^2)^1.5:

yyy[j, 0] := yyy[j, 0]+(y[k, 0]-y[j, 0])/((x[k, 0]-x[j, 0])^2+(y[k, 0]-y[j,
0])^2+(z[k, 0]-z[j, 0])^2)^1.5:

zzz[j, 0] := zzz[j, 0]+(z[k, 0]-z[j, 0])/((x[k, 0]-x[j, 0])^2+(y[k, 0]-y[j,
0])^2+(z[k, 0]-z[j, 0])^2)^1.5:

else

xxx[j, 0] := xxx[j, 0]+(x[k, 0]-x[j, 0])/dmin^3:

yyy[j, 0] := yyy[j, 0]+(y[k, 0]-y[j, 0])/dmin^3:

zzz[j, 0] := zzz[j, 0]+(z[k, 0]-z[j, 0])/dmin^3:

end if:

end do:

for k from j+1 to np do

if (x[k, 0]-x[j, 0])^2+(y[k, 0]-y[j, 0])^2+(z[k, 0]-z[j, 0])^2 > dmin^2 then

xxx[j, 0] := xxx[j, 0]+(x[k, 0]-x[j, 0])/((x[k, 0]-x[j, 0])^2+(y[k, 0]-y[j,
0])^2+(z[k, 0]-z[j, 0])^2)^1.5:

yyy[j, 0] := yyy[j, 0]+(y[k, 0]-y[j, 0])/((x[k, 0]-x[j, 0])^2+(y[k, 0]-y[j,
0])^2+(z[k, 0]-z[j, 0])^2)^1.5:

zzz[j, 0] := zzz[j, 0]+(z[k, 0]-z[j, 0])/((x[k, 0]-x[j, 0])^2+(y[k, 0]-y[j,
0])^2+(z[k, 0]-z[j, 0])^2)^1.5:

else

xxx[j, 0] := xxx[j, 0]+(x[k, 0]-x[j, 0])/dmin^3:

yyy[j, 0] := yyy[j, 0]+(y[k, 0]-y[j, 0])/dmin^3:
```

```
zzz[j, 0] := zzz[j, 0]+(z[k, 0]-z[j, 0])/dmin^3:

end if:

end do:

xxx[j, 0] := xxx[j, 0]*G*m/(Gamma[j, 0]+v[j, 0]^2*Gamma[j, 0]^3):

yyy[j, 0] := yyy[j, 0]*G*m/(Gamma[j, 0]+v[j, 0]^2*Gamma[j, 0]^3):

zzz[j, 0] := zzz[j, 0]*G*m/(Gamma[j, 0]+v[j, 0]^2*Gamma[j, 0]^3):

ddd[j, 0] := vector([xxx[j, 0], yyy[j, 0], zzz[j, 0]]):

end do:

for j from 0 to np do

if xxx[j, 0]^2+yyy[j, 0]^2+zzz[j, 0]^2 = 0 then

xx[j, 0] := 0:

yy[j, 0] := 0:

zz[j, 0] := 0:

else

xx[j, 0] := -xxx[j, 0]*v[j, 0]/(xxx[j, 0]^2+yyy[j, 0]^2+zzz[j, 0]^2)^.5:

yy[j, 0] := -yyy[j, 0]*v[j, 0]/(xxx[j, 0]^2+yyy[j, 0]^2+zzz[j, 0]^2)^.5:

zz[j, 0] := -zzz[j, 0]*v[j, 0]/(xxx[j, 0]^2+yyy[j, 0]^2+zzz[j, 0]^2)^.5:

end if:

dd[j, 0] := vector([xx[j, 0], yy[j, 0], zz[j, 0]]):

h[j, 0] := dotprod(dd[j, 0], d[j, 0])/(x[j, 0]^2+y[j, 0]^2+z[j, 0]^2):

end do:
```

```
px := 0:

py := 0:

pz := 0:

for j from 0 to np do

px := px+xx[j, 0]:

py := py+yy[j, 0]:

pz := pz+zz[j, 0]:

end do:

px := m*px/np;

py := m*py/np;

pz := m*pz/np;

for i from 0 to ni do

for j from 0 to np do

x[j, i] := x[j, i-1]+xx[j, i-1]*ts+(1/2)*xxx[j, i-1]*ts^2:

y[j, i] := y[j, i-1]+yy[j, i-1]*ts+(1/2)*yyy[j, i-1]*ts^2:

z[j, i] := z[j, i-1]+zz[j, i-1]*ts+(1/2)*zzz[j, i-1]*ts^2:

d[j, i] := vector([x[j, i], y[j, i], z[j, i]]):

xx[j, i] := (xx[j, i-1]+xxx[j, i-1]*ts)/(1+(xx[j, i-1]^2+yy[j, i-1]^2+zz[j,
i-1]^2)^.5*(xxx[j, i-1]^2+yyy[j, i-1]^2+zzz[j, i-1]^2)^.5*ts):

yy[j, i] := (yy[j, i-1]+yyy[j, i-1]*ts)/(1+(xx[j, i-1]^2+yy[j, i-1]^2+zz[j,
i-1]^2)^.5*(xxx[j, i-1]^2+yyy[j, i-1]^2+zzz[j, i-1]^2)^.5*ts):
```

```
zz[j, i] := (zz[j, i-1]+zzz[j, i-1]*ts)/(1+(xx[j, i-1]^2+yy[j, i-1]^2+zz[j,
i-1]^2)^.5*(xxx[j, i-1]^2+yyy[j, i-1]^2+zzz[j, i-1]^2)^.5*ts):

dd[j, i] := vector([xx[j, i], yy[j, i], zz[j, i]]):

v[j, i] := (xx[j, i]^2+yy[j, i]^2+zz[j, i]^2)^.5:

Gamma[j, i] := 1/(1-v[j, i]^2)^.5:

h[j, i] := dotprod(dd[j, i], d[j, i])/(x[j, i]^2+y[j, i]^2+z[j, i]^2):

end do:

for j from 0 to np do

xxx[j, i] := 0:

yyy[j, i] := 0:

zzz[j, i] := 0:

for k to j-1 do

if (x[k, i]-x[j, i])^2+(y[k, i]-y[j, i])^2+(z[k, i]-z[j, i])^2 > dmin^2 then

xxx[j, i] := xxx[j, i]+(x[k, i]-x[j, i])/((x[k, i]-x[j, i])^2+(y[k, i]-y[j,
i])^2+(z[k, i]-z[j, i])^2)^1.5:

yyy[j, i] := yyy[j, i]+(y[k, i]-y[j, i])/((x[k, i]-x[j, i])^2+(y[k, i]-y[j,
i])^2+(z[k, i]-z[j, i])^2)^1.5:

zzz[j, i] := zzz[j, i]+(z[k, i]-z[j, i])/((x[k, i]-x[j, i])^2+(y[k, i]-y[j,
i])^2+(z[k, i]-z[j, i])^2)^1.5:

else

xxx[j, i] := xxx[j, i]+(x[k, i]-x[j, i])/dmin^3:

yyy[j, i] := yyy[j, i]+(y[k, i]-y[j, i])/dmin^3:

zzz[j, i] := zzz[j, i]+(z[k, i]-z[j, i])/dmin^3:
```

```
end if:

end do:

for k from j+1 to np do

if (x[k, i]-x[j, i])^2+(y[k, i]-y[j, i])^2+(z[k, i]-z[j, i])^2 > dmin^2 then

xxx[j, i] := xxx[j, i]+(x[k, i]-x[j, i])/((x[k, i]-x[j, i])^2+(y[k, i]-y[j,
i])^2+(z[k, i]-z[j, i])^2)^1.5:

yyy[j, i] := yyy[j, i]+(y[k, i]-y[j, i])/((x[k, i]-x[j, i])^2+(y[k, i]-y[j,
i])^2+(z[k, i]-z[j, i])^2)^1.5:

zzz[j, i] := zzz[j, i]+(z[k, i]-z[j, i])/((x[k, i]-x[j, i])^2+(y[k, i]-y[j,
i])^2+(z[k, i]-z[j, i])^2)^1.5:

else

xxx[j, i] := xxx[j, i]+(x[k, i]-x[j, i])/dmin^3:

yyy[j, i] := yyy[j, i]+(y[k, i]-y[j, i])/dmin^3:

zzz[j, i] := zzz[j, i]+(z[k, i]-z[j, i])/dmin^3:

end if:

end do:

xxx[j, i] := xxx[j, i]*G*m/(Gamma[j, i]+v[j, i]^2*Gamma[j, i]^3):

yyy[j, i] := yyy[j, i]*G*m/(Gamma[j, i]+v[j, i]^2*Gamma[j, i]^3):

zzz[j, i] := zzz[j, i]*G*m/(Gamma[j, i]+v[j, i]^2*Gamma[j, i]^3):

ddd[j, i] := vector([xxx[j, i], yyy[j, i], zzz[j, i]]):

end do:

end do:
```

```
for i from 0 to ni do

R[i] := 0:

RR[i] := 0:

RRR[i] := 0:

h[i] := 0:

for j from 0 to np do

R[i] := R[i]+(x[j, i]^2+y[j, i]^2+z[j, i]^2)^.5:

RR[i] := RR[i]+(xx[j, i]*x[j, i]+yy[j, i]*y[j, i]+zz[j, i]*z[j, i])/(x[j,
i]^2+y[j, i]^2+z[j, i]^2)^.5:

RRR[i] := RRR[i]+(xxx[j, i]*x[j, i]+yyy[j, i]*y[j, i]+zzz[j, i]*z[j, i])/(x[j,
i]^2+y[j, i]^2+z[j, i]^2)^.5:

h[i] := h[i]+(xx[j, i]*x[j, i]+yy[j, i]*y[j, i]+zz[j, i]*z[j, i])/(x[j,
i]^2+y[j, i]^2+z[j, i]^2):

end do:

R[i] := R[i]/np:

RR[i] := RR[i]/np:

RRR[i] := RRR[i]/np:

h[i] := h[i]/np:

end do:

npoints := np:

for i from 0 to ni do

Omega[i] := 0:
```

```
for j from 0 to np do

Omega[i] := Omega[i]+(xxx[j, i]*x[j, i]+yyy[j, i]*y[j, i]+zzz[j, i]*z[j,
i])/((x[j, i]^2+y[j, i]^2+z[j, i]^2)*h[j,i]^2)+3^3*G*M/(4^3*h[j,i]^2*R[i]^3):

end do:

Omega[i] := Omega[i]/npoints:

end do:

Om := .26:

points0a := {seq([q, Omega[q]], q = 0 .. ni)}; plot0a := pointplot(points0a,
style = point, color = red); display(plot0a, labels = [iterations, Omega], view
= [0 .. ni, -0.5e-1 .. 100]):

points1 := {seq([((1+(xx[p, 0]*x[p, 0]+yy[p, 0]*y[p, 0]+zz[p, 0]*z[p, 0])/(x[p,
0]^2+y[p, 0]^2+z[p, 0]^2)^0.5)/(1-(xx[p, 0]*x[p, 0]+yy[p,0]*y[p,0]+zz[p, 0]*z[p,
0])/(x[p, 0]^2+y[p, 0]^2+z[p, 0]^2)^0.5))^0.5-1, (x[p, 0]^2+y[p, 0]^2+z[p,
0]^2)^0.5*(xx[p, 0]*x[p, 0]+yy[p, 0]*y[p, 0]+zz[p, 0]*z[p, 0])*c/((x[p,
0]^2+y[p, 0]^2+z[p, 0]^2)^0.5*H)], p = 1 .. np)}:

plot1a := pointplot(points1, style = point, color = red):

plot1b := plot(((1+p)^2-1)*c/(((1+p)^2 + 1)*H), p = 0 .. 2, color = black,
linestyle = dash):

plot1c := plot(2*(1-(1+p)^(-1/2.0))*c/H, p = 0 .. 2, color = black, linestyle =
dot):

plot1d := plot(2*c*(Om*p+(Om-2)*((Om*p+1)^.5-1))/(H*Om^2*(1+p)), p = 0 .. 2,
color = black); plot1e := plot((p+p^2)*c/((1+p)*H), p = 0 .. 2, color = blue):

display({plot1a, plot1b, plot1c, plot1d, plot1e}, labels = [z, distance(Mpc)]);

points2 := {seq([((1+(xx[p, ni]*x[p, ni]+yy[p, ni]*y[p, ni]+zz[p, ni]*z[p,
ni])/(x[p, ni]^2+y[p, ni]^2+z[p, ni]^2)^0.5)/(1-(xx[p, ni]*x[p, ni]+yy[p,
ni]*y[p, ni]+zz[p, ni]*z[p, ni])/(x[p, ni]^2+y[p, ni]^2+z[p, ni]^2)^0.5))^0.5-1,
(x[p, ni]^2+y[p, ni]^2+z[p, ni]^2)^0.5*(xx[p, ni]*x[p, ni]+yy[p, ni]*y[p,
```

```
ni]+zz[p, ni]*z[p, ni])*c/((x[p, ni]^2+y[p, ni]^2+z[p, ni]^2)^0.5*H)], p = 1 ..
np)}:

plot2a := pointplot(points2, style = point, color = red):

plot2b := plot(((1+p)^2-1)*c/(((1+p)^2+1)*H), p = 0.5e-2 .. 2, color = black,
linestyle = dash):

plot2c := plot(2*(1-(1+p)^(-1/2.0))*c/H, p = 0.5e-2 .. 2, color = black,
linestyle = dot):

plot2d := plot(2*c*(Om*p+(Om-2)*((Om*p+1)^.5-1))/(H*Om^2*(1+p)), p = 0.5e-2 ..
2, color = black):}

display({plot2a, plot2b, plot2c, plot2d}, labels = [z, distance(Mpc)], axis[1,
2] = [mode = log]):

points3 := {seq([p, RR[p]], p = 0 .. ni)}:

plot3a := pointplot(points3, style = point, color = red):

display({plot3a}, labels = [iterations, velocity*'c'], view = [0 .. ni, -1 ..
1]);
```

References

[1] Riess A., et al., *The Astrophysical Journal* 116, 1009 (1998).

[2] Perlmutter S., et al., *The Astrophysical Journal* 517, 565 (1999).

[3] d'Inverno R., *Introducting Einsteins relativity*, Oxford University Press Inc. (1992). Pages 322-323.

[4] Nasa, *WMAP Highlights*.
URL: http://map.gsfc.nasa.gov

[5] Geller M., Huchra J., *Science* 246, 897-903 (1989).

[6] Department of Physics and Astronomy, University of Tennessee, *Astrowiki*.
URL: http://csep10.phys.utk.edu/astr162/lect/cosmology/growth.html

[7] Nasa, *Astronomy Picture of the Day*.
URL: http://apod.nasa.gov/apod/ap991219.html

[8] Penzias A., Wilson R., *The Astrophysical Journal Letters* 142, 419-421 (1965).

[9] Nasa, *Imagine the Universe!*.
URL: http://imagine.gsfc.nasa.gov/docs/features/news/26may04.html

[10] Peebles P., *Principles of Physical Cosmology*, Princeton University Press (1993).

[11] Hansson J., Lindkvist J., *The Open Astronomy Journal* 3, 145-149 (2010).
arXiv:0906.3403v1. [physics.gen-ph]