TRAJECTORY ESTIMATION AND CONTROL OF AUTONOMOUS GUIDED VEHICLES

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CONTENTS

The licentiate thesis comprises the following parts:


B. Andersson, U., "A trajectory estimator for autonomous guided vehicles - Based on measurements of directions to identical beacons and on the observer concept".

C. Andersson, U., "A control strategy for autonomous guided vehicles - Based on drive path specification by sequences of points".

INTRODUCTION

If a vehicle should be not just a vehicle, but an Autonomous Guided Vehicle (AGV), it is necessary that the vehicle by itself can figure out where it is in its own local universe and guide itself to some predefined position. Methods that make it possible to solve these problems, i.e. the navigation and the control of the vehicle, are presented in this thesis.

An optical anglemeter that measures angles to identical beacons is described in A together with a trajectory estimator and controllers for steering and driving a tricycle AGV. The trajectory estimator that estimates the position and the heading of the vehicle is based on stochastic models of the vehicle and the measurements to the identical beacons.

In B, a trajectory estimator is presented that is based on the anglemeter described in A and on deterministic models of the vehicle and the measurements to the identical beacons.

A control strategy for guiding the AGV along a memorized drive path is described in C. The strategy results in controllers for steering and driving a tricycle AGV. The drive path considered consists of straight line segments specified by a sequence of points.
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AGV navigation by angle measurements
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We describe an optical navigation system for the navigation and control of an autonomous guided vehicle (AGV). The navigation system consists of a low-power laser, a rotating mirror and the necessary optics. It is used to measure the angles to several identical reflective beacons. The position and heading of the AGV is recursively updated each time a valid angle is measured. It is easy to define and change the drive path which is a list of coordinates. The AGV follows straight lines between these coordinates. The system has been tested on an AGV prototype.
1 INTRODUCTION

In this paper we describe how an optical directional measuring system is used for the navigation of an autonomous guided vehicle (AGV). The directional measuring system consists of a low-power laser, a rotating mirror and the necessary optics. The measuring system is called anglemeter throughout this paper. The anglemeter is used to determine the direction to reference points, consisting of identical stripes of retro-reflective tape. The position and heading of the vehicle is determined recursively by using the angles measured to these reflectors. The vehicle used has three wheels where the single front wheel is used both for steering and traction.

For a non-moving vehicle, at least three reflectors have to be visible in order to determine the position. This requires that we know from which reflector each measured angle originates. Since the reflectors have no identity marks, we have to associate each angle with a reflector. There might also be false reflections from other objects. These false detections have to be detected and discarded.

If the initial position is unknown, there is no way that the association problem can be solved if only three reflectors are used. Therefore we have to use more reflectors than three to get redundancy in the pattern of measured angles. Our previous work concerned this topic - a method of how to find the initial position. We also made a prototype of the anglemeter and tested the system on an AGV prototype [1,2].

When the vehicle is operating in a production environment, only a fraction of the reflectors will be visible from its current position. Therefore many reflectors are needed for the determination of the position with high accuracy anywhere in the area of operation.

The main advantages of having passive reflectors of the type we use, are the flexibility and the economy. It is easy to change the transportation routes in a factory, compared with wires in the floor or painted lines. A change in the routes can be caused by a change in production or, simply, the planned route is obstructed. Since every reflector strip is inexpensive, the cost of the system is essentially proportional to the number of AGV's - not the present or future working area. The same navigation system can also be used to operate, say, a cleaning machine. The present system can be used outdoors if the surface it travels on is reasonably flat, or if the beacons are not too far away. Applications might be in mines, in agriculture, when cleaning airfields, for the positioning of a dredging boat etc.

In our current work we have taken the dynamics of the system into consideration. We have restricted our study to a single vehicle and describe the navigation system and guidance law that we use. A complete system will consist of several vehicles coordinated by a supervisory computer via radio communication.

1.1 Outline of the paper

The vehicle and the anglemeter are briefly described and analyzed in section two. We also give some suggestions on how to locate the reflectors.

Section three deals mainly with estimating the position when the vehicle is moving while angle measurements are made. In [1,2] we assumed that the speed of the vehicle was very low so three subsequent angles could be considered to be measured from the same point. The rotational speed of the anglemeter is 1 Hz. We now present a method (based on Kalman filtering) for updating the position each time an angle is measured. It is possible to use a high speed vehicle, typically 0.5-2 m/s. The association problem is solved preliminary using a windowing technique. To determine the initial position we use the method from [2]. We then give an overview of the different components in the navigation and guidance system.

In section four we turn to the control problem. The drive path of the vehicle consists of straight line segments between given points. The list of points can for instance be generated with a CAD-program on a personal computer. It is then simple to change the drive path that the vehicle is to follow.

Simulation results are presented in section 5.

2 SYSTEM DESCRIPTION

2.1 The vehicle

A picture of our vehicle is shown in fig 1. The upper part, which we call the "tower" is connected to the "frame" through thin legs leaving a 40 mm wide slit between them. This allows the anglemeter which is mounted in the center of the frame to have almost 360° of horizontal field of view. The dimensions of the vehicle are (w,l,h) 0.5m, 1.3m, 1.4m. The combined traction and steer unit is placed in front of the anglemeter in the frame. Both motors in it are of the permanent magnet 24 V DC type.
2.2 The anglemeter

There are many methods to measure angles to reflective beacons, our method take advantage of the known constraints in the measuring situation. It will gain orders of magnitude in receiver signal to noise ratio compared to other systems which use a vertical fan-shaped illuminating beam and a corresponding receiver field of view.

A picture of our prototype is shown in fig 2. A rotating mirror deflects a 1 mW laser beam to sweep in a horizontal plane. Before the laser beam hits the rotating mirror it travels through a hole in another deflecting mirror which is part of the optical receiver. The receiver is optically coaxial with the outgoing laser beam. The lens in the receiver has a focal length of 100 mm and a maximum aperture of 18 mm. The detector, which is placed on the optical axis in the focal plane of the lens, is a silicon photodiode with a diameter of 0.1 mm.

Our vehicle is moving on a reasonably flat floor, which means that we do not need to waste our illuminating laser power by spreading it out in a fan. We know at what height the laser beam will hit the wall and put our beacons at that height. This means that we can restrict the vertical field of view of the optical receiver in the anglemeter to the same order as the divergence of the illuminating laser beam. In this way we minimize the background noise and the probability of catching false beacons from strong external light sources. The divergence of the laser is 1 mrad. The discussion above is of course valid only if the scanning beam plane is parallel with the floor. The vertical length of the reflective stripes makes the anglemeter tolerate non-ideal i.e. real floors.
The improved signal to noise ratio will influence four parameters positively:

- Longer range
- Higher rotational speed of the mirror
- Narrower reflective strips
- Smaller error in measured angle

The reason to have mechanically moving parts in the anglemeter, which naturally makes it more fragile than e.g. a system built around CCD-cameras, is the precision level we want to reach. Our demand on almost 360° of horizontal field of view takes several CCD-cameras. The resulting adjustments and calibration of them and their non-ideal imaging optics will be very difficult.

The most important part of the anglemeter is the precision incremental encoder. The rotating mirror is attached to its axis. The error in measured angle to a beacon originates from:

- Errors in the incremental encoder
- Alignment errors
- Noise in the receiver
- Partly blocked beacons

The incremental encoder outputs two 90° shifted pulsetrains and a zero pulse. The pulsetrains contain 9600 pulses/revolution which gives a resolution of better than 0.2 mrad after electronic processing. The manufacturer does not state any absolute non-accuracy but we believe it is of the same order as the resolution. Its contribution to the total error is therefore negligible.

There are several alignments to be made in the anglemeter which can cause errors in the measured angle. The misalignment between the mirror rotational axis and the laser beam is the dominant one and will be discussed in some detail.

Fig 3 shows the rotating mirror, the laser beam, an attached coordinate system and the symbols which will be used. The mirror rotates around the vertical Z-axis. Its rotational angle is \( \gamma \), which also is the angle of the reflected beam with no alignment error. The incoming laser beam can be assumed to lie in the XZ-plane and to hit the mirror in the origin of the coordinate system without loss of generality. The alignment error is \( \beta \) and the error in \( \gamma \) is \( \Delta \gamma \). Observe that the reflected laser beam does not lie in the XY-plane and that \( \Delta \gamma \) is the angle between the nominally reflected beam and the actual beams projection in the XY-plane.

Let \( n_0 \) be a unit vector normal to the mirror surface when \( \gamma = 0 \).

\[
\begin{align*}
n_0 &= \frac{1}{\sqrt{2}} \cdot (1, \ 0, \ -1)^T \\
\end{align*}
\]

This can be transformed to the rotating vector \( n(\gamma) \) by premultiplying with the rotational matrix \( R(\gamma) \).

\[
\begin{align*}
n(\gamma) &= R(\gamma) \cdot n_0 \\
\end{align*}
\]

The rotational matrix is given by

\[
\begin{align*}
R(\gamma) &= \begin{pmatrix}
c\gamma & -s\gamma & 0 \\
s\gamma & c\gamma & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\end{align*}
\]

where we have used \( c\gamma \) and \( s\gamma \) as shorts for \( \cos(\gamma) \) and \( \sin(\gamma) \). This gives
\[ n(\gamma) = \frac{1}{\sqrt{2}} \cdot (c\gamma, \ s\gamma, \ -1) \]  

(2.4)

The reflection of the laser beam can be modeled with a rotational matrix \( A \), which is defined by

\[ A(\gamma) = I - 2n(\gamma) \cdot n^T(\gamma) \]  

(2.5)

This gives

\[
A(\gamma) = \begin{pmatrix}
    s^2\gamma & -c\gamma \cdot s\gamma & c\gamma \\
    -s\gamma \cdot c\gamma & c^2\gamma & s\gamma \\
    c\gamma & s\gamma & 0
\end{pmatrix}
\]  

(2.6)

A unit vector \( p_{in} \) parallel with the incoming beam can be found by inspection

\[ p_{in} = (s\beta, 0, c\beta) \]  

(2.7)

A unit vector \( p_{out} \) parallel with the reflected beam is formed by premultiplying \( p_{in} \) with \( A \).

\[ p_{out} = A \cdot p_{in} \]  

(2.8)

Which becomes

\[ p_{out} = (s\gamma \cdot s\gamma \cdot s\beta + c\gamma \cdot c\beta, -s\gamma \cdot c\gamma \cdot s\beta + s\gamma \cdot c\beta, c\gamma \cdot s\beta)^T \]  

(2.9)

Finally the projection of \( p_{out} \) in the XY-plane is \( p_{outp} \)

\[ p_{outp} = (s\gamma \cdot s\gamma \cdot s\beta + c\gamma \cdot c\beta, -s\gamma \cdot c\gamma \cdot s\beta + s\gamma \cdot c\beta, 0)^T \]  

(2.10)

this can also be written as

\[ p_{outp} = k \cdot (\cos(\gamma + \Delta\gamma), \sin(\gamma + \Delta\gamma), 0)^T \]  

(2.11)

After some manipulations where we have assumed that \( \beta \) and \( \Delta\gamma \) are small we arrive at the result

\[ \Delta\gamma = -\sin(\gamma) \cdot \beta \]  

(2.12)

This is a systematic contribution to the total error in the measured angle and might also be the dominating one if the alignment is not done with utmost care.

Our present beacons have a horizontal width of 20 mm and one can ask the following question. To what part of the beacons do we measure our angles? The mirror rotates in the positive sense so that the laser beam crosses the beacon from right to left. Due to our high signal to noise ratio we can have a rather low threshold level in the receiver without introducing false detections as a result of electronic noise. Experiments have shown that the angle is measured to a point less than 1 mm from the right edge of the reflective tape. This is at a range of 10 m and at a rotational speed of 1 Hz. Thus the contribution to the error angle from the electronic noise is negligible.

From the discussion above it follows that if the right side of a beacon is blocked due to some obstacle we will get an error which could be significant. We would be better off if the beacon were completely blocked! The error is random with a positive mean.

2.3 Reflectors maps

A large number of reflectors are needed in a production environment. It would be almost an impossible task to find the coordinates of all the reflectors with high accuracy in a global coordinate system. Therefore it is wiser to divide the area of operation into smaller areas. Different rooms in a factory is a simple example of this division. The area near a docking station is another. The approximate location of the local rooms in the global system can easily be found. The reflectors are then localized within these local coordinate systems with high accuracy. We use the anglemeter to do the localization of the reflectors.
If the reflectors are located in a local room with too much symmetry, there can be several solutions when the initial position is determined. Therefore the reflectors should be located randomly. We can however allow a symmetrical location of the reflectors in for instance a narrow corridor which the AGV only drives through, without docking at any specific points. Without knowing the actual position of the beacons, the symmetry in the measured angles could be used for navigation when the vehicle is passing the corridor.

3 ESTIMATING THE TRAJECTORY OF THE AGV

The position is updated each time a valid angle is measured. This means that all angles will be measured from different positions of the AGV. We therefore have to model the vehicle’s motion between the measurements. In this section we start by deriving a motion model which will be used in the estimator design. We have chosen a first-order model instead of the more common second order models where acceleration is taken into account. To compensate for this model simplification, we assume that the values of the speed and steer angle are contaminated with additional noise. The control laws are also designed such that the changes in the set values to the servos between two sampling instants are small. This helps to justify our reduced order model.

The motion model can be used to update the position even if there are no angles available. This will however lead to large accumulated errors. We therefore need the angles to make corrections. But the measured angles have to be associated with reflectors or discarded as false. An association method is presented where we use the estimated position from the filter.

Before the filter can be used, the initial position has to be found. The method used at present will be described at the end of this section.

3.1 Motion model

We will study planar motion for a vehicle that moves with the translational velocity \( u_1 \) and the angular velocity \( u_2 \). The location of the AGV is described by the coordinates \( x, y \) and its orientation by the angle \( \theta \) to the x-axis. In the description we have chosen the midpoint of the rear axis as reference point on the vehicle, cf. fig 4. The angle \( \theta \) is also the heading of the AGV.

The differential equations for the motion are

\[
\begin{align*}
x &= u_1 \cos \theta \\
y &= u_1 \sin \theta \\
\dot{\theta} &= u_2
\end{align*}
\]  

(3.1)

On the vehicle we can control the speed of the front wheel, \( v \), and the steer angle, \( \alpha \). In fig 4 we can see that there will be an instantaneous center of rotation for the motion of the vehicle.

![Instantaneous center of rotation](image)

\( \rho = \frac{L}{\tan \alpha} \)

\( \rho = \text{instantaneous radius of curvature} \)

Fig 4. When the steer angle is constant, the motion of a three wheeled vehicle is along a segment of a circle.

Since the vehicle is a tricycle, we will get the following relations for our input signals \( u_1 \) and \( u_2 \)

\[
\begin{align*}
u_1 &= v \cos \alpha \\
u_2 &= \frac{v}{L} \sin \alpha
\end{align*}
\]  

(3.2)
where $L$ is the distance between the steer axis and the rear axis. It is also possible to derive expressions for the input signals when the rear wheel movements are measured. In the future we will use incremental encoders to measure the wheel movements. This will increase the accuracy of the update of the motion model between the sampling instants. At present we do the assumptions that the input signals $u_1$ and $u_2$ are constant during the sampling interval. This means that we have assumed that the speed and steer angle only changes at the sampling instants and remain constant during the sampling interval, $T$. The equations of motion can be integrated between the sampling instants with constant input signals. This gives the non-linear discrete time model

$$x(k + 1) = x(k) + \frac{u_1}{u_2} (\sin(\theta(k) + u_2 T) - \sin \theta(k))$$

$$y(k + 1) = y(k) + \frac{u_1}{u_2} (-\cos(\theta(k) + u_2 T) + \cos \theta(k))$$

$$\theta(k + 1) = \theta(k) + u_2 T$$

Throughout this paper we will use $k$ to denote the value of a variable at the discrete time instants $t_i$ when sampling is made. For convenience, this will be omitted for the input signals.

The geometrical interpretation of (3.3), is that the motion is along a segment of a circle. For small changes $u_2 T$ in the heading, we can approximate the trajectory with a straight line.

$$x(k + 1) = x(k) + u_1 T \cos \theta(k)$$

$$y(k + 1) = y(k) + u_1 T \sin \theta(k)$$

$$\theta(k + 1) = \theta(k) + u_2 T$$

With a proper control law, the steer angle often will have a small variation around a nominal value needed to follow the current segment. The steer servo also has a short time constant. The neglected dynamics in the servos will introduce some uncertainties in the input signals. We will take these model errors into account by including noise signals in the discrete time model, i.e. we replace the idealization in (3.2) with the additive noise model

$$u_1 = (v + w_1) \cos(\alpha + w_2)$$

$$u_2 = \frac{(v + w_1)}{L} \sin(\alpha + w_2)$$

This can now be inserted in (3.3) or (3.4) where we, after straightforward identification of terms, can find the coefficients for the noise terms. Only the first order terms are used. For sampling intervals of 0.2 s or shorter, the numerical values of the coefficients in the two cases are practically the same. We therefore use the coefficients from the second linearized case. We will now have a model of the type

$$x(k + 1) = f_1(x(k), y(k), \theta(k), u_1) + g_{11} w_1 + g_{12} w_2$$

$$y(k + 1) = f_2(x(k), y(k), \theta(k), u_1) + g_{21} w_1 + g_{22} w_2$$

$$\theta(k + 1) = f_3(x(k), y(k), \theta(k), u_2) + g_{31} w_1 + g_{32} w_2$$

where for instance

$$g_{11} = T \cos \alpha(k) \cos \theta(k)$$

The noise signals $w_1$ and $w_2$ are assumed to be independent zero mean white noise processes with covariance matrix $Q$.

We can write this motion model in a more compact way by introducing the state vector $X = (x,y,\theta)^T$ and noise vector $w = (w_1, w_2)^T$. 

$$X(k + 1) = f(X(k), u_1, u_2) + G(k)w(k)$$

where

$$X = (x,y,\theta)^T$$

$$w = (w_1, w_2)^T$$

$$G(k) = \begin{pmatrix} g_{11} & 0 & 0 \\
0 & g_{21} & 0 \\
0 & 0 & g_{31} \end{pmatrix}$$
3.2 Measurement model

Reflector \( i \) is located at \((x_i,y_i)\). The index \( i \) is used to denote the current reflector. The position of the midpoint of the rear axis is \((x(k),y(k),0(k))\). Using fig 5 we see that the angle to this reflector is

\[ \gamma_i(k) = \theta(k) + \arctan \left( \frac{y_i - y(k) - d \sin \theta(k)}{x_i - x(k) - d \cos \theta(k)} \right) + \nu(k) \]  

(3.9)

where \( d \) is the distance between the anglemeter and the rear axis. We can write this as

\[ \gamma_i(k) = h_i(X(k)) + \nu(k) \]  

(3.10)

We assume that the sequence \( \{\nu(k)\} \) is white noise with zero mean and covariance \( R \) and uncorrelated with the process noise. The real measurement noise was analyzed in section 2.2. There are also errors in the reflector coordinates. The white noise assumption on \( \{\nu(k)\} \) has been chosen to simplify the estimator design.

![Fig 5. Principle of angular measurements and variation in the angles to four reflectors when the vehicle follows a straight line with the speed 0.5 m/s.](image)

The mirror in the anglemeter rotates with one revolution per second. Suppose that the vehicle is moving straight forward in a room with four reflectors. In fig 5 we show the variation in the angles to the reflectors. The instantaneous angle \( \gamma(t) \) of the anglemeter is also shown in the same figure. It is only possible to measure an angle to a reflector at those time instants when we have a crossing between the anglemeter curve and the corresponding reflector curve. If we get a detection or not, depends on if the reflector is obstructed or not. There can also be other objects that occasionally give false reflections. The detections will occur at a non-uniform sampling rate.

3.3 Estimator design

The position is updated with a discrete extended Kalman filter. Using the motion model (3.6) and measurement model (3.10), the design is straightforward. We will not rederive the filter equations here. They can be found in for instance [3] or [4] and in our forthcoming report [5]. In this paper we only describe the principles of the filter.

Suppose we want to estimate the state \((x,y,\theta)\) at time \( t_k \). The previous state estimate was made at time \( t_{k-1} \) and determined by the measurements up to and including that time. We denote this estimate by \( \hat{X}(k | k) \). This estimate and the motion model is used to predict the state at the time when the next sampling is made. The predicted state is notated \( \hat{X}(k+1 | k) \) and is given by (3.3), or for short sampling intervals or small steer angles by (3.4). In general we can write this as

\[ \hat{X}(k+1 | k) = f(\hat{X}(k | k),u_1,u_2) \]  

(3.11)
At this stage we have to determine from which reflector the angle originates. At present we use a preliminary association method which is described in the next section. Let us assume that we have associated the angle \( \gamma \) with reflector \( j \). The association problem is to make sure that \( j = i \), where \( i \) is the true reflector number. No matter if the association was successful or not, the predicted measurement at time \( t_{k+1} \) will be

\[
\hat{\gamma}_j(k+1 | k) = h_j(\hat{X}(k+1 | k))
\]  

(3.12)

We then correct the prediction of the state according to

\[
\dot{X}(k+1 | k+1) = \dot{X}(k+1 | k) + K(k+1)(\gamma_j(k+1) - \hat{\gamma}_j(k+1 | k))
\]

(3.13)

where \( K(k) = (K_1(k), K_2(k), K_3(k))^T \). If no angle is measured at time \( t_{k+1} \), then the gain vector is \( K(k) = 0 \), otherwise it is determined by linearizing the nonlinear system, given by (3.6) and (3.10), around a nominal value. In our case we choose the estimated state as nominal value. A Kalman filter is then designed for the linearized system. We then use the calculated filter gain on our nonlinear process.

### 3.4 Association of measured angles with reflectors

When an angle is measured, it has to be associated with a reflector or discarded. Only the angles that can be associated with reflectors are used for the update of the position. At present we use an association method based on knowledge of an approximate value of the position and orientation. This is determined using the motion model (3.11) with the set values of the control signals. The predicted angles to a number of reflectors is then determined with (3.12). If the measured angle falls into an angular tolerance band around the predicted angle to one reflector, we assume that this is the one.

This association routine will work satisfactorily if we only have 6-7 reflectors in the room. The corresponding window to reflectors close to the vehicle can be allowed to be rather large, cf. fig 5. But if it is made too large, we will have problems with false detections.

### 3.5 Estimating the initial position

In this version of the system, the initial position is determined with a method proposed in [2]. From the unknown position the vehicle will only be able to measure the angle to perhaps 60-70% of the reflectors in the room. The rest will be obstructed by different objects. The position can be calculated using triangulation if we know the angles to three reflectors. So we start with assuming that the first three angles comes from the first three reflectors. This gives a position in the room. If this position is correct, there should be reflectors located in the directions given by the rest of the angles. In this way we try all possible combinations of angles and reflectors, until we find a position where we can associate all angles with reflectors.

To reduce the number of possible combinations, we assume that the number of reflectors in a room is limited to 15. All reflectors are located on the walls so the order between them never change. We also assume that there are no false detections among the measured angles. If there is a false value, the proposed method will fail. The vehicle then has to be moved and the procedure repeated. However, when we have made tests on our experimental setup, this problem seldom occurs. When the vehicle has found the initial position, false detections cause little problem.

### 3.6 System overview

An overview of the navigation and the control system is given in fig 6. We have already described the hardware and the navigation system. We will now briefly summarize each component in the system. In the next section we will turn to the control of the AGV, i.e. how the set values of the speed and steering angle are determined. These set values are used in the position estimator to predict the position of the AGV at the time-instant when an angle is measured. The angle originates from reflector \( i \). The predicted position and the map of reflector coordinates are then used to associate the measured angle with reflector number \( j \) and correct the estimated position. The estimated position is compared with the desired position and new set values of the control signals are determined. In this figure we can note that we do not need any measurements of the rear wheels movements. This will however be implemented in a future version of the system to increase the accuracy further.
4 CONTROL LAWS

The variables that are controlled, are the speed and the steer angle of the single front wheel. The control laws are designed to take the following requirements into account:

A. The vehicle should follow the drive path.
B. The speed should be as high as possible.
C. The change of the controlled variables from one sampling to another should lie within certain limits.

The first and second points are obvious. One reason for the third requirement is that the acceleration of the vehicle can cause slipping of some wheel. The estimator does not model the dynamics in the servos or slipping. If there are large changes in the set values to the servos or slipping occurs, this may result in poor estimates of the position and heading. Another reason is that large changes of the set values to the servos cause wear of the mechanical components of the vehicle.

4.1 Guidance control

The drive path is given in the memory of the onboard computer as a list of coordinates $(x, y)$. It is defined by drawing straight lines between neighboring coordinates [6]. One such line is referred to as a segment. Since the vehicle is supposed to follow the segments, it is natural to transform the position and heading received from the estimator to a position $(x_s, y_s)$ and heading $\theta_s$ in a local coordinate system around the segment $s$ which the vehicle is following. $x_s(k)$ is the orthogonal projection of the position of the front wheel onto segment $s$, $y_s(k)$ is the perpendicular distance from the segment to the front wheel and $\theta_s(k)$ is the heading of the vehicle relative to the segment. This information is used in the guidance controller. The set value to the steer servo is given by

$$\alpha_s(k) = A\tan 2(-y_s^*(k), d_s) - \theta_s^*(k)$$

The interpretation of (4.1) is that the steer wheel is aimed at a point on the segment which is the distance $d_s$ away from the projection $x_s^*$. The aiming point is chosen such that large changes in the set value of the steer angle are avoided. $\alpha_{max}$ is the maximum allowed steer angle.

When the vehicle starts to follow the drive path, the distance $d_s$ is small if the vehicle is far away from the segment so that the vehicle will travel almost perpendicular to it. When the vehicle is close to the segment and almost parallel to it, $d_s$ can not be too small. If that is the case, or the speed compared to the sampling time is large, there will be oscillatory behavior in the control signal since the control is done in discrete time.
The way we define the drive path has the advantage that it is easy to design and that it will occupy a small amount of memory in the computer. The disadvantage is the sharp corners between the segments. It is not desirable that the vehicle makes such sharp turns. The vehicle starts to follow the new segment before the end of the old is reached to prevent overshoots in the path the vehicle is taking. If we call the distance from the end of the segment where the vehicle starts to follow the new segment \( r^* \) and the length of the segment \( l \), we have the condition for switching between segments

\[
x^*_r(k) \geq l - r^*
\]  

(4.2)

The angle between segment \( s \) and segment \( s+1 \) is \( \lambda_r \). \( r^* \) is calculated as

\[
l^* = c \cdot |\lambda_r|, \quad -\pi \leq \lambda_r \leq \pi
\]  

(4.3)

where \( c \) is chosen such that the vehicle makes a smooth turn when it starts to follow the new segment, \( s+1 \). To prevent a large change of the steer angle when a switch to a new segment is made, \( d \), depends on how the vehicle is located relative to the new segment and also on the steer angle at the time instant when the segment change is made. Fig 7 visualizes (4.1) and shows the aiming distance as a function of \( y^* \).

Fig 7. Here equation (4.1) is visualized. The guidance controller calculates at each sampling instant a set value to the steer servo such that the steer wheel is aimed the distance \( d \), away from the projection of the vehicle’s position on the segment. In this figure we see the vehicle at the sampling instant when it leaves segment \( s-1 \) and starts to follow segment \( s \). The segments are defined in the global coordinate system by the coordinates \((x_{s-1}, y_{s-1}), (x_s, y_s)\) and \((x_{s+1}, y_{s+1})\). To the right we can see the aiming distance \( d \), on segment \( s \) as a function of the front wheel’s perpendicular distance to it.

In fig 7 we see the vehicle at the sampling instant when it leaves segment \( s-1 \) and starts to follow segment \( s \). The aiming distance is chosen at this sampling instant such that it is the distance between the projection \( x^*_r(k) \) on the new segment, (in this case segment \( s \)), and the point where the tangent of the front wheel crosses the x-axis of the new segment before the set value has been changed. This value of \( d \) is called \( d_{\text{aim}} \) in the right figure, where we can see the aiming distance \( d \), on segment \( s \) as a function of the front wheel’s perpendicular distance to it. \( y^*_{\text{aim}} \) is the value \( y^*_r(k) \) when the vehicle starts to follow the new segment. \( d \), can be positive or negative depending on how the vehicle is located relative to the segment and the steer angle when a switch is made to a new segment. \( d_{\text{aim}} \) is the aiming distance when the perpendicular distance to the segment is zero. When the steer angle and heading to the segment are small one can linearize and discretize the closed loop system, assuming no dynamics in the servos. It turns out that \( 1/d \) and \( 1 \) are the feedback parameters in a state-feedback regulator. \( d_{\text{aim}} \) is the value of \( d \), that gives the linearized closed loop system proper poles. Choosing \( d_{\text{aim}} \) as in the right figure prevents large changes in the steer angle at switching time and at any other sampling instant if the sampling time compared to the speed is low.

### 4.2 Speed control

The linear and the angular accelerations of the vehicle depend on the steer angle, the speed of the front wheel and their derivatives with respect to time. One possible control strategy is to limit the acceleration to prevent slipping by choosing suitable set values to the servos. We have chosen to calculate the set value to the steer servo according to (4.1), which leaves the set value to the speed servo as the only
remaining variable left for limiting the acceleration. The guidance controller is designed such that the derivative of the steer angle is low. This property cooperates with the ambition of the speed controller to limit the acceleration, and also allows a higher speed since the acceleration partly depends on the derivative of the steer angle.

One can use models of the vehicle and the servos to predict the acceleration that a certain set value would give rise to and chose a set value to the speed servo that will not cause slipping. However, estimating the acceleration is not an easy task. Therefore we limit the acceleration using the rule of thumb.

The set value to the speed servo is calculated as a function of the set value to the steer servo as shown in fig 8. The angular velocity of the vehicle gives rise to forces acting perpendicular to the wheels. If these forces exceed certain values, slipping occurs. Therefore the speed has to be reduced when the steer angle is large, at least when the maximum allowed speed is high.

There is also a restriction in the change of the set value to the speed servo from the previous sampling instant to the current sampling instant according to

\[ |v_r(k) - v_r(k-1)| \leq \Delta v_{\text{lim}} \]  

This is to limit the acceleration of the vehicle. This function is mainly active at start-up. At that time the speed is 0 m/s, and if the steer angle is 0°, the set value to the speed servo will be maximum speed according to fig 8. This can lead to slipping of the front wheel.

Calculation of the set value to the speed servo according to fig 8 and (4.4) guarantees that the speed control is smooth if the guidance control is smooth.

5 SIMULATION RESULTS

The navigation and the control system have been simulated and tested on our AGV prototype with the simulation and control program REGSIM, which is developed at Luleå University of Technology [7]. When simulating the system, we assumed that the speed and steerwheel servos in fig 6 were first order systems. The time constant was set to 0.2 s for the speed servo and 0.1 s for the steerwheel servo. The set value of the speed was set to 0.5 m/s. Equations (3.1) and (3.2) were used to simulate the vehicle dynamics.

The room has the dimensions 9x5.5 m². There are eleven reflectors on the walls, marked as dash-dotted lines in the fig 9, and five tables that the AGV should avoid to move. Since the mirror rotates with 1 Hz, the mean time between angle measurements would be approximately 0.1 s if only a few of the reflectors are obstructed. To simplify the simulation routines we assumed that angles were measured with the sampling interval 0.1 s. In tests on the prototype we have a non-uniform sampling interval.

In fig 9 and fig 10 we see the result from two different simulations. We assumed that the initial position had 50 mm error in the x- and y-coordinates and 0.05 rad error in the heading. In reality the initial position will be determined with high accuracy since it is determined using a lot of angles measured from one point.

In fig 9 we see the result when no angles were used for the update of the position. Neither the initial errors nor the accumulated errors due to model errors were removed. The estimated trajectory of the AGV is shown with a dashed line. We can also see the function of the guidance controller. The initial position is not on the first segment. The guidance controller will make the estimated position follow a smooth path until it reaches the segment. A change to the next segment is made before the end of a segment is reached. The control laws for guidance control and speed control are further described in [8].

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In fig 10 we see the result when measured angles were used to correct the estimated position. We assumed that the measured angles had been associated with the correct reflectors. Errors of the same magnitude as the assumed initial errors can however occur when the AGV is moving if an erroneous association is made. In this simulation the initial errors are reduced. This indicates that the system can handle up to around 10% false detections and erroneous associations. The errors in position is less than 50 mm during the 30 s simulation. The errors can be reduced by increasing the rotational speed of the anglemeter, i.e. have a shorter sampling interval, and measuring the rear wheel movements.

**Fig 9.** The estimated and simulated trajectory differs when no angles are used for the update of the position.

**Fig 10.** In this figure angles are used. We can also see how the vehicle follows and changes between segments.

6 SUMMARY

We have described an optical navigation system for an AGV based on directional measurements to several identical beacons. The position and heading of the AGV is updated recursively each time a valid angle is measured. The drive path that the AGV should follow consists of straight line segments between given points. This gives a very flexible system where it is easy to change the transportation routes. We also described the control laws that are used for segment following, segment change and speed control.

The system has been tested on an AGV prototype in the room shown in fig 9 and fig 10. It is however difficult to present any results from these tests, since we do not have any way to measure the position of the AGV when it is moving, except using the position estimator. At present we use a personal computer for all the calculations. This increases the computational times. Therefore we have only tested the system with the speed 0.3 m/s. In a future version we will use a Motorola 68020 with a 68881 floating-point coprocessor.

Our present research concerns a thorough error analysis of the position estimator, a new method to determine the initial position and a robust association method. The anglemeter will be redesigned to enable a rotational speed of at least 5 Hz.

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8 REFERENCES


A TRAJECTORY ESTIMATOR FOR AUTONOMOUS GUIDED VEHICLES

Based on measurements of directions to identical beacons and on the observer concept

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Key Words - Autonomous Guided Vehicle; AGV; trajectory estimator; time-varying observer; measurements of directions; identical beacons; multi-output system.

Abstract - In this report, a trajectory estimator for estimating the two-dimensional position and heading of an Autonomous Guided Vehicle (AGV) is presented. The estimator is based on directional measurements to several identical beacons, and on the observer concept. The minimum number of measurements needed to stabilize the error system is three. It is shown that because of the time-varying nature of the models describing the AGV and the measurements, it is not sufficient to only stabilize the error system over each period of three consecutive measurements without considering the structure of the error system matrix. It is proved for a third order system that in addition, by selecting the gains of the observer so that the matrix of each error system is diagonal or either upper or lower triangular, it is possible to guarantee the convergence of the estimation error. Simulations of an AGV show good results.
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1 INTRODUCTION

In this report, a method to estimate the position and the heading of a vehicle in a two-dimensional space is described. The method is based on directional measurements to several identical beacons.

The models for the measurements that are used in the design of the estimator, are based on an optical anglemeter that measures angles to identical reflectors. The system is described in [1,2,3]. When identical beacons are used, the navigation becomes more complex than if beacons with identities are used. If identical beacons are used, the measured angle has to be associated to a particular reflector before the measurement can be used to estimate the position and the heading. This is not a problem if the beacons have identities. The association problem is not dealt with in this report. In [4], different association methods are described.

The estimator is based on the observer concept, which implies a deterministic approach to the estimation problem. In [3,4], an estimator based on stochastic models of the vehicle and the measurements to the reflectors is described. Because of the time-varying nature of the models describing the vehicle and the measurements, the standard method in observer design to only stabilize the error system over each sampling period by suitable gains of the observer can not be used. A method to overcome this difficulty is presented.

1.1 Outline of the report

Some standard results in the estimation theory for linear deterministic systems that are essential to this report, are presented in chapter 2. In appendix 7.1 some properties of eigenvalues of matrices are given. In chapter 3 we turn to the trajectory estimator. Section 3.1 gives models of the vehicle and the measurements. In section 3.2 the insufficiency in only assigning the eigenvalues of the error system is shown. Section 3.3 presents a method that makes it possible to overcome this insufficiency. In section 3.4 the gain calculations based on the method in section 3.3, are presented. Section 3.4 concerns the implementation of the estimator. In chapter 4, simulation results for three case studies are shown.
2 THE CURRENT OBSERVER

A deterministic discrete-time state-space representation of a linear process is

\[ x(k + 1) = F(k) \cdot x(k) + B(k) \cdot u(k) \]  \hspace{1cm} (2.1a)
\[ y(k) = H^T(k) \cdot x(k) \]  \hspace{1cm} (2.1b)

where \( x(k) \) is the \([n,1]\) state (variable)-vector of the process, \( u(k) \) is the \([m,1]\) control-vector and \( y(k) \) is the \([p,1]\) measurement-vector. If the process is time-invariant then \( F(k) = F \), \( B(k) = B \) and \( H(k) = H \). \( k \) denotes the sampling instant.

A well known technique to estimate the state-vector of a time-invariant deterministic system, is to use an observer. The type of observer that is developed later in this report is based on the so called current observer \([7,8]\), where the current measurement is used to obtain the current state estimate.

**Time-invariant systems**

The estimate of the state-vector at sampling instant \( k \), based on measurements at the same sampling instant is

\[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K \cdot (y_k - \hat{y}_{k|k-1}) \]  \hspace{1cm} (2.2a)
\[ \hat{y}_{k|k-1} = H^T \cdot \hat{x}_{k|k-1} \]  \hspace{1cm} (2.2b)
\[ \hat{x}_{k|k-1} = F \cdot \hat{x}_{k-1|k-1} + B \cdot u_{k-1} \]  \hspace{1cm} (2.2c)

where \( K \) is the gain of the observer. The subscript \([k|k]\) denotes the value of a variable at the current sampling instant based on measurement information at the same sampling instant. Consequently, the subscript \([k|k-1]\) denotes the value of a variable at the current sampling instant based on measurement information at the previous sampling instant.

The estimation error \( \hat{x}_k \) is defined by

\[ \hat{x}_k = x_k - \hat{x}_{k|k} \]  \hspace{1cm} (2.3)

Using (2.1) and (2.2)

\[ \hat{x}_k = E \cdot \hat{x}_{k-1} \]  \hspace{1cm} (2.4a)

where

\[ E = F - K \cdot H^T \cdot F \]  \hspace{1cm} (2.4b)

If the gains remains unchanged then the error at sampling instant \( k-1 \) propagates to sampling instant \( q \) as

\[ \hat{x}_q = E^{(q+1-k)} \cdot \hat{x}_{k-1}, \hspace{0.5cm} q \geq k - 1 \]  \hspace{1cm} (2.5)

If all eigenvalues of \( E \) lies inside the unit circle, that is if

\[ |\lambda_i(E)| < 1 \hspace{1cm} i = 1, \ldots, n \]  \hspace{1cm} (2.6a)
then

$$\lim_{q \to \infty} E^{(q+1-k)} = 0$$  \hspace{1cm} (2.6b)$$

The closer to the origin the eigenvalues of E are, the faster the limit in (2.6b) goes to zero.

If it is possible to find a gain K of the observer such that the error system is asymptotically stable, i.e. \( |\lambda_q(E)| < 1 \), then the influence of old errors on new errors decreases as time goes by. The key problem in the design of an observer is therefore to find a gain that stabilizes the error system, i.e. gives the matrix E in (2.4) desired eigenvalues inside the unit circle. If all eigenvalues of the error system are in the origin, the observer is called a dead-beat observer which has the property that it eliminates an impulse disturbance in equal many sampling periods as the order of the system [8].

If the observability matrix, O, of the process has rank n, and the system matrix, F, has rank n, then it is always possible to find a stabilizing gain [7,9]. Formulated mathematically the conditions are

$$\text{rank}(O) = n$$  \hspace{1cm} (2.7a)

$$\text{rank}(F) = n$$  \hspace{1cm} (2.7b)

$$O = [H \quad F^T H \quad \ldots \quad (F^{n-1})^T H]^T$$  \hspace{1cm} (2.7c)

The observability matrix is simply the relation between the state-vector, x(k), and the measurements, y(k), y(k+1), ... , y(k+n-1). That is

$$Y(k + n - 1) = O \cdot x(k)$$  \hspace{1cm} (2.8a)

$$Y(k + n - 1) = [y^T(k) \quad y^T(k + 1) \quad \ldots \quad y^T(k + n - 1)]^T$$  \hspace{1cm} (2.8b)

If Y(k+n-1) is known then it is possible to calculate the state-vector at sampling instant k if \( \text{rank}(O) = n \), which is the same as to say that the process is observable.

**Time-varying systems**

Up to now we have only discussed time-invariant processes. If the process in question is time-variant, the observer design becomes difficult. Since the process changes at each sampling instant, the gain K(k) has to be evaluated so that the eigenvalues of the error system over the period \( T(k) = t(k) - t(k - 1) \) has all eigenvalues inside the unit circle, where \( t(\cdot) \) denotes a instant in time. Observe that the sampling periods T(k), k=0,1,2,3,..., not necessarily has to be the equal. Unfortunately, this condition is not sufficient to guarantee the asymptotic stability of the total error system over all periods. For a time-variant system, the propagation of the estimation error over one sampling period is given by

$$\bar{x}_k = E(k) \cdot \bar{x}_{k-1}$$  \hspace{1cm} (2.9a)
2 THE CURRENT OBSERVER

where

\[ E(k) = F(k - 1) - K(k) \cdot H^T(k) \cdot F(k - 1) \]  \hspace{1cm} (2.9b)

Defining

\[ \prod_{i=k}^{q} E(i) = E(q) \cdot E(q - 1) \cdot \ldots \cdot E(k), \quad q \geq k \]  \hspace{1cm} (2.10)

The propagation of the estimation error \( \tilde{x}_{k-1} \) over \( q \) periods can be written as

\[ \tilde{x}_q = \prod_{i=k}^{q} E(i) \cdot \tilde{x}_{k-1} \]  \hspace{1cm} (2.11)

The property

\[ \lim_{q \to \infty} \prod_{i=k}^{q} E(i) = 0 \]  \hspace{1cm} (2.12)

is not in general guaranteed for arbitrary matrix sequences even if

\[ |\lambda_i(E(j))| < 1 \quad i = 1, \ldots, n \quad j = k, \ldots, q \]  \hspace{1cm} (2.13)

However, the condition (2.12) can be guaranteed if all matrices \( E(\cdot) \) have the same eigenvectors or are all diagonal or either upper or lower triangular and if (2.13) holds. If (2.12) does not hold, then the observer will never recover from an estimation error. This is demonstrated in the following example.

**Example 1**

A process can be described as

\[ x_{k+1} = F \cdot x_k \]
\[ y_k = H_k^T \cdot x_k \]

\[ F = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} \]
\[ H_k = \begin{cases} H^A & \text{even} \\ H^B & \text{odd} \end{cases} \quad H^A = \begin{bmatrix} 1 \\ 1/4 \end{bmatrix}, \quad H^B = \begin{bmatrix} 1 \\ 17/4 \end{bmatrix} \]

The measurements come from two different sensors, each active every second sampling instant. Suppose that an observer has been designed so that the two matrices

\[ E^A = F - K^A \cdot H^{AT} \cdot F, \quad E^B = F - K^B \cdot H^{BT} \cdot F \]

of the error systems

\[ \tilde{x}_{k+1} = E^A \cdot \tilde{x}_k \quad k + 1 \text{ even}, \quad \tilde{x}_{k+1} = E^B \cdot \tilde{x}_k \quad k + 1 \text{ odd} \]
have all eigenvalues in \( z = 0.5 \). This implies that

\[
K^A^T = \begin{bmatrix} 1/4 & 1/2 \\ \end{bmatrix}, \quad K^B^T = \begin{bmatrix} -11/8 & 1/2 \\ \end{bmatrix}
\]

If the observer should be able to recover from an estimation error it has to satisfy the condition

\[
\prod_i (\lambda_i(E^A \cdot E^B)) < 1 \quad i = 1, 2
\]

In this case \( \lambda_1(E^A \cdot E^B) = -0.030 \) and \( \lambda_2(E^A \cdot E^B) = -2.111 \). Thus, the observer will never recover from an estimation error.
3 TRAJECTORY ESTIMATOR

In this chapter we present an estimator to estimate the trajectory of an AGV, defined by the states \((x, y, \theta)\), where \((x, y)\) is the position and \(\theta\) is the heading in a two-dimensional space. The estimator is based on the current observer concept presented in chapter 2. Since the linearity of the process model is assumed in the observer design, the linear models for the motion of the vehicle and the measurements to the reflectors are derived in section 3.1. The models are valid for the prototype AGV described in [1,2,3]. The estimator design presented in section 3.2, section 3.3 and section 3.4 are based on the linear models. In section 3.5, a combination of a linear and a non-linear estimator is presented.

3.1 Models of the vehicle and the measurements

In this chapter, non-linear and linear models of the motion of the vehicle and the measurements to the reflectors are given. The models are derived in detail in [3,4].

Fig 3.1 The position of the AGV is described with the \(x\)- and \(y\)-coordinates for the midpoint of the rear axis and the heading \(\theta\) with respect to the \(x\)-axis. When the steer angle \(\alpha\) of the front wheel is constant, the motion of the three wheeled vehicle is along a segment of a circle.
3 TRAJECTORY ESTIMATOR

Fig 3.2 The angles to the reflectors are measured using a rotating laser scanner.

Motion models
If the steer angle, $\alpha$, and the speed, $v$, of the front wheel are constant during $T$ seconds, then it is possible to describe the motion of the vehicle with an exact discrete time model. The model is

$$
L \frac{v(k)T}{\tan(\alpha(k))} x(k+1) = x(k) + \tan(\alpha(k)) \left( \sin \left( \theta(k) + \frac{v(k)T}{L} \sin(\alpha(k)) \right) - \sin(\theta(k)) \right)
$$

$$
y(k+1) = y(k) + \frac{L}{\tan(\alpha(k))} \left( -\cos \left( \theta(k) + \frac{v(k)T}{L} \sin(\alpha(k)) \right) + \cos(\theta(k)) \right)
$$

$$
\theta(k+1) = \theta(k) + \frac{v(k)T}{L} \sin(\alpha(k))
$$

By introducing the state-vector $X = (x, y, \theta)^T$, the non-linear motion model can be written as

$$
X(k+1) = f(X(k), \alpha(k), v(k))
$$

The servo systems that handle the steer and the drive functions of the vehicle, are dynamical systems with small time-constants compared to the time-constant of the vehicle. This implies that the model (3.2) is not an exact model of the real vehicle. However, simulations show that it is a sufficiently good model for analysis.

A linear motion model is given by a first order linearization of (3.2). The non-linear model is linearized around a nominal value, $\bar{X}(k)$, of the state-vector, $X(k)$, which gives

$$
\delta X(k+1) = F(k) \cdot \delta X(k)
$$

$$
\delta X(k) = X(k) - \bar{X}(k)
$$

where $F(k)$ is
\[ F(k) = \begin{bmatrix} \frac{\delta f(X)}{\delta X} \bigg|_{X = \bar{X}(k)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & f_{13} \\ 0 & 1 & f_{23} \\ 0 & 0 & 1 \end{bmatrix} \] (3.4a)

\[ f_{13} = \frac{L}{\tan(\alpha(k))} \left( \cos(\theta(k) + \frac{v(k)T}{L} \sin(\alpha(k))) - \cos(\theta(k)) \right) \] (3.4b)

\[ f_{23} = -\frac{L}{\tan(\alpha(k))} \left( -\sin(\theta(k) + \frac{v(k)T}{L} \sin(\alpha(k))) + \sin(\theta(k)) \right) \] (3.4c)

If \( v(k)T \sin(\alpha(k))/L \) is small, then (3.4b) and (3.4c) are approximately equal to

\[ f_{13} = -v(k)T \cos(\alpha(k)) \sin(\theta(k)) \] (3.4d)

\[ f_{23} = v(k)T \cos(\alpha(k)) \cos(\theta(k)) \] (3.4e)

The linear model in (3.3) describes the perturbation from the nominal value of the state-vector.

**Measurement models**

Using the geometry in fig 3.2, the angle to reflector \( i \) is given as

\[ \gamma^i(k) = \arctan\left( \frac{y_i - y(k) - d \sin(\theta(k))}{x_i - x(k) - d \cos(\theta(k))} \right) - \theta(k) \] (3.5a)

which, omitting the superscript \( i \), can be written as

\[ \gamma(k) = h(X(k)) \] (3.5b)

The linear measurement equation is given by a first order linearization of (3.5) around a nominal value, \( \bar{X}(k) \), of the state-vector, \( X(k) \), which gives

\[ z(k) = H^T(k) \cdot \delta X(k) \] (3.6a)

where \( \delta X(k) \) are according to (3.3b) and \( z(k) \) is using (3.5b)

\[ z(k) = h(X(k)) - h(\bar{X}(k)) \] (3.6b)

The entries of the output-vector

\[ H(k) = \begin{bmatrix} \frac{\delta h(X)}{\delta X} \bigg|_{X = \bar{X}(k)} \end{bmatrix} = [h_x \ h_y \ h_i]^T \] (3.6c)

are

\[ h_x = \frac{s_y}{s_x^2 + s_y^2} \] (3.6d)
3 TRAJECTORY ESTIMATOR

\[ h_y = \frac{-s_x}{s_x^2 + s_y^2} \]  
\[ h_i = -\frac{d(s_x \cos(\bar{\theta}(k)) + s_y \sin(\bar{\theta}(k)))}{s_x^2 + s_y^2} - 1 \]  

(3.6e)  
(3.6f)

For convenience we have introduced the distance between the reflector and the midpoint of the rear axis, which is defined by

\[ s_x = x_i - \bar{x}(k) - d \cos(\bar{\theta}(k)) \]  
\[ s_y = y_i - \bar{y}(k) - d \sin(\bar{\theta}(k)) \]

(3.7a)  
(3.7b)

3.2 Assigning the eigenvalues of the error system

In this section the insufficiency in only assigning the eigenvalues of the error system to guarantee the convergence of the estimation error is shown. This is because of the time-varying nature of the system.

To start with assume that the vehicle is standing still, i.e. \( F(k) = I \) in (3.4a), and that \( H'(k) = H' \) in (3.6a), where \( r \) is a particular reflector. To succeed with the design of an observer for estimating the state-vector of the AGV, it is necessary that the conditions in (2.7) holds. If they don't, it is impossible to stabilize the error system and the observer will never recover from an estimation error. If we check (3.4a), we see that the system matrix, \( F(k) \), always has full rank, i.e. \( \text{rank}(F(k)) = 3 \). Thus (2.7b) always holds for the model of the AGV.

If we measure at each sampling instant the angle to the same reflector, i.e. only one reflector is visible to the vehicle, we get the observability matrix

\[ O = [H, H, H]^T \]  

(3.8)

which is a rank one matrix, and hence (2.7a) does not hold. It is not possible to stabilize the error system

\[ \delta X(k) = E \cdot \delta X(k-1) \]  
\[ E = I - K \cdot H^T \]

(3.9)

with a suitable gain of the observer. Two eigenvalues remains at \( z=1 \), which are the eigenvalues of the system matrix \( F(k) \), whether or not the vehicle is moving.

Instead of having only one visible reflector, assume that the vehicle is travelling in a room with three reflectors, A, B and C. Then, provided no reflectors are hidden, the measurement at every third sampling instant originates from the same reflector. We have the following model of the vehicle and the measurements
\[ \delta X(k + 1) = \delta X(k) \]  
\[ z(k) = H^T(k) \cdot \delta X(k) \]  
\[ H^T(k) = \begin{cases} 
H^A_T & k = 0, 3, 6, 9, \ldots \\
H^B_T & k = 1, 4, 7, 10, \ldots \\
H^C_T & k = 2, 5, 8, 11, \ldots 
\end{cases} \]  
which gives the relation between the measurements \( z(k), z(k + 1), z(k + 2) \) and the state-vector \( \delta X(k) \)

\[ \begin{bmatrix} z(k) & z(k + 1) & z(k + 2) \end{bmatrix}^T = O \cdot \delta X(k) \]

With three reflectors visible to the vehicle, it is possible to have an observability matrix with full rank. The rank depends on how the reflectors are located relative to the vehicle. The output-vector, \( H(r) \), to a particular reflector is given by the linearization of the measurement equation to the reflector around a nominal value of the state-vector. This procedure is described in section 3.1. If, for instance, the three reflectors and the laser-scanner are in the corners of a square, one output-vector, \( H^r \), \( r = A, B, C \), is a linear combination of the two others. In this case the rank of the observability matrix is 2. If the reflectors are 90\(^\circ\) apart and are at the same distance from the scanner, then the output-vectors are not linear combinations of each other. In this case the observability matrix has full rank.

A full-rank observability matrix implies that it is possible to stabilize the error system over three sampling instants, i.e. the gains \( K_A, K_B \) and \( K_C \) in the observer, corresponding to the output-vectors \( H_A, H_B \) and \( H_C \) can be designed so that an estimation error at some sampling instant decline when it propagates through the error system. The error system over three sampling instants is

\[ \delta X(k + 2) = E(k + 2) \cdot \delta X(k - 1) \]

\[ E(k) = \begin{cases} 
E^C \cdot E^B \cdot E^A & k = 0, 3, 6, 9, \ldots \\
E^A \cdot E^C \cdot E^B & k = 1, 4, 7, 10, \ldots \\
E^B \cdot E^A \cdot E^C & k = 2, 5, 8, 11, \ldots 
\end{cases} \]

\[ E^C \cdot E^B \cdot E^A = [I - K^C \cdot H^{CT}] \cdot [I - K^B \cdot H^{BT}] \cdot [I - K^A \cdot H^{AT}] \]

\[ E^A \cdot E^C \cdot E^B = [I - K^A \cdot H^{AT}] \cdot [I - K^C \cdot H^{CT}] \cdot [I - K^B \cdot H^{BT}] \]
\[ E^B \cdot E^A \cdot E^C = [I - K^B \cdot H^{B\top}] \cdot [I - K^A \cdot H^{A\top}] \cdot [I - K^C \cdot H^{C\top}] \]

If the laserbeam makes \( q \) revolutions, an initial estimation error propagates as

\[ \delta \hat{x}(3q) = (E^C \cdot E^B \cdot E^A)^q \cdot \delta \hat{x}(0) \quad (3.13) \]

If

\[ |\lambda_i(E^C \cdot E^B \cdot E^A)| < 1 \quad , \quad i = 1, 2, 3 \]

then

\[ \lim_{q \to \infty} (E^C \cdot E^B \cdot E^A)^q = 0 \]

i.e. the influence of the initial error on the estimation error at sampling instant \( 3q \), decline as time goes by.

If rank \((O)\)=3, there are several combinations of gains \( K^A, K^B \) and \( K^C \) that stabilizes the error system. In principle, it is possible to select two of the gains arbitrary, except the zero gains, and stabilize the system with the third gain. This freedom in the design of the gain is a typical property of multi-output systems if only the eigenvalues of the error systems are considered [9]. A further discussion on this degree of freedom is presented in section 3.3.

In reality, there are more than three reflectors visible to the vehicle. If that is the case, it is still possible to stabilize the error system over three sampling instants provided that the observability matrix has full rank for every consecutive three-combination of reflectors. However, this may lead to instability of the error system over one revolution of the laserbeam. If that is the case, the observer will never recover from an estimation error. This is illustrated below.

Suppose that instead of three are four visible reflectors, A,B,C and D, and that the observability matrix has full rank for every consecutive three-combination of reflectors. The error system is

\[ \delta \hat{x}(k + 3) = \overline{E}(k + 3) \cdot \delta \hat{x}(k - 1) \quad (3.14) \]

\[
\overline{E}(k) = \begin{cases} 
E^D \cdot E^C \cdot E^B \cdot E^A & k = 0, 4, 8, 12, \ldots \\
E^A \cdot E^D \cdot E^C \cdot E^B & k = 1, 5, 9, 13, \ldots \\
E^B \cdot E^A \cdot E^D \cdot E^C & k = 2, 6, 10, 14, \ldots \\
E^C \cdot E^B \cdot E^A \cdot E^D & k = 3, 7, 11, 15, \ldots 
\end{cases}
\]

One possible criteria to stabilize the error system over three sampling instants, is to select the gains so that
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\[
| \lambda_i(E^C \cdot E^B \cdot E^A) | < 1, \quad i = 1, 2, 3 \quad (3.15a)
\]

\[
| \lambda_i(E^D \cdot E^C \cdot E^B) | < 1, \quad i = 1, 2, 3 \quad (3.15b)
\]

An initial error propagates through the error system during \( q \) revolutions of the laser-beam as

\[
\delta \hat{x}(4q) = (E^D \cdot E^C \cdot E^B \cdot E^A)^q \cdot \delta \hat{x}(0) \quad (3.16a)
\]

Even though (3.15) holds, there is nothing that says that

\[
| \lambda_i(E^D \cdot E^C \cdot E^B \cdot E^A) | < 1, \quad i = 1, 2, 3 \quad (3.16b)
\]

which has to be the case if the observer should be of any use. In the light of the previous discussion, the idea of stabilizing the error system over a revolution of the laserbeam is quite obvious. This may be a good idea if the number of reflectors are limited, and if the same reflectors are visible at each revolution of the beam.

Again, suppose that there are four visible reflectors. Assume that it is possible to select the gains so that (3.16b) holds. If (3.16b) holds and one reflector, say reflector \( D \), is hidden, then the propagation of an initial error over \( q \) revolutions is

\[
\delta \hat{x}(3q) = (E^C \cdot E^B \cdot E^A)^q \cdot \delta \hat{x}(0) \quad (3.17)
\]

which may or may not be a stable system.

The insufficiency in the method to only assign the eigenvalues of the error system is further illustrated when the vehicle moves. Then, even if the same reflectors are seen from scan to scan and the error system over each scan is stable, it is impossible to guarantee the convergence of the estimation error. This is because the system becomes time-varying, which implies that the matrix of the error system over one revolution is not the same from one scan to another. If there are \( r \) visible reflectors, the propagation of an initial error over \( q \) revolutions is

\[
\delta \hat{x}(rq) = \prod_{i=1}^{q} \bar{E}(ri) \cdot \delta \hat{x}(0) \quad (3.18)
\]

\[
\bar{E}(rj) = \prod_{i=1}^{r} E(r(j-1)+i), \quad j = 1, \ldots, q
\]

If not

\[
\bar{E}(rq) = \bar{E}(r(q-1)) = \ldots = \bar{E}(r)
\]

in (3.18), or all matrices have the same eigenvectors or are all diagonal or either upper or lower triangular, it is impossible to say anything about the convergence of the estimation error from the eigenvalues of each separate matrix \( \bar{E}(\cdot) \).
3.3 Assigning the structure of the error system matrix

It is clear from the discussion in section 3.2 that it is not sufficient to only assign the eigenvalues of the error system to make estimation errors disappear. The insufficiency in such a strategy is that it is only possible to guarantee an asymptotically stable error system over every period

$$T(k + s) = t(k + s) - t(k)$$  \hspace{1cm} (3.19)

where $s$ is the number of measurements that are used to stabilize the error system. From the discussion in section 3.2, it is clear that $\text{min}(s) = 3$. The error system for one period, when the movement of the vehicle is also taken into consideration, is

$$E(k + j) = [F(k + j - 1) - K(k + j) \cdot H^T(k + j) \cdot F(k + 1 - 1)] \cdot \delta X(k + j - 1), \hspace{1cm} 0 \leq j \leq s - 1$$  \hspace{1cm} (3.20)

In (3.20) it is assumed that there are sufficiently many non-linear combinations of output vectors $H(.)$, corresponding to different reflectors, that makes it possible to stabilize the error system.

To succeed in constructing an useful observer, it is necessary to have control over the resulting error system when two or more 'one-period' error systems interacts. The discussion below will clarify this statement.

Assume $q$ error systems, called $1, 2, \ldots$ and $q$, which are

$$\delta X(k_1 + s_1 - 1) = \prod_{i=k_1}^{k_1 + s_1 - 1} E(i) \cdot \delta X(k_1 - 1)$$  \hspace{1cm} (3.21)

$$\delta X(k_2 + s_2 - 1) = \prod_{i=k_2}^{k_2 + s_2 - 1} E(i) \cdot \delta X(k_2 - 1)$$

$$\vdots$$

$$\delta X(k_q + s_q - 1) = \prod_{i=k_q}^{k_q + s_q - 1} E(i) \cdot \delta X(k_q - 1)$$

where

$$k_2 = k_1 + s_1 \hspace{0.5cm} k_3 = k_2 + s_2 \hspace{0.5cm} \cdots \hspace{0.5cm} k_q = k_{q-1} + s_q$$

i.e. the period $T(k_q + s_q - 1)$ comes immediately after the period $T(k_{q-1} + s_{q-1} - 1)$ which comes immediately after the period $T(k_{q-2} + s_{q-2} - 1)$ etc. This implies that an estimation error at $t(k_1 - 1)$ propagates to $t(k_q + s_q - 1)$ as
\[ \delta X(k_1 + s_q - 1) = \prod_{j=1}^{q} \prod_{i=k_j}^{k_{j-1}+1} E(i) \cdot \delta X(k_i - 1) \]  

(3.22)

\[ k_q + s_q = k_1 + \sum_{i=1}^{q} s_i \]

If the observer should be of any use, then from (3.22)

\[ \lim_{q \to \infty} \prod_{j=1}^{q} \prod_{i=k_j}^{k_{j-1}+1} E(i) = 0 \]  

(3.23)

(3.23) can not be guaranteed in general. However, for certain structures of the matrices \( E(\cdot) \), (3.23) can be guaranteed. This is investigated in the following lemma.

**Lemma 1**

If \( E(k), k=1,2,\ldots, \) are either upper or lower \([3,3]\) triangular matrices such that

1. all diagonal elements
   \[ | e_{ii}(k) | < \lambda < 1, \quad i = 1,2,3 \]  
   (3.24a)

2. all off-diagonal elements in the non-zero triangle
   \[ | e_{ij}(k) | < \eta < \infty \]  
   (3.24b) \quad i \neq j

Then

\[ \lim_{k \to \infty} \prod_{i=1}^{k} E(i) = 0 \]  

(3.25)

**Proof**

Define

\[ P(k) = \prod_{i=1}^{k} E(i) \]  

(3.26a)

Then

\[ P(k) = E(k)P(k-1) \]  

(3.26b)

Without loss of generality assume the matrices \( E(\cdot) \) in (3.26a) to be upper triangular. The matrix \( P(k) \) in (3.26) will then also be upper triangular. The entries of \( P(k) \) can be expressed in the entries of \( E(k) \) and \( P(k-1) \) as

\[ P_{jj}(k) = E_{jj}(k)P_{jj}(k-1), \quad j = 1,2,3 \]  

(3.27a)

\[ P_{12}(k) = E_{12}(k)P_{12}(k-1) + E_{13}(k)P_{23}(k-1) \]  

(3.27b)

\[ P_{23}(k) = E_{23}(k)P_{23}(k-1) \]  

(3.27c)

\[ P_{13}(k) = E_{13}(k)P_{13}(k-1) + E_{12}(k)P_{23}(k-1) + E_{13}(k)P_{33}(k-1) \]  

(3.27d)

By induction it is proved that
$$|P_\gamma(k)| \leq \lambda^k$$  \hspace{1cm} (3.28a)

$$|P_{12}(k)| \leq k\eta \lambda^{k-1}$$  \hspace{1cm} (3.28b)

$$|P_{23}(k)| \leq k\eta \lambda^{k-1}$$  \hspace{1cm} (3.28c)

$$|P_{13}(k)| \leq k\eta \lambda^{k-1} + \frac{k}{2} (k-1)\eta^2 \lambda^{k-2}$$  \hspace{1cm} (3.28d)

This is true for \(k=1\) by assumptions. If it is true for \(k=m\) then for \(k=m+1\)

$$|P_\gamma(m+1)| \leq |E_\gamma(m+1)| \cdot |P_\gamma(m)| \leq \lambda \lambda^m = \lambda^{m+1}$$

$$|P_{12}(m+1)| \leq |E_{11}(m+1)| \cdot |P_{12}(m)| + |E_{12}(m+1)| \cdot |P_{22}(m)| \leq$$

$$\leq \lambda (m\eta \lambda^{m-1}) \cdot \eta \lambda^m = (m+1)\eta \lambda^m$$

The same bound holds for \(|P_{23}(m+1)|\) and

$$|P_{13}(m+1)| \leq |E_{11}(m+1)| \cdot |P_{13}(m)| + |E_{12}(m+1)| \cdot |P_{23}(m)| +$$

$$+ |E_{13}(m+1)| \cdot |P_{33}(m)| \leq$$

$$\leq \lambda \left( m \eta \lambda^{m-1} + \frac{m}{2} (m-1) \eta^2 \lambda^{m-2} \right) + \eta (m \eta \lambda^{m-1}) + \eta \lambda^m =$$

$$= (m+1)\eta \lambda^m + \eta \lambda^m \left[ \frac{m}{2} (m-1) + m \right] =$$

$$= (m+1)\eta \lambda^m + \frac{(m+1)}{2} m \eta^2 \lambda^{m-1}$$

This completes the induction.

From (3.24) and (3.28)

$$\lim_{k \to \infty} P_\gamma(k) = 0$$  \hspace{1cm} (3.29a)

$$\lim_{k \to \infty} P_{12}(k) = 0$$  \hspace{1cm} (3.29b)

$$\lim_{k \to \infty} P_{23}(k) = 0$$  \hspace{1cm} (3.29c)

$$\lim_{k \to \infty} P_{13}(k) = 0$$  \hspace{1cm} (3.29d)

$$\Rightarrow \lim_{k \to \infty} \prod_{i=1}^{k} E(i) = 0$$  \hspace{1cm} (3.30)
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The results in lemma 1 can be generalized to \([n,n]\) matrices. The elements in \(P(k)\) are bounded from above by

\[
|P_{ij}(k)| \leq \sum_{l=0}^{\min(i,j)} \eta_l^l f_l(k) \lambda_i^{l-1}
\]

where \(f_l(k)\) is a polynomial in \(k\) to the power \(l\).

Using this bound, if the order \(n\) of the system is finite

\[
\lim_{k \to \infty} \prod_{i=1}^{k} E_{n,n}(i) = 0
\]

3.4 The gains of the estimator

The obvious conclusion from the results in lemma 1 in section 3.3 is to, if possible, select the gains of the observer so that all 'one-period' error systems are diagonal or triangular of the same kind. The minimum number of measurements needed, to different reflectors, is three. In an error system over three sampling instants, we have to determine three gains, each containing three entries. The resulting error system matrix contains nine entries. Thus, we have nine equations, specified by the entries in the matrix of the error system, and nine unknowns, the entries in the three gain vectors. In principle, this system of equations is possible to solve if all equations are linearly independent.

The error system over three sampling instants, starting at \(k=1\), is

\[
\dot{\tilde{X}}_{k+2} = E_{k+2} \cdot E_{k+1} \cdot E_k \cdot \dot{\tilde{X}}_{k-1}
\]

\[
E_{k+2} = [F_{k+1} - K_{k+2} \cdot H_{k+2}^T \cdot F_{k+1}]
\]

\[
E_{k+1} = [F_k - K_{k+1} \cdot H_{k+1}^T \cdot F_k]
\]

\[
E_k = [F_{k-1} - K_k \cdot H_k^T \cdot F_{k-1}]
\]

Assume that the measurements are done to reflector A, B and C, and for the sake of simplicity introduce

\[
E^C = [F^C - K^C \cdot H^{CT} \cdot F^C] = [F_{k+1} - K_{k+2} \cdot H_{k+2}^T \cdot F_{k+1}]
\]

\[
E^B = [F^B - K^B \cdot H^{BT} \cdot F^B] = [F_k - K_{k+1} \cdot H_{k+1}^T \cdot F_k]
\]

\[
E^A = [F^A - K^A \cdot H^{AT} \cdot F^A] = [F_{k-1} - K_k \cdot H_k^T \cdot F_{k-1}]
\]

If the expressions in (3.34) are used and all matrix-multiplications in (3.33) are carried out, we get

(3.35)

where \( F_{CBA} \) is a shortening for \( F^C \cdot F^B \cdot F^A \) etc. In (3.35) there are four terms containing more than one gain. This non-linearity makes it very difficult to find the gains needed to get a resulting diagonal matrix \( E_{CBA} \) in (3.35). However in the four terms there are three scalar-products, i.e. \([1,1]\) matrices

\[ H^C F^C K^B , H^B F^B K^A , H^C F^C K^A \]

(3.36)

If those products are zero, the remaining terms in (3.35) are linear in the gains. If the gains are selected so that the all the scalar products are zero, we are left with only six equations to assign the entries in \( E_{CBA} \) with. Fortunately, this is enough because it is not necessary that \( E_{CBA} \) is diagonal, it can also be triangular which follows from lemma 1. To make \( E_{CBA} \) triangular with control over the eigenvalues, we need to assign the diagonal elements and the off-diagonal elements, for instance the lower ones.

If the scalar-products are zero, what is left of (3.35) is

\[ E_{CBA} = F_{CBA} - K^C H^C F^C K^B H^B F^A - K^A H^A T \]

(3.37)

\[ K^B = F^C K^B , K^A = F^C K^A \]

\[ H^C F^C K^B = H^C F^C K^A , H^B F^B K^A = H^B F^B K^A \]

\[ H^C F^C K^A = H^C F^C K^A , H^A T = H^A T \]

The side-conditions can be expressed in terms of the gains in (3.37) as

\[ H^C K^B = 0 \]

(3.38a)

\[ H^C K^A = 0 \]

(3.38b)

\[ H^B F^C K^A = H^B F^C K^A \]

(3.38c)

Introduce

\[ K^{ij} = [k_1^{ij} k_2^{ij} k_3^{ij}]^T \]

(3.39a)

\[ H^{kl} = [h_1^{kl} h_2^{kl} h_3^{kl}] \]

(3.39b)
and use the fact that
\[ h_{ni}^{(k)} = h_n^k, \quad m = x, y \]  
(3.40)

The result in (3.40) is clear from an inspection of (3.4a). Now identify the diagonal elements and the lower triangular elements of \( E_{\text{CBA}} - F_{\text{CBA}} \) in (3.37) and the side-conditions in (3.38). Then it is possible to express the relation between the gains in (3.37) and the terms to be identified as a matrix equation, \( Mx = b \), where \( x \) contains the entries in the gains and \( b \) contains the terms to be identified. We have

\[ M \cdot x = b \]  
(3.41)

\[ M = \begin{bmatrix} M_{11} & M_{12} \\ 0 & M_{22} \end{bmatrix} \]

\[ M_{11} = \begin{bmatrix} h_x^A & h_x^B & h_x^C & 0 & 0 & 0 \\ 0 & 0 & 0 & h_x^A & h_x^B & h_x^C \\ 0 & 0 & 0 & h_x^A & h_x^B & h_x^C \\ h_x^B & 0 & 0 & h_y^B & 0 & 0 \\ h_x^C & 0 & 0 & h_y^C & 0 & 0 \\ 0 & h_x^C & 0 & 0 & h_y^C & 0 \end{bmatrix} \]

\[ M_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ h_x^{B/C} & 0 & 0 \\ h_x^C & 0 & 0 \\ 0 & h_x^C & 0 \end{bmatrix} \]

\[ M_{22} = \begin{bmatrix} h_x^A & h_x^B & h_x^C \\ h_x^A & h_x^B & h_x^C \\ h_x^A & h_x^B & h_x^C \\ h_{t_i}^{A/C} & h_{t_i}^{B/BA} & h_{t_i}^{C/CBA} \end{bmatrix} \]

\[ x = [k_A^{A/\text{CB}} k_{i1}^{B/C} k_1^C k_2^{A/\text{CB}} k_2^{B/C} k_2^C k_3^{A/\text{CB}} k_3^{B/C} k_3^C]^T \]

\[ b = [1 - e_{11}^{\text{CBA}} 0 1 - e_{22}^{\text{CBA}} 0 0 0 0 0 0 1 - e_{33}^{\text{CBA}}]^T \]

If \( M \) in (3.41) is an invertible matrix, then the gains in (3.37) are

\[ x = M^{-1} \cdot b \]  
(3.42)
Suppose that the inverse exists, then from [9]

$$M^{-1} = \begin{bmatrix} M_{11}^{-1} & -M_{11}^{-1}M_{12}M_{22}^{-1} \\ 0 & M_{22}^{-1} \end{bmatrix}$$

(3.43)

The submatrix $M_{22}$ of $M$ can be expressed as

$$M_{22} = [H^T F^A \quad H^T F^B \quad H^C F^C]^T$$

(3.44)

which reminds of the observability matrix when the vehicle is measuring all angles from the same position, i.e. the vehicle is standing still. If the vectors $H^{rT} = H^T F^r$, $r=A, B, C$, are linearly independent then the inverse of $M_{22}$ exists. If the output vectors $H^A, H^B$ and $H^C$ are linearly independent it is most likely that also $H^{A/A}, H^{B/B}$ and $H^{C/C}$ are linearly independent if the vehicle does not move much between the measurements. A small movement of the vehicle between the measurements shows up as small values of the off-diagonal elements in the system matrix $F(\cdot)$.

The conditions for the inverse of the submatrix $M_{11}$ of $M$ to exist is clearly that $h_x^C \neq 0$ and, which is not very clear from an inspection of $M_{11}$, $(h_x^B h_x^C - h_x^B h_x^C) \neq 0$. The interpretation of the later condition is that the line of sight from the scanner to reflector B, and the line of sight to reflector C can not be the same. In reality, this can not occur. If the line of sights are the same to two reflectors, one reflector hides the other reflector to the scanner, and a measurement to the hidden reflector can not be done. However, it is possible that the line of sights are almost parallel, which implies that the matrix becomes numerically ill-conditioned. It is more difficult to find a physical interpretation of the first condition. The condition implies that the line of sight from the scanner to reflector C can not be parallel with the x-axis. Inspection of the linearized measurement equation in (3.6) clarifies the discussion above about the interpretation of the conditions for invertibility.

If $M_{11}^{-1}$ and $M_{22}$ exists, then the gains in (3.37) are

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}^{A/CB} = \begin{bmatrix} 0 & 0 & n_{13}^{A/CB} \\ 0 & 0 & n_{23} \\ 0 & 0 & n_{33} \end{bmatrix} \cdot \begin{bmatrix} 1 - e_{11}^{CB} \\ 1 - e_{22} \\ 1 - e_{33} \end{bmatrix}$$

(3.48a)

$$n_{13}^{A/CB} = \frac{(h_x^B h_x^C - h_x^B h_x^C)h_x^C}{\det(M_{22})}$$

$$n_{23}^{A/CB} = -\frac{(h_x^B h_x^C - h_x^B h_x^C)h_x^C}{\det(M_{22})}$$

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\[
\begin{align*}
n_{33}^{A/\text{CB}} &= \frac{(h_x^A h_y^C - h_x^A h_z^C)}{\det(M_{22})} \\
\begin{bmatrix}
k_1^{B/C} \\
k_2^{B/C} \\
k_3^{B/C}
\end{bmatrix} &= \begin{bmatrix} 0 & n_{12} & n_{13} \\
0 & n_{22} & n_{23} \\
0 & 0 & n_{33}
\end{bmatrix}^{B/C} \begin{bmatrix} 1 - e_{11} \\
1 - e_{22} \\
1 - e_{33}
\end{bmatrix}^{CBA}
\end{align*}
\]

\[
\begin{align*}
n_{12}^{B/C} &= \frac{h_y^C}{(h_x^A h_y^C - h_x^A h_z^C)} \\
n_{13}^{B/C} &= \frac{(h_x^A h_y^C - h_x^A h_z^C) \cdot (h_x^B h_y^C - h_x^{B/C^{-1}} h_y^C)}{\det(M_{22}) \cdot (h_x^B h_y^C - h_x^B h_z^C)} \\
n_{22}^{B/C} &= \frac{-h_x^C}{(h_x^B h_y^C - h_x^B h_z^C)} \\
n_{23}^{B/C} &= \frac{(h_x^A h_y^C - h_x^A h_z^C) \cdot (h_x^B h_y^C - h_x^{B/C^{-1}} h_y^C)}{\det(M_{22}) \cdot (h_x^B h_y^C - h_x^B h_z^C)} \\
n_{33}^{B/C} &= \frac{-(h_x^A h_y^C - h_x^A h_z^C)}{\det(M_{22})} \\
\begin{bmatrix}
k_1^{C} \\
k_2^{C} \\
k_3^{C}
\end{bmatrix} &= \begin{bmatrix} n_{11} & n_{12} & n_{13} \\
0 & n_{22} & n_{23} \\
0 & 0 & n_{33}
\end{bmatrix}^{C} \begin{bmatrix} 1 - e_{11} \\
1 - e_{22} \\
1 - e_{33}
\end{bmatrix}^{CBA}
\end{align*}
\]

\[
\begin{align*}
n_{11}^{C} &= \frac{1}{h_x^C} \\
n_{12}^{C} &= \frac{-h_x^B \cdot h_y^C}{h_x^C \cdot (h_x^B h_y^C - h_x^B h_z^C)} \\
n_{13}^{C} &= \frac{(h_x^A h_y^B - h_x^A h_z^B) \cdot (h_x^B h_y^C - h_x^{B/C^{-1}} h_y^C)}{\det(M_{22}) \cdot (h_x^B h_y^C - h_x^B h_z^C)} \\
n_{22}^{C} &= \frac{h_x^B}{(h_x^B h_y^C - h_x^B h_z^C)} \\
n_{23}^{C} &= \frac{-(h_x^A h_y^B - h_x^A h_z^B) \cdot (h_x^B h_y^C - h_x^{B/C^{-1}} h_y^C)}{\det(M_{22}) \cdot (h_x^B h_y^C - h_x^B h_z^C)} \\
n_{33}^{C} &= \frac{1}{h_x^C}
\end{align*}
\]

20
\[ n_{35}^{C} = \frac{(h_{y}^{A}h_{y}^{B} - h_{x}^{A}h_{x}^{B})}{\text{det}(M_{22})} \]

The inverse of \( M_{11} \) is derived with the aid of the program package MACSYMA [10]. Using the shortening in (3.37) and the expressions (3.45a,b), \( K^A \) and \( K^B \) are

\[
K^A = (F^{CB})^{-1} \cdot K^{A/CB} \tag{3.46}
\]

\[
K^B = (F^{C})^{-1} \cdot K^{B/IC}
\]

which are what we are looking for.

Actually, if the vehicle is measuring all angles from the same position and the diagonal entries of \( E_{cBA} \) in (3.37) are equal, then it is possible to show that even the upper off-diagonal entries of \( E_{cBA} \) becomes zero i.e. \( E_{cBA} \) becomes diagonal.

### 3.5 Implementation of the estimator

The ideas in the last section (3.4) imply that the estimate of the state-vector can only be corrected with measurements at every third sampling instant.

Assume that the latest estimate of the state-vector that is corrected with measurements is done at sampling instant \( k-3 \). At \( k-2 \), \( k-1 \) and \( k \), measurements to respective reflector A, B and C are done. This implies that at the current sampling instant, \( k \), the old estimates of the state-vector at \( k-2 \) and at \( k-1 \), and the current estimate are corrected with measurements to reflector A, B and C respectively. The gains of the observer are calculated according to (3.45) and (3.46), provided that the inverse of \( M \) in (3.41) exists. To calculate the gains it is necessary to know the system matrix, \( F \), at \( k-3 \), \( k-2 \) and \( k-1 \), and the output-vector, \( H \), at \( k-2 \), \( k-1 \) and \( k \). To determine those, it is necessary to know the state-vector at \( k-3 \), \( k-2 \), \( k-1 \) and at \( k \). The best guess of the state-vector at each sampling instant are predictions that are based on the latest corrected estimate, i.e. the estimate at \( k-3 \). If we use (3.2) the predictions are

\[
\hat{X}_{k+i-2|k-3} = f(\hat{X}_{k+i-3|k-3}, \alpha_{k+i-3}, \nu_{k+i-3}) \tag{3.47a}
\]

and if we use (3.4a) and (3.6c) we get the system matrix and output-vector at respective sampling instant as

\[
F'(k+i-3) = \left[ \frac{\delta f(X)}{\delta X} \right]_{X=\hat{X}_{k+i-3|k-3}} \tag{3.47b}
\]

\[
H'(k+i-2) = \left[ \frac{\delta h(X)}{\delta X} \right]_{X=\hat{X}_{k+i-2|k-3}} \tag{3.47c}
\]
where \( r = A \) when \( i = 0 \), \( r = B \) when \( i = 1 \) and \( r = C \) when \( i = 2 \). When the gains are determined, the estimate of the state-vector at \( k-2 \), \( k-1 \) and \( k \) are corrected with the measurements according to

\[
\dot{X}_{k-2|k-2} = f(X_{k-3|k-3}, \alpha_{k-3}, v_{k-3}) + K^A(k-2) \cdot (\gamma^A(k-2) - \hat{\gamma}^A(k-2))
\]

\[
\dot{X}_{k-1|k-1} = f(X_{k-2|k-2}, \alpha_{k-2}, v_{k-2}) + K^B(k-1) \cdot (\gamma^B(k-1) - \hat{\gamma}^B(k-1))
\]

\[
\dot{X}_{k|k} = f(X_{k-1|k-1}, \alpha_{k-1}, v_{k-1}) + K^C(k) \cdot (\gamma^C(k) - \hat{\gamma}^C(k))
\]

Note in (3.48) that the non-linear motion model in (3.2) and the non-linear measurement equation in (3.5) are used. This is a modification of the standard observer in section 3.2. It is better to use the non-linear models since they give better descriptions of the vehicle and the measurements than the corresponding linear models.

When the next three consecutive measurements are done the procedure described above is repeated.

If the matrix \( M \) in (3.41) at some sampling instant becomes singular or close to singular, the gains can not be determined according to (3.45). If this is the case, the only thing to do is to put the gains to zero and wait for three consecutive measurements that makes \( M \) invertible. The effect of putting the gains to zero on the convergence of the estimation error is not serious if it does not happen to often. What happens if the gains are zero at some sampling instant is that the error system matrix at that sampling instant equals the corresponding system matrix. If the vehicle does not move much between the measurements, the system matrix is approximately the identity matrix as pointed out in the end of section 3.4. The only effect in principle on the convergence of the estimation error, is therefore a delay of equally many sampling instants as the number of times the gains are zero. Of course for this discussion to hold, the gains can not be zero to often.

It may seem like a drawback that the current measurement can only be used at every third sampling instant to correct the state-vector at the same time. However, this is not a drawback in reality since the measurements comes very dense in time if not too many reflectors are hidden. If the control-loop that makes the vehicle follow the drive path is sampled with 10 Hz. and if the revolution speed of the laserbeam is 5 rev/s., then with 12 visible reflectors the estimates of the state-vector are corrected with measurements twice every sampling period of the control-loop. This is unnecessary compared to what is needed for good control of the vehicle. If the motion model of the vehicle is good, it is possible to get satisfactory control action even if the estimates are corrected with measurements once every 10 sampling periods of the control-loop. While three useful measurements to reflectors are collected, the estimates needed for control are simply those of the motion model.
4 SIMULATION RESULTS

In chapter 3, theory for linear time-varying systems are developed. Since the vehicle and the measurements is not only a time-varying system but also a non-linear system, pure mathematical analysis of the performance of the system becomes difficult. One way to make it possible to investigate the performance in a case like this, is to simulate the system in a computer and analyze the simulation result. The simulations presented in this chapter are done with the simulation package REGSIM [11]. Three case studies are done. In section 4.1, a non-moving vehicle is studied. In section 4.2, an open-loop controlled vehicle is studied. The last section, 4.3, is a study when the estimate of the state-vector is used to calculate the setvalues to the servos.

In fig. 4.1, we can see a drawing of a room of dimension 8.9x5.45 [mxm] in which the simulated vehicle moves. The triangles on the walls marks the locations of the reflectors. The dashed areas close to the walls indicates obstacles. A vehicle is also located in the room. The size of the vehicle is the same as the size of the prototype AGV described in [1,2,3].

![Diagram of a room](image)

**Fig 4.1. The room in which the simulated vehicle moves.**

The measurements to the reflectors are done periodically with 6.67 Hz. to simplify the implementation of the estimator. Measurements to every second reflector are omitted during one revolution of the laserbeam to simulate hidden reflectors. Since there are 11 reflectors in the room, measurements to the same reflectors are done every second revolution of the laserbeam.
The condition for the possibility to calculate the gains according to (3.45), is that the inverse of \( M \) in (3.41) exists. To prevent the inverse of \( M \) to become numerically ill-conditioned, the following conditions are used in the gain calculations

\[
|\text{det}(M_2)| < 0.01 \tag{4.1a}
\]
\[
|h_2^B h_2^C - h_2^B h_2^C| < 0.01 \tag{4.1b}
\]
\[
|h_2^C| < 0.01 \tag{4.1c}
\]

If any of the conditions in (4.1) holds, the gains of the observer are simply put to zero. The numerical values in the conditions are determined by a number of simulations.

The following data are used for the simulated vehicle. The distance between the front wheel and the rear wheel axis, \( L \) in fig. 3.1, is 0.83 m., and the distance between the scanner and the rear wheel axis, \( d \) in fig. 3.2, is 0.57 m. This data corresponds to the prototype AGV described in [1,2,3].

4.1 Non-moving vehicle

The position and the heading of the vehicle in the simulations presented in this section are \((x, y, \theta) = (3, 4.5, 5.4978)\), where the origin are in the lower left corner of the room in fig. 4.1 and the x-axis is in the direction of the horizontal wall. The units are for the position [m], and for the heading [rad]. The initial errors are \(C_E(0), \dot{5},-(0),ö(0)).\)

Fig. 4.1.1 and fig. 4.1.2 shows simulations when the three eigenvalues of the error system matrix all are the same. In fig. 4.1.1, the eigenvalues are 0.7 and in fig. 4.1.2, the eigenvalues are 0. The convergence of the initial error is faster for the dead-beat observer, but the dead-beat observer is also more sensitive to disturbances which shows up as ripple when the signals have settled. In fig. 4.1.3 the eigenvalues are \(\lambda_1 = 0.7, \lambda_2 = 0.5 \text{ and } \lambda_3 = 0.3\). As opposed to the simulations in fig. 4.1.1 and fig. 4.1.2, the total error in the position and the heading increases in the beginning of the simulation. This phenomenon can be explained as follows.

To guarantee immediate decay, so that for all vectors \(\delta X(k)\)

\[
||\delta X(k + 3)|| < ||\delta X(k)|| \tag{4.2a}
\]

i.e.

\[
||\overline{E}(k + 3)\delta X(k)|| < ||\delta X(k)|| \tag{4.2b}
\]

it is necessary that

\[
\delta X^T(k)(\overline{E}^T(k + 3)\overline{E}(k + 3) - I)\delta X(k) < 0 \tag{4.3}
\]
i.e. the matrix $\overline{E}^T(k+3)\overline{E}(k+3) - I$ is negative definite. This is a stronger requirement than the condition $|\lambda_i| < 1$ on the eigenvalues.

In section 3.4, it is pointed out that if all eigenvalues of the error system matrix are equal, then the resulting error system matrix becomes diagonal if the gains are calculated according to (3.45). If $\overline{E}(\cdot)$ is diagonal then (4.3) holds. On the other hand if $\overline{E}(\cdot)$ has distinct eigenvalues, which is the case in the simulation in fig. 4.1.3, then $\overline{E}(\cdot)$ becomes triangular and because of this, (4.3) is not guaranteed in general.

The estimates might go astray if the errors increases to much in the beginning, since the observer design is based on linearized models around the estimated state-vector. Therefore, if the estimator should be able to recover from large estimation errors, it is important that the eigenvalues of the error system matrix are equal. They also have to be close to 1, since a fast observer, i.e. the eigenvalues of the error system are close to the origin, weights the measurements more in the estimates than a slow observer which relies more on the motion model.

Fig 4.1.1 $\lambda_1 = \lambda_2 = \lambda_3 = 0.7$
Fig 4.1.2 $\lambda_1 = \lambda_2 = \lambda_3 = 0$
Fig 4.1.3 $\lambda_1 = 0.7 \lambda_2 = 0.5 \lambda_3 = 0.3$
4.2 Open-loop controlled vehicle

Fig. 4.2.1 shows the results when the vehicle drives along the diagonal in the room that starts in the upper left corner and ends in the lower right corner. The speed is 1 m/s, and the eigenvalues of the error system matrix are all 0.7. The initial position and the initial heading of the vehicle are \((x(0), y(0), \theta(0)) = (2, 4.225, 5.734)\), and the initial estimation errors are \((\hat{x}(0), \hat{y}(0), \hat{\theta}(0)) = (0.2, 0.225, 0.2362)\).

It is interesting to note that as long as there is an estimation error in the heading, the estimation error in the position grows during the sampling instants when the measurements are not used to correct the estimate of the state-vector. At two times, the errors continue to grow for six consecutive sampling instants. This is because one or more of the singularity conditions in (4.1) are fulfilled.

![Graphs showing simulation results](image)

Fig. 4.2.1 \(\lambda_1 = \lambda_2 = \lambda_3 = 0.7\) \(v=1.0\) m/s.

4.3 Feedback-controlled vehicle

In the simulation in this section, the estimate of the state-vector is used to calculate setvalues to the steer servo and to the speed servo. The control-loop is sampled with the same frequency as the measurements are done. The servos are modeled as fast second-order systems according to
A(s) = \frac{100}{s^2 + 20s + 100} \quad \text{(4.4a)}

V(s) = \frac{10(s + 10)}{s^2 + 20s + 100} \quad \text{(4.4b)}

where the subscript \( r \) denotes the setvalue. \( A(s) \) is the steer angle of the front wheel and \( V(s) \) is the speed of the front wheel. Only the setvalues to the servos are used in the estimator. Instead of using the setvalues calculated at the previous sampling instant to estimate the state-vector at the current sampling instant, the average of the setvalues at the previous sampling instant and at the sampling instant before that are used [4]. In all simulations the initial position, the initial heading and the initial estimation errors are the same as for the simulations in section 4.1.

Fig. 4.3.1 shows the estimation errors, the inputs to the servos and the outputs from the servos, when all eigenvalues of the error system matrix are 0.7. When the speed drops from 0.3 m/s, the vehicle turns. When \( t \geq 27 \) s, the speed is constant 0.1 m/s.

In fig. 4.3.2., the eigenvalues of the error system matrix are \( \lambda_1 = 0.85 \), \( \lambda_2 = 0.7 \) and \( \lambda_3 = 0.55 \). The errors grows in the beginning of the simulation in the same way as the errors in fig. 4.1.3., which is explained by the triangular structure of the matrices of the error systems.

Compared to the simulations when the vehicle stand still, the errors fluctuate more around zero in the simulations when the vehicle moves. One explanation to this, is that the motion between the measurements is not modeled exactly.
Fig 4.3.1 $\lambda_1 = \lambda_2 = \lambda_3 = 0.7$
Fig 4.3.2 $\lambda_1 = 0.85 \lambda_2 = 0.7 \lambda_3 = 0.55$
5 SUMMARY

It has been shown that to guarantee the convergence of the estimation error for a time-varying system, it is not sufficient to only stabilize each one-period error system by suitable gains of the observer without considering the structure of the error system matrix. It is proved for a third order system that in addition, by selecting the gains of the observer so that the matrix of each error system becomes diagonal or either upper or lower triangular, it is possible to guarantee the convergence of the estimation error.

Linearized models of the vehicle and the measurements has been used in the design of the observer. Simulations of an AGV shows good results if the estimation errors are in the region where the linear models are valid, and if the gains of the observer are selected so that each one-period error system is stabilized and each error system matrix is upper triangular or diagonal. Even if the gains occasionally are zero because of numerical problems, the estimation errors decays.

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7 APPENDIX

7.1 Properties of the eigenvalues of matrices

Let
E(j), j=1,2 be \([n,n]\) matrices

\(\lambda_{ij}\) - \(i\):Th. eigenvalue of \(E(j)\)

\(x_{ij}\) - eigenvector corresponding to \(\lambda_{ij}\)

1. If matrix \(E(j)\) is asymptotically stable, i.e. \(|\lambda_{ij}| < 1\) then
   a. \(\lambda_i(E^q(j)) = \lambda_{ij}^q\)
   b. \(\lim_{q \to \infty} E^q(j) = 0\)

2. If eigenvector \(x_{ij}\) of \(E(l)\) is also an eigenvector of \(E(j)\) then
   \(\lambda_i(E(l) \cdot E(j)) = \lambda_i(E(l)) \cdot \lambda_i(E(j))\)

3. If \(E(j)\) and \(E(l)\) have common eigenvectors then
   \(\lambda_i(E(l) \cdot E(j)) = \lambda_i(E(l)) \cdot \lambda_i(E(j))\) \(n = 1, \ldots, n\)

Further if \(E(j)\) and \(E(l)\) are asymptotically stable then
   a. \(|\lambda_i(E(l) \cdot E(j))| < 1\)
   b. \(\lim_{q \to \infty} (E(l) \cdot E(j))^q = 0\)

4. If \(E(l)\) and \(E(j)\) are both either upper or lower triangular matrices then
   \(\lambda_i(E(l) \cdot E(j)) = \lambda_i(E(l)) \cdot \lambda_i(E(j))\)
8 REFERENCES


A CONTROL STRATEGY FOR AUTONOMOUS GUIDED VEHICLES

Based on drive path specification by sequences of points

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Key Words - Autonomous Guided Vehicle; AGV; control strategy; steering controller; driving controller; segmented drive path; memorized drive path.

Abstract - In this report we study a control strategy for guiding an Autonomous Guided Vehicle (AGV) along a memorized drive path. The drive path considered consists of straight line segments defined by a sequence of points. A sequence of points is simple and flexible to use. A detour around an obstacle is for instance easy to specify. The strategy results in controllers for steering and driving a tricycle AGV. An inherent property of the controllers is a resulting smooth path without overshoots and oscillations around the drive path. By specifying instants in time when the vehicle should reach checkpoints on the segments, it is possible to make the vehicle travel the drive path in predefined time.
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1 INTRODUCTION

In this report we study a control strategy for guiding an Autonomous Guided Vehicle (AGV) along a memorized drive path. The vehicle in study is a tricycle, where the single front wheel is used for both steering and driving.

The drive path considered consists of straight lines specified by a sequence of points in a frame in the surroundings of the vehicle. One such line is referred to as a segment, and the frame is referred to as the global frame and could for instance be a room if the vehicle operates indoor.

The advantage with the type of drive path we consider, is that it is very easy to design. A detour around an obstacle is for instance easy to calculate once the obstacle is detected. The disadvantage with using segments is the sharp corners between them. It is not desirable that the vehicle makes such sharp turns. One design requirement of the controllers has therefore been that the vehicle should take a smooth turn without an overshoot when it starts to follow a new segment.

In literature, linear controllers seem to be frequently used for the steering of AGVs when the errors in the displacement and in the heading to the drive path are small \([1,2,3,4,5,6]\). When a segmented drive path is used together with a linear steer controller, it is necessary to shift the mode in the controller from linear control to 'turn' control when a switch to a new segment is done \([1,2]\). At least when the angle between two consecutive segments is large.

Our approach to the steering problem is to use the same steer controller no matter what the magnitude of the errors are. This implies that there only is one mode in the steer controller. The speed controller have two modes. When the errors in the displacement and in the heading are small, the speed is selected so that the vehicle reaches checkpoints on the segments at predefined time instants, which makes it possible for the vehicle to travel the drive path in predefined time. If the vehicle is performing maneuvers, for instance when it switch to a new segment, there might be large changes in the steer angle from one sampling instant to another due to a too high speed. The mode in the speed controller is shifted to predictive control when larger errors in the displacement and in the heading are detected. The predictive controller selects a setvalue to the speed servo so that the predicted change in the setvalue to the steer servo at next sampling instant does not exceed a certain value.

In this work, neither the path planning problem, i.e. how to design the drive path connecting the docking stations, nor the problem of redefining the drive path when an obstacle is detected, is investigated. Instead it is assumed that the path planning is done and a completely specified path is available for the control law calculation. It is also assumed that there is no risk of slipping between the wheels of the vehicle and the surface. Other assumptions made are that the vehicle travels on a reasonably flat surface and that the two-dimensional position and heading estimates are available from other subsystems.
The controller for steering has been implemented in an AGV prototype [7].

1.1 Outline of the report

In chapter 2 we describe the total AGV system. In section 2.1, a block diagram description of the AGV system is found together with a brief introduction to the theory presented in chapter 3. In section 2.2, the dynamical models of the AGV used for simulation and analysis are given. In chapter 3 we turn to the control problem. In section 3.1 the control law for steering is deduced. A stability analysis of linearized models of the vehicle and the steer controller is done in section 3.2. In section 3.3, we study how a switch to a new segment should be done. Up to section 3.3 it is assumed that the point of the vehicle that should follow the drive path is the contact point between the front wheel and the surface. This assumption is relaxed in section 3.4. In section 3.5, the speed controller is dealt. In the last section of chapter 3, section 3.6, there is a discussion on how the information needed in the drive path specification for the control law calculations should be determined.
2 SYSTEM DESCRIPTION

In section 2.1, a block diagram description of the total AGV system is found together with a brief introduction to the theory presented in chapter 3. In section 2.2, the dynamical models of the AGV used for simulation and analysis are given.

2.1 Preliminaries

![Block diagram of the total AGV system.](image)

**Fig 1a. Block diagram of the total AGV system.**

Fig. 1a is a block diagram representation of the total AGV system. The three leftmost blocks in the diagram are specific for the controllers described in chapter 3. The blocks connected with thick arrows will later be referred to as the closed-loop system. To get an overall picture of the whole system in the block diagram, we will briefly investigate it block by block.

The control laws generate set values, $\alpha_r$ and $v_r$, to the steer servo and the speed servo. There are two DC-motors used for steering and driving the AGV, which are controlled by a P-controller and a PI-controller respectively. The outputs from the servos, which are the steer angle, $\alpha$, and the speed of the steer wheel, $v$, will make the vehicle move in some way. The relation between the outputs from the servos and the way the vehicle moves is illustrated in fig. 1b.

The state (variable)-vector, $x$, of the vehicle, which contains the position and the heading, will cause sensors to produce information, $Z$, that is used together with the set values in a trajectory estimator that estimates the state-variables. The sensor information relevant for the estimation problem could for instance be rear wheel movements or angles measured to beacons in known positions in the global frame \([1,7,8,9,10]\). It is possible to also include the derivatives of the position and the heading in the state-vector, but they are omitted since they are not needed for the control law calculations.
The estimates of the position and heading have to be transformed to a position, \( (x', y') \), and a heading, \( \theta' \), in a local frame of the segment that the vehicle is following, because this information is needed for the control law calculations and to decide when to switch to the next segment in the drive path. To do this transformation it is also necessary to know the coordinates of the drive path. The difference between the global and local position is illustrated in fig. 1c. Formulas for the transformation of the position and the heading from one frame to another is found in appendix 6.1.

The drive path specification consists of not only the coordinates of the drive path but also information of the maximum allowed speed on each segment, the condition for when the vehicle should start to follow a new segment, what time it should reach the checkpoint on each segment, if the vehicle should stop and report to a supervisory system on certain segments etc.

**Fig 1b. The position of the AGV is described with the \( x \)- and \( y \)-coordinates for the front wheel and the heading \( \theta \) with respect to the \( x \)-axis. When the steer angle \( \alpha \) is constant, the motion of the tricycle is along a segment of a circle.**
Fig 1c. The position of the front wheel and the heading of the vehicle in the global frame are \((x,y,\theta)\). The figure illustrates how the these state-variables are projected onto a local frame of segment \(s\). The local frame has its origin in the starting coordinate of the segment in the global frame.

Fig 1d. This sequence illustrates the behavior of the control law for guiding the AGV along the drive path during three consecutive sampling instants. Notice how the 'rabbit' is accelerating when the AGV reaches the segment.

Fig. 1d illustrates the function of the control law for guiding the vehicle along the drive path during three consecutive sampling instants. A one wheeled vehicle is considered here for the sake of simplicity. The difference in the path taken compared to that of the tricycle is that the path will be straight lines and not segments of circles, which is the case for the tricycle when the steer angle is different from zero, see fig. 1b.
2 SYSTEM DESCRIPTION

The function of the controller is such that it makes the steer wheel aim at the aiming point on the drive path at each sampling instant. In the figure we can see the line of sight to the aiming point, dotted line, and the path the vehicle has taken, solid line. Compare this guidance technique with a dog, the vehicle, chasing a rabbit, the aiming point.

2.2 Models used for the dynamics of the vehicle and the servos

In the simulations presented in this report we make the somewhat unrealistic assumption that the position and heading in the global frame are directly measurable. This because we want to illustrate the strength and weakness of the controllers and not the combined strength and weakness of the controllers and the estimator.

We will base the analysis and simulations on a continuous time motion model describing the motion of the front wheel. The model is given by

\[
\begin{align*}
\dot{x} &= v \cdot \cos (\alpha + \theta) \\
\dot{y} &= v \cdot \sin (\alpha + \theta) \\
\dot{\theta} &= \frac{v}{L} \cdot \sin (\alpha)
\end{align*}
\]

(2.1a) (2.1b) (2.1c)

In (2.1), slipping of the wheels is not modeled.

The DC-motors in the servos are modeled as first order systems. The motor in the speed servo is controlled by a PI-controller, and the motor in the steer servo is controlled by a P-controller. The models of the servos corresponds to the servos in the AGV prototype described in [7,8,9,10]. In reality the velocities of the motors are limited. In the simulations, no limitations of the velocities are used.

The simulations of the AGV is done with the simulation program REGSIM [11]. The default values of the parameters used for simulation are length of vehicle 0.83 m, time constant of the DC-motor in the steer servo 0.05 s, time constant of the DC-motor in the speed servo 0.1 s. Gain of P-controller 5, gain of PI-controller 1 and integral time in PI-controller 0.1 s. The maximum allowed speed of the front wheel 0.5 m/s, and the sampling frequency of the control-loop 5 Hz.
3 CONTROL STRATEGY

The requirements that are taken into consideration in the design of the control laws are that the vehicle should follow the drive path smoothly without overshoots and oscillations around it, and that the vehicle should travel the drive path in predefined time.

In section 3.1, the control law for steering is deduced. A stability analysis of linearized models of the vehicle and the steer controller is done in section 3.2. In section 3.3, we study how a switch to a new segment should be done. Up to section 3.3 it is assumed that the point of the vehicle that should follow the drive path is the contact point between the front wheel and the surface. This assumption is relaxed in section 3.4. In section 3.5, the speed controller is dealt. In the last section, section 3.6, there is a discussion on how the information needed in the drive path specification for the control law calculations should be determined.

3.1 Following a single segment

Since it is stipulated that the vehicle should follow the drive path, it is necessary to decide how this should be done, and formulate the strategy mathematically such that it can be implemented as algorithms in the computer.

To start with, we can formulate the strategy a human being would use when he want to reach a specific object by walking towards it as

```
LOOP
  1. Look at the object that should be reached.
  2. Walk in that direction.
UNTIL stop-condition.
```

This strategy can be formulated for the vehicle as

```
LOOP
  1. Turn the steer wheel such that it aims at the specific point on the drive path that we want to reach.
  2. Drive.
UNTIL stop-condition.
```

If this strategy could be formulated mathematically then the only remaining problem would be how to select the aiming point such that the design requirements are fulfilled. Fortunately, this strategy can be mathematically formulated as
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\[
\bar{\alpha}_s(k) = \text{Atan2}(-y_s^*(k), d_s) - \theta_s^*(k)
\]

\[
\alpha_s(k) = \text{sat} (\bar{\alpha}_s(k))
\]

The interpretation of (3.1) is that the steer wheel is aimed at a point (the aiming point) on the x-axis of the segment which is the (aiming) distance \(d_s\) away from the projection \(x_s^*\). \(\bar{\alpha}_s(k)\) is the unsaturated value of the set value. The operator \(\text{Atan2}\) is just a four-quadrant arctan function. The principal function of the control law is illustrated in fig. 1d.

We can intuitively get a feeling for how the vehicle behaves for different choices of the aiming distance, without analyzing the closed-loop system mathematically.

- If the aiming distance has a negative sign, the result will be that the vehicle follows the drive path in the negative direction of it.

- If the vehicle is some distance away from the segment and if the aiming distance is zero, the vehicle will travel perpendicular towards the segment.

- If the aiming distance is zero or close to zero, and the vehicle is on the segment and almost parallel to it, the result will be that the vehicle oscillates around the drive path. This because the control is done in discrete time. Observe that the contact point of the steer wheel coinciding with the aiming point on the drive path does not imply that the vehicle stops, unless there is some interaction between the steer controller and the speed controller. The oscillating behavior can be reduced by decreasing the speed or by shortening the sampling interval or, as is most natural, by choosing a larger aiming distance.

- On the other hand, if the aiming distance is very large and the vehicle is not on the segment and/or not parallel to it, the result will be that it takes quite some time before the vehicle is on the segment and parallel to it.

The conclusions one can make from the third observation, is that one can never allow the vehicle to reach the aiming point on the segment if the control should be successfully.
Fig 2. The path the vehicle takes when it is controlled by the control law in (3.1). The initial position and heading to the segment is \((s^*, y^*, \theta^*) = (0, -0.82, 1.57)\). To the right, the aiming distance \(d\), as a function of the perpendicular distance from the segment to the vehicle, \(y_s\), is given. \(d^{\text{close}}\) is the aiming distance when the perpendicular distance from the segment to the vehicle is 0 m. In this simulation \(d^{\text{close}}\) is 0.3 m.

The simulation results in fig. 2 indicates that it is possible to fulfill the design requirements with a proper choice of the aiming distance. The vehicle takes a smooth path without an overshoot when it starts to follow the drive path and it does not oscillate around the segment. As times goes by, the errors in the position and the heading decreases.
3.2 Stability analysis of the linearized closed-loop system

A mathematically useful tool for investigating the properties of the closed-loop system is stability analysis. In this case we have a non-linear process model and a non-linear control law. This makes stability analysis a little bit difficult. However, when the vehicle is on the segment and almost parallel to it, it is possible to get good approximations of both the motion model and the steering controller by linearization.

If we linearize (2.1) around small values of α and θ, and assume that the position and heading to a segment are modeled, then the position and heading are given by

\[ y = v \cdot (\theta - \alpha), \quad \theta = \frac{v}{L} \cdot \alpha \]  

(3.2)

A state-space representation of (3.2) is

\[ z = A \cdot z + b \cdot \alpha \]  

(3.3)

\[ A = \begin{pmatrix} 0 & v \\ 0 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} v \\ v/L \end{pmatrix}, \quad z = \begin{pmatrix} y \\ \theta \end{pmatrix} \]

Since the input α originates from a discrete time controller, we have to discretize (3.3) to make it possible to analyze the closed loop system. If we assume that v and α are constant during the sampling interval h, which implies that we assume no dynamics in the servos, we have the discrete time system

\[ z(k + 1) = \Phi(k) \cdot z(k) + \Gamma(k) \cdot \alpha(k) \]  

(3.4)

\[ \Phi(k) = e^{Ah} = \begin{pmatrix} 1 & v(k)h \\ 0 & 1 \end{pmatrix}, \quad \Gamma(k) = \int_0^h e^{As}bds = \begin{pmatrix} v(k)h \cdot (1 + v(k)h/(2L)) \\ v(k)h/L \end{pmatrix} \]

Since we have assumed that there are no dynamics in the servos, we have \( \alpha_s(k) = \alpha(k) \), where \( \alpha_s(k) \) is given by (3.1). Linearization of (3.1) gives the state-feedback controller

\[ \alpha(k) = \alpha_{set} - (1/d_s, 1) \cdot \begin{pmatrix} y(k) \\ \theta(k) \end{pmatrix} \]  

(3.5)

where \( \alpha_{set} \) is 0 rad if the aiming distance is in the positive direction of the segment and \( \pi \) rad if the aiming distance is in the negative direction. (3.4) and (3.5) gives the linearized and discretized closed loop system as

\[ z(k + 1) = \Omega(k) \cdot z(k) + \Gamma(k) \cdot \alpha_{set} \]  

(3.6)

\[ \Omega(k) = \begin{pmatrix} 1 - v(k)h/d_s \cdot (1 + v(k)h/(2L)) & -(v(k)h)^2/(2L) \\ -v(k)h/(d_sL) & 1 - v(k)h/L \end{pmatrix} \]  

(3.7)

The characteristic equation of (3.6) is given with the dimensionless variables \( u_1 = v(k)h/L \) and \( u_2 = L/d_s \) by
\[ \lambda^2 + a_1 \cdot \lambda + a_2 = 0 \] (3.8a)

\[ a_1 = u_2 u_1 \left( 1 + \frac{u_1}{2} \right) + u_1 - 2 \] (3.8b)

\[ a_2 = u_2 u_1 \left( \frac{u_1}{2} - 1 \right) + 1 - u_1 \] (3.8c)

The values of \( \lambda \) that satisfies (3.8) are the eigenvalues of \( \Omega(k) \) in (3.7), i.e. the poles of the closed loop system.

**Fig 3.** The closed loop system is asymptotically stable if the poles lies within the unit circle. Which is equivalent with that \((a_1, a_2)\) in (3.8a) lie within the triangle.

If we assume that \( u_1 \) and \( u_2 \) in (3.8b&c) can take any values, one might expect that \( a_1 \) and \( a_2 \) also can take any values and thereby the closed loop system can have arbitrary poles. However, this is not the case which is shown below.

If we substitute \( u_2 \) in (3.8c) with \( u_1 \) in (3.8b) expressed in \( u_1 \) and \( a_1 \), we get

\[ a_2 = \frac{(2 + a_1) \cdot u_1 - 2 \cdot (1 + a_1) - u_1^2}{(2 + u_1)} \] (3.9)

If we study fig. 3.1 we see that it is necessary that \(-2 < a_1 < 2\) for the closed-loop system to be asymptotically stable. If we select a value of \( a_1 \) in that interval it should be possible to select values of \( u_1 \) in (3.9) that makes \( a_2 \) take any values inside the triangle in fig. 3 for arbitrary poles.

\( \delta a_2 / \delta u_1 = 0 \) gives the values of \( u_1 \) that maximizes and minimizes (3.9). Inserting those values of \( u_1 \), into (3.9), assuming \( u_1 \) can not be negative, gives the extreme values of \( a_2 \) as a function of \( a_1 \), according to
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\[ a_2^{\text{(max)}} = \frac{(11 + a_1) \cdot \sqrt{8 + 2a_1} - 8a_1 + 32}{\sqrt{8 + 2a_1}} \]

\[ a_2^{\text{(min)}} = -a_1 - 1 \]

The assumption that \( u_i \) can not be negative implies that the front wheel can only rotate in one direction around its axis.

Fig 4. The shadowed region is possible to reach by choosing values of \( u_i \) and \( u_2 \) in (3.8). \((a_1, a_2)\) on the curve \( a_2 = a_1^{1/4} \) implies that the closed-loop system has a double-pole. Since the shadowed region lies to the left of the curve \( a_2 = a_1^{1/4} \) it is not possible to have complex poles.

From fig. 4 it is clear that the stability condition is

\[ a_1 < 1 + a_2 \]  \hspace{1cm} (3.10)

(3.8b&c) and (3.10) gives, after some manipulations, the stability conditions expressed in \( u_i \) and \( u_2 \) as

\[ 0 \leq u_i < 2 \]
\[ 0 \leq u_2 < \frac{2 - u_1}{u_1} \]

Thus, the linearized and discretized closed loop system is stable if

\[ 0 \leq v(k)h < 2L \]  \hspace{1cm} (3.11a)
\[ d_s > \frac{Lv(k)h}{2L - v(k)h} \]  \hspace{1cm} (3.11b)
These results verifies what was indicated earlier, namely that the aiming distance can not be to small and that the speed times the sampling interval can not be to large. It is also interesting to note from (3.11a) that a vehicle with a larger distance between the rear-axis and the front wheel allows, in stability sense, a larger product $v(k)h$, than a vehicle with a smaller distance between the rear-axis and the front wheel.

### 3.2.1 Choice of the aiming distance based on pole-placement analysis

The vehicle is under the assumptions made driving with small steer angles. If we assume the speed to be the maximum speed, we are left with the aiming distance as the only parameter to affect the poles. One design requirement is that the vehicle should not oscillate around the drive path. This is equivalent to the demand that the linear closed-loop system should be at least critically damped. Which is the same as to say that the closed-loop system should have both poles on the real-axis. From fig.4, it is clear that the system only can have poles on the real axis. The only possible double-pole is that of the open-loop system, located in $z=1$, which is out of the question for obvious reasons. If a system has poles on the negative real-axis, there is a significant risk of 'ringing' in the output from the system, i.e. the output will jump up and down with a decreasing amplitude if the closed-loop system is asymptotically stable [12]. In this case, a small value of the aiming distance may cause one of the poles to have negative sign.

The results of the pole-placement analysis can be summarized as:

- Keep the sampling time times the speed as small as possible.
- Select the aiming distance such that both poles are positive.

In the simulation presented in fig. 2, $\nu_1 = 0.5 \cdot 0.2 / 0.83 = 0.12$ and $\nu_2 = 0.83 / 0.3 = 2.77$. This gives $\lambda_1 = 0.89$ and $\lambda_2 = 0.64$, which corresponds to a relative damping of 1.06.
3.3 Switching between segments

The disadvantage in defining the drive path as a sequence of points connected with straight line segments is, as pointed out earlier, the corners between the segments. One design requirement is that the vehicle should follow the drive path smoothly (which is hard to formulate precisely). Allowing only very small angles between consecutive segments is one way of fulfilling this requirement. This restriction may lead to drive paths defined with enormous sets of coordinates. Therefore it is desirable to allow large angles between consecutive segments. It is also desirable, for the sake of simplicity, to use the same control law for steering when the vehicle turns from one segment to another, as when it follows a single segment. If the same control law should be used, we need to:

- find a condition for when to start following a new segment.
- select the aiming distance on the new segment.

The condition for switching between segments, must be such that with a proper selection of the aiming distance, the vehicle takes a smooth path without an overshoot. To make this possible, the vehicle must start to follow the new segment before the end of the old segment is reached. If we call the distance from the end of the segment along the x-axis where the vehicle starts to follow the new segment \( l_s \), and the length of the segment \( l_i \), we have the condition for switching between segments

\[
x_s^*(k) \geq l_s - l_i^e
\]  

(3.12)

Clearly if \( l_i^e \) is too small, there is a risk that the vehicle makes an overshoot in the beginning of the new segment, because of the dynamics of the vehicle and the servos.

Suppose that the vehicle starts to follow the new segment on a distance away from it that makes it possible for the vehicle to take a smooth path, and that we calculate the aiming distance according to fig. 2. The path the vehicle then takes is smooth without an overshoot. The problem is that there might be a large change in the steer angle at the sampling instant when it starts to follow the new segment. This because the steer wheel might aim in a totally different direction other than what the aiming point on the new segment tells. The solution to this problem is simply to place the aiming point where the steer wheel aims at this sampling instant. A mathematical formulation of this is

\[
d_s^{stari} = -y_s^{stari} \cdot \tan[\text{sign}(\theta_s^i(k)) \cdot \pi/2 - \theta_s^i(k) - \alpha_s(k)] , \quad -\pi \leq \theta_s^i(k) \leq \pi
\]  

(3.13)
where $d_s^{\text{act}}$ and $y_s^{\text{act}}$ are the values of the aiming distance and the perpendicular distance from the segment at the sampling time when the vehicle starts to follow the new segment. $\alpha(k)$ is the steer angle before the set value has been changed at sampling instant $k$. Observe that $d_s^{\text{act}}$ not necessarily has to be zero, it can even be negative. Fig. 5 gives an example of the later case.

**Fig 5.** The vehicle at the sampling instant when it leaves segment $s-1$ and starts to follow segment $s$, and the aiming distance $d$, where $d_s^{\text{act}}$ is calculated according to (3.13).

**Fig 6.** The path the vehicle takes when it turns from one segment to another segment rotated $\pi/2$ rad to the starting segment. The aiming distance in (3.1) is calculated according to (3.13). To the upper right, the set value to the steer servo during the travel is shown, and to the lower right we can see the difference in the set value from one sampling instant to another. Note that this difference is never greater than 0.1 rad. $r$ in (3.12) is 0.86 m.
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From fig. 6 it is clear that calculating $d_i$ according to (3.13) prevents large changes in the steer angle at switching time, and at any other sampling instant if $y_i^{\text{max}}$ is not too small. A smaller value of $y_i^{\text{max}}$ would have resulted in larger changes in the steer angle, but also in a smaller deviation from the drive path.

If $d_i^{\text{max}}$ is calculated according to (3.13) and if the two consecutive segments are almost parallel, the quotient $|d_i^{\text{low}} - d_i^{\text{max}}| y_i^{\text{max}}$ might be very large. This can cause instability, and therefore the maximum allowed quotient has to be bounded.
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3.4 Steering an arbitrary point on the mid-axis of the vehicle

Up to now we have assumed that the point of the vehicle that should follow the drive path is the contact point between the front wheel and the surface. This maybe the most natural point to control, but it could also be of interest to control other points of the vehicle.

If we introduce a fictive front wheel located somewhere on the line connecting the rear-axis and the front wheel, it is easy to modify the steer controller to control any other point on the mid-axis of the vehicle, except that in the crossing of the rear-axis and the mid-axis.

All points of the vehicle have the same instantaneous center of rotation. A vector describing the direction of the speed of a wheel is perpendicular to a vector starting in the instantaneous center of rotation and going through the center of the wheel, if the wheel does not slip. Since we assume no slipping, we have the following relation between the steer angle of the fictive wheel, $\alpha'_f$, and the steer angle of the real front wheel, $\alpha$,

$$L \cdot \tan(\alpha'_f) = L_f \cdot \tan(\alpha_r) \tag{3.14}$$

where $L$ is the distance between the rear-axis and the front wheel, see fig. 1b, and $L_f$ is the distance between the rear-axis and the fictive front wheel.

The modification needed to control an arbitrary point is that the position of the vehicle that is feed into the controller is the position of the fictive wheel. The resulting setvalue, $\alpha'_r$, is transformed to a setvalue, $\alpha_r$, of the real front wheel according to

$$\alpha_r = \text{Atan}\left[ \frac{L}{L_f} \cdot \tan(\alpha'_f) \right] \tag{3.15}$$

By inspection of (3.15) we see that it is impossible to locate the fictive wheel on the rear-axis, since $L_f$ in that case equals 0 m.

If we change the control to a point closer to the rear-axis than the front wheel, the maneuverability of the vehicle increases. Unfortunately, this may also lead to an unstable system if the speed is not reduced compared to when the front wheel is controlled, which is clear from an inspection of (3.11a).
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3.5 Speed control

The speed of the vehicle will affect the stability, see (3.11), and also the time it takes to travel the drive path. A strategy of the speed control is to select the set value to the speed servo so that the stability is guarantied and that the travel time is the desired one. Obvious there has to be a compromise between the two demands.

When the errors in the distance from the segment to the vehicle and in the heading to the segment are small, the speed is selected so that the vehicle reaches checkpoints on the segments at predefined time instants, which makes it possible for the vehicle to travel the drive path in predefined time. If the vehicle is performing maneuvers, for instance when it switch to a new segment, there might large changes in the steer angle from one sampling instant to another, due to a too high speed. The mode in the speed controller is shifted to predictive control when larger errors are detected. The predictive controller selects a setvalue to the speed servo so that the predicted change in the setvalue to the steer servo at next sampling instant does not exceed a certain value. This reduces the risk of oscillations. In addition, there also is a possibility to restrict the change of the set value to the speed servo from the previous sampling instant to the current sampling instant according to

\[ |v_r(k) - v_r(k-1)| \leq \Delta v_{\text{max}} \] (3.16)

This is to prevent slipping a and to avoid mechanical wear. Also it improves the estimates of the state-vector [10].

3.5.1 Time control

The checkpoint on each segment is located where the vehicle turns to the next segment. Call the instant in time when we want the vehicle to reach the checkpoint for \( t^* \), and the position on the segment where the vehicle switch to the next segment for \( x^* \). The relation between \( t^* \) and \( x^* \) are

\[ x^* - x^* = \bar{v}(k) \cdot (t^* - t) \] (3.17)

where \( \bar{v}(k) \) is the average speed the vehicle has to drive with to reach the checkpoint in time. \( t \) is the total travelling time of the vehicle. We assume that \( t < t^* \). The set value to the speed servo is then \( v_r(k) = \bar{v}(k) \), if \( v_{\text{min}} \leq \bar{v}(k) \leq v_{\text{max}} \).

3.5.2 Predictive control

The set value to the speed servo should be selected so that it prevents the system to oscillate or, in the worst case, go unstable. An indication of that this undesirable behavior is going to happen, shows up as large changes in the steer angle from one
sampling to another. Therefore the set value to the speed servo is selected such that 
\[ |\Delta \alpha_\text{v}(k+1)| \leq \Delta \alpha\text{max} \], where \(\Delta \alpha_\text{v}(k+1)\) is a prediction of the change in the set value to the steer servo at sampling instant \(k+1\). If we use (3.1) we get

\[
\Delta \alpha_\text{v}(k+1) = A\tan 2(-y_\text{s}(k+1), \hat{\alpha}_\text{v}(k+1)) - \hat{\theta}_\text{s}(k+1) -
\]

\[
-A\tan 2(-y_\text{s}(k), d_\text{s}(k)) + \theta_\text{s}(k)
\]

(3.18)

If we integrate (2.1c) over a sampling period \(T = t_{k+1} - t_k\), assuming the speed and steer angle to be constant and equal to the set values, we get

\[
\theta(k+1) = \theta(k) + \Delta \theta(k)
\]

(3.19a)

\[
\Delta \theta(k) = \frac{v_\text{s}(k) \cdot T}{L} \cdot \sin(\alpha_\text{s}(k))
\]

(3.19b)

which is a prediction of the heading at \(k+1\). The aiming distances \(d_\text{s}(k)\) and \(d_\text{s}(k+1)\) are, if the vehicle is close to the segment, given by

\[
d_\text{s}(k) = d_\text{s}^\text{close} + k \cdot y_\text{s}(k)
\]

(3.20a)

\[
d_\text{s}(k+1) = d_\text{s}^\text{close} + k \cdot y_\text{s}(k+1)
\]

(3.20b)

where \(k = -\left(\frac{d_\text{s}^\text{close} - d_\text{s}^\text{max}}{y_\text{s}^\text{max}}\right)_1\), see (3.13). If the vehicle is far away from the segment, say more than 1 m., the aiming distance is 0 m. The motion of the vehicle is along a segment of a circle with radius \(L/\sin(\alpha_\text{s}(k))\), if the steer angle is \(\alpha_\text{s}(k)\) during the sampling interval. A prediction of \(y_\text{s}(k+1)\) is then

\[
y_\text{s}(k+1) = y_\text{s}(k) - v_\text{s}(k) \cdot T \cdot \cos(\alpha_\text{v}) + \Delta y(k)
\]

(3.21)

\[
a_\text{v} = A\tan 2(d_\text{s}(k), y_\text{s}(k))
\]

\[
\Delta y(k) = \frac{L}{\sin(\alpha_\text{s}(k))} \left[\sqrt{1 + \Delta \theta^2(k)} - 1\right]
\]

where \(\Delta y(k) \to 0\) as \(\alpha_\text{s}(k) \to 0\). The predicted change in the set value to the steer angle is

\[
\Delta \alpha_\text{v}(k+1) = A\tan 2(-y_\text{s}(k+1), \hat{\alpha}_\text{v}(k+1)) - A\tan 2(-y_\text{s}(k), d_\text{s}(k)) - \Delta \theta(k)
\]

(3.22)

where \(\hat{y}_\text{s}(k+1)\) is given by (3.21) and \(\Delta \theta(k)\) by (3.19b).

A number of values in the interval \([v_\text{max}, v_\text{min}]\) that are candidates to become the set value, are tried in (3.22). The search for a set value starts with \(v_\text{max}\) and continues
with the second largest value etc. The search ends when a value that satisfies 
$|\Delta \alpha_{s}(k + 1)| \leq \Delta \alpha_{max}$ is found. That value is picked out to be the set value to the speed servo at sampling instant $k$.

(3.22) is a rough prediction mainly because the dynamics of the servos are neglected. It has the advantage that it is easy to find the set value to the speed servo that limits the predicted change in the set value to the steer servo at the next sampling instant.

Fig. 7 shows the simulation results when the vehicle drives along a segment with the controlled point 0.075 m. in front of the rear-axis and with the set value to the speed servo constant 0.5 m/s. The steer angle, $\alpha(t)$, and the speed, $v(t)$, of the servos are recorded together with the change in the set value to the steer servo from one sampling instant to another, $\Delta \alpha_{s}(k) = \alpha_{s}(k) - \alpha_{s}(k - 1)$. The initial errors to the segment are $(y_s(0), \delta_y(0)) = (-0.35, \pi/2)$. The initial error in the distance from the segment to the controlled point is also $y_s^{max}$ in (3.13), i.e. $y_s^{max} = 0.35$ m. The aiming distance when the vehicle is on the segment, $e_{4y}$, is 0.3 m.

The maximum change in the set value to the steer servo is approximately 0.15 rad. and occurs when the heading of the vehicle to the segment is around $\pi/4$ rad. The system is compared to the simulations in fig. 2 and fig. 6 closer to the stability margin. The stability conditions according to (3.11) are, $v(k)h < 0.15$ m. and $d_{max} > 0.15$ m. There is a quite significant ringing in the set value when the vehicle is close to the segment, which is explained by that the linearized closed-loop system has a pole with negative sign in $z = -0.61$. The ringing shows up as oscillations in the steer angle around 0 rad.

In fig. 8, the same simulation as in fig. 7 is shown, but with the difference that the set value to the speed servo is calculated such that the predicted change in the set value to the steer servo according to (3.22), is less than $+/− 0.075$ rad. The maximum allowed speed is the same as for the simulation in fig. 7 and the minimum allowed speed is 0 m/s. 10 equal distributed values in the interval $\{v_{max}, v_{min}\}$ are candidates to become the set value to the speed servo. There is no limitation in the change in the set value to the speed servo from one sampling instant to another.

The changes in the set value to the steer servo from one sampling instant to another are smaller, compared to the changes when the set value to the speed servo is constant 0.5 m/s. The changes are never greater than the maximum allowed change.

It may seem that the control action is unnecessary careful in the interval 4 to 8 seconds, which is the interval when the heading to the segment is around $\pi/4$. Compared to the simulation in fig. 7, where the corresponding interval is 2 to 4 seconds, this careful action is clearly preferable.

Another difference compared to the previous simulation, is that the amplitude of the ringing in the set value to the steer servo is much lower. This because the errors to the segment are small when the speed is increased to maximum speed.

The price to pay for the careful control is that the segment takes 20 seconds to travel instead of 14.5 seconds, which is the time it takes if the set value to the speed servo is constant 0.5 m/s.
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Fig 7. The setvalue to the speed servo is constant 0.5 m/s.

Fig 8. The setvalue to the speed servo is calculated so that the predicted change in the set value to the steer servo is less than +/− 0.075 rad.
3.6 Discussion on drive path specification

The drive path specification contains

1. coordinates for the drive path

and to each segment there is a specification on

2. maximum allowed speed

3. condition for when to switch to the next segment in the drive path

4. at what time the vehicle should reach the check point

5. if the end station is connected to the segment

6. if the vehicle should report to the supervisory system and wait for an order

7. the minimum allowed distance to an object before a report to the supervisory system should be done

Some of the information needed in the specification of the drive path can hardly be determined by pure mathematical analysis. Therefore there have to be an interaction between mathematical analysis, simulations and practical experiments to determine the information needed. A few guidelines to what should be considered when the drive path specification is determined are given below.

Other considerations than the stability conditions that have to be taken into account when the maximum allowed speed is determined are if the surface is slippery, if there are more than one vehicle operating the segment at the same time and how accurate the navigation should be. A low speed of the vehicle results in a better estimate of the state-vector, i.e. the position and the heading of the vehicle, compared to a high speed of the vehicle [10].

The distance from the end of the segment when the vehicle should start to follow the next segment, \( t \) in (3.12), should depend on the maximum allowed speed of the vehicle, the maximum allowed change in the setvalue to the steer servo from one sampling instant to another, the angle between two consecutive segments and on the surroundings to the drive path. If, for instance, the two segments are connecting two corridors that are perpendicular to each other, it is not advisable to select \( t \) so that the vehicle (tries to) take a short cut through the walls.

If an obstacle is detected close to the vehicle, the speed should be put to zero and a report should be done to the supervisory system that take action. For instance recalculates the drive path so that the vehicle avoids the obstacle [14].
4 SUMMARY

A control strategy for guiding an AGV along a segmented drive path has been deduced. The strategy has resulted in controllers for steering and driving. The simplicity of the drive path demands some care when the parameters of the controllers are designed. Analysis and simulations have shown that properly selected parameters, combined with the dynamics of the AGV, implies that the vehicle takes a smooth path without overshoots and oscillations when it follows the drive path. By specifying instants in time when the vehicle should reach checkpoints on the segments, it is possible to make the vehicle travel the drive path in predefined time.

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6 APPENDIX

6.1 Transformation of position and heading

Here we describe how the global position and heading can be transformed to a local position and heading in a frame of a segment, and vice versa. Fig. 1c gives an example of a global position and heading, \((x,y,\theta)\), and a local position and heading, \((x'_s,y'_s,\theta'_s)\), in the frame of segment \(s\).

The heading \(\theta\) in the global frame is transformed to a heading in the local frame of segment \(s\) according to

\[
\theta'_s = \theta - \arctan2((y_{s+1} - y_s), (x_{s+1} - x_s))
\]

If we assume the position \(p' = (x'_s, y'_s)\) in the local frame to be known (the reason for this assumption will become clear later), we can transform it using homogeneous transformation to a position \(p = (x, y)\) in the global frame according to

\[
\begin{pmatrix}
  p \\
  1
\end{pmatrix}
= \begin{pmatrix}
  R^* & p_{org}^* \\
  0 & 1
\end{pmatrix}
\begin{pmatrix}
  p'_s \\
  1
\end{pmatrix}
\]

where \(p_{org}^*\) is a vector describing the origin of the local frame in the global frame [13]. That is

\[
p_{org}^* = (x'_s, y'_s)\]

\(R^*\) is a rotation matrix describing the rotation of the local frame of segment \(s\) in the global frame. It can be expressed as

\[
R^* = \frac{1}{l_s} \begin{pmatrix}
  (x_{s+1} - x_s) & (y_s - y_{s+1}) \\
  (y_{s+1} - y_s) & (x_{s+1} - x_s)
\end{pmatrix}
\]

where \(l_s\) is the length of the segment. Actually, we are not interested in transforming the position and heading in the local frame to a position and heading in the global frame. Instead we are interested in the inverse transformation. We would like to find a transformation of the type

\[
\begin{pmatrix}
  p' \\
  1
\end{pmatrix}
= \begin{pmatrix}
  R & p_{org} \\
  0 & 1
\end{pmatrix}
\begin{pmatrix}
  p \\
  1
\end{pmatrix}
\]

By using the properties of homogeneous transformations we have
\[ R^* = -Rp^* \]

This gives
\[
R = \frac{1}{l_e} \begin{pmatrix} (x_{s+1} - x_s) & (y_{s+1} - y_s) \\ (y_s - y_{s+1}) & (x_{s+1} - x_s) \end{pmatrix}
\]

\[ p_{org} = \frac{1}{l_e} \cdot (y_s \cdot (y_s - y_{s+1}) - x_s \cdot (x_{s+1} - x_s)\right) \]

By inspection of the last expression it is easy to understand the assumption made above.
7 REFERENCES


