Progressive Landslides in Long Natural Slopes

Formation, potential extension and configuration of finished slides in strain-softening soils

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Preface and Acknowledgements

In the late 1960’s and the early 1970’s a number of large planar landslides took place in southwestern Sweden. On inspecting the sites of some of these slides, I observed that the topography of the finished slides seemed to be inconsistent with the failure mechanism based on ideal-plastic limit equilibrium, by which practicing engineers generally predict potential slide hazards.

Therefore, in my capacity during the years 1970 – 91 of heading the engineering department of Skanska Väst AB (then a subsidiary of a leading contracting company in Sweden), I conducted a research program focused on the possible effects of brittle failure mechanisms in landslides, which had occurred in deformation softening clays. A computer software for incorporating the effects of deformation softening into the analysis of slope stability was developed.

The progress of this work was presented to a larger audience in a number of separate publications in Swedish and English during the period 1978 to 1989. However, the various reports reflected different aspects of the problem of brittle failures in soils as well as different stages in the development of an engineering approach.

The purpose of the present report is to synthesize the essential principles, ideas and findings that resulted from this research and motivated the above mentioned publications.

In 1997 the bodies mentioned below granted funding for a research project with the following three objectives:

1) Establishing a report giving a coherent account of the various issues involved in brittle slope failures i.e.
   - limitations as to the applicability of the ideal-plastic failure type of analysis
   - defining the different phases of a progressive failure event
   - detailing and exemplifying the basic equations of the applied analytical model
   - identification of factors and circumstances conducive to brittle slope failures
   - practical recommendations regarding procedures for investigating slope stability in deformation softening soils.

2) Updating existing computer software in Basic to a Windows environment

3) Applying the analytical model on a few well documented landslides, and examining the viability of the method of analysis by checking if the computational results match or explain the actual slide events.

The organizations supporting this research program are:

- the Swedish Council for Building Research (BFR)
- the Development Fund of the Swedish Construction Industry (SBUF)
- the Division of Structural Engineering, Luleå University of Technology, Luleå
- Skanska AB
- Congeo AB, Mölndal
An advisory reference group was appointed for the project consisting of the following members:

- **Elvin Ottosson**  Chief engineer, SGI (the Swedish Geotechnical Institute)
- **Per Evert Bengtsson**  Research engineer (Avd.dir.), SGI
- **Göran Sällfors**  Professor, Chalmers University of Technology
- **Jan Hartlén**  Doctor of technology, JH Geo Consulting HB
- **Ingmar Svensk**  Civil engineer, Skanska Teknik AB
- **Ulf Ericsson**  Civil engineer, NCC Teknik AB

I wish to express my gratitude to the members of the reference group for having read the manuscript of the report (or parts thereof) and for the advising and critical comments made. In particular I must thank Per Evert Bengtsson (SGI) for his detailed, constructive and knowledgeable criticism. Also, his rapid implementation of the proposed differential equations in Chapter 4 into Excel facilitated checking of numbers in the numerical examples in the report. Editorial comments by Jan Lindgren (SGI) have been of considerable value.

I am indebted to professor Lennart Elfgren (head of the Department of Civil and Mining Engineering, Luleå University of Technology) for not only initiating the current project but also for his constant and inspiring support. Working with him and with professor Krister Cederwall and their co-workers and students as a part-time adjunct professor at the Division of Structural Engineering during the years 1980 – 1998, gave me the opportunity to consider crack formation and strain softening also in young concrete.

Special acknowledgements are directed to Ingmar Svensk, Anders Hansson and Lars Nordström (Skanska Teknik AB). I am much obliged to Ingmar for his wholehearted support and for his active contributions to raising the necessary funding for this project. Much credit must be given to Anders for performing a major part of the computer analyses in connection with the case records described in the report and to Lars for having drawn most of the figures.

Furthermore, I want to express my deep appreciation to my former colleagues at Skanska Teknik AB for their various contributions in the 1980-ties to the research work that has led forward to the present study, and in the absence of which the current project would not have been possible. In this context, I feel obliged to mention the names of the civil engineers Hasse Gustås, Ingvar Olofsson, Ingmar Svensk and Jan Olofsson.

I also take this opportunity to extend my gratitude to Bernt Bernander (former Assistant Administrator of the United Nations Development Program (UNDP), New York, for dedicating considerable time to reading the manuscript and for valuable editorial and linguistic advice. Thanks are also due to Phil Curtis, SYCON and Dr of engineering Keith Rush, M.Sc., (LTA Earthworks Ltd, South Africa) for having read and commented on the manuscript.

I am also grateful to many a colleague, who in discussion or even by contesting the novelty in some of my reasoning, have contributed to what I believe to be a further step in the development of concepts of brittle failures in natural slopes.

Last but not least, I am deeply grateful to my wife Sonja for the patience she has shown me and to my work, considering the many long hours I have spent compiling this report.

Mölndal in April 2000

Stig Bernander
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Foreword

Landslides in extensive natural slopes constitute a latent menace in many populated areas of the world and often have disastrous consequences. Ground movements of this kind are triggered by many different agents and by complex mechanisms. They are, therefore, not always well understood, even by engineers of the profession. For instance, the large landslides in Sweden at Surte 1950 and at Tuve 1977, which inflicted excessive damage to housing areas, led to lively debates among geotechnical engineers as to the correct explanations of the remarkable features of these slides.

Discussion and open-mindedness are, of course, in general desirable, but in this context, one may say, these virtues are indicative of uncertainty among the professionals as to how to interpret and understand the mechanisms actually at work in these slides. Although, in hindsight, investigators of landslides usually manage to find round-about ways of explaining events in a landslide, they rarely do so in such a way that future slides of the kind can be identified and prevented.

In 1979, the Swedish Geotechnical Institute (SGI) invited a number of experts in Sweden on landslides to present their ideas about the causes and the mechanisms that formed the Tuve slide, where some 800 meters of ground was displaced, causing the death of 9 people and the total destruction of 65 homes. Nine reports were published in a booklet named SGI Report No 10.

The different contributions exposed a variety of explanations of the slide, and the main impression from this report was that it revealed a rather confusing state-of-the-art in this particular field of geotechnical engineering. The subsequent SGI Report No 18, (1981), in which chapters 11 and 12 deal with the institute's own ideas about the mechanisms in the Tuve slide, did not either, in the authors opinion, explain the remarkable features of the finished slide.

The evident lack of success of engineers in soil mechanics to offer rational and plausible explanations of large landslides relates to the fact that the normally used analytical concept is basically inaccurate from a structure-mechanics point of view. This concept, based on ideal plasticity of soils, simply has limited validity in many practical situations. The fact that no generally accepted or unambiguous explanations have been given of some of the most spectacular slides in Scandinavia obviously raises doubts regarding the validity, not only of other investigations of similar landslides made in hindsight, but more importantly of the validity of predictions of potential landslide hazards.

The following presentation deals with progressive failure mechanisms in large planar landslides in deformation softening soils. These brittle failure processes often provide plausible and logical explanations of initiation, propagation and final configuration of slides in such slopes. The analytical method applied here provides the means to making a more accurate assessment of the risk of local failure. Further, it makes it possible to estimate the length of the passive zone (the tongue of the slide, which may ensue) and to predict how far this zone is likely to propagate into less sloping inherently stable ground. Apart from its direct damage potential, the upheaval in the passive zone provides the prerequisite for the loss of ground and settlements in the up-slope active zones of the slide area, which are often of an even more disastrous nature.
Scope

The present report is focused on the implications of deformation softening in soils to the stability of long natural slopes. The analytical tool used in this study is a two-dimensional finite difference model (FDM), by which the main deformations within and outside the potentially sliding soil mass are taken into account. The computational method is specifically adapted to what is here denoted as ‘downward progressive slope failure’. Cf Chapter 4.

The time factor in slope stability is addressed by way of defining the constitutive relationships in accordance with the timing of the currently studied event.

The fact that the shear deformations are determined in two directions allows modeling of the entire incipient failure zone as a thick structural layer, and not just as a discrete shear band (or a slip surface). This is a crucial feature in the used analytical approach.

The result of applying the proposed method of analysis to long natural slopes with sensitive soils entails recognition of the risk of progressive failure formation. Case records dealing with slides, where indications and/or documentation of progressive failure mechanisms have been identified, are treated in a special chapter.

Other related topics treated in this report are:

- Limitations with regard to the validity of the conventional approach based on ideal-plastic limit equilibrium. (Chapter 2.)
- Different phases of a landslide in a downward progressive failure event. (Chapter 3.)
- Case records. (Chapter 5.)
- Factors conducive to brittle behavior in slope failure. (Chapter 6.)
- Agents likely to trigger progressive slope failures. (Chapter 7.)
- Principles and procedures for investigating landslide hazards related to potential risk of progressive failure formation. (Chapter 8.)

It may be noted that, although factors conducive to brittleness in soils are treated to some extent in this report, it is not within its scope to address methods and procedures for establishing and documenting the constitutive stress/deformation properties of soils. This task is left to the soil mechanics engineer himself and to future R & D.

**Key phrases:** Applicability of conventional ideal-plastic failure analysis, deformation softening, configuration of finished slides, different phases of a downward progressive slide, analytical modeling of progressive failure, analogy to buckling stability, case records, the slope of the Surte slide - a ‘time bomb’ ticking through millennia, brittle behavior of soils, brittleness due to nature of loading, geometric brittleness, effect of time factor, in situ earth pressures, effects of seasonal redistribution of in situ earth pressures, exemplification etc

VIII
Summary in Swedish:

Progressiva skred i långsträckta naturliga slänter

Orsaker, förlopp och utbredning hos skred i deformationsmjuknande jordar

Allmänt


Vid konventionell skredanalys (i rapporten benämnd I-PFA = Ideal-plastic failure analysis) bortser man, som det kan förelåna, för enkelhets skull från såväl deformationerna inom den potentiella glidkroppen som från de relativa deformationerna mellan densamma och under brotttytan liggande fastare material. Detta innebär att man i praktiken tillskriver jordmaterialet obegränsat ideal-plastiska egenskaper, något som beträffande lösare lorer sällan gäller i verkligheten.

Ett viktigt tema i föreliggande rapport är att bristande överensstämmelse mellan teori och praktik på detta område av geotekniken just harrör från det faktum, att många jordarter är utpräglad deformationer-mjuknande inom ramen för de skjuvdeformationer och de försjutningar i förhållande till underlaget som kan förekomma i den blivande brottzonen vid begynnande skred. Detta gäller i synnerhet vid utsträckta flaskred i sensitiva jordar.

Analys under hänsynstagande till relevanta deformationer

I rapporten ställes möjligheten av progressiv brottbildning i fokus, något som motiveras av ett antal inträffade skred med, enligt författarens mening, uppenbara indikationer på att spröda brottmekanismer varit för handen. En numerisk beräkningsmetod baserad på finita differenser (FDM) tillämpas vid analysen av deformationsmjuknandet inverkan på släntstabiliteten.

Förfarandet liknar konventionell analys i så motto att den presumtiva glidyans sträckning antages vara känt. Icke desto mindre avviker den föreslagna analysmetoden från gångse metoder i flera betydelsefulla avseenden enligt nedan:

- Under det att man vid gångse beräkningsmetoder (I-PFA) begränsar sig till att studera jämvikten hos den tänkta glidkroppen i sin helhet, tillämpas jämviktvillkoret vid analys av progressiv brottbildning (Pr FA) på vart och ett av de vertikala element i vilka glidkroppen indelats.
- Vidare tas hänsyn till deformationerna inom och utom den presumtiva glidkroppen. Härvid tillses att de axiella deformationerna i slänttäckningen p g a ändringar i jordtrycksfördelningen är förenliga med skjuvdeformationerna i de vertikala elementen i varje sektion. Härigenom kan skjuvspänningsfördelningen av t.ex. lokala tilläggslaster bestämmas, även som på vilken
längd skjuvhållfastheten i brottzonen kan mobiliseras i släntriktningen för upptagande av denna last. Eftersom FDM - analysen är två-dimensionell kan den begynnande brottzonen modelleras i sin helhet och ej endast som ett diskret 'skjuv-band' eller glidyta.

- Jordens egenskaper vid skjuvning definieras medelst ett fullständigt spännings-
deformationssamband och ej endast med enstaka värden på skjuvhållfastheten såsom vid gängse beräkningsmetoder. De konstitutiva sambanden indelas i två skilda stadier (stadium I och II), vilket på så sätt simulerar förhållandena före respektive efter utbildandet av en diskret glidyta. De konstitutiva sambanden kan varieras och anpassas allt efter de i slänten rådande förhållandena.

- Genom att relatera nämnda spännings/deformationss-egenskaper till olika tidshorisonter för påförandet av tilläggs slaster eller till tidsförhållandena vid andra skredutlösande orsaker, kan hänsyn till tidsfaktorn införas i analysen.

- Olika typer av lastfördelning samt specifika förhållandena i släntens och fasta bottens morfologi som ökar benägenheten för progressiv brottbildning, kan beaktas.

- Ehurstroottzonens höjdläge i varje enskild beräkning antages vara given, så erhålls skredets slutliga utbredning i släntriktningen och passivzonens längd som resultat av beräkningarna.

**Konsekvenser av hänsynstagande till deformationerna vid skredanalys enligt föreslagen FDM- metod.**

Den föreslagna analysmetoden belyser nödvändigheten av att beakta deformationerna vid skred i långa slänter med deformationsmjuknande jord samt att underlåtenhet därvidlag kan leda till allvarlig felbedömning av, dels risken för lokalt brott i slänten och dels omfattningen hos det totalskred som kan utlösas vid ett dylikt lokalt brott. Analysen möjliggör identifiering av de verkligt kritiska förhållandena i en slänt med hänsyn tagen till lastfördelning, geometri och lokala egenskaper hos jordmaterialet. Risk för progressivt brott föreligger om jordens resthållfasthet (cr) i någon del av en slänt vid någon tidpunkt kan komma att understiga rådande in situ spännings d v s

\[ c_R(t,x) < \tau(x) \] (Beteckningar enligt 'Notations')

Den omständigheten att skjuvspänningsarna p g a en lokal last endast kan mobiliseras på en begränsad sträcka räknat från lastens angreppspunkt, kan i många fall vara av avgörande betydelse. På ett avstånd definierat som Lcr (enligt avsnitt 3.3) från en koncentrerad tilläggslast blir nämligen dess inverkan på spännings, jordtryck och deformationer försumbar, något som utesluter eller reducerar möjligheten av att utnyttja ökande passiva jordtryck längre ner i slänten för stabilisering av denna last i **initieringsskedet**. Man kan uttrycka förhållandet så, att jorden nedanför den sektion som definieras av avståndet Lcr från den lokala tilläggslasten, inte 'vet om' eller känner av när brott vid lastens angreppspunkt är förestående. Vid omfördelnings av jordtrycken i samband med progressivt brott kan dock fullt passivt motstånd mobiliseras.

En viktig konsekvens av den begränsade möjligheten att mobilisera passiva jordtryck längre ned i slänten blir, beroende på graden av deformationsmjuknande, att brottmotståndet långs
plan parallella med markytan eller längs med fast botten parallella sedimentplan är avsevärt mindre än motståndet baserat på glidytor som utmynnar i slutningen närmare lasten.
Det bör noteras härvidlag att detta förhållande gäller även vid höga värden på förhållandet mellan resthållfasthet och odränerad skjuvhållfasthet (cH/cH0) d v s initialt även under ideal-plastiska betingelser, eftersom betydande förskjutningar erfordras för att kunna mobilisera passiva jordtryck.
Detta förhållande kullkastar en utbredt föreställning att ideal-plastisk analys - trots eventuellt erkända brister - ändock skulle äga god tillämplighet vid faststående av s.k. "initialskred", varmed man i allmänhet avser instabilitet med avseende på någon lokal glidyta.
Nämnda avvikelse mellan utvärdering av initierande brottorsak under hänsynstagande till deformationerna å ena sidan och resultat från konventionella beräkningar å den andra, kan dessutom bli än allvarligare vid dränerad analys. Detta sammanhänger med att höga porvattenövertryck med större sannolikhet utbredes sig längs sedimentskiktten än i vinkel mot desamma.

Den föreslagna FDM modellen för framåtgripande progressiv brottbildning medger också hänsynstagande till deformationer under den presumtiva glidytan. Emellertid, som framgår av ovanstående, medför de begränsade möjligheterna att mobilisera passiva tryck längre ner i slutningen att brot i slänter uppvisar en markerad trend till att följa sedimentlagren och/eller i stort sett lutningen hos fast botten till avsevärt djup under markytan. Vid Tuve skredet synes exempelvis glidytan i huvudsak vara parallell med fast botten ända ned till c:a 35 m:s djup. Beaktande av deformationerna under brottzon torde därför i många fall ej ha någon större inverkan på resultaten av analysen.

En annan parameter av betydelse i detta sammanhang utgöres av relationen mellan den kritiska längden (Lcrit) och släntens totala längd (L). Detta förhållande (Lcrit/L) kan sägas utgöra ett mätt på tillämpligheten av konventionell analys i en aktuell situation, i synnerhet då det är fråga om påförande av lokala tilläggslastar. (Jfr exempelvis skreden vid Surte, Tuve, Bekkelaget och Trestyckeavetnet i kapitel 5, Case Records).
Stabilitetsundersökningar i längre slänter bör med hänsyn härtilt regelmässigt inbegripa åtminstone en ungefärlig uppskattnings av den kritiska längden med avseende på den aktuella belastningen.

**Faktorer som inverkar på benägenheten till sprödbrott i naturliga slänter**

Deformationsanalys enligt kapitel 4 i denna rapport visar klart att även andra förhållanden än jordens sprödhet kan ha stor inverkan på benägenheten till progressiv brottbildning. Till dessa faktorer, som belyses särskilt i kapitlen 6 och 7, kan räknas:

- Markytans, sedimentskiktens och fasta bottens geometri – 'geometrisk sprödhet'
- Typ och läge av påförd belastning eller störning
- Tidsförhållanden för dito
- Hydrologiska förhållanden och hydrologisk historia
Säkerhetsfaktorer

I samband med den föreslagna skredanalysen vid vilken deformationerna beaktas, blir gängse sätt att definiera brottsäkerheten utan fysikalisk mening. Följaktligen måste i dessa sammanhang säkerheten mot brott omformuleras med hänsyn till de kriterier som är avgörande för uppkomst och utveckling av progressiv brottbildning.

Varför tillämpa progressiv brottanalys?

Stabilitetsförhållanden i en naturlig slänt är nära förbundna med dess geologiska och hydrologiska historia. Många lerslänter i Västsverige är uppbyggda av glaciala och postglaciala sediment som höjt sig ur det regredierande havet under efteristiden. Allteftersom marken rest sig över havsytan har jordens hållfasthet och jordtrycken i slänten, genom konsolidering och kryoprørelser, kommit att gradvis anpassa sig till de ökande påfrestningarna, som blivit följden av sjunkande grundvattenytor, klimatologiska variationer, kemiska förändringar och urläckning.

Följaktligen är varje naturlig slänt stabil i den meningen att den existerat under årtusenden och med hänsyn till att densamma under denna tidsrymd med åtminstone någon marginal klarat extrema situationer med höga porvattenövertryck bör ’säkerhetsfaktorn’ under normalt rådande betingelser vara större än 1.

Den avgörande frågeställningen för geoteknikerns bedömning av risken för skred blir då hur stabiliteten kommer att påverkas av tilläggstroller eller störningskällor, för vilka tidshorisonten mätts i timmar, dagar, veckor eller månader i stället för århundraden respektive årtusenden?

Och slutligen, vad blir följderna om en lokal instabilitet skulle uppkomma p g a ovan nämnda störningskällor? Kommer det lokala brottet bara att resultera i en markspricka vid släntkrönet eller leder det till ett katastrofalt skred varvid hundratals meter av horisontell, eller i och för sig stabil mark, undergår våldsamma hävningar och förskjutningar.

Analys enligt kapitel 4 med beaktande av deformationerna i slänten erbjuder just strukturmekaniskt logiska förklaringar till varför ett antal katastrofala skred i Skandinavien kunnat utlösas p g a vad man bedömt som förhållandevis små störningseffekter. Ifrågavarande slänter har förblivit stabila under tusental år sedan marken en gång höjde sig ur det post-glaciala havet. Ändock har väldiga markförskjutningar och markhävningar med vidsträckt utbredning över svagt sluttande mark ofta Inträffat i samband med mindre mänskliga ingrepp av lokal natur.

Progressiv brottanalys visar emellertid att detta är precis vad som kan hända även vid en obetydlig störning av ett ömtåligt parti i en dylik slänt, något som bör vara av stort värde vid kartering av skredrisk.

Som nämnts medför hänsynstagandet till ett jordmaterials deformationsmjuknande i allmänhet betydligt större beräknad risk för skred p g a koncentrerad last respektive lokala störningsmoment än vid tillämpning av konventionell analys baserad på ideal-plastiska egenskaper hos jorden. Detta gäller även om jordens konstitutiva egenskaper varieras inom vida gränser inom ramen för vad som kan anses rimligt.

Beräkningar

Ehu nu beräkningarna enligt den i kapitel 4 föreslagna metoden för analys av progressiva skred i princip är tämligen enkla, kan de för många geotekniker förfalla komplicerade jämfört med
gängse metoder för bedömning av släntstabilitet. Det gäller exempelvis att välja tillämpliga rimliga konstitutiva samband för den aktuella jordarten, varvid tidsramen för påförande av tilläggslasten, hydrologiska förhållanden, OCR och huvudspänningstillstånd utgör några av de inverkande betingelserna.

Men vill man verkligen uppnå pålitliga förutsägelser beträffande risk i avseende på människoliv, samhällsekonomiska konsekvenser och egendom måste man, enligt författarens mening, adressera dessa svårigheter.

Såsom framgår av beräkningsexemplet i kapitel 4 och appendix I medför handberäkningar, ehuru enkla i princip, mycket omfattande beräkningsarbete. Med hjälp av datorkraft blir dock tidsåtgången för beräkningarnas genomförande obetydlig. Sedan man väl definierat och matat in ingående parametrar rör sig den egentliga beräkningstiden om sekunder.

Den extra arbetsinsats, som geoteknikern måste ägna släntstabilitetsundersökningar enligt föreliggande metodik utgöres således till mycket ringa del av ökat beräkningsarbete. Den huvudsakliga utmaningen ligger i att kunna utnyttja möjligheterna till att studera hur en slänts stabilitet påverkas av ett antal faktorer, vilkas inverkan ‘par definition’ ej kan identifieras medelst konventionella metoder som grundar sig på ideal-plastiska egenskaper hos förkommande jordarter. Denna inverkan måste i stället baseras på jordens deformationsmjuknande egenskaper och släntens geometri.
Symbols and Notations

Greek letters:

\( \alpha \) Coefficient defining the elevation of the earth pressure resultant
\( \beta, \beta_x \) Slope gradient at coordinate \( x \)
\( \gamma, \gamma_{(x,z)}, \gamma_{xz} \) Deviator strain, (angular strain) as a function of \( x \) and \( z \)
\( \gamma_{el} \) Deviator strain, (angular strain) at elastic limit
\( \gamma_f \) Deviator strain, (angular strain) at failure stress
\( \varphi' = \varphi_i \) Angle of internal friction, drained conditions
\( \delta_x = \delta(x) \) Down-slope displacement
\( \delta_N \) Down-slope displacement in terms of axial deformation generated by forces \( N_x \)
\( \delta_t \) Down-slope displacement in terms of deviator deformation
\( \Delta \delta \) Differential of \( \delta \)
\( \Delta x \) Differential of \( x \) coordinate
\( \delta_{s, \delta_{s}}(x), \delta_{s,x} \) Post peak slip deformation in the slip surface in relation to the sub-ground
\( \delta_{s_{(CR)}} \) Post peak slip in slip surface at ultimate residual shear strength \( c_R \)
\( \delta_{s_{100}} \) Post peak slip in slip surface = 100 mm = 0.1 m
\( \delta_{s_{300}} \) Post peak slip in slip surface = 300 mm = 0.3 m
\( \delta_{ave} \) Average down-slope displacement of the soil above the potential slip surface
\( \alpha H_x \) Level at which the down-slope displacement (\( \delta_{ave} \)) is valid
\( \varepsilon \) Longitudinal strain
\( \nu \) Poisson’s ratio
\( \Omega \) Coefficient relating the modulus of elasticity to the undrained shear strength
\( \rho, \rho(z) \) Soil density (Mg/m\(^3\))
\( \sigma_1 \) Major principal stress in tests
\( \sigma_3 \) Minor principal stress in tests
\( \sigma_v \) Vertical normal stress
\( \sigma_h \) Horizontal (down-slope) normal stress
\( \sigma_{s, \sigma(x), \sigma_x} \) Mean incremental down-slope axial stress corresponding to \( N \)
\( \tau_{el} \) Shear stress (deviator stress) at elastic limit
\( \tau_o, \tau_{o(x,0)}, \tau_{ox,0} \) In situ shear stress at failure plane \( (z = 0) \)
\( \tau, \tau(x,0), \tau_{xo,0} \) Total shear stress at failure plane \( (z = 0) \)
\( \tau_o, \tau_{o(x,z), \tau_{ox,z}} \) In situ shear stress
\( \tau, \tau(x,z), \tau_{xz} \) Total shear stress (deviator stress)

Roman letters:

\( b, b(x), b_x \) Width of element
\( c \) Shear strength of clay
\( c_o \) Adhesion for \( \varphi' = 0 \) - drained shear strength
\( c_u = c_u(\gamma) \) Undrained shear strength
\( c_{u,mean} \) Mean undrained shear strength of the soil above the failure plane
\( c_R = c_R(x) = c_{uR} \) Residual shear strength at a point \( (x) \) for post peak slip of \( \delta_{s,x} \) in slip surface
\( c_R(t,x) \) Residual shear strength at a point \( (x) \) at time \( (t) \)
\( c' \) Drained shear strength
\( g \) 9.81 m/sec\(^2\)
\( k_o \) Ratio of \( \sigma_h \) to \( \sigma_v \)
\( K_o \) Ratio of minor to major principal stress
q(x)  Additional vertical load
\( t(x) \)  Additional horizontal load
w  Natural water content (%)  
\( w_L \)  Liquid limit (%)  
\( w_P \)  Limit of plasticity (%)  
\( x \)  Horizontal (or down-slope) coordinate  
\( z \)  Vertical coordinate  

\( E_x, E_x(x), E_x \)  Down-slope earth pressure at point \( x \), i.e. \( E_x = E_{ox} + N_x \) or \( E_x = E_{ox} + \Delta E_x \)  
\( E_{ox}, E_{ox}(x), E_{ox} \)  In situ earth pressure at point \( x \)  
\( D_w \)  Submerged depth (when slope borders river or lake)  
\( E_{\text{Rankine}} \)  Critical down-slope earth pressure resistance at passive Rankine failure  
\( \Delta E = N \)  Incremental down-slope earth pressure at point \( x \) due to additional loading  
\( E_{el,0} \)  Elastic modulus of structural element at \( z = 0 \)  
\( E_{el}, E_{el,\text{mean}} \)  Mean secant elastic modulus in down-slope compression of a vertical structural element \( H\Delta x \), i.e. \( E_{el,\text{mean}} = \Omega \cdot c_{u,\text{mean}} \)  
\( F_s \)  Safety factor  
\( G \)  Elastic modulus in shear  
\( G_0 \)  Elastic modulus in shear of structural element at elevation \( z = 0 \)  
\( H, H(x) \)  Height of element, (from slip surface to ground surface)  
\( L_{cr} \)  Limit length of mobilization of shear stress at \( N_{cr} \)  
\( L_{\text{instab}} \)  Limit length at which slope fails for \( N_i = 0 \)  
\( L_p = L_{E=E(Rankine)} \)  Length of the passive Rankine zone at the foot of the slope  
\( N_i, N(x), N_x \)  Earth pressure increment due to additional load or progressive failure formation at point \( x \)  
\( N_i \)  Load effect of agent initiating local slope failure  
\( N_{cr} \)  Critical load effect initiating local slope failure  

**Abbreviations:**

I-PF  Ideal-plastic failure  
I-PFA  Ideal-plastic failure analysis  
Pr F  Progressive failure  
PrFA  Progressive failure analysis  
SGI  Swedish Geotechnical Institute
1. Introduction – historical background

1.1 Historical background


Terzaghi & Peck (1948) emphasized the risk of progressive failures in brittle soils, but when exemplifying these phenomena in normally consolidated soft clays they seem to be limited to bottle-neck slides, clay flows, successively retrogressive slides and hydraulic failures in loose sands or silts. Considering the enormous scope of the writings of these two authors, their rather modest contributions in this field of soil mechanics may indicate that they did not regard brittleness in normally consolidated clays as an important problem in the sense conceived in this report.

In the late 1960’s Bjerrum (1967) lectured on retrogressive brittle failures in cemented tertiary clays. However, in response to a direct question by the author of this report as to whether progressive failure formation could also be a conceivable issue in normally consolidated Scandinavian clays, Bjerrum firmly stated that in his opinion this was not the case.

Kjellman (1954) discussed ‘progressive failure mechanisms’ in connection with large Swedish landslides and some important features of his failure concept coincide in principle with the progressive failure mechanism on large landslides suggested in his paper. However, although he discussed the effects of down-slope axial deformations on deformation softening in the slip surface proper, Kjellman did not model the deformations in the highly strained zones adjacent to the failure plane. In the light of the present report, this means that his approach would have seriously exaggerated the risk of incidence of progressive failure phenomena. In the article referred to, no quantitative analyses were made of key issues in this context, e.g. the relative horizontal deformations within the sliding body. Critical parameters such as safety criteria and other phenomena arising from considering these deformations were not either identified. He also did not address the rate (time) dependency of shear strength and the role of the time factor in this context.

However, interestingly Kjellman argued that progressive failure formation should not be limited only to sensitive clays as they may be liable to occur also in many other types of normally consolidated soft clays. The author of this report subscribes to this opinion.

With special reference to Skempton’s and Bjerrum’s reports on slope failures in over-consolidated clays and clay shales, Christian and Whitman (1969) proposed a method of analysis for a specific mode of retrogressive (or upward progressive) failure, in which the sliding soil mass moves as an integral block owing to failing down-slope support. The paper is interesting in this context because it addresses some of the issues highlighted in the present
report. However, their one-dimensional model is very simplistic, the slope gradient and depth of the sliding soil block being assumed to be constant in the model.

The most problematic feature of this approach is may be the fact that the stress-deformation relationship, which defines the properties of the shear band connecting the sliding soil body to the sub-base, cannot be derived directly from basic conventional soil parameters without the use of specific 'field observations'. It is therefore difficult to conceive how this crucial parameter is to be established in practice, for instance when investigating potential slope failures.

P. Hansbo et al (1984), de Beer & van Impe (1984) and Wiberg et al (1989) made studies of progressive failures in slopes. All of these studies were essentially based on the simplistic approach adopted by Bernander & Olofsson (1981) for investigative purposes. Here the shear deformations are presumed to be limited to a specific sensitive soil stratum of given thickness. However, the basic weakness of this approach is that the integral shear deformation of this layer is difficult to define, as the one-dimensional model provides no information about the distribution of the deformations within the failure zone.

In order to address this difficulty, an improved version of the analytical model was later developed, whereby the shear deformations in the entire failure zone are modeled in a two-dimensional analysis, applying a constitutive shear stress/deformation relationship. Bernander et al (1984,1989).

A valuable contribution to the analysis of progressive landslides has been presented by S.Y. Chen, X.S. Zhang and W.S. Tang (1997). Their article features many of the important issues dealt with in the subsequent report. The safety factor is here defined as the ratio between the peak strength and the mobilized mean shear stress, which in turn emerges from an analysis considering the deformations in the slip surface together with a constitutive stress deformation relationship. The approach resembles that applied by Skempton and other workers, which is discussed in more detail in the next section. The progressive failure process is then studied by gradual increase of the relative displacements along the slip surface. The method offers a good understanding of the progressive failure mechanisms. However, as Chen et al point out themselves, the accuracy of the results may be affected by the fact that the development of the failure process is linked with the distribution of the incremental displacements, which are not uniquely defined by the given method.

The progressive failure development is by Chen et al denoted as 'dynamic' in the sense that the method of analysis models a progressively changing scenario. However, as time and inertia forces do not enter into their computations, the failure mode is not dynamic in a mechanical sense, as is for instance demonstrated in the Figures 5:1.9 to 5:1.17 in Chapter 5 of this report.

Also, this one-dimensional method of analysis lacks some of the main characteristics of the method of analysis described in the present report, as defined in Section 1:3 below.

Alén (1996,1998) has proposed what was denoted as a 'shear beam model', which in some essential respects is similar to the model adopted by Bernander et al (1981, 1984, 1989) and therefore exhibits several features in common with the approach implemented in this report. The model allows studying effects of progressive failure, particularly in steep slopes. The global safety factor is, however, still defined as a weighted mean value of local safety margins, roughly in accordance with formulations used by other investigators of progressive failure formation.

Tiande, Chongwu and Shengzhi (1999) have proposed a model for progressive failure by which the strain softening of the soil is defined in a time related Maxwell model. In this approach, the effect of strain softening on failure propagation is calculated considering its
effects on inter-slice forces. The model, which is applied to failures propagating up-slope, appears to be intended for steep slopes and over-consolidated clays.

1.2 Definitions of ‘progressive failure’

In this context it may be of interest to observe that the term ‘progressive failure’ often has different meanings for individual researchers. In some papers, the term designates a failure process, which is progressive in a spatial sense, i.e. the slip surface formation starts at some point in the incipient slide and propagates towards the limiting boundaries. The gradual loss of shear strength of the soil is then mainly expressed in terms of the development of displacements. The analysis should therefore consider, at least in some measure, the relative deformations in the failure zone, such as in Christian & Whitman (1969), Chen et al and in the papers by Bernander (1981-1989) noted at the end of this chapter. In case records of slides in strongly over-consolidated clays, as for instance described by Skempton in his famous Rankine Lecture (1964) and Tjandra et al (1999), the mechanical processes leading to deformation softening and failure propagation are essentially governed by time, often in terms of decades. However, the safety factors are in these papers still defined as a mean strength to mean stress ratio ($c\tau_c/\tau_{mean}$), in accordance with the normal practice in conventional analyses based on limit plastic equilibrium. Hence, the loss of mean shear strength in the forming slip plane and the consequential risk of sliding are manifest primarily as functions of time or time related displacement.

Furthermore, the term ‘progressive failure’ is sometimes used as opposed to ‘retrogressive failure’, whereas in other contexts, the expression only refers to the mechanism leading to a ‘retrogressive failure’. For instance, the types of slide referred to in Skempton (1964), Christian & Whitman (1969) and Tjandra et al (1999), are undoubtedly set off by failing support at the toe of the slopes, especially considering the fact that stress concentrations tend to build up at the toes of steep slopes. These slide categories may by some be regarded as being ‘upwards progressive’ or retrogressive, whereas others, like Skempton, designate them as ‘progressive’. Progressive failure in the sense adopted in the present report is defined in the following section.

1.3 Features characterizing the present report

In the late 1960’s and the early 1970’s a number of large planar landslides took place in southwestern Sweden, some of which are accounted for in more detail in Chapter 5. On inspecting the sites of some of these slides, the author of this report observed that the topography of the finished slides was actually inconsistent with the failure mechanism based on ideal-plastic limit equilibrium, by which practicing engineers still tend to predict potential slide hazards. This particular issue is dealt with in more detail in Section 2.4. In this context, suffice it to say that the upheaval of the passive zone provides clear evidence of immense unbalanced down-slope forces acting during the slide. The enormity of these forces, which may readily be estimated by back analysis of a slide, is totally inconsistent with an ideal-plastic failure process.

All the landslides described in the present report have taken place in quaternary deposits of normally consolidated or slightly over-consolidated, more or less sensitive clays, in which the
implications of deformation softening are generally radically different from those in highly over-consolidated clays.

In addition, the landslides dealt with in Chapter 5 have been triggered by specific additional loading or disturbing agents, which are basically local in time and space and usually brought about by human activities.

(By contrast, in the type of slides documented by e.g. Skempton in highly over-consolidated clays, the total load is gravitational and essentially invariable, as well as being more evenly distributed. Hence, the main cause of slope failure is there related to the time dependent long-term deformation softening, and not to any decisive effect of additional loading at the slide event. The magnitude and distribution of earth pressures in the slope do not apparently form a crucial part of the analysis).

In the subsequent report, the magnitude and distribution of earth pressures along the slope, including those of the in situ state, are target results of the analysis, and constitute the key parameters in the assessment of safety factors against local as well as global slope failures. Moreover, the distribution of shear stresses and down-slope displacements are accounted for. It has therefore seemed appropriate to define the term 'progressive failure' as a failure propagating along the potential slip surface in strict accordance with a requirement for compatibility of displacements within and outside the potentially sliding body of soil, as calculated using relevant constitutive stress/deformation relationships. In doing so, the deformations in the entire failure zone (i.e. not only in the shear band) are accounted for by two-dimensional modeling of the crucial deformations in the potentially sliding soil mass. This is one of the unique features of the analytical approach proposed in the present report.

Another key circumstance highlighted in the report is the limited distance down-slope of a local load along which additional shear stresses in the potential failure can be mobilized. Thus at some distance from the point of load application, the effect on the earth pressures of this load is no longer felt. This has the crucial implication of reducing the possibility of utilizing increased earth pressures in less sloping ground as a means of stabilizing additional loading further up-slope.

Furthermore, five distinct phases in the progressive failure development are defined in Chapter 3. New formulations of the safety factors, which are related to the specific key issues in progressive failure analysis, are given. (Cf Bernander & Gustás, 1984).

1.4 Earlier publications by the author on the current topic

The difficulties experienced by the author in explaining the development and the final configuration of large planar landslides led to specific research and studies in this field of geotechnical engineering. The progress of this work was presented to a larger audience in some fifteen publications in Swedish and English during the period 1978 to 1989. The various reports reflect different aspects of the problem of brittle failures in soils as well as different stages in the development of an engineering approach. Notable contributions were made to the ICSMFE conferences 1981, 1985 and 1989. An important phase of the development of the analytical approach was presented at the Symposium on Landslides in Toronto, 1984.

The object of the present report is to synthesize the essential principles, ideas and findings that motivated the above publications. Although many of the principal contents of this report have been published earlier, reference in the text to earlier publications by this author will be restricted to a minimum.
2. On the applicability of ideal-plastic failure analysis (I-PFA) to strain-softening clays

2.1 General

Analysis of stability in natural slopes is, in engineering practice, normally based on the supposition of ideally plastic properties of the soil, whereby the equilibrium of a solidified rigid body is determined assuming

a) full effective shear strength, or estimated mean strength, to be mobilized simultaneously at failure along the slip surfaces limiting the moving soil mass;

b) uniform mobilization of shear stresses in terms of a mean stress level along these surfaces in the service condition.

![Diagram of shear deformation and shear stresses in a vertical plane of a potentially sliding soil mass.](image)

**Figure 2.1.1** Shear deformation and shear stresses in a vertical plane of a potentially sliding soil mass. In ideal-plastic failure analysis (I-PFA), the effects of deformations on stress distribution are disregarded.

The safety factor \( F_s \) is then defined as

\[
F_s = \frac{c_u}{\tau_{\text{mean}}} \quad \text{.........................2:1}
\]

Where \( c_u \) denotes the undrained shear strength of the soil and

\( \tau_{\text{mean}} \) denotes the mean ambient shear stress in the slip surface

Under drained conditions the drained shear strength \( c_d = c' + \sigma' \cdot \tan \varphi' \) is used.

From a structure-mechanical point of view, this methodology is strongly simplified, as the deformations within and outside the sliding body are not considered in the analysis. (Figure 2.1.1) This means that, already by definition, the way in which distribution of load, in situ stresses, stiffness properties and geometry affect the stress distribution in the potential slip surface cannot be considered. For instance, the crucial state and development of the in situ down-slope earth pressures do not affect the results of the calculations. Admittedly, the I-PF analysis may not claim to model the true behavior in the serviceability limit state but it does claim to provide a defined degree of safety against slope failure.

But also in terms of soil mechanics, there are a number of questionable approximations, which may undermine the validity of conventional analysis of slope stability. The undrained shear strength of clays is, for all practical purposes, normally regarded as a material property.
This is done notwithstanding the recognized facts, that both shear strength and ductility of clays intimately depend on a number of ambient conditions in the soil structure. Such conditions are, for instance, the state and magnitude of principal stresses, the effective stress situation (OCR), the level of deviatory strain and deformation etc. The timing and the rate of loading are also of paramount importance to the strength characteristics of soft clays. For instance, laboratory shear tests according to current practices are carried out at strain rates in the range of 0.3 to 0.5 % per hour. However, a major landslide such as the Tuve slide (see Section 5.1) covering some eight hundred meters in length of ground, may well start as a local acceleration of an ongoing creep deformation, but the final phases of the event take place within a few minutes in sensitive clays. This is, therefore, the time range for the mechanisms governing development, propagation, final extent and morphology of the slide. It stands to reason that only the response of the soil in the actual timing of the different phases of a slope failure can have any relevance to the prediction of its consequences with regard to the extension and ultimate degree of catastrophe.

In conclusion, only laboratory tests related to the actually occurring rates of deformation will allow valid predictions with regard to the failure mechanisms, the failure propagation and the ultimate spread and morphology (i.e. the degree of disaster) of a finished slide.

The shear and deformation properties of brittle clays are further discussed in Chapter 6.

2.2 Prerequisite conditions for the validity of ideal-plastic failure analysis (I-PFA) in engineering practice

If conventional analysis based on full plastic behavior in soils is to apply, at least one of the following conditions must be fulfilled:

1) The soil in the failure zone can be subjected to virtually unlimited deformation without substantial loss of strength.

2) The deformations within the sliding body due to additional load are small compared to the strain range ($\Delta \gamma$ in Fig. 2.2.1) within which the assumed shear strength is valid - i.e. the sliding body being considered as practically rigid.

3) The distribution of the incremental stress provoking a potential slope failure conforms with the distribution of in situ stresses and shear capacity in the failure zone.

Condition No 1 normally applies to drained conditions in both clays and cohesionless soils. Also for undrained conditions, this requirement may be considered as being met in engineering practice for normally consolidated plastic clays, the water contents of which do not deviate far from the liquid limit ($w_l$), provided the length of the potential sliding body has reasonable proportions (say 30 to 100 m, depending on the degree of deformation softening).
Figure 2.2.1 Examples of shear stress/deformation relationships

A) Elastic / ideally plastic material
B) Tough clay at low strain rate – drained conditions
C) Sensitive deformation softening clay
D) Loose saturated silts, sands or silty sands. Sands or gravels with interstices filled with underconsolidated clayey material.

Condition No. 2 usually applies, even in sensitive clays, when the length to height ratio as well as the extension of the sliding body is reasonable as, for instance, is the case in the design of retention walls, sheet piling excavations, slopes < 20 to 50 m etc. However, in highly sensitive clays, it is recommended that the deformations of the retaining structures be limited.

Condition No. 3 may also be fulfilled in many natural slopes, considering that long term creep in a slope is likely to result in a condition where the stress levels (τ / c_u) are virtually constant along the potential failure zone on account of gradual adaptation of both in situ earth pressures in the slope and shear stresses in highly strained zones in the soil profile. However, for reasons explained in the following, a crucial requirement in this context must be that any incremental load must induce a stress field conforming to the in situ conditions in the potential critical zones. In practice, this may apply when the additional stress is induced by e.g. a fill placed evenly over the area susceptible to sliding. It may also apply when pore water pressures tend to rise by the same amount in all of the potential slide area.

2.3 Accuracy of basic assumptions with regard to the application of I-PFA

In the section above, some essential prerequisites for applying conventional stability analysis in practical engineering have been listed. However, it is evident that, in many instances, the likelihood of these conditions being fulfilled is extremely small. In fact, as will be demonstrated later, it can be shown by computation that conventional analysis based on ideal plasticity may, in markedly deformation softening soils, entail highly erroneous safety factors.
against failure. It may also result in total misjudgment of the eventual spread and potential degree of disaster resulting from a local slope failure. In the following, the applicability of conditions 1 through 3 to sensitive clays will be scrutinized.

![Graph showing stress/strain curves for consolidated, undrained vane tests at different strain rates (Aas, 1966)](image)

**Figure 2:3.1** Stress/strain curves for consolidated, undrained vane tests at different strain rates (Aas, 1966)

![Graph showing vane resistance or cone resistance](image)

**Figure 2:3.2** Typical test results from consolidated undrained direct shear tests on a soft Swedish clay. Note that deformation on the horizontal axis is represented both in terms of angular strain and slip movement in mm. (Bernander & Svensk, 1985)

a) In sensitive, deformation softening clays, normally with water contents significantly above the liquid limit, condition (1) about unlimited ductility without substantial loss of strength is not likely to be satisfied under undrained circumstances. Figures 2:3.1 and 2:3.2 illustrate how stress/strain relationships and residual shear strengths may dramatically be affected by deformation and rate of loading as well as by the effective stress ratio (OCR).

b) The probability of condition (2) being met at all times in respect of sufficiently small differential deformations within the sliding soil volume must also be considered to be extremely small.
Assuming for instance that the maximum horizontal deviatory strain immediately prior to shear failure in the soil is $\gamma_f$, and that the width of a slide is $b \text{ m}$, then the differential downslope displacement may well amount to at least $\Delta\delta = \gamma_f \cdot b/4$, before the lateral boundaries of the slide manifest themselves and the soil mass begins to move as an integral block. (Cf Figure 2:3.3 a)).

Putting e.g. $\gamma_f = 10\%$ and $b = 50\text{ m}$, then $\Delta\delta$ will be in the order of $1.25\text{ m}$.

When investigating a slide, involving some 500 by 180 $\text{ m}^2$'s of ground, at the construction site of the Kotmale dam (Sri Lanka), the author documented (1981) a differential displacement across the slide area of about 7 $\text{ m}$ at a stage when failure at boundary DF in Figure 2:3.3b was incipient and signs of impending rupture were still hardly detectable. This observation corresponds to a maximum horizontal deviatory strain ranging from about $\gamma_f = 3.7,0/2.180 \approx 6\%$ to $4.7,0/2.180 \approx 8\%$, depending on the distribution of strain across the line BE.

![Figure 2:3.3](image)

**Figure 2:3.3 a)** Conceivable range of downslope displacement prior to the actual slide movement of the soil mass as an integral block.

b) Documented differences in downslope displacement in a slide at the Kotmale dam site (Sri Lanka)

The implication of the observation above, is that the downslope movement relative to the sub-base in a potentially sliding soil mass may locally adopt any value between a displacement corresponding to maximum angular strain ($\gamma_f$) and slip deformations in the order of several meters prior to the formation of the lateral boundaries of the slide - i.e. before the soil mass actually assumes the global kinematic behavior of the assumed analytical model.

This implies in turn that the maximum shear stress, which can be mobilized along major portions of the slip surface area is actually limited by the residual shear strength ($c_R$) - i.e. a condition seriously invalidating the use in markedly strain softening soils of conventional analysis based on peak shear strength.

c) The prerequisite condition no (3) states that the stress field due to the incremental load causing a slope failure should at least in some measure conform to the in situ stress situation. This condition may be highly contentious, because landslides in western Sweden are - more often than not - triggered by agents, the loading effects of which are far from being evenly distributed over the area of the prospective slide. In fact, a considerable number of landslides
have been set off by local human operations such as pile driving, heavy vibratory equipment, local up-slope earth fills, stock piling of waste material, earthwork affecting hydrologic regimes, etc.

Figure 2.3.4 Shear stress field from local fill at the crest of a slope.
Curve I: Shear stress ($\tau_{ox}$) - corresponding to slope gradient
Curve II: Shear stress ($\tau_{ox} + \frac{dE}{dx}$) – corrected for earth pressure distribution ($E_{ox}$) in the in situ condition.

Figure 2.3.4 illustrates the short-term shear stress situation in a case, where a local fill has been placed near the crest of a sloping ground. Excluding the possibility of failure along a local slip circle surfacing in the slope, (which in any case later will be shown not to be critical), the instantaneous effect of the fill will be the generation of local shear stresses immediately down-slope of the fill.

Experience shows that any slide, that may ensue from this loading situation will in sensitive clays often engage the whole slope. Hence, if a stress analysis based on a long slip surface is made, then condition (3) is obviously not fulfilled. (Cf case records in Chapter 5). Now, if the fill is established over a long period of time, the absence of condition (3) may not be a problem because of creep and excess pore water pressure dissipation due to drainage. However, if the fill is applied at such a rate that undrained conditions prevail, then analysis according to the ideal-plastic approach is no longer applicable.
2.4 Relationship between the features of a finished slide and the mechanisms acting during the slide

The final morphology of landslides in Scandinavia often exhibits extensive zones at the foot of the slope or on the valley floor where the ground has heaved in passive Rankine failure. As demonstrated below, this feature is not compatible with the ideal - plastic failure concept.

Figure 2:4.1 Earth pressure development in a uniform slope at failure

If the laws of force equilibrium are applied to the soil element shown in Figure 2:4.1, we get:

\[ N + \Delta N = N + \rho g H \Delta x \cos \beta \cdot \sin \beta - c_u(\gamma) \cdot \Delta x \]

\[ \Delta N = \rho g H (\sin 2\beta)/2 \cdot \Delta x - c_u(\gamma) \cdot \Delta x = [\tau_0 - c_u(\gamma)] \cdot \Delta x \]

\[ \text{............................. 2:2} \]

where:

- \( c_u(\gamma) \) = the shear strength of the soil as defined by the stress strain curves P or D
- \( \tau_0 \) = prevailing stress due to down-slope forces

Other notations according to Figure 2:4.1.

Case a) Ideal-plastic failure (I-PF, Curve P in Figure 2:4.1)

It follows directly from Equation 2:2 that, in the case of ideal-plastic failure, \( c_u(\gamma) = c_u^{\max} \) for all values of \( \gamma > \gamma_F \) and \( F_c = c_u^{\max}/\tau_0 = 1 \). Thus substituting \( c_u(\gamma) = c_u^{\max} \) by \( \tau_0 \) in Equation 2:2 it is evident that \( \Delta N \approx 0 \) for all large post-failure deformations. This means that no significant build-up of earth pressures (\( \Sigma \Delta N \)) can take place down the slope.

Case b) Deformation-softening failure (Curve D in Figure 2:4.1)

If, on the other hand, the soil exhibits deformation-softening properties as for instance according to curve D, then

\( \Delta N > 0 \) from the very moment \( \gamma \) exceeds \( \gamma_F \). Hence:

\[ N = \int [\tau_0 - c_u(\gamma)] \cdot dx > 0 \] as soon as \( c_u(\gamma) < \tau_0 \).
The force increment $N$ may thus bring about a significant build-up of the static down-slope earth pressures as well as an acceleration of the soil masses. Both these phenomena originate from the inherent strain-softening properties of the soil.

The inevitable conclusion must be that the build-up of static earth pressures and the accumulation of kinetic energy during a landslide are conceivable only when the failure process deviates substantially from ideal-plastic behavior. Hence, when landslides exhibit evidence of passive Rankine failure having taken place over large areas of mildly sloping ground at the foot of a slope, the failure mechanism is clearly the result of significant deformation-softening in the progressive phase of the slide. In the Tuve slide, for instance, some 60% of the area affected by the main slide covered the almost horizontal valley floor.

In short, already the final appearance of a landslide offers important clues allowing of an explanation of the event.

![Diagram](image)

**Figure 2:4.2a** Slide in *ideal-plastic* soil featuring a small passive Rankine zone and insignificant build-up of down-slope forces. (Bernander, 1984).

![Diagram](image)

**Figure 2:4.2b** Slide in *deformation-softening* soil featuring an extensive passive Rankine zone due to massive build-up of down-slope static and dynamic earth pressures. (Bernander, 1984).
2.5 Conclusions - Progressive or brittle slope failures

The discussion in Section 2.3 above implies that, in markedly deformation softening soils the relative displacements between the soil mass involved in a potential slide and its sub-base vary significantly. Hence, while the shear stresses in parts of the failure zone range from zero to full shear strength, residual shear strength may be valid in other portions of the failure plane.

The compelling conclusion must be that the possibility exists of brittle or progressive failure mechanisms. Strain softening in one zone of a potentially unstable soil mass will strain, or lean on, neighboring zones, which themselves may undergo strain softening and so on. It is, therefore, reasonable to assume that prevailing conditions in a slope, may now and again, involve such sets of parameters that this interactive strain softening mechanism leads to global instability, forming a veritable landslide.

In consequence, the prospect of progressive failure must be addressed in the analysis of slope stability, particularly, but not only, in sensitive clays.

It should be observed in this context that, although factors conducive to brittleness in soils are treated to some extent in Chapter 6, it is not within the scope of this report to address methods and procedures for establishing and documenting the constitutive stress/deformation properties of soils. This task will be up to R & D and the soil mechanics engineer himself to perform.

The subsequent sections of this report highlight the impact of deformation softening on landslide hazards and present an analytical method of predicting the risk of local instability in a slope. Another important aim is to make a reasonable estimate of the final spread of the slide that may ensue.

As will be demonstrated in the following chapters, a slope with deformation softening soil layers, though reliably stable under long term drained conditions, may readily fail due to the effect of any powerful agent capable of inducing undrained local failure in some highly strained part of the slope.

2.5.1 Implications of progressive failure analysis (Pr FA) for design philosophy

In the opinion of the author, progressive failure analysis entails the following advantages:

- It models the mechanisms at hand more accurately than conventional analysis, allowing more reliable predictions of the ultimate consequences of a local slope failure. Many features of slides in sensitive clays cannot, by definition, be explained or understood in terms of the ideal-plastic failure concept.
  In fact, the formulation of valid constitutive (stress/deformation) relationships is a prerequisite for reliable investigation of landslide hazards in deformation softening soil of any kind.

- By means of progressive failure analysis, the truly most critical conditions in a slope can be identified, enabling preventive or remedial measures to focus on the pertinent issues.

- A better understanding of the mechanisms leading to global failure in a slope will induce geotechnicians to focus R&D and exploratory investigations on topics and soil parameters that are truly relevant in this context.
For instance, the Tuve slide described in Section 5.1 substantiates the importance of the statements above. Although *conventional analysis predicts*, by safety factors of about 2.4 to 3.0, that the slide could not extend a distance of about 270 m over the almost horizontal valley ground, this was, indeed, what actually occurred. By contrast, progressive failure analysis indicates clearly that the vast spread of the tongue of the slide on level ground was precisely what *should be expected*. The same applies to the great slide at Surte (1950), Section 5.2.

In this context, reference may also be made to the slide at Bekkelaget (Norway) described in Section 5.3, where the slide actually took place along the slip surface that rendered the highest safety factor according to current analytical I-PF approach.
3. Exemplification – different phases of downward progressive failures in natural slopes

3.1 General

The modeling of brittle failures in natural slopes is an issue of considerable complexity, and it is not within the scope of this report to deal with all aspects of the problem. The analytical model presented in this section is primarily tailored for slopes in sensitive normally consolidated clays, but applies in principle to any material that is markedly deformation-softening in shear.

3.2 Different types of progressive failure

Progressive failures in natural slopes may be classified as

a) **Downward progressive landslides**, where an initial local instability in the upper part of a slope propagates down the slope generating a major increase in horizontal earth pressures in ground with lesser gradients further down-slope. If, there and then, the total pressure exceeds passive **Rankine** resistance, a global ground displacement takes place involving, typically, also inherently stable ground at the foot of the slope. Downward progressive landslides are characterized by significant growth of the down-slope normal stress (σn).

b) **Upward progressive or retrogressive slides**, where local instability in the lower part of the slope propagates up-slope eventually resulting, at first, in a global monolithic displacement of the entire soil mass, which, however, often disintegrates in various active failure modes, e.g. piece by piece retrogressive slides or earth flows. Thus, retrogressive slides are characterized by significant decrease of the normal stress (σn), which may even become negative by suction i.e. tensile. A typical failure mode here is the ‘column failure’ described by Janbu, 1979.

c) **Laterally progressive slides**, where local instability anywhere in a slope propagates sideways along the elevation contours. In this case the destabilizing forces are transmitted to initially stable parts of the potential slide area by horizontal shear in vertical planes in the direction of the slide. Slides with significantly larger width than length in the direction of movement are likely to be of a laterally progressive nature.

Laterally progressive slides can be controlled by ensuring adequate safety against failure for the most critical section in the direction of the slope.

Many major landslides combine all the categories a, b and c (Cf the Rissa slide, Norway, Gregersen, (1981)).

3.3 Stability conditions in slopes susceptible to downward progressive failure

3.3.1 Different stages in the development of a progressive slide - limiting criteria

Most landslides in Sweden, where progressive failure mechanisms may have operated, belong to the category a) ‘downward progressive slides’. In consequence, the analytical model presented in this context is focused on this specific type of brittle failure.
In order to facilitate the understanding of the analytical model presented in Chapter 4, the specific features and stages of a progressive failure in a slope are highlighted in the following example. Reference is here made to Figures 2.3.4 and 4.2.1, which illustrate some of the main principles applied in the example. (For denotations not defined here, the general list of denotations applies).

Downward progressive failures in natural slopes exhibit several distinct phases. Thus, the Figures 3.3.2 to 3.3.5 illustrate different critical stages in the development of a down-slope progressive landslide due to deformation softening. Figure 3.3.1 shows the stress/strain (deformation) relationship assumed to be valid for the sensitive soil in the incipient failure zone.

![Diagram showing stress/deformation relationship](image)

**Figure 3.3.1** Assumed types of stress/deformation relationship \( \tau(\gamma) \) or \( c_R(\delta) \) of the soil in the example. Curves 1 and 2 exemplify such relationships at different rates of loading. \( c_R \) is the large deformation residual value of \( c_R(\delta) \).

For the sake of simplicity, the slope gradient is in this example taken to be constant and the ground below the presupposed failure zone to consist of firm soil. The ratio of horizontal to vertical stresses \( k_h = c_h/c_v \) is also presumed to be constant. As will be evident later, these simplifications do not affect the phenomena to be highlighted in the example.

In the example, the agent initiating the slope failure consists of an earth fill placed in such timing that the soil response is undrained.

**Stability conditions prior to local failure**

The weight of the earth fill generates an earth pressure increment of \( N_i \) at the toe of the earth fill as shown in Figure 3.3.2. As the fill is being placed, the increasing force \( (N_i) \) gradually mobilizes the remaining shear capacity in terms of stress \( (c_u - \tau) \) in the potential failure zone, i.e. shear capacity, which is not being exploited for stabilizing the sloping ground in the in situ condition. Figure 3.3.2 displays a situation where the shear strength \( (c_u) \) and the shear strain \( (\gamma) \) are fully mobilized at point A. Compatibility between the deviatoric deformation, mainly in the failure zone, and the related down-slope displacements due to the additional earth
pressures demands that the shear stress (τ) abates as the distance from the point of application of the force (N_i) increases. Hence, in Figure 3:3.2 the coordinate (x_i) defines the limited length along which the shear resistance required to balance the force N_i can be mobilized at this stage.

(Comment: It may be pointed out already in this context, that the line defined by the in situ stress τ_o (x) actually constitutes an asymptote to the curve τ (x), the point x, defined by the differential (τ_x-τ_o,x)=0 being theoretically located at an infinite distance from A. In practice, this difficulty is overcome by locating origo (i.e. x = 0) at a point where (τ_x-τ_o) has a defined, but negligible value).

![Figure 3:3.2 Stability situation prior to local failure, i.e. for γ_x ≤ γ_F. N_x denotes the additional earth pressure force induced by N_i caused by an earth fill. (N_i ≤ N_cr).](image)

Post peak stress stability phase – critical length, 1st limit state of static equilibrium

However, further growth of the force (N_i) initiates local failure at A, and from this point on, the deformation in the failure zone between the points A and B in Figure 3:3.3 will consist of an additional component arising from the slip developing from now on in the failure plane. Figure 3:3.3 depicts the situation where the in situ stress (τ_o) is just balanced by the current post peak residual shear strength (c_R,x_2) i.e. where (c_R,x_2 - τ_o,x) = 0.

At this point, all available shear capacity is mobilized, and the stabilizing resistance (N_2=N_cr) has attained its maximum value possible.

N_cr = ∫₀^x₂ (τ(x) - τ_o(x))dx  (for 0 ≤ x ≤ x₂)  

where τ(x) ranges from τ_o,ν→ c_u→ τ_o,x₂ = c_R,x₂

The maximum length (x₂) corresponding to N_cr, along which shear strengths in excess of the in situ stress (τ_o) can be mobilized is in the following denoted as the critical length (L_cr), as further deformation at A generates negative values of (c_R,x - τ_o,x). This implies that unbalanced down-slope forces may start acting at A. Thus, the critical length L_cr represents in some measure the maximum length of a slide, induced by local loading, that can be studied on the basis of ideal-plastic soil properties with any prospects of attaining acceptable accuracy.
The general condition that has to be fulfilled, lest a dynamic progressive failure (Pr F) take place, is therefore:

\[ c_R(x) - \tau_0(x) > 0 \quad \text{for all values of } x > x_2 \quad \ldots \ldots \ldots 3.2 \]

![Figure 3.3.3](image)

**Figure 3.3.3** Effect of increasing the down-slope force \( N_i \) beyond the value corresponding to the shear strength at point A. When \( c_R(x) - \tau_0(x) = 0 \), the maximum resistance \( N_{c2} = N_{cr} \) is reached. In the figure the 'critical length' \( L_{cr} = x_2 \) is shown.

Another key criterion governing the possible occurrence of a progressive failure, is that the earth pressure \( (E_{cr} = N_{cr} + E_0(x_2)) \) required to provoke failure in a zone of limited length \( (L_{cr}) \), oriented parallel to the ground surface or to the firm bottom, must be smaller than the resistance along a failure plane \( A_o-C \) (Figure 3.3.3), i.e.

\[ E_{cr}(x_2) = N_{cr} + E_0(x_2) = N_{cr} + k_o \cdot \rho \cdot g \cdot H^2 / 2 \leq N_{A_0-C} \approx \rho \cdot g \cdot H^2 / 2 + 2 \cdot \sqrt{2} \cdot \sigma_l H \cdot c_o(z) \cdot dz \quad \ldots \ldots 3.3 \]

or

\[ N_{cr} < (1 - k_o) \cdot \rho \cdot g \cdot H^2 / 2 + 2 \cdot \sqrt{2} \cdot \sigma_l H \cdot c_o(z) \cdot dz \quad \ldots \ldots 3.3a \]

where \( N_{A_0-C} \) is the force required to provoke failure along the plane \( A_o-C \)

The steeper the gradient and the more the soil is deformation softening, the more the value of \( N_{cr} \) tends to fall below the resistance in a failure plane \( (A_o-C) \) surfacing in the steeper part of a slope. Analyses indicate that, in sensitive soils, the condition 3:3 (or 3:3a) is normally fulfilled, even in gently sloping ground. For the slope studied in the example in Appendix I, \( N_{cr} \) is for instance = 113.7 kN/m, while the value of the force \( N_{A_0-C} \) (for \( k_o = 0.7 \)) may be estimated at 2100 kN/m \( >>> 113.7 \) kN/m.

The inverse conclusion to be drawn from this is that, when applying I-PF analysis, short slip surfaces engaging only the steeper part of a long slope cannot be used for predicting the risk of the initial failure. This is simply because such failure modes will not represent the lowest resistance against slide formation.
Dynamic phase 1 – progressive failure and associated redistribution of forces

Due to the build-up of unbalanced forces, the soil mass immediately down-slope of the fill passes into a stage characterized by dynamic displacements down-slope of point A and increasing slip deformation in the failure surface. Importantly, the increasing deformation extends the zone in the slope, where the in situ down-slope forces defined by the shear stress ($\tau_0$), are no longer balanced by the post-peak residual shear strength, which soon attains the residual value of ($c_R$). (Cf. Figure 3:3.4). The unbalanced down-slope driving force $N_D$ may be written as

$$N_D(x > x_2) = N_1 + \int_{x_2}^x [\tau_0(x) - c_R(x)] \, dx$$

The increasing force $N_D$ causes the failure front in the shear band to propagate fast down-slope producing a significant change of the earth pressure distribution in the slope. This movement should, however, not be understood as a regular slide but rather as a progressive pressure wave, by which unbalanced forces in the zones subject to deformation softening are transmitted to less sloping ground further down the gradient.

The maximum velocity at which pressure changes in the slope can propagate is $v_{\text{max}} = \sqrt{E_{\text{el}}/\rho}$. If, for example, $E_{\text{el}}$ is set at 200 $c_u$, then $v \approx 52$ m/sec. However, friction and time dependent processes in the rupture zones are likely to effectively slow down the speed of failure propagation and thus reduce the dynamic effects of the progressive failure.

Post progressive failure stage - possible second state of equilibrium

As the progressive failure (or the pressure wave) travels into less sloping or horizontal ground

Figure 3:3.4 Possible second state of equilibrium following the redistribution of earth pressures resulting from progressive failure.
the value of maximum resistance $N_{cr}$ increases dramatically with falling values of $\tau_o$, whereby a second stage of static equilibrium becomes possible, i.e.

$$N_D(x_3) = N_i + \int [\tau_o(x) - c_R(x)]dx < N_{cr} \quad \text{See Figure 3:3.4} \quad \ldots \ldots \ldots 3:4a$$

Here, $N_{cr}$ applies to the conditions at the foot of the slope. The progressive failure in the dynamic phase I may thus be understood as a redistribution of unbalanced forces from less stable areas to more stable ones further down the slope.

It is therefore important to note that the displacements ($\delta$) at this stage are limited to the effect of axial compression induced by the additional force $N_i$ and the deformation softening. This means that the total displacement at A may then merely be in a range of a few centimeters, decimeters or even meters, subject to the extent of the potential slide.

The timing of the progressive phase is subject to many factors. In very sensitive clays it is believed to be a matter of tens of seconds. (Cf slide at Rävekärr, 1971, Section 5.5)

In conclusion, the progressive failure in dynamic phase I represents a transmission of unbalanced shear forces in steeper ground to more stable, less sloping parts of the gradient where a dramatic growth of the total earth pressures may ensue.

$$E_x = (E_o)_x + N_x \quad \ldots \ldots \ldots 3:5$$

where $N_x$ denotes the earth pressure increment resulting from the force $N_D(x_3)$

Now, in case the maximum earth pressure ($E_x$) after the progressive failure event remains less than the earth pressure resistance in a passive Rankine state, then the soil mass of the potential slide contained between $0 < x < x_3$ will remain in equilibrium and retain its monolithic structure.

$$E_x^{\max} = [E_o(x) + N(x)]^{\max} = [k_o \cdot \rho \cdot g \cdot H^2/2 + N(x)]^{\max} < E_p^{\text{Rankine}} = \rho \cdot g \cdot H^2/2 + \int \rho_2 \cdot c_u(z)dz \quad 3:5a$$

or

$$N_x^{\max} < (1 - k_o) \cdot \rho \cdot g \cdot H^2/2 + \rho_2 \cdot c_u(z)dz \quad \ldots \ldots \ldots 3:5a_1$$

Equation 3:5a: shows the importance of making a reasonable assessment of the value of $k_o$. Considering time dependent creep, the value of $k_o$ is likely to assume rather high values in the zones at the foot of the slope, where gradient decreases. Thus, if for instance $k_o = 1$, then the criterion will be $N_x^{\max} < \rho_2 \cdot c_u(z)dz$

If the condition according to Equation 3:5a is valid, the slide event is terminated. This means that the fill generating the force $N_i$, is merely displaced a limited distance $\delta x_3$ corresponding to the axial down-slope compression of the soil. The only result is a local active failure up-slope of the fill, as well as an associated vertical offset of the same order as the displacement $\delta x_3$. Cf slide at Rävekärr, 1971, Section 5.5.

Dynamic phase II – the actual slide event

On the other hand, the build-up of down-slope pressure may well far exceed passive Rankine resistance, i.e.

$$E_x^{\max} = [E_o(x) + N(x)]^{\max} > E_p^{\text{Rankine}} = \rho \cdot g \cdot H^2/2 + \rho_2 \cdot c_u(z)dz \quad \ldots \ldots \ldots 3:5b$$
\[ N_x^{\text{max}} > (1 - k_o) \cdot p \cdot g \cdot H^2/2 + \int_{0}^{H} 2 \cdot c_u(z) \cdot dz \]

Equation 3.5 b (or b1) constitutes the critical criterion for the occurrence of a major slide due to deformation softening. The consequences will be dramatic if the passive resistance is exceeded over some distance in the lower part of the slope, as the potentially sliding soil volume will then disintegrate in a passive Rankine failure state, causing the ground surface to heave.

More important still, the ground heave is a prerequisite condition for the now inherently unstable up-slope soil masses to start moving down-slope at an accelerating rate. It is at this point that the slide enters its truly dynamic phase, in which further events are governed by Newtons laws of motion.

(See Section 5.1, the Tuve slide, dynamic analysis and Section 5.2, the Surte slide).

In this context, it is important to note that the increasing sliding rates tend to further reduce the residual shear resistance in the slip surfaces already formed, thus further amplifying the unbalanced force \( N_D \). This applies also to that part of the failure zone, which is engaged in passive failure.

The additional growth of the force \( N_D \), together with the dynamic inertia forces in the retardation phase, require that even more of the initially stable, less sloping ground has to be engaged in order to attain a final state of equilibrium. (See Figure 3:3.5).

The final extension of the slide, based on static equilibrium, can be estimated by applying a residual shear strength \( c_R \), which is compatible with the relative velocity at which the soil actually slides over the slip surfaces. Therefore, if we really want to predict the potential extent or degree of disaster of a slide, it is imperative to use quite a different set of soil parameters from the ones used for the evaluation of \( N_{cr} \), governing the safety factor against the initiation of a local failure. (Cf. Chapters 6 to 8).

3.32 Synopsis

The main conditions, under which local instability in a slope may develop into a progressive failure eventually leading to global collapse, are:

a) The differential \((c_{R,x} - \tau_{o,x})\) becomes less than zero (< 0) in the zone of local instability, i.e.

\[ N_i > N_{cr} \]

It is important to note at this point that also for values of \((c_{R,x} - \tau_{o,x}) > 0\), progressive failure will be induced by increasing values of \(N_i\). However, the vital difference is that the additional load \(N_i\) under such conditions is no longer limited to a specific critical value but tends to rise as the failure is ‘forced’ down the slope. The ultimate limit of \(N_i\) is then more related to the passive Rankine resistance.

b) The earth pressure \(E(x) = N_{cr} + E_o(x)\) is smaller than the resistance along a failure plane surfacing in the slope. (e.g. the failure plane \(A_o-C\) shown in Figure 3:3.3), i.e.:

\[ E(x) = N_{cr} + E_o(x) < N_{Ao-C}, \text{ where } N_{Ao-C} \text{ is the force required to provoke failure along the plane } A_o-C \]
c) The example presented above indicates, and further analysis will document, that in a fully developed downward progressive slide, three possible limit states of equilibrium and two intervening dynamic phases may be discerned:

1 - A primary state of static equilibrium, where the force induced by the initiating agent is less than the force defined as \( N_{cr} \). (See Figures 3:3.2 and 3:3.3), i.e.

\[
N_i \leq N_{cr}
\]

1a - If, however, \( N_i \) exceeds \( N_{cr} \), a downward progressive failure generates a dynamic transmission of unbalanced up-slope forces entailing earth pressure increments in more stable ground further down the slope.

2 - A second state of static equilibrium may occur if, subsequent to the down-slope force transmission, the maximum earth pressures remain below passive Rankine resistance, i.e.

\[
E_{max} = [E_o(x) + N(x)]_{max} < E_p^{Rankine}
\]

In this case, the progressive failure will only result in moderate displacements with a local, active Rankine failure zone up-slope of the agent initiating the local failure.

2a - If, on the other hand, the resulting maximum down-slope earth pressures exceed the passive Rankine resistance, i.e.

\[
E_{max} = (E_o(x)+ N(x))_{max} > E_p^{Rankine}
\]

a dynamic phase constituting the major slide event takes place, resulting in large displacements of soil masses down the slope.

3 - Final equilibrium of the finished slide

![Diagram](image)

**Figure 3:3.5** Conditions at the far end of a downward progressive landslide.

Final equilibrium is attained, when the static forces and the forces of inertia are balanced by the passive resistance of the heaved ground.

i.e. \( N_i + N_D + N_{inertia} < E_p^{Rankine}\text{ (heave)} \)

\[..........................3:6\]

See Figure 3:3.5 above.
The heave of the ground may be estimated by equating the potential energy \( w \) released by the slide to the energy required to raise the center of gravity of the soil masses in the passive zone, i.e.

\[
w_{\text{potential energy}} = \sum\left[ p \cdot g \cdot H_x^{\text{heave}} \cdot \Delta x \right] \cdot \left( H_x^{\text{heave}} - H_x \right)/2
\]  

\[
\text{3.7}
\]

The potential energy \( w \) has to be determined by iteration procedures.

(Note that Equation 3.7 does not consider the energy lost in the slip surfaces during the passive Rankine failure process. In very sensitive soils, this energy is likely to be small compared to the total energy released in the main slide event.)

As shown in Figure 3.3.5, the actual limit of the slide extends a distance \( x_a \) often far beyond the limit of the passive Rankine zone. The length of \( x_a \) may be estimated by Equation 3.8

\[
E_p^{\text{Rankine}} = E_0 + \int_{A}^{x} k_s \cdot dx + \int_{B}^{x_a} c_r(x) \cdot dx
\]  

\[
\text{3.8}
\]

3.33 Safety factors - new formulations

The discussion above indicates that the conventional definition of the safety factor against slope failure is devoid of any meaning in terms of predicting progressive failures. Instead, the following criteria are considered pertinent. (Bernander & Gustås, 1984)

With regard to preventing initial local failure, the potential triggering force \( N_t \) should not exceed the local maximum stabilizing resistance \( N_{cr} \), i.e. the safety factor

\[
F_s = \frac{N_{cr}}{N_t} > 1
\]  

\[
\text{3.7}
\]

or, if the additional loads \( (q,t) \) shown in Figure 4:2.1 are also considered,

\[
F_s = \left( \frac{N_t(q,t)}{N_{cr}(q,t)} \right) > 1
\]  

\[
\text{3.7a}
\]

where \( (N_t(q,t))_{cr} \) denotes a critical combination of the additional loads

With regard to the global failure that may result from a local instability releasing a progressive failure, the ultimate earth pressure after the redistribution of forces must not exceed passive Rankine resistance, i.e.

\[
F_s = \frac{E_p^{\text{Rankine}}}{(E_{cr} + N_s)_{\text{max}}} > 1
\]  

\[
\text{3.8}
\]

or

\[
F_s = \left[ (1 - k_u) \cdot p \cdot g \cdot H^2/2 + \int_{A}^{H} 2 \cdot c_r(z) \cdot dz \right] / N_{s_{\text{max}}} > 1
\]  

\[
\text{3.8a}
\]

3.34 Slope failure in sensitive soils - a problem analogous to buckling

It is thus possible to conclude that once the initiating force \( N_t \) exceeds the value of \( N_{cr} \), static equilibrium is no longer mechanically possible. The work performed by the force initiating progressive failure can be expressed as

\[
W_{cr} = \int N_{cr} \cdot d\delta_s = \int^{\infty}_{0} N_{cr} \cdot d\delta_x
\]  

\[
\text{3.9}
\]

Where both \( N_s \) and \( \delta_x \) are functions of the coordinate \( x \).
As may be concluded from Figures 3:3.4 and 4:2.4a, a case may arise where the force required to set off a progressive failure is equal to zero. Hence, Equation 3:9 applies also to the case, when a forced deformation corresponding to \( N_x = 0 \), is applied, i.e.

\[
W_{cr} = \int L_{\text{instab}} N_x \, d\delta_x
\]

\[\cdots \cdots \cdots 3:9a\]

Interestingly, Equation 3:9a signifies that when a certain *forced* displacement (\( \delta_{N_x} \)) is applied, e.g. by the driving of soil displacing piles, the slope will fail despite the fact that there may be no sustained active force \( N_x \) to maintain the failure process. In other words, the failure criterion here is related to the total energy or the deformation generated by the agent causing the initiating failure. (See Figure 4:2.4a, where the length \( L_{\text{instab}} \) in Equation 3:9a is defined.) Accordingly, slope stability may be considered as a problem analogous to 'buckling stability' in the true structure-mechanical sense of the word. As in the case of an axially loaded strut, static equilibrium is no longer possible once the limit critical load is reached at a certain initial or applied mid-point deflection.

The stability of a slope may therefore metaphorically be thought of as being similar to the stability of a ball placed in a depression on top of a cone as shown in Figure 3:3.6. Here, if the work applied to the ball by a displacing agent \( N_x \) is greater than \( W = w \cdot \Delta h \), the ball is lifted over the rim of the depression and static equilibrium is out of the question.

![Figure 3:3.6 Symbolic representation of slope stability in deformation softening soils.](image)

The value of \( N_{cr} \) is in the analogy related to the depth and the steepest gradient of the sides of the depression (\( N_{cr, \text{max}} = w \cdot \tan \beta \)), while the steep (\( \alpha \)) of the exterior sides of the cone may be said to correspond to the degree of deformation softening in the soil. Hence, in this model, ideal-plastic equilibrium will correspond to the special case when the gradient of the sides of the cone becomes so infinitely small that the exterior surfaces of the cone form a horizontal plane \( A''-A''' \) as indicated in Figure 3:3.6
3:4 Conclusions

A local slope failure may lead to the total collapse of not only the entire slope, but also of large areas of adjoining inherently stable ground subject to a number of factors such as

- soil sensitivity
- ground and slope morphology
- profile of slip surface
- distribution of incremental loads
- type and timing of the agents initiating failure
- hydrological conditions, etc

For reference, see Chapter 5 dealing with a number of case records, where the morphology of the finished slide in the opinion of the author is only conceivable using a progressive failure model.

It is of particular interest to note that the ratio between the critical length \( L_{cr} \) (corresponding to \( N_{cr} \)) and the length of the prospective slide (\( L \)) offers a clear indication regarding the degree of applicability of the conventional ideal-plastic approach with respect to concentrated additional loading. For low values of \( L_{cr}/L \), analyses based on ideal-plastic soil properties must yield poor predictions of slope stability. If in a specific case, \( L_{cr} \) amounts for instance to only 0.3\( L \), it is obvious that an I-PF analysis, engaging a slip surface of the full length \( L \), cannot possibly yield adequate predictions of safety against slope failures induced by local disturbance agents. (See e.g. the slides at Surte, Tuve and Bekkelaget in Chapter 5, Case records). Hence, any slope stability investigation of longer natural slopes should at least include a rough assessment of the critical length \( L_{cr} \).

It is sometimes argued that there is not much cause for concern about the manner in which the ultimate disintegration of a slope takes place, provided the initiating local failure can be identified and prevented. However, preventive measures against landslide hazards must address the precise critical situation or problem in a slope.

It is therefore important to note that Pr F analysis, as opposed to conventional I-PF analysis, indicates that local slip circles surfacing in sloping ground, even assuming high values of the brittleness ratio (\( c_p/c_u \)), rarely constitute the critical failure modes in slopes with deformation softening soils. (Cf Chapter 5 and Sections 3.31 and 4.6).

Regardless of this, there is, for obvious reasons, always a need for predicting the possible extent and degree of disaster that may result from local inherent instabilities or man-made destabilizing changes in a slope. For instance, the risk involved by pile driving activities, which released the great 240 000 m\(^2\) slide at Surte would, according to calculations made by the author, have been readily exposed by Pr F analysis. (See Section 5.2)

The comprehensiveness and the cost of measures to secure a slope against failure must, of course, be related to the risk and the stakes involved. In some instances, the consequences of a progressive slope failure may be negligible, whereas in other cases they may entail disaster.

*In much the same way as the considerable energy released by a gunshot can be controlled by applying the safety catch, the enormous energy set free by a landslide in deformation softening soil can be safely secured by appropriate measures - provided the mechanisms of the slide are sufficiently well understood.*

*Even so, it is imperative to know what will actually happen if the gunshot were to go off by accident.*
4. An analytical FDM-model for downward progressive slides - theory

4.1 General

The model for progressive failure analysis described below is a further development of an earlier approach published at the Xth ICSMFE (1981). The improved model was first presented at a poster session at the XIth ICSMFE (1985), and was later available in a paper to the XIIth ICSMFE in Rio de Janeiro (1989).

Ideally, slope stability analysis should define unambiguously the critical conditions in a slope directly on the basis of input data. However, such an analysis would, apart from being very complicated, in most cases also be unnecessarily laborious, as the critical failure planes can often be reasonably well identified by the morphology of a slope and the stratification of the soils. In fact, the analysis proposed below predicts that failure planes primarily tend to develop along the steep of firm bottom, even to great depth below the ground surface.

The approach to slope stability analysis presented below does not form an integral ‘closed’ analysis with the critical failure planes emerging as a result of the computations. The method of procedure resembles conventional limit plastic equilibrium modeling in so far as the potential failure planes are presumed as being known. The most critical conditions are therefore - as in conventional stability calculations - found by iteration and ‘trial and error’ procedures.

Nevertheless, the proposed analysis differs from ideal-plastic limit equilibrium methods in a number of important ways:

a - Whereas, in the ideal-plastic failure approach, the equilibrium of the entire potential sliding body of soil is investigated, the Pr F- analysis focuses on the equilibrium of each individual element into which the body is subdivided.

b - Furthermore, the main deformations within and outside the potentially sliding soil mass may be considered. Hence, the axial displacements in the slide direction due to earth pressure changes in the slope are at all times maintained compatible with the shear deformations of the discrete vertical elements. In doing so, it is possible to consider the distribution of shear stresses and the extent to which the shear capacity can be mobilized along the potential failure plane. The shear deformations are defined by constitutive shear/deformation relationships. The differential equations are integrated and solved numerically.

c - The shear properties of the soil are defined by a full non-linear stress/strain curve and not only by a discrete shear strength parameter, as in normal limit equilibrium calculations. This constitutive relationship is separated into two stages (I and II), simulating the conditions prior to, and after the formation of a slip surface - e.g. as in direct shear tests of the kind described by Bernander & Svensk (1985). The stress/deformation relationships may be chosen so as to suit arbitrary conditions in different parts of the slope.

d - By using different stress/deformation relationships, relating to different timings of the stress (load) application events, the time factor can effectively be included in the analysis.

e - Local horizontal or vertical loads, as well as local conditions in the slope morphology, which may be conducive to failure formation, can be considered.
Although some failure planes are assumed to be given, the final extent of the failure plane and the length of the passive zone emerge as results of the computations.

4.2 Soil model - derivation of formulae

4.21 Basic assumptions - drainage conditions

A landslide may begin as a drained or undrained *local* failure depending on the timing of the agents causing the limited zone of instability. The soil strength parameters applied to define the critical conditions susceptible of initiating a progressive failure in the slope must therefore be selected in accordance with the character of the additional loading being investigated. However, once a progressive failure has begun, shear resistance is mobilized at such high rates that undrained response in the soil is valid.

Hence, although total stress parameters are used in the structure-mechanical analysis of the slide events, the strength parameters (including the constitutive stress-deformation relationship) are based on the consolidated-*undrained* or *drained* behavior of the soil, whichever of these conditions is appropriate. Ground water conditions and possible artesian conditions enter into the analysis by the OCR - ratio.

Furthermore, the soils of the entire slope profile are taken as being saturated, which means that the seepage pressures due to percolation of ground water down the slope are accounted for, even in cases with highly permeable soil strata.

If the slope is partially submerged, the stabilizing effect of horizontal hydraulic pressure can be considered in the model.

4.22 Basic assumptions in the analytical model

Some of the general principles and notations applied in the adopted model for slope failure are shown in Figure 4:2.1 below.

![Figure 4:2.1 Notations and general principles](image-url)
The basic mathematical approach used is that of finite differences in a two-dimensional FDM model. Geometrically, however, any desired three-dimensional shape of the sliding body can be accommodated. As indicated in Figure 4.2.2, the potentially sliding soil volume is subdivided into discrete vertical elements of length Δx in the direction of movement. The coordinate (x) is, in the derivation below, oriented along the slip surface, which is justified as long as cos β ≈ 1. For instance, with a gradient of 1:8, cos β is 0.992. However, in the computer version referred to in Section 4.5, the x-coordinate is aligned horizontally.

Each vertical slice is further subdivided into a number of rectangular elements of height Δz in the z-direction, thus permitting modeling of the deviatory deformations within and outside the soil profile, and in particular, the deformations in the failure zone adjoining the actual slip surface. This is an important feature because the incipient failure zone contributes to a major part of the shear displacement of a vertical soil element, even prior to the formation of a regular shear band or failure plane.

The denotations used in the subsequent derivation of Equations 4:1 to 4:5 are defined in Figure 4:2.2 and as follows:

- δₓ: average down-slope displacement of the soil above the potential slip surface
- αHₓ: level at which the down-slope displacement is considered to be valid
- E₀(x): in situ earth pressure at point x
- N(x): earth pressure increment due to additional load or to progressive failure formation
- E(x) = E₀(x) + N(x)
- τ(x,z): total shear stress in section x at elevation z
- τ(x,0): total shear stress at failure plane (z = 0)
- τ₀(x,z): in situ shear in section x stress at elevation z
- τ₀(x,0): in situ shear stress at failure plane (z = 0)
- γ(x,z): deviator strain in point (x,z)
- σ(x): mean incremental down-slope axial stress
- q(x): additional vertical load
- t(x): additional load in the down-slope direction

![Figure 4:2.2 Soil model – denotations (From Bernander et al, 1989)](image-url)
4.23 Basic differential equations

Derivation of formulae valid in stage I, i.e. for values of $\gamma(x) < \gamma_f$

Equilibrium of an element $[H(x) \cdot b(x) \cdot \Delta x]$ in the down-slope direction requires that

\[
\Delta N = [\tau(x, o) - \tau_o(x, o)] \cdot b(x) \Delta x - q(x) \cdot b(x) \sin \beta(x) \Delta x - t(x) \cdot b(x) \Delta x \quad \text{.....} \quad 4.1
\]

The in situ shear stress in the slip surface may be written as

\[
\tau_o(x, o) = \sum_o \frac{\gamma_o}{\alpha} (\sigma - \tau_o(x, o) \Delta z - g \cdot \rho_w \cdot b(x) \cdot \Delta E_o(x)/(b(x) \Delta x)) \quad \text{.....} \quad 4.2
\]

(Note: $x$ is positive in the up-slope direction, implying that $\Delta E_o$ is negative for decreasing earth pressure in the direction of $x$.)

The axial compression of an element in the $x$ direction may be written as

\[
\Delta \delta_N = (N + \Delta N/2) \cdot \Delta x / [E_{el} \cdot (H(x) \cdot b(x))] \quad \text{.....} \quad 4.3
\]

where $\Delta \delta_N$ is the incremental mean down-slope displacement due to the compression of an element.

However, the total mean down-slope displacement ($\delta_N$), to which a vertical element is subjected, must be compatible with the shear deformation of the same element relative to the ground below the slip surface. This condition may be expressed as

\[
\delta(x) = \sum_o \frac{\alpha H(x)}{\gamma_o} [(\tau(x, z)/G(x, z, \tau) - \tau_o(x, z)/G(x, z, \tau))] \cdot \Delta z + \delta(x, o) \quad \text{.....} \quad 4.4
\]

The compatibility criterion with regard to down-slope displacement demands that

\[
\delta(x) = \sum_o \gamma \cdot (\Delta \delta_N) = \delta(x) \quad \text{When} \quad \gamma(x, z) < \gamma_f, \quad \text{then} \quad \delta(x, o) = 0 \quad \text{.....} \quad 4.5
\]

The known constitutive relationship defined by the shear stress/deformation curve is expressed as

\[
\tau(x, z) = \phi'(\gamma(x, z), \delta_s, d\delta_s/dt) \quad \text{or inversely,} \quad \gamma(x, z, \delta_s, d\delta_s/dt) = \phi_1(\tau(x, z)) \quad \text{.....} \quad 4.6 \text{ a}
\]
Stage I: $\delta_N = \delta_t$
Stage II: $\delta_N = \delta_{t,cr} + \delta_S = \delta_{t,cr} - \delta_{t,cr} + \delta_S$

**Figure 4.2.3** The down-slope displacement of a soil element must be compatible with the shear deformation of the same element in relation to the sub-ground.

Thus, the shear stress $\tau(x,z)$ is a function of the deviatoric strain $\gamma(x,z)$ and the displacement $\delta_S$ in the slip surface. If this function is known, the differential Equations 4.1 to 4.6 can be integrated numerically yielding the states of stress, strains and displacements for any chosen mode of mobilizing the resistance to failure propagation - and that in any chosen portion of the slope. See Figure 4.2.4.

**Figure 4.2.4a** Principal results from the progressive failure analysis according Equation 4.1 to 4.6.

- notations.

**Figure 4.2.4b** Time dependent stress-strain relationship $\tau = \phi(\gamma, \delta_S, d\delta_S/dt)$. Laboratory test curve compared with the same curve translated to the real dimensions of the soil structure. Note the apparent difference in brittleness. Curves 1 and 2 exemplify stress/deformation relationships at different rates of loading.
Derivation of equation valid in stage II, i.e. for values of $\delta_s(x) > \delta_s(z R)$

When the residual shear strength is attained in the slip surface, the Equation 4.1a is substituted for Equation 4.1.

$$\Delta N = [c_R(x,0) - \tau_0(x,0)] b(x) - q(x) - b(x) \cdot \sin \beta(x) - t(x) - b(x)$$  \hspace{1cm} 4.1a

where $\tau_0$ is defined as before

$$\tau_0(x,0) = \sum \gamma_0 h(z) g \cdot p(z) \cdot A(z) \cdot \sin \beta(z) - g \cdot p w D w(z) \cdot \sin \beta(z) - A e(z) / b(z)$$

and $c_R(x,0)$ = the residual large deformation, and/or if applicable, the high deformation rate strength of the soil at $z = 0$.

Moduli of elasticity

Hook's modulus of elasticity ($E_{el}$) enters into the analysis (Equation 4.3) when evaluating the displacement of a vertical section in the down-slope direction. Referring to the constitutive relationship shown in Figure 4.4.2, the initial shear modulus, which is valid below the elastic limit defined by $(\tau_{el})$ and $(\gamma_{el})$, can be expressed as

$$G_{el} = \frac{\tau_{el}}{\gamma_{el}}$$

and the corresponding E-modulus is then according to basic theory

$$E_{el} = 2(1 + v) \cdot G_{el} = \Omega \cdot c_u$$

where $\Omega$ is a coefficient relating the E-modulus to the undrained shear strength.

As the elastic modulus in cohesive materials is often expressed in terms of the shear strength, the sought mean elastic modulus ($E_{el}$) may thus be put as

$$E_{el,mean} = \Omega \cdot c_{u,mean} = \Omega \cdot 1 / H \int_0^H c_u(z) \, dz$$  \hspace{1cm} 4.7

For example, if in a specific case $c_{u,z=0} = 30 \text{ kN/m}^2$, $c_{u,mean} = 25 \text{ kN/m}^2$, $\tau_{el} = 20 \text{ kN/m}^2$, $\gamma_{el} = 1 \%$ and $v = 0.5$ then

$$G_{el,0} = 200/0.01 = 2000 \text{ kN/m}^2$$

$$E_{el,0} = 2(1 + 0.5) \cdot 2000 = 6000 \text{ kN/m}^2 = 200 \cdot c_u \quad \text{(i.e. } \Omega = 200)$$

$$E_{el,mean} = 200 \cdot c_{u,mean} = 200 \cdot 25 = 5000 \text{ kN/m}^2$$

Thus, for low shear stresses in the elastic or quasi-elastic range, the problem of time dependency of the moduli is not acute, since the ratio of $G/E$ will largely be independent of time.

However, at high shear stress levels in the potential failure zone, time dependency of the deviatory strains is more pronounced while the displacements determined by $E_{el,mean}$ may still be basically elastic. Hence, time dependent phenomena in the failure zone have to be modeled by appropriate constitutive relationships, selected in accordance with the time range of the event or agent jeopardizing the stability of the slope.

Regarding the value of ‘$\alpha$’ in equation 4.4

The value of $\alpha$ in Equation 4.4 may require special consideration. The analytical model illustrated in Figures 4.2.2 and 4.2.3 can be defined as a structure composed of a compression member in the slope direction connected to the sub-base by a number of ‘shear keys’. Hence, in this model, the soil masses in the sloping ground are stabilized jointly by compression along the slope and by shear in the vertical shear keys.
Now if, hypothetically, this compression member of massive soil were to be replaced by a strut or a beam with the same stiffness as the soil, compatibility would require that the strut be located at the elevation of the earth pressure resultant. Thus, the value of $\alpha$ is taken to be equal to $z_R/H$, where $z_R$ denotes the $z$-coordinate of the earth pressure force resultant. The fact that $z_R$ and $\alpha$ are not constant along the slope may constitute a complication in the proposed analysis. However, the variation of the value of $\alpha$ is not very significant. For instance, in the case of normally consolidated Swedish clays, the following values of $\alpha$ would be valid for total stress analysis:

<table>
<thead>
<tr>
<th></th>
<th>$H = 10$ m</th>
<th>$H = 20$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>At active earth pressure</td>
<td>$\alpha = 0.27$</td>
<td>$\alpha = 0.30$</td>
</tr>
<tr>
<td>At values of $k_0 \approx 1.0$</td>
<td>$\alpha = 0.33$</td>
<td>$\alpha = 0.33$</td>
</tr>
<tr>
<td>At passive earth pressure</td>
<td>$\alpha = 0.37$</td>
<td>$\alpha = 0.35$</td>
</tr>
</tbody>
</table>

Furthermore, the outcome of the analysis is insensitive to variations of $\alpha$, which is partly due to the non-linear behavior of soils at high stress levels. (See further comment at the end of the following paragraph).

In conclusion, although the method of analysis can accommodate any $\alpha$-value deemed to be appropriate, a value of 0.33 can in practice be applied within the ranges of normally prevailing earth pressures in slopes.

*Regarding the distribution of shear load*

Another point of interest in this context is the vertical distribution of shear load in the vertical elements constituting the shear keys. Shear in these elements originates mainly from four different sources, namely

a) slope inclination;

b) possible direct effect of additional loads at the studied section;

c) change of earth pressure distribution along the slope resulting from loads acting at some distance away from the section studied;

d) forced deformation associated with down-slope displacements - also emanating from loads acting away from the section studied.

In the dominant case (a), the shear load increases basically linearly with depth. In case (b), the distribution of shear load depends on the type and point of application of the load. In case (c), the effect of distant loads may be assumed to materialize as changes of the values of $k_0$, equally entailing linear shear growth ($dE/dz$) with depth. In case (d), forced deformation of the 'shear keys' tends to generate linear shear on account of the fact that shear strength and stiffness of the soil normally builds up linearly with depth.

Again, the critical results from the slope stability analysis are markedly insensitive to inaccuracy as regards shear load distribution in the zone limited by the coordinates $z = 0$ and $z = z_R = 0.33H$. The reason for this is also related to the non-linear behavior of soils at high stress levels and, of course when applicable, even more so to the effects of slip in the failure surface.

In other words, large deformations in the incipient failure zone or at the developed slip surface tend to eclipse the consequences of possible inaccuracies with regard to the shear load distribution with depth. For practical purposes, therefore, this relationship may be assumed to be linear.
4.3 Calculations - method of procedure

The aim of this exercise is to determine the additional forces (N, \(q\), and t), that will induce stresses and deformations of any significance at a chosen location (x=0) further down the slope, taking into account acting forces, displacements and relevant stress/deformation relationships. The additional forces may typically be located in a critical or steep part of the slope. As discussed in more detail in Section 3.3 and Chapter 8, this type of computation is made in order to detect the critical limit conditions with regard to the stability of a slope. One of these limit criteria is, for instance, defined by the set of values of the load increments (\(N_\sigma\), \(q_\omega\) or \(t_\omega\)) likely to trigger a progressive failure.

Integration of the differential Equations 4:1 to 4:6 can be made by the following step-by-step method, which may be used for manual as well as computer analysis.

**Step 1:**

Step 1a  Beginning at some point x = 0, which is selected to suit the aim of the analysis, the shear stress is increased by a value \(\Delta \tau\), so that \(\tau_1 = \tau_0 + \Delta \tau_1\). The value of \(\tau_0\) is defined by Equation 4.2. The abscissa of the studied section is then \(x_1 = 0 + \Delta x_1\).

The choice of the location for the point x = 0 may be regarded as the down-slope boundary condition, as it constitutes the one point where the sought parameters \(N_x, \delta_N, \delta_t\) and \(\tau - \tau_0\) are known, all being \(\approx 0\). The significance of where in the slope the point x = 0 is located is therefore that the subsequent computations will yield the additional force \(N_x\) that must not be exceeded at the location defined by (x), lest the point with zero displacement (\(\delta = 0\)) propagate beyond x = 0. The up-slope boundary condition is that \(N_x\) shall correspond to the additional force \(N_1\) at the upper limit of the presumptive slide.

(Comment: Again, it should be observed that the line defined by the in situ stress \(\tau_0(x)\) actually constitutes an asymptote to the curve \(\tau(x)\), the point \(x\), defined by the differential \((\tau(x) - \tau_0) = 0\) being theoretically located at an infinite distance from A. This difficulty is overcome by locating origo (i.e. \(x = 0\)) at a point where \((\tau(x) - \tau_0)\) has a defined, but negligible value.

Step 1b  Equation 4:1 gives the value of \(\Delta N_1 = N_1\) in terms of \(\Delta x_1\).

Step 1c  Equation 4:3 yields the corresponding value of the displacement \(\delta_{N1}\), while \(\delta_{11}\) is computed from Equation 4:4a.

Step 1d  The value of \(\Delta x_1\) is then obtained by the compatibility criterion (Equation 4:5), which is solved with respect to \(\Delta x_1\).

Step 1e  \(\Delta N_1\) may then be computed from Equation 4:1 and \(\delta_N\) from Equation 4:3.

Step 1f  The analyzed section is then advanced a distance of \(\Delta x_1\), i.e. \(x_2 = x_1 + \Delta x_2\).

**Step 2**  From this point and on, the calculation proceeds by repeating steps 1a) to 1f) for each vertical element and by advancing in steps of suitably chosen (or fixed) values of either \(\Delta x\) or \(\Delta \tau\). As the values of \(\delta_N\) and \(\delta_t\) may now be expressed in terms of the assumed values of the shear stress increment \(\Delta \tau\) and/or \(\Delta x\), the correlating values of \(\Delta x\) and \(\Delta \tau\) in each step cycle have to be found by iteration so that the compatibility equation 4:5 is satisfied, i.e. \(\delta_N = \Sigma \delta_N (\Delta \delta_N) = \delta_2\), or just simply \(\Delta \delta_N = \Delta \delta_t\).

The computation procedure is demonstrated in Section 4.4 as well as in the practical example given in Appendix I.
4.4 Exemplification of the numerical procedure for a calculation step involving one slope element of length $\Delta x$

The objective in this section is to demonstrate the numerical method of solving the Equations 4.1 to 4.6 by an iterative procedure. The calculations may appear prohibitively laborious, but it should be realized that using computers, the time required to perform the computational work is insignificant.

Assumed data in the current example:

- $\rho g = 15.5 \text{ kN/m}^3$
- $k_{\alpha,n} = k_{\alpha,n+1} = \sigma_y/\sigma_v = \text{constant}$
- $\Delta E_v/\Delta x \approx 0$
- $H = 18.0 \text{ m}$
- $b = 1 \text{ m}$
- $\beta_n = \beta_{n+1} = 2.866^\circ$
- $q(x) = 0$
- $t(x) = 0$

4.41 Constitutive relationships:

The general constitutive relationship $\tau_{xz} = \phi (\gamma_{xz}, \delta_F, d\delta_F/dt)$ in Equation. 4.6, may in the range $0 < \gamma < \gamma_f$ be defined by the inverse expression $\gamma_{xz} = \phi_1 (\tau_{xz})$ ....... 4.6a

Assumed data in the current example are:

- $c_{u,z=0} = 32 \text{ kN/m}^2$
- $c_r/ c_u = 0.40$
- $\tau_{el} = 20 \text{ kN/m}^2$
- $G_{el,0} = 1333 \text{ kN/m}^2$
- $c_{u,z=18} = 16 \text{ kN/m}^2$
- $\gamma_f = 3.3\%$
- $\gamma_{el} = 1.50\%$
- $E_{el,0} = 125c_u = 4000 \text{ kN/m}^2$
- $c_{u,\text{mean}} = 24 \text{ kN/m}^2$
- $E_{el,\text{mean}} = 125c_u,\text{mean} = 3000 \text{ kN/m}^2$
- $(E_{el} = G_{el}2(1+v))$

Elastic range.

In the range $0 < \gamma_{xz} < \gamma_{el}$ (for $0 < \tau_{xz} < \tau_{el}$), the relationship between shear stress and deviator strain is taken to be linear.

\[
\tau_x = G \cdot \gamma_x \quad \text{or} \quad \gamma_x = \tau_x/G
\]

.......................... I:1

\[
\Delta \gamma_{xz} = \Delta \tau_{xz} / G
\]

.......................... I:1a

where $G = \tau_{el}/\gamma_{el}$

($\tau_{el}$ and $\gamma_{el}$ denote shear stress and shear strain at the elastic limit as defined in Figure 4.4.2)

![Figure 4.4.1 Section of the slope being analyzed in the example.](image-url)
Figure 4:4.2 Constitutive shear stress/deformation relationships. It may be noted here that the ratio of $\tau_{el}/c_u$ is assumed to be constant when $c_u$ varies with the coordinate $(z)$. (Note: The parabolic relationship to the power of 2, which is used here only for practical reasons may be substituted for any other desired relationship, deemed as appropriate by the investigating engineer. However, the issue has little impact on the essential results of the analysis.)

Non-linear range $\gamma_{el} < \gamma_{xz} < \gamma_f$

In the non-linear range, where $\gamma_{el} < \gamma_{xz} < \gamma_f$ (i.e. for $\tau_{el} < \tau_{xz} < c_u$), the relationship between shear stress and deviatory strain is taken to be a 2nd power parabolic relationship with its vertex in the point $(\gamma_{el}, c_u)$ as in Figure 4:4.2.

As shown in Appendix I, equation I: 4 then applies:

$$\Delta \gamma_{xz} = (\gamma_f - \gamma_{el}) \left[1 - \left(\tau_{(n+1)z} - \tau_{el}\right)/(c_u - \tau_{el})\right]^{1/2} - \left[1 - \left(\tau_{(n+1)z} - \tau_{el}\right)/(c_u - \tau_{el})\right]^{1/2}$$

(Equ. I:4)

where $\tau_{(n)z}$ and $\tau_{(n+1)z}$ denote the shear stresses in elements $(n)$ and $(n+1)$.

In the transition range between linear and non-linear behavior, the combined expression in equation I:4 a is valid.

$$\Delta \gamma_{xz} = (\tau_{el} - \tau_{(n)z})/G + (\gamma_f - \gamma_{el})\left[1 - \left(\tau_{(n+1)z} - \tau_{el}\right)/(c_u - \tau_{el})\right]^{1/2}$$

(Equ. I:4 a)

4.42 Calculation procedure

In this particular example, $c_u(z)$ in the failure zone is taken to be constant with height. In a more general case $c_u(z)$ may be varied arbitrarily, observing only that the ratio of $\tau_{el}(z)/c_u(z)$ is kept constant.

However if necessary, each individual element may in principle be attributed its own specific properties.

Assume that in the course of the preceding computation step, the following results have been obtained at location $x = x_n$ and that $x$ is advanced by 4 m such that $x_{n+1} = x_n + 4$ m:

Results from step No (a):

$$x = x_n, \quad \tau_{o}(x_n) = 13.93 \text{ kN/m}^2, \quad N_x = 126.9 \text{ kN/m}$$

$$\tau_{o}(x_n) = 24.50 \text{ kN/m}^2, \quad \delta_N = \delta_\tau = 0.03827 \text{ m}$$

Using Equation 4:2

$$\tau_{o}(x_{n+1}) = [\sum H_{o}(x) \cdot g \cdot \rho (z) \cdot 1 \cdot \Delta z] \cdot \sin \beta(x) - \Delta E_o(x)/b(x) \cdot \Delta x =$$

$$= H \cdot g \cdot \rho \cdot \sin \beta(x) - \Delta E_o(x)/\Delta x = 18 \cdot 15.5 \cdot \sin (2.866^\circ) + 0 = 13.93 \text{ kN/m}^2$$
Step No (n+1) (i.e. step 2) Advance x by $\Delta x = 4$ m \[ x_{n+1} = x_n + 4 \text{ m} \]

Iteration No 1: Try $\Delta t = 3.0 \text{ kN/m}^2 \rightarrow \tau_x (x_{n+1},o) = 27.5 \text{ kN/m}^2$

Step 2b $\Delta N = (\tau(x_{n+2},o) + \tau(x_n,o))/2 - \tau_0(x_o) \cdot \Delta x = 48.28 \text{ kN/m}$ (Equ. 4:1)

\[ \frac{N_{n+1}}{N_n + \Delta N} = \frac{126.90 + 48.28}{2} = 75.18 \text{ kN/m} \]

Step 2c $\Delta \delta_N = (126.90 + 48.28)/2 - 4/3000/18 = 0.01119 \text{ m}$ (Equ. 4:3)

$\delta_N = \Sigma \Delta \delta_N = 0.03827 + 0.01119 = 0.04946 \text{ m}$

Proceed to calculate $\delta_t$ in Table 4:4.1 using Equation 4:6 (i.e. according to Equation I.1a, I.4 or I.4 a). $\tau_0$ and $\tau$ vary linearly with $z$.

Table 4:4.1 $x = x_{n+1}, \Delta t = 3.0 \text{ kN/m}^2$

<table>
<thead>
<tr>
<th>z (m)</th>
<th>$\tau_0(x_{n+1},z)$</th>
<th>$\tau(x_n,z)$</th>
<th>$\Delta t$</th>
<th>$\tau(x_{n+1},z)$</th>
<th>$\Delta \gamma_{x,z} \cdot 10^7$</th>
<th>$\Delta z$</th>
<th>$\Delta \gamma_{x,z} \cdot \Delta z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13.93</td>
<td>24.50</td>
<td>3.000</td>
<td>27.500</td>
<td>0.1107</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>12.38</td>
<td>21.78</td>
<td>2.667</td>
<td>24.447</td>
<td>0.0873</td>
<td>2.0</td>
<td>0.01980</td>
</tr>
<tr>
<td>2</td>
<td>21.78</td>
<td>19.06</td>
<td>2.333</td>
<td>21.391</td>
<td>0.0795</td>
<td>2.0</td>
<td>0.01668</td>
</tr>
<tr>
<td>3</td>
<td>19.06</td>
<td>16.33</td>
<td>2.000</td>
<td>18.335</td>
<td>0.0679</td>
<td>2.0</td>
<td>0.01474</td>
</tr>
</tbody>
</table>

Result from iteration No 1:

$\delta_t = 0.05122 > \delta_N = 0.04946 \text{ m}$

Hence, Equation 4:5 is not satisfied. Try another value of $\Delta t$.

Iteration No 2: Try $\Delta t = 2.0 \text{ kN/m}^2 \rightarrow \tau_x (x_{n+1},o) = 26.5 \text{ kN/m}^2$

$\Delta N = [(26.5 + 24.5)/2 - 13.93] = 46.28 \text{ kN/m}$ (Equ. 4:1)

$N = 126.9 + 46.28 = 173.18 \text{ kN/m}$

$\Delta \delta_N = (126.90 + 46.28)/2 - 4/3000/18 = 0.01111 \text{ m}$ (Equ. 4:3)

$\delta_N = \Sigma \Delta \delta_N = 0.03827 + 0.01111 = 0.04938 \text{ m}$

Proceed to calculate $\delta_t$ in Table 4:4.2 using Equation I.1a, I.4 and I.4 a.

Table 4:4.2 $x = x_{n+1}, \Delta t = 2.0 \text{ kN/m}^2$

<table>
<thead>
<tr>
<th>z (m)</th>
<th>$\tau_0(x_{n+1},z)$</th>
<th>$\tau(x_n,z)$</th>
<th>$\Delta t$</th>
<th>$\tau(x_{n+1},z)$</th>
<th>$\Delta \gamma_{x,z} \cdot 10^7$</th>
<th>$\Delta z$</th>
<th>$\Delta \gamma_{x,z} \cdot \Delta z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13.93</td>
<td>24.50</td>
<td>2.000</td>
<td>26.500</td>
<td>0.0991</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>12.38</td>
<td>21.78</td>
<td>1.778</td>
<td>23.558</td>
<td>0.0792</td>
<td>2.0</td>
<td>0.01782</td>
</tr>
<tr>
<td>2</td>
<td>21.78</td>
<td>19.06</td>
<td>1.556</td>
<td>20.616</td>
<td>0.0734</td>
<td>2.0</td>
<td>0.01526</td>
</tr>
<tr>
<td>3</td>
<td>19.06</td>
<td>16.33</td>
<td>1.333</td>
<td>17.663</td>
<td>0.0629</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\Delta \delta_t = \Sigma \Delta \gamma_{x,z} \cdot \Delta z = 0.04671 \text{ m}$

Result from iteration No 2:

$\delta_t = 0.04671 \text{ m} < \delta_N = 0.04938 \text{ m}$

(Equ. 4:5)
Equation 4:5 is not yet satisfied. Try another value of $\Delta \tau$ by proportioning between previous results.

**Iteration No 3:** Try $\Delta \tau = 2.610$ kN/m$^2 \rightarrow \tau(x_{n+1},0) = 27.11$ kN/m$^2$

\[
\Delta N = [(27.11 + 24.5/2) - 13.93] \times 4 = 47.50 \text{ kN/m} \quad \text{(Equ. 4.1)}
\]

\[
N = 126.90 + 47.50 = 174.40 \text{ kN/m}
\]

\[
\Delta \delta_N = (126.90 + 47.50/2) \times 4/3000/18 = 0.01116 \text{ m} \quad \text{(Equ. 4.3)}
\]

\[
\delta_N = \sum \Delta \delta_N = 0.03827 + 0.01116 = 0.04943 \text{ m}
\]

Proceed to calculate $\delta_t$ in Table 4:4.3 using Equation I:1a, I:4 and I:4 a:

**Table 4:4.3** $x = x_{n+1}$, $\Delta \tau = 2.610$ kN/m$^2$

<table>
<thead>
<tr>
<th>z (m)</th>
<th>$\tau_0(x_{n+1},z)$</th>
<th>$\tau(x_n,z)$</th>
<th>$\Delta \tau$</th>
<th>$\tau(x_{n+1},z)$</th>
<th>$\Delta y_{x,z'}$</th>
<th>$\Delta z$</th>
<th>$\Delta y_{x,z'} \times \Delta z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13.93</td>
<td>24.50</td>
<td>2.610</td>
<td>27.113</td>
<td>0.1060</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12.38</td>
<td>21.78</td>
<td>2.320</td>
<td>24.100</td>
<td>0.0841</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10.83</td>
<td>19.06</td>
<td>2.030</td>
<td>21.090</td>
<td>0.0771</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9.29</td>
<td>16.33</td>
<td>1.740</td>
<td>18.073</td>
<td>0.0659</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\Delta \delta_t = 0.04943$

Result from iteration No 3:

$\delta_t = 0.04943$ m $= \delta_N = 0.04943$ m

Hence, Equation 4:5 is satisfied.

Thus, the final results from step No $(n+1)$, i.e. from $x = x_n$ to $x = x_{n+1}$ are:

$x_{n+1} = x_n + 4$ m $\tau_0(x_n) = 24.50$ kN/m$^2$, $N_x = 174.4$ kN/m

$\tau(x_{n+1}) = 27.11$ kN/m$^2$, $\delta_N = \delta_t = 0.04943$ m

### 4.5 Objectives and overall procedures for performing stability investigations according to Section 4.

(For more detail see Chapter 8)

In Figure 4:2.4a, the principal parameters derived from the computations are shown. The down-slope force $N_x$ denotes, as already mentioned, the earth pressure increment, that may not be exceeded at the section defined by the coordinate $x$, lest the displacements induced by $N_x$ propagate beyond the starting point ($x \approx 0$) of the calculations.

$N_x$ will, therefore, assume different values depending on the extent to which, as defined by the point of reference $x = 0$, the resistance down-slope of $N_x$ is mobilized. Hence, any chosen portion of the slope may be analyzed for any selected sets of failure planes.

The different stages and limit conditions of a downward progressive landslide have been detailed in Chapter 3, according to which the analysis must focus on the two possible conditions of equilibrium before and after force redistribution due to progressive failure. Another important condition is the in situ 'creep state', which as suggested in Chapter 8, may also be studied as a slow progressive failure by the method given in Chapter 4.
Safety criteria with regard to locally initiated failure

The limit value of the force $N_i$, induced by the agent initiating local slope failure, is defined by $N_{cr}$, as shown in Figure 3:3.3. The value of $N_{cr}$ is computed by finding the position of the reference point ($x = 0$), which will result in a value of $\tau_x = c_{Rx} = \tau_{ox}$ i.e. when $(\tau - \tau_o) = 0$ at the location of $N_i$. This necessitates a procedure of combined ‘trial and error’ and interpolation. Also, as in conventional slope stability analysis, several different failure planes may have to be investigated.

Once the value of $N_{cr}$ at the location of the force $N_i$ has been established, the safety against local failure may be expressed as

$$F_s = \frac{N_{cr}}{N_i} \quad \text{(Equation 3:7)}$$

or, if the additional loads (q and t) shown in Figures 4:2.1 and 4:2.2 are also considered,

$$F_s = \frac{(N_oq_0t)_{cr}}{(N_oq_0t)} \quad \text{(Equation 3:7a)}$$

where $(N_o, q, t)_{cr}$ denotes a critical combination of the additional loads.

Criteria with regard to global slope failure

If local slope failure does occur for some reason, the ensuing progressive failure results in transmission of the unbalanced up-slope forces to more stable ground conditions further down

Figure 4: 5.1 Graphic display from a computer analysis of the ‘post-progressive’ stage of equilibrium based on a ratio of $c_\theta/c_\phi = 0.42$. In this case – actually the Tuve slide – passive Rankine resistance is exceeded already by the static forces in the virtually horizontal ground, entailing global slope failure. (Bernander et al, 1979, 1989).
the slope. A new, at least transient, state of equilibrium is then possible, as demonstrated in Chapter 3.

At this point in the calculations, the reference point (x=0) is defined by a boundary condition requiring that the computed value of \( N_z \) must be equal to \( F_s \cdot N_i \), precisely at its point of action (\( F_s \) being the required safety factor).

Again, this exercise constitutes an iterative process of the kind mentioned above.

The critical limit condition to be satisfied for the 'post-progressive failure' equilibrium, lest global failure take place, is then (according to Section 3.3):

\[
E_x^{\text{max}} = (E_0(x) + N(x))^{\text{max}} < \frac{E_p}{\text{Rankine}}
\]

i.e. the safety factor is \( F_s = \frac{E_p}{E_x^{\text{max}}} \)

or

\[
F_s = \frac{[(1-k_e)\cdot H^2/2+\sigma_cH\cdot c_v(\cdot)(\cdot)\cdot dz]/N_z^{\text{max}}}{1}
\]

(Equation 3.8 a)

**Computer programs**

As mentioned, the presented calculation method involves iterative procedures, which make manual calculations very onerous. However, this problem is readily overcome with computers, and the time needed to carry out the numerical solutions to Equations 4:1 to 4:6 is insignificant.

Computer programs following the outlines given above were devised at the Department of Design and Engineering at Skanska West AB (Gothenburg) as early as in 1980. A revised two-dimensional version was developed in 1984. An updated version of the 1984 computer

---

**Figure 4.5.2** Graphic display showing options and some of the main input data in the present version (2000) of computer software developed in 1984.
program in C++ is available from June, 2000.

Several studies of existing and failed slopes were carried out already in the 80-ies. In two cases of the existing slopes investigated, preventive measures were taken in order to ameliorate the safety factor with particular regard to progressive failure formation.

4.6 Conclusions from Chapter 4.

The proposed model for studying downward progressive failures in natural slopes has in practical applications conducted by the author proved to be a useful tool for evaluating in situ stresses, the additional effects of superimposed loading, as well as for assessing the limit conditions affecting potential slope failure.

In each scenario, the stress/strain response of the soil may be related to the timing of the current load increment, enabling a study of the stress history of a slope, irrespective of whether it has failed or not.

It is of particular interest that the analysis makes it possible to study the long term in situ distribution of earth pressures, yielding values of the inter slice forces and of $k_o$ ($=\sigma_v/\sigma_u$), on which engineers engaged in soil stability problems have long focused their interest. (Cf e.g. Janbu’s method of constant stress levels, 1979).

The critical length ($L_{cr}$) corresponding to $N_{cr}$ indicates in some measure the maximum length of a slide, that can be studied on the basis of the ideal-plastic soil properties with any prospects of attaining reasonable accuracy. The fact, that there may be a limit to the distance down-slope of a local load, along which additional shear stresses in the potential failure can be mobilized, has another crucial implication. At a length of $L_{cr}$ from the point of load application, the effect of the load is no longer felt in terms of slope, earth pressure and deformation. In the short term, this circumstance rules out or effectively limits the possibility of utilizing earth pressure resistance (e.g. in passive Rankine state) in less sloping ground for stabilizing the local up-slope load. See Figure 4:6.1

$$N_{cr} = \sigma \int_c^{\infty} (\tau - \tau_0) \, dx \ll E_p^{BC} - E_o = (1-k_o)yH^2/2 + 2\sqrt{(1+c_o/c_u) \cdot c_u \cdot \text{mean}}$$

Figure 4:6.1 Maximum passive earth pressure, which can be mobilized along the failure plane BD if deformations are considered. Note that $N_{cr} \ll E_p^{BC} - E_o$. (Bernander 1981)
The *important* implication of not being able to utilize available passive resistance further down the slope is that the failure resistance along planes, which are oriented in the direction of firm bottom or along sedimentary layers, is *considerably* less than that based on slip circles *surfacing in sloping ground* closer to the local load.

Thus, shorter failure planes and slip circles, i.e. failure modes for which I-PF analysis may still be valid as such, seldom constitute the critical failure modes in long slopes of deformation softening soil. This incongruity between PrF and I-PF analyses tends to become more pronounced in drained analyses, as extreme pore water pressures are more likely to spread *along* sedimentary layers than *across* (or at some angle to) the same. It should be observed that the above may even apply to high values of the brittleness ratio \((c_p/c_d)\), in fact it applies *initially even* in the ideal-plastic failure state, if deformations are considered in the analysis.

As the differences in resistance dealt with above may be dramatic in many cases, the conclusion must be that not even the *initial slides* in long slopes of brittle soil can be reliably predicted by computations based on ideal-plastic analysis using local slip circles or failure planes of less extent.

Hence, consideration of deformation softening in slope stability assessments generally results in considerably *higher computed risk of slope failure* than that emerging from the conventional ideal-plastic approach.

This applies even to wide variation (within reason) of chosen constitutional relationships. The *cardinal issue* here is whether the conditions in the slope are such that a local disturbance agent is *susceptible of inducing a critical degree of deformation softening* in the soil or not. Common disturbance agents are additional loading, forced deformations (e.g. due to piling) and extreme excess pore water pressure regimes.

These circumstances should be contemplated whenever soils exhibiting markedly deformation softening behavior are encountered.

*Deformations below the slip surface*

The proposed FDM-model for analysis of progressive downward slope failures enables consideration also of deformations below the assumed failure plane. However, as mentioned above, the fact that passive resistance further down-slope cannot be mobilized at a distance greater than \(L_{cr}\) for stabilizing local additional loads predicts that failure planes primarily tend to develop in the direction of the steep of firm bottom, even to great depth below the ground surface. E.g. in the about 500 m long main slide at Tuve, the failure zone followed the direction of firm bottom down to about 35 m below the valley floor.

In many cases, therefore, there is no particular need of considering the deformations below the slip surface, which is why the computer program, referred to in Section 4.5, addresses this issue only in an approximate way.

Hence, in the computer program, the deformations below the slip surface may be taken into account as follows:

1) With reference to Figure 4:2.2, establish the shear deformation in the failure zone below as well as above the assumed potential slip surface in a soil column of length \(\Delta x\). This exercise is then repeated for relevant locations along the potential slide.

2) The effect of the additional deviatoric deformation can then be accounted for by modifying the shear deformation in the failure zone by an amplification factor based on item 1) above. The value of this factor is then to be given in the in-put data of the computer
program. Denoting the deformations above and below the potential failure surface \( \delta_{cu} \) and \( \delta_{co} \) respectively, the factor to be inserted is \( \delta_{cu} + \delta_{co} / \delta_{co} \).

**Final remarks**

The progressive failure analysis described in this chapter may impress practicing geotechnical engineers as an almost prohibitive complication of slope stability analysis. The constitutive relationships of the sensitive soils have to be known reasonably well - depending as they do on many factors. The timing and state of principal stresses are among those of greatest importance.

But, complexity of analysis must be balanced against the imperative of making valid predictions of *risk in terms of human life, property, social and economic values.*

As may be concluded from the calculation examples in Section 4.4 and Appendix I, manual calculation is, although possible, too laborious to be practicable. However, the numerical computations can easily be performed using computers. Once appropriate in-put data are established, the time to carry out a complete study of a loading case is insignificant.

The additional effort devoted to slope stability investigations along these lines consists, therefore, only to a minor extent of increased computational work. The major challenge lies in exploiting the enhanced possibilities of identifying the effects on slope stability of a number of factors that cannot be obtained using the conventional ideal-plastic failure approach.
5. Case records

The following presentation of case records of landslides, mainly in southwestern Sweden, is not intended to be exhaustive or to account for all relevant data and circumstances. Instead, it will focus on features and characteristics that in opinion of the author prove or suggest that deformation-softening mechanisms have governed the initiation, development and final configuration of the slides. The case records also serve to illustrate the shortcomings of an analysis based uniquely on limit state plastic equilibrium and to substantiate the need for a different type of analytical

Figure 5:1.1 The landslide at Tuve, 1977. Topography of the valley before the slide and boundaries of the slide area.
approach when predicting slope stability in markedly strain softening soils, such as e.g. the one dealt with in Chapter 4.
In most of these cases, even for the famous Surte and Tuve slides, official explanations hitherto of the slides are in the opinion of the author incomplete or inconclusive.

5.1 The landslide at Tuve (1977), Sweden

The landslide at Tuve, near Gothenburg took place on the 30th of November, 1977, just after four o’clock in the afternoon at a time, which must have effectively reduced the death toll since people had not yet returned from work or from school. In all, the slide resulted in nine deaths, the total destruction of 65 family houses and a drastic change of topography of some 270 000 m² of ground. Settlements in the active zone of about 10 m, horizontal displacements of up to 200 m and upheaval in the passive zone of about 5 m over a distance of ≈ 300 m were recorded.

The total length of the slide was about 800 m. Two main phases could be identified, namely an initial slide event encompassing the ground east of line B-B in Figure 5.1.1 and a secondary retrogressive stage covering the area west of line B-B. The initial slide is presumed to having been triggered by a local instability in the steepest portions of the slope, i.e. near and up-slope of the Tuve Church road.

The length of the main slide measured some 500 m with a maximum width of the passive zone of ≈ 600 m. According to SGI Report No 18, the main slide “occurred suddenly and the events that followed took place in rapid succession”. The total duration was estimated at approximately 5 minutes.

\[ c_{u(e)} = 0.12 \cdot \sigma_e' \]  
(Empirical shear strength as assumed by SGI in Report No 18)

\[ E_{p1} << R = \int c_{ux} \cdot b_x \cdot dx - w \cdot \sin \beta + E_{p2} \]

Horizontal scale 1:400

E.g. for \( \beta = 0 \), \( b_1 = 400 \text{ m} \), \( b_2 = 600 \text{ m} \) and \( L = 270 \text{ m} \)

Vertical scale 1:200

For \( H = 20 \text{ m} \), \( E_{\text{Rankine}} \approx 1520 \text{ MN} \ll R_{\text{BDF}} \approx 4560 \text{ MN} \)

\( F_s = 4560/1520 \approx 3.0 \)

\( H = 35 \text{ m} \), \( E_{\text{Rankine}} \approx 4520 \text{ MN} \ll R_{\text{BDF}} \approx 10655 \text{ MN} \)

\( F_s = 10655/4520 \approx 2.4 \)

**Figure 5.1.2** Section through the main slide. (Distorted vertical scale). - Forces required to provoking plastic failure over the valley floor according to I-PF analysis. \( (c_{u(e)} = 0.12 \cdot \sigma_e') \)

A striking feature, which may be seen on the aerial photograph (Figure 5.1.3) is that about 80% of the area engaged in the main slide consists of a passive Rankine upheaval zone in almost horizontal ground. Referring to the discussion in Section 2.4 regarding the relationship between the features of a finished slide and insufficient plasticity of the soil, it must be
concluded that the slide events have been governed by the marked deformation-softening properties of the soil. Nonetheless, SGI Report No 18 explains the Tuve slide only in terms of ideal-plastic failure analysis (I-PFA). Assuming normal growth of shear resistance with depth, safety factors of 2.0 to 2.3 (undrained analyses) and 2.6 (drained analysis) were computed at first. (Figure 5:1.4).

![Map of Tuve slide]

**Figure 5: 1.3** Aerial photograph of the Tuve slide. From SGI Report No 18.

However, as these results did not explain why the slide occurred, further stability investigations were then focused on what was termed ‘empirical shear strength’ values, using the lowest alternative of drained and undrained shear strengths in active and direct shear zones. This empirical strength is here denoted as $c_{ud(e)}$.

Thus by setting the undrained shear strength from direct shear tests (CKC-U-DSS tests according to the terminology of Ladd & Foot, 1974) as low as $c_{ud(e)} = 0.12 \cdot \sigma'\gamma'$, safety factors of about 1.0 to 1.13 were obtained for slip surfaces ranging in length from 80 to 180 m, and located in the upper third of the area involved in the main slide (Figure 5:1.5).

This approach raises a number of questions, which have been examined in some detail in a critical study of Chapters 11 and 12 of SGI Report No 18. (Bernander, 1983). For one thing, it may be questionable whether direct undrained shear tests using stress ranges of $0 < \tau < c_{ud(e)}$ are representative of the in situ conditions at Tuve, where the stress range at failure would be more like $\tau_c < \tau < c_{ult}$, i.e. assuming the presumed slip surface to be the only mode of failure that is kinematically possible. (See Figure 5:1.6).
Hence, compatibility between the said loading conditions may not exist, as there are important differences between principal stresses, drainage conditions and stress paths in the

Figure 5:1.4 From SGI Report No 18 – results from stability calculations for a section through part of the 500 m long main slide assuming normal increase of shear strength with depth.

Figure 5:1.5 From SGI Report No 18 – results from stability calculations for a section through part of the 500 m long main slide (Section A-A in Figure 5:1.2) assuming 'empirical shear strengths' in deep lean clay layers.

two cases. For instance, the soil material in situ is, in reality, consolidated also for the prevailing shear stress $\tau_0$, which in addition may mean that the soil in the slope structure is consolidated for a somewhat different $K_o$ value than in tests with the stress range of

$0 < \tau < c_{ul(0)}$.

In tests carried out at Skanska’s geotechnical laboratory in Gothenburg, the difference in shear strengths between consolidated undrained CK$_u$-DSS tests and what may be termed as consolidated pre-sheared undrained (CK$_p$PU-DSS) tests was measured. For a clay with a liquid limit of 50% and $\tau_0 = 0.10 - \sigma_c'$, the following results were recorded: $c_{ul} = 0.18 - \sigma_c'$ and $c_{ull} = 0.26 - \sigma_c'$ i.e. $c_{ull}/c_{ul} = 1.44$ (See Figure 5:1.6)
Another indication that a value of \( c_{u(s)} = 0.12 \sigma_c \) may be too conservative an assumption is that it corresponds to a slope gradient of only about 3°. Considering that most natural slopes

I) Normal direct shear test

\[ \sigma_s + c_u \]

\[ \text{Drained} \]

\[ \text{Undrained} \]

\[ K_0 \text{- consolidated undrained} \]

\[ \text{direct shear tests CK}_0\text{-DSS (Ladd)} \]

II) Direct shear test corresponding to slope conditions

\[ \tau + \sigma_t + \Delta \tau \]

\[ \text{Drained} \]

\[ \text{Undrained} \]

\[ K_0 \text{- consolidated undrained} \]

\[ \text{direct shear test (CK}_0\text{P-DSS, Bernander)} \]

**Figure 5:1.6** Difference between results from normal CK\(_0\)U-DSS tests and pre-sheared CK\(_0\)PU-DSS tests. The latter tests would correspond more to the states of stress in the soil of the macro structure when loaded to failure.

contain seams of sands, silts and low plasticity clays, few of them would be able to stand steeper than 3° with a safety factor of \( \approx 1 \).

The above reasoning implies that the safety factors related to the slip surfaces, which were identified as being critical according to SGI Report No 18, may in the opinion of the author actually have been in the range of 1.5 instead of about 1. If that is the case, the role in the main slide of the slip surfaces shown in Figure 5:1.5, (involving roughly a third of the main slide), still remains unexplained on the basis of I-PF analyses.

Another crucial question that may be raised in this context is whether the strain rates in the laboratory tests performed are compatible with the in situ conditions or with the slide events.

*The Tuve slide explained in terms of progressive failure*

Regardless of the above discussion, there is no doubt that the steepest up-slope portions of the main slide (close to Tuve church road) had low factors of safety as presumed by SGI. This condition was also documented by Sällfors, (1979) and Bernander & Olofsson, (1981).
Spread of the slide over the valley floor

However, even accepting the low shear strength of $c_{w(d)} = 0.12\cdot\sigma_c$ attributed to the silty clays in SGI Report No 18, the report does not offer any explanation as to why most of the area involved in the actual slide consisted of virtually horizontal ground. Admittedly, Chapter 11 of the report contains some verbal reference to the possibility of progressive failure and dynamic effects, but in this respect there is no quantitative explanation of the global slide event.

It is, for instance, in the opinion of the author of this report not consistent, as in the Report No 18, to assume unlimited plastic properties in the soil and then simultaneously explain paradoxical phenomena in the slide by referring to dynamic effects, which are not quantified. Because, if the computed safety factors in the order of $F_s \approx 1$ (shown in Figure 5:1.5) are to represent any physical reality, it follows that the sensitive lean clays, as they must have been deformed in the in situ macro structure, possessed almost unlimited plasticity. This in turn means that, as apparent from the discussion in Section 2.4, the enormous build-up of forces and kinetic energy required to generate the 300 m long passive zone, have not been accounted for. The vast spread of the slide over horizontal ground must therefore be attributed to deformation softening and progressive failure mechanisms.

The numbers shown in Figure 5:1.2 indicate for instance that, using I-PF analysis, the main slide could not propagate beyond point C (the New Tuve Rd). This is due to the fact, that the passive Rankine resistance ($E_{pl}$) is very much smaller than the resistance $E_{BDF}$ along a failure plane defined by BDF.

Notably, this would apply even if the low shear strength of $c_{w(d)} = 0.12\cdot\sigma_c$ were used. Hence, by ‘safety factors’ of about 2.4 to 3.0, Figure 5:1.2 suggests that (applying conventional analysis) the event, which actually took place, could not really happen.

In conclusion, the Tuve slide raises serious doubts with regard to the application of ideal-plastic limit equilibrium methods (I-PFA) when predicting failure in long slopes of sensitive soils.

Instead, in the opinion of the author, the inevitable conclusion is that significant deformation softening of the soil must have governed the entire main slide, which was triggered by a local instability in the steepest portions of the slope, i.e. near and up-slope of the Tuve Church road. (Bernander & Olofsson, SGI Report No 10, 1981.)

Progressive failure analysis of the Tuve slide

This course of events has been substantiated by progressive failure analyses performed according to the principles outlined in Chapter 4, offering good predictions of the extent and final appearance of the Tuve slide. Referring to figure 4:4.2 or to I: 1.2, the following in-put data were used in the analysis, the results of which are shown in Tables 5:1.1 to 5:1.3 and Figure 5:1. 7.

In the in situ state condition:

\[ \begin{align*} 
c_{w} / c_{u} & = 1.00 \\ 
\gamma_d & = 2.5 \% \\ 
\gamma_f & = 7.5 \% \\ 
G_0 & = \tau_d / \gamma_d \\ 
E_o & = 2(1+\nu) \\ 
G_0 & = 3G_0 \approx 1440 \text{ kN/m}^2 \\ 
\tau_d & = 12 \text{ kN/m}^2 \\ 
k_o & = 0.55 \\ 
E_{\text{el,mean}} & = 60 \text{ kN/m}^2 \\ 
c_{u,\text{mean}} & = 60 \text{ kN/m}^2 \\ 
\end{align*} \]

Note: In all calculations in Chapter 5, the curved portion of the constitutive relationship from $\gamma_d$ to $\gamma_f$ is a function of $x^k$ with vertex at $(c_u, \gamma_f)$ and connecting tangentially at ($\tau_d, \gamma_d$).
In the disturbance condition:
\[
c_R/c_u = 0.80 \quad \gamma_d = 2 \% \quad \gamma_f = 6 \% \quad \delta_{cr} = 0.3 \text{ m} \quad G_o^* = \tau_d/\gamma_d = 810 \text{ kN/m}^2
\]
\[
c_u = 27^* \rightarrow 33^** \text{ kN/m}^2 \quad \tau_d = 16.2 \text{ kN/m}^2 \quad E_{el,lo} = 3G_o = 90c_u \approx 2430 \text{ kN/m}^2
\]
\[
k_o^{***} \quad E_{el,mean} = 90c_u^\text{mean}
\]

In the global failure condition:
\[
c_R/c_u = 0.40 \quad \gamma_d = 1 \% \quad \gamma_f = 3 \% \quad \delta_{cr} = 0.3 \text{ m} \quad G_o^{**} = \tau_d/\gamma_d = 1800 \text{ kN/m}^2
\]
\[
c_u = 30^* \rightarrow 36^** \text{ kN/m}^2 \quad \tau_d = 18 \text{ kN/m}^2 \quad E_{el,lo} = 3G_o \approx 5400 \text{ kN/m}^2
\]
\[
k_o^{***} \quad E_{el,mean} = 150c_u^\text{mean}
\]

* Mean values applying to the initiation zone.  ** Mean values applying to the down-slope failure zone.  *** As computed in the in-situ condition

It may be observed that shear strengths and E-moduli are varied along the slope as may be applicable.

In the disturbance condition, the shear deformations in the failure zone have been taken to be largely one-sided in relation to the incipient slip surface, which in the higher parts of the slope follows the direction of the steep of firm bottom. However, at global failure, they have been assumed to be symmetrical with respect to the potential failure plane in the lower parts of the slope.

Results from the Pr F analysis

Table 5.1.1 The Tuve slide - results from Pr F analysis

(L = distance from upper end of slide or point of application of critical load)

In situ state condition:
\[
c_R/c_u = 1.0 \quad N_{max} = 557 \text{ kN/m} \quad L = 250 \text{ m} \quad E_o = 4135 \text{ kN/m} \quad k_o = 0.64
\]
\[
(\delta_{\text{creep}} = 4.52 \text{ m})
\]

Disturbance condition a) - Force initiated failure
\[
c_R/c_u = 0.60 \quad N_{cr} = 77.4 \text{ kN/m} \quad L_{cr} = 96 \text{ m} \quad E_x = 2376 \text{ kN/m} \quad \text{at} \quad x = L_{cr}
\]
\[
\delta_{cr} = 0.058 \text{ m}
\]

Disturbance condition b) – Deformation initiated failure
\[
c_R/c_u = 0.60 \quad N_{cr} = 0 \text{ kN/m} \quad L_{\text{instab}} = 124 \text{ m} \quad E_x = 2774 \text{ kN/m} \quad \text{at} \quad x = L_{\text{instab}}
\]
\[
\delta_{\text{instab}} = 0.090 \text{ m} \quad \text{*(As defined in Figure 4.2.4a)}
\]

The in situ state condition

In the steepest part of the slope, available shear strengths do not match the in situ shear stress in terms of \(\tau_s = \rho - g \cdot H \cdot \sin \beta\). According to Equation 4.2 (Chapter 4) this implies that in the in situ condition the soil masses were shored up by incremental earth pressures in less inclined ground further down the slope i.e. \(k_o\) increasing from 0.55 to (maximum) 0.64. See Table 5.1.1 and Figure 5.1.7.

Disturbance condition - Force initiated failure

One of the main implications of applying the analysis according to Chapter 4 on a slope like that of Tuve is the fact that the length over which shear stresses and deformations can be induced by local load effects (\(N_i\)) is limited. (Cf. Section 4.6.)
In consequence, as the deformations related to \( N_t = N_{cr} \) in this case do not materialize beyond the distance of \( L_{cr} = 96 \, \text{m} \) down-slope of the load, passive resistance there cannot be utilized for stabilization of an additional local load \( N_{cr} \) considering that progressive failure is then already imminent. The implication of this is that progressive failure analysis denies the possibility of exploiting the contribution to slope stability of the potential down-slope passive resistance related to the rising part of long slip surfaces such as, for instance, those shown in figures 5:1.4 and 5:1.5. This applies in particular to long slopes such as the one at Tuve, where the soils were considered to be very sensitive.

Thus, if the passive component related to the slip surfaces in figure 5:1.5 cannot be used for stabilizing the forces initiating the main slide in the steep part at the Old Tuve Church Road, the computed safety factors based on I-PF analysis shown in the figures have no physical meaning.

It may be noted in this context that the evaluation of the load effect capable of initiating failure is also affected by the reasoning above. If, for example, the weight of fill which would (hypothetically) be required to provoke slope failure is computed using conventional short slip surfaces on one hand and using the methods in Chapter 4 on the other, the outcome is radically different. E.g. according to Table 5:1.1, the critical load \( (N_{cr}) \) sufficient to initiate local failure in the steepest part of the slope, amounts to \( 77 \, \text{kN/m} \).

Now, although the Tuve slide is not believed to have been brought about solely by the weight of an applied fill, it may still be of interest to observe that the value of \( N_{cr} \) only corresponds to a distributed load on the ground surface of about \( q_{cr} \approx 4.6 \, \text{kN/m}^2 \), i.e. assuming undrained conditions.

By contrast, ideal-plastic failure analysis based on local slip circles, indicates a corresponding minimum value of \( q_{cr} \approx 97 \, \text{kN/m}^2 \), i.e. a disparity that can be expressed by a factor of more than 20. (Cf. Figure 5:2.3 related to the Surte slide.)

Hence, according to I-PF analysis \( q_{failure(min)} = 97.0 \, \text{kN/m}^2 \) (Circular slip surface)

According to Pr F analysis \( q_{failure} = 77.4/17 = 4.6 \, \text{kN/m}^2 \) (Slip surface in the direction of the steep of firm bottom)

**Sensitivity study**

The effect of varying some of the in-put parameters on \( N_{cr} \), \( L_{cr} \), \( \delta_{cr} \), \( q_{failure} \) (etc) is demonstrated in Table 5:1.2 below.

**Table 5:1.2 The Tuve slide – variation of parameters**

<table>
<thead>
<tr>
<th>( c_R/c_u )</th>
<th>( E_{el,mean} )</th>
<th>( \gamma_t )</th>
<th>( \gamma_f )</th>
<th>( N_{cr} )</th>
<th>( L_{cr} )</th>
<th>( \delta_{cr} )</th>
<th>( L_{instab} )</th>
<th>( \delta_{instab} )</th>
<th>( q_{failure} )</th>
<th>( q_{failure} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kN/m²</td>
<td>%</td>
<td>%</td>
<td>kN/m</td>
<td>m</td>
<td>m</td>
<td>m</td>
<td>m</td>
<td>kN/m² (Pr FA)</td>
<td>kN/m² (I-PFA)</td>
</tr>
<tr>
<td>0.70</td>
<td>90 ( c_u,\text{mean} )</td>
<td>2</td>
<td>6</td>
<td>80.2</td>
<td>96</td>
<td>0.057</td>
<td>135</td>
<td>0.101</td>
<td>5.3</td>
<td>97</td>
</tr>
<tr>
<td>0.60</td>
<td>90 ( c_u,\text{mean} )</td>
<td>2</td>
<td>6</td>
<td>77.4</td>
<td>96</td>
<td>0.058</td>
<td>124</td>
<td>0.090</td>
<td>4.6</td>
<td>97</td>
</tr>
<tr>
<td>0.50</td>
<td>90 ( c_u,\text{mean} )</td>
<td>2</td>
<td>6</td>
<td>73.0</td>
<td>90</td>
<td>0.048</td>
<td>116</td>
<td>0.079</td>
<td>4.3</td>
<td>97</td>
</tr>
<tr>
<td>0.40</td>
<td>90 ( c_u,\text{mean} )</td>
<td>2</td>
<td>6</td>
<td>71.9</td>
<td>89</td>
<td>0.045</td>
<td>112</td>
<td>0.072</td>
<td>4.2</td>
<td>97</td>
</tr>
<tr>
<td>0.30</td>
<td>90 ( c_u,\text{mean} )</td>
<td>2</td>
<td>6</td>
<td>69.1</td>
<td>88</td>
<td>0.043</td>
<td>109</td>
<td>0.068</td>
<td>4.1</td>
<td>97</td>
</tr>
<tr>
<td>0.30</td>
<td><strong>180 ( c_u,\text{mean} )</strong></td>
<td><strong>1</strong></td>
<td><strong>3</strong></td>
<td><strong>76.2</strong></td>
<td><strong>93</strong></td>
<td><strong>0.026</strong></td>
<td><strong>122</strong></td>
<td><strong>0.045</strong></td>
<td><strong>4.4</strong></td>
<td><strong>97</strong></td>
</tr>
<tr>
<td>0.30</td>
<td><strong>200 ( c_u,\text{mean} )</strong></td>
<td><strong>2</strong></td>
<td><strong>6</strong></td>
<td><strong>104.8</strong></td>
<td><strong>137</strong></td>
<td><strong>0.043</strong></td>
<td><strong>164</strong></td>
<td><strong>0.068</strong></td>
<td><strong>6.2</strong></td>
<td><strong>97</strong></td>
</tr>
</tbody>
</table>
As may be seen from Table 5:1.2, changing the input parameters defining the progressive failure analysis obviously affect the results of the analysis. Even so, it is interesting to note that the values of the critical force $N_{cr}$, the critical length $L_{cr}$ or the critical load $q_{failure}$ are remarkably insensitive to variation of the values of $c_R/c_u$ – ratio, $\gamma_d$ and $\gamma_f$. In the current case, for instance, the magnitude of $N_{cr}$ deviates from the mean value by only $\pm 8\%$ within a range of $c_R/c_u$ between 0.3 and 0.7.

This is a circumstance, which effectively contributes to the viability of progressive failure analysis when evaluating the risk of local instability in slopes. In particular, it may be noted that the disparity between FPF and I-PF analyses as highlighted above is valid for wide amplitudes of the pertinent parameters. The crucial factor in this context relates to the issue as to whether deformations are taken into account or not.

**Conclusion:** Despite the fact that the slope at Tuve had been stable through millennia, it was nevertheless, according to Pr F analysis, extremely sensitive to additional short-term loads or disturbance agents inducing undrained soil behavior and acting in critical sections of the slope.

**Global failure condition:**

Figure 5:1.7 displays the calculated earth pressure distribution, shear stresses and displacements for a slip surface in accordance with boring logs in SGI Report No 18. The figure represents the situation at the end of the progressive redistribution phase, in which unbalanced shear forces in the steeper parts of the slope have been transmitted further downslope, resulting in a tremendous build-up of earth pressures down in the valley. (Cf Figure 3:3.4 and Figures 5:1.3 and 5:1.7 in this chapter).

**Table 5:1.3 The Tuve slide - results from Pr F analysis**

<table>
<thead>
<tr>
<th>Global failure condition:</th>
<th>Case I</th>
<th>Global failure condition:</th>
<th>Case II</th>
<th>Global failure condition:</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_R/c_u$ = 0.10-0.30</td>
<td>$N_{max}$ = 7582 kN/m</td>
<td>$E_{el}/G = 3$</td>
<td>$E_{el}/G = 3$</td>
<td>$E_{el}/G = 3$</td>
<td>$E_{el}/G = 3$</td>
</tr>
<tr>
<td></td>
<td>$E_{max}$ = 13472 kN/m</td>
<td>$E_{Rankine} = 12852$ kN/m (varies)</td>
<td>$E_{max}$ = 12705 kN/m</td>
<td>$E_{Rankine} = 12852$ kN/m (varies)</td>
<td>$E_{max}$ = 12060 kN/m</td>
</tr>
<tr>
<td>$(E_{Rankine}/E)_{min}$ = 0.84</td>
<td>$E_{el} = 150 c_u, \text{mean}$</td>
<td>$L = 456$ m</td>
<td>$L = 456$ m</td>
<td>$L = 456$ m</td>
<td>$L = 456$ m</td>
</tr>
<tr>
<td></td>
<td>$L_{E=E(R)\text{Rankine}}$ = 450 m #</td>
<td>$L_{E=E(R)\text{Rankine}}$ = 300 m #</td>
<td>$L_{E=E(R)\text{Rankine}}$ = 0 m #</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

# Length over which passive Rankine pressure is exceeded.

It should be observed that the earth pressures are calculated under the assumption that the potentially sliding soil volume retains its geometrical shape before its possible disintegration in passive Rankine failure. Hence, in cases where the resulting maximum earth pressure $E_{max}$ exceeds $E_{Rankine(max)}$, the computed earth pressure scenario represents a highly transient situation, which immediately merges into the dynamics of the slide proper as described in Chapter 3.

The significance of the earth pressure distribution in this transient stage of equilibrium is that...
it constitutes a measure of the catastrophe that may ensue if the critical load in the
disturbance condition is exceeded, i.e. will the progressive failure result in a veritable
landslide or not?

Figure 5:1.7 Static earth pressure distribution in the Tuve slide subsequent to the progressive
failure phase but prior to the slide proper resulting in disintegration and heave in passive
failure. The figure indicates that the spread of the passive zone over almost horizontal ground
need only to be ascribed to the static forces developed in the dynamic progressive phase I of
the ground movement as explained in Section 3.3. Case I, c_R /c_u = 0.10-0.30.
Curve A, E_{(x)} = In situ earth pressure prior to local failure, kN/m
Curve B, N(x) = Earth pressure increment due to Pr F redistribution, kN/m
Curve C, E(x) = E_{(x)} + N(x) = Earth pressure after Pr F redistribution, kN/m
Curve D, E_{Rankine} = Passive Rankine resistance, kN/m
Curve E, \tau_{0(x)} = In situ shear stress distribution before progressive failure, kN/m^2
Curve F, \tau (x) = Shear stress distribution after progressive failure, kN/m^2
The calculations for Case I in table 5.1.3 are based on residual shear strengths in some proportion to the magnitude of displacement in the progressive failure phase, thus varying from $c_R = 0.10\ c_0$ to $c_R = 0.40\ c_0$ in different locations in the slope. Considering the large displacements ($\approx 25\ m$) and the velocities involved already in the progressive phase, these values of $c_R$ may be considered as high for the current type of normally consolidated sensitive clays.

As can be seen in Figure 5.1.7, the earth pressures resulting from the progressive failure redistribution of forces, entail that passive resistance is exceeded over about 450 m of gently sloping ground. This condition would inevitably lead to total disintegration and heave in the lower areas of the slope and the valley, thus initiating the dynamic phase of the slide proper.

**Sensitivity studies**

The effects of changing the $c_R /c_0$ – ratios from 0.10-0.40 to 0.20-0.40 and to 0.30-0.40 is evident from the figures given in Table 5.1.3 above. Whereas the maximum earth pressure exceeds passive Rankine resistance over a distance of 450 m in Case I, no static exceeding of passive resistance occurs in case III, signifying that global failure with excessive heave of the passive zone is not likely to take place in the latter case.

The effect on the global failure condition of doubling the $E_{el,\ mean}/G_0$ – ratio (i.e. reducing the compressibility of the soil mass) is insignificant. Thus for $c_R /c_0=0.10-0.40$ and $E_{el,\ mean}/G_0=6$ instead of 3, the following values result:

- $E_{max}$ becomes 13474 kN/m at $L = 456\ m$ instead of 13678 kN/m and
- $L-E_{E=\ E(Rankine)} \approx 430\ m$ instead of 450 m.

Moreover, the effect of putting the constitutive parameters $\gamma_t = 1\%$ instead of 2% and $\gamma_f = 3\%$ instead of 6% is virtually zero. This follows from the assumption that the shear modulus and the modulus of elasticity are interrelated as well as being time dependent in a similar way.

**Conclusions with regard to the Tuve slide**

The progressive failure analysis performed indicates that the slope was *extremely vulnerable* to additional short-term loading and unprecedented disturbance related to human activities of various kinds. The analysis also provides a *logical* and *quantitatively consistent explanation* of the vast spread of the slide over horizontal ground. The analysis highlights the fact that displacements in a slide are not confined to its directly visible topographical manifestations. The earth movements beyond the apparent down-slope boundary can be considerable over several hundreds of meters. (Cf. also Rävëkkärr, Section 5.5)

As mentioned, the causes of the Tuve slide are in the SGI Report No 18 attributed to disturbance generated by high ground water pressures in combination with the weight of a road embankment applied many years before. Manmade changes in the hydrological regime were believed to have contributed to local instability.
5:11 Dynamic effects in a progressive landslide like that at Tuve

With the intent of studying the dynamic effects in a downward progressive slide, qualitatively and quantitatively, a study was carried out at Skanska Engineering Department (Gothenburg). (Bernander & Gustás, 1984). Although the slope studied was fictitious, the geometry, the mean gradient and soil parameters were chosen so that, within reason, they could be considered to be valid for the conditions assumed in the mentioned analysis of the Tuve slide performed by Bernander & Olofsson, 1981.

The analysis consisted of dynamic step by step numerical calculations in the time domain applying Newton’s laws of motion. The time interval between the discrete steps in the computations was about 1 second. The results of the investigation proved to be very instructive, and the different dramatic phases of the slide event were illustrated on a display board. (See Figures 5:1.8 to 5:1.16 below). Although not previously published, these figures have been presented as photographic slides in the course of discussions at the Nordic Geotechnical Conference (NGM 84), at the Symposium on Landslides (Toronto, 1984), and at a poster session of XI th ICSMFE (San Fransisco, 1985). The figures depict various stages of a progressive slope failure resulting in a fully developed landslide event. In the figures escalating earth pressure development is simulated with different shades of yellow → green → dark green. Black color indicates passive Rankine resistance being exceeded.

The dynamic effects on the active zone as well as on the final spread of the slide can be estimated by comparing the Figures 5:1.12 and 5.1.16. According to the calculations, the time to failure is about 22 seconds. It is conceivable that in reality time dependent fracture and disintegration processes prolong the slide events thus substantially reducing the dynamic forces of inertia and kinetic energy, which are proportional to the square of the rate of displacement.

\[
E = 333 \cdot C_u = 10000 \text{ kN/m}^2
\]
\[
\rho \cdot g = 17 \text{ kN/m}^2
\]
\[
C_u = 30 \text{ kN/m}^2
\]
\[
C' / C_u = 0.42
\]

\[H = 20 \text{ m} \]
\[L = 300 \text{ m} \]
\[\beta = 4.2^\circ \]
\[\beta = 0^\circ \]

Figure 5:1.8 Main data of the slope studied
Figure 5:1.9  Progressive failure released by the additional force $N_i = N_e$, dynamic phase I.

Time $t = 1.3$ sec. Deepening colors or shades indicate growth of down slope earth pressures.

- Acceleration $= 0.02 \text{ m/sec}^2$
- Max. velocity $\approx 0 \text{ m/sec}$
- Kinetic energy $\approx 0 \text{ kNm}$
- Total length, $L = 100 \text{ m}$

Figure 5:1.10  Progressive failure propagation down-slope, dynamic phase I.

Time $t = 3.2 \text{ m/sec}$

- Deepening colors or shades indicate growth of down slope earth pressures.
- Acceleration $= 0.20 \text{ m/sec}^2$
- Max. velocity $= 1.3 \text{ m/sec}$
- Kinetic energy $= 780 \text{ kNm}$
- Total length, $L = 250 \text{ m}$
**Figure 5:1.11** Progressive failure propagation down-slope, dynamic phase I continued.

**Time t = 4.3 sec**

Deepening colors or shades indicate growth of down slope earth pressures.

Acceleration = 0.18 m/sec$^2$  Max. velocity = 1.7 m/sec

Kinetic energy = 10 300 kNm  Total length, L = 425 m

---

**Figure 5:1.12** Progressive failure propagation down-slope, dynamic phase I.

**Time t = 4.7 m/sec**

Deepening colors or shades indicate growth of down slope earth pressures.

Acceleration = 0.12 m/sec$^2$  Max. velocity = 2.1 m/sec

Kinetic energy = 17500 kNm  Total length, L = 475 m
Figure 5.1.13 Progressive failure propagation accomplished and dynamic phase II begins. **Time** t = 5.7 sec. Deepening colors or shades indicate growth of down slope earth pressures. If, however, \( E_p^{max} \) had been < \( E_p^{Rankine} \), then the ground movement would have terminated at this point. (See slide at Rävekkär Section 5.5)

- Acceleration = 0 m/sec\(^2\)  
- Max. velocity = 2.5 m/sec  
- Kinetic energy = 24 600 kNm  
- Total length, L = 540 m

---

Figure 5.1.14 Incipient heave in the passive zone, dynamic phase II, **Time** t = 10 sec. Deepening colors or shades indicate growth of down slope earth pressures.

- Acceleration = 0.08 m/sec\(^2\)  
- Max. velocity = 3.7 m/sec  
- Kinetic energy = 33 000 kNm  
- Total length, L = 540 m
Figure 5:1.15 Heave in the passive zone due to static build-up of earth pressure forces almost completed. Observe development of the active zone. 

\[ \text{Time } t = 14.2 \text{ sec} \]
\[ \text{Acceleration (retardation)} = -0.08 \text{ m/sec}^2 \]
\[ \text{Max. velocity} = 4.9 \text{ m/sec} \]
\[ \text{Kinetic energy} = 42500 \text{ kNm} \]
\[ \text{Total length, } L = 540 \text{ m} \]

Figure 5:1.16 Retardation phase. Heave in the passive zone due to dynamic (inertia) forces. Deepening colors or shades indicate growth of down slope earth pressures.

\[ \text{Time } t = 18 \text{ sec} \]
\[ \text{Retardation} = 0.15 \text{ m/sec}^2 \]
\[ \text{Max. velocity} = 2.5 \text{ m/sec} \]
\[ \text{Kinetic energy} = 25000 \text{ kNm} \]
\[ \text{Total length, } L = 630 \text{ m} \]
**Figure 5:1.17** The slide is completed and has reached its final spread. Time $t = 22$ sec

Deepening colors or shades indicate magnitude of down slope earth pressures.

- Acceleration $= 0$ m/sec$^2$
- Max. velocity $= 0$ m/sec
- Kinetic energy $= 0$ kNm
- Total length, $L = 720$ m
5.2 The landslide at Surte (1950), Sweden

The second largest landslide in Sweden in recent times took place at Surte, 10 km North of Gothenburg soon after 8 a.m. on 29 September 1950. In the time lapse of about 3 minutes, some 240 000 m$^3$ of ground encompassing about 4 million m$^3$ of soil moved westwards blocking the Göta River, sweeping away 31 dwelling houses and 10 outhouses. A railway and a highway were displaced distances varying from 50 to 150 m. The length of the slide in an east-westerly direction (i.e. perpendicularly to the river), was about 600 m. The almost uniform width was about 400 m. The height of the riverbank was insignificant, and within a distance of ca 300 m from the river, the slope of the ground was only 1:100 to 1:60. Further east, i.e. in the area below what was to become the scarp of the initial slide, the gradient amounted to 1:8 to 1:5. (See Figure 5:2.2)

![Figure 5:2.1 Aerial view of the large landslide at Surte in the valley of the Göta River about 10 km north of the city of Gothenburg.](image)

The slide was observed by a number of people inside and outside the slide limits, but as is often the case in dramatic events of this kind, most eyewitnesses only registered incidents, which were local in time and space. However, one witness positioned outside the slide area gave an exceptionally coherent, continuous and time-wise extended description of the main events of the slide. This must be considered to be of great value to anyone who seeks to reproduce the main events in order to understand the causes and the mechanisms of the slide. Mr Ture Berntsson sums up his impressions as follows:

"The whole ground was moving rather slowly at a speed that can approximately be compared to that of the Bohus ferry. The movement did not proceed at the same speed all the time - the speed increased progressively and the movement finally ceased when the ground piled up against the opposite side of
the river. Then the ground rose and folded. However, folding had already begun during the first stage of the movement. …" 

Another important witness, Mrs Hjórdís Svensson, standing in her kitchen and facing the south, said among other the following:

"She first noticed that a pile driving machine and the ground around it started to subside and the men engaged in pile driving started to run away. Then she observed that the houses beyond were also moving. …….. The movement was wavelike and smooth. The houses seemed to sail along. "

Figure 5:2.2 Plan and section A-A of the slide area showing elevation contours and a longitudinal section of the slide. (From Jakobson, 1952). The point marked (P) on the plan is the location where piling operations were going on at the time the slide occurred. (This point was not indicated in the source document. Section B-B marks the section analyzed in Figure 5:2.4 and was also not shown on the original).

The landslide at Surte was treated in two comprehensive reports by Jakobson et al, 1952, and by Caldenius & Lundström, 1956. The thorough field and laboratory investigations made in connection with these reports constitute valuable contributions to the knowledge of the behavior of the clays involved in the slide. However, in respect of the explanations of the causes and the mechanisms that formed the slide, both of these reports must in the opinion of the author be regarded as inconclusive, and at least from an official standpoint, the Surte slide may still be deemed as being unexplained.
The reasons for this are as follows:

a) The two reports are contradictory on essential issues, e.g. the pore water pressure situation, causes of the initial slide, sequence of slide events and slide mechanisms.

b) In both reports, computational analysis of the main or initial slide is based on ideal plastic limit equilibrium, i.e. differential deformations within the potentially sliding bodies are not considered. However, the validity of the concept of ideal-plasticity in these soft and sensitive clays over several hundreds of meters long failure surfaces is unlikely, especially when applied as by Jakobson to the main slide of length \( \approx 400 \) m. Lundström, albeit somewhat crudely, considers effects of strain softening in attempting to explain the spread of what he terms the 'progressive passive slide' over horizontal ground. The term 'progressive' is, however, by him used in a different sense than that adopted in the present report.

c) Other circumstances present, as discussed below.

Jakobson presumes that the soil volume in the main slide, excluding the retrogressive after slides, moved as a block towards the river. He further finds that the crucial reason for the slide was the presence of elevated artesian pore water pressure heads in the order of 7 m in the failure zone, resulting from high precipitation in the years 1949 and 1950. Such high pressures were recorded after the slide. The analytical model is plausible as such, but the problem with this explanation is that Jakobson assumes without substantiation, that these high artesian pressures existed prior to the slide event. This assumption was not ever documented and, incidentally, contested by Lundström.

Jakobson seems to have overlooked the fact that when soft and sensitive clays, which are basically collapsible, are sheared, excess pore water pressures are generated by the disturbance and maintained over long periods of time on account of the weight of the overburden.

Accordingly, pore pressure measurements in the ground just outside the slide limits exhibited only about a half of the artesian heads mentioned above, and even these values were probably induced by the slide, considering the proximity of the location of the pore pressure gauges to the slide limit.

Although Jakobson appears to have been aware of the fact that during the thousands of years the steep part of the slope had existed, more extreme ground water conditions must have prevailed time and time again. However, he does not present any argument as to why the slide happened on the 29 September, 1950.

Moreover, in his discussion of the immediate causes of the slide, Jakobson makes no reference to the fact that piling operations were ongoing in a steep section of the sloping ground. This is noteworthy because the pile driving activity was the only known disturbance that conceivably had never taken place before in the slope. All other houses involved were actually founded on slabs without the use of piles.

However, it may be noted here that Jakobson, in response to critical comments on his report by Löfquist (1952), as well as in the animated debate that followed (1953), argued that the immense spread of the slide may have been due to progressive failure formation. He then in the opinion of the author rightly attributed this failure process to gradual loss of shear strength as the slide propagated - however, without any supporting analytical documentation.
In his review of Jakobson's report, Löfquist contends that the remarkable spread of the Surte slide must have been due to a near total loss of frictional resistance in a presupposed thin stratum of fine sand. In this layer, considerably higher pore water pressures than even those assumed by Jakobson were presumed to prevail. Although also Löfquist's model for slope failure is viable in theory, his approach must be regarded as rather speculative, as neither artesian pressures of this magnitude nor any continuous seam of fine sand was ever documented. Löfquist does not either present any argument explaining why the artesian pressures in the sand layer should have been higher than ever before at the time of the slide event.

In contrast to Jakobson, Lundström asserts that the slide developed as a rather complicated series of smaller local slides with circular slip surfaces. Hence, he argues that an initial slide first took place in the steeper part of the gradient. From deliberations with regard to the kinetic energy of the local slide, he maintains that this first slide to some extent affected the practically horizontal ground, in displacing it towards the Göta River. Nevertheless, he concludes that this impact was not sufficient to provoke a continuance of the slide movement all the way to the river. Then, in order explain the further progression of the earth movement, he suggests that inertia forces originating from the retrogressive after-slides acted on the immense soil masses in the almost horizontal part of the valley, completing the passive heave of the ground as far as the river bank. The ground near the riverbank then in turn became unstable, ending the sequence of ground displacements by eventually blocking the riverbed in a major circular slide of the conventional type. Although Lundström's reasoning appears complicated, circumstantial and mechanically disputable, his explanation of the Surte slide has from the author's point of view the merit of recognizing inertia forces as an important feature of slide mechanisms.

Now, kinetic energies and forces of inertia are time dependent dynamic phenomena and cannot, therefore, be added algebraically unless they are perfectly concurrent. The author's difficulty in accepting Lundström's concept consists partly in the way that he compounds the dynamic effects of the retrogressive after-slides to those of the initial slide when explaining the passive heave of the almost horizontal ground and the riverbed. Essentially, his reasoning implies that the retrogressive slides would have taken place simultaneously. Considering the nature of, and records from similar slides, this assumption seems improbable. Lundström, however, correctly in the author's view, ascribes the initiation of the slide to the ongoing piling activities.

In conclusion, neither Jakobson's nor Lundström's explanation of the Surte slide is acceptable in the author's opinion. Especially in the latter case, it is difficult to conceive how any geotechnician, in a similar situation, would be able to predict the risk and the events of a potential slide on the basis of a complex series of slip circles as in Lundström's concept.

In an article written in Swedish (1997), Lundström has elaborated somewhat on his 1956 theory of the Surte slide events. However, his presentation does not address the possibility of progressive failure in accordance with concepts that have appeared in soil mechanics literature since 1956. In the absence of a coherent integral analysis in the time domain of the combined static and dynamic forces and related accelerations and velocities covering the total duration of the slide events, his explanation of the slide remains inconclusive in the author's opinion.
Authors explanation of the Surte slide

Fortunately for the art of slope stability analysis, the issues are not in the opinion of the author as erratic or complicated as indicated by the concept outlined above. An investigation of the Surte landslide has now been carried out along the lines suggested in Chapter 4 of this report, i.e. considering the deformations in the volume of sliding soil. The results of the analysis reveal unambiguously that neither high artesian pressures nor effects of kinetic energy are necessary prerequisites for the formation of the large passive zone, extending some 300 m over almost horizontal ground.

Quantitative, progressive failure analysis (Pr FA) shows

a) that the limited possible length of mobilization of shear stress, defined as $L_{cr}$ in Chapters 3 and 4, is conducive to the formation of failure planes parallel to the ground (or to layers of sedimentation).

b) that merely the static build-up and redistribution of forces in the progressive dynamic phase of the initial slide are sufficient to make the slide propagate all the way down to the river. Cf Table 5:2.2 and Figure 5:2.4. Dynamic forces tend to extend the passive zone and enhance the heave effect, in principle as shown in Figure 5:1.16.

The Surte slide can, therefore, readily be explained as a fully developed progressive failure of the kind described in Section 3.3. The slide events may be understood as having been similar to those depicted in the series of Figures 5:1.8 through 5:1.16. This, of course, does not exclude the possibility of artesian pore water pressures having contributed to the dramatic consequences of the progressive failure.

Several sensitivity analyses based on reasonable variations of crucial parameters all show that, once the initial local failure at the piling site had formed, the stability of the entire slope was inexorably lost. (Cf. Section 3.34).

With reference to the specific way of defining the constitutive relationship given in Figures 4:4:2 and 1:1.2, the following values of the characteristic parameters have been used in this study.

In the in situ state condition:

$\gamma_d = 2.5\% \quad \gamma_f = 7.5\% \quad G_{el,o} = \tau_{el}/\gamma_{el} = 480 \text{ kN/m}^2$

$c_u * = 24 \text{ kN/m}^2 \quad \tau_{el} = 12 \text{ kN/m}^2 \quad E_{el,o} = 2(1+\nu)G_{el,o} = 60 c_u = 1440 \text{ kN/m}^2$

$\rho_g = 15.5 \text{ kN/m}^3 \quad k_o = 0.55 \text{ (for horizontal ground)} \quad \nu = 0.5 \quad E_{el,mean} = 60 c_{u,mean}$

In the disturbance condition – failure initiation

$c_R / c_u = 0.80 \quad \gamma_d = 2\% \quad \gamma_f = 6\% \quad \delta_{cr} = 0.3 \text{ m} \quad 3G_{el,o} = \tau_{el}/\gamma_{el} = 900 \text{ kN/m}^2$

$c_u * = 30 \text{ kN/m}^2 \quad \tau_{el} = 18 \text{ kN/m}^2 \quad E_{el,o} = 3G_{el,o} = 90 c_u = 2700 \text{ kN/m}^2$

$\rho_g = 15.5 \text{ kN/m}^3 \quad k_o^{max} = 0.594 \text{ (computed)} \quad E_{el,mean} = 90 c_{u,mean}$

In the global failure condition:

$c_R / c_u = 0.35-0.20 \quad \gamma_d = 1\% \quad \gamma_f = 3\% \quad \delta_{cr} = 0.3 \text{ m} \quad 3G_{el,o} = \tau_{el}/\gamma_{el} = 2400 \text{ kN/m}^2$

$c_{u,cr} *** = 35 \text{ kN/m}^2 \quad \tau_{el} = 24 \text{ kN/m}^2 \quad E_{el,o} = 3G_{el,o} = 206 c_{u,cr} = 7200 \text{ kN/m}^2$

$k_o$ (as computed in the in situ condition) \quad $E_{el,mean} = 206 c_{u,mean}$

* Mean values applying to the initiation zone.

** Mean values applying to the down-slope failure zone.

Note: In all calculations in Chapter 5, the curved portion of the constitutive relationship from $\gamma_d$ to $\gamma_f$ is a function of $x^a$ with vertex at $(c_u, \gamma_f)$ and connecting tangentially at $(\tau_{el}, \gamma_{el})$. 
Deformations in the incipient failure zone are assumed to be symmetrical above and below the potential failure plane.
It may be observed that shear strengths and E-moduli are varied along the slope as interpreted from soils investigation data given in the report by Jakobson, 1952.

Results of the Pr F analysis

The results of the Pr F computations are shown in Table 5:2.1 below. The global failure condition, subsequent to the redistribution (related to progressive failure) of up-slope unbalanced forces to ground with lesser gradients further down-slope, is illustrated in Figure 5:2.4.

Table 5:2.1 The Surte slide - results from Pr F analysis
(L = distance from upper end of slide or point of application of critical load)

<table>
<thead>
<tr>
<th>In situ state condition:</th>
<th>c_R /c_u = 1.0</th>
<th>N_max = 138 kN/m</th>
<th>L = 120 m</th>
<th>E_max = 1673 kN/m</th>
<th>k_o = 0.594</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disturbance condition a) - Force initiated failure</td>
<td>c_R /c_u = 0.60</td>
<td>N_cr = 192 kN/m</td>
<td>L_cr = 114 m</td>
<td>E_max = 1665 kN/m</td>
<td>at L = 0 m</td>
</tr>
<tr>
<td>δ_cr = 0.145 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Disturbance condition b) - Deformation initiated failure</td>
<td>c_R /c_u = 0.60</td>
<td>N_cr = 0 kN/m</td>
<td>L_cr = m</td>
<td>E_max = 1770 kN/m</td>
<td>at L =50 m</td>
</tr>
<tr>
<td>δ_instab = 0.292 m</td>
<td>L_instab = 162 m (As defined in Figure 4:2.4a)</td>
<td></td>
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</table>

The in situ state condition

In the steepest part of the slope, available shear strengths do not match the in situ shear stress in terms of τo = ρ·g·H·sinβ. According to Equation 4:2 (Chapter 4) this implies that in the in situ condition the soil masses were supported by incremental earth pressures in less inclined ground further down the slope. See Table 5:2.1

Disturbance conditions

Force initiated failure

The critical load (N_cr), sufficient to initiate local failure in the steepest part of the slope, amounts to 192 kN/m.
Although the Surte slide was not engendered by the weight of an applied fill, it may still be of interest to note that this value of N_cr only corresponds to a rapidly applied distributed load on the ground surface of about q_cr ≈ 11 kN/m², (i.e. assuming undrained conditions).* By contrast, ideal-plastic failure analysis based on local slip circles indicates a corresponding minimum value of q_cr ≈ 90 kN/m², i.e. a difference that can be expressed by a factor of more than 8. See Figure 5:2.3 below.

* It may be observed in this context that slowly applied loads (drained conditions) would, using progressive failure analysis, also result in much higher values of q_cr.
\[ q_{cr} (Pr FA) = q_{cr} ABDF \approx 11 \text{ kN/m}^2 \ll q_{cr} (I-PFA) = q_{cr} ABC \approx 90 \text{ kN/m}^2 \]
\[ q_{cr} (Pr FA) = q_{cr} ABDF \approx 11 \text{ kN/m}^2 \ll q_{cr} (I-PFA) = q_{cr} ABDE \approx 180 \text{ kN/m}^2 \]

**Figure 5.2.3** Comparison between progressive failure analysis (Pr FA) and ideal-plastic failure analyses (I-PFA) with regard to a critical load \( q_{cr} \) due to an earth fill. (Undrained conditions are presumed.)

In the opinion of the author, this important discrepancy between the results from the ideal-plastic equilibrium approach on one hand, and analyses considering deformation softening on the other, is the main reason why downward progressive slides of the type dealt with in this report have long eluded logically coherent explanations. Cf Section 4.6.

In the disturbance condition, the computed resistance is mostly related to stage I, i.e. prior to the formation of a discrete failure band or slip surface. At this stage, both the modulus of elasticity and the shear modulus are time dependent in a similar way. In consequence, the analysis is not very sensitive to the time factor considering that the ratio of \( E/G \) is largely constant and is not likely to vary widely. (Cf. Section 5.1, **Sensitivity studies**).

However, in order to investigate the response of the analysis to variation of the compressibility of the soil mass, the effect of doubling the ratio of \( E/G \) has been computed and, other conditions unchanged, found to be as follows:

\[ c_R / c_u = 0.60 \quad N_{cr} = 274 \text{ kN/m} \quad L_{cr} = 163 \text{ m} \quad E_{max} = 1670 \text{ kN/m} \text{ at } L = 0 \]

Hence, doubling the ratio of \( E/G \) brings about increases of \( N_{cr} \) and \( L_{cr} \) by 43 %, while the value of \( E_{max} \) is virtually unaffected. It may be observed in this context that a 43% increase of \( N_{cr} \) has little impact on the issue highlighted in Figure 5.2.3. (In fact, the issue would remain unchanged even for much higher values of the compression modulus of the soil in the slope).

**Deformation initiated failure**

As demonstrated in Section 3.34 and Figure 4.2.4a, there exists a critical value of forced deformation \( \delta_{instab} \), which may result in global slope failure, even in the absence of a sustained force pursuing and maintaining the failure process. In reality, such a situation can
arise when driving soil-displacing piles, in which case the force causing the displacement only gives rise to internal stresses.

As already mentioned, the Surte slide is for good reasons believed to have been triggered by pile driving for the foundation of a family house, which was going on at the time of the slide event. Table 5:2.1 gives a critical value of $\delta_{\text{instab}} \approx 0.3$ m. However, as the number of piles in the foundation was not sufficient to generate a down-slope movement of this magnitude, it may be concluded that soil displacement cannot have been the sole disturbance initiating the Surte slide.

It is thus very likely that the piling activities also induced locally high pore water pressures and tendency to liquefaction in local seams of coarser moraine out-wash in the clay formation. Such coarse strata commonly intermix with the clay sediments in ancient steep shores of the regressing post-glacial seas. (Cf. slide at Råvekårr, Section 5.5)

Also in this condition doubling of the values of the $E/G$ ratio has a moderate impact on the issue highlighted in Figure 5:2.3. Thus, $L_{\text{instab}}$ is increased by 41 %, while $E_{\text{max}}$ is raised only by 8 %.

$$c_R/c_u = 0.60 \quad N_{cr} = 0 \text{ kN/m} \quad L_{cr} = \text{---- m} \quad E_{\text{max}} = 1911 \text{ kN/m at } L = 65 \text{ m} \quad \delta_{\text{instab}} = 0.289 \text{ m} \quad L_{\text{instab}} = 228 \text{ m} \quad \text{(As defined in Figure 4.2.4a)}$$

**Global failure condition:**

Figure 5:2.4 displays calculated earth pressures, shear stresses and displacements for the slip surface defined by Jakobson (1952) in the slope at Surte. The global failure condition is illustrated in the figure, representing the situation at the end of the progressive redistribution phase, in which unbalanced shear forces in the steepest parts of the slope have been transmitted further down-slope, resulting in massive build-up of earth pressures in less sloping ground. (See Section 3.3, Figure 3:3.4).

**Table 5:2.2 The Surte slide - results from Pr F analysis**

(L = distance from upper end of slide or point of application of critical load)

| Global failure condition: - Case I | $c_R/c_u = 0.35-0.20$ | $N_{max} = 3112 \text{ kN/m}$ | $E_{\text{max}} = 4969 \text{ kN/m}$ | $E_{\text{Rankine(max)}} = 3900 \text{ kN/m}$ | $L = 260 \text{ m}$ | $L_{E> E_{\text{Rankine}}} = 420 \text{ m}$ #
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<tr>
<td>$E_{\text{Rankine}}/E_{\text{max}} = 0.785$</td>
<td>$E_{dl} = 206 c_u, \text{mean}$</td>
<td>$E_{dl} = 206 c_u, \text{mean}$</td>
<td>$E_{dl} = 206 c_u, \text{mean}$</td>
<td>$E_{dl} = 206 c_u, \text{mean}$</td>
<td>$E_{dl} = 206 c_u, \text{mean}$</td>
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| Global failure condition: - Case II | $c_R/c_u = 0.40-0.25$ | $N_{max} = 2682 \text{ kN/m}$ | $E_{\text{max}} = 4554 \text{ kN/m}$ | $E_{\text{Rankine(max)}} = 3900 \text{ kN/m}$ | $L = 260 \text{ m}$ | $L_{E> E_{\text{Rankine}}} = 234 \text{ m}$ #
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<tr>
<td>$E_{\text{Rankine}}/E_{\text{max}} = 0.856$</td>
<td>$E_{dl} = 206 c_u, \text{mean}$</td>
<td>$E_{dl} = 206 c_u, \text{mean}$</td>
<td>$E_{dl} = 206 c_u, \text{mean}$</td>
<td>$E_{dl} = 206 c_u, \text{mean}$</td>
<td>$E_{dl} = 206 c_u, \text{mean}$</td>
<td>$E_{dl} = 206 c_u, \text{mean}$</td>
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| Global failure condition: - Case III | $c_R/c_u = 0.40-0.25$ | $N_{max} = 2682 \text{ kN/m}$ | $E_{\text{max}} = 4558 \text{ kN/m}$ | $E_{\text{Rankine(max)}} = 3900 \text{ kN/m}$ | $L = 260 \text{ m}$ | $L_{E> E_{\text{Rankine}}} = 225 \text{ m}$ #
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<tbody>
<tr>
<td>$E_{\text{Rankine}}/E_{\text{max}} = 0.855$</td>
<td>$E_{dl} = 412 c_u, \text{mean}$</td>
<td>$E_{dl} = 412 c_u, \text{mean}$</td>
<td>$E_{dl} = 412 c_u, \text{mean}$</td>
<td>$E_{dl} = 412 c_u, \text{mean}$</td>
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* # Length over which passive Rankine pressure is exceeded.

It should be observed that the earth pressures are calculated under the assumption that the potentially slicing soil volume retains its geometrical shape before its possible disintegration in passive Rankine failure. Hence, in cases where the resulting maximum earth pressure $E_{\text{max}}$ exceeds $E_{\text{Rankine(max)}}$, the computed earth pressure scenario represents a highly transient
situation, which immediately merges into the dynamics of the slide proper as described in Chapter 3.

Figure 5:2.4 Static earth pressure distribution in the Surte slide subsequent to the progressive failure phase but prior to the slide proper resulting in disintegration and heave in passive failure. The figure indicates that the static forces developed in the dynamic progressive phase of the ground movement suffice to explain the spread of the passive zone over almost horizontal ground. Cf. Section 3.3.

Global failure condition: Case I, \( c_R/c_u = 0.35\text{-}0.20 \), \( E_{el} = 206 \, c_{u,\text{mean}} \)

Curve A, \( E_0(x) \) = In situ earth pressure prior to local failure, kN/m

Curve B, \( N(x) \) = Earth pressure increment due to Pr F redistribution, kN/m

Curve C, \( E(x) = E_0(x) + N(x) \) = Earth pressure after Pr F redistribution, kN/m

Curve D, \( E_{p,\text{Rankine}} \) = Passive Rankine resistance, kN/m

Curve E, \( \tau_0(x) \) = In situ shear stress distribution before progressive failure, kN/m²

Curve F, \( \tau(x) \) = Shear stress distribution after progressive failure, kN/m²
The *significance* of the earth pressure distribution in this transient stage of equilibrium is that it constitutes a *measure of the catastrophe* that may ensue if the critical load in the disturbance condition is exceeded, i.e. will the progressive failure result in a veritable landslide or not?

The calculations in case 1 in Table 5:2.2 are based on residual shear strengths in proportion to the magnitude of displacement in the progressive failure phase, varying from \( c_R = 0.35 \ c_u \) to \( c_R = 0.20 \ c_u \) in different locations in the slope. Considering the large displacements (\( \approx 15 \) m) and the velocities involved already in the progressive phase, these values of \( c_R \) must be considered as high.

As may be seen in Figure 5:2.4, the earth pressures resulting from the progressive failure redistribution of forces, entail that passive resistance is exceeded over about 420 m of gently sloping ground. Even apart from dynamic effects in the progressive phase, this static condition inevitably leads to total disintegration and heave in the lower areas of the slope and the valley thus initiating the dynamic phase of the slide proper.

**Sensitivity studies**

The effect of changing the \( c_R / c_u \) – ratios from 0.35-0.20 to 0.40-0.25 is evidenced by the figures given in Table 5:2.2 above. The maximum earth pressure decreases from 4969 kN/m to 4554 kN/m i.e. by a factor of 0.92, whereas the length of the potential passive zone is reduced from 420 m to 234 m.

However, for values of \( c_R / c_u > 0.6 \), the value of \( E_{\text{max}} \) no longer exceeds \( E_{\text{Rankine}} \) implying that global failure with excessive heave of the passive zone would not be likely to take place.

The effect on the global failure condition of *doubling* the \( E_{\text{el,mean}} / G_o \) – ratio (i.e. reducing the compressibility of the potentially sliding soil mass) is insignificant as far as the maximum earth thrust is concerned. The effect on the length of the passive zone in heave is moderate. Thus for \( c_R / c_u = 0.40-0.25 \) and \( E_{\text{el,mean}} / G_o = 6 \) instead of 3, the following values result:

\[
E_{\text{max}} \text{ becomes } 4558 \text{ kN/m instead of } 4554 \text{ kN/m and } \frac{L_{E > E_{\text{Rankine}}}}{L_{E > E_{\text{Rankine}}}} = 225 \text{ m instead of 265 m.}
\]

**Conclusions with regard to the Surte slide**

The main cause of the Surte slide is attributed to the disturbance generated by the pile driving activities in the steep slope at the upper limit of the main slide. A consequence is then that if, hypothetically, the piling operations had not taken place in 1950 the slope may have remained to this day in the state, in which it had existed for thousands of years.

A slope in this condition may therefore be thought of as a *‘time bomb’* ticking through the millennia. One of the main objectives in a regional program for surveying potential landslide hazards must, accordingly, be to *identify in-built, dormant disasters* of this kind. By definition, this cannot be achieved using the conventional ideal-plastic failure approach.
5.3 The landslide at Bekkelaget (1953), Norway

The landslide at Bekkelaget, close to the Oslo fjord, has been documented by Bjerrum & Eide, 1955. The slide, which took place in quick clay, encompassed some 20 000 m² of ground, of which about 70 % had a gradient of only 2 to 3° in the north-easterly direction towards the fjord. The event was triggered by the placement of an earth fill designed to widen an existing road running parallel to a railway track.

Figure 5:3.1 Main characteristics of the slide at Bekkelaget, Norway. Computed safety factors for three different potential failure surfaces. (Aas 1983).

Figure 5:3.1 shows the major features of the slide area together with computed safety factors for three of the investigated slip surfaces as reported by Aas 1983. From a conventional design point of view, the odd circumstance here is the fact that, although the shortest slip surface has a safety factor of only 0.87 (< 1), the slide actually took place along the longest slip surface with the highest safety factor of 1.32. This is, however, entirely consistent with progressive failure analyses and highlights the fact that the resistance along a shear band conforming to the ground surface will, in strain softening soils, often be considerably smaller than the resistance along failure planes surfacing in the sloping ground. In markedly deformation softening soils, this even applies when the gradient is small. (See Equation 3:3).
5.4 The landslide at Rollsbo (1967), Sweden

A landslide involving some 20 000 m² of ground took place in 1967 at Rollsbo, about 20 km north of Gothenburg. Figure 5:4.1 shows a section through the slide. Although conventional calculations based on I-PFA indicated a minimum safety factor of 2.3, the slide was triggered while sand drains were being driven at the up-slope end of the area involved in the slide. The slide is of particular interest because the driving of the sand drains and the soil conditions before and after the slide event were unusually well documented. In view of the soil conditions, the initiation agent and the specific features of this slide, there is little doubt in the authors' mind about this slide being a clear case of a downward progressive failure.

![Diagram of the landslide at Rollsbo](image)

**Figure 5:4.1** Section through the landslide at Rollsbo, (Kungälv), 1967

5.5 The slide movement at Rävekärr (1971), Sweden

The slide took place at Rävekärr, 8 km South of Gothenburg in the gently sloping ground of a side valley to the Mölndal River valley. Figure 5: 5.1 shows plan and a representative section through the 550 m wide slide area. This slide movement is reported, not because of significant damage having occurred, but for its interesting circumstances and features. Thus, in 1971 a small piling project for a family house was started. When the fifth pile was being driven a crack in the ground suddenly appeared. The crack propagated swiftly at a speed, judged by an eyewitness to be about the pace of a running person, some 130 m northwards where it halted against an outcrop of firm ground. In the opposite direction, the ground crack passed through an area of family housing following the contour lines of the slope and came to a stop some 420 m from where it started. However, the final width of the crack and the related vertical off-set due to local active failure remained at only 0.2 to 0.3 m. The total area involved in various degrees of documented down-slope movement of this magnitude, was about 150 000 m².

Although the ground down-slope of the crack was displaced, no passive zone with associated heave could be detected, implying that the crack must have originated from the redistribution of forces in accordance with the first dynamic phase of a progressive failure as described in Section 3.3. This redistribution implies that elevated ground, which was largely stabilized by shear forces prior to the slide event, now received the necessary additional support from increased earth pressures further downhill.
With reference to Section 3.3, (dynamic phase I), the slide at Rävekärre thus represents a case, where the earth pressure increase in the ‘post progressive’ state of equilibrium remains smaller than the passive Rankine resistance at the foot of the slope, as defined by equation 3:3a.

Figure 5: 5.1 Section through slide area at Rävekärre. Observe the gentle slope gradient. Slip surfaces were documented at depths of 5 - 7 m in the upper part of the slide and angular deformations were recorded at 13 m and 33 m depth in the lower parts of the valley. Pile driving took place at the point marked x. (Löfquist 1973).

The slide movement at Rävekärre was reported by Löfquist, (1973). Löfquist classified the slide as a ‘clay slide by hydraulic uplift’, i.e. caused by reduced vertical effective stresses in the clay or in seams of silt and silty sand. Löfquist’s explanation is probably very correct for the initiation zone of the slide movement, as the soil profiles at the crack location contained discrete seams of moraine out-wash that had contaminated the clay during the sedimentation process.

In his appraisal, Löfquist presents a number of hypothetical aspects on slide mechanisms, some of which in the author’s opinion are speculative. For example, his theory of progressive degradation of soil strength over time due to creep deformation does not, in the author’s opinion, apply to normally or slightly over-consolidated soils. It disregards, for instance, the effects of re-consolidation and the basic behavior of cohesive and cohesionless soils as documented in consolidated/undrained direct shear tests, (CU-DSS tests).
The area affected by the piling activity can be estimated at some 100 m², which is only a minute fraction (i.e. 1/1500) of the total area of about 150 000 m² involved in the slide. In view of this, and the fact that the slope had existed for many centuries it seems unlikely that high artesian conditions constituted a major agent in the earth movement. If that had been the case, the slide would, in all probability, have taken place long before at some previous extreme hydrological situation in the history of the slope and not in connection with the driving of a few piles.

However, although Löfquist rightly maintains that high pore water pressures due to rainfall or piling operations may be conducive to slope failure, back-analyses according to the method described in Chapter 4 show that the crack and associated displacements can readily be explained by the redistribution of forces generated by progressive failure. It is therefore not necessary to resort to unlikely events such as a sudden rise of pore water pressures, related to pile driving, in layers of silt over the entire vast area involved. Moreover, continuous silt or sand layers in the soil profile, consistent with the dimensions of the slide area, were not documented. Peak pore water pressures were not either recorded at the time.

The observed velocity at which the crack propagated parallel to the contour lines offers an interesting clue to the timing of Pr F formation, i.e. as to how fast the related stress redistribution wave can travel down the slope. Hence, the slide at Rävekärre indicates that the time range for a progressive failure to take place is a matter of tens of seconds.

In conclusion, the ground movement at Rävekärre may be classified as an unfinished landslide, where, due to the low gradient and/or moderate sensitivity of the soil, the progressive failure did not terminate in any upheaval in passive Rankine failure.

*Other unfinished slides suspected of being of a similar nature:*

On September 28, 1905, a similar almost 1 km long crack is reported to have formed from Klockaregården in the north to Åtestupan in the south, not far from a hill called Ramberget on the island of Hisingen, Gothenburg. Many houses were damaged by ground subsidence.

Another unfinished slide of this type, causing a crack 50 to 100 mm wide and some 150 m long, occurred at the Björlanda Road (Hisingen) in Gothenburg proper in 1972. In this case, also, the ground movement was triggered by pile driving in the outskirts of the slide area.
5.6 The landslide at Tre-styckevattnet (1990), Sweden.

Not far from the city of Uddevalla, about 80 km north of Gothenburg, another slide exhibiting the typical features of a progressive failure took place in connection with the construction of a berm designed to provide additional stability to an embankment for the E6 highway. Figure 5:6.1 shows a plan of the slide area while a section through the slide is given in Figure 5:6.2. The ground, above the water level of the lake, actually involved in the slide movement measured about 70 m in width and 140 m in length. How far the slide extended into the lake was not recorded. The lateral limits of the slide were essentially parallel to one another. The original ground gradient from the foot of the highway embankment down to Lake Trestyckevattnet was uniform and remarkably small (~1°), thus strongly indicating that the cause of the slide was related to the ongoing construction work rather than to inherent instability.

The embankment for the highway proper was not involved in the slide, as it had been founded on compacted rock waste fill replacing excavated loose soils down to competent ground.

The roughly 5 m high berm had been placed already in the fall of 1989, but was completed about a year later with a finishing layer of topsoil for vegetation. The topsoil was placed using bulldozers and compacted by means of a heavy vibratory roller. It was at this point, when only two or three loads of topsoil remained to be leveled and compacted that the slide took place. The heavy berm had thus been stable for more than a year, and during this period the underlying soil had been subject to drainage and consolidation effects. It seems, therefore, unlikely that the slide was initiated solely by the weight of the thin layer of humus-rich topsoil constituting only about 5% of the total weight of fill applied more than a year before.

Figure 5:6.1 Plan and section of the slide area. The slide proper measured about 70 m in width and at least 140 m in length.
Hence, in the opinion of the author, the impact of the heavy vibratory roller on the subsoil must be assumed to have been the crucial factor in the slide initiation process.

Be this as it may, the cardinal issue here is that the failure mode is not compatible with ideal-plastic equilibrium analysis, according to which the critical failure mode would follow a slip surface of the kind indicated as ABC in Figure 5:6:2. Instead, at least 140 m of almost horizontal ground was displaced towards the lake, thus overcoming not only the resistance along the horizontal slip surface but also the lateral shear resistance in the two 140 m long sides of the sliding soil volume.

Although no casualties or damage to housing resulted from the slide, a group of geotechnical engineers involved in the road project decided that the unusual features and circumstances of the slide merited closer investigation. The group included representatives of the following bodies:
The Swedish National Road Administration,
The Swedish Geotechnical Institute, (SGI local division),
KM Consulting AB,
Skanska Teknik AB, (Contractors Engineering Division).

It was agreed within the group that the slide area should be surveyed and that sufficient documentation of the ground profile and of soil properties should be secured immediately in order to allow future studies of the slide.

Soil conditions

The vegetation in the slide area consisted of full-grown pine and spruce, some of which had been felled in connection with the construction of the highway. Under a top layer of humus, the soil was made up of peat to a depth varying between 4 and 7 m, the smallest value being valid at the edge of the berm.

The soil underlying the peat was composed of soft sensitive clay with a water content of almost invariably about 60% with some local peaks of ≈70%. The liquid limit was typically 40 to 47% in the failure zone. In two bore holes located about 80 m from the edge of the berm, the liquid limit at the critical level was locally as low as 25% in layers classified as sandy respectively silty clay. The sensitivity number (cone tests) in the failure zone varied between 90 and 103 with an estimated mean value of 95.

The depth to firm bottom below the clay formation varied from 13 m to 20 m, the lower value applying close to the berm limit.

Back analysis

The failure zone has been identified by the conspicuous drops in the sensitivity of the tested clay samples and has been located to the upper strata of the sensitive clay formation. The depth of the slip surface below ground level is thus about 7 m. The following input data have been assumed for the Pr F analysis:

In the in situ state condition:

\[
\begin{align*}
\frac{c_R}{c_u} &= 0.94 & \gamma_f &= 2.5\% & \gamma_f &= 7.5\% & \delta_{cr} &= 0.3\ m, & G_{el,0} &= \tau_{el}/\gamma_f = 240\ kN/m^2 \\
\gamma_{el} &= 9.9\ kN/m^2 & \tau_{el} &= 6.0\ kN/m^2 & E_{el,0} &= 50-c_u,\text{mean} = 500\ kN/m^2
\end{align*}
\]

In the disturbance condition:

\[
\begin{align*}
\frac{c_R}{c_u} &= 0.80 & \gamma_{el} &= 2\% & \gamma_f &= 6\% & \delta_{cr} &= 0.3\ m, & G_{el,0} &= \tau_{el}/\gamma_f = 350\ kN/m^2 \\
\gamma_{el} &= 11.0\ kN/m^2 & \tau_{el} &= 7\ kN/m^2 & E_{el,0} &= 100-c_u,\text{mean} = 1100\ kN/m^2
\end{align*}
\]
In the global failure condition:
\[ c_R/c_u = 0.40 \quad \gamma_{el} = 1 \% \quad \gamma_f = 3 \% \quad \delta = 0.3 \text{ m} \quad G_{el,o} = \tau_{el}/\gamma_{el} = 1000 \text{ kN/m}^2 \]
\[ c_u** = 14 \text{ kN/m}^2 \quad \tau_{el} = 10 \text{ kN/m}^2 \quad E_{el,o} = 200 \cdot c_u, \text{mean} = 2200 \text{ kN/m}^2 \]

* Mean values applying to the initiation zone.
** Mean value applying to the extended failure zone.

Note: In all calculations in Chapter 5, the curved portion of the constitutive relationship from \( \gamma_{el} \) to \( \gamma_f \) is a function of \( x^2 \) with vertex at \((c_u, \gamma_f)\) and connecting tangentially at \((\tau_{el}, \gamma_{el})\).

Results

The in situ state condition:
\[ c_R/c_u = 1.00 \quad N_{cr} = \quad \text{kN/m} \quad L_{cr} = 144 \text{ m} \quad E_o = 661 \text{ kN/m} \quad k_o^{max} = 1.84 \]
\[ c_u* = 9.9 \text{ kN/m}^2 \quad c_R = 9.9 \text{ kN/m}^2 \]

As may be concluded from the relationship between Curve A and Curve D in Figure 5:6.2, the Pr F analysis indicates that, although the slope is shown to be globally stable after the placement of the earth berm, passive Rankine resistance is exceeded over a distance of some 50 m down-slope of crest of the fill. It therefore seems quite likely that the ground surface actually heaved in passive failure already when the fill was being placed. This phenomenon may have been very gradual, thus escaping much attention or causing alarm. Measurements after the slide documented a heave of the ground in this area of \( \approx 1 \text{ m} \), but it has not been established if this rise existed before the final great slide in 1990.

An explanation of the actual ground resistance here being higher than computed may be that the peat formation was heavily interlaced with thick roots from the tall pine and spruce trees. This was evidenced at the lateral boundaries of the slide, where thick broken roots protruded from exposed surfaces.

Furthermore, the viscous character of the peat layer may have effectively mitigated strain concentrations and transmission of forces to the underlying sensitive clay stratum.

Anyway, the ground remained stable for more than a year thence.

It is important to note in this context that, as was the case at Bekkelaget, a safety factor based on conventional slip circle analysis was according to the soil strength parameters in the soils investigation also less than 1.

The disturbance condition:
\[ c_R/c_u = 0.80 \quad N_{cr} = 40 \text{ kN/m} \quad L_{cr} = 95 \text{ m} \quad E_o = 661 \text{ kN/m} \quad E_o+N = 701 \text{ kN/m} \]
\[ c_u* = 11 \text{ kN/m}^2 \quad c_R = 8.8 \text{ kN/m}^2 \quad F_s = N_{cr}/N_i \approx 1.0 \]

The critical load according to the Pr F analysis required to trigger the planar progressive failure is \( N_{cr} = 40 \text{ kN/m} \). This corresponds to a distributed surface load of \( q_{cr} \approx 6.7 \text{ kN/m}^2 \), which in turn is equal to the weight of a soil layer of about 0.4 m.

The global failure condition:
\[ c_R/c_u = 0.40 \quad N_{max} = 448 \text{ kN/m} \quad L_{\text{Rankine}} = 300 \text{ m} \quad E_{\text{max}} = E_o+N_{80} = 972 \text{ kN/m} \]
\[ c_u** = 14 \text{ kN/m}^2 \quad L_{N,\text{max}} = 104 \text{ m} \quad E_{\text{Rankine}} = 700 \text{ kN/m} \quad L_{E,\text{max}} = 80 \text{ m} \]
\[ c_R = 5.6 \text{ kN/m}^2 \quad F_s = E_{\text{Rankine}}/E_{\text{max}} = 0.72 \]
Figure 5:6.2 Shear stress and static earth pressure distribution in the slide at Trestyckevatnet subsequent to the progressive failure phase but prior to the global failure. The figure indicates that the spread of the passive zone over almost horizontal ground need only to be ascribed to the static forces developed in the progressive phase I of the ground movement as explained in Section 3.3.

Curve A, $E_0(x)$ = In situ earth pressure prior to placing of topsoil layer, kN/m
Curve B, $N(x)$ = Earth pressure increment due to Pr F redistribution, kN/m
Curve C, $E(x)$ = $E_0(x) + N(x)$ = Earth pressure after Pr F redistribution, kN/m
Curve D, $E_p^{\text{Rankine}}$ = Passive Rankine resistance, kN/m
Curve E, $\tau_0(x)$ = In situ shear stress distribution before progressive failure, kN/m²
Curve F, $\tau(x)$ = Shear stress distribution after progressive failure, kN/m²
Curve G, $\delta$ = Displacement, m
Figure 5:6.2 shows shear stresses and earth pressures before and after progressive failure has taken place. As mentioned, the curve A representing the in situ condition indicates a low factor of safety already before the application and compaction of the topsoil layer.

These activities apparently induced deformations entailing softening of the clay below and down-slope of the berm causing the residual shear strength to become less than 0.8 $c_R$. A gradual drop in shear strength to e.g. 0.4 $c_R$ would according to the analysis by a wide margin explain the remarkable spread of the slide all the way down into the lake. The fact that most of the sliding soil consisted of peat makes it difficult to estimate the correct compressibility of the material in terms of an E - modulus for the global failure analysis. The chosen values, which would apply to very soft clay, may therefore appear as too high. However, using lower values of the E-modulus would in the calculations only further promote the prospect of the formation of progressive failure.

Conclusions with regard to the slide at Trestykevatnet

The analysis suggests that the slide at Trestykevatnet was initiated already in the final stages of constructing the berm in 1989. However, for likely reasons given above, the slope remained globally stable until the topsoil layer was placed more than a year later.

As the safety factor at this stage was close to 1, the application of a comparatively insignificant load from the 0.3 m thick topsoil layer set off the large slide extending probably far into the lake Trestykevatnet. In the opinion of the author, the decisive disturbance effect initiating the progressive failure entailing the final global slide has to be attributed to the use of vibratory compaction equipment.

As mentioned, the safety factor based on conventional slip circle analysis was also less than 1. However, the crucial issue in this context is that the slide did not in fact develop along any kind of local slip circle such as ABC in Figure 5:6.2. Instead, about 300 by 70 m$^2$ of ground above and below the water line, was displaced. This fact highlights the inadequacy of conventional ideal-plastic analysis for predicting the development, events and outcome of landslides in deformation softening clays.
5.7 Conclusions from 'Case Records'

It is evident from the discussion in Section 2.2 that all extensive landslides do not necessarily occur as a result of progressive failure (Pr F) formation. However, the landslides listed in Table 5:8.1 are all believed by the author to belong to the downward progressive failure category for the following reasons:

a) Analyses based on ideal-plastic limit equilibrium principles do not explain the extent or other features of the slides.

b) The slides were triggered by known, locally acting agents such as earth fills, undercutting erosion, pile driving, heavy vibratory soil compaction, sometimes but far from always in combination with any documented elevated pore water pressures due to spells of high rainfall.

Unprecedented types of loading inducing undrained behavior in deformation softening soils may be particularly conducive to initiation of progressive failures in natural slopes, and that irrespective of the fact that they have existed in a stable condition for thousands of years.

c) The finished landslides feature vast areas of gently sloping ground having heaved as a result of exceeding passive Rankine resistance. As proven in Section 2.4 and illustrated in Figure 2:4.2 b, this is typical of the immense release of potential energy and the build-up of static as well as dynamic forces associated with slides in deformation softening soils.

d) The analogy of slope stability in deformation softening soils to 'buckling stability', as described in Section 3.34 is striking in these case records.

Thus vast areas of stable ground may become engaged in extensive landslides triggered by some seemingly trivial local disturbance agent. Progressive failure analysis has the potential of identifying dormant catastrophes of this kind.
Table 5.8.1 Landslides in sensitive clays classified by the author as progressive failures

<table>
<thead>
<tr>
<th>Location of slide</th>
<th>Year</th>
<th>Total length</th>
<th>Length of passive zone, m</th>
<th>Total slide area, m²</th>
<th>Type of slide</th>
<th>Locally acting triggering agent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Svärta</td>
<td>1938</td>
<td>160</td>
<td>130</td>
<td>21000</td>
<td>Downward progressive</td>
<td>Earth fill, up-slope</td>
</tr>
<tr>
<td>Sköttorp</td>
<td>1946</td>
<td></td>
<td></td>
<td></td>
<td>Upward progressive</td>
<td>Undercutting, down-slope</td>
</tr>
<tr>
<td>Surte</td>
<td>1950</td>
<td>600</td>
<td>400</td>
<td>300 000</td>
<td>Downward progressive</td>
<td>Piling activity, up-slope</td>
</tr>
<tr>
<td>Bekkelaget, Oslo M.y, Norway</td>
<td>1953</td>
<td>160</td>
<td>100</td>
<td>20 000</td>
<td>Downward progressive</td>
<td>Earth fill, up-slope</td>
</tr>
<tr>
<td>Rollsbo, Kungälv Municipality</td>
<td>1967</td>
<td></td>
<td></td>
<td>21 000</td>
<td>Downward progressive</td>
<td>Driving sand drains, up-slope</td>
</tr>
<tr>
<td>Rödbo, Kungälv Municipality</td>
<td>196x</td>
<td></td>
<td></td>
<td></td>
<td>Downward progressive</td>
<td>Fill of tunnel spoil, up-slope</td>
</tr>
<tr>
<td>Jordbro, V:a Haninge M:y</td>
<td>196x</td>
<td></td>
<td></td>
<td></td>
<td>Downward progressive</td>
<td>Earth fill, up-slope</td>
</tr>
<tr>
<td>Rävekärr, Mölndal M:y</td>
<td>1971</td>
<td>c:a 300</td>
<td>250</td>
<td>150000</td>
<td>Downward progressive</td>
<td>Pile driving, up-slope</td>
</tr>
<tr>
<td>Sem , Norway</td>
<td>1974</td>
<td>120</td>
<td></td>
<td></td>
<td>Downward progressive</td>
<td>Earth fill, up-slope</td>
</tr>
<tr>
<td>Tuve, Gothenburg</td>
<td>1977</td>
<td>800</td>
<td>300</td>
<td>270 000</td>
<td>Downward progressive</td>
<td>Earth fill, up-slope</td>
</tr>
<tr>
<td>Rissa, Norway, slide area C</td>
<td>1978</td>
<td>800</td>
<td>300</td>
<td>270 000</td>
<td>Downward progressive</td>
<td>Disturbance, upslope</td>
</tr>
<tr>
<td>Rissa, Norway, slide D</td>
<td>1978</td>
<td></td>
<td></td>
<td></td>
<td>Upward progressive</td>
<td>Disturbance, down-slope</td>
</tr>
<tr>
<td>Tre-styckevattnet, Uddevalla M:y</td>
<td>1990</td>
<td>400</td>
<td>ca 300</td>
<td>ca 21 000</td>
<td>Downward progressive</td>
<td>Fill + vibration, up-slope</td>
</tr>
</tbody>
</table>

* According to Gregersen, 1981

Figure 5.8.1  The landslide at Svärta, 1938
6. Factors conducive to the brittleness of slope failures

6.1 Britteness due to inherent properties of the soil

6.11 Sensitive soft clays

Although this report focuses on the effects on slope stability of deformation softening in soils, this topic will not be dealt with in great detail here. The reason for this is that the brittle behavior of sensitive soft clays is a rather well documented phenomenon in soil mechanics, and it is not within the scope of this report to elaborate on this issue. Valuable contributions to the knowledge of the properties of soft Scandinavian clays have, for instance, been made by R Larsson (1977) in the SGI Report No 4, named “Basic Behavior of Scandinavian Soft Clays” and by Karlsrud, Aas & Gregersen in their State-of-the-art Report to the Toronto Symposium on Land-slides (1984).

However, in this context the following points will be emphasized:

1) In order to study brittle slope failures, it is necessary to know not only the shear strength of the soil but also its entire stress/strain behavior and in particular the shear strength at large deformations ($c_R$). Hence, the constitutive relationship has to model also the conditions subsequent to the formation of a slip surface. Now, the residual shear resistance at slip is highly dependent on displacement and, especially, the rate of displacement. These parameters are not readily revealed by current soil testing procedures, and new methods for laboratory testing and/or testing in the field will have to be developed.

For instance, according to current procedures, clays in Sweden are sheared in DSS tests at a standard rate of 0.3 to 0.5 % per hour. It is obvious that such tests are of little value when estimating the probable outcome of local slope failures, where the maximum rates of displacements in the resulting slide may be in the order of tens of centimeters or meters per second.

2) As regards the formation of failure planes in small soil samples, it may be observed that the use of rubber membranes is prone to distort the stress/deformation relationships. This is because membranes contribute to the residual shear strength of soft clays and tend to postpone and control the formation of slip surfaces. One way of avoiding this problem is to confine the specimens in DSS tests by means of horizontal rings, which are freely movable relative to one another. (Bernander & Svensk, 1985).

In respect of the effects of timing, see Section 6.4 below.

6:12 Sensitivity due to low plasticity

The sensitivity of normally consolidated clays is readily identified by the ratio of the natural water content to the liquid limit ($w/w_L$), or more specifically by the liquidity index: $I_L=(w-w_F)/(w_L-w_F)$, where $w_F$ is the plasticity limit.

Loosely compacted silts and sands, with relative densities well below the critical relative density ($D_c$), are of course prone to deformation softening or even liquefaction. In slopes with ongoing creep, layers of such materials are likely to be pre-sheared so that a state of critical density prevails, i.e. a condition not particularly conducive to strain softening in shear.

However, even soils of critical density may tend to liquefy when subjected to vibrations from compaction equipment or pressure wave radiation associated with blows from pile driving gear.
6.13 Brittleness related to over-consolidation

Deformation softening is not limited only to sensitive normally consolidated clays. Over-consolidated clays tend to adopt useable shear strengths, which are related to the current vertical effective stress in the formation. (Ladd & Foot, 1974). Thus, even at moderate values of the over-consolidation ratio (OCR), clays may exhibit strain softening. In consequence, exceptionally high pore water pressures must be considered as being conducive to brittle soil behavior.

Highly over-consolidated clays are known to show brittle behavior, e.g. London clay (Skempton, 1964).

Such britteness is even more pronounced in over-consolidated cemented clays, as fissures in the jointed clay, which are oriented along the failure surface may be, or develop to be, slicken-sided, thus forming continuous or discontinuous planes of weakness. In a report (1967), Bjerrum studied numerically the possibility of brittle retrogressive failure in slopes of cemented tertiary clays.


The effects on shear strength and brittleness of clays, linked with the state of stress have long been recognized in soil mechanics literature. The shear stress/deformation behavior may vary widely with the ratio of horizontal normal stress to vertical stress, \( k_o = 
\sigma_h/\sigma_v \). The fact that \( k_o \) tends to adopt low values in the active zone, usually located in the upper and steeper parts of a slope, means that brittleness is often concurrent with high mobilization of shear strength. This setting is an important factor promoting the formation of downward progressive failures. (See Bernander, ‘Active Earth Pressure Build-up ….’) 1981). (Janbu, 1979).

6.15 Slide development as a function of brittleness index.

The brittleness index was defined by Bishop, (1967) as \( B_t = 1 - c_R/c_u \). The higher the value of \( B_t \), the more potential energy is released in the failure process, (Cf. Section 2.4). The effect is clearly illustrated in Figure 6:1.1 where a slope has been analyzed assuming five different values of the ratio \( c_R/c_u \). When the ratio \( c_R/c_u \) ranges from 0.3 to 0.7, the force required to initiate a slide varies between 95 to 120 kN/m. Accordingly, the value of \( N_{cr} \) is not extremely sensitive to the \( c_R \) to \( c_u \) ratio. By contrast, the risk and extent of global slope failure is radically affected by this parameter.

Considering that the corresponding ratios between \( q_{sfr}^{1-PFA} \) and \( q_{sfr}^{FrF} \) in Table 6:1 range from about 11 to 9, it may be concluded that conventional analysis greatly underestimates the risk of slope failure in deformation softening soils and that even for a wider range of the \( c_R/c_u \) values.

Figure 6:1.1 indicates that a local up-slope failure at \( c_R/c_u \) ratios greater than 0.7 will not generate static passive earth pressures sufficient to provoke disintegration and excessive heave of the ground further down the slope. A progressive failure would in such cases therefore only result in creep movements or limited displacements, settlements and cracks at the upper limit of the slide to be. (Compare the earth movement at Rävekärr in Section 5.5).

By contrast, in the scenarios based on \( c_R/c_u \) ratios less than 0.6, passive resistance is exceeded already by static build-up of forces, entailing massive heave and associated earth movements over long distances, as evidenced in most of the slides listed in Chapter 5, ‘Case records’.
The high velocities in the second dynamic phase of the slides tend to further reduce the residual shear strength $c_R$, enhancing the static and dynamic forces that determine the features of the finished global slope failure.

\[ \frac{c_R}{c_u} = 0.30 - 0.70, \quad \gamma_d = 2\% \quad \gamma_f = 6\% \quad \delta_{cr} = 0.3 \text{ m} \quad G_o = \frac{\tau_{el}}{\gamma_d} = 750 \text{ kN/m}^2 \]

$\tau_{el} = 15 \text{ kN/m}^2 \quad E_{slo} = 3G_o \quad = 100 - c_u = 2250 \text{ kN/m}^2$

$\rho \cdot g = 15.5 \text{ kN/m}^3 \quad k_o^{\text{max}} = 0.594 \text{ (computed)} \quad E_{\text{mean}} = 100 - c_{u,\text{mean}} = 2250 \text{ kN/m}^2$

**Figure 6:1.1** The diagram illustrates the static build-up of down-slope earth pressures for five different scenarios based on the $c_R/c_u$ ratios of 0.3, 0.4, 0.5, 0.6 and 0.7. Note that at $c_R/c_u = 0.5$, the length of the potential passive zone amounts to some 270 m. (In principle according to Bernander & Gustás, 1984.)

**Results – sensitivity study**

The results of varying the $c_R/c_u$ ratio are shown in table 6:1

<table>
<thead>
<tr>
<th>$c_R/c_u$</th>
<th>$N_{cr}$</th>
<th>$L_{cr}$</th>
<th>$E_{max}$</th>
<th>$E_{Rankine}$</th>
<th>$L_{E=E(Rankine)}$</th>
<th>$q_{PrF}^*$</th>
<th>$q_{I-PFA}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>120</td>
<td>117</td>
<td>904</td>
<td>1250</td>
<td>0 #</td>
<td>12.0</td>
<td>110</td>
</tr>
<tr>
<td>0.6</td>
<td>105</td>
<td>108</td>
<td>1180</td>
<td>1250</td>
<td>60 #</td>
<td>10.5</td>
<td>110</td>
</tr>
<tr>
<td>0.5</td>
<td>100</td>
<td>107</td>
<td>1600</td>
<td>1250</td>
<td>210 #</td>
<td>10.0</td>
<td>110</td>
</tr>
<tr>
<td>0.4</td>
<td>97</td>
<td>105</td>
<td>2130</td>
<td>1250</td>
<td>290 #</td>
<td>9.7</td>
<td>110</td>
</tr>
<tr>
<td>0.3</td>
<td>95</td>
<td>103</td>
<td>2800</td>
<td>1250</td>
<td>410 #</td>
<td>9.5</td>
<td>110</td>
</tr>
</tbody>
</table>

* These parameters denote the magnitude of the evenly distributed load on the ground surface that would initiate failure according to PrF and I-PFA analyses respectively.
Conclusion  It is evident from Figure 6.1.1 and Table 6:1 that if we really aspire to predict the possible outcome of a local instability in a slope, it is imperative that we devise methods designed to document the true soil behavior under the conditions that actually prevail during earth movements of this kind. The figure also indicates that progressive failure is conceivable in soft normally consolidated clays of moderate sensitivity, as postulated by Kjellman, 1954.

6.2  Britteness due to slope morphology – 'geometric brittleness'

![Diagram of Slopes a and b](image)

Figure 6.2.1a  **Slope a**: The ground and the assumed failure plane slope *linearly* from point A to point B

Figure 6.2.1b  **Slope b**: The ground and the assumed failure plane form *parabolic* curves from point A to point B. (In principle from Bernander & Svensk, 1982)
The risk of brittle failures in natural slopes is by no means restricted to the degree of deformation softening in the soil. Geometry and morphology of the soil profile greatly contribute to brittle behavior in the formation of slides.

Figures 6:2.1 a and b show two slopes with different profiles, in which all other relevant parameters are taken to be identical, including the elevations of points A and B and the distances between them. The parameters being identical, the safety factors against slope failure based on ideal-plastic failure analysis (I-PFA) are practically the same, despite the fact that the ground surface gradients, and the profiles of the assumed failure planes, vary differently between points A and B in the two cases.

In contrast to I-PFA, progressive analysis according to Chapter 4 reveals that the potential risk of disaster is very different in cases a and b. Hence, in terms of local instability, the safety factor at the $c_R/c_b$ ratio of 0.4 according to Equation 3:7 ($F_s^1 = N_{cr}/N_1$) is:

<table>
<thead>
<tr>
<th>Table 6:2.1</th>
<th>$c_R/c_b$</th>
<th>$F_s^1$</th>
<th>$L_{cr}$</th>
<th>$\delta_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>In the case of slope a (figure a)</td>
<td>0.4</td>
<td>286/N_i</td>
<td>88 m</td>
<td>0.093 m</td>
</tr>
<tr>
<td>In the case of slope b (figure b)</td>
<td>0.4</td>
<td>151/N_i</td>
<td>84 m</td>
<td>0.096 m</td>
</tr>
<tr>
<td>($\rho - g = 16 \text{ kN/m}^3$</td>
<td>$c_b = 25 \text{ kN/m}^2$</td>
<td>$E_{cl} = 75 \text{ c_{u,mean}}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In conclusion, the safety factor against slide initiation is about 2 times higher for slope a, than for slope b.

Moreover, with regard to the scale of disaster that might ensue in case a local failure is triggered by some unforeseen disturbance, the risk of an extensive passive zone developing is considerably greater in slope b than in slope a. (See Table 6:2.2). The criterion with regard to possible disintegration of the ground at the foot of the slope is according to Equation 3:8:

$$F_s^{II} = \frac{E_p}{E_x^{max}}$$

<table>
<thead>
<tr>
<th>Table 6:2.2</th>
<th>$c_R/c_b$</th>
<th>$E_p^{\text{Rankine}}/E_x^{\text{max}}$</th>
<th>$F_s^{II}$</th>
<th>Estimated length of potential passive zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hence, for slope a</td>
<td>0.4</td>
<td>1300/1287</td>
<td>1.01</td>
<td>0 m</td>
</tr>
<tr>
<td>for slope b</td>
<td>0.4</td>
<td>1300/1476</td>
<td>0.88</td>
<td>145 m</td>
</tr>
</tbody>
</table>

Impact of sedimentary structure

The structure of the soil strata formed in the sedimentation process obviously has a decisive impact on the development of landslides.

A specific, but not infrequent, situation arises when, as a result of out-wash from adjacent moraine or sandy beds, inclusions of coarse sediments have been laid down during the deposition of a clay formation. Discrete coarse layers of this kind often tend to occur in steep slopes that, at some epoch in the past, have constituted a shoreline environment. The presence of such layers of coarser collapsible material may, located as they often are in the steep upper parts of slopes, be highly conducive to progressive failure formation by the impact of vibrations. (Compare Sections 5.2, 5.5 and 5.6, concerning the ground movement at Rävekkärr, the great Surte slide and the slide at Trestyckevattnet).

Conclusions: Slope geometry and the morphology of sedimentary layers have a decisive impact on the formation of failure in slopes of deformation softening soils. Although the
safety factors of the two slopes shown in Figures 6:2.1a and b are virtually identical according to the conventional ideal-plastic concept, the inherent risks in terms of human life, social economy and property are dramatically different in the two investigated slopes a and b. It seems therefore evident that an analysis of slope stability on the basis of ideal-plastic behavior of clays should not be applied, unless the residual shear strength at large deformations is actually documented to be sufficiently high.

**Effect of morphology on creep deformations**

The geometry of a slope and the structure of the sub-ground significantly influence the effect of creep deformations on the values of $k_o = \sigma'_{s} / \sigma'_{v}$. The following conclusions were reached by the author of this report in a study of the effect of creep deformations in slopes. (‘Active Earth Pressure Build-up, a trigger mechanism... etc’, Bernander, 1981.)

a) Creep deformations in *uniform* slopes do not affect the distribution of strain and earth pressures or the distribution of the related $k_o$-values.

b) In contrast, creep deformations in *non-uniform* slopes can be shown to influence the accumulation of strain according to the law:

$$\varepsilon_x = \text{const} \cdot \beta_x^{-n} \cdot \frac{d\beta_x}{dx}$$

where $\beta_x$ is the slope gradient of firm bottom and $d\beta_x/dx$ is the curvature. $n$ is a value defining creep rate as a function of shear stress level, i.e. in principle as derived from Mitchell & Singh, 1968.

This implies for instance that with a value of $n = 2$, the impact of creep on $k_o$ values and earth pressures is most important in those parts of a slope, where the product of gradient and curvature has a maximum. Thus, increase of $k_o$ due to creep is typically most significant on the up-slope side of the foot of a slope.

A way of understanding this phenomenon in a qualitative sense, is to note that, as creep deformations in steep portions of a slope are faster and greater than those in gently sloping ground, tension or compression tends to occur in the transition zones between areas of markedly different slope inclinations.

c) The down-slope strains are associated with vertical strains causing either settlement or heave in the zones where strains accumulate.

As indicated by Equation 3:5b1 (or 3:8a) in Section 3.3, the magnitude and distribution of $k_o$ has an important impact on the risk of landslide formation.

**Conclusions:** Creep phenomena significantly affect the proneness to slope failure. (Cf. Chapter 8, Section 8.21, Assessment of $k_o$-values). Slow creep movements often tend to attenuate the effect of slope geometry on slope stability, whereas rapid creep in the active zone may be conducive to failure formation.

Another important implication of the occasional significant build-up of in situ earth pressures, and related higher values of $k_o$, is the consequential over-consolidation, in relation to the prevailing vertical effective stresses of the soil near the toe of a slope. The effect of this condition tends to even out the shear strengths in the soil profile with depth. The slope at Surte (Section 5.2, Case records) exemplifies this type of over-consolidation.
6.2 Britteness related to distribution and location of applied incremental loading

In the study of large planar landslides using I-PF analysis, the distribution of vertical and/or horizontal additional loading has little or no impact on the safety factor resulting from the calculations. This can of course only hold true if the soil is ideally plastic in reality, which is rarely the case even in moderately sensitive normally consolidated soft clays. By contrast, in deformation softening soils, the effect on slope stability of concentrated loading is to a great deal what progressive failure is all about. In other words, structures of brittle material react in a more brittle way to concentrated loads than to evenly distributed loads.

If for instance the force $N_i$, induced by the local fill in the example given in Section 3.3, had instead been evenly distributed over a major portion of the slope, the outcome of the analysis would have been radically different. However, the difference can readily be defined and quantified using the analytical model recommended in Chapter 4.

Load distribution is accordingly a cardinal issue in the analysis of slope stability in sensitive soils and must be addressed if we aspire to make reliable predictions of potential slope failures.

6.4 Britteness related to the timing of applied incremental loading

Another important factor in landslide mechanisms is the timing of the incremental load triggering a potential slope failure. Time-related effects on slide initiation can be accommodated in the Pr F analysis given in Chapter 4 by selecting the constitutive relationship (defined in Equation 4:6) in such a way that it is consistent with the current rate of load application.

For instance, pile-driving operations would normally require the use of undrained response in the soil, while drained soil parameters would be applicable in the case of a slowly constructed earth fill or refuse dump.

In general, long term stress deformation relationships tend to increase the critical length ($L_{cr}$) and the critical force ($N_{cr}$).

6.5 Britteness related to hydrological conditions

Although it is not within the scope of this report to engage in the above topic, it may in this context be important to remind of the fact, that it is as vital to consider the hydrological conditions in progressive failure analysis as in conventional methods of assessing slope stability.

Reference should also be made to the influence of climatic history and seasonal variations. Long spells of dry weather and extended periods of extreme precipitation tend to change the in situ earth pressures in a slope, subject to the manner in which the hydrological conditions act on creep movements. Ground water regimes may, for instance, substantially affect the drained shear strength parameters in different parts of the slope, thus creating changes of the distribution of the $k_o$ values over time.

As the values of $k_o$ influence the risk of global failure according to Equation 3:8 a, climatic history may well be a condition worth studying in this context. Also this phenomenon can readily be investigated in the proposed model for Pr F analysis of slope stability.
7. Agents prone to triggering progressive slope failure

7.1 General

It is not the author’s purpose to deal extensively with this subject. However, experience over the years has shown that some phenomena tend to be present or occur repeatedly in connection with downward progressive landslide events.

7.2 Failure initiation by natural causes

Natural phenomena susceptible of triggering landslides are:

- High pore water pressure build-up due to extreme precipitation
- Reduced effective shear strength because of high pore water pressures in discrete more permeable layers in the formation – in extreme cases liquefaction of collapsible soil
- Down-slope undercutting by erosion
- Change of soil properties due to leaching
- Change of soil properties due to chemical action affecting the cat-ions in clayey soils, thus entailing gradual loss of bonding between soil particles.

Although many landslides have actually occurred during, or subsequent to, periods of extreme precipitation, the problem with this disturbance agent is that it can in most cases be argued that the current adverse pore water pressure regime has most probably been exceeded over and over again in the past. It is therefore reasonable to assume that, when landslides eventuate during extremely wet spells, there must have been at least one other more recent contributing factor. This factor may of course be man-made but can also be an effect produced by any of the other agents mentioned above.

Bernander (1981) proposed a self-generated process named ‘Active Pressure Build-up - a Triggering Mechanism in Landslides in Sensitive Clays’, by which the additional forces emanate from the effects of accelerated creep on the horizontal stresses in the active transition zone between moving and firm ground. But even this failure mechanism requires an additional factor to explain why failure has not occurred before in the past millennia.

In many slides occurring during extreme wet periods, it has in effect been possible to identify or at least suspect the presence of other contributing agents, such as earth fills or subsoil construction work, that might have changed the hydrological regime in the upper part of the slope.

7.3 Failure initiation by man-made intervention

All of the slides treated in Chapter 5 ‘Case records’, except the Tuve slide and the slide at Sköttorp, took place while construction work was actually going on.

In SGI Report No 18 regarding the Tuve slide, the triggering mechanism is ascribed to the possible joint effect of a road embankment constructed some years before, a conceivable man-made modification of the hydraulic regime, and exceptionally high pore water pressures at the upper limit of the main slide.

The remaining 11 slides in Table 5:1 were undoubtedly caused directly by either earth construction work or pile driving. However, this does not mean that exceptionally high pore
water pressures did not contribute to some extent, which in the nature of things can rarely be determined after the event.
Man-made interventions susceptible of setting off progressive landslides may be listed as follows:

- Stockpiling of heavy materials, earth fills, construction of road embankments
- Excavation work straining the initiation zone in the lateral direction
- Soil compaction using heavy vibrators
- Installation of soil displacing sand drains
- Driving of soil displacing steel pipe or prefabricated concrete piles
- Sub-ground construction disturbing the prevailing ground water regime.

It stands to reason, that extreme hydrologic conditions are always potentially a contributing factor. In many instances, however, it might appear to be a question of semantics, whether a certain slide was released by this or that factor. Clearly, on the other hand, where man-made operations were actually ongoing at the time of the slide event, the slopes referred to above had in fact been standing for hundreds or thousands of years.
For instance, the critical, steep part of the slope at Surte had prevailed ever since it emerged from the sea some thousands of years ago. Yet, only driving of a few pre-cast piles for the foundation of a small house was evidently, as in many of the other documented cases, sufficient to trigger this catastrophic event. (Cf. Section 8.2, History of a slope.)

It would serve little purpose to elaborate here on the specific effects of the various man-made disturbances listed above, as it will be up to the investigator to assess their possible impact in each particular case.

Pile driving

Pile driving should be regarded as a particularly risky operation, because its action is threefold.
Firstly, the piles actually displace the soil in the down-slope direction by amounts varying from a few centimetres to several decimetres, thus possibly initiating slip surface formation and strain softening. This phenomenon alone is a powerful triggering mechanism. (See comments on this subject in Section 3.34).
Secondly, high pore water pressures are generated in clayey material. Several tenths of meters of excess pore water head have been recorded in connection with large piling jobs.
Thirdly, pressure wave radiation from hammer-blows or vibrating hammers tend to promote deformation softening or liquefaction even in soils of moderate relative density – not to mention the effect on possible strata of loose, collapsible cohesionless soils.
8. Principles and procedures for investigating potential landslides in slopes related to progressive failure mechanisms

8.1 General comments

Many a geotechnical engineer may ask himself why he should abandon the simple method of analysis offered by the concept of ideal-plastic limit equilibrium in favor of a significantly more complicated procedure, such as the one outlined in this chapter and Chapters 3 and 4?

The answer to this question is as follows:

Analyses based on unlimited ductility of the soil has limited validity in many practical applications, especially in long slopes made up of soft clays. It is, therefore, of vital importance to use methods by which down-slope displacements and shear deformations in the potentially sliding soil volume are accounted for, and linked with the deformations of the sub-base by means of constitutive stress/deformation relationships, for instance as advocated in Chapter 4. Such analyses make it possible to assess how the distribution of shear stresses, of earth pressures and displacements are affected by arbitrarily applied additional loading.

A crucial circumstance emerging from this type of analysis is that, contrary to the implications of ideal-plastic conditions, shear stresses due to concentrated loading may only be mobilized over a limited length (L_w) prior to the initiation of a slip surface, subsequent further deformation softening and formation of progressive failure.

Another important general condition revealed in this analytical modeling is the tendency of failures in sloping ground to propagate along planes parallel to firm bottom or along sedimentary layers, rather than along circular failure planes surfacing in the slope. Thus, slip circles emerging in sloping ground seldom turn out to be the critical ones in soft clays, and especially not in sensitive clays.

As mentioned in the Foreword, SGI Report No 10 contained nine diverging explanations by different authors of the great Tuve slide, which if SGI's own version in Report No 18 is included, adds up to some 10 different approaches to defining the causes of the slide. The Surte slide was treated in two comprehensive reports as well as by other workers. All these accounts were, as mentioned earlier, contradictory on essential issues and, in the opinion of the author of this report, inconclusive.

In contrast to this, analyses of the Surte and the Tuve slides in line with Chapters 3 and 4 provide straightforward explanations of the widespread passive zones in almost horizontal ground. This feature cannot be explained on the basis of ideal-plastic soil properties, but emerges as a direct, compelling result from analyses based on deformation softening.

It follows, therefore, that when considering slope stability, the compatibility of deformations, e.g. along the principles highlighted in Chapter 4, must be addressed whenever a soil does not meet the ductility requirements for the current application.

In practice, this means that progressive failure analysis should be performed in all investigations of long slopes made up of soils exhibiting significant loss of shear strength at large deformations.

The procedure advocated below is exemplified in the studies regarding the slides at Surte, Tuve and Trestyckevattnet given in Chapter 5, Case Records.
8.2 Investigation procedure

History of a slope

The stability conditions in natural slopes are closely related to their geological and hydrological history. Hence, many clay slopes in western Sweden are made up of glacial and post-glacial sediments that emerged from the regressing sea after the last glaciation. As the ground by degrees rose above the sea level, the strength properties of the soils and the earth pressures in the slope have, by way of consolidation and creep, gradually accommodated to the increasing loads. These may have resulted from the retreating free water level, falling ground water tables, varying climatic conditions and chemical deterioration.

In consequence, every existing slope is inherently stable by some unknown factor of safety, which in view of extreme high ground water events during past millennia, at least by some measure may be assumed to exceed the value of 1 under currently prevailing conditions. Now, the vital issue for the engineer responsible of investigating the stability of the slope is then: How will the slope respond to additional loading or disturbance effects, for which the ‘time horizon’ is measured in terms of days, weeks or months instead of hundreds or thousands of years?

Finally, if local failure is conceivable, what degree of catastrophe is likely to ensue? Will local instability just result in a minor crack in the ground up-slope or terminate in a colossal landslide displacing hundreds of meters of horizontal ground over large distances?

The proposed analysis according to Chapter 4 provides a means of studying these questions.

8.20 Assessment of Critical length

Referring to Sections 3.4 and 4.6 concerning the significance of the ratio between the critical length \( L_{cr} \) and the length of the prospective slide \( L \), it is recommended that any slope stability investigation of longer natural slopes should begin with, at least, a rough assessment of the critical length \( L_{cr} \). This applies particularly in cases of concentrated additional loading. Low values of \( L_{cr}/L \), signal risk of progressive failure formation in respect of concentrated loading effects and thus constitute a measure of the adequacy of applying the conventional ideal-plastic approach to the specific slope under investigation.

The fact, that the distance down-slope of a local load, along which the additional shear stresses in the potential failure zone can be mobilized, is limited has a crucial implication in this context. Thus, at a distance of \( L_{cr} \) (as defined in Section 3.3) from the point of load application, the effect of the additional load is no longer felt in terms of earth pressures and displacements. This circumstance rules out or reduces the possibility of utilizing earth pressure resistance (e.g. in passive Rankine state) in less sloping ground downhill for the stabilization of local up-slope loads. See section 8.22 below.

The important implication of not being able to utilize passive resistance further down the slope is that the failure resistance along planes parallel to the ground profile, along firm bottom or sedimentary layers may - subject to the degree of strain softening - be considerably less than the resistance based on failure planes surfacing in slope closer to the local load. (Cf. Section 4.6)
8.21 **In situ condition** - Assessment of in situ $k_o$ – values

As indicated above in *History of a slope*, the safety factor against slope failure and disintegration of the potential passive zone significantly depends on the prevailing in situ distribution of earth pressures. Hence, an estimate of the values of $k_o$ in Equation 3:8 should be made.

These values may be chosen empirically on the basis of past experience. However, they may also be calculated according to Chapter 4, in which case a reasonable long term stress/deformation relationship is applied; the basic idea being that creep in a slope can be regarded as an extremely *slow progressive failure*. In the absence of specific tests related to a creep situation, long term shear strength and ideal-plastic properties in the soil may be presumed. (Cf. Section 6.2, ‘Effects of morphology on creep deformations’.)

The input value of $k_o$ may, in this phase of the analysis be taken as the $k_o$-value considered to be valid for the current soil in horizontal ground, i.e. typically 0.5 to 0.7 for normally consolidated clays.

The boundary condition in this calculation is that the force $N_i$ must be consistent with the conditions at the upper limit of the potentially sliding soil volume. (See Figures 4:5.1, 5:1.7 and 5:2.4, regarding curves for in situ $E_i(x)$).

8.22 **Disturbance condition** - assessment of the critical load susceptible of initiating progressive failure

Progressive failure is conceivable *only if in part of the slope*, at some *point in time*, the *residual shear strength* falls below the *prevailing in situ shear stress* because of some additional loading effect or disturbance i.e.

$$c_R(x) - \tau_o(x) \leq 0 \quad \text{(Equ.3:2)}$$

The critical additional load ($N_{cr}$) can be calculated using the values of $k_o$ established as above by repeating the procedure in accordance with Section 4.3. The object of this exercise is to define how far from the applied loads the shear stresses in the failure zone can be mobilized, before the prevailing in situ stress $\tau_o$ exceeds the available post-peak shear strength, $c_R(x)$. The boundary criterion for this part of the analysis is therefore, that the condition defined by the differential $\tau_o - c_R = 0$, shall apply at the upper limit of the potential slide. (See Figure 3:3.3.)

Another critical condition is the deformation $\delta_{cr}$ at which the slope fails even if there is no sustained force pursuing the incipient failure. Driving of soil displacing piles constitutes a case, when this criterion may be applicable. (Cf Section 3.34).

$$\delta_N < \delta_{instab}$$

The constitutive relationship to be used in this study must be compatible with the type and rate of the applied loading. Pile driving may for instance produce undrained response in the soil, while for a slowly built up embankment or stockpile, drained parameters may be appropriate.

The main results from this phase of the stability investigation are
- Critical additional load, $N_{cr}$
- Shear stress distribution and critical lengths, $L_{cr}$, $L_{instab}$
- Critical displacements at the upper slide limit, $\delta_{cr}$.

(See Section 3.3, Stability conditions prior to local failure).
If the current combination of the loads $N_i$, $q$, and $t$, as defined in Figures 4.2.1 and 4.2.2 exceed the corresponding critical combination of these loads, then a progressive failure is triggered.

The safety factor against such a failure is

$$F_s^i = \frac{N_{cr}(N_i, q, t)}{(N_i, q, t)} \quad \text{(Equation 3.7)}$$

The procedure is exemplified in detail in Appendix I.

Note: At this point another check should be made with regard to the possibility of failure along a plane surfacing immediately down-slope of the area subjected to the additional loading: $E_\sigma = E_0 + \Delta N_\sigma < k_o \cdot \gamma \cdot H^2 / 2 + 2 \cdot \sqrt{2} \cdot \phi \cdot c_u \cdot dz \quad \text{(Equation 3.3)}$

This criterion defines one of the prerequisite conditions for the formation of progressive failure. If the resistance along a failure plane parallel to the ground surface or along firm bottom is lower than the local passive resistance, the failure will propagate along that plane. As mentioned above, this condition is usually fulfilled in sensitive soils and sloping ground, as slip planes surfacing in the slope then do not constitute the most critical failure modes.

Thus shorter failure planes and slip circles, i.e. failure modes for which ideal plastic analysis may still be valid as such, seldom constitute the critical failure modes in long slopes of deformations softening soil. This fact invalidates in many applications the use of the conventional ideal-plastic approach for defining the risk of slide initiation. (Cf. Sections 3.32 b), 3.4, and 4.6 as well as Sections 5.1 and 5.2 'Case records').

Inversely, provided the condition $[c_R(x) - c_u(x) \leq 0]$ applies, the permissible load effect computed on the basis of progressive failure formation is generally significantly smaller than the corresponding load established by conventional ideal-plastic analysis, even when using failure planes of moderate length. Frequently, the load effects from the two types of analysis are not even in the same order of magnitude. It may be noted that this condition tends to become more pronounced in drained analysis considering that high pore water heads are more likely to spread along layers in the sedimentary soil structure than across (or at some angle to) the same. This applies in principle also to high values of the brittleness ratio ($c_R/c_u$).

**Choice of $c_R/c_u$ - value**

The values of $N_{cr}$ and $L_{cr}$ depend on the $c_R/c_u$ - ratio but, as indicated in Sections 5.1, 5.2 and Figure 6:1.1, they are not particularly sensitive even to important variations of the brittleness number. Therefore, when investigating a long slope of marked deformation softening soils, it may be wise to adopt a reasonably low value for $c_R/c_u$. The reason for this is that, although higher values close to 1.0 (i.e. the ideal-plastic condition) may well be valid in a stable long term situation, short term disturbance agents causing accelerated creep deformations can readily entail drastic reductions of the $c_R/c_u$ - ratio.

The choice of input values of $c_R/c_u$ in the disturbance situation must therefore always be made by the investigating engineer with due consideration to all specific factors applying in the current case.

8.23 Global failure condition – assessment of the possibility of a second state of equilibrium subsequent to progressive failure.

The objective in this part of the study is to ascertain if there is a possible state of static equilibrium subsequent to the redistribution of earth pressures effectuated by a dynamic
progressive failure. In this phase it is assumed that the potentially sliding soil mass, at least momentarily, retains its geometrical shape before possible disintegration in passive Rankine failure. The significance of this second stage of equilibrium is that it constitutes a measure of what may happen if the critical load according to the foregoing section is exceeded, i.e. will the progressive failure result in a veritable landslide or not?

\[
F^H_s = \frac{E^{\text{Rankine}}}{(E_0 + N_{cr})}
\]  
(Equation 3.8)

It is recommended that such investigations be carried out irrespective of the degree of risk with regard to local failure, as it must always be of vital interest to estimate the consequences of such a failure being triggered due to unknown or uncontrolled circumstances. It should be observed that the sought stage of equilibrium may be real or hypothetical depending on whether the build-up of earth pressures in the gentler gradients at the foot of the slope exceed passive resistance or not. (Cf. equation 3.8)

If there, the passive resistance is not exceeded, the incipient landslide will come to an end resulting in limited displacements at the upper end of the slope studied. (Compare with the slide at Rävekärr, Section 5.5).

On the other hand, if the computed static earth pressures exceed passive resistance, the second stage of equilibrium will be transient or hypothetical - the progressive phase merging into the truly dynamic phase of the slide. At this point, the heave of the ground in the passive zone provides a prerequisite for the soil masses further up the slope to move downwards at an accelerated pace. The landslide proper is set in motion.

Note: If additional, more accurate, predictions of the slide events are deemed necessary, they will have to be made on the basis of Newton's laws of motion as exemplified in Section 5.1.

8.24 Final comments

Slope stability analysis, as outlined in this report, has among other the following merits:

- the study pinpoints weak features in the slope structure, making it possible to assess distribution of shear stresses and quantification of deformations;
- appropriate safety factors may be defined;
- the ratio of total length (of the investigated part of the slope) to critical length, (i.e. \( L_{\text{total}}/L_{cr} \)) provides a quick indication of to what extent conventional I-PF analysis is applicable in the current case;
- it discloses indispensable information about the risk and the consequences associated with a local instability, making it possible to estimate the degree of potential catastrophe involved. Such information is by definition not accessible in the ideal-plastic limit equilibrium approach.

As mentioned in Section 5.2, Pr F analyses as per Chapter 4, show conclusively that the slope at Surte harbored a primordial weakness, allowing a disturbance caused by a minor piling job to trigger a 600 m long landslide in a residential area. Notably, the effect of the piling job was at the time considered to be inconsequential by most engineers concerned, including several of the experts who subsequently investigated the causes of the disaster.
9. Summary

General

The spread and final morphology of many landslides in Scandinavian soft clays cannot be explained on the basis of the - in practical engineering - currently applied concept of ideal-plastic limit equilibrium. Possibly because of this, the discrepancy between actual slide events and the results of back-analyses provides a prolific background for all sorts of imaginative and speculative failure concepts. For instance, the great Tuve slide near Gothenburg generated about 10 different explanations of the slide events by engineers of the profession.

In the conventional ‘ideal-plastic failure’ analysis (referred to in this report as I-PFA), deformations inside and outside the studied soil volume are disregarded entirely, as it may seem for the sake of simplicity. This means that the soil is presumed to be ideally plastic for all practical purposes.

An important contention in this report is that poor correlation between theory and practice in this particular field of geotechnical engineering derives from the fact that many soils are markedly softening in the range of the differential deformations actually occurring between a potentially sliding soil mass and the sub-ground. The same applies to differences of deformation that may develop within the sliding body. This relates in particular to potentially extensive slides in sensitive clays.

Analysis considering relevant deformations in the slope

The report accordingly focuses on the possibility of progressive or brittle failures occurring in slopes with deformation softening soils. A finite difference method (FDM) is applied for the numerical analysis of progressive failure (Pr F) formation.

The procedure resembles that of conventional I-PF modeling in so far as potential failure planes are presumed as being known. The most critical conditions are therefore - as in conventional stability calculations - found by ‘trial and error’ and iteration procedures. Nevertheless, the proposed analysis differs from ideal-plastic limit equilibrium methods in a number of key aspects:

- Whereas in the ideal-plastic failure approach, the equilibrium of the entire potential sliding body of soil is investigated, the Pr F- analysis focuses on the equilibrium of each individual vertical element into which the body is subdivided.

- Moreover, the main deformations within and outside the potentially sliding soil mass are considered. Hence, axial displacements in the slide direction due to earth pressure changes in the slope are at all times maintained compatible with the shear deformations of the discrete vertical elements. In doing so, it is possible to define the distribution of shear stresses from concentrated loads and the extent to which shear capacity can be mobilized along the potential failure plane. The fact that the shear deformations are determined in two directions allows modeling of the entire incipient failure zone as a thick structural layer, and not just as a discrete shear band (or a slip surface). This is a crucial feature in the used analytical approach.

- The shear properties of the soil are defined by a complete non-linear stress/strain curve and not just by a discrete shear strength parameter as in normal limit equilibrium calculations. This constitutive relationship is divided into two stages (I and II), simulating the conditions
before and after the formation of a slip surface. The stress/deformation relationships may be chosen so as to suit the conditions in different parts of the slope.

- By relating stress/deformation relationships to different timings of loading or rates of stress application, the time factor can effectively be incorporated into the analysis.
- Local horizontal or vertical loads as well as local conditions in the slope morphology that may be conducive to failure formation, can be considered.
- Although the elevation of a failure plane is presumed as being known, the final extent of the failure plane and the length of the passive zone in failure emerge as results of the computations.

**Implications of the proposed FDM deformation analysis of slope stability**

Progressive failure, due to some additional loading effect or disturbance, is conceivable only if in part of the slope, at some point in time, the residual shear strength (c<sub>R</sub>) falls below the prevailing in situ shear stress (τ<sub>o</sub>) i.e. c<sub>R</sub>(x) - τ<sub>o</sub>(x) ≤ 0.

The analysis proposed highlights the necessity of considering deformations in potentially extensive slides in long slopes, and indicates that neglecting to do so may in many cases result in total misjudgment of a stability situation. The results of the analysis make it possible to identify the most critical conditions in a slope, allowing preventive measures to focus on pertinent issues related to distribution and timing of loads, geometry, varying soil properties etc.

The fact that the distance, down-slope of a local load, along which the additional shear stresses in the potential failure zone can be mobilized, is limited has a crucial consequence. At a distance of L<sub>cr</sub> (as defined in Section 3.3) from the point of load application, the effect of the additional load is no longer felt in terms of earth pressures and displacements. This circumstance rules out or reduces the possibility of utilizing earth pressure resistance (prior to progressive failure) in less sloping ground further downhill for the stabilization of local upslope loads. (However, in the post progressive failure stage, passive Rankine resistance can be mobilized and in some cases prevent further slide development).

The implication of not being able to mobilize passive resistance further down the slope is of great importance in the formation of initial failure. It means that the resistances against failure along planes parallel to the ground, to firm bottom or along weak sedimentary strata are, subject to the degree of strain softening, considerably less than the resistances based on failure planes surfacing in the sloping ground closer to the local load. Notably, the proposed analysis considering deformations shows that this tendency may also apply to high values of the brittleness ratio (c<sub>r</sub>/c<sub>o</sub>), in fact, initially it even applies when ideal-plastic properties are ascribed to the soil. This is evident in view of the fact that mobilization of passive resistance requires sizable displacement.

Thus, shorter failure planes and slip circles, i.e. failure modes for which the ideal-plastic approach may well be valid as such, seldom constitute the most critical failure modes in long slopes of deformations softening soil. This circumstance invalidates in many applications the use of the conventional ideal-plastic approach for identifying the initiating slide effect. The discrepancy in this respect tends to become more pronounced in drained analysis considering that high pore water heads are more likely to spread along sedimentary layers than across (or at some angle to) the same.

The proposed FDM-model for analysis of downward progressive slope failures enables consideration also of deformations below the assumed failure plane. However, as indicated
above, the fact that passive resistance further down-slope cannot be mobilized at a distance greater than $L_{cr}$ for stabilizing local additional loads predicts that *failure planes primarily tend to develop along the steep of firm bottom*, even to great depth below the ground surface.

Another result of interest is that the ratio between the critical length $L_{cr}$ and the total length of the prospective slide ($L$), offers an indication regarding the validity, in a current case, of analyses based on ideal-plastic soil properties. This applies particularly with regard to local additional loading effects. For low values of $L_{cr}/L$, such analyses are prone to yield poor predictions of slope stability. (See e.g. the slides at Surte, Tuve and Bekkelaget in Chapter 5, Case records).

In consequence, slope stability investigations of longer natural slopes in deformation softening soils should at least include a rough assessment of the critical length $L_{cr}$.

*Factors conducive to brittle failure in slopes*

Progressive failure analysis according to Chapter 4 also highlights the fact that there are several conditions, other than the inherent brittleness of the in situ soil, that are conducive to brittle slope failures. These conditions, which are specially dealt with in Chapters 6 and 7 are *inter alia*:
- Ground and slope morphology and profile of slip surface – ‘geometric brittleness’
- nature and distribution of applied incremental loading
- type, location and timing of the agents initiating failure
- hydrological conditions and hydrological history.

*Safety factors*

In the context of progressive failure analyses of landslide hazards, the conventional safety factors currently used in stability investigations are devoid of physical meaning. Hence, new formulations, which address the critical conditions in slopes with regard to formation of progressive failure, are proposed.

*Why apply progressive failure analysis?*

The stability conditions in natural slopes are closely related to their geological and hydrological history. Hence, many clay slopes in western Sweden are made up of glacial and post-glacial sediments that emerged from the regressing sea after the last glaciation. As the ground gradually rose above the sea level, the strength properties of the soils and the earth pressures in the slope gradually accommodated to the increasing loads by way of consolidation and creep. These loading effects may have resulted from retreating free water levels, falling ground water tables, varying climatic conditions and chemical deterioration. In consequence, every existing slope is inherently stable by some unknown factor of safety, which in view of extreme high ground water events during past millennia, at least by some measure may be assumed to exceed the value of ‘1’ under currently prevailing conditions.

Now, the vital challenge to the engineer responsible of investigating the stability of a slope is to study how the slope will respond to *additional loading* applied at rates, for which the ‘time horizon’ is measured in terms of days, weeks or months instead of hundreds or thousands of years?
Finally, if local failure does take place, what *degree of catastrophe* is likely to ensue? Will local instability just result in a minor crack in the ground up-slope or terminate in a colossal landslide displacing hundreds of meters of horizontal ground over large distances? The proposed analysis according to Chapter 4 provides a means of studying these questions.

Progressive failure analyses explain in a straightforward way why, in many Scandinavian landslides, local disturbances caused by human activity have developed into huge catastrophic slides, involving hundreds of meters of inherently stable ground. For instance, the specific morphology of the finished Tuve and the Surte slides, featuring immense passive zones in almost horizontal ground, materializes as a clear result from the computational analyses. The fact that prediction of the final extent of a potential landslide can be made is of great importance for assessing the risks and stakes involved, making it possible to evaluate the comprehensiveness and cost of reasonable measures designed to forestall landslide hazards. Another important feature in this analysis is the ability to pinpoint and predict the possible consequences of man-made interference in critical portions of a slope.

Consideration of deformation softening in slope stability assessments generally results in considerably higher computed risk of slope failure than that emerging from the conventional ideal-plastic approach. This applies even to wide variation (within reason) of chosen constitutional relationships, the *decisive issue* being whether the conditions in the slope are such that a local disturbance agent is susceptible of inducing a *critical state of deformation softening* in the soil or not. Common disturbance agents are additional loading, forced deformations (e.g. due to piling) and extreme excess pore water pressure regimes. These circumstances should be contemplated whenever soils exhibiting markedly deformation softening behavior are encountered.

Although not difficult in principle, progressive failure analysis, as described in Chapter 4, may appear as an excessive complication of slope stability analysis to practicing geotechnical engineers. The constitutive relationships of the sensitive soils have to be known reasonably well, depending as they do of various factors, among which the timing of loading and state of principal stresses are of significant importance.

*However, if we are serious in the purpose of making valid predictions of risk in terms of human life, property and other social economic losses then, in the opinion of the author of this report, these complications should be addressed.*

As may be concluded from the calculations demonstrated in Section 4.4 and in Appendix I, hand calculations are, albeit being simple in principle, too laborious to be practicable. However, using computers, the time needed to perform the numerical computations, is insignificant. Once the appropriate in-put data have been established, the complete computational study of a loading case, related to a specific failure plane, is a matter of only a few seconds.

The additional effort that may have to be dedicated to investigations of slope stability along these lines is, therefore, only to a *minor* extent made up of increased computational work.

*The principal challenge lies in exploiting the enhanced possibilities of identifying the effects on slope stability of a number of factors, which by definition cannot be studied using the conventional ideal-plastic failure (I-PF) approach.*
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<tr>
<td>Hirata T</td>
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<td>Non-associated Plasticity for Soils, Concrete and Rock</td>
<td>Heron, Vol. 29 (Special No).</td>
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Appendix I  Exemplification of calculation procedure

General comments

The example presented below serves to demonstrate a calculation procedure and does not recommend or prescribe generally applicable laws of soil behavior. In fact, one of the main advantages of the approach described is that it can accommodate any defined shear stress/deformation property in the soil, that the geotechnical engineer may wish to apply to the situation studied.

For demonstration purposes, essential sections of the example have been computed manually. Because of limited space, all iterative computations cannot, however, possibly be shown and have been performed in a computer spreadsheet. Thus, many of the repetitive steps and iterations are only presented as input data and results. Nevertheless, the interested engineer should readily be able to follow the computation procedure and how calculations in progressive failure are actually performed under the Finite Difference Method (FDM).

The numerical calculations are easily carried out in a computer program such as Excel, but when dealing with slope stability on a regular basis, the advantage in time and cost of programming an integrated computer version of this FDM is unquestionable. It should be born in mind that, once data have been inserted, the numerical computations take little time.

I:1  Calculation of local stability – \( N_{cr} \), \( L_{cr} \) and associated stresses and deformations

I:11  Slope data

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{slope_data.png}
\caption{Assumed data in the current example: \( \rho g = 17 \text{ kN/m}^3 \), \( k_o = \sigma_0/\sigma_v \approx \text{constant} \). \( \Delta E_o/\Delta x \approx 0 \).}
\end{figure}

\begin{itemize}
\item \( \rho g = 17 \text{ kN/m}^3 \)
\item \( k_o = \sigma_0/\sigma_v \approx \text{constant} \)
\item \( \Delta E_o/\Delta x \approx 0 \)
\end{itemize}

\begin{itemize}
\item \( H = 20 \text{ m} \)
\item \( b = 1 \text{ m} \)
\item \( \beta = 4.2^\circ \)
\end{itemize}

Figure I: 1.1 The upper part of the slope, which is analyzed in the example. The gradient corresponds approximately to the mean gradient of the upper part of the main slide at Tuve.
It is important to note that the simplifications, implying the use of fixed values for some of the variable parameters in the example with respect to the co-ordinate (x), were introduced here only in order to make calculations and results more transparent to the reader. In fact, varying these parameters arbitrarily is accommodated by the applied differential equations and does not complicate the computations significantly, especially if a computerized version is used. Just about any desired configuration of ground surface and of failure planes can thus be treated.

I: 12 Constitutive shear deformation relationships

The general constitutive relationship \( \tau_{xz} = \phi (\gamma_{xz}, \delta_F, \text{d} \delta_F/\text{d}t) \), as in equation 4.6, may in the range \( 0 < \gamma < \gamma_f \) be defined by the inverse expression \( \gamma_{xz} = \phi_1 (\tau_{xz}) \) ..........4.6a

Assumed data in the current example:

\[
\begin{align*}
\sigma_u &= 30 \text{ kN/m}^2 & c_{uR}/c_u &= 0.50 & \tau_{el} &= 20 \text{ kN/m}^2 & E_{el} &= 44.4 \cdot c_{u,\text{mean}} = 1000 \text{ kN/m}^2 \\
c_s &= 15 \text{ kN/m}^2 & \gamma_f &= 7.5\% & \gamma_{el} &= 3.75\% & G_{el} &= \tau_{el}/\gamma_{el} = 533 \text{ kN/m}^2 \\
c_{u,\text{mean}} &= 22.5 \text{ kN/m}^2
\end{align*}
\]

Elastic range
In the range \( 0 < \gamma_{xz} < \gamma_{el} \) (i.e. for \( 0 < \tau_{xz} < \tau_{el} \)), the relationship between shear stress and deviator strain is taken to be linear.

\[
\begin{align*}
\tau_x &= G \cdot \gamma_x & \gamma_x &= \tau_x/G \\
\Delta\gamma_{xz} &= \Delta\tau_{xz}/G
\end{align*}
\]

where \( G = \tau_{el}/\gamma_{el} \) (\( \tau_{el} \) and \( \gamma_{el} \) are shear stress and shear strain at the elastic limit as defined in figure I:1.2)

Non-linear range \( \gamma \)
In the non-linear range, where \( \gamma_{el} < \gamma_{xz} < \gamma_f \) (i.e. for \( \tau_{el} < \tau_{xz} < \sigma_u \)), the relationship between shear stress and deviatoric strain is taken to be a 2nd power parabolic relationship with its vertex at point \( (\gamma_f, c_u) \), as in Figure I:1.2.

![Figure I:1.2 Constitutive shear stress / deformation relationships. It may be noted here that the ratio of \( \tau_{el}/c_u \) is assumed to be constant when \( c_u \) varies with the coordinate (z).](image-url)
Hence
\[ \tau_{xz} - \tau_{el} = 2 \left( \c_{u} - \tau_{el} \right) \cdot \left[ \frac{\gamma_{x}\gamma_{x} - \gamma_{y}\gamma_{y}}{\gamma_{x} - \gamma_{y}} \right] - \left( \c_{u} - \tau_{el} \right) \left[ \frac{\gamma_{x}\gamma_{x} - \gamma_{y}\gamma_{y}}{\gamma_{x} - \gamma_{y}} \right] \]
\[ \text{or} \quad \left( \gamma_{x} - \gamma_{y} \right)^2 - 2 \left[ \frac{\gamma_{x}\gamma_{x} - \gamma_{y}\gamma_{y}}{\gamma_{x} - \gamma_{y}} \right] + \left( \tau_{xz} - \tau_{el} \right) \left( \c_{u} - \tau_{el} \right) = 0 \]
\[ \text{or} \quad \left( \gamma_{x} - \gamma_{y} \right)^2 - 2 \left[ \frac{\gamma_{x}\gamma_{x} - \gamma_{y}\gamma_{y}}{\gamma_{x} - \gamma_{y}} \right] + \left( \tau_{xz} - \tau_{el} \right) \left( \c_{u} - \tau_{el} \right) = 0 \]

The solution to equation 1.2a is
\[ \left( \gamma_{x} - \gamma_{y} \right)^2 - 2 \left[ \frac{\gamma_{x}\gamma_{x} - \gamma_{y}\gamma_{y}}{\gamma_{x} - \gamma_{y}} \right] + \left( \tau_{xz} - \tau_{el} \right) \left( \c_{u} - \tau_{el} \right) = 0 \]
\[ \text{or} \quad \left( \gamma_{x} - \gamma_{y} \right)^2 - 2 \left[ \frac{\gamma_{x}\gamma_{x} - \gamma_{y}\gamma_{y}}{\gamma_{x} - \gamma_{y}} \right] + \left( \tau_{xz} - \tau_{el} \right) \left( \c_{u} - \tau_{el} \right) = 0 \]

Equation 2b may be transformed to:
\[ \gamma_{x} = \gamma_{y} - \frac{1 - \left( \tau_{xz} - \tau_{el} \right) / \left( \c_{u} - \tau_{el} \right)}{1 / \left( \c_{u} - \tau_{el} \right)} \]
\[ \text{or} \quad \gamma_{x} = \gamma_{y} - \frac{1 - \left( \tau_{xz} - \tau_{el} \right) / \left( \c_{u} - \tau_{el} \right)}{1 / \left( \c_{u} - \tau_{el} \right)} \]

\[ \gamma_{x} = \gamma_{y} - \frac{1 - \left( \tau_{xz} - \tau_{el} \right) / \left( \c_{u} - \tau_{el} \right)}{1 / \left( \c_{u} - \tau_{el} \right)} \]

Check: For \( \tau_{xz} = \tau_{el} \to \gamma_{x,z} = \gamma_{el} \) and
\[ \gamma_{x,z} = \gamma_{y} - \frac{1 - \left( \tau_{xz} - \tau_{el} \right) / \left( \c_{u} - \tau_{el} \right)}{1 / \left( \c_{u} - \tau_{el} \right)} \]

In order to establish continuity between the two curves at the point defining the elastic limit, the following condition must be satisfied:
\[ \frac{\gamma_{el}}{\gamma_{x,z}} = \frac{2 \left( \c_{u} - \tau_{el} \right) / \left( \gamma_{y} - \gamma_{x} \right)}{2 \left( \c_{u} - \tau_{el} \right) / \left( \gamma_{y} - \gamma_{x} \right)} \]
\[ \text{i.e.} \quad \gamma_{el} = \gamma_{y} - \tau_{el} / \left( 2 \c_{u} - \tau_{el} \right) \]
\[ \gamma_{el} = \gamma_{y} - \tau_{el} / \left( 2 \c_{u} - \tau_{el} \right) \]

E.g. in the current case, \( \gamma_{el} = 0.075-20/(2.30-20) = 0.0375 = 3.75 \% \)

Differentiating equation 1.2c with respect to \( \tau_{xz} \), we get
\[ \Delta\gamma_{x,z} = \left( \frac{\gamma_{y} - \gamma_{x}}{\gamma_{x} - \gamma_{y}} \right) \left[ 1 - \left( \tau_{x(z,n)} - \tau_{el} \right) / \left( \c_{u} - \tau_{el} \right) \right]^{1/2} - \left[ 1 - \left( \tau_{x(z,n+1)} - \tau_{el} \right) / \left( \c_{u} - \tau_{el} \right) \right]^{1/2} \]
\[ \Delta\gamma_{x,z} = \left( \frac{\gamma_{y} - \gamma_{x}}{\gamma_{x} - \gamma_{y}} \right) \left[ 1 - \left( \tau_{x(z,n)} - \tau_{el} \right) / \left( \c_{u} - \tau_{el} \right) \right]^{1/2} - \left[ 1 - \left( \tau_{x(z,n+1)} - \tau_{el} \right) / \left( \c_{u} - \tau_{el} \right) \right]^{1/2} \]

Where \( \tau_{x(z,n)} \) and \( \tau_{x(z,n+1)} \) denote the shear stresses in elements \( (n) \) and \( (n+1) \)

(Note: The parabolic relationship to the power of 2, which is used here for practical reasons may of course be replaced by any other relationship considered appropriate by the investigating engineer. However, the issue has little bearing on the results of the analysis.)

Combined elastic and non-linear range

When the current stress range spans across the transition point between elastic and non-linear behavior, then the following expression derived from equations 1.1a and 1.4, is used:
\[ \Delta\gamma_{x,z} = \left( \gamma_{x,z} - \gamma_{x(z,n)} \right) / \left( \gamma_{x} - \gamma_{y} \right) \left[ 1 - \left( \tau_{x(z,n)} - \tau_{el} \right) / \left( \c_{u} - \tau_{el} \right) \right]^{1/2} \]

Post peak non-linear range: \( \gamma_{nom} > \gamma_{f} \) i.e. \( \c_{u} > \tau_{z} > \c_{R}^{min} \) \( (0 < \delta_{S,z} < \delta_{CR}) \)

The post peak shear strength (= mobilised shear stress) \( \c_{R,x} \) is now set as a function of \( \delta_{S,z} \) according to figure 1.12. Hence for \( 0 < \delta_{S,z} < \delta_{CR} \), (where \( \delta_{CR} \) in this case is taken to be \( 0.30 \) m).

According to figure 1.12, we derive
\[ \delta_{S,z} / \delta_{CR} = \left( \c_{u} - \tau_{x} \right) / \left( \c_{u} - \c_{R} \right) \]

or
\[ \delta_{S,z} / \delta_{CR} = \left( \c_{u} - \tau_{x} \right) / \left( \c_{u} - \c_{R} \right) \]
\[ \delta_{S}(\tau_x) = \delta_{cR} \cdot (c_u - \tau_x)/(c_u - c_{ur}) \quad \text{...............I: 5a} \]

where \( \delta_{S} \) (x) is the total slip.

\[ \delta_{cR} = \text{the slip at which minimum residual shear strength } c_{R} \text{ is attained} \]

\[ \delta_{S}(c_{R}) = \text{elastic rebound at failure plane due to un-loading from } c_u \text{ to } c_{u(x)} \]

\[ \delta^{el}_{cR} = \text{elastic rebound at failure plane when unloading from } c_u \text{ to } c_{ur} \]

Thus \( \delta^{el}_{cR} = \sum_{i=1}^{n} \frac{(c_{u(x)} - c_{R,i})}{G \cdot dz} \). (See Figure I:1.2)

Check: for \( \tau_x = c_{ur} \rightarrow \delta_{S}(c_{R}) = \delta_{cR} \) and for \( \tau_x = c_u \rightarrow \delta_{S}(\tau_x) = 0 \)

**I:13 Calculation of critical initial load \((N_{o})\)**

Using equation 4.2, the value of \( \tau_{ox} \) is determined \((b = 1 \text{ m})\)

\[ \tau_{o(x,0)} = \left[ \sum_{i=1}^{n} H_{i(x)} \cdot g \cdot p(z) \cdot 1 \cdot \Delta z \right] \cdot \sin \beta(x) - \Delta E_{o}(x)/ (b(x) \cdot \Delta x) \]

\[ = H \cdot g \cdot p \cdot \sin \beta(x) - \Delta E_{o}(x)/ \Delta x \]

\[ = 20 \cdot 17 \cdot \sin (4.2^\circ) + 0 \approx 25 \text{ kN/m}^2 \]

**Step 1:** \( x = 0 \), Set \( \Delta \tau_1 = 0.5 \text{ kN/m}^2 \) at \( x_1 \) \( \tau_{o(x,0)} = 25 \text{ kN/m}^2 \) \( \tau (x_1,0) = 25.5 \text{ kN/m}^2 \)

**Step 1**

**Step 2**

![Diagram](image)

**Figure I:13 Illustration of the first two calculation steps.**

Applying equation 4.1 and 4.2

\[ \Delta N = [(\tau(x,o) - \tau(o,o))/2 - \tau_{o(x,o)}] \cdot b(x) \cdot \Delta x - q(x) \cdot b(x) \cdot \sin \beta(x) \cdot \Delta x - t(x) \cdot b(x) \cdot \Delta x \]

\[ = 0.5 \cdot 1/2 \cdot \Delta x - 0 - 0 \quad = 0.25 \cdot \Delta x \text{ kN/m} \]

Apply equation 4.3,

\[ \Delta N = (N + N/2) \cdot \Delta x /[E_{ef} \cdot H(x) \cdot b(x)] \]

\[ = (0 + 0.25 \cdot 1 \cdot \Delta x_1 /2) \cdot \Delta x_1 / [1000 \cdot 20] \quad = 0.125 \cdot \Delta x_1^2 / 20000 \text{ (m)} \]

Now apply equation 4.4a \((\tau > \tau_{el})\)

\[ \delta_{\tau} = \sum_{i=1}^{13} [\Delta \gamma_{xz}] \Delta z + \delta_{S(x,o)}, \]

where \( \Delta \gamma_{xz} = (\gamma_f - \gamma_{el}) \cdot [[1 - (\tau_{x_i-1} - \tau_{el})/(c_u - \tau_{el})]^{1/2} - [1 - (\tau_{x_i} - \tau_{el})/(c_u - \tau_{el})]^{1/2}] \quad \text{...............I:4} \]
In the elastic zone for \( \tau_e < 20 \text{kN/m}^2 \), equation I:1a applies i.e.

\[ \Delta \gamma_{xz} = \Delta \tau_{xz} / G, \quad \text{where} \quad G = 2(c_u - \tau_0) / (\gamma_f - \gamma_a) = 2 \times 10 / 0.0375 = 533.3 \text{kN/m}^2 \]

As long as \( \gamma_{xz} < \gamma_f \) then \( \delta_8(x, o) = 0 \)

**Table I:1**

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Now, in this initial step (Step 1), \( \Delta \delta_N = \delta_t \). By equating the results (Equation 4:5)

\[ \Sigma \Delta \delta_N = 0.125 \cdot \Delta x^2 / 20000 \quad \delta_t = 0.00586 \text{m} \]

\[ \Delta x^2 = 20000 \cdot 0.00586 / 0.125 \rightarrow \Delta x = 30.62 \text{ m} \]

\[ N_x = 0 + (0.5 \cdot 30.62 - 1)/2 \quad = 7.66 \text{ kN/m} \]

**Results from step 1:**

\( x = 0 \quad \tau_0 = 25 \text{kN/m}^2 \quad N_x = 0 \text{kN/m} \quad \delta = 0.0000 \text{ m} \)

\( x_1 = 30.62 \text{ m} \quad \tau_0 (x_1) = 25 \text{kN/m}^2, \quad \tau_3 (x_1) = 25.5 \text{kN/m}^2, \quad N_x = 7.66 \text{kN/m} \quad \delta = 0.00586 \text{ m} \)

**Step No 2** Advance \( x \) by \( \Delta x = 18 \text{ m} \) i.e. \( x_2 = 30.62 + 18 = 48.62 \text{ m} \)

\( \tau_0 (x_2, 0) = 25 \text{kN/m}^2, \quad \tau_3 (x_2, 0) = 25.5 \text{kN/m}^2 \)

**Iteration No 1:** Try \( \Delta \tau = 1.0 \text{kN/m}^2 \) \( \rightarrow \Delta \tau (x_2, 0) = 26.5 \text{kN/m}^2 \)

\[ \Delta N = [(26.5 + 25.5)/2 - 25] \cdot 18 = 18 \text{kN/m} \quad \text{(Equation 4:3)} \]

\[ \Delta \delta_N = (7.66+18/2) - 18/1000 / 20 = 0.01499 \text{ m} \quad \text{(Equation 4:4)} \]

\[ \delta_N = \Sigma \Delta \delta_N = 0.00586 + 0.01499 = 0.02085 \text{ m} \]

Proceed to calculate \( \delta_t \) using Equation 4:6 (i.e. according to Equations I:1, I:4 and I:4a in Table I:2).
Table I.2 \( x = x_2, \Delta \tau_2 = 1.0 \text{kN/m}^2 \)

<table>
<thead>
<tr>
<th>z (m)</th>
<th>( \tau_0(x,z) )</th>
<th>( \tau(x_2,z) )</th>
<th>( \Delta \tau )</th>
<th>( \tau(\Delta x_{i-1}, z) )</th>
<th>( \Delta \tau_x )</th>
<th>( \Delta z )</th>
<th>( \Delta \tau_{x,z} )</th>
<th>( \Delta z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25.00</td>
<td>25.50</td>
<td>1.00</td>
<td>26.50</td>
<td>26.00</td>
<td>0.297</td>
<td></td>
<td>x100</td>
</tr>
<tr>
<td>0.5</td>
<td>23.75</td>
<td>24.23</td>
<td>0.95</td>
<td>25.18</td>
<td>24.70</td>
<td>0.245</td>
<td>1.0</td>
<td>0.271</td>
</tr>
<tr>
<td>1</td>
<td>22.50</td>
<td>22.95</td>
<td>0.90</td>
<td>23.85</td>
<td>23.40</td>
<td>0.208</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>21.25</td>
<td>21.68</td>
<td>0.85</td>
<td>22.53</td>
<td>22.10</td>
<td>0.179</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20.00</td>
<td>20.40</td>
<td>0.80</td>
<td>21.20</td>
<td>20.80</td>
<td>0.156</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>18.75</td>
<td>19.13</td>
<td>0.75</td>
<td>19.88</td>
<td>19.50</td>
<td>0.141</td>
<td></td>
<td>0.149</td>
</tr>
<tr>
<td>3</td>
<td>17.50</td>
<td>17.85</td>
<td>0.70</td>
<td>18.55</td>
<td>18.20</td>
<td>0.131</td>
<td></td>
<td>0.136</td>
</tr>
<tr>
<td>3.5</td>
<td>16.75</td>
<td>17.09</td>
<td>0.67</td>
<td>17.76</td>
<td>17.42</td>
<td>0.126</td>
<td></td>
<td>0.077</td>
</tr>
</tbody>
</table>

\[ \Delta \delta_x = \Sigma_{i=1}^{13} \Delta \tau_{x,z} \times \Delta z = 0.01220 \text{m} \]

Result from Iteration No 1:
\( \delta_{N} = 0.00586 + 0.01220 = 0.01806 \text{m} \neq 0.2085 \text{m} \)
Equation 4.5 is not satisfied. Try another value of \( \Delta \tau_2 \).

Iteration No 2
The procedure in Iteration 1 is repeated assuming \( \Delta \tau_2 = 1.6 \text{kN/m}^2 \).

\[ \tau_0(x_2,0) = 25 \text{kN/m}^2, \quad \tau_x(x_2,0) = 25.5 + 1.6 = 26.1 \text{kN/m}^2, \]
\[ \Delta N = [(27.1+ 25.5)/2 - 25] \times 18 = 23.4 \text{kN/m} \quad (\text{Equation 4.3}) \]
\[ N_{\Delta z} = (7.66 + 23.4) = 31.06 \text{kN/m} \]
\[ \Delta \delta_N = (7.66 + 23.4)/2 \times 18/1000/20 = 0.01742 \text{m} \quad (\text{Equation 4.4}) \]
\[ \delta_N = \Sigma \Delta \delta_N = 0.00586 + 0.01742 = 0.02328 \text{m} \]

Proceed to calculate \( \delta_t \) using Equations 1.1, 4.4 and 1.4a as in Table I.1 (not shown), which gives a value for \( \Delta \delta_t = \Sigma_{i=1}^{13} \Delta \tau_{x,z} \times \Delta z = 0.0199 \text{m} \).
\[ \delta_t = \Sigma \Delta \delta_t = 0.00586 + 0.0199 = 0.2575 \text{m} \]

Again, Equation 4.4 is not quite satisfied, as \( \delta_t = 0.2575 \text{m} \neq 0.2085 \text{m} \).
However, interpolation from the results of the two previous iterations indicates that the correct value is \( \Delta \tau_2 = 1.32 \text{kN/m}^2 \).

Iteration No 3
Check: The procedure is repeated assuming \( \Delta \tau_2 = 1.32 \text{kN/m}^2 \).

\[ \tau_0(x_2,0) = 25 \text{kN/m}^2, \quad \tau_x(x_2,0) = 25.50 + 1.32 = 26.82 \text{kN/m}^2, \]
\[ \Delta N = [(26.82 + 25.5)/2 - 25] \times 18 = 20.88 \text{kN/m} \quad (\text{Equation 4.3}) \]
\[ N_{\Delta z} = (7.66 + 20.88) = 28.54 \text{kN/m} \]
\[ \Delta \delta_N = (7.66 + 20.52)/2 \times 18/1000/2 = 0.01629 \text{m} \quad (\text{Equation 4.4}) \]
\[ \delta_N = \Sigma \Delta \delta_N = 0.00586 + 0.01629 = 0.02215 \text{m} \]
Table 1:2  \( x = x_2 = 31.7 \text{ m}, \Delta \tau_2 = 1.32 \text{ kN/m}^2 \)

<table>
<thead>
<tr>
<th>z (m)</th>
<th>( \tau_0(x,z) )</th>
<th>( \tau(x_1,z) )</th>
<th>( \Delta \tau )</th>
<th>( \tau(x_2,z) )</th>
<th>( \tau(\Delta x_{2,3},z) )</th>
<th>( \Delta \gamma_{x_2} )</th>
<th>( \Delta z )</th>
<th>( \Delta \gamma_{x_2,\Delta z} )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>25.50</td>
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<td>26.82</td>
<td>26.16</td>
<td>( \times 100 )</td>
<td>0.401</td>
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<tr>
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<td></td>
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<td></td>
<td>( \text{Equation 9.4} )</td>
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</tr>
<tr>
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<td></td>
<td></td>
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</tr>
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<td></td>
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<td>0.88</td>
<td>17.97</td>
<td>17.63</td>
<td>0.166</td>
<td>( \Delta \delta_t = 0.01627 \text{ m} )</td>
<td></td>
</tr>
</tbody>
</table>

Result from Iteration No 3:
\( \delta_N = \Sigma \Delta \delta = 0.00586 + 0.01627 = 0.2213 \text{ m} \)

As \( \delta_N = 0.02213 \text{ m} = \delta_N = 0.02215 \text{ m} \), equation 4.5 is satisfied.

Results from Step 2:

\[ x_1 = 30.62 \text{ m} \quad \tau_0 = 25 \text{ kN/m}^2 \]
\[ x_2 = 48.62 \text{ m} \quad \tau_x(x_1) = 25.50 \text{ kN/m}^2 \quad N(x_1) = 7.66 \text{ kN/m} \quad \delta(x_1) = 0.00586 \text{ m} \]
\[ \tau_x(x_2) = 26.82 \text{ kN/m}^2 \quad N(x_2) = 28.54 \text{ kN/m} \quad \delta(x_2) = 0.02214 \text{ m} \]

Step 3  \( \Delta x = 10 \text{ m} \),  \( x = 48.62 + 10 = 58.62 \text{ m} \)

In the following, the calculation is continued using an Excel program, based on the expressions and following the procedure hitherto applied in this section.

Thus repeating the procedures in Step 2, and the computations in Step 3, the following obtains:

\[ \tau_o(x_{2,0}) = 25 \text{ kN/m}^2 \quad \tau_x(x_{2,0}) = 26.82 \text{ kN/m}^2 \]
\[ \Delta \tau_o = 1.51 \text{ kN/m}^2 \quad \tau_x(x_3,0) = 26.82 + 1.51 = 28.33 \text{ kN/m}^2, \]
\[ \Delta N_x = [(28.33+26.82)/2-25]*10 = 25.75 \text{ kN/m} \]  \( \text{Equation 4.3} \)
\[ \Delta \delta_N = (28.54 + 25.75)/2-1000/20 = 0.02071 \text{ m} \]  \( \text{Equation 4.4} \)
\[ \delta_N = \Sigma \Delta \delta_N = \delta_t = \Sigma \Delta \delta_N = 0.02214 + 0.02071 = 0.04285 \text{ m} \]  \( \text{Equation 4.5} \)

Results from Step 3:

\[ x_2 = 48.62 \text{ m} \quad \tau_0 = 25 \text{ kN/m}^2 \]
\[ x_3 = 58.62 \text{ m} \quad \tau_x(x_2) = 26.82 \text{ kN/m}^2 \quad N(x_2) = 28.54 \text{ kN/m}, \quad \delta(x_2) = 0.02214 \text{ m} \]
\[ \tau_x(x_3) = 28.33 \text{ kN/m}^2 \quad N(x_3) = 54.29 \text{ kN/m} \quad \delta(x_3) = 0.04285 \text{ m} \]
Step 4: \( x = 58.62 + 6 = 64.62 \) m

Repeating the procedures in Step 3 the computations in Step 4, produce:
\[
\begin{align*}
\tau_o (x_4,0) &= 25 \text{ kN/m}^2 \\
\tau_o (x_3,0) &= 28.33 \text{ kN/m}^2 \\
\Delta \tau_o &= 1.22 \text{ kN/m}^2 \\
\tau_o (x_4,0) &= 28.33 + 1.22 = 29.55 \text{ kN/m}^2,
\end{align*}
\]
\[
\begin{align*}
\Delta N &= [(29.55 + 28.33)/2 - 25] \cdot 6 = 23.64 \text{ kN/m} \quad \text{(Equ. 4.3)} \\
N_{x_4} &= (54.29 + 23.64) = 77.93 \text{ kN/m} \\
\Delta \delta_N &= (59.29 + 23.64/2) \cdot 6/1000/20 = 0.01983 \text{ m} \quad \text{(Equ. 4.4)} \\
\delta_N &= \sum \Delta \delta_N = \delta_t = 0.04285 + 0.01983 = 0.06268 \text{ m}
\end{align*}
\]

Results from Step 4:
\[
\begin{align*}
x_3 &= 58.62 \text{ m} \quad \tau_o = 25 \text{ kN/m}^2 \\
x_4 &= 64.62 \text{ m} \quad \tau_o (x_3) = 28.33 \text{ kN/m}^2, \quad N (x_3) = 54.29 \text{ kN/m}, \quad \delta (x_3) = 0.04285 \text{ m} \\
\tau_o (x_4) &= 29.55 \text{ kN/m}^2 \quad N (x_4) = 77.93 \text{ kN/m} \quad \delta (x_4) = 0.06268 \text{ m}
\end{align*}
\]

Step 5: Assume \( \Delta x = 2.50 \) m \( x = 64.62 + 2.5 = 67.12 \) m

\[
\begin{align*}
\tau_o (x_4,0) &= 25 \text{ kN/m}^2 \\
\tau_o (x_4,0) &= 29.55 \text{ kN/m}^2 \\
\Delta \tau_o &= 0.45 \text{ kN/m}^2 \\
\tau_o (x_5,0) &= 29.55 + 0.45 = 30.00 \text{ kN/m}^2,
\end{align*}
\]
\[
\begin{align*}
\Delta N &= [(30+29.55)/2 - 25] \cdot 2.5 = 12.01 \text{ kN/m} \\
N_{x_5} &= 77.93 + 12.01 = 89.93 \text{ kN/m} \\
\Delta \delta &= (77.93 + 11.94/2) \cdot 2.5/1000/20 = 0.01056 \text{ m} \\
\delta_N &= \sum \Delta \delta_N = 0.06268 + 0.01056 = 0.07324 \text{ m} \\
\delta_t &= \sum \Delta \delta_t = 0.06268 + 0.01055 = 0.07323 \text{ m} \quad \text{i.e. } \delta_N \approx \delta_t \\
\text{Slip deformation } \delta_s &= 0.0000 \text{ m}
\end{align*}
\]

Results from Step 5:
\[
\begin{align*}
x_4 &= 64.62 \text{ m}, \quad \Delta x = 2.50 \text{ m}, \quad x_5 = 67.12 \text{ m}, \quad \tau_o = 25 \text{ kN/m}^2 \\
\tau_o (x_4) &= 29.55 \text{ kN/m}^2, \quad N (x_4) = 77.93 \text{ kN/m}, \quad \delta (x_4) = 0.06268 \text{ m} \\
\tau_o (x_5) &= 30.00 \text{ kN/m}^2, \quad N (x_5) = 89.93 \text{ kN/m} \quad \delta (x_5) = 0.07324 \text{ m}
\end{align*}
\]

Note: At this point in the computation, the shear strength \( c_s = 30 \text{ kN/m}^2 \) is fully mobilised and a slip surface will be forming. The shear/deformation relationship is from now on expressed also in terms of the slip \( \delta_s \) according to Equations 1.5 and 1.6. In other respects, the calculation proceeds in principle as before. For increasing \( x \)-values, shear stress will decline and correspond to the current value of the residual shear strength.

Step 6:

In Equation 4.3 and 4.4, slip deformation \( \delta_s \) in the failure plane has to be added. Hence \( \delta_t = \Sigma_o^{1/3} \left[ \Delta \gamma_{xz} \cdot \Delta z + \delta_s (x_o) \right] \), where \( \Delta \gamma_{xz} \) is defined as before in equation 1.4, 1.4a and 1.1a,
observing however that the mobilized shear stress is now equal to the post peak shear strength at large deformations according to figure I:1.2, and that $\delta_S$ enters into the analysis.

For current post-peak range, the constitutive relationship is taken according to equation I:5 a

\[ \delta_S(\tau) = \delta_{cR} \left( \frac{c_u - \tau}{(c_u - c_{ur})} \right) \]  

(See figure I:1.2)

**Iteration No 1**

Let \( \Delta x = 3.50 \) \( \Rightarrow \) \( x = 67.14 + 3.5 = 70.64 \) m

Assume \( \Delta \tau = -1.00 \) kN/m²

\( \tau_x(x_2,o) = 25 \) kN/m²

\( \tau_x(x_2,o) = 30.00 - 1.00 = 29.00 \) kN/m²,

\[ \Delta N = [(30+29)/2- 25]-3.5 = 15.75 \text{ kN/m} \quad \text{(Equ. 4.3)} \]

\[ N_x = 89.93+ 15.75 = 105.68 \text{ kN/m} \]

\[ \Delta \delta_N = (89.93 + 15.75)/2-3.5/1000/20 = 0.0171 \text{ m} \quad \text{(Equ. 4.4)} \]

From computation Step 5: \( \delta_N = \delta - \Delta \delta_N = 0.07324 \text{ m} \)

\[ \delta_N = \sum \Delta \delta_N = 0.07324 + 0.00171 = 0.09035 \text{ m} \]

As earlier the compatibility criterion demands that \( \delta_N = \delta_x \) or \( \Delta \delta_N = \Delta \delta_x \)

\[ \delta_x = \delta_{ca} - \delta_{ca}(x) + \delta_S(x) \]

where \( \delta_{ca} = \) shear deformation at fully mobilised shear strength \( c_u \) (at \( x = x_S \)) and

\[ \delta_{ca}(x) = \text{elastic rebound due to the deloading from } c_u \text{ to } c_R(x) \]

\[ \delta_S(x) = \text{slip (including slip due to elastic rebound.)} \]

The elastic rebound may be expressed as

\[ \delta_{ca}(x) = \sum \omega = 6.6 \left[ (\tau(x_0) - \tau(x_n)) + \Delta z = 1/2: \Delta \tau + \Delta \tau (H-\alpha H)/H / \gamma_{pl} / \tau_{pl} : \alpha H \right] \]

\[ \text{0.5}[1.00+1.00-(20-6.6)/20] \times 0.0375/20-6.6 = 0.01033 \text{ m} \]

\[ \sum \Delta \delta_x(x) = 0.01033 \text{ m} \]

The slip deformation is according to Equation I:5 a

\[ \delta_S(x) = \delta_{ca} \left( \frac{c_u - \tau}{(c_u - c_{ur})} \right) = 0.30 \times (30-29)/(30-15) = 0.02000 \text{ m} \]

Hence, \( \delta_x = \delta_{ca} - \Delta \delta_{ca}(x) + \delta_S(x) = 0.07324 - 0.01033 + 0.02000 = 0.08291 \text{ m} \)

Thus, the compatibility condition is not satisfied, as \( \delta_N = 0.09035 \text{ m} \neq \delta_x = 0.08291 \text{ m} \).

**Iteration No 2**

Repeating the iterative procedure, it is soon found that the compatibility criterion is full-filled if

\( \Delta \tau = -1.758 \) kN/m²

\( \tau_x(x_1,o) = 30.00 \) kN/m²

\( \tau_x(x_2,o) = 30.00 - 1.758 = 28.24 \) kN/m²,

\[ \Delta N = [(30+28.24)/2- 25]-3.5 = 14.42 \text{ kN/m} \quad \text{(Equ. 4.3)} \]

\[ N = 89.93+ 14.42 = 104.36 \text{ kN/m} \]

\[ \Delta \delta_N = (89.93 + 14.42)/2-3.5/1000/20 = 0.01700 \text{ m} \quad \text{(Equ. 4.4)} \]

From computation Step 5: \( \delta_N = \delta_{ca} \)

\[ \delta_N = \sum \Delta \delta_N = 0.07324 + 0.01700 = 0.09024 \text{ m} \]
As before, the compatibility criterion demands that $\delta_N = \delta_t$ or $\Delta \delta_N = \Delta \delta_t$

The elastic rebound may be expressed as

$$\Delta \delta_{t, el}(x) = \sum_o^{AH} \frac{\Delta z = 1/2 \cdot [\Delta \tau + \Delta \tau (H-H)H]}{H} \cdot \Delta \delta_{t, el}(x) = 0.01817 \text{ m}$$

$$\Sigma \Delta \delta_{t, el}(x) = 0.01817 \text{ m}$$

The slip deformation is, according to Equ. 1.5 a,

$$\delta_s(x) = \delta_c r \cdot (c_\alpha - c_\omega)/(c_u - c_\omega) = 0.30 \cdot (30-28.242)/(30-15) = 0.03516 \text{ m}$$

Hence, $\delta_t = \delta_t^{cu} - \Delta \delta_{t, el}(x) + \delta_s(x) = 0.07324 - 0.01817 + 0.03516 = 0.09023 \text{ m}$

Thus, the compatibility condition is satisfied, as $\delta_N = 0.09024 \text{ m} \approx \delta_t = 0.09023 \text{ m}$

**Results from Step 6:**

$$x_3 = 61.7 \text{ m}, \quad \Delta x = 3.50 \text{ m}, \quad x_6 = 70.64 \text{ m}, \quad \tau_0 = 25 \text{ kN/m}^2$$

$$\tau_\tau (x_3) = 30.00 \text{ kN/m}^2, \quad N(x_3) = 89.93 \text{ kN/m}, \quad \delta(x_3) = 0.07324 \text{ m}$$

$$\tau_\tau (x_6) = 28.24 \text{ kN/m}^2, \quad N(x_6) = 104.36 \text{ kN/m}, \quad \delta(x_6) = 0.09024 \text{ m}$$

**Step 7:**

The iterative procedure in Step 6 is now continued. Iteration carried out as in Step 6 and the correct value of $\Delta \tau$ is found to be: $\Delta \tau = -1.675 \text{ kN/m}^2$

Let $\Delta x = 3.00 \text{ m}$

$$x_7 = 70.62 + 3 = 73.62 \text{ m}$$

Hence, assuming $\Delta \tau = -1.675 \text{ kN/m}^2$

$$\tau(\tau_6, o) = 28.242 \text{ kN/m}^2$$

$$\tau_\tau (x_6, o) = 25 \text{ kN/m}^2, \quad \tau(x_\tau, o) = 28.242 - 1.675 = 26.567 \text{ kN/m}^2$$

$$\Delta N = [(28.242 + 26.567)/2 - 25] \cdot 3 = 7.21 \text{ kN/m}$$ (Equ. 4:3)

$$N_7 = 104.36 + 7.21 = 111.57 \text{ kN/m}$$ (Equ. 4:4)

$$\Delta \delta_N = (104.36 + 7.21/2) \cdot 3/1000 = 0.01619 \text{ m}$$

From computation step 6: $\delta_N = \delta_t^{cu} = 0.09024 \text{ m}$

$$\delta_N = \Sigma \Delta \delta_N = 0.09024 + 0.01619 = 0.10643 \text{ m}$$

As before, the compatibility criterion demands that $\delta_N = \delta_t$

The elastic rebound is

$$\Delta \delta_{t, el}(x) = \sum_o^{AH} \frac{\Delta z = 1/2 \cdot [\Delta \tau + \Delta \tau (H-H)H]}{H} \cdot \Delta \delta_{t, el}(x) = 0.01817 \text{ m}$$

$$\Sigma \Delta \delta_{t, el}(x) = 0.01817 + 0.017304 = 0.03547 \text{ m}$$
The slip deformation is according to Eqn. 1.5 a,
\[ \delta_S(c_{Rx}) = \delta_{CR} \cdot (c_u - c_{uR}) / (c_{uR} - c_{uR}) = 0.30 \cdot (30-26.567)/(30-15) = 0.06866 \text{ m} \]
Hence, \[ \delta_t = \delta_{t,el}^{cu} - \Sigma \Delta \delta_t, el(x) + \delta_S(x) = 0.07324 - 0.03547 + 0.06866 = 0.10643 \text{ m} \]

The compatibility condition is now satisfied, as \[ \delta_N = 0.10381 \text{ m} = \delta_t = 0.10381 \text{ m} \]

**Results from Step 7:** \( \Delta \tau = -1.675 \text{ kN/m}^2 \)

\[
\begin{align*}
X_6 &= 70.62 \text{ m, } \Delta x = 3.00 \text{ m, } x_7 &= 73.62 \text{ m, } \tau_0 = 25 \text{ kN/m}^2 \\
\tau_x(x_6) &= 28,242 \text{ kN/m}^2, \quad N(x_6) = 104.36 \text{ kN/m, } \delta(x_6) = 0.09024 \text{ m} \\
\tau_x(x_7) &= 26.567 \text{ kN/m}^2, \quad N(x_7) = 111.57 \text{ kN/m, } \delta(x_7) = 0.10643 \text{ m}
\end{align*}
\]

**Step 8:**

The procedure in Steps 6 to 7 is continued. Iteration is carried out as in Step 6 and the correct values of \( \Delta x \) and \( \Delta \tau \) are found.

Let \( \Delta x = 2.69 \text{ m } x_7 = 73.62 + 2.69 = 76.31 \text{ m} \)

Hence, assume \( \Delta \tau = -1.567 \text{ kN/m}^2 \)

\[
\begin{align*}
\tau_x(x_7, 0) &= 26.567 \text{ kN/m}^2 \\
\tau_0(x_8, 0) &= 25 \text{ kN/m}^2 \\
\tau_x(x_8, 0) &= 26.567 - 1.567 = 25.000 \text{ kN/m}^2,
\end{align*}
\]

\[
\begin{align*}
\Delta N &= [(26.567 + 25.00)/2 - 25] \cdot 2.69 = 2.11 \text{ kN/m} \quad (\text{Eqn. 4:3}) \\
N_{x8} &= 111.571 + 2.11 = 113.68 \text{ kN/m} \\
\Delta \delta_N &= (111.571 + 2.11/2) \cdot 2.69/100/20 = 0.01515 \text{ m} \quad (\text{Eqn. 4:4}) \\
\delta_N &= \Sigma \Delta \delta_N = 0.10643 + 0.01515 = 0.12158 \text{ m}
\end{align*}
\]

The compatibility criterion demands that \( \delta_N = \delta_t \)

The elastic rebound is

\[
\begin{align*}
\Delta \delta_{t,el}(x) &= \Sigma \alpha_{QH} \cdot \delta_{CR} \cdot (c_u - c_{uR}) / (c_{uR} - c_{uR}) \\
&= 0.5 \cdot [1.567 + 1.567 \cdot (30-25.000)/(30-15)] \cdot 0.01619 = 0.03547 + 0.01619 = 0.05166 \text{ m}
\end{align*}
\]

The slip deformation is according to equation I:5 a,

\[
\begin{align*}
\delta_S(c_{Rx}) &= \delta_{CR} \cdot (c_u - c_{uR}) / (c_{uR} - c_{uR}) = 0.30 \cdot (30-25.000)/(30-15) = 0.10000 \text{ m} \\
\delta_t &= \delta_{t,el}^{cu} - \Sigma \Delta \delta_{t,el}(x) + \delta_S(x) = 0.07324 - 0.05166 + 0.10000 = 0.12158 \text{ m}
\end{align*}
\]

The compatibility condition is satisfied for \( \delta_N = 0.12158 \text{ m} = \delta_t = 0.12158 \text{ m} \)

**Results from Step 8:** \( \Delta \tau = -1.567 \text{ kN/m}^2 \)

\[
\begin{align*}
x_7 &= 73.64 \text{ m, } \Delta x = 2.69 \text{ m, } x_8 &= 76.33 \text{ m, } \tau_0 = 25 \text{ kN/m}^2 \\
\tau_x(x_7) &= 26.567 \text{ kN/m}^2, \quad N(x_7) = 111.57 \text{ kN/m, } \delta(x_7) = 0.10643 \text{ m} \\
\tau_x(x_8) &= 25.000 \text{ kN/m}^2, \quad N(x_8) = 113.68 \text{ kN/m, } \delta(x_8) = 0.12158 \text{ m}
\end{align*}
\]

**Important results:** \( N_{max} = N_{cr} = 113.68 \text{ kN/m} \quad \delta_{cr} = 0.12158 \text{ m} \)
Note: At this point in the analysis, shear stress (and residual shear strength) \( \tau_x(x_9) = c_\alpha(x_9) \) is equal to the in situ stress \( \tau_0 = 25 \text{ kN/m}^2 \). This means that all the available shear resistance in the range \( (c_\alpha - \tau_0) \) is mobilized.

The maximum down-slope resistance \( N_{cr}(x) = N(x_9) = 113.68 \text{ kN/m} \) is now attained. Hence, if a load \( N_i = 100 \text{ kN/m} \) is applied at this location the safety factor against progressive failure formation would be:

\[
Fs = \frac{N_{cr}(x_9)}{N_i} = \frac{113.68}{100} = 1.1368
\]

Accordingly, slope stability may be thought of as a phenomenon analogous to buckling stability, as in the case of an axially loaded strut. When the limit critical load is reached at a certain deflection, stable equilibrium is no longer possible

The value of \( N_{cr} \), having been determined, that part of the analysis, which relates to local slope stability and the risk of an initial slide is completed.

Nevertheless, for illustrative purposes, the iterative procedure in Steps 6 to 9 are, in this example continued until the value of \( N \) is equal to zero.

Again, by iteration as carried out as in Step 6, the correct value of \( \Delta \tau \) is found.

Step 9:

The procedures in Steps 6 to 8 are continued. Iteration carried out as in Step 6, and the correct value of \( \Delta \tau \) is found.

Let \( \Delta x = 4.00 \text{ m} \)

\[
\begin{align*}
x_9 &= 76.33 + 4.00 = 80.33 \text{ m} \\
\tau_x(x_8,0) &= 25.000 \text{ kN/m}^2 \\
\tau_x(x_9,0) &= 25.000 - 2.304 = 22.696 \text{ kN/m}^2,
\end{align*}
\]

\[
\begin{align*}
\Delta N &= [(22.696+25.000)/2 - 25].4.00 = -4.61 \text{ kN/m} \quad \text{ (Equ. 4.3)} \\
N_\phi &= 113.68 - 4.61 = 109.07 \text{ kN/m} \\
\Delta \delta_N &= (113.68 - 4.61/2).4.00/1000/20 = 0.02227 \text{ m} \quad \text{ (Equ. 4.4)} \\
\delta_N &= \Sigma \Delta \delta_N \quad \text{ 0.12158 m} \\
&= 0.14385 \text{ m}
\end{align*}
\]

The elastic rebound may be expressed as

\[
\begin{align*}
\Delta \delta_{x, \text{el}}(x) &= 2.304 \cdot 0.5 \cdot [1+\{(20-6.6)/20\}-0.0375/20-6.6] = 0.023807 \text{ m} \\
\Sigma \Delta \delta_{x, \text{el}}(x) &= 0.05166 + 0.02381 = 0.07547\text{m}
\end{align*}
\]

Slip deformation is according to Equation 1.5 a,

\[
\delta_S(c_{\text{ex}}) = \delta_{\text{cR}} \cdot (c_\alpha - c_{\text{ux}})/(c_\alpha - c_{\text{UR}}) = 0.30 \cdot (30 - 22.696)/(30 - 15) = 0.14608 \text{ m}
\]

Hence, \( \delta_t = \delta_{x, \text{el}} + \Sigma \delta_{x, \text{el}}(x) + \delta_S(x) = 0.07324 - 0.07547 + 0.14608 = 0.14385 \text{ m} \)

The compatibility condition is satisfied as \( \delta_N = 0.14385 \text{ m} = \delta_t = 0.14385 \text{ m} \)
Results from Step 9:

\[ x_8 = 76.33 \text{ m}, \ \Delta x = 4.00 \text{ m}, \ \delta (x_8) = 0.12158 \text{ m} \]
\[ \tau_x (x_8) = 25.000 \text{ kN/m}^2, \ N(x_8) = 113.68 \text{ kN/m}, \ \delta (x_8) = 0.12158 \text{ m} \]
\[ \tau_x (x_9) = 22.696 \text{ kN/m}^2, \ N(x_9) = 109.07 \text{ kN/m} \]
\[ \delta (x_9) = 0.14385 \text{ m} \]

Step 10:

Let \( \Delta x = 5.67 \text{ m} \)
\[ x_9 = 80.33 + 5.67 = 86.00 \text{ m} \]
Hence, assume \( \Delta \tau = -2.887 \text{ kN/m}^2 \)
\[ \tau_x (x_9,0) = 22.696 \text{ kN/m}^2 \]
\[ \tau_x (x_{10},0) = 25 \text{ kN/m}^2 \]
\[ \tau_x (x_{10},0) = 22.696 - 2.887 = 19.809 \text{ kN/m}^2 \]
\[ \Delta N = [(22.696 + 19.809)/2 - 25] \times 5.67 = -21.25 \text{ kN/m} \]
\( N_{x_{10}} = 109.07 - 21.25 = 87.82 \text{ kN/m} \)
\[ \Delta \delta_N = (109.07 - 21.25)/2 - 5.67/1000/20 = 0.02791 \text{ m} \]

From computation Step 9:
\[ \delta_N = \delta_{\tau^a} = 0.14385 \text{ m} \]
\[ \delta_N = \Sigma \Delta \delta_N = 0.14385 + 0.02791 = 0.17176 \text{ m} \]

The elastic rebound may be expressed as
\[ \Delta \delta_{\tau, el}(x) = 2.887 \times 0.5 [1 + (20 - 6.6)/20] \times 0.0375/20 - 6.6 = 0.029832 \text{ m} \]
\[ \Sigma \Delta \delta_{\tau, el}(x) = 0.07547 + 0.029832 = 0.10530 \text{ m} \]

Slip deformation is according to Equation 1.5a,
\[ \delta_R (c_R) = \delta_{CR} - (c_R - c_M)/(c_M - c_R) = 0.30 - (30 - 19.809)/(30 - 19.809) = 0.20382 \text{ m} \]
Hence, \[ \delta_{\tau} = \delta_{\tau^a} - \Sigma \Delta \delta_{\tau, el}(x) + \delta_R (x) = 0.07324 - 0.10530 + 0.20382 = 0.17176 \text{ m} \]

Thus, the compatibility condition is satisfied, as \( \delta_N = 0.17176 \text{ m} = \delta_{\tau} = 0.17176 \text{ m} \)

Results from step 10:
\[ \Delta \tau = -2.887 \text{ kN/m}^2 \]
\[ x_8 = 80.33 \text{ m}, \ \Delta x = 5.67 \text{ m}, \ x_9 = 86.00 \text{ m} \]
\[ \tau_x (x_9) = 22.696 \text{ kN/m}^2 \]
\[ N(x_8) = 109.07 \text{ kN/m}, \ \delta (x_8) = 0.14385 \text{ m} \]
\[ \tau_x (x_{10}) = 19.809 \text{ kN/m}^2 \]
\[ N(x_{10}) = 87.82 \text{ kN/m} \]
\[ \delta (x_9) = 0.17176 \text{ m} \]

Step 11:

Let \( \Delta x = 5.0 \text{ m} \)
\[ x_9 = 86.00 + 5.00 = 91.00 \text{ m} \]
Hence, assume \( \Delta \tau = -1.875 \text{ kN/m}^2 \)
\[ \tau_x (x_{10},0) = 19.809 \text{ kN/m}^2 \]
\[ \tau_x (x_{11},0) = 19.809 - 1.875 = 17.934 \text{ kN/m}^2 \]
\[ \Delta N = [(19.809 + 17.934)/2 - 25] \times 5.0 = -30.64 \text{ kN/m} \]
\[ N_{x_{10}} = 87.82 - 30.64 = 57.18 \text{ kN/m} \]
\[ \Delta \delta_N = (87.82 - 30.64/2) \times 5.0/1000/20 = 0.01813 \text{ m} \]

From computation Step 10:
\[ \delta_N = \delta_{\tau^a} = 0.17176 \text{ m} \]
\[ \delta_N = \Sigma \Delta \delta_N = 0.17176 + 0.01813 = 0.18989 \text{ m} \]
The elastic rebound may be expressed as

\[ \Delta \delta_{r, el}(x) = 1.875 - 0.5 \left[ 1 + \frac{(20 - 6.6)}{20} \right] 0.0375/20 - 6.6 = 0.019374 \text{ m} \]

\[ \Sigma \Delta \delta_{r, el}(x) = 0.10530 + 0.01937 = 0.12467 \text{ m} \]

Slip deformation is according to Equation 1.5 a,

\[ \delta_{s}(c_{R}) = \delta_{cR} \cdot \frac{(c_{u} - c_{uR})}{(c_{u} - c_{R})} = 0.30 \cdot \frac{(30 - 17.934)}{(30 - 15)} = 0.24132 \text{ m} \]

Hence, \( \delta_{r} = \delta_{r}^{cu} - \Sigma \Delta \delta_{r, el}(x) + \delta_{s}(x) = 0.07324 - 0.12467 + 0.24132 = 0.18989 \text{ m} \)

Thus, the compatibility condition is satisfied, as \( \delta_{N} = 0.18989 \text{ m} = \delta_{r} = 0.18989 \text{ m} \)

**Results from Step 11:**

\[ x_{10} = 86.0 \text{ m}, \quad \Delta x = 5.00 \text{ m}, \quad x_{11} = 91.00 \text{ m} \]

\[ \tau_{x}(x_{10}) = 19.809 \text{ kN/m}^{2}, \quad N(x_{10}) = 87.82 \text{ kN/m}, \quad \delta(x_{10}) = 0.18989 \text{ m} \]

\[ \tau_{x}(x_{11}) = 17.934 \text{ kN/m}^{2}, \quad N(x_{11}) = 51.18 \text{ kN/m}, \quad \delta(x_{11}) = 0.19348 \text{ m} \]

**Step No 12**

Let \( \Delta x = 4.0 \text{ m} \)

Assume \( \Delta \tau = -0.873 \text{ kN/m}^{2} \)

\[ \tau_{x}(x_{11}, o) = 17.934 \text{ kN/m}^{2} \]

\[ \tau_{0}(x_{11}, o) = 25 \text{ kN/m}^{2} \]

\[ \tau_{x}(x_{12}, o) = 17.934 - 0.873 = 17.061 \text{ kN/m}^{2} \]

\[ \Delta N = \left[ (17.934 + 17.061)/2 - 25 \right] 4.0 = -30.01 \text{ kN/m} \quad \text{(Equ. 4.3)} \]

\[ N_{o} = 57.18 - 30.01 = 27.17 \text{ kN/m} \]

\[ \Delta \delta_{N} = (57.18 - 30.01/2) 4.0/1000/20 = 0.008435 \text{ m} \quad \text{(Equ. 4.4)} \]

From computation Step 11 \( \delta_{N} = \delta_{r}^{cu} = 0.18989 \text{ m} \)

\[ \delta_{N} = \Sigma \Delta \delta_{N} = 0.18989 + 0.00844 = 0.19833 \text{ m} \]

The elastic rebound may be expressed as

\[ \delta_{r, el}(x) = 0.873 - 0.5 \left[ 1 + \frac{(20 - 6.6)}{20} \right] 0.0375/20 - 6.6 = 0.00902 \text{ m} \]

\[ \Sigma \Delta \delta_{r, el}(x) = 0.12467 + 0.00902 = 0.13369 \text{ m} \]

Slip deformation is according to Equation 1.5 a,

\[ \delta_{s}(c_{R}) = \delta_{cR} \cdot \frac{(c_{u} - c_{uR})}{(c_{u} - c_{R})} = 0.30 \cdot \frac{(30 - 17.934)}{(30 - 15)} = 0.25878 \text{ m} \]

Hence, \( \delta_{r} = \delta_{r}^{cu} - \Sigma \Delta \delta_{r, el}(x) + \delta_{s}(x) = 0.07324 - 0.13369 + 0.25878 = 0.19833 \text{ m} \)

Thus, the compatibility condition is satisfied, as \( \delta_{N} = 0.19833 \text{ m} = \delta_{r} = 0.19833 \text{ m} \)

**Results from Step 12:**

\[ x_{11} = 91.0 \text{ m}, \quad \Delta x = 4.00 \text{ m}, \quad x_{12} = 95.00 \text{ m} \]

\[ \tau_{x}(x_{11}) = 17.934 \text{ kN/m}^{2}, \quad N(x_{11}) = 57.18 \text{ kN/m}, \quad \delta(x_{11}) = 0.18989 \text{ m} \]

\[ \tau_{x}(x_{12}) = 17.061 \text{ kN/m}^{2}, \quad N(x_{12}) = 27.17 \text{ kN/m}, \quad \delta(x_{12}) = 0.19833 \text{ m} \]
Step 13:

Let $\Delta x = 3.5$ m  \hspace{1cm} x_{13} = 95.00 + 3.5 = 98.5$ m

Hence, assume $\Delta \tau = -0.235$ kN/m$^2$  \hspace{1cm} $\tau_x(x_{12},0) = 17.061$ kN/m$^2$

$\tau_0(x_{13},0) = 25$ kN/m$^2$  \hspace{1cm} $\tau_x(x_{13},0) = 17.061 - 0.235 = 16.826$ kN/m$^2$

$\Delta N = [(17.061 + 16.826)/2 - 25] \cdot 3.50 = -28.20$ kN/m  \hspace{1cm} (Equ. 4.3)

$N_{20} = 27.17 - 28.20 = -1.03$ kN/m

$\Delta \delta_N = (27.17 - 28.20/2) \cdot 3.50/1000/20 = 0.00229$ m  \hspace{1cm} (Equ. 4.4)

From computation Step 12: \hspace{1cm} $\delta_N = \delta_c = 0.19832$ m

$\delta_N = \sum \Delta \delta_N = 0.19832 + 0.00229 = 0.20061$ m

The elastic rebound may be expressed as

$\delta_{c_{el}}(x) = 0.235 \cdot 0.5 \cdot [(1 + (20-6.6)/20) \cdot 0.0375/20-6.6] = -0.002428$ m

$\sum \Delta \delta_{c_{el}}(x) = 0.13369 + 0.00243 = 0.13613$ m

Slip deformation is according to Equation 9.5 a,

$\delta_s(c_{Ra}) = \delta_{cR} \cdot (c_u - c_{adr})/(c_u - c_{adr}) = 0.30 \cdot (30-16.826)/(30-15) = 0.26348$ m

Hence, $\delta_c = \delta_c^{cu} - \sum \delta_{c_{el}}(x) + \delta_s(x) = 0.07324 - 0.13613 + 0.26348 = 0.20060$ m

Thus, the compatibility condition is satisfied, as $\delta_N = 0.20061$ m $\approx \delta_c = 0.20060$ m

Results from Step 13:

$x_{12} = 95.00$ m, $\Delta x = 3.5$ m, $x_{13} = 98.50$ m  \hspace{1cm} $\tau_0 = 25$ kN/m$^2$

$\tau_x(x_{12}) = 17.061$ kN/m$^2$, $N(x_{12}) = 27.17$ kN/m, $\delta(x_{12}) = 0.19832$ m

$\tau_x(x_{13}) = 16.826$ kN/m$^2$, $N(x_{13}) = -1.03$ kN/m, $\delta(x_{13}) = 0.20060$ m

At this point in the study of forces, stresses and displacements in the slope, $N(x_{13}) \approx 0$. This implies that if a forced displacement of $\delta(x_{13}) = 0.2006$ m were to be applied, the slope would fail even with no additional load, i.e., $N_i = 0$.

Also, this condition is analogous to the buckling of a strut, i.e., when failing under its own weight due to an applied (or initial) limit deflection.

The energy required to provoke such a 'buckling' failure is:

$\text{Energy} = \int N_x \, d\delta_{c_{el}}$, \hspace{1cm} integrated from $x = 0$ to $x = x_{13}$

The limiting boundary condition defined by $N_x = 0$ is of particular interest when studying the in situ earth pressure distribution in a slope subjected to creep deformation. Creep is then regarded as a long term failure process.
Figure I:14 Computed additional earth pressures, shear stresses and displacements in the slope subject to analysis.

I:2 Calculation of redistribution of earth pressures subsequent to progressive failure.

If the analysis, exemplified in Section I:1, indicates that local stability is insufficient, proceed to study the second possible state of equilibrium, which may exist subsequent to a progressive failure event. It is crucial to investigate this state of equilibrium in order to be able to predict the risk of the further development of a potential slide.

This second state of possible equilibrium is found by repeating the exercise shown in Section I:1. The starting point, defined by \( x = 0 \) (i.e. the down-slope boundary condition), is now located much further down the slope, usually a considerable distance into less sloping ground. The correct location for the point \( x = 0 \) is the one, which satisfies the up-slope boundary
condition, when \( N_x = N_i \) at the precise location of the additional force \( N_i \) (or this force multiplied by a safety factor \( F_s N_i \)).

The outcome of this exercise then yields the magnitude of down-slope earth pressures, and the crucial issue consists in how these forces relate to passive Rankine resistance in the zone at the foot of the slope where the gradient of the ground declines markedly. See Section 3.3).

The computations required to determine the post progressive failure equilibrium may seem extremely laborious but, again, it should be borne in mind that each investigation of a suitable location for \( x = 0 \) is facilitated by the use of computers.

Typical results from analyses of the post progressive failure stage are shown on Figures 4:5.1, 5:1.7, 5:2.3, 6:1.1 and 6:2.1. When the earth pressures \( (E_{ox}+ N_d) \) exceed the down-slope passive resistance, subsequent global slope failure is inevitable.
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