Mechanics of Microdamage Development and Stiffness Degradation in Fiber Composites

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Preface

This thesis and the work herein presented have been performed at the Division of Polymer Engineering at Luleå University of Technology during the period between October 2005 and November 2007.

The work would of course not have been possible without my supervisor Prof. Janis Varna. I would like to thank him for his support, supervision, dedication and for being an ambitious and strongly contributing colleague in all papers herein presented. He has a sense for mathematical and numerical details and he has been curious and interested in my work even though he has many other things on his mind as head of the division. I would also like to thank my assistant supervisor Doc. Roberts Joffe and my other colleagues for their support and sense of humour.

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Johannes Eitzenberger
Abstract

Microdamage in composites reduces its performance and durability and thus its usefulness. The common subject in all papers (A,B,C,D) included in this thesis is distributed microdamage. The materials considered in the papers are a Hemp/Lignin natural composite and glass/carbon fiber reinforced plastics composites. The focus is on how the microdamage affects the performance in terms of creep strain and stiffness.

The papers are preceded by an introduction to long and short fiber composites as well as to stress transfer models. Knowledge about the axial fiber stress distribution in aligned fiber composites loaded in tension in the fiber direction is important since the axial fiber stress control the size of fiber cracks. The size of each fiber crack controls the degree of stiffness reduction. The stiffness reduction also depends on the amount of fiber cracks and on the presence of other types of microdamage like matrix cracks and fiber/matrix interface debonding.

In Paper A a nonlinear viscoelastic viscoplastic model of a Hemp/Lignin composite is generalized by including stiffness reduction, and thus the degree of microdamage, in the composite (when loaded in the axial direction). Schapery’s model is used to model the nonlinear viscoelasticity whereas the viscoplastic strain is described by a nonlinear function presented by Zapas and Crissman. In order to include stiffness reduction due to damage, Schapery’s model is modified by incorporating a maximum strain-state dependent function reflecting the elastic modulus reduction with increasing strain measured in tensile tests. The model successfully describes the main features for the investigated material and shows good accuracy within the considered stress range.

In Paper B the stiffness reduction of a unidirectional (UD) composite containing fiber breaks with partial interface debonding is analyzed. The analysis is performed by studying how the average crack opening displacement (COD) depends on fiber and matrix properties, fiber content and debond length. The COD is normalized with respect to the size of the fiber crack and to the far field stress in the fiber. In contrast to other performed analysis an analytical relationship is developed which links the entire stiffness matrix of the damaged UD composite with the COD and the crack sliding displacement (CSD). However, the CSD is excluded from the analysis since it is found by parametric inspection that it does not affect the longitudinal stiffness. Some trends regarding the COD dependence on the different properties can be extracted from available approximate analytical stress transfer models. To obtain more reliable results, in the current analysis these dependences are extracted from extensive FEM based parametric analysis performed on a model consisting of three concentric
cylinders: a) broken fiber; b) matrix cylinder around it; c) large effective composite cylinder surrounding them. This model is used since it is more adequate than unit cell models considering only fiber and matrix. The cracks, which are only in the fibers, are distributed in such a way that they are non-interactive.

It is shown that the parameters that affect the COD the most are the ratio of the longitudinal fiber modulus and matrix modulus, the fiber content and the debond length. These relationships are described by simple fitting functions which excellently fit the numerical results. These simple functions are merged into one relationship describing the COD’s dependence on the relevant parameters. Simulations performed for carbon and glass fiber polymer composites show that the relative longitudinal stiffness reduction in the carbon fiber composite is slightly larger than in the glass fiber composite. This trend holds for all considered debond lengths and is related to higher longitudinal fiber and matrix modulus ratio in the carbon fiber composite leading to larger crack openings and larger stress perturbation zones. It is shown that the stiffness reduction depends on the debond length.

In Paper C the analysis performed in Paper B is continued by studying how the COD is affected when the cracks are interactive. It is shown that the effect on the COD in the glass fiber composite is negligible. However, the effect on the COD in the carbon fiber composite is significant. This difference is related to higher longitudinal fiber and matrix modulus ratio for the carbon fiber composite.

In Paper D the same model is used to analyse the strain energy release rate related to the debond crack growth along the fiber. The energy release rate is calculated using the virtual crack closure technique applied to displacement and stress field in the vicinity of the debond crack tip calculated using refined FE model. It is shown that the energy release rate is larger for very short debonds. It reduces to a constant value indicating a stable debond crack growth after its initiation. It is shown that the strain energy release rate in the plateau region also can be calculated using a simple analytical model based on the self-similar crack growth assumption. When the stress state perturbations related to debonds at both fiber ends start to interact, the energy release rate decreases. In a future work the obtained relationships for the energy release rate will be incorporated in a microdamage evolution model describing the statistics of fiber breaks and debond growth in fatigue loading conditions.
Appended papers

**Paper A**

**Paper B**
Varna J., Eitzenberger J. Modeling UD composite stiffness reduction due to multiple fiber breaks and interface debonding. 6th International Symposium on Advanced Composites, may 2007.

**Paper C**

**Paper D**
Varna J., Eitzenberger J. Modeling energy release rate for debond crack growth along fiber in UD composites with broken fibers. *To be submitted.*
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1. Introduction

Microdamage in composites reduces its performance and durability and thus their usefulness. The common subject of the papers presented in this thesis is stiffness reduction of composites loaded in tension. One of the materials studied is a Hemp/Lignin composite which is a randomly oriented short fiber composite. Other materials studied are a Carbon/Epoxy composite and a Glass/Epoxy composite which both are unidirectional (UD) continuous fiber composites.

The stiffness is reduced because of microdamage and the amount (of microdamage) grows with increasing load applied to the composite. In UD continuous fiber composites loaded in tension the stiffness reduction is related to microdamage such as fiber breaks. A fiber break in its turn can cause a debond crack at the interface between the fiber and the matrix or a matrix crack. The debond crack is caused by a high shear stress at the interface. The higher the shear stress is the larger is the probability for a debond crack to initiate. A strong interface adhesion prevents debonding and thus increases the probability for the fiber crack to continue as a crack in the matrix (or only cause shear yielding of the matrix) instead of as a debond crack.

The larger the size of each crack is and the more cracks there are (and thus more debonds and matrix cracks) the larger is the stiffness reduction. A fiber break in its turn is caused by a large axial stress in the fiber. The larger the axial fiber stress is the larger is the probability of fiber failure. The larger the applied stress to the composite is (and thus the axial fiber stress) the larger is the displacement of the crack surfaces (COD) of a broken fiber. The debond length govern the stiffness reduction since the longer the debond crack is the larger is the COD. The number of cracks, and thus the amount of microdamage, grows with increasing stress applied to the composite.

The purpose of the fibers is to reinforce the matrix (polymer). The majority of the stress applied to the composite is transferred from the matrix to the fibers. In order for the fibers to perform well as reinforcement a good adhesion at the interface is needed. The stress in transferred via each fiber/matrix interface using a relatively small region of the fiber (at the fiber ends). This transfer of stress causes high shear stresses in the matrix close to the fiber and at the interface. The axial fiber stress grows from zero at the fiber end to reach its maximum at the center of the fiber. The shear stress at the interface behaves in the opposite way; it has its maximum at the fiber ends and reaches zero at the center of the fiber (if not earlier).
In many situations the knowledge about the axial fiber stress distribution is needed. This is because it can be used to estimate the COD which in its turn can be used to estimate the stiffness reduction of the composite. The larger the COD (size of the crack) and the more cracks there are the larger is the stiffness reduction. The axial fiber stress distribution at the fiber ends is not only of interest for UD short fiber composites where the fibers are embedded in the matrix but also for UD continuous fiber composites. This is because the continuous fibers break down to shorter fibers as a consequence of increasing load applied to the composite. In other words, two new fiber ends are created at each fiber failure.

The knowledge about the degree of stiffness reduction can be attained using different approaches. It can be attained using a pure analytical approach which means that knowledge about the axial fiber stress distribution is needed. The stress distribution can be estimated using many different models that describe how the stress is transferred from the matrix to the fiber at the fiber end. The difference between these models is which assumptions that are made and thus how close to the real life the modeled stress distribution is. Further, as the COD depends on the load applied to the composite and thus on the axial fiber stress, an expression for the COD is needed. The last relation that is needed is an expression for the stiffness containing the COD and the crack density. When this is known, then an analytical expression for the stiffness reduction is attained. The stiffness reduction can as well be estimated by combining analytical models with numerical methods. This is the case in one of the papers in this thesis where the COD is estimated using finite element modeling (FEM) and an expression for the stiffness reduction containing the COD and the crack density is analytically derived.

The simplest models do not include a partially debonded interface while the more complicated models do. If debonding is included then the friction between the matrix and the fiber has to be considered in the model. If it is assumed that there is no friction then the axial fiber stress is zero at the position of the debond tip. However, a good model considers friction at the interface which is how it is in reality. In that case the axial fiber stress is not zero at the debond tip.

The contents of this thesis are as follows. An introduction to fiber composites is given in chapter 2. Since fiber composites with different fiber geometry are studied in the papers an introduction to continuous fiber composites (chapter 3) and short fiber composites (chapter 4) is presented. Different stress transfer models and some numerical methods are presented in chapter 5. The objectives of the work presented in the papers are given in chapter 6 followed by summaries of the appended papers in chapter 7. Conclusions and some words about future work are presented in chapter 8.
2. Fiber composites

Fiber composites are materials that are reinforced with fibers. The material (matrix) is normally a synthetic polymer (for example epoxy) or a natural polymer (for example lignin). The fibers are normally made of carbon, glass, or an organic material (for example hemp or flax). The fibers can work as reinforcement since the load is transferred from the matrix to the fibers via the interface. The geometrical factors that separate different fiber composites are: a) the volume fraction of the fibers; b) the length of the fibers; c) the distribution of the fibers; d) the orientation of the fibers.

If a composite has continuous fibers (fibers going from one side to the opposite side) that are aligned (oriented in the same direction) then it is a unidirectional (UD) continuous composite. (It can also be called an aligned continuous composite.) A composite can consist of several layers where each layer has its own fiber orientation. This kind of composite is called a laminate, which often is used as skins in sandwich structures. A layer can consist of not only aligned fibers but also fibers with different orientations. They can for example have random orientations or be orthogonal to each other like in woven fabrics or non-crimp fabrics (NCF).

The maximum theoretical fiber volume fraction a composite can have is 91% which occurs when the fibers are packed in a hexagonal manner. The fiber volume fraction in everyday use is 45-65% but it is possible to reach more than 70%.

A composite can have different fiber structure as seen in Fig 2.1. The composite can consist of short or long fibers where long fibers include continuous fibers. Short fibers are fibers with a length of about 10-100 fiber diameters. The fibers can be either parallel or randomly oriented. In Fig 2.2 the classification of fiber composites is shown.

![Figure 2.1. Schematics of different fiber structures in composites.](image)
When a composite is loaded in tension with increasing load the composite will eventually fail (macroscopically). The failure is preceded by the initiation and evolution of microdamage. There are different microdamage modes. A part of the matrix can fail, fibers can fail, and there can be fiber/matrix interface debonding. Due to this kind of microdamage a composite undergoes stiffness reduction when loaded in tension. In other words, the elastic modulus in the loading direction will decrease. The strength of a UD (aligned) composite strongly depends on the fiber volume fraction. The higher the fiber volume fraction is, the higher is the strength.

3. Long fiber composites

The stiffness (longitudinal modulus, \( E_c \)) of a UD continuous composite is given by

\[
E_c = E_f V_f + E_m V_m
\]

where \( V \) is the volume fraction and \( E \) is elastic modulus. Eq (1) can be converted to

\[
\sigma_c = \sigma_f V_f + \sigma_m V_m
\]

using Hooke’s law. It is valid that \( V_f + V_m = 1 \) if it is assumed that no voids (air-pockets in the composite) are present and thus \( V_r = 0 \). Eq. (1) and (2) are based on “rule of mixture” and an assumption is that the longitudinal strain in the fibers and in the matrix is the same as for the composite as a whole (constant strain model).

In Fig 3.1(a) schematics of a UD continuous composite loaded in tension are shown. The fibers are first to fail since the fibers have lower failure strain than the matrix (i.e. \( \varepsilon^*_{f} < \varepsilon^*_m \)). (These failure strains are valid throughout this document.)
(The opposite $\varepsilon_f^* > \varepsilon_m^*$ is more common in case of a ceramic matrix where the matrix is very brittle.) Assume that the UD continuous fiber composite is under a high tensional stress in the fiber direction. Since $\varepsilon_f^* < \varepsilon_m^*$ a lot of fibers will break down to shorter fibers. The breaking of fibers will eventually reach saturation since the stress in the fibers no longer can reach the fiber failure stress. In other words, the fiber crack density has a limit. The composite has more or less become a short fiber composite. Further, the sequence of microdamage modes is shown in Fig 3.1(a). At first there is a fiber break (where the crack is a penny-shaped (circular) crack), then the fiber crack either causes a debond crack or a crack in the matrix (if not only matrix yielding).

![Figure 3.1. Sequence of microdamage modes involving fiber break, debonding and matrix failure (a). A typical fracture surface preceded by fiber pull-out (b).](image)

When many fibers have failed and a lot of debonding and matrix failure have occurred then there will be a complete failure of the composite by coalescence (see Fig 3.1(b)). Coalescence means that different cracks will merge into one. This is because the fibers that are closest to a broken fiber will experience an increase in stress as seen in Fig 3.2. This means that the fibers that are close to this broken fiber have a higher probability to fail. In Fig 3.2 the stress in different parts of a fiber (marked with F) are shown. Notice the peak in fiber stress at the position where the neighbouring fibers are broken. It can also be seen that the strength is changing somewhat randomly along the fiber.
It can also be seen in Fig 3.2 that there is a region at each fiber break where there is a perturbation in the axial fiber stress. This is because the stress is zero at the fiber end and needs a certain distance to reach the unperturbed stress level. The higher the ratio is between the elastic modulus of the fiber and the matrix the longer is this perturbation zone.

Longitudinal failure models are based on statistical strength distribution. This is because the fibers fail at a somewhat random stress. This is due to surface flaws/defects which cause the fibers to break in a brittle manner at an axial stress that can be described with statistics. In other words, the fibers have a statistical fiber strength distribution. The distribution of strength resembles the Weibull distribution and this is why Weibull distribution is normally used to describe at which stress level the fibers most probably break. Weibull distribution is the same as normal distribution except that the peak can have any position in the current interval which means that the distribution can be non-symmetric. An example of how the strength can be distributed is shown in Fig 3.3. It can be seen that the distribution resembles a Weibull distribution where the peak it shifted to higher stresses. Most of the fibers break at the stress where the curve has its peak (in other words where the failure frequency is as highest).
The average fiber strength can be described by
\[
\overline{\sigma}_f = \sigma_0 l^{-1/\beta} \Gamma(1+1/\beta)
\]  
(3)

where \(\sigma_0\) and \(\beta\) are the Weibull parameters which are obtained by some fitting procedure. The parameter \(l\) is the fiber length and \(\Gamma\) is the gamma function.

The distribution of the fiber strength (as in Fig 3.3) can be attained using
\[
P_f(\sigma_f) = 1 - \exp\left[-l\left(\frac{\sigma_f}{\sigma_0}\right)^\beta\right]
\]  
(4)

where \(P_f\) is the probability of failure of a fiber with the axial fiber stress \(\sigma_f\). The probability lies between 0 and 1.

The strength of a UD continuous composite in tension can not be described directly using (2) saying that
\[
\sigma^*_L = \sigma^*_f V_f + \sigma^*_m V_m
\]  
(5)

where the superscript (*) means the strength. Eq. (2) is only valid as long as both the matrix and the fibers are unbroken. Consider following assumptions: all fibers fail at the same strain (\(\varepsilon_f^*\)) which means that they have the same (uniform) strength; the fibers break before the matrix (\(\varepsilon_f^* < \varepsilon_m^*\)); and \(\varepsilon_f = \varepsilon_m = \varepsilon_c\) (as in the constant strain model). Depending on the fiber volume fraction \((V_f)\) the forthcoming failure of the composite is either stable or unstable as shown in Fig 3.4. If \(V_f < V_f^*\) then the fibers break first (since \(\varepsilon_f^* < \varepsilon_m^*\)) at \(\sigma^*_f V_f + \sigma^*_m (1-V_f)\) and then after the composite stress has increased to \(\sigma^* m(1-V_f)\) the matrix fails and thus the whole composite. This failure process is thus a stable process. If \(V_f > V_f^*\) then
the fibers break first (at \( \sigma_f' V_f + \sigma_m'(1-V_f) \)) which means that the stress that the fibers were carrying now has to be carried by the matrix alone. Since the stress is too high for the matrix (see Fig 3.4) to carry the matrix fails directly and thus the whole composite. This failure process is thus an unstable process.

![Figure 3.4. Composite strength (thick line) versus fiber volume fraction.](image)

The strength of the composite is thus

\[
\sigma^*_c = \begin{cases} 
\sigma^*_m (1-V_f), & V_f < V^*_f \\
\sigma^*_f V_f + \sigma^*_m (1-V_f), & V_f > V^*_f 
\end{cases} 
\tag{6}
\]

The strength of a composite can thus be estimated either under the assumption that the fibers have a uniform strength \( \sigma_f' = E_f \varepsilon_f \) or that the strength is statistically distributed as in (3).

4. Short fiber composites

In unidirectional and random long fiber composites the effects associated with fiber ends can be neglected since these effects are only acting on a small fraction on the fiber length (these effects can not be neglected if fracture processes are considered.) However, for short fiber composites these effects are important and thus the focus is on the effect of fiber length. Short fibers are fibers where the fiber-end effects can not be neglected which means that the limit between a long and a short fiber is not well-defined. However, a definition could be: fibers where the axial fiber stress can not reach the failure stress of the fiber due to its length.

The failure modes in short fiber composites are more or less the same as for long fiber composites. In other words, a crack can be created in the matrix, fibers can
break, and there can be an interface debonding between fiber and matrix. The stress applied to the composite can be expressed by the rule of mixture as

$$\sigma_c = V_f \sigma_{f(\text{av})} + V_m \sigma_{m(\text{av})}$$  (7)

where $\sigma_{(\text{av})}$ is the average stress defined as

$$\sigma_{\text{av}} = \frac{1}{V} \int \sigma dv$$  (8)

where $V$ is the volume of the composite. The stress-strain relationship (assuming that it is elastic) is

$$\sigma_c = E_c \epsilon_c$$  (9)

where $\sigma_c$ and $\epsilon_c$ are the stress and strain applied to the composite. The average stress and strain are defined as the applied stress and strain which makes it possible to write (9) as

$$\sigma_{c(\text{av})} = E_c \epsilon_{c(\text{av})}$$  (10)

The average fiber stress ($\sigma_{f(\text{av})}$) needed in (7) is

$$\sigma_{f(\text{av})} = \frac{1}{l} \int_{-l/2}^{l/2} \sigma_f(x) dx$$  (11)

and the average stress in the matrix (also needed in (7)) is

$$\sigma_{m(\text{av})} = E_m \epsilon_c$$  (12)

which means that the average strain in the matrix is the same as the applied strain to the composite.

4.1 Aligned short fiber composites

Aligned (or oriented) short fiber composites have a fiber structure as exampled in Fig 4.1.
In aligned composites having fibers that are shorter than the critical length $l_c$ (defined in (21)) the fibers can not break (since the fiber failure stress can not be reached). When such a composite is macroscopically failing and the fracture surfaces are being created then the fibers that are between the fracture surfaces becomes pulled out from the matrix (fiber pull-out). (Pull-out occurs since the fiber can not break any further.) In composites with fibers that are longer than $l_c$ then some fibers will break before the fracture surfaces are being created and pull-out occurs. If the fracture surfaces are studied after failure then it can be seen that many fibers are protruding from the matrix, and with a distance of at most half the fiber length. The degree of smoothness of the fracture surfaces can vary between different composites.

A common assumption for an aligned short fiber composite which is loaded in the fiber direction is that the stress is transferred from the matrix to each fiber according to

$$\frac{d\sigma_f}{dx} = \tau \frac{2}{r}$$

where $\tau$ is the shear stress at the interface and $r$ is the fiber radius. The assumption for this relation is that the interface adhesion is perfect. If it is assumed that the shear stress is constant ($\tau_c$) then the integration of (13) gives

$$\sigma_f = x \frac{2\tau_c}{r}$$

which means that the axial fiber stress changes linearly at the fiber ends. (Relation (14) will be applied throughout this chapter) Worth mentioning is that after the stress is transferred from the matrix to the fibers then the shear stress is zero which means that the axial fiber stress is constant which is supported by (13). This level of constant stress is called plateau value ($\sigma_f(lim)$). The distance of the fiber that is needed for the stress to be transferred is called transfer length ($l_t$) and is defined as
\[ l_t = \frac{r}{2\tau_c} \sigma_{f(lim)} \] (15)

The transfer length is the same as the length of the stress perturbation zone mentioned in the previous chapter. Using (7) –(12) the stiffness in the tensile direction can be expressed as

\[ E_c = \eta_l E_f V_f + E_m V_m \] (16)

where \( \eta_l \) is the length correction factor according to

\[ \eta_l = 1 - \frac{2}{\beta l} \tanh \frac{\beta l}{2} \] (17)

in the shear lag model (where it is assumed that the bonding between fiber and matrix is perfect). (When the fiber stress \( \sigma_f(x) \) is integrated over the fiber length in (11) to attain \( \sigma_{f(avg)} \) used to derive (16) then the shear stress is not assumed to be constant. This assumption \( \tau = \tau_c \) was however made in order to get (14).) The parameter \( \beta \) in (17) is the shear lag parameter. Note that \( \eta_l \) is between 0 and 1 depending on the fiber length \( l \). This means that the shorter the fibers are the less they are reinforcing the matrix. (If they are really short then \( \eta_l = 0 \) and (16) becomes \( E_c = E_m V_m \).

If the fibers are long enough then \( \eta_l = 1 \) and (16) becomes the same expression as (1). The stiffness can also be expressed by the Halpin equation according to

\[ E_c = E_m \frac{1+\xi \eta V_f}{1-\eta V_f} \] (18)

where parameters \( \xi \) and \( \eta \) are defined as

\[ \xi = \frac{l}{r} \quad \text{and} \quad \eta = \frac{E_f / E_m - 1}{E_f / E_m + \xi} \] (19)

If now the strength of the composite \( (\sigma_c^*) \) are considered it can be expressed as

\[ \sigma_c^* = \begin{cases} \frac{l \tau_c}{2r} V_f + \sigma_m^* V_m, & l < l_c \quad \text{(matrix failure)} \\ \sigma_f^* \left(1 - \frac{l}{2l}\right) V_f + \sigma_m^* V_m, & l > l_c \quad \text{(fiber failure)} \end{cases} \] (20)
where $l_c$ is the critical fiber length defined as

$$l_c = \sigma_f^* \frac{r}{\tau_c}$$  \hspace{1cm} (21)

Fibers that are shorter than the critical length never break since the stress can not reach the failure stress of the fibers ($\sigma_f^*$). This means that a composite with fibers that are shorter than the critical fiber length will fail due to matrix failure (and not fiber failure since they are too short to break). On the other hand, if the fibers are longer than the critical fiber length then the stress in the fibers can reach $\sigma_f^*$ and the fibers will break. (It should be mentioned that the fibers have different lengths in a short fiber composite in real life which means that the fiber lengths are distributed over some length interval.)

For a fiber in a UD continuous composite the average stress is $E_f \varepsilon_c$ where $\varepsilon_c$ is the strain applied to the composite. The average stress in a fiber in an aligned short fiber composites is different and is

$$\sigma_{f(\text{av})} = \begin{cases} \frac{l \tau_c}{2r}, & l < l_c \\ \sigma_{f(\text{lim})} \left(1 - \frac{l}{l_c}\right), & l > l_c \end{cases}$$  \hspace{1cm} (22)

where $\sigma_{f(\text{lim})}$ is the plateau stress

$$\sigma_{f(\text{lim})} = \frac{\sigma_c}{E_c} E_f$$  \hspace{1cm} (23)

The ratio $\sigma_c/E_c$ in (23) is the strain applied to the composite. The average stress in a fiber at the moment when the composite fails is

$$\sigma_{f(\text{av})} = \begin{cases} \frac{l \tau_c}{2r}, & l < l_c \\ \sigma_f^* \left(1 - \frac{l}{2l_c}\right), & l > l_c \end{cases}$$  \hspace{1cm} (24)

which supports the fact that a fiber shorter than $l_c$ never breaks. An example of how the fiber length affects the strength of a composite is shown in Fig 4.2. It can be seen that the fiber strength strongly depends on the fiber length. It can also be
seen that fibers that are longer than about 50mm is reinforcing the composite well (in this particular glass fiber epoxy composite).

Figure 4.2. Effect of fiber length on strength of an aligned glass fiber epoxy composite (from Hancock & Cuthbertson 1970).

4.2 Randomly oriented short fiber composites
Randomly oriented short fiber composites have a fiber structure as exampled in Fig 4.3.

Figure 4.3 Schematic representation of a section through a randomly oriented short fiber composite.

Since the fibers have different orientations the elastic modulus of the composite in the tensile direction will be similar to (16) but with an extra factor $\eta_\theta$ according to

$$E_c = \eta \eta_\theta E_f V_f + E_m V_m$$  \hspace{1cm} (25)

which is the stiffness according to Krenchel’s model. The parameter $\eta_\theta$ is an orientation efficiency factor and $\eta_\theta = 3/8$ for in-plane random fiber orientations and $\eta_\theta = 1/5$ for three-dimensional random fiber orientations. If $\eta_\theta = 1$ (which corresponds to the case when the fibers are aligned in the tensile direction) then (25) is the same as (16).
5. Stress transfer models at fiber ends and fiber breaks

There are different approaches to establish the distribution of the axial fiber stress. The fiber(s) considered here are oriented in the direction of the applied load (as in Fig 4.1). The analytical models that exist are either based on elasticity analysis (force equilibrium) or on variational mechanics analysis. A well known analytical model based on elasticity analyses is the shear lag model. Another approach is to use a numerical approach (for example a finite element approach (FEM)).

As mentioned in the introductory chapter the simplest models do not include a partially debonded interface while the more complicated models do. Some authors [2,3] have derived one-dimensional analytical models for a partially debonded interface, based on the shear-lag theory. Two-dimensional analytical models have also been derived [4] in order to improve the 1-D models. (There are three-dimensional analytical models as well.) If debonding is included then a good model considers the friction between the matrix and the fiber. This has been done by [5] where Coulomb’s friction law is used to simulate the friction between fiber and matrix. In contrast to the shear lag model, they use a variational mechanics analysis approach based on the principle of minimum complementary energy. If it is assumed to be no friction at the debonded interface then the axial fiber stress is zero at the position of the debond tip. However, in reality there is friction at the interface, which then means that the axial fiber stress is non-zero at the debond tip.

5.1. The shear lag model and its variations

There are different variations of the shear lag model. In other words, there are different ways to describe the axial fiber stress distribution. Even though they describe the stress differently they are all built on the same equation (13). Equation (13) relates the change in the axial fiber stress with the axial coordinate (x) to the shear stress at the interface. (Recall that a fiber-end can either be a fiber in an UD short fiber composite or a broken fiber in a UD continuous fiber composite.) As can be seen in (13) the equation has two unknown parameters; the axial fiber stress (σf) and the shear stress at the interface (τ). This means that one more relation is needed in order to find σf (and thus τ). This extra relation is the relation that describes the shear stress at the interface. The stress distribution is then found by taking the (second) derivative of (13) with respect to x and then solve the arising second-order differential equation. The difference between the variations of the shear lag model is how the extra relation describes the shear stress and what kind of boundary conditions that are used.

Since (13) is fundamental for the shear lag model the relation needs a closer presentation. Consider Fig 5.1 where a fiber is embedded in a matrix in a UD
composite loaded in tension in the fiber direction. The fibers are enforcing the composite since most of the applied stress is transferred from the matrix to the fibers. As seen in Fig 5.1 (left) there are large displacements of the matrix around the fiber-end. This gives a large shear stress at the interface. In Fig 5.1 (right) two elements taken from the FEM-mesh (left) are shown. The upper element is undeformed and the lower element shows large displacements which imply a large tensile strain but in particular a large shear strain (and thus a large tensile stress and shear stress). Further, the shear stress has its maximum at the fiber end and gradually reaches zero further into the fiber. The axial fiber stress grows from zero at the fiber end to reach its maximum (plateau value) at the mid-point of the fiber. At the same time as the axial fiber stress has reached its maximum the shear stress has reached zero. This means that all the axial stress is then transferred to the fiber.

Figure 5.1. Deformation of the matrix around a fiber end (left). An undeformed element and a highly deformed element taken from the FEM-mesh (right). (From [1].)

In Figure 5.2 a small section of the fiber in Fig 5.1 is presented. Since this element is in force balance is it possible to write an equation for this equilibrium which is given in (26).
The simplification of (26) leads to (13).

5.1.1 The plastic model (Kelly’s model)
If the shear stress is assumed to be constant ($\tau_c$), first assumed by Kelly and Tyson [6], then the integration of (13) gives (14). This model is also called the constant shear stress model. The assumption that gives a constant shear stress is that the matrix deforms plastically (and not elastically).

5.1.2 The elastic model (Cox’s model)
The most widely used stress transfer model is the shear lag model originally proposed by Cox [7]. The basic assumptions are that there is no shear strain in the fiber, the interfacial adhesion is perfect, there is no load transferred across the fiber end, and that both fiber and matrix behaves like linear elastic and isotropic solids. This model is the standard shear lag model. Since it is neglected that stress is transferred across the fiber end this model is well suited for broken fibers in a UD continuous composite where the fiber ends are not embedded in the matrix. The axial fiber stress according to Cox’s model is

$$\sigma_f(x) + \tau(2r \pi dx) = (\sigma_f + d\sigma_f)(\pi r^2)$$  \hspace{1cm} (26)

The boundary conditions for $\sigma_f$ in (27) are that $\sigma_f(x = -l/2) = \sigma_f(x = l/2) = 0$ meaning that the stress is zero at the fiber ends. The parameter $R$ in (28) is the radial distance to a position in the matrix where the strain-field no longer is affected by the presence of the fiber. In other words, a position where the tensile

$$\sigma_f = \frac{E_f \varepsilon}{1 - \frac{\cosh \beta x}{\cosh \frac{\beta l}{2r}}} \left(1 - \frac{\cosh \frac{\beta l}{2r}}{\cosh \frac{\beta l}{2r}}\right)$$  \hspace{1cm} (27)

where $\beta$ is

$$\beta = \sqrt{\frac{2G_m}{E_f \ln(R/r)}}$$  \hspace{1cm} (28)
strain in the matrix can be well approximated by the strain applied to the composite \((\varepsilon_c)\). The ratio \(R/r\) in (28) can be approximated by \((1/V_f)^{1/3}\) (where \(V_f\) is the fiber volume fraction as before). Further, the shear stress at the interface is attained using (13) together with (27) which give

\[
\tau_f = -E_f \varepsilon_f \frac{\beta \sinh \frac{\beta x}{r}}{2 \cosh \frac{\beta l}{2r}}
\]  

(29)

The appearance of the axial fiber stress and the shear stress is shown in Fig 5.3. In (a) the fiber stress is shown for two different fiber lengths; one with length \(50r\) (fiber radius) and one with \(5r\). The longer fiber reaches the plateau value while the other is too short for the stress to be entirely transferred to the fiber. This fact can be supported by Fig 5.3 (b) where the shear stress for the short fiber is not stable at zero stress at any position in the fiber. This is however not the case for the longer fiber where the shear stress levels out at zero stress. Finally, it can be seen that the shear stress is as largest at the fiber ends.

If it is assumed that load actually is transferred across the fiber end (which it normally is for a fiber embedded in a matrix) then the axial fiber stress distribution is like in Fig 5.4 (as modified shear lag). It can be seen that the stress is non-zero at the fiber end. This variation of Cox’s model is well suited for the fiber in Fig 5.1 since the fiber end is embedded in the matrix.
5.1.3 The partially elastic model (Piggott’s model)

The elastic model (Cox’s model) is in most cases not realistic. This is because the matrix at the fiber ends behaves in a non-elastic manner due to the high shear stresses that are acting there (cf. Fig 5.1). The high shear stresses cause matrix yielding or debonding. Piggott [2] included this non-linearity in his model.

5.1.4 Shear lag model in single fiber fragmentation test

In the single fiber fragmentation test (SFFT) (introduced by Kelly and Tyson [8]) a single fiber is embedded in a matrix as shown in Fig 5.5. There are two main phenomena in SFFT due to existing fiber cracks. The first phenomenon is debonding between the fiber and the matrix. This happens if the interface is weaker than the matrix. In this case the fiber fragment ends will slip and the shear stresses in this region are transmitted by friction. The second phenomenon happens when the interface is stronger than the matrix. Then the fiber crack instead creates a matrix crack which normally is a conical crack, or a combination of a conical and a flat crack.

The elastic conditions in SFFT are the same as for a single fiber in a UD fiber composite. This means that the expression for the stress distribution of the axial fiber stress is the same in these two cases (and the shear stress as well). If for example Cox’s model is used then (27) describes the stress distribution in SFFT. There is however one difference between these two cases and it is in the geometry. The fiber in SFFT has no neighbouring fibers and the fiber volume fraction is thus as good as zero. This is in contrast to the fiber in a UD composite which has neighbouring fibers and a fiber volume fraction far from zero. This difference in geometry gives large differences in the ratio $R/r$ (needed in (28)) and
thus numerical differences in the axial fiber stress and the shear stress. For details on single fiber fragmentation tests see for example [8] and [9].

![Figure 5.5. Specimen used in single fiber fragmentation test (SFFT).](image)

5.2 Variational mechanics model
The distribution of the axial fiber stress and the shear stress at the interface in SFFT according to Nairn [10] are

\[ \sigma_f = \psi_0(\rho)(1 - \phi) \quad (30) \]

and

\[ \tau = \frac{\xi}{2} \frac{d\sigma_f}{dx} \quad (31) \]

where

\[ \psi_0(\rho) = -\frac{C_{15}\sigma_0(\rho) + C_{36}\sigma_w + D_T}{C_{33}} \quad (32) \]

and

\[ \phi = \left( \frac{\beta \cosh \alpha \xi - \alpha \cosh \beta \xi}{\sinh \alpha \rho - \sinh \beta \rho} \right) \left( \frac{1}{\beta \coth \alpha \rho - \alpha \coth \beta \rho} \right) \quad (33) \]

where \( \rho = l/2, \xi = r, \) and \( \alpha, \beta \) are indirect functions of \( C_{ij} \) which in its turn is a function of the elastic properties of fiber and matrix. (The third term \( (D_T) \) in (32) describes thermal effects.) Note that (31) is the same as (13). The stress distribution (30) is established using an energy approach (the principle of minimum complementary energy). For a complete description of the expressions (30) – (33) see [10].

5.3 Numerical approaches
If the analytical solution to a problem is known then there is no need to use numerical methods (if not for verification). Since an analytical solution is unknown for most of the problems the utilization of a numerical method is necessary. If there is information available about the solution (but not the analytical solution) then boundary element modelling (BEM) can be used. Other methods similar to BEM are the finite difference method (FDM) and the finite volume method (FVM). The utilization of FDM and FVM in solid physics has decreased over the years in contrast to BEM that has increased in use. If there is no information available about the solution (or insufficient information) then finite element modelling (FEM) can be used.
5.3.1 Boundary element method (BEM)
BEM is a numerical method that only needs elements for the boundary in contrast to FEM which need elements for the whole domain. This means that fewer elements are needed in BEM and thus the computational time is much shorter. Other advantages are that infinite and semi-infinite domains can be treated and the accuracy in problems involving stress concentrations is higher (compared to FEM). The major disadvantage is that a so called fundamental solution is needed. Another disadvantage is the difficulty in treatment of inhomogeneous and non-linear problems. For fundamentals of BEM see for example [11-13]. BEM has been used to analyze the single fiber fragmentation test in for example [14] and [15].

5.3.2 Finite element method (FEM)
If no information about the solution is available (or insufficient information) then finite element modelling (FEM) can be used. This is the advantage with FEM. Since the boundary as well as the interior is used in FEM the disadvantage is consequently the computational time which can be quite long for problems where many elements are needed.

6. Objectives

The objectives of the work presented in the papers are:

- To generalize a nonlinear viscoelastic viscoplastic model of a Hemp/Lignin composite by including its stiffness reduction and thus the degree of microdamage in the composite (Paper A).

- To describe and simulate the stiffness reduction of a unidirectional (UD) composite containing fiber breaks with partial interface debonding (Paper B).

- To find an expression for the average crack opening displacement (COD) (which is needed to simulate the stiffness reduction) in the UD composite when the fiber cracks are non-interacting (Paper B).

- To find an expression for the average crack opening displacement (COD) when the fiber cracks are interacting (Paper C).

- To find an expression for the strain energy release rate related to the debond crack growth along the broken fibers in the UD composite (Paper D).
7. Summary of appended papers

Paper A

*Nonlinear Viscoelastic Viscoplastic Material Model Including Stiffness Degradation for Hemp/Lignin Composites*

A nonlinear viscoelastic viscoplastic model of a Hemp/Lignin composite is generalized by including stiffness reduction, and thus the degree of microdamage, in the composite (when loaded in tension in the axial direction). The stiffness reduction was attained by repeatedly apply a load-unload ramp to the specimen to introduce damage, followed by low stress load-unload ramp to measure the elastic modulus and after each cycle increase the maximum strain. Some of these stress load-unload ramps are shown in Fig A.1(a) and the resulting stiffness reduction $E_x/E_0$ is shown in Fig A.1 (b).

![Tensile stress-strain curves (a) and resulting stiffness reduction with increasing strain for three specimens from tensile tests and regression line that determines the function $d(\varepsilon_{\text{max}})$](b) (b).

Further, schapery’s model is used to model the nonlinear viscoelasticity whereas the viscoplastic strain is described by a nonlinear function presented by Zapas and Crissman. In order to include stiffness reduction due to damage, Schapery’s model is modified by incorporating a maximum strain-state dependent function $d(\varepsilon_{\text{max}})$ reflecting the elastic modulus reduction with increasing strain measured in tensile tests. This function $d(\varepsilon_{\text{max}})$ is determined by the regression line in Fig A.1(b) according to

$$d(\varepsilon_{\text{max}}) = \begin{cases} 1 & \text{never loaded above 0.3 \%} \\ \frac{1}{1 - 0.116\varepsilon_{\text{max}} + 1.033} & \text{otherwise} \end{cases} \quad (A1)$$
and modifies the general nonlinear constitutive equation of viscoelasticity and viscoplasticity in the case of uniaxial loading according to

\[
\epsilon = d(\epsilon_{\text{max}}) \left( \epsilon_0 + g_1 \Delta S (\psi - \psi') \frac{d(g_2 \sigma)}{d\tau} d\tau + \epsilon_{\text{pl}}(t, \sigma) \right)
\]

(A2)

The model successfully describes the main features for the investigated material and shows good accuracy within the considered stress range. This is supported by Fig A.2 where experimental values are plotted with model prediction.

![Figure A.2. Strain response to a linear loading, unloading, and loading ramp, model prediction (solid line) and experimental values for two specimens (dots).](image)

**Paper B**

*Modeling UD composite stiffness reduction due to multiple fiber breaks and interface debonding*

The stiffness reduction of a unidirectional (UD) composite containing fiber breaks with partial interface debonding is analyzed. The analysis is performed by studying how the average crack opening displacement (COD) depends on fiber and matrix properties, fiber content and debond length. The COD is normalized with respect to the size of the fiber crack and to the far field stress in the fiber. In contrast to other performed analysis an analytical relationship is developed which links the entire stiffness matrix ([Q]_{RVE}) of the damaged UD composite with the COD \( (\epsilon_{1an}) \) and the crack sliding displacement (CSD \( (\epsilon_{2an}) \)) according to (B1) and (B2).

\[
[Q]_{RVE} = \left[ [I] + V_f \rho \frac{E_f}{E_m} [Q]_f [U]_f [Q]_f [H]_f^T [S]^{RVE}_{\text{RVE}} \right]^{-1} [Q]_0^{RVE}
\]

(B1)
$$\begin{bmatrix} U \end{bmatrix}_f = \begin{bmatrix} u_{1,an} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & u_{2,an} \frac{E_{\mu}}{G_{\mu,T}} \end{bmatrix}$$ \hspace{1cm} (B2)$$

However, the CSD is excluded from the analysis since it is found by parametric inspection that it does not affect the longitudinal stiffness. Some trends regarding the COD dependence on the different properties can be extracted from available approximate analytical stress transfer models. To obtain more reliable results, in the current analysis these dependences are extracted from extensive FEM based parametric analysis performed on a model consisting of three concentric cylinders: a) broken fiber; b) matrix cylinder around it; c) large effective composite cylinder surrounding them. This model is used since it is more adequate than unit cell models considering only fiber and matrix. The cracks, which are only in the fibers, are distributed in such a way that they are non-interactive.

It is shown that the parameters that affect the COD the most are the ratio of the longitudinal fiber modulus and matrix modulus, the fiber content and the debond length. These relationships are described by a simple fitting function (B3) which excellently fit the numerical results. These simple functions are merged into one relationship (B3) describing the COD’s dependence on the relevant parameters.

$$u_{1,an} = 1.5u_{1,an}^{b} + \frac{l_{ap}}{r_{f}}$$ \hspace{1cm} (B3)

Relation (B3) expresses the COD in the case of debonding and depends on the COD for the bonded case (B4).

$$u_{1,an}^{b} = A \left( \frac{E_{\mu}}{E_{m}} \right)^n$$ \hspace{1cm} (B4)

Simulations performed for carbon and glass fiber polymer composites show that the relative longitudinal stiffness reduction in the carbon fiber composite is slightly larger than in the glass fiber composite (see Fig B.1.)
Figure B.1. Longitudinal modulus reduction in UD composite in a normalized form as a function of the number of fiber breaks in one fiber: a) CF/EP composite, b) GF/EP composite.

This trend holds for all considered debond lengths and is related to higher longitudinal fiber and matrix modulus ratio in the carbon fiber composite leading to larger crack openings and larger stress perturbation zones. It is shown that the stiffness reduction depends on the debond length.

**Paper C**

*Modeling fiber crack opening displacement in UD composites with partially debonded fibers*

The analysis performed in Paper B is continued by studying how the COD is affected when the cracks are interactive. It is shown that the effect on the COD in the glass fiber composite is negligible (Fig C.1(b)). However, the effect on the COD in the carbon fiber composite is significant (Fig C.1(a)).
Figure C.1. The NACOD (= $u_{1am}$) versus fiber length for the carbon fiber composite (a) and the glass fiber composite (b) for different debond lengths.

This difference in behaviour is related to how the axial fiber stress distribution responds to crack interactions (Fig C.2). It can be seen in Fig C.2 that the axial fiber stress distribution of the carbon fiber composite strongly responds to the crack distance while the stress glass fiber composite is hardly affected. The difference in stress distributions are related to the longitudinal fiber and matrix modulus ratio. The higher the ratio is, the stronger the stress distribution depends on the crack distance. The stress in the carbon fiber case has a stronger dependence on the crack distance since carbon fiber has a much larger longitudinal elastic modulus than glass fiber.

![Figure C.2. Axial fiber stress in the carbon fiber composite (a) and the glass fiber composite (b) for different fiber lengths.](image)

Finally, it is demonstrated using a simple shear lag model that the qualitative trends of $u_{1am}$ dependence on geometrical and material parameters can be described fairly well whereas numerical values have 20-40% error.

**Paper D**

*Modeling energy release rate for debond crack growth along fiber in UD composites with broken fibers*

The same model (as in Paper B and C) is used to analyse the strain energy release rate related to the debond crack growth along the fiber. The energy release rate is calculated using the virtual crack closure technique applied to displacement and stress field in the vicinity of the debond crack tip. The displacement and the stress field are calculated using refined FE model. It is shown that the energy release
rate is larger for very short debonds. It reduces to a constant value indicating a stable debond crack growth after its initiation (Fig D.1).

![Graph](a)

![Graph](b)

Figure D.1. Strain energy release rate $G_{II}$ versus normalized debond length $l_d / r_f$ in composites with a fiber volume fraction of 45% and 55% for carbon/epoxy (a) and glass/epoxy (b).

It is shown that the strain energy release rate in the plateau region also can be calculated using a simple analytical model based on the self-similar crack growth assumption. When the stress state perturbations related to debonds at both fiber ends start to interact, the energy release rate decreases (Fig D.2).

![Graph](a)

![Graph](b)

Figure D.2. Strain energy release rate $G_{II}$ for debond growth in the carbon/epoxy composite (a) and glass/epoxy (b) versus normalized fiber length $2L_f / r_f$ for different debond lengths.

Further, the strain energy release rate decreases with increasing debond length (Fig D.3). As can be seen in Fig D.2 and D.3 the carbon/epoxy composite is affected more than the glass/epoxy composite. This is related to respective axial fiber stress distribution and respective longitudinal fiber and matrix modulus ratio in the same way as in Paper C.
Figure D.3. The interaction effect on strain energy release rate $G_{II}$ for debond growth in the carbon/epoxy composite (a) and glass/epoxy (b) versus normalized debond length $l_d/r_f$ for different fiber lengths.

8. Future work

In a future work the obtained relationships for the energy release rate will be incorporated in a microdamage evolution model describing the statistics of fiber breaks and debond growth in fatigue loading conditions.

9. References


Paper A
Nonlinear Viscoelastic Viscoplastic Material Model Including Stiffness Degradation for Hemp/Lignin Composites

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Abstract
In repeating tensile tests with increasing maximum strain for every loading cycle the hemp/lignin composites clearly showed a nonlinear behavior and hysteresis loops in loading and unloading. The explanation for this behavior is the inherent viscoelastic nature for this type of material, but also noticeable stiffness degradation with increasing strain level. Creep tests performed at different stress levels revealed a nonlinear viscoelastic response and after recovery viscoplastic strain was detected for high stress levels. Schapery’s model has been used to model nonlinear viscoelasticity whereas viscoplastic strain is described by a nonlinear function presented by Zapas and Crissman. In a creep test this function leads to a power law with respect to time and stress.

In order to include stiffness reduction due to damage Schapery’s model has been modified by incorporating a maximum strain-state dependent function reflecting the elastic modulus reduction with increasing strain measured in tensile tests. A generalized incremental model of the constitutive equation for viscoelastic case has been used to validate the developed material model in a linear stress controlled loading and unloading ramp. The model successfully describes the main features for the investigated material and shows good accuracy within the considered stress range.

Keywords: Natural fiber composite, NFC, hemp, lignin, nonlinear viscoelasticity, creep, viscoplasticity, recovery, stiffness degradation, damage, load ramp

1. Introduction
Plant fibers as reinforcement in composites have received much attention over the recent years [1-4]. Natural fibers, such as hemp, flax, jute and sisal are renewable and biodegradable cellulosic materials which offer relatively good specific mechanical properties. Drawbacks are high moisture absorption and poor adhesion with non-polar polymer matrices. Natural fiber composites with thermoplastic matrix are extensively used already today in the automotive industry where the fiber acts mainly as filler material in non-structural interior panels. Natural fiber composites used for structural purposes do exist, but then usually with oil-based synthetic thermoset matrices which of course limits the...
environmental benefits. The future goal will be to develop environmental friendly high performance composites which are recyclable and come from renewable resources. A complete biodegradable system may be obtained if the matrix material also comes from a renewable resource. Examples of such materials are lignophenolics, starch and polylactic acid (PLA). Several studies suggest that there are biodegradable composite systems which indeed show promising results. Oksman et al. [5] have reported that flax fiber composites with PLA matrix can compete with and even outperform flax/polypropylene composites in terms of mechanical properties.

Lignin is a complex non-crystalline aromatic macromolecule readily extracted in vast quantities from the paper industry. Studies have suggested that it is possible to replace part of phenol by lignin in phenolic thermoset matrices without loss of mechanical properties [6]. However, there seem to be little work done on mechanical performance of the hemp/lignin composite system in particular.

A very important feature of natural fiber composites is that the mechanical properties of both fibers and the polymer matrix are time dependent. Therefore, natural fiber composites experience complex time dependent stress-strain behavior with loading rate effects and hysteresis loops. This behavior is due to viscoelastic effects of both constituents and may also include micro damage evolution resulting in stiffness degradation and development of irreversible viscoplastic strains. Mechanical properties of natural fiber composites have been studied previously by several authors. For example, composites made from wood fibers and thermoset matrices have been studied [7,8]. In [9] flax/polypropylene composites were characterized in terms of viscoelastic behavior.

In the present study repeating tensile tests with increasing maximum strain for every loading cycle of hemp/lignin composites showed that the elastic modulus was reduced which indicates damage accumulation. High stress creep tests gave a nonlinear viscoelastic response and irreversible strains could be measured after recovery. Hence, the composite has to be described as a nonlinear viscoelastic viscoplastic material that also experience stiffness degradation. A maximum strain-state dependent function reflecting the elastic modulus reduction with increasing strain measured in tensile tests will be incorporated in the material model.

A general thermodynamically consistent theory of nonlinear viscoelastic and nonlinear viscoplastic materials was developed by Schapery [10] and it has been used in simulations by several authors [11,12]. The model contains three stress dependent functions which characterize the nonlinearity (actually they also depend on temperature and humidity). In the case of linear viscoelasticity and in
fixed conditions these stress dependent functions are equal to 1 and the data reduction scheme is significantly simplified. A methodology to determine the nonlinearity parameters for materials which obey power law time dependence was described by Lou and Schapery [13]. However, a better fit to experimental data is often achieved if the viscoelastic creep compliance in form of Prony series is used instead [9,14]. The optimal set of experiments needed to determine the stress dependent functions in the material model and development of reliable methodology for data reduction is still an issue for debate. The representation of the viscoplastic term which is used in the following work is described by a nonlinear function presented by Zapas and Crissman [15].

The objectives of the presented paper are (i) to determine the hemp/lignin composites stiffness dependence on maximum strain in tensile tests, (ii) to analyze the viscoelastic and viscoplastic properties of this material, (iii) to present a nonlinear viscoelastic viscoplastic material model including stiffness degradation, (iv) to validate the developed model in a linear stress controlled loading and unloading ramp using an incremental form of the constitutive equation. It will be shown that the material can be characterized using only a few specimens and that the stress dependent nonlinearity functions may be expressed via simple polynomial functions.

2. Theory

2.1. Constitutive model

Lou and Schapery [13] presented a general nonlinear constitutive equation of viscoelasticity in the case of uniaxial loading. The same constitutive equation with an additional term for viscoplastic strain accumulation \( \varepsilon_{pl}(t, \sigma) \) was used in [9] where flax/polypropylene composites were characterized using different forms of the viscoelastic creep compliance. No stiffness degradation was detected and thus nonlinear viscoelasticity and viscoplasticity were the mechanisms responsible for the observed behavior.

In the present study however, tensile tests revealed that the hemp/lignin specimens do indeed experience stiffness degradation. The constitutive equation in this case has therefore been slightly modified by incorporation of a maximum strain-state dependent function \( d(\varepsilon_{max}) \) which reflects the elastic modulus reduction with increasing strain.

\[
\varepsilon = d(\varepsilon_{max}) \left( \varepsilon_0 + g_1 \int_0^\tau \Delta S(\psi - \psi') \frac{d(g_2(\sigma))}{d\tau} d\tau + \varepsilon_{pl}(t, \sigma) \right)
\] (1)
In Eq. (1) integration is over “reduced time” according to,

\[ \psi = \int_{0}^{t} \frac{dt'}{a_{\sigma}} \quad \text{and} \quad \psi' = \int_{0}^{t} \frac{dt'}{a_{\sigma}} \]  \hspace{1cm} (2)

\( \varepsilon_0 \) represents the initial strain which may be nonlinear with respect to stress. \( \Delta S(\psi) \) is the transient component of the linear viscoelastic creep compliance. \( g_1 \) and \( g_2 \) are stress dependent material properties. \( a_{\sigma} \) is the shift factor, which in fixed conditions is a function of stress only. For sufficiently small stresses \( g_1 = g_2 = a_{\sigma} = 1 \), and thus Eq. (1) turns into the strain-stress relationship for linear viscoelastic viscoplastic materials. In the thermodynamic analysis presented by Schapery [10] it was shown that the viscoelastic creep compliance does not depend on the applied stress level and it may therefore be determined using loads in the linear region. Viscoelastic creep compliance in the form of Prony series was obtained,

\[ \Delta S(\psi) = \sum_{m} C_m \left( 1 - \exp \left( -\frac{\psi}{\tau_m} \right) \right) \]  \hspace{1cm} (3)

\( C_m \) are constants and \( \tau_m \) are called retardation times. The retardation times are chosen arbitrary, but the highest \( \tau_m \) should at least cover the time for the conducted creep test. A good approximation to experimental data may be achieved if the retardation times are spread uniformly over the logarithmic time scale, typically with a factor of ten between them.

In a creep test the stress is constant until some time instant \( t_1 \) whereby the stress is removed and the recovery period begins according to \( \sigma = \sigma[H(t) - H(t-t_1)] \). \( H(t) \) is the Heaviside step function. The expression (1) may therefore be divided into creep strain and recovery strain in a creep test. Together with the creep compliance from Eq. (3) we obtain the following form of creep strain and recovery strain respectively:

\[ \varepsilon_c = d(\varepsilon_{\max}) \left( \varepsilon_0 + g_1 g_2 \sigma \sum_{m} C_m \left( 1 - \exp \left( -\frac{t}{a_{\sigma} \tau_m} \right) \right) + \varepsilon_p(t, \sigma) \right) \]  \hspace{1cm} (4)
\[ \varepsilon_r = d\varepsilon_{\text{max}} \cdot \left( g_2 \sigma \sum_m C_m \left( 1 - \exp \left( -\frac{t_{1t}}{a_1 \tau_m} \right) \right) \exp \left( -\frac{t - t_{1t}}{\tau_m} \right) + \varepsilon_{\text{pl}}(t_1, \sigma) \right) \] (5)

### 2.2. Viscoplastic strain

The viscoplastic strain is expressed via a nonlinear function presented by Zapas and Crissman [15] according to,

\[ \varepsilon_{\text{pl}} = C_{\text{pl}} \left\{ \int_0^t \sigma(\tau)^M d\tau \right\}^m \] (6)

\( C_{\text{pl}}, M \) and \( m \) are constants and must be determined experimentally. The following discussion is a short summary of the procedure needed for the parameter identification which is described in detail in [9,14]. First the \textit{time dependence} of viscoplastic strains is determined by performing creep tests at a fixed stress level so that integration of Eq. (6) is trivial. After strain recovery the remaining irreversible strain corresponding to the loading period is measured. Several creep tests with different lengths are performed and the developed viscoplastic strains are summed. The viscoplastic strain after \( k \) steps of creep loading at the \textit{same stress level} \( \sigma_0 \) will be,

\[ \varepsilon_{\text{pl}}^{1+2+\ldots+k} = C_{\text{pl}} \sigma_0^M (t_1 + t_2 + \ldots + t_k)^m \] (7)

The development of viscoplastic strains at fixed stress should thus follow a power law in time with coefficient \( B = C_{\text{pl}} \sigma_0^M m \) and constant \( m \) which are determined as the best fit in logarithmic axes. Furthermore, only one specimen is necessary to obtain the time dependence of viscoplastic strains at a certain fixed stress level.

The \textit{stress dependence} of viscoplastic strains has to be obtained performing creep tests of the same length at several stress levels. In result \( B \) is obtained as a function of stress and the best fit in logarithmic axes yields the required constants \( C_{\text{pl}} \) and \( M \).

### 2.3. Incremental form of the constitutive equation

In structural analysis with nonuniform and complex stress state the material model has to be implemented in FE codes which require an incremental form of Eq. (1). In present case the material model will be validated in a linear loading and unloading ramp.
Substitution of Eq. (3) in (1) and integration gives,

\[ ε(t) = d(ε_{\text{max}}) \cdot \left( ε_0(σ) + g_1(σ)g_2(σ)σ \sum_{m} C_m - g_1(σ) \sum_{m} ε^m(ψ) + ε_{\text{pl}}(σ, t) \right) \]  

where

\[ ε^m = \int_{0}^{ψ} C_m e^{-\frac{ψ-ψ^0}{τ_m}} \frac{d\varepsilon^m(σ)}{dψ} dψ \]  

The integral in Eq. (9) may be calculated in time instant \( t_k \) using the previously determined value at time \( t_{k-1} \) where \( t_k = t_{k-1} + Δt \). Equation (2) gives the relation between the time increment and \( Δψ \) according to,

\[ Δψ = \frac{1}{a_σ} Δt \quad \text{and} \quad ψ_{k+1} = ψ_k + Δψ \]

The recursive expression for Eq. (9) becomes,

\[ ε^m(t_k) = e^{\frac{Δψ}{τ_m}} ε^m(t_{k-1}) + C_m \left( 1 - e^{\frac{Δψ}{τ_m}} \right) R(t_{k-1})τ_m \]

In (11)

\[ R(t_{k-1}) = \left. \frac{d[g_2(σ)]}{dσ} \right|_{σ_{t-1}} \cdot \frac{1}{a_σ} \left. \frac{dσ}{dt} \right|_t \]

The routine for simulation of \( ε(t) \) is as follows; (i) choose the time increment \( Δt \) (preferably small and constant); (ii) for the time instant \( t_{k-1} \) find \( Δψ \), \( R(t_{k-1}) \) and calculate all \( ε^m(t_k) \) using \( ε^m(t_{k-1}) \) from the previous step (they are zero in the 0-step); (iii) now use Eq. (8) to calculate \( ε(t) \) and then repeat the steps.

3. Experimental
Each specimen was a 150x10x4 mm dog-bone shaped specimen. The specimen designation was H302030 where H stands for hemp and the numbers stands for weight fraction of fibers (30%), weight fraction of flame retardant (20%) and finally the weight fraction of plasticizer (30%). The matrix is lignin (20%). The
composite material studied herein is still in an early development stage and we are thus prevented to further discuss any information about constituent morphology. The material was chosen because at this stage it had the features that we wished to describe with our model regarding viscoelasticity, viscoplasticity and stiffness degradation.

The reduction in elastic modulus was established using a universal Instron 4411 tensile testing machine. Load was measured by a standard 5 kN load cell and strain was measured by an Instron 2630-100 series extensometer with 50 mm gauge length. Cross-head speed was set to 5mm/min. The recorded data was load (kN) and displacement (mm). All tests were performed at RT and with relative humidity of 25 – 35%. The approach to get the stiffness reduction was to repeatedly apply a load-unload ramp to introduce damage followed by low stress load-unload ramp to measure the elastic modulus and after each cycle increase the maximum strain. Due to viscoelastic effects the specimens had to recover after introducing damage for some short period of time before the elastic modulus was measured. The specimens were not removed from the grips during the recovery. The first maximum strain was 0.2% and the following maximum strains were increments of approximately 0.1% until failure. The non-damaging stiffness determination ramps were loaded to 0.2% in strain.

In the data reduction the elastic modulus, i.e. the slope of the stress-strain curve, was estimated between 0.05 and 0.15% in strain. Due to viscoelastic and viscoplastic effects the load-displacement curves were slightly shifted to higher strain during the test. This made it necessary to translate the used strain interval (0.05-0.15%) for the first stress-strain curve to corresponding stress interval. This stress interval was then used for the rest of the curves to establish each slope. The sampling rate was 20 points per second which gave about 25 points for establishing the modulus. The modulus was calculated for the loading part as well as for the unloading part of the curve and then taken as the average. A few examples of stress-strain curves are shown in Figure 1.
The creep tests were performed by hanging of weights which results in tensile loading applied to the specimens. The strain was measured with standard extensometers. One specimen was first subjected to 10, 20 and 30 min of creep loading at 9 MPa with following recovery in order to estimate the viscoplastic time dependence at a fixed stress level. The viscoelastic properties were later determined from the same specimen by subject it to 1 hour of creep loading at stress levels 9, 6 and 3 MPa respectively. Another specimen was subjected to 10, 20 and 30 min of creep loading at 6 MPa for estimation of the viscoplastic stress dependence. A small degradation in stiffness was detected afterwards which indicates that viscoplastic strains due to damage might have developed. Unfortunately we were unable to measure these very small viscoplastic strains with sufficient accuracy. The limit for viscoplastic strain development was therefore set to 6 MPa. The same specimen was also tested in creep at 10 MPa (10, 20 and 30 min). A third specimen was tested in creep at 11 MPa and experienced creep rupture after less than 8 min of loading. The fracture surface revealed a dry region with many voids.

4. Results and discussion

4.1. Stiffness degradation
The procedure to get the reduced modulus has been described in the experimental part. The initial modulus $E_0$ is determined from the first loading cycle corresponding to 0.20% in strain. Several tests revealed that the initial modulus
(i.e. Young’s modulus) was in the range 2.3 – 2.8 GPa. Tensile strength was 14 – 16 MPa. Figure 2 shows the reduced modulus normalized with respect to the initial modulus for strains up to 0.9%. No stiffness degradation could be seen for strain values lower than 0.3% and consequently this strain level was therefore set as limit for stiffness reduction. At a strain level of 0.9% the stiffness reduction was roughly 7-8%.

\[
\frac{E_x}{E_0} = -0.116\epsilon + 1.033
\]

Figure 2. Stiffness reduction with increasing strain for three specimens from tensile tests and regression line that determines the function \(d(\epsilon_{\text{max}})\).

The regression line in Figure 2 determines the maximum strain-state dependent function,

\[
d(\epsilon_{\text{max}}) = \begin{cases} 
1 & \text{never loaded above 0.3 \%} \\
\frac{1}{-0.116\epsilon_{\text{max}} + 1.033} & \text{otherwise}
\end{cases} 
\]

(13)

The stiffness degradation is manifested through micro damage evolution. In Figure 3 in situ micrographs of poorly impregnated regions in one of the tested hemp/lignin specimens are shown. All cracks are visible independently of the magnitude of the load, which direction is horizontally. The micrographs give an understanding of the condition of the surface. There are many small randomly oriented surface cracks (a). Their contribution to the stiffness reduction is most likely negligible. The large crack (b) opens more if the specimen is loaded (than if
not loaded) which means that it is more a volume crack than a surface crack. This motivates that the large crack is one of the contributors to the stiffness reduction.

Figure 3. An in situ hemp/lignin specimen.

4.2. Viscoplastic strain
The following analysis of viscoplastic properties for the hemp/lignin composites is limited to tension case. The behavior in compression can certainly be different. The viscoplastic strains were measured as the remaining recovery strains after 6 times the loading period in creep. The viscoplastic strains accumulated in all steps were summed and plotted against time in log-log scale according to the description in section 2.2. The slope of the trend line was straight which means that the development of viscoplastic strains at fixed stress indeed follows a power law with high accuracy. However, the power law assumption in Eq. (7) states that the exponent \( m \) is constant for all stress levels and certainly that was not true in the present case. At 10 MPa the exponent is higher than at 9 MPa. In light of this result the decision was to use an average value of \( m \) in the forthcoming calculations. Experimental results and model predictions for viscoplastic time dependence at fixed stress using an averaged \( m \) value is shown in Figure 4. The corresponding values of \( B \) are also shown.
Figure 4. The development of viscoplastic strains at fixed stress levels: Experimental data at 9 MPa (♦), 10 MPa (●) creep tests and model predictions using the averaged $m$ value (×) for both stress levels compared to “true” model predictions (solid lines).

The stress dependence for development of viscoplastic strains was obtained by assuming a strain value of 0.001% at 6 MPa in order to have three values of $B$ (remember that we were unable to measure the very small viscoplastic strains at this stress level and at 11 MPa the specimen experienced creep rupture). Consequently, values $M = 20.7$ and $C_{pl} = 1.65 \times 10^{-13}$ (for time in s, stress in MPa and strain in %) were determined.

4.3. Nonlinear viscoelastic model with Prony series

The following analysis of viscoelastic properties for the hemp/lignin composites is limited to tension case. Only one specimen has been used to analyze the viscoelasticity which is a preferable strategy since the data reduction procedure otherwise easily becomes both tedious and impractical and may contain some artificial trends when averages are used.

After recovery in the 9 MPa creep test a small irreversible strain was detected. Thus viscoplastic strains are developing during the test and the creep strain is a sum of viscoelastic and viscoplastic strains. In order to obtain a pure nonlinear viscoelastic response the viscoplastic strains that develop must therefore be subtracted from experimental data. Following the work in [14] the development of viscoplastic strains in the current creep test is expressed by,
\[ \varepsilon_{pl}(t) = \left[ \left( \varepsilon_{pl}^{1+2+\ldots+k} \right)^{1/m} \frac{t}{t_k} + \left( \varepsilon_{pl}^{1+2+\ldots+(k-1)} \right)^{1/m} \left( 1 - \frac{t}{t_k} \right) \right]^{m} - \varepsilon_{pl}^{1+2+\ldots+(k-1)} \]  \hspace{1cm} (14) 

\( t_k \) is the length of the creep period in question. \( m = 0.49 \) was determined in the previous section. The specimen had experienced creep loading three times at 9 MPa in the viscoplastic characterization procedure earlier with a total duration of one hour so \( \varepsilon_{pl}^{1+2+3} = 0.043\% \) was known from the previous creep tests. After the 9 MPa creep test in the viscoelastic characterization procedure \( \varepsilon_{pl}^{1+2+3+4} = 0.055\% \) was measured and Eq. (14) could be used to calculate the pure nonlinear viscoelastic response for this stress level. No viscoplastic strains developed during the 6- and 3 MPa tests. Tensile tests before and after the viscoplastic characterization procedure showed that the stiffness of the specimen was reduced by approximately 8\% and measurements after the 9-, 6- and 3 MPa creep tests showed no further stiffness degradation. Thus \( d(\varepsilon_{\text{max}}) = 1.08 \) was used in the following calculations.

The first step in the development of the nonlinear viscoelastic model is to determine the parameters in the linear viscoelastic range (i.e. determine \( \tau_m \) and \( C_m \)). The retardation times \( \tau_m \) were chosen to be uniformly spread over the logarithmic time scale and with the highest value covering the time for the creep tests in question. The coefficients \( C_m \) were determined from experimental recovery data for the 3 MPa creep test with the assumption that we are within the region of linear response (all stress dependent nonlinearity functions are thus equal to 1) and the result is shown in Table 1. This procedure, and also the forthcoming calculations of the stress dependent nonlinearity functions were performed using the method of least squares written in MATLAB code.

<table>
<thead>
<tr>
<th>Table 1. Coefficients in Prony series.</th>
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<tr>
<td>m</td>
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Next step will be to determine the stress dependent nonlinearity functions from recovery and creep data and the methodology will be the same for all stress levels:
$a_\sigma$ is altered in the recovery strain expression (5) in time interval $t_1 < t < 2t_1$ and for every value of $a_\sigma$ we obtain the corresponding value of $g_2$ directly via method of least squares. This procedure is continued until we find the value of $a_\sigma$ which gives a nice visual fit to experimental data. Equation (4) is then used to find $\varepsilon_0$ and $g_1 \cdot g_2$ from experimental creep data for the same stress level. When all the nonlinearity parameters have been determined for all stress levels they may be approximated with simple analytical functions. Figure 5a) shows the initial strain $\varepsilon_0$ as a function of stress and b) the nonlinearity values for the creep tests of 3-, 6- and 9 MPa and their approximations as function of stress.
In order to check whether the approximations lead to acceptable results the model is compared to experimental creep and recovery data as shown in Figure 6. Clearly, the accuracy of the model based on Prony series and polynomial approximation of the stress dependent functions is good for all stress levels in the considered stress range.

The developed model may be used to predict the nonlinear viscoelastic behavior for this type of composite (with similar properties) for any stress from 0-9 MPa. However, we must emphasize that all the viscoelastic time dependences have been obtained from creep tests on one single specimen and so far the model is applied to that particular specimen. It is therefore crucial for the characterization that the specimen is representative for the analyzed material. The specimen was singled out on the basis that its elastic properties were intermediate in this group of specimens. The accuracy of the developed model is best understood comparing the simulated creep curves with other specimens not used in the data reduction and it has been checked that the variation of properties between different specimens is larger than the small deviation shown in Figure 6a).

**4.3. Model validation in linear loading and unloading ramp**

The developed viscoelastic viscoplastic material model including stiffness degradation was used to simulate the composite behavior in a linear loading and unloading scheme: constant load rate of 0.01 MPa/s up to 9 MPa, unloading with 0.01 MPa/s and finally loading with 0.02 MPa/s up to 13 MPa according to the solid line in Figure 7. The dashed line in Figure 7 shows the shift in time which is necessary when the viscoplastic strains are calculated. According to Eq. (6) the integration is over a continuous function from time \( t = 0 \). However, in this case we have no viscoplastic strains developing for stresses lower than 6 MPa.
(between $0 < t < t_1$ and $t_3 < t < t_4$). The stress as a function of time (with time coordinate $t'$) for the different parts of the loading ramp which needs to be integrated is also shown in Figure 7.

Figure 7. Linear loading and unloading ramps and the required time shift for viscoplastic strain calculation.

The incremental form of the constitutive equation was programmed using MATLAB code following the routine described in section 2.3. A time step of 0.1 sec was used. Smaller steps were not considered since the difference in result from using a time step of 1 sec was less than 1%. The elastic (initial) strain, the viscoelastic- and viscoplastic strains are all calculated separately and then added. Finally the total strain due to stiffness degradation is calculated with the requirement that $d(\varepsilon_{\text{max}}) = 1$ before $\varepsilon(t)$ passes 0.3% the first time and then $d(\varepsilon_{\text{max}}) > 1$ following Eq. (13). Since $d(\varepsilon_{\text{max}})$ is a function of the highest previously known strain state it will increase until $t_2$ and then have a constant value of $d(\varepsilon(t_2))$ until next time it reaches a higher strain value (around $t = 2250$ sec and $\sigma = 8.92$ MPa). Figure 8 shows the result of the simulation. The largest part of the time dependent strain is clearly nonlinear viscoelastic. The viscoplasticity and stiffness reduction do not contribute much to the total strain except in the end of the loading ramp where we have high stresses and actually are very close to rupture for these composites.
Figure 8. The contribution of nonlinear elastic strain and nonlinear viscoelastic strain compared to total strain (viscoelastic strain, viscoplastic strain and stiffness reduction) in the linear loading and unloading ramp.

The model and its strain response to the linear loading and unloading ramp was also validated by comparing it to experimental data for two specimens, see Figure 9. The accuracy of the model is remarkably good and it captures most features from the experimental curves except for a small discrepancy in the unloading part. The model also seems to predict the behavior of the composites rather well even for the very high stresses prior to rupture which is surprising since we are well outside the region for which it was designed. Figure 10 on the other hand illustrates the problem that might occur if we apply the model to a specimen tested in creep at 11 MPa. The specimen experienced secondary creep and ruptured after only 8 minutes. Clearly the creep mechanisms are different at this high stress level.
Figure 9. Strain response to the linear loading and unloading ramp, model prediction (solid line) and experimental values for two specimens (dots).

Figure 10. Creep strain at 11 MPa and model prediction.

5. Conclusions
The analysis of time dependent properties for the hemp/lignin composites has been limited to tension case. Creep tests performed on the composites showed that
they may be described as a nonlinear viscoelastic material for stresses higher than 3 MPa. For stresses higher than 6 MPa the material may be described as nonlinear viscoelastic and viscoplastic. The material also showed micro damage evolution which resulted in reduction of elastic modulus for strain levels higher than 0.3%.

The largest part of the time dependent strain is viscoelastic and this material behavior has been modeled using the theory of nonlinear viscoelasticity developed by Schapery. The stress dependent nonlinearity functions in Schapery’s expression was successfully described by simple polynomial functions. Prony series was used to describe viscoelastic creep compliance. The viscoplastic strain was described by a nonlinear function originally presented by Zapas and Crissman. This function contains three constants which must be determined from experiments. It was found that one of these “constants” was in fact not a constant, but rather a function of stress. Therefore, this term was approximated using an average value in the considered stress range. A maximum strain-state dependent function reflecting the elastic modulus reduction with increasing strain has also been incorporated in the material model.

The developed model has been validated in a linear loading and unloading ramp using an incremental form of the constitutive equation. The accuracy of the model is remarkably good within the stress range for which it is designed and captures most features of the compared experimental curves. For very high stresses when we are well outside the model stress range (and actually close to rupture) the model description gets inaccurate.

It was shown that the time dependent properties of the material may be characterized using only a few specimens. It was therefore crucial for the characterization procedure that the specimens were representative for the analyzed material. The specimens used in this study were chosen on the basis that their elastic properties were intermediate in this group of specimens.

6. References


Paper B
Modeling UD composite stiffness reduction due to multiple fiber breaks and interface debonding

Paper COMP07-073

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ABSTRACT

Stiffness of a unidirectional composite (UD) containing fiber breaks with partial debonding is analyzed. Using divergence theorem exact relationships are obtained which link the entire stiffness matrix of the damaged UD composite with two robust parameters from the local solution: average opening displacement (COD) of the fiber break and its sliding displacement (CSD). Both are normalized with respect to the size of the fiber crack and to the far field stress in the fiber. The effect of the partial fiber/matrix debonding at the interface is included in the model through increasing crack opening.

Using parametric inspection of obtained expressions it is shown that CSD does not affect the longitudinal stiffness and the COD effect on transverse and shear modulus and Poisson’s ratio of the damaged composite is negligible. It motivated the decision to focus on COD and the longitudinal modulus. In this paper cracks are considered as non-interactive which makes the stiffness predictions conservative in a large fiber crack density region.

The dependence of COD on fiber and matrix properties, fiber content, debond length etc is described by simple fitting functions obtained from extensive FEM based parametric analysis. FEM analysis is performed using a model which consists of three concentric cylinders: a) broken fiber; b) matrix cylinder; c) large cylinder of effective composite cylinder. The observed trends are described by simple fitting functions which with a high accuracy describe COD’s of perfectly bonded and partially debonded cracks.

Simulations performed for carbon and glass fiber polymer composites show the significance of debond length on the stiffness reduction.
1. Introduction

Multiple fiber fractures are observed in unidirectional (UD) long fiber composites loaded in fiber direction. The number of fiber cracks grows with increasing load and each individual fiber crack can be accompanied by fiber/matrix debonding in the proximity of the crack. An alternative is matrix yielding at the interface in this high shear stress region. The fiber fracture is governed by the statistical nature of the fiber strength distribution and the globally and locally (stress concentrations due to breaks in neighbouring fibers) increasing stress. The debond growth along the interface in fatigue and/or increasing macroscopic loading conditions is most probably governed by fracture mechanics parameters.

In this paper the damage evolution modelling is not considered. Realizing the briefly listed reasons for the existing damage state and leaving the simulation of it for a further study the analysis here is focused on determination of the effect of the damage state on elastic properties of the UD composite.

A large number of research papers have been published on description of the stress transfer from matrix into fiber at the fiber break. Analyses have been analytical, see for example [1-4] as well as numerical [3] with more focus on short fibers or on the description of the single fiber fragmentation test. These studies are relevant to the topic of this paper because the average stress in a fiber which may be calculated from the stress distribution is directly linked to the elastic modulus of the damaged composite.

In contrast to the papers which deal with stress distribution models, in this paper we develop relationships which link the entire stiffness matrix of the damaged UD composite with two robust parameters from the local solution: average opening displacement (COD) of the fiber break and its sliding displacement (CSD) both normalized with respect to the size and to the far field stress in the fiber.

In order to use these expressions we need to know the dependence of COD, CSD on fiber and matrix properties, fiber content, debond length etc. This information can be obtained from available analytical stress transfer models. However in this paper we extract these dependences from FEM parametric analysis performed on a model consisting of three concentric cylinders: a) broken fiber; b) matrix cylinder around it; c) large effective composite cylinder surrounding them. The observed trends are described by simple fitting functions which with a high accuracy describe COD’s of perfectly bonded and partially debonded cracks. The analysis in this paper is limited by opening displacements only and considering cracks as non-interactive which makes the stiffness predictions conservative.
2. Stiffness reduction modelling

The stiffness matrix of the UD composite shown in Fig. 1 is given by the relationship between stresses, \( \{\sigma\}_{RVE} \) applied to the UD composite RVE and the corresponding strain response, \( \{\varepsilon\}_{RVE} \).

\[
\{\sigma\}_{RVE} = [Q(D)]^{RVE} \{\varepsilon\}_{RVE}
\]

(1)

Figure 1. Schematic showing of the UD composite with randomly distributed fiber breaks.

\( D \) represents the damage state with fiber breaks which may be accompanied by fiber/matrix interface debonding. Two types of cylindrical elements may be recognized in the Representative Volume Element (RVE) of a UD composite. The first element group is fibers with orientation which coincides with the global axis of the composite RVE. Obviously in the case of a UD composite local and global coordinate systems coincide. The second is matrix regions which also have cylindrical form. Each element is characterized by its volume fraction in the RVE \( (fV, mV) \), geometry of the element (circular cross-section of the fiber with radius \( r_f \)), constituent stiffness matrices \([Q]_f\) and \([Q]_m\) which are defined by a linear elastic constitutive law

\[
\{\sigma\}_k = [Q]_{k} \{\varepsilon\}_{k} \quad k=f,m
\]

(2)

In (2) \( \{\sigma\}_k, \{\varepsilon\}_k \) are strain and stress vectors for the k-th element.

According to (A4) and (A5)

\[
\{\sigma\}_RVE = \sum_k V_k \{\sigma\}_k \ = V_f \{\sigma\}_f + V_m \{\sigma\}_m
\]

(3)

Using averaged Hook’s law (A9) in (3) we obtain
Replacing in (4) volume averaged strain (over the element) by boundary averaged and Vakulenko-Kachanov tensor (A7) we obtain

\[
\{\sigma\}_{RVE} = \sum_k V_k [Q]_k \{\varepsilon\}^a_k
\]  

Eq. (5) may be used to represent cracks in both, fibers and in the matrix. In the present study we will consider only fiber cracks, thus assuming that \(\{\beta\}_m = 0\). The effect of the debonded interface at fiber breaks will be represented by increasing displacements of the crack surfaces.

The boundary averaged strain of an element is expressed through its surface displacements, see (A6). Hence in elastic problem it is a linear combination of strains applied to the RVE

\[
\{\varepsilon\}^{ba}_k = [H]_k \{\varepsilon\}_{RVE} \quad \text{or} \quad \begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix}^{ba}_k = \begin{bmatrix} H_{11} & H_{12} & H_{16} \\ H_{21} & H_{22} & H_{26} \\ H_{61} & H_{62} & H_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix}^{RVE}
\]  

Elements of the H-matrix are very complex functions of element properties, internal geometry of the RVE which may be obtained from numerical solution of the local problem. Generally speaking the H-matrix may change with damage evolving inside of the k-th element, which could be presented as

\[
[H]_k = [H]_k^R + [H]_k^D
\]  

However, in the present study we assume that the second term in (7) is relatively small and neglect it. Hence

\[
[H]_k = [H]_k^R
\]  

The \(\{\beta\}_f\) vector for the fiber is expressed through crack face displacements which are proportional to the number of fiber breaks \(N\) represented by normalized crack density \(\rho_n = \frac{N}{L} r_f\), proportional to the far-field stress in the fiber \(\{\sigma_0\}_f\) (average fiber stress at the same applied load to the undamaged RVE in the absence of fiber break). Hence, it can be written in the following form

\[
56
\]
\[
\{\beta\}_{f} = -\frac{\rho_{n}}{E_{\mu}}[U]_{f}\{\sigma_{0}\}_{f}
\]  
(9)

The longitudinal fiber modulus \(E_{\mu}\) is introduced (9) to have \([U]_{f}\) dimensionless.

The “far field” stress \(\{\sigma_{0}\}_{f}\) can be expressed through the strain applied to RVE of an undamaged composite in the following form

\[
\{\sigma_{0}\}_{f} = [Q]_{f}\{\varepsilon_{0}\}_{f} = [Q]_{f}\{\varepsilon_{0}^{ba}\} = [Q]_{f}[H]\{\varepsilon\}_{RVE}^{0}
\]  
(10)

Now

\[
\{\beta\}_{f} = -\frac{\rho_{n}}{E_{\mu}}[U]_{f}[Q]_{f}[H]\{\varepsilon\}_{RVE}^{0}
\]  
(11)

Substituting (6) and (11) in (5) we obtain

\[
\{\sigma\}_{RVE} = \sum_{k}V_{k}[Q]_{k}[H]_{k}\{\varepsilon\}_{RVE} - V_{f}\frac{\rho_{n}}{E_{\mu}}[Q]_{f}[U]_{f}[Q]_{f}[H]\{\varepsilon\}_{RVE}^{0}
\]  
(12)

From (12) the stiffness matrix of the undamaged composite is

\[
[Q]_{0}^{RVE} = \sum_{k}V_{k}[Q]_{k}[H]_{k}
\]  
(13)

Since for the applied stress \(\{\sigma\}_{RVE}\) the strain response of the undamaged composite will be

\[
\{\varepsilon\}_{0}^{RVE} = [S]_{0}^{RVE}\{\sigma\}_{RVE}
\]  
(14)

we obtain from (12)

\[
\{\sigma\}_{RVE} = \left([I] + V_{f}\frac{\rho_{n}}{E_{\mu}}[Q]_{f}[U]_{f}[Q]_{f}[H]_{f}[S]_{0}^{RVE}\right)^{-1}[Q]_{0}^{RVE}\{\varepsilon\}_{RVE}
\]  
(15)

Obviously the stiffness of the damaged composite is

\[
[Q]_{RVE} = \left([I] + V_{f}\frac{\rho_{n}}{E_{\mu}}[Q]_{f}[U]_{f}[Q]_{f}[H]_{f}[S]_{0}^{RVE}\right)^{-1}[Q]_{0}^{RVE}
\]  
(16)

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3. Form of the \([H]_k\) matrix

Obviously, an accurate determination of \([H]_k\) is one of the main difficulties in stiffness prediction of undamaged and damaged UD composites. The \([H]_k\) matrix is formulated in the local axes of an element (fiber and matrix cylinders) and it defines the relationship between \(k\)-th element boundary averaged strains and the strains applied to the RVE.

\[
\{\varepsilon^\text{b,a}\}_{k} = [H]_k \{\varepsilon\}_\text{RVE}
\]  

(17)

In UD composite each element has a form of a cylinder (with an axis in the L-direction) extending in this direction over the whole length of the RVE. Due to this feature some conclusions regarding the form of the \([H]_k\) -matrix can be formulated.

If only \(\varepsilon^\text{RVE}_L\) is applied to the RVE, the strain in each element in the L-direction is equal to the applied strain. Consequently, the elements \(H^f_{11} = H^m_{11} = 1\). Applying to the RVE \(\varepsilon^L_T\) only (with zero longitudinal strain and shear strain), the average strain in the L-direction in the element is equal to this strain component applied to the RVE (it is zero). Hence \(H^f_{12} = H^m_{12} = 0\). We assume also that only applied RVE shear strains may cause non-zero boundary averaged shear strains in the element leading to \(H^f_{16} = H^m_{26} = H^f_{61} = H^m_{62} = 0\).

The boundary averaged strain in the T-direction in the cylindrical element at applied \(\varepsilon^L_T\) is a function of all parameters. The dependence of \(H^f_{22}\) on geometrical and stiffness parameters comes from solution of a complex 3-D problem. The above qualitative analysis leads to the following form of the H-matrix for fiber and for matrix

\[
[H]_f = \begin{bmatrix}
1 & 0 & 0 & 0 \\
H^f_{21} & H^f_{22} & 0 & 0 \\
0 & 0 & H^f_{66}
\end{bmatrix}
\quad
[H]_m = \begin{bmatrix}
1 & 0 & 0 & 0 \\
H^m_{21} & H^m_{22} & 0 & 0 \\
0 & 0 & H^m_{66}
\end{bmatrix}
\]  

(18)

H-matrix constants in fiber and in the resin are not independent. One relationship between them may be obtained using the request of thermodynamics that the RVE stiffness matrix of the undamaged UD composite, see (13) has to be symmetric.

Two more relationships come from the rule of mixtures (ROM) which exists between the RVE strains and boundary averaged strains in elements. The
longitudinal strain ROM is satisfied automatically due to iso-strain state. The transverse strain ROM leads to two independent conditions. The formulated three conditions for $H_{22}^f, H_{21}^f, H_{21}^m, H_{22}^m$ are used to express three of them through $H_{22}^f$.

$$H_{21}^f = (H_{22}^f - 1) \frac{Q_{12}^f - Q_{12}^m}{Q_{22}^f - Q_{22}^m}$$  \hspace{1cm} (19)$$  

$$H_{21}^m = \frac{V}{V_m} H_{21}^f$$  \hspace{1cm} (20)$$  

$$H_{22}^m = \frac{1 - V_f H_{22}^f}{V_m} \quad H_{66}^m = \frac{1 - V_f H_{66}^f}{V_m}$$  \hspace{1cm} (21)$$

As a very rough approximation to determine $H_{22}^f$ we can use constant stress models (CSM) used in material mechanics. According to the constant stress model

$$\varepsilon_{T}^{T} = V_{f} \varepsilon_{T}^{f} + V_{m} \varepsilon_{T}^{m} \quad \frac{\varepsilon_{T}^{m}}{\varepsilon_{T}^{f}} = \frac{E_f}{E_m}$$  \hspace{1cm} (22)$$  

From here

$$\varepsilon_{T}^{RVE} = \varepsilon_{T}^{f} \left( V_{f} + V_{m} \frac{E_f}{E_m} \right)$$  \hspace{1cm} (23)$$  

and

$$H_{22}^f = \frac{1}{V_f + V_m \frac{E_f}{E_m}}$$  \hspace{1cm} (24)$$

Similar assumptions regarding shear stresses lead to

$$H_{66}^f = \frac{1}{V_f + V_m \frac{G_{fT}}{G_m}}$$  \hspace{1cm} (25)$$

Certainly more accurate estimations may be obtained using Concentric Cylinder Assembly model [5,6] or numerical solutions.
4. Crack face displacement matrix $[U]_f$

The $[U]_f$ matrix is defined by (9) and its explicit form can be obtained analyzing \{\beta\}_f which is according to (A8) related to crack face displacements. In the local coordinate system the normal vector has on the crack face only one component

$$n_1 = \pm 1 \quad n_2 = n_3 = 0$$  \quad (26)

Using definition we obtain

$$\beta_{11} = 2 \rho_n \frac{u_{1a}}{r_f} \quad \beta_{22} = 0 \quad \beta_{12} = -\rho_n \frac{u_{2a}}{r_f}$$  \quad (27)

In (27) $u_{1a}$ and $u_{2a}$ are average displacements of fiber crack faces in the longitudinal and transverse direction respectively

$$u_{1a} = \frac{2}{r_f^2} \int_0^{r_f} \frac{\Delta u_1(r)}{2} rdr \quad u_{2a} = \frac{2}{r_f^2} \int_0^{r_f} \frac{\Delta u_2(r)}{2} rdr$$  \quad (28)

In (28) $\Delta u$ is the displacement gap between two crack faces. Introducing crack face displacements normalized with respect to the crack radius and the far field average stress in the fiber as

$$u_{1am} = \frac{u_{1a}}{r_f} E_{\beta} \sigma_{lf}^0 \quad u_{2am} = \frac{u_{2a}}{r_f} G_{\beta,T} \sigma_{lTf}^0$$  \quad (29)

we can rewrite (26) as

$$\begin{bmatrix} \beta_{11} \\ \beta_{22} \\ 2\beta_{12} \end{bmatrix} = -2 \rho_n \begin{bmatrix} u_{1am} E_{\beta} \\ 0 \\ u_{2am} G_{\beta,T} \end{bmatrix} = -2 \rho_n \begin{bmatrix} u_{1am} 0 0 \\ 0 0 0 \\ 0 0 u_{2am} \end{bmatrix} \begin{bmatrix} \sigma_{lf}^0 \\ \sigma_{Tf}^0 \\ \sigma_{lTf}^0 \end{bmatrix}$$  \quad (30)

Comparing (9) with (30) we see that $[U]_f$ which enters stiffness reduction expressions is
\[
[U]_f = \begin{bmatrix}
 u_{1,n} & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & u_{2,n} \frac{E_f}{G_fT}
\end{bmatrix}
\] (31)

5. Factors affecting the normalized average crack opening displacement (NACOD)

The value of \( u_{1,n} \) depends on the interface quality. If the fiber is debonded the crack opening may be significantly larger. NACOD depends also on fiber volume fraction, elastic properties etc. Micromechanical models developed to describe stress distribution in fiber fragments in SFF test may be used to evaluate the COD. As an alternative to approximate analytical models in this study we performed FEM calculations in axi-symmetrical formulation to calculate the COD. The opening of the penny-shaped crack in longitudinal tension (applied strain 1%) was analyzed considering three concentric cylinder model. The broken fiber is surrounded by a cylindrical matrix zone which is its turn embedded in a large cylindrical block of the effective composite, see Fig. 2. The surrounding composite cylinder is necessary to describe more adequate the effect of neighbouring fibers on the local stress distribution. The effective composite properties were calculated using rule of mixtures and Halpin-Tsai expressions. Thus the fiber break effect on effective composite properties in the outer cylinder was not included assuming that the NACOD is not sensitive to about 10% changes in the properties in this cylinder. This argument was also used to justify the use of simplified elastic properties expressions in this region instead of more accurate models like Hashin’s concentric cylinder assembly model [2,3].

FEM calculations are performed in ANSYS using an axisymmetric formulation. Element type used is the plane element, PLANE82, which is a 2-D, second order element with relatively high accuracy. A non-uniform mesh consisting of both triangular and rectangular elements was used. To obtain higher accuracy a refined mesh (of triangular elements) was used in the vicinity of the crack tip and at the end of the debonding zone.
Figure 2 Schematic showing of the model geometry consisting of cylindrical fiber surrounded by matrix cylinder which is embedded in the composite with effective properties: a) perfectly bonded interface; b) partially debonded interface with debond length $l_{deb}$.

There is a symmetry condition on $z = 0$, $r \in [0, R]$, where $R$ is the outer radius of the fiber-matrix-composite system. The axisymmetry is around the $z$-axis. Nodes on the side $r = R$, $z \in [0, L]$, are coupled in the $r$-direction. ($L = 90r_f$ is the length of the fiber-matrix-composite system in the axial direction which represents one half of the distance between two fiber cracks.) The thickness of the effective composite cylinder is $r_c = 5r_f$. The size of the matrix cylinder depends on the fiber volume fraction $V_f$. Constant displacement is applied in the $z$-direction on $z=L$, $r \in [0, R]$.

The NACOD was analyzed for isolated cracks ($L/r_f = 90$) which are far enough from each other to exclude the stress field interaction effects on the $u_{lam}$. However, expressions presented above are not limited by the condition of non-interactive cracks. If several cracks in the same fiber would be interacting the NACOD would be smaller. Interaction between cracks in different fibers can also be analyzed but this is out of the scope of this paper and can not be performed using the three cylinder model used here.

Sensitivity analysis was performed to identify some elastic properties of constituents which were expected to have small influence on COD and which could be fixed in the following parametric analysis. Transversally isotropic carbon fiber with 5 elastic constants was used. All (independent) properties are changed with 25%. The elastic modulus $E_{f}$ affects the COD with about 10% and $E_{ft}$ affects the COD with about 2%. The shear modulus $G_{fT}$ affects the COD
with about 2%. The Poisson’s ratios $\nu_{\mu T}$ and $\nu_{\mu\phi}$ affect the COD with less than 1%. As far as the elastic properties of the matrix concerns, the Poisson’s ratio $\nu_m$ affects the COD with less than 1%. Although the elastic modulus, $E_m$, is an independent variable there is no need for a sensitivity analysis because in the parametric analysis all elastic moduli will be normalized with respect to it.

Similar conclusions regarding the significance of Poisson’s ratios were obtained for glass fiber composites. It was decided to exclude these parameters from the list of parameters and to assign to them fixed values $\nu_{\mu T} = 0.2$, $\nu_{\mu\phi} = 0.45$, $E_m = 3\text{GPa}$ $\nu_m = 0.4$

The modulus ratio $\frac{E_{\mu T}}{E_m}$ and the fiber volume fraction $V_f$ were used as the main parameters in the analysis. A fiber content range $0.45 \leq V_f \leq 0.55$ which has practical significance was considered. To account for variability of the radial modulus $E_{\mu T}$ and shear modulus $G_{\mu T}$ the four fiber materials with properties given in Table 1 were used.

<table>
<thead>
<tr>
<th>Property</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\mu T}$ (GPa)</td>
<td>20</td>
<td>30</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>$G_{\mu T}$ (GPa)</td>
<td>30</td>
<td>30</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

First the **perfect bonding case** at the fiber/matrix interface shown in Fig 2a) was analyzed. The $u_{i\mu m}^b$ (index $b$ is used to denote the perfectly bonded case) dependence on the fiber/matrix modulus ratio for Material 1 is shown in Fig. 3. Obviously, the stiffness ratio has much larger effect on $u_{i\mu m}$ than the variation in fiber volume fraction.
Presenting these data in log-log axes as shown in Fig. 4 we obtain a very straight line with slope which very weakly depends on the fiber content. This result proves that a power law with respect to modulus ratio can give an adequate description of the relationship

$$u_{1,m}^h = A \left( \frac{E_{fL}}{E_m} \right)^n$$

(32)

Similar results for Material 2 to 4 show slight dependence of $n$ on the ratio $E_{fL} / G_{fL}$. The described dependences are fitted with a high accuracy with the following linear relationship
\[ n = 0.5148 + 0.0143 \frac{E_f}{G_{RT}} - 0.066V_f \]  

(33)

\( A \) in (32) is a function of fiber content \( V_f \) and of other elastic properties of the fiber normalized with respect to the matrix modulus

\[ E_{fT}^n = \frac{E_f}{E_m} \quad G_{RT}^n = \frac{G_{RT}}{E_m} \]  

(34)

Analysis of FEM results showed that \( \log A \) is a linear function of \( V_f \). The dependence on normalized parameters (34) was presented by polynomial expansion

\[- \log A = V_f \left(0.194 - 0.0018E_{fT}^n\right) + a_0 + a_1E_{fT}^n + a_2G_{RT}^n + a_3E_{fT}^nG_{RT}^n \]  

(35)

\[ a_0 = 0.0941 \quad a_1 = 0.00927 \quad a_2 = 0.00387 \quad a_3 = -0.000234 \]  

(36)

The predicting accuracy of (32)-(36) is demonstrated in Fig. 5 and 6 which shows excellent agreement with FEM values represented by symbols.

As a final check of the application range we apply the obtained fitting law to glass fiber composite case (\( E_f = 70 GPa, \nu_f = 0.2 \)) which due to fiber isotropy is outside the region which was used for fitting parameter determination. In this case, see Fig. 7 the predicted normalized average COD’s are about 10% lower than the FEM values. This means that the fitting functions are more accurate for...
anisotropic fibers. However, as it will be shown in following the interface debonding introduces much larger opening displacements and the obtained 10% underestimation for glass fiber case is acceptable.

![Graph showing normalized average COD predictions](image)

Figure 6. Normalized average COD Predictions according to (32)-(36) (solid lines) and according to FEM (symbols).

![Bar graph showing normalized average COD](image)

Figure 7. Accuracy of predictions for glass fiber composite

In most cases the interface at the fiber break is partially debonded and the debond length $l_{deb}$, see Fig.3b), grows with increasing load or with the number of cycles in fatigue. Calculations were performed only for $V_f = 0.45$. The used carbon fiber properties are

$$E_{fL} = 500 GPa \quad E_{fT} = 30 GPa \quad G_{fLT} = 20 GPa$$

(37)
The effect of the debond length $l_{deb}$ on the carbon fiber crack opening profile is shown in Fig. 8. Obviously the COD significantly increases with the debond length and the coordinate dependence is smaller.

![Figure 8 Crack opening displacement profiles at 1% applied strain for different normalized length of debond $l_{deb}$. Friction is ignored.](image)

Calculations were performed for the following four cases: 1) without contact elements thus allowing for material interpenetration (results in Fig 8); 2) with contact elements but with zero friction. The opening profile almost coincides with the case 1); 3) contact elements with friction coefficient $k = 0.2$ (Coulomb's friction); 4) contact elements with friction coefficient $k = 0.4$. Calculations were performed without account for thermal effects. Hence, the friction is caused by differences in Poisson’s ratios only.

![Figure 9 Normalized average COD dependence on the normalized debond length $l_{deb}/r_f$ in carbon fiber/epoxy composite for solution without contact elements and for solution with contact elements and three values of frictional coefficient $k$.](image)
The normalized average COD as a function of the debond length is presented in Fig. 9 for carbon fiber composite and in Fig. 10 for glass fiber composite. The average normalized COD, $u_{\text{1an}}$, is a linear function of the debond length for debond length $l_{\text{deb}} = 0.5r_f$. Results are almost coinciding for no-contact case and for contact elements with $k = 0$. The presence of friction reduces the slope by 5-10%. The slope is rather insensitive to $k$ change in the region $0.2 \leq k \leq 0.4$ used in calculations. The slope in the frictional case is very close to one which leads to the following choice of fitting function for COD in this region

$$u_{\text{1an}} = \frac{l_{\text{deb}}}{r_f} + D$$

where $D$ depends on material properties. It is equal to the normalized average COD at $l_{\text{deb}} = 0.5r_f$. $D$ is much larger for CF case than for GF case.

Figure 10. Normalized average COD dependence on the normalized debond length $l_{\text{deb}} / r_f$ in glass fiber/epoxy composite. Solution without contact elements and solutions with contact elements and three values of frictional coefficient $k$.

Comparing the $u_{\text{1an}}$ values at $l_{\text{deb}} / r_f = 0.5$ with the value $u_{\text{1an}}^b$ for the bonded case, $l_{\text{deb}} = 0$ considered earlier we obtain that the ratio is almost the same for carbon fiber and glass fiber case (1.51 and 1.47 respectively). Based on this observation we roughly assume that this ratio is material independent and equal to 1.5 and $D$ can be written as

$$D = 1.5u_{\text{1an}}^b$$

(39)
Substitution in (38) leads to

\[ u_{\text{int}} = 1.5 u_{\text{lan}}^{+} + \frac{f_{d}}{r_{f}} \]  

(40)

The validity of this expression for fiber contents different than 0.45 has not been checked. This function was used to predict the normalized average opening displacement of a fiber crack. The predictions and values from FEM for friction coefficient \( k = 0.2 \) are presented in Fig. 11 and they are in a very good agreement. Eq (40) will be used in the following section to predict stiffness reduction in UD composite.

![Normalized average COD of fiber cracks with debonds. Predicted (40) (lines) and FEM with contact elements and friction coefficient \( k = 0.2 \) (symbols).](image)

Figure 11. Normalized average COD of fiber cracks with debonds. Predicted (40) (lines) and FEM with contact elements and friction coefficient \( k = 0.2 \) (symbols).

Detailed FEM analysis of fiber crack displacements for debonds \( 0 < l_{\text{deb}} < 0.5 r_{f} \) was not performed. The following expression may be obtained in this region if we assume linear COD dependence on the debond length

\[ u_{\text{lan}} = u_{\text{lan}}^{+} + \left( u_{\text{lan}}^{+} + 1 \right) \frac{l_{d}}{r_{f}} \]  

(41)

6. Stiffness reduction due to fiber breaks in a unidirectional layer.

The fiber crack face sliding displacements as well as the normal displacements of the debond cracks were not investigated in this study. However, as checked by putting arbitrary values in the model, they mainly affect the transverse and shear properties of the UD composite and their effect on the composite longitudinal modulus, is negligible.
The main focus in this paper is the combined effect of fiber breaks and the accompanied debonding on the longitudinal modulus of the composite. This property is affected by the resultant fiber crack opening displacement, COD, only. On the other hand from the performed calculations using the developed model follows that COD has a negligible effect on other stiffness properties than the longitudinal modulus.

Stiffness reduction predictions were performed for carbon fiber composite with fiber properties \( E_f = 300 \text{GPa}, \quad E_T = 30 \text{GPa}, \quad G_{LT} = 20 \text{GPa}, \quad \nu_{LT} = 0.2 \) and for glass fiber composite with fiber properties \( E_f = 70 \text{GPa}, \quad \nu_f = 0.2 \). Matrix was the same for both composites with \( E_m = 3 \text{GPa}, \quad \nu_m = 0.4 \). Fiber content in the composites was \( V_f = 0.5 \) and fiber radius \( r_f = 4 \mu m \).

The statistical fiber cracking evolution with load was not simulated and neither the debond crack growth with load or number of cycles. This type of analysis would give the stiffness changes as a function of the load history.

In this paper the main emphasis is on understanding of the significance of fiber breaks. Hence the damage state in following predictions is used as an input parameter. The number of cracks in all fibers of the RVE can be assumed the same. Predictions are presented in Fig. 12 as the normalized longitudinal modulus reduction versus the fiber break density in one fiber measured as the number of fiber breaks per 1cm. The representation of the damage state by crack density does not limit the accuracy because the cracks are considered non-interactive and hence the COD does not depend on the particular location of the fiber crack. Therefore the result is the same for evenly spaced cracks and for arbitrary distributed. The highest number of breaks (20cr/cm correspond to fiber fragment length 0f 0.5mm where the stress perturbations from both ends of the fiber fragment start to interact and the used expressions for COD’s are overestimated. The results obtained for three values of the normalized debond length \( l_{deb} / r_f \) show that the debond length has a great significance for the stiffness reduction. The relative modulus reduction in carbon fiber composites is larger due to a larger opening of the fiber crack which is a consequence of higher fiber and matrix modulus ratio leading also to larger extent of the stress transfer zone.
Figure 12 Longitudinal modulus reduction in UD composite in a normalized form as a function of the number of fiber breaks in one fiber: a) CF/EP composite, b) GF/EP composite.

Conclusions
Stiffness reduction in unidirectional composites due to fiber breaks with a partial interface debonding can be analyzed by the presented model which is based on exact expressions relating the composite stiffness with the normalized average displacements of the fiber crack faces.

Only the opening displacements were studied leaving the sliding effect to further studies. It was found by using arbitrary values that fiber crack face sliding does not affect the longitudinal modulus of the composite. The axial sliding of the debond crack faces is already included through the increased opening of the fiber crack. On the other hand the crack opening does not affect the Poisson’s ratio, transverse modulus and shear modulus of the composite.

The main parameters affecting the normalized fiber crack opening have been analyzed using FEM and it is found that the fiber longitudinal modulus, fiber content and the debond length are of the highest significance. These relationships are described by simple functions which excellently fit the numerical results. The effect of other less important parameters is also included in these fitting expressions.

It was found that the relative longitudinal modulus reduction in carbon fiber composite is slightly higher than in glass fiber composite. This trend holds for all considered debond lengths and is related to higher longitudinal modulus/matrix modulus ratio for carbon fibers leading to larger crack openings and larger stress perturbation zones.
Appendix

Average stresses and strains over the domain with volume $V$ are defined as follows

\[
\{\sigma\}^a = \frac{1}{V} \{\sigma\} dV \quad (A1)
\]

\[
\{\varepsilon\}^a = \frac{1}{V} \{\varepsilon\} dV \quad (A2)
\]

Superscript $a$ denotes the volume average.

Boundary average stress, which has the meaning of the macroscopic stress applied to the external surface $S_E$ of a domain with volume $V$ (in present study it is the average stress applied to the RVE of the UD composite, $V = V_{RVE}$, $S_E = S_{E(RVE)}$), is defined as

\[
\sigma_{ij}^{RVE} = \frac{1}{V_{RVE}} \int_{S_{E_RVE}} \sigma_{ik} n_k x_j dS \quad (A3)
\]

It is easy to prove [1] that the boundary averaged stress is equal to the boundary averaged stress. Hence

\[
\{\sigma\}^{RVE} = \{\sigma\}^a \quad (A4)
\]

Since volume integral over volume $V$ can be written as a sum of integrals over subdomains of this volume (elements) with volume $V_k$, one can write

\[
\{\sigma\}^a = \sum_k V_k \{\sigma\}^a_k \quad (A5)
\]

In (A5) $V_k$ is the volume fraction of the k-th sub domain. Boundary averaged strains may be introduced as

\[
\varepsilon_{ij}^{ba} = \frac{1}{V} \int_{S_{E_RVE}} \left( u_i n_j + u_j n_i \right) dS \quad (A6)
\]

Also the volume averaged strains are equal to boundary averaged strains [7]. Since the boundary includes also crack surface ($S = S_E + S_C$) this equality may be written in the following vectorial form
Here the upper index “ba” stays for “boundary average” over the external boundary. It describes the apparent strain due to the deformation of the outer boundaries. For RVE it is the applied strain. For sub-domains of the RVE (elements) its determinations is in general a complex problem. For elements like layers in laminates, where iso-strain hypothesis is valid this strain is equal to the strain applied to the laminate. In (A7) \( \beta \) is the Vakulenko-Kachanov tensor defined by

\[
\beta_{ij} = \frac{1}{V_s} \int \frac{1}{2} (u_i n_j + u_j n_i) dS
\]  

(A8)

where \( S_c \) is the total surface of cracks in the domain under consideration, \( u_i \) are displacements of the points on the crack surface, \( n_i \) is outer normal to the crack surface, in domain with no cracks, \( \beta_{ij} \) is zero.

The stress-strain relationship of the k-th element (1) can be averaged over this element, leading to

\[
\{ \sigma \}^a_k = \mathcal{Q}_k \{ \varepsilon \}^a_k
\]

(A9)

References

Modeling fiber crack opening displacement in UD composites with partially debonded fibers.

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ABSTRACT
The opening displacements of a fiber crack in UD composite due to tensile loading in the fiber direction are analyzed. This study is motivated by previous research which showed that the stiffness reduction in unidirectional composites due to increasing number of fiber breaks with a partial interface debonding can be predicted using exact expressions containing the normalized average displacements of the fiber crack faces.

FEM based parametric analysis is performed in this paper to study the average value of the fiber crack opening displacement in a normalized form (NACOD). It is shown to be a robust parameter dependent on the fiber and matrix properties, fiber content in the composite and the fiber fragment length and the debond length. The data are described by simple fitting functions. For example it was found that the NACOD is a linear function of the debond length and follows a power law with respect to fiber/matrix elastic modulus ratio.

The potential of simple analytical models in NACOD determination is assessed, relating the average COD to the average stress in the fiber fragment. Using a simple shear lag model it is demonstrated that the qualitative trends of NACOD dependence on geometrical and material parameters can be described fairly well whereas numerical values have 20-40% accuracy.

1. Introduction
Multiple fiber fractures may be observed in unidirectional (UD) long fiber composites loaded in fiber direction, see Fig. 1 where a representative volume element (RVE) of the composite is shown. The fiber strength follows the statistical distribution given by Weibull. This means that there are many randomly distributed week locations in fibers in the UD composite where the fiber strength is significantly below the average fiber strength.

Therefore the number of fiber cracks grows with increasing load. Due to stress transfer across the fiber/matrix interface, the stress in the fiber at a certain distance from the fiber break has almost recovered its far-field value and multiple fiber breaks (fragmentation) are possible. Each individual fiber crack can be
accompanied by fiber/matrix debonding initiated at the fiber crack tip and
growing with increasing loading and/or the number of cycles in fatigue. The
debond growth along the interface in fatigue and/or increasing macroscopic
loading conditions is most probably governed by fracture mechanics parameters.
An alternative mode of degradation in the interface region is matrix yielding at the
interface in the high shear stress region at the fiber crack tip.

The interface crack (debond) initiation from the fiber crack and growth,
consequences of what are considered in this paper, is only one among many
possible scenario of damage development, see for example [1]. AS another
example, fiber crack propagation in the resin is analysed in [2]. An alternative
sequence in fatigue may be crack initiation in the resin with further debond crack
growth along the fiber which leads to magnified role of surface defects in the fiber
resulting in fiber breaks at lower stresses than for perfectly bonded fibers.

In this paper the damage evolution modelling is not considered. The analysis here
is focused on parameters characterizing the existing damage state with
significance for elastic properties of the UD composite. The decrease of elastic
properties of the UD composites due to broken fibers with partially debonded
interface is related to reduced average stress in these fibers which leads to lower
macro-stress at the same applied macro-strain. Another way to express the macro
– micro state relationship used in this paper is with fiber crack face opening
displacements (COD) and sliding displacements (CSD). It is easy to realize that if
these displacements would be zero these fibers would act as before break and
elastic macro-properties would not be affected. If these displacements are large
the stress state in the fiber fragment is lower and the composite properties are
reduced more.

The fiber break is assumed to be a penny-shaped crack in the fiber which has a
normal parallel to the fiber direction and which covers the whole cross section of
the fiber. The debond initiated from the fiber crack is a fiber/matrix interface
crack characterized by the debond length \( l_d \) (measured from the fiber crack tip)
which does not depend on the angular coordinate. Obviously fibers with partially
debonded interface will exhibit larger fiber crack face displacements. This model
with axial symmetry of the geometry and boundary conditions on the fiber
boundaries becomes three-dimensional if the presence of surrounding fibers with
their exact locations is accounted for.
In order to keep a reasonable level of complexity the cylindrical symmetry or even one-dimensional approaches are used. A large number of research papers have been published on description of the stress transfer from matrix into fiber at the fiber break. Analyses have been analytical, see for example [3-5] as well as numerical [4] with more focus on short fibers or on the description of the single fiber fragmentation test. These studies are relevant to the topic of this paper because the average stress in a fiber which may be calculated from the stress distribution is directly linked to the COD and CSD which we in this paper consider as the most important parameters for elastic modulus of the damaged composite with a given damage state.

In a previous paper [6] we developed exact relationships which link the entire stiffness matrix of the damaged UD composite with two robust parameters from the local solution: average COD of the fiber break (with or without debond) and its average sliding displacement CSD both normalized with respect to the size and to the far field stress in the fiber. It was obtained that the stiffness matrix of a damaged UD composite may be written as

\[
[Q]^{UD} = \left[ I + V_f \frac{\rho_n}{E_f} [Q]_f [U]_f [Q]_f [H]^p [S]^{UD}_0 \right]^{-1} [Q]^{UD}_0
\]  

(1)

In (1) \( \rho_n \) is “density” of fiber breaks in one fiber with length \( L \) written in normalized form

\[
\rho_n = \frac{N}{L} r_f
\]  

(2)

The stiffness \([Q]^{UD}\) of the damaged UD composite depends on the fiber volume fraction \( V_f \) in the composite, the fiber stiffness matrix \([Q]_f\), the stiffness and compliance matrices of the undamaged composite, \([Q]^{UD}_0\), \([S]^{UD}_0\). \( E_f \) is the
longitudinal modulus of the fiber. The $[H]_f^y$ – matrix defines the link in the local coordinate system between average fiber strains and strains applied to the RVE in the undamaged composite

$$\{\varepsilon_i\} = [H]_f \{\varepsilon\}_RVE \tag{3}$$

Elements of the H-matrix are very complex functions of material properties and internal geometry of the RVE which may be obtained from numerical solution of the local problem. In the first approximation this matrix may be extracted from the rule of mixtures and constant stress models.

The matrix $[U]_f$ which enters stiffness reduction expressions is defined as

$$[U]_f = 2 \begin{bmatrix} u_{1am} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{E_{\beta}}{G_{\beta T}} u_{2am} \end{bmatrix} \tag{4}$$

Parameters in matrix (4) are crack face displacements normalized with respect to the crack radius and the far field stress in the fiber

$$u_{1am} = \frac{u_{1a}}{r_f} \frac{E_{\beta}}{\sigma_{fLf}} \quad u_{2am} = \frac{u_{2a}}{r_f} \frac{G_{\beta T}}{\sigma_{fLf}} \tag{5}$$

In (5) $u_{1a}$ and $u_{2a}$ are average displacements of fiber crack faces in the longitudinal and transverse direction respectively

$$u_{1a} = \frac{2}{r_f^2} \int_0^{r_f} \frac{\Delta u_1(r)}{2} r dr \quad u_{2a} = \frac{2}{r_f^2} \int_0^{r_f} \frac{\Delta u_2(r)}{2} r dr \tag{6}$$

In (6) $\Delta u$ is the displacement gap between two crack faces.

Using arbitrary values of $u_{2am}$ in (1) it was found that fiber crack face sliding CSD does not affect the longitudinal modulus of the composite. The axial sliding of the debond crack faces is already included through the increased opening displacement of the fiber crack. On the other hand it was found that the crack opening does not affect the Poisson’s ratio, transverse modulus and shear modulus.
of the composite. In other words the UD composite longitudinal modulus is COD
governed whereas other elastic properties are CSD governed.

In order to use the above expressions we need to know the dependence of COD,
CSD on fiber and matrix properties, fiber content, debond length etc. This
information can be extracted from available approximate analytical stress transfer
models. However the accuracy of these results is not known.

The objective of this paper is to study these dependencies in a most accurate way
using FE parametric analysis. The analysis is limited by fiber crack opening
displacements only. In the axi-symmetrical FE analysis we use a model
representing the UD composite with a broken and partially debonded fiber by a
three concentric cylinder assembly as shown in Fig.2: a) broken fiber in the
middle; b) matrix cylinder around it; c) large effective composite cylinder
surrounding them. The results of the parametric analysis are described by simple
fitting functions which with a high accuracy describe COD’s of perfectly bonded
and partially debonded cracks.

2. FEM analysis of factors affecting NACOD

2.1 Description of the FEM model and sensitivity analysis

In the previous section it was shown, see equations (1) and (4), that the crack
opening displacement enters the stiffness expressions in an averaged and
normalized form given by $u_{1\text{av}}$, which may be calculated from the COD profile
using (5) and (6). We will use the term “NACOD” (normalized average crack
opening displacement) in text and in figures to refer to $u_{1\text{av}}$.

The value of $u_{1\text{av}}$ depends on the interface quality. If the fiber is debonded in a
certain region with length $l_d$ starting from the fiber crack tip, the fiber crack
opening may be significantly larger. NACOD may depend also on fiber volume
fraction, elastic properties etc. In this study we perform FEM calculations in axi-
symmetrical formulation to calculate the COD. As an alternative micromechanical
models developed to describe stress distribution in fiber fragments in single fiber
fragmentation test (SFF) [3,5] may be used to evaluate the COD. The opening of
the penny-shaped fiber crack in longitudinal tension (applied axial strain 1% in all
presented results) was analyzed considering three concentric cylinder model
shown in Fig 2. The broken fiber is surrounded by a cylindrical matrix zone which
is in its turn embedded in a large cylindrical block of the effective composite. The
surrounding composite cylinder is necessary to describe adequately the integral
effect of neighbouring fibers and matrix between them on the local stress
distribution. The effective composite properties were calculated using rule of
mixtures and Halpin-Tsai expressions for undamaged composite. Thus the effective composite properties in the outer cylinder were not reduced to include the effect of fiber breaks, assuming that the NACOD is not sensitive to about 10% changes in the properties in this cylinder. This argument which was validated using FEM was also used to justify the use of simplified expressions for elastic properties in this region instead of using more accurate models like Hashin’s concentric cylinder assembly model [7,8].

FEM calculations are performed using the commercial code ANSYS in an axisymmetric formulation. The PLANE82 plane element, which is a 2-D, second order element with relatively high accuracy was used in a non-uniform mesh consisting of both triangular and rectangular elements. To obtain higher accuracy a refined mesh (of triangular elements) was used in the vicinity of the crack tip and at the end of the debond zone.

There is a symmetry condition on $z = 0$, $r \in [r_f, R]$, where $R$ is the outer radius of the fiber-matrix-composite system. The axisymmetry is with respect to the $z$-axis. Displacement in nodes on the side $r = R$, $z \in [0, L_f]$, are coupled in the $r$-direction. ($L_f = 90r_f$ is the nominal length of the fiber-matrix-composite system in the axial direction which represents one half of the distance between two fiber cracks which is $2L_f$.) The thickness of the effective composite cylinder is $r_c = 5r_f$. The size of the matrix cylinder depends on the fiber volume fraction $V_f$.  

\[ r_c = 5r_f \]

\[ r_f \]

Figure 2. Schematic showing of the model geometry consisting of cylindrical fiber surrounded by matrix cylinder which is embedded in the composite with effective properties: (a) perfectly bonded fiber/matrix interface; (b) partially debonded interface with debond length $l_d$. 
Constant displacement is applied in the $z$-direction at $z = L_f$, $r \in [0, R]$. In all calculations $r_f = 4 \mu m$.

The NACOD was first analyzed for isolated cracks ($L_f / r_f = 90$) which are far enough from each other to exclude the stress field interaction effects on $u_{1\omega}$. However, expressions (1) to (6) presented above for stiffness reduction are not limited by the condition of non-interactive cracks. If cracks in the same fiber would be interacting the NACOD would be smaller. Interaction between cracks in different fibers can also be analyzed but this is out of the scope of this paper and can not be performed using the three concentric cylinder model used here.

Sensitivity analysis was performed to identify some elastic properties of constituents which were expected to have small influence on COD and which could be kept constant in the following parametric analysis. Transversally isotropic carbon fiber with 5 elastic constants was used. To check the sensitivity each property (one at a time) is changed with 25%. It was found that the change in the fiber elastic modulus $E_{fl}$ affects the NACOD with about 10% and $E_{ff}$ affects the NACOD with about 2%. The change in shear modulus $G_{flT}$ affects the NACOD with about 2% whereas the Poisson’s ratios $\nu_{flT}$ and $\nu_{ffT}$ affect the COD with less than 1%. Although the elastic modulus, $E_m$, is an independent variable there is no need for a sensitivity analysis because in the parametric analysis all elastic moduli are normalized with respect to it.

Similar conclusions regarding the significance of Poisson’s ratios were obtained for glass fiber composites. It was decided to exclude these parameters from the list of parameters and to assign to them fixed values $\nu_{flT} = 0.2$, $\nu_{ffT} = 0.45$, $E_m = 3 GPa$, $\nu_m = 0.4$.

The modulus ratio $E_{fl}/E_m$ and the fiber volume fraction $V_f$ were used as the main parameters in the analysis. A fiber content range $0.45 \leq V_f \leq 0.55$ which has practical significance was considered.

To address for variability of the radial modulus $E_{ff}$ and shear modulus $G_{flT}$ the following four fiber materials with properties given in Table 1 were used.
Table 1. Elastic properties for materials M1 to M4.

<table>
<thead>
<tr>
<th>Property</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{fT}$ (GPa)</td>
<td>20</td>
<td>30</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>$G_{fT}$ (GPa)</td>
<td>30</td>
<td>30</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

2.2 NACOD in the perfect bonding case

First the case of perfect bonding at the fiber/matrix interface shown in Fig 2) was analyzed. The index $b$ is used to denote the NACOD in the perfectly bonded case. The $u_{lam}^b$ dependence on the fiber/matrix modulus ratio for Material 1 is shown in Fig. 3. Obviously, the stiffness ratio has much larger effect on $u_{lam}^b$ than the variation in fiber volume fraction.

![Figure 3. Normalized average COD dependence on fiber/matrix axial modulus ratio in the case of perfect bonding.](image)

In log-log axes, see Fig.4, this relationship is represented by a very straight line with slope which very weakly depends on the fiber content. This means that a power law with respect to modulus ratio is an adequate description of the dependency

$$u_{lam}^b = A \left( \frac{E_{fl}}{E_m} \right)^n$$  \hspace{1cm} (7)

The slope in Fig.4 represents the value of $n$ in the power law.
Results for Material 2 to 4 show slight dependence of $n$ on the ratio $E_{fT}/G_{\beta,T}$ and on the fiber content which was fitted with the following linear relationship

\[ n = 0.5148 + 0.0143 \frac{E_{fT}}{G_{\beta,T}} - 0.066V_f \]  

(8)

In (8) parameter $A$ is a function of fiber content $V_f$ and of the following normalized elastic properties of the fiber

\[ E_{fT}^n = E_{fT}/E_m \quad G_{\beta,T}^n = G_{\beta,T}/E_m \]  

(9)

FEM results show that $\log A$ is a linear function of $V_f$. The dependence on normalized parameters (9) is fitted by polynomial expansion

\[-\log A = V_f \left(0.194 - 0.0018E_{fT}^n\right) + a_0 + a_1E_{fT}^n + a_2G_{\beta,T}^n + a_3E_{fT}^nG_{\beta,T}^n \]  

(10)

\[ a_0 = 0.0941 \quad a_1 = 0.00927 \quad a_2 = 0.00387 \quad a_3 = -0.000234 \]  

(11)
Figure 5. Predicted normalized COD versus calculated using FEM. Predictions are presented by solid lines for $V_f = 0.45$ and $0.55$.

The predicting accuracy of (7)-(11) is demonstrated in Fig. 5 and 6: the agreement with FEM values represented by symbols is excellent. To estimate the application range we apply the obtained fitting law to glass fiber (GF) composite ($E_f = 70GPa$, $\nu_f = 0.2$) which due to fiber isotropy is outside the region used for fitting parameter determination. In this case, see Fig. 7 the predicted normalized average COD’s are about 10% lower than the FEM values. This means that the fitting functions are more accurate for anisotropic fibers. However, as it will be shown in following the interface debonding introduces much larger opening displacements and the obtained 10% underestimation is acceptable.

Figure 6. Normalized average COD predictions for material 2 and 3 according to (7)-(11) (solid lines) and according to FEM (symbols).
The cause for lower accuracy of the fitting functions in the GF case is in the (10) used linear approximation with respect to the normalised fiber transverse modulus and the in-plane shear modulus. They are relatively much smaller for the analysed cases than for the GF composite. An alternative is to simplify the fitting functions given by (8)-(11) by including the dependence on the above parameters in an average sense. The simplification may be performed by applying average values of the parameters used in the fitting and determining new constants in this way. This procedure leads to

\[
\begin{align*}
n &= 0.5291 - 0.066 V_f \\
- \log A &= 0.1796 V_f + 0.1842
\end{align*}
\]

(12)

Using these expressions we obtain for GF composite NACOD values given in Table 2.

Table 2. NACOD for GF composite calculated using (7) and (12) and the shear lag model.

<table>
<thead>
<tr>
<th>( V_f )</th>
<th>0.45</th>
<th>0.50</th>
<th>0.55</th>
</tr>
</thead>
<tbody>
<tr>
<td>(7) and (12)</td>
<td>2.62</td>
<td>2.54</td>
<td>2.46</td>
</tr>
<tr>
<td>Shear lag m.</td>
<td>3.61</td>
<td>3.37</td>
<td>3.13</td>
</tr>
</tbody>
</table>

The values in Table 2 overestimate the values presented in Fig. 7 and the accuracy is not better than with the more detailed fitting functions derived for anisotropic fiber case. However, when applied to CF case (for example Material 1 in Fig. 5) the accuracy of the simplified fitting functions is sufficiently good, see Table 3. The introduced error is about 1% except the 500 GPa case where it is about 2%.
Table 3. Accuracy of fitting for CF composite calculated using (32) and (37)

<table>
<thead>
<tr>
<th>$E_{\text{fL}}$ (GPa)</th>
<th>70</th>
<th>150</th>
<th>230</th>
<th>300</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM</td>
<td>2.43</td>
<td>3.57</td>
<td>4.39</td>
<td>4.98</td>
<td>6.34</td>
</tr>
<tr>
<td>Fitting</td>
<td>2.46</td>
<td>3.58</td>
<td>4.42</td>
<td>5.04</td>
<td>6.49</td>
</tr>
</tbody>
</table>

2.3 NACOD for partially debonded long fibers

In most cases the fiber/matrix interface at the fiber break is partially debonded and the debond length $l_d$, see Fig.2b, grows with increasing load or with the number of cycles in fatigue. Calculations were performed only for carbon fiber (Material 4) with properties

$$
E_{\text{fL}} = 500\text{GPa} \quad E_{\text{fT}} = 30\text{GPa} \quad G_{\text{fL,T}} = 20\text{GPa}
$$

(13)

The effect of the debond length $l_{\text{deb}}$ on the fiber crack opening profile is shown in Fig.8 and 9. Obviously the COD significantly increases with the debond length and the coordinate dependence is smaller.

![Figure 8](image)

(a) Carbon/Epoxy $V_f = 0.45$

(b) Carbon/Epoxy $V_f = 0.55$

Figure 8. Crack opening displacement profiles in CF/EP composite for several values of debond length $l_d$. Friction is ignored and fiber volume fraction, $V_f = 0.45$ (a) and $V_f = 0.55$ (b).
Figure 9. Crack opening displacement profiles in GF/EP for several values of debond length $l_{db}$. Friction is ignored and fiber volume fraction, $Vf = 0.45$ (a) and $Vf = 0.55$ (b).

Calculations were performed for the following four cases: 1) without contact elements thus allowing for material interpenetration (results in Fig 8 and 9); 2) with contact elements but with zero friction. The COD profile almost coincides with the case 1); 3) contact elements with friction coefficient $k = 0.2$ (Coulomb's friction); 4) contact elements with friction coefficient $k = 0.4$. Calculations were performed without account for thermal effects. Hence, the friction is caused by differences in Poisson's ratios only.

Figure 10. Normalized averaged COD dependence on the normalized debond length $l_{d}/r_f$ in CF composite with $Vf = 0.45$. Solution without contact elements and solutions with contact elements and values of frictional coefficient $k$. All data (a) excluding $l_{1,tm}$ (b). The equations appear in the same order as the contact settings in the legend.
The normalized average COD as a function of the debond length is presented in Fig. 10 and 11 for CF composite and in Fig. 12 and 13 for GF composite. The average normalized COD, $u_{an}$, is a linear function of the debond length for debond lengths larger than $l_{deb} = 0.5r_f$.

$$u_{an} = B \frac{l_{deb}}{r_f} + D$$  \hspace{1cm} (14)

Results are almost coinciding for no-contact case and for contact elements with $k = 0$. The presence of friction reduces the slope by 5-10%. The slope is rather insensitive to $k$ change in the region $0.2 \leq k \leq 0.4$ used in calculations. The average slope in the frictional cases is very close to one and the value $B = 1$ was chosen for the fitting function in this region

$$u_{an} = \frac{l_{deb}}{r_f} + D$$  \hspace{1cm} (15)

where $D$ depends on material properties. It is the value where the fitting curve intersects the vertical (NACOD) axis. $D$ is much larger for CF case than for GF case.
Figure 12. NACOD dependence on the normalized debond length $l_d/r_f$ in GF composite with $V_f = 0.45$. Solution without contact elements, with contact elements and frictional coefficient $k$. All values (a) excluding $u_{lam}^b$ (b).

Figure 13. NACOD dependence on the normalized debond length $l_d/r_f$ in GF composite with $V_f = 0.55$. Solution without contact elements, with contact elements and frictional coefficient $k$. All values (a) excluding $u_{lam}^b$ (b).

Comparing $D$ with the value $u_{lam}^b$ for the bonded case, $l_d = 0$ considered earlier one can see that the ratio is almost the same for carbon fiber and glass fiber case (1.56 respective 1.53 for $V_f = 0.45$). For fiber content 0.55 the ratio is 1.49 and 1.51 respectively. Based on this observation we roughly assume that this ratio is material independent and equals to 1.5. Then $D$ can be written as

$$D = Xu_{lam}^b$$

$$X = 1.5$$

(16)

Substitution in (15) leads to
\begin{equation}
    u_{1am} = 1.5u_{1am}^b + \frac{f_d}{r_f}
\end{equation}

This function was used to predict the normalized average opening displacement of a fiber crack accompanied by a partial fiber/matrix interface debonding. The predictions and values from FEM for friction coefficient $k = 0.2$ and volume fraction, $V_f = 0.45$ and 0.55 are presented in Fig. 14 and they are in a good agreement.

![Figure 14. NACOD: predicted (17) and FEM with contact elements and $k = 0.2$. Volume fraction, $V_f = 0.45$ (a) and $V_f = 0.55$ (b).](image)

As expected predictions according to (17) are in a very good agreement with FEM also for $V_f = 0.5$, see Fig.15.

![Figure 15. NACOD of fiber cracks with debonds. Predicted (17) and FEM with $k = 0.2$.](image)
2.4 Interactive fiber cracks

The NACOD for non-interacting cracks (long fiber fragments) can be calculated using (15), (16). For interacting cracks, i.e. shorter fiber fragment length, the changes of the factor $X$ in (16) and the factor (slope) in front of $l_d/r_f$ in (17) are shown in Fig. 16. It can be seen in (a) that the factor $X$ does not change more than by 2%. The slope change is larger for CF composite and decreases by more than 15% for $L_f = 22r_f$. However, the deviation from the initial slope is noticeable only for $2L_f/l_d < 60$.

![Figure 16. Parameters in (15), (16) versus fiber length in composites with Vf = 55%. Average of $k = 0$ and $k = 0.2$.](image)

For interacting cracks the NACOD is smaller for carbon fiber as shown in Fig. 17. To be more specific, the NACOD decreases (2-10% depending on debond length) with decreasing fiber length. For glass fiber the NACOD is unchanged, see Fig. 14. (This is true except for $l_d = 10R_f$ and $k = 0.2$ which decreases (3%) with decreasing fiber length.)

![Figure 17. The NACOD versus fiber length for CF (a) and GF (b) composites.](image)
The crack interaction effects on NACOD reflect the interaction of stress perturbations from both fiber fragment ends. In Fig. 18 and 19 the axial fiber stress distribution in CF respective GF which is perfectly bonded to the matrix is shown for three different fiber lengths. It can be seen that for the shortest CF fiber fragment the stress plateau region does not exist and the maximum value is significantly lower than the far-field value. In contrast, the stress curve for the shortest GF reaches maximum value which is very close to the far-field value. This explains why the interaction effect on NACOD in Fig. 17 a) for CF is much larger than for GF in and Fig. 17 b). Theoretical aspects of the relationship between the average stress in the fiber and the NACOD will be analysed in next section.

Figure 18. Axial fiber stress in CF composite with Vf = 55% and ld = 0 versus normalized coordinate (a) versus axial coordinate (b).

Figure 19. Axial fiber stress in GF composite with Vf = 55% and ld = 0 versus normalized coordinate (a) versus axial coordinate (b).
The fiber volume fraction, $V_f$, in Fig. 18 and 19 is 55% but since the fiber stress is only weakly dependent on $V_f$ the conclusions hold also for other volume fractions.

The difference in the decrease of peak value of the axial fiber stress between carbon fiber and glass fiber can be explained by the higher ratio of the axial fiber modulus and the matrix modulus in the CF case. For CF the ratio is 500/3 and for GF the ratio is 70/3. For high modulus ratio the distance to reach the far-field fiber stress is larger. Hence the stress in glass fiber should reach the far-field value in a shorter distance than in the carbon fiber, which is confirmed by Fig. 18, 19. The axial fiber stress distribution in a partially debonded fiber in a composite with $V_f=0.55$ is shown in Fig. 20, 21 together with distribution in the bonded case for carbon fiber respective glass fiber. The small vertical markings indicate where the debond starts/ends.

![Graphs showing axial fiber stress in carbon and glass fibers](image)

(a) Carbon fiber stress distribution
(b) Glass fiber stress distribution

Figure 20. Axial fiber stress in carbon fiber as a function of axial coordinate.

![Graphs showing axial fiber stress in carbon and glass fibers](image)

(a) Carbon fiber stress distribution
(b) Glass fiber stress distribution

Figure 21. Axial fiber stress in glass fiber as a function of axial coordinate.
As one can see the axial stress in the debonded region is not equal to zero even if the coefficient of friction is zero as in the presented calculation. The axial stress starts to increase before the z-coordinate reach the debond crack tip. In the presence of frictional shear stresses at the interface the average fiber stress increase rate is proportional to the shear stress. For example constant shear stress in the friction zone would lead to linearly increasing fiber stress.

3. Assessment of shear lag model in its potential for NACOD calculation
In this section we will perform simple theoretical analysis to reveal the relationship between the average stress in the fiber fragment and the normalized average crack opening displacement which is the subject of our study. Then the obtained relationship will be used for a particular case when fiber stress distribution is given by shear lag model. The NACOD calculation will be performed according to this model and the obtained results will be analysed in comparison with FEM based values discussed in the previous section.

3.1 Theoretical aspects
The deformation of the fiber fragment (fiber region between two breaks) can be visualised as shown in Fig. 22 for a perfectly bonded fiber case.

As the result of solution we obtain the stress, strain and displacement distributions. The axial displacement distribution in the fiber and in the matrix is denoted respectively by

\[ u_f(r,z) \quad u_m(r,z) \]  

(18)
The values of these displacements at \( z = L_f \) are

\[
\begin{align*}
    u_f^0(r) &= u_f(r, z = L_f) \\
    u_m^0 &= u_m(r, z = L_f)
\end{align*}
\] (19)

At \( z = L_f \) the axial displacement in the matrix \( u_m^0 \) is constant whereas the displacement in the fiber has a certain profile \( u_f^0(r) \) and

\[
    u_t(r) = u_m^0 - u_f^0(r)
\] (20)

is the crack opening displacement (COD) introduced in section 1.

The objective of this section is to find simple relationship between the average normalized COD (NACOD) and the average fiber stress. Using the strain displacement relationships

\[
\begin{align*}
    \frac{\partial u_m}{\partial z} &= \epsilon_{mz} \\
    \frac{\partial u_f}{\partial z} &= \epsilon_{fz}
\end{align*}
\] (21)

We can write

\[
\begin{align*}
    u_f^0(r) &= \int_0^{L_f} \epsilon_{fz}(r, z)dz \\
    u_m^0 &= L_f \epsilon_c
\end{align*}
\] (22) (23)

where \( \epsilon_c \) is the average strain in the composite (equal to the average matrix strain). From (20), (22) and (23)

\[
    u_t(r) = L_f \epsilon_c - \int_0^{L_f} \epsilon_{fz}(r, z)dz
\] (24)

Integration over \( r \) leads to the following expression for average COD

\[
    u_t(r) = L_f \epsilon_c - \frac{2}{r_f^2} \int_0^{r_f} rdr \int_0^{L_f} \epsilon_{fz}(r, z)dz
\] (25)

Substituting in (25) the \( \epsilon_{fz} \) expression via stresses
\[ \varepsilon_f = \frac{\sigma_f}{E_f} - \frac{V_{fz}}{E_f} \sigma_f - \frac{V_{fn}}{E_f} \sigma_f \]  

(26)

In the particular case when only the axial stress is kept in (26) we obtain

\[ u_{1a} = L_f \varepsilon_c - \frac{1}{E_f} L_f \sigma_f^{av} \]  

(26)

The final expression for the average COD is

\[ u_{1a} = L_f \frac{\sigma_f^0 - \sigma_f^{av}}{E_f} \]  

(27)

In (27) \( \sigma_f^0 \) is the far-field stress in the fiber. Index z is omitted in the notation for the average stress. From (27) using (5) we obtain for NACOD

\[ u_{1am} = \frac{L_f}{r_f} \left( 1 - \frac{\sigma_f^{av}}{\sigma_f^0} \right) \]  

(28)

which is the searched relationship between the NACOD and the stress in the fiber.

In order to use (28) we have to use a stress solution obtained numerically or analytically. As an example we choose the simple version of the shear lag model presented by Cox [3] and in a more modern way explained in many text books, see [9] for example.

The stress distribution in the fiber considered in the coordinate system shown in Fig. 22 is given by

\[ \sigma_f(z) = E_f \varepsilon_c \left( 1 - \cosh \frac{\beta z}{L_f} \right) \]  

(29)

In (29) the shear lag parameter is defined as

\[ \beta = \sqrt{\frac{2G_m}{E_f r_f^2 \ln \left( \frac{R}{r_f} \right)}} \]  

(30)
In (30) $G_m$ is the shear modulus of the matrix and $R$ is related to the fiber volume fraction in the composite via

$$V_f = \left( \frac{r_f}{R} \right)^2 \quad (31)$$

Another form of the fiber stress which includes a dimensionless shear lag parameter

$$\bar{\beta} = \sqrt{\frac{2G_m}{E_f \ln \left( \frac{R}{r_f} \right)}} \quad (32)$$

is

$$\sigma_f(z) = E_f \varepsilon_c \left[ 1 - \frac{\cosh \bar{\beta} \frac{z}{r_f}}{\cosh \bar{\beta} \frac{L_f}{r_f}} \right] \quad (33)$$

From (33) follows expression for average fiber stress

$$\sigma_f^m = \sigma_f^0 \left[ 1 - \frac{r_f}{\beta L_f} \tanh \bar{\beta} \frac{L_f}{r_f} \right] \quad (34)$$

Using the average stress expression (34) in (28) we obtain

$$u_{\text{tan}} = \frac{1}{\bar{\beta}} \tanh \bar{\beta} \frac{L_f}{r_f} \quad (35)$$

Equation (28) is applicable also in the case of partially debonded interface. The stress distribution in the fiber is certainly different and more complex. Here we consider a very simple approach based on the analysis of the stress distributions presented in (20), (21). It appears that if the friction is not included the stress in the fiber in the debonded zone is almost zero. Assuming it zero in the debonded zone and calculating the stress in the bonded zone using the same shear lag model just for a fiber fragment equal to the bonded part we obtain
\[ \sigma_f(z) = E_p \varepsilon_c \left( 1 - \frac{\cosh \beta \frac{z}{r_f}}{\cosh \beta \frac{L_f - l_d}{r_f}} \right) \]  

(36)

The average stress in the fiber (including the part with zero stress) may be calculated leading to the following expression for NACOD

\[ u_{tan} = \frac{l_d}{r_f} + \frac{1}{\beta} \tanh \left( \frac{L_f - l_d}{r_f} \right) \]  

(37)

3.2 Simulation results and analysis

We start the analysis with the stress distribution given by the shear lag model. In Fig. 23 the stress distribution profile according to FEM (on the symmetry axis) and according to FEM are shown. The stress growth rate is different (lower when the shear lag model is used) but surprisingly the distance over which the stress is recovered is very similar for long and for short fibers. This is an indication that the very simple shear lag models may lead to rather good predictions in cases where the behaviour “in average” is of importance.

Figure 23 Stress distribution in a CF fiber close to the fiber crack in a composite with \( V_f = 0.55 \) in the perfectly bonded case. Long fiber fragment (a); short fragment (b).

The trends are the same in the glass fiber case presented in Fig. 24. The stress transfer zone which in a glass fiber case is much smaller than for carbon fiber may be accurately estimated using the shear lag model.
Figure 24 Stress distribution in a GF fiber close to the fiber crack in a composite with $V_f = 0.55$ in the perfectly bonded case. Long fiber fragment (a); short fragment (b).

The stress distribution in the partially debonded fiber fragment using the above described shear lag solution (zero stress in the debonded zone and in the bonded zone the stress is calculated according to the shear lag model with fiber length reduced by the debond length) is shown in Fig. 25.

Figure 25. Fiber stress distribution in a partially debonded fiber with length $L_f = 22r_f$ with the debond length $l_d = 10r_f$ for glass fiber (a) and carbon fiber (b) composite with $V_f = 0.55$.

The stress distribution in the short GF case, Fig 25 (b), is described rather accurately by the shear lag model. As discussed before it is related to shorter stress transfer zone due to smaller fiber and matrix longitudinal modulus ratio. In
the carbon fiber case, Fig 25 (a) the accuracy is much lower. In both cases the stress level in the middle of the short fiber fragment is overestimated.

The average normalized COD (NACOD) calculated in the perfect bonding case, see (35), is given in Table 2 for glass fiber. The shear lag model overestimates the NACOD by 30-30%. The accuracy of predicted NACOD in CF can be evaluated analysing data in Fig. 26.

![Figure 26. The normalized average fiber crack opening in carbon fiber in composite with $V_f = 0.55$ in the perfect bonding case ($l_d = 0$) calculated using the shear lag model.](image)

The used shear lag model in this case overestimates the NACOD values by about 30%. This result is not very promising because the stiffness reduction simulated using the shear lag values would also be overestimated by 30%. However, in practice the fiber breaks are followed by debonds and the assessment of the accuracy of the shear lag model in this case is most important.

![Figure 27. The average normalized COD dependence on the debond length in fiber with length $L_f = 90r_f$.](image)
The NACOD calculated according to (37) is shown in Fig. 27. The slope of the debond length dependence is the same as in the fitting expression (17) and the accuracy is acceptable. The error is not larger than 10%. Using the shear lag model the NACOD in the large debond region is underestimated. The explanation of this trend lies in the stress distributions shown in Fig. 25 where one can see that the average fiber stress values from the shear lag model are higher than the FEM values. The difference is larger in the carbon fiber case.

Finally, we can assess the shear lag model in its ability to describe the effect of the fiber length on the NACOD values. The results presented in Fig. 28 show that the interaction is slightly underestimated by the shear lag model and the NACOD values are too low. However, the fiber length starting with which the length effects are noticeable is described adequately.

![Figure 28. The dependence of the average normalized COD on the length of the fiber fragment in carbon fiber composite with $V_f = 0.55$. Results from FE analysis and the shear lag model.](image)

The results presented in this section show that analytical models have a good potential in describing the averaged parameters of the stress state. Even the used very simple shear lag model was able to describe the main trends in the average stress and in the average crack opening displacement. More sophisticated models would give more accurate numerical values but at the expense of more complex calculations.

4. Conclusions
In the previous paper it was shown that the stiffness reduction in unidirectional composites due to increasing number of fiber breaks with a partial interface debonding can be analyzed using exact expressions containing the normalized average displacements of the fiber crack faces. The average crack opening
displacement in a normalized form (NACOD) is a robust parameter which depends on the fiber and matrix properties, fiber content in the composite and the fiber fragment and the debond length.

These relationships in this paper are established using FEM based parametric analysis. The data are described by simple fitting functions which excellently fit the numerical results. For example it was found that the fiber crack average opening is a linear function of the debond length.

Using the FEM values as the “correct” values the potential of simple analytical models in NACOD determination is assessed, relating the average COD to the average stress in the fiber fragment. Using a simple shear lag model it is demonstrated that the qualitative trends of NACOD dependence on geometrical and material parameters can be described fairly well whereas numerical values have 20-40% error. It is concluded that improved analytical models would render sufficient accuracy to calculate the NACOD needed for stiffness predictions in UD composites.

5. References
Paper D
Modeling energy release rate for debond crack growth along fiber in UD composites with broken fibers

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Abstract
The strain energy release rate related to debond crack growth along the fiber/matrix interface in a broken fiber in a unidirectional composite is analysed using FEM and also using simple analytical considerations based on self-similar debond crack propagation. The analysis is based on a model with axial symmetry consisting of three concentric cylinders: broken fiber in the middle is surrounded by matrix cylinder which is embedded in a large block of effective composite with properties calculated using rule of mixtures and Halpin-Tsai expressions. It is shown that the fiber elastic properties have a huge effect on the released energy, whereas fiber content in the composite has effect only if the debond is small. The interaction between debonds approaching from both fiber fragment ends is investigated and related to material properties and geometrical parameters. It is shown that the self-similar debond propagation model gives rather good results but the they are slightly overestimating the strain energy release rate which may be partially explained by interaction effects not included in the analytical model.

1. Introduction
Since the fiber strain to failure in polymer matrix fiber reinforced unidirectional (UD) composites is lower than the matrix strain to failure the first failure event in tensile loading in these composites is fiber failures. Due to statistical distribution of fiber strength, different positions along the fiber have different strength values. In result of stress transfer over the fiber/matrix interface the stress level in the fiber after a certain distance from the fiber crack recovers its far-field value and multiple fiber breaks in the same fiber are possible with increasing load.

Very often the fiber failure is an unstable phenomenon and the energy released during this event is larger than required. The excess of energy may go to initiation of the fiber/matrix debond at the tip of the fiber crack which is assumed to be a penny-shaped cracks transverse to the fiber axis. The fiber debonding can be considered as an interface crack growth along the fiber and fracture mechanics may be used for the crack evolution analysis. In other words the debonding is a creation of free fiber surface growing along the fiber surface in the axial direction. The debond initiation (transition from “no debond” state to “debond” state) is very
complex process and due to lack of relevant information it is not suitable for fracture mechanics treatment. For long debonds there is a very complex stress state at the fiber crack, plateau region away from the fiber crack and away from the debond crack tip and the debond tip region with stress singularity. Due to further debond crack growth the plateau region becomes larger. Long debond cracks propagate in a self-similar manner meaning that when the crack grows the local stress profile at the crack front shifts along the fiber axis without changes in the shape and in the value. For very long debonds (and short fiber fragments) the debonds from both fiber ends start to interact and the self-similarity is lost.

In order to analyse the interface crack growth by fracture mechanics the energy release rate related to debond propagation has to be calculated. It has been done previously for debond growth analysis along a single fiber fragment in so called single fiber fragmentation (SFF) test. The used methods cover a wide spectrum from approximate analytical to numerical based on finite elements (FE) or boundary elements (BE) [1-7]. The variational model based on minimization of the complementary energy [1] is probably one of the best analytical solutions but the accuracy is achieved in rather complex calculation routine. The most careful numerical analysis of the local stress state at the debond cracks tip in terms of stress intensity factors and degree of singularity has been performed in [6] using BE method. Unfortunately this method at present is limited to isotropic constituents and, hence, not applicable for carbon fibers. Generally speaking, most of the described approaches may be adapted for dealing with partially debonded broken fiber surrounded by matrix in a composite. However a systematic parametric analysis of the energy release rate due to debond growth in composite as affected by constituent properties, geometrical parameters is not available.

The objective of this paper is to perform the abovementioned parametric analysis using FE and to identify the most significant parameters influencing the strain energy release rate. A three concentric cylinder assembly model is considered. A broken fiber in the middle is surrounded by a matrix cylinder and the interface is partially debonded, see Fig.1. This fiber /resin block is embedded in outer “effective composite” cylinder.

The energy release rate is calculated using the virtual crack closure technique [8]. The solution in the plateau region is compared with simple exact solution based on the self-similar crack propagation in this region.
2. Mode II energy release rate $G_{II}$

2.1 The crack closure technique

The unit cell of the composite with a partially debonded fiber is shown in Fig.1. The radius of the transversally isotropic fiber is $r_f$. The outer radius of the matrix cylinder is directly related to the fiber content in the composite by

$$V_f = \left( \frac{r_f}{r_m} \right)^2$$  \hspace{1cm} (1)

To represent the infinite effective composite surrounding the fiber/matrix unit, the outer radius of the cylinder assembly $R$ is large.

The main statement in the crack closure technique [8] is that the energy released due to the crack surface growth by $dA$ is equal to the work required to close this newly created surface from size $A + dA$ back to size $A$. For the debond crack

$$dA = 2\pi r_f dl_d$$  \hspace{1cm} (2)

![Figure 1. Schematic showing the three cylinder geometry with the partially debonded fiber in the middle. The stress distribution in the front of the debond crack and the displacement profile behind the crack are indicated.](image)

In the particular case of the debond crack the radial stresses on the fiber surface are compressive. It is due to larger Poisson’s ratio of the matrix and also due to higher thermal expansion coefficient (if thermal stresses are accounted for). This means that the crack propagation in the analysed problem is in Mode II. Effects related to friction at the interface are neglected in the present analysis.
Points at the debonded surfaces of a crack with size $l_d + dl_d$ are sliding with respect to each other the relative displacement being

$$u^{l_d+dl_d}(z, r = r_f) - u^{l_d+dl_d}_{zzz}(z, r = r_f) \quad z \in [0, l_d + dl_d]$$

(3)

To close the crack by $dl_d$ we have to apply an increasing tangential traction at each point $z \in [l_d, l_d + dl_d]$ to move it back by $u^{l_d+dl_d}(z)$. When it is done the value of the traction in point $z$ equals to $\sigma^{l_d}_{zz}(z)dz$, where $\sigma^{l_d}_{zz}(z)$ is the shear stress in front of a crack with size $l_d$. The work performed equals to

$$dW = \frac{1}{2} u^{l_d+dl_d}(z)\sigma^{l_d}_{zz}(z)dz$$

(4)

In the virtual crack closure technique the assumption is that the sliding displacement field at the tip of the crack with size $l_d + dl_d$ is the same as at the tip of the debond with size $l_d$

$$u^{l_d+dl_d}(z) = u^{l_d}(z - dl_d)$$

(5)

This assumption is based on the assumed self-similarity of the crack growth between $l_d$ and $l_d + dl_d$ which is a good assumption as long as $dl_d$ is “small”. The benefit of this assumption is that only one stress state calculation for a given debond length $l_d$ is required. From (4) and (5) follows expression for the work performed to close the crack by $dl_d$

$$W = 2\pi \gamma_f \frac{1}{2} l_d^{l_d+dl_d} u^{l_d}(z - dl_d)\sigma^{l_d}_{zz}(z)dz$$

(6)

Changing the origin to $l_d$ by introducing $z' = z - l_d$, equation (6) turns to

$$W = 2\pi \gamma_f \frac{1}{2} dl_d^{l_d} u^{l_d}(z' + l_d - dl_d)\sigma^{l_d}_{zz}(z' + l_d)dz$$

(7)

The energy release rate ($G$) is defined as $G_{II} = \frac{W}{2\pi \gamma_f dl_d}$ which using (7) gives
In numerical calculations $dl_d$ is usually finite and the calculated energy release rate value depends on the integration distance $dl_d$. The calculated value is called “energy release rate over distance $dl_d$”. This quantity denoted $G_d^{dl_d}$ is analysed in the present paper.

### 2.2 Self-similar debond crack growth region

If the debond length is several times larger than the fiber radius, the stress state at the fiber crack is not interacting with the stress state at the debond crack tip. Additionally assuming that the fiber fragment is long enough and the interaction with the debond approaching from the other end of the fiber is negligible one may state that the debond is growing in a self similar manner. It means that due to debond growth by $dl_d$ the stress perturbation region at the debond tip is shifted in the $z$-direction by $dl_d$ without any changes in the stress profiles and values. From other hand the complex stress state region at the fiber crack tip remains unchanged by this increase of the debond. Thinking in terms of the change of the strain energy of the whole system we can observe that the energy change analysis is very straightforward: a region with the volume $\pi R^2 dl_d$ which previously had the strain energy as for long three cylinder assembly with perfectly bonded interfaces is now replaced by the same volume where the fiber cylinder is separated (debonded) from the rest of cylinders. Denoting the strain energies for these two states by indexes 0 and 1 we obtain

$$\Delta U = U_1 - U_0 = \frac{\pi R^2 dl_d}{2} \varepsilon_0^2 (E_1 - E_0)$$  \(9\)

In (9) $E_0$ and $E_1$ is the elastic longitudinal modulus of the considered part of the cylinder assembly in the initial state (perfect bonding) and in the final state (debonded fiber). The elastic modulus change between two cases may be calculated using FEM but there is also an exact analytical solution available [9,10,11]. This solution is used to calculate the elastic moduli. Additional assumption made is that in the debonded case the radial stresses due to the presence of the debonded fiber inside the assembly may be neglected. The strain energy of the debonded fiber in the zero friction case is equal to zero. The strain energy release rate in the self-similar debond growth region is calculated as

$$G_d(l_d) = \lim_{dl_d \to 0} \frac{1}{2 dl_d} \int_0^{dl_d} \left[ u^b(z' + l_d - dl_d) \sigma^b_{zz}(z' + l_d) \right] dz$$  \(8\)
\[ G_{II} = \frac{\Delta U}{2\pi r_f dl_d} \]  
(10)

leading to

\[ G_{II} = \frac{R^2}{4r_f} \varepsilon_0^2 (E_0 - E_1) \]  
(11)

As an alternative to the concentric cylinder assembly model the rule of mixtures can be used to calculate elastic moduli in (11).

3. Results and Discussion

Calculations were performed for carbon fiber and for glass fibers in polymeric matrix. The used elastic properties of the matrix are

\[ E_m = 3 \text{ GPa} \quad \nu_m = 0.4 \]  
(12)

The isotropic glass fiber has properties

\[ E_f = 70 \text{ GPa}, \quad \nu_f = 0.2 \]  
(13)

The elastic properties of the transversally isotropic carbon fiber are as follows

\[ E_{ll} = 500 \text{ GPa}, \quad E_{lt} = 30 \text{ GPa}, \quad G_{llT} = 20 \text{ GPa}, \quad \nu_{llT} = 0.2, \quad \nu_{ltT} = 0.45 \]  
(14)

The properties of the effective composites were calculated using the rule of mixtures for longitudinal modulus and Poisson’s ratio and Halpin-Tsai relationships for transverse modulus and shear modulus.

The fiber radius used in calculations was \( r_f = 4 \mu m \). The thickness of the matrix cylinder was calculated from the fiber volume fraction in the composite using (1) and is varying with \( V_f \). The thickness of the effective composite cylinder was \( 5r_f = 20 \mu m \).

FEM calculations were performed on one half of the fiber fragment using the commercial code ANSYS in an axi-symmetric formulation. The PLANE82 plane element, which is a 2-D, second order element with relatively high accuracy was used in a non-uniform mesh consisting of both triangular and rectangular elements. To obtain higher accuracy a refined mesh (of triangular elements) was used in the vicinity of the crack tip and at the end of the debond zone.
Symmetry condition was applied on \( z = 0, \ r \in [r_f, R], \) where \( R \) is the outer radius of the fiber-matrix-composite system. The axial symmetry is with respect to the \( z \)-axis. Displacement in nodes on the side \( r = R, \ z \in [0, L_f] \), are coupled in the \( r \)-direction. \( (L_f = 90r_f) \) is the length of the fiber-matrix-composite system in the axial direction which represents one half of the distance between two fiber cracks which is \( 2L_f. \) Constant displacement is applied in the \( z \)-direction at \( z = L_f, \ r \in [0, R] \). The applied axial strain to the assembly was \( \varepsilon_0 = 1\% . \)

An example of one of the quantities needed in order to calculate \( G_{II} \) is shown in Fig. 2. Axial coordinate \( z = 20 \mu m \) corresponds to the debond tip. It can be seen that the displacement of the fiber (in the axial direction) is almost the same along the whole fiber surface in the studied coordinate interval. However, the tangential displacement of the surface of the matrix at the tip of the debonded zone is twice as large as the displacement far from \( (r_f/2) \) the tip of the debond zone. Accordingly, the strong coordinate dependence of the relative motion of the fiber and the matrix \( (u_f - u_m) \) at the fiber/matrix interface with the axial coordinate is due to the displacement of the surface of matrix (at the fiber/matrix interface).

![Figure 2. Sliding displacements at both debond faces and the relative sliding given by (3). Carbon/epoxy composite with \( V_f = 0.55, \ 2L_f = 180r_f, \ l_d = 5r_f, \) friction coefficient \( k = 0 . \)](image)

The energy release rate \( G_{II} \) was calculated according to (8) and its dependence on the length of the integration region \( dl_d \) is shown in Fig.3. The results for carbon/epoxy and \( V_f = 0.45 \) are presented versus the normalized integration distance \( dl_d / r_f \). For small values of the ratio the calculated values decrease. This is the result of insufficient accuracy of the used mesh in the local singular
stress state region. In other words FEM is not the best tool to analyse local stress singularities. For large values of the ratio the integration region is far outside the local stress region and the calculated values do not have the meaning of strain energy release rate for a small crack increase. This means that the optimum value is somewhere in between. As a compromise the value corresponding to $dl_d / r_f = 0.1$ has been used to calculate $G_{II}$ throughout this paper.

In Fig. 4 the energy release rate $G_{II}$ when composite volume fraction $V_f = 0.45$ is compared with $V_f = 0.55$ for carbon/epoxy and glass/epoxy. It can be seen that for medium to large debond lengths $G_{II}$ is about the same independently of $V_f$. For short debond lengths, close to the thickness of the fiber, the values of $G_{II}$ for the two different fiber volume fractions are different.

![Figure 3. Strain energy release rate $G_{II}$ as the function of the integration length $dl_d / r_f$ for long fiber carbon/epoxy with $V_f = 0.45$.](image)

![Figure 4. Strain energy release rate $G_{II}$ versus normalized debond length $l_d / r_f$ in composites with $V_f = 0.45$ and 0.55 for carbon/epoxy (a) and glass/epoxy (b).](image)
For both materials the strain energy release rate is larger in this region and the increase is larger for composites with higher fiber content.

The fiber in this calculation was sufficiently long \((L_f = 90r_f)\) insuring that the crack does not interacting with the symmetrical crack on the other end of the fragment.

In Fig. 5 and Fig. 6 the effect of the fiber fragment length on the calculated values of the strain energy release rate \(G_{II}\) is presented. The \(G_{II}\) dependence on normalized debond length \(l_d/r_f\) is shown for different fiber lengths for carbon/epoxy respective glass/epoxy. It can be seen in Fig. 5 that in carbon fiber composite the \(G_{II}\) decreases with increasing debond length. This rather linear trend is observed for all fiber lengths. However, the shorter the fiber is the larger is the dependence on the debond length. This is understandable because the debond length constitutes a larger part of a shorter fiber than of a longer fiber. For short fiber fragments the intact part of the fiber is much smaller and the interaction with the debond approaching from the other fragment end is larger. According to Fig. 5 there is no plateau region in the strain energy release rate which means that the interaction in carbon fiber case starts with very short debond length even for the longest fiber fragment. The decrease of \(G_{II}\) (going from very small debond length to \(10r_f\)) is in the range between 22 and 43% depending on fiber length.

![Figure 5](image_url)

Figure 5. The interaction effect on strain energy release rate \(G_{II}\) for debond growth in carbon/epoxy composite versus normalized debond length \(l_d/r_f\) for different fiber lengths when \(V_f = 0.55\).
It can be seen in Fig. 6 that the overall trend is the same for debond growth in glass/epoxy as it is for carbon/epoxy. The $G_{II}$ decreases with increasing debond length for all fiber lengths. The shorter the fiber is the larger is the dependence on the debond length. The decrease in $G_{II}$ is between 21 and 24% depending on fiber length. This means that the decrease is smaller for glass/epoxy than for carbon/epoxy. In other words, the interaction between debonds from both fiber fragment ends is smaller in glass fiber composite case.

![Figure 6. The interaction effect on strain energy release rate $G_{II}$ for debond growth in glass/epoxy composite versus normalized debond length $l_d/r_f$ for different fiber lengths when $V_f = 0.55$.](image)

To gain a deeper insight in the nature of the interaction leading to the demonstrated overall trend of $G_{II}$ decrease with increasing debond length the data from Fig. 5 and Fig. 6 are presented as function of fiber length for fixed length of the debond, see Fig. 7 and Fig. 8. It can be seen in Fig. 7 for carbon/epoxy that the $G_{II}$ decreases with decreasing fiber length for all debond lengths. The larger the debond length is the larger is the dependence on the fiber length. The reasons for that are explained above. The decrease in $G_{II}$ going from fiber length $180r_f$ to $44r_f$ is between 10 and 34% depending on the debond length.
Figure 7. Strain energy release rate $G_{II}$ for debond growth in carbon/epoxy composite versus normalized fiber length $2L_f/r_f$ for different debond lengths when $V_f = 0.55$.

The strain energy release rate in glass/epoxy composite, which can be seen in Fig. 8, follows the same trends as in carbon/epoxy. However, the dependence on the fiber length in glass/epoxy is not as strong as in carbon/epoxy. The decreases in $G_{II}$ going from fiber length $180r_f$ to $44r_f$ is between 1 and 5% depending on the debond length.

Figure 8. Strain energy release rate $G_{II}$ for debond growth in glass/epoxy composite versus normalized fiber length $2L_f/r_f$ for different debond lengths when $V_f = 0.55$.

The fact that $G_{II}$ for debond growth in glass/epoxy composite has a weaker dependence on the fiber length compared to carbon/epoxy is related to the differences in the stress distribution (plateau value) in both fibers as shown in Fig. 9 and Fig. 10. It can be seen that the decrease of the plateau value and the length
of this zone with decreasing fiber length is much smaller for glass/epoxy (Fig. 10) than for carbon/epoxy (Fig. 9). The difference of the axial fiber stress between carbon/epoxy and glass/epoxy can be explained by the difference in elastic modulus. The higher the ratio $E_{fz} / E_m$, the longer is the distance needed to reach the plateau value in the axial fiber stress. For carbon/epoxy the ratio is $500/3$ and for glass/epoxy the ratio is $70/3$. Thus, due to lower modulus the rate of the stress recovery is higher in glass fiber case which leads to smaller stress perturbation zone which in turn results in the weaker dependence for $G_{II}$ on the fiber length.

![Graphs showing axial fiber stress distribution](image1)

**Figure 9.** Axial fiber stress distribution when $V_f = 55\%$ and $L_d = 0$ in carbon fiber versus normalized axial coordinate $z/L_f$ (a) versus axial coordinate (b).

![Graphs showing axial fiber stress distribution](image2)

**Figure 10.** Axial fiber stress distribution when $V_f = 55\%$ and $L_d = 0$ in glass fiber versus normalized axial coordinate $z/L_f$ (a) versus axial coordinate (b).

Using the expression (11) for strain energy release rate which is valid in the region of self–similar debond crack growth we obtain the following $G_{II}$ values


   For carbon fiber composite:
   
   \[ V_f = 0.45 \quad G_{II} = 50 \text{ J/m}^2 \]
   \[ V_f = 0.55 \quad G_{II} = 50 \text{ J/m}^2 \]  
   \[ \quad \text{ (15)} \]
For glass fiber composite
\[ V_f = 0.45 \quad G_{II} = 7 \text{ J/m}^2 \]
\[ V_f = 0.55 \quad G_{II} = 7 \text{ J/m}^2 \]

(16)

2. Using Rule of mixtures
The results with the used accuracy coincide with results from cylinder assembly model.

The obtained values for the self-similar debond cracks do not depend on the fiber content in the composite. For long debonds this result was also obtained from FEM. The numerical values are slightly higher than obtained by FEM. One explanation for this is that according to FEM there always was an interaction between cracks lowering the \( G_{II} \) values. This interaction is not accounted for in the self-similar crack model. Another reason for differences may be that in the concentric cylinder model we have neglected the compressive radial pressure from the debonded fiber to the matrix/effective composite system. It has to be noted that the calculated values of \( G_{II} \) are proportional to the fiber modulus, see (13) and (14).

4. Conclusions
The strain energy release rate in Mode II related to fiber/matrix interface debond growth along the fiber surface in unidirectional composites is analysed using FEM. The parametric analysis performed to reveal the significance of constituent properties, fiber volume fraction fiber fragment length and the debond length on the \( G_{II} \) lead to following conclusions.

- The \( G_{II} \) is proportional to the fiber modulus and is much larger in carbon fiber composite
- The fiber volume fraction has no effect on the \( G_{II} \) for long debonds whereas for short debonds it is larger for lower volume fraction \( V_f \)
- The interaction between debonds from both fiber fragment ends decreases the \( G_{II} \) values this effect being stronger in carbon fiber composites. The difference is caused by higher stress recovery rate in glass fiber composites due to lower elastic modulus.
- The self-similar debond propagation model with strain energy changes calculated using concentric cylinder assembly solution give a good approximation of the \( G_{II} \). However, the numerical values are by 5-10% higher than obtained using FEM

The obtained results will be used to simulate debond growth during quasi-static and fatigue loading
5. References


