Modeling the Lunar Plasma Wake

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Licentiate thesis

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Abstract

This thesis discusses the solar wind interaction with the Moon and the formation of the lunar plasma wake from a kinetic perspective. The Moon is essentially a non-conducting body which has a tenuous atmosphere and no global magnetic fields. The solar wind plasma impacts directly the lunar day-side and is absorbed by the lunar surface. This creates a plasma void and forms a wake at the night side of the Moon.

We study the properties and structure of the lunar wake for typical solar wind conditions using a three-dimensional hybrid plasma solver. Also, we study the solar wind proton velocity space distribution functions at close distances to the Moon in the lunar wake and investigate the effects of lunar surface plasma absorption and non-isothermal solar wind velocity space distribution functions on the solar wind protons there.

Finally, we compare the simulation results with the observations and show that a hybrid model of plasma can explain the kinetic aspects of the lunar wake and we investigate the effects of the lunar surface plasma absorption and non-isothermal solar wind velocity distribution on the solar wind proton properties there.
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List of included papers

Paper I

The interaction between the Moon and the solar wind
M. Holmström, S. Fatemi, Y. Futaana, and H. Nilsson
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Paper II

The effects of Lunar surface plasma absorption and solar wind temperature anisotropies on the solar wind proton velocity space distributions in the low-altitude Lunar plasma wake
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CONTENTS

1 Introduction 1

2 Kinetic Theory of the Solar Wind Plasma 5
   2.1 Boltzmann and Vlasov Equations 6
   2.2 Moments of the Distribution Function 7
   2.3 Distribution Functions 9
      2.3.1 Maxwellian Distribution 9
      2.3.2 Kappa Distribution 9
      2.3.3 Bi-Maxwellian Distribution 11
   2.4 Solar wind Plasma Distribution Functions 12
   2.5 Solar wind Plasma Instabilities 13

3 Hybrid Model of Plasmas 15
   3.1 Hybrid Approximations 16
   3.2 The Hybrid Equations 18
   3.3 Particle Injection and Boundary Conditions 21
   3.4 The Backward Liouville Method 22

4 Solar Wind Interaction with the Moon 23
   4.1 The Lunar Dayside-Plasma Interaction 24
   4.2 The Lunar Plasma Wake and its Structure 24

5 Summary of Papers 27

Physical Constants 29

Bibliography 31
The solar wind is a multi-species, almost collision-less plasma, flowing supersonically outward from the Sun, and carrying the interplanetary magnetic field (IMF) into the heliosphere. Our solar system contains millions of different size bodies, including the planets and their satellites, comets and asteroids that interact with the solar wind plasma. The interaction of the solar wind with these different bodies depends on the characteristics of these objects such as the body sizes, whether or not they have a significant atmosphere, and the strength of their intrinsic magnetic fields.

Since our solar system was formed, the Sun has been continuously ejecting the solar wind plasma and it has been interacting with the different solar system objects. Apart from the solar wind plasma, solar radiation is another source that interacts with the different objects. Studying the solar wind plasma and radiation interaction with solar system objects help us to understand planetary evolution and gain valuable knowledge to estimate their future. Understanding life formation on our beautiful planet, the Earth, and finding similar evidences of life in the outer space have been the human ambitions for thousands of years. Although we believe that we are currently in the golden era of space and technology, we are not able to answer many of the fundamental questions even about the planet that we have been living on for so many years. Therefore, studying the solar wind interaction...
Chapter 1. *Introduction*

with the different objects will lead us to answer some of the basic questions about life, its future on the Earth, habitability of other objects than the Earth, and the possibility of finding life similar or perhaps of different kinds than that of the Earth in outer space.

The solar wind interaction with the solar system objects are categorized into four groups: Earth type, Venus type, Lunar type, and Comet type [1].

(1) **Earth type:** The Earth is surrounded by its intrinsic magnetic field which creates the magnetosphere. The magnetosphere acts as an obstacle to the solar wind plasma flow and diverts most of the solar wind around it. However, since the solar wind plasma contains charged particles, they interact with the magnetospheric field lines and penetrate into the magnetosphere, mainly from the magnetospheric poles where the field lines are open and merge to the IMF. Other solar system planets such as Mercury and all the giants (Jupiter, Saturn, Uranus and Neptune) also have global magnetic fields.

(2) **Venus type:** Venus, although it does not have any global magnetic field, has a dense neutral atmosphere. The photoionization of neutrals by solar extreme ultraviolet radiation generates a significant ionosphere which acts as an obstacle to the solar wind flow. Titan’s (a kronian satellite) interaction with the solar wind is another example of a Venus type interaction.

(3) **Lunar type:** The Earth’s Moon does not have any significant atmosphere and no global magnetic fields but small scale crustal magnetic anomalies. Therefore, the solar wind plasma directly impacts the Lunar surface and is absorbed and neutralized there. Another example of the Lunar type solar wind interaction is Phobos, the Martian satellite.

(4) **Comet type:** Comets usually have negligible intrinsic magnetic field, but their nuclei contains of ice and dust. While the comets are far away from the Sun, their interaction with the solar wind is similar to that of the Lunar type, but when they get close to the Sun, their nuclei are heated and evaporate water. Therefore an extensive atmosphere is formed around a comet which extends thousands of kilometers behind it. Photoionization of cometary neutrals forms an ionosphere close to the nucleus which deflects the solar wind flow.

Several techniques including observations, theoretical models and numerical simulations, have been used to study the solar wind interaction with the different objects. In the rest of this chapter we only explain the modeling techniques to
study the solar wind interaction with Lunar type objects and then in the following chapters of this thesis we choose one of the modeling approaches to study the kinetics of the solar wind plasma interaction with the Moon.

The most commonly applied models to study the solar wind interaction are magnetohydrodynamics (MHD), full particle-in-cell (PIC) and hybrid models. In the MHD model, the plasma is treated as a charge neutral fluid. Therefore, the kinetic properties of plasma can not be examined using this approach. In PIC models, the different plasma species are treated as kinetic particles; therefore, the kinetic nature of plasma species can be examined using this approach. Another kinetic/particle simulation approach is the hybrid model, which treats the ions as kinetic particles and the electrons as a mass-less fluid; therefore, it takes computationally less time than the PIC models [2].

In order to study the kinetic properties of the solar wind plasma in the lunar environment, a kinetic (particle) simulation approach is used. Moreover, to obtain a full velocity space distribution of the particles, a three-dimensional (3D) model is needed. Other kinetic simulation approaches than the hybrid model of plasma, e.g. PIC model, are computationally expensive for 3D modeling of the solar wind interaction with the Moon, therefore we choose a self-consistent 3D hybrid model of plasma [3] to study the kinetics of the solar wind interaction with the Moon.

In this thesis we briefly discuss the kinetic properties of the solar wind plasma (Chapter 2), then we explain the hybrid model of plasma (Chapter 3) and we give a global understanding about the solar wind interaction with the Moon (Chapter 4). Finally, in the published papers, we study the lunar wake and the solar wind protons kinetic properties using the hybrid plasma solver.
KINETIC THEORY OF THE SOLAR WIND PLASMA

Fluid theory can explain the majority of the observed plasma phenomena, and it is sufficiently accurate when the scales of our interests are much larger than the scales involving the plasma species. However, there are some phenomena which have scales smaller than the fluid scales and fluid theory can not capture them. Therefore, kinetic theory is required which considers the velocity space distribution function \( f_s \) for each particle species \( s \).

Consider a position, \( \mathbf{r} = [x, y, z] \), and a velocity, \( \mathbf{v} = [v_x, v_y, v_z] \), at time \( t \). A velocity space distribution function (VSDF) is a function of these seven independent variables: \( f_s = f_s(\mathbf{r}, \mathbf{v}, t) \) and it defines the number density of particles of specie \( s \) that at time \( t \) present in an infinitesimally small phase space volume \([\Delta \mathbf{r}, \Delta \mathbf{v}]\), located at a phase space point \([\mathbf{r}, \mathbf{v}]\). Therefore, the integral of \( f_s(\mathbf{r}, \mathbf{v}, t) \) over all velocity space gives the number density of particles of specie \( s \).

\[
n_s(\mathbf{r}, t) = \int f_s(\mathbf{r}, \mathbf{v}, t) \, d\mathbf{v} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_s(x, y, z, v_x, v_y, v_z, t) \, dv_x \, dv_y \, dv_z
\]

(2.1)

where \( n_s(\mathbf{r}, t) \) is the number density of specie \( s \) at time \( t \) and position \( \mathbf{r} \).

Each specie of plasma has a set of properties, listed in Table 2.1,

where \( \mathbf{B} \) is the magnetic field, \( k_b \), \( \epsilon_0 \) and \( c \) are the Boltzmann constant, permittivity of free space and the speed of light, respectively.
Table 2.1: Different plasma specie properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass</td>
<td>$m_s$</td>
</tr>
<tr>
<td>charge</td>
<td>$q_s$</td>
</tr>
<tr>
<td>number density</td>
<td>$n_s$</td>
</tr>
<tr>
<td>charge density</td>
<td>$\rho_s = q_s n_s$</td>
</tr>
<tr>
<td>bulk velocity</td>
<td>$u_s$</td>
</tr>
<tr>
<td>kinetic temperature</td>
<td>$T_s$</td>
</tr>
<tr>
<td>thermal speed</td>
<td>$v_{th_s} = (2k_B T_s / m_s)^{1/2}$</td>
</tr>
<tr>
<td>gyro-frequency</td>
<td>$\Omega_s = q_s B / m_s$</td>
</tr>
<tr>
<td>plasma frequency</td>
<td>$\omega_s = (n_s q_s^2 / \epsilon_0 m_s)^{1/2}$</td>
</tr>
<tr>
<td>gyro-radius</td>
<td>$r_{gs} = u_s / \Omega_s$</td>
</tr>
<tr>
<td>Debye length</td>
<td>$\lambda_{Ds} = (\epsilon_0 k_B T_s / n_s q_s^2)^{1/2} \approx 0.71 v_{th_s} / \omega_s$</td>
</tr>
<tr>
<td>inertial length</td>
<td>$\delta_s = c / \omega_s$</td>
</tr>
</tbody>
</table>

2.1 Boltzmann and Vlasov Equations

The rate of change of $f_s$ due to an explicit time variation is

$$\frac{df_s}{dt} = \lim_{\Delta t \to 0} \frac{f_s(r + \Delta r, v + \Delta v, t + \Delta t) - f_s(r, v, t)}{\Delta t}$$

(2.2)

When $\Delta t$ is small enough, Taylor expansion of Equation 2.2 can be taken and it gives

$$\frac{df_s}{dt} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} [f_s(r, v, t) + \Delta r \cdot \nabla f_s + \Delta v \cdot \nabla_v f_s + \Delta t \frac{\partial f_s}{\partial t} - f_s(r, v, t)]$$

(2.3)

where $\nabla$ is the gradient operator in configuration space, $\nabla_v$ is the gradient operator in the velocity space and all high orders of the Taylor expansion are neglected as $\Delta t \to 0$. Then we get a fundamental equation, known as the Boltzmann equation of kinetic theory

$$\frac{df_s}{dt} = v_s \cdot \nabla f_s + a_s \cdot \nabla_v f_s + \frac{\partial f_s}{\partial t}$$

(2.4)

where $v_s = dr/dt$ and $a_s = dv_s/dt$. $a_s$ is the acceleration term and is in our case obtained from the Lorentz force

$$F_s = m_s a_s = q_s (E + v_s \times B)$$

(2.5)

where $E$ and $B$ are electric and magnetic fields, respectively.

The left-hand side of the Boltzmann Equation 2.4 is the collisional term and in a collision-less plasma it will be zero. The collision-less form of the Boltzmann
Chapter 2. Kinetic Theory of the Solar Wind Plasma

The Vlasov Equation

During the last decades, some efforts have been done to solve the Boltzmann and Vlasov equations, both theoretically and numerically, but most of the successful solutions have been restricted to a few specialized, or low dimensional, problems [4]. In many applications, we are only interested in a limited number of macroscopic variables of the distributions and we do not need to know all the details of the distribution function. These measurable variables are functions of position and can be obtained by integrating the distribution function over the velocity space domain.

The general approach to obtain the $\beta$-th moment of a single particle distribution function $f_s$ is

$$\Psi_\beta = \int v^\beta f_s(r, v, t) \, dv$$

(2.7)

where $\Psi$ is the moment (macroscopic variable) of the distribution, and $dv = dv_x \, dv_y \, dv_z$ [1].

The first four moments of a single particle distribution function are as follows

- **Zero-th moment ($\beta = 0$)** is the number density of specie $s$
  $$\Psi_0 = n_s(r, t) = \int f_s(r, v, t) \, dv$$
  (2.8)
  with the SI unit of $m^{-3}$.

- **First moment ($\beta = 1$)** is the particle flux of specie $s$
  $$\Psi_1 = \Phi_s(r, t) = \int v \, f_s(r, v, t) \, dv$$
  (2.9)
Chapter 2. Kinetic Theory of the Solar Wind Plasma

Flux is a vector quantity and has units of \( m^{-2} s^{-1} \) and the bulk flow velocity of particles of type \( s \) is expressed by

\[
\mathbf{u}_s(\mathbf{r}, t) = \Phi_s(\mathbf{r}, t)/n_s(\mathbf{r}, t)
\]

(2.10)

• Second moment \((\beta = 2)\) is the particle pressure tensor of specie \( s \)

\[
\Psi_2 = \mathbf{P}_s(\mathbf{r}, t) = m_s \int c c f_s(\mathbf{r}, \mathbf{v}, t) \, d\mathbf{v}
\]

(2.11)

where \( c = \mathbf{v} - \mathbf{u}_s \) and is the random velocity. \( P_s \) is a 3 \( \times \) 3 matrix and in component notation, each of its \((i, j)\) elements are given as

\[
P_{s,ij}(\mathbf{r}, t) = m_s \int c_i c_j f_s(\mathbf{r}, \mathbf{v}, t) \, d\mathbf{v}
\]

(2.12)

and a scalar pressure \( p_s \) is defined as one third of the trace of \( P_{s,ij} \)

\[
p_s = \frac{1}{3} \sum_{j=1}^{3} P_{s,j,j}
\]

(2.13)

Kinetic temperature of specie \( s \) can be given in a form of a 3 \( \times \) 3 matrix of

\[
\mathbf{T}_s(\mathbf{r}, t) = \mathbf{P}_s(\mathbf{r}, t)/n_s(\mathbf{r}, t)k_b
\]

(2.14)

where \( k_b \) is the Boltzmann constant.

• Third moment \((\beta = 3)\) is the heat flux of specie \( s \) and is given by

\[
\Psi_3 = \mathbf{Q}_s(\mathbf{r}, t) = \frac{1}{2} m_s \int c c^2 f_s(\mathbf{r}, \mathbf{v}, t) \, d\mathbf{v}
\]

(2.15)

which has the SI unit of \( W m^{-2} \).

(for more details about the moments of the distribution functions and the their higher orders, see [1, 5])
Chapter 2. *Kinetic Theory of the Solar Wind Plasma*

### 2.3 Distribution Functions

There exist a variety of different velocity space distribution functions, but here we only address the important ones which relate to solar wind plasma distribution functions.

#### 2.3.1 Maxwellian Distribution

One of the well-studied distribution functions is the Maxwellian distribution. It is also known as Maxwell-Boltzmann distribution and is defined as

\[
 f^M_s = n_s \left( \frac{m_s}{2\pi k_b T_s} \right)^{3/2} \exp \left( -\frac{m_s(v - u_s)^2}{2k_b T_s} \right)
\]  

The Maxwellian distribution is an isothermal distribution and its vector quantity moments are symmetric, with respect to the bulk velocity, along all three dimensions in velocity space. Figure 2.1 shows an example of a two-dimensional drifting Maxwellian distribution, in one- and two-dimension slices. The bulk flow velocity can be determined from the peak of the distribution and here in this example is \( u_x = 400 \text{ km/s} \) and \( u_y = 200 \text{ km/s} \). Since the distribution is isothermal, it has equal thermal speed \( v_{th} \) in all directions. \( f^M_{\text{max}} \) is the maximum of VSDF and the thermal speed (the most probable speed) is the velocity difference between \( f^M_{\text{max}} \) and \( f^M_{\text{max}}/e \), where \( e \) is the Napier’s constant.

#### 2.3.2 Kappa Distribution

Another well-known distribution function is the \( \kappa \)-distribution and it is defined as

\[
 f^\kappa_s = \frac{n_s}{2\pi (\kappa v^2)^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2) \Gamma(3/2)} \left( 1 + \frac{(v - u_s)^2}{\kappa v^2} \right)^{-(\kappa + 1)}
\]  

where

\[
 \Gamma(x) = (x - 1)!
\]

\[
 \Gamma(x + 1/2) = \frac{1 \times 3 \times 5 \times \ldots \times (2x - 1)}{2^x} \sqrt{\pi}
\]

\[
 v_u = \sqrt{\frac{(2\kappa - 3)}{\kappa}} \frac{k_b T_s}{m_s}
\]
Chapter 2. Kinetic Theory of the Solar Wind Plasma

Figure 2.1: Two-dimensional drifting Maxwellian VSDF for protons as a one-dimensional spectra (left) and a two-dimensional contour plot (right). The color bar and the dashed contours in the right show the VSDF in linear and logarithmic scales, respectively. The bulk flow velocity is \( u_b = [400, 200] \) km/s, and since the distribution is isothermal, thermal speed \( v_{th} \) is equal along the different directions and is here around 60 km/s.

Figure 2.2: Comparison between the one-dimensional spectra for different \( \kappa \) values (\( \kappa = \{2, 4, 8\} \)) for \( \kappa \)-distributions and for a Maxwellian distribution function. All the distributions have the same number density and kinetic temperature. The horizontal axis shows the velocity and the vertical axis shows the VSDF values in logarithmic scale.

In general, the \( \kappa \)-distribution predicts higher energy for the particles at the tail of the distribution compared to the Maxwellian but it tends to a Maxwellian distribution when \( \kappa \to \infty \). This can be seen clearly from Figure 2.2.
Chapter 2. Kinetic Theory of the Solar Wind Plasma

2.3.3 Bi-Maxwellian Distribution

A Bi-Maxwellian distribution is a non-isothermal VSDF. In contrast to the Maxwellian distribution, particles in a Bi-Maxwellian distribution can have different thermal speed in different directions in the velocity space. This makes the distribution function anisotropic.

The Bi-Maxwellian distribution is defined as

\[
f^B_s = n_s \left( \frac{m_s}{2\pi k_b T_{s||}} \right)^{1/2} \left( \frac{m_s}{2\pi k_b T_{s\perp}} \right) \exp \left( -\frac{m_s c_{s||}^2}{2k_b T_{s||}} - \frac{m_s c_{s\perp}^2}{2k_b T_{s\perp}} \right)
\]

\[
T_s = \frac{T_{s||} + 2T_{s\perp}}{3}
\]

(2.18)

where the directional subscripts denote directions relative to the background magnetic field \((B_0)\) and \(c_{||} = v_{||} - u_{s||}\) and \(c_{\perp} = v_{\perp} - u_{s\perp}\).

Figure 2.3 shows an example of a two-dimensional Bi-Maxwellian distribution function for protons with \(n_s = 5 \times 10^6\ \text{m}^{-3}\), \(u_{s||} = 400\ \text{km/s}\), \(u_{s\perp} = 200\ \text{km/s}\), \(T_s \simeq 17.5\ \text{eV}\) and \(T_{s||}/T_{s\perp} = 2/3\).

**Figure 2.3:** Two-dimensional Bi-Maxwellian velocity distribution function for protons as a one-dimensional spectra (left) and as a two-dimensional contour plot (right). The color bar and the dashed contours in the right show the VSDF in linear and logarithmic scales, respectively. The bulk flow velocity is \(u_s = [400, 200]\ \text{km/s}\), and the distribution has different thermal speed in parallel \(v_{th||}\) and perpendicular \(v_{th\perp}\) to the magnetic field. Parallel temperature \(T_{s||}\) in this example is higher than the perpendicular temperature \(T_{s\perp}\).
2.4 Solar wind Plasma Distribution Functions

The solar wind is a multi-species but weakly collisional medium. Solar wind particles mean free path depends on the distance to the Sun, therefore the Coulomb collision frequency is low enough that it sometimes can be neglected at 1 AU (1 Astronomical Unit = distance between the Earth and the Sun). As a consequence of this low collision rate in the solar wind, temperature anisotropies evolve with heat fluxes along the background magnetic field. However, even a few collisions per AU prevents formation of an extremely large temperature anisotropy in the solar wind plasma distributions [6].

Solar wind plasma distributions have been extensively studied using observations and simulations [6–10]. In the low-speed solar wind (<400 km/s), protons have isotropic cores in their distributions and total temperature anisotropies with $T_\parallel > T_\perp$, while in the high-speed solar wind (>600 km/s), the total temperature anisotropies are pronounced with $T_\perp > T_\parallel$ and anisotropic cores [6, 7]. As the solar wind plasma speed increases, the distribution functions departure more towards anisotropic distributions and this is as a result of the increasing heat flux and less Coulomb collision rates than for lower solar wind speeds [6–8].

Solar wind helium ions in the low-speed solar wind, somewhat similar to the protons, have total temperature anisotropies with $T_\parallel > T_\perp$, while for high-speed solar winds, in contrast to the solar wind protons, they have small temperature anisotropy with $T_\parallel \gtrsim T_\perp$ [8].

Solar wind electron distribution functions consist of two parts: a core and a hot halo. These two parts have density ratio of $n_H/n_C \approx 0.05$ and temperature ratio of $T_H/T_C \approx 6$ at 1 AU. Both core and halo temperatures ($T_C$ and $T_H$) vary together and they are generally lower in the high-speed solar wind than in the low-speed regions [11].

Often, the interplanetary solar wind plasmas have non-isotropic distribution functions and this is due to the very low collision frequency for the inhomogeneous plasma. Maxwellian distribution has been observed in very low-speed and high density plasma where the collision frequency is high, e.g in the planetary magnetosphere/ ionosphere or on some occasions in the interplanetary plasma.
Chapter 2. Kinetic Theory of the Solar Wind Plasma

2.5 Solar wind Plasma Instabilities

As the solar wind distribution functions departure from isotropic to anisotropic as a consequence of the plasma expansion, the first question arising is whether the anisotropic distributions are stable or not. The second question is if there are signs of instabilities, which modes do the instabilities have and are they effective enough to relax the distributions or not. Here, in this section, we try to answer these questions but we only focus on the solar wind protons.

As the solar wind plasmas expand in the heliosphere, particles distribution functions are far from the Maxwellian distribution. In such cases, particles and specifically the solar wind protons departure from adiabatic behavior and the adiabatic invariant is no longer conserved.

Departures from isotropic distributions towards anisotropic and non-isothermal distributions are a possible source of free energy for many instabilities \[12\]. A non-isothermal distribution arises the electromagnetic proton cyclotron and mirror instabilities if \( R > 0 \) \[13, 14\] and Alfvén and fire-hose instabilities if \( R < 0 \) \[12, 14\], where \( R = T_{\perp} / T_{\parallel} - 1 \).

Theory and simulation that agree with observations show that there are strong constraints on the protons temperature anisotropies

\[
R = \frac{T_{\perp}}{T_{\parallel}} - 1 = \frac{S}{\beta_{\parallel}} \tag{2.20}
\]

\[
\beta_{\parallel} = 2\mu_0 n_p k_b T_{\parallel}/B_0^2 \tag{2.21}
\]

due to the enhanced magnetic field fluctuations from the instabilities which scatter the protons and change the distribution functions from non-isothermal to isothermal distributions \[13–16\]. \( S \) and \( \alpha \) are linear theory fitting parameters to the observations, \( \beta_{\parallel} \) is called the proton plasma beta parallel to the background magnetic field \( B_0 \), \( n_p \) is the protons number density, \( k_b \) is Boltzmann’s constant and \( \mu_0 \) is the permeability of free space.

Observations show that 40% to 80% of the time \( R < 0 \) in the solar wind which is in agreement with the conservation of the first adiabatic invariant \[6, 10, 13\]. WIND spacecraft observations show that the best fit measured value for \( R \) is given by \((S, \alpha) = (1.21 \pm 0.26, 0.76 \pm 0.14)\) \[14\] while hybrid simulations of plasmas predict
Chapter 2. *Kinetic Theory of the Solar Wind Plasma*

\( S \simeq 1 \) and \( \alpha \simeq 0.74 \) [17] (In Chapter 3 we see more about hybrid models).

The growth of magnetic fluctuations which occurs because of the instabilities leads to wave-particle scattering and increases the effective collision rates and finally \( R \to 0 \) and eventually the plasma \( \beta_{||p} \) changes [17]. Therefore the solar wind proton temperature anisotropy range is constrained by the kinetic instabilities, especially when \( \beta_{||p} > 1 \).

Table 2.2 summarizes the possible unstable micro-instabilities which can occur for the solar wind protons [12–17].

**Table 2.2:** Possible solar wind micro-instabilities.

| \( \beta_{||p} \leq 1 \) | \( \beta_{||p} \geq 1 \) |
|--------------------------|--------------------------|
| \( R < 0 \)               | Alfven                   |
| \( R > 0 \)               | Cyclotron, Mirror        |

In most of the solar wind proton distribution studies, Bi-Maxwellian distributions fit the observations and can explain the anisotropy features of the distributions [13–15, 17]. However, there are higher order moments of the distributions which include the heat flux and stress tensor, e.g., 16-Moments distribution function, that can explain the observed data [9, 18], but usually for simplicity the lower order moments of the distributions are considered to study the macroscopic properties of the solar wind plasmas.
Hybrid model of plasma is a self-consistent kinetic modeling approach that involves solving Maxwell’s equations, listed in Table 3.1, for positively charged particles while the electrons are treated as a mass-less fluid.

Generally in PIC or hybrid plasma solvers there are a number of basic steps in the calculations that have to be made [4]. These steps are illustrated in Figure 3.1 and they are as follows.

Plasma species, their mass and mass per charge ratio are defined and each of them are represented as particles, the rest are considered as fluids. The subset of Maxwell’s equations (electromagnetic or electrostatic) to be solved is decided. The geometry of the simulation/calculation, initial values and the boundary conditions for the problem should be defined. Now the particle species are advanced in small amounts of time, $\Delta t$, and their new positions and velocities in space and time are obtained. The sources (plasma density and current) are collected to solve for the fields and once the new fields have been obtained, the particles can be moved again and the steps shown inside the dashed box in Figure 3.1 are repeated until a final time is reached. Finally the results are analyzed through appropriate diagnostics [4, 19].
3.1 Hybrid Approximations

Several assumptions are considered in a hybrid solver which are mostly in common for all kinetic solvers [2, 3].

1. Quasi-neutrality

\[ \rho = \sum \rho_e = \rho_e + \sum \rho_i = 0 \quad (3.1) \]
where \( \rho \) is the charge density of \( M \) different particle species \( s \) that contain electrons, \( e \), and \( M - 1 \) different ions, \( i \). This assumption implies that \( \nabla \cdot \mathbf{J} = 0 \), where \( \mathbf{J} = \sum \mathbf{J}_s \) is the total current density, and removes most electrostatic instabilities and it is only valid for grid resolutions larger than the Debye length \( \lambda_D \).

2. Darwin approximation

If we assume a quantity \( Q \) can be separated into transverse, \( Q_T \), and longitudinal, \( Q_L \), parts (\( Q = Q_T + Q_L \)) such that \( \nabla \cdot Q_T = 0 \) and \( \nabla \times Q_L = 0 \), Ampère’s law can be decomposed into two parts, a divergence-free and a curl-free part [19].

\[
\nabla \times \mathbf{B}_T = \mu_0 \left( \mathbf{J}_T + \epsilon_0 \frac{\partial \mathbf{E}_T}{\partial t} \right), \quad \text{and} \quad 0 = \mu_0 \left( \mathbf{J}_L + \epsilon_0 \frac{\partial \mathbf{E}_L}{\partial t} \right)
\]

(3.2)

where \( \mu_0 \) and \( \epsilon_0 \) are the permeability of free space and the vacuum permittivity, respectively.

In the Darwin approximation, the transverse displacement current \( \partial \mathbf{E}_T / \partial t \) is neglected. This removes both relativistic phenomena and light waves (high frequency waves). Then Ampère’s law is simply reduced to

\[
\nabla \times \mathbf{B} = \mu_0 \mathbf{J}
\]

(3.3)

Since in a hybrid model electrons are treated as mass-less fluid, the electrons current, \( \mathbf{J}_e \), and velocity, \( \mathbf{u}_e \), are calculated from the charge neutrality approximation as follows

\[
\mathbf{J}_e = \mathbf{J} - \sum \mathbf{J}_i
\]

(3.4)

\[
\mathbf{u}_e = \frac{\mathbf{J}_e}{\rho_e} = \frac{\mathbf{J} - \sum \mathbf{J}_i}{\rho - \sum \rho_i}
\]

(3.5)

3. Adiabatic pressure

Electron pressure, \( p_e \), can be assumed to be adiabatic. Then the electron pressure is related to the electron charge density by \( p_e \propto |\rho_e|^\gamma \); where \( \gamma \) is the adiabatic index and usually \( \gamma = 5/3 \) [20].
4. Massless electrons
We know that \( m_e/m_i \ll 1 \), then we can assume that \( m_e = 0 \). In this assumption, the plasma mass density is only the ion’s mass density, and the electron gyro frequency and the electron plasma frequency are removed from the calculations because of the electron’s zero mass.

5. Faraday’s law
Faraday’s law is used to advance the magnetic field in time

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]  

(3.6)

### 3.2 The Hybrid Equations

Ions positions, \( \mathbf{r}_i \) and velocities, \( \mathbf{v}_i \) are obtained by solving the equation of motion which is an ordinary differential equation (ODE)

\[
\frac{d}{dt} \begin{bmatrix} \mathbf{v}_i \\ \mathbf{r}_i \end{bmatrix} = \begin{bmatrix} \mathbf{a}_i \\ \mathbf{v}_i \end{bmatrix}
\]

(3.7)

where \( \mathbf{a}_i = \frac{q_i}{m_i}(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) \) and \( \mathbf{E} \) and \( \mathbf{B} \) are electric and magnetic fields, respectively and they are given by

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]

(3.8)

\[
\mathbf{E} = \sum_{p_i} \left( -\sum \mathbf{J}_i \times \mathbf{B} + \rho_0^{-1} (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p_e \right) + \frac{\eta \mu_0}{\mu_0} \nabla \times \mathbf{B}
\]

(3.9)

where \( p_e = n_e k_b T_e \) is the electron pressure, \( \eta \) [Sm\(^{-1}\)] is the resistivity, and \( \mu_0 = 4\pi \cdot 10^{-7} \) [Hm\(^{-1}\)] is the magnetic constant.

We have a complete set of hybrid equations to solve the system. We assume that the simulation geometry, initial and boundary conditions are known and we aim to solve the equations by following the cycle surrounded by the dashed lines in Figure 3.1.
Chapter 3. Hybrid Model of Plasmas

1. Moving particles
To advance the particles positions and velocity, we use a commonly used integration method called the leap-frog method. Using the leapfrog method, Equation 3.7 is replaced by the finite difference equation

\[
\begin{bmatrix}
\frac{v_{\text{new}} - v_{\text{old}}}{\Delta t} \\
\frac{r_{\text{new}} - r_{\text{old}}}{\Delta t}
\end{bmatrix} = \begin{bmatrix}
a_{\text{old}} \\
v_{\text{new}}
\end{bmatrix}
\]

(3.10)

The method is shown in Figure 3.2. We define full-integer grids at \( t = j\Delta t \) and half-integer grids at \( t = (j + 1/2)\Delta t \), where \( j \) is an integer number. In this method
1. a particle’s position and velocity at \( t = 0 \) (initial condition) are known \([r(0), v(0)]\).
2. push \( v(0) \) back from \( t = 0 \) to \( t = -\Delta t/2 \) to obtain \( v(-\Delta t/2) \) using the acceleration term \( a(0) \).
3. set \( v_{\text{old}} = v(-\Delta t/2) \) and \( t = \Delta t \).
4. compute the new velocity and new position from Equation 3.10. Then \( r_{\text{new}} = r(t) \) and \( v_{\text{new}} = v(t + \Delta t/2) \).
5. compute \( a_{\text{new}} = \frac{q_i}{m_i} (E + v_{\text{new}} \times B) \).
6. advance the time \( t \leftarrow t + \Delta t \), set \( a_{\text{old}} = a_{\text{new}}, v_{\text{old}} = v_{\text{new}}, r_{\text{old}} = r_{\text{new}} \) and goto step (4).

Note that the particle’s position is advanced on the full-integer grid while the particle’s velocity is advanced on the half-integer grid.

2. Collecting sources
The sources are the charge density and the current density which are used to solve
for the electric and magnetic fields. Here we find the charge density first, and then compute the current density.

For simplicity we assume a one-dimensional spatial domain along the $X$-axis (Figure 3.3). We discretize the domain into equal length cells of size $\Delta x$. When we move particles in time and space, a charged particle at position $x_p$ has a charge density distribution $q_s/\Delta x$ in the range $x_p-\Delta x/2 \leq x_p < x_p + \Delta x/2$. On the other hand, the cell around each grid point at $X_k$ is $X_k - \Delta x/2 \leq X_k < X_k + \Delta x/2$ and shown in Figure 3.3. The charge of the particle is split to the closest grid points, proportional to the areas covered in each cell. The numerical assignment of the charge $q_s$ to the adjacent grid points $X_k$ and $X_{k+1}$ is as follows.

$$\rho(X_k) = q_s (X_{k+1} - x_p)/\Delta x$$  \hspace{0.5cm} (3.11)  

$$\rho(X_{k+1}) = q_s (x_p - X_k)/\Delta x$$  \hspace{0.5cm} (3.12)

![Figure 3.3: Length weighting method in computing charge density](image)

The total current density, $\mathbf{J}$, is obtained from the simplified Ampère’s law (Equation 3.3). Ion current density, $\sum \mathbf{J}_i$, is deposited on the grid from $\sum q_i \mathbf{v}_i$. Then $\mathbf{J}_e = \mathbf{J} - \sum \mathbf{J}_i$. Both the charge density and the current density are advanced in time on the half-integer grid, similar to the velocity.

3. Solving fields

Several methods exist to solve for the electric and magnetic fields using Equations 3.8, 3.9, but the most common one is the general predictor-corrector loop. In this method we assume that electric field $\mathbf{E}(t)$ and magnetic field $\mathbf{B}(t)$ at a moment of time $t$ are known. We have already collected the charge density $\rho(t+\Delta t/2)$ and the current density $\sum \mathbf{J}_i(t + \Delta t/2)$, then we can compute the electric and magnetic fields from predictor-corrector scheme by

1. Using Faraday’s law (Equation 3.8), $\mathbf{B}(t)$ is advanced to $\mathbf{B}(t + \Delta t/2)$:

$$\mathbf{B}(t + \Delta t/2) = \mathbf{B}(t) - \frac{\Delta t}{2} \nabla \times \mathbf{E}(t)$$  \hspace{0.5cm} (3.13)
Chapter 3. Hybrid Model of Plasmas

(2) We see from Equation 3.9 that the electric field \( E(t) \) is a function of \( B(t), n_i(t) \) and \( J_i(t) \), then

\[
E(t + \Delta t/2) = F( B(t + \Delta t/2), n_i(t + \Delta t/2), J(t + \Delta t/2) )
\] (3.14)

where \( F \) denotes the function on the right-hand side of Equation 3.9.

(3) We predict both the electric field \( E_p \) and the magnetic field \( B_p \) at time \( t = t + \Delta t \):

\[
E_p(t + \Delta t) = 2E(t + \Delta t/2) - E(t)
\] (3.15)

\[
B_p(t + \Delta t) = B(t + \Delta t/2) - \frac{\Delta t}{2} \nabla \times E_p(t + \Delta t)
\] (3.16)

(4) The particles are advanced one time step using the predicted fields in order to collect the predicted sources \( \rho_p(t + 3\Delta t/2) \) and \( J_p(t + 3\Delta t/2) \).

(5) \( E_p(t + 3\Delta t/2) \) and \( B_p(t + 3\Delta t/2) \) are computed similar to step (3) using the predicted sources.

(6) Finally, \( E(t + \Delta t) \) and \( B(t + \Delta t) \) are determined:

\[
E(t + \Delta t) = \frac{E_p(t + 3\Delta t/2) + E(t + \Delta t/2)}{2}
\] (3.17)

\[
B(t + \Delta t) = B(t + \Delta t/2) - \frac{\Delta t}{2} \nabla \times E(t + \Delta t)
\] (3.18)

For more details refer to [3, 19].

3.3 Particle Injection and Boundary Conditions

We divide the 3D simulation domain into a Cartesian grid with cubic cells. Of course there are more methods, e.g., adaptive grids, but they are out of the scope of this thesis. One side of the simulation domain is assumed to be the inflow boundary and one or more sides can be assumed as outflow boundaries. The charged particles are injected into the simulation domain at the inflow boundary and are removed from the system at the outflow boundaries. We can also define some of the simulation boundaries as the periodic boundaries, which means that if a particle goes out for example from \(+x\) direction, it comes back into the system from \(-x\) direction. Particles are equally distributed in space at the inflow boundary with a Maxwellian velocity space distribution in a cell. The type of the
velocity distribution is arbitrary and they can have any type of the distributions mentioned in the previous chapter, but since the number of the particles in a cell are too few (usually in a 3D particle model are not more than a few hundreds), most distributions will be indistinguishable from a Maxwellian distribution.

When a particle model is applied to study the plasma interaction with an atmosphere-less object, e.g., the Moon, a critical boundary condition occurs at the surface of the object where the plasma impacts the surface. Since the plasma is mostly neutralized and absorbed by the surface, the impacted particles should be removed from the simulation domain. If they are removed suddenly at the moment they impact the surface, numerical oscillations can occur in the system and affect the solution. One method is to reduce the weight of the charged particles that hit the surface of the object by a factor $f_{obs}$ after each time step while they are inside the object (the mass and charge of each particle are reduced), as described in [20].

3.4 The Backward Liouville Method

Analyzing velocity space distributions in PIC models poses a problem. Accurate representation of the velocity space might require thousands of particles per cell, resulting in billions of particles in total for a simulation. Storing all these particles for later analysis of velocity space distributions in different regions is infeasible. Several approaches are possible to analyze velocity space distribution functions (VSDF) from such PIC simulations [2]. A solution that gives arbitrarily high velocity space resolution for a PIC method is to use the Backward Liouville (BL) method. The BL method has been used with electric and magnetic fields from fluid solvers [21, 22], but not with fields from PIC solvers. The idea behind the BL method is that the VSDF at any location can be computed by integration backward in time until we reach a position with known VSDF. This enhances the velocity space resolution and we can compute the VSDF at any arbitrary position in our simulation domain.
SOLAR WIND INTERACTION WITH THE MOON

As we previously mentioned, the solar wind interaction can be categorized into four different types: Earth type, Venus type, Lunar type and Comet type. Here, in this chapter, we focus on the Lunar type interaction and study the characteristics of the solar wind interaction with the Earth’s Moon.

The atomic and molecular particles on the lunar surface, which might come from lunar interior particle release, sputtering, chemical, thermal, and meteoritic sources, either reach the lunar escape velocity and leave the lunar exosphere, or collide and bond to the surface [23]. This creates a tenuous atmosphere/ionosphere around the Moon that is essentially collision-less.

Both the electrical conductivity and the surface magnetic field of the Moon were reviewed in [24]. The effect of the lunar conductivity appears as perturbations of the solar wind magnetic fields passing through the Moon and is evident in the lunar night–side. Ample observations at the lunar surface and on the lunar night–side during the Apollo era indicated that the IMF passes the lunar body almost unaltered, hence, the lunar conductivity is very low and can be neglected [24].

Small-scale lunar crustal magnetic anomalies which extend from a few kilometers altitude above the surface to tens of kilometers, with magnetic field strength from hundreds of nano-Tesla on the lunar surface to a few nano-Tesla at ~100 km altitude above the surface, will affect the lunar plasma environment [25–27].
Chapter 4. Solar Wind Interaction with the Moon

In summary, the Moon is a non-conductor and almost atmosphere-less body which has some crustal magnetic fields. As the solar wind passes the Moon, the solar wind plasma interacts directly with the lunar day–side, is absorbed and neutralized by the lunar surface, and forms a plasma void and a wake downstream at the night side of the Moon [28]. The morphology of the solar wind interaction with the lunar day–side is different than that of the night–side, therefore we explain them here separately, but we focus on the lunar night–side.

4.1 The Lunar Dayside-Plasma Interaction

As the solar wind protons impact the lunar surface, ∼1% remain charged and backscatter/reflect from the lunar surface [29] and ∼20% reflect as energetic neutral atoms (ENA) after they gained electrons [30]. The back-scattered protons from the lunar surface are picked-up by the solar wind convective electric field, accelerate to higher energies than the prevailing solar wind protons [29], and form a partial ring velocity distribution with large initial velocities in the velocity phase space [31]. These picked-up protons may penetrate deep into the lunar night–side and affect the local plasma there [32].

4.2 The Lunar Plasma Wake and its Structure

Solar wind plasma absorption by the lunar surface leaves a plasma cavity and forms a wake structure behind the Moon which is filled in through plasma thermal expansion and the effect of the space charge electric field [33, 34]. As the solar wind plasma expands into the wake, the magnetic field starts to gradually increase [20, 35] as a consequence of a diamagnetic current system generated by the pressure gradient across the wake boundary [36]. In addition, a rarefaction wave propagates outward perpendicular to the solar wind flow direction [20, 35, 37] with fast magnetosonic speed from the wake boundary due to the filling of the wake by electron and replacing them by undisturbed solar wind electrons [20, 37]. This forms the wake edges with decreased electric and magnetic fields and plasma density depletions [20, 33, 37, 38].

The electrons stream ahead of the ions into the wake, making charge separation and causing an electric potential difference between the plasma in the wake and
Chapter 4. Solar Wind Interaction with the Moon

the ambient solar wind [37]. The generated ambipolar electric field accelerates the ions into the wake to cancel the charge separation developed by electrons and that reduces the electric potential difference, increases the plasma density and decreases the electric and magnetic fields downstream in the wake [33].

The wake structure is affected by a large number of variables including the solar wind parameters [39, 40], IMF strength and orientation [20, 38, 40], electron and ion dynamics [33, 37], lunar crustal magnetic anomalies [27] and perhaps more unknown parameters.

Downstream from the Moon, further than 3 lunar radii ($R_L \approx 1730$ km), observations and simulations indicate that ion beams counterstream to refill the wake along the magnetic field lines with higher energies than the ambient solar wind energy [33, 38, 41] and with unstable distributions which grows ion-beam instabilities [37]. Electrostatic PIC simulation of the lunar wake shows that the ambipolar electric field extends only about 5$R_L$ behind the Moon and generates accelerated beams at the low altitudes ($< 5R_L$) that then are convected downstream in the wake. Significant electrostatic instabilities are evident far in the wake ($> 10R_L$) that ruptures the ion beams and increases their number densities [33, 37]. Moreover, observations show an extreme ion temperature anisotropy with $T_{\perp}/T_{\parallel} \sim 10$, where the directional subscripts denote directions relative to the background magnetic field, and proton distribution stable to the cyclotron instability in the far wake ($> 15R_L$) [42] which can be a result of the conservation of the adiabatic invariants [13, 14] and presumed to be a lunar wake feature [42].

Some of the downstream lunar wake features have also been observed at very low altitudes above the Moon. The enhanced magnetic fields at the central wake and reduced fields at the flanks, electron density depletion, electron temperature increase and electrostatic potential drop at the central wake were observed at low-altitudes ($\sim 100$ km) almost similarly to the downstream observations [40]. Moreover, the occasional occurrences of high energy protons in the low altitude lunar wake were also observed by Apollo 12 and 14 [43], Selene [Kaguya] [32, 44], Chandrayaan-1 [45] and Chang`E-1 [46] satellites. In contrast to the ion beams in the far tail, protons at low altitudes get access to the wake from both parallel and perpendicular directions to the IMF and make four different types of entries:

1. The gyrating solar wind protons enter the lunar wake perpendicular to the direction of the IMF as a result of ambipolar processes, known as Type-I entry [44].
2. Scattered protons from the lunar day-side are picked-up by the solar
wind and enter deep into the wake perpendicular to the IMF, known as Type-II entry [32]. (3) The scattered protons at lower deflection angles on the day-side are accelerated close to the polar terminator, perhaps by the same procedure as the Type-I, and enter the lunar night-side perpendicular to the magnetic field lines [46]. (4) The solar wind protons intrude into the wake along the magnetic field lines [45].

However, both the perpendicular and parallel entries of the solar wind protons into the wake at close distances to the Moon can be related to the ambipolar electric field around the Moon. Electric fields larger than 0.5 mV/m are needed around the lunar wake boundary to accelerate the solar wind protons to the energies $\gtrsim 1.3$ times higher than the solar wind energy. Such a large electric field at $\sim 100$ km altitude above the Moon, although assumed in some of the theoretical models [44, 47], have not been observed yet.

All in all, we see that the Moon-solar wind interaction is more complex than previously thought [27]. Considering the small scale lunar magnetic anomalies [25, 26], surface potential charging [48], physics of the lunar wake and its dependences on the solar wind parameters [20, 38, 40], existence of different wave modes in the wake [20, 49, 50] and their interaction with plasma, the occurrence of unstable plasma velocity distributions [34, 51, 52] and the generation of the instabilities [42] as well as many other features such as the lunar swirls [53] and formation of the magnetic anomalies [54, 55] are evidences for this complexity. Perhaps in the future, more accurate observations and simulations will help us to understand more details about the solar wind interaction with the Moon.
CHAPTER FIVE

SUMMARY OF PAPERS

Paper I

The interaction between the Moon and the solar wind.
This presents the solar wind interaction with the Moon using a three-dimensional hybrid model of plasma. First, the hybrid model is explained and then, the lunar wake structure and its dependences on the IMF direction are discussed in details. Finally, the results are compared with the WIND spacecraft observations and an agreement is shown between the model results and the observations.

Paper II

The effects of Lunar surface plasma absorption and solar wind temperature anisotropies on the solar wind proton velocity space distributions in the low-altitude Lunar plasma wake.
This paper describes the effects of the lunar surface plasma absorption and bi-Maxwellian solar wind protons VSDFs to change the solar wind protons moments of the distributions at 100 km altitude above the Moon in the lunar night-side. In this study, we used a self-consistent three-dimensional hybrid model of plasma but to improve the velocity space resolution we applied the Backward Liouville
algorithm. We compared the results with Chandrayaan-1 satellite observations and we discussed both the agreements and disagreements between the simulation results and observations.
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BIBLIOGRAPHY


Bibliography


Paper 1
The interaction between the Moon and the solar wind

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Abstract

We study the interaction between the Moon and the solar wind using a three-dimensional hybrid plasma solver. The proton fluxes and electromagnetic fields are presented for typical solar wind conditions with different magnetic field directions. We find two different wake structures for an interplanetary magnetic field that is perpendicular to the solar wind flow, and for one that is parallel to the flow. The wake for intermediate magnetic field directions will be a mix of these two extreme conditions. Several features are consistent with a fluid interaction, e.g., the presence of a rarefaction cone, and an increased magnetic field in the wake. There are however several kinetic features of the interaction. We find kinks in the magnetic field at the wake boundary. There are also density and magnetic field variations in the far wake, maybe from an ion beam instability related to the wake refill. The results are compared to observations by the WIND spacecraft during a wake crossing. The model magnetic field and ion velocities are in agreement with the measurements. The density and the electron temperature in the central wake are not as well captured by the model, probably from the lack of electron physics in the hybrid model.

1 Introduction

Bodies that lack a significant atmosphere and internal magnetic fields, such as the Moon and asteroids, can to a first approximation be considered passive absorbers of the solar wind. The solar wind ions and electrons directly impact the surface of these bodies due to

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The interaction between the Moon and the solar wind

The interaction between the Moon and the solar wind, the lack of atmosphere and the interplanetary magnetic field passes through the obstacle relatively undisturbed because the bodies are assumed to be non-conductive. Since the solar wind is absorbed by the body, a wake is created behind the object. This wake is gradually filled by solar wind plasma downstream of the body, through thermal expansion and the resulting ambipolar electric field, along the magnetic field lines. For a review of the Moon–solar wind interaction, including recent findings, see Halekas et al. (2010), who also point out the fact that we can view the near lunar space as a plasma laboratory. Although the Moon lacks any significant atmosphere and ionosphere there are many interesting features to this seemingly simple solar wind interaction. To understand the global interaction between the solar wind and the moon we need computer models that can give us a three-dimensional picture that complements the in-situ plasma observations.

Magnetohydrodynamic (MHD) modeling of the Moon–solar wind interaction has a long history, starting with the work of Spreiter et al. (1970), who analytically treated the special case of an interplanetary magnetic field (IMF) aligned with the solar wind flow direction by finding a transformation of the MHD equations into a hydrodynamic problem. However, there are many kinetic processes that cannot be described by fluid models, e.g., the non-Maxwellian particle populations in the wake region, so particle models should capture more of the physical processes. An approximation of the refill of the lunar wake is the general one-dimensional problem where plasma expands into a vacuum (Widner, 1970; Samir, 1983; Mora, 2003), and such models have specifically been applied to the refill of the lunar wake (Farrell, 1998; Birch and Chapman, 2001). A nice property of such models is that even if they are one-dimensional, they can to some degree approximate a two-dimensional model of the lunar wake, where time corresponds to distance downstream the wake, i.e., the one-dimensional model is applied perpendicular to the solar wind flow and convects with the flow. A disadvantage of one-dimensional models, e.g., along $x$, is that the magnetic field component along the $x$-axis is forced to be constant, from the requirement that $\nabla \cdot \mathbf{B} = 0$. This is a severe restriction for modeling the lunar wake, where the field component across the wake can increase. Due to the computational complexity, particle in cell (PIC) models, that include electrons as particles, are for the foreseeable future limited to at most two spatial dimensions, so what has been investigated using such models is the interaction of an infinite cylinder with the solar wind (Harnett and Wingglee,
The interaction between the Moon and the solar wind

Hybrid models, where ions are particles and electrons a fluid, are less computational demanding that full PIC models and are suitable for studying processes of size comparable to the ion inertial length and gyro radius. A two-dimensional hybrid model by Trávníček et al. (2005) focused mostly on wave activity in the lunar wake. The Moon–solar wind interaction is however a fully three-dimensional problem, since the IMF component perpendicular to the solar wind flow introduces asymmetry into an otherwise cylindrical symmetric problem. A three dimensional hybrid model by Kallio (2005) had a fairly coarse grid on a small region, and did not handle the wake region in a self consistent way (the electric field was specified in that region). Recently another three dimensional hybrid model has been presented in Wiehle et al. (2011). They compare observations by one of the ARTEMIS spacecrafts to a model run with time dependent inflow conditions, and find a good agreement. The time dependence enables comparison with observations even during changing solar wind conditions, but makes it more difficult to see how the wake forms differently depending on the solar wind conditions.

Here we also apply a three dimensional hybrid model to the Moon–solar wind interaction, but for steady inflow conditions. This allow us to study how the wake depends on the direction of the IMF, and in what follows we describe the method and present a comprehensive global overview of the different plasma quantities in the wake.

2 Model

In the hybrid approximation, ions are treated as particles, and electrons as a massless fluid. In what follows we use SI units. The trajectory of an ion, \( \mathbf{r}(t) \) and \( \mathbf{v}(t) \), with charge \( q \) and mass \( m \), is computed from the Lorentz force,

\[
\frac{d\mathbf{r}}{dt} = \mathbf{v}, \quad \frac{d\mathbf{v}}{dt} = \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}),
\]

where \( \mathbf{E} = \mathbf{E}(\mathbf{r}, t) \) is the electric field, and \( \mathbf{B} = \mathbf{B}(\mathbf{r}, t) \) is the magnetic field. The electric field is given by

\[
\mathbf{E} = \frac{1}{\rho} \left( -\mathbf{J}_f \times \mathbf{B} + \mu_0^{-1} (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p_e \right)
\]  

where

\[
\rho = \int \frac{1}{\mu} |\mathbf{v}| \, d\nu.
\]
where \( \rho_I \) is the ion charge density, \( J_I \) is the ion current, \( p_e \) is the electron pressure, and \( \mu_0 = 4\pi \cdot 10^{-7} \text{[Hm}^{-1}] \) is the magnetic constant. We assume that the electrons are an ideal gas, then \( p_e = n_e k T_e \), so the pressure is directly related to temperature (\( k \) is Boltzmann’s constant). There are several ways to handle the electron pressure (p. 8790, Winske and Quest, 1986). Here we assume that \( p_e \) is adiabatic (small collision frequency), with an adiabatic index, \( \gamma = 5/3 \). Then the relative change in electron pressure is related to the relative change in electron density by \( p_e \propto |\rho_e|^{\gamma} \) (Winske and Quest, 1986), and we have that

\[
\frac{p_e}{p_{e0}} = \left( \frac{n_e}{n_{e0}} \right)^\gamma,
\]

where the zero subscript denote reference values (here the solar wind values at the inflow boundary). From charge neutrality and \( p_e = n_e k T_e \) we can derive that

\[
p_e = A \rho_I^\gamma \text{ with } A = \frac{k}{\pi} \rho_I^{-\gamma} T_e
\]

where \( \gamma = 1 \) corresponds to assuming that \( T_e \) is constant, and \( \gamma = 0 \) gives a constant \( p_e \).

Finally, Faraday’s law is used to advance the magnetic field in time,

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}.
\]

We use a cell-centered representation of the magnetic field on a uniform grid. All spatial derivatives are discretized using standard second order finite difference stencils. Time advancement is done by a predictor-corrector leapfrog method with subcycling of the field update, denoted cyclic leapfrog (CL) by Matthews (1994). An advantage of the discretization is that the divergence of the magnetic field is zero, down to round off errors. Also, the discretization conserves energy very well (Holmström, 2011). The ion macroparticles (each representing a large number of real particles) are deposited on the grid by a cloud-in-cell method (linear weighting), and interpolation of the fields to the particle positions are done by the corresponding linear interpolation. Initial particle positions are drawn from a uniform distribution, and initial particle velocities from a drifting Maxwellian distribution. Further details on the hybrid model can be found in
The interaction between the Moon and the solar wind

Holmström (2010).

The implementation of the algorithm was done in the FLASH software framework, developed at the University of Chicago (Fryxell et al., 2000), that implements a block-structured adaptive (or uniform) Cartesian grid and is parallelized using the Message-Passing Interface (MPI) library for communication. Further details on the FLASH hybrid solver can be found in Holmström (2011). The general solver will be included in future releases of FLASH.

A crucial point in modeling the interaction between the Moon and the solar wind is how to choose the inner boundary conditions, at the surface of the moon. Here we model the Moon as an absorber of solar wind. Ideally we should then remove all ions that hit the surface of the Moon, but that would introduce a discontinuity in the charge density, from solar wind values outside the sphere, to zero inside the sphere. However, numerical differentiation of discontinuous functions on a grid will lead to oscillations and instabilities in the fields. Therefore we choose to keep also ions inside the Moon, but we gradually reduce their weight by a factor $f_{obs}$ after each time step while they are inside the Moon (the mass and charge of each particle are reduced). Thus, the Moon acts as a gradual sink for ions, and this approach also avoids the problem of $\rho_I = 0$ in Eq. 1. However, we do not plot these reduced ions in what follows, only protons that have never hit the Moon are used when computing the various statistics. This can be seen as a form of smoothing, done in a self-consistent way. Often smoothing of fields is used in hybrid solvers, but it is then done in an ad-hoc way, as an extra step. The downside of our approach is that we introduce a numerical parameter, but experience shows that the solution will not change by much when reducing $f_{obs}$, indicating that the solution is converging. The reduction parameter cannot be chosen too small however, since a steep gradient in ion density will cause oscillations in the magnetic field. We have verified numerically that this approach keeps the IMF constant inside the Moon, and thus that no spurious currents are present there. There are no inner boundary conditions on the electro-magnetic fields.

The coordinate system is centered at the Moon, with the $x$-axis is directed toward the Sun, so the solar wind flows opposite to the $x$-axis. The IMF is in the $xy$-plane, and we will from now on denote it as the IMF plane. This defines a plane of symmetry, since the only asymmetry in the problem stems from a non-zero component of the IMF perpendicular to
The interaction between the Moon and the solar wind

Figure 1: The proton number density for an IMF along the direction (1, 1, 0). This also illustrates the coordinate system and the simulation geometry. The cuts are the planes $x = -10000$ km, $x = -30000$ km, $y = 0$, and $z = 0$. These cutting planes are the same that later are used in Fig. 2-4.

The $x$-axis. The solar wind consists only of protons (and the massless electron fluid), and we have a rectangular simulation domain divided into cells. The $+x$-face of the domain is an inflow boundary where we have a layer of shadow cells outside the domain that are filled with macroparticles drawn from the solar wind proton velocity distribution, and where the solar wind magnetic field is set. The $-x$-face is an outflow boundary where particles exiting the domain are removed, and where the magnetic field is extrapolated from the interior. The four other faces have periodic boundary conditions for fields and particles.

The simulation domain is divided into a Cartesian grid with cubic cells of size $160$ km. The geometry of the simulation domain is shown in Fig. 1. The extent of the simulation box is $[-32.40, 4.08] \times [-8.32, 8.32] \times [-8.32, 8.32] \cdot 10^3$ km $\approx [-18, 2.4] \times [-4.8, 4.8]^2 R_L$, with the Moon as a sphere of radius $R_L = 1730$ km at the origin. The total number of simulation macroparticles is about 300 million, with 121 particles per cell on average.

Here we have used values of $\gamma = 5/3$, $f_{obs} = 0.965$, a time step of 0.08 s, and a final time of 170 s, when the solution has reached a steady state. The plots in what follows show the solution at the final time. Five subcycles of the CL algorithm are used here when updating the magnetic field, and the execution time is about 13 hours on 384 CPU cores.
3 Results

In what follows, we have used typical solar wind conditions at 1 AU (Table 4.1, Kivelson and Russel, 1995), unless otherwise noted. The solar wind velocity is 450 km/s, the number density is 7.1 cm$^{-3}$, and the ion temperature is $1.2 \cdot 10^5$ K. The electron temperature is $1.4 \cdot 10^5$ K, and the IMF is in the $xy$-plane with a magnitude of 7 nT. For these plasma parameters, the ion inertial length is 85 km, the Alfvén velocity is 58 km/s, the thermal proton gyro radius is 66 km (the gyro time is 9 s), and the ion plasma beta is 0.6.

3.1 Overview

In Fig. 2 the number density (three first rows) and the ion velocities (three last rows) are shown, each for three different IMF directions (at 0, 45 and 90 degrees to the $x$-axis, in the $xy$-plane). The most obvious feature of the interaction between the Moon and the solar wind is that a wake is formed behind the Moon, visible as a region of low density. We also see how the refill of the wake depends on the IMF direction. The wake refills along the magnetic field lines, making the wake shorter when the component of the IMF that is perpendicular to the solar wind is larger (Fig. 2.1). When the IMF is anti-parallel to the solar wind flow there is hardly any refill of the wake over the length of the simulation domain, also the flow then is cylindrical symmetric around the Moon–Sun axis. That the wake refill occur along the magnetic field lines is also visible in the cuts of the far wake (Fig. 2.1d), with the refill region as a rectangle aligned with the IMF.

Another notable feature is the density rarefaction cone that emanate from the terminator region. This region of low density propagate mostly above the poles in the case of a perpendicular IMF (Fig. 2.1), indicating that it propagates perpendicular to the magnetic field lines. This will be discussed in more detail when we look at the associated reduction in magnetic field strength (Fig. 3).

We now examine the ion velocities, computed from the ion current and the charge density as $\mathbf{J}_i/\rho$, as is usual for particle in cell models, since the ion current density and the ion charge density are quantities that are available on the computational grid. The small white region downstream of the Moon is a region without any particles that has not hit the Moon, thus no velocities can be computed there. The $x$-velocity is shown as the deviation from the solar wind velocity, when the IMF is perpendicular to the flow, in
The interaction between the Moon and the solar wind

Figure 2: Proton number density and velocities in different planes, for different upstream IMF conditions. Arrows show the direction of the IMF. Top three rows show number density for different upstream IMF directions. Along the directions \((x, y) = (0, 1), (1, 1),\) and \((1, 0),\) respectively. The geometry is repeated in the lower three rows \((4-6)\) for the proton velocities \(u_x - u_{sw}, u_y,\) and \(u_z.\) The columns are from the left, cuts in the planes \(z = 0\) (seen from \(+z\)), \(y = 0\) (seen from \(+y\)), \(x = -10000\ km,\) and \(x = -30000\ km\) (seen from \(+x\)). Fig. 1 provides a three-dimensional illustration of the cutting planes. The two vertical lines in 1-3b show the position of the cuts perpendicular to the \(x\)-axis.

Fig. 2.4. We see a slight slow down of the solar wind in the rarefaction cone. There is also a slow down behind the \(+z\) hemisphere (and a speed up behind the \(−z\) hemisphere) seen in Fig. 2.4b. This is probably a geometric effect, where part of the velocity distribution function is emptied by collisions with the lunar surface. The \(y\)-velocity (Fig. 2.5) shows how the wake symmetrically refills from both sides of the wake. The ions move inward not only in the central part of the wake, but also in the rarefaction cone. For the case of an aligned IMF, the \(z\)-velocity (Fig. 2.6) also shows a flow in the rarefaction cone toward the wake center. There we also see a flow of ions at the wake boundary in the IMF plane, and there seem to be a small kinetic effect of alternating upward and downward going ions in
the upper and lower part of the wake (Fig. 2.6b). This seems associated with the wave pattern in density inside the rarefaction cone (Fig. 2.1-3), possibly caused by oscillations at the wake boundary.

The magnetic field magnitude (three first rows) is shown for three different IMF directions in Fig. 3, where also each of the vector components (rows 4-9) are shown for two different IMF directions (perpendicular and aligned to the solar wind flow).

The most prominent feature is the enhanced magnetic field in the wake. For a perpendicular IMF (Fig. 3.1) this region spreads out in the IMF plane, in the same region where we in the density saw the wake refill. For the aligned IMF the magnetic field increase is larger, and confined to the narrow wake (Fig. 3.3). There are also structured fluctuations in the far wake region in the IMF plane. Then there is a cone of lower field strength in the case of an aligned IMF (Fig. 3.3), corresponding to the cone of reduced density (Fig. 2.3). For the perpendicular IMF the cone is partial, and only extends perpendicular to the IMF, toward +z and −z (Fig. 2.1). Between the central region of enhanced field and the cone of lower field, there are field fluctuations parallel to the cone, as also seen in the density (Fig. 2.1b, 2.2b, 2.3b). Examining these density plots, the waves seem to originate at the wake boundary and could be caused by oscillations at this interface between high and low ion density.

The magnetic field z-component for the case of a perpendicular IMF (Fig. 3.8) exhibits a pinching of the field, consistent with earlier predictions (Fig. 3, Owen, 1996) and model observations (Kallio, 2005). The field is bent toward the IMF plane in the wake (Fig. 3.8c, 3.8d). However, the x-component of the magnetic field is also perturbed (Fig. 3.4). Close to the IMF plane, the field is bent toward the moon on one side of the wake, and away from the moon on the other side of the wake (Fig. 3.4a). Note also that these bends in the magnetic field are not confined to central parts of the wake, but expand along with the rarefaction in magnetic field (Fig. 3.1d) and density (Fig. 2.1d). Thus, for a perpendicular IMF the magnetic field in the wake is not only bent toward the IMF plane, but also toward the moon, as shown in Fig. 4.

The Mach cone of decreased density and magnetic field makes an angle to the solar wind flow direction of about 10 degrees, as seen in Fig. 2 and Fig. 3. Since the outer edge of the cone is diffuse, it is impossible to give a more exact angle. The ion-acoustic wave
Figure 3: The magnetic field magnitude, and components in different planes, for different upstream IMF conditions. Arrows show the direction of the IMF. Top three rows show magnetic field magnitude for different upstream IMF directions. Then the magnetic field $x$-component for an IMF perpendicular to and opposite the solar wind flow is shown on row 4 and 5. Similarly, the $y$-component is shown on row 6 and 7, and the $z$-component on row 8 and 9. The geometry of the cuts in the different columns are the same as in Fig. 2. The colorbar is for the row immediately to the left, and for all following rows, until the next colorbar.
The interaction between the Moon and the solar wind

Figure 4: Magnetic field lines 10000 km behind the Moon for an IMF along the y-axis (same as in Fig. 3.1). (a) seen along the +z-axis, and (b) along the -z-axis. The color scale shows the magnitude of the magnetic field similarly to Fig. 3. To enhance the bending of the field the magnetic field x-component was multiplied by 4, and the z-component by 2, before drawing the field lines.

The velocity is \( v_S \approx \sqrt{2\gamma k_B T_e/m_i} \approx 58 \text{ km/s} \), and the Alfvén velocity is also \( v_A \approx 58 \text{ km/s} \) for these solar wind conditions, corresponding to angles of 7 degrees. We see that the density depletion travel perpendicular to the magnetic field. This together with the fact that the propagation velocity is larger than both the ion-acoustic and the Alfvén velocity suggests that the fast magnetosonic wave, with a phase velocity of \( \sqrt{v_A^2 + v_S^2} \) leading to an angle of 10 degrees, is responsible for the creation of the Mach cone in the hybrid model. This is consistent with the observation of Wiehle et al. (2011) that all three MHD modes (fast, Alfvén and slow) will be manifested in the lunar wake.

In Fig. 5 the electric field magnitude (top row) is shown along with the three vector components (three lower rows) for the case of an IMF perpendicular to the solar wind flow direction. What is shown is the electric field with the solar wind convective electric field, \( \mathbf{E}_{sw} = -\mathbf{u}_{sw} \times \mathbf{B}_{sw} \) (directed along the z-axis) subtracted. We do not show the electric field for the case of an IMF aligned with the x-axis, since it is indistinguishable from zero using the same color scale.
The interaction between the Moon and the solar wind

Figure 5: The electric field magnitude, and the three components in different planes, for an IMF that is perpendicular to the solar wind flow direction. Arrows show the direction of the IMF. For the magnitude and z-component, the solar wind convective electric field, \( \mathbf{E}_{\text{sw}} = -\mathbf{u}_{\text{sw}} \times \mathbf{B}_{\text{sw}} \), has been subtracted. The cuts in the different columns are the same as in Fig. 2 and 3.

We can note that the regions of enhanced electric field magnitude coincide with those of enhanced magnetic field magnitude shown in Fig. 3.1. Also, there are similarities in the components of the electric and magnetic fields. \( E_y \) is very similar to \( B_z \) (Fig. 3.8), and correspondingly \( E_x \) is similar to \( B_y \) (Fig. 3.8).

3.2 Comparison with WIND observations

We here compare the hybrid model results with observations by the WIND spacecraft on December 27, 1994, when it traversed the lunar wake approximately perpendicular to the solar wind flow, in the plane of the IMF, at a distance of \( x = -6.5 \) R\(_L\) \( \approx -11300 \) km. Ogilvie et al. (Fig. 1, 1996) plots several plasma parameters during the wake crossing. Apparently the solar wind conditions changed during the observation. This is most apparent in the plasma number density that decreased by more than a factor of two from the undisturbed solar wind on the inbound trajectory to the solar wind on the outbound trajectory. Therefore we make two runs of the hybrid model for the two sets of solar wind conditions, here denoted before and after. The solar wind conditions used for the before (after) case are as follows. A solar wind velocity of 470 (500) km/s, a number density of
The interaction between the Moon and the solar wind

5 (2) *cm*⁻³, an ion temperature of 0.9 (0.5) *·*10⁵ K, and an electron temperature of 1.5 (2) *·*10⁵ K. The IMF was (6.39, 2.33, 0) nT for the before case, and (5.57, 5.02, 0) nT for the after case. The time step was 0.08 (0.05) s and *f*₁₈₆ = 0.98(0.985). Since the plasma parameters are different in the two cases, these numerical parameters have different values to ensure the stability of the solutions.

![Graph showing the hybrid model magnetic field magnitude (nT) along y at x = -6.5 Rₜ = 11245 km for two different upstream solar wind conditions labeled before and after, compared to WIND observations from Ogilvie et al. (Fig. 1, 1996). The vertical lines show the location of the optical shadow, and the x-axis show time on December 27, 1994, as in the original plot.]

Figure 6: The hybrid model magnetic field magnitude (nT) along y at x = -6.5 Rₜ = 11245 km for two different upstream solar wind conditions labeled before and after, compared to WIND observations from Ogilvie et al. (Fig. 1, 1996). The vertical lines show the location of the optical shadow, and the x-axis show time on December 27, 1994, as in the original plot.

In Fig. 6 we show the model magnetic field magnitudes, compared with the observation by WIND. The observed magnetic field magnitude matches well to the model values, if we assume that there was a change in the upstream solar wind conditions around wake entry (there we switch from comparing the before model results to comparing the after model results with the observation). Before the wake crossing there is a drop in the observed magnetic field strength that is also seen in the model. This is the rarefaction wave that we saw in Fig. 3 earlier. The IMF in the before case has the largest component along the x-axis and therefore the magnetic field magnitude should best match Fig. 3.3a, with a pronounced rarefaction cone. Then there is an increase in the central wake which is reproduced in the after model. Followed by a gradual decrease at wake exit, but no rarefaction wave. For the after case the IMF is at about 45° away from the x-axis and thus correspond to Fig. 3.2a, with a much weaker rarefaction cone. In the observation

I–13
The interaction between the Moon and the solar wind

there are some large variations further away from the wake that are not present in the model. This could be upstream solar wind disturbances.

In Fig. 7 the proton velocity observed by WIND is compared to the velocity in the two hybrid model runs. Again we have fairly good agreement between the model and observations, assuming a shift in solar wind conditions around wake entry. There are more variations in the observed velocities, but the locations of the velocity changes, and the slopes match the model. In the central wake there is however no data from the model since the model density is too low. This can be seen when we compare the proton number density from the model with observations in Fig. 8.

We see that the general features of the observed density profile across the wake is found in the model, including a shift of the minimum density toward the outbound part of the trajectory, if we assume the shift in solar wind conditions around wake entry. However, the drop off in density is quicker in the hybrid model, and is much below the observations in the central part of the wake. The cause of this could be the incomplete treatment of electron dynamics in a hybrid model. Since the electrons are a massless charge neutralizing fluid, there are no forces from the electrons on the ions, except for the electron pressure gradient term in the definition of the electric field (Eq. 1). In reality, the refill of the
wake is governed by the electrons. Due to their higher thermal velocity they will speed ahead of the ions to refill the wake, setting up an ambipolar electric field due to the charge separation that will accelerate the ions into the wake (Halekas et al., 2010).

Similarly, the lack of electron dynamics in the hybrid model will affect the electron temperature, $T_e$, in the wake. Observations show an increased electron temperature in the wake (Fig. 1, Ogilvie et al., 1996). But from Eq. 2, and the ideal gas law, $T_e \sim \rho_i^{-1} = \rho_i^{2/3}$ for $\gamma = 5/3$, so $T_e$ in the hybrid model is then lower in the wake.

### 4 Discussion

Since the only deviation from axial symmetry is the component of the IMF that is perpendicular to the solar wind flow, we can see that the two extreme cases are when the IMF is perpendicular to the flow, and when the IMF is aligned with the flow. For a constant IMF magnitude, the flow and field for cases when the IMF is at an angle to the flow will be a mixture of these extreme cases, as seen in Fig. 2.2 and Fig. 3.2.

There are many processes related to electron dynamics that are not captured by a hybrid model. The process of plasma expansion into a vacuum has a lot of details, and instabilities that can form (Birch and Chapman, 2001). Also, the region near the lunar surface probably has strong electric fields, and charge separation effects (Farrell et al.,
The interaction between the Moon and the solar wind

2008). However using the hybrid approximation it is possible to model the global three-dimensional interaction between the Moon and the solar wind, so full PIC models and hybrid models are complementary tools in the study of this interaction. Also, the Moon–solar wind interaction, although seemingly simple, is complex enough to be a good test of the applicability of different models; fluid, hybrid, and PIC models.

We have here considered the simplest possible model of the Moon in the solar wind — a spherical sink of solar wind protons. There are many physical processes that make the real interaction more complex than that. Two examples are magnetic anomalies that deflect the solar wind and will cause downstream disturbances (Lue et al., 2011), and the reflection of solar wind protons by the dayside lunar surface (Holmström et al., 2010).

5 Summary and conclusions

We have presented a fully self consistent three-dimensional hybrid plasma model of the interaction between the Moon and the solar wind and shown the resulting global ion fluxes and fields for typical solar wind conditions and different IMF directions. Magnetic field enhancement in the wake, and a rarefaction wave in density and magnetic field are seen in the model. The two extreme cases are that of an IMF perpendicular to the solar wind flow, and that of an IMF that is parallel to the solar wind flow. We find that the wake for intermediate IMF directions can be seen as a superposition of these two states. Some kinetic effects were also observed in the model. There are density and field variation in the far wake, maybe caused by an ion beam instability from the refill of the wake. Visible are also kinks in the magnetic field at the inner boundary of the rarefaction cone for an IMF perpendicular to the solar wind flow direction.

Observations by the WIND spacecraft were fairly well reproduced by the hybrid model, showing that a hybrid model with the moon as a spherical sink of ions is able to represent many global features of the interaction. However, the model central wake density and electron temperature were different from observations, probably due to the lack of electron dynamics in the hybrid model.
The interaction between the Moon and the solar wind

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References


The interaction between the Moon and the solar wind


Paper II
The effects of Lunar surface plasma absorption and solar wind temperature anisotropies on the solar wind proton velocity space distributions in the low-altitude Lunar plasma wake

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Abstract

We study the solar wind proton velocity space distribution on the lunar night-side at low-altitudes (∼100 km) above the lunar surface using a three-dimensional hybrid plasma solver when the Moon is in the unperturbed solar wind. As the solar wind plasma expands in the interplanetary medium, solar wind proton velocity space distributions depart from isotropic to anisotropic distributions. Moreover, when the solar wind encounters a passive obstacle, such as the Moon, without any strong magnetic field and no atmosphere, solar wind protons that impact the obstacle’s surface are absorbed and removed from the velocity space distribution function. We examine the effects of a solar wind bi-Maxwellian velocity space distribution function and the lunar surface plasma absorption on the solar wind protons entry in the direction parallel to the interplanetary magnetic field lines into the low-altitude lunar plasma wake. We model the proton velocity space distributions there and compare with observations by the SARA (Sub-keV Atom Reflection Analyzer) instrument on board Chandrayaan-1. We also present a Backward Liouville method for particle-in-cell solvers that improves velocity space resolution. We show that the model can explain the reported observations of higher proton energy and lower density than the ambient solar wind in the low-altitude lunar wake. Additionally, a large temperature anisotropy is found at close distances to the Moon on the lunar night-side as a consequence of the lunar surface plasma absorption effect.

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1 Introduction

The Moon is a non-conducting obstacle to the solar wind plasma without any considerable atmosphere, and no global magnetic fields but small scale crustal magnetic anomalies. The solar wind plasma is absorbed by the lunar surface, creating a plasma void and forming a wake downstream the night side of the Moon [Lyon et al. 1967].

Downstream from the Moon, further than 3 lunar radii, \( R_L \), where \( R_L \approx 1730 \) km, observations and simulations indicate low density accelerated ion beams refilling the wake along the interplanetary magnetic field lines (IMF) [Ogilvie et al. 1996, Farrell et al. 1998, Halekas et al. 2011], which can be explained through one-dimensional (1-D) plasma expansion theory [Ogilvie et al. 1996, Halekas et al. 2011].

Likewise, Futaana et al. [2010] using the SARA instrument on-board the Chandrayaan-1 satellite observed a low density population of solar wind protons at 100 km altitude above the surface of the Moon on the lunar night-side accelerating into the wake parallel to the IMF with higher energy than the prevailing solar wind plasma. High energy protons at close distances to the Moon were also observed recently by the SELENE (Kaguya) and Chang’E-1 satellites, but those protons are either back-scattered/reflected protons from the lunar surface or solar wind protons entering the wake perpendicular to the IMF direction as a consequence of the ambipolar electric field and/or gyration around the magnetic field lines [Nishino et al. 2009a;b, Wang et al. 2010].

Futaana et al. [2010] discussed that the prediction of 1-D plasma expansion theory was not enough to explain the parallel entry of high energy protons at low-altitudes in the wake. They proposed that this disagreement might be an effect of the lunar surface plasma absorption and lunar surface electric potential. However, the processes that explain the solar wind proton entries parallel to the IMF at low-altitudes lunar wake are still an open question. Therefore, the impetus for the present work is to examine possible mechanisms using a self-consistent 3-D hybrid model of plasma [Holmström 2010]. In practice, the number of particles per cell in a particle model at close distances to an obstacle’s night-side are too small to examine the particles kinetic properties there. Consequently, to improve the statistics and enhance the velocity space resolution, a Backward Liouville (BL) method [Marchand 2010] is applied for the first time to a particle model, where the electric and magnetic fields of the interaction region are obtained from the hybrid model.
simulation results, and ion trajectories are computed backward in time using these fields.

We also investigate the effect of non-isothermal (bi-Maxwellian) solar wind velocity space distribution functions (VSDF) and the role of lunar surface plasma absorption on low-altitude proton VSDF in the lunar wake. A bi-Maxwellian VSDF has temperature ratio of $R \neq 0$ where $R$ is defined by the temperatures parallel ($T_{||}$) and perpendicular ($T_{\perp}$) to the background magnetic field as $R = T_{\perp}/T_{||} - 1$ [Gary et al. 2001, Kasper et al. 2003]. Gary et al. [2001] using ACE spacecraft observations at $\sim$1 AU showed that 40% to 80% of the time $R < 0$ in the solar wind. They also showed that there are constraints on the values of $R$ due to plasma instabilities.

Solar wind electrons and ions impact the lunar surface and are absorbed and removed from their respective VSDFs [Whang 1968]. This generates a density depletion in the lunar wake and an associated charge separation between electrons and ions which creates an electrostatic potential and drives an electric current in the wake [Bale 1997, Bale et al. 1997]. Recently, Harada et al. [2010] using SELENE (Kaguya) data discussed the existence of a strong electric field around the Moon as a consequence of the lunar surface plasma absorption on the electron VSDF.

Accounting for both the solar wind bi-Maxwellian VSDF and the lunar surface plasma absorption, we study the intrusion of the solar wind protons into the wake with higher velocities than the ambient solar wind velocity parallel to the IMF and compare the simulation results with Chandrayaan-1 observations.

2 Model

Modeling of collision-less plasmas are often done using fluid magnetohydrodynamics (MHD) models. However, the MHD fluid approximation is questionable when the gyro radius of the ions are not small compared to the spatial region that is studied. Also, since fluid models do not resolve velocity space, they cannot accurately represent processes such as plasma-vacuum expansion and counter-streaming ion beams, that occur near the Moon. On the other hand, kinetic models that discretize the full velocity space, or full particle-in-cell (PIC) models that treat ions and electrons as particles, are too computational expensive for global 3-D modeling of the Moon-solar wind interaction. For problems where the ion time- and spatial scales are of interest, hybrid models provide a compromise. In
such models, the ions are treated as discrete particles, while the electrons are treated as a fluid (often massless). This means that the electron time- and spatial scales do not need to be resolved, and enables us to model the solar wind interaction with the Moon using currently available computational resources.

2.1 The Hybrid Equations

In the hybrid approximation, ions are treated as particles, and electrons as a massless fluid. In what follows we use SI units. The trajectory of an ion, \( r(t) \) and \( v(t) \), with charge \( q \) and mass \( m \), is computed from the Lorentz force,

\[
\frac{dv}{dt} = q(E + v \times B), \quad \frac{dr}{dt} = v \tag{1}
\]

where \( E = E(r,t) \) is the electric field, and \( B = B(r,t) \) is the magnetic field. The electric field is given by

\[
E = \frac{1}{\rho_I} \left( -J_I \times B + \mu_0^{-1} (\nabla \times B) \times B - \nabla p_e \right) + \frac{\eta}{\mu_0} \nabla \times B \tag{2}
\]

where \( \rho_I \) is the ion charge density, \( J_I \) is the ion current, \( p_e \) is the electron pressure, \( \eta \) \([\text{Sm}^{-1}]\) is the resistivity, and \( \mu_0 = 4\pi \cdot 10^{-7} \) \([\text{Hm}^{-1}]\) is the magnetic constant.

We assume that the electrons are an ideal gas, then \( p_e = n_e k_b T_e \), so the pressure is directly related to temperature \( (k_b \text{ is Boltzmann’s constant}) \), and we assume that \( p_e \) is adiabatic (small collision frequency), with an adiabatic index, \( \gamma = 5/3 \). Then the relative change in electron pressure is related to the relative change in electron density by \( p_e \propto |\rho_e|^\gamma \). Finally, Faraday’s law is used to advance the magnetic field in time,

\[
\frac{\partial B}{\partial t} = -\nabla \times E.
\]

Further details on the hybrid model can be found in Holmström [2010], and on the implementation in Holmström [2011a].

A crucial point in modeling the interaction between the Moon and the solar wind is how to choose the inner boundary conditions, at the surface of the Moon. Here we model the Moon as an absorber of the solar wind, and reduce the weight of the ion macro-particles that hit the surface of the moon by a factor \( f_{obs} \) after each time step while they
Modeling the Solar Wind Protons in the Low-Altitude Lunar Wake

are inside the Moon (the mass and charge of each particle are reduced), as described in Holmström et al. [2011b]. Thus, the Moon acts as a gradual sink for ions. However, only ions that have never hit the Moon are used when computing the various ion statistics in what follows.

2.2 Coordinate System and Simulation Box

The coordinate system, as shown in Figure 1, is centered at the Moon, with the $x$-axis directed towards the Sun, so the solar wind flows opposite to the $x$-axis. The $+y$-axis lies at a right angle to the $x$-axis and the $z$-axis completes right handed system. We also define longitude ($\lambda$) and latitude ($\Lambda$) in this coordinate system. Longitude ($0^\circ \leq \lambda < 360^\circ$) is the vector projected angle on the $xy$-plane measured from $+x$-axis counterclockwise when viewed from $+z$-axis and latitude ($-90^\circ \leq \Lambda \leq +90^\circ$) is the angle between the $xy$-plane and the vector.

In what follows, we assume that the IMF is in the $xy$-plane, and we will from now on denote it as the IMF plane. This defines a plane that breaks symmetry, since the only asymmetry in the problem stems from a non-zero component of the IMF perpendicular to the $x$-axis. The solar wind consists only of protons (and the massless electron fluid), and we have a rectangular simulation domain divided into cells. The $+x$-face of the domain is an inflow boundary where we have a layer of shadow cells outside the domain that are filled with macro-particles drawn from the Maxwellian solar wind proton VSDF, and where the solar wind magnetic field is set. The $-x$-face is an outflow boundary where particles exiting the domain are removed, and where the magnetic field is extrapolated from the interior. The four other faces have periodic boundary conditions for fields and particles.

The simulation domain is divided into a Cartesian grid with cubic cells of size 62.5 km. The simulation box, as shown in Figure 1, is a cube with 5000 km sides ($\approx 2.9 R_L$), with the Moon as a sphere of radius $R_L = 1730$ km at the origin. The total number of simulation macro-particles is about 100 million, with $\sim 200$ particles per cell on average. Here we have used values of $\gamma = 5/3$, $f_{obs} = 0.984$, a time step of 0.05 s, and a final time of 30 s, when the solution has reached a steady state. Five subcycles are used here when updating the magnetic field.
Figure 1: Illustration of coordinate system and simulation geometry. The solar wind is flowing along the $-x$-axis and the IMF points towards $+y$-axis and is perpendicular to the solar wind velocity. The direction and magnitude of the solar wind and IMF are constant and do not change during the simulation. The $xy$-plane is called the IMF plane here. Two blue curves show the simulated orbits at 100 km altitude above the lunar surface on the lunar night-side: orbit (A) is perpendicular to the IMF plane, $50^\circ$ behind the terminator and represents the Chandrayaan-1 orbit [Futaana et al. 2010] and orbit (B) is in the IMF plane.

2.3 A Backward Liouville Algorithm

Analyzing velocity space distributions in PIC models poses a problem. Accurate representation of the velocity space might require thousands of particles per cell, resulting in billions of particles in total for a simulation. Storing all these particles for later analysis of velocity space distributions in different regions is infeasible. Several approaches are possible to analyze VSDFs from such PIC simulations [Ledvina et al. 2008]. One can in advance decide on certain locations of interest, and only store particles in the vicinity of these locations (virtual probes). The disadvantage is that to get the VSDF at any
other location requires a new run of the simulation, with the new locations for collecting particles.

Another approach is to use test particle in the fields computed by the PIC solver. This assumes that the fields are in a steady state (or the fields at all time steps need to be stored). The disadvantage is that a large amount of test particles need to be launched to get good resolution of the velocity space at any specific location [Ledvina et al. 2008].

A related problem is that how well the hybrid method resolves velocity space depends on the number of macro-particles per cell that are used. This poses a problem in regions of low density, since if we use equally weighted macro-particles the number of particles in a low density cell will be small, thus giving a low resolution of velocity space, e.g., using 100 particles per cell in the solar wind, in a region with a density of 1/100 of the solar wind density we will have about one macro-particle per cell.

A solution that gives arbitrarily high velocity space resolution for a PIC method, without storing any particles, is to use the Backward Liouville (BL) method. The BL method has been used with electric and magnetic fields from fluid solvers [Mackay et al. 2008, Marchand 2010], but not with fields from PIC solvers. The idea behind the BL method is that the VSDF at any location can be computed by integration backward in time until we reach a position with known VSDF. For charged particles the trajectory integration requires that we know the electric and magnetic fields along the trajectory, e.g., from a hybrid plasma solver. We have here assumed that the plasma flow has reached a steady state and we only use the fields at one time to compute the trajectories. If that assumption is not valid it would be required that the fields at every time step is available. The steady state assumption can be tested by repeating the computations using the fields from several different times, to verify that the solution does not change significantly. This has been verified for the runs done in this paper.

More precisely, the BL method used can be stated as follows. A particle trajectory is given by the particle’s position and velocity as a function of time, \( r_i(t) \) and \( v_i(t) \). The phase space density (VSDF) along the trajectory is \( f(r_i(t), v_i(t)) \) \([m/s]^{-3}m^{-3}\). Assume a particle production function, \( P(r, v) \) \([m/s]^{-3}m^{-3}s^{-1}\], that can be a total sum of different source and loss processes. From now on we will use the notation \( f(t) = f(r_i(t), v_i(t)) \) and \( P(t) = P(r_i(t), v_i(t)) \) as shorthand for these quantities along a trajectory. The change in
$f$ along a trajectory during a time step $\Delta t$ is

$$f(t + \Delta t) = f(t) + \Delta t \, P(t), \text{ so } \frac{df}{dt} = P.$$  

Thus, the distribution function at times $t = T_0$ and $t = T_1$ are related by

$$f(T_1) - f(T_0) = \int_{T_0}^{T_1} P(t) \, dt.$$  

This implies that if we seek the distribution function at time $T_1$, we can integrate the production function backward in time along the trajectory to a point where the distribution function is known, at $t = T_0$, e.g., at an inflow boundary. Then the distribution function at $t = T_1$ is given by, written out in full,

$$f(r_i(T_1), v_i(T_1)) = f(r_i(T_0), v_i(T_0)) + \int_{T_0}^{T_1} P(r_i(t), v_i(t)) \, dt. \tag{3}$$

In our case at hand there is no production of ions, then

$$f(r_i(T_1), v_i(T_1)) = f(r_i(T_0), v_i(T_0)) \tag{4}$$

So we simply start the backward in time trajectory integration at the velocity space position of interest and integrate until we reach a position where we know the distribution function. In our present study the known distributions are at the solar wind inflow boundary (e.g., Maxwellian) and at the Lunar surface (zero outgoing flux).

By integrating the VSDF in velocity space we can compute any moments of the velocity distribution at any position using the BL method. To numerically integrate the distribution function, we define a Cartesian grid in velocity space with cell centers at $v_{i,j,k} = (i\Delta v_x, j\Delta v_y, k\Delta v_z)$, where $i$, $j$, and $k$ are integers, chosen such that the grid covers the region of velocity space where the VSDF is non-zero, or larger than some threshold value. Thus, velocity space is divided into cells of volume $\Delta v_x \times \Delta v_y \times \Delta v_z$ and we can compute approximate moments of the VSDF by summation over this grid. The value of the VSDF at each cell center, $f(r, v_{i,j,k})$, is approximated using Eq. (4) where we numerically integrate backward in time until we reach a point with known VSDF.
In particular, the first three kinetic moments are approximated by

\[ n(r) \approx \sum_{i,j,k} f(r, v_{i,j,k}) \Delta v_x \Delta v_y \Delta v_z \]  
\[ u(r) \approx \frac{1}{n(r)} \sum_{i,j,k} v_{i,j,k} f(r, v_{i,j,k}) \Delta v_x \Delta v_y \Delta v_z \]  
\[ p(r) \approx \frac{1}{3} \sum_{i,j,k} m |v_{i,j,k} - u|^2 f(r, v_{i,j,k}) \Delta v_x \Delta v_y \Delta v_z \]

where \( n, u, p \) and \( m \) are the particles number density, bulk flow velocity, kinetic pressure and particles mass, respectively [Cravens 2004].

The previously defined Cartesian velocity space grid can be used to obtain the different moments of the VSDF, but in order to obtain an energy or a velocity spectrum of particles, we need to integrate over velocity magnitude, \( v = |v| \), which is complicated on a Cartesian grid. Therefore, we define a spherical coordinate system \((v, \phi, \theta)\) in velocity space, where \( v \) is the velocity magnitude \( v = |v| \), \( \phi \) is the velocity vector azimuth angle \( (0 \leq \phi < 2\pi) \), and \( \theta \) is the velocity vector elevation angle \( (-\pi/2 \leq \theta \leq \pi/2) \). In this coordinate system we define a grid \( v_{l,m} = (v, l \Delta \phi, m \Delta \theta) \), with integer values \( 0 \leq l < l_{max} \), and \( 0 \leq m < m_{max} \). Thus, the sphere in velocity space with radius \( v \) is divided into cells with area \( \approx \Delta \phi \times \sin \theta \Delta \theta \). In each cell the distribution function is approximated by its value at the cell center, \( f(r, v_{l,m}) \), computed from Eq. (4). Then, the velocity spectrum can be computed as

\[ g(r, v) = \int_{0}^{2\pi} \int_{0}^{\pi} f(r, v) v^2 \sin \theta \ d\theta \ d\phi \approx \sum_{l,m} f(r, v_{l,m}) v^2 \sin \theta \Delta \theta \Delta \phi. \]

In what follows, the Cartesian velocity grid is constructed using \( \Delta v_x = \Delta v_y = \Delta v_z = 4 \) km/s, and the spherical velocity grid using \( \Delta v = 2 \) km/s, \( \Delta \phi = \Delta \theta = 2\pi/1080 \).

We can note that it is more natural to use the BL method with fields from a PIC solver than with fields from a fluid solver. The fields are computed from the trajectories of particles in a PIC solver, in the same way that the BL method uses trajectories, the only difference being that the sample of trajectories are different in the BL computation.
compared to the original PIC trajectories. Also, we can use the same trajectory integration method, in this case leapfrog, and the same interpolation of fields to particle positions, in this case linear interpolation, for the BL method as for the PIC solver. We can also note that the BL method implies that all information from a PIC simulation is contained in the fields (as a function of time) and the initial and boundary conditions. Given this information the VSDF can be computed at any place, and at any instance of time, using the BL method. We can also note that the BL method requires that the problem is collision-less. As soon as we have collisions, the BL method is not easily applicable.

3 Non-isothermal Solar Wind Velocity Distributions

During the supersonic expansion of the solar wind, starting at the solar corona, the solar wind proton VSDF gradually changes from a Maxwellian distribution to bi-Maxwellian distribution at $\sim 1$ AU [Marsch et al. 1982a, Gary et al. 2001]. A Bi-Maxwellian VSDF has an anisotropic temperature of $R \neq 0$. This anisotropy triggers the electromagnetic proton cyclotron and mirror instabilities if $R > 0$ and fire-hose instabilities if $R < 0$ [Gary et al. 2001, Kasper et al. 2003]. Theory and simulation, that agree with observations, imply that there are strong constraints on the proton temperature anisotropies in the form of Eq. 8 due to the enhanced magnetic field fluctuations from the instabilities which scatter the protons and change the VSDFs from non-isothermal to isothermal distributions [Gary et al. 2001, Kasper et al. 2003, Hellinger et al. 2003]. The constraints for onset of the proton instabilities can be written as [Kasper et al. 2003]:

$$ R = \frac{T_\perp}{T_||} - 1 = -\frac{S}{\beta_||} $$

where $\beta_|| = 2\mu_0n_pk_bT_||/B_0^2$ is the plasma parallel beta, $S$ and $\alpha$ are linear theory fitting parameters to the observations, $n_p$ is the protons number density, $k_b$ is Boltzmann constant and $B_0$ is the background magnetic field.

Gary et al. [2001] using the ACE spacecraft measurements concluded that 40% to 80% of the time $R < 0$ in the solar wind which is in agreement with the conservation of the first adiabatic invariant. Also, Kasper et al. [2003] using WIND/SWE measurements at $\sim 1$ AU showed that the solar wind protons most probable temperature anisotropy is $R \simeq -0.3$.
and $\beta || \simeq 0.8$ for low speed solar wind ($u_{sw} < 500$ km/s). Hence, we can assume the solar wind VSDF with $R < 0$ near the Moon, but we need to make sure that the fire-hose instabilities do not occur in the range of anisotropy we choose for our studies.

Kasper et al. [2003] using theory and WIND measurements showed that there is a threshold for resonant fire-hose instability using bi-Maxwellian proton VSDF for $R < 0$. They found the best fit to the measurements for the fire-hose instability constraints corresponds to $(S, \alpha) = (1.21 \pm 0.26, 0.76 \pm 0.14)$. Table 1 shows the upstream solar wind parameters we apply in our model and if we assume a bi-Maxwellian VSDF for the upstream solar wind with $R = -0.8$, then $\beta || \simeq 1.36$. Applying the fitting parameters and our $\beta ||$ into Eq. 8 gives that $R$ can not be less than $-0.9$, therefore the minimum temperature anisotropy we assume in our studies is $R = -0.8$. A solar wind temperature anisotropy with $R = -0.8$ is below the threshold level of the occurrence of the fire-hose instability for the assumed conditions of the upstream solar wind in Table 1 and such anisotropy has been observed [Kasper et al. 2003].

Table 1: Upstream solar wind simulation parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{sw}$</td>
<td>$-330 \  \hat{x} \ \text{km/s}$</td>
</tr>
<tr>
<td>$B_{sw}$</td>
<td>$+3 \  \hat{y} \ \text{nT}$</td>
</tr>
<tr>
<td>$n_p$</td>
<td>$1.70 \ \text{cm}^{-3}$</td>
</tr>
<tr>
<td>$v_{thp}$</td>
<td>$40 \ \text{km/s}$</td>
</tr>
<tr>
<td>$T_p$</td>
<td>$8.35 \ \text{eV}$</td>
</tr>
<tr>
<td>$T_e$</td>
<td>$15.50 \ \text{eV}$</td>
</tr>
</tbody>
</table>
4 Model Results

Futaana et al. [2010] using the SARA instrument on-board Chandrayaan-1 observed solar wind protons with an energy of ∼1.3 times the prevailing solar wind energy on a 100 km polar orbit, 50° behind the terminator and near the lunar night-side equatorial plane when the Moon is in the unperturbed solar wind. The density ratio between these night-side protons and the solar wind protons is 0.05-0.2% and their parallel and perpendicular velocity to the IMF is ∼300 km/s when the IMF is in the xy-plane and is perpendicular to the solar wind plasma flow [Futaana et al. 2010]. The processes by which these protons enter into the lunar wake, as well as the mechanisms changing their velocities are not well understood. We use the 3-D hybrid model to examine the role of the non-isothermal (bi-Maxwellian) VSDF and the effect of the lunar surface plasma absorption on these protons and to study their moments of the velocity distributions (kinetic properties) in details.

Figure 1 illustrates the geometry of the simulation domain. The solar wind and IMF directions and magnitudes are fixed during the simulations and they do not change. We choose two half orbits on the lunar night-side at 100 km above the surface of the Moon, similar to Chandrayaan-1’s orbit altitude, marked as orbit A and orbit B in the figure. Orbit A is in a plane perpendicular to the IMF plane, 50° behind the terminator, and orbit B is located on the IMF plane. A virtual observer is moving from the lunar south pole (Λ = −90°) towards the north pole (Λ = +90°) along orbit A and from longitude λ = 90° to λ = 270° along orbit B. The upstream solar wind parameters applied in the model are listed in Table 1, which are similar to those reported by Futaana et al. [2010], and are all constant during the simulation. We assume two different VSDFs for the upstream solar wind protons. A Maxwellian with $R = 0$ and a bi-Maxwellian with $R = −0.8$ which are defined in the Appendix. The total proton temperature is the same for the two distribution functions and is calculated from Eq. 11 in the Appendix for the bi-Maxwellian VSDF. Since the number of meta–particles per cell in a particle model is too small (∼200 particles per cell in our case), applying the non-Maxwellian distribution function to the inflow solar wind protons in the hybrid model does not affect the results compared to the Maxwellian assumption. Therefore, the bi-Maxwellian VSDF is only applied in the BL method for the solar wind boundary condition.

Figure 2 compares the proton energy spectrum along orbits A and B (left and right
Figure 2: Solar wind protons energy along orbits A (left column) and B (right column) for Maxwellian $R = 0$ (top panels) and bi-Maxwellian $R = −0.8$ (bottom panels) upstream solar wind VSDFs. The orbital geometries are defined in Figure 1. The horizontal axes show latitudes ($\Lambda$) along orbit A and longitudes ($\lambda$) along orbit B in the simulation coordinate system. Contour lines denote the normalized VSDF ($g/g_{sw}$) and their levels decrease logarithmically from the innermost ($10^0$) to the outermost ($10^{-8}$) contour line. The horizontal dashed lines show the upstream solar wind energy and the vertical dotted lines show the lunar night-side equatorial ($\Lambda = 0^\circ$) and the lunar mid-night ($\lambda = 180^\circ$) passages for orbit A and B, respectively.

panels) for Maxwellian (top panels) and bi-Maxwellian (bottom panels) upstream solar wind VSDFs. The top-left panel shows that the Maxwellian distribution function cannot explain the high energy protons observed by Chandrayaan-1 at the lunar night-side equator ($\Lambda = 0^\circ$), and also shows energy decrease there, while the protons energy using bi-Maxwellian VSDF (bottom-left panel) at the night-side equator is $\sim 1.25$ times the ambient solar wind energy. This energy increase is almost in agreement with the Chandrayaan-1 observation. The bottom-right panel shows a rise in the protons energy from bi-Maxwellian distribution when the observer moves on the parallel plane to the IMF from the lunar terminator ($\lambda = 90^\circ$, $270^\circ$) towards the mid–night meridian ($\lambda = 180^\circ$), but such an enhancement cannot be seen from the Maxwellian distribution assumption (top-right panel).

Two-dimensional slices of the solar wind proton VSDFs in the Cartesian $v_x$-$v_y$ plane are illustrated for orbit A in Figure 3. As the observer moves from the poles (panels a and c) towards the equator (panels b and d), the majority of the incoming solar wind protons are absorbed by the lunar surface (white regions) and small percentage of them penetrate deep into the night-side equator at 100 km altitude and thereby the shape of the distribution function changes. We see from panels (a) and (c) that the lunar surface
Figure 3: Two-dimensional slices of the solar wind proton VSDFs in the Cartesian $v_x$-$v_y$ plane for orbit A when the observer is at the south pole (panels a and c) and in the lunar night-side equator (panels b and d) for the Maxwellian $R = 0$ (top panels) and bi-Maxwellian $R = -0.8$ (bottom panels) upstream solar wind proton VSDFs. The color shows the normalized VSDF ($g/g_{sw}$) in logarithmic scale, dashed contour lines show the values below the color bar minimum range, horizontal and vertical axes denote the protons perpendicular and parallel velocities to the IMF plane, respectively, and the vertical and horizontal dashed lines mark the upstream solar wind velocities. The white areas which are not covered by the contours are part of the distribution functions that are removed by the lunar surface proton impact.

(white areas) removes part of the distributions tail, but the distributions core remained unchanged, while this is opposite for the distributions at the night-side equatorial plane (panels b and d). Panel (d) shows that the proton velocity perpendicular to the IMF ($v_y$) is not that much smaller in the equatorial plane than the upstream solar wind velocity when we assume a bi-Maxwellian VSDF, while we see a large reduction ($\sim 100$ km/s) in the perpendicular velocity for the Maxwellian VSDF in panel (b). Both panels (b) and (d) show higher parallel velocity ($v_y$) at the equator than the upstream solar wind, but
Figure 4: Contours show the normalized VSDF ($g/g_{sw}$) of incoming solar wind protons to an observer in the spherical velocity space for a velocity magnitude that the velocity spectrum is in the maximum in the lunar night-side equator. The observer is in the lunar night-side equatorial plane for orbit A and the upstream solar wind VSDF is bi-Maxwellian. The axes show the direction of incoming protons to the observer. The dashed line contours show the normalized VSDF below the color bar minimum range and the solid line contour corresponds to zero phase space density and surrounds the directions blocked by the Moon.

The velocity increase is more pronounced for bi-Maxwellian distribution when the velocity increases from 0 at the poles to $\sim 200$ km/s at the equator. The parallel velocity from the Maxwellian distribution at the equator is in agreement with the 1-D plasma expansion theory calculated by Futaana et al. [2010]. Note that the night-side protons have lower phase space density than the upstream solar wind and their $v_z$ velocity is almost zero and is not shown here.

The direction of incoming solar wind protons to a virtual observer on orbit A at the lunar night-side in the IMF plane is shown in Figure 4. This illustrates the possible directions of incoming protons at such position. The upstream solar wind velocity has $180^\circ$ azimuth angle and $0^\circ$ elevation angle in a spherical coordinate system explained in Section 2.3. We see that the majority of the protons incoming to the observer are deflected from the solar wind direction and enter the wake parallel to the IMF from the flank and the deflection angle coincides with the velocities obtained from Figure 3-d. The solid
contour surrounds the Moon with zero phase space density and it shows how the Moon, as an obstacle to the solar wind flow, is seen by an observer on the lunar night side.

We showed in Figure 3 that the VSDF changes as the observer moves from the poles towards the wake, hence, the moments of the solar wind proton VSDF change. Shown in Figure 5 are some solar wind protons plasma parameters along orbit A (left column) and orbit B (right column). It is the first three calculated moments of the distribution (Eq. 5, 6, and 7), along the orbits for a Maxwellian $R = 0$ (dashed lines) and for a bi-Maxwellian $R = -0.8$ (solid lines) upstream solar wind VSDFs. The calculated proton densities (panel A) along orbit A at the lunar night-side equator ($\Lambda = 0^\circ$) is about three orders of magnitude higher for the bi-Maxwellian distribution than for the Maxwellian one and it is almost one order of magnitude less than the Chandrayaan-1 observations reported by Futaana et al. [2010]. There, it was found that the incoming protons get access to the lunar wake with a velocity perpendicular to the IMF that was similar to that of the upstream solar wind velocity and a parallel velocity of approximately 300 km/s at the lunar night-side equator. Panels B and C show that in the night-side equator ($\Lambda = 0^\circ$) the Maxwellian VSDF results in a much lower perpendicular velocity than the solar wind velocity and the parallel velocity is half of the Chandrayaan-1 observations, while the bi-Maxwellian distribution shows that the perpendicular velocity is almost the same as the solar wind upstream velocity and the parallel velocity is $2/3$ that of the Chandrayaan-1 observation. We also see these velocity changes clearly from Figure 3. Panel D shows that the protons bulk flow velocity increases in the lunar wake when the upstream solar wind VSDF is bi-Maxwellian and this increases their energy there too. The protons total kinetic temperature, as shown in panel E, decreases in the lunar wake for both distributions, but the temperature reduction is more pronounced for the bi-Maxwellian than for the Maxwellian distribution. This temperature reduction is in contrast to the observations far downstream in the wake where the ions temperature is constant [Ogilvie et al. 1996], but the ion temperature has not been reported for low-altitudes yet. As one can expect from the distributions shown in Figure 3 right panels, we see a large temperature anisotropy with $R > 0$ in the lunar wake for both of the upstream solar wind VSDFs which occurs due to the lunar plasma surface absorption. Panel F shows that the temperature anisotropy obtained from the Maxwellian distribution is almost two times larger than for the bi-
Modeling the Solar Wind Protons in the Low-Altitude Lunar Wake

Figure 5: Solar wind protons plasma parameters obtained from moment calculation of the velocity distribution functions along orbit A (left column) and orbit B (right column) for Maxwellian (dashed lines) and bi-Maxwellian (solid lines) upstream solar wind VSDFs. When protons number density in the lunar night-side is below eight orders of magnitude than the upstream solar wind density, the moments of the distributions are not calculated and that area is masked for orbit B for $150^\circ \leq \lambda \leq 210^\circ$.

Maxwellian at the lunar night-side equator for orbit A.

We calculated the moments of the distributions for orbit B, similar to those of orbit A, and the results are shown in Figure 5, right column. Since the model predicts very low proton density near the mid-night meridian ($\lambda = 180^\circ$), we do not show the protons density below eight orders of magnitude less than the solar wind density which occurs for $150^\circ \leq \lambda \leq 210^\circ$. Hence, we do not show any moments of the distributions there. Panel I shows two ion beams coming towards the wake from the flanks symmetrically along the magnetic field lines with higher velocities than the prevailing solar wind velocity. The symmetry of the lunar wake refill along the IMF direction is clearly seen from all panels.
for orbit B. A large temperature anisotropy of about $R \approx 8.0$ (or $T_\perp/T_\parallel \approx 9.0$) for the Maxwellian distribution is evident in panel L at $\sim 30^\circ$ behind the terminator.

5 Discussion

Lunar wake simulations and observations indicate that the ambipolar electric field plays a major role for proton entry into the wake far downstream of the Moon [Ogilvie et al. 1996, Farrell et al. 1998]. PIC simulations by Farrell et al. [2008] showed the existence of a large electric field around the lunar wake boundary even at close distances to the Moon which can lead to the entry of solar wind protons into the lunar wake parallel to the IMF direction, but this has not been observed yet. In addition, lunar surface charging, although large in the lunar shadow [Halekas et al. 2010], vanishes at distances larger than a few Debye length ($\lambda_D$) above the surface ($\lambda_D \ll 1$ km in the lunar plasma environment). Therefore, lunar surface charging is mainly applicable to very low altitude processes such as the dust transport at a few hundred meters above the surface, but not at 100 km altitude and higher than that. Hence, we did not include it in the model. Thus, it is not easy to identify the mechanisms for proton entry into the low-altitude lunar wake.

By modeling, we proposed and examined two processes in this paper to study the proton VSDF and their moments of distributions in the lunar wake: (1) Solar wind protons bi-Maxwellian VSDF and (2) Lunar surface plasma absorption.

Our modeling results show that a bi-Maxwellian VSDF can explain high energy proton fluxes with low number density which move in parallel to the IMF direction at low altitudes close to the Moon in the lunar wake as a result of the long tail of the distribution along the IMF. Although the modeling results do not exactly match with Chandrayaan-1/SARA observations reported by Futaana et al. [2010], we see that the solar wind protons enter the wake with a velocity component parallel to the IMF and their energy increase because of the changes in the shape of their respective VSDF. On the other hand, there are some uncertainties in the Chandrayaan-1/SARA observations. Futaana et al. [2010] showed that the night-side solar wind protons velocity is approximately 300 km/s in both directions, perpendicular and parallel to the IMF. This velocity corresponds to an energy of approximately 900 eV, while the energy observed by SARA is about 700 eV, similar to the energy we obtain from assuming a bi-Maxwellian solar wind, but our simulated
velocity parallel to the IMF is nearly 100 km/s less than that of the SARA’s observation. In addition, there was no magnetometer on-board Chandrayaan-1 and the solar wind magnetic field variations were obtained from the ACE Magnetic Field Experiment (MAG) [Smith et al. 1998] and then time-shifted to the Moon [Futaana et al. 2010]. This makes it difficult to know the effects of IMF variations on proton entry into the lunar wake. Figure 4 in Futaana et al. [2010] shows ACE/MAG data. It shows that the IMF was highly fluctuating a few minutes before SARA observed the high energy protons on the lunar night-side, if we assume exactly one hour propagation time for the IMF from ACE to the Moon. Hence, there could be a relation between those IMF variations and the observed night-side protons. Furthermore, Chandrayaan-1 during that event passed close by the strongest lunar near-side magnetic anomaly, known as the Reiner Gamma. Perhaps the influences of the magnetic anomalies, in combination with the lunar surface plasma absorption, the solar wind bi-Maxwellian distribution, and the ambipolar electric field around the lunar wake all affect the proton entry into the wake. Therefore, the effects of the lunar magnetic anomalies need further investigations.

The lunar surface plasma absorption, clearly shown in Figure 3, removes the gyrating charged particles from their respective VSDFs and makes a different distribution at the lunar night-side compared to the undisturbed solar wind VSDFs. Therefore, the protons number density, velocity and temperature change. This effect is more pronounced close to the obstacle when the majority of the solar wind ions and electrons are absorbed by the lunar surface and only a few of them get access deep into the lunar wake.

We found a large temperature anisotropy on the lunar night side as a consequence of the lunar surface plasma absorption effect. The maximum temperature anisotropy occurs along orbit B when proton density is about 1% of the solar wind density. This anisotropy may generate an instability which may propagate downwards in the tail and affect the lunar plasma environment far in the lunar tail. Clack et al. [2004] observed a large temperature anisotropy $T_\perp/T_\parallel \simeq 10$, almost similar to what we found, in the far downstream in the wake ($>15R_L$). Moreover, Bale et al. [1997] observed unstable particle distributions and wave propagations at $8R_L$. These large distance temperature anisotropies and unstable distributions might be related to the perturbations from the temperature anisotropies at lower heights that propagate downstream in the wake. The generation of any instabilities
due to the temperature anisotropy and their direction of propagation in the lunar wake needs further investigations.

6 Conclusions

By using a hybrid model of plasma and BL algorithm, we investigated the effects of surface plasma absorption and non-isothermal solar wind VSDF on the solar wind proton intrusion into the deep lunar wake at 100 km altitude above the Moon. The surface plasma absorption effect on the solar wind plasma VSDF is very strong for the near lunar wake. The solar wind is absorbed by surface impact, and is removed from the VSDF of the particles and generates a different distribution function in the night-side. High velocity, low density and low temperature solar wind protons with large temperature anisotropy at low-altitudes in the lunar wake can be the consequences of the lunar surface plasma absorption and the non-isothermal VSDF of the solar wind. The agreement between the simulation and the observations indicate that the BL method for a particle model provides high velocity space resolution, therefore we can study the particles kinetic properties in details. These results can be applied for any passive obstacles to the solar wind, e.g., asteroids.

A Velocity Space Distribution Functions

Maxwellian velocity distribution function. One of the well-studied distribution functions is the Maxwellian distribution which is also known as Maxwell-Boltzmann distribution. It is an isothermal distribution and its vector quantity moments are symmetric along all three dimensions in velocity space. The Maxwellian distribution is defined as

\[ f_M(r, v, t) = n \left( \frac{m}{2\pi k_b T} \right)^{3/2} \exp \left( -\frac{m(v - u)^2}{2k_b T} \right) \]  

where \( r \) and \( v \) are particle’s position and velocity vector, respectively. \( k_b \) is the Boltzmann constant, \( m, n \) and \( T \) are particle’s mass, number density and temperature, respectively. \( u \) is the bulk flow velocity [Cravens 2004].

Bi-Maxwellian velocity distribution function. The Bi-Maxwellian distribution is
a non-isothermal distribution function. In contrast to the Maxwellian distribution, particle populations from a Bi-Maxwellian distribution can have different thermal speed in different directions. This makes the distribution function anisotropic. The Bi-Maxwellian distribution is defined as

\[ f_{BM}(r, v, t) = n \left( \frac{m}{2\pi k_b T_{||}} \right)^{1/2} \left( \frac{m}{2\pi k_b T_{\perp}} \right) \exp \left( -\frac{m(v_{||} - u_{||})^2}{2k_b T_{||}} - \frac{m(v_{\perp} - u_{\perp})^2}{2k_b T_{\perp}} \right) \]

where the total kinetic temperature is

\[ T = \frac{T_{||} + 2T_{\perp}}{3} \]

and the directional subscripts denote directions relative to the background magnetic field [Demars et al. 1979].

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References


Modeling the Solar Wind Protons in the Low-Altitude Lunar Wake
