DOCTORAL THESIS

Studies of the flow pattern in short term hot water storages

by
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Studies of the flow pattern in short term hot water storages

av

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Abstract

This thesis relates to the study of thermally stratified hot water storages used in district heating systems and is an extension of the doctoral theses of Jan Dahl and Roger Hermansson. The thermal performance of a stratified water storage is reduced by different phenomena such as heat transfer to the surroundings and mixing between hot and cold water during the charging of the tank. The study of the flow field in the storage tank gives a better understanding of these phenomena.

The work described here is composed of two parts. The first concerns the development of a Particle Tracking Velocimetry (PTV) technique, described in Paper I, which can be used to measure the velocity field in a model water heat storage. Paper II presents a study of phenomena affecting the accuracy of the PTV technique. The inaccuracy of the velocity measurements due to the limited resolution of the video camera increases rapidly if the time-step between pictures is too small. The number of incorrect identifications is also an important factor for the determination of the time-step. A method of selecting the appropriate time-step has been proposed in Paper III. With the help of a procedure to remove incorrect identifications, a method for evaluation of the velocity field aimed at reducing errors caused by resolution inaccuracy and incorrect particle identifications has been developed.

The second part of this thesis concerns studies of flow patterns in a model water heat storage. Using the PTV technique presented previously, the natural convection boundary layer has been observed and measured in Paper IV. Flow phenomena caused by heat losses to the surrounding are found to have an important effect on the nature of the gradient zone. The gravity current resulting from the inflow of hot water in the tank filled with cold water has been studied in Paper V. A numerical simulation has been performed. The mixing has been quantified through an efficiency based on exergy. Phenomena leading to a loss of efficiency during the formation of the thermocline have been identified. The conditions of vortex formation at the edge of the inlet plate during the formation of the thermocline are investigated in Paper VI. A range of critical Richardson numbers, based on a comparison of experimental results, numerical simulation and theoretical analysis has been proposed.
Preface

This thesis has been carried out for three quarters of my time at the Division of Energy Engineering at Luleå University of Technology and for the rest of the time at the Multiphase Flow and Heat Transfer group at the Energy, Theoretical and Applied Mechanics Laboratory in Nancy, France. This work has been financially supported by the Swedish Ministry of Education for developing the co-operation between the two laboratories.

I would like to express my gratitude to my supervisors Professor Björn Kjellström in Luleå and Lecturer Salah Skali in Nancy for their support and encouragement during my studies.

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Luleå, May 1998

Veber Pascal
List of Papers

This thesis is based on the following papers:


III Veber P; Linnaeus S (1998) An improved Particle Tracking Velocimetry technique. Submitted for publication


V Veber P; Skali S; Kjellström B (1998) Gravity currents at the inlet of thermally stratified storage tanks. Submitted for publication

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Introduction

Thermal storage is an effective energy management technique appropriate to even out variations in demand or production of heat. Heat generated during off-peak periods can be charged into the storage and used during the following peak load periods. This technique allows a more effective use of the base load units improving the average efficiency. The construction of additional units to meet the peak load demand can also be avoided.

The most common storage medium is water due to its abundance, low cost, high heat capacity and non-toxicity. Stratified water tanks are often used as thermal storages in district heating systems. The principle of this type of thermal storage is based on use of the difference in density to keep the hot and the cold water separated. This option is simple, requires low investment and needs little maintenance which makes it attractive.

However, since the separation force between the hot and cold water volumes is small, some mixing between hot and cold water will always occur which reduces the heating capacity of the storage tank. The region where the density difference creates a vertical gradient of temperature is called the thermocline. The performance of a thermally stratified storage tank is mainly influenced by:

- Mixing at the inlet/outlet during charging/discharging. The losses, caused by this, are depending of the geometry of the inlet diffuser, the flow rate and the temperature conditions.
- Internal heat transfer across the thermocline. The diffusion of heat from the hot region to the cold region of the tank is proportional to the contact area and the temperature difference between these two regions.
- Vertical conduction in the tank wall. This phenomena is depending of the design of the insulation and the walls.
- Heat transfer to the surroundings. The prediction of the heat transfer on annual basis is depending of the degree of loading and the surrounding temperature and will vary over the year.
- Variations in charging temperature.
Introduction

Research about some of the phenomena that influence the performance of water heat storage was started at the Division of Energy Engineering of the University of Luleå in 1986 by J. Dahl and R. Hermansson. In order to study these phenomena, a plexiglass model storage tank was built and a technique to visualize the flow field was developed. The work presented in this thesis is a continuation of this research and has been focussed on increasing the understanding of flow phenomena that may be important for the loss of performance of a stratified storage tank.

Particle Tracking Velocimetry technique

The Particle Tracking Velocimetry (PTV) technique is presented in Paper I. The PTV technique consists of continuous recordings of the in-plane movements of particles in a light sheet in seeded water using an ordinary video camera. The light sheet is created with a laser and a cylindrical lens and the particles have almost the same density as the water. This technique allows continuous recording of transient flows and subsequent analysis of the interesting parts of the flow.

The pictures are transferred from a video recorder to a computer through a frame grabber. A number of frames with an appropriate time difference are then analysed. Files that contain the co-ordinates for all particle spots that are within a band of size and sufficiently illuminated after a thresholding of light are stored. It is possible to superimpose a number of frames to display particle traces. These traces are an effective tool to study the flow field.

A computer tracking procedure determines the magnitude and direction of the particle velocities. This tracking procedure requires selection of a maximum search area radius and a time-step between two consecutive pictures. These parameters have to be adapted to the velocity field studied. This requires some a priori understanding of the flow pattern. The possibility to choose the time-step between two consecutive pictures after completion of the experiment constitutes an important advantage compared to other techniques that require the time intervals to be determined before or during the experiment.
The accuracy of the PTV technique is investigated in *Paper II*. The resolution error due to the CCD (Charge Coupled Device) camera sampling of the imaged light intensity distribution scattered from particles in the light sheet has been studied (Wernet and Pline 1993). The subpixel accuracy has been estimated to be ±1/3 pixel for the particle image size observed in the water heat storage experiments. Thus in order to obtain a resolution error smaller than 10%, the distance covered by the particle has to be larger than 5 pixels.

During the image processing, the errors due to the calibration of the picture, the thresholding of light and the transfer of the pictures from the video recorder to the computer have been evaluated roughly to 10%.

The last part of the PTV technique, the evaluation of the velocities, is disturbed by appearance of incorrect identifications of particle tracks. A strong increase in the tracking error, defined as the ratio of incorrect identifications to the number of evaluations, has been observed when the distance covered by a particle exceeds 6 pixels. The choice of the time-step is essential if a high accuracy is desirable, since the distance covered by a particle between two pictures is increasing with it. A too small time-step can result in a high resolution error and a too large time-step in a high number of incorrect identifications.

A new procedure for removing incorrect velocity vectors has been developed and presented in *Paper III*. This procedure has been tested on a simulated velocity field. The results show that it is possible to quantify the variation of the search area radius versus the particle density for a given percentage of incorrect identifications. However the choice of the time-step could still lead to a velocity field with a too high number of incorrect identifications to be treated by the procedure. An automatic tracking procedure aimed at reducing the intervention of the operator is needed.

A preliminary evaluation is performed with fixed parameters. The time-step is the smallest possible based on the video-camera and computer features and the search area radius is set to 6 pixels due to the strong increase of incorrect identifications for higher distance. This preliminary evaluation provides values of the particle density and of the maximum velocity.
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With the particle density calculated in the preliminary evaluation, it is possible to determine the search area radius resulting in 20% incorrect identifications. The time-step in the final evaluation is then determined with the maximum velocity obtained in the preliminary evaluation. Thus in the final evaluation, the resolution error is reduced and most of the incorrect identifications can still be removed. The time-step is better adapted to the velocity field studied and more vectors are identified. This new automatic tracking procedure should allow a wider use of the PTV technique.

The PTV technique has been successfully used to measure velocities in a water heat storage tank model and the results are presented in the next paragraph.

Study of the effects of heat transfer to the surroundings and mixing at the inlet on the thermal performance of a water heat storage tank model

The following figure illustrates schematically the development of the exergy efficiency, defined as the ratio between the exergy of water in the storage divided by exergy supplied during charging, for a storage initially filled with cold water which is charged with hot water between time 0 and t₃.

![Figure: Evolution of the exergy efficiency versus time](image-url)
For \( t < t_1 \), the initial pass of the hot water along the top of the tank results in a strong decrease of the efficiency due to the mixing between hot and cold water. For \( t > t_1 \) the hot water has reached the tank wall and the thermocline starts to establish. Thus the mixing is reduced and the efficiency of the tank is rising. Different phenomena can affect the formation of the thermocline such as mixing between hot and cold water when the hot water reaches the wall or the centre of the tank and is redirected downwards. For \( t > t_2 \), the thermocline is established and the inflow of hot water at the inlet has no more effect on the performance of the tank. The efficiency is increasing slowly due to the increasing volume of hot water above the thermocline.

In the period up to \( t_2 \), the flow conditions at the inlet have a dominating influence on the exergy losses. From time \( t_2 \), the continuous losses in form of heat conduction across the thermocline, heat conduction in the walls and heat exchange with the surroundings are the main contributions to the exergy losses. The exergy efficiency of the storage will nevertheless continue to increase as long as charging continues since these losses are much smaller than the charging rate. When the charging is complete, the exergy efficiency will slowly decrease as a consequence of the continuous losses. The rate of efficiency loss can be expected to increase when the stratification is impaired by the breakthrough of the thermocline by the wall boundary layer at time \( t_4 \).

The heat exchange with the surroundings tends to average the temperature in the tank, decreasing the efficiency of the tank. A one-dimensional model of the stratification decay with heat exchange through the walls of the tank has been compared to experimental data and showed fairly good agreement (Jaluria and Gupta 1982 and Al-Najem 1993). The model results indicate that the heat loss to the surroundings is the major factor in destroying the stratification in bare-walls storage tanks. The only discrepancy with the experimental data is due to the thermal diffusion in the middle region of the tank, which amounts to about 10\% of the total heat loss.

The effects of the heat transfer to the surroundings on the flow field in a storage tank model have been investigated in Paper IV with the help of the PTV technique. The study of the natural convection boundary layer due to the heat...
transfer from the storage tank to the surroundings shows that when the flow through the storage tank is stopped during standby conditions, a convection cell appears starting from the top of the storage to the thermocline, where the buoyancy forces redirect the flow towards the centre and into a stream of water going upwards. As the boundary layer is continuously running into the thermocline, near the wall the temperature gradient gets weaker. After some hours, the downward flow along the wall will penetrate through the thermocline resulting in a decrease of the temperature above the thermocline and thus a loss of efficiency.

Quantification of the effects of the penetration of the thermocline by the wall boundary layer flow and formation of a convection cell that includes the entire tank on the storage capacity has not been undertaken in this work. Identification of this phenomenon is however important for improvement of the understanding of the performance of thermal storage tanks. A better knowledge of the behaviour of the convection cell in real scale storage will permit to take appropriate measure to stop it or at least reduce its effects.

Most of the research performed in the Division of Energy Engineering of the University of Luleå has been focussed on the mixing of hot and cold water during the charging or the discharging of the storage tank, see Dahl (1993) and Hermansson (1993). This mixing decreases the performance of the storage as the mixed water is partly lost for the storage.

Previous studies have shown that the behaviour of the hot water during the beginning of the charging which determines the amount of mixing is depending on the Richardson number, $R_i$, or the Froude number, $F_r$:

$$R_i = \frac{(g' \cdot h)}{U^2} \text{ and } F_r = \frac{1}{\sqrt{R_i}}$$

where $U$ is the inlet velocity, $g' = g \cdot \beta \cdot (T_0 - T_1)$ is the reduced gravity, $g$ is the gravity acceleration, $\beta$ is the coefficient of thermal expansion, $T_0$ is the temperature of the hot water, $T_1$ is the temperature of the cold water and $h$ is a characteristic length. The Richardson number is used in the previous studies of thermal storage tank and is prefered in this part of the thesis. However in Paper V
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concerning gravity currents, the Froude number is given due to its general use in the literature on gravity currents. Holness (1988) and Hollands and Lightstone (1989) suggested to take the distance between the inlet and the outlet as characteristic length in the Richardson number. However the distance between the inlet and the outlet has no relevant effect on the mixing. The mixing occurs during the beginning of the charging and has no more influence on the exergy efficiency of the tank after that the thermocline is established and is going down in the tank. The inlet height as characteristic length used by Yoo et al (1986) seems to be a more logical choice. Hermansson proposed also to use as velocity in the inlet Richardson number the inlet velocity multiply by the ratio of the inlet diameter to the tank diameter to take into account the geometry of the tank and inlet plate. In this work, h is taken as the inlet height and is marked by $R_i$, the inlet Richardson number. The velocity is taken as the inlet velocity as only one geometry modification has been made. Yoo et al gave a review of the flow field behaviour in a rectangular tank for different values of the inlet Richardson number. For an inlet Richardson number over 0.25, the hot water injected into the tank filled with cold water can be represented as a gravity current with a head followed by a main body. When the gravity current has bounced at the wall, i.e. $t > t^*$, a reverse current begins to flow in the direction of the inlet. The temperature in this reverse current is nearly the same as the cold water temperature. Thus the bottom of the thermocline is formed by the initial pass of the hot water across the tank. For an inlet Richardson number lower than 0.25, the flow is more chaotic and produces more mixing. To compare experiments, Yoo et al used different parameters as thermocline thickness, temperature gradient at the centre of the thermocline and temperature difference between inlet water and the water above the thermocline. This temperature difference is in fact the best measure of the mixing. The decrease of the average temperature above the thermocline for low inlet Richardson number illustrates quite well the problems occurring for low Richardson number.

The flow field observed at the inlet by Yoo and al for $t < t^*$ described as a gravity current has been studied in Paper V. The horizontal spreading of gravity currents is driven by the horizontal pressure gradient due to the density difference. In a
gravity current, the gravity force is higher than the inertia force. In the opposite case, a horizontally discharging buoyant jet is observed. The main effects of the buoyancy are an increase of the lateral mixing and a reduction of the vertical spreading (Mc Guirk and Rodi 1979). Benjamin (1968) analysed gravity currents using perfect fluid theory. Completing this theoretical approach, experiments conducted by Simpson and Britter (1979) have shown that mixing occurs at the head of gravity currents due to shear instability between the two fluids and gravitational instability of less dense fluid overrun by the gravity current. The mixing is located at the head of the gravity current for high Reynolds number flows, $Re = U \cdot h/v$ with $U$ the velocity of the front, $h$ the current thickness and $v$ the kinematic viscosity (Britter 1979).

For low Reynolds numbers the driving gravity force is initially balanced by the inertia force as long as the inertia force is large compared to the total viscous drag (Didden and Maxworthy 1982). This is the gravity-inertia regime. As the time progresses, the viscous force becomes dominating and a gravity-viscous regime establishes. The spreading rate of gravity currents was determined separately for the gravity-inertia regime and the gravity-viscous regime. A model has been developed to predict velocity and temperature profiles in the gravity-inertia regime (Nakos 1994). Huppert (1982) concluded that the head of gravity currents has lost its influence on the spreading rate for low Reynolds numbers. A numerical simulation of the spreading of gravity currents at the top of the storage tank model has been performed in Paper V and compared with a fairly good agreement to experimental results presented by Hermanson (1993) and Didden and Maxworthy (1982). For the range of Froude number and Reynolds number studied in the numerical simulation, the mixing of hot and cold water has been quantified through an efficiency based on the exergy and related to the shape of the gravity current. The mixing causes a decrease of the exergy efficiency until the hot water reaches the wall and the thermocline has been established, i.e. $t < t_1$. For the storage tank model investigated, the exergy efficiency is increasing with the temperature difference between the hot and cold water as expected. The study of the evolutions of the exergy efficiency for different flow rates has shown that there exists an optimal flow rate. For low flow rates, the gravity current has no
head wave and mixing occurs all along the gravity current. As the flow rate is increased, the mixing is more and more occurring in the neighbourhood of the head. The exergy loss is minimal when the mixing is reduced along the main body and the mixing at the head is not fully developed as for high flow rates. In this case, the mixing at the head is increasing resulting in a drop of the exergy efficiency.

For $t > t_1$, during the formation of the thermocline, variations in the exergy efficiency for the different flow rates have also been observed. A study has been undertaken in Paper VI to analyse the flow field near the edge of the inlet plate during the formation of the thermocline. The effects of the buoyancy force on the flow field near the inlet plate during the formation of the thermocline are investigated. During the formation of the thermocline, the temperature gradient is located at the level of the inlet plate and the buoyancy force acts directly on the flow field at the edge of the inlet plate. Thus the injected hot water, confined between the top of the storage tank and the inlet plate, may or may not generate a toroidal vortex in the tank, depending on the temperature difference and velocity. By using potential flow theory for a two-dimensional model, a critical inlet Richardson number, $R_i=2.3$ for the geometry of the model storage tank, has been determined above which the formation of the vortex is delayed due to the action of the buoyancy force until the thermocline has progressed past the inlet plate. Experiments achieved in the model storage presented in Paper IV and a numerical simulation presented in Paper V have been compared to the analytical result. A reasonable agreement between them for the value of the critical inlet Richardson number has been found. Further work is required to evaluate the effects of this phenomenon on mixing between hot and cold water during the charging of the storage.

The present work has shown the importance of the inlet velocity and temperature difference for the flow field in the storage tank and thus for the exergy efficiency of the storage tank. A more complete study should be undertaken to analyse entirely the flow field during the formation of the thermocline and its effects on the mixing.
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Scientific contributions

This work has resulted in:

- improvement of the knowledge on inaccuracies and errors occurring during the different steps of the PTV technique
- a new procedure for evaluation of the velocity field from video records of particle movements aimed at reducing errors caused by resolution inaccuracy and incorrect particle identifications
- visualization of the breakdown of the thermocline by the natural convection boundary layer due to heat exchange with the surroundings
- quantification of the mixing in gravity currents at the inlet of the storage tank through an exergy efficiency and identification of a minimum for the exergy loss for the range of flow rates studied
- identification of phenomena leading to a loss of efficiency during the formation of the thermocline
- determination of the critical inlet Richardson number for the formation of a vortex at the edge of an inlet plate of a storage tank

References

Introduction


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Yoo J; Wildin MW; Truman CR (1986) Initial formation of a thermocline in stratified thermal storage tanks. Ashrae Transactions 92: 280-292
Paper I
Use of Video-based Particle Image Velocimetry technique for studies of velocity fields in a water heat storage vessel

J. Dahl, R. Hermansson, S.-E. Tiberg, P. Veber

Abstract A video-based Particle Image Velocimetry technique has been developed. The technique is particularly suitable for measurement of small velocities, below 3 cm/s. It has proved to be useful for the documentation of non-stationary velocity fields in a scaled-down model of a water heat storage vessel. An ordinary video camera is used to record the in-plane movements of particles in a light sheet in seeded water. The hardware used, the experimental method and the accuracy of the method are discussed. The use of two commercially available software packages (NIH-Image and IGOR) for the analyses is described. Examples of velocity fields are presented, showing that the measuring technique can be used for studies of mixing near the inlet of the storage vessel and exchange of water between the boundary layer and the core.

1 Introduction
In most applications of heat storage where water is used as the storage medium it is essential to maintain a high degree of thermal stratification (Dahl and Hermansson 1988). Many investigators have shown that the overall efficiency of storage systems is improved with thermal stratification, especially in solar applications (Han et al. 1980).

Degradation of the thermal stratification is caused primarily by mixing of hot and cold water during the charging of the storage vessel and by natural convection along the walls when heat is lost to the surroundings.

The present study has two objectives, the first of which is to adapt the video-based PIV-technique for use in studying flow fields in water heat storage vessels. The second is to generate data on velocity fields which could be used for modelling of the thermal behaviour in water heat storage vessels and for comparisons with numerical calculations of the flow conditions.

In this paper the emphasis has been placed on presentation and evaluation of the PIV-technique.

2 The Particle Image Velocimetry technique
A comprehensive description of PIV-techniques applied to velocity measurements in fluids may be found in Hinsch (1993), Dudderer et al (1988), and Adrian (1991). Moreover a short summary of such applications is presented in Schmidt and Löfler (1995).

Digital PIV or DPIV is a method where a video recording of the light sheet is made instead of a photographic record of the tracer particles in the flow. Records are obtained with the frequency of the video system, and data are stored and then evaluated digitally. Schmidt and Löfler used a CCD (Charge Couple Device) camera for recording of double-exposed pictures. They used image processing based on auto correlation of image sections to capture the velocity fields.

2.1 PIV-technique used in this study
The method used in the present work was based on continuous recordings with an ordinary video camera. Commercially available software was used for image processing and evaluation of the velocity field. This technique enables recording of a complete experiment on video tape and allows interesting parts of the transient flow to be analysed afterwards. By using a frame-grabber, an optional number of frames with appropriate time differences can be captured from the video tape and stored in the computer. To visualize the flow, particle trace pictures can easily be created without ambiguity concerning the direction of the flow.

The experiments were performed in a model of a water heat storage vessel, see Sect. 3 for details. The particles in a vertical symmetry plane inside the heat storage vessel were illuminated with a light sheet, created with a laser and a cylindrical lens (Fig. 1).

It was possible to observe the separate particles even if the view covered the whole storage vessel. To obtain good accuracy in the evaluation of velocities, the camera (a Panasonic S-VHS, NV-MS1, recording 25 frames/s) was normally positioned quite close to the storage vessel. A 450 mm focal length close-up lens was mounted on the video camera lens, to enable focusing on the light sheet to cover an area of up to 7 cm times 10 cm. Each close-up view then covered roughly 10% of the light sheet. It was therefore necessary to change the view to record movements in the whole storage vessel, while still maintaining high resolution.

The fact that the walls of the storage vessel were cylindrical and the refraction index differed from that of water meant that it was not possible to see the light sheet up to the wall. The camera was normally positioned on a line through the centre of the storage vessel, perpendicular to the light sheet. To enable observations in the boundary layer, the camera had to be placed at an angle 5-10° to that line. This decreased the area...
that could be focused on and thereby reproduced in a good way. The size of the area depended on the degree of zooming, the focal length of the close-up lens used and the aperture. The useful part of the picture was then reduced by up to 50% in the horizontal direction.

It is essential for the PIV-methods in general that the particles follow the fluid in an inertia-free way. The particles used in this study were grained from a material offered under the name of Ploiliite AC, a styrene - acrylic monomer with specified density \( \rho_p = 1.03 \cdot 10^3 \text{ kg/m}^3 \). Buoyancy tests in salt solutions of different concentrations indicated the density to be \( \rho_p = 1.034 \cdot 10^3 \text{ kg/m}^3 \). Two different size fractions were used, 38–76 \( \mu \text{m} \) and 20–36 \( \mu \text{m} \). Both fractions gave good results and the smaller particles were only used for studies of natural convection, when the falling velocity is of importance. The maximum particle concentration was about 20 particles per \( \text{cm}^2 \) of the light sheet.

The evaluation of velocity fields was performed in four steps. First the particle movements were recorded on tape with a video camera. A pre-determined number of frames with a certain time delay were then read from a video recorder into the computer through a frame-grabber, Quick Capture Data Translation board with a 6-bit grey scale resolution. This reading was handled by the Public Domain program NIH-Image (Rasband 1993). The pictures were further processed in this program as discussed below, and the co-ordinates for the particles were calculated and stored on file.

Determination of velocities, to be further discussed below, was finally performed within the program IGOR (WaveMetrics 1992).

### 2.2 Image processing

The program utilized for image processing is the powerful NIH-Image (version 1.49) for PAL from the National Institute of Health Research Branch (NIMH). The frame-grabber used has a maximum reading capacity of 25 frames per second. If full-sized pictures are captured they can be transferred to the computer memory at a rate of 15 frames per second. This rate can be increased if the size of the captured part of the image is reduced. The number of frames to be read and the real time delay between frames is set from the NIH-Image menu according to the velocity range to study, and the pictures are displayed on the computer screen simultaneously with the reading.

The number of frames that can be stored in the computer is determined by the size of its primary memory and the size of the pictures (400 kbytes for a full-size picture). When frames are stored in the computer memory it is possible to eliminate noise by filtering.

Using NIH-Image it is possible to choose what size and brightness of particle spots should be kept for further evaluation. The size is normally chosen between 5 and 25 pixels and the threshold level between 100 and 200 in a range of 256 to end up with an appropriate number of particle spots to analyse. The particles are not resolved on the video film, and therefore the size of these spots will not show the accurate size of the particle but instead be more related to the amount of scattered light.

If a number of frames are superimposed it is possible to display particle traces, where each exposure is given its own colour or grey scale. These traces are an effective tool for showing the flow and are created with little time effort. Examples of pictures generated in this way are shown in Figs. 3 and 4.

The image processing program is finally used to determine the co-ordinates for all the particle spots that are within a band of size and sufficiently illuminated to be accepted as predictable particles. These co-ordinates are determined within 0.5 pixels with the present software and stored on file for further analysis of velocities.

### 2.3 Evaluation of velocities

The magnitude and direction of velocities were calculated and displayed in the program IGOR 1.26, a commercially available graphing and data analysis tool from WaveMetrics. The co-ordinate data sets, normally 5 to 10 files, were read into IGOR and the further calculations were performed in a number of macros developed as part of this study.

The main problem, which requires most computer time, is to determine which spots in all images represent the same particle captured at different real times. The procedure used can be described in the following way.

Let there be \( n \) files containing the co-ordinates for the particles on \( n \) frames, captured at consecutive and equal time steps. For each particle in frame two, vectors are calculated representing the movements from all particle positions in frame one and three to this particle in frame two (Fig. 2).

\[
\vec{F}_1 = (x_i - x_{i-1}, y_i - y_{i-1}) \quad \text{and} \quad \vec{F}_2 = (x_i^{n-1} - x_i, y_i^{n-1} - y_i)
\]

Vectors that represent possible translations due to \( \vec{s}_{\text{max}} \) are kept for further evaluation. These vectors, \( \vec{s}_1, \vec{s}_2, \vec{s}_3 \), are further compared due to change of length and change of direction. If the following conditions are fulfilled a particle trace is found.

\[
|\vec{s}_1 - \vec{s}_2| \leq \Delta s_{\text{max}} \quad \text{and} \quad \left| \arctan \frac{s_{2y}}{s_{2x}} - \arctan \frac{s_{1y}}{s_{1x}} \right| \leq \alpha_{\text{max}}
\]

If there is more than one particle within these limits, the best fitted particle with regard to direction and velocity is accepted for the true particle trace. A velocity vector is calculated...
Fig. 2. Particle tracking based on three time separated frames. For each particle in frame two \((P^k)\) the distance to particles in frame one \((P^{k-1})\) and three \((P^{k+1})\) are calculated, \(S_i\) and \(S_j\). A possible trace is detected when given conditions on maximum displacement \((S_{\max})\), change of displacement \((\Delta S_{\max})\) and change of direction \((\alpha_{\max})\) are fulfilled which represents the velocity at the position for the particle in frame two.

The limits are used to save computer time and are easily determined from particle trace pictures achieved in Image, since the geometric scale in the figure is known. If the velocity scale to be expected is not known, it is necessary to start from the smallest possible time step, \(1/25\) s, to ensure that the largest velocities that are possible to detect are captured.

See for example Fig. 3a, where the highest velocity is detected near the inlet at the upper left corner. The displacement during one time step and change of displacement between consecutive time steps for the particles at the inlet are determined with an accuracy on sub-pixel level after zooming on this area. Maximum displacement is found to be \(S_{\max} = 1.4\) mm and maximum change of displacement \(\Delta S_{\max} = 0.5\) mm during 0.2 s.

The procedure is repeated through Fig. 3b–3c. Figure 3c is suitable to evaluate the small velocities in the lower part of the picture using the same limits as above.

Nevertheless, if an inappropriate choice of limits is made, for example if the limits adopted are much larger, the amount of particles inside the large circle will be too large for the computer to analyse. The result after considerable computer time will be randomly spread velocity vectors. On the other hand, if lower limits are used for Fig. 3a, only the smallest velocities will be detected, and with poor accuracy due to small displacements. Both results will show that the main flow pattern is not captured when comparisons are made with the particle trace pictures, and a better choice should be made.

The process described above for particles in frame two, is repeated for all particles in frame three and so on up to

Fig. 3a–d. Particle trajectories achieved near the inlet in the upper left corner of the picture after 9 min 30 sec of charging at 200 ml/min. Temperature difference \(2\) °C between the incoming water and the water in the storage vessel. Particles measuring size 38–76 \(\mu m\) moving from light to dark gray. a Time separation between frames 0.2 s; b Time separation between frames 1.0 s; c Time separation between frames 5.0 s; d The resulting velocity field after evaluation of particle traces. Dots represent unmatched particles.
The velocity vectors are stored and separately displayed in graphic form (Figs. 3d, 3a).

The evaluation of velocities can be enhanced by dividing the viewed area into sub-areas before running the IGOR program. This separation is accomplished inside NIH-Image and the sub-areas are chosen according to one of three criteria: the sub-area should have homogeneous illumination or homogeneous velocities, or contain an appropriate number of particles.

2.4 Accuracy of the method

The overall accuracy of this method within its limitations is determined by the inaccuracy in every step involved in the determination of the velocities. There are essentially four types of errors, which relate to deviations between flow and particle trajectories, limited resolution of the recordings, geometrical positioning and finally interpretation of particle traces. Each type of error and its effects will be discussed below.

Calculations with Stoke’s theory and performed experiments indicate a falling velocity of less than about 20 μm/s for the smaller fraction of particles and 100 μm/s for the bigger ones. The error is considered to be of minor influence everywhere in the storage vessel except in the core, where the mean velocity is of the same magnitude as the falling velocity.

The camera has a specified horizontal resolution of 460 lines and a vertical resolution of 625 lines, and the specified resolution for the frame-grabber is 768 pixels in the horizontal and 512 pixels in the vertical direction.

Tracing errors are difficult to quantify at this stage since they are related to the experience of the user, and they will be dealt with in a future article.

An estimate of the best accuracy that can be achieved with the method could be as follows. The estimate is made for a close-up view of an area 3 cm times 2 cm, and a maximal detectable translation of 20 particle diameters, here 1.2 mm, is used. Translation of particles is considered to be known within 1 pixel, since the centre of gravity is determined on sub-pixel level. Resolution will be poorest in the horizontal direction, where one pixel corresponds to 0.066 mm, giving an error corresponding to 5.5% of the highest velocities.

Calibration of distances introduces a possible error of 5% at the most. These two errors add up to a maximum error of about 10% of maximal detected velocity for a close-up view.

3 Model heat storage vessel used for evaluation of the video-based PIV-technique

Detection of velocity fields with the PIV-technique requires that the inside of the storage vessel can be illuminated with a light sheet to visualize the movement of the seeded water. Therefore the storage vessel must have transparent walls. In order to allow illumination of a whole cross-section, it must not be too large and with the equipment used the maximum size is about 50 cm.

The model storage vessel should have a geometry as close to that of a real storage vessel as possible, that is cylindrical shape. Nevertheless, it must be possible to make video recordings with the smallest possible optical distortion of the picture. This is more difficult to achieve for the cylindrical shape.

The storage vessel used is made of Plexiglas, has a cylindrical shape and a vertical axis. It has a height of 420 mm, a diameter of 200 mm and a wall thickness of 3 mm, giving a total volume of 12,5 dm³. The storage vessel is connected to a network of circulation tubes to enable different types of experiments like charging, discharging and mixing to homogeneous temperatures.

In order to minimize optical distortion, the storage vessel is surrounded by water contained in a rectangular tank that measures 400 mm x 250 mm x 420 mm. Its total volume is 42 dm³ and the thickness of the glass walls 8 mm. This gives media with the same refraction index on both sides of the cylindrical wall and provides a plane wall towards the camera. The surrounding tank also makes it possible to simulate different conditions that affect the heat losses from the storage vessel, since the temperature level and water flow can be controlled in both water volumes.

4 Measured velocity fields in a stratified model heat storage vessel

The performance of the video-based PIV-technique can best be illustrated by some examples of measured velocity fields. The measurements focused both on studies of the mixing that occurs near the inlet and on the exchange of water between the boundary layer at the wall and the core of the storage vessel.

In all figures showing particle traces discussed below the movements illustrated are from light to dark grey.

4.1 Evaluation of flow pattern by use of different time steps

Figure 3 shows a sequence of particle traces achieved near the inlet during charging of the storage vessel with a flow rate of 200 ml/min, giving an inlet mean velocity of 5.0 mm/s. The figure shows traces, at the same position in the storage vessel, achieved from 10 pictures separated in time in Fig. 3a by 0.2 s, in Fig. 3b by 1 s and in Fig. 3c by 5 s.

The particle traces shown represent a velocity field with large spatial variations. It is therefore suitable to illustrate the benefit of choosing several different time steps for evaluation of the velocity vectors. Figure 3a shows the field very close to the inlet where we find the highest velocities. Figure 3b shows the medium velocity flow near the wall and how this flow partly changes direction towards the centre of the storage vessel, due to buoyancy forces.

The time separation (5 s) used in Fig. 3c gives particle traces only for very low velocities, while particles seem to be randomly distributed in areas with higher velocities.

The velocity field visualized above was used to evaluate velocity vectors as described earlier. The tracing picture was divided up into sub-areas where different time steps were used, due to the large range of velocities in the area. The maximum velocities were about 7 mm/s and the smallest detected velocities in the area about 200 μm/s. The resulting velocity vectors are shown in Fig. 3d.

The flow pattern at the inlet is typical for a back step and is mainly caused by separation behind the inlet edge.
Fig. 4a–d. Particle-trajectories in the boundary layer near the wall during stationary conditions. Distances from the top of the storage vessel are marked in the separate pictures. Temperature difference to the surroundings 1.2 °C. Particles measuring 20–38 μm were used in the experiment.

4.2 Evaluation of homogeneous flow pattern by superposing pictures

The possibilities of using the present PIV-technique for studies of natural convection boundary layers can be illustrated by a test where the gradient zone is the storage vessel was allowed to deteriorate. The temperature difference between the inside and the outside of the wall was about 1.2 °C when the sequences shown were recorded and the overall heat transfer coefficient was estimated to be 40 W/m²K.

There was a weak stratification above the gradient zone and the effect of this on the boundary layer can clearly be seen in the particle trace pictures. Figures 4a–d show the particle traces, achieved at four separate levels above and in the gradient zone. Near the top the water in the core is moving upwards as shown in Fig. 4a and is fed into the boundary layer flow. Further down in the storage vessel the weak stratification will decrease the velocities in the inner part of the boundary layer and this water is directed into the core as shown in Fig. 4b.

In Fig. 4c, the stratification is stronger and the buoyancy effect even more manifest. Water is continuously being tapped from this boundary layer and the thickness of the layer becomes

Fig. 5. a The result of three different evaluations of velocities, separated in real time to give better field information. Field recorded after 3 h standing still in the same experiment as in Figure 4. Particles measuring 20–38 μm. Dots represent unmatched particles; b Detected velocities in the boundary layer versus distance to the wall. Least mean square fit of a polynomial of the third degree added.
smaller. In the next picture the flow is stagnating, affected by the buoyancy forces (Fig. 4d).

When the velocity field is comparably stable as in the boundary layer, it is possible to evaluate many series of pictures centred around the same real time. In this way a number of velocity plots can be added, giving a better estimate of the velocity field. It is likely that the spots registered in different series of frames are spots of the same particles that have moved a short distance. Such an evaluation was performed, in the same experiment as the one shown in Fig. 4 after approximately 3 hours of stagnant conditions in the storage vessel.

Figure 5a shows the resulting vector plot from three sets of velocity plots. The main flow is clearly directed vertically but there are some vectors with horizontal components. These could arise from real horizontal movements of the particles, but it is also possible that the horizontal vectors are caused by erroneous interpretation of the traces.

An approximation of the velocity profile in this boundary layer is achieved by plotting vertical velocity components versus distance to the wall (Fig. 5b). One can see the step-by-step change of velocities due to resolution achieved in image processing. This could to some extent be improved by a more narrow view.

The best least mean square fit was found for a polynomial function of the third degree, which is normally used in theory to represent the laminar velocity profile.

5 Potential for PIV in the study of heat storage vessels

The flow pattern in water heat storage vessels is characterized by very low velocities which change with time during charging, storing and discharging. Studies of the flow therefore require a method which can measure low velocities, and there is need for a method that permits continuous recording during a complete storage cycle.

The video-based PIV-method has been found to be very suitable for studies of the flow in water heat storage vessels. It provides clear particle tracks, an effective tool to study the flow. On the video tape, pictures are recorded every 1/25 s and be read into the computer separated in time form 0.04 s up to several seconds. This means that very small velocities can be detected, and by reading pictures from the same real time event with dissimilar time separation, different velocity levels can be detected. It is also possible to perform animations with the stored pictures, a powerful means of showing flows that are too slow to be observed in real time.

The greatest advantage of this technique is that the whole experiment can be documented for later analyses. It is possible to analyse the same real time interval many times and with different time delays between the captured frames, thereby enabling detection of velocities within a wide range. A very important feature is that the time intervals between video frames being analysed can be determined after the experiment. Other techniques require the time intervals to be determined during the experiment. With these techniques, the use of different time intervals means that the exposures must be repeated. In experiments with transient flows the flow may then have changed.

Another advantage of the method used in this study is that there is no direction ambiguity. The order of the captured pictures is known and the particle spots can be given different colours for each captured time step.

The main weakness of the method is that velocities above 0.03 m/s cannot be measured as a result of problems with particle tracing. Another weakness is the accuracy of the method, mainly due to limited resolution in captured frames, calibration of distances in the storage vessel and the fact that the particles might not follow the flow due to different densities.

There seems to be a good potential for future development of this technique for visualization and measurements of fluid flows. The main problem up to now has been the evaluation of velocity vectors from the data set of co-ordinates achieved from Image. Preliminary results show that this can be performed much faster with new routines and by using a SUN-workstation. The process can then be run many times to give more complete velocity fields. A new release of IMAGE also indicates much better resolution in determination of the centre of gravity for particle spots.

It could also be possible to use more than three spots of the same particle to detect a trace. In this way one would obtain fewer velocity vectors from each run but instead have more accurate values. For high velocities it would be useful to look at the possibilities of using high-speed video recordings, or of using multiple exposures on each frame of the video film.

PIV-technique based on video recordings of particles in seeded water is a powerful method for studies of fluid flow and velocity fields in water heat storage vessels. The combination of particle traces and vector plots have proven to be an efficient tool in gaining knowledge about phenomena which affect the thermal behaviour.

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Study of the phenomena affecting the accuracy of a video-based Particle Tracking Velocimetry technique

P. Veber, J. Dahl, R. Hermansson

Abstract The development of a video-based Particle Tracking Velocimetry (PTV) technique has focused on the problem of the accuracy of this method. The PTV-method can be decomposed into three parts: the recording of the experiment, the image processing and the evaluation of the velocities. The accuracy of each stage has been studied. Inaccuracies due to resolution, length scale, light intensity and distortion of the x and y direction are analysed. One of the main factors influencing the accuracy is the selection of the time difference between frames. During the evaluation of velocities, incorrect identifications of particles may occur. The relation between the time-step of the frames and the percentage of incorrect identifications has been shown. The percentage of false identifications increases with the size of the time-step. The resolution accuracy is however improved when the time-step is increased. An adequate selection of the time-step has to be made to obtain a high resolution accuracy and a limited number of incorrect identifications.

1 Background
A video-based Particle Tracking Velocimetry technique has been presented by Dahl et al. (1995) who used it for studies of the velocity field in a water heat storage vessel. The technique is suitable for the determination of non-stationary velocity fields, particularly for small velocities. The method is based on continuous recordings with an ordinary video camera of the in-plane movements of particles in a light sheet in water seeded with particles. The pictures are transferred from a video recorder to a computer through a frame grabber. An optional number of frames with appropriate time differences is analysed by the software NIH-Image. Files are stored that contain the co-ordinates for all particle spots that are within a band of size and sufficiently illuminated after a thresholding of light. Particle trace pictures are created to visualize the flow. A computerized tracking procedure determines the magnitude and direction of velocities.

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Fig. 1. Particle tracking based on three time-separated frames. For each particle in frame one ($P^*$), the distances to particles in frame two ($P^{*+1}$) are calculated, $|S_i|$. If the condition of maximum displacement $S_{\text{max}}$ is fulfilled, particles in frame three ($P^{*+2}$) located in the circle centred at the possible position of the particle and of radius $|S_i|/2$ are selected.

Application of this tracking procedure requires that suitable values for the maximum velocity and time-step can be selected. This requires some a priori understanding of the velocity field to be studied. The value of the maximum velocity is deduced from measurement of the maximum distance covered by a particle between two consecutive particle trace pictures. The judgement of the evaluator is then essential. The choice is important for the accuracy obtained in the result. A too high maximum velocity will increase the number of incorrect identifications and a too small maximum velocity will reduce the number of correct identifications. The choice of time-step depends on factors that will be discussed later on.

2 Resolution error

The resolution error due to the CCD (Charge Coupled Device) camera sampling of the imaged light intensity distribution scattered from particles in the light sheet has been studied previously (Wernet and Pline 1993). The particle image centroid error is a function of parameters linked to the particle image, such as the image particle radius, the pixel size, and the Gaussian peak location on the CCD array, and to other factors increasing the error, such as CCD read noise and variation in pixel intensity. In the case of Wernet and Pline (1993), the subpixel accuracy has been estimated to be $\pm \frac{1}{2}$ pixel for a mean particle image size of 6 pixels in diameter. The particle image size observed in the water heat storage experiments is approximately of 15 pixels in diameter. The subpixel accuracy then deteriorates to an estimated value of $\pm \frac{1}{4}$ pixel. According to Wernet and Pline (1993), the error in the particle displacement is $\sqrt{2}$ times the particle centroid error, so the displacement accuracy would be $\pm \frac{\sqrt{2}}{4}$ pixel. The variation of the ratio between the particle displacement error and the displacement covered by the particle between two consecutive pictures is presented in Fig. 2 as a function of the displacement. The large error for displacements smaller than 2 pixels is obvious. In order to obtain an error smaller than 10%, the distance covered by the particle has to be larger than 5 pixels.

3 Image processing accuracy

To identify the errors occurring during the reading of the video recording and the creation of the files containing the coordinates of the particle image centroid in each picture, a film recording was made of a record-player with three points, at a radius of 13.5, 20 and 22.2 mm from the centre of the turntable, see Fig. 3, rotating at a specific speed measured by an accurate time-indicator. By sending the pictures of these points, moving with a well-defined velocity, from the video-recorder to the computer through the frame grabber, it has been possible to identify different errors that occur and in some cases also to quantify these errors. It has also been possible to compare the velocity obtained from the time-indicator to the velocity resulting from the image processing. The same errors were detected for the three radii. The following discussion is therefore limited to the results for the second radius.

Fig. 2. Resolution error for the distance covered by a particle between two consecutive pictures versus the distance covered by a particle.

Fig. 3. Record-player image
The software Image allows the selection of the time-step between pictures and of the number of pictures. For the record-player, the point velocity (~75 mm/s) is high compared with the velocities in the small-scale water heat storage tank (~10 mm/s), and the video-camera time-step is adequate for the velocity field.

The algorithm used to analyse the pictures and to estimate the centroid of the point image requires some inputs, such as the time-step between pictures, the number of pixels per millimetre defined by the set-scale function, and the threshold of light intensity to reduce the disturbance due to the background and to limit the number of points to be analysed.

3.1 Elimination of possible errors caused by features of the camera and frame grabber

Some errors can be avoided by properly accounting for peculiarities of the camera and frame grabber. The time-step has to be a multiple of the video-camera time-step, in our case 3/5 s. If the time-step T is chosen so that \( n \cdot \frac{3}{5} < T < (n+1) \cdot \frac{3}{5} \), the frame grabber will alternate time differences between pictures of \( n \cdot \frac{3}{5} \) s and \( (n+1) \cdot \frac{3}{5} \). The time-step between collected frames will not be constant and the velocity measurements distorted. Another reason for inaccuracy comes from the fact that the frame-grabber first reads the even lines during 30 s and the odd lines during the other 35 s. During the first reading, the point images have moved. The final picture is the superposition of two half-pictures representing the same particle displaced by the distance covered during \( \frac{3}{5} \) s, illustrated in Fig. 4. The centroid estimated by the software would be a combination of the centroids of the particles in both half-images. The use of an algorithm separating the two half-pictures and reconstituting a full picture is necessary for a correct evaluation of the particle image centroid.

3.2 Image processing errors

For the set-scale function, a well-defined distance must be measured. For the experiment with the record player, the diameter of the turntable was used. The contours of the imaged object to be used for setting of the scale must be as precise as possible. The error on the distance measured is a source of inaccuracy for the PTV-technique.

3.2.1 Observations

Figure 5 illustrates the variations of the distance covered by the centroid of an image particle during \( \frac{3}{5} \) s over two laps in the record player experiment, one lap representing 43 points. The video-camera is positioned vertically and perpendicular to the record player. We observe two sorts of inaccuracies, one random and an other periodic since maximum and minimum are repeated periodically.

3.2.2 Analysis

With a uniform illumination, a unique threshold level for the total picture can be used, but even with this precaution, the selection of the threshold level depends on the operator. The three threshold levels used in this analysis are adapted for the processing of the pictures. As illustrated in Fig. 6, the size of the particle image decreases when the level of the threshold increases. A random variation in the estimated centroid co-ordinates has been observed, up to \( \pm \frac{1}{2} \) pixels for one of the co-ordinates. This value is the maximum variation that has been observed between two levels of threshold for the same image particle. A value of the error can be evaluated to \( \pm \frac{1}{2} \) pixels. As mentioned previously, the error in the particle displacement is \( \sqrt{2} \) times the image particle centroid error. So the displacement inaccuracy due to the thresholding comes up to \( \pm \frac{1}{2} \) pixel. This high error could be due to the integration period of the video-camera (\( \frac{3}{5} \) s) that results in an image particle that is not round.

![Fig. 4. a Relative movement of the even and odd pictures during the reading of the picture; b a full picture of the point has been reconstituted by an algorithm starting from one of the half-pictures](image)

![Fig. 5. Variation of the distance covered by point 2 between two pictures separated by \( \frac{3}{5} \) second over two laps and fitted sinus curve](image)

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![Fig. 6. Variations of estimated centroid co-ordinates and size of point image with threshold level](image)
Experiments but 1.1%.

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performed with different angles between the video-camera and the record-player. The x-radius of the record-player (0.2-0.7 radians). The x-radius of the record-player have been performed, see Table 1.

An experiment with the video-camera turned of \( \pi/2 \) radians has showed that the turntable is actually round but the turntable image is oval.

Experiments with different angles between the video-camera and the record-player have been performed, see Table 1. These experiments should increase the amplitude of the sinus and confirm that the error is sinusoidal. Two experiments have been realized with an angle in the plane between the video-camera and the normal to the record-player. Figure 7 shows the decrease of the minimum due to the cosines factor on the y co-ordinate. The small decrease of the maximum might be due to an inaccuracy in the x plane angle between the video-camera and the normal to the record-player.

For an angle in the x plane, the maximum and minimum are phase shifted \( \pi/2 \) radians. The x-radius of the record-player appears smaller than the y-radius.

3.2.3 Interpretations

Two errors have been observed during the transfer from the video-recorder to the computer:

- A random error due the thresholding of the pictures. This is related to the illumination and dependent on the decisions made by the operator.
- A periodic error, caused by distortion of the picture so that the y radius becomes smaller than the x radius. The difference between the displacement in the x direction and the y direction can be estimated to 2% of the displacement. If the error observed would be due to a non-perpendicular position of the video-camera, the angle in the y plane between the video-camera and the normal to the record-player should be of 9°. This angle is too large for not have been noticed. This distortion could be due to the difference in the number of pixels between the video-camera and the frame-grabber.

- The difference between the velocity obtained from the time-indicator and the velocity resulting from the image processing, around 1%, is quite smaller than the addition of the image processing errors, around 10%.

4 Tracking error

During the tracking process, incorrect identifications may occur due to the recognition of a particle track from a random particle location. An algorithm to remove the false identifications has been successfully used (Wernet and Pline 1993) but there is no information available on the parameters influencing the possibility of false identifications.

The above-mentioned tracking algorithm requires two manual data inputs: the maximum particle velocity in the picture studied and the time-step between two consecutive pictures. The maximum velocity is evaluated from the particle trajectories and is related to the a priori understanding of the velocity field. The choice of the time-step is more arbitrary depending on the operator. The operator should always keep in mind that the resolution accuracy increases with the distance between the positions of a particle in two consecutive pictures, see Fig. 2. In order to minimize resolution errors, the time-step should therefore be the highest possible. When the time-step increases, the distance covered between two pictures and the radius of the search area will also increase. The possibility of making false identifications then increases. It is desirable that the number of false identifications is minimized. An optimal choice for the time-step reduces this problem and allows a good resolution accuracy. A rough idea of the best fitted time-step will help the operator in this choice.

Incorrect identifications were quantified in three experiments for a range of time-steps. The three experiments were realized in a small-scale water heat storage tank made of Plexiglas (Dahl et al. 1995). The first experiment had the purpose of studying the inlet of the tank and the formation of a vortex. The velocities were about 1.5 mm/s and the flow field representing this vortex should have been propitious for false identifications. Two evaluations were carried out after the beginning of the charging of the storage vessel. During the second experiment the phenomena under the inlet plate were studied. The velocities were lower (1.25 mm/s) and the flow downwards more regular. The purpose of the last experiment was to observe phenomena near the wall, particularly the natural convection boundary layer, for standstill conditions (stagnant flow), low velocities (0.2-0.7 mm/s) and regular flow. Four evaluations were performed, close to the wall and in the centre of the storage vessel at different times. The seven evaluations of the three experiments represent the range of velocity fields evaluated for the study of the heat water storage behaviour. Each set of data has been evaluated with different

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Fig. 7. Comparison of the fitted sinus curves for five experiments realized with different angles between the video-camera and the record-player.

Fitting a sinus curve to the points representing the particle displacement between two pictures gives a better understanding of the periodic phenomena. The maximum displacement is observed when the points are moving in the x direction and the minimum displacement when the points are moving in the y direction. The image of the turntable is not round but oval. An experiment with the video-camera turned of \( \pi/2 \) radians has showed that the turntable is actually round but the turntable image is oval.

For an angle in the x plane, the maximum and minimum are phase shifted \( \pi/2 \) radians. The x-radius of the record-player appears smaller than the y-radius.

The difference between the velocity obtained from the time-indicator and the velocity resulting from the image processing, around 1%, is quite smaller than the addition of the image processing errors, around 10%.
time-steps adapted to the value of the velocity in the experiment considered. Starting from a value of the time-step resulting in few incorrect identifications, the time-step is increased by a sample spacing. For small time-steps (< 1 s), the sample spacing is 0.12 s, for higher time-steps (> 1 s), the sample spacing is 0.2 s.

To identify incorrect velocity vectors, the velocity field resulting from the tracking processing is compared with the particle trajectories from the image processing. In Fig. 8, the false identifications have been marked by thinner lines. The evolution of the ratio of false identifications to the number of evaluations, the tracking error, is presented in Fig. 9 as a function of the time-step. The tracking error has a tendency to increase strongly for time-steps beyond a critical value. In Fig. 10, the tracking error is presented as a function of the average distance in pixels covered by a particle between two consecutive pictures. A large increase in the tracking error is observed when the average distance is in the range of 4–6 pixels. This range of values can be explained by different factors, such as the flow field configuration, out-of-plane movements, the choice of the threshold level and the precision of the maximum velocity value. For the maximum distance covered by a particle between two consecutive pictures, the corresponding range is 7–15 pixels.

For an average distance of 5 pixels covered by a particle between two consecutive pictures, the resolution inaccuracy is about 10%, see Fig. 2, and the tracking error is smaller than 10%. As mentioned previously, the tracking error can be reduced by the use of algorithms to remove incorrect identifications. A tracking error of 20% used by Wernet and Pline (1993) seems to be more realistic to obtain a high resolution.
accuracy. In this case, the average distance covered is in the range of 5 to 8 pixels and the resolution inaccuracy reduced to 7.5%. This inaccuracy could be even more reduced if a higher percentage of tracking error is acceptable.

The time-step for which the tracking error amounts to 10% is presented in Fig. 11 versus the average velocity in pixels/second calculated from the correct velocities obtained after the tracking processing. In the same figure, the values of the time-step for 20% of tracking errors have been added and are close to the values for 10%. A small change in the time-step has a large impact on the tracking error.

The influence of the particle density on the maximum search area radius is illustrated in Fig. 12. The maximum search area radius for 10% tracking error decreases when the density of particles increases. Indeed if the search area radius remained the same, the number of particles in the search area would increase with the density and the possibility of getting a random particle location that could be identified as the particle position after the time-step $T$ would be higher. The outlier, for a density of $1.3\cdot10^{-3}$ particle/pixel$^2$, seems to be due to an underestimation of the density. In nearly half of the picture, no particles could be observed due to a shadow in the corner of the storage vessel. The density should be quite higher than the calculated mean density and the point should be moved to the right in the graph.

The last error studied, the tracking error, increases when the time-step increases. An optimum value for the time-step has to be found which gives an acceptable resolution error and a limited number of incorrect identifications. For example, a tracking error of 20% results in a resolution error of 7.5% for the distance covered by a particle.

A maximum value of 17% is a good approximation of the total error from the recording of the experiments to the evaluation of the velocity field. This value is dependent on the allowed resolution and threshold error, the inaccuracy of the measure of a scale object, the correction of the $x$ and $y$ dimensions and the tracking error that can be handled.

A general conclusion for this study concerns the relation between the resolution inaccuracy and, to a lesser extent,
the inaccuracy due to the thresholding and the tracking error. A careful analysis of the time-step has to be undertaken. A high value reduces resolution inaccuracy. At the same time, incorrect identifications during the tracking processing increase with the time-step value. Depending on the percentage of incorrect identifications that can be treated by algorithms removing incorrect identifications, a rough idea of the best fitted time-step can be evaluated from the average velocity of the velocity field studied. Starting from this time-step the operator has to adjust this value to his experimental conditions (e.g. flow field, threshold, etc.) and to his requirements (e.g. low resolution inaccuracy, high number of evaluated velocities, etc.).

References
Paper III
An improved Particle Tracking Velocimetry technique

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Abstract The use of a Particle Tracking Velocimetry (PTV) technique has shown that the choices of the time-step between two consecutive pictures and of the search area radius during the evaluation of the velocities are essential to obtain a velocity field with a low resolution error and a small number of incorrect vectors. The resolution accuracy is increasing with the size of the time-step but the the number of incorrect identifications occurring during the tracking process is also increasing. An automatic tracking procedure has been developed which aims at optimising the resolution accuracy for a number of incorrect identifications under 20% of the total number of vectors identified. A procedure to remove the incorrect identifications has been developed and the experiences from practical use of this procedure are summarised.

1. Introduction

A Particle Tracking Velocimetry (PTV) technique has been developed at the Division of Energy Engineering (Dahl et al 1995). This method has been successfully used to measure velocities in a water heat storage model.

The PTV technique consists of continuous recordings of the in-plane movements of particles in a light sheet in seeded water with an ordinary video camera. The pictures are then transferred from a video recorder to a computer through a frame grabber. A number of frames with an appropriate time difference are then analysed. Files that contain the co-ordinates for all particle spots that are within a band of size and sufficiently illuminated after a thresholding of light are stored. A computer tracking procedure determines the magnitude and direction of the particle velocities. This tracking procedure requires selection of a maximum velocity allowed and a time-step
between two consecutive pictures. These parameters have to be adapted to the velocity field studied. This dependence on the experience of the user and the need for some \textit{a priori} knowledge of the velocity field for these parameters are the main drawbacks in the wide use of this technique. Procedures to avoid this dependence have been presented by Guezennece et al (1994) and Wernet and Pline (1993). However the maximum particle displacement between two frames was taken as a constant depending of the flow conditions.

In the present research, a PTV procedure is proposed where the maximum displacement varies with the flow system. The first task was to develop a procedure for removing incorrect velocity vectors. Algorithms for this task have been investigated by Wernet and Pline (1993) and Cowen and Monismith (1997). The algorithm developed in this study has been evaluated by means of simulated velocity fields. The use of synthetic data is common for evaluation of new PTV process and errors (Hassan and Philip 1997; Okamoto et al 1995; Guezennece and Kiritsis 1990).

The second task was to develop an automatic tracking procedure aimed at reducing the intervention of the operator. Two steps procedures are often used to reduce the number of incorrect identifications (Guezennece and Kiritsis 1990; Cowen and Monismith 1997). A preliminary evaluation is necessary to provide data for a second evaluation. In the final evaluation presented in this work, the time-step and the search area radius are determined so that the resolution error for the distance covered by a particle between two consecutive pictures is reduced, and the incorrect identifications can still be removed. With this technique, the PTV technique is easier to handle and gives results that are more amenable to analysis.

2. Procedure to remove incorrect particle identifications

For each supposed identification of a vector $v$, the $m$ most close lying vectors $w_1, w_2, ..., w_m$ are selected. The average value

$$<w> = \frac{\sum_{i=1}^{m} w_i}{m}$$  \hspace{1cm} (1)
and the standard deviation

\[ \Delta w = \sqrt{\frac{\sum_{i=1}^{m} |w_i - \langle w \rangle|^2}{(m-1)}} \]  \hspace{1cm} (2)

of each vector is calculated. If the difference \(|v - \langle w \rangle|\) is greater than \(c \cdot \Delta w\), \(c\) a chosen constant, the vector is removed. After all the vectors have been checked, the whole process is repeated until no more vectors are removed. Thus we end up with a "self-consistent" set of vectors in which no one differs too greatly from its neighbours. It is noted that one or more of the comparison vectors may be themselves incorrectly identified. One may ask if this can cause the calculated average value to be shifted so much that the vector in question is removed, although it is in fact correct. Suppose that one of the comparative vectors is incorrect. Let \(D\) be the absolute value of its deviation from the correct velocity in the corresponding point. The calculated average velocity will be shifted by \(D/m\), but the standard deviation will typically be broadened by the much larger amount \(D/m\). Thus the current vector will not be removed if it is correctly identified.

Another question is if two incorrectly identified vectors can assist each other to slip through the test by means of broadening the standard deviation. Suppose that an incorrectly identified vector \(v\), deviating by \(D\) from the correct velocity, has caused another incorrect vector \(u\) to pass the test. Since \(v\) has caused an increased standard deviation of at most \(D/\sqrt{m}\), \(u\) cannot deviate more than \(cD/\sqrt{m}\) from the correct value in that point. When \(v\) is checked, \(u\) cannot increase the standard deviation by more than \(cD/m\). To save \(v\) from being removed, an increased standard deviation of \(D/c\) would be necessary. Hence, if \(c^2 < m\), \(v\) and \(u\) can not help each other to slip through the test. Furthermore, in the next testing round, also \(u\) will be removed unless additional incorrect vectors exist in the neighbourhood.

In the calculations presented later \(m = 10\) and \(c = 1.5\) were used. These values allow removing most of the incorrect identifications, with a few correct identifications also removed.
3. Testing the procedure with synthetic data

In order to test the particle tracking procedure, a program for generating simulated velocity fields was developed. This made it possible to study the dependence of the number of false identifications on the particle density, and to what extent the false identifications are removed.

3.1. Numerical velocity field

The simulation program generates three consecutive pictures of a set of randomly distributed particles in the velocity field

\[ v(x, y) = ae_x + bye_y, \tag{3} \]

where \( a \) and \( b \) are adjustable constants. The particle trajectories become the exponential curves

\[
\begin{align*}
    x(t) &= x_0 + at \\
    y(t) &= y_0 e^{bt}
\end{align*}
\tag{4}
\]

In order to simulate the uncertainty of the particle positions in a real experiment (Veber et al 1997), a random (Gauss distributed) perturbation was added to the coordinates of each particle. The two-dimensional Gauss distribution is created from uniformly distributed random numbers. A polar representation \((r, \theta)\) was chosen since this leads to simpler analytical expressions. Thus \( \theta \) is uniformly distributed between 0 and \( 2\pi \), while \( r \) will have the normalized probability density

\[ \rho(r) = \frac{2}{\sigma^2} re^{-r^2/\sigma^2}, \tag{5} \]
where $\sigma$ is the standard deviation. Given a random number $\xi$ which is uniformly distributed between 0 and 1, a Gauss distributed radius can be generated by the transformation

$$\rho(r)dr = d\xi.$$  \hfill (6)

Solving this differential equation with the obvious boundary condition $r(0) = 0$ gives

$$r = \sigma \sqrt{-\ln(1 - \xi)}. \hfill (7)$$

Of course, since $1 - \xi$ is also uniformly distributed between 0 and 1, it is possible to calculate $r$ from

$$r = \sigma \sqrt{-\ln \xi}. \hfill (8)$$

Here the standard deviation was chosen equal to the subpixel accuracy, which is estimated to be $\frac{1}{3}$ pixel for the particles used in our experiment.

Fig. 1 shows the program output for a simulated velocity field generated with the procedure described above. Fig. 2 represents the same simulated velocity field after the procedure to remove incorrect identifications has been applied. 108 out of 574 vectors have been removed. 82 vectors were incorrect identifications and 26 correct identifications. 6 false identifications are left in Fig. 2. Thus most of the incorrect identifications have been removed. The number of correct vectors removed, 5% of the correct identifications, is considered acceptable.

3.2. Relation between incorrect identifications, particle density and search area

The simulation program allows the choice of particle density in the area to be analysed. For each particle density simulated, the search area radius of the tracking program was increased, from a radius giving a low number of incorrect identifications (5%) to a radius giving a high number of incorrect identifications.
The number of incorrect identifications was counted for each search area radius. Fig. 3 represents the percentage of incorrect identifications versus the search area for different particle densities. A straight line fitted to the curves shows that for a fixed particle density the number of incorrect identifications is roughly proportional to the search area for up to 30% incorrect identifications. These results have been compared to those resulting from experiments (Dahl et al 1995). Fig. 4 represents the variations of the particle image density versus the search area radius resulting in a velocity field containing 10% of incorrect identifications. The particle image density in the experiments is far from uniform. A way to calculate an effective density is described in 4.1.2. Good agreement between the simulation and the experiments has been found. The simulation program represents quite well the behaviour of the particles in the experiments.

In the experimental results, the appearance or disappearance of particles due to out of plane movements from the laser sheet can result in incorrect identifications. However, since the discrepancy between the experiments and the simulations are minor, incorrect identifications due to out of plane movements appear to be negligible.

It is possible to fit a polynomial of second degree to the curve representing the simulation results. This can be used to estimate the search area radius that results in 10% of incorrect identifications for a given particle density. The same analysis has been done for 20% of incorrect identifications. Fig. 5 represents the variations of the search area radius resulting in a velocity field containing 10% respective 20% of incorrect identifications versus the particle density. It is observed that the polynomials of second degree showed good agreement with the data.

4. Procedure for an automatic tracking processing

A drawback of the PTV technique is that parameters such as light threshold, time-step and maximum velocity are dependent on decisions made by the operator. It will be important for a wider use of this technique to reduce the number of such parameters.
In the new procedure described here, the choice of the light threshold in the image processing is still left to the operator. Guezenne and Kiritsis (1990) have proposed an algorithm to choose an appropriate threshold. The evaluation of the velocity field is divided in two parts. The preliminary evaluation with fixed parameters gives information about the velocity field studied, including the particle density and maximum velocity. This information is then used to obtain parameters for the final evaluation. In this final evaluation, the resolution error is reduced and most of the incorrect identifications are removed.

4.1. Preliminary evaluation

4.1.1. Choice of Parameters

A preliminary evaluation of the velocity field is made with fixed values for the time-step and search radius. The time-step is the smallest possible based on the video-camera features, and the memory capacity of the computer. In the experiments analysed in this study, the time-step of the video-camera is 0.04 s. It was nevertheless necessary to use a time-step of 0.08 s due to limitations in the computer memory capacity.

The search area radius in the tracking procedure is fixed at 6 pixels. This search area radius is adapted to the maximum velocity that can be studied with this technique, around 1 cm/s, the percentage of incorrect identifications that can be removed with the procedure described previously, and the density range studied. A strong increase in identification of incorrect vectors has been observed for a distance covered by a particle between two consecutive pictures exceeding 4 to 6 pixels (Veber et al 1997). A search area radius of 6 pixels allows the procedure to remove false identifications to operate in suitable conditions.
4.1.2. Results generated

The preliminary evaluation generates data on the velocity field studied, the particle density and the maximum velocity. Incorrect particle identifications are removed before the maximum velocity is calculated.

The distribution of particle images is often far from uniform due to illumination disparities. Thus the average density, i.e. the total number of particles divided by the area, would not be appropriate for estimating the number of incorrect identifications. Instead, an effective density has been calculated by using the average distance between each particle and its nearest neighbour.

For each particle the distance $a_i$ to its nearest neighbour is found. The average value, given by

$$<a> = \frac{1}{n} \sum_{i=1}^{n} a_i$$

is calculated. The effective particle density $\rho$ is taken to be the density of a uniform particle distribution for which the average value of the distance to the nearest neighbour is $<a>$.

To obtain the relation between $\rho$ and $<a>$, $n$ particles uniformly distributed in a circular disc of radius $R$ can be considered. It should be noted that the particles are distributed independently of each other. Therefore, instead of studying the distance between two particles, the distance between a given point and the nearest particle can be studied. Hence, the average distance between the centre of the disc and the nearest particle can be calculated. Ultimately the limit when $n$ and $R$ tend to infinity such that the density remains constant can be determined.

Let $r_i$ be the distances between the $i$th particle and the centre. The average distance between the centre of the disk and the nearest particle is

$$<a> = \left(\frac{2}{R^2}\right)^n \int_0^R \int_0^R \int_0^R \int_0^{R-r_r-r_{\text{min}}} dr_r d_r d_{r_2} \ldots d_r d_{r_r}.$$
This integral is most easily evaluated by dividing it into $n!$ contributions arising from the different ways in which the radii can be ordered relative each other. Chosing for simplicity the case when $0 < r_n < r_{n-1} < \ldots < r_1 < R$, so that $r_{\text{min}} = r_n$, gives

$$
< a > = n! \left( \frac{2}{R^2} \right)^n \int_0^R \int_0^{r_1} \int_0^{r_2} \ldots \int_0^{r_n} r_n \, dr_n \, r_{n-1} \, dr_{n-1} \ldots r_1 \, dr_1 = \frac{2^n n!}{(2n + 1)!} R.
$$

(11)

The large-$n$ limit of this expression can be obtained by Stirling's formula

$$
n! \rightarrow \sqrt{2\pi n} n^n e^{-n}
$$

(12)

with

$$
(2n + 1)! = \frac{(2n + 1)!}{2^n n!} = \frac{2n + 1}{2^n} \frac{(2n)!}{n!}.
$$

(13)

Substitution of (12) and (13) into (11) gives

$$
< a > \rightarrow \frac{1}{2} \sqrt{\frac{\pi R^2}{n}} = \frac{1}{2\sqrt{\rho}}.
$$

(14)

The effective density will thus be given by

$$
\rho = \frac{1}{4 < a >^2}.
$$

(15)

By this method, the particle density of the velocity field can be calculated. With a time-step of 0.08 s, the inaccuracy in the calculated velocity field is very high. For example, with a calibration of 10 pixels/mm and a velocity of 2 mm/s, the inaccuracy in the velocity is around 30%. A new evaluation with a larger time-step has to be performed to decrease the inaccuracy in the velocity.
4.2. Final evaluation

An increase of the time-step improves the velocity accuracy and also increases the number of incorrect identifications. The procedure described in chapter 2. allows to remove up to 20% of incorrect identifications. In 3.2, it was shown that the variations of the search area radius resulting in 20% of incorrect identifications versus the particle density are represented by a polynomial of second degree

\[ r_{\text{search}} = (a_0 + \text{dens} \cdot (a_1 + \text{dens} \cdot a_2)) \]  

with

\[ a_0 = 26 \text{ pixels} \]
\[ a_1 = -8200 \text{ pixel}^3/\text{particle} \]
\[ a_2 = 1 \cdot 10^6 \text{ pixel}^5/\text{particle}^2 \]

Thus with the particle density calculated in the preliminary evaluation, it is possible to determine the search area radius resulting in 20% incorrect identifications. The maximum velocity obtained in the preliminary evaluation is used to determine the time-step between two consecutive pictures, using

\[ \Delta t = \frac{r_{\text{search}}}{v_{\text{max}}} \]  

With the time-step and search area radius now known, a final evaluation is performed. This gives a velocity field with around 20% of incorrect identifications. Most of them can be removed by using the procedure described previously. The use of two velocity evaluations is time consuming but will give an optimal compromise between velocity accuracy and the number of incorrect identifications.
4.3. Practical test of the new procedure

In order to test this new procedure, experimental data obtained in a water heat storage (Dahl et al 1995) were used. The procedure has been tested for two of the experiments, namely the velocity field between the inlet plate and the wall of the storage vessel and the velocity field in a wall boundary layer. In the first of these experiments, the velocity field is characterized by the formation of a vortex at the edge of the inlet plate. Fig. 6 represents the velocity field after the preliminary evaluation. It is possible to determine the particle density and the maximum velocity in this velocity field. From these values, a new time-step of 0.52 s and a search area radius of 12.5 pixels can be determined.

Fig. 7 represents the final evaluation with these values. It can be observed that quite a larger number of vectors have been identified, not only in the vortex but also around the vortex. For an average velocity of 1.25 mm/s obtained in the final evaluation and a calibration of 6.6 pixels/mm, the average displacement for a time-step of 0.08 s is 0.7 pixels. The displacement accuracy is around 0.5 pixels for the particles used. Particle displacements of approximately 0.5 pixels are difficult to identify as they are of the same order as the displacement accuracy. Most of them are considered as incorrect and are not represented in the first velocity field. With the time-step calculated after the preliminary evaluation, which is 6 times higher, the displacement covered by the particles is much larger than the displacement accuracy, and the tracking procedure identifies more particle traces.

For the vectors identified during the preliminary evaluation, the accuracy has been improved. From an average error of 30% or higher, an error under 10% was obtained. Fig. 8 shows the velocity field before the incorrect identifications have been removed. The parameters are the same as in the final evaluation. The percentage of false vectors is even higher than 20%, due to the large size of the picture. Many incorrect identifications appear outside the vortex in the zone of low velocity.

In the second experiment, the velocity field in a natural convection boundary layer close to the storage vessel wall was studied. The velocities measured are much lower than in the experiment discussed before, and are, on average 0.2 mm/s. With a time-step of 0.08 s, most of the particle tracings are not identified as their displacement is
under 0.5 pixels. Thus, a larger time-step must be used. The first time-step for which the displacement is sufficient compared to the displacement accuracy is 0.64 s. A necessary precaution is to check that the average particle displacement is higher than the displacement accuracy.

The velocity field obtained for this time-step is shown in Fig. 9. From the particle density and the maximum velocity resulting from this evaluation, a final evaluation has been done, represented in Fig. 10. Even if there is a strong velocity gradient between the boundary layer and the center of the storage, the tracking procedure has succeeded in representing correctly the velocity field.

5. Conclusions

Incorrect identifications occurring during the tracking procedure are removed by a new procedure. Each identification is compared to the ten most close lying vectors. Any identification that differs significantly from the most close lying vectors is removed. Velocity fields with a percentage of incorrect identifications up to 30% can be processed. This procedure has been tested on simulated velocity fields. The results show that it is possible to quantify the variation of the search area radius versus particle density for a given percentage of incorrect identifications.

An automatic tracking procedure has been developed. Two evaluations are performed. The first analysis provides values of the particle density and of the maximum velocity. This data is analysed and used to determine the time-step between two consecutive pictures and the search area radius for a final evaluation. In the final evaluation, the resolution error for the distance covered by a particle between two consecutive pictures is reduced, but the number of incorrect identifications has increased. Most of them are removed by the procedure described previously.

Experiments performed in a water heat storage have been used to test this new procedure. The results show that the procedure is effective. The intervention of the operator is reduced. The resolution error has been decreased. The time-step is better adapted to the average velocity of the velocity field studied, and thus, more vectors are identified. The new automatic tracking procedure should allow a wider use of the PTV technique.
References


Fig. 1: Simulated velocity field. Incorrect velocity vectors are represented by dotted lines.

Fig. 2: Simulated velocity field after that the procedure to remove incorrect identifications has been applied. Correct identifications that have been removed are represented by dotted lines.
Fig. 3: Number of incorrect identifications versus the search area for five particle densities

Fig. 4: Particle density resulting in 10% incorrect identifications versus the search area radius for simulated velocity fields and experiments
Fig. 5: Search area radius resulting in 10% respective 20% incorrect identifications versus the particle density for simulated velocity fields. Fitted polynomials of second degree added.
Fig. 6: Velocity field after the preliminary evaluation with fixed parameters

Fig. 7: Velocity field after the final evaluation with parameters determined from the analysis of the preliminary evaluation
Fig. 8: Velocity field calculated with the same parameters as in Fig. 7. before the procedure to remove incorrect identifications has been applied.
Fig. 9: Velocity field after the preliminary evaluation with fixed parameters

Fig. 10: Velocity field after the final evaluation with parameters determined from the analysis of the preliminary evaluation
STUDIES OF VELOCITY FIELDS IN A WATER HEAT STORAGE WITH A VIDEO BASED PARTICLE TRACKING VELOCIMETRY TECHNIQUE

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ABSTRACT
Thermal storage in water has become very common the last decades in many countries. Short term water heat storage systems play an important role in combined heat and power plants, in the process industry, in solar thermal systems and for domestic hot water production.

The water volume must be thermally stratified to avoid degradation of heat quality. The temperature distribution in the storage is however the integrated result of a complicated process where many different phenomena act together. Only temperature measurements will not be sufficient to explain and evaluate the thermal behaviour of a storage. Increased knowledge about the velocity field is needed.

A video based Particle Tracking Velocimetry technique has been developed and used to document the non-stationary velocity field in a water heat storage. An ordinary video camera is used to record the in plane movements of particles in a light sheet in seeded water.

The technique offers a powerful method for studies of fluid flow and velocity field. The combination of particle traces and vector plots have proven to be an efficient tool in gaining knowledge about crucial phenomena.

Velocity fields are presented, showing mixing near the inlet, natural convection at the wall and exchange of water between the boundary layer and the core, and their impact on the stratification in the storage.

INTRODUCTION
It is commonly accepted that in almost every applications of heat storage where water is used as the storage medium the water volume should be thermally stratified. The overall efficiency of a system including a heat storage is improved by increasing the degree of stratification, especially in solar applications, as shown for instance by (Han, Wu and Christensson 1980).

Degradation of the thermal stratification in storages used in heat and power plants is caused primarily by mixing of hot and cold water during the charging of the storage, by heat diffusion from the hot to the cold part and by natural convection along the walls when heat is lost to the surroundings.

In solar applications the main problem is the varying temperatures during charging which cause severe mixing in the storage. Heat exchangers inside the storage are also common in solar applications and for production of domestic hot water. However they cause mixing by inducing natural convection. Mixing of hot and cold water due to forced convection near the inlet has been studied by the authors in a previous work based on temperature measurements in a model storage. It was concluded there that the mixing near the inlet is related to a modified Richardson number (Dahl and Hermansson 1988a). It was not possible to determine the correct length and velocity scales that should be used to predict the mixing in other storages. This requires determination of the velocity field close to the inlet.

Natural convection near the wall caused by heat losses to the surrounding has also been studied earlier by the authors (Dahl and Hermansson 1988) using two-component Laser Doppler Anemometry. Vertical and tangential velocities in the boundary layer were then recorded. The effect on the thermal stratification is however determined by the exchange of water between the boundary layer and the core. The radial velocity component, which affects this process could not be measured with this technique.

The need for more complete mapping of the flow field led to the use of the PIV-technique (PIV stands for Particle Image Velocimetry) based on multiple exposed photos of particles in seeded water. (Dahl, Hermansson, Gren, Benckert 1991). This technique gave information
about the interchange of water but only a small area near the wall could be studied, since the evaluation of the Young's fringes was very time-consuming. Another problem with this technique is that the appropriate time delay between exposures is determined by the magnitude of velocities to study and has to be set before recording. The velocity range that can be detected is quite narrow and a bad choice of the time delay will give poor results. The experiment must then be repeated.

It was therefore of interest to find another measuring technique that would make it possible to record movements in the whole storage during a complete charging cycle. The new technique must give good resolution in space and enable accurate determination of velocities in a wide range, from a few mm/s up to the maximum velocity near the inlet in the laboratory storage, a few cm/s. Video based Particle Tracking Velocimetry (PTV) where particle trace pictures are created by adding many frames from a video recording of the flow with seed particles appeared to be a step in this direction.

The objective of this paper is to present some of the results achieved in experiments performed with a model heat storage and to briefly describe the PTV method used in the experiments.

**EXPERIMENTAL SETUP**

**The model storage**

The storage vessel used is made of Plexiglas, has cylindrical shape and a vertical axis. It has a height of 420 mm, diameter of 200 mm and wall thickness 3 mm, giving a total volume of 12.5 dm³. The vessel is connected to a network of circulation tubes to enable different types of experiments like charging, discharging and mixing to homogenous temperatures.

In order to minimise optical distortion, the storage is surrounded by water contained in a rectangular tank that measures 400 mm x 250 mm x 420 mm. Its total volume is 42 dm³ and the thickness of the glass walls 8 mm (Fig. 1a). This gives media with the same refraction index on both sides of the cylindrical wall and provides a plane wall towards the camera. The surrounding tank also makes it possible to simulate different conditions that affect the heat losses from the storage, since the temperature level and water flow can be controlled in both water volumes.

The top inlet/outlet of the storage is made of a circular plate with diameter 100 mm (Fig. 1b). The water enters the storage from a vertical pipe at the top. To make sure that the water is uniformly distributed in radial direction, a circular joint is mounted on the plate to provide a narrow slot for the incoming water. The size of the slot can be varied by moving the pipe and plate in vertical direction. Most of the experiments were performed with a 2 mm slot, giving an inlet velocity of 5 mm/s for a flow rate of 12.5 dm³/h. This corresponds to a one hour loading cycle.

The bottom inlet/outlet is also made of a circular plate of diameter 100 mm. The width of the slot has been maintained at 5 mm during these experiments.

To get the same boundary conditions all around the storage, the temperatures have to be evened in horizontal direction on the outside of the storage. The water fed to the outside is therefore introduced at the top in horizontal direction.

**Temperature measurements and control**

All the heat to the system is supplied by an accurate electric primary heater, Lauda Immersion Thermostat mS/2 with 20 dm³ water store, enabling accurate temperature regulation (within 0.1°C). The cold water withdrawn from the bottom of the storage is heated in two steps, first by a heat exchanger inside the primary heater and then by another in the open tank (Fig. 1a).

The water on the outside of the storage can also be heated in two steps. In this way the temperature of the water, fed to the outside of the storage, differs only a few tenths of a degree from the temperature of the water entering the storage. This is necessary since the two water volumes are separated only by the Plexiglas wall, giving a high overall heat transfer coefficient. The loading rates can be regulated to give the same mean velocity on both sides so that the gradient zones will move in parallel. This also makes it possible to perform experiments with nearly no natural convection along the walls of the storage.

The temperature sensors used are thermocouples of the Copper-Constantan type. The gauges are connected to a DataTaker, DT100 Laboratory Logger, which scan all the gauges with certain time intervals. The calculated temperatures are stored on a Personal Computer. The reference junction for all the gauges is in an Aluminium block inside the DataTaker and the temperature of this block is measured with an LM335 temperature sensor.

All the gauges were separately calibrated in the range of 15 to 55°C before the experiments started. The inaccuracy is therefore considered to be less than 0.2°C during these experiments.
PTV-TECHNIQUE USED IN THIS STUDY

A detailed description is presented in (Dahl, Hermansson et al. 1995). The technique applied in this work enables continuous recordings with a video camera of inplane movements of particles in seeded water. The complete experiment can be analyzed afterwards and interesting parts of the transient flow can be evaluated thoroughly.

To visualize the flow, particles (Pliolite AC) with diameter 20-70 μm and density 1.03±10^3 kg/m^3 were used. A light sheet was obtained by using a 5W Argon Laser and a cylindrical lens (Fig 2a).

![Figure 2a. Particle Tracking Velocimetry Equipment.](image)

The evaluation of the velocity field was performed as follows. First the particle movements were recorded on tape.

A pre-determined number of frames with a certain time delay were then read into the computer through a frame grabber with a maximum reading capacity of 25 pictures /second. The number of frames to be read and the time delay between frames was set from the image processing software, NIH-Image (Rusband 1993) according to the velocity range to study.

A number of frames were superimposed to display particle traces. The coordinates were then determined for particles spots with acceptable sizes and sufficiently illuminated to be accepted as predictable particles.

Finally the magnitude and direction of the velocities were calculated on a SUN work station with special routines developed in this work. Vector fields were in the IGOR software (Wavemetrics 1992).

The procedure is illustrated in figure 2b and can be described in the following way. The main problem is to determine which spot in all images represent the same particle captured at different real time steps. Let there be n files containing the coordinates on n frames. For each particle in frame two vectors are calculated representing all the movements from all particles in frame one and three to this particle in frame two. Vectors that represent possible translations due to conditions illustrated in figure 2b are stored in the computer and the vector that fits best due to the limitations represent the velocity field in the certain point.

![Figure 2b. Particle tracking based on three time-separated frames.](image)

The overall accuracy of the method is within 10 % and related to different types of errors as, deviation between flow and particle trajectories, limited resolution of the recordings, geometrical positioning and interpretation of particle traces.

MEASURED VELOCITY FIELDS IN THE STRATIFIED MODEL HEAT STORAGE

Flow pattern during charging

![Figure 3a. Visualization of the flow at the inlet. Ri = 1.64.](image)

Figures 3a and 3b are the result from the addition of 20 pictures, separated by a time step of 0.2s, achieved near the inlet during charging of the storage. Figure 3a is captured from the video-tape after roughly 5 minutes and 30 seconds charging with flow rate 210 ml/min equal to a one hour charging cycle. The inlet temperature was 34.8°C and initial temperature in the storage 22.6°C. Figures 3b shows the fluid flow of an experiment achieved with flow rate of 180 ml/min, an inlet temperature of 38.7°C and an initial temperature of 21.4°C, after 6 minutes and 30 seconds charging. As the flow rate is not the same for the two experiments, the time of capture for the two pictures has to be chosen so that the gradient zone is roughly at the same level in the storage for the two experiments.
Figure 3b. Visualization of the flow at the inlet. $R_i = 3.35$.

In figure 3a, it is possible to observe a vortex from the edge of the inlet plate up to the wall of the storage. Figure 3b shows a flow widening at the edge of the inlet and following the contour of the inlet. The Richardson number enables a prediction of the initial mixing:

$$R_i = \frac{g \beta \Delta T \cdot h}{v^2}$$

where
- $g$ = acceleration of gravity
- $\beta$ = coefficient of volumetric expansion
- $\Delta T$ = temperature difference between incoming water and the water in the storage
- $h$ = a length scale for the mixing
- $v$ = a velocity scale for the mixing ($\equiv 5 \text{ mm/s}$)

The inlet slot is chosen as the length scale for the mixing and the inlet velocity as the velocity scale.

The values of the Richardson number for the two experiments are 1.64 and 3.35 respectively. This difference is mainly due to the temperature difference between the inlet temperature and the initial temperature causing stronger buoyancy forces in the second experiment.

**Flow pattern during discharging**

Figure 4 shows results of measurements near the inlet, at the bottom of the storage, during a discharging experiment where colder water was entering at the bottom. Figure 4a shows traces achieved from 10 pictures, separated by 1.0s. The sequences are captured from the video-tape after roughly 20 min discharging, at a flow rate of 180 ml/min. Inlet temperature was 24°C and initial temperature in the storage 39°C.

The inlet slot is chosen as the length scale for the mixing and the inlet velocity as the velocity scale.

The values of the Richardson number for the two experiments are 1.64 and 3.35 respectively. This difference is mainly due to the temperature difference between the inlet temperature and the initial temperature causing stronger buoyancy forces in the second experiment.

**Flow pattern during discharging**

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Figure 4a was divided into 6 separate parts before the calculation of velocity vectors shown in figure 4b. This was motivated by the large range of velocities, inhomogeneous illumination and a high number of particles.

The velocity vectors in figure 4b show the strong effect from the gradient zone. This can be seen in the left part of the picture where the vertical velocities are redirected horizontally by buoyancy forces. The water at the inlet is supplied to the storage in horizontal direction at the bottom and meets the water stream from the boundary layer. Both streams counteract and result in a vertical up-wards flow near the inlet. This vertical flow first appeared close to the wall and then gradually moved towards the inlet. This can be explained by the growth of the boundary layer when the gradient zone moves upwards.
The boundary layer flow

It is possible to use the PTV-technique to study the natural convection boundary layers at the wall as shown in an experiment where the storage, initially at 22.6°C, was charged with water of 34.8°C. The charging was stopped when the gradient zone was at mid-level. The velocity measurements were performed during stand still conditions with no flow through the storage and the gradient zone was allowed to deteriorate. The heat losses were comparably high, due to an overall heat transfer coefficient of about 40 W/m²K and a temperature difference between the inside and the outside of the storage of about 1.2°C.

Figure 5a shows the velocity field in the boundary layer from the top down to the gradient zone. The temperature distribution with temperatures shown as point values, at the levels where gauges are sited can be found in figure 5b. Since the stand still had to be preceded by a charging, in this case during 35 min to get a gradient zone in the middle of the storage, the elapsed time are given from the beginning of the charging cycle.

In a level from top down to about 3 cm one can see the water in the core of the storage moving upwards and towards the wall (Fig. 5a) where water is fed into the top of boundary layer. Further down buoyancy forces due to the weak stratification above the gradient zone will decrease the velocity in the inner part of the boundary layer. This flow is redirected towards the centre and the water is fed into the stream of water going upwards.

Near the gradient zone the stratification is stronger and the buoyancy effect even more manifested. The boundary layer is here continuously tapped on water and the thickness of the layer becomes smaller in the lower part. Even further down the flow is stagnating, affected by the buoyancy forces.

The experiment was continued for many hours maintaining roughly the same temperature difference to the surroundings. This was achieved by circulating and cooling the water on the outside of the storage.

The temperature variations for the gauges above and below the gradient zone show that the temperature gradient gets weaker and weaker and finally becomes zero (Fig. 6a). A boundary layer will establish again during this decrease of the temperature gradient and becomes stronger, the weaker the gradient gets. Figure 6b shows the velocity field, after 6 h stand still, at the level where the strongest temperature gradient initially was established. A boundary layer is developed since the gradient zone has deteriorated as shown in figure 6a.

From the observations of the boundary flow it is possible to determine the velocity distribution inside the boundary layer.

Since the flow is comparably stable it is possible to evaluate many series of pictures centred around the same real time and thus taking advantage of the video-recording. A number of velocity plots can be added, giving a better estimate of the velocity field. Such an evaluation was performed, in the same experiment as the one shown in figure 5, after approximately 2 hour of stagnant conditions in the storage.

Figure 7a shows the resulting vector plot from five sets of velocity plots obtained 16 to 20 cm from the top of the storage. The temperature difference to the surroundings was then about 1.2 °C and the storage was weakly stratified at this level as shown by figure 6a.

An approximation of the velocity profile in this boundary layer is achieved by plotting vertical velocity components versus distance to the wall (Fig. 7b,c). This result is achieved from a view of 4 cm in vertical direction. Each dot in figure 7b represents a vector in the upper half of figure 7a and dots in figure 7c corresponding values from the lower part. The fitted polynomial of third degree indicate higher maximum velocity and wider boundary layer in the upper part of the view, figure 7b, which is due to the weaker temperature gradient at this level.

Laminar theory (Holeman 1990) predicts for an isothermal core and an isothermal wall a boundary layer that has higher maximum velocities. The discrepancy can be explained by the stratification in the storage which has a great impact on the boundary layer behaviour as discussed above.

DISCUSSION

The design of heat storages of high efficiency demands detailed knowledge about the main phenomena that cause loss of heat quality in the storage. Mixing of hot and cold water during charging and discharging of the storage, diffusion of heat across the gradient zone and heat losses to the surroundings that cause natural convection and mixing in the storage are crucial phenomena that has to be considered in the optimization. This is however not always done.

The PTV technique offers a method to increase the knowledge about these phenomena and thus bringing the understanding of the thermal behaviour of heat storages a step forward. The method is found to be very suitable for such studies and it gives both qualitative and quantitative information about the flow field.

The mixing near the inlets can be visualized and quantified which makes it possible to relate the flow field to the initial mixing and find the correct scales to use in the Richardson number.

Convection at the wall can be studied as well as the exchange of water between the boundary layer and the core of the storage and its impact on the thermal stratification in the storage can be investigated.

The results can be used to develop thermal models for storage vessels that account for these phenomena. The PTV method will also be used to compare numerical simulations of the flow field with experimental results, another way to increase the knowledge about heat storages.

REFERENCES


Figure 5a. Flow near the wall during stagnant conditions. Temperature difference to the surroundings is 1.2 °C. The field is recorded 55 min to 1 h 7 min from the start of the experiment. Vector scale: 1 cm in the scale of the figure = 1 mm/s.

Figure 5b. Vertical temperature distribution in the core of the storage at the time when fig 5a was recorded.

Figure 6a. Temperature history at three levels in the storage during the stand still experiment.

Figure 6b. The gradient zone has deteriorated and a boundary layer is established after 6 h of stagnant conditions. Vector scale: 1 cm in the scale of the figure = 2 mm/s.
Figure 7a. The result of five different evaluations of velocities, separated in real time to give better information. The field is recorded after 2 h stand still in the same experiment as in fig 5 and 6.

Fig 7b. Detected vertical velocity components in the boundary layer in the upper half of fig 7a. Least mean square fit of polynomial of third degree.

Fig 7c. Detected vertical velocity components in the boundary layer in the lower half of fig 7a. Least mean square fit of polynomial of third degree.
Gravity currents at the inlet of thermally stratified storage tanks

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Abstract A numerical simulation of the charging of hot water in a thermally stratified storage tank has been performed and compared to experimental results. The mixing in the gravity current resulting from the inflow of hot water in the cold water is studied. The flow rate and the temperature difference between the hot and cold water have been varied and their effects on the shape of the gravity current and mixing have been observed. The effects of this mixing on the performance of the storage tank has been evaluated by an exergy efficiency. The evolution of the exergy efficiency with the time is quite similar for a constant flow rate and different temperature differences. On the other hand for a constant temperature difference, an optimal flow rate for the exergy efficiency has been found. This results is enhanced for higher diameter of the storage tank.

1. Introduction

Thermally stratified water storages are commonly used in district heating systems. This technique is appropriate to even out variations in demand or production of heat. The principle of thermally stratified water storage is based on use of the difference in density to keep the hot and cold water separated. The charging is achieved by injecting hot water uniformly in the radial direction at the top of the tank. However, mixing between hot and cold water occurs which reduces the heating capacity of the storage tank. One of the main phenomena affecting the thermal performance of the tank is the mixing at the inlet during charging (Dahl et al 1995). Similarly for chilled water storage in cooling systems, the charging of chilled water in warm water induces mixing at the inlet.
The hot water can be injected into the tank filled with cold water as a gravity current to reduce the mixing between hot and cold water. The motive force of gravity currents is provided by the horizontal pressure gradient due to density difference. The gravity force is balanced by inertia and viscous forces. For the gravity force to be dominant over the inertia force, the Froude number \( Fr = \left( \frac{g'}{h} \right)^{1/2} \) has to be of order unity or smaller (\( U \) is velocity at the front, \( g' = g \cdot \left( \frac{\rho_0 - \rho_i}{\rho_i} \right) / \rho_i \) is the reduced gravity, \( g \) is the acceleration due to the gravity, \( \rho_0 \) is the density of the gravity current, \( \rho_i \) is the density of ambient fluid and \( h \) is a characteristic length, the current thickness).

Gravity currents have been subject to several theoretical and experimental studies. Yoo et al (1986) gave a review of the flow field behaviour for different values of the inlet Froude number, \( Fr_i \), and observed that for inlet Froude numbers above 2, the inflow of hot water became increasingly chaotic inducing mixing and entrainment. The characteristic length in the inlet Froude number is the inlet height and the velocity is the inlet velocity.

Mc Guirk and Rodi (1979) considered buoyant jets, where the inertia force is larger than the gravity force. It was found that the main effects of the buoyancy were an increase of the lateral mixing and a reduction of the vertical spreading.

Benjamin (1968) studied steady gravity currents using perfect fluid theory. Mixing of the fluids and effects of viscosity were ignored. Benjamin analysed theoretically the shape of gravity currents described as a head wave followed by a rearward side. Completing this theoretical approach, experiments conducted by Simpson and Britter (1979) have shown that mixing occurs at the head of gravity currents due to shear instability between the two fluids and gravitational instability of less dense fluid overrun by the gravity current. This last phenomenon is suppressed in Britter and Simpson (1980) to make an analysis of the shear instability possible.

Britter (1979) studied the position of gravity currents versus time and concluded that mixing occurs only at the head of the gravity current for high Reynolds number flows, \( Re = \frac{U \cdot h}{\nu} \) (\( \nu \) is the kinematic viscosity). Huppert and Simpson (1980) considered the effect of the total depth on the mixing at the head and found it negligible for a total depth exceeding ten times the current thickness.
Low Reynolds number flows were considered by Didden and Maxworthy (1982) and Huppert (1982). They found that the gravity force is initially balanced by inertia force and then as the viscous force becomes larger than the inertia force, a gravity-viscous regime sets up. The spreading rate of gravity currents was determined separately for the gravity-inertia regime and the gravity-viscous regime. Huppert concluded that the head of gravity currents has no influence on the spreading rate for low Reynolds numbers, contrary to gravity currents at high Reynolds number which are controlled by conditions at the head. Simpson and Britter also noticed that no head wave could be observed for Reynolds numbers smaller than 10.

The aim of this paper is to determine the effects of the mixing in gravity currents on the thermal performance of storage tanks by numerical simulation of a storage tank model. To evaluate the effect of the different types of mixing, an efficiency based on the exergy has been defined. The spreading of gravity currents at the top of the tank has been considered and compared to experimental results presented by Didden and Maxworthy (1982). For the range of Froude number and Reynolds number studied, the mixing of hot and cold water has been related to the shape of the gravity current.

2. Geometry and model

A three-dimensional, transient numerical simulation of a storage tank model has been performed with the commercially available software PHOENICS. The geometry of the tank, shown in Fig. 1, is similar to that of the Plexiglas model used in the experiments presented by Hermansson (1993) and Dahl et al (1995). The height of the tank is 420 mm and the diameter 194 mm. The inlet plate is made of Plexiglas with a diameter of 97 mm and a thickness of 8 mm. The inlet and outlet plates are positioned 2 mm from the top and bottom of the tank. The hot water is entering vertically through an opening of diameter 20 mm at the top center of the tank and is redirected in the radial direction by the inlet plate. The cold water is leaving by an opening of the same diameter at the bottom of the tank.
The governing equations are the conservation equations for mass, momentum and energy in their incompressible laminar form. The density is assumed constant except in the buoyancy term of the momentum equation where the Boussinesq approximation is used.

The heat losses to the surroundings are neglected so the walls are considered adiabatic. The velocity is fixed at the inlet. The pressure at the outlet is set as constant. The tank is assumed to be filled with cold water at the initial time.

The computational domain in the orthoradial direction is restricted to one cell of one radian due to the axisymmetry of the problem. A mesh of 5000 points was initially used. Some of the calculations performed with a larger tank diameter have required a larger number of points, up to 12000. The time-step was selected depending on the flow rate, from 0.003 s for the highest flow rate to 0.025 s for the lowest.

3. Range of conditions studied

Most of the calculations were made for dimensions of the simulated storage tank equal to those of the model storage tank used by Hermansson (1993). Calculations were also made for a larger tank diameter of 485 mm.

The conditions selected for the cases analysed are listed in Table 1. The inlet Froude number is in between 0.3 and 3.7 corresponding to a range of flow rates in between 3 ml/s and 24 ml/s. The inlet Reynolds number varied between 10 and 80. The temperature of the cold water was set to 20.6°C for all the simulations. The maximum value of the hot water temperature was 50°C.

4. Computational results

4.1. Temperature evolutions

The predicted temperature evolutions are plotted in Fig. 2. The temperature evolutions show how the temperature gradient zone, called thermocline, is going
down in the storage during the charging of hot water. The experimental temperature evolutions obtained by Hermansson for five temperature gauges at different heights in the storage tank are also shown in Fig. 2. The temperature rises when the hot water reaches the gauge location and stops when the thermocline has passed the gauge. The first numerical simulation presented in Table 1 has been performed with the same flow, inlet and outlet temperatures as those used in the experiment. A fairly good agreement between experimental and calculated temperature evolutions is observed. Some of the differences between the experimental and predicted temperatures have been discussed previously by Hermansson. The small vertical temperature gradient observed in the experiment at the top of the storage tank before the beginning of the charging and later on above the thermocline are due to the heat transfer to the surrounding that could not be totally prevented. The predicted temperatures also rise faster than the experimental temperatures. This difference is due to mixing between hot and cold water in the tubing before the inlet which takes place in the experiment. The mixed water enters first in the tank delaying the temperature rise. Furthermore inaccuracies in the measured flow rate in the experiment may have contributed to the delay between the experimental and calculated temperature evolutions.

4.2. Velocity fields

In order to validate the numerical prediction of the velocity field, the results presented by Didden and Maxworthy (1982) have been used. Didden and Maxworthy have compared the order of magnitude of the different forces involved in the spreading of gravity currents and deduced the spreading relations for the gravity-inertia regime and the gravity-viscous regime. The transition time $t_1$ between the gravity-inertia regime and the gravity-viscous regime can be obtained from $t_1 = (q/g' \cdot v)^{1/2}$ (q the flow rate). For the gravity-inertia regime occurring for time $t << t_1$, the radius of the gravity current is given by

$$R \sim (g' \cdot q)^{1/4} \cdot t^{3/4}$$
For $t >> t_i$, the viscous forces become dominant over the inertia force and the radius of the gravity current is then given by

$$ R \sim (g' q^3 / \nu)^{1/8} \cdot t^{1/2} $$

This relation is valid for a bottom gravity current which is similar to a gravity current at the top of a storage tank. This theoretical investigation has been verified by experimental measurements of the front propagation of dyed fluid. The analysis has been focused on studies of the gravity-viscous regime since the gravity-inertia regime lasts only for a few seconds for the lowest flow rate studied. The diameter of the model storage tank used in Hermansons experiments was too small for studies of the spreading rate of a gravity current over a long period as the gravity current will reach the wall after only 30 s for the lowest flow rate used. Thus a larger tank diameter of 485 mm was analysed.

In order to study the spreading rate of gravity currents, the predicted position of a temperature contour has been followed as a function of time. The contour representing the temperature of the ambient fluid increased by one tenth of a degree Celsius was chosen for study for case 8, 9 and 10. The choice of a temperature contour closer to the cold water temperature as measure of the evolution of the front gave the same results.

To compare cases performed with different flow rates, a reference time corresponding to the same volume of hot water entered in the storage tank has been introduced. Case 0 was chosen as reference. For this case, the storage tank is filled with hot water in nearly one hour. The reference time $t_{ref}$ is given by:

$$ t_{ref} = \frac{t \cdot q}{q_{ref}} $$

where

$t$=charging time
$q$=flow rate
$q_{ref}$=flow rate of case 0
Fig. 3 shows the logarithmic plot of the radius $R$ versus time $t_{ref}$ for case 8, 9 and 10. Straight lines have been fitted to the predicted values. The time dependence for case 9 and 10 is found to be $t_{ref}^{0.5}$. This corresponds to Didden and Maxworthy results.

However case 8 shows a time dependence of $t_{ref}^{0.58}$ which differs from Didden and Maxworthy theoretical relation. The difference between the results presented by Didden and Maxworthy and the predicted values is too large to be due to the radial heat conduction in the gravity current. It has to be noticed that for case 8, the inlet Reynolds number is 10 and that the lowest inlet Reynolds number in the experiments performed by Didden and Maxworthy was 30. Thus it seems that for lower inlet Reynolds number, the theoretical relation of Didden and Maxworthy does not represent completely the behaviour of the gravity current. This variation in the time dependence of the front of the gravity current corresponds with the disappearance of the head of the gravity current. This phenomena will be discussed in the next paragraph. The spreading rate of the gravity current of the lowest flow rate of the three cases studied has to be investigated more precisely in a future work.

Thus in conclusion, the numerical simulation represents fairly well the charging of a water heat storage tank compared to previous experimental results.

4.3. Mixing in the gravity current

We have considered three cases to study the mixing of hot and cold water during the initial pass of a gravity current at the top of a storage tank.

Fig. 4, 5 and 6 show the evolution of $\bar{T}$, $\bar{T} = \Delta T \cdot \bar{V}$ with $\Delta T$ the temperature difference between the hot and cold water and $\bar{V}$ the velocity of the water, for cases 0, 2 and 4 at a reference time equal to 16 s. The variable $\bar{T}$ gives an overview of the heat transport in the tank. Five contours of temperature representing the temperature distribution in the tank have been added. The temperature difference between the hot and cold water is the same for the three cases. The flow
rate in case 4, respectively case 2, is eight times, respectively four times, the flow rate of case 0. The Froude number varies from 0.5 for case 0 to 3.7 for case 4.

The shape of the front temperature contour for case 4 shown in Fig. 6 is similar to the description of Benjamin (1968), a head followed by main body. The temperature contours in the main body show a strong temperature gradient which indicates that nearly no mixing has occurred in the rearward side of the gravity current. On the other hand, the temperature gradient is weaker at the head of the gravity current. Thus most of the mixing occurs at the head for the highest flow rate. This result is similar to that presented by Britter (1979).

When the flow rate is reduced, the head of the gravity current is not as much marked as shown in Fig. 5 or even no more observed in Fig. 4. The disappearance of the gravity current's head was expected from the results presented by Simpson and Britter (1979). It can be noticed that the temperature gradient behind the head is weaker and weaker as the flow rate is reduced. Thus the mixing is no more localised at the head of the gravity current but occurs all over the gravity current for low flow rates.

It may be concluded that the mixing occurs mainly at the head of gravity currents for the highest flow rates simulated in this study. The characteristic head disappears for smaller Reynolds number and the mixing is distributed over the length of the gravity current.

An estimate of the effects of the mixing at the head or all along the gravity current on the thermal performance of the storage tank is presented in the next paragraph.

4.4. Exergy efficiency

For evaluation of the effect of inlet mixing on the storage performance, the exergy efficiency, \( \eta_{ex} \), used by Hermansson (1993) has been chosen as a performance parameter. The exergy efficiency is calculated as:
\[ \eta_{\text{ex}} = \frac{\sum_{\text{cell}} Q_{\text{cell}} \cdot (1 - T_{\text{ref}}/T_{\text{cell}})}{\sum_{\text{cell}} Q_{\text{cell}} \cdot (1 - T_{\text{ref}}/T_{\text{inl}})} \]

where

\[ Q_{\text{cell}} = m_{\text{water}} \cdot c_p \cdot (T_{\text{cell}} - T_{\text{cold}}) = \text{actual energy content in each cell} \]

\[ m_{\text{water}} = \text{mass of water in each cell} \]

\[ c_p = \text{specific heat of the water} \]

\[ T_{\text{cold}} = \text{temperature of the cold water} \]

\[ T_{\text{ref}} = \text{reference temperature for exergy} \]

\[ T_{\text{inl}} = \text{inlet temperature} \]

The reference temperature is chosen as the initial temperature in the tank, i.e. \( T_{\text{ref}} = T_{\text{cold}} \).

The exergy efficiency as function of time from start of charging has been calculated for the cases presented in Table 1.

Fig. 7 shows the exergy efficiency for four simulations performed with the same flow rate, 3 ml/s. The temperature difference between hot and cold water is increased from 2.4°C to 29.4°C. The inlet Froude number varies from 0.3 to 1.5 and the inlet Reynolds number is close to 10. A high similarity between the curves can be noticed.

The exergy efficiency decreases rapidly to a minimum value reached after 30s. This minimum is lower for lower inlet temperature. The minimum exergy efficiency is reached after the initial pass of the gravity current along the top of the storage tank, when the gravity current has reached the wall and a distinct current flows back to the inlet under the initial gravity current as shown in Fig. 8.

The initial pass of the gravity current has been described as the top of the thermocline by Yoo et al (1986). Thus the formation of the thermocline starts just after the hot water has reached the wall. The mixing is reduced and the exergy efficiency rises. When the thermocline is established, further charging of hot water just pushes this zone downwards. The exergy efficiency is improved due to the increasing volume of hot water above the thermocline.
After the thermocline formation, the curves shown in Fig. 7 are parallel. For temperature differences between the hot and cold water from 2.4°C to 29.4°C, the increase of the exergy efficiency during the next 200 s is around 7%.

It can be concluded that the mixing of hot and cold water occurs mainly during the initial pass of the gravity current along the top of the storage tank. When this initial pass of the gravity current reaches the wall, the mixing is reduced and the formation of the thermocline prevents the mixing of hot and cold water. This results in an strong exergy efficiency rise. When the thermocline is finally established, the hot water enters in the storage tank water above the thermocline. There is no mixing with the cold water and the exergy efficiency increases with the time.

The five simulations 0, 1, 2, 3 and 4 with different flow rates and the same temperature difference between the hot and cold water, 19.4°C where made in order to study the effect of the Reynolds number. The flow rate was varied from 3 ml/s to 24 ml/s, giving a range of Reynolds number from 10 to 80 and Froude numbers in between 0.5 and 3.7. In Fig. 9, the exergy efficiencies of cases 0, 1, 2, 3 and 4 have been plotted versus \( t_{ref} \), the time necessary for the same volume of hot water as in case 0 to enter the storage tank. The general shapes of the curves are similar to those shown in Fig. 7. The exergy efficiency drops at the beginning of the charging to a minimum value. After reaching this minimum, the exergy efficiency increases first rapidly and then gradually slower.

However the exergy loss has no monotonous variation with the flow rate but has a minimum for a flow rate in between 6 and 12 ml/s.

As explained previously, the minima for each case are reached when the gravity current bumps against the wall and a reverse current flows back under the initial pass of the gravity current. For the lowest flow rate and also lowest Reynolds number studied, the gravity current has no characteristic head as shown in Fig. 4.

The mixing occurs therefore all along the length of the gravity current. The consequence is a high total mixing and a low exergy efficiency. As the flow rate is increased, the gravity current is more and more controlled by phenomena occurring at its head. The mixing is localised at the head and just behind it as shown in Fig. 5 and the total mixing is reduced. For higher flow rates, more hot
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water is mixed at the head, see Fig. 6, resulting in a drop of the exergy efficiency. Thus the optimal exergy efficiency during the first minutes of the charging is obtained for a gravity current partially controlled by conditions at the head.

The wider hollow of the curve representing case 4 is due to the high velocity of the gravity current bumping in the wall. The gravity current is redirected vertically as shown in Fig. 10 inducing mixing between hot and cold water.

The exergy efficiency is slightly decreasing for cases 1, 2 and 3 after 150 s of the charging. The thermocline is located just under the inlet plate. In Figs. 11 and 12, it can be observed that the hot water is reaching the centre of the tank and redirected downwards. This vertical flow can result in mixing with the cold water as previously when the hot water was reaching the wall of the storage tank. The velocity of the hot water is increasing close to the centre of the tank due to the cylindrical geometry. Thus for case 3 presented in Fig. 12, the temperature contours close to the centre of the tank are no more horizontal but show that mixing has occurred reducing the exergy efficiency. For lower velocity as in Fig. 11, the temperature contours are still horizontal and no mixing is observed.

The effects of the tank diameter on the exergy efficiency have also been studied by increasing the diameter of the tank to 485 mm. Fig. 13 shows the evolution of the exergy efficiency for cases 8, 9 and 10 performed with the same temperature difference, 18.9°C and three flow rates. We can observe that the dips in the exergy efficiency curves are much larger than calculated with the previous diameter, 194 mm. The minimum exergy efficiencies are lower than the values obtained for the smallest diameter due the larger volume of the thermocline resulting from the increase of the tank diameter. The mixing between hot and cold water is lasting longer as the gravity current has to cover a longer distance during its initial pass.

The decrease of exergy efficiency due to the increase of the diameter is obvious for the lowest flow rate as the minimal value varies from 73.5% for case 0 to 50% for case 8. This can be compared to cases 2 and 9 for which the exergy efficiency decreases only from 77% to 69%. The drop of exergy efficiency due to a higher diameter is larger for gravity currents at low Reynolds numbers. This enhances the differences between gravity current with a mixing at the head as cases 2 and 9 and gravity currents current where no head is made out as cases 0 and 8.
5. Conclusion

The problem of the inflow of hot water in cold water in a thermally stratified storage tank has been investigated numerically. For the range of Froude number and Reynolds number considered in this study, the hot water enters the tank as a gravity current. The calculations performed in this study have confirmed that the shape of gravity currents is depending on the flow rate and temperature difference between the hot and cold water. The mixing between the hot and cold water is also affected by these parameters. The effects of this mixing on the thermal performance of the storage tank has been evaluated by an efficiency based on the exergy. The mixing causes a decrease of the exergy efficiency until the hot water reaches the wall and the thermocline has been established.

For the storage tank model investigated here, it was determined that an increase of the temperature difference between the hot and cold water results in a higher exergy efficiency as expected. On the other hand for a constant temperature difference, an optimal flow rate for the exergy efficiency has been found. For low flow rates, the gravity current has no head wave and mixing occurs all along the gravity current. The total mixing is higher compared to higher flow rates where the mixing occurs only in the neighbourhood of the head of the gravity current. This results in a higher exergy efficiency for the higher flow rate. However the gravity current is more and more controlled by the head as the flow rate is increased and the mixing at the head is also increasing resulting in a drop of the exergy efficiency. These results are enhanced for a higher diameter of the storage tank due to the longer distance to cover for the gravity current during its initial pass.

References


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Table 1: Numerical simulation parameters. The calculations have been performed with an inlet height of 2 mm.
Fig. 1: Geometry of the tank model
Fig. 2: Experimental and predicted temperature evolutions at five heights in the tank model. The gauges are located at 7, 20, 40, 70 and 120 mm from the top of the tank.

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Fig. 8: Vector field of T for case 0 with a flow rate of 3 ml/s and a temperature difference of 18.9°C after tref=40 s of charging. Five contours of temperature representing the temperature distribution in the tank have also been plotted.
Fig. 9: Evolution of the exergy efficiency versus tref for cases 0, 1, 2, 3 and 4. The temperature difference between the hot and cold water is the same for the five cases, 18.9°C. The flow rate varies from 3 ml/s to 24 ml/s.

Fig. 10: Vector field of T for case 4 with a flow rate of 24 ml/s and a temperature difference of 18.9°C after tref=40 s of charging. Five contours of temperature representing the temperature distribution in the tank have also been plotted.
Fig. 11: Vector field of $T$ for case 0 with a flow rate of 3 ml/s and a temperature difference of 18.9°C after $t_{ref}=180$ s of charging. Five contours of temperature representing the temperature distribution in the tank have also been plotted.

Fig. 12: Vector field of $T$ for case 3 with a flow rate of 18 ml/s and a temperature difference of 18.9°C after $t_{ref}=180$ s of charging. Five contours of temperature representing the temperature distribution in the tank have also been plotted.
Fig. 13: Evolution of the exergy efficiency versus tref for cases 8, 9 and 10. The temperature difference between the hot and cold water is the same for the three cases, 18.9°C. The flow rate varies from 3 ml/s to 24 ml/s. The diameter of the tank is 485 mm.
Effect of buoyancy force on a toroidal vortex behind a circular step in a semi-infinite medium

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Abstract The flow field at the edge of the inlet plate in a hot water storage tank is affected by the buoyancy force due to the difference of density between the hot and cold water. During the formation of the thermocline, a vortex can be generated due to a step phenomenon. A two-dimensional theoretical investigation of the conditions required for vortex formation has been compared to experimental data and numerical simulation. The results show that there exists a critical Richardson number above which the formation of the vortex is delayed by the buoyancy force until the gradient zone has established and gone further down in the storage.

1. Introduction

Studies of the loss of thermal performance in thermally stratified storage tanks have led to the analysis of the mixing between hot and cold water near the inlet diffuser during the charging. The hot water is uniformly injected in the radial direction at the top centre of the tank filled with cold water. The inflow of hot water into the cold water can be considered as a gravity current which has been analysed by Benjamin (1968). In a gravity current, the gravity force is dominant over the inertia force. For the case of low velocities, Didden and Maxworthy (1982) have studied the spreading rates of gravity currents in plane and axisymmetric flow geometries. Yoo et al (1986) described the first pass of the gravity current along the top of the storage tank followed by the formation of the temperature gradient zone called thermocline. The thermocline starts to establish when the gravity current has bounced to the wall and a reverse current begin to
flow in the direction of the inlet plate. Yoo et al concluded that an inlet Richardson number of 0.25 is a the minimum value for optimal charging. The inlet Richardson number is defined as $R_i = g' \cdot b / u_{in}^2$ with $u_{in}$ the inlet velocity, $g' = g \cdot (\rho_h - \rho) / \rho$ the reduced gravity, $g$ the acceleration due to the gravity, $\rho_h$ the density of the incoming water, $\rho$ the density of ambient fluid and $b$ a characteristic length taken as the inlet height. A numerical simulation of the charging of a storage tank has been presented by Veber and Skali (1998). The mixing occurring during the first pass of the gravity current along the top of the storage tank and during the formation of the thermocline has been studied and related to the thermal performance of the tank.

In the present study, the effects of the buoyancy force on the flow field near the inlet plate during the formation of the thermocline are investigated. When the gravity current has reached the wall of the storage tank and the thermocline has started to establish, the temperature gradient is located at the level of the inlet plate and the buoyancy force acts directly on the flow field at the edge of the inlet plate.

Outside of Stokes regime, the injected hot water, confined between the top of the storage tank and the inlet plate, may or may not generate a toroidal vortex in the tank, depending on its temperature and velocity and the temperature of the ambient cold water. This vortex affects the quantity of hot water mixed with cold water during the formation of the thermocline and thus the thermal performance of the storage. By using potential flow theory for a two-dimensional model, it is possible to evaluate a critical inlet Richardson number above which the formation of the vortex is delayed due to the action of the buoyancy force until the thermocline has progressed past the inlet plate. This model can not account for all the phenomena occurring in the three-dimensional case but is a first theoretical approach to the problem. Experiments achieved in a small scale storage have been compared to the analytical results. Numerical simulations have been performed and confirms the results obtained with the experiments.
2. Theoretical analysis

2.1. Complex potential of the flow

For a perfect fluid of constant density, a two-dimensional model for the flow field around the inlet plate, is proposed, with the geometry shown in Fig. 1. The complex potential of the flow around the inlet plate is defined by the equation (Paterson 1992)

\[ z = w + \exp(w) \]  

(1)

where \( w = \phi + i\psi \) is the complex potential and \( z = x + iy \). We only consider the negative values of \( y \) i.e. \( y \leq 0 \).

The streamline \( \psi = 0 \) is located at \( y = 0 \) representing the top of the storage tank. The streamline \( \psi = -\pi \) represents an inlet plate set at \( y = -\pi \) from the top of the storage. The edge of the inlet plate is located at \( x = -1 \).

For \( \phi \) large and negative and \( \psi \) between \( -\pi \) and 0, we get \( w(z) = z \). Thus between the top of the storage and the inlet plate, for \( x \) large and negative, the stream is uniform.

For \( \phi \) large and positive and \( \psi \) between \( -\frac{1}{2} \pi \) and 0, we get \( w(z) = \ln(z) \) so that the origin has the potential of a source of volume flow rate \( 2\pi \).

The potential flow studied corresponds to a flow confined between the top of the tank and the inlet plate. This potential flow represents fairly well the flow occurring at the edge of the inlet plate during the formation of the thermocline and the first moment of the descent of the thermocline.

In the case of an inlet opening height \( b \) and an inlet velocity at the edge of the plate \( u_{in} \), the complex potential is given by

\[ z = \frac{w}{u_{in}} + \exp\left(\frac{\pi w}{bu_{in}}\right) \]  

(2)
2.2. Equation of motion

The flow described previously in 2.1 creates a circulation. The inlet plate can be replaced by a vortex sheet of bound vorticity \( \omega \) (Saffman 1992). The hydrodynamic impulse of the bound vorticity is

\[
I_v = \iiint \rho r \wedge k \omega dx dy
\] (3)

As the volume of the plate is zero, \( I_v = I_B \) which is the virtual momentum of the plate. \( I_B \) is the impulsive force applied to the plate to set it in motion against the inertia of the fluid. In this study, it is the impulsive force required to maintain the plate at the same position.

The equation of motion for this region is

\[
\frac{dI_v}{dt} = \iiint F dx dy + \iiint \rho u \wedge k \omega dx dy
\] (4)

where \( \iiint F dx dy \) is the external force required to keep the inlet plate stationary.

The second term, the vortex force, can be replaced by

\[
\iiint \rho u \wedge k \omega dx dy = \oint \rho \left( \frac{1}{2} u^2 n - u(u.n) \right) ds
\] (5)

where the contour integral on the right-hand is around any circuit such that there is no vorticity between it and the plate. \( n \) is the normal exterior of the contour integral and ds is an element of the contour integral.

Equation (5) can be applied to the rectangle defined by the top of the tank \( S_1 \), the walls \( S_3 \) and \( S_4 \) and an imposed boundary in the storage tank \( S_2 \), see Fig. 1.

When considering only the vertical component of the impulsive force, there is no contribution to the integral from the surfaces \( S_3 \) and \( S_4 \). The contribution to the vertical component from \( S_1 \) is
where \( u \) is the velocity along the top of the tank and \( x_p \) is the abscissa of the edge of the inlet plate, defined by

\[
x_p = \frac{b}{\pi} \ln\left(\frac{b}{\pi}\right) - \frac{b}{\pi}
\]  

An expression for \( u \) can be obtained by taking the derivative of the complex potential \( w \). From this we get

\[
u = \frac{u_{in}}{\left(1 + \frac{\pi}{b} \exp\left(\frac{\pi \phi}{u_{in} b}\right)\right)}
\]  

and

\[
x = \frac{\phi}{u_{in}} + \exp\left(\frac{\pi \phi}{u_{in} b}\right)
\]  

where \( y=0 \) and \( \psi = 0 \) at the top of the storage. Differentiating equation (9) we get

\[
dx = \left(\frac{1}{u_{in}} + \frac{\pi}{(u_{in} b)} \exp\left(\frac{\pi \phi}{u_{in} b}\right)\right) d\phi
\]  

Substituting equations (8) and (10) into equation (6), we get

\[
I = \int_{\phi_a}^{\phi_c} \frac{1}{2} pu(\phi) d\phi
\]  

where \( \phi_c \) and \( \phi_a \) are defined from
\[ c + x_p = \frac{\phi_c}{u_{in}} + \exp\left(\frac{\pi\phi_c}{u_{in}b}\right) \]

\[ -a + x_p = \frac{\phi_{-a}}{u_{in}} + \exp\left(\frac{\pi\phi_{-a}}{u_{in}b}\right) \]

Evaluating the integral in equation (11) then gives

\[ I = \frac{\rho u_{in}^2}{2} \left[ (k_c - k_{-a}) - \frac{b}{\pi} \ln \left( \frac{1 + \frac{\pi}{b} \exp\left(\frac{\pi k_c}{b}\right)}{1 + \frac{\pi}{b} \exp\left(\frac{\pi k_{-a}}{b}\right)} \right) \right] \]

where \( k_c = \frac{\phi_c}{u_{in}} \) and \( k_{-a} = \frac{\phi_{-a}}{u_{in}} \). For clarity, let \( \alpha = \alpha(a, b, c) \) denote the expression within the brackets.

Then for the vertical component due to the circulation on \( S_2 \) we have

\[ J = \int_{-a+x_p}^{c+x_p} \frac{1}{2}\rho(u^2 - v^2)dx \]

where \( u \) is the horizontal component and \( v \) is the vertical component of the velocity along \( S_2 \).

Far from the inlet plate, from the flow conservation we get \( v = u_{in}\frac{b}{(a + c)} \), with \( u = 0 \) along \( S_2 \).

Therefore

\[ J = \frac{\rho u_{in}^2}{2} \left( \frac{b^2}{a + c} \right) \]

For the equation of motion (4) we then get
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\[
\left( \frac{dI_y}{dt} \right)_y = \left( \int \int Fdxdy \right)_y + \alpha \frac{\rho u_{in}^2}{2} + \frac{\rho u_{in}^2}{2} \left( \frac{b^2}{a+c} \right)
\]  \hspace{1cm} (16)

where \( \left( \int \int Fdxdy \right)_y \) is the vertical component of the external force required to keep the inlet plate stationary.

The vortex sheet created is not an equilibrium solution due to the singularity at the tip. It can be drawn out by the infinite velocity at the tips into a spiral.

2.3. Stratified flow

The theory involving bound vorticity and vortex force can be used to describe a stratified flow if the Boussinesq approximation is used for the density (Saffman 1992). During the formation of the thermocline, the buoyancy force applied on the inlet plate by the fluid is \((g\Delta \rho \beta) a_j\). To keep the inlet plate stationary, an opposing force has to be applied. Thus we get:

\[
\left( \int \int Fdxdy \right)_y = -(g\Delta \rho \beta) a_j = -(\rho g \beta \Delta T b) a_j
\]  \hspace{1cm} (17)

where \(\rho\) is the density of the ambient fluid, \(g\) is the gravitational acceleration, \(\beta\) is the coefficient of thermal expansion, and \(\Delta T\) is the temperature difference between hot and cold water. Substituting equation (17) in equation (16), we get

\[
\left( \frac{dI_y}{dt} \right)_y = \rho u_{in}^2 \left( \frac{\alpha}{2} + \frac{b^2}{2(a+c)} \left( \frac{g\beta \Delta T b}{u_{in}^2} \right) \right)
\]  \hspace{1cm} (18)

Noting that the inlet Richardson number is defined by \(\text{Ri}_i = \frac{g\beta \Delta T b}{u_{in}^2}\), equation (18) can be rewritten as
with $\alpha$ depending only of the geometry of the storage. This equation is applicable when the buoyancy force is applied on the plate and the flow field is defined as previously.

2.4. Variation of the vertical component of the impulsive force $(I_v)_y$

Examination of equation (19) reveals that:

- if $\text{Ri}_i < \frac{\alpha + \frac{b^2}{(a + c)}}{2a}$, the impulse of the bound vorticity $(I_v)_y$ increases with the time and a vortex forms at the edge of the inlet plate.

- if $\text{Ri}_i > \frac{\alpha + \frac{b^2}{(a + c)}}{2a}$, $(I_v)_y$ decreases with the time. The force due to the buoyancy is larger than the vortex force, and the formation of the vortex is delayed until the gradient zone has moved downwards, away from the inlet.

2.5. Application to an experimental hot water storage tank

In the laboratory model presented in the next paragraph, the storage vessel is cylindrical and the radial velocity along the inlet plate is inversely proportional to the radial distance. This effect in the complex potential model is partly taken into account by using the mean velocity along the inlet plate instead of the inlet velocity.

For the inlet geometry shown in Fig. 2, the mean velocity along the inlet plate is
\[ u_m = \frac{u_{\text{in}} a \ln \left( \frac{a}{d} \right)}{(a - d)} \]

where \( d \) is the radius of the incoming water pipe at the centre of the storage tank model. For \( a = c = 0.05 \text{ m} \), \( b = 0.002 \text{ m} \), \( d = 0.01 \text{ m} \), we have

\[ u_m = 2.0 u_{\text{in}} \]

Substituting \( u_m \) in \( \alpha(a, b, c) \) instead of \( u_{\text{in}} \), we obtain

\[ \alpha = 0.23 \]

thus \( \text{Ri}_i = \frac{\alpha + \frac{b^2}{(a + c)}}{2a} = 2.3 \) is the critical inlet Richardson number above which no vortex should form. In fact, in most of cases we have \( a, c >> b \), thus \( \alpha >> \frac{b^2}{(a + c)} \), the contribution from \( S_1 \) to the vortex force is generally much higher than the one from \( S_2 \) due to the flow conservation.

3. Experimental results

A number of charging experiments (Dahl et al 1995) has been conducted in a laboratory model storage tank made of Plexiglas. The tank has a height of 420 mm and an inner diameter of 194 mm. The inlet plate has a diameter of 97 mm and a thickness of 8 mm. The inlet and outlet plates are set 2 mm from the top and bottom, of the tank. The hot water is entering vertically through an opening of diameter 20 mm at the top centre of the tank and is redirected in the radial direction by the inlet plate. The inlet configuration is shown Fig. 2. The cold water is leaving by an opening of the same diameter at the bottom of the tank.

By using a Particle Tracking Velocimetry technique presented in Dahl et al (1995), the velocity field in the upper part of the storage was visualized. It was possible to
observe whether a vortex formed during the first minutes of each experiment. The experimental parameters are presented in Table 1.

The observed flow fields are shown in Figs. 3-6. The pictures represent particle movements in a light sheet in seeded water resulting from the addition of a number of pictures separated by a chosen time step. The pictures have been made at a time representing approximately the same volume of hot water entered in the storage tank for the four experiments.

Figs. 3 and 4 represent experiments with a low inlet Richardson number, where the formation of a vortex was observed. This vortex extends from the edge of the plate to the wall at the lowest inlet Richardson number, and is confined near the inlet plate for the inlet Richardson number of 0.5. In Figs. 5 and 6 for Richardson number of 4.8 and 4.7, it can be noticed that no vortex appeared, but a widening of the flow following the shape of the inlet plate is observed. The formation of the vortex is stopped during the establishment of the thermocline confirming the analytical results. For the experiments, it appears that the critical inlet Richardson number is somewhere between 0.5 and 4.7.

4. Numerical results

A three-dimensional numerical simulation of the storage performed with the commercially available software PHOENICS, allows further evaluation of the flow field close to the inlet plate. The geometry of the simulated tank is similar to that of the Plexiglas model presented in the previous paragraph. The numerical simulation has been presented in more details by Veber and Skali (1998). The results of the calculations performed have been compared to the temperatures evolution at different heights in the experimental tank and shown a fairly good agreement. However a delay between the experimental and numerical temperature evolutions has been observed. This delay is mainly due to mixing occurring in the tubing located before the inlet in the experimental set-up. To take into account this difference, the numerical velocity fields have been represented when the temperature at 7 mm from the top of the tank, position of the first
temperature gauge in the experimental set-up, reaches the same level as in the experiment.

Four simulations with different inlet Richardson numbers were carried out, as detailed in Table 2. The velocity field from the edge of the inlet plate to the wall are shown in Figs. 7-11. Five contours of temperature have been added in order to give an overview of the position of the thermocline. In Figs. 7-10, the five temperature contours are located at the level of the inlet plate and just under it. Thus the thermocline acts directly on the flow field at the edge of the inlet plate.

In Figs. 7, 8 and 9 for the lowest inlet Richardson numbers, a vortex has appeared at the edge of the inlet plate. Fig. 7 represents a simulation with conditions close to the experimental conditions of Fig. 3. The vortex in the experiment is of larger dimension than the one observed in the simulation. For Fig. 8, that is quite similar to the experiment observed in Fig. 4, the vortex is of small dimension and stays close to the inlet plate edge. For a higher Richardson number, as shown in Fig. 9, which is due to a larger temperature difference, the formation of a vortex is also observed. However in Fig. 10, the flow is following the inlet plate contour and no vortex is observed during the formation of the gradient zone. Later when the gradient zone has progressed downwards in the storage as shown in Fig. 11 where only two temperature contours are still visible, a vortex is able to form due to the disappearance of the buoyancy force on the inlet plate.

The critical Richardson number is within the range 1.2 and 4.8. This shows a good agreement with the experimental results.

5. Discussion

The formation of a toroidal vortex behind a circular step can be suppressed by buoyancy force. A two-dimensional theoretical model based on potential flow theory indicates a critical inlet Richardson number of 2.3 for the geometry of the model storage tank used here. Above this value the buoyancy force hinders the formation of the vortex. When the gradient zone has passed the inlet plate, such that the buoyancy force is no longer acting on the inlet plate, the vortex is able to form.
Experimental results obtained using a Particle Tracking Velocimetry technique show that the critical value of the inlet Richardson number for which a vortex has been observed during the formation of the thermocline is in the range 0.5 and 4.7.

The range of inlet Richardson number delimiting the formation of a vortex for the numerical simulation is between 1.2 and 4.8.

A reasonable agreement between a theoretical model, experimental results and numerical simulation has been found. It has been possible to define a critical inlet Richardson number or a range of inlet Richardson number delimiting the formation of a vortex at the edge of the inlet plate. The theoretical model can be applied to other geometries to determine the critical Richardson number for the formation of the vortex.

It will be difficult to have experimental and numerical values closer to the theoretical value for various reasons, including the following:

- the two-dimensional theoretical model can not represent all the phenomena occurring in the three-dimensional experimental storage. As mentioned previously, an average inlet velocity was used as the radial velocity along the inlet plate.
- the inlet in the experimental model as shown in Fig. 2, is not simply composed of a plate, but also incorporates a circular joint to ensure that the water is uniformly distributed in the radial direction. The effects of this joint on the flow field have not been studied.
- the theoretical model is based on the complex potential of the flow, so all viscosity effects are neglected.

However for inlet Richardson number above the critical value, the formation of the thermocline will not be affected by the vortex. Further work is required to evaluate the effects of this phenomenon on the level of mixing between hot and cold water during the charging of the storage and on the thermal performance of the storage tank.
References


Table 1: Experimental parameters. $R_{i}$ is the inlet Richardson number

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Inlet velocity</th>
<th>Inlet temperature</th>
<th>Initial temperature</th>
<th>$R_{i}$</th>
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</thead>
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<td>(mm/s)</td>
<td>(°C)</td>
<td>(°C)</td>
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<td>1</td>
<td>10.8</td>
<td>25.1</td>
<td>23</td>
<td>0.09</td>
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<td>2</td>
<td>5.4</td>
<td>24</td>
<td>21</td>
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<td>24.5</td>
<td>4.7</td>
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Table 2: Numerical simulations parameters. $R_{i}$ is the inlet Richardson number

<table>
<thead>
<tr>
<th>Case</th>
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<th>Inlet temperature</th>
<th>Initial temperature</th>
<th>$R_{i}$</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>(mm/s)</td>
<td>(°C)</td>
<td>(°C)</td>
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<tr>
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<td>4.9</td>
<td>39.5</td>
<td>20.6</td>
<td>4.8</td>
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</tbody>
</table>
Fig. 1: Two-dimensional storage tank model. $u_{in}$ is the inlet velocity, $b$ the inlet slot height, $a$ the length of the inlet plate, $c$ the distance to the wall and $x_p$ the abscissa of the edge of the inlet plate. $S_1$, $S_2$, $S_3$ and $S_4$ are the integration lines in equation (5)

Fig. 2: Configuration of the inlet in the experimental set-up
Top of the storage tank
distance between the inlet plate and the wall
4.85mm

Fig. 3: Observation of the flow field at the edge of the inlet plate for $R_i=0.09$.
The pictures results from the addition of 40 frames separated by 0.2s

Fig. 4: Observation of the flow field at the edge of the inlet plate for $R_i=0.5$.
The picture results from the addition of 20 frames separated by 0.2s
Inlet plate

Distance between the inlet plate and the wall: 4.85 mm

Fig. 5: Observation of the flow field at the edge of the inlet plate for $R_i=4.8$. The picture results from the addition of 20 frames separated by 0.2s.

Fig. 6: Observation of the flow field at the edge of the inlet plate for $R_i=4.7$. The picture results from the addition of 60 frames separated by 0.2s.
Fig. 7: Case 1. Velocity field at the edge of the inlet plate from a numerical simulation performed on PHOENICS, plotted on MATLAB for an inlet Richardson number of 0.09 after 45 seconds of charging. Five contours of temperature have also been plotted.
Fig. 8: Case 2. Velocity field at the edge of the inlet plate from a numerical simulation performed on PHOENICS, plotted on MATLAB for an inlet Richardson number of 0.5 after 100 seconds of charging. Five contours of temperature have also been plotted.
Fig. 9: Case 3. Velocity field at the edge of the inlet plate from a numerical simulation performed on PHOENICS, plotted on MATLAB for an inlet Richardson number of 1.2 after 50 seconds of charging. Five contours of temperature have also been plotted.
Fig. 10: Case 4. Velocity field at the edge of the inlet plate from a numerical simulation performed on PHOENICS, plotted on MATLAB for a inlet Richardson number of 4.8 after 100 seconds of charging. Five contours of temperature have also been plotted.
Fig. 11: Case 4. Velocity field at the edge of the inlet plate from a numerical simulation performed on PHOENICS, plotted on MATLAB for a inlet Richardson number of 4.8 after 400 seconds of charging. Five contours of temperature have also been plotted.