Numerical Modelling of the Rock Fracture Process Under Mechanical Loading

by

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Licentiate Thesis

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Preface

This thesis consists of two parts: on the one hand the research and development (R & D) for and the calibration of the rock and tool interaction code (R-T\textsuperscript{2D}), and on the other hand the preliminary industrial application of R-T\textsuperscript{2D} in rock fragmentation by mechanical loading. Some of the research in the first part was conducted in China when I was studying at Northeastern University for a Master’s degree. The financial support from LKAB’s Foundation for the Promotion of Research and Education at Luleå University of Technology, Trelleborg AB’s Research and Education Foundation, the Foundation for Technology Transfer, Arne S. Lundbergs Foundation, and the Knowledge Foundation is greatly appreciated.

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Abstract

The fracture of rock has been the subject of extensive research in mechanical fragmentation. With the rapid development of computing power, interactive computer graphics and topological data structure, numerical tools have become a good means of gaining some insight into the problem of rock fracture. However, most of the commercial programs are not so robust that they can model the progressive fracture process while taking the rock fabric characteristics (heterogeneity) into consideration. Consequently, a novel numerical code called the rock and tool interaction code (R-T^2D) has been developed to understand the progressive fracture process of heterogeneous rock. The research and development (R & D) for and the calibration of the R-T^2D code (Paper A and B) mainly consist of (1) the characterization of rock heterogeneity, (2) mesoscopic constitutive laws, non-linear behaviour and associated seismicity, (3) the Mohr-Coulomb and the double elliptic strength criterion, and (4) the calibration of the R-T^2D code by simulating basic rock mechanics experiments.

In the second part of the thesis (Paper C and D), preliminary industrial applications of the R-T^2D code in mechanical fragmentation are conducted. In rock cutting, the peculiarities of cutting heterogeneous brittle materials are investigated. Besides, the fracture processes induced by cutters with different back rake angles are examined and compared with each other. In indentation, research on indentation-induced fractures is performed using the R-T^2D code. The influences of confining pressure on the formation of side cracks are discussed. In simultaneous loading by multiple indenters, the propagation, interaction and coalescence of side cracks induced by neighbouring indenters are simulated to check how the side cracks propagate, interact and coalesce to form large rock chips. Finally, the indexing-effect in simultaneous loading is discussed.

The major results for the two parts can be summarized as follows: (1) On the basis of the Weibull distribution, a heterogeneous material model is proposed to characterize the heterogeneity of rock, and in this model rock is specified by a few characteristic parameters: the homogeneous index and the elemental seed parameters. For any specific rock, the homogeneous index can be determined on the basis of the defect distribution of the microstructure and the elemental parameters can be obtained from laboratory tests. (2) Rock heterogeneity has an important influence on the crack initiation location and the subsequent propagation path. (3) Compared with the traditional Mohr-Coulomb and Hoek-Brown strength criterion, the double elliptic strength criterion is more useful for modelling the fracture occurring in mechanical fragmentation and can represent the transition from brittle failure to ductile cataclastic failure with increasing confining pressure. (4) A series of numerical experiments including both intact rock and notched rock were conducted using the R-T^2D code to obtain the physical-mechanical properties and fracture toughness, and to simulate the crack initiation and propagation, as well as the whole progressive fracture process. The developed R-T^2D code seems to have built a bridge between the physical-mechanical parameters and fracture toughness. The detailed visually shown stress distribution and redistribution, crack nucleation and initiation, stable and unstable crack propagation, interaction and coalescence, and corresponding load-displacement curves can be proposed as benchmarks for numerical programs for crack propagation. (5) In mechanical fragmentation, a crushed zone is always available near the tool. The crushed zone has an important influence on the chipping process and energy utilization. The crushed zone is in fact the zone with a high density of microcracks, and some of the rock in this zone behaves in a ductile manner with stress satisfying the ductile failure surface of the double elliptic strength criterion. (6) A simple description and qualitative model of the rock fragmentation process induced by truncated indenters are summarized as follows. Little damage to the rock is observed at the linear-elastic deformation stage. Then conical Hertzian cracks are initiated adjacent to both corners of the truncated indenter and propagated in the...
well-known conical Hertzian manner. As the loading displacement increases, some of the elements in the high confining pressure zone immediately under the indenter fail in the ductile cataclastic mode, with the stress satisfying the ductile failure surface of the double elliptic strength criterion. With the cataclastic failures and tensile conical cracks releasing the confining pressure, the elements in the confining pressure zone are compressed into failure and the crushed zone comes into being. In the crushed zone, microcracking is pervasive. The intensity of the microcracking within the zone increases and a re-compaction behaviour occurs with increasing loading displacement. Associated with this microcracked region there is a volumetric expansion and a tensile stress field, which drives side cracks to propagate in a curvilinear path. It is thought that the curvilinear path is caused by the heterogeneity of the rock. With an increased loading displacement, the side cracks rapidly propagate and intersect with the free rock surface to form rock chips. (7) The confining pressure has an important influence on the failure mode in indentation tests. As the confining pressure increases, a small but noticeable increase in the indentation strength is measured. With decreasing confining pressure, of particular interest is a change in the rock failure mode when the confining stress is reduced below a critical value. Instead of the usual formation of rock chips adjacent to the punch, vertical cracks are propagated beneath the punch, causing the specimen to be split in half when the confining pressure on the sample is less than a critical value. This result is of practical interest. In boring hard rock at low confining stresses, the creation of such tensile fractures beneath an indenter may serve to fragment the rock sufficiently to facilitate its removal. (8) The simultaneous loading of the rock surface by multiple indenters seems to provide a possibility of forming larger rock chips, controlling the direction of subsurface cracks and consuming a minimum total specific energy. (9) The results simulated by the R-T$^{2D}$ code reproduce the progressive process of rock fragmentation under mechanical loading: the build-up of the stress field, the formation of the crushed zone, surface chipping, and the formation of the crater and subsurface cracks. Therefore, the R-T$^{2D}$ code is indeed a valuable numerical tool for research on rock fracture which can be utilized to improve our understanding of rock-tool interaction and the rock failure mechanisms under the action of mechanical tools, which, in turn, will be useful in assisting the design of fragmentation equipment and fragmentation operations.

On the basis of the above research, a number of interesting problems are discussed and future research is suggested in the last part of the thesis.

**Key words**: Rock fracture, numerical simulation, heterogeneity, fracture toughness, strength criterion, mechanical fragmentation, cutting, indentation.
The thesis comprises a summary and the following four papers.


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Papers A-D
1 Introduction

1.1 R & D for and calibration of the R-T²D code

The fracture and failure of rock have been the subject of extensive research in mechanical fragmentation, the results of which have led to a number of comprehensive texts (Lindqvist, 1982; Whittaker et al, 1992; Kou, 1995; Tan, 1996; Mishnaevsky, 1998). This extensive work has provided an understanding of rock as a heterogeneous brittle material that fractures via the formation, growth and coalescence of microcracks. Though experimental observations have provided a great deal of insight into the complicated fracture process, the mechanism of rock fracture under mechanical loading is not clear and the evolution of the progressive fracture process cannot be visually shown at this moment. Besides, it is too expensive to conduct a large number of experiments. The use of linear-elastic fracture mechanics (LEFM) to describe these mechanisms analytically is arduous (Schlangen and Garboczi, 1997), since the fracture patterns consist of a main crack with various branches, secondary cracks and microcracks. Moreover, analytical models have to be simplified and sometimes this simplification ignores important factors influencing the material behaviour. Heterogeneity is an example of one such factor for rocks. During the past few years, with the rapid development of computing power, interactive computer graphics and topological data structure, numerical tools have become a good means of gaining some insight into the problem of rock fracture.

Numerical simulation of the fracture process in rock requires robust numerical methods that can allow the efficient resolution of multiple interacting cracks and rigorous fracture models that can reflect the material fabric characteristics. In fact, a large number of numerical methods and fracture models have been developed for research on the rock fracture process. The numerical methods most widely used for analysis of the rock fracture process in rock engineering are the finite difference method (FDM), finite element method (FEM), boundary element method (BEM) and discrete element method (DEM).

FLAC (Itasca, 1995) is the outstanding representative of the finite difference method (FDM). There are nine basic constitutive models provided in FLAC, Version 3.3 (Itasca, 1995), arranged into null, elastic and plastic model groups. The plastic model group contains some models, e.g. the Mohr-Coulomb model and the strain-softening/hardening model, which allow rock fracture to be simulated. McKinnon and Barra (1998) used the strain-softening constitutive law to model primary fracture initiation and growth with FLAC.

As for the finite element method (FEM), Saouma and Kleinosky (1984) simulated crack initiation and subsequent crack propagation in the process of major chip formation with their finite element program – SICRAP (simulation of crack propagation program). Cracker (Swenson and Ingraffea, 1988) is a finite element code used for modelling the dynamic propagation of a discrete crack. FRAN2D (Wawrzynek and Ingraffea, 1989) is a two-dimensional crack propagation simulator using the finite element method. Alehossein and Hood (1996) used DIANA (1993) and NUMA (1995), which are also finite element methods, to simulate crack problems in indentation.

Recent advances in the boundary element method (BEM) have resulted in a new generation of the method for the solution of crack problems in fracture mechanics. In the boundary element method, the displacement discontinuity method (DDM) is mainly used to model fracture. Tan et al (1996) used the DDM coupled with a splitting model to simulate side crack propagation in indentation. Blair and Cook (1998) coupled DDM in their non-linear rule-based model to analyse compressive fracture in rock using statistical techniques.

The formulation and development of the distinct element method have been progressing for a period of more than 30 years, beginning with the initial presentation by Cundall (1971). The method was originally
created as a two-dimensional representation of a jointed rock mass, but has been extended to applications in studies on microscopic mechanisms in granular material (Cundall and Strack, 1983), and crack development in rock and concrete (Plesha and Aifantis, 1983; Lorig and Cundall, 1987). The representative codes of the distinct element method are UDEC (Cundall, 1980; Cundall and Hart, 1985), 3DEC (Cundall, 1988; Hart et al., 1988), DIBS (Walton, 1980), 3DSHEAR (Walton et al., 1988), PFC (Itasca, 1995) and DDA (Shi, 1988).

Rock fracture models, such as the energy model (Glucklich, 1963; Karihaloo, 1984; Shen and Stephansson, 1994), the stress model (Murrell and Digby, 1970; Paul and Mirandy, 1976; Tiros and Catz, 1981), the lattice model (Herrmann and Roux, 1992; Schlangen and Van Mier, 1992; Schlangen and Garboczi, 1997; Place and Mora, 1999), the RFPA model (Tang, 1997; Tang et al., 1998; Tang et al., 2000), the cellular automaton model (Wilson et al., 1996; Psakhie et al., 1999) and so on, have been developed to accompany the numerical methods. Wang and Shrive (1995) have provided a good detailed review and summary of typical existing models for brittle fracture in compression. These models are versatile and able to handle complex geometry and material behaviour. In these models, the behaviour of the materials, i.e. the rocks, is usually assumed to range from simple linear-elastic to non-linear behaviour. The analyses range from static and quasi-static to dynamic analyses.

The fracture process of heterogeneous rock and rocklike material is so complex that it is not surprising that different numerical methods and fracture models have been developed. A number of idealizations have been used in the above numerical methods and fracture models which may limit their correspondence to the complex fracture processes occurring in the loading of heterogeneous brittle materials. Although all these models have improved our understanding of rock fracture, the discrepancy between prediction and experiments is still significant. At this stage of investigation, few of the above studies can visually simulate the progressive fracture process including crack initiation, propagation, interaction and coalescence. The most important factor of all is that most of them have not related their work to the uniquely heterogeneous character of rock. Moreover, none of them can determine input parameters on the basis of the specific rock studied.

It appears that the success of modelling brittle fracture in heterogeneous material depends on the full understanding of fracture mechanisms, the soundness of a universal fracture criterion and the effectiveness of the numerical technique used. At present the finite element method is the most mature method of the above four numerical methods. Preliminary researches (Tang, 1997; Tang et al., 1998; Kou et al., 1999; Tang et al., 2000; Liu et al., 2001a-c; Kou et al., 2001; Liu et al., 2002a-c) indicate that the RFPA (Rock Failure Process Analysis) model (Tang 1997) is appropriate for research on the rock fracture problem. Therefore, in this thesis a novel numerical program called the rock and tool interaction code (R-T 2D) is developed on the basis of the finite element method (FEM) and the RFPA model to investigate the rock fracture process under mechanical loading.

This thesis is an extension of previous work conducted in China. The investigation dealing with the research and development (R & D) for and the calibration of the R-T 2D code consists of four parts: 1) the characterization of rock heterogeneity; 2) mesoscopic constitutive laws, non-linear behaviour and associated seismicity; 3) the Mohr-Coulomb and the double elliptic strength criterion; and 4) the calibration of the R-T 2D code by simulating basic rock mechanics tests.

1.2 Preliminary application of the R-T 2D code in mechanical fragmentation

Rock fragmentation by mechanical tools is the preferred technique of effective rock removal because of the continuity of the process, the directly fragmenting rock mode involved and the smaller energy loss incurred. The technique is employed in one way or another in most of today’s mining and tunnelling machines. As drilling and mining environments are becoming increasingly severe, improved understanding
of rock behaviour under mechanical loading and optimisation of cutter or drilling design are necessary in order to hold down costs.

Rock cutting and indentation constitute the fundamental mechanisms for most mechanical excavation methods. In rock cutting and indentation, a tool is forced into the rock, and the rock near the tool is crushed into small or large fragments, leaving subsurface cracks in the remaining rock wall. Research analysing the stress and failure occurring in mechanical fragmentation captures the basics and enables a more detailed study of the physical process. The results of this type of work are important both for the understanding of rock fragmentation and for the evaluation of the damage to the remaining rock. However, the analysis of mechanical fragmentation has mostly concerned the tool penetration and the rock failure at the surface, mainly because rocks are non-transparent and it is difficult to trace the propagation of the rock fracture and fragmentation within the rock.

Lindqvist (1982), Cook et al (1984), Howarth and Bridge (1988) were among the first to conduct crack discrimination. The most informative results were found in the work performed by Lindqvist et al (1994). A general picture of fractures in rock under mechanical tools was summarized. It was found that rock fragmentation by mechanical tools was mainly a chipping process. Therefore, it was of great interest to follow up closely the process of chipping in order to achieve a better understanding of the mechanisms. In order to conduct research on the chipping process in mechanical fragmentation, the theories of information, fractals and fuzzy sets were introduced by Mishnaevsky (1998). However, at present the stress state and crack propagation behaviour involved in the chipping process are too complex to be studied with a theoretical analysis method. The failure process may, however, be traceable in detail by applying appropriate numerical methods. Gradually mature computational mechanics theories and high-powered personal computers have made this possible. From this point of view, there is a clear incentive to develop numerical methods that can simulate the process of material failure under mechanical tools, e.g. cutters and indenters. Numerical models that can accurately simulate rock-tool interaction and failure mechanisms, including chip formation, can be used in parametric studies to determine the optimal tool (cutter or indenter) designs, reducing much of the expensive and time-consuming experimental work which would otherwise have to be carried out to assess the performance of a particular tool design.

As a matter of fact, several attempts have been made to simulate the rock fracture process under mechanical tools with numerical methods. The three main numerical methods, the finite element method (FEM), the boundary element method (BEM) and the discrete element method (DEM), have been used to calculate the stress field and simulate the fracture under a mechanical tool by many researchers, such as Lawn and Swain (1975), Lawn and Wilshaw (1975), Hood (1997), Cook et al (1984), Saouma and Kleinosky (1984), Chen and Alehossein (1996), Alehossein and Hood (1996), and Tan et al (1997). These simulations coupled with fracture models have been capable of detecting some mechanisms of rock fragmentation under mechanical tools. In the simulations using an elastic model, the rock has been assumed to behave as an ideal, isotropic elastic material. Due to the fact that in-elastic deformation within the crushed and damaged rock has not been taken into account, the simulations have over-predicted the peak load at the onset of chip formation (Chen and Alehossein, 1996) and ignored its effects on the indentation stress field afterwards. In some of the above simulations, a blister model (Yoeff, 1982) has been introduced to describe the crushed and damaged zone. However, not only is the shape of the crushed and damaged zone uncertain, but also the blister model needs to be validated further. Moreover, most of the above works have not taken the heterogeneity of rocks into consideration. The most important drawback of all is that none of the above simulations has concerned the progressive formation process of the whole crack system in mechanical fragmentation, and almost none of them has considered the interaction between the stress fields induced by neighbouring tools in simultaneous loading.

At present numerical methods based on FEM coupled with a fracture model are promising methods for simulating indentation-induced fracture. In this thesis the progressive failure process of rock under
mechanical loading is simulated using a computer program called R-T\textsuperscript{2D} (the rock and tool interaction code), developed in the first part of this thesis, which is based on FEM and the RFPA model (Tang, 1997). The simulated results reproduce all of the fragmentation processes in mechanical fragmentation: the build-up of the stress field, the formation of the crushed zone, surface chipping, crater formation and the formation of subsurface cracks. According to the simulated results, some qualitative and quantitative analyses are performed.
2 R & D for and calibration of the R-T$^{2D}$ code

The numerical approach called R-T$^{2D}$ (the rock and tool interaction code) has been developed on the basis of FEM (the finite element method) and the RFPA (rock failure process analysis) model. The basic elements of the RFPA model can be summarized as follows (Tang, 1997): 1) by introducing the heterogeneity of rock properties into the model, the RFPA model can simulate the non-linear deformation of a quasi-brittle rock with an ideal brittle constitutive law for the local material; 2) by introducing a reduction of the material parameters after element failure, the RFPA model can simulate strain-softening and discontinuum mechanics problems in a continuum mechanics mode; 3) by recording the event-rate of failed elements, the RFPA model can simulate seismicities associated with the progressive fracture process.

2.1 Characterization of rock heterogeneity

The actual goal of numerical simulation is to simulate fracture in a real material. Real materials, such as rock, are in general random and inhomogeneous, and therefore their fracture will be influenced by the heterogeneous microstructure of the material. Heterogeneity has to be implemented to simulate the fracture process correctly. Different techniques have been used in the past to implement heterogeneity (Schlangen and Garboczi, 1997). These techniques all have their own application scale for which they will give good results. The different options include: 1) Randomly assigning different properties, possibly following some kind of distribution, to the elements in the numerical model (Batrouni and Hansen, 1988; Tang, 1997); 2) Using a mesh with a random geometry, but equal properties for the elements (Garboczi and Day, 1995); 3) Generating a microstructure and projecting this on regular elements, assigning different properties to the elements depending on their position (Schlangen and Van Mier, 1992; Schlangen and Garboczi, 1997); 4) Using a combination of a random geometry and a generated grain structure (Bazant et al, 1990). However, none of these techniques correlate their parameters describing heterogeneity with the practical physical-mechanical parameters of the specific rock. To solve this problem, a heterogeneous material model is proposed in Paper A.

A common building and decorative stone, granite, is used in Paper A as a representative of heterogeneous rock. The specific granite used comes from Umeå in Sweden and is a fine- and even-grained granite with a massive structure. In order to model the mechanical behaviour and the fracture process of the real heterogeneous rock material, integrated image analysis and numerical modelling have been applied fully on-line to simulate the fracture behaviour of the granite specimen: image acquisition, image processing, meshing, and computation of stress, strains and displacements, and crack initiation, propagation, coalescence and interaction. Fig. 1 shows the successive steps of image acquisition with a microscope, automatic image analysis and numerical modelling for the representative volume element (RVE) from the heterogeneous granite specimen. Then the homogenisation theory is used to construct a numerical model to simulate the mechanical behaviour and fracture process under a uniaxial compressive load.

However, homogenisation modelling needs detailed microstructure observation. Moreover, because of limited computer capacity and calculation speed, there is a scale problem when modelling large-scale macroscopic specimens or rock masses in the field. Therefore, maybe it is better to use the statistical method to build a numerical model on the basis of the statistical analysis of microstructures. Fig. 2 shows a comparison between the force-displacement curves resulting from the homogenisation modelling and the statistical modelling. As can be seen from the figure, the results from the homogenisation modelling and the statistical modelling show an almost identical curve in the linear-elastic deformation stage and non-linear-
elastic deformation stage before the peak force. Therefore, the two specimens have almost the same macroscopic elastic modulus and compressive strength.

Fig. 1 Successive steps of the integrated image analysis and numerical modelling

a) Micrograph of a representative volume element (RVE) under a polarized microscope

b) View after image processing, segmentation and polygons

c) Numerical modelling of an RVE built by the R-T2D code
On the basis of the comparison between the homogenisation modelling and the statistical modelling, it is observed that, in order to deal with real random microstructures in numerical simulation, rock heterogeneity can be characterized better with statistical approaches. Here, the Weibull statistical distribution (Weibull, 1951; Hudson and Fairhurst, 1969) is used to characterize the rock heterogeneity. The two-parameter Weibull distribution can be expressed as follows:

\[
p(\sigma) = \begin{cases} 
\frac{m\sigma^{m-1}}{\sigma_0^m} \exp \left[ -\left( \frac{\sigma}{\sigma_0} \right)^m \right], & \sigma \geq 0 \\
0, & \sigma < 0
\end{cases}
\]

where,
- \( \sigma_0 \) = The scale parameter
- \( m \) = The shape parameter

The mathematical expectation and dispersion of the two-parameter Weibull distribution are respectively:

\[
E(\sigma) = \sigma_0 \Gamma\left(1 + \frac{1}{m}\right)
\]

(2)

\[
D(\sigma) = \sigma_0^2 \left[ \Gamma\left(1 + \frac{2}{m}\right) - \Gamma^2\left(1 + \frac{1}{m}\right) \right]
\]

(3)

where \( \Gamma \) is the Gamma function. Fig. 3 shows the relationship between the scaled mathematical expectation \( E(\sigma)/\sigma_0 \), the dispersion \( D(\sigma)/\sigma_0^2 \) and the shape parameter \( m \). As can be seen from the figure, with the shape parameter \( m \) increasing, the scaled mathematical expectation approaches 1, i.e. the scale parameter \( \sigma_0 \) is approximately the mean value of the Weibull distribution. When \( m \) trends to infinity, the mathematical dispersion, i.e. the variance, trends to zero. Therefore, in order to describe the heterogeneity of rock, we can define the shape parameter \( m \) as the homogeneous index and the scale parameter \( \sigma_0 \) as the seed parameter. In numerical simulation, we build a two-dimensional element network model, in which the elemental
parameters, such as the critical strength $\sigma_c$, the elastic modulus $E_c$, etc., follow the Weibull distribution law with the homogeneous index $m$ and seed parameters $\sigma_0, E_0$, etc. Fig. 4 shows the Weibull distribution with the typical homogeneous index $m$ and the seed parameter $\sigma_0 = 200$.

Then a sample space is formed after we specify these elemental parameters following the Weibull distribution. Moreover, even if the parameters of the elements have the same distribution function, the spatial distribution may be different. Rocks consisting of these elements with the same characteristic parameters have almost the same macroscopic mechanical behaviour, e.g. peak strength. However, they may have different failure patterns. The great difference lies in the details of the mesoscopic structure, which represent the disorder of the mesoscopic structure. The special discrete characteristics of heterogeneous rock or rocklike brittle material are exactly reflected by the disorder of the mesoscopic structures. Usually the disorder of the probability distribution in physical space can be achieved by the Monte Carlo method. The simplest method is to generate a series of random data which have a uniform distribution between 0 and 1. Because the Weibull distribution is non-monotone, the integral distribution function is derived as follows:
\[ Q(\sigma) = \int_0^\sigma \varphi(x)dx = 1 - \exp \left[ -\left( \frac{\sigma}{\sigma_0} \right)^m \right] \] 

(4)

The random data between 0 and 1 which are generated by the Monte Carlo method have certain values in the \( y \)-coordinate of the integral distribution function, as shown in Fig. 5. Then the corresponding values in the \( x \)-coordinate can be found, which are the values that we are looking for. Therefore, a series of random data map a series of element parameters, which can be given to elements in a finite element network. The method based on statistics and randomicity satisfies the requirements of heterogeneity and randomicity of element parameters in a finite element network of rock material.

Therefore, on the basis of the Weibull statistical distribution, we characterize the heterogeneity of rock as the following two types of characteristic parameters: (1) the homogeneous index \( m \) and (2) the elemental seed parameters, i.e. the mean value of the main physical-mechanical parameters (critical strength, \( \sigma_0 \), elastic modulus, \( E_0 \), etc) of the elements. For any specific rock, the homogeneous index can be determined on the basis of the defect distribution of the microstructure and the elemental seed parameters can be obtained from laboratory tests.

2.2 Mesoscopic constitutive law, non-linear behaviour and associated seismicity

According to the RFPA model (Tang, 1997), heterogeneity is the source of non-linearity. It has been shown that the global non-linear behaviour observed in brittle rock can be simulated with brittle-elastic elements if heterogeneity is considered (Tang et al, 2000). Therefore, in R-T^2D, linear mesoscopic constitutive laws have been introduced for all the elements, and the elements are given different strength and elastic constant parameters depending on the heterogeneity of the rock materials. This makes the model simple but successful in simulating non-linear rock behaviour and the associated seismic events. During the loading process, the finite element method is used to compute the stress and deformation in each element, and the strength criterion is used to examine whether or not the elements undergo a phase transition. An external load is slowly applied on the constructed model step by step. When in a certain step the stresses in some elements satisfy the strength criterion, the elements are damaged and become weak according to the rules specified by the strength criterion. The stress and deformation distributions throughout the model are then adjusted instantaneously after each rupture to reach the equilibrium state. At positions with an increased stress due to stress redistribution, the stress may exceed the critical value and further ruptures are caused. The process is repeated until no failure elements are present. The external load is then increased further. In this way the system develops a macroscopic fracture. Thus the code links the micromechanical model to the continuum damage model and ultimately to macrostructure failure. Energy is stored in the element during the loading process and is released as acoustic emissions (AE) through the onset of elemental failure. Due to the interaction induced by stress redistribution and long-range deformation, a single important element failure may cause an avalanche of additional failures in neighbouring elements, leading to a chain reaction releasing more energies.

Accordingly, R-T^2D, a Microsoft Windows application, is a package for analysis of the progress of rock failure under mechanical loading. It has been developed by considering the deformation of an elastic material containing an initial random distribution of micro-features. As the load is applied, the fractures will grow, interact and coalesce. As a result, the behaviour becomes non-linear and in addition macroscopic fractures are formed. A user-friendly pre- and post-processor is integrated to generate the finite element mesh and prepare the input data. The analysed results may be displayed graphically as an “animation” to aid users in understanding the mechanics of the failure process.
2.3 Mohr-Coulomb and double elliptic strength criterion

The Mohr-Coulomb criterion and the Hoek-Brown criterion are the two most popular strength criteria in mining and geotechnical engineering. The RPFA model coupled with these criteria has successfully investigated some problems (Tang et al., 1998; Liu, 2000; Fu, 2000; Zhu, 2000; Tang et al., 2000 a-d; Liu et al., 2001 a-c; Tang et al., 2001; Kou et al., 2001; Yang, 2001). However, the traditional Mohr-Coulomb criterion and Hoek-Brown strength criterion are only valid for the brittle part of the rock failure envelope (Verhoef and Ockeloen, 1996; Zhao, 2000). Rock cutting and indentation experiments have shown that very high confining pressures exist near the tips of the tools, maybe with values above the brittle-ductile transition stress known from triaxial tests (Lindqvist and Lai, 1983; Verhoef and Ockeloen, 1996). In this case, a cap strength criterion may be an appropriate choice. In fact, constitutive equations under the category of cap plasticity models have been formulated by numerous authors (Faruque, 1987). However, most of them are based on soil or concrete and may not be applicable to rock.

In the current simulations, as shown in Fig. 6, a double elliptic cap model, proposed by Yu (1998), is adopted. The double elliptic cap model can be expressed in terms of the generalized shear stress \( \tau_g = (\sigma_1 - \sigma_3)/2 \), where \( \sigma_1 \) and \( \sigma_3 \) are the maximum and minimum principal stress respectively, and the generalized normal stress \( \sigma_g = (\sigma_1 + \sigma_3)/2 \) as follows:

\[
\begin{align*}
\sigma_g^2 + 4\tau_g^2 + b\sigma_g &= c \\
\sigma_g^2 + 4\tau_g^2 + b'\sigma_g &= c
\end{align*}
\]

where \( a, b, a', b', \) and \( c \) are constant parameters. The constant parameters can be defined according to the uniaxial tensile strength \( \sigma_t \), the uniaxial compressive strength \( \sigma_c \), the transition point \( (\sigma_g^*, \tau_g^*) \) from brittle to ductile failure and the hydrostatic pressure \( p_0 \), as shown in Fig. 6. Hoek and Brown (1980) chose the relationship \( \sigma_1 = 3.4\sigma_3 \) (\( \tau_g = k\sigma_g \), where \( k = 6/11 \), in a generalized shear and normal stress presentation).
as the best approximation of the brittle-ductile transition. Therefore, the limiting stress conditions can be represented as the following two groups of equations.

\[
\begin{align*}
\sigma_g^2 + 4\tau_g^2 + b\sigma_g &= c, & \tau_g \geq k\sigma_g \\
\sigma_g^2 + 4\tau_g^2 + b\sigma_g &= c, & \tau_g < k\sigma_g \\
\end{align*}
\]

(6)

The brittle failure face of the double elliptic criterion will represent rock failure in the shear mode or tensile mode, just as the modified Mohr-Coulomb (including tensile cut-off) or Hoek-Brown strength criterion at a low confining pressure. The ductile failure surface will represent rock failure in the ductile cataclastic mode at a high confining pressure. The major advantages of the proposed model are that the double elliptic strength criterion avoids the tip angle in the tension area in the linear failure criterion, and the failure criterion and yield cap model have a unified mathematical form, in which the ductile yield face is interrelated with the brittle failure surface.

2.4 Calibration of the R-T2D code by simulating basic rock mechanics experiments

In Section 2.1, heterogeneous rock material has been characterized according to a few characteristic parameters: the elemental seed parameters (the critical strength \(\sigma_0\) or elastic modulus \(E_0\) of mesoscopic elements, etc) and the homogeneous index \(m\). For any specific rock, the seed parameters can be obtained from the main physical-mechanical parameters in laboratory tests and the homogeneous index can be determined on the basis of the defect distribution of the microstructure (Orlovskaja et al, 1997; Curtis and Juszczyk, 1998; Davies, 2001). Therefore, we can use these characteristic parameters as the input parameters of R-T2D to simulate the fracture behaviour of heterogeneous rock materials.

In Paper B, a series of numerical specimens with the same seed parameters – e.g. critical strength (\(\sigma_0 = 200\) MPa) and elastic modulus (\(E_0 = 60\) GPa), etc – but different homogeneous indices, \(m\), which vary between 1.1 and 200, are numerically uniaxially compressed. The geometry of the constructed specimens follows that suggested by ISRM (International Society of Rock Mechanics, 1979) for the uniaxial compressive strength test, i.e. the diameter is 54 mm and the ratio between the height and the diameter is 2.5. Besides the above characteristic parameters, before performing finite element analysis, some other parameters are needed to specify the rock, such as the Poisson’s ratio (0.25) and the ratio (10%) between the tensile strength and the compressive strength of the element.

Fig. 7 Scaled macroscopic strength and elastic modulus corresponding to different homogeneous indices
The simulated macroscopic specimen parameters corresponding to the characteristic parameters of mesoscopic elements are shown in Fig. 7. From the figure, we can observe that the dimensionless uniaxial compressive strength and dimensionless elastic modulus approach 1 with the homogeneous index increasing. In other words, the simulated uniaxial compressive strength and elastic modulus of the specimen approach the specified seed parameters of the elements. As can be seen from the figure, with a high homogeneous index, \( m \) (larger than 200), our heterogeneous model becomes the traditional homogeneous model. If we fit the curves with the appropriate function, we can obtain the following two empirical formulas representing the relationship between the macroscopic parameters (compressive strength, \( \sigma_c \), elastic modulus, \( E_c \)) of the specimen and the characteristic parameters (homogeneous index, \( m \), critical strength, \( \sigma_0 \) and elastic modulus, \( E_0 \)) of the mesoscopic elements:

\[
\frac{\sigma_c}{\sigma_0} = 0.85928 - 0.80668 \exp\left(-\frac{m}{10.68877}\right) \tag{7}
\]

\[
\frac{E_c}{E_0} = 1.02453 - 0.62081 \exp\left(-\frac{m}{2.59074}\right) \tag{8}
\]

where \( \sigma_c \) and \( E_c \) are the simulated uniaxial compressive strength and elastic modulus, respectively, \( \sigma_0 \) and \( E_0 \) are the characteristic compressive strength and characteristic elastic modulus of the mesoscopic elements, and \( m \) is the homogeneous index. The uniaxial compressive strength and elastic modulus corresponding to any specific rock in practice can be obtained from laboratory tests, and the homogeneous index can be determined on the basis of the defect distribution of the microstructure (Orlovskaja et al, 1997; Curtis and Juszczyk, 1998; Davies, 2001). Therefore, we can use these two empirical formulas to determine the input parameters (the characteristic parameters of elements) for the R-T2D software package.

In the laboratory, basic rock mechanics experiments are conducted to obtain the physical-mechanical properties and fracture toughness of rock materials. In Paper B, a great variety of numerical specimens for the uniaxial compressive strength (UCS) test, the Brazilian tensile strength (BTS) test, diametral compression of a notched Brazilian disc (NBD), and the three-point bending (3PB) and four-point shearing (4PS) tests are constructed with the same characteristic parameters as those defined in Section 2.1 to simulate the progressive fracture process and determine the local physical-mechanical properties and fracture toughness. In building these models, the following main characteristic parameters of specimens are used: the homogeneous index is \( m = 2 \), the compressive strength is \( \sigma_c = 180 \) MPa and the elastic modulus is \( E_c = 60 \) GPa. We further assume here that the Poisson’s ratio is 0.25 and a fixed ratio (0.1) exists between the tensile strength and the uniaxial compressive strength of the rocks. On the basis of these parameters, we can use granite as the reference rock, whose main physical-mechanical parameters and fracture toughness are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \sigma_c ) (MPa)</th>
<th>( \sigma_t ) (MPa)</th>
<th>( E ) (GPa)</th>
<th>( K_{IC} ) (Mpa.m(^{1/2}))</th>
<th>( K_{IIC} ) (Mpa.m(^{1/2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Granite</td>
<td>175</td>
<td>15.6</td>
<td>62.5</td>
<td>1.72</td>
<td>1.75</td>
</tr>
</tbody>
</table>

\( ^a \) Newhurst granite in Whittaker et al (1992)
Table 2 Final failure modes simulated by the R-T $^{2D}$ code and a comparison with the experimental (or theoretical) results

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Numerically Simulated Results</th>
<th>Experimental (or Theoretical) Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
</tr>
<tr>
<td>BTS</td>
<td><img src="image3" alt="Image" /> <img src="image4" alt="Image" /> <img src="image5" alt="Image" /> <img src="image6" alt="Image" /> <img src="image7" alt="Image" /> <img src="image8" alt="Image" /> <img src="image9" alt="Image" /> <img src="image10" alt="Image" /></td>
<td><img src="image11" alt="Image" /> <img src="image12" alt="Image" /> <img src="image13" alt="Image" /> <img src="image14" alt="Image" /> <img src="image15" alt="Image" /> <img src="image16" alt="Image" /> <img src="image17" alt="Image" /> <img src="image18" alt="Image" /></td>
</tr>
<tr>
<td>NBD-I</td>
<td><img src="image19" alt="Image" /> <img src="image20" alt="Image" /> <img src="image21" alt="Image" /> <img src="image22" alt="Image" /> <img src="image23" alt="Image" /> <img src="image24" alt="Image" /> <img src="image25" alt="Image" /> <img src="image26" alt="Image" /></td>
<td><img src="image27" alt="Image" /> <img src="image28" alt="Image" /> <img src="image29" alt="Image" /> <img src="image30" alt="Image" /> <img src="image31" alt="Image" /> <img src="image32" alt="Image" /> <img src="image33" alt="Image" /> <img src="image34" alt="Image" /></td>
</tr>
<tr>
<td>NBD-II</td>
<td><img src="image35" alt="Image" /> <img src="image36" alt="Image" /> <img src="image37" alt="Image" /> <img src="image38" alt="Image" /> <img src="image39" alt="Image" /> <img src="image40" alt="Image" /> <img src="image41" alt="Image" /> <img src="image42" alt="Image" /></td>
<td><img src="image43" alt="Image" /> <img src="image44" alt="Image" /> <img src="image45" alt="Image" /> <img src="image46" alt="Image" /> <img src="image47" alt="Image" /> <img src="image48" alt="Image" /> <img src="image49" alt="Image" /> <img src="image50" alt="Image" /></td>
</tr>
</tbody>
</table>

(Gomez et al, 2001)

(Jia et al, 1996)

(Chen et al, 1998)
Table 2 (Continued)

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Numerically Simulated Results</th>
<th>Experimental (or Theoretical) Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>NBD-Mixed</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /> (Landis, 1999)</td>
</tr>
<tr>
<td>S3PB</td>
<td><img src="image3" alt="Image" /></td>
<td><img src="image4" alt="Image" /> (Landis, 1999)</td>
</tr>
<tr>
<td>4PB</td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /> (Rao, 1999)</td>
</tr>
<tr>
<td>A3PB</td>
<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /> (Xeidakis, 1997)</td>
</tr>
</tbody>
</table>

Table 2 shows the final failure modes in the various test techniques and a comparison between the numerically simulated results and the experimental (or theoretical) results. To investigate the fracture problem, fracture mechanics needs a well pre-fabricated notch. There is no need for a pre-fabricated notch in the R-T²D code, although it also works very well when there is a well pre-fabricated notch. Rock heterogeneity acts as a stress concentrator under loading. A crack is created at a point where the principal stresses reach the tensile strength or the compressive strength. Therefore, the code can be applied to un-notched test geometries such as the geometry of the uniaxial compressive test, the Brazilian test and so on. Of course, it also works perfectly when there is a pre-fabricated notch. In Paper B we used it to study the diametral compression of a notched Brazilian disc (NBD) specimen, the bending and shearing loading of a notched beam. The detailed visually shown stress distribution and redistribution, crack nucleation and initiation, stable and unstable crack propagation, interaction and coalescence, and corresponding load-
displacement curves are found to be in good agreement with experimental results in the literature, and can be proposed as benchmarks for numerical programs for crack propagation.

Table 3 Simulated physical-mechanical parameters and fracture toughness

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Tests</th>
<th>$\sigma_c$ (MPa)</th>
<th>$\sigma_t$ (MPa)</th>
<th>$E$ (GPa)</th>
<th>$K_{IC}$ (Mpa.m$^{1/2}$)</th>
<th>$K_{II}$ (Mpa.m$^{1/2}$)</th>
<th>Mixed-mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>UCS</td>
<td>183.5</td>
<td>59.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>BTS</td>
<td>18.7</td>
<td>1.74</td>
<td>1.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>NBD-I</td>
<td>1.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>NBD-II</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.78</td>
</tr>
<tr>
<td></td>
<td>NBD-Mixed</td>
<td>1.68</td>
<td>2.72</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3PB-I</td>
<td>3.74</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.11</td>
</tr>
<tr>
<td></td>
<td>4PB-II</td>
<td>2.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>3PB-Mixed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3 summarizes the simulated physical-mechanical parameters and fracture toughness for the numerical specimens with the same characteristic parameters. The results obtained using the different methods can be compared with each other. For example, the ratio between the uniaxial compressive strength and the tensile strength is $\sigma_c/\sigma_t = 183.5/18.7 = 9.8$, which is consistent with the statement that the ratio varies between 8 and 12 for most rocks. The mode-I fracture toughness values obtained from the BTS test (1.74), the NBD test in pure mode I (1.70) and the NBD test in mixed mode I-II (1.68) are close to each other. The ratio between the mode-II fracture toughness and the mode-I fracture toughness is 2.78/1.70 = 1.64, which is consistent with most of the experimental results, which show that the pure mode-II fracture toughness is usually larger than the pure mode-I fracture toughness (Ingraffea, 1981; Whittaker et al, 1992; Al-Shayea et al, 2000). The slight difference was caused by the spatial distribution of heterogeneity, and the effects of size, shape, thickness, notch length and so on. Moreover, most of the simulated physical-mechanical parameters and fracture toughness values are close to those of the reference granite, whose physical-mechanical properties and fracture toughness were previously shown in Table 1. On the basis of the different test methods and a large number of obtained data, we can now be confident that rock can be specified according to the characteristic parameters, the built heterogeneous model is reasonable and the developed R-T$^{2D}$ code is a valuable numerical tool for research on the progressive fracture process of heterogeneous material. Besides, from Table 3, we have observed that the fracture toughness data from diametral compression of the NBD specimen show little variance compared with those from the bending and shearing tests. Therefore, taking the size effects, simple configuration and the loading procedure into consideration, the disc type specimen may be more useful in measuring fracture toughness.
3 Preliminary application of the R-T\textsuperscript{2D} code in mechanical fragmentation

Mechanical fragmentation is the preferred technique of effective rock removal because of the continuity of the process, the directly fragmenting rock mode involved and the smaller energy loss incurred. The technique is employed in one way or another in most of today’s mining and tunnelling machines. In Paper C and D, cutting and indentation, the fundamental mechanisms in mechanical fragmentation, have been investigated using the R-T\textsuperscript{2D} code.

3.1 Numerical modelling of the fracture process in cutting heterogeneous brittle rock

Numerical modelling of the fracture process in cutting heterogeneous brittle rock is conducted in Paper C. Here, just the results are reported.

3.1.1 Numerical model

Rock cutting is simplified to a two-dimensional plane strain problem, as shown in Fig. 8. The gray scales represent different values of Young's modulus. The brighter the colour, the higher is the value of the Young's modulus of the element. The steel cutter is modelled as a quasi-isotropic homogeneous material with a high strength compared with the rock, whereas the rock is modelled as a heterogeneous material. An external displacement in the horizontal direction (0.005 mm/step) is applied to the steel cutter from the left side, and the upper side is kept at the same horizontal level without any movement in the vertical direction. The whole model is divided into 250 × 125 (= 31,250) mesoscopic finite elements.

3.1.2 Quasi-photoelastic stress fringe pattern

In the initial stage of cutting rock, the rock is deformed as an elastic body. Therefore, before performing failure analysis, the quasi-photoelastic stress fringe patterns induced in the cutter and homogeneous and heterogeneous rocks are simulated, as shown in Fig. 9. It is apparent that the stress fields in rock consist of three zones: the interaction zone, the confining pressure zone and the zone outside the confining pressure zone. The interaction zone is caused by a mismatch between the elastic moduli of the rock and cutter. The rock in the confining pressure zone is generally in a state of rather high tri-axial compression. In the post-failure analyses, it is found that the confining pressure zone defines, to a great extent, the shape and size of
the so-called crushed zone. The stress contours in the cutter, which are caused by the reaction from the rock, are also clearly shown in the fringe patterns. The high stress concentration in the corners often causes tools to break or wear. The stress distribution coupled with the heterogeneity will determine the sites of crack initiation and the directions of crack propagation in the following failure process.

3.1.3 Chipping process

Fig. 10 shows the development of the chipping process and the progressive evolution of the stress fields as the cutting displacement increases. When the cutter is applied on the rock, a high stress concentration is induced in the rock near the cutter, and the rock far from the cutter is almost unaffected. As shown in Fig. 10 (a), cracks sprout first at the lower edge of the cutter and propagate, dipping into the rock at approximately 45° in the direction of the cutter displacement. The mechanism of the calculated cracks is tensile failure. That is because the tensile strength of brittle material is far lower than the compressive strength. Ahead of the cutter, the rock is highly stressed and it is in a triaxial state due to the high confining pressure. Therefore, the rock in this zone does not fail at the initial stage of cutting. As the cutter displacement increases, tensile cracks sprout at the upper edges of the cutter, and at the same time the cracks initiated from the lower edge of the cutter propagate downward, as shown in Fig. 10 (b). With continuous loading displacement, compressive failure occurs ahead of the confining pressure zone, as shown in Fig. 10 (c, d). With the tensile failure and compressive failure releasing the confinement for the tri-axial compressive zone, the rock in the initial confining pressure zone is compressed into failure and the crushed zone gradually comes into being with a dimension similar to that of the cutter-rock interface, as shown in Fig. 10 (e). According to experimental results (Verhoef and Ockeloen, 1996), due to the very high confining pressure, the stresses of the elements in the confining zone may fail in the ductile cataclastic mode with the stresses satisfying the ductile face of the double elliptic strength criterion. However, in the present simulation, the stresses of few elements satisfy the ductile failure surface of the double elliptic strength criterion. Maybe this is because the depth of the cut is not great enough or there is no confinement on the rock. From the simulated results, it is found that in fact the crushed zone is the zone with a high density of microcracks, and it is not necessary to consider the crushed zone as some separate body which interacts with the tool and the rock massif. This kind of investigation has also been conducted by other researchers (Kutuzov et al, 1969; Simonov and Vyskrebtsov, 1975; Mishnaevsky, 1999).

The crushed zone has an important influence on the subsequent development of the crack system, which changes the transferring direction of the force applied by the cutter. As shown in Fig. 10 (a-d), before the formation of the crushed zone, all of the cracks show a tendency to dip down into the rock. Associated with
Fig. 10 Fracture process in cutting (maximum principal stress distribution)
the crushed zone there must be a volumetric expansion and a tensile stress field, which result in tensile crack propagation and the formation of chipping cracks. Therefore, after the formation of the crushed zone, two main categories of cracks, chipping cracks and subsurface cracks, come into being, as shown in Fig. 10 (e). As the displacement increases, one of the chipping cracks rapidly intersects with the free surface and small pieces of rock are chipped, as shown in Fig. 10 (f). At the same time, the other main chipping crack propagates in a curvilinear path but approximately parallel to the free surface of the rock. The subsurface cracks propagate in a curvilinear path to dip into the rock at a certain angle in the direction of the cutter. It is thought that the curvilinear path is caused by the heterogeneity of the rock. The mechanism of the chipping crack is tensile failure mainly induced by tensile stress associated with the crushed zone. With the continuous increase in the loading displacement, the main chipping crack, which is driven by tensile stress, propagates and bifurcates as shown in Fig. 10 (g). As the loading displacement increases, the bifurcated chipping cracks propagate and interact as shown in Fig. 10 (h). At the same time, more and more cracks form around the crushed zone, which makes the crushed zone enlarge and its boundary become vague. With a continuous increase in the loading displacement, the bifurcated chipping cracks coalesce and form a single major chipping crack again, as shown in Fig. 10 (i). As the chipping cracks propagate and intersect with the free surface, the major chip is expected to come into being as shown in Fig. 10 (j). The cracks which propagate to dip into the rock form the subsurface cracks after the formation of the major chip.

According to the simulated results, when cutting heterogeneous brittle material, the process of chip formation is caused by a complicated stress state, with the elements mainly failing in a tensile mode or a mixed shear and tensile mode and sub-fracture occurring in a dominant tensile mode. The subsurface fractures are actually the bifurcation of some major tensile fracture. Therefore, in cutting brittle materials, as differentiated from cutting plastic materials, the mechanism of chip removal is not shear, but tensile or mixed-mode crack growth. Similar phenomena were also noted by Mishnaevsky (1998).

![Graphs showing load-displacement and AE-displacement](image-url)

**Fig. 11** Load-displacement curve and associated seismicity (letters corresponding to plates in Fig. 10)**
3.1.4 Force-loading displacement response and associated seismicity

The load displacement response, as shown in Fig. 11 (i), is actually the reflection of the fractures ahead of the cutter. On the basis of the assumption that there is consistency between the damage and the seismicity associated with the fracture process, the acoustic emission (AE) phenomenon during the fracture process is also observed, as shown in Fig. 11 (ii). From the two curves, we know that the load displacement response is therefore closely related to the fractures in the rock induced by the cutter. This can be made clear when we compare Fig. 11 (i, ii) with Fig. 10. The increasing load at the very beginning causes quasi-elastic deformation of the rock surface, as shown in Fig. 10 and 11 marked by (a). At this stage, the load displacement relation is quasi-linear and, correspondingly, there is almost no acoustic emission in the rock, except for that caused by a very small amount of damage occurring. This quasi-elastic deformation is then followed by the deformation of the crushed zone and cracked zones. At this stage, the movement of the rock fragments in the crushed zone is restrained by the surrounding intact rock. In order to break the rock, continuous loading is required. This would cause intense comminution of the rock fragments if the confinement were sufficiently high. However, in the present case the moving tendency of the rock fragments in the crushed and cracked zone pushes the surrounding rock upwards, due to a smaller confinement on the upper side, and results in chipping cracks. During the process of rock fracturing the cutter gradually penetrates the rock. When the chipping cracks reach the rock surface, the surrounding rock, which offered the upper confinement at the earlier stage, together with part of the rock underneath the cutter, moves away forming the chips. The movement of the cutter meets less resistance in this case. The force acting on the cutter is thus reduced to a rather low value, as shown in Fig. 11, and the cutter penetrates the rock to a greater depth with a decreasing load. After that the rock fragments can be removed with a very small load.

It is clear from the curve of the AE counts that this load drop is caused by a sudden increase in the AE counts (or the number of failed elements). Generally, every big increase in the AE counts results in a big drop. The load-displacement curve for the cutting process obtained from the simulation shows that the simulated chipping process is similar to the chipping process found through experimental observation of brittle cutting (Deketh et al, 1998). It is also somewhat similar to the results obtained from indentation experiments carried out in the laboratory (Kou et al, 1996). In addition, from the above-described simulation, it is easy to understand that the cutting depth is mainly dependent on the load magnitude, the macro-mechanical properties of the rock, and the heterogeneity of the rock. However, it is also influenced by the cutter geometry, which characterises the load distribution.

3.1.5 Results for rock fragmentation by cutters with different back rake angles

The stress fringe patterns induced by cutters with different back rake angles (20°, 0° and -20°) are shown in Fig. 12. As can be seen in the fringe patterns, the far field stress distributions are almost the same for the three cutters. This mechanical behaviour is in accordance with the well-known Saint Venant principle. However, each of the three cutters exhibits a different stress distribution in an area immediately adjacent to the cutter. Finally, this difference in the stress distribution causes a completely different cutting efficiency. This further indicates the importance of stress redistribution during the failure process in cutting and the importance of the crushed zone.

Fig. 13 shows the final failure patterns induced by cutters with different rake angles. Fig. 14 shows the corresponding elastic energy release (ENR) and accumulated ENR in the crushed zones and the whole specimens. According to the load-displacement curves (Fig. 11), the work necessary for fragmenting the rock can be calculated for the three different rake angles. According to the elastic energy release (ENR-) displacement curves (Fig. 14), the useful energy can be calculated for the three different rake angles.
According to the ENR-displacement curves in the crushed zone (Fig. 14), the energy consumed by the formation of the crushed zone can be calculated. From the final fracture pictures (Fig. 13), the area of rock spalled by the chipping crack can be calculated. According to the simulated results, the energy utilisation ratios are 14%, 18% and 21% respectively for the $20^\circ$, $0^\circ$ and $-20^\circ$ rake angles, which are greater than the experimental observation results, because the influences of the loading rate, friction, temperature, etc. are not considered. The energies consumed by the formation of the crushed zone are 41%, 71% and 63% of the useful energies respectively, which are close to the experimental observation results. The specific energies for chipping are 199, 123 and 49 kJ/m$^3$ respectively for the three cutters with rake angles of $20^\circ$, $0^\circ$ and $-20^\circ$. When comparing the three different rake angles, it is reasonable to conclude that a rock cutter with a rake angle of $-20$ degrees is the most efficient of the three cutters. This is a reasonable conclusion because the wedging action of sharp cutters produces tensile stresses in the rock in a fairly direct manner. This advantage of a sharp cutter begins to disappear, however, as the geometry of these tools changes from a sharp to a blunt shape because of friction. For applications that require the machining of stronger rocks, it is often necessary to increase the wedge angle of a drag bit in order to increase the tool strength.

Fig. 12 Quasi-photoelastic stress fringe patterns induced by cutters with different rake angles
Fig. 13 Failure patterns with different rake angles

(i) Rake angle: 20 degrees
(ii) Rake angle: 0 degrees
(iii) Rake angle: -20 degrees

Fig. 14 Elastic energy release (ENR), accumulated ENR and displacement curve

i. ENR-displacement in whole specimen (Rake angle: $20^\circ$)
Fig. 14 (Continued)

ii. ENR-displacement in crushed zone (Rake angle: 20°)

iii. ENR-displacement in whole specimen (Rake angle: 0°)

iv. ENR-displacement in crushed zone (Rake angle: 0°)

v. ENR-displacement in crushed zone (Rake angle: -20°)
3.2 Numerical modelling of the rock fragmentation process induced by indenters

The rock fragmentation process induced by a single indenter and multiple indenters is investigated in Paper D. Here, just the results are reported.

Fig. 14 (Continued)

Fig. 15 Numerical simulation models: a) Single indenter, b) Double indenters
3.2.1 Numerical model

The indentation problem is simplified to a plain strain condition. A plane passing through the central axis of the indenter is considered, as shown in Fig. 15 (a) and (b). The grey scales represent different values of Young’s modulus. The brighter the colour, the higher is the value of the Young’s modulus of the element. The indenter is simulated as a homogeneous material whose elastic modulus is a few times higher than that of rock in order to safeguard against permanent deformation of the indenter. The rock specimen is considered as a heterogeneous material. Both the indenter and the rock are divided into many mesoscopic elements. In the case of the single indenter, the model is discretized into 150*250 = 37,500 elements. In the case of simultaneous loading with double indenters, the model is divided into 150*300 = 45,000 elements. A displacement increment (0.005 mm/step) is applied on the indenters and a confining pressure (20 MPa) acts on both the lateral sides of the constructed rock specimen.

3.2.2 Quasi-photoelastic stress fringe pattern

The first requirement of any soundly-based simulation of indentation fracture is a detailed knowledge of the stress field within the loaded system. Our foremost aim here is to investigate the distribution of the elastic stress component primarily responsible for the operation of fracture processes. At the same time, it is also important that we should pay special attention to the distribution of the hydrostatic or high confining pressure zone, for this zone will determine the extent of the irreversible deformation within the field. The solutions for the stress field arising when an isotropic, linear-elastic half-space is subjected to a normal point load were first given in 1885 by Boussinesq, who invented the famous Boussinesq elastic field. The stress field arising when an isotropic, linear-elastic half-space is subjected to normal loading by a smooth spherical indenter is called the ideal Hertzian elastic field. Fig. 16 shows the simulated quasi-photoelastic stress fringe pattern induced by a single indenter when the rock is considered as homogeneous material. According to the simulated results, the indentation stress field is not uniform. The stresses are extremely large close to the loading point and decrease rapidly with increasing distance from the loading point. The stress distributions are symmetrical about the axis. A very high stress field is induced in the regions both immediately underneath the punch and on either side of the punch. Lindqvist (1982) analyzed the one-point stress field presented by Lawn and Swain (1975) and used it to predict the cone crack and median crack. However, Lindqvist (1975) pointed out that the Boussinesq elastic stress field gives no evidence of the evolution of tensile surface (lateral) cracks creating the indentation crater, which is considered to be responsible for rock fragmentation in indentation. Therefore, it is necessary for us to perform failure analysis in the following.

Fig. 16 Simulated quasi-photoelastic stress fringe pattern caused by a truncated indenter
Fig. 17 Simulated results for the rock fragmentation process induced by a single indenter (Stress distribution)
3.2.3 Chipping process

When the indenter acts on the rock, a high stress zone, which corresponds to the highlight zone in Fig. 17, comes into being immediately beneath the indenter. A fan-shaped stress field is radiated outside the highly stressed zone. Far from the highly stressed zone, the stress field is like water-waves due to the heterogeneity of the rock and the confining pressure, as shown in Fig. 17 (A). As the stress intensity builds up with an increasing load, one or more of the flaws nucleates a crack around the two corners of the truncated indenter. It is interesting to find that, although the rock immediately beneath the indenter is highly stressed, it does not fail primarily because of the high confining pressure. On the contrary, cracks initiate first on both corners of the truncated indenter to form cone cracks. The cone cracks lose their symmetrical shapes because of the rock heterogeneity, as shown in Fig. 17 (B). With the loading displacement increasing, the cone cracks driven by tensile stresses run downward along the stress trajectories of the maximum principal stresses in the well-known conical Hertzian mode. At the same time, due to increasing stress, the elements immediately beneath the indenter fail. Some of them fail, even if there is a high confining pressure, in the ductile catastatic mode with the stresses satisfying the ductile failure surface of the double elliptic strength criterion. Others are compressed into failure because the formation of cone cracks and ductile catastatic failure release the confining pressure, as shown in Fig. 17 (C). The crushed zone gradually comes into being as the elements in the high confining pressure zone fail. The formation of the crushed zone has an important influence on the direction of the cracks. Before the formation of the crushed zone, all of the cracks propagate downwards, which is expected to form cone cracks or subsurface cracks, as shown in Fig. 17 (B, C). During the formation of the crushed zone, some cracks bifurcated from the cone cracks propagate almost parallel to the free surface and some new cracks are initiated from the crushed zone, as shown in Fig. 17 (D). This is because the crushed zone has a changeable shape and volume. Yoffe (1982) provided a blister model to explain this behaviour. With the loading displacement increasing, a re-compaction of the crushed zone occurs. During the re-compacting process, the crushed zone expands to both sides, which drives the cracks initiated from the crushed zone or bifurcated from the cone cracks to propagate approximately parallel to the free surface expected to form side cracks. Because of the heterogeneity, the side cracks on both sides have no symmetric shapes and propagate in a curvilinear path. In this simulation, the propagating velocity of the side cracks on the right side is faster than that on the left side, as shown in Fig. 17 (E). As the loading displacement increases, the side cracks on the left side, which are dormant at first, begin to propagate forward. At the same time, the side cracks on the right side propagate stably, as shown in Fig. 17 (F). With the loading displacement increasing, the side cracks on both sides of the indenter propagate stably and almost parallel to the free surface, but in a curvilinear path. In addition, some cracks propagate downwards at an angle of about 45 degrees to form cone cracks. Some bifurcated cracks propagate downwards and are expected to form subsurface cracks, as shown in Fig. 17 (G). As the penetration displacement increases, the side cracks on both sides of the indenter propagate stably to form almost symmetrical shapes. At the same time, some discrete cracks initiate under the crushed zone and are expected to form median cracks, as shown in Fig. 17 (H). With the loading displacement increasing, the side crack on the right side of the indenter accelerates and propagates unstably to form chips, as shown in Fig. 17 (I). As the loading displacement increases, the side crack on the left side of the indenter is expected to form chips also. At the same time, more discrete cracks initiate under the indenter. Some of them coalesce to form subsurface cracks, as shown in Fig. 17 (J).

3.2.4 Force-penetration curve and associated seismicity

Fig. 18 shows the simulated force-penetration curve and associated seismicity during the rock fragmentation process induced by a single indenter. The lateral pressure gradually acts on both boundaries.
of the rock specimen. At point $A_1$ in the force-penetration curve, the lateral pressure attains its maximum value: 20 MPa. The letters A-J correspond to the slides marked with the letters A-J in Fig. 17. The associated seismicity, which is usually called the acoustic emission (AE) phenomenon in rock mechanics, is obtained by recording the counts of the failure elements. From the picture it can be seen that, although there are microcracks, as shown by the AE counts, the force-penetration curve has almost a linear shape in the initial loading stage, i.e. the curve between point ($A_1$) and point (A), which is the linear-elastic deformation stage of the load displacement curve. As shown in Fig. 17 (A), little damage to the rock occurs until this point. With the loading displacement increasing, the force-penetration curve attains its first peak value: point (B). As shown in Fig. 17 (B), at this point a conical Hertzian crack is initiated at the punch corners. As the elements immediately beneath the indenter fail, the force-penetration curve falls off to its first trough at point (C), where the crushed zone comes into being. As the loading displacement increases, a re-compaction behaviour occurs in the crushed zone immediately beneath the indenter, and the loading displacement curve climbs up. As shown in Fig. 18 from point (D) to point (I), small corrugations are the main characteristic of the force-penetration curve. The small corrugated variations in the force, which occur prior to a major breakout (chipping), are indications of the propagation of cracks, the crushing of microstructural grains and the formation of small chips. The large drop in the force occurs after point (I). At this time, a major chip forms on the right side of the indenter, which indicates that a substantial part of the rock has now been separated or unloaded. Further penetration by the indenter repeats the peak-trough behaviour in a cyclic pattern as shown in Fig. 18 between point (I) and point (J). Another major chip on the left side of the indenter is expected to form. Accompanying the major chip, there is also a large acoustic emission (AE) count. As shown in Fig. 18, the small corrugations and large peak-trough are the two main features of the force-penetration curve. The peak force for the next chip is normally greater, due to an increase in both the depth of penetration and the contact area of indentation. It is found that this corrugated peak-trough (saw-tooth) behaviour is typical for most indenter types.

Fig. 18 Force-penetration curve during rock fragmentation by single indenter

3.2.5 Comparison between the numerical and the experimental results

Fig. 19 shows a comparison between the experimental and the numerically simulated results. After the comparison, a general picture of fractures in rock under indentation can be summarized. Underneath the indenter there is a crater, a disintegrated and sometimes partly re-compacted zone, a crushed but basically confined zone, and a cracked zone. Outside the cracked zone there are mainly three kinds of cracks: median cracks, radial cracks, and side cracks according to their positions related to the indenter. The crater is formed due to tool penetration and rock removal. The re-compacted zone lies immediately under the
indenter and is caused by the re-compaction of the crushed rock. The crushed zone is in fact the zone with a high density of microcracks, which make the boundary between the crushed zone and the crack zone become vague. The median cracks and radial cracks are not obvious in some cases. Side cracks are initiated in the vicinity of the crushed zone or bifurcated from the cone cracks and propagate nearly parallel to the free surface, and side cracks are mainly responsible for rock fragmentation in indentation.

### 3.2.6 Results for rock fragmentation induced by double indenters

Fig. 20 (A) shows the simulated quasi-photoelastic stress fringe pattern induced by double indenters when rock is regarded as a homogeneous material. Comparing Fig. 16 with Fig. 20, we will find that there are some new features in the stress contours induced by double indenters. According to the simulated elastic stress field caused by a single indenter as shown in Fig. 16, this kind of stress field is expected to form cone cracks or subsurface cracks. However, there is no evidence of the formation of side cracks, which are considered to be responsible for the formation of chipping in the rock fragmentation process in indentation.

As shown in Fig. 20 (A), the directions of the maximum principal stresses seem to be influenced to a small extent immediately near the indenters. The interaction of the principal stress induced by each of the two indenters is clear between the two indenters. The stress trajectories change their directions in the rock between the indenters. Rock chips are expected to form between the two indenters. With the distance from the indenters increasing, the stresses slowly converge to a combined field. In the far region the stress field appears to originate from a load distributed over the area between the two indenters, giving stress directions similar to the stress field induced by a single indenter, as shown in Fig. 16.
Fig. 21 shows the rock fracture process induced by double indenters. At the first stage of loading, the respective stress fields induced by the double indenters are equal to those induced by the single indenter. The rocks immediately under the indenters are highly stressed. The stresses decrease rapidly with increasing distance from the loading points. However, with increasing loading displacement, the interference of the two stress fields induced by the two indenters is more and more obvious in the rock between the two indenters, as shown in Fig. 21 (A). Cone cracks are initiated first around the two corners of both indenters, as shown in Fig. 21 (B). As shown in Fig. 21 (C), before the formation of the crushed zone, the cone cracks bifurcate and propagate approximately parallel to the free surface expected to form side cracks. Because of the heterogeneity, the crack systems under the two indenters are not completely identical. However, similarities exist between the two crack systems. As the loading displacement increases, the crushed zones gradually come into being immediately under the indenter, and the side cracks are driven by tensile stresses associated with the expansions of the crushed zone, as shown in Fig. 8 (D, E). With the loading displacement increasing, more and more side cracks are initiated from the two crushed zones, as shown in Fig. 21 (F). With a continuous loading displacement, the side cracks propagate stably forward and the subsurface cracks are richly developed under the indenters. Because of the heterogeneity, the side cracks propagate in a curvilinear path. As shown in Fig. 21 (G), the interactions between the side cracks are more and more obvious as the tips of the side cracks become closer and closer. With the increasing loading displacement, the side cracks coalesce in a complicated manner and the rock between the two indenters is chipped, as shown in Fig. 21 (H). The chipped rock has a complicated geometrical shape. As the loading displacement increases, the crack systems are well developed under the two indenters, as shown in Fig. 21 (I). More cracks are initiated from the crushed zone. Some of them propagate to form side cracks and some of them dip into the rock forming subsurface cracks. At the same time, some discrete cracks are initiated under the crushed zones to form radial and median cracks. With increasing loading displacement, more and more side cracks interact and coalesce. Therefore, after the major chips, some small chips occur as shown in Fig. 21 (J).
Fig. 21 Simulated result for the rock fragmentation process induced by double indenters (stress distribution)
Fig. 22 shows the recorded force-penetration curve associated with the fragmentation process induced by double indenters, and this curve is similar to that for the fragmentation process induced by a single indenter. This means that almost the same amount of energy is consumed. However, the chipped rock volume is larger than that in the fragmentation induced by a single indenter. Therefore, simultaneous loading of the rock surface by multiple indenters with an appropriate line spacing seems to provide a possibility of forming larger rock chips, controlling the direction of subsurface cracks and consuming a minimum total specific energy.

![Force-penetration curve during rock fragmentation by double indenters](image-url)
4 Discussions

4.1 Influence of heterogeneity on rock behaviour

According to the proposed heterogeneous model, the macro specimen parameters and the elemental seed parameters have been correlated with the homogeneous index through Eqs. 7 and 8. We have defined the homogeneous index and the elemental seed parameters as characteristic parameters. In loading numerical specimens with the same characteristic parameters using the same testing method, similar resultant stress-strain curves are observed. However, the final fracture patterns may be different depending on the spatial random distribution. For any specific rock, we can conduct a UCS and a BTS test to obtain the uniaxial compressive strength, elastic modulus and so on. The homogeneous index can be determined on the basis of the defect distribution of the microstructure. Therefore, the proposed heterogeneous model is reasonable and applicable.

![Stress-displacement curve](image)

Fig. 23 Stress-displacement curve induced in specimens with the same mean values but different homogeneity

In loading a numerical specimen with a lower homogeneous index, there is a big difference between the macro parameters and the elemental seed parameters. This is reasonable, since elements with a lower strength will fail earlier during loading. The resultant stress-strain curve has a typical linear-elastic deformation stage, a non-linear deformation stage, a post-failure stage and a residual strength stage, as
shown in Fig. 23. The higher the value of the homogeneous index is, the higher are the macro parameters of the specimen. When the homogeneous index is very high (larger than 200), the heterogeneous model will become the traditional homogeneous material model. As a result, the curve becomes more linear, the strength loss is sharper and no obvious nucleation sites of failure events are observed. Therefore, the R-T\textsuperscript{2D} code, through the inclusion of rock heterogeneity, can simulate the complete stress-strain curve and associated seismicity.

In a series of numerical experiments conducted in Paper B, corresponding to every heterogeneous specimen, a homogeneous specimen with the same geometry is built to obtain the quasi-photoelastic fringe patterns. The stress distribution and redistribution in the heterogeneous specimen are shown in the subsequent fracture process analysis. It is found that rock heterogeneity has a strong influence on the stress distribution and re-distribution, which results in a tortuous crack propagation path. In front of the crack propagation direction, if the strength of the element is not so high, the crack will propagate straight, which corresponds to the transgranular failure observed with SEM in experiments. If the strength of the elements is very high, the crack will propagate round elements with a high strength and a tortuous crack propagation path comes into being, which can be considered as the intergranular failure observed with SEM in experiments. Therefore, the R-T\textsuperscript{2D} code can simulate the tortuous crack propagation path which is caused by rock heterogeneity.

4.2 Physical-mechanical properties and fracture toughness

Rock failure results from the propagation of one or more cracks. In theoretical studies and engineering applications, rock failure is investigated from two points of view: strength theory and fracture mechanics. It has long been known that there is a quantitative relationship between the physical-mechanical parameters and fracture toughness of rock. For example, Whittaker et al (1992) obtained some approximate relations between fracture toughness and tensile strength, compressive strength, point load strength, hardness, and the velocity of acoustic waves in rock on the basis of experimental data from many references. However, those formulas have very low regression coefficients. Recently, Zhang (2002) obtained an empirical relation between the mode-I fracture toughness and the tensile strength of rock with a higher regression coefficient. However, this kind of relation does not help us understand the mechanism occurring in nature.

Usually the notched specimen is studied through the principles of fracture mechanics and the intact rock specimen is often studied using conventional rock mechanics principles. In the series of numerical experiments mentioned above, we have simulated the fracture processes of both intact rock and notched rock, although the R-T\textsuperscript{2D} code uses characterized physical-mechanical parameters as input parameters. It is found that R-T\textsuperscript{2D} can also simulate very well the fracture toughness and the progressive fracture process of the notched specimen in terms of fracture mechanics. The simulated physical-mechanical parameters and fracture toughness for the numerical specimens with the same characteristic parameters in Paper B are summarized in Table 3. The numerical results obtained using the different methods are comparable with each other. For example, the ratio between the uniaxial compressive strength and the tensile strength is $\sigma_c/\sigma_t = 183.5/18.7 = 9.8$, which is consistent with the statement that the ratio varies between 8 and 12 for most rocks. The mode-I fracture toughness obtained from the BTS test is 1.74, that from the NBD test in pure mode I is 1.70, and that from the NBD test in mixed mode I-II is 1.68. The ratio between the mode-II fracture toughness and the mode-I fracture toughness is 2.78/1.70 = 1.64, which is consistent with most of the experimental results, which indicate that the pure mode-II fracture toughness is usually larger than the pure mode-I fracture toughness (Whittaker et al, 1992; Al-Shayea et al, 2000; Ingraffea, 1981). The slight difference was caused by the spatial distribution of heterogeneity, and the effects of size, shape, thickness,
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Notch length and so on. Moreover, most of the simulated physical-mechanical parameters and fracture toughness values are close to those of the reference granite, whose physical-mechanical properties and fracture toughness were previously shown in Table 1. Besides, the comparison between the results from diametral compression of NBD specimens and those from the bending and shearing tests shows that the disc type specimens are more suitable for measuring the fracture toughness of rock.

Table 4 Predicted results for fracture toughness based on physical-mechanical properties

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values from relationships</th>
<th>Adopted Equations$^a$</th>
<th>Input values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{IC}$ vs $\sigma_c$</td>
<td>$K_{IC} = 1.81$</td>
<td>$K_{IC} = 0.708 + 0.006\sigma_c$</td>
<td>$\sigma_c = 183.5$</td>
</tr>
<tr>
<td>$K_{IC}$ vs $\sigma_t$</td>
<td>$K_{IC} = 1.03$</td>
<td>$K_{IC} = 0.114 + 0.005\sigma_t$</td>
<td>$\sigma_t = 18.7$</td>
</tr>
<tr>
<td>$K_{IC}$ vs $E$</td>
<td>$K_{IC} = 2.27$</td>
<td>$K_{IC} = 0.27 + 0.107\sigma_t$</td>
<td>$E = 59.5$</td>
</tr>
</tbody>
</table>

$^a$ Adopted equations in Whittaker et al (1992)

In the following, the simulated results are used to check the empirical relationship between the physical-mechanical properties and fracture toughness proposed by Whittaker et al (1992), as shown in Table 4, although these formulas have very low regression coefficients. In the table, the symbols are defined as follows: $K_{IC}$ is the mode-I fracture toughness, $K_{IIc}$ is the mode-II fracture toughness, $\sigma_c$ is the uniaxial compressive strength, $\sigma_t$ is the tensile strength and $E$ is the elastic modulus. It is found that the predicted fracture toughness data are far more reasonable compared with those of the reference granite and our numerical simulations. Moreover, all of the predicted values for the mode-II fracture toughness based on the relations are smaller than the values for the mode-I fracture toughness, which is contradicted by most of the experimental results, which usually indicate that the mode-II fracture toughness is two or three times higher than the mode-I fracture toughness (Rao, 1999). Therefore, a prediction based on these empirical relationships is not reliable. Considering the difficulty in determining the fracture toughness of rock in the laboratory, maybe it is a good option to obtain the fracture toughness by using a numerical method after we have determined the physical-mechanical properties. The developed R-T$^{2D}$ code builds a bridge between the fracture toughness and the physical-mechanical properties of rock.

4.3 Failure modes in pure mode-II and mixed-mode I-II loading

Classical fracture mechanics defines three basic modes of fracture, i.e. mode I (opening), mode II (sliding) and mode III (tearing). In our research, we find that in mixed-mode I-II loading and even in pure mode-II loading, tensile failure is the overwhelming failure mode, as shown in Paper B. The same phenomenon has also been observed in many examples of experimental research (Tapponnier and Brace, 1976; Swartz and Taha, 1990; Whittaker et al, 1992; Schlangen and Van Mier, 1992; Rao, 1999). On the basis of observations using the scanning electron microscope (SEM), Tapponnier and Brace (1976) conclude that most microcracks are tensile in nature.
However, Rao (1999) argued that classical fracture mechanics defines three basic modes of fracture from the loading point of view. It is assumed that the fracture propagates in the pre-fabricated crack plane. That is true for most metal materials. For heterogeneous rock materials, however, shear loading on a pre-fabricated crack is not certain to result in mode-II fracture, i.e. shear fracture propagating in the pre-fabricated crack plane, because of the rock heterogeneity and the high ratio between the compressive strength and the tensile strength. Traditional testing methods for $K_{IIC}$ (NBD and 4PS tests), which have been developed on the basis of the shear loading rather than the mode-II fracture mechanism, usually lead to $K_{IIC}$ values of rock smaller than or close to $K_{IC}$ values (Ingraffea, 1981; Huang and Wang, 1985; Whittaker et al, 1992). Obviously, it is contradictory to fact that the shear strength of rock should be much larger than its tensile strength. Our numerical analyses also indicate that, even under pure mode-II loading, tensile stresses inevitably exist around the notch tips in the same order of magnitude as compressive stresses and cause crack propagation in the direction parallel to the maximum principal stress. Similar results are found in the literature (Swartz and Taha, 1990; Schlangen and Van Mier, 1992). The crack develops in the direction perpendicular to the maximum tensile stress, which is characteristic of mode-I fracture rather than mode-II fracture. Maybe that is why none of these tests has been accepted as a standard testing method for determining the $K_{IIC}$ of rock.

4.4 Acoustic emission (AE) and the fracture process zone (FPZ)

Linear-elastic fracture mechanics (LEFM) is usually used to describe the fracture extension and the fracture mechanism under essentially linear-elastic (i.e. ideally brittle) conditions. However, during a crack propagation process, ahead of the crack tip there is always a non-linear zone which is referred to as the fracture process zone (FPZ) in rock. LEFM is usually invalid in this case, unless the fracture process zone is so small that the condition of small-scale yielding (SSY) is satisfied. Therefore, it is important to understand the mechanisms that develop the fracture process zone and what happens in it. Mihashi (1987) considers that around the branches of the main crack there are closed microcracks and that in front of them opening microcracks exist. Wittmann (1992) separated an inner microcracking zone from a surrounding isolated microcracking zone. Moreover, Otsuka and Date (2000) summarized that these concepts are valuable, but did not define the fracture process zone completely.

The load-displacement relationships measured in the notched specimen are shown in Fig. 18, Fig. 24, Fig. 30, Fig. 34, Fig. 41 and Fig. 48 in Paper B. AE inspections were made at the lettered points, A, B, C, etc, on the curves. In the figures, the number of failure events and the number of accumulated AE events are also plotted. After the maximum load, the failure events increase rapidly. The measured AE source locations in Paper B show that AE events are observed around the notch tip even in the first loading period. The AE events at other places are caused by heterogeneity and the loading configuration. The majority of AE events are located in the period around the peak load and show a tendency to concentrate around the notch tip. After this period, the AE map shows a more widely spread distribution. Moreover, the energy released in the failure of elements can be calculated approximately according to the element strengths, and then the magnitude of the energy release can be calculated. In the above-mentioned figures in Paper B, the red circles represent the seismicities caused by tensile failure in the current loading step, the green circles represent the seismicities caused by compressive failure, and the black circles record the accumulative seismicities in the previous steps. The diameters of the circles represent the relative magnitudes of the AE released energy. It is obvious that the higher energy events tend to localize near the notch tip. As a result, it is considered that these events are related directly to the fracture of rock and that this area associated with most of the energies could be called an FPZ. In this area, more densely distributed events are observed. Our numerically simulated results are consistent with the results which were observed by Otsuka and Date (2000) through an
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AE technique and which indicate that the microcrack zone corresponds to an FPZ or at least must be a main part of an FPZ.

4.5 Crushed zone in mechanical fragmentation

According to the simulated results in Paper C, Paper D and the review by Mishnaevsky (1995), a zone of highly fractured and inelastically deformed rock, which is usually called the crushed zone, is always available near the tools in mechanical fragmentation. The crushed zone has an important influence on the chipping process and energy utilization. In this zone microcracking is pervasive and rocks behave in a ductile manner. Associated with this microcracked or dilatant region there must be a volumetric expansion, which results in tensile crack propagation to the surface to form rock chips. The ductile behaviour of brittle materials under a sufficiently high confining pressure has also been observed by many researchers (Lindqvist and Lai, 1983; Mishnaevsky, 1995). However, in mechanical fragmentation the very nature of the inelastic deformation zone remains a highly contentious issue. It is not a simple matter to establish which of the two basic and competing processes, shear-induced flow (either plastic or viscous) or pressure-induced densification (compaction of an “open” microstructure), dominates within the small contact zone in rock materials. Yoffe (1982) provided a blister model to describe the behaviour of the crushed zone. In our simulations, it can be shown that in the regions immediately underneath the punch, the elements in the rock specimen are generally in a state of triaxial compression. According to Cook et al’s (1984) simulation using the Mohr-Coulomb criterion, it is interesting to note that immediately beneath the punch the rock does not fail because of confining pressure. However, according to the experiments conducted by Lindqvist and Lai (1983) and Cook et al (1984), the compressive strength of the rocks in the regions beneath the punch has been reached and therefore failure occurs.

In order to assess the significance of the state of stress at a given point underneath the indenter, the double elliptic strength criterion is adopted in our simulations. This assessment is achieved by the computation of the Mohr circle from the principal stresses at that point. When this circle falls outside the brittle failure envelope of the double elliptic criterion, the rock at that point is considered to have exceeded the strength of the rock and brittle failure occurs. When this circle falls outside the ductile yield envelope of the double elliptic criterion, the rock at that point is considered to have exceeded the strength of the rock and ductile cataclastic failure occurs. According to the simulation, it is found that the stresses of some elements immediately beneath the indenter satisfy the ductile failure surface and the elements fail in the ductile cataclastic failure mode, which contributes to the formation of the crushed zone. With increasing loading displacement, a re-compaction behaviour occurs in the crushed zone immediately beneath the indenter. However, in rock cutting simulation, the stresses of few elements satisfy the ductile failure surface of the double elliptic strength criterion. Maybe this is because the depth of the cut is not great enough or there is no confinement on the rock.

According to previous research in mechanical rock fragmentation, about 70% (Artsimovich et al, 1978) to 85% (Protassov, 1985) of the energy is consumed by the formation of the crushed zone. Lindqvist (1982) has shown that the energy loss associated with friction, microcracking and so on concerns 89% of the transmitted energy in disc cutting, whereas the useful energy is about 2-3%. In rock cutting simulation, the effective energy utilized to fragment the rock is 21%. The energy consumed by the crushed zone is 63%. The calculated result for the energy ratio consumed by the crushed zone is close to the results of previous observations. However, in our calculation, the useful energy is larger than the observed result. Maybe this is because the simulation does not consider the influence of the loading rate, temperature and friction, etc.
4.6 Influence of confining pressure on the failure mode in indentation

During the research in Paper D, we found that the lateral pressure had an important influence on the formation of side cracks, which are considered to be responsible for the rock fragmentation in indentation. The force-penetration curves for the three different lateral pressure values, 5, 10 and 20 MPa, are shown in Fig. 24. This figure shows a small but noticeable increase in the indentation strength with increased confinement. With decreasing confining pressure, decreases in the indentation strength are not of great interest, which can be observed in many tri-axial tests. Of particular interest is a change in the rock failure process when the confining stress is reduced below a critical value. Instead of the usual formation of rock chips adjacent to the punch, vertical cracks are propagated beneath the punch, causing the specimen to be split when the confining pressure on the sample is less than 20 MPa. Fig. 25 shows the simulated result when the lateral pressure remains 10 MPa.

Cook et al (1984) conducted a series of indentation tests using flat-bottomed circular punches to load the flat surfaces of the rock sample and presented similar results. In the punching tests at low confining stresses, the specimens failed in tension across a plane passing through the axis of the punch. Such failures may be a result of the finite size of the laboratory specimens and may not occur if a semi-infinite surface of rock is loaded by the punch. However, this result is of practical interest. In boring hard rock at low confining stresses, the creation of such tensile fractures beneath an indenter may serve to fragment the rock sufficiently to facilitate its removal. According to the simulation, it is evident that at a low confining stress
the region of tensile failure extends completely beneath the punch. As the confining stress increases, tensile failure is restricted to a region adjacent to the punch corners, and with the confining pressure decreasing, this region extends downwards in an approximately vertical direction as the punch load increases. The numerical simulation is validated by the experiments conducted by Cook et al (1984).

4.7 Interaction between neighbouring indenters in multiple indentations

During simultaneous loading by multiple indenters, the rock between the indenters can be additionally chipped owing to the interaction between neighbouring systems of cracks. This means that there is a possibility of increasing the removing capacity of a multi-indenter tool by using the interaction between the long cracks. In simultaneous loading by multiple indenters, the separation between the centre-lines of adjacent indenters is defined as the line spacing ($S$). This line spacing is very important in the design and operation of machines for rock drilling because of the interaction between adjacent side splays of chipping grooves. It seems that in simultaneous loading with an appropriate line spacing, the interaction of the stress fields induced by two indenters gives a possible combined crack at great depth to form large rock chips. Fig.
26 shows the simulated quasi-photoelastic fringe patterns induced by double indenters with different line spacings ($S=18a$, $S=12a$ and $S=6a$). A comparison of these pictures shows that when the line spacing is small, as shown in Fig. 26 C, the stress distributions induced by the double indenters are similar to those caused by a single indenter. As the line spacing increases, the indenters will act independently in the zones adjacent to the indenters and interact with each other in the zone between the indenters (Fig. 26 B). With the appropriate line spacing, as shown in Fig. 26 A, the interaction between the two indenters gives a possible combined crack at great depth to form large rock chips.

![Fig. 27 Rock fragmentation induced by double indenters with different line spacings](image)

Fig. 27 shows the final failure modes corresponding to the line spacings shown in Fig. 26. With a small line spacing, the coalescence occurs at a shallower depth and the fragment will therefore be smaller. With a large line spacing, there will be no crack coalescence and the multiple indenters act on the rocks separately. With the appropriate line spacing, the crack systems induced by the two indenters will interact with each other and the long cracks will finally coalesce to give a possible combined crack at great depth to form large rock chips. Using the blister model (Yoeff, 1982) to describe the behaviour of the crushed zones, by placing the initial side crack seeds at specific locations and in specific directions, Tan et al (1997) also simulated the
coalescence of side cracks in rock fragmentation by two indenters simultaneously. They concluded that only cracks at specific locations and orientations can generate crack coalescence, while others will have no development or miss each other. Moreover, the maximum spacing found is about $5d$, where $d$ is the diameter of the crushed zone according to the blister model.

![Fig. 28 Force-penetration curve in simultaneous loading by double indenters with different line spacings](image-url)

Fig. 28 shows the simulated force-penetration curves corresponding to the three different line spacings ($S=6a$, $12a$ and $18a$). The curves have almost the same shape, which means that the work performed by the indenters in the three cases is almost the same. However, the chipped depth and the volumes of the detached rock in the three cases are different, as shown in Fig. 27. This means that during indentation by multiple indenters, the volume of detached material can be increased by the interaction of crack systems without an increase in the loading energy; i.e. the specific energy can be smaller. This effect is caused by the fact that long cracks in two crack systems induced by the indenters can join together due to the interaction of their stress fields. Therefore, the simultaneous loading of the rock surface by multiple indenters with the appropriate line spacing seems to provide a possibility of forming larger rock chips, controlling the direction of subsurface cracks and consuming a minimum total specific energy.

In simultaneous loading, the line spacing between the indenters can be formulated as follows:

$$S = 2l + 2K_s a$$

(9)

where $S$ is the line spacing, $l$ is the linear length of the long side crack, $a$ is the indenter diameter and $K_s$ is the coefficient of the indenter shape: $K_s=1$ for a blunt indenter (Kou, 1995), with which the cracks are nucleated near the side faces of the indenter; $K_s=0.8$ (Mishnaevsky, 1998) for a spherical indenter, with which the cracks are initiated in the rock near $0.8a$; and $K_s=0$ (Kou, 1995) for a sharp indenter, with which the cracks are nucleated near the indenter axis. Moreover, the chipped depth $d$ in the rock between the two indenters is equal to the maximum depth of the side crack propagation.
5 Conclusions

1 In order to model the complicated fracture behaviour of heterogeneous rock under loading, a method integrating microscopic observation, image analysis and numerical modelling can be used. Microstructure information concerning a heterogeneous rock can be obtained from a sequence of physical or optical cuts by microscopic observation. Image analysis can be used to simplify the microstructure, identify the mineral composition and obtain the morphological information. Then a numerical model can be constructed by subsequent meshing to study the influence of morphological details and their evolution during the fracture process.

2 Considering the research scale and computer capacity, rock heterogeneity can be characterized better with the statistical approach. On the basis of the Weibull distribution, a heterogeneous material model is proposed to characterize the heterogeneity of rock, and in this model rock is specified according to a few characteristic parameters: the homogeneous index and the elemental seed parameters. For any specific rock in practice, the seed parameters, such as the uniaxial compressive strength, elastic modulus, etc, can be obtained from laboratory tests and the homogeneous index can be determined on the basis of the defect distribution of the microstructure.

3 Using these characteristic parameters as input parameters, a numerical code called the rock and tool interaction code (R-T2D) is developed to investigate progressive rock fracture. A series of numerical experiments including both intact rock and notched rock are conducted to obtain the physical-mechanical properties and fracture toughness, and to simulate the crack initiation and propagation and the whole progressive fracture process. It seems that the R-T2D code is indeed a valuable numerical tool for research on the progressive rock fracture process. Although the R-T2D code uses the characteristic parameters as input parameters, reasonable values for the fracture toughness are obtained. Therefore, the developed R-T2D code seems to have built a bridge between the physical-mechanical parameters and fracture toughness. Moreover, the detailed visually shown stress distribution and redistribution, crack nucleation and initiation, stable and unstable crack propagation, interaction and coalescence, and corresponding load-displacement curves are found to be in good agreement with experimental results in the literature and can be proposed as benchmarks for numerical programs for crack propagation.

4 Rock heterogeneity has an important influence on the crack initiation location and the subsequent propagation path. It is found that in crystalline heterogeneous materials such as granite, grain boundaries act as the predominant source of stress concentrating cracks. At the locations of strong mineral grains such as quartz, the first detectable fracturing starts at the grain boundaries. However, although grain boundaries are the preferred sites of microcracks, intragranular cracks also occur at the locations of weak mineral constituents such as feldspar and mica grains, where harder minerals induce a point load on neighbouring soft minerals. In the localization of cracks, secondary fracturing crosses the strong mineral grains to form a macroscopic shear band.

5 Compared with the traditional Mohr-Coulomb and Hoek-Brown strength criterion, the double elliptic strength criterion is more useful for modelling the fracture occurring in mechanical fragmentation in that it can represent the transition from brittle failure to ductile cataclastic failure with increasing confining pressure. The brittle failure face of the double elliptic criterion will represent rock failure in the shear mode or tensile mode, just as the modified Mohr-Coulomb strength criterion (including tensile cut-off) or Hoek-Brown strength criterion at a low confining pressure. The ductile failure surface will represent rock failure in the ductile cataclastic mode at a high confining pressure.

6 In mechanical fragmentation, a crushed zone is always available near the tool. The crushed zone has an important influence on the chipping process and energy utilization. According to our simulations, the crushed zone is in fact the zone with a high density of microcracks. In this zone microcracking is
In the optimum design of cutter tools, the shape of the tool is an important factor. In the research conducted for this thesis, the influence of the geometrical shape of tools with different back rake angles, $-20^\circ$, $0^\circ$ and $20^\circ$, has been investigated. The effective energy utilisation ratio for cutters with different back rake angles, $-20^\circ$, $0^\circ$ and $20^\circ$, is 14%, 18% and 21% respectively. The energy consumed by the formation of the crushed zone is 41%, 71% and 63% respectively. The specific energies for chipping are 199, 123 and 49 $kJ/m^3$ respectively for the three cutters with rake angles of $20^\circ$, $0^\circ$ and $-20^\circ$. When comparing the three different rake angles, it is reasonable to conclude that a rock cutter with a rake angle of $-20$ degrees is the most efficient of the three cutters.

According to the simulated results, a simple description and qualitative model of the rock fragmentation process induced by truncated indenters can be given as follows. Little damage to the rock is observed at the linear-elastic deformation stage. Then conical Hertzian cracks are initiated adjacent to both corners of the truncated indenter and propagated in the well-known conical Hertzian manner. As the loading displacement increases, some of the elements in the high confining pressure zone immediately under the indenter fail in the ductile cataclastic mode, with the stress satisfying the ductile failure surface of the double elliptic strength criterion. With the cataclastic failures and tensile conical cracks releasing the confining pressure, the elements in the confining pressure zone are compressed into failure and the crushed zone comes into being. In the crushed zone, microcracking is pervasive. The intensity of the microcracking within the zone increases and a re-compaction behaviour occurs with increasing loading displacement. Associated with this microcracked region there is a volumetric expansion and a tensile stress field, which drives side cracks to propagate in a curvilinear path. It is thought that the curvilinear path is caused by the heterogeneity of the rock. With an increased loading displacement, the side cracks rapidly propagate and intersect with the free rock surface to form rock chips.

In indentation tests, the confining pressure has an important influence on the failure mode. As the confining pressure increases, a small but noticeable increase in the indentation strength is measured. With decreasing confining pressure, decreases in the indentation strength are not of great interest, which can be observed in many tri-axial tests. Of particular interest is a change in the rock failure process when the confining stress is reduced below a critical value. Instead of the usual formation of rock chips adjacent to the punch, vertical cracks are propagated beneath the punch, causing the specimen to be split in half when the confining pressure on the sample is less than a critical value. This result is of practical interest. In boring hard rock at low confining stresses, the creation of such tensile fractures beneath an indenter may serve to fragment the rock sufficiently to facilitate its removal.

The simultaneous loading of the rock surface by multiple indenters seems to provide a possibility of forming larger rock chips, controlling the direction of subsurface cracks and consuming a minimum total specific energy. In the design and operation of machines for rock drilling, the line spacing between the indenters is a very important factor because of the interaction between adjacent side splays of chipping grooves. With a small line spacing, the cracks induced by the double indenters will behave in a manner similar to those caused by a single indenter. With the appropriate line spacing, the side cracks caused by the two indenters will propagate, interact and coalesce, giving a possible combined crack at great depth to form large rock chips.

As drilling and mining environments are becoming increasingly severe, improvements through drilling optimisation are needed to hold down costs. Numerical models that can accurately simulate tool-rock interaction and failure mechanisms, including chip formation and interaction between adjacent indenters, could be used in parametric studies to determine the optimal indenter designs, reducing much of the expensive and time-consuming experimental work which would otherwise have to be carried out to
assess the performance of indenter designs. The results simulated by the R-T$^{2D}$ code reproduce the progressive process of rock fragmentation under mechanical loading: the build-up of the stress field, the formation of the crushed zone, surface chipping, and the formation of the crater and subsurface cracks. The R-T$^{2D}$ code can be utilized to improve our understanding of rock-tool interaction and the rock failure mechanisms under the action of mechanical tools, which, in turn, will be useful in assisting the design of fragmentation equipment and fragmentation operations.
6 Suggestion for further research

6.1 Numerical simulation of specific rocks

In this thesis, we have characterized the heterogeneity of rock according to the following two types of characteristic parameters: (1) the homogeneous index $m$ and (2) the elemental seed parameters. For any specific rock in practice, the seed parameters can be obtained from the main physical-mechanical parameters in laboratory tests and the homogeneous index can be determined on the basis of the defect distribution of the microstructure (Orlovskaja et al, 1997; Curtis and Juszczyk, 1998; Davies, 2001). Further researches are needed to determine the homogeneous index.

6.2 Consideration of dynamic effects

In the rock fragmentation industry, blasting is another popular method widely employed in the areas of excavation and construction. As for the simulation of rock blasting, dynamic effects should be considered. Some researches (Zhang, 2001) have indicated that dynamic factors have an important influence on rock fracture. When the loading rate is up to and over $10^6$ MPa m$^{1/2}$ S$^{-1}$ or when the inertia effects must be considered, a quasi-static assumption may not be suitable and methods for measuring dynamic effects are needed. In this case, a dynamic wave propagation module should be developed in the R-T$^{2D}$ code.

6.3 Coupling of the thermo-hydro-mechanical (THM) process

Rock fragmentation is in fact a coupled thermo-hydro-mechanical (THM) process. The simulation of rock fragmentation will be made more practical if the THM process is combined into the R-T$^{2D}$ code.

6.4 Three-dimensional (3D) numerical model

With the development of computing power, interactive computer graphics and topological data structure, the two-dimensional model should be extended to create a three-dimensional model, so that the numerical simulation may come closer and closer to the situation in practice.

6.5 Quantitative relationship between the applied force and the resultant crack length in mechanical fragmentation

In mechanical fragmentation, there is a big difference between the theoretical crack length and the actual crack length. In order to improve the present theoretical relationship and make it more apt for practical applications, it is imperative to develop a series of numerical simulations which would consider rock confinement, contact conditions and the effect of the loading rate. Moreover, relatively new theoretical researches, numerical studies and field measurements are needed to determine the indentation force and the crack length in rock.
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