Time Planning under Uncertainty in a Mining Environment

by

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ABSTRACT

This study was initiated within the national research programme G2000, which is sponsored by the Swedish government, mines and manufacturers. Its overall goal is to propose enhanced time planning technique.

The planning process in many mines is often based on manual routines using uncertain time data without the application of statistical techniques.

It is shown that the deterministic time planning technique, which is used in most Swedish mines, results in optimistically biased estimates of the event times (e.g. the project completion time).

The approach is therefore to suggest a computerized probabilistic time planning model to decrease the amount of development buffers and increase the utilization of resources, thus reducing the costs in underground mining. The technique is applied to the Oscar project, LKAB Kiruna.

Time planning with stochastic project networks is generally described, including simulation and resource scheduling. The computer program "PertSim" has been developed as a tool. Models to optimize the extent of development buffers are constructed as well as models to balance development and production activities.
The thesis underlines that

* Time planning should be computerized to give an easily updated successively adapted time plan.

* Probabilistic planning is superior to deterministic planning, the latter is generally too optimistic.

* Efforts should be put into a more accurate estimation of time data, including distributions.

* Rock mechanical aspects and ore quality control should be considered more thoroughly in time planning.

* Numerical results for the Oscar case indicate the potential, with probabilistic planning, for substantial savings (many MSEK per year).
PREFACE

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1. INTRODUCTION

1.1 Background

This thesis is a part of a project that has been performed to propose modern techniques in mine planning. It is part of the G2000-programme, sponsored by the National Swedish Board for Technical Development, Swedish mines and manufacturers. The project is specifically applied to the Oscar project, LKAB Kiruna.

Today the planning process in many mines seems to be done on a rather basic level. The tools are often based on manual routines, not using modern knowledge in planning technique. The knowledge of statistics is often limited to the calculation of the mean value. This gives small appreciation for the "stochastic" aspects of planning, for instance the probability to complete a project on time.

Frequent deviations from plan will occur due to this lack of knowledge. There are a number of causes for this:

(i) unrealistic schedules, including incorrect estimation of activity times

(ii) unforeseen events - rock falls, strikes etc

(iii) policy changes

This thesis is primarily concerned with deviations from plan due to (i).
Activity times are normally specified in deterministic terms (i.e. each activity is assigned a precise duration). In practice activity times cannot be defined with such precision, but may assume any value within a range. This results in a distribution of possible completion dates. A technique for scheduling, utilizing stochastic activity times, would indicate both the range of possible completion times, and the risk of exceeding the planned project duration.

Because of the large degree of uncertainty associated with completion dates estimated using deterministic activity times, it is necessary to schedule completion of development well in advance of the start of production in order to avoid any loss of production due to late completion of development. The difference between the planned date of completion of development and start of production is termed the development buffer.

Development buffers tie up large amounts of capital in assets which are not immediately utilized. Substantial cost savings may therefore be achieved by minimizing the size of development buffers. This thesis investigates a technique for determining the minimum size of development buffers with an acceptable degree of risk of a production shortfall.

Techniques to obtain optimum balance between development and production activities are also described.

1.2 Goals

The overall goal of this work is, through developed mine planning, to increase the utilisation of equipment, manpower and capital resources.
This overall goal has been divided into a set of sub-goals, which are

1) Develop and introduce a technique to plan with stochastic time data.

2) Present a general approach on how to determine the optimum size of a development buffer.

3) Show methods to optimize the use of resources in development vs production activities, i.e keep a optimum balance between those two activity types.

1.3 Thesis Disposition and Execution

The thesis consists of four parts which are listed below.

1.3.1 Introduction

The purpose of this section is to give a brief overview and introduction to the thesis.

1.3.2 Literature Review

The literature review is divided into two parts, the first part (chapter 2) describes different time planning techniques together with some aspects of Monte Carlo simulation that is considered important for this work. Time planning under uncertainty is discussed in more general terms together with some comments regarding the handling of time data.
The second part (chapter 3) covers how these techniques have been used, or made available for the not so specialised user. This is done by giving some examples of available software categories for project management, a few examples are mentioned. The use of simulation languages has also been considered. Papers that describe approaches related to this work have been summarised. This section may be omitted by the reader whose main concern is the applications of the theory.
1.3.3 Bridge between theory and application

A computer program has been developed to form a bridge between the described theory and the applications. The program is tailored to fit the needs of the above stated goals. It is briefly described in chapter 4. This chapter is not needed to understand the applications of the theory, and can thus be omitted by the reader.

1.3.4 Applications

This last section (chapter 5-9) deals with the application of the selected techniques, especially on the Oscar project at LKAB's Kiruna mine, Sweden. The purpose of the applications is to reach the goals stated in section 1.2.
2. TIME PLANNING WITH PROJECT NETWORKS

2.1 General principles of time scheduling

The following sections present a brief introduction to the most common approaches to time scheduling using network models, and a review of their current status. The text in this chapter and in appendix 2.2 - 2.7 is partly based on the following textbooks: Bratley, Bennet and Schrage (1983), Elmaghraby (1977), Hammersley and Handscomb (1964), Law and Kelton (1982), Lichtenberg (1984), Mihrar (1972), Pritsker (1986), Taha (1982). A short dictionary of technical terms is presented in appendix 2.1.

2.1.1 Gantt charts

The Gantt chart time planning technique is not a network technique, but is included in this chapter as it is the technique that most Swedish mine planners are familiar with. The technique involves the construction of bar charts, see fig 2.1.

![Gantt Chart Diagram]

Fig 2.1 Gantt Chart
The bars, which represent activities, are drawn parallel to the time axis of the project as fig 2.1 shows. The duration of, and dependencies between, those activities is normally based on experience and not subjected to any optimizing procedure. The durations are normally deterministic estimates.

It is of course possible to present any time schedule as a gantt chart irrespective of how it was constructed.

2.1.2 CPM (Critical Path Method)

The simplest form of time modelling with networks is the critical path method (CPM). The network consists of a number of activities. For each activity there is a time estimate of its duration and also information about what other activities it is dependent upon. This type of model will provide an estimate of the project completion time only, and will not in any way consider the uncertainties in the time estimates. The critical path method will also indicate the different time slacks for the activities together with start- and completion times for all activities. This method is sufficient when there is no need to consider the uncertainties in time estimates, but this is generally not the case. The method can, however, be used as a planning tool if the user corrects the planned activity data that do not coincide with the real (observed) data during the execution of the project and then recalculates the network.
2.1.3 PERT (Project Evaluation and Review Technique)

A step towards dealing with uncertainties in time estimates came with the PERT-technique. In this technique every duration is represented by a statistical distribution. The method uses the same approach as CPM to calculate project completion time and the different slack times. The difference is that the time values used in the calculation are now expected values from a statistical distribution and not estimated values. The next step is to determine the degree of confidence that can be placed in the calculated completion time. This is achieved by summing the variances for the different activities along the critical path to yield the total variance for the project completion time. If there exists more than one critical path, the path with greatest variance is chosen.

Fig 2.2. An example of a project network.
The original, and most frequently used, distribution for the activity duration is the beta-distribution. It is approximated by three time values, these are optimistic, pessimistic and most likely time. Variance and expected value can easily be calculated from the three time values with a simple formula, see appendix 2.4. If the different activities can be considered as independent, and the path is sufficiently long, then it is possible to assume that the project completion time is normally distributed in accordance with the central limit theorem, see section 2.2.3.

2.1.4 CI (Criticality Indexing)

The above mentioned PERT-technique does not, however, allow for the situation when there are other paths in the network besides the critical path which are capable of influencing the project completion time. The extent of this influence depends primarily on the difference in completion times and variances for the involved paths. If, for instance, there exists one path which is marginally too short to become the critical path but has a much greater variance than the critical path. It is then very likely that this path will be, in reality, the critical path almost as often as the calculated critical path.

Fig 2.3. Comparison between density functions representing different paths in the same network.
The concept of criticality indexing has been introduced to compensate for this possible source of error. This technique is presented by Van Slyke (1963). The criticality index gives information about the number of times a certain activity lies on the critical path (i.e. the probability for an activity to be critical), if due consideration is given to the uncertainty in time estimates. The criticality index varies between 0-1. The most straightforward method to estimate the criticality index is by simulation. This technique is briefly presented in section 2.2.

Another time planning approach which uses the criticality index concept is described in Lichtenberg (1984), some of that philosophy is also presented in appendix 2.7.

2.2 Simulation

2.2.1 General description

In network problems of great complexity, like stochastic project networks, the number of possible outcomes is so large (i.e. an infinite amount), that they can not all be individually considered. Even if the time distributions are transformed into sets of discrete values, it would be an enormous task to evaluate every possible combination to obtain the analytically correct solution to the problem. The general solution technique based on multivariate integration is not feasible in practice due to its enormous complexity. This technique is discussed by Burt and Garman (1971). Simulation is an alternative method to arrive at an acceptable estimate of this solution, but with just a fraction of the computational effort.

The kind of simulation that is referred to in this text (the crude Monte Carlo type) implies a sampling process from stochastic distributions. This process will be briefly presented below.
The first step is to construct a numerical model (e.g. linear equation) of the phenomena to be studied. This model must, among other things, include a set of stochastic variables (distributions), from which it is possible to calculate a result for the model.

One way of achieving this result would be to substitute these stochastic variables with their corresponding mean values and to solve the model (e.g. linear equation) in a deterministic manner. This procedure will not however, take into account the possible interaction between different divergent (deviation from mean value) variable values, and will thus give no appreciation of the reliability of the result. An indication of the reliability of a result will however, be obtained by using the simulation process.

Simulation is a technique to obtain a number of samples of the model result by assigning different values to the variables within their permitted ranges (i.e. different initial conditions). These sampled results will provide the frequency data to which a distribution can be fitted. The different variable values (i.e. start conditions) are generated by letting the variables assume random values corresponding to their respective distributions. The precision of the result can be measured by the variance. This variance will decrease when this procedure is repeated and will asymptotically converge, rather slowly, to some value. The precision can however be improved by performing the simulation in loops, see Taha (1982). This is done by using the mean value of a set of results from one simulation (level 1) as one sample in a second simulation (level 2). The result from the second level simulation will then be used as the ultimate result. The increased precision is due to the fact that the distribution of mean values has a much reduced variance compared to the original level.
Fig 2.4. The fluctuations of the results from different simulation runs as a function of the number of samples.

2.2.2 Simulation in network planning

The simulation procedure in probabilistic project networks is based on the fact that the activity times are defined as statistical distributions, which are described by certain parameters. Together with a random generator, these distributions are then used to determine the time values, which represent the activity time for that particular evaluation of the network. Each network determined and calculated this way represents one sample in a simulation experiment. If the network is sufficiently large, these samples can be assumed to be normally distributed according to the central limit theorem, see section 2.2.3. The estimation of the activity time distributions will be dealt with further in section 2.3.
2.2.3 The central limit theorem

In brief, the central limit theorem states that, if a random variable arises as the sum of a large number of relatively insignificant distributions, then the probability density function (pdf) for the variable \( X \) will be normally distributed with a mean \( \mu \) and a variance \( \sigma^2 \).

Thus

\[ X \sim N(\mu, \sigma^2) \]

In the project network case the random variable is the project completion time.

2.2.4 Criticality indexing using simulation

The criticality indices may also be calculated using the simulation procedure. The activity durations are simulated for a number of alternative activity networks, i.e. with different activity times. The number of times an activity lies on the critical path, in relation to the total number of simulations, indicates how critical this particular activity is (criticality index). The different project completion times that have been calculated are stored during the simulation. Afterwards it is possible to determine the distribution for the project completion time from these data. The choice of distribution for the different activity durations is, of course, optional.

2.2.5 Testing of random generator and simulation results

The simulation results, (i.e. the project completion times), should be tested to verify that the results belong to a normal distribution. For this purpose there exists "goodness of fit" tests.
The random number generator should also be tested, to verify that it yields uniformly distributed random numbers, see appendix 2.6. The testing of the random generator is the responsibility of the software developer.

a) The chi-square test

One approach to test a distribution is using the chi-square test. It determines the probability that a given sequence follows a certain distribution. This is done by dividing the appropriately derived distribution function into a number of sub-intervals. For each interval the theoretical number of occurrences is calculated. They are then compared with the experimental (simulation or a random generator) values. The standard formula is given below.

\[ V = \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i} = \sum_{i=1}^{N} \frac{O_i^2}{E_i} - N \]

- \( O_i \) = Number of observed occurrences (completion times)
- \( E_i \) = Number of expected occurrences (not a discrete value)
- \( N \) = Number of samples (intervals)
- \( V \) = The computed (random) test variable

The test variable should have a large value but, particularly in the case of a random number generator not improbably large.

b) The Kolmogorov - Smirnov test

This test is considered to be more objective than the chi-square test since no "intervals" have to be chosen. The test-variable used is a measure of the maximum absolute difference between the theoretical and the experimental cumulative distribution function.
c) Comments

The above tests were performed, in this study, on a relatively large (75 activities) project network, and it was found that the distribution fairly well fitted a normal distribution, see appendix 2.5. The test has also been performed on the adopted random generator which confirmed that the results were uniformly distributed, see appendix 2.6.

It should be noted that even when the random numbers are properly behaved, there are a certain number of independent tests that fail.

2.3  Time scheduling under uncertainty

2.3.1 Sources of uncertainty in time plans

There are a number of uncertainty factors associated with the construction of a time plan. Some of these uncertainties are presented in the following sections.

a) Changes of plans due to new "ideological" conditions

One uncertainty when trying to adhere to time plans, is that people have the tendency to change them for various reasons. This can be due to policy changes, new ideas etc.
b) Changes of plans due to new physical conditions

If mining, as an operation, is studied one can see that the plans are often disrupted by unforeseen events, not due to time and capacity data for the mining operations, but due to lack of control of the surrounding rock. These unforeseen events would include, for example, the caving of a drift or an erroneous geological interpretation (e.g. the ore is not where it was anticipated), making it necessary to alter the plan. These kind of problems are often hard to foresee and might cause large deviations from the original plan. The most problematic characteristic of these kind of deviations is that they not only influence the activity durations, but also the sequence of events. Furthermore, there is a significant possibility that new, i.e not planned, activities have to be performed in order to complete the project.

Example case:
Part of an ore body is planned to be mined in two stopes between the hangingwall and the footwall. Production drilling is performed in both stopes before production starts. The mining of the first stope induces a stress release in the second. This causes movements along planes of weakness which intersects a number of drill holes. Because it is now no longer possible to charge the bottoms of all the holes, some of them have to be redrilled.

It is for obvious reasons very hard to make accurate predictions that can handle even the kind of "disturbances" mentioned above. Disruption of plans will probably then be impossible to avoid. The best solution to handle this kind of problem is to computerize the mine plan in order to facilitate replanning of the remainder of the project. Today, throughout the mining industry, this is usually not done. If changes of this type occur, the replanning extends only over a limited part of the remaining project. It is very unusual to reschedule the entire project in the light of the changes.
The work presented in this thesis does not cover uncertainties in planning due to unforeseen events other than in a general way. The work deals principally with the effects of uncertainty in the duration of the activities.

c) General delays caused by external events

There is always a certain risk that the project meets with some sort of general delays as strikes, fire in another part of the mine etc. Those kinds of problems delay the plan, but do not alter it.

d) The uncertainty in the estimation of activity duration

It is almost never possible to make a deterministic, (single value), estimate of an activity duration, and then expect to be able to conform to it. The time necessary to perform a single activity is very sensitive to minor disturbances such as machine break downs, anomalies in rock conditions, jamming of the drilling rod etc. These kinds of deviations are very hard to foresee as specific incidents. They can however be accommodated by substituting the activity time estimate by a statistical distribution, or simply an estimate of the mean and standard deviation.

e) Comments

The points of possible uncertainties mentioned above can be divided into three groups. They are

1) Structural changes of the plan, the time data is still valid. Point a and b above.

2) Overall unforeseen delays, both the plan structure and time data are, however, still valid. Point c above.
3) Changes of the time data, the structure of the plan is still valid. Point d above.

The emphasis of this thesis is on point 3.

2.3.2 Collection and handling of uncertain time data

In order to establish the "uncertainty" parameters for an activity time, it is first necessary to collect information about them. This may be obtained in several ways:

a) By interviewing persons experienced in the planning process.

b) Compilation of existing data in a form which enables conclusions to be drawn regarding the expected activity time and range.

c) Follow up plans and perform a statistical analysis of activity times.

d) To perform specially designed work (time) studies.

The estimation of activity time distributions can be divided into two parts. The first part is to locate the "limits" of the distribution, i.e. the smallest and largest conceivable time values that are fairly realistic. The second part is to decide which distribution type the activity time belongs to. This can be difficult, especially if the estimation is based on experience values rather than analysis. The experience is, however, that the most usual distribution seems to be the lognormal distribution, see Schunnesson (1989) and Lindström (1989). More complex activities might, however, tend to be normally distributed as a consequence of the central limit theorem. Some other possible distributions are, together with the lognormal, mentioned in appendix 2.4.
2.4 Scheduling under resource constraints

2.4.1 Introduction

The CPM calculations only consider the restrictions that are imposed by the logical sequence of the activities. Any limitations due to resource constraints are ignored, i.e. unlimited resources are assumed. In most cases there are however resource constraints of some form, hence the need for resource scheduling.

The objective in resource scheduling is to construct a work schedule which either completes the work at the earliest possible date using the resources available, or makes the most efficient use of resources to complete the work by a fixed date. The latter procedure is often referred to as resource levelling.

A resource can be defined as an entity which is necessary for the completion of an activity, and might be available only in a limited quantity. Examples from various types of resources are presented below.

Machinery; special kinds of equipment that are necessary for the intended operation.

Specialists; personnel with special education that is needed for a specific task.

Manpower; if the amount of available men is limited, personnel will be a limiting resource.

Material; e.g. when the delivery of a special material is limited to a certain amount per time unit.
Space: if two activities have to be performed at the same place, and there is not enough space to carry them out simultaneously.

Capital: if a certain amount of money is required but not always available then capital will be a limiting resource.

2.4.2 Objectives of resource scheduling

As stated above there is more than one objective possible when resource scheduling, the most important ones are mentioned below.

a) Resource levelling

Resource levelling is the procedure used to make the best utilization of the resources, i.e. to minimize the number of resources required, to reach a certain deadline. This technique is particularly useful when there is a need to shorten the project completion time.

b) Minimum project duration

This is the perhaps most obvious objective of resource scheduling. That is, to minimize the total project completion time under a known availability of resources.
2.4.3 Techniques

a) Introduction

There exist several different approaches to deal with the resource scheduling problem, i.e. to complete the project as fast as possible with the available resources. These approaches may be classified as either analytical or heuristic techniques. Analytical ones are, however, mathematically very complex. For complex networks, the only feasible approach in practice is to use a heuristic method.

b) The heuristic approach

Heuristic methods use empirical decision rules which produce a better than average resource allocation, but not necessarily the best. The rules can be written in formal terms and are therefore easy to implement in computer software. The decision rules, in their turn, can be applied in a number of different ways. Three major groups (techniques) can be distinguished.

All three groups treat the project in a series of discrete time intervals, for each of which an availability of resources may be specified. They consider which activities could be scheduled in the given interval, and schedule as many as possible. The activities that can not be scheduled are postponed to the next interval. This process continues until all activities have been scheduled.

The first, and also the least complicated group, is sometimes referred to as the serial algorithm group. The chosen decision rule(s) are used to sort all activities into a definite sequence which are then scanned for possible scheduling. As the scheduling proceeds, activities already scheduled are removed from the sequence. However, no changes are made from the initial order of the sequence.
The second group, using what is sometimes called serial-parallel algorithms, begins by scheduling exactly as in the case above. The sequence of activities that remains to be scheduled is reviewed and, if necessary, altered in the light of the effects of those activities already scheduled.

The third group, often referred to as the parallel algorithms group, considers all the activities that are available for scheduling in a certain time interval. These activities are sorted, (or resorted), according to the given decision rules, before scheduling of each new activity. This approach is often regarded as the most efficient.

2.5 Comments on the estimation of the project completion time

2.5.1 Introduction

It is often not understood why a project is not completed on schedule even though nothing has gone wrong and all deviations from planned activity times have been small and within reasonable limits.

This is due to a poorly understood phenomena, the interaction between uncertainties. The fact that a network with more than one path will have an expected completion time longer than the longest path is not often known, compare with section 2.1.4. This is easily shown with a very simple experiment.
Consider a pair of random variables with the same expected value. If one generates a value from each variable and assigns the largest of those to a test variable, this test variable will then get a mean value larger than the expected value of either of the random variables. If, for instance, the random variables are uniformly distributed between 1-5, (expected value = 3), then the mean of the test variable (T) will be about 3.65, see below.

\[ E(R_{1i}) = 3, \text{ uniformly distributed in the range 1-5} \]
\[ E(R_{2i}) = 3, \text{ uniformly distributed in the range 1-5} \]

\[ \lim_{N \to \infty} T = \frac{1}{N} \sum_{i=1}^{N} \text{ Largest of } R_{1i} \text{ and } R_{2i} \approx 3.65 \]

![Diagram of network with activity times and probabilities](image)

**Fig 2.5.** The determination of the expected completion time for a very simple network.

This is obvious if one realises that there is just 50 % probability that one of the random variables will get a value below 3, and thus just 25 % probability that the value of the whole realisation will lie below 3.

The phenomena described above will of course influence the results given by the simulation procedure and is therefore of essential importance to this thesis. The simulation will, however, give an unbiased estimate of the project completion time.
The impact of the interaction of uncertainties in activity times is of considerable importance when estimating the project completion time, but is very often neglected.

2.5.2 The project completion time before resource scheduling

As described earlier, the distribution of project completion times following simulation may be described by three values. These are the mean value, the variance for a single network and the variance for a single run. If the network is large enough the completion time can be considered normally distributed in accordance with the central limit theorem. This means that the mean value and the variance for a single network can be used for probability calculations, i.e. to calculate the probability of completion within a certain time etc. The variance for a single run is a measure of how good an estimate of the expected value the mean value is.

The vital question now is, what network size is large enough? This is very hard to predict due to the fact that a few activities might have much more influence over the completion time than the rest. Assuming that none of the activities are very significant one has to consider the length of the most critical paths and the number of such paths. As a general rule there needs to be at least 10-15 independent random variables to make their sum normally distributed. This would imply that the separate paths should have a lengths of at least 10 activities. This number can probably be somewhat decreased with parallel paths. An interesting test parameter for application of the central limit theorem might be the total number of activities in the network. This is however not investigated fully yet owing to the references. It should, however, be considered when using this technique.
2.5.3 The project completion time after resource scheduling

A problem arises with the estimate of the completion time after resource scheduling has been performed. The resource scheduling procedure invalidates the whole estimate of project completion time by introducing artificial slack times. The only thing that remains valid from the original simulation is the estimate of the individual activity times. It is of course possible to assume that the new completion time is the mean value and that the old variance is in some way (constant or proportional) related to the new one. This will probably give a biased estimate in the same way as PERT gives a biased estimate of the original completion time, the variance will probably vary depending on the artificial slacks that have been introduced.

Another approach would be to perform a separate simulation following resource scheduling and then use the sequence introduced by the scheduling algorithms to estimate the project completion time. This is, however, not an altogether satisfying solution either. The problem is that this solution will probably give instances when the utilization of different resources are over 100 % because of the uncertainty in times. These situations would in real life induce a reorganizing of the resources, and in that way invalidate the results.

Therefore, the best way to incorporate scarce resources in the estimate of the probability density function for project completion time might be to include additional dependencies in the project network. Those dependencies would then compensate for the effect of scarce resources. The time estimates for activity durations that are given by mine planners often have the problem with scarce resources implied in the estimates. This is usually a problem, (i.e. a possible cause of error), but might in cases like this be utilized in an acceptable way.
3. PROJECT MANAGEMENT SOFTWARE, TECHNIQUES AND APPLICATIONS

This section reviews the various categories of software available for planning and control and presents examples of their application.

3.1 Software

3.1.1 Project management packages

There exists on the market a wide variety of software packages for project planning and control. A comprehensive list of packages is available in Föreningen Projektplan (1987). Some information is also given in Wood (1988). Most of the software deal, however, not with the area of time planning under uncertainty, but deterministic time planning. The software may be classified into three categories of uncertainty which can be incorporated in the activity times i.e.

* Deterministic planning
* Semi-stochastic planning
* Stochastic planning

In the following sections examples of software will be discussed. Addresses to obtain further information about the software is given in appendix 3.1. Only those aspects concerning time planning will be considered.

There is apparently considerable confusion about the meaning of the terms CPM and PERT in the software industry, especially PERT. Both terms are frequently applied incorrectly, the prospective buyer should check which technique is in fact used.
a) Deterministic Planning

Deterministic planning does not in any way consider the uncertainty in time data. The software in this group is mostly based on the standard CPM technique. This means that the calculated time data are limited to earliest start times and slacks for the activities. The programs might also incorporate some form of resource levelling or scheduling function.

One example of this type of package is Timeline. It uses a standard CPM approach with the possibility to perform resource scheduling. This type of program works well as a planning tool if the actual execution times are entered into the program as the project proceeds. From a strategic point of view, however, it has a number of draw backs, especially if the project completion date is critical. This will undoubtedly be biased towards an optimistic estimate, see section 2.5.1.

b) Semi-stochastic Planning

Software in this category mainly utilizes the PERT technique. Stochastic time data are entered for each activity, but not all of the data are used at one time. The time distribution used is usually limited to the beta-distribution. This kind of model will almost certainly give a biased (optimistic) estimate of project completion time. As only one path at a time is considered, see sections 2.1.3 and 2.1.4.

c) Stochastic Planning

This group includes the straight-forward Monte Carlo simulation approach. All data about the activity time distributions are used in order to obtain an unbiased estimate of the project completion time. Consequently, this is the preferred approach to time planning under uncertainty.
The Artemis 2000 system has PAN (Probabilistic Activity Networks) as an optional extension. This PAN-extension has the capability to perform a probabilistic analysis using Monte Carlo simulation. It allows the user to choose from five different distributions. The program also has resource scheduling and levelling functions.

PGL-timing is a program which is based on the successive principle, see appendix 2.7. The program is intended to be a tool for the planner who has a sound knowledge of this technique.

The PertSim program, which has been used in this thesis, belongs to this group which uses the stochastic approach. PertSim is presented in chapter 4.

3.1.2 Simulation languages

There exists so called simulation languages which are specially designed for different kinds of simulation applications. Simulation languages provide a modeling framework which enables the user to put his own simulation model into this supportive environment. However, it is not that easy to do this in a general form, it is more likely that the user has to write a new "program" for each case. Furthermore, the use of a simulation language might have the disadvantage that the users "world view" will be coloured by the employed language.

One such language is SLAM II which is presented in Pritsker (1986) and Pritsker et al (1989). In SLAM the model is represented by a set of standard symbols, (nodes and branches), interconnected into a network structure. The model is then translated into an equivalent set of SLAM statements for execution on the computer.

Bratley (1983) and Law (1982) give a good overview over the most common simulation languages as SIMSCRIPT II, SIMULA, GPSS and GASP IV.
3.2 Applications

A large number of rather theoretical papers on the subject of the project completion time distribution are available in the literature. These are often performed on networks of rather limited size, or are out of date as a result of the rapid development of personal computer applications in this area. The studies presented below describe the application of the above techniques within the area covered by this thesis.

The paper of Gardner (1987) is directed to the mine planner and gives the basic information about Monte Carlo simulation and how it can be applied in the mining industry. Information on how to deal with different kinds of probability distributions is given in short terms. The paper points out the possibility to use this kind of simulation models to make sensitivity analysis and to answer questions of "what if" character by altering one variable at a time.

Bandopadhyay S and Sundararajan A (1987) show how the criticality index concept can be applied to a mining project. The paper stresses the fact that there may be a number of critical paths depending on the particular set of sampled activity times. It also points out how important it is that the project manager is aware of the consequences of uncertainty, i.e. that the concept of probability is fully understood, and the superiority of the CI approach compared with the PERT approach. The paper also discusses resource scheduling and levelling.
Shtub (1986) deals with the trade-off between the net present cost of a project and the probability to complete it on schedule. The heuristic procedure presented is designed to be used as an extension to CPM analysis. The procedure, using the calculated slack times, tries to delay activities with a high value (expensive) one at a time. At each iteration a Monte Carlo-type simulation is used to approximate the probability of not completing the project on time. This probability is stored along with the net present cost of the project. The result of the analysis is a set of points on the plane representing the probability of not completing the project on time versus the net present cost of the project, where each point corresponds to a specific schedule. The management can then choose the most appropriate schedule for implementation.

3.3 Conclusions

3.3.1 Software

There exist a number of packages for time planning. Most of them, however, do not consider the uncertainties in the activity times and consequently they are unsuitable for the investigation of project scheduling using stochastic techniques.

Of the software which deals with uncertainties in time data, most packages tend to limit the user to the approach implemented by the developer. They are generally rather expensive and more suitable to run on mainframes than PCs.

The use of simulation languages are found to be too complex for the ordinary mining engineer.
3.3.2 Applications

Little work appears to have been done on the practical application of project planning using stochastic networks, particularly with reference to the mining industry. Most papers appear to be theoretically orientated and consider relatively small networks.
4. DEVELOPMENT OF A COMPUTER PROGRAM FOR PROJECT SCHEDULING USING STOCHASTIC TECHNIQUES

The computer program PertSim was written by the author, in cooperation with Lena Andersson, to investigate the stochastic aspects of time planning. The authors contribution to development of the program concerns those aspects dealing with time and resource scheduling, whereas those aspects dealing with cost and production scheduling were developed by Lena Andersson, and will be reported in a separate thesis.

4.1 Program development objectives

The primary objective for the development of a stochastic project scheduling program was to acquire a tool which could be applied to fulfill the thesis objectives outlined in section 1.2.

The decision to develop the code rather than adapt an existing package was based on the following considerations:

(i) The substantial purchase cost of commercial packages with no guarantee that it would be possible to modify the package.

(ii) Access to the source code was necessary in order to incorporate the routines required to achieve the specific objectives.

(iii) In those instances where the source code could be acquired, usually at additional cost, it would be difficult to understand the code of such a sophisticated package to the degree necessary to enable it to be modified.

(iv) Development of the code gives a fuller understanding of the procedures involved and provides the opportunity to evaluate alternative techniques.
A citation from Bratley et al. (1983) concerning simulation might also be adequate in this section. They say that "for any important large-scale real application we would write the programs in a standard general-purpose language, and avoid all the simulation languages we know. The reason is simple: we would not be comfortable writing simulation programs in a language whose behavior we are not able to understand and predict in detail".

4.2 Program overview

A special PertSim manual will also be presented at a later stage by the authors, Almgren and Andersson (1989).

4.2.1 General outline

The PertSim program is written in Turbo Pascal 5.0 and is designed to run primarily on an IBM AT or any compatible computer.

The program is based on the CPM/PERT/CI techniques described above and calculates project completion time, slack times for the different activities, and criticality indices. The estimates of event times and the project completion time are based on Monte Carlo simulation. The program also determines the parameters of a normal distribution which describes the project completion time based on the simulation results. It is therefore possible to estimate the probability of completing the project at a given time.
Input data to the program consists of the activities together with the information about how they are interrelated, i.e., their dependencies. The program will check the information in order to find any illegal or unnecessary data such as redundant dependences, loops in the network, illegal time parameters etc. The validated data will, at a later stage, be used to create an activity network, which forms the basis for the calculations of time slack times etc.

4.2.2 Implemented distributions

A number of alternative functions have been incorporated to describe the distribution of activity times. These are listed below together with the required parameters.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>optimistic time</td>
</tr>
<tr>
<td></td>
<td>pessimistic time</td>
</tr>
<tr>
<td>Triangular</td>
<td>optimistic time</td>
</tr>
<tr>
<td></td>
<td>most likely time (mode)</td>
</tr>
<tr>
<td></td>
<td>pessimistic time</td>
</tr>
<tr>
<td>Normal</td>
<td>mean time</td>
</tr>
<tr>
<td></td>
<td>standard deviation</td>
</tr>
<tr>
<td>Lognormal</td>
<td>mean time</td>
</tr>
<tr>
<td></td>
<td>standard deviation</td>
</tr>
<tr>
<td>Modified exponential</td>
<td>minimum time</td>
</tr>
<tr>
<td></td>
<td>mean time</td>
</tr>
<tr>
<td>Weibull</td>
<td>shape</td>
</tr>
<tr>
<td></td>
<td>scale</td>
</tr>
</tbody>
</table>

The distribution functions are described in more detail in appendix 2.4.
4.2.3 Simulation method

The technique implemented in this program is a straight-forward Monte Carlo technique. The procedure described in section 2.2.1 to decrease the variance of the completion time by simulation in loops has been implemented.

4.2.4 Resource Scheduling

a) Scheduling algorithm

An heuristic, parallel approach was adopted for use in PertSim while it, owing to the references, is considered to be the most efficient one, see section 2.4.3. Two sets of heuristic rules have been used. This gives the user the opportunity to choose the specific criteria which are most appropriate for the particular application. The resource scheduling algorithm is presented in a simplified form in appendix 4.1.

4.3 Results

The results can be divided into two categories, namely before and after resource scheduling.

4.3.1 Results before resource scheduling

The program calculates mean duration, earliest start time, last completion time, total float, free float and criticality index for each activity. It also gives the project completion time, the variance of the project completion time and the variance for just one realization of the project. These figures then form a basis for subsequent probability calculations.
4.3.2 Results after resource scheduling

The results after the resource scheduling are start and finish times for the activities together with activity slacks. This activity slack should not be confused with the other time floats calculated prior to resource scheduling. The slack here is the amount of time an activity can be delayed without interfering with any other activity. This slack depends on how the resource allocation has been done. A new project completion time has also been derived.
5. **THE PLANNING AND EXECUTION OF THE OSCAR PROJECT**

For several reasons, the Oscar project has been chosen as a suitable mining project to evaluate the concept named "time planning under uncertainty". The most important of these are listed below.

i) The project was of suitable size and complexity.

ii) It was in operation during the project time.

iii) The rock mechanics of the project were very closely monitored as a part of the G2000 program.

5.1 **The Oscar project, a short description**

The project was designed as a test run of a new sublevel stoping design. It is a sublevel stoping operation consisting of five modules (stopes), see fig 5.1, and amounts to a total tonnage of about 1.2 Mton iron ore.

![Horizontal section of Oscar with module definitions.](image)

**Fig. 5.1.** Horizontal section of Oscar with module definitions.
Each module is drilled (ø115 mm) and blasted from a drilling level down towards a loading level 60 m below, see fig 5.2a. The production from each module starts by opening a primary room against a raise. After the primary production, the rest of the module is blasted in one single mass blast. The modules were planned to be mined in order B–C–E–A–D, see fig 5.1.

Fig 5.2a. Vertical section of the Oscar project.

Fig 5.2b. Horizontal section of the drilling level.
Fig 5.2c. Horizontal section of the loading level.

5.2 Plan vs reality

It should be noted that the amount of the delays presented below is related to the certain activities and not to the entire project in general.

5.2.1 Development

Due to poor rock conditions there were large difficulties with the reinforcement of the loading level in the initial stages. Among other things, special concrete reinforcements caused a delay of about 3 weeks.

There was also a problem with the ventilation on the drilling level which delayed the loading about one hour/round. This caused a total delay of some 3 weeks. This was owing to an initial inadequate layout and improved after a supplementary drift was driven.
Fig 5.3. Supplementary drift on drilling level.

Failure due to poor installation of cable reinforcement of the roof on the drilling level occurred, which caused some caving and a 3 weeks delay. The failure was due to poor adherence between the cable and the rock, which was induced by a new method to apply the concrete. New cables had to be installed.

The drilling of the opening raises went smoothly.

The production drilling had lower capacity in the beginning than what had been assumed, which caused some delay. This was because the drilling rig was a new prototype and had some teething troubles (i.e. problems because the equipment were new and not previously used).

Additional reinforcement of the loading areas took two extra weeks.

Roof-drilling on the loading level went according to plan.

However, the roof-drilling on the drilling level got scheduled at the same time as a raise. This was not possible to achieve. The reason for this was that both operations needed access to the same drift. The consequence was one week delay and was due to lack of communication.
5.2.2 Production

During the production period there where some problems because there was not any need for the ore quality available in Oscar, so the loading was directed elsewhere. This brought about 2-3 weeks during which no loading was done.

There were also problems with the loading because the personnel were inexperienced, which led to about two weeks delay.

The position of a borehole for one of the opening blasts in module B was misjudged, which made the burden to large. This gave some problems that delayed the production start by one week.

Progressive caving (failure) of the roof in the primary room in module B (the first mined) occurred. The caving did not stop until there was just 1-2 m up to the drilling level. This gave no particular delays but introduced a certain amount of uncertainty.

Fig 5.4. Caving in primary room module B.
At the drilling level there was a large cave caused by lack of control of the surrounding rock. This caving cut off the access to the production holes for one of the modules (module A) and caused a lot of problems and delays, see below.

![Caving area](image)

**Fig 5.5a.** Horizontal section of caving at drilling level.

**Fig 5.5b.** Vertical section of caving at drilling level.

i) New reinforcements were installed in a neighbouring drift, which took one week.

ii) A new access to the module that had been cut off was driven. This also took one week.
iii) There were no activities on the drilling level, while decisions about changing the mining sequence and whether or not to enter module A were taken. This stop lasted for two weeks.

iv) That incident made it necessary to drill extra holes to make it possible to extract the ore in module A. The redrilling caused a stop in loading from one module and the charging of another one. This operation took about seven weeks. The blasting in module A was limited to a slot just above the loading level to make blocking of the module possible.

There were additional bolting of the pillar between module D and E to protect the stability of E. This took two days.

The loading of module D was stopped because it induced movements on the drilling level of module E. The charging of module E was then delayed for five weeks because the drill holes had to be cleaned (i.e. gravel and stones were removed). The loading of module D could be resumed after module E had been massblasted.

For quality reasons the ore was "saved" for production at a later stage, the loading of module E was therefore discontinuous.

5.2.3 Mining sequence

The original plan called for mining to be done in the sequence B-C-E-A-D. At a later stage it was discovered that the hanging wall boundary in module A was located further into the module than what had been initially believed. The consequence of this was that the A and D modules were put together into one module.
It was also decided, after the large caving in module D-A, see fig 5.5, that module D should be mined before module E, and that module A should be mined by block caving.

5.3 Comments

The delays presented in section 5.2 are summarized here in more numerical terms to facilitate a better overview of the problems. The classification might be a bit crude but represents the main principal problem types. In reality the ore quality type ("grade control") belongs under "planning and layout".

<table>
<thead>
<tr>
<th>Problem type</th>
<th>Mean duration (wdays)</th>
<th>Nr of Occurrences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock mechanically induced</td>
<td>2.5</td>
<td>2.7</td>
</tr>
<tr>
<td>Equipment + technique related</td>
<td>2.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Planning and layout</td>
<td>2.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Grade control</td>
<td>0.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Inexperience</td>
<td>0.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 5.1. Different types of deviations from the original plan in the Oscar project.

By looking at the table 5.1 above it is possible to conclude that most of the major delays are caused by unforeseen rock instabilities. This is most apparent in the production phase when the rock is exposed to severe stresses due to the removal of supporting rock and vibrations from blasts in the vicinity. The massblasts subject the neighbouring rock to a lot of strain.
The other large cause of deviations from the mining plan was the ore quality problem, which occurred during the production phase. Today, in practice, the ore quality planning seems to be done mainly at the production stage, which makes it necessary to take this kind of action, i.e. disrupting the plan, to remain within the quality restraints.
6. **PLANNING WITH STOCHASTIC TIME DATA**

6.1 **Introduction**

This section will, in short terms, describe the current technique used in the Kiruna mine. The mining methods used are sublevel caving and sublevel stoping (see section 5.1).

The first consideration in the planning process is to make an estimate of how much ore can be disposed of in the market. This is done by a separate group in the company, and forms the basis for a production plan which lies well above (10-15 % in tonnage) the sale plan. The difference forms a sort of buffer (uncertain investment), which eventually might be sold on the spot market or on other markets.

The second consideration is the ore quality problem. Different qualities of the ore are to be sold at different times during the year. This must be dealt with in the planning phase.

6.2 **The currently used technique**

The first step in the actual planning process is the "operational planning", see also Bäcktorp (1989), which, for a three year period, decides which areas are to be mined and when. These plans then form the basis for the development planning. This planning includes some geometric planning (production holes in the sublevel caving operation) as well as the time planning. The layouts are made separately, but are strictly constrained by certain geometries. A coarse timeplan is made for both sublevel- caving and stoping. However, a more detailed timeplan was made for the Oscar stoping operation described in chapter 5. The caving operation is planned weekly for each mining area. Generally, the ore quality controls are a complicating factor for both of the methods.
The timeplans used in the sublevel stoping operations are Gantt charts (bar charts), see section 2.1.1. They are based on estimates of the activity times and the planners knowledge about dependencies between activities. No special tool is used to optimize the timeplan (minimize the project completion time), and all time data is deterministic. The charts are hand-written by the planner and might range over a two year period.

### 6.3 Shortfalls in the currently used technique

#### 6.3.1 Planning

The time plans are written manually which means a lot of work if they need rewriting. In reality, this is never or seldom done. This means that it will be harder and harder, as the original plan becomes obsolete, for the planner to get a good general view over the project, which will probably produce a plan of inferior quality. There will be more mistakes in the plans, (e.g. a resource is planned to be at two locations at the same time or two activities are scheduled at the same place at the same time etc.). The optimization of the plan will also be more difficult.

Today the planning process is not done with stochastic time data. In general, this leads to optimistic estimates of the time that it will take to arrive at different events in the project, see section 2.5.1. This means that the resources might be needed at other times than what was planned. This might disrupt the whole plan if the resources at the moment they are needed are scheduled for other assignments. New unforeseen delays are introduced.
In today's mining industry the departments of rock mechanics in the companies often function as some sort of fire-brigade. They are not sent for until after the problems have occurred, and their task is just to limit the consequences of the problems. As could be seen in section 5.3 most of the larger delays were caused by rock mechanical problems, (i.e lack of control of the surrounding rock), even though most of them were encountered during the production phase. More emphasis should be put on the rock mechanical work in the planning stage. This should be done in order to avoid those kind of problems as much as possible.

The different ore quality characteristics are not coordinated well enough with the overall time plan. The quality parameters are as important as the volume parameters and should be considered much more carefully in the long range time planning.

6.3.2 Execution

A problem from the planners point of view is when there is a lack of control of the utilized resources. This problem occurred in the Oscar case when no loading was done due to ore quality regulations, see section 5.2.2. The same problem might also occur if the rock-reinforcement is done by an external group etc. A mine planner that can not control the on-going activities in his domain cannot be expected to meet the time schedule accurately.

The disruption of plans due to ore quality regulations are a very serious problem that surface in the execution phase. This depends on the need for certain qualities at certain times. Those demands are then met without much concern of the overall time plan.
6.4 Possible approach to include stochastic time data

6.4.1 Introduction

Our existence is a stochastic process. The use of deterministic models is often a simplification of reality. These are made to reach a certain goal with a small effort. The level of this goal can many times be discussed. Therefore, to get a reliable plan, it is necessary to include stochastic time data. The plan is also dependent on being as constant as possible within the defined stochastic limits. Given below are some suggestions on how to achieve a good and reliable plan.

6.4.2 Some general suggestions

The timeplans have to be developed in a computer environment to ensure that they are easy to correct and adjust. They should be stochastic.

To ensure that the plan is as constant as possible, it is of vital importance to have a sound layout. Rock mechanical aspects are of major importance for the endurance of the plan. The planning of layouts and sequences should always be to reach as safe a working environment as possible. These facts demand some sort of a rock mechanical pre-investigation and analysis before the actual planning of the project.

The control of the project should, as much as possible, be in the hands of the person that planned it. This is to avoid cases as the one when no loading capacity was made available because the loader was needed elsewhere, see section 6.3.2.

The different ore qualities should be given consideration at an early stage in the time planning procedure. This would make it possible to have a more continuous production on the scheduled time.
6.4.3 Suggestion of planning sequence

The following steps should be performed when planning for a mine, or planning and executing a mining project. This is to achieve a plan with good accuracy when it comes to estimating different event times, and also to get control over the production in terms of ore quality and quantity. A block is defined as a mining area of limited size. The Oscar project would be considered as one block. The determination of the project completion time distribution is the basis, and thus of primary importance to the validity, of the plan. A suggestion on how to do this is presented in the following section.

a) Determination of the probability density function for the project completion time

The uncertainty of the project completion time depends primarily on four factors. They are (see also section 2.3.1)

i) Uncertain time data
ii) Large unexpected rock mechanics problems
iii) External influence
iv) Policy changes

(i) Uncertain time data

Previous sections of this thesis have dealt principally with the uncertainty in time data for different activities, and it is this factor that is modelled in the simulation model. The model gives the probability density function (pdf) only if the project follows the plan (i.e. no deviations from plan due to points 2-4). This factor therefore covers all normal mining engineering delays, but not the more extreme delays resulting from caving or strikes etc.
The estimates of project completion time derived from the simulation model of the project are assumed to be normally distributed, see section 2.2.2, and will here be denoted as

\[ TP = \text{estimated completion time for the project} \]
\[ \sigma_{TP} = \text{standard deviation of } TP \]
\[ TP_{DET} = \text{deterministic project time} \]

(ii) **Large unexpected rock mechanics problems**

Large rock mechanics problems are defined as problems that either disrupt the structure of the plan, or induce an abnormal prolongation of an activity. The latter almost always turn out to be of the former category anyway, as new activities often have to be introduced (e.g. if a drift caves some additional loading and rock reinforcement have to be performed). The estimation of this factor has to be based on earlier experiences, as well as on a rock mechanics pre-investigation. The latter approach is most important if the present mining conditions have never been experienced before, for instance when starting up a new mining area or initial tests with a new mining method.

The factor is assumed to be normally distributed and the labelling is presented below.

\[ TR = \text{estimated delay caused by large rock mechanics problems} \]
\[ \sigma_{TR} = \text{standard deviation of } TR \]
It should be noted that, in the model, there should be a reasonable chance of performing a project without any delays caused by large rock mechanics problems. This implies that approximately,

$$3\sigma_{TR} \cdot TR$$

(6.1)
i.e. the pdf should be cut by $t=0$ on a relevant level, in theory the pdf is always cut by $t=0$

Fig 6.1a. Pdf for activity duration were $TR = 3\sigma_{TR}$

This assumption is, however, a bit dangerous because it indicates that there exists a possibility of negative delays. Therefore, it might be advisable to assume normal distribution only when the standard deviation is within reasonable limits.
A fair estimate of these could be

$$3\sigma_{TR} \leq TR \leq 4\sigma_{TR}.$$  \hfill (6.2)

Fig 6.1b. Pdf for activity duration were $TR = 4\sigma_{TR}$

If this is not possible, it might be better to use a distribution, which is defined over a finite interval.

(iii) External influence

External influence might be very hard to predict, especially with respect to delays caused by accidents in other parts of the mine (e.g. fire on the haulage level etc.). Experience based data have to be used to estimate how frequently this type of event occurs. It might be somewhat easier to predict events like strikes. If, for instance the wage negotiations are predicted to be tough, then it might be more probable that a strike will occur etc.
This delay factor is assumed to be normally distributed and the labelling is presented below. The same reservations apply as in the case above.

\[ TE = \text{estimated delay caused by external influence} \]
\[ \sigma_{TE} = \text{standard deviation of } TE \]

(iv) Policy changes

A change in metal prices, mining philosophies and so forth can drastically alter the project situation. In the most serious case, the whole project will be distorted and the old project plan invalidated. It is out of the scope of this work to include these kind of uncertainties. New metal prices might be foreseen, but the consequences they have on the project plan are almost impossible to predict. It is more or less useless to try to establish some sort of distribution for this uncertainty factor because of the unpredictability in the possible disturbance factors. However, this policy change factor might be used as safety factor, which might be adequate if it comes to policy changes in smaller scale. The labelling is presented below.

\[ TPOL = \text{delay caused by policy changes (safety factor)} \]

This factor should be excluded if the project is of standard type.

(v) The overall estimation of the project completion time

The overall project completion time might be calculated as the sum of the different distributions, see below.

\[ T = TP + TR + TE + TPOL \quad (6.3) \]
\[ \sigma_T = \sqrt{\sigma_{TP}^2 + \sigma_{TR}^2 + \sigma_{TE}^2} \quad (6.4) \]
It is very probable that the overall estimate (T) is normally distributed, even if the factors TR, TE and TPOL are not. The value of TP, (which is normally distributed), should be predominant, and thus have the strongest influence on the final distribution.

b) For the mine

The mine should be divided into a number of production blocks, based upon the estimated demands of the future (e.g. 5 years).

A. Put the blocks into a stochastic network model together with qualities and quantities. This means a separate time plan with the production of whole modules as single activities. The plan should include grades and tonnages as well as the durations of development and production.

B. Each block should then be carefully planned, see appendix 5.1 and the section below, and produce as a result, among other things, an estimate of the project (block) completion time distribution, see also section 6.4.3 a. Then those figures are used to update the overall production plan (paragraph A above). There is much to be gained if the blocks are designed in a similar way. This means that the same time parameters might be used for several different blocks.

Plan the blocks individually, see below, adjust the plan with the project completion time distributions given by the block plans.

C. Try to keep the plan as accurate as possible. Therefore, if possible, continue to develop the plan by considering new market surveys. Corrections for new information about grades and tonnages should also be included. Try to make it a continuous process.
c) For each block

A layout is drawn, based on a geological and rock mechanics survey of the area in question.

1. Make a computerized, stochastic network model (i.e. time plan) which supplies criticality indices and a project completion time distribution, see section 6.4.3.a.

2. Proceed according to plan. Special care is to be taken to activities with criticality indices above zero because they are more likely to delay the completion of the block than the other activities.

3a. Update the plan as carefully as possible. The observed (experienced) times for performing the activities are entered as deterministic, or a new smaller network is made. An updated plan is always more accurate.

3b. Keep record of the observed activity durations. Observed data can be used to update stochastic time data at a later stage of the project or used when making new time plans.

4. Iterate between the paragraphs 2-3 until the project is finished. Also update the overall (for the mine) network plan.
Define all production blocks including time, ore quantity and ore quality parameters.

Put the blocks into a general production plan, which is a stochastic network model.

Plan block \( <i> \) more carefully with a more detailed stochastic network. Update the general plan.

Continue until all blocks are planned.

Mining of block \( <i> \) may start when it is in accordance with the general plan. 

\[ i = i + 1 \]

Mining of blocks according to general plan. Update general plan when necessary.

**Fig. 6.2.** Flow sheet illustrating principal planning procedure.
d) Simplified planning sequence

To simplify and perhaps get a planning sequence that are more realistic to accomplish, one may omit paragraph C and the paragraphs 3a, 3b and 4. Of course this will give a decrease in accuracy, but it will also save some work if planning personnel are a scarce resource. However, Paragraph 3a should be kept, if the plan are to be used in the daily planning process.

6.5 Conclusions

The following points are of importance to achieve an accurate planning.

i) Stochastic planning is superior to deterministic planning. A deterministic plan always, (except in the case when the network consists of a straight line), gives an optimistic bias of the expected completion time. This was also pointed out in section 2.5.1.

ii) Some effort should be put into the estimation of different activity times and their distributions. This is necessary to ensure the reliability of the model.

iii) Ore quality restrictions should be considered at an early stage in the planning process to ensure the continuity of the plan.

iv) A sound layout with respect to rock mechanics is of major importance for the reliability of the plan.
v) It is of vital interest to follow the established plan as much as possible. This does not mean that deviations in activity durations are not permitted. They are, within reasonable limits, accommodated in the stochastic model. However, the project network should be kept as intact as possible. Some flexibility must of course be maintained.

vi) The planner should, as much as possible, be responsible for the performance of the included activities. If other personnel have to make decision about those activities, also their goal should be to meet the established time plan.

vii) Determination when it comes to following the established plans is essential on all stages of the hierarchy. It is never possible to keep to a plan if not all people concerned are determined to do so.
7. DETERMINATION OF OPTIMUM SIZE OF DEVELOPMENT BUFFER, THEORETICAL APPROACH

7.1 Introduction

To avoid the risk of production losses, the mining companies create buffers by developing production areas far in advance of production, incurring large capital costs* as a result, see section 1.1. In order to decrease the development buffer, the range of possible project completion times has to be estimated. This problem will be addressed in general terms in this chapter. The theory will then be applied on the particular case of the Oscar project, see chapter 5, in chapter 8.

* In this thesis the cost of this capital expenditure is represented by the interest charge and is henceforth termed the interest cost.

7.2 Determination of optimum development buffer for a single block

The procedure for determining the optimum size of development buffer can be divided into four main steps. They are

a) Determination of the probability density function (pdf) for the project completion time, which is described in section 6.4.3.a.

b) Determination of the costs of capital tied up in the development buffer.

c) Determination of the cost of production losses.

d) Calculation of the expected additional cost of development buffer.
7.2.1 Interest costs for development buffer

Capital invested in drifts, production holes etc do not yield interest. This means a lack of income that must be considered as a cost. In this simplified study the interest cost is assumed to depend primarily on the amount of money invested, the time the production area is under development or developed but not used and the interest rate. These variables are denoted as presented below,

\[ K_i = \text{The total amount of capital invested in a specific mining area} \]

\[ I_r = \text{The interest rate} \]

\[ \text{PT} = \text{Planned time, the estimate used in production planning by the planner} \]

\[ \text{RT} = \text{One arbitrary (random) realization of the project time, a "real" rather than a planned project time.} \]

A "year" is given in corresponding time units.

The basic equation for the calculation of the interest costs is given below,

\[ \text{COST} = K \times ((1+I)^t - 1) \]  \hspace{1cm} (7.1)

The total interest cost for the development work is easily calculated with a simple integral, see below, the cost of development without interest is assumed to be linear. This is done for both the planned case (PT) and the real one (RT).
The planned interest cost at the time $PT$ can be expressed as

$$\text{Cost}_{PT} = \int_{0}^{K_i} \left( \frac{(1+I_r)(PT-t)/year}{PT} \right) - 1) \times dt - \int_{0}^{K_i} dk \quad (7.2a)$$

$$= \frac{K_i \times (1+I_r)(PT-t)/year}{PT} - \int_{0}^{K_i} dk \quad (7.2b)$$

$$= \frac{K_i \times ((1+I_r)(PT)/year - 1)}{PT \times \ln(1+I_r)} - K_i \quad (7.2c)$$

The real interest cost at the time $RT$ can be derived in a similar way

$$\text{Cost}_{RT} = \frac{K_i \times ((1+I_r)(RT)/year - 1)}{RT \times \ln(1+I_r)} - K_i \quad (7.3)$$

The difference in cost is

$$\text{CostDvlp} = \text{Cost}_{RT} - \text{Cost}_{PT} \quad (7.4)$$

Observe that this function gives a negative additional interest cost if the project is completed faster than planned, (i.e. $PT > RT$). This is due to a shorter investment period than planned.
There is also a waiting stage between the times RT and PT (i.e. if PT>RT) which is not considered in the model above, assuming that the production is not started in advance (i.e. before PT). This gives rise to an additional interest cost for the development, see equations 7.5 and 7.6.

\[
\text{If } PT > RT \text{ then} \\
AddCostDvlp = (K_i + Cost_{RT}) \times ((1+I_r)(PT-RT)/\text{year}) -1) \quad (7.5)
\]

\[
\text{If } PT \leq RT \text{ then} \\
AddCostDvlp = 0 \quad (7.6)
\]

The total interest cost for the development buffer at the time the development is finished (RT) is therefore:

\[
\text{CostBuffer} = \text{CostDvlp} + AddCostDvlp \quad (7.7)
\]

Observe the intervals for the two sections of the AddCostDvlp function.

### 7.2.2 Costs due to production losses

Ore that is not produced cannot be sold, which means a reduction in income. This reduction is considered as a cost. The amount of this cost depends on the planned production volume and the extent of the delay. In extreme cases the mining companies may incur penalties if they are unable to deliver on time. Such situations are not addressed in this work. They may, however, be included if they are considered to be of importance. The parameters used, (in addition to PT and RT),

\[C_p = \text{The cost of production loss per time unit}\]

which gives

\[
\text{CostProd} = C_p \times (RT - PT) \quad \text{if } RT > PT \quad (7.8)
\]

\[
\text{CostProd} = 0 \quad \text{if } RT \leq PT \quad (7.9)
\]
7.2.3 Optimum size of development buffer

The optimum development buffer depends primarily on the balance between the cost of maintaining the buffer and the cost of production losses. However, it can be defined in other ways. Two alternatives are presented below.

a) Approach 1

The goal with this approach is perhaps a very simple and obvious one. It is, with a reasonable degree of certainty, to avoid a delay of the start of production. No costs are considered. The buffer necessary is defined as the time difference between the deterministic project time estimate, which normally is the same as the planned time, and the point in time that gives the confidence level of 98 % of completing the project on schedule. The figure 98 %, or rather 97.9 %, is chosen because it is easily calculated. The buffer time can therefore be expressed as:

\[
BT_1 = T_{98\%} - TP_{DET} \quad \text{Development buffer} \quad (7.10)
\]

where

\[
T_{98\%} = T + 2 \times \sigma_T \quad (7.11)
\]

The value 2 in equation 7.11 is derived from a normal distribution with a 97.9 % confidence level.

This means that the time used in the strategic planning should be \( T_{98\%} \) instead of \( PT \).
b) Approach 2

In this case the goal is to minimize the expected additional cost of the project. Only the costs of the development buffer and of production losses are now considered. The expected cost is defined as the mean cost for a certain planned project time \( (P_T^*) \), if the project was to be performed a large number of times.

Minimize Cost for all \( P_T^* \) where

\[
\text{Cost} = f(R_T) = \text{CostBuffer} + \text{CostProd} \quad (7.12)
\]

One approach is to calculate a simulation estimate (where \( R_T \) is varied) of the variable "Cost" for each \( P_T^* \). As this is not feasible, a selection of suitable \( P_T^* \)-values are chosen around \( T \). These simulation estimates will then form a curve, with a more or less distinct minimum, in the Cost-\( P_T^* \) - plane, see fig 7.3.
Fig 7.3. Ideal Cost - PT curve were $PT^*_\text{min}$, $TP_{\text{DET}}$ and $\text{Cost}_{\text{min}}$ are indicated.

The distance (time) between the time value of this minimum, $PT^*_\text{min}$, and $TP_{\text{DET}}$ defines the optimum development buffer.

$$BT_2 = PT^*_\text{min} - TP_{\text{DET}}$$ (7.13)

This means that the time used in the strategic planning should be $PT^*_\text{min}$ instead of $PT$.

The simulation is easily implemented in a simple computer program, see appendix 7.1. This technique also offers the possibility to perform different kinds of sensitivity analyses (e.g. to vary interest rate or the cost rate of production loss etc.).
7.2.4 Comments

One point that could be criticized is the simplicity in the cost model approach. The model can, of course, be more elaborate and not assume the linearity in the cash flow. One example of this is the drifting operation, which is very cost intensive and also performed early in the project. Therefore, another approach could be to use a more realistic cash flow model than the linear model. This could be done by using the time plan and assess different costs for the activities. In this way we obtain a more refined cash flow model. The main argument for using the simple model instead, is because it is the stage after the completion of the development, i.e. all costs are paid, that is of major importance.

Another difficulty is the estimation of possible delays from the paragraphs 2-4 (section 6.4.3a). These parameters should be used with care, otherwise there is a risk that they might have a too significant an influence on the result, which is undesirable. It might be advantageous to introduce models to assist the planner to make better estimates of these parameters.

7.3 The buffer requirement for a larger system

7.3.1 Introduction

The next logical step is to extrapolate the block case to a complete mining system. This is in order to study the amount of advance development that is necessary when an entire mine’s production is based on mining blocks as for instance the Oscar project.
7.3.2 Approach

a) Used concepts and parameters

This study suggests that the mine should be divided into a number of production lines. Each production line consists of a number of blocks (stopes) that are mined in sequence, one such sequence is here defined as a production line, see fig 7.4.

Fig.7.4. Several production lines consisting of a number of blocks.

Production for each block starts when the previous block changes from regular production into recovery of pillars etc. This time also determines when the development should start. The development is planned to start to allow adequate time for development plus the buffer period, see fig 7.5.

Fig 7.5. Principal figure of one production line consisting of several activities.
This study is based on the following parameters,

**Time parameters (valid for only one block)**

- $TP_{DET}$ = Time for development (wdays)
- $BT$ = Buffer time, derived as in section 7.2.4 (wdays)
- $ProdT$ = Time for production (wdays)
- $CycleT$ = Time for one cycle (wdays)
- $\sigma_{PR}$ = Standard deviation of $ProdT$ (wdays)

**Mass parameters (Mton)**

- $ReqProd$ = Total amount of production requested per year
- $LineCap$ = Production per year from one production line
- $BlockTon$ = Total amount of ore in one block
- $RegTon$ = Amount of ore mined in regular production
- $RecTon$ = Amount of ore mined after the ordinary production

Note that production of ore from the development phase is not included.

**Other parameters**

- $NoLines$ = Number of production lines

The following simple equation is assumed to be valid,

$$BlockTon = RegTon + RecTon$$  \hspace{1cm} (7.14)
(b) Study of a single production line

The mining cycle for one production line can be divided into the following parts, see also fig 7.5. The blocks in the line are mined in the order ..., j-1, j, j+1, ... etc.

1. Development of block \((j)\) started at time determined by end of production of block \((j-1)\), \(T_{DET}\) and \(BT\).
2. Buffer time \((j)\)
3. Start of production of block \((j)\)
4.a End of production \((j)\)
4.b Start of production of block \((j+1)\)
4.c Start of recovery production \((j)\)

and so forth

If the inequality \((T_{DET} + BT) \leq ProdT\) is satisfied, which means that the development of block \((i,j+1)\) can entirely take place during the production of block \((i,j)\), then the cycle time \((CycleT)\) will be the same as the production time \((ProdT)\).

Note: \(i\) denotes the number of the production line

The primary interest is to determine the amount of ore that can be produced from one production line per year (LineCap). This is easily calculated

\[
\text{LineCap} = \frac{\text{year} \times \text{BlockTon}}{\text{CycleT}} \quad \text{(7.15)}
\]

c) Study of the whole mine system

The study of the whole mining system can be divided into a number of sub-sequent steps, which are presented below.
i) **Number of production lines**

The mine is assumed to consist of "identical", i.e. in terms of times and tonnages, blocks. The number of lines (NoLines) required is then easily calculated. One additional line is added as a safety margin and to cover decrease in production at changes between blocks in lines etc, see equation 7.16.

\[
\text{NoLines} = \frac{\text{ReqProd}}{\text{LineCap}} + 1 \quad (7.16)
\]

NoLines is rounded upwards

ii) **Buffer volume**

The time of the cycle will be the same as the production time (ProdT). The buffer volume can be estimated as in equation 7.17.

\[
\text{Buffer volume} = \frac{(\text{TPDEm} + \text{BT})}{\text{CycieT}} \times \text{NoLines} \times \text{BlockTon} 
\quad (7.17)
\]
iii) Life time of buffer volume

The "life time" of the buffer is on average given by the following expression:

\[ \text{Life time buffer volume} = BT + \frac{TR_{DEP}}{2} \]  \hspace{1cm} (7.18)

iv) Cost of buffer volume

The cost of the buffer volume is easily calculated, see equation 7.19.

\[ \text{Cost} = \frac{\text{Buffer volume} \times \text{Block cost}}{\text{Block Ton}} \]  \hspace{1cm} (7.19)

d) Comments

It is essential that the different production lines are not all driven simultaneously, but are staggered. This will ensure a more even production rate and at the same time make it possible to use a more limited amount of equipment (resources) for the pillar recovery, and also for the development.

A special time plan might have to be introduced for the resources that not are tied to just one production line. Small groups of production lines with a joint resource supply can be established to simplify the resource scheduling.
8. DETERMINATION OF OPTIMUM SIZE OF DEVELOPMENT BUFFER, THE OSCAR CASE

8.1 Introduction

The approach presented in section 7.2 will in this chapter be applied to the particular Oscar case. This application is intended to give a general indication of the figures that might arise, for the Oscar case, as well as other sublevel stoping operations of similar design.

The program PertSim presented in chapter 4 has been used in the calculations of the project completion time distribution.

8.1.1 Assumptions

Some assumptions have been made in the following calculations. These are,

* The concept of overtime is not included.

* No resource scheduling has been carried out on the network as indicated in section 2.5.3.

8.2 Estimations of optimum development buffer

The estimation of the optimum development buffer for the Oscar case involves three main steps. These are

a) Gathering of relevant data
b) Numerical modelling
c) Interpretation of results
8.2.1 Gathering of relevant data

The relevant data consist of two main parts, namely time data and cost data, see section 7.2.

a) Time data

The unit for time is "wdays" (working days) throughout this chapter. It is assumed that one week consists of five working days, and one year of 240 working days.

i) Uncertain time data

The Oscar project consists of a number of activities that are tied together in a project network. Each activity has an individual time distribution of its duration. These activity distributions are, because of the lack of accurate information, assumed to be limited by three values, optimistic, pessimistic and most likely time. The time values were supplied by the mine planning engineer who carried out the planning for the Oscar project and are based on statistics, (i.e. mean values of operation capacities), and experience. The distributions are all assumed to be uniform (i.e. the most likely times are never used). This assumption may seem coarse, but it can, to some extent, be validated by the facts presented below.

1. Many of the activities are in reality lognormally distributed, see Schunnesson (1989) and Lindström (1989). This means that there exists a tail on the right hand side of the distribution density curve. The uniform distribution, with its large variance, gives some compensation for the exclusion of this tail.
2. Comparison made between a network with triangular distributions and a network with uniform ones indicates that the span between optimistic- and pessimistic time are of more importance to the project completion time than the choice of distribution, see appendix 8.1. The choice of distribution primarily influences the variance. However, a smaller variance throughout the network gives a shorter project completion time.

3. The uniform distribution gives a fairly conservative estimate, considering it is based on just two (or three) time values. This should be remembered when interpreting the final analysis results.

These assumptions stated above are probably an anathema to the eyes of a statistician but nevertheless necessary due to the lack of data. No segregation between different types of distributions has been made due to the same lack of knowledge.

These time distributions are then modelled as described earlier, see section 6.4.3a. The model for the Oscar project yielded the following results, see also appendix 8.2.

\[
\begin{align*}
TP_{DET} &= 196 \ (466) \ \text{wdays (in parenthesis - production included)} \\
TP &= 210 \ (486) \ \text{wdays} \\
\sigma_{TP} &= 15 \ (21) \ \text{wdays}
\end{align*}
\]
ii) Large rock mechanics problems

Unpredictable and large rock mechanics problems are reported to be rare. It is usually possible to foresee this kind of problem from experience of mining levels immediately above. Section 5.2.1 presents a list of the actual problems experienced during the development of the Oscar area. The impact of these experiences has been reduced due to the fact that the Oscar project was to some extent a rock mechanics experiment. It was also the first project of its kind and had therefore no earlier experience to rely on. The following estimate is made.

\[ TR = 10 \text{ wdays} \]
\[ \sigma_{TR} = 3 \text{ wdays} \]

iii) External influence

Delays due to external influence is also reported to be very rare. An estimate of this parameter is therefore low.

\[ TE = 3 \text{ wdays} \]
\[ \sigma_{TE} = 1 \text{ wday} \]

iv) Policy changes

Policy changes are very hard to predict, and the experience on which to base predictions vary from case to case. In this example (the Oscar project), a more general indication of the buffer requirement for sublevel stoping in the Kiruna mine is given. No policy changes are therefore predicted.

\[ TPOL = 0 \text{ wdays} \]
b) Economics data

The economics data used was provided by LKAB and is presented below.

\[ K_i = 11.4 \text{ MSEK (7.5 MSEK drifting and 3.9 MSEK drilling)} \]
\[ I_r = 11.0 \% \quad \text{(Interest rate)} \]
\[ C_p = 0.165 \text{ MSEK/wday (Cost of production losses)} \]

It should also be mentioned that the cost of production losses could be reduced significantly if there exists an alternative loading area. This can, however, be considered as using an extra buffer and is not as inexpensive as it may seem. A decrease in the cost of production losses will also decrease the need for development buffer.

8.2.2 Numerical modelling

The calculations of TP is performed with the program PertSim described in chapter 4. The economics modelling is performed with another simple computer program, which implements the algorithms and equations described in sections 7.2.1 and 7.2.2, see appendix 7.1.

Three cases are considered, see below

\( TP<>0 \) and \( \sigma_{TP}<0 \) for all cases

Case 1: \( TR = 0; TE = 0; TPOL = 0; \) Costs are not considered
Case 2: \( TR = 0; TE = 0; TPOL = 0; \)
Case 3: \( TR <> 0; TE <> 0; (TPOL <> 0); \)
a) Case 1:

This case is the implementation of approach 1 described in section 7.2.3a. It can be viewed as a comparison between a deterministic and a probabilistic approach. In this case \( T_{DET} \) represents the deterministic date and TP the probabilistic completion date. Observe that the probability to complete the project on TP wdays is just 50%.

\[
T_{DET} = 196 \text{ wdays}
\]

Notice that \( \text{Prob}[RT \leq 196] = 16\% \)

\[
\begin{align*}
TP &= 210 \text{ wdays} \\
\sigma_{TP} &= 15 \text{ wdays}
\end{align*}
\]

\[ \Rightarrow T_{98\%} = 240 \text{ wdays} \]

**Fig 8.1.** The pdf of the Oscar completion time (time for start of production) with the deterministic time and the 98% confidence level indicated

Equation 7.10 gives

\[ \Rightarrow BT1 = T_{98\%} - T_{DET} = 44 \text{ wdays} \ (22\%) \]

The results above indicate that a buffer time of 44 wdays (9 weeks = 2 months) is adequate.
What is even more interesting is that there is only a 16% probability to complete the project on schedule (i.e. if a deterministic timeplan is used). As most timeplans in use in the mines are deterministic this figure could indicate why so many deviations from the plan occur. One could go even further and assert that plans that go according to schedule are erroneous, (or too pessimistic). As stated earlier, there is never more than a 50% probability of concluding a deterministic time plan on schedule.

b) Case 2:

This case is an implementation of approach 2, see section 7.2.3b, where no other uncertainties than those in the project network are considered. The calculated values that describe the probabilistic project completion time are,

\[
\begin{align*}
TP_{DET} & = 196 \text{ wdays} \\
T & = 210 \text{ wdays} \\
\sigma_T & = 15 \text{ wdays}
\end{align*}
\]

Fig. 8.2. Additional cost in case 2 as a function of the planned time (\(TP_{DET} + BT\))
\[ PT^*_\text{min} = 243 \text{ wdays} \]
\[ \text{Cost}_{\text{min}} = 0.09 \text{ MSEK} \]
\[ \text{Cost}_{\text{DET}} = 2.7 \text{ MSEK} \]

Equation 7.13 gives
\[ BT_2 = PT^*_\text{min} - TP_{\text{DET}} = 47 \text{ wdays} (24\%) \]

This case suggests a buffer time of 47 wdays (about 9 weeks or 2 months). It should also be observed that the minimum cost \( (\text{Cost}_{\text{min}}) \) is not equal to zero. This is due to the influence of the uncertainty \( (i.e. \sigma_T=0 \implies \text{Cost}_{\text{min}}=0) \).

c) Case 3:

Case 3 is also an implementation of approach 2, see section 7.2.3b. In this approach some consideration has been given to the fact that incidents outside the project network might influence the project completion time.

\[ TP_{\text{DET}} = 196 \text{ wdays} \]
\[ T = 223 \text{ wdays} \ (\text{see section 8.2.1}) \]
\[ \sigma_T = 19 \text{ wdays} \]
Additional Cost (MSEK)

250 255 260 265 270 275 280

Planned Project Time (wdays)

Fig. 8.3. Additional cost in case 3 as a function of the planned time \((TP_{DET} + BT)\)

\[ \Rightarrow \text{PT}^*_{\text{min}} = 264 \text{ wdays} \]
\[ \text{Cost}^*_{\text{min}} = 0.12 \text{ MSEK} \]
\[ \text{Cost}_{DET} = 4.7 \text{ MSEK} \]

Equation 7.13 gives

\[ \Rightarrow BT_2 = \text{PT}^*_{\text{min}} - TP_{DET} = 68 \text{ wdays (35\%)} \]

The buffer need according to this model is much higher, 70 wdays (14 weeks), than indicated in the previous models. This is mainly due to the increase in variance.

8.2.3 Recommended buffer time as a function of the standard deviation

As can be seen in the figure 8.4 below the buffer requirement increases with the standard deviation. This implies that the requirement can be decreased if more precise estimates of activity durations can be made.
An increase in expected minimum cost is also an obvious consequence of increasing standard deviation.

Assumption number 2 presented in section 8.2.1b should of course also be viewed in the light of the implications given above.

8.2.4 Recommended buffer time as a function of the interest rate

The buffer requirement will decrease with an increasing interest rate. However, the cost of production losses is dominant (i.e. $C_p$ is much larger than the interest cost), as can be seen in figure 8.5. Consequently, the interest rate has very little influence on the required buffer time.
8.2.5 Comparison between a deterministic vs. probabilistic model

It is of interest to study the difference between the output from a deterministic and a probabilistic version of the "economical model". The deterministic model will of course give the optimum in $T_{P_{DET}}$ with $Cost_{min}$ equal to zero.

Fig. 8.6. Comparison between deterministic and stochastic cost function.
The difference between the curves (fig 8.6) when the PT* becomes large depends on the difference in expected value of TP (i.e. 196 vs. 210).

### 8.2.6 Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Buffertime</th>
<th>Cost\textsubscript{min}</th>
<th>Cost\textsubscript{DET}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>44 wdays</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Case 2</td>
<td>47 wdays</td>
<td>0.09 MSEK</td>
<td>2.6 MSEK</td>
</tr>
<tr>
<td>Case 3</td>
<td>68 wdays</td>
<td>0.12 MSEK</td>
<td>4.7 MSEK</td>
</tr>
</tbody>
</table>

As can be seen from the results presented above, the optimum development buffer for a case like the Oscar project is about 2-3 months. The results should give an indication of the order of the buffer requirement. It can also be seen that the expected cost for deterministic time (Cost\textsubscript{DET}) as the planned time (PT) is much higher than the minimum cost (Cost\textsubscript{min}).

### 8.2.7 Comments

The model described is a simplification of the actual situation, but does anyhow point out some qualitative aspects which are important to understand in the time planning procedure. The value of these should not be underestimated, but considered with care.

The results above can of course be invalidated if the control over the development activities is inadequate. It is therefore important that the attitude towards possible consequences of taken measures does not grow lax. This might be a risk as the action of men is often governed by routine.
8.3 The buffer requirement for a larger system

8.3.1 Introduction

The next step is to extrapolate the Oscar case to a complete mining system. This is in order to study the amount of development that is necessary when an entire mine’s production is based on blocks like the Oscar project.

8.3.2 Number of production lines

Parameters of interest are presented below. The time values adopted are the same as those derived in section 8.2.1.

\[ \begin{align*}
TP_{DE} &= 196 \text{ wdays} \\
ProD_{T} &= 271 \text{ wdays} \\
\sigma_{PR} &= 10 \text{ wdays} \\
BlockTon &= 1.14 \text{ Mton} \\
RegTon &= 1.0 \text{ Mton} \\
RecTon &= 0.14 \text{ Mton}
\end{align*} \]

Equation 7.15 gives

\[ \text{LineCap} = \frac{240 \times 1.14}{271} = 1.01 \text{ Mton} \]

The required production for the mining system must be known. The assumed value is that of the Kiruna mine, see below.

\[ \text{ReqProd} = 19.0 \text{ Mton} \]

this, inserted in equation 7.16, gives, (read the integer value of the expression inside the brackets),

\[ \text{NoLines} = \text{int} \left( \frac{19.0 + 1 + 1}{1.01} \right) = 20 \]

\[ * \] where int means integer value of
The required number of production lines is estimated to 20. Theoretically this gives space for an additional production capacity of 1.2 Mton per year. Currently there are about 16 production lines (machine sites) in the Kiruna mine.

8.3.3 Development buffer volume

a) Buffer volume

The production line capacity will be calculated for two cases. These cases correspond to two of the cases presented in section 8.2. They are,

Case 2: BT = 47 wdays
Case 3: BT = 68 wdays

The development for both case 2 and 3 satisfies the inequality $(TP_{DET} + BT) \leq ProdT$. This means that the development of block $(i,j+1)$ (where $i$ denotes the number of the production line) can entirely take place during the production of block $(i,j)$. Equation 7.17 is used to calculate the buffer volumes.

Case 2:
Buffer volume $= \frac{(196 + 47) \times 20 \times 1.14}{271} = 20.4$ Mton

Case 3:
Buffer volume $= \frac{(196+68) \times 20 \times 1.14}{271} = 22.2$ Mton
Fig 8.7. Buffer volume vs. buffer time.

Interesting in figure 8.7 is that the buffer volume never reaches zero. A certain buffer volume is apparently unavoidable.

When there is a certain risk that resources will be a limiting factor because development might take place at two places in the same production line. This risk will of course increase with increasing (positive) difference between $(TP_{DET} + BT) > ProdT$.

What is stated above is, however, not entirely true, because all development is not finished when the production can start.

Equation 7.18 is used to calculate the life time of the buffer volume for the different cases.

Case 2 ==> 145 wdays
Case 3 ==> 166 wdays
Today ==> 240 wdays

The figures above are of great interest, because presently the average buffer duration is about one year. Also note that the buffer time is of greater importance for the life time of the buffer volume than the actual development time.
b) Economic consequences

Today the Kiruna mine has a buffer volume of about 35 Mton in which they have invested about 265 MSEK (215 MSEK drifting and 50 MSEK drilling). In a block like Oscar they invest about 7.5 MSEK in drifting and about 3.9 MSEK in drilling, the total amount is about 11.4 MSEK.

The cost of the buffer volume will thus be, see equation 7.19,

Case 2: \( \text{Cost} = \frac{20.4 \times 11.4}{1.14} = 204 \text{ MSEK} \)

Case 3: \( \text{Cost} = \frac{22.2 \times 11.4}{1.14} = 222 \text{ MSEK} \)

Today: 265 MSEK

This would give the following interest costs per year (240 wdays), see equation 7.1,

Case 2: \( 204 \times ((1+0.11)^{\frac{145}{240}} - 1) = 13.3 \text{ MSEK} \)

Case 3: \( 222 \times ((1+0.11)^{\frac{166}{240}} - 1) = 16.6 \text{ MSEK} \)

Today: \( 1 \times 0.11 \times 265 = 29.2 \text{ MSEK} \)
Three important facts should be noted when considering the figures above.

1) The Oscar project is the first of its kind. There is probably a certain potential to reduce the costs if this kind of mining becomes routine.

2) The scale of Oscar is also smaller, (i.e. fewer tons per meter of drift), than the sublevel caving that it is compared with.

3) Currently the Kiruna mine uses 16 production lines, which means that 16 "loading places" are sufficient to support the total production of the mine. This means that the production time (ProdT) for the case described above probably could be shortened somewhat and thus possibly reduce the buffer volume even more. That is if the reduction is large enough to allow for a decrease in the number of production lines.

c) Comments

The results above indicate that there is a large potential for reducing the interest cost by optimizing the use of development buffers. The reliability of the results presented, depend very much on the strict control of the planning and execution of the development and production schedule.
8.4 **Conclusions**

8.4.1 **Planning aspects**

The following conclusions regarding general time planning can be drawn from the preceding study,

Incidents outside the project network (e.g. large rock mechanics problems) have a large influence on the development buffer requirement, see "case 3". Therefore, it is of interest to be able to reduce these type of disturbances to a minimum, and also to be able to estimate their duration. Section 8.3.3b indicates that about 3-4 MSEK per year can be saved just in interest costs.

The real potential for this kind of technique is when each block is part of a larger system and not just an individual unit. The technique should than be applied to the whole system, as well as the individual blocks, see section 6.4.3.

8.4.2 **Numerical results for the Oscar case**

Note that the figures given below are approximate, but should give a fair indication of the order of magnitude of the required buffer.

The buffer requirement for a single block appears to be in the vicinity of 2-3 months. However, this figure cannot be generalized to the whole mine, (compare with section 8.3.)

With the currently used planning technique (deterministic), the probability of being able to start the production on time is only about 16 %.
The minimum expected extra cost is about 25 times higher, (12 MSEK instead of 0.5 MSEK), for the deterministic case than for the probabilistic case.

This work indicates that there is a large potential for economic savings resulting from a more strict planning organisation utilizing development buffers and a stochastic technique. The capital tied up in development could be reduced by ca 50 MSEK (20%) and the interest costs by ca 15 MSEK/year (50%) for the entire Kiruna mine with a production volume of roughly 19 Mtons per year.
9. **OPTIMUM BALANCE BETWEEN DEVELOPMENT AND PRODUCTION ACTIVITIES**

9.1 **Introduction**

The optimum balance between development and production activities is achieved when the development proceeds at the same rate as the production and with the minimum amount of resources. This means neither creating buffers nor consuming them. This chapter will discuss this problem and propose some alternative approaches to solve it.

The best strategy to change production rate is another aspect of the same problem. A change in rate (and volume) will inevitably displace the balance between development and production. Therefore this problem will also be considered.

The ideas presented in this chapter are fully compatible with the planning approach that was presented in section 6.4.3. and it is recommended that they be used in conjunction with that approach.

9.2 **The use of resources in development vs production**

9.2.1 **Objective**

As stated in the introduction the main objective should be to try to reach the optimum development buffer in as smooth and natural way as possible in order to minimize cost. This means that the demand to achieve a certain buffer time should not constrain the overall timeplan (i.e. of one production system) more than necessary. This should be done with as small amount of equipment (resources) as possible. The minimum production volume must of course always be maintained.
9.2.2 Strategies

Two main approaches to study and optimize the balance between development and production activities are considered. A method of combining them is also described.

a) Strategy 1

i) introduction

One way is to study the extent of the over-lapping between the different production lines, see section 7.4, and see how much influence it has on the resource utilization. It is easily concluded that if all lines were started simultaneously, it would give a large initial demand for development resources, which would be followed by a period of low development and production intensity (i.e. during the buffer times) and subsequently a high utilization of the production resources and so forth. This would require a rather large amount of resources, which would be utilized very inefficiently. Some of the production lines can of course start at the same time (i.e. in parallel).
However, if the extent of the over-lapping is varied and all steps except production, (i.e. development, buffer time and recovery mining), are considered as resources and levelled as such (see section 2.4.2a), the result should indicate some sort of balance between the basic activity types. Note that production cannot be levelled because that might give too low a production volume.

The risk by using a levelling approach, as presented in chapter 2, is that the production level cannot be guaranteed, not even when production resources are excluded. A better result might be obtained using a trial and error experiment with the resource scheduling approach presented in section 2.4.2b, i.e. to vary the amount of available resources while trying to minimize project completion time.

Another alternative, and perhaps more reliable, would be to make a systematic search procedure, where a large number of possible solutions are tested against a criterion. Some of the plans are then rejected and the "best" one of those remaining is chosen. The disadvantage is that this procedure might be very time consuming. It may therefore be necessary to bring down the accuracy and reliability of the model to gain speed. However, if the solution presented by the model is not satisfactory, it is always possible to change one of the search parameters and make a new attempt.

The first step is then to study how the utilization of the development resources depends on the durations of the different activities. Note that the recovery activities are judged to be of less importance because no activities are dependent on them. The number of production resources will of course always be the same as the number of lines, in order to keep the production at the required level.
ii) The number of utilized resource groups

The number of production groups (loaders) used must, as mentioned earlier, be the same as the number of production lines, in order to maintain the required production level. However, the minimum number of development groups depends on the quotient between the lengths of the development and the production phases, (e.g. a shorter development phase decreases the need for development resources). This relation can be described as,

\[
\text{NoDev}_{\text{DET}} = \text{int} \left( \frac{\text{TP}_{\text{DET}}}{\text{TP}_{\text{DET}}} \right) \times \text{NoLines} + 1
\]  

(9.1)

*) integer value of

Note that the figure 1 is used in the rounding procedure.

where

\[
\begin{align*}
\text{NoDev}_{\text{DET}} &= \text{The minimum number of development resources} \\
\text{TP}_{\text{DET}} &= \text{Deterministic development time} \\
\text{NoLines} &= \text{Number of production lines}
\end{align*}
\]

However, the formula above does not take the probabilistic aspects into consideration, it uses the deterministic development time. It must thus be more appropriate to use the probabilistic estimate of the development time (T). This assumption will, in the computer model, consume some of the buffer time (i.e. T-TP_{DET}). The new formula is,

\[
\text{NoDev}_{\text{PROB}} = \text{int} \left( \frac{T}{\text{ProdT}} \right) \times \text{NoLines} + 1
\]  

(9.2)

It is assumed that the effects on the resource utilization by those development phases that are extended to a longer time than T (50 % of the cases) will be cancelled out by the activities which are completed in a shorter time (also 50 %). It should also be noted that the approach presented above assumes that the development resources are more or less confined within their production line except for this last case.
The recovery resources should also be given some attention so that it is still possible to perform the recovery production. However, as no activities depend on the recovery it is enough if the equation given below can be satisfied.

\[
\text{NoRecGrp} = \text{int} \left\lfloor \frac{\text{RecT} \times \text{NoLines} + 1}{\text{year}} \right\rfloor \quad (9.3)
\]

where

\[
\text{NoRecGrp} = \text{the number of recovery resources}
\]

iii) Numerical values

The same set of values as in chapter 7 is used (buffer case 2), see below.

\begin{align*}
\text{TP}_{\text{DET}} &= 196 \ \text{wdays} \\
\text{T} &= 210 \ \text{wdays} \\
\text{BT} &= 47 \ \text{wdays} \\
\text{ProdT} &= 271 \ \text{wdays} \\
\text{RecT} &= 30 \ \text{wdays} \\
\text{NoLines} &= 20
\end{align*}

They are used in the equations presented above, which yields the following results,

\begin{align*}
\text{NoDev}_{\text{DET}} &= \text{int} \left\lfloor \frac{196 \times 20 + 1}{271} \right\rfloor = 15 \text{ resource groups} \\
\text{NoDev}_{\text{PROB}} &= \text{int} \left\lfloor \frac{210 \times 20 + 1}{271} \right\rfloor = 16 \text{ resource groups} \\
\text{NoRecGrp} &= \text{int} \left\lfloor \frac{30 \times 20 + 1}{240} \right\rfloor = 3 \text{ recovery groups}
\end{align*}
The simplest way to solve this problem is by performing a computerized search procedure, see above, which enables the user to test a large number of cases in a comparatively quickly. Two standard parameters have been chosen to represent the pattern of starts of each line. This simplification is made to make the computation task more reasonable. The origin of the parameters will now be explained.

The production lines have been divided into a number of groups, which all start at the same time (i.e. 0 wdays). The size of those groups is the first parameter (Gsize). The lines in each group have been arranged in a staggered pattern, which means that the start of each line within the group is displaced in relation to its predecessor. This displacement is assumed to be constant, and is given by the second parameter (denoted $S_{inc}$).

\[ Gsize = 3 \]

Fig 9.1. Principal figure of Gsize and $S_{inc}$.

These parameters together with the time values given above have been modelled in a simple computer model. The criteria was, for both cases (det. and prob.), that they should utilize the same number of production resources as there were production lines, while the utilization of the development resources should be as low as possible. The utilization of the recovery resources was considered of less importance, since they are at the end of each cycle and can, if necessary, be delayed slightly while waiting for resources.
iv) An arbitrary case

Two arbitrary, but realistic, parameter values will be used to demonstrate the importance of how they are selected. The other values represent the deterministic case with an optimum result of using 15 resource groups. The test parameters are,

\[
\begin{align*}
\text{Gsize} & = 4 \\
\text{Sinc} & = 120 \text{ wdays}
\end{align*}
\]

![Diagram](image)

**Fig 9.2.** The utilization as a function of time for an arbitrary case.

In this case the utilization of the development resources is, as can be seen in figure 9.2, very uneven. Other effects of the choice of parameters might be that it takes a long time to achieve full production.

v) The Oscar case

It should be remembered that the development, in a case like Oscar, is not necessary complete when the production starts. This assumption has been made for the sake of simplicity, though it is possible to include this in the search model. The easiest approach is to classify the resource requirements after start of production as buffer resources.
The following results were obtained, see also appendix 9.1 (time plans).

\[ \text{NoDevDET}^* = 15 \quad \text{if} \quad Gsize = 4 \]
\[ Sinc = 70 \text{ wdays} \]

\[ \text{NoDevPROB}^* = 16 \quad \text{if} \quad Gsize = 5 \]
\[ Sinc = 60 \text{ wdays} \]

Note that other solutions might also exist.

**Fig. 9.3a.** The utilization function for the deterministic model.

**Fig. 9.3b.** The utilization function for the probabilistic model.
The utilization has been plotted in the figure 9.3 above. Note that they both indicate an uniform production from all 20 lines, which was required. The negative slopes on the plots depend on the fact that the extent of the plotted models is limited to 5 levels (i.e. each line consists of five blocks in sequence - ref. fig 9.2). The following features are of interest:

i) Full production is achieved rapidly (452 <det> cf. 483 <prob> wdays),

ii) The time to complete the mining from full production to zero production is 240 cf. 272 wdays.

iii) A steady production rate is reached.

An alternative solution to the probabilistic case is presented, which demonstrates that there might be more than one "good" solution, see below.

Gsize = 10
Sinc = 80 wdays

Fig 9.4. Alternative utilization function for the probabilistic case.
This plan has a lower utilization of the recovery resources but needs more time to reach full production.

b) **Strategy 2**

Another approach is to compare the cost, or lack of profit, for a longer development time versus the cost of using additional equipment. However, the dominant cost that varies with time, if the project is properly planned, is the interest cost, assuming that no changes are made in the number of production lines. The interest cost for a Oscar-block is plotted below, see figure 9.5.

![Graph](image)

**Fig. 9.5.** Capital cost (interest cost) as a function of development time.

A planned reduction of the development time of one month (20 wdays), which is not an insignificant reduction, would considering this aspect only save about 0.05 MSEK. This represents a saving of 1 MSEK for 20 production lines. It is evident that this factor is not of major importance.
i) Alternative 1

One approach is to consider just one block or project. Costs are assigned to the different resources, the project is scheduled and the total cost calculated. This procedure can then be performed for a number of possible "resource banks". The alternative with the lowest cost is chosen.

ii) Alternative 2

Another way would be the traditional approach in project scheduling with networks, see Taha (1982). This technique considers how much each activity can be shortened if more resources (costs) are made available (i.e. the activity durations are not constant). A number of schedules are made with varying degree of influence. The schedule with the lowest overall cost, or largest profit, is chosen. However, as this method does not recognize slacks due to lack of resources it cannot reduce them. Instead it tries to reduce activity durations by applying a larger amount of resources, which is often not feasible in a mining environment due to narrow drifts etc. Therefore, this method is not judged to be suitable for the present application.

It must also be considered that a longer development or production time might yield too small a production volume, with the need of additional production lines required. The cost of extra production lines might be a significant burden to that particular alternative. It is therefore of interest to be able to shorten the cycle-time as much as possible in order to be able to reduce the amount of production lines.

Special care is needed because the use of resource scheduled time plans might influence the extent of the buffer time, see also section 2.5.3. There is also a certain risk of sub-optimizing if strategy 2 is applied without consideration of the overall plan.
c) Strategy 3

The two strategies presented above could also be combined to one, and in that way make use of the advantages of both methods. The procedure is briefly described below. It is, however, not within the scope of this work to illustrate this strategy with an example.

![Flow sheet illustrating strategy 3.](image-url)

**Fig 9.6.** Flow sheet illustrating strategy 3.
General principal (approach)

1) Create a standard network for the mining block. Include relevant resource needs.

2) Create a specific "resource bank"

3) Create a resource scheduled time plan for the mining area (block) in question.

4) Make an optimized overall plan following the guide-lines given in strategy 1.

5) Repeat paragraphs 2-4 for all feasible resource banks in order to get the plan with the lowest cost (highest profit).

6) Choose the alternative with the best combination resource bank - overall timeplan.
9.2.3 Comments

It is of course possible to use the same or an analogous technique for other mining methods. The important thing is the different steps, which starting with the construction of the project network, continue with the calculation of a local (block) timeplan and the estimation of the buffer time and end with the establishing of a logical "time pattern".

9.3 Strategy to make changes of production volume

9.3.1 Objective

The problem in this case is to change the production volume with as low cost as possible, which means that a new optimum balance between development and production has to be found. How to achieve this change, in a smooth way, is also a part of the problem.

9.3.2 Strategy

a) introduction

One should first try to localise the optimum amount of change measured in production volume. This amount will most depend on the increase (or decrease) in the number of necessary production lines, since each additional line will cause a dent in the cost and utilization functions. It is therefore to be recommended that the amount of change is given in the terms of additional production lines (integer values). This assumes that the original production volume is determined in a corresponding way. If this is not the case, then it is a good opportunity to adjust the plan to a more suitable production volume. Two aspects should thus be considered,
a) The present number of resources, especially development groups. This approach is interesting when increasing the production.

b) The present configuration of the time pattern, i.e. the way the time schedules for the different production lines are related.

b) Present number of resource groups

It should first be considered what amount of increase that is possible to achieve with the available amount of development resources. The maximum number of production lines can be calculated from equation 9.4, see equation 9.2.

\[
\text{MaxNoLines} = \text{int} \left( \frac{\text{NoDevPROB} \times \text{ProdT}}{\text{T}} \right)
\]

(9.4)

In the Oscar case for instance, how much could the production volume be increased if the number of development resources was increased by two resource groups to 18 groups.

\[
\text{MaxNoLines} = \text{int} \left( \frac{18 \times 271}{210} \right) = 23
\]

This would give a production increase of

\[
\Delta \text{Prod} = (23-20) \times \text{BlockTon} = 2 \times 1.14 = 2.3 \text{ Mton}
\]

The number of recovery groups should also be considered. The number required is calculated from equation 9.3,

\[
\text{NoRecGrp} = \text{int} \left( \frac{30 \times 23 +1}{240} \right) = 3
\]

No additional recovery groups are required
c) The present configuration of the time pattern

The timing should also be chosen, so that it is possible to accommodate the change into the old pattern without complications. If this is not possible, a new pattern has to be created, and the transition from the old one might create problems.

9.3.3 Comments

It is wise to choose an initial time pattern that is flexible and allows for changes of the production volume without too many complications. This means that a simple analysis of the chosen pattern should be carried in order to avoid inflexible ones.

9.4 Conclusions

The best way to obtain the optimum balance between development and production activities can be summarized into two points, they are,

1. Include buffer times as a part of the overall plan.

2. Make an overall plan that has as low a development intensity as possible, but which supports the full production volume. This is easiest to achieve if the planning is made well in advance, since a lower development intensity means a longer time until full production is reached.

In practice, the ideal conditions assumed above, e.g. identical blocks, will not exist. This fact might of course complicate the optimization algorithms and increase the amount of required input data. The important thing is that the basic philosophy is still valid. This also means that the ideas are adaptable to other mining methods.
It has also been shown that changes in the production volume should be performed in a way that enables the new plan to reach the optimum balance between development and production activities. Otherwise there is a large risk for low utilization of available resources, with higher costs as a result.
10. CONCLUSIONS

10.1 Introduction

The aim of this chapter is to answer to the sub-goals stated in section 1.2. This is mostly done by summarising the work in, and conclusions of, chapter 6-9. More detailed information is presented in the conclusions in each chapter, i.e. chapter 6-9.

10.2 Development and introduction of probabilistic time planning technique

Stochastic time data are easily applied to a network time planning technique by using Monte Carlo simulation. This has been done in the PertSim program, which was used for the investigations made in this thesis.

The major advantage of the approach taken, over a deterministic technique, is that this method gives unbiased estimates of the event times (e.g. the project completion time). Deterministic time plans gives invariably too optimistic event time estimates.

The largest threats against the reliability of the results from a probabilistic model are,

i) Bad input data, i.e. incorrect estimates of the activity times.

ii) Ore quality restrictions that is not in line with the plan.

iii) Large rock mechanics problems

iv) Changes in the time plan due to the human factor
10.3 Determination of optimum development buffer

The optimum development buffer has been estimated by minimizing the additional cost caused by the buffer. The following parameters have been regarded,

i) The statistical distribution of the project completion time.

ii) Interest cost of capital invested in the development buffer.

iii) Cost of production losses due to inability to produce ore on schedule, i.e. the development is not finished on time.

The optimum development buffer for the Oscar case, see chapter 5, was estimated to 2-3 months. The probability of finishing the development on time, assuming deterministic planning, was about 16%.

The Oscar case was extrapolated to the production tonnage of the Kiruna mine. This was done by dividing the mine into production lines, which consist of Oscar-type blocks. The buffer size for each block was as suggested above.

The results indicate, for the Kiruna mine, that the interest costs could be cut with ca 15 MSEK/year, and that the capital tied up in development could be cut with ca 50 MSEK.
10.4 Optimization of resource utilization

The resource utilization may be optimized by scheduling the different production lines in such fashion that the different lines, in as high degree as possible, utilize different resource types, i.e. balance between development and production activities. The mining should thus be done so that several production lines may share the same group of development resources, production resources etc.

The calculated buffer times should be included in this overall plan, this is to ensure that the same resource group is not required by more than one production line at a time.

Changes of production volume should be done in a way that enables the new plan to reach a balance between the utilization of development and production resources.
11. **RECOMMENDATIONS**

11.1 **Introduction**

The general recommendation is to implement a more recent and sophisticated time planning technique for planning mining activities. The technique used at present, Gantt-charts etc, originate from the days of world war one. Since then techniques have developed significantly. This chapter will summarize some of the most important aspects of the utilization of a more modern and advanced time planning technique than that currently in use. Some of the possible problems when introducing the new technique are also discussed.

The recommendations below are divided in two groups, namely "special recommendations" and "general recommendations and possible problems". There is also a section which gives some ideas about possible areas of further work.

11.2 **Special recommendations**

The following points summarize of recommendations stated earlier in this thesis, especially section 6.4.

a) **Computerize the time planning process**

Manual time plans are very seldom updated in a proper way. They do not facilitate an easy optimization of the time plan, but conserve the old ways of planning (i.e. the "we have always done it this way..." - phenomena).
This problem could be avoided if the time planning procedure were computerized. Such a model is easy to update and always calculates the time plan with shortest completion time. A much better overview over the project and its critical activities is also gained.

b) Use stochastic instead of deterministic plans

Deterministic time plans always give an optimistic bias of the project completion time, which is the reason why plans are seldom adhered to.

A stochastic simulation model (not PERT) would give an unbiased estimate of the project completion time. Such models are of necessity computerized and fit well under point 1 above.

c) Follow-up of activity times

It is necessary to have reliable time data to get an accurate time plan.

Activity times or durations should therefore be followed up in order to improve the estimation of the activity time-distributions and distribution parameters.

d) Reduce the two main non-network disturbances

There are two types of problem that are not accounted for in the stochastic network model, they are

- large rock mechanics problems
- grade control problems
The planning should be based on rock mechanically sound layouts. Investigations to verify the stability of the layouts should be performed. Large problems should be included in the time plan if they are foreseen with some confidence (e.g. from similar areas mined earlier) and can not be avoided.

Ore qualities and quantities should be included in the plan at an early stage.

11.3 General recommendation and possible problems

To fully realise the benefits of a good time planning system it has to be thoroughly implemented. The general recommendation presented in this section is therefore more or less dependent on the introduction of the points presented in the section above. Then the planning system might be both reliable and produce a substantial reduction in costs, see section 7.5.2. The planning strategy should also facilitate future changes of the production volume as much as possible, which means that the plan must have some inbuilt flexibility.

The general idea is to combine the planning technique presented in section 6.4.3 with the production philosophy presented in section 7.4 and section 8.2. This can, however, not be done instantly due to several "problems" that have to be overcome, some of which are pointed out below.

i) A lot of mine development has already been started or completed according to the current planning system in the Kiruna mine. The main part is planned for sublevel caving, which in it self is no argument against the presented technique. The information about the mining blocks must, however, be compiled and converted into an appropriate format.
ii) The planning organisation probably must be partly reorganized, because new kinds of assignments have to be performed, and different types of information will be needed. New routines have to be established.

iii) This technique introduces a new way of approaching mine planning. Consequently, some problems maybe encountered in its introduction on mines due to the human nature. The introduction must be based on awareness of psychological constraints.

11.4 Proposals for future research and studies

This section presents some possible subjects for further work that are regarded to be of interest.

i) To make the same type of investigation regarding optimum development buffer as the one in this thesis, but with higher accuracy. This can be done by using more accurate time data, and to use a more advanced cost model which is directly connected with the time plan.

ii) To estimate the project completion time after resource scheduling, see section 2.5.3. This is a difficult problem due to the resource constrained slack times that are introduced by the scheduling procedure.
12. REFERENCES


Schunnesson Håkan, Produktionsstatistik och kapacitetsprognoser för bergborrning - Fältförsök i Oscar området Kiirunavaara, Rapport koncept, Tekniska Högskolan i Luleå, To be Published, 1989.


Dictionary of relevant technical words

This appendix consist of a list of the technical terms most frequently used.

CI, Criticality index, The probability for an activity of being critical.

CPM, Critical Path Method, a project planning technique using deterministic networks, see section 2.1.2.

Deterministic, to use single determined values, opposed to probabilistic.

Discrete, consisting of distinct parts, using a finite or a countable infinite set of values, opposed to continuos.

Distribution, the arrangement of a set of numbers classified according to some property as frequence, (or to some other criterion as time or location).

Event, a moment of time, an incident of importance, the start or end of one or more activities.

Expected value, the value that are expected to be the mean value if a test were to be performed.

Free Float, it represents the largest possible delay of an event i without delaying the start of any other activities.

Heuristic, to proceed along empirical lines, using rules of thumb.

Mean value (mean), the sum of i values divided by i.
Monte Carlo method, a statistical procedure where mathematical operations are performed on random numbers.

PERT, Project Evaluation and Review Technique, a project planning technique using stochastic networks, see section 2.1.3.

Probabilistic, to follow the doctrine that certainty in knowledge is impossible and that probability is a sufficient basis for action and belief.

Probability density function, related to probability distribution function, e.g. probability = f(value).

Pdf, see probability density function

Probability distribution function, mathematical function describing a distribution, e.g. frequency = f(value).

Run, here - a set of Monte Carlo realizations used as a sample in a Monte Carlo simulation of higher degree.

Sample, part taken to show nature of whole

Slack, float, the play of freedom (in time) between, for instance, two activities, see also free float and total float.

Standard deviation, the square root of the variance, a measure of the spread of the data in the same order of size as the mean value etc.

Stochastic, random

Total Float, it represents the largest possible delay of an event i without delaying the whole project.

Variance, the variance is a measure of the spread of the data.
SOME VARIANCE REDUCING TECHNIQUES

It can sometimes be problematic to reach a good estimate of the average project completion time because the variance is too large. If the assessment of the completion time is the main reason to perform the simulation, it might be a good idea to use some variance reducing technique. The method of reducing the variance by simulation in loops is described in section 2.2.1, and will therefore not be considered in this section. The methods described below are becoming obsolete due to the rapid development of personal computers. They will therefore only be presented very briefly. The main reference for this appendix is Burt and Garman (1971:1).

Antithetic Variates

This technique uses the same random number series twice. The first time \( R_i \) (=random number \( i \)) and the second time \( (1-R_i) \). This causes a negative covariance between the two realizations which makes this procedure more than twice as effective as the double number of realizations. This is shown by Burt and Garman (1971:1). The effect is most pronounced when the different activities have symmetric density functions.
Stratification

The density functions of the activities are each divided into $k$ intervals. Then $k$ parallel simulations are run in such a manner that for each realization, a sample from each interval is assigned to one of the $k$ processes. The assignment of intervals for each activity to each process is random. This stratification leads to a dependence between the $k$ simulated processes. The samples for each process are, however, still independent samples from each distribution, and are not serially correlated.

Control Variates

A control variate is sought, which is positively correlated with the network completion time, and whose complete density function is known. Such a network may be found by applying the series-parallel reduction technique presented by Martin (1965). Both the "real" network and the control network is then calculated with the same series of random numbers. By comparing the simulation results for the control network and the known control variate, it is then possible to "correct" the real network.

Regression

This can be described as a process of weighting together the time estimates, derived using different techniques, to form a new estimate.
Conditional Sampling

The network should first in some way be reduced as much as possible to an equivalent network, one approach of achieving this has been described by Martin (1965). If all paths through a network were independent, then it would be possible to express the distribution function for the total completion time as the product of the distribution functions for the different paths. They are, however, not independent. Therefore all paths through the equivalent network must be examined, and one unique activity per path selected (i.e. if such exists). If the rest of the activities now are considered as "constants", then the paths will be independent. Only these constants need now be sampled for each realization. This method will reduce the required number of samples per realization by a number somewhat less than the total number of paths in the network. This method is presented more thoroughly by Burt and Garman (1971:2).

Conditional Sampling with Cutsets

A cutset is a path independent set of arcs (activities) in a network. If one selects the cutset that contains the most elements (activities), and utilizes it in the conditional Monte Carlo formulation in place of the set of unique arcs, then one is guaranteed to have the minimum number of arcs to condition upon. This results in an improvement over the unique arch approach. Thus, if conditioning upon non-cutset arcs, each network path can be treated as an independent random variable. Special consideration must, however, be given to the case when the cutset arc is on more than one path. The method is described by Sigal et al (1979), who also describe how to implement criticality indices with this technique.
The techniques for reducing variance described in this section can be divided in four groups.

The first group contains the antithetic variate and the stratification approaches. These provide more efficient ways of using random numbers by introducing dependencies between different realizations, thus decreasing the variance of the project completion time. These are probably the most interesting techniques for use on a modern PC.

The second group includes the control variate technique. This approach requires that the network is reduced to a simpler one, followed by the not entirely trivial task of determining the exact distribution of this simplified network.

The two conditional sampling approaches form the third group. Simplification of the network to a smaller equivalent network is also necessary using this technique to increase the efficiency of the method. The method then uses the whole distribution for some activities instead of sampling them.

The last group consists of the regression technique which is not a group in the same sense as the others, but as a matter of fact it is more a method of combining them. There also exists of course other means of combining the techniques above in order to obtain a more efficient sampling procedure (e.g. using antithetic variates in the stratification process).

It also exists technique to derive the performance measure (e.g. the mean shortest path of a stochastic network) together with its sensitivities (with respect to certain parameters) simultaneously during the performance of a Monte Carlo experiment, see Rubinstein (1989).
CHOICE OF RANDOM GENERATOR

The use of a good random generator is always of great importance when performing a simulation. Many in use today show non-random characteristics when they are put to the test. The desired generator should produce a virtually infinite sequence of statistically independent random numbers, uniformly distributed between 0 and 1. Park and Miller (1988) recommend a multiplicative linear congruential generator, (also known as a Lehmer generator), with multiplier 16807 and prime modulus $2^{31} - 1$. This can also be written as

$$f(z) = 16807z \mod 2147483647$$

This generator is reported to have been tested thoroughly and found to be a full period generating function where the full period sequence \(\ldots, z_1, z_2, \ldots, z_{m-1}, z_1\) is random. The implemented random generator has also been tested as described in section 2.2.5, see appendix 2.6. It has therefore been used throughout this investigation.
Distribution functions

a) Uniform (Rectangular) distribution

A time interval for the activity time is given. All times within this interval are equally probable. The uniform distribution is used when there is a large uncertainty, or when the duration of the activity can be regarded as totally random. This distribution has the advantage that it is very easy to sample from, and is the distribution that is implemented in the random number generator in most computer software. However, many random number generators in use do not appear to be particularly random.

\[
f(x) = \frac{1}{b-a}
\]

Fig. 1. Example of uniform distribution.

b) Triangular distribution

Three time values are given, two of them define the boundaries for the interval and the third is the most likely time (mode). The probability for the different values varies linearly from the outer values (P=0) to the most likely time (P=max).
c) Normal distribution

As mentioned in sections 2.2.2 and 2.2.3 the normal distribution arises as a sum of number of independent random variables, which perhaps makes it the most widely used of all distributions. The normal distribution has the advantage that it is defined by only two parameters. The major drawback is that theoretically, it allows infinitely small and large values. Several alternative procedures may be used for sampling. The central limit theorem can be applied but is a bit circumstantial. Other techniques are Box-Müller and Marsaglia's methods, see Råde (1987).
d) Lognormal distribution

The lognormal distribution is the distribution of a random variable whose natural logarithm follows the normal distribution. By use of the central limit theorem, it can be shown that the distribution of the product of independent positive random variables approaches a lognormal distribution.
e) "Modified Exponential"

This is an exponential distribution that is moved to the right along the x-axis. The difference from the ordinary exponential distribution is that the minimum value in this case may be greater than zero.

\[ f(x) \]

\[ \text{Fig. 5. Example of modified exponential distribution.} \]

f) Beta distribution

This is the traditional distribution used in PERT-analysis. Three values are estimated, the optimistic, pessimistic and most likely times, which form the basis for the beta-distribution. It is defined over a finite interval and is therefore frequently used to describe situations which have a finite range. Although it can not be unambiguously expressed with only three values, expressions for expected value and variance can be deduced if certain assumptions are made. The simple formulae below are used in the traditional PERT technique.

\[
E = \frac{a+4m+b}{6} \quad a = \text{optimistic time}
\]

\[
\sigma^2 = \frac{(b-a)^2}{36} \quad m = \text{most likely time}
\]

\[
b = \text{pessimistic time}
\]
g) Erlang distribution

This is the distribution recommended by Lichtenberg, see Lichtenberg (1984), when using the successive principle. It is derived as the sum of independent and identically distributed exponential random variables, and therefore represents a special case of the gamma distribution. It has a similar form to the beta-distribution and is also difficult to sample from. The simplified expressions for expected value and variance are presented below.

\[
E = \frac{a + 3m + b}{5} \\
\sigma^2 = \frac{(b - a)^2}{25}
\]

- \(a\) = optimistic time
- \(m\) = most likely time
- \(b\) = pessimistic time
2.4 (6)

Fig. 7. Example of Erlang distribution.

h) Weibull distribution

The Weibull distribution is applicable to the assessment of systemic reliability. The time between the failures of components in a system is likely to be a random variable of the Weibull family. A Weibull distribution is described by two parameters on shape and one scale parameter. The case when the shape parameter has the value one is the same as an ordinary exponential distribution.

Fig. 8. Example of Weibull distribution.
Appendix 2.5

Test for normality of project completion time

Chisquare Test

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Chisquare = 13.299 with 16 d.f. Sig. level = 0.650782

Kolomogorov-Smirnov test

Estimated KOLMOGOROV statistic DPLUS = 0.0355985
Estimated KOLMOGOROV statistic DMINUS = 0.0189898
Estimated overall statistic DN = 0.0355985
Approximate significance level = 0.998713
## Test of random generator

### Chisquare Test

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<th>Chisquare</th>
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Chisquare = 17.8449 with 23 d.f. Sig. level = 0.765846

### Kolmogorov-Smirnov test

Estimated KOLMOGOROV statistic DPLUS = 0.0142783
Estimated KOLMOGOROV statistic DMINUS = 0.0236062
Estimated overall statistic DN = 0.0236062
Approximate significance level = 0.999525
The successive principle (Lichtenberg)

Lichtenberg (1984) describes a method to reduce the variance of the estimate of the project completion time at the planning stage. The method can be compared with the principle that no chain is stronger than its weakest link. The technique involves the sub-division of the activities with the largest variances into a series of sub-activities. It can be shown, (see below), that if

a) The expected duration of the series is equal to the duration of the parent activity, and

b) The ranges of the sub activities add up to the range of the parent activity,

then the variance of the terminal event is necessarily decreased. This can very easily be illustrated for the case of a uniformly distributed activity that is sub-divided into two sub-activities. The parent activity varies between $a$ and $b$, and the two sub-activities vary between $a_1-b_1$, and $a_2-b_2$. The variance, $\sigma^2$, of a uniform distribution can be described as

$$\sigma^2 = \frac{(b-a)^2}{12} \quad (1)$$

The expression is of course valid for the parent activity as well as the sub-activities. Hence the variance for the series of sub-activities is

$$\sigma_*^2 = \frac{1}{12} \left[ (b_1-a_1)^2 + (b_2-a_2)^2 \right] \quad (2)$$
The second condition (b) above gives the equation

\[(b-a) = (b_1-a_1)+(b_2-a_2)\]  \hspace{1cm} (3)

If then equation 3 is substituted in equation 1, it will give the following expression

\[\sigma^2 = \frac{1}{12} [(b_1-a_1)+(b_2-a_2)]^2\]
\[= \frac{1}{12} [(b_1-a_1)^2+(b_2-a_2)^2+2(b_1-a_1)(b_2-a_2)]\]

Which gives

\[\sigma^2 - \sigma^2_* = \frac{(b_1-a_1)(b_2-a_2)}{6}\]  \hspace{1cm} (Difference in variance)

It is obvious that this difference must be equal to, or greater than zero if, as by definition, \(b_x \geq a_x\).

The activities are sub-divided until no more suitable activities (for division) exist, or until the required accuracy has been reached.

This simple demonstration gives an indication of the potential reduction in the variance using this technique and how it is achieved.

The method (the successive principle) also offers the possibility to include uncertainty factors that are not connected directly to any particular activity, including such parameters as bad weather, poor management etc.
Short information on software models

Timeline

Symantec Corp.
10201 Torre Ave.
Cupertino, CA 95014
US
Telephone: (408) 253-9600

Artemis 2000

Artemis 2000 is marketed by Metier Management Systems. Further information may be obtained from the following addresses.

Metier Management Systems Sweden AB
Korta Gatan 4
S-171 54 SOLNA
SWEDEN

Among several regional headquarters

SCANDINAVIA
Norway, Oslo
Telephone: (02) 54 22 02
Telex: 72942 METIE N

USA & CANADA
Texas, Houston
Telephone: (713) 956 7511
Telex: 774367 METIER HOU
PGL-Timing

PGL-Timing is marketed by Innovation-Strategi-Management. Further information may be obtained from the following addresses.

ISM Svenska AB
Box 4179
S-203 13 MALMÖ
SWEDEN
Telephone: 040 - 11 63 10
Telefax: 040 - 11 68 70

ISM
Jesper Glahn ApS
19, Store Strandstraede
DK-1255 Copenhagen K
DENMARK
Telephone: 451 - 14 22 14
Resource scheduling algorithm used in the PertSim program

An heuristic, parallel approach was adopted for use in PertSim. Two sets of heuristic rules have been used. The resource scheduling algorithm is presented below, see also fig 1.

a) The algorithm

1. Set clock=0, start scheduling. "Clock" keeps track of the current "scheduling time".

2. Locate those activities that can be started, i.e those where all predecessors have been completed. These activities are stored in a list, referred to as the "AvailList".

3. Sort the AvailList according to the primary criterion selected. The properties of the primary criterion will be explained in section (b).

4. Select an activity from AvailList. Check if there are resources enough to schedule it.

If sufficient resources exist, assign the necessary resources to the activity and schedule it. Proceed to point 5.

If the amount of available resources is insufficient, go through the activities that already have been scheduled and check if any of those can be "unscheduled", i.e fulfil the secondary criterion, see section (c). If a suitable candidate is identified then unschedule it, and schedule the activity from AvailList, proceed to point 5.
Proceed to the next activity in AvailList. Continue until an activity which can be scheduled is found, or, until the last activity in the AvailList is reached. If any of those two conditions are fulfilled then proceed to point 5.

5. If any activity has been scheduled under point 4 then proceed to point 6, otherwise get new "event time" and reset clock. Proceed to point 6.

An event time is a point in time when an activity, under current conditions, can be started or completed.

6. Check if all activities are scheduled. If they are, then scheduling is complete. If any activities remain proceed to point 4.
Fig 1. Flow Sheet presenting resource scheduling principal.

b) Primary scheduling criteria

The various primary criteria are used to sort the activities that are available for scheduling at a certain time into order of priority, see point 3 above. The five primary criteria incorporated in the PertSim program can be divided into two groups. The first group consists of the early start ("Earl.Start") criterion, and the second group includes the remaining alternatives.
The "Earl.Start" criterion sorts the activities according to their earliest start time in the CPM calculations. The activity with the earliest start time is assigned highest priority. The criterion does not in any way consider the float times, in contrast with the alternatives in the second group. However this criterion can sometimes give a shorter project completion time than the other alternatives.

The second group contains the sorting keys Free Float (FF), Total Float (TF) and Criticality index (CI). Small float goes before larger, high CI goes before lower. All three keys are considered simultaneously. The first key is the most important. The second will only be used if two activities have the same argument in the first key etc. All of the alternatives in this group will all, in different degrees, attempt to schedule the most critical activities first. Four different sequences of the keys are used in the program.

c) Secondary scheduling criteria

The secondary criterion is used when an activity that could not be scheduled, due to shortage of resources, is compared with activities already scheduled. If the unscheduled activity has a higher priority than the scheduled activity, according to the specified criterion, the activities will be exchanged i.e. the unscheduled activity will be scheduled in place of the currently scheduled activity. The different available criteria are described below.

Free Float. This criterion considers the free float, which is the time an activity can be delayed without delaying any other activities. The relation between "clock" and the activities early start is also considered, see below.
Schedule activity 1 first if the following condition is fulfilled.

\[ FF_1 - (\text{clock} - ES_1) < FF_2 - (\text{clock} - ES_2) \]
\[ FF_1 + ES_1 < FF_2 + ES_2 \]

The activity with the lowest value for the free float parameter is scheduled first.

**Total Float.** Total float is a measure of how much the activity can be delayed without increasing the project completion time. This criterion works in the same way as the free float criterion. The activity with lowest value for the total float parameter is scheduled first.

**Criticality Index.** Criticality index describes how frequently an activity lies on the critical path and is expressed in percent. The activity with highest criticality index is scheduled first.

**Minimum activity duration.** Completion the shortest activities first provides the earliest opportunity to revise a decision.

**Number of successors,** The program checks how many activities are in any way dependent on the current activity, i.e. the number of activities in the network after the one considered. The activity with most successors is scheduled first.

**Longest remaining path.** The program calculates the longest remaining path, including the duration of the current activity. The activity with the longest remaining path will be scheduled first.
4.1 (6)

Maximum resource utilization. This criterion tries to utilize the resources at the beginning of the project. If the activities with very high resource demand are performed at an early stage, there may be a greater degree of flexibility towards the end of the project. The decision criterion is described below.

\[
\text{parameter}_1 = \sum_{r=1}^{n} (\text{Available resources}(r) - \text{Activity}_1 \text{ demand})^2
\]

where \( n \) = number of available resources

Minimum resource utilization. This criterion is actually the inverse of the "Max. Resource util" criterion. This criterion often gives smoother utilization curves than the other criteria.

First In First Served, This is, in fact, no criteria at all. This "criterion" results in the scheduling of the resources solely according to the primary criterion.

Priority Number, This criterion is based on criticality index and the uncertainty in time estimates. It is analogous with Lichtenberg’s priority number presented by Lichtenberg (1984), (see below). A high priority number indicates a high uncertainty. It might be the best criterion to use if the primary goal of the resource scheduling is to achieve a "safe" time schedule and not necessarily fastest. The criterion is defined below.

Lichtenberg’s priority number (KV)

\[
\begin{align*}
\text{KV} &= \text{CI} \times s^2 \\
\text{KV} &= \text{CI} \times k_1 \times (b-a)^2 \\
/\text{KV} &= /\text{CI} \times /k_1 \times (b-a) \\
\text{KV}_* &= /\text{CI} \times (b-a)
\end{align*}
\]
Priority number used in PertSim

\[ \text{Prio} = (\text{CI} + 1)^{0.5} \times (b - a) \] (2c)

The additional 1 is to give the priority number a meaning even when the criticality index is zero, which frequently occurs.

Min.Proj.Time (1-11), The program calculates through all the criteria described above, and selects that which gives the shortest project completion time.
Flow sheet describing cost simulation model implemented in computer program

Define interval for PT *, no of samples for each PT *, time and cost parameters.

Calculate PT * <i>.

Take sample from project completion time distribution

Calculate additional development cost using equations 7.5 - 7.13.

Has the sampling been completed for this PT *, i.e. PT * <i>?

- Yes
  - Calculate average additional development cost for PT * <i>.
  - Has the average additional cost been calculated for all PT *?
    - Yes
      - Chose the PT * with the lowest average additional cost as the optimum PT.
    - No
      - No
Comparison between the use of uniform and triangular distributions in a project network.

The uniform and the triangular distributions have the same optimistic and pessimistic times for all activities. The most likely times used with the triangular distributions correspond to the times used in the deterministic model. The network modelled is the same as the one in appendix 7.3.

<table>
<thead>
<tr>
<th>Model</th>
<th>Prod.start (first module)</th>
<th>Tot.time (all develop.)</th>
<th>Std.Dev. (all dev.)</th>
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<td>241</td>
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<tr>
<td>uniform</td>
<td>210</td>
<td>245</td>
<td>15</td>
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</table>

Table. Characteristica triangular vs. uniform distributions

The choice of distribution has, as can be seen in the table above, rather large effect on the standard deviation. However, the effect on the total time is not so obvious. The effect the probability to start the production on schedule because of the decrease of the time until production start will not change in a drastic way. This is owing to the fact that also the standard deviation is lower for the triangular case, the pdf-bell will thus be more narrow.
### Deterministic time plan for the Oscar project

<table>
<thead>
<tr>
<th>Activity:</th>
<th>Dur</th>
<th>ES</th>
<th>LC</th>
<th>TF</th>
<th>FF</th>
<th>CI</th>
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| production b             | 0.0 | 196.0| 237.0| 41.0| 41.0| 0   |
| L 289 cav.drill.         | 13.0| 199.0| 212.0| 0.0 | 0.0 | 100 |
| L 292 cav.drill.         | 10.0| 203.0| 237.0| 24.0| 24.0| 0   |
| L 294 blasting           | 25.0| 205.0| 237.0| 7.0 | 0.0 | 0   |
| L 290 blasting           | 25.0| 212.0| 237.0| 0.0 | 0.0 | 100 |

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Total project completion time (last run): 237.00

Number of runs: 1
Samples per run: 1
Distribution: Deterministic
Average project completion time: 237.00 ± 0.00
Standard deviation single network: 0.00
Abbreviations:

Dur = Average activity duration
ES =Earliest start of activity
LC =Latest completion of activity
TF =Total Float
FF =Free Float
CI =Criticality Index

In front of row:

D = Drilling level
L = Loading level
## Probabilistic time plan for the Oscar project

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Total project completion time (last run): 242.48

Number of runs: 10
Samples per run: 50
Distribution: Stochastic
Average project completion time: 244.87 ± 1.83
Standard deviation single network: 14.88

Note that the standard deviation given above is not identical with the standard deviation used in the thesis. The value used in the thesis was derived from a somewhat modified value that ended with the start of production.
Abbreviations:

Dur = Average activity duration
ES  = Earliest start of activity
LC  = Latest completion of activity
TF  = Total Float
FF  = Free Float
CI  = Criticality Index

In front of row:

D = Drilling level
L = Loading level