DOCTORAL THESIS

Fatigue Damage Mechanisms in Polymer Matrix Composites

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Summary

Polymer matrix composites are finding increased use in structural applications, in particular for aerospace and automotive purposes. Mechanical fatigue is the most common type of failure of structures in service. The relative importance of fatigue has yet to be reflected in design where static conditions still prevail. The fatigue behavior of composite materials is conventionally characterized by a Wöhler or S-N curve. For every new material with a new lay-up, altered constituents or different processing procedure, a whole new set of fatigue life tests has to be repeated for such a characterization. If the active fatigue damage micromechanisms and the influence of the constituent properties and interface were known, it would be possible, at least qualitatively, to predict the macroscopic fatigue behavior. A study of the fatigue damage mechanisms would also give indications of the weakest microstructural element, which is useful information in materials selection for improvement in service properties. In tensile fatigue of a multidirectional laminate, the critical elements are the longitudinal plies which are the last to fail. Although failure of neighboring off-axis plies as well as delamination will influence the fatigue process, an understanding of the behavior of the longitudinal plies forms an important foundation. Effects of plies of other directions may then be interpreted based on this foundation. Fatigue of longitudinal plies is therefore focused on in the present study.

The underlying fatigue damage mechanisms were investigated for unidirectional 0° carbon fiber reinforced plastics (CFRP) and glass fiber reinforced polypropylene (GF/PP) in tension-tension fatigue. By use of a surface replication technique the evolution of fatigue damage could intermittently be monitored during the course of fatigue testing. In the CFRPs, the matrix was an epoxy resin or polyetheretherketone (PEEK). In the GF/PP system, the matrix was modified with maleic anhydride (MA) to achieve a stronger fiber-matrix interface. The macroscopic fatigue behavior was characterized by fatigue life diagrams. A statistical method has been devised to systematically characterize fatigue life data in terms of fatigue life diagrams. On the microscopic level, the CF/epoxy and GF/MA-PP composites have relatively strong
interfaces and showed localized and scarce fiber breaks from which matrix cracks propagated perpendicular to the fiber direction. In CF/epoxy, fiber bridged cracks with squeezed fiber tips appeared. Conversely, CF/PEEK and GF/PP have weaker interfaces, and the principal mechanisms were extensive and distributed debonding or longitudinal matrix cracking followed by further fiber breakage. Macroscopically, the weak interface composites showed shorter fatigue lives and more rapid fatigue degradation. This suggests that higher interfacial strengths lead to improved fatigue performance.

Modeling studies were undertaken for the two observed mechanisms; debonding from a fiber break, and fiber bridged cracking. The stochastic breakage of fibers next to a growing debond was parametrically investigated with a shear lag model. The stress profile in the surviving fibers becomes attenuated and more distributed as the debonds grow. This results in longer axial distances between fiber breaks, and hence a more jagged and uneven crack propagation. A larger variability in strength along the fibers has basically the same break distributing effect. With a more homogeneous stress distribution caused by long debonds, the variability in fiber stress at failure of the intact fibers decreases. This can explain the experimentally observed lower scatter in fatigue life of composites exhibiting a more homogeneous distribution of damage caused by debonding.

Furthermore, the experimental results of fiber bridged cracking was modeled with a fracture mechanics approach. The crack growth curve can be plotted in terms of the effective stress intensity factor where the contribution of the cohesive crack surface forces from the bridging fibers are taken into account. This curve falls somewhat closer to that of the neat matrix material compared to the unbridged crack, but the difference is still considerable. Besides the fiber bridging, there should therefore be other active toughening mechanisms that slows the crack propagation down to account for the fatigue resistant behavior of the tested material.

In fatigue of multidirectional laminates, tension-compression loading has shown to be more detrimental than tension-tension loading. The reason for this behavior has not been entirely clarified. The adverse effect of the compressive load excursions is partly caused by the formation of transverse cracks. This was verified by counting transverse cracks in cross-ply laminates. Since debonding is the subcritical mechanism which leads to transverse cracking and eventually influences ultimate failure, the debonding was studied in low cycle fatigue of a single transverse fiber. In tension, contact zones developed at the crack tips for sufficiently large debonds. Due to the inherent geometry and the mismatch in elastic properties of the constituents, an opening zone appeared at the crack tips of the debond in compression. This was also verified by finite element analysis. Since debond propagation
is more susceptible to mode I loading, the sensitivity to tension-compression loading is explained by the effective opening zone in compression.
To my parents,
Ebba and Anders
Den mätta dagen, den är aldrig störst.
Den bästa dagen är en dag av törst.
Nog finns det mål och mening i vår färd –
men det är vägen som är mödan värd.
Det bästa målet är en nattlång rast,
där elden tänds och brödet bryts i hast.
På ställen där man sover blott en gång,
blir sömnen trygg och drömmen full av sång.
Bryt upp, bryt upp! Den nya dagen gryr.
Oändligt är vårt stora äventyr.

Karin Boye, 1900-1941.
Preface

This thesis forms a partial fulfilment of the requirements for the doctoral degree at Luleå University of Technology. At Swedish universities, the theses are usually made up by a set of scientific articles covering the research field. However, this thesis is written as a monograph to provide a more lucid and self-contained style, since its topic is well-confined.

The thesis is divided into six chapters. Chapter 1 gives a brief review and background of the problem of fatigue in polymer matrix composites, and outlines a motivation of the present study. In Chapter 2, a statistical method is presented, which can be used as a tool to systematically compare the fatigue life properties of different composite materials in terms of the fatigue life diagram. Chapter 3 contains an experimental study of the fatigue damage mechanisms on a microscopic level in carbon and glass fiber reinforced plastics. The macroscopic fatigue performance is also characterized, and interpreted with the observed micromechanisms. Chapter 4 covers two models for the propagation of the experimentally observed fatigue damage. The first model deals with the influence of interfacial debonding and fiber strength variability on further fiber breakage. The second model describes experimental results of the growth of fiber bridged cracks. In Chapter 5, a study on the mechanisms in tension-compression fatigue of transverse plies is presented. The effects of reversed cycle loading on debond growth and transverse crack formation have been investigated. A mechanism is proposed which promotes the adverse effects of compressive fatigue loading, that ultimately contributes to failure in multidirectional laminates containing transverse plies. Concluding remarks and suggestions for future developments are made in Chapter 6.

In brief, this thesis deals with fatigue damage mechanisms on the microscopic level in polymer matrix composites, and how the macroscopic fatigue behavior is affected by microstructural properties. This has led to a modeling investigation of the observed fundamental mechanisms, in order to shed some light on the interaction of constituent properties and damage propagation. Hopefully, the present work contributes to the understanding of fatigue in polymer matrix composites, which has become an increasingly important
issue in structural design of light weight applications.

First of all, I would like to sincerely thank my advisor, Professor Lars Berglund, for his inspiring enthusiasm and zest for all kinds of scientific endeavors. It has been a privilege to work in his group, and I will look back with nostalgia to the time I have spent at the Division of Polymer Engineering as a graduate student. Secondly, I wish to thank my co-advisor, Professor Ramesh Talreja, for being my host during my stay at the School of Aerospace Engineering at Georgia Institute of Technology, and for enlightening me on the intricacies of fatigue in composites — sometimes repeatedly on the same issue when I have drifted off into delusion. I am also grateful to Dr. Anders Sjögren for keeping up the spirits and for dexterous laboratory work with cumbersome tension-compression tests of microcomposites. Ms. Stéphanie Baty and Mr. Norbert Cabrera are two talented former students who, during their master’s thesis work, did small wonders in the solid mechanics laboratory. I wish to thank Dr. Ton Peijs who procured the glass fiber/polypropylene material, and helped to interpret the results, and Dr. Sören Östlund for fruitful discussions on fiber bridged cracking. Financial support from the Swedish Research Council for Engineering Sciences (TFR) is acknowledged.

To accomplish this thesis, the scientific input has been a necessary, but not a sufficient condition. The social input, which has conveyed an enjoyable working atmosphere, has been equally important. Fredrik Thuvander is a colleague who has never turned down the opportunity for a cup of coffee and a chat, for which I am grateful. Also, I thank Roberts Joffe for an interminable supply of Petka and Chapayev stories, and Mats Ericson for other kinds of jokes. Kristina Stenlund has expeditiously cut the red tape, always with a smile. Johan Lindhagen and Lennart Wallström are two cheerful peers, who are always eager to discuss important topics such as skiing and fishing. Leif Asp assured a rapid acclimatization to life as a graduate student. I am indebted to all the members of the Division of Polymer Engineering and fellow graduate students of the Department of Materials and Manufacturing Engineering, past and present, who have provided an excellent ambiance. I am also grateful for support from friends at other departments at Luleå University of Technology and at SICOMP, as well as everyone I forgot to mention here. Finally, I wish to thank to my parents, to whom I dedicate this piece of work.

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E. KRISTOFER GAMSTEDT
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Chapter 1

Introduction

1.1 Preamble

Composite materials offer some exciting advantages over more traditional metallic materials. Their stiffness to weight and strength to weight ratios can be superior to other known structural materials. Modern advanced composites are a success story in the current age of engineered materials. Applications range from ski sticks, tennis rackets, parts in biomedical implants, and reinforcement of highway bridges to advanced aircraft and space vehicles. However, the story could and should be even more successful. More widely diverse applications suffer from difficulties in recycling to questions of long-term durability and the inability to accurately predict life. This work will address the problem of damage accumulation and eventual failure in a composite material through repeated cyclic loading, i.e. fatigue of composites.

Higher demands for superior mechanical properties, durability and weight savings provide impetus for the development of composite materials. Fiber reinforced composite materials consist of fibers of high strength and modulus embedded in a matrix. In this form, the composite produces a combination of properties that can not be achieved with the constituents acting alone. The fibers are the main load carrying members, while the surrounding matrix keeps them in the desired location and orientation, protects them from environmental degradation, and transfers the load between them. For general reading on mechanical properties of composite materials, the textbooks by Agarwal & Broutman (1990), and Hull & Clyne (1996) are recommended. The quest for improved mechanical properties is closely linked with the processing science of composites. Mechanical tests do not only provide essential information for safe design of composite structures, but can also give indi-
cations of how to change the processing parameters to achieve even better service properties. On processing and manufacturing of composite applications, see e.g. Åström (1997) or Gutowski (1997).

Composites usually consist of a matrix reinforced by strong fibers. The surrounding matrix consists of either a metallic, ceramic or polymeric material depending on the application. For use at temperatures below 250°C, polymer matrix composites (PMCs) is commonly the strongest contender. Other composite material categories are metal matrix composites (MMCs), and ceramic matrix composites (CMCs). High performance composites for structural applications usually have continuous high strength fibers in the main load direction. Depending on the cost and usage, the fibers may be made of carbon, glass, Kevlar etc. The superior specific modulus and strength of carbon fibers have led to a more widespread use in aerospace application of those fibers, whereas glass fibers prevail in large volume applications in e.g. the automotive industry where cost efficiency is more important than performance. Carbon fiber reinforced plastics and glass fiber reinforced polypropylene are the object materials in this work.

1.2 Fatigue of composites

Fatigue of materials has enormous practical implications. It has been estimated that over 80% of all service failures of structural materials can be traced to fatigue (Dauskardt et al., 1993). Nevertheless, design criteria are predominantly based on conservative static properties. Structures designed with fatigue criteria can be lighter and more optimized. Reliable predictions in structural fatigue can only be made when fatigue of materials is well understood. Fatigue of isotropic materials has been investigated for a long time, and the process is fairly well understood and documented (see e.g. Suresh, 1991). However, the fatigue process in anisotropic and heterogeneous materials such as composites has only recently become subject to extensive research due to the relatively newly upsurge in load bearing applications for this type of materials. On fatigue of materials in general, see e.g. Suresh (1991), Carlson & Kardomateas (1996), or Ellyin (1997).

The aerospace industry, which is the dominating user of high performance composite materials, has during the last decade put higher demands on the manufacturers: higher strains to failure with maintained strengths and materials that can sustain higher temperatures are desired. Improved toughness can be achieved by rubber modification of the conventional epoxy matrices or by use of a thermoplastic matrix such as PEEK (polyetheretherketone). However, by meeting these demands, the fatigue properties of the improved
composites have in many cases become worse (Curtis, 1987; Simonds et al., 1989; Baron, 1992). The reason for this has not been clarified.

Composite laminates are formed by stacking several plies (or laminae) in a specified sequence. For uniaxial loading, each unidirectional ply has a fiber direction given by the angle between the fibers and the load direction. In this way the mechanical properties of the composite can be optimized for the application in mind. The fiber orientation angle of a unidirectional laminate with the highest strength as well as largest strain to failure is 0°. Therefore, the 0° ply is the last ply to fail in a multidirectional laminate, in fatigue as well as under static conditions. It has been shown by numerous researches (Dickson et al., 1989; Talreja, 1993; Liu & Lessard, 1994; Andersen et al., 1996) that the fatigue life of multidirectional laminates containing plies with the fibers in the load direction (i.e. 0° or longitudinal) is controlled by the fatigue behavior of the unconstrained 0° plies. For a particular type of fatigue loading of a multidirectional laminate, the number of cycles to failure falls close to or could even be indistinguishable from the data of 0° laminates for the same loading conditions (cf. Figure 1.1). An explanation for this could be that the lifetime of unidirectional off-axis plies are significantly lower than that of 0° plies (Hashin, 1973; Awerbuch & Hahn, 1981), and that the constraint of the off-axis plies themselves and the evolving cracks therein have a minor influence on the stress state in the 0° plies (Subramanian et al., 1995). In cases where subcritical cracks in the off-axis plies do have a significant effect on the local stress in the adjacent 0° plies, the stacking dependent lifetime of the entire laminate can be estimated from the fatigue properties of the 0° plies given the evolution of redistribution of the localized stress state with time (Reifsnider & Stinchcomb, 1986).

Considering this relative importance of the 0° plies, it may be concluded that an analysis of the fatigue behavior of unidirectional 0° plies can serve as a basis and a starting point in the understanding of the fatigue behavior of continuous fiber composites with a general lay-up. The focus of this thesis is placed on longitudinal 0° laminates, except in the penultimate chapter.

Most research efforts in fatigue of composites have been directed towards uniaxial tension-tension cycling only, primarily because of the difficulty to prevent buckling failure of thin laminates and in gripping of the specimens. Also, since the scope of this study is mainly on fatigue mechanisms, a well-defined and relatively simple system is desirable. The high complexity which arises from interaction of different damage mechanisms in compression (e.g. fiber microbuckling and debonding) and in tension (e.g. fiber breaks and matrix cracking) makes it difficult to assess the development and the kinetics of one particular kind of mechanism under controlled conditions. Therefore, tension-tension fatigue with a stress ratio of approximately $R = \sigma_{\text{min}}/\sigma_{\text{max}} =$
0.1 is mainly treated here. The minimum stress, $\sigma_{\text{min}}$, was chosen to be somewhat larger than zero to ensure that excursions into the compressive regime were avoided. Chapter 5 deals with a study of tension-compression fatigue of transverse plies.


### 1.3 Damage mechanisms and fatigue life diagrams

The fatigue life follows as a consequence of the fatigue damage mechanisms; such as fiber breakage, debonding, matrix yielding, crack coalescence, fiber bridged cracking *etc.* The rates and distributions of active fatigue damage mechanisms are dependent on the local stress state. The type of applied loading, the microstructure and damage distribution influence the local stress...
1.3. Damage mechanisms and fatigue life diagrams

In uniaxial fatigue tests there are three major modes of controlled cyclic loading. Fatigue tests may be controlled by displacement, energy or load. In the displacement controlled fatigue tests the displacement levels are maintained constant during the fatigue. The global stress decreases as damage accumulates in the material. A disadvantage of this test is that the stress state relaxes radically when slipping occurs at the grips and when delamination develops at the interface between the composite specimen and the end tabs. Furthermore, for an equal maximum cyclic stress this test exhibits longer fatigue lives than do energy or load controlled tests.

In energy controlled testing, the strain energy per cycle remains constant throughout the test. This test yields an intermediate fatigue life. An elaborate control system must be conceived which may be susceptible to instability when perturbed.

Load controlled testing maintains constant load levels, and has generally the shortest life. Load controlled tests do not demonstrate any particular technical difficulties, and is probably the best candidate to represent the actual fatigue life in service conditions. From now on, all fatigue tests considered will be load controlled with a constant global stress level.

The so-called Wöhler or S-N diagrams describe the applied global stress level (usually the peak value of a sinusoidal stress) with respect to the number of cycles to failure. For composite materials composed of unidirectional plies exposed to tension-tension cyclic loading, the fatigue behavior becomes more tractable to interpretation if the S-N diagram is replotted with the maximum strain attained in the first load cycle on the ordinate and the logarithm of the number of load cycles to failure on the abscissa. This type of fatigue life diagram was first introduced by Talreja (1981a). The strain level in a longitudinal ply is globally the same for fibers and matrix, and both the static strain to failure and the fatigue limit can be ascribed to strain properties of the constituents. If the fatigue life were described in terms of stress, this reasoning would not be so straightforward. In a multidirectional laminate, the different plies are likewise subjected to the same global strains. The fatigue behavior of the individual plies can then directly be connected to the fatigue life curve of the unidirectional composite with the corresponding orientation angle.

Furthermore, the maximum initial strain is a quantity that is linked to the damage state reached in the very first load cycle, and there is reason to believe that any progression of damage during the course of fatigue is determined from this initial damage state. A baseline fatigue life diagram pertains to a unidirectional 0° composite loaded in the fiber direction in tension-tension fatigue, and is depicted in Figure 1.2. The fatigue life diagram consists of three distinct regions, each assumed to be associated to a different set of
operative damage mechanisms. For notational simplicity, these regions are enumerated as Region I, II and III in order of decreasing strain level.

![Fatigue life diagram of longitudinal composites in tension-tension fatigue.](image)

**Figure 1.2.** Fatigue life diagram of longitudinal composites in tension-tension fatigue.

**Region I** Generically, the strain level of this region coincides with the scatter band of the static strain to failure, since virtually all the load is carried by the fibers. The operative mechanisms are similar to those in static failure and can be regarded as chaotic in the sense that it is next to impossible to predict the lifetime distribution due to the sensitivity to the slightest variation in the state of damage developed in the first loading cycle. Static fracture is a stochastic process controlled by the fiber strength distribution and stress concentrations from fiber breaks. For instance, Batdorf (1982), Bader (1988), and Marston & Neumeister (1998) have investigated static fracture of composites experimentally and by modeling of the stochastic fiber break progression. In contrast to metal matrix composites, the scatter band of Region I for composites with polymer matrices is horizontal and does not show any progressiveness (Talreja, 1995).

**Region II** This region is also called the progressive region. Candidate damage mechanisms responsible for the downward slope of this region are fiber bridged cracking, and debond propagation. Lorenzo & Hahn (1986) have studied the microscopic fatigue mechanisms with respect to the applied stress for carbon fiber bundles embedded in epoxy matrices. They initially observed distributed isolated fiber breaks followed by zones of transverse accumulation of fiber breaks across the bundles. The same process has been noticed for glass fiber composites by Dharan (1975). These studies were
made on laboratory manufactured specimens. The damage evolution was only qualitatively characterized for one group of materials. The evolution of single damage sites was not traced during the course of fatigue cycling.

Razvan et al. (1990) investigated the macroscopic fatigue damage mechanisms on center notched quasi-isotropic carbon fiber/epoxy composite laminates, and observed different damage mechanisms depending on the applied fatigue load level. At high load levels, initiation triggered mechanisms dominated, e.g. fiber breakage, and transverse cracking. At lower load levels, the active mechanisms were progressive, i.e. they associated damage was continuously propagating, such as delamination and splitting. Also on a more local level in a longitudinal ply, initiation mechanisms should prevail at higher load levels, and propagation mechanisms are controlling the damage evolution at lower load levels. A reasonable hypothesis, which is supported by the present study, would therefore be that fiber breakage is the main mechanism at high load levels, and matrix or interface crack propagation is the main mechanism at low load levels.

As stated in the previous section, fibers with higher strains to failure and tougher matrices have an adverse effect on the fatigue performance of the composite, i.e. the slope is greater of the scatter band in Region II. Talreja (1995) has pointed out trends in fatigue life performance depending on these constituent properties. These influences are schematically outlined in Figure 1.3.

Region III In this region, the damage is constrained and arrested from further growth by the fibers. As will be shown later in this report, small-scale debonding can have a crack retarding or arresting effect in Region III. Final failure is not reached within a feasibly large number of cycles (typically $10^6$ to $10^7$). Above Region III, there is generally an asymptotic horizontal scatter band of Region II for high cycle fatigue. Two plausible reasons of the existence of Region III can be that (i) the applied strain level is so low that the driving force for propagation of the progressive damage mechanism does not reach above its threshold value, and that (ii) an effective crack arrest mechanism obstructs further fatigue degradation.

In the following chapter, a scheme will be proposed how to quantitatively deduce a fatigue life diagram from a set of strain and life data. The appearance of the fatigue life diagram (scatter, location and slope of Region II etc.) can be described by characteristic values, which can be used to compare differences in fatigue behavior of different material systems. The influence of material properties on the fatigue performance can thus be investigated in
a more systematic manner. A statistical approach is a necessity, since the relative brittleness of the composite caused by the carbon or glass fibers leads to relatively large scatters in life and strength.

1.4 Experimental methods

In the quest for accurate design methodologies and materials improvement, the fatigue behavior of a composite material should be carefully characterized on a wide front line, ranging from molecular analysis of the fiber-matrix interface, over damage mechanisms, to macroscopic fatigue life behavior. Full use of the existing arsenal of suitable experimental techniques should be evoked. Some of these will be briefly presented in this section.

The two classic mechanical fatigue test methods are (i) fatigue life measurements for different load levels which result in a Wöhler diagram or the more refined fatigue life diagram discussed in the previous section, and (ii) measurement of crack propagation rate with respect to the applied load or stress intensity factor.

Fatigue life measurements

Large numbers of load cycles to failure require a servohydraulic tensile machine. The sample may be set up in a special rig to achieve a particular stress state (biaxial, flexural, pure shear etc.) and controlled in one of the aforementioned modes, i.e. displacement, energy or load control. The loading
waveform is usually sinusoidal, triangular or random with a given spectrum. Cyclic uniaxial tension-tension fatigue is considered here. For a fixed stress ratio, $R = \sigma_{\text{min}}/\sigma_{\text{max}}$, the variable parameter of the global stress state is the maximum stress, $\sigma_{\text{max}}$, from which the initial maximum strain, $\varepsilon_{\text{max}}$, can be calculated. With fibers along the load direction the decrease in stiffness is generally negligible. A fatigue life diagram as illustrated in Figure 1.2 can be formed from the sets of values of $\varepsilon_{\text{max}}$ and concomitant fatigue lives, $\log N_f$. To account for the mean stress level effects and a variable stress ratio, the so-called Haigh or constant life diagram can be conceived. In this case, the specimen must be supported on all sides to prevent buckling when compressive loads are applied ($R < 0$).

The results from this method reflect the macroscopic fatigue behavior, and include the initiation, propagation and ultimate failure (or termination) processes of fatigue damage. Due to the large scatter in fatigue lives of composite materials, the number of fatigue tests should be large in order to characterize the fatigue performance with some degree of confidence. To quantify the operative mechanisms, non-destructive evaluation (NDE) techniques should be employed during the course of fatigue testing.

**Crack propagation measurements**

This procedure is of micromechanical nature. The crack propagation rate of an existing crack is measured on a microscopic level for varying load levels. The rate is usually plotted with respect to the mode I stress intensity factor range, $\Delta K$, or the corresponding energy release rate range, $\Delta G$, in a log-log diagram, and generally takes a sigmoidal shape (see Figure 1.4). Three regimes can be identified. In Regime A, the crack growth is initiated. Below a certain threshold value, $\Delta K_{\text{th}}$, the cracks remain dormant or grow at undetectable rates. Determination of $\Delta K_{\text{th}}$ is time consuming; if no crack advance is detected for at least $10^7$ cycles, a crack growth threshold is considered to have been reached. Regime B is called the Paris regime, where the Paris power law relationship for fatigue crack propagation is valid, viz.,

$$\frac{da}{dN} = C(\Delta K)^m; \quad (1.1)$$

where $a$ is the crack length, and $C$ and $m$ are material parameters that characterize the fatigue crack growth rate. Equation (1.1) can be refined to account for different toughening mechanisms (Suresh, 1991). Regime C is associated to static behavior, and pertains to the range of high $\Delta K$ values where crack growth rates increase rapidly causing catastrophic failure at $\Delta K_c$. 
CHAPTER 1. INTRODUCTION

Figure 1.4. Schematic picture of the different regimes of fatigue crack propagation.

Single, double edge notched (SEN and DEN) or double cantilever beam (DCB) specimens are the most commonly used for composites. DEN specimens have been used to determine the crack propagation rate in cross-ply laminates (Mandell & Meier, 1975). DCB specimens are more suitable for delamination growth rate measurements. For longitudinal coupons, the crack does not always grow in a plane perpendicular to the fiber direction due to the marked anisotropy of the material. In fact, the crack tip radius of prefabricated notches are in general so large that the mode I stress intensity factor, $K_I$, does not attain a threshold value to make the crack progress. For longitudinal composites, it would be more likely that debonding along the fibers would occur at the crack tip (cf. Chapter 3). However, successful experiments on longitudinal SEN specimens of metal matrix composites (Bakuckas & Johnson, 1992) and model polymer matrix composites with fiber bundles acting as individual fibers (Botsis et al., 1995) have been performed.

Both of these basic methods require extensive testing and the results are only inherent to the material tested. Extrapolation to fatigue properties of untested material systems is very risky. A better understanding of the underlying mechanisms and their kinetics could serve as an input in predicting the lifetime and crack propagation rate of a material measured with the two classic methods. The aim is to somehow link the static and fatigue properties of the constituents, their interface and the fiber geometry to the fatigue life.
by incorporation of models of the operative mechanisms. This is a formidable
task, and an identification and quantification of the mechanisms would only
serve as a first step. Some of the advantages and drawbacks of suitable
experimental methods in a fatigue context will be outlined. Since damage
mechanisms are of primary concern in this work, continuously monitored
non-destructive evaluation (NDE) will be highlighted. Damage mechanisms
evolve progressively during the course of the fatigue testing, and destructive
testing (e.g. residual strength measurements, Awerbuch & Hahn, 1977, deepy
techniques and penetrant enhanced x-ray radiography, Reifsnider & Jamison,
1982) will not give any information on how damage grows with the number
of applied load cycles since the material is demolished when tested. For an
in-depth and more complete overview, see the textbooks on experimental
techniques for composite materials by Whitney et al. (1984), Carlsson &
Pipes (1987) or Pendleton & Tuttle (1989). Other NDE techniques appro­
priate for composite materials include for instance adiabatic thermoelastic
measurements, shearography, Moiré and speckle interferometry.

**Acoustic emission**

The emission, which consists of stress waves resulting from energy release
due to initiation or operation of failure mechanisms (fiber failure, interface
debonding etc.), can be picked up by piezoelectric sensors. The stress waves
are converted into electric signals, which in turn may be correlated and identi­
fied with a particular failure mechanism. This correlation must be performed
for every new material and stacking sequence, where direct observation of the
emerging types of damage is linked with the emitted signals. The background
noise may shade off the weaker signals from less energetic mechanisms. De­
tection of a single fiber breakage is technically feasible, but unambiguous
identification can not always be realized when simultaneous emissions are re­
ceived. The spatial location of the damage sites may be roughly determined
by attachment of several sensors on the specimen. In fatigue of unidirectional
composites, the acoustic emission (AE) method has been used to register the
number of AE events with respect to the number of elapsed cycles (Bhat &
Murthy, 1993).

**Stiffness monitoring**

Measurement of the stiffness degradation gives a macroscopic reflection of the
fatigue damage development on the microscopic level. If fatigue damage is
distributed in the composite material at the earlier stages of the fatigue life,
the measured stiffness (e.g. axial Young’s modulus and major Poisson’s ratio)
can be directly linked to the damage state by continuum damage mechanics (see Talreja, 1994). In certain design situations, the failure criterion can be formulated as decrease in Young’s modulus below a critical value.

The strain is continuously registered by a computer through an extensometer. The secant modulus is calculated and presented as a function of time. The stress-strain hysteresis and the adherent loss factor can also be monitored. This type of dynamic mechanical analysis has been done for carbon fiber composites at different temperatures to characterize internal damage (Osiroff & Stinchcomb, 1992).

The stiffness monitoring method is not suitable for longitudinal composite laminates, or laminates which is dominated by longitudinal plies, since the stiff load-bearing fibers are the last to fail. In laminates containing off-axis plies, the subcritical transverse cracks often have a measurable influence on the total stiffness (Jamison et al., 1984).

**Surface replication**

A polymeric tape, usually made of cellulose acetate, is turned malleable by soaking it in an acetone based solution, and pressed onto the surface of the composite. The acetone softened tape flows into the surface and upon drying provides a high fidelity impression of the surface topology on the micrometer level. The replica is covered with a thin carbon or gold coating to improve the contrasts when analyzed by optical or scanning electron microscopy. This procedure is repeatedly performed at different stages of the fatigue life in order to study the development of fatigue damage (see Chapter 3). Great care has to be taken to keep the sample and replica clean, because even a tiny amount of debris will make the structures on the replica undetectable at high magnification. Naturally, damage processes within the bulk composite can only be indirectly deduced from what is observed to take place on the surface.

**Ultrasonic scanning**

The specimen is submerged in a water tank between a glass plate at the bottom and a transducer at the water surface of the tank. An ultrasonic pulse is emitted from the transducer, and the wave is partially reflected at the interfaces within the composite material. The peak from the reflection at the glass plate is analyzed and used as a measure of the attenuation in the material. A two-dimensional through-thickness image of the composite material is produced from the reflected acoustic signal. Delaminations and larger voids are readily detected, but smaller entities like fiber breaks and
debonds can not be discovered since the resolution of the method is limited by
the emitted wavelength, and the scattering and diffraction at the interfaces
obscure the details. Since the sample must be exposed to water each time
an ultrasonic scan is made, chemical degradation will affect the material to
some extent (in particular at glass fiber surfaces).

Ultrasonic C-scan has been used to investigate the sequential damage de­
velopment under fatigue loading in cross-ply carbon-epoxy laminates (Haque
et al., 1993). Incipient cracks were almost immediately detected in the trans­
verse plies, followed by splits in the longitudinal plies and local delamination
at the intersection points.

**X-ray tomography**

X-ray transmission radiography with an enhancing penetrant is widely used
for the detection of larger damage in multidirectional laminates (see Simonds
et al. (1989) for fatigue in laminates with drilled center holes). Radiography
takes everything into account that the x-ray beam encounters while going
through the material and gives a plane projection of delaminations, pores,
transverse cracks etc. Flaws perpendicular to the beam are therefore liable
to escape detection. However, x-ray tomography confers a three-dimensional
reconstruction of the interior of the material with a maintained resolution.

When x-ray tomography is used, a collimated beam goes through the
tested material and is received by an array of detectors on the opposite side.
The object is gradually rotated through an angle of 180°. The attenuation
of the intensity of the x-ray beam is measured and acquired by a computer
which reconstructs the interior structure of the sample. The procedure is
time consuming since the sample must be fully scanned for each increment
in the rotation angle. However, improvements in resolution and scanning
time are to be expected in the future. Considerable activities are undertaken
in the development of this method for medical purposes.

For x-ray radiography and tomography of composite materials, a contrast
enhancing penetrant may be of benefit. Due to the small differences in density
between the composite constituents and damage cavities within the material,
a high density penetrant like zinc iodide would render the cavities more
visible provided they are connected to the surface and can be filled. Since
a penetrant would alter the composition and mechanical properties of the
composite, it cannot be used intermittently during fatigue.

X-ray tomography has been used to study the development of fatigue
damage in notched quasi-isotropic laminates (Bathias & Bagnasso, 1992;
Eriksson et al., 1997). Since the resolution of the medical tomograph was in
the order of 1 mm, the smallest crack entities detected were delaminations
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CHAPTER 1. INTRODUCTION

and transverse cracks.

Fractography

This method does not belong to the NDE family, but can be used post mortem to identify the failure mode (cf. Bascom & Gweon, 1989). The fracture surface is sputtered with gold or evaporized with carbon, and preferably analyzed by scanning electron microscopy (SEM). The initiation region is first located, and then carefully examined for any indications of which damage processes that have been active. Since longitudinal carbon fiber composites are relatively brittle, a small amount of contiguous fiber breaks is sufficient to make the specimen fail catastrophically even at moderate stresses. Locating the initiation spot is therefore an arduous task, which is further aggravated by the extensive splitting at final failure. The elevated strain energy is suddenly released at failure, and the subsequent dynamic motion causes a multitude of longitudinal splits which make the fracture surface uneven and difficult to analyze. However, fractography has successfully been used in examining compressive fatigue failure in three point bending (Dillon & Buggy, 1995).

Laser Raman spectroscopy

During the last decade, laser Raman spectroscopy (LRS) has gained considerable popularity in experimental analysis of composite micromechanics. The underlying principle is that the strain dependency of the frequency of the atomic vibrations in a crystalline material can be probed with LRS. Therefore, by loading a composite, the local strains in the visible fibers can be determined from the resulting frequency shift of the Raman spectrum. The fibers should have a crystalline microstructure, i.e. aramid and most carbon fibers are well suited, but amorphous glass fibers cannot be directly analyzed. Rigged with an optical microscope, the focused laser beam can be directed onto a spot on one of the superficial fibers. The stress profile can be determined by scanning the along the length of the fiber. LRS has been successfully used to measure the fiber stress or strain in single fiber/epoxy coupons at a resolution of 1–2 μm. From the stress profiles of adjacent fibers, it is possible to infer the type of damage present between the fibers, e.g. debonding, matrix yielding or cracking. Since it is an optical technique, it does not require physical contact with the specimen and can therefore be used remotely.

For example, Melanitis & Galiotis (1993) have used LRS to quantify the interfacial properties single a fiber carbon/epoxy composite under fragmentation. A model composite with a monolayer of equidistant carbon fibers in
1.5. Modeling

An epoxy matrix was analyzed, from which the interfacial shear stresses were deduced by van den Heuvel et al. (1997). Bennett & Young (1997) investigated the influence of a fiber bridged crack on the fiber stresses in an aramid fiber/epoxy monolayer microcomposite. Marston et al. (1997) extended the technique for application on high volume fraction carbon fiber/epoxy composites.

The differences in features and performance of the above techniques are summarized in Table 1.1. Their abilities of detecting fatigue damage in longitudinal composites (fiber breaks, debonds and small matrix cracks) are indicated. Comparing the damage detection techniques in this overview, the advantages of the surface replication method for intermittent micromechanical monitoring become evident. LRS is a powerful complement for stress state quantification. Results from the surface replication technique applied to longitudinal carbon fiber composites are presented in Chapter 3.

<table>
<thead>
<tr>
<th>Experimental technique</th>
<th>Test &quot;volume&quot;</th>
<th>Fiber break</th>
<th>Debond</th>
<th>Matrix crack</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acoustic emission</td>
<td>entire volume</td>
<td>yes</td>
<td>possible</td>
<td>no</td>
</tr>
<tr>
<td>Stiffness monitoring</td>
<td>entire volume</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Surface replication</td>
<td>surface</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Ultrasonic scanning</td>
<td>through thickness</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>X-ray tomography</td>
<td>entire volume</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Fractography</td>
<td>fracture surface</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Laser Raman</td>
<td>surface</td>
<td>yes</td>
<td>possible</td>
<td>possible</td>
</tr>
</tbody>
</table>

1.5 Modeling

The ultimate objective of fatigue modeling is to predict the fatigue life of a structure. A mechanical analysis of fatigue is made difficult by the interaction of simultaneously progressing fatigue mechanisms at different rates and dependency on loading. The mechanisms themselves are found also in monotonic static loading, but their different rates and their ensuing mutual influence result in dissimilar failure modes (Owen, 1980). Therefore, the fatigue processes of the individual constituents give limited information on how the composite will behave when exposed to fatigue. The brittle carbon and glass fibers show hardly any intrinsic fatigue degradation in an inert environment, whereas the polymeric matrices are sensitive to fatigue.
Most methodologies for life prediction of composite materials contain some statistical treatment of the experimental data. The significant scatter in fatigue life data is a result of the complexity of the mechanisms, the heterogeneous microstructure and the relative brittleness of the material. More precise life predictions can be made for homogeneous isotropic materials such as metals since failure is caused by nucleation and deterministic progression of one single dominant crack. Moreover, in actual structural applications, the loading history may be irregular or have overload situations which make fatigue damage and life predictions less accurate. Small variations in the manufacturing process and in-service conditions may alter the fatigue properties and induce larger variations in fatigue life. For design purposes for composite structures, reliability is a key issue, and a statistical approach is a necessity albeit the source of statistical variation is not known.

The fatigue models for life predictions can be divided into two groups; empirical models based on macroscopic measurements, and mechanistic models accounting for microscopic damage mechanisms. The former group is the most widely used for composites, and requires ample testing. The latter group, mechanistic modeling, is experimentally cumbersome but gives a more clear picture of what actually occurs in the material. Some features of the principal models for life predictions in composite materials subjected to uniaxial fatigue are concisely described below. Models which describe distributed fatigue damage accumulation but do not include failure criteria are not discussed here. Review articles on fatigue modeling and life prediction with numerous references have been provided by Sendeckyj (1990) and Andersons (1994).

**Empirical modeling**

This group of models is also referred to as semi-empirical, macroscopic or phenomenologic models. Their foundation is the $S$-$N$ relationship, i.e. the applied stress level and the corresponding lifetime. The fatigue life can be predicted for a given load situation within the range of stresses encompassed by the $S$-$N$ relationship. The empirical models are as a rule material dependent, and in most cases also dependent on the stacking sequence. In the development of these models, the micromechanics of the damage accumulation is generally not taken into consideration. Since the micromechanics of fatigue degradation in composite materials is not liable to amenable modeling, life predictions generally inclines towards empirical models at present.

The first models to be conceived dealt with load situations with constant amplitude. These were later on generalized to account for variable amplitudes by a damage accumulation approach. For statistical treatment
of these models, the assumption of equal rank in static strength and fatigue life is widely adopted.

**Constant amplitude models**

Originally, the S-N relation was naively calculated from linear regression of the fatigue life data for constant load amplitudes. A linear regression scheme requires a normal distribution of the fatigue life. However, the normal distribution is a mediocre representation of the scatter in fatigue life. The Weibull distribution is generally accepted as the best representative to describe fatigue life of composites (Hwang & Han, 1989). Procedures to generate linear S-N curves from fatigue lives characterized by a two-parameter Weibull distribution have been put forth by Whitney (1981).

More refined models with non-linear S-N curves, are usually obtained by linking the fatigue life to an observable damage metric, $D$. The evolution of the damage metric $D$ is monotonic during fatigue. Typically, $D$ represents axial Young’s modulus, residual strength, or damage size. This degradation can be represented by a differential equation:

$$\frac{dD}{dN} = f(\sigma, R, D),$$

where $f$ is a function the applied maximum stress, $\sigma$, stress ratio, $R$, and the damage metric, $D$. Kachanov (1986) introduced the damage variable $D$ (originally $\omega$) as a scalar. A tensorial representation with a more detailed description of the damage state can be found in work by Lemaitre & Chaboche (1990). However, for many practical applications, a scalar damage variable is a more suitable choice, since the damage state relates to a physical quantity which is macroscopically measurable. Good agreement between experimental data and model predictions has been achieved with $D$ as e.g. residual strength (Sendeckyj, 1981), residual stiffness (Hwang & Han, 1989; Brøndsted et al., 1996), and delaminated area (Beaumont, 1987).

The residual strength degradation model is the most frequently employed in the literature. The degradation expressed in Equation (1.2) is rewritten in terms of a monotonically decreasing residual strength. The degradation rate is typically assumed to obey a power law of the form

$$\frac{d\sigma_r}{dN} = \frac{-A\sigma^\alpha}{\alpha\sigma_r^{\alpha-1}},$$

where $\sigma_r$ is the residual strength, and $A$ and $\alpha$ are fitting parameters. The degradation function on the left-hand side of Equation (1.3) is chosen ad hoc. The applicability of a multitude of functions of similar kind has been tested
by Hwang & Han (1989). The specimen fails when the residual strength decreases to the applied maximum stress, i.e. $N = N_f$ when $\sigma_r = \sigma$. The value of $N_f$ is obtained by integration of Equation (1.3).

Unidirectional $0^\circ$ laminates are known to have a more abrupt decrease of the residual strength at the point of failure than the smoothly decreasing strength given by the above degradation equation (Chou & Croman, 1979; Adam et al., 1986; Ma et al., 1995). The ‘sudden death’ model formulates an abrupt decrease of the residual strength at failure, which usually means as a consequence that the residual stiffness remains constant until the point of failure. The degradation processes until failure of the ‘sudden death’ model and the residual strength model are illustrated in Figure 1.5.

![Figure 1.5. Schematic illustration of the residual strength degradation and sudden death model in an S-N diagram.](image)

For competing failure modes, e.g. tensile and compressive failure in tension-compression cycling, a system of differential equations of the type in Equation 1.3 must be considered (Whitney, 1983). Each equation corresponds to a specific failure mode. Indications of competing failure modes generating clusters in the S-N diagram have also been observed for unidirectional composites in tension-tension loading (Barnard et al., 1985).

**Variable amplitude models**

The stress amplitude of a composite member may vary due to stress concentration, stiffness change in neighboring plies, or application of block or spectrum loading. The constant amplitude models should in this case be modified to account for variable amplitudes. When the stress changes from
one level to another, the damage state remains momentarily constant. The damage state uniquely defines position in the fatigue life, and damage accumulates progressively during fatigue. The variable amplitude models can also be termed cumulative damage models.

The classic example of this approach is the Palmgren-Miner rule, in which the damage metric is set to the fractional life at the different cyclic stress levels, and failure is assumed to occur when the damage sum exceeds 1, i.e.

\[ D = \sum_i d_i = \sum_i \frac{N_i}{N_{fi}}, \]  

(1.4)

where \( N_i \) and \( N_{fi} \) are the number of cycles performed and cycles to failure at stress level \( i \). The Palmgren-Miner failure criterion is irrespective of the order of application of stress level blocks, and is known to yield non-conservative predictions for composites (Curtis, 1989a; Adam et al., 1994). Testing in compression-compression fatigue followed by tension-tension fatigue is more detrimental than the opposite order (Adam et al., 1994). This calls for a non-linear cumulative damage rule which accounts for load block order. The recursive Marco-Starkey model is more suitable for composite materials in which the damage mechanisms are highly dependent on the applied stress level. Furthermore, one type of damage may incite enhanced activity of another damage mechanism, and the order of block loading is hence of primary concern. The Marco-Starkey model, which is based on the iso-damage concept and is a generalization of the linear Palmgren-Miner model, can be expressed as

\[ d_i = \left( \frac{N_i}{N_{fi}} + d_{i-1}^{\frac{\gamma_i}{\gamma}} \right)^{\gamma_i}, \]  

(1.5)

where \( \gamma_i \) is a function of stress amplitude, and \( d_0 = 0 \) reflects the pristine state. Failure occurs when \( d_i = 1 \). Similar non-linear cumulative damage models have been proposed by Hashin (1985), Reifsnider & Stinchcomb (1986), and Gao (1994).

For continuous variation in the stress amplitude level, Equations (1.4) and (1.5) may be adapted to a differential form. Non-linear damage accumulation rules have successfully been used to predict the fatigue life of multidirectional laminates containing at least one 0° ply (Reifsnider & Stinchcomb, 1986; Diao et al., 1995). The global stress level has here a constant amplitude. As subcritical cracks in the off-axis plies develop, the local stress state in the 0° changes continuously. When the 0° ply fails, the entire laminate fails. The 0° ply constitutes the critical element of the laminate. The critical element models are based on the cumulative damage models since the local stress amplitude varies. Experiments and satisfying predictions of the critical element model underscores the relative importance of the 0° ply (Gao,
A critical element model considering localized damage in the 0° plies close to the transverse cracks in the adjacent plies would be a more physical description. However, delaminations and longitudinal splitting in the region of the critical model makes it difficult to predict the evolution of the local stress amplitude.

One of the problems with the empirical variable amplitude models is that the chosen damage metric, usually a scalar determined from macroscopic testing, does not reflect the true damage state of the composite. A set of damage parameters that relate in more detail to the current damage state would be more adequate. This can only be done by identifying the dominating fatigue damage mechanisms, and characterizing their growth rates and critical values as a function of load level.

**Strength life equal rank**

The equal rank of strength and life is an important notion and plays a significant role in fatigue modeling of composites. It is postulated that if a sample of components could be tested for both static strength and fatigue life expectancy, each individual member would occupy the same rank in both strength and fatigue life data sets. The percentiles in strength, \( \sigma_p \), and life, \( N_{fp} \), are thus directly linked by the fatigue life relationship:

\[
\frac{\sigma}{\sigma_p} = F(N_{fp}),
\]

where \( \sigma \) is the applied stress, and \( F \) is a function dependent on the fatigue life. By fatigue life measurements and estimation of the distributions of strength and life, \( F \) in Equation (1.6) can determined (Barnard et al., 1988). Similarly, the life distribution can be deduced from the fatigue life curve and the strength distribution (Barnard et al., 1985). With this above scheme, the deterministic cumulative damage models can be modified for the analysis of life distributions (Hwang & Han, 1987). Wang et al. (1981) showed that proof tests can ascertain a minimum life with a reasonable confidence. This supports the assumption of equal rank.

An argument against the equal rank assumption is that the damage mechanisms differ in fatigue at lower stress levels and in static testing. It can therefore not be excluded that the fatigue degradation and subsequent failure may occur earlier for specimens with lower static strengths. However, for stress levels close to the static failure stress where similarities in failure mechanisms are notable, the assumption could be considered valid. The strength life equal rank assumption seems to be valid in practice, but its general truth from a scientific viewpoint is more questionable.
Mechanistic modeling

The above empirical models are based on a macroscopic approach. A micromechanical model would be based on statistics of fiber failure and fracture mechanics where the inhomogeneous microstructure is taken into account. There are several advantages with mechanistic models. If they represent an accurate description of the physical fatigue degradation, they can be generalized to other composite materials, provided the microstructural properties have been quantified. This means that less testing is necessary, and that parametric studies can easily be undertaken for performance optimization. Since the fatigue damage accumulation in composites is a complex phenomenon with interacting mechanisms, the modeling is not as straightforward as for homogeneous materials. Early work considered composite laminates as an anisotropic homogeneous material, and anisotropic fracture mechanics was applied (Corten, 1972). Notions from fracture mechanics can be extended to the ply level, and to describe the delamination process in particular (Carlsson, 1993). Detailed micromechanical models which describe individual fiber breaks, and the ensuing stress redistribution, have been presented in parametric studies, or as a description of experimental results of microcomposites. These models pertain to static loading (Bader, 1988; Marston & Neumeister, 1998), or to creep (Phoenix et al., 1988; Du & McMeeking, 1995; Fabeny & Curtin, 1996). Once the fatigue mechanisms have been understood, it should be feasible to extend the models to fatigue conditions.

For a material that shows fatigue degradation, a cycle dependent progressive mechanism should be active. This mechanism gives rise to some amount of irreversible damage accumulation for each load cycle. Matrix or interfacial crack growth, and viscoelasticity can be envisaged in a PMC material, which would typically grow according to a power law, e.g. Equation (1.2). By integration of such a relation from the initiation size to a current size, an explicit expression in terms of number of elapsed cycles can be obtained. However, the coalescence of distributed damage sites and synergism with e.g. fiber breakage imply a large set of differential equations, and a numerical approach is necessary. Recently, Zhou & Curtin (1995) and Ibnabdelljalil & Curtin (1997) have adopted a lattice Green function method previously used in modeling in solid state physics describe damage accumulation in a unidirectional composite by fiber breakage and matrix cracking. It would be useful if this group of micromechanical models could be extended to account for fatigue. However, this would first require careful experimental work to quantify the microstructural and kinetic parameters for a wide range of materials.
Design methodology

In an engineering perspective, the primary purpose of a model is to incorporate it in a design method for reliable and accurate prediction of damage growth and life. Two design methods prevail in practice; (i) safe life, and (ii) damage tolerance design (Curtis & Dorey, 1986). Safe life design is based on detailed statistics on failure probabilities, and requires a considerable amount of test data. Damage tolerance design assumes the presence of small flaws of a certain detectable size. The material has to be characterized in terms of damage propagation rate and critical flaw size. The structure is continuously inspected at least twice before an undetected flaw can become critical. If no flaw is detected on inspection, the hypothetical flaw size is reset to the initial detectable size again. In a homogeneous material, the flaws are generally distinct large cracks which can readily be measured. In composites, there is not yet enough knowledge of which flaws to monitor, their propagation rates, and when they become critical and lead to final failure. The more diffuse damage state in composites necessitates a careful experimental investigation on the fatigue damage mechanisms in polymer matrix composites to identify these main parameters. A mechanistic model based on such a characterization can serve as input in a more general fatigue design methodology.

1.6 Objective of study

Newer polymer composite materials with tougher matrices have proved to be more susceptible to fatigue degradation. The reason for this is not altogether understood. To contribute to the clarification this issue, a mechanistic path has been embarked in this work. The aim is to map the fundamental fatigue damage mechanisms in $0^\circ$ plies, and to propose reasons for the inferior performance of tough matrix composites.

Pertinent materials used in primarily aeronautical and automotive applications have been investigated: carbon fiber reinforced plastics with tough matrices for high performance applications, and glass fiber reinforced polypropylene for structural applications in large volume. The mechanical test method is constant load amplitude fatigue in tension-tension loading under ambient conditions. This is to confine the problem, and keep the independent parameters to a limited number. Off-axis plies are excluded, since the critical element for ultimate fatigue failure is the longitudinal ply. It is anticipated that fatigue of multidirectional laminates can be better interpreted once the fatigue mechanisms of the controlling unidirectional plies are understood.

Cracking in transverse plies also contributes to failure, in particular in
tension-compression fatigue loading. A study of the responsible incipient micromechanisms would indicate the weak constituent element. Tests of model composite specimens and an elementary finite element analysis were therefore undertaken.

The long-term aim is to develop fracture mechanics models for the damage development based on experimentally observed mechanisms. Tentative models of the observed mechanisms have been outlined, within the purpose of a parametric investigation and assessment of operative mechanisms. Furthermore, a partial goal was to conceive a quantitative statistical tool based on the fatigue life diagram. Such a tool can be used to investigate the influence of material properties on the fatigue behavior in a more systematic manner.
Chapter 2
Fatigue life diagrams

2.1 Physical background

In order to quantitatively compare the influence of material properties on the fatigue life performance, a well-defined method is necessary. Due to the large variation in time to failure of composite materials subjected to fatigue, a statistical approach must be undertaken. A statistical treatment would also result in a possibility of reliability assessment. Reliability is an important issue in design against fatigue, even though over-conservative design methods based on the static properties or fatigue limit prevail in practice. A statistical fatigue life model is also prompted by the aim for more optimized design with lighter and more slender structures.

Fatigue life data for composite materials are generally presented in a Wöhler diagram or $S$-$N$ curve, and the fatigue degradation is commonly characterized by linear $S$-log $N$ curve deduced from linear regression of the data. This represents a relatively crude description, since the fatigue behavior composites is known to exhibit different regions dependent on the applied load level, each pertinent to different damage mechanisms.

It is not the author’s contention to contribute with a conjectural model to the plethora of empiric fatigue life relations. The proposed procedure is based on the fatigue life diagram and its rationale of underlying mechanisms. The fatigue life diagrams introduced by Talreja (1981a) have shown themselves useful as a framework in interpreting the fatigue properties of uni-directional composite materials in tension-tension loading. It contains three distinct regions (see Figure 2.1), each relevant to different fatigue damage mechanisms. Region I is associated with the static failure; multiple random fiber breakage. Region II pertains to a progressive mechanism – hence the sloping characteristic. Region III is the fatigue limit of the composite. Below
this limit, failure does not occur prior to a large number of cycles where the fatigue test was stopped, typically at $10^6$ cycles. Fiber-matrix debonding and crack arresting by neighboring fibers may inhibit damage accumulation and subsequent failure (see Chapter 3). A fatigue life model described by the appearance in Figure 2.1 is inferred from experimental observations and mechanistic reasoning (Talreja, 1993).

A better goodness-of-fit may be obtained by adapting the shape of the fatigue life curve to every new set of data over the entire range of stress amplitudes, but such an approach is not suitable for material comparison and demands several fitting parameters. Since the division of fatigue life data into three regions in the fatigue life diagram has proved to describe the fatigue behavior well, and serve as a framework for fatigue interpretation, it will also be advocated here.

In the fatigue life diagram, strain instead of the more commonly used stress is used as the independent variable. Both fibers and matrix would globally be subjected to the same longitudinal strain, whereas stresses in the two phases would differ depending on the volume fraction and the elastic moduli. Most fatigue tests are load controlled, and the peak strain value would give a measure of the damage state in the material at the first application of load. Further damage accumulation would depend directly on the initial damage state, and therefore also on the initial strain amplitude. Furthermore, the level of Region I is strongly influenced by the strain to failure of the fiber, and the fatigue limit of the matrix material in strain represents an upper bound of the level of Region III at the fatigue limit of the composite.
2.1. PHYSICAL BACKGROUND

(Talreja, 1981a). It is therefore sensible to select initial peak strain for the ordinate.

It would be interesting to investigate how microstructural parameters (such as matrix ductility, fiber stiffness, interfacial adhesion etc.) influence the slope and position of Region II. For instance, it can be quantitatively shown that an increase matrix ductility in carbon fiber reinforced plastics results in a steeper slope in Region II (Gamstedt & Talreja, 1997), with a well-defined description of the fatigue life diagram. The fatigue life diagrams can also be used for interpretation of the fatigue behavior in unidirectional metal matrix composites (Majumdar & Newaz, 1995; Brindley & Bartolotta, 1995; Talreja, 1995), and in unidirectional ceramic matrix composites (Talreja, 1990).

A characteristic feature in the fatigue behavior of composite materials is the relatively large scatter in life compared to e.g. metallic materials (Curtis, 1989b). The source of the large scatter is manifold. Since longitudinal composite laminates are brittle, their strength is dependent on the distribution in inherent flaw size. This results in a relatively large scatter in strength, and an even more pronounced scatter in fatigue life.

If the composite microstructure with disparate constituent properties has a size scale in the same order as the propagating crack, the growth rate is erratic and refuses to be characterized by a Paris power law. This has been observed for composites with spherical fillers (Gadkaree & Salee, 1983) and unidirectional model composites (Botsis & Beldica, 1994/95). This kind of behavior results in a large variation in lifetime. Steady state crack growth in homogeneous materials allow more deterministic and accurate life predictions. In a composite, it is likely that a larger part of the total fatigue life is expended by the initiation and erratic damage accumulation compared to the duration of steady state crack growth. This uneven relative contribution is indirectly supported by experimental observations. Almost all acoustic emission activity from fiber breakage (Jamison, 1986; Bhat & Murthy, 1993) and measurable longitudinal stiffness degradation (Awerbuch & Hahn, 1981; Piggott & Lam, 1991) take place in the last few percent of the total life prior to ultimate failure. If the lifetime were dominated by steady state growth of damage entities, progressive fiber breakage would occur, and the stiffness would gradually degrade. A previous study (Gamstedt & Talreja, 1997) with a surface replica technique showed that cracks grow at an irregular pace, and are comparable in size to the heterogeneous microstructure. Furthermore, the critical flaw size for brittle $0^\circ$ composites is small for a certain stress level compared to tougher materials. As presented in the following chapter, not more than 6 to 7 contiguous fiber breaks were found at the surface of the CFRP materials during fatigue. No multiplets of greater order than three
were detected by Jamison (1982) in the 0° plies of cross-ply laminates subjected to fatigue. Prior to static failure of carbon fiber tows embedded in epoxy resins, the maximum number of adjacent fiber breaks encountered by Marston & Neumeister (1998) was 6. By statistical modeling of fiber breakage in a unidirectional composite, Bader (1988) and Wisnom & Green (1995) estimated ~ 8 as critical multiplet order for CFRPs. A critical flaw size of no more than a few order of magnitude larger than the size of the fiber diameter could mean that the onset of the steady state crack growth would never occur. The initiation and unpredictable non-parisian crack growth would then be the contributors to the total life.

Whitney (1981) developed a procedure to generate linear $S$-$N$ curves provided with a two-parameter Weibull distribution for the time to failure at a specific stress level. Many coupons were tested at a few stress levels, and the life distribution parameters were estimated at these levels by data pooling. Subsequently, the linear $S$-$N$ curve was obtained by fitting to the fatigue life scale parameter. In the present work, the shape and the slope of the fatigue curve are emphasized, in contrast to the life distribution at a few discrete stress levels. Therefore, the fatigue tests were here performed at evenly dispersed load levels. Each specimen was tested at a different stress level. An estimation of reliability at a given stress level can nevertheless be made for an experimental set with only one sample on each stress level by use of data pooling.

The basic concepts of statistical methods in strength of materials and structural mechanics can for example be found in the textbooks of Bolotin (1969) and Provan (1987). For fatigue and life estimation, see e.g. Little & Jebe (1975).

### 2.2 Statistical preliminaries

To estimate the position of the scatter bands in a fatigue life diagram some statistical tools are necessary. These tools are provided in this section. They are based on assumptions of plausible distribution functions of the measured data and methods of estimating their parameters in order to obtain confidence intervals of the scatter bands.

### Distributions

The Weibull distribution is known to give a good description of the scatter of static strength of materials. Waloddi Weibull introduced this distribution
in 1939. The probability density function takes the form

\[ p_r(\sigma) = \frac{\beta}{\sigma_0^\beta} (\sigma - \sigma_u)^{\beta-1} \exp \left\{ - (\frac{\sigma - \sigma_u}{\sigma_0})^\beta \right\}, \]  

(2.1)

where \( \beta \) is the shape parameter, \( \sigma_0 \) is the scale parameter, \( \sigma_u \) is the location parameter, and \( \sigma_u \leq \sigma < +\infty \). With the static strength of the material as the random variable, it is stipulated that \( \beta, \sigma_0 \) and \( \sigma_u \) are characteristic constants of the material with positive values. The associated cumulative distribution function, or probability of failure, has a more clear form,

\[ P_f(\sigma) = \int_{\sigma_u}^{\sigma} p_r(\xi) \, d\xi = 1 - \exp \left\{ - (\frac{\sigma - \sigma_u}{\sigma_0})^\beta \right\}. \]  

(2.2)

The corresponding reliability function, or probability of survival, is expressed as

\[ P_s(\sigma) = 1 - P_f(\sigma) = \exp \left\{ - (\frac{\sigma - \sigma_u}{\sigma_0})^\beta \right\}. \]  

(2.3)

The expectancy or mean of the Weibull distribution is

\[ \langle \sigma \rangle = \sigma_0 \Gamma \left( 1 + \frac{1}{\beta} \right) + \sigma_u, \]  

(2.4)

where \( \Gamma \) is the gamma function. The location parameter represents a maximum stress level the specimens can endure without failure. A two parameter variant of the Weibull distribution can be devised by removal of the location parameter, \( \sigma_u = 0 \), and has the cumulative distribution function

\[ P_f(\sigma) = 1 - \exp \left\{ - (\frac{\sigma}{\sigma_0})^\beta \right\}. \]  

(2.5)

General articles on the properties of the Weibull distribution and its application to strength of materials have been compiled by Hallinan (1993) and Lindquist (1994).

For most brittle materials, it is assumed that \( \sigma_u = 0 \), i.e. the strength takes a two parameter Weibull distribution. It can be shown that the two parameters, \( m \) and \( \sigma_0 \), can be related to the distribution of pre-existing flaws (Jayatilaka & Trustrum, 1977).

Since the objective is to estimate the distribution in fatigue life, a conjectural choice of its distribution type is undesirable. In order to find the fatigue life distribution, a presumptive kinetic model of the residual strength degradation is commonly formulated (Whitney, 1983; Sendeckyj, 1990; Diao et al., 1995). Another approach is based on the Paris law of fatigue crack
propagation (Halpin, 1973). The latter is physically more sound since it is based on the underlying mechanism, in this case the growth of one single crack. This relates better to metallic materials than to composites, where the fatigue damage is more distributed and complex. However, Talreja (1987) generalized the notion of fatigue crack propagation of a single hypothetical crack entity to account for fatigue damage propagation in composites. The damaged zones in the composite may be conceptually substituted by a single crack which releases the same amount of the stored elastic energy as that released combinely by the various damage mechanisms in all the localized damage zones in the composite. It is assumed that the characteristic dimension of the single conceptual crack is given by the scalar damage metric $D$, which may be taken as the Kachanov damage variable, $\omega$ (Kachanov, 1986). A schematic picture of the mapping from the complex damage state in a composite to the elementary case with a single crack subjected to tensile normal load is presented in Figure 2.2. It should be noted that the crack dimension, $D$, does not reflect a physical and measurable value. It is an abstraction used to determine a relation between the distributions in static strength and fatigue life, and between the applied load amplitude and the expected life, respectively.

![Figure 2.2. Mapping of fatigue damage in a composite to a conceptual single crack.](image)

At a given state of damage described by $D$, the residual strength can be expressed as

$$\sigma_{\text{res}} = \frac{K_c}{Y\sqrt{D}},$$

(2.6)

where $K_c$ is a material constant characterizing the material toughness, and $Y$ is a geometric factor, which is assumed to be independent of $D$ since the damage sites causing failure generally are small in composites compared to the specimen dimensions (e.g. Jamison, 1982; Bader, 1988; Marston &

The rate of damage growth can now be expressed by the rate of increase in the crack size $D$ according to the same pattern as the Paris law of fatigue crack growth, namely

$$\frac{dD}{dN} = C \left( \sigma Y \sqrt{D} \right)^m,$$

where $N$ is the number of elapsed load cycles, $\sigma$ is the applied stress level for zero-tension loading, and $C$ and $m$ are parameters that characterize the crack propagation rate. By combination of Equations (2.6) and (2.7), a rate equation for the degradation of the residual strength is obtained,

$$\frac{d\sigma_{res}}{dN} = -\frac{C(\sigma K_c)^m}{2} \left( \frac{Y}{K_c} \right)^2 \sigma_{res}^{3-m},$$

which is a separable differential equation, that yields

$$\sigma_{res}^{m-2}(N) = \sigma_e^{m-2} - \frac{C(m-2)(\sigma K_c)^m}{2} \left( \frac{Y}{K_c} \right)^2 N,$$

where $\sigma_e$ is the initial static strength. This kinetic equation for the residual strength degradation can be simplified by a substitution of variables. Let the exponent be $c = m - 2$, and the degradation coefficient be

$$k = \frac{C(m-2)K_c^m}{2} \left( \frac{Y}{K_c} \right)^2,$$

which results in the kinetic relation

$$\sigma_{res}^c(N) = \sigma_e^c - k \sigma_e^{c+2} N,$$

which is of similar type as ad hoc power laws for the degradation of residual strength frequently used in the literature. Through the conceptual mapping from the complex damage state in composites to a hypothetic single crack, Equation (2.11) possesses traits of physicality.

The cumulative distribution function of the residual strength is obtained from Equations (2.5) and (2.11) as

$$P_f(\sigma_{res}) = 1 - \exp \left\{ - \left( \frac{\sigma_{res}^c + k \sigma_e^{c+2} N}{\sigma_0^c} \right)^{\beta/c} \right\}.$$

The residual strength decreases monotonically and $\sigma_{res} = \sigma$ at failure, where $N = N_f$. The cumulative distribution function of the fatigue life can then be expressed as

$$P_f(N_f) = 1 - \exp \left\{ - \left( \frac{N_f + k^{-1} \sigma^{-2}}{k^{-1}[\sigma_0/\sigma]^c \sigma^{-2}} \right)^{\beta/c} \right\}.$$
Comparing with Equation (2.2), the fatigue life distribution is identified as a three-parameter Weibull distribution. However, the shift term \( k^{-1} \sigma^{-2} \) can be neglected for high cycle fatigue (low stress levels), and a two-parameter Weibull distribution presents itself,

\[
P_f(N_f) = 1 - \exp \left\{ - \left( \frac{N_f}{N_0} \right)^{\beta/c} \right\},
\]

where the scale parameter is

\[
N_0 = \frac{\sigma^c_0}{k \sigma^{c+2}}.
\]

Recapitulating the strength degradation model in Equation (2.11) and the assumption of a two-parameter Weibull distribution for the initial strength in Equation (2.5), it followed that the life has a two-parameter Weibull distribution for high cycle fatigue. This distribution function will be used in the estimation of the distribution parameters.

The shape parameter in the life distribution in Equation (2.14), \( \beta/c \), is independent of the applied stress level. This is also generally assumed when pooling experimental data for a more accurate determination of fatigue reliability (Whitney, 1981, 1983; Tai et al., 1995). However, Tai et al. measured a decrease in the shape parameter with increasing stress level. This conforms with intuition, since a larger scatter in life is expected at longer lives close to the fatigue limit where the progressive damage mechanisms are suppressed. If there is a uniform mechanism in Region II whose propagation can be described by a power law, the distribution in fatigue life has a homoscedastic property. The shape parameter of the fatigue life distribution is constant, and independent of the applied strain level. Experimental results indicates a constant shape parameter within this region in tension-tension loading (Gathercole, 1994). The pooling of fatigue data from different load amplitudes may be extended to encompass different materials after a sensible normalization (Echtermeyer et al., 1996b), which is useful for design studies when exact materials data are not available.

The scale parameter in Equation (2.15) is on the contrary dependent on the stress level. Since \( N_f = N_0 \) represents a constant probability of survival of 37%, Equation (2.15) can be plotted as an \( S-N \) curve with a constant level of probability, \( \text{viz.} \)

\[
\log N_0 = -m \log \sigma + \log \frac{\sigma_0^{m-2}}{k}.
\]

The curve would be linear when \( \log \sigma \) is plotted with respect to \( \log N_f \), \( \text{i.e.} \) the fatigue degradation is described by a Basquin relation. Likewise, the
curve would be linear in a fatigue life diagram with $\log \varepsilon$ versus $\log N_f$ if the stiffness of the composite remains approximately the same throughout the fatigue test. This ‘sudden death’ type of behavior is common for composites containing unidirectional $0^\circ$ plies (Chou & Croman, 1979; Adam, 1986; Harris, 1986; Ma et al., 1995). The linear Basquin relation has shown to best fit experimental fatigue life data for certain composites (Echtermeyer et al., 1996b).

Another distribution function that is widely used for describing the fatigue life, and to a lesser extent also strength, is the log-normal distribution. The scheme outlined below can readily be modified for use of a log-normal distribution, or an arbitrary distribution function. The log-normal probability density function is

$$p_f(x) = \frac{1}{\sigma_0 x \sqrt{2\pi}} \exp \left\{ -\frac{(\ln x - \mu)^2}{2\sigma_0^2} \right\},$$  \hspace{1cm} (2.17)

where $x > 0$. The mean or expectancy is $\exp(\mu + \sigma_0^2/2)$. The corresponding cumulative distribution function is

$$P_f(x) = \Phi \left( \frac{\ln x - \mu}{\sigma_0} \right),$$  \hspace{1cm} (2.18)

where $\Phi(\cdot)$ is the cumulative distribution function of the standardized normal distribution. The three-parameter counterpart of the log-normal distribution can be obtained by a shift of $x$ in Equation (2.17), i.e. by substitution of $x$ with $x - x_u$. Within certain limits, there is no appreciable difference between the log-normal and Weibull distribution for fatigue data (Jacoby & Nowack, 1972), although the choice of the Weibull distribution is recommended for strength characterization (Hwang & Han, 1987). The log-normal distribution has the benefit of having parameters that are easily determined, e.g. the maximum likelihood estimate of the mean logarithmic life equals the sample mean, and the maximum likelihood estimate of the standard deviation is explicitly determined and consistent with the sample standard deviation (Nelson, 1982). Also, with a log-normal distribution, confidence and tolerance bounds are readily obtained from amply tabulated values of non-central $t$ distributions, whereas for e.g. Weibull distributions, one would have to resort to Monte Carlo simulations to obtain the corresponding bound values (Ronold & Echtermeyer, 1996).

The use of a three-parameter Weibull distribution (see Chou & Croman, 1979, and Wang et al., 1981) in the description of the fatigue life is more stringent. It is based on the condition that the specimen subjected to fatigue survives the first cycle, i.e. the applied stress is lower than the static
strength. However, a two-parameter Weibull distribution will sufficiently serve its purpose for practical applications (Trustrum & Jayatilaka, 1979), and will be employed here.

**Parameter estimation**

The maximum likelihood (ML) method can be used e.g. to estimate the shape parameter, $\beta$, and the scale parameter, $\sigma_0$, of the two-parameter Weibull distribution in Equation (2.5). This is a versatile and generally applicable method frequently used in reliability and life analysis (Bain, 1978). Furthermore, it has the advantage of being applicable to multiply censored or truncated data (Nelson, 1982). The censored data considered in this application are of type I, i.e. the experiments were terminated after a fix time, typically $10^6$ cycles or at premature failure not caused by fatigue of the material under investigation (e.g. tab failure).

The maximum-likelihood estimators are biased for small sample sizes. Increasing the sample size generally reduces the bias. As the number of samples approaches infinity, the estimator becomes unbiased, i.e. the estimator is consistent. The extent of the bias can be computed by Monte Carlo simulations, which can be used *a posteriori* to correct the estimated value (Sendeckyj, 1981).

Other candidate methods are e.g. the moment (Bard, 1974), multiple regression (Sheikh & Younas, 1995) and standardized variable estimation (Talreja, 1981b). The latter has proved to render more accurate and efficient estimates for three-parameter Weibull distributions than the moment and maximum likelihood methods.

Let $\sigma_1, \sigma_2, \ldots, \sigma_r$ be complete measured values and $\sigma_{r+1}, \sigma_{r+2}, \ldots, \sigma_n$ censored measured values of the random variable for the specimen strength which has a two-parameter Weibull distribution. The probability density function $p_t(\sigma; \sigma_0, \beta)$ is given by the unshifted form of Equation (2.1) where $\sigma_u = 0$. The reliability function $P_s(\sigma; \sigma_0, \beta)$ is similarly obtained by setting $\sigma_u = 0$ in Equation (2.3). The likelihood function of this set of samples is defined as

$$L(\sigma_0, \beta) = \prod_{i=1}^r p_t(\sigma_i; \sigma_0, \beta) \prod_{j=r+1}^n P_s(\sigma_j; \sigma_0, \beta).$$

The maximum likelihood estimators, $\hat{\sigma}_0$ and $\hat{\beta}$, maximize the likelihood function. Essentially, $\hat{\sigma}_0$ and $\hat{\beta}$ maximize the probability of occurrence of the sample results. The estimators satisfy the likelihood equation system

$$\frac{\partial \ln L(\sigma_0, \beta)}{\partial \sigma_0} \bigg|_{\sigma_0 = \hat{\sigma}_0} = 0, \quad \text{and} \quad \frac{\partial \ln L(\sigma_0, \beta)}{\partial \beta} \bigg|_{\beta = \hat{\beta}} = 0.$$
For distribution functions with more than two unknown parameters the likelihood equation system is generalized to the same number of equations.

The maximum likelihood method may yield erroneous estimates from local maxima of the likelihood function in Equation (2.19). Unreasonable estimates can instantly be discarded. For a closer examination, a distribution curve (the cumulative distribution function \( \text{versus} \) sampled values of the random variable) should be plotted as a conviction that the correct estimates have been obtained. The matrix procedure employed by Spindel & Haibach (1979), or an equivalent contour plot, gives information on the location of the maximum with the defined ranges of the parameters.

Another case is when data are censored from both ends, e.g. in the estimation of applied stress or strain for a certain lifetime interval. Assume that \( r \) specimens with the applied stresses \( \sigma_1, \sigma_2, \ldots, \sigma_q \) have failed at \( N_f < N_{\text{lim}} \). Let the number of specimens that have failed in the interval \( N_{\text{lim}} \leq N_f < N_{\text{end}} \) be \( r - q \) with stress levels \( \sigma_{q+1}, \sigma_{q+2}, \ldots, \sigma_r \). The \( n - r \) specimens have not failed at \( N_f = N_{\text{end}} \), and their stresses are \( \sigma_{r+1}, \sigma_{r+2}, \ldots, \sigma_n \). This set of data is multiply censored and contains a set of complete data. Even though the stress level \( \sigma \) is not a random variable, its distribution parameters for a fix interval of lifetime can be obtained provided that the stress in the interval is independent of the fatigue life. The distribution parameters can be determined by maximizing the likelihood function

\[
L(\sigma_0, \beta) = \prod_{i=1}^{q} P_f(\sigma_i; \sigma_0, \beta) \prod_{j=q+1}^{r} p_f(\sigma_j; \sigma_0, \beta) \prod_{k=r+1}^{n} P_s(\sigma_k; \sigma_0, \beta) \quad (2.21)
\]

with respect to \( \sigma_0 \) and \( \beta \). As in the previous case, the maximum likelihood estimators \( \hat{\sigma}_0 \) and \( \hat{\beta} \) satisfy Equations (2.20).

The fatigue limit is the stress or strain value below which failure never occurs. The stress level close to the fatigue limit would have a three parameter distribution, e.g. the Weibull distribution \( P_f(\sigma; \sigma_u, \sigma_0, \beta) \) as in Equation (2.2), where the location parameter \( \sigma_u \) is the fatigue limit. It can be argued whether a true fatigue limit exists for a certain material or not. Some composite materials seem to have a fatigue limit (Grimes, 1977), while others do not possess a fatigue limit up \( 10^8 \) cycles (Echtermeyer et al., 1996a). It is not realistic to let fatigue tests run for longer than a certain lifetime at reasonable frequencies (say \( 10^8 \) cycles). Also, no structures are designed to last for all eternity, and the existence of a fatigue limit is not an issue from a practical viewpoint. Furthermore, the generalization of Equation (2.21) makes the determination of the parameters more costly and far less efficient. It is therefore assumed that the fatigue limit is described by a two parameter Weibull distribution, in conformity with the distributions of the static strength and fatigue life expressed in Equations (2.5) and (2.14).
From a practical standpoint, it is more interesting to obtain the strength or lifetime for a given probability of survival than the corresponding mean, mode or median. When the parameters have been estimated, confidence intervals of the random variable can be calculated from Equation (2.3). The lower and upper \( p \)th percentiles are

\[
\sigma_p = \sigma_0 (-\ln R_p)^{1/\beta},
\]

where \( R_p = 1 - p \) or \( p \), respectively. The probability that a random sample takes a value in the interval bounded by the lower and upper percentiles is \( 1 - 2p \). For few fatigue data, there is some inherent variability in the estimates that cannot be neglected, and the confidence bounds should be complemented with tolerance bounds for design purposes (Ronold & Echtermeyer, 1996).

![Figure 2.3. Static, complete and censored data in a fatigue life diagram.](image)

### 2.3 Deduction scheme

The generation of the scatter bands in the fatigue life diagram is made in three steps, each corresponding to a different region. There are also three types of data; (i) static strengths, (ii) complete strain level-life data from specimens that have failed during the course of the fatigue test, and (iii) censored data from specimens that never failed (run-outs) or failed prematurely due to \( e.g. \) tab delamination. These data can be plotted in a fatigue life diagram, maximum initial logarithmic strain with respect the logarithmic life
The static and fatigue limit bands related to Region I and III, respectively, are determined first. The scatter band of the progressive region, Region II, is obtained by estimation of the slope and variance from data not contained by the two other regions. The estimations are easily performed with a software package for symbolic mathematics. The Mathematica™ software (Wolfram Research, Inc.) was used here. The deduction scheme is schematically outlined in Figure 2.4.

A. Region I

The horizontal scatter band in Region I reflects a similitude in mechanisms to those present in static failure. The static scatter band is determined only from the static strength data. It is assumed that the static strength has a two parameter Weibull distribution, which has been experimentally validated for other brittle materials (Trustrum & Jayatilaka, 1979). The maximum likelihood estimates of the scale and shape parameters are determined from Equations (2.20). Next, the boundaries of the scatter band for a certain level of reliability is obtained from Equation (2.22). For consistency, the same level of reliability should be used for all three regions in the fatigue life diagram. The confidence intervals bounded by $e_{1,p}$ and $e_{1,1-p}$ are plotted as horizontal lines in the fatigue life diagram as depicted in Figure 2.4 a.

B. Region III

The estimation of the horizontal scatter band in Region II just above Region III at the fatigue limit may pose some problems. Scant number of data, of which a fair share is censored, imposes caution in the interpretation of the estimates. Also, the very existence of a fatigue limit is a matter of
controversy. A verification is not feasible due to perpetual testing times. However, a scatter band may be devised where the fatigue life curve levels off asymptotically. The fatigue limit is sometimes used as a design criterion, i.e., the applied stress or strain should not exceed the fatigue limit. The so-called staircase or up-and-down method may be used to determine the fatigue limit (Little, 1975).

The plotted fatigue life data is first visually examined to determine the existence of a horizontal scatter band in Region III. If no such region can be discerned, all remaining data belong to Region II which is dealt with in step C.

The transition from Region II to Region III must first be identified. It is ideally characterized by a discontinuity in the slope of the fatigue life curve. This transitional curve kink is generally termed the endurance point. Models with smooth transitions at the endurance points have also been established, but they require an extra parameter which has to be estimated from the same set of data (Spindel & Haibach, 1981; Ling & Pan, 1997). Due to a large scatter in material data, the location of the endurance point is only roughly determined (cf. schematic illustration in Figure 2.3). However, if there is ample data in the horizontal fatigue limit scatter band, an approximate choice of endurance point does not mean the estimators will be misjudging.

At the fatigue limit in the horizontal part of Region II immediately above Region III, the fatigue life data in the interval \( N_{\text{lim}} \leq N_f < N_{\text{end}} \) are considered. The failures before this interval, and the failures expected after the interval can also be used as censored data for more accurate parameter estimates. The distribution parameters governing the fatigue limit can be determined by maximizing the likelihood function given by Equation (2.21) where the product runs over all observations. The distribution parameters yield from the solution of Equations (2.20). The scatter band lines from the preset percentile are then obtained from Equation (2.22). The lower and upper percentiles are denoted \( \varepsilon_{\text{III},p} \) and \( \varepsilon_{\text{III},1-p} \), respectively and drawn in Figure 2.4 b.

C. Region II

The fatigue life data points that do not fall within the determined scatter bands of Region I and in the horizontal band above Region III belong to the set of data that will be used to determine the sloping scatter band of Region II. Also here, the lifetime is believed to be governed by proper mechanisms. A progressive crack as described in Equation (2.7) manifests itself in a Basquin fatigue relation,

\[
\log N_f = k \log \varepsilon + l,
\]  

(2.23)
where $k$ and $l$ denote the slope and the intercept with the life axis in the log-log fatigue life diagram. This relation is analogous to the scale parameter relation in Equation (2.16) deduced from the conceptual crack propagation model. Visual inspection of the plotted fatigue life data usually supports the linear relation in Equation (2.23). The fatigue life at a certain strain amplitude level, $N_f$, has the distribution found in Equation (2.14). By redefinition of the shape parameter $\beta$, the fatigue life at a certain stress level has the following cumulative distribution function:

$$F(N_f) = 1 - \exp \left\{ - \left( \frac{N_f}{N_0} \right)^\beta \right\}$$  \hspace{1cm} (2.24)

The shape parameter, $N_0$, signifies a constant probability of survival. In a previous report (Gamstedt, 1995), a statistical deduction scheme was presented how to construct fatigue life diagram. It was assumed that the logarithmic life, $\log N_f$, was Weibull distributed. Here, the fatigue life, $N_f$, takes a Weibull distribution which is more consistent with experimental data, and relies on a deduced fatigue degradation model. This is to remedy the previously presented model.

A description of iso-survival lines by a Basquin relation can be achieved by letting $\log N_{0i} = k \log \varepsilon_i + l$ for each initial strain level $\varepsilon_i$. As a result, all the data in Region II are pooled to render a common shape parameter, $m$, and a good fit of the linear Basquin relation. Previously, data pooling has also been used in fatigue life assessment of composite materials without consideration of the strain level-life relation (Whitney, 1981; Talreja 1981b). There are now three unknown distribution parameters; $k$, $l$ and $\beta$. The Equations (2.20) must be modified to give all three estimators. The first equation is rewritten as

$$\sum_{i=1}^{n} \frac{\partial[\ln L(N_i; N_{0i}, \beta)]}{\partial N_{0i}} \frac{dN_{0i}}{dk} \bigg|_{k=k} = 0,$$

and

$$\sum_{i=1}^{n} \frac{\partial[\ln L(N_i; N_{0i}, \beta)]}{\partial N_{0i}} \frac{dN_{0i}}{dl} \bigg|_{l=l} = 0,$$

where $dN_{0i}/dk = 10^l \varepsilon_i^k \ln \varepsilon_i$, $dN_{0i}/dl = 10^l \varepsilon_i^k \ln 10$, and $L(N_i; N_{0i}, \beta)$ is the factor of the likelihood function that corresponds to sample $i$. The second equation in (2.20) remains unaltered. From these three differential equations, the estimates $\hat{k}$, $\hat{l}$ and $\hat{\beta}$ can be determined.

Given the values of the estimators $\hat{k}$, $\hat{l}$ and $\hat{\beta}$, the strain levels of the boundaries of the adjacent Regions I and III, $\varepsilon_{Lp}$ and $\varepsilon_{III,1-p}$, are used as
input in Equation (2.22) to determine that confidence intervals in life of Region II,
\[
\log N_{I,p} = (\hat{k} \log \epsilon_{I,p} + \hat{i}) (-\ln R_p)^{1/\delta},
\] (2.27)
and
\[
\log N_{III,p} = (\hat{k} \log \epsilon_{III,1-p} + \hat{i}) (-\ln R_p)^{1/\delta},
\] (2.28)
where \(R_p = p\) or \(1 - p\). The data pairs \((N_{I,p}, \epsilon_{I,p})\), \((N_{I,1-p}, \epsilon_{I,p})\), \((N_{III,p}, \epsilon_{III,1-p})\)
and \((N_{III,1-p}, \epsilon_{III,1-p})\) are subsequently used to plot the scatter band of Region II as illustrated in Figure 2.4 c. In the above analysis, it is assumed that the applied load level is constant. For variable amplitude loading, the reliability \(R_p\) can be expressed in terms of the distribution of the load spectrum (Murty et al., 1995).

The transition between Regions I and II is distinctly drawn as a kink. The mechanisms associated to Region I is believed to be different from those in Region II. In Region I, stochastic fiber breakage as in quasi-static loading is expected, whereas a progressive damage mechanism has to be active in the sloping Region II. Conversely, the transition from the sloping scatter band to the horizontal scatter band above Region III is drawn as a smooth transition, since the mechanisms are expected to be basically the same. The difference lies in the driving force for damage propagation which does not exceed its threshold for further growth at the fatigue limit, or that a crack arrest mechanism sets in which obstructs propagation. This will be discussed further in the two following Chapters 3 and 4.

This completes the procedure of generation of scatter bands in the fatigue life diagram. Since the percentile \(p\) can be chosen arbitrarily, the above scheme can be used to estimate the reliability of tested material. Basically, a variant of a \(P-S-N\) (reliability—stress—life) curve with geometric constraints and strain as the independent variable has been generated. For design purposes, the reliability model ought also to include full confidence and tolerance bound analysis.

2.4 Descriptive example

The fatigue life diagram pertains primarily to unidirectional 0° composite laminates in tension-tension fatigue loading. To illustrate the applicability for this type of material, the described method for generation of fatigue life diagrams will be exemplified from experimental fatigue life data.
Material and experimental procedure

Unidirectional CFRP specimens were made with a $[0_4]_T$ stacking sequence of F3900 prepreg plies from Hexcel with Hercules’ IM7 intermediate modulus fibers. The prepreg was cured in an autoclave according to the recommendations of the manufacturer. The coupon width was 12.7 mm and the gage length was $\sim 130$ mm (ASTM D3039). Glass/epoxy end tabs were glued onto the specimens with an adhesive epoxy film. The edges were polished with 1000 grit SiC papers. For brittle materials like longitudinal composite laminates, it is important to indicate the tested material volume in this type of investigation. Since a larger test volume would result in a decrease in the static strength, the fatigue life would also be reduced (Barnard, 1988).

A servohydraulic tensile machine was used for the static tests and load controlled tension-tension fatigue tests. A sinusoidal load waveform at 10 Hz was applied. The stress ratio was $R = \sigma_{\text{min}}/\sigma_{\text{max}} = 0.1$ for all stress levels.

Generation of fatigue life diagram

For reasons of completeness, the full fatigue life data are listed in Table 2.2 containing maximum initial strain, number of cycles to failure and failure category. Totally 8 static tests and 34 fatigue tests were performed. During the fatigue tests, 6 specimens failed prematurely by delamination between the end tab and the composite material, and 4 were suspended at $10^6$ cycles.

<table>
<thead>
<tr>
<th>Region</th>
<th>Estimators</th>
<th>Percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$\hat{\varepsilon}_0 = 1.283%$</td>
<td>$\varepsilon_{I,1} - p = 1.291%$</td>
</tr>
<tr>
<td></td>
<td>$\hat{\beta} = 144.1$</td>
<td>$\varepsilon_{I,P} = 1.263%$</td>
</tr>
<tr>
<td>II</td>
<td>$k = -23.76$</td>
<td>$N_{I,p} = 0.949$</td>
</tr>
<tr>
<td></td>
<td>$l = 3.738$</td>
<td>$N_{III,p} = 12,406$</td>
</tr>
<tr>
<td></td>
<td>$\hat{\beta} = 0.822$</td>
<td>$N_{III,1} - p = 529,465$</td>
</tr>
<tr>
<td>III</td>
<td>$\hat{\varepsilon}_0 = 0.0.786%$</td>
<td>$\varepsilon_{III,1} - p = 0.861%$</td>
</tr>
<tr>
<td></td>
<td>$\hat{\beta} = 34.1$</td>
<td>$\varepsilon_{III,P} = 0.786%$</td>
</tr>
</tbody>
</table>

A confidence level of 80% was chosen for the calculation of scatter band widths, i.e. the 10% and 90% percentiles constitute the boundaries of the scatter bands ($p = 0.1$). In Region III, the endurance point was set to $N_{\text{lim}} = 10^5$ cycles and $N_{\text{end}}$ was set to the censoring time $10^6$ cycles, after graphical inspection. The results of the computations are listed in Table 2.1.
In Region III, the endurance point was set to $N_{\text{lim}} = 10^5$ and $N_{\text{end}}$ was set to the censoring time $10^6$, after graphical inspection. The experimental data is plotted together with the scatter bands of the three regions in Figure 2.5.

![Fatigue life diagram with accompanying scatter bands for the tested composite material.](image)

**Figure 2.5.** Fatigue life diagram with accompanying scatter bands for the tested composite material.

In summary, the proposed deduction scheme for fatigue life diagrams works fairly well, and can be used as a quantitative tool to systematically compare the influence of material properties on the fatigue performance. The basic quantities are the distribution parameters for the three regions in the fatigue life diagram, from which scatter bands with an arbitrary confidence level can be calculated. In the absence of a full-fledged detailed micromechanical model to predict the distribution in fatigue life, the complex damage state of a composite is conceptually mapped to an elementary crack in a monolithic material. In this way, a link is established between the distribution in static strength and fatigue life. In the subsequent chapter, comparisons will be made of fatigue life diagrams for composites with different constituent properties.
### Table 2.2. Experimental fatigue life data.

<table>
<thead>
<tr>
<th>Strain, $\varepsilon_{\text{max}}$ [%]</th>
<th>Life, $N_f$</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.290</td>
<td>0.25</td>
<td>static</td>
</tr>
<tr>
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</tr>
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<td>0.755</td>
<td>$10^6$</td>
<td>run-out</td>
</tr>
<tr>
<td>0.678</td>
<td>$10^6$</td>
<td>run-out</td>
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</tbody>
</table>
Chapter 3

Damage mechanisms in longitudinal plies

3.1 A mechanistic approach

If possible, a model which describes a physical process should be based on its fundamental mechanisms. In contrast to phenomenological models, a physical model gives a better description, deeper understanding, and can be generalized to similar processes. A mechanism based model that accurately describes the physical reality of fatigue degradation can then confidently be incorporated in a design tool. Microstructural tailoring for optimal performance would also be feasible. Due to limited insight into the fatigue damage mechanisms in composites with polymer matrices, the majority of the fatigue models proposed in the literature have been empirical and at worst inclined towards curve fitting. As a first step in the conception of physical fatigue models, the operative damage mechanisms should be identified and carefully mapped for a variety of materials with different constituent properties.

In this chapter, experimental results on the fatigue damage mechanisms in four unidirectional composite materials are presented. Two CFRP materials were investigated, namely carbon fiber reinforced epoxy and carbon fiber reinforced polyetheretherketone (PEEK), both with the same brand of carbon fibers. Two GRP materials were also tested; glass fiber reinforced polypropylene, with and without maleic anhydride modification of the matrix. The purpose of this particular choice of materials was to examine the influence on fatigue of (i) glass vs. carbon fibers, (ii) brittle vs. ductile matrix, and (iii) weak vs. strong interface.

Fatigue damage mechanisms of unidirectional $0^\circ$ composites have been experimentally investigated by Dharan (1975), Curtis (1991), and Lorenzo &
Hahn (1986). Dharan studied filament wound glass fiber/epoxy with a fiber volume fraction of 48%. Progressive fiber breakage emanating from matrix cracks or individual fiber breaks was observed. Debonding along the fiber-matrix interface entailed the progressive fiber breakage in some cases. Curtis focused on longitudinal splitting originating from a drilled hole in the center of specimens of glass and carbon fiber reinforced plastics. Lorenzo and Hahn investigated fatigue processes in glass and carbon fiber bundles molded into a tough and a more brittle epoxy resin. Zones of transverse accumulation of fiber breaks followed by interfacial debonding for the tougher matrix system were detected. The cited works lacked a thorough discussion on how to interpret the mechanisms in terms of microstructural properties, and how to establish a link between the features of the observed micromechanisms and the macroscopic fatigue performance. Only Curtis's material pertained to commercial systems, but the emphasis was placed on macroscopic fatigue degradation such as through-thickness splitting arising from a notch. This calls for a more meticulous investigation of the fatigue processes at the micrometer level concurrent with a macroscopic fatigue life characterization of composite materials for service applications.

3.2 Experimental procedures

Materials

Unidirectional CF/epoxy specimens were fabricated with a [0\(_4\)]\(_T\) stacking sequence of AS4/8552 prepreg plies from Hercules. The matrix is a rubber modified epoxy system. The CF/PEEK were manufactured with the same lay-up of prepreg tapes of APC-2 from ICI Fiberite. The fibers in the APC-2 material were AS4 carbon fibers, i.e. the same brand of fibers as used in the CF/epoxy material.

The thermoplastic polypropylene (PP) and maleic anhydride grafted polypropylene (MA-PP) reinforced by continuous longitudinal glass fibers (GF) have also been investigated. The E-glass fiber type is identical for the two materials but the matrix has been modified to achieve a more efficient adhesion to the fibers. The fibers were PPG 854-1200 from PPG Industries Fibre Glass BV, and were treated with a PP compatible sizing. The MA-PP matrix was made by blending an isotactic PP homopolymer (VM 6100, Shell Chemicals) with a commercial maleic-anhydride grafted PP (Polybond 3002, BP Chemicals). The weight proportions of the blend was 90% homopolymer and 10% of the MA grafted PP, which have showed optimal static strength properties (Rijsdijk et al., 1993).
3.2. Experimental Procedures

Processing

The CF/epoxy prepreg material was autoclave cured according to the manufacturer’s recommendations at 180°C and 6 bar for 120 minutes. During the autoclave process, a well polished plate of stainless steel was placed inside the vacuum bag directly on top of the release film covering the stacked prepreg plies. This was to enhance the surface smoothness and to prevent the usual texture on the upper surface of the composite from the release fabric or surface bleeder. As a result, more excess resin was bled than for a conventional cure process without a pressure plate, and a fiber volume fraction of ~ 63% was obtained.

The CF/PEEK prepregs were stacked and covered with smooth release coated aluminum foils, and heated between thick steel plates for 10 minutes at 400°C. Thereafter, the laminate and plates were immediately brought to a hot press which held a temperature of 260°C. The laminate was press molded for 10 minutes, whereupon the heating system was turned off, and the composite was cooled down under pressure. The thickness of the steel plates were 2 cm to assure a moderate cooling rate. The manufacturer recommends a cooling rate of 7 to 700°C/min. Since the cooling rate affects the degree of crystallinity, the static mechanical properties of the CF/PEEK will also be dependent on the cooling rate (Talbott et al., 1987). The cooling rate is also known to noticeably influence the fatigue life of unidirectional CF/PEEK composites (Moore, 1993). Since the exact cooling rate could not be determined, the degree of crystallinity was measured instead with differential scanning calorimetry (DSC), and found to be 36%. However, the degree of crystallinity does not fully reflect the morphology of the polymer matrix. The thermal processing history (isothermal cooling, annealing etc.) influences the formation of transcrysalline zones initiated at the fiber surfaces, which also affects the fatigue life (Tregub et al., 1994). The transcrysalline interphase between the fiber and bulk semi-crysalline polymer matrix has shown to have profound effects on some ultimate properties of composites (Thomason & van Rooyen, 1992). By permanganic etching, the amorphous phase can be dissolved to reveal the crystalline morphology of PEEK (Olley et al., 1986). In the present study, pieces of the processed CF/PEEK material was mounted in phenolic resin, and exposed to a solution of permanganic etchant based on orthophosphoric acid for 50 minutes. This reagent is a 2% solution of potassium permanganate in a mixture of 4 volumes of orthophosphoric acid (approximately 90%) and 1 volume of water. The approximate volume fraction of fibers was determined from the etched cross-sectional surface by image analysis, and found to attain ~ 60%.

It should also be mentioned that multidirectional PMC laminates pro-
cessed at relatively high temperature, such as CF/PEEK, risk to develop damage when cooled down to ambient temperature due to the high residual thermal stresses.

Unidirectional 0° GF/PP and GF/MA-PP laminates were manufactured by winding layers of glass fibers onto a 200 × 200 mm steel mandrel with alternating sheets of the polymer film (see Figure 3.1). The steel mandrel was covered by release coated Mylar™ sheets to avoid adhesion. After film stacking, the wound mandrel was let to dry in an oven at 90°C for one hour. Next, the mandrel was placed in a heated press at 25 bar and 200°C for one hour. The laminates were cooled in the press to minimize curvature. The volume fraction of fibers was determined both by burning of the matrix and image analysis, and found to range between 55% and 60%.

The specimens were cut with an abrasive water jet to dimensions recommended by ASTM standard D3039; a 127 mm gage length and 12.7 mm width. The laminate thickness was 0.50 and 1.0 mm for the CFRP and GRP, respectively. Tapered end tabs of glass weave reinforced epoxy were bonded to the laminates by an adhesive epoxy film cured at 125°C for 2 hours. Tab delamination occurred occasionally during fatigue testing, in particular for the CF/PEEK material. The bond is known to improve by shot blasting in the end regions where the tabs are fixed (Dickson et al., 1985). The specimen edges were subsequently polished with successively finer silicon carbide papers ending with 2400 grit. This conveyed edges with a glossy and scratch free appearance.
3.2. EXPERIMENTAL PROCEDURES

Testing

The specimens were subjected to quasi-static and load controlled fatigue tests in an Instron 1272 tensile machine with a dynamic hydraulic load cell of type 2513-504 and hydraulic grips. For the quasi-static tests, a cross-head stroke rate of 0.4 mm/min was used. The longitudinal and transverse strains were registered on a computer with a Sandner A10-2.5 extensometer with a gage length of 10 mm and precision strain gages of type CEA-06-125UN-350 from Measurements Group, respectively. The closed loop fatigue tests consisted of a sinusoidal load with a frequency of 10 Hz for the CFRPs, and a triangular wave form with a constant absolute strain rate of $10^{-2}$ s\(^{-1}\) for the GRPs. The stress ratio was continually $R = 0.1$ for various strain levels. By the strain level is here meant the initial (first cycle) peak strain which was calculated from the applied peak load. The chosen fatigue load rates did not result in any significant autogeneous heating. A thermocouple was on occasions applied on the specimen surface during testing, and never showed any temperature increase in excess of 2°C for all materials. Global isothermal testing conditions can therefore be assumed. All tests were performed at room temperature.

During the course of the fatigue tests, a sequence of surface replicas were intermittently taken at the same position in the middle of a specimen surface. Since longitudinal splitting tends to occur at the edges of unidirectional CFRP coupons due principally to fiber misalignment, the replicas were taken on the unpolished surface across the width. A cellulose acetate film was plasticized with an acetone based solution and thereafter pressed onto a predesignated position on the specimen surface for 2 minutes. As the solution evaporated, the polymer film was allowed to harden. Before each replication, the surface spot was carefully cleansed with ethanol and dried. The replicas were stored under microscope glass on which a weight was placed to prevent them from curling. The peak load was applied during the replication to allow the cracks to open and render them more visible. A carbon coating with a thickness of 30 to 50 nm was sputtered on the replica samples to improve the image resolution in the optical microscope (Goodhew, 1985). The development of fatigue damage was mapped by locating a large damage site on the replica with the highest number of elapsed cycles, and relocating the same site on the previous replicas from the same set with lower number of cycles. Since the damage sites sometimes were dispersed and rare, a map with reference points such as scratches, misaligned fibers etc. had to be drawn in order to relocate the position of the fatigue damage. Moreover, repeated applications on dummy samples were done to confirm that the replica solvent did not degrade the composite surface.
Fracture surfaces were studied in a scanning electron microscope (SEM). The surfaces were coated with a thin gold layer in a Blazers SCD050 sputter coater. The SEM was a Cambridge Camscan SH-80D operating at accelerating voltage of 20 kV. Optical microscopy was performed with a built-in microscope of a Matzusawa MXT-α microhardness tester connected to a computer for image analysis.

3.3 Carbon fiber reinforced plastics

Background

Carbon fiber reinforced plastics (CFRPs) are high performance composite materials used primarily in aerospace applications. Conventionally, the carbon fibers are embedded in an epoxy matrix. During the last decade, demands raised for applications at higher temperatures of polymer composites have resulted in the development of thermoplastic matrix composites. Other incentives to replace epoxy based matrices with thermoplastics are cheaper processing, improved toughness, and reduced health hazards and storage problems due to polymer reactivity. The most successful contenders are polyimide (PI), polyetherimide (PEI) and in particular polyetheretherketone (PEEK). Delamination is one of the most important objectives why CF/PEEK is used in applications (Simonds et al., 1989). For the neat matrix materials, PEEK shows a much higher resistance to fatigue crack growth than epoxies, but when reinforced by carbon fibers the difference in resistance to delamination is significantly reduced (Hojo et al., 1994). Yee (1987) explains this behavior with the constraints imposed by the fibers on the yielding matrix zones. Interlaminar fracture toughness is influenced by the rigid fibers, which create a triaxial stress state, and thus also affects the fracture process. Shear yielding is suppressed in favor of a more brittle fracture failure. For low rates of delamination propagation in mode II, Russell and Street (1987) even measured a higher resistance to delamination for CF/epoxy than for CF/PEEK. Furthermore, in fatigue of unidirectional 0° specimens, Curtis (1987) found that CF/PEEK shows inferior fatigue properties with steeper slopes in S-N curves compared to CF/epoxy, which was later assumedly accounted for by the propensity to longitudinal splitting from artificially introduced holes in the CF/PEEK laminates (Curtis, 1991). However, one should be aware that longitudinal splitting is a macroscopic feature and fatigue failure is generally locally initiated on a smaller scale. It is likely, though, that observed longitudinal splitting CF/PEEK is preceded and initiated by intrinsic micromechanisms which are responsible for the inferior fatigue per-
Table 3.1. Static longitudinal properties of the CFRPs.

<table>
<thead>
<tr>
<th>Property</th>
<th>CF/epoxy</th>
<th>CF/PEEK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress to failure, $\sigma_u$</td>
<td>1.63 GPa</td>
<td>1.99 GPa</td>
</tr>
<tr>
<td>Strain to failure, $\varepsilon_u$</td>
<td>1.27%</td>
<td>1.50%</td>
</tr>
<tr>
<td>Young’s modulus, $E$</td>
<td>129 GPa</td>
<td>133 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
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</table>

formance of CF/PEEK. The following experimental study was undertaken as an attempt to shed some light on these micromechanisms. In this work, the development of fatigue damage has therefore been examined on a microscopic level for two CFRPs with a common type of carbon fiber; (i) one with an epoxy based matrix, and (ii) the other with a PEEK matrix. The observed mechanisms of the two material systems were put in relation with their respective fatigue life properties. Previous experimental investigations on laboratory manufactured model CFRP specimens have identified overall fatigue mechanisms (Lorenzo & Hahn, 1986), whereas in the present study, the evolution of individual damage sites was monitored, and focus is placed on commercial CFRPs used in service applications.

Results and discussion

The static tests resulted in the properties listed in Table 3.1. The average values of the properties are indicated. The standard deviation was within 5% from the mean for the elastic values, and somewhat larger for the strength values. The Young’s modulus determined from the static tests was used to calculate the initial peak strain value (strain level) of the fatigue tests. The set of strain levels were chosen with reference to the static strain to failure. The static strength properties of these laminates clearly shows better performance for the CF/PEEK. Next step is to investigate if they are retained under fatigue conditions.

In Figure 3.2, the fatigue life data of the CF/epoxy and CF/PEEK composites are plotted. In this plot, the differences in static and fatigue behavior are visualized. The static strain to failure is lower and has a wider scatter for CF/epoxy. A similar trend has been observed by Baron et al. (1987), who measured higher strains to failure for $0^\circ$ composites with more ductile epoxy matrices.

Since the static strain to failure of $0^\circ$ composites are primarily governed by the strain to failure of the fibers, and the fiber types of the two composite
systems were the same, it might have been expected that the two sets of static data would have overlapped. Since this is not the case, the difference could be attributed to batch-to-batch variability in properties of the AS4 carbon fibers. Another contributing effect could be the effective stress distribution and relaxation at fiber breaks from the more ductile PEEK matrix. This is supported by fragmentation tests where the ineffective length at fiber breaks was greater, and the maximum interfacial shear stress was smaller in CF/PEEK than in CF/epoxy (Vautey & Favre, 1990), which implies a weaker interface or a higher inclination to shear yielding in the matrix along the fiber. These mechanisms would allow the material to further deformation by the accumulation of debond or yield damage. Furthermore, CF/PEEK show a more pronounced fatigue degradation with a steeper slope, which has also been observed by Kim & Hartness (1985) and Curtis (1987). Harris et al. (1990) found that the slope of Region II became steeper with increasing matrix toughness for a variety of CFRP materials. The question that presents itself immediately is which underlying micromechanisms are responsible for the inferior fatigue performance of CF/PEEK. If these mechanisms were known, indications of how to remedy the fatigue sensitivity can be made. A tentative mechanistic explanation based on observations of surface damage development will be put forth shortly.

In Table 3.2, the statistical estimators of the fatigue life diagrams of
the CFRPs are tabulated. The parameter values were calculated by use of the statistical method presented in the previous chapter. With these values, the fatigue life diagrams in Figure 3.3 were plotted with scatter bands bounded by 10% and 90% percentiles. The wide scatter is not only limited to the static strain to failure for the CF/epoxy. Also the fatigue life shows considerable variability. This is indicative of a brittle behavior also under fatigue conditions. The fatigue limit is slightly lower for the CF/PEEK material. The fatigue limit of some unidirectional CF/epoxy composites seem to be controlled by the fatigue limit of the neat matrix (Talreja, 1981a). However, the fatigue limit in initial peak strain of PEEK is about 2.5% (Dominghaus, 1993), which is significantly higher than the measured fatigue limit of the CF/PEEK composite which is about 0.75%. It should still be appreciated that the fatigue limit of the matrix influences the long term properties of the composite, since the progressive damage has to be dependent on the matrix or its interface to the fibers, and the propagation of this damage would depend on the propagation threshold in the matrix material.

During the fatigue experiments, the cycling was stopped at certain intervals and replicas were taken. The replicas were taken at different strain levels, in order to investigate if there is a difference in damage mechanisms in different regions in the fatigue life diagram. The prime objective of this study was to map the operative damage mechanisms rather than to give a detailed data base on the fatigue life performance. The fatigue life relation was resorted to for an appropriate choice of strain level at which the replicas were taken, and for a fair characterization of the macroscopic fatigue behavior. The presented replicas are typical, and do not represent a particular assortment with special features. The micrographs of the replicas were taken
Figure 3.3. Fatigue life diagrams of CF/epoxy and CF/PEEK.

at a high magnification (individual fiber breaks are clearly seen). All micrographs are provided with a length scale. As ubiquitous reference, the fiber diameter is \( \sim 5 \) \( \mu \text{m} \). The number of elapsed cycles for each replica is likewise indicated in the micrographs.

The observed mechanisms on the surface cannot be readily generalized to the interior of the material. However, the stress concentration from a surface flaw may prove to be more severe than that from a flaw of comparable dimensions in the interior. For example, the mode I stress intensity factor of a semi-circular surface crack in an infinite elastic body is 30% higher than that of an embedded circular (penny shaped) crack of equal area (see e.g. Ewalds & Wanhill, 1984). Furthermore, the amount of defects is believed to be more abundant on the surface than in the interior, since scratches could not be completely avoided during handling. A study of the surface fatigue mechanisms is therefore further justified if it could be corroborated that final failure were induced from the a surface crack. The choice of the surface as the object of study would then not merely be of experimental necessity (cf. Section 1.4), but also of physical significance. Butler et al. (1988) observed a considerable increase in lifetime for specimens whose edges had been well polished. This indicates that fatigue failure is initiated from the edge surface of the specimens with inferior finish quality. Experimental investigation of fatigue failure modes have indicated that ultimate failure originates at the surface for various MMCs and short fiber PMCs (Majumdar & Newaz, 1995; Soboyejo et al., 1995; Casado et al.; 1997).

Typically, the replication is made on the coupon edges, but since segments
tend to split off from the laminate at the edges during fatigue, the plane surface was chosen as replica target. These sprawling segments form due to misalignment of the fibers, and slight off-axis loading. For the same reason, waisted specimens of unidirectional specimens frequently leads to shear failures at the waists, which trigger split growth in fatigue (Curtis, 1989a). All replicas of the surface, except for the pristine state, were taken when the peak load was applied statically. This opened the cracks and rendered them visible with a finite crack opening displacement. The cracks closed completely when the load was removed, and became almost undetectable.

In Figure 3.4, a sequence of micrographs of a propagating fiber bridged crack in CF/epoxy is shown. The crack tips take a deformed squeezed shape after some time of loading. As the crack grows transverse to the load and fiber direction, it encounters the neighboring fibers, grows past them, and acquires a squeezed profile at the tips. Similar crack shapes have been observed in unidirectional MMCs with fiber bridging (Davidson, 1992; Zheng & Ghonem, 1995). The squeezed shape is due to the cohesive traction from the bridging fibers. With the replica technique, individual bridging fibers could not be directly discerned, except in special cases with extremely superficial fibers (cf. Figure 3.5). A schematic picture illustrating the principal appearance of a fiber bridged crack which has been initiated from a single fiber break is found in Figure 3.6. The fiber bridged cracks originated from flaws or fiber breaks. No fiber breaks were detected in the pristine state, but as soon as load was applied, a number of distributed fiber breaks appeared. Since the brittle fibers have a distribution in strength along their lengths, they will break at weak segments even at moderate applied stresses. From these fiber breaks, matrix cracks originated, and propagated to form fiber bridged cracks. The cracks were scarce, and localized. In Figures 3.7 and 3.8, sequences of propagating cracks are presented. Tendencies of coalescence were only observed for cracks within the extreme vicinity. Cracks striving to merge were not observed for larger interspaces than those in Figures 3.7 and 3.8. Otherwise, the cracks could be regarded as individual and independent entities during the main part of the fatigue tests. Judging by the crack lengths, the maximum number of contiguous broken fibers in the same plane along the surface was about 6 or 7. This figure is in agreement with a number of experimental investigations (Jamison, 1982; Marston & Neumeister, 1998) and theoretical estimations (Bader, 1988; Wisnom & Green, 1995) for similar types of composite materials.

The crack length at the surface was measured for a number of fiber bridged cracks of the type presented in Figure 3.4. In Figure 3.9, it is noticed that the crack growth rates are decreasing. Ghosn et al. (1992) have also observed decelerating growths of fiber bridged crack in fatigue. In a homogeneous ma-
Figure 3.4. Fiber bridged crack propagation in CF/epoxy ($\varepsilon = 1.08\%$).
Figure 3.5. Fiber bridged crack in CF/epoxy ($\varepsilon = 1.08\%$) at 1,000 cycles.

Figure 3.6. Schematic picture a fiber bridged crack originating from a single fiber break.
Figure 3.7. Propagation of colinear cracks in CF/epoxy ($\varepsilon = 1.21\%$).
Figure 3.8. Tendency for crack coalescence in CF/epoxy ($\varepsilon = 1.21\%$).
terial, fatigue cracks are expected to propagate with an accelerating rate until catastrophic failure takes place. Since propagation is usually governed by the stress intensity at the crack tip, and the stress intensity factor increases with increasing crack length, an accelerating growth rate is anticipated. The observed decelerating growth pattern must then be ascribed to crack retarding mechanisms which pertain to the heterogeneous composite microstructure. A contributing mechanism would be the fiber bridging itself. The cohesive action by the bridging fibers would diminish the crack tip stress intensity. Cox & Lo (1992) have shown that for a through thickness bridged crack, the effective stress intensity factor increases monotonically for increasing crack length, but has an upper asymptotic limit, which means that the crack would attain a constant steady-state growth rate. Since a deceleration was measured, additional crack retarding mechanisms are likely to be active. These could include crack front bowing, fiber-matrix debonding, matrix yielding, although they were not explicitly detected. The effects of bridging on fatigue crack propagation will be studied in some more detail in Section 4.3.

No protruding difference in mechanisms were seen in the set of replicas taken at the strain levels 1.21% (Region I) and 1.08% (Region II), in spite of their belonging to different regions. However, the tested samples were too few to make any conclusive statements on the similitude in mechanisms in the static and progressive regions. Zok et al. (1995) observed fiber bridged cracking from a notch in a unidirectional MMC in Region II. Increasing the stress level to Region I, they observed the same mechanism, but with a gradual transition to distributed stochastic fiber breakage.

In Region III, below the fatigue limit, there was a transition in mecha-
nisms. In Figure 3.10, a stationary crack encompassing about 4 fibers at the surface is found. Debonding commenced after a few million of cycles at one crack tip. Extensive shear yielding occurred at the other crack tip. In the last two replicas, matrix shear bands are visible. These are incipient cusps formed perpendicular to the maximum principal shear stress (Purslow, 1986). However, debond formation at the crack tips was far more frequent than matrix shear yielding. Figure 3.11 presents a crack of similar dimensions from which debonds grew from both crack tips. It is noteworthy that the debonds progress at different rates, and have a jagged and irregular appearance, which indicate an erratic and unpredictable growth rate.

A slant surface scratch has produced the line of fiber breaks in Figure 3.12. The cracks at the fiber breaks are seen to merge and become confined through debonding. Noticeable is the increase in crack opening displacement as the number of cycles increases. The increase in crack opening can be explained by interfacial debonding. When the specimen was unloaded, the cracks closed fully.

The typical damage entity in Region III was a single fiber break from which debonds grew. An example of this damage type is shown in Figure 3.13. The debond length can readily be measured with an image analysis program, and plotted with respect to the number of cycles. Such a plot of debond propagation from a single fiber break is found in Figure 3.14. The four curves refer to one debond length each, i.e. the distance from the fiber break to the debond crack tip in all four quadrants. Notable is that the mode II debond growth retards and apparently arrests after about two million cycles, which can be explained by the increase in frictional stresses at the debonded interfaces with increasing debond lengths. The rapid debond growth at the onset is also enhance by a mode I contribution (Chan & Davidson, 1989). If the debond rate is a necessary input in a damage accumulation model, it could be quantified with this type of measurement in terms of the Paris power law for fatigue crack propagation. However, the initial mode mixity, frictional constraint and stochastic growth at the bimaterial interface necessitates fairly detailed stress analysis. Spearing et al. (1991) have adopted a Paris law model to describe delamination growth in notched laminates. Curtis (1991) used the same method to correlate the stress level to through thickness longitudinal splits originating from small drilled holes in unidirectional laminates.

Since the debonding came to a halt, and did not cause additional fiber breakage, it acted as a crack arresting mechanism. This mechanism was observed below the fatigue limit in Region III, whereas progressive fiber bridged cracking was active in Region II. A transition in fatigue mechanisms from Region II to III as a matrix crack approaches an neighboring fiber is schematically depicted in Figure 3.15. The prevailing of the two mechanisms precludes
Figure 3.10. Debond growth and shear yielding from a set of plane fiber breaks in CF/epoxy ($\varepsilon = 0.89\%$).
Figure 3.11. Debonding from a plane crack of fiber breaks in CF/epoxy ($\varepsilon = 0.8\%$).
Figure 3.12. Crack coalescence by debonding from a slant surface scratch in CF/epoxy ($\varepsilon = 0.89\%$).
3.3. CARBON FIBER REINFORCED PLASTICS

Figure 3.13. Debond growth from a single fiber break in CF/epoxy ($\varepsilon = 0.89\%$).
Figure 3.14. Debond propagation from a single fiber break in CF/epoxy ($\varepsilon = 0.89\%$).
the other, and is dependent on the applied strain level. Once debonding sets in, progressive fiber bridged cracking, which ultimately leads to failure, is suppressed. Therefore, debonding can be regarded as a crack arresting mechanism, that will inhibit crack growth perpendicular to the load direction. The driving force further crack propagation is thus diffused by crack bifurcation. In a general sense, this mechanism is sometimes known as the Cook-Gordon mechanism (Cook & Gordon, 1964). Cox & Marshall (1991b) have observed the same type of transition for notched MMCs. However, the mechanism transition was not controlled by the applied strain level, but mainly by the interfacial friction. Analytical work of the two-dimensional problem indicate that the interfacial toughness and the elastic mismatch are the controlling parameters for the static case (He & Hutchinson, 1989; Ahmad, 1991). The crack interfacial deflection is obviously beneficial in the sense that it blunts the crack and eventually effectively arrests further crack propagation. The onset appears to occur at the fatigue limit. A more thorough experimental investigation of this phenomenon is prompted by the wide use of the fatigue limit in design for infinite life.
Returning to the relatively fatigue resistance macroscopic behavior of 
CF/epoxy presented in Figure 3.2, it can be inferred that the observed mech­
nisms in CF/epoxy, localized fiber bridged crack and crack arrest by limited 
debonding, contribute to the macroscopic behavior. Experimental results 
from investigations of the underlying mechanisms responsible for the fatigue 
sensitivity of CF/PEEK will be presented next.

As pointed out earlier, the heat treatment of composites with semi­
crystalline matrices influences the properties considerably. The crystallization 
process is time and temperature dependent, and is affected by the pres­
ence of fibers. The microstructural morphology will in turn influence the 
active damage mechanisms, and hence also the macroscopic properties. In 
particular for CF/PEEK, there is extensive experimental proof of the depen­
dency of heat treatment on fatigue properties (Folkes et al., 1993; Moore, 
1993; Tregub et al., 1994). In order to qualitatively characterize the mi­
crostructure and matrix morphology of the CF/PEEK laminates used in this 
work, polished samples of the composite were exposed to a permanganic etchant to dissolve the amorphous phase of the matrix. On the ply level, 
the fiber distribution was found to be fairly uniform, and the plies could perceptibly be discerned in cross section (see Figure 3.16).
In Figure 3.17, micrographs of etched sections of CF/PEEK samples are shown. The matrix morphology resembles that of isothermally cooled samples of CF/PEEK reported by Tregub *et al.* (1993). They noticed that the matrix morphology of quenched composites was more homogeneous and amorphous, with no apparent signs of crystallites. The quenched samples showed inferior fatigue performance, and poor fiber-matrix adhesion. Since the laminates in the present study were cooled at a slow rate, the crystallization was given sufficient time to form a strong transcrystalline phase next to the fiber surface, which improves fatigue resistance. By DSC, the degree of crystallinity was found to attain the relatively high value of 36%, which can be compared to the maximum achievable crystallinity of 48%.

The replicas of the CF/PEEK material did not show any evident difference in mechanisms at different strain levels, although the propagation rate differed. In Figure 3.18, a series of replicas with fatigue damage growth in CF/PEEK is shown. The mechanisms showed a completely different pattern compared to CF/epoxy. The surface was rougher before testing commenced. From the wide spread pre-existing damage, matrix cracks or debonding grew predominantly along the fiber direction. These debonds were abundant and ubiquitous. The cracks grew in a random accelerating-decelerating manner, and became more pronounced during fatigue, which could be explained by the continuous wear of the crack surfaces. In contrast to CF/epoxy, successive breakage of fibers was detected during the course of the fatigue tests, which has also been observed by Kim & Hartness (1985). The longitudinal
Cracks did not come to a halt and were substantially larger than the debonds observed in Region III in the CF/epoxy samples.

The difference in mechanisms in CF/epoxy and CF/PEEK is illustrated schematically in Figure 3.19. The CF/epoxy shows a brittle behavior, with small localized cracks that propagate perpendicular to the fiber direction, and become bridged by the adjacent fibers. By brittleness is here meant the ensemble of mechanical properties of a material which is incapable of inelastic deformation by the accumulation of distributed irreversible damage. Conversely, the CF/PEEK has a tougher behavior in the sense that it is more damage tolerant. The damage accumulates on a larger scale in distributed multiple longitudinal cracks, with considerably more fiber breaks. The CF/PEEK shows a propensity to shear induced matrix cracking or debonding in the axial direction. This may be due to a lower matrix shear yield stress or a weaker interface than in CF/epoxy. Vautey & Favre (1990) performed single fiber fragmentation tests on CF/epoxy and CF/PEEK, and noticed lower interfacial shear stresses along a larger ineffective lengths in CF/PEEK. This means that CF/PEEK has a weaker interface, and is more susceptible to relaxation of the interfacial shear stresses by means of debonding, shear yielding or longitudinal matrix cracking. The growth of these types of damage will result in a stress redistribution in the neighboring fibers. Since the brittle fibers have a distribution in strength along their lengths, a weak segment may be overloaded and fail. From these fiber breaks, further longitudinal cracking proceeds, and results in new fiber breaks etc. The damage accumulation is an on-going, continuous and distributed process, which results in a sloping region in the fatigue life diagram. Conversely, CF/epoxy has few damage sites on a small scale and therefore a more brittle behavior.
3.3. **Carbon Fiber Reinforced Plastics**

A brittle material either fails or is undamaged upon loading. Such a material has a flat horizontal scatter band in the fatigue life diagram. These trends with brittleness vs. ability to progressively accumulate damage is clearly manifested in the fatigue life data of Figure 3.2.

In Figures 3.20 and 3.21, micrographs from scanning electron fractography of CF/epoxy and CF/PEEK are shown, respectively. Neither the macroscopic features of the failed specimens, nor the microscopic fractography showed any difference between specimens that failed under static and high cycle fatigue loading. Awerbuch & Hahn (1981) did not find any fractographic differences between unidirectional CF/epoxy specimens that failed under static loading and those that failed under fatigue loading conditions. It is expected that the critical flaw size is small, since less than a maximum of 10 contiguous broken fibers were observed on the replicas, and these damage sites were far apart and few. Since the critical flaw size is small, the fracture surface is dominated by catastrophic static fracture rather than non-critical fatigue cracks. This can explain the resemblance of the *post-mortem* fracture features. The entire fracture surface was scanned to locate the initiatory defect. Such a defect would have the size of the critical flaw for the applied stress level, but no point of initiation was encountered. The identification of a primary fracture plane was impeded by the unevenness of the fracture surface. For background on fractographic techniques and interpretations of fracture of CFRPs, see Bascom & Gweon (1989).

![Figure 3.19. Damage state in CF/epoxy and CF/PEEK prior to failure.](image)
CHAPTER 3. DAMAGE MECHANISMS IN LONGITUDINAL PLIES

Figure 3.20. Scanning electron micrograph of CF/epoxy fracture surfaces from (a) static failure, and (b) fatigue failure.

Figure 3.21. Scanning electron micrograph of CF/PEEK fracture surfaces from (a) static failure, and (b) fatigue failure.
Brittle static failure of longitudinal CFRP specimens generally manifests diverging radials from the primary fracture origin (Purslow, 1981). Some radials were observed on the fracture surfaces, but they were intertwined and did not emanate from the same source. Another feature was the zones of plane fracture separated by longitudinal splits on the macroscopic level. The splits along the fibers formed during the dynamic disruptive motion at separation. Post failure damage arise from the abrupt release of elevated strain energy. The splits were more numerous with a brush like appearance in CF/PEEK compared to a more planar brittle failure for CF/epoxy. This was also observed by Pannkoke & Wagner (1991) for fatigue at low temperatures. This difference in appearance between the materials is also a confirmation of the weaker interface with more debonding in CF/PEEK. Diao et al. (1997) observed a more brush like fracture surface of unidirectional commingled CF/PEEK subjected to fatigue, and a more planar fracture for static failure, which suggests antecedent debonding or longitudinal cracking as a fatigue damage mechanism.

3.4 Glass fiber reinforced polypropylene

Background

In recent years, increased attention has been directed towards composite structures with thermoplastic matrices. In many cases, they offer advantages over thermosets in handling, processing, damage tolerance, environmental resistance and recycling. In combination with glass fibers, thermoplastic composites may present a cost effective alternative to glass fiber/epoxy composites. Glass mat reinforced thermoplastic (GMT) materials are widely used in automotive applications, and the use of continuous fiber reinforcement for load carrying structures is steadily increasing. A strong contender in this market area is glass fiber reinforced polypropylene. The polymer may be modified with maleic anhydride for improved adhesion. In previous studies the macroscopic mechanical properties of these materials have been examined under static (Rijsdijk et al., 1993) and fatigue loading (van den Oever & Peijs, 1997). The most protruding effect of the modification with maleic anhydride in the composite is a stronger fiber-matrix interface. For a weak interface debonds are likely to develop. Owen (1980) has pointed out the influence of fatigue on the incipient formation of debonds leading to larger scale matrix cracks, and eventually to failure. In the present section, the effects of interfacial adhesion on fatigue performance in combination with the underlying micromechanisms for polypropylene composite systems are
Polypropylene is an apolar polymer, and has a limited affinity to fiber sizings. Maleic anhydride grafted onto the polymer results in polarity, and a more efficient interaction with the fiber sizing. The bond can be the result of chemical reaction (Xanthos, 1988), or physisorption with interdiffusion of the polymer chains into the sizing and local dipole-dipole bonding (Mäder & Freitag, 1990). These bonds are illustrated in Figure 3.22. Hydrogen bonding to the oxygen of the maleic anhydride endgroup is another likely bond type. Reaction mechanisms of the grafting of maleic anhydride onto polypropylene have been described by Lin (1993) and De Roover et al. (1995). The interfacial bond can be improved also by other means. Thomason & Schoolenberg (1994) investigated the influence of different surface coatings on the interfacial shear strength. Most modifications result in an increase in polarity at the interface. A disadvantage is that the polarity of the matrix makes it hydrophilic, which could facilitate water sorption at the interface. Water is known to degrade the glass fibers by stress corrosion (Michalske & Bunker, 1987). Jones et al. (1984) measured deteriorated fatigue performance after treatment in boiling water of composites. This effect was particularly strong for composites with glass fibers. In acidic environments the stress corrosion of the glass fibers leads to substantially shorter lifetimes (Caddock et al., 1990).

Figure 3.22. Possible interfacial bonding after maleic anhydride modification.
3.4. Glass Fiber Reinforced Polypropylene

Table 3.3. Static longitudinal properties of the GRPs.

<table>
<thead>
<tr>
<th>Property</th>
<th>GF/PP</th>
<th>GF/MA-PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress to failure, $\sigma_u$</td>
<td>1.02 GPa</td>
<td>1.06 GPa</td>
</tr>
<tr>
<td>Strain to failure, $\varepsilon_u$</td>
<td>2.27%</td>
<td>2.30%</td>
</tr>
<tr>
<td>Young’s modulus, $E$</td>
<td>45.1 GPa</td>
<td>46.3 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>0.31</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Results and discussions

The static properties are listed in Table 3.3. The values of all four samples of each material were within 5% of the mean. There is no significant difference between the two materials in static mechanical properties. Rijsdijk et al. (1993) detected slightly larger differences, in particular for properties that are governed by the interface, such as transverse, shear and compressive strength. Longitudinal static strength is fiber dominated, and the interfacial properties plays a minor role. If the similitude in properties of longitudinal laminates is not maintained under fatigue conditions, it can be inferred that the interface affects the fatigue degradation mechanisms.

The macroscopic fatigue behavior has been characterized in terms of stiffness reduction and fatigue life diagrams. The longitudinal Young’s modulus degraded more rapidly for GF/PP (see Figure 3.23), which indicates a higher degree of damage growth and accumulation. The GF/MA-PP exhibited a ‘sudden death’ behavior, where the stiffness remained virtually constant until imminent failure. This implies localized small scale damage, which can be explained by the more effective fiber-matrix bonding through maleic anhydride modification and the ensuing embrittlement. The fatigue tests for replications and stiffness measurements were performed at a stress amplitude of ~ 60% of the ultimate tensile strength (UTS).

The fatigue life data in Figure 3.24 show that the improvement in static strength is negligible, and the fatigue life is prolonged by as much as a decade with the stronger interface by modification of the PP matrix with maleic anhydride. A corresponding shift to longer fatigue lives as well as a retention of stiffness have been observed for a CF/epoxy system with improved interfacial properties (Subramanian et al., 1995). No distinct regions can be discerned in Figure 3.24 since all data points seem to belong to the sloping scatter band associated with a progressive fatigue mechanism. The static strength is here basically controlled by the fibers, hence the overlapping for low cycles in the fatigue life curve.
The shorter fatigue lives of GF/PP for equal strain amplitudes suggest a more rapid growth of the critical damage site. The weaker interfacial bond presents itself as a possible reason for the enhanced damage propagation rate.

The statistical tool described in Chapter 2 has been employed with the data in Figure 3.24 as input to conceive fatigue life diagrams. Since no clear fatigue limit can be perceived with run-outs and a clear kink in the data plot, Region III was excluded from the analysis. The estimated parameters are listed in Table 3.4, and the fatigue life diagrams are found in Figure 3.25. The scatter bands represent 10% and 90% probabilities of failure. The scatter in fatigue life is indicated by the shape parameter $\beta$ for Region II, and shows that the scatter is larger for the GF/MA-PP material. A large variability in fatigue life is generally associated with brittle and damage intolerant materials.

A typical sequence of replicas of the fatigue damage development in GF/PP is found in Figure 3.26, and in lower magnification, in Figure 3.27. The fatigue damage in the GF/PP is characterized by large distributed debonds originating from flaws or fiber breaks, while the damage in GF/MA-PP is given by localized small cracks perpendicular to the load direction. The former grow in an accelerating-decelerating manner, and the debonds become more pronounced with thicker and darker appearance on the replicas during fatigue. This is due to attrition of the crack surface asperities as the local
3.4. Glass Fiber Reinforced Polypropylene

Figure 3.24. Fatigue life data for GF/PP and GF/MA-PP (van den Oever and Peijs, 1997).

Table 3.4. Calculated characteristic fatigue life estimators for 0° GF/PP and GF/MA-PP laminates.

<table>
<thead>
<tr>
<th>Region</th>
<th>Estimator</th>
<th>GF/PP</th>
<th>GF/MA-PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$\dot{e}_0$</td>
<td>2.29%</td>
<td>2.33%</td>
</tr>
<tr>
<td></td>
<td>$\dot{\beta}$</td>
<td>51.9</td>
<td>44.6</td>
</tr>
<tr>
<td>II</td>
<td>$\dot{k}$</td>
<td>-11.6</td>
<td>-11.0</td>
</tr>
<tr>
<td></td>
<td>$\dot{l}$</td>
<td>4.37</td>
<td>5.12</td>
</tr>
<tr>
<td></td>
<td>$\dot{\beta}$</td>
<td>1.81</td>
<td>1.08</td>
</tr>
</tbody>
</table>
loading is mainly in mode II. The mismatch in elastic properties of the fiber and the surrounding matrix results in shear loading. When the effective frictional constraint has become low enough, the interfacial crack will propagate some further distance before being arrested by a microstructural obstacle. There is a large variability in interfacial properties on the micrometer size scale which will further enhance the erratic debond growth pattern. The debond mechanism is not critical per se, since it does not directly result in ultimate failure of longitudinal composites, but as the debonds grow, the overload in the adjacent fibers redistributes. When the redistributed overload exceeds the local strength of a weak fiber segment, it will fail and serve as an initiation point for the growth of a new debonds etc.

The fatigue damage in GF/MA-PP also initiate from fiber breaks or flaws, but takes a more tortuous crack path, and are arrested by the neighboring fibers. Replicas of fatigue crack growth in GF/MA-PP are found in Figures 3.28 and 3.29. The load and fiber directions are vertically oriented in all replicas. The difference in fatigue damage mechanisms observed on the replica pictures for GF/PP and GF/MA-PP is schematically depicted in Figure 3.30. In virtue of the stronger interface in GF/MA-PP, debonding is suppressed during fatigue. This makes the material more brittle, and notch sensitive, i.e. the debonds have a crack blunting effect, and moderate the overload imposed on intact fibers in the vicinity of damage sites. Without debonding, it is therefore expected that the variability in fatigue life would be larger, since the material would be more flaw sensitive. This inference is confirmed by the estimated fatigue life shape parameters presented in Ta-
3.4. Glass Fiber Reinforced Polypropylene

Figure 3.26. Fatigue damage growth in GF/PP (high magnification).

Figure 3.27. Fatigue damage growth in GF/PP (low magnification).
Fractographic analysis shows convincing indications of a stronger interfacial bond for the GF/MA-PP material, since the fibers were covered with residual polymer matrix. The stronger interfacial bond is due to the affinity of the maleic anhydride groups to the fiber sizing. The GF/PP did not show any residual polymer on the fibers. The interfacial bond may here be limited to compressive residual stresses, and interdiffusion of molecular segments into the sizing, with weak van der Waals bonds on the molecular level. Scanning electron micrographs of fibers at the fracture surface of failed specimens are shown in Figure 3.31.

The failed GF/PP specimens were more brush-like and rougher in appearance with straggling fibers than those that failed under static loading.
Figure 3.29. Fatigue damage growth in GF/MA-PP.
This fractographic difference is another indication that debonding is a prevalent fatigue mechanism in GF/PP. There was no conspicuous difference in the fracture surfaces of GF/MA-PP subjected to fatigue and static tensile tests. Both of them were relatively even, which suggests localized small-scale damage, and is in concert with the replica observations. Comparing failures of the two composites, the GF/PP specimens showed notably more sprawling and uneven fractures both in static and fatigue loading. In Figure 3.32, the two extreme cases of macroscopic fracture appearance of failed unidirectional longitudinal composites are sketched. Composites with strong interfaces and efficient load transfer between fibers tend to fracture in a brittle manner as shown in Figure 3.32a, whereas in composites with a weak interface, the fibers fail with higher integrity, and tend to show brush-like fractures as sketched in Figure 3.32b. Even though the tested specimens did not fully resemble the limit cases drawn in Figure 3.32, the GF/MA-PP specimens failed in a more brittle fashion, and the fracture GF/PP specimens appeared more brush like.

On the molecular level, grafting MA groups to the PP chains results in a more efficient interfacial bond. This is shown by fractographic pictures with adhering polymer matrix caused by cohesive failure of GF/MA-PP. The stronger interface suppresses debond propagation during fatigue which has been observed by a replica technique throughout fatigue testing. In GF/PP, the widely distributed and progressively growing debonds with accompanying fiber breakage led to a more rapid stiffness reduction. This degradation indicates a higher rate of damage accumulation, and subsequently a shorter fatigue life. Successive breakage of the load carrying fibers was observed in
Figure 3.31. Fractographic scanning electron micrographs for GF/PP and GF/MA-PP.
CHAPTER 3. DAMAGE MECHANISMS IN LONGITUDINAL PLYS

Figure 3.32. Appearance of failed coupons; (a) brittle, and (b) brush-like failure mode.

the GF/PP composite, from which debonds grew that made the ineffective lengths of the fibers increase, and hence the gradual reduction of the longitudinal stiffness. In this context, a qualitative link from chemical modification on the molecular level, over mesoscale damage mechanisms, to macroscopic fatigue life properties can be established. This microstructure-property connection is portrayed in Figure 3.33.

Modeling endeavors must be focused in a way that reflects the operative damage mechanisms. A local load sharing model with debonds growing according to a phenomenological power law is amenable for GF/PP. The distribution of fiber strength result in distributed fiber breaks in the composite from which debonds may initiate. The following redistribution of stresses will cause further fiber breakage etc. This type of model has successfully been used for creep in unidirectional microcomposites consisting of 6 or 7 fibers (Phoenix et al., 1988; Otani et al., 1991). In GF/MA-PP, more localized damage development is present without apparent signs of debonding. Nevertheless, a similar modeling approach may be adopted were a growing longitudinal zone of irreversible deformation in the matrix occurs. The difference of the two materials lies in the nature of this zone, which was not directly observable for the GF/MA-PP. Cycle-dependent matrix yielding is a likely process for composites with strong interfacial adhesion. Its propagation
### 3.4. Glass Fiber Reinforced Polypropylene

<table>
<thead>
<tr>
<th></th>
<th>GF/PP</th>
<th>GF/MA-PP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Molecular bonding</strong></td>
<td>Polypropylene homopolymer</td>
<td>Maleic anhydride grafted polypropylene</td>
</tr>
<tr>
<td></td>
<td>Weak van der Waals bonds</td>
<td>Polarity by MA grafting: dipole-dipole &amp; covalent bonding</td>
</tr>
<tr>
<td><strong>Fiber-matrix interface</strong></td>
<td>Weak interface</td>
<td>Strong interface</td>
</tr>
<tr>
<td><strong>Fractography: adhesion</strong></td>
<td>Adhesive failure</td>
<td>Cohesive failure</td>
</tr>
<tr>
<td><strong>Fatigue damage mechanisms</strong></td>
<td>Distributed and progressive debond growth, fiber breakage</td>
<td>Localized small-scale cracks perpendicular to the fiber direction</td>
</tr>
<tr>
<td><strong>Stiffness degradation</strong></td>
<td>Steady degradation</td>
<td>'Sudden death'</td>
</tr>
<tr>
<td></td>
<td>GF/PP</td>
<td>GF/MA-PP</td>
</tr>
<tr>
<td><strong>Fatigue life performance</strong></td>
<td>Shorter fatigue lives</td>
<td>Longer fatigue lives</td>
</tr>
<tr>
<td></td>
<td>GF/PP</td>
<td>GF/MA-PP</td>
</tr>
</tbody>
</table>

**Figure 3.33.** Schematic illustration of micro-macro relation.
rate should be low, since the damage was localized and planar. The strong interface also results in a more effective load transfer between the fibers, which in turn means that the critical cluster size at catastrophic failure is small. A model of the fatigue damage growth would be useful for a parametric study where the influence of constituent properties can be investigated, which would be valuable in materials engineering. A first parametric model could consist of a single fiber rupture from which debonds grow symmetrically, and give rise to a continuously changing stress field in the adjacent fibers. This changing stress may cause further fiber failure at different locations depending on the local strength distribution. Section 4.2 will be devoted to a parametric study of this damage process.

3.5 Modeling impetus

As declared earlier, an experimental investigation of the fundamental fatigue damage mechanisms is prompted by the need of physically accurate models. Once the operative mechanisms have been mapped, reliable models can be conceived on the basis of the observations. Based on the same results, it should also be possible to judge if the modeling assumptions are sound.

Two main mechanisms were observed in the tested composites. The first one was debonding or longitudinal matrix cracking, which occurred in the composites with the weaker interfaces, i.e. CF/PEEK and GF/PP. These longitudinal cracks propagated progressively during fatigue, and led to new fiber breaks in a sequential manner. The simplest case in such a scenario is a single fiber break from which debonds grow along the fiber, which subsequently results in rupture of neighboring fibers. The second mechanism was transverse propagation of a matrix crack from broken fibers, which was found in the materials with strong interfaces, i.e. CF/epoxy and GF/MA-PP. In the CFRP, the matrix crack grew past the neighboring fibers and became bridged by the neighboring fibers, whereupon it acquired a deformed shape with squeezed crack tips.

Tentative models of the two main mechanisms, debonding from a fiber break and fiber bridged cracking, will be addressed in the next chapter. These modeling efforts concern the influence of fiber strength variability and debond propagation on further fiber breakage, which can qualitatively be linked to the macroscopic behavior. The second model deals with experimental results of the fiber bridged cracks in the CF/epoxy material. The effect of fiber bridging on the propagation rate is studied, from which the influence of other undetected toughening mechanisms can be evaluated.

In summary of the observed mechanisms, the progressive damage mecha-
nisms in CF/epoxy and CF/PEEK show important differences. In CF/epoxy, the matrix cracks often start from fiber breaks, and grow to become bridged by adjacent fibers. The fiber bridging mechanism causes deceleration of the transverse crack growth rate. The cracks remain sparsely distributed for most of the fatigue life, suggesting negligible crack interaction during the growth process. Below the fatigue limit, limited debonding reduces and eventually arrests crack growth, allowing insufficient fiber breakage to result in failure of the composite. The CF/PEEK composite shows extensive matrix and interfacial cracking along the fiber direction, leading to crack link-up and the consequent damage accumulation.

Furthermore, the shorter fatigue lives and more rapid reduction of the Young’s modulus of the GF/PP composite are explained by its relatively poor fiber-matrix adhesion, which leads to wide-spread debond growth from fiber breaks and flaws. The growing debonds result in overloads on the surviving adjacent fiber segments, which in turn lead to further fiber breakage with debonding etc. Conversely, the GF/MA-PP has a more brittle behavior with localized damage and non-propagating mechanisms, viz. small matrix cracks form from flaws, and are generally arrested by adjacent fibers. This results in a more fatigue resistant behavior for the GF/MA-PP material. The propensity to debond propagation caused by a weaker interface has an adverse effect on the fatigue performance.

In spite of the large difference in material composition, the CFRPs and GRPs show similarities in mechanisms depending on the interfacial strength, or the tendency for longitudinal crack propagation. It can be concluded that these properties play a fundamental role in fatigue. The transverse or shear properties of composite laminates are known to be controlled by the interfacial properties. In static tension loading of the critical member of a composite laminate, the 0° ply, the strength is generally believed to be dominated by the fiber properties. The presented results show that the interface and the matrix play an important role in fatigue.
Chapter 4

Modeling of damage growth

4.1 Mechanism based modeling approaches

In order to model the development of fatigue damage in unidirectional composites, the detailed mechanisms on the microscopic level have to be considered. Composite failure, both under static and fatigue conditions, is intrinsically localized, i.e. failure occurs in the weakest volume element. In this element, the microstructure is generally in the same size scale as the failure inducing damage. Individual fiber breaks, debonds, bridged cracks, yielded matrix zones etc. have to be considered. Conversely, if the gradual degradation of a macroscopic average property, say the axial Young’s modulus, is investigated, an approach based on a homogenized composite with effective smeared-out damage may be adopted. Continuum damage mechanics can be used in this case where the fatigue damage is evenly distributed in the composite (see Talreja, 1994). Since lifetime prediction is the ultimate goal in the extension of this study, failure and localized damage are considered. This means that it is necessary to take the heterogeneous microstructure of the composite into account in the analysis of fatigue damage growth.

Simple damage scenarios that have been experimentally identified should be the first to be modeled, which typically involve single damage sites. Next, this can be extended to more complex situations with e.g. cluster formation and coalescence, and volume scaling. Modeling of fatigue damage accumulation and life prediction based on e.g. degradation of stiffness, strength or life from a macroscopic viewpoint is not considered here. This traditional macroscopic approach is well-investigated by others (to name a few, see Halpin et al., 1973; Hashin & Rotem, 1973; Hashin, 1985; Reifsnider & Stinchcomb, 1986; Beaumont, 1987; Andersons, 1994; Lee et al., 1996, Brøndsted et al., 1996). One of the potential benefits of microscopic modeling of fatigue degra-
dation is that judicious parametric studies can indicate improvements in the constituent properties for microstructural tailoring and optimum macroscopic performance.

There are two main approaches to model damage accumulation in composites through the successive breakage of individual fibers depending on how the load is redistributed among the surviving fiber segments when a fiber is broken. In composites with weak interfaces, the global load sharing (GLS) concept can be used. It assumes that all remaining intact fibers share the load equally in the plane of the fiber breaks, which is locally analogous to a loose fiber bundle with no stress transfer between contiguous fibers. For PMCs, this is generally a poor presumption, since the interfacial adhesion is usually fairly efficient, and the relatively high stress transfer ability results in localized damage with small critical flaws at catastrophic failure. Instead, local load sharing (LLS) can be adopted, in which only the neighboring fibers to a fiber break are assumed to share the additional load according to an empirical or physical stress redistribution function. The LLS scheme becomes more detailed, and generally mathematically more intricate, but forms a more physical description which is more consistent with experimental data. The LLS approach will implicitly be advocated for the debond model in the present chapter.

The degree of material anisotropy is very high for unidirectional composites. For example, the transverse strength is significantly lower than the tensile strength. Even if the composite is loaded in the fiber direction, cracks may grow longitudinally in mode II along the fiber-matrix interface due to low interfacial strength and the large mismatch in elastic properties. This trend prevails for short fiber composites also. Friedrich et al. (1986) tested injection molded short fiber composites in fatigue, and noticed crack propagation in the principal material direction independent of the load direction. Xian & Choy (1994) tested notched CF/bismaleimide in fatigue, and observed mode II propagation in the fiber direction for 0° specimens. In composites with long continuous fibers in the load direction, debonds may initiate from the distributed fiber breaks, and propagate under fatigue. The growing debond continuously changes the local load of the neighboring fibers. When a weak element of an intact fiber is over-loaded, it may fail and give rise to new debonds, etc. In this manner, fiber breaks will be accumulated, and the material will eventually fail. This scenario can be combined with cycle or time dependent growth of a yield or creep zone in front of the interfacial debond (Beyerlein & Phoenix, 1997a). Due to the growth of debonds and yield zones, the fatigue crack will have a non-planar fracture surface. Depending on the direction of matrix crack growth, different patterns of damage growth occur. If the matrix crack grows perpendicular to the load and fiber direction, a
fiber bridged crack will form. If the material is prone to debond or yielding in the longitudinal direction, there will be non-planar successive fiber breakage. This is supported by the experimental results presented in the previous chapter. The two cases are schematically depicted in Figure 4.1, namely debonding with yielding and fiber bridged cracking. Both scenarios have been observed experimentally, where bridging took place in carbon fiber/epoxy, and accumulation of debond-induced fiber breaks was the dominating mechanism in carbon fiber/PEEK (Gamstedt & Talreja, 1997). Other experimental observations in PMCs of debonding in fatigue have been made by e.g. Dharan (1975), Lorenzo & Hahn (1986), Shih & Ebert (1987) and Vauthier et al. (1996), and observations of fiber bridged cracking in fatigue have been made by e.g. Ritchie et al. (1989), Wilson & Wilson (1991), Baron (1992), Botsis et al. (1995) and Luke et al. (1996). Even though the two scenarios are based on the same basic submechanisms (fiber breakage influenced by matrix crack propagation, matrix yielding and debonding), they are treated separately here since the observed evolving fatigue damage states are quite different from one another. For debonding and subsequent fiber breakage, the damage is relatively evenly distributed in the composite, whereas for fiber bridged cracking, the damage sites are localized and relatively scarce. The two following sections are devoted to the two damage cases, respectively.
4.2 Debond propagation and fiber failure

Debonding in fatigue

As seen in the previous chapter, one important feature of the rapid fatigue degradation in CF/PEEK and GF/MA-PP is growing debonds. They emanate from distributed fiber breaks or flaws and grow longitudinally along the fiber direction (e.g. Lorenzo & Hahn, 1986; Shih & Ebert, 1987; Horsthemeyer & Staab, 1990; Vauthier et al., 1996; Abeles Couillard & Schwartz, 1997; Gamstedt & Talreja, 1997; Gamstedt et al., 1997). During debond growth, the local stress profile in the surrounding fibers will gradually change, and a weak segment may become overstressed leading to fiber rupture, followed by further debonding and fiber rupture etc. until final failure. The evolution of stress transfer during fatigue debond growth and the resulting successive breakage of fibers is a key mechanism for debond prone PMCs. In order to model this behavior and ultimately predict fatigue life, the fundamental and basic configurations must first be investigated and understood.

A simple case with one fiber break from which uniform debonds grow along the fiber will be investigated parametrically to shed some light on the interaction between stochastic fiber breakage and debond propagation. From the results some qualitative conclusions on macroscopic fatigue behavior can be drawn.

In fatigue, the progression of any damage, debonding subsumed, should be described by a kinetic equation to predict damage development. A kinetic relationship for debond growth should be measured experimentally. The growth rates from the replica measurements of the type presented in Figure retard is relevant to itself only, as the stress state and interfacial toughness vary from fiber to fiber, and along each fiber. The growing debonds in the CF/PEEK and GF/PP materials showed an irregular accelerating-decelerating growth pattern, which could be explained by a variability in interface properties along the fibers. This behavior would require a stochastic growth model. Furthermore, these measurements were made on the surface only, and debond propagation can not be expected to behave similarly in the interior of the composite. Fatigue crack propagation along a bimaterial interface in mode II with crack surface wear and constraints from a three dimensional composite geometry is a formidable modeling problem to tackle. However, some reassuring work in this direction has been presented (Chan & Davidson, 1989; Larson, 1995; Hsueh, 1996). No kinetic law for the debond propagation will be considered here, since the conceived model is not a direct description of experimental results, but rather aiming for a parametric study to assess influences of debonding and fiber strength variability on fiber break-
4.2. DEBOND PROPAGATION AND FIBER FAILURE

age under fatigue conditions. However, for future attempts to life predictions a kinetic law is an exigency.

Since the advent of laser Raman spectroscopy (LRS), the composite research community has been endowed with a powerful tool to study local stresses and strains in composites reinforced by fibers with crystalline or turbostratic morphologies. Most LRS investigations have focused on model microcomposites with a single fiber or a monolayer of distributed fibers. A useful application is to characterize the interfacial properties for the single fiber test (Galiotis, 1991; Andrews & Young, 1993), and the influence of these properties on stress transfer between fibers for the monolayer test (van den Heuvel et al., 1997). These systems have a very low volume fraction of fibers, and a direct transferability of the quantified material data to commercial composite systems intended for service applications might be questionable. Some LRS investigations have been made on composite material with high volume fractions of fibers (Marston et al., 1997), but as for almost all other non-destructive evaluation techniques, the experiment is confined to a superficial layer of fibers at the surface. Nevertheless, the LRS technique provides an excellent method to verify the assumptions made in analysis and to identify microstructural parameters.

A two-dimensional shear lag approach will be used to model the composite stress state. It is first best to gain understanding of the micromechanical interactions of fiber ruptures and debond propagation in a two dimensions before extending the analysis to three dimensions. Furthermore, early experimental work has shown similar failure behavior in single monolayers and in composites composed of stacked multiple layers (Herring et al., 1973; Jones & Goree, 1983).

Shear lag model

A fundamental scenario would be a single fiber break with propagating debonds between the broken fiber and the two adjacent fibers. The behavior of this elementary case would in a general sense reflect the behavior of a more complex damage state with multiple fiber breaks and growing debonds. Therefore, the case with a single fiber break with debonding will be subject to scrutiny to isolate the debonding process and the subsequent fiber rupture in a well-defined and simple configuration. A schematic picture is shown in Figure 4.2, and micrographs from experiments on a CFRP is found in Figure singlefiber. At the outset, there is a single fiber break at the origin in fiber number 0. The length of the debonds in all four quadrants is \( a \) with the fiber break at the origin. The debonds are in matrix bays number 0 and \(-1\). The stress profile of the fibers adjacent to the broken fiber will be
investigated in some more detail, since it is generally the next one to be broken. In order to calculate these stresses, a shear lag model can be used. The two-dimensional shear lag model developed by Beyerlein & Phoenix (1996a) was chosen to calculate the stress concentration profile in a fiber adjacent to a single fiber break. The main features of the model and the specifics of the present configuration are outlined here. For details, the reader is referred to the cited original work. As is typical for most shear lag models, the fibers are only supporting axial loads, and the matrix transfers the loads in the fibers through shear only. The fibers are arranged equidistantly in an infinite two-dimensional array. Poisson contractions are altogether neglected.

Firstly, the constitutive relation can be expressed in dimensional units. The distance from the fiber break plane is $x$, the fiber spacing is $w$, and $h$ is the ply thickness. The constitutive equation of the axial deformation of the fibers can be expressed as

$$p_n(x) = E_f A \frac{du_n(x)}{dx},$$  \hspace{1cm} (4.1)

where $E_f$ is the Young's modulus of the fibers, $A$ is the cross-sectional area of a fiber, $u_n(x)$ is the axial displacement in fiber number $n$. The constitutive law for matrix shear deformation is

$$\tau_n(x) = G_m \frac{u_{n+1}(x) - u_n(x)}{w},$$  \hspace{1cm} (4.2)
where \(\tau_n(x)\) is the shear stress profile in matrix bay number \(n\), and \(G_m\) is the matrix shear modulus. Combining the two constitutive laws with the condition of static equilibrium, the following differential equation presents itself:

\[ E_f A \frac{d^2 u_n(x)}{dx^2} = - \frac{G_m h}{w} [u_{n+1}(x) - 2u_n(x) + u_{n-1}(x)]. \] (4.3)

Secondly, a normalization would render this differential equation dimensionless, and the solution would be independent of the material properties and the composite geometric parameters. The fiber displacement is assigned to be

\[ U_n(\xi) = \frac{u_n(x)}{p^* \sqrt{E_f A G_m h}}, \] (4.4)

where the dimensionless axial coordinate is

\[ \xi = \frac{x}{\sqrt{E_f A w G_m h}}, \] (4.5)

and the scaling factor is

\[ p^* = \sigma^* \sqrt{\frac{w E_f A h}{G_m}}, \] (4.6)

where \(\sigma^*\) can be arbitrarily chosen to keep the order of magnitude of the stresses close to unity. The fiber stress becomes

\[ P_n(\xi) = \frac{p_n(x)}{p^*}, \] (4.7)

and the matrix shear stress and shear strain become identical in the elastic regime,

\[ T_n(\xi) = \Gamma_n(\xi) = U_{n+1}(\xi) - U_n(\xi). \] (4.8)

In order to obtain real physical dimensional values, the normalized non-dimensional quantities must be converted back to \(p_n(x)\) and \(\tau_n(x)\). To give an idea of the size scale, the value \(\xi = 1\) would correspond to \(x = 70\) to 130 \(\mu\)m for standard values of the constituent elastic properties and volume fractions of conventional polymer matrix composites with long continuous fibers (data from Hull & Clyne; 1996).

The differential equation (4.3) can then be re-expressed in a non-dimensional form as

\[ \frac{d^2 U_n(\xi)}{d\xi^2} + U_{n+1}(\xi) - 2U_n(\xi) + U_n(\xi) = 0, \] (4.9)
from which solution, the fiber stress can be calculated,

\[ P_n(\xi) = \frac{dU_n(\xi)}{d\xi}. \]  

(4.10)

As outlined by Phoenix and co-workers, the determination of the stress state in the fibers can be reduced to the solution of two basic problems: the stress redistribution due to (i) an isolated fiber break, and due to (ii) an isolated shear load couple. The numerical problem will be confined to a description of only the damaged regions (fiber break and debonds), from which the stress state in the entire volume can be formulated. In e.g. a finite element approach, the determination of the stress redistribution would demand a discretization and solution of the stress state in the entire volume. A finite element solution is also dependent on the choice of elements and mesh.

For the single fiber break at the origin, the boundary conditions are that \( P_n(\pm\infty) = 1 \) for the far-field applied load, and \( P_0(0) = 0 \) at the fiber break. Under these conditions, the break influence functions in the elastic case have been solved by Hedgepeth & Van Dyke (1967). They solved the isolated break problem analytically by a discrete Fourier transformation of Equation (4.9). The stress along fiber \( \eta_j \) at position \( \xi_j \) imposed by a unit compressive stress at the fiber break (which will later be compensated with a unit applied tensile stress) was determined to be

\[ \Lambda = -\frac{1}{2} \int_0^\pi \sin \frac{\theta}{2} \exp \left( -2|\xi| \sin \frac{\theta}{2} \right) d\theta. \]  

(4.11)

Likewise, the shear stress and strain in matrix bay \( n_k \) at position \( \xi_k \) was shown to be

\[ \Omega_k = \frac{\text{sgn} \xi_k}{4} \int_0^\pi \{ \cos(n_k + 1)\theta - \cos n_k\theta \} \exp \left( -2|\xi_k| \sin \frac{\theta}{2} \right) d\theta, \]  

(4.12)

where \( \text{sgn}(\cdot) \) is the signum function, defined as +1 for a positive or zero argument, and -1 for a negative argument.

The influence of isolated shear couples has been deduced by Beyerlein & Phoenix (1996a), who used a technique of quadratic influence superposition to ascribe inelastic properties from yielding and debonding to the matrix. In the present basic damage configuration, a simplified formulation with a set of shear couple point loads will be employed, but the deduction scheme from the aforementioned work will be used. In the shear couple case, the boundary conditions of unit compressive load for the fiber break case are exchanged by \( P_n(\xi) = -\frac{1}{2} \) and \( P_{n+1}(\xi) = \frac{1}{2} \), which correspond to a unit shear couple.
in matrix bay \( n \) at position \( \xi \). The stress at the origin, i.e., the would-be position of the fiber break, due to an imposed shear load couple in matrix bay \( n_k \) at position \( \xi_k \) was determined to be

\[
\Phi_k = \frac{\text{sgn} \xi_k}{2\pi} \int_0^{\pi} \{ \cos n_k \theta - \cos(n_k + 1)\theta \} \exp \left( -2|\xi_k| \sin \frac{\theta}{2} \right) d\theta. \tag{4.13}
\]

Correspondingly, the shear stress/strain in matrix bay \( n \) at \( \xi \) caused by an applied unit shear couple in bay \( n_k \) at \( \xi_k \) became

\[
\Psi_{lk} = -\frac{1}{\pi} \int_0^{\pi} \cos(n_l - n_k)\theta \sin \frac{\theta}{2} \exp \left( -2|\xi_k - \xi_l| \sin \frac{\theta}{2} \right) d\theta + \delta_{lk}, \tag{4.14}
\]

where \( \delta_{lk} \) is Kronecker’s delta function, which is defined as \( \delta_{lk} = 1 \) if \( l = k \) and 0 otherwise.

The influence functions \( \Phi \) and \( \Psi \) can be used to determine the local stresses in the composite imposed by debonds. In the debonded region, the stress transfer ability of the matrix is reduced, and limited to frictional shear stresses. In fatigue, the debonded surfaces are continuously sliding against one another due to the repeated loading and unloading. The ensuing wear and attrition of the crack surface asperities make the frictional shear forces decrease continually during fatigue. Therefore, it is here assumed that there are no shear stresses in the debonded regions, \( T = 0 \). Since the measured fatigue debonds reported in the previous chapter were large compared to the size scale of the microstructure, the effect from a yielded zone at the debond crack tip is here neglected. However, it should be appreciated that in an elastic material, there is a stress singularity at the crack tips, which results in local yielding in the physical material. This implies an attenuation of the stress concentration in the adjacent fiber, whose effect decreases as the debond length increases. Moreover, since the matrix material is constrained by stiffer surrounding fibers, the stress state in the matrix will be triaxial, which subdues large scale yielding even for ductile polymers.

Along the debonded strips in matrix bays \(-1\) and \(0\) in Figure 4.2, a distribution of \( N \) shear couples are evenly dispersed to counteract the elastic response from other stress sources. To each shear couple \( k \) is a weight factor \( K_{k,m} \) allotted to account for its relative contribution. Similarly, the functions \( \Lambda \) and \( \Omega \) are used to determine the local stresses contributed by the fiber break. The corresponding weight factor of the fiber break is denoted \( K_f \). Combining the influences from both the fiber break and the debonded regions, the complete stress state in the composite can be established through the determination of the weight factors:

\[
\begin{pmatrix}
K_f \\
K_m
\end{pmatrix} = \begin{pmatrix}
\Lambda & \Phi \\
\Omega & \Psi
\end{pmatrix}^{-1} \begin{pmatrix}
-1 \\
0
\end{pmatrix}, \tag{4.15}
\]
where $-1$ represents the unit compressive load at the fiber break, $\mathbf{0}$ is an $N$ dimensional vector assigning zero shear stress in the debonded regions. The elements $\Lambda$ and $K_f$ are scalars since only a single fiber break is considered. The submatrices $\Phi$, $\Omega$, and $\Psi$ are of dimensions $1 \times N$, $N \times 1$ and $N \times N$, respectively. The weight factor vector $K_m$ is $N$ dimensional, the elements of which pertain each to an individual shear couple. Given the weight factors from the solution of Equation (4.15), the stress profile along any fiber can be calculated.

In like manner to the expressions of $\Lambda$ and $\Omega$ in Equations (4.11) and (4.12), the contribution from the fiber break and a matrix shear couple onto the stress profile in the adjacent fiber can be resolved. The adjacent fiber number 1 along the positive $\xi$-axis is selected, although the same stress profile is present in the other three quadrants due to symmetry. The stress profile in fiber 1 imposed by the fiber break is then expressed as

$$
\lambda(\xi) = -\frac{1}{2} \int_{0}^{\pi} \cos \theta \sin \frac{\theta}{2} \exp \left(-2|\xi| \sin \frac{\theta}{2}\right) \, d\theta,
$$

and the stress profile in the same fiber imposed by a shear couple in matrix bay $n_k$ at position $\xi_k$ is by translation invariance expressed as

$$
\omega_k(\xi) = -\frac{\text{sgn}(\xi - \xi_k)}{2\pi} \int_{0}^{\pi} \left\{ \cos(n_k - 1)\theta - \cos n_k\theta \right\} \times \exp \left(-2|\xi - \xi_k| \sin \frac{\theta}{2}\right) \, d\theta.
$$

The stress profiles of the adjacent fibers are $P_1(|\xi|) = P_{-1}(|\xi|)$. The stress profile in the selected quadrant is denoted $\Sigma(\xi) = P_1(\xi)$ for $\xi \geq 0$, which also corresponds to the stress concentration factor since the applied load is unity. With the stress factors $K_f$ and $K_m$ from Equation (4.15), the stress concentration factor is

$$
\Sigma(\xi) = 1 + K_f \lambda(\xi) + \sum_{k=1}^{N} K_{k,m} \omega_k(\xi),
$$

where the first term 1 represents the influence from the applied load which completely relieves the unit compressive stress at the fiber break.

The Matlab™ software (The MathWorks, Inc.) was used for the numerical calculations, where 10 to 20 shear couples were uniformly distributed along each of the four debonded lengths, $|\xi| \leq \alpha$, where the debond length $\alpha$ was varied between 0 and 2.25. In Figure 4.3, the stress concentration factor is plotted with respect to the distance to the fiber break plane for various debond lengths. Experimental investigations by means of laser Raman
4.2. DEBOND PROPAGATION AND FIBER FAILURE

Figure 4.3. Stress concentration factor for different debond lengths.
spectroscopy by e.g. van den Heuvel et al. (1997), Marston et al. (1997) and Chohan & Galiotis (1997) support the calculated stress concentration in the adjacent fiber. The elastic case with no debonding gives the highest stress concentration in the adjacent fiber with a value of $\Sigma = 1.3334$. The stress concentration factors for full elastic and finite element solutions in three dimensions is generically the same, but takes smaller maximum values than the above two-dimensional shear lag solution (Nedelek Wisnom, 1994a). It is doubtful if the elastic case with no debonds exists, since dynamic effects from sudden release of stored elastic energy on fiber breakage should cause some additional damage besides the fiber break itself. The fatigue propagation of the debonds may therefore start at a finite non-zero debond length, $a_0 > 0$. As the debond grows, the peak value decreases and moves away from the plane of the fiber break, and roughly follows the position of the debond crack tip. The shapes of the stress profiles for different debond lengths are not unlike those obtained by Nedelek & Wisnom (1994b) in a three-dimensional axisymmetric finite element investigation. Lagoudas et al. (1989) made a numerical investigation of the overstress profiles near broken fibers using a shear lag model with a viscoelastic creeping matrix, and obtained a broadening of the stress profiles as the viscoelastic relaxation of the matrix shear stresses progressed. Bennett & Young (1997) have presented experimental data on moving peak axial fiber stresses with growing debonds by laser Raman spectroscopy. The peak value of the stress concentration factor in Figure 4.3 evens out, and seems result in a small overload, $\Sigma > 1$, which is presented in Figure 4.4.

The fiber break and its debonds give rise to a notably non-uniform stress profile along the adjacent fiber. As the debond propagates, the stress profile changes, which leads to relaxation of some of the previously overloaded parts of the fiber, while other parts close to the debond crack tip become more overloaded than heretofore. This continuous change in stress profile may eventually lead to rupture of the fiber at a point with an unprecedented overload. Increasing debond length will definitely result in an increasing probability of failure of the neighboring fibers. As experimentally observed and as discussed earlier, there is an abundance of fiber breaks, or clusters of fiber breaks in real composites with weak interfaces. From these flaws, debonds grow, and the cumulative likelihood that adjacent fibers will break is increasing monotonically. As successive fiber breakage sets out, further debonding and subsequent fiber breakage will continue in a self-escalating manner. Eventually, a rapid catastrophic fiber failure process will take place, and ultimate failure ensues. Returning to the original issue with a single fiber break with growing debonds, the failure of the neighboring fibers, and therefore also of the composite as a whole, is a stochastic process, and a
Figure 4.4. Maximum stress concentration factor for different debond lengths.
statistical approach is a necessity.

One of the drawbacks of a simple shear lag model is that the stress concentration factor, $\Sigma$, is independent of the interfiber spacing, $w$. Chohan & Galiotis (1996) measured the stress concentration factor next to fiber breaks in a monolayered model composite by means of laser Raman spectroscopy and observed a decreasing stress redistribution with increasing fiber spacing. For large interfiber distances, the influence from adjacent fiber breaks was negligible. In contrast, for fibers placed immediately next to one another, the stress concentrations were not far from those predicted by the shear lag model in spite of its independence of fiber spacing. The distribution of fiber breaks in a composite monolayer was investigated by van den Heuvel et al. (1996), and the same effect manifested itself in spatially distributed fiber breaks for large spacings, and planar and aligned fiber breaks for close spacings. For standard composite materials with high volume fractions of fibers, $V_f > 0.5$, the fibers are virtually positioned next to the neighboring fibers in closer regime than what is feasible with the commonly used manufacturing technique of composite monolayers developed by Wagner & Steenbakkers (1989). Therefore, any error stemming from the adoption the present spacing independent model is neglected.

**Statistical analysis of fiber failure**

The statistical failure of the fiber adjacent to the broken and debonded fiber will be analyzed in particular. The fibers considered here are carbon or glass fibers, which are frequently used in PMCs. They are known to have a strength that obeys reasonably well the Weibull distribution with the cumulative probability of failure

$$P_f(\Sigma_f) = 1 - \exp \left\{ -L \left( \frac{\Sigma_f}{\Sigma_0} \right)^\beta \right\}, \quad (4.19)$$

where $\beta$ is the shape parameter, $\Sigma_0$ is the scale parameter for unit fiber length (i.e. $L = 1$). The shape parameter can be interpreted physically as a measure of the flaw size distribution of the fiber (Jayatilaka & Trustrum, 1977). It should be mentioned that linear scaling with $L$ may not prove to be adequate for composite fibers over a large length range (Gutans & Tamužs, 1984; Watson & Smith, 1985). When the weakest link assumptions do not hold, the dependency on length in Equation (4.19) can be formulated as a power law with $L^\gamma$ in lieu of $L$. A reason for this behavior can for instance be a variation in fiber cross-sectional area along its length. Typical values for the exponent $\gamma$ are 0.6 for AS4 carbon fibers (Beyerlein & Phoenix, 1996b) and
for aramid fibers (Phoenix et al., 1988), and 0.9 for HS Celion 1000 carbon fibers (Bader & Priest, 1982; Watson & Smith, 1985), and a range from 0.38 to 0.67 for various carbon fibers of Soviet origin (Gutans & Tamužs, 1984). Asloun et al. (1989) suggested a linear logarithmic dependence on gage length on the mean strength based on experimental results from carbon fibers. Scaling with $\gamma = 1$ has been found to be quite appropriate with gage lengths within the same order of magnitude (Otani et al., 1991). This linear behavior with $\gamma = 1$ as in Equation (4.19) will be used here. In that case, the distribution of single fiber strength may readily be obtained from the load displacement curve of fiber bundles rather than tedious testing the fiber individually (Chi et al., 1984; R'Mili et al., 1996), provided there is full integrity of the fibers in the bundle with no stress transfer.

The Weibull distribution in Equation (4.19) is a two parameter distribution ($\beta$ and $\Sigma_0$). Sometimes a three parameter Weibull distribution is employed for characterization of the scatter in strength of brittle materials. The parameter $\Sigma_f$ is the substituted with $\Sigma_f - \Sigma_u$ for $\Sigma_f \geq \Sigma_u$, where $\Sigma_u$ is the location parameter below which an applied load never leads to failure, and is equivalent to the theoretical strength of the fiber in the absence of any internal defects or surface flaws. It has been recommended to use a two-parameter Weibull distribution for the strength of brittle materials, which gives a higher goodness-of-fit with the Smirnov-Kolmogorov statistic and higher consistency in estimation than does a three-parameter Weibull distribution (Trustrum & Jayatilaka, 1979). The two-parameter Weibull distribution will be used here.

The associated expectancy or mean value of the strength of a fiber segment with length $L$ is

$$\bar{\Sigma}_f = \frac{\Sigma_0 \Gamma \left(1 + \frac{1}{\beta}\right)}{L^{1/\beta}},$$  \hspace{1cm} (4.20)

where $\Gamma(\cdot)$ is the Gamma function.

In order to investigate the influence of scatter in fiber strength on the rupture of the fiber adjacent to the originally broken and debonded fiber, the mean strength, $\bar{\Sigma}_f$, was kept constant for a number of different shape parameters, $\beta$. The selected values are $\beta = 2, 6 \text{ and } 10$, which represent experimentally obtained values for carbon and glass fibers (Hull, 1981). By choosing the stress normalization parameter $\sigma^*$ in Equation (4.6) so that $\Sigma_0 = 1$ for the middle value of the shape parameter, $\beta = 6$, the mean strength is $\bar{\Sigma}_f = \Gamma(1 + \frac{1}{6})$ for unit gage length. The scale parameter for other
shape parameters can then be calculated with

\[ \Sigma_0(\beta) = \frac{\Sigma_i}{\Gamma \left( 1 + \frac{1}{\beta} \right)} \]  

(4.21)

for \( L = 1 \). In this way a parametric study of the influence of scatter in fiber strength on the breakage of the adjacent fiber can be undertaken without altering the mean fiber strength.

The investigated fiber segment can be partitioned into subsegments of length \( \delta \). The probability of survival of such a subsegment is

\[ P_s(\delta, \Sigma) = \exp \left\{ -\delta \left[ \frac{\Sigma}{\Sigma_0(\beta)} \right]^{\beta} \right\}, \]  

(4.22)

for a constant stress \( \Sigma \). By use of the weakest link concept, the probability of failure of a fiber of length \( \xi_0 = n \delta \) subjected to a non-uniform stress along its length can be estimated, if the fiber is divided into \( n \) pieces of length \( \delta \). The gage length is here chosen to be \( \xi_0 = 6 \) since the debonded stress profiles seem to be fairly close to the asymptotic value of the elastic stress profile at this point (cf. Figure 4.3). The probability of failure is

\[ P_f(\Sigma(\alpha, \xi)) = 1 - \lim_{n \to \infty} \prod_{i=0}^{n} \exp \left\{ -\xi_0 \left[ \frac{\Sigma(\alpha, i \xi_0/n)}{\Sigma_0(\beta)} \right]^{\beta} \right\}, \]  

(4.23)

which yields

\[ P_f(\Sigma(\alpha, \xi)) = 1 - \exp \left\{ -\int_0^{\xi_0} \left[ \frac{\Sigma(\alpha, \xi)}{\Sigma_0(\beta)} \right]^{\beta} d\xi \right\}, \]  

(4.24)

where \( \Sigma(\alpha, \xi) \) is the stress concentration at \( \xi \) in the fiber adjacent to the broken fiber for a unit applied load, and with a varying debond length, \( \alpha \). As the debond grows during fatigue, the stress profile along the considered fiber changes throughout the test. The fiber is subjected to a continuously changing proof test. In an elastic material, it is the unprecedented stress at each given point that is critical, and that may cause failure, i.e. the Kaiser principle is assumed to be valid. The envelop or maximum stress profile from the point where there was no debonding up to a debond length \( \alpha \) is

\[ \Sigma_e(\alpha, \xi) = \max_{0 \leq a \leq \alpha} \Sigma(a, \xi). \]  

(4.25)
Figure 4.5. Weibull plot of the probability of fiber failure with respect to debond length and fiber strength variability.
The probability of failure with respect to debond length is presented in a Weibull plot in Figure 4.5 for a unit applied load, and shape parameters $\beta = 2, 6$ and 10. The history and spatially dependent stress profile envelop, $\Sigma_e$, in Equation (4.25) was used to determine the probability of failure prior to a certain given debond extent. For all strength distributions, the probability of failure is increasing with increasing debond lengths, which is a necessary condition for a progressive damage mechanism with successive fiber failures. Even though debonding is not a critical damage mechanism per se, it has an important influence on fiber breakage. Since the fibers are the main load carrying constituents in structural PMCs, the failure of the composite can only be caused by failure of its fibers. The load redistribution from the growing debonds in an ensemble of fibers leads to an accumulation of fiber breaks which may finally cause final composite failure. In this way, debonding act as a progressive damage mechanism ultimately contributing to final failure.

The probability of failure is significantly larger for higher values of $\beta$. With a more narrow distribution of fiber strength, the composite shows higher sensitivity to local stress concentrations. The notch-insensitivity of composites with fibers with a wide strength distribution has been investigated by Monte Carlo simulation by Beyerlein & Phoenix (1997b). The simulations presumed length invariant strength and no debonding. The flaw tolerance for small values of $\beta$ was particularly large for small notches or cracks. A fiber type with a larger variability in strength provides a toughening effect due to its relative insensitivity to high overloads. This effect can be further enhanced by alteration of the path of progression of fiber fractures up to instability, as will be discussed later.

As the debond grows, and the stress profile in the neighboring fiber changes, so will the expectancy of the distance from the original fiber break to the secondary break, $\xi_f$. With no debond, the stress concentration is relatively localized to the plane of the original fiber break, but as the debond propagates, the stress concentration profile becomes more distributed along the $\xi$-axis, and rupture is expected further away from the first break. With a narrow distribution of fiber strength, planar fiber breakage would be more likely. To investigate this supposition, a Monte Carlo simulation was undertaken. Through inversion of Equation (4.19), the strength of fiber segment $i$ of length $\delta$ is

$$\Sigma_i = \Sigma_0(\beta) \left( -\frac{\ln Z_i}{\delta} \right)^{1/\beta},$$

(4.26)

where $Z_i$ is a generated random variable from the uniform distribution between 0 and 1. Each of the 100 assigned fiber elements was then compared
to the local stress, and the position of the weakest element with lowest positive strength to stress ratio was registered for 100,000 iterations for each stress profile and shape parameter. To isolate the effect of the damage inflicted stress concentration from the far field applied stress, the local stress was somewhat unrigourously defined by subtraction of the applied stress from the present maximum stress profile, \( \Sigma_e(\alpha, \xi) - 1 \). In this manner, only fiber failures caused by the stress concentration from the present damage were generated. The mean value of the fiber break distance, \( \xi_f \), was thus calculated for each debond length and fiber strength distribution. The simulation was repeated a number of times, and the maximum test-to-test difference was found to be no greater than 2%. The results of the simulations are presented in Figure 4.6. As the debonds grow, the mean distance to the second fiber break increases. With debonding, the overload on the adjacent fiber becomes more spatially distributed and fiber breaks tend to occur further away from the previous break. A more jagged and tortuous path between fiber breaks would then be expected for composites that are prone to debonding. In CFRPs, more planar fracture surfaces were observed in fatigue of composites with a strong interface (Hahn, 1979). In fatigue of GF/PP composite with a strong and weak interface, the fracture surface has shown straggling appearance for the weaker interface (van den Oever & Peijs, 1997). As presented in the previous chapter, the fatigue micromechanisms were investigated in the same GF/PP material, and abundant debonding occurred in the weak interface material which accounts for the uneven broom like fracture surface. This manifests itself in a more uneven, brush-like failure mode of the composite specimens, whereas composites with stronger interfaces, \( e.g. \) CF/epoxy, display a more planar fracture surface (see schematic picture in Figure 3.32). This pattern seem to hold also for certain MMCs. Naik et al. (1991) observed distributed non-planar fiber breaks for titanium matrix composites with weak interfaces, whereas those with stronger interfaces showed localized and planar fiber breakage.

Furthermore, since debonding is a progressive and time dependent mechanism, its probability to grow to larger extents will increase for longer fatigue lives at lower load amplitudes. This concords with the observations of CF/epoxy presented in the previous chapter, which showed that large scale debonding sets on at low amplitudes close to the fatigue limit. In model composites with carbon and glass fiber bundles embedded epoxy matrices, Lorenzo & Hahn (1986) observed isolated fiber breaks followed by debonding at low and intermediate amplitudes, whereas planar fiber breakage was more prevalent at high load amplitudes. Incipient debonding and subsequent longitudinal macroscopic splitting has been found to be key mechanisms at low amplitudes, and planar fiber breakage at high amplitudes in GF/epoxy spec-
Figure 4.6. Mean distance to secondary fiber break with respect to debond length.

imens (Barnard et al., 1985). After final failure in fatigue, CFRP specimens have shown a more uneven and straggling fracture surface for low amplitude loading, and more planar cracks for higher amplitudes (Charewicz & Daniel, 1986).

Figure 4.6 also shows that larger fiber strength variability, i.e. smaller values of \( \beta \), leads to longer mean distance between fiber breaks. With a wider distribution in strength, the composite becomes less notch sensitive, i.e. less susceptible to failure at local stress concentrations. Scanning electron microscopy of fracture surfaces of unidirectional CFRPs indicates that scatter in fiber strength together with debond propagation are responsible for a more winding and distributed crack propagation (Theocaris & Stassinakis, 1981). In the extreme case with perfectly brittle fibers where the fibers have a uniform constant strength \( \beta \to \infty \), the fibers will always fail in the same plane as the original fiber break, if it would fail at all. This results in a planar fracture surface. The influence of irreversible growth of debonds would not result in any fatigue degradation, rather than link-up of already existing fiber breaks. In contrast, a large variability in fiber strength results in longer average distances between fiber breaks, which means a more uneven
and zigzagging link-up between breaks. This is the same feature that arises for debonding, and at low fatigue amplitudes. Schematic pictures of this relation is presented in Figure 4.7. How damage propagates from a notch or a flaw depends on the stochastic failure of fibers, which in turn depends on the nature of the load distribution caused by growing debonds. Figure 4.7a illustrates distributed fiber breakage linked by propagating debonds. This scenario is prevalent for composites that are prone to debonding, have wide distributions in fiber strength, and are subjected to high cycle fatigue. The other scenario is the planar brittle type of fracture in Figure 4.7b, which prevails in materials that are resistant to debond growth, especially for low cycle fatigue. A low variability in fiber strength also favors this type of behavior.

The variability in the applied stress to cause failure in the adjacent fiber will vary depending on the stress profile and the distribution of fiber strength. A measure of the variability in applied stress that has resulted in fiber failure is the standard deviation, which is the square root of the variance, $s^2 =$
The standard deviation in fiber strength is given by

$$s = \left( \int_0^{\xi_0} \left[ \frac{\sigma_e(\alpha, \xi)}{\Sigma_0(\beta)} \right]^\beta \, d\xi \right)^{1/\beta},$$

and is plotted in Figure 4.8. Naturally, the standard deviation of failure of the adjacent fiber is larger for fibers with larger scatter in strength, irrespective of the applied stress profile. More noteworthy is the trend that the variation in fiber strength decreases as the debonds grow. This is because $\Sigma_e(\alpha, \xi)$ becomes more distributed along $\xi$ for larger $\alpha$, which can be seen in Figure 4.3. The overstress becomes more evenly shared along the fiber, and thus turns the event of fiber failure less susceptible to local variations in fiber strength.

Returning to Figure 4.7a where the more widespread state of damage with ample stress soothing debonds result in a more homogeneous stress field, further damage propagation through fiber breakage would be more predictable with less scatter. In Figure 4.7b, the damage is more localized with higher local stress concentrations. This implies larger variations in fiber breakage and crack propagation. This trend has been observed by Beyerlein & Phoenix (1997b) in simulation of fracture of a one-dimensional composite, where large notch lengths with more overstressed fibers resulted in a relatively lower scatter in composite strength. Fernando et al. (1988) measured less scatter in fatigue life for carbon/aramid fiber composite hybrids compared to the neat CFRPs. The incorporation of the low modulus aramid fibers with high extensibility provides a mechanism of deflecting and arresting cracks, which leads to more damage tolerant material able to accumulate more distributed fiber breaks (Peijs & de Kok, 1993), and hence a more uniform stress state. As presented in the previous chapter, CF/PEEK showed less scatter in strength as well as in fatigue life than did CF/epoxy. In CF/PEEK, the fiber breaks were distributed by debonding, whereas the fiber breaks and bridged cracks were highly localized for the CF/epoxy material. Kim & Hartness (1984) have made corroboratory observations with more fiber breaks in CF/PEEK compared to CF/epoxy in fatigue. Chan & Davidson (1989) studied fatigue damage accumulation from a notch in MMCs, and observed fatigue crack paths as shown in Figure 4.7b for the as received composite, and crack paths as in Figure 4.7a for the heat treated composite with oxidized and degraded interface which led to enhanced debonding.

The distribution of damage by debonding makes the debond propagation more deterministic and reduces the scatter in strength and lifetime. An
analogy can be made by considering a brittle ceramic where failure is controlled by the largest flaw, and therefore shows a large scatter in strength. In contrast to this, a ductile polymer material undergoes global yielding under deformation, and the yield stress shows very little variation from one test sample to another. Local load sharing or localization of damage in general leads to wide scatter in strength and fatigue life, but global load sharing or distributed damage accumulation results in a narrow scatter in strength and life.

In this context, a qualitative reasoning indicates that there is an optimum in static and fatigue properties with respect to interfacial debonding and scatter in fiber strength. Firstly, with a high rate of debond propagation, the stress transfer between fibers becomes inefficient. In the extreme case the composite will act as a loose bundle with no stress transfer. Fatigue sensitivity is the disadvantage for rapid debond propagation. With no or little debonding, the composite is less sensitive to fatigue (cf. CF/epoxy and GF/MA-PP in the previous chapter). However, it is notch-sensitive

Figure 4.8. Standard deviation of strength with respect to debond length.
with tradeoffs in static strength due to its brittleness. Secondly, a wide distribution of fiber strength allows spatially distributed fiber breaks in the composite which renders it more damage tolerant. However, the coalescence by debonding or yielding and subsequent failure are prominent in fatigue. The deterministic case with no distribution in fiber strength would result in planar crack growth as previously discussed. This has a deteriorating effect on the static properties, since stronger fiber segments may not be present at the crack tip to alter the crack path. Wagner & Steenbakkers (1989) investigated damage propagation in composite monolayers and found aligned planar fiber breaks with aramid fibers which possess a relatively narrow distribution in strength, whereas the E-glass composite with its wide strength distribution showed multiply distributed fiber breaks. It should be appreciated that fiber breakage and debonding are mutually influencing mechanisms, and can not be separated as their synergistic growth leads to final failure.

Finally, the coefficient of variation, i.e. the standard deviation divided by the mean fiber strength, remains the same as for a uniformly loaded fiber, viz.

\[
\frac{s}{\langle \Sigma_f \rangle} = \sqrt{\frac{\Gamma \left( 1 + \frac{2}{\beta} \right)}{\Gamma^2 \left( 1 + \frac{1}{\beta} \right)}} - 1. \tag{4.28}
\]

An important feature of the coefficient of variation is that it is independent of the tested length and of the non-uniform stress profile. The shape parameter, \( \beta \), is the only dependency.

**Synopsis on debonding and fiber failure**

There is ample experimental evidence that debonding is an active fatigue damage mechanism in polymer matrix composites. It is subcritical in the sense that it does not directly result in final failure. Fibers must be broken to cause ultimate separation. However, the growing debonds redistribute the load in the neighboring fibers, and eventually a weak segment in one of these fibers may fail when it is overloaded. Subsequently, further debonding and fiber failures can ensue, until ultimate failure occurs. A parametric study was undertaken to investigate debond propagation and its influence on stochastic fiber breakage. A shear lag model has been adapted for a single broken fiber with propagating debonds. This constitutes a simple case to illustrate a fundamental type of damage propagation in fatigue of composite with weak interfaces. The shear lag model was normalized to yield dimensionless results
independent of material properties. The inelastic response from the zero shear stresses along the debonds were compensated by applied shear couples. From damage influence functions (Beyerlein & Phoenix, 1996a), the stress in the fiber adjacent to the broken fiber was calculated with respect to the length of the debonds. A peak stress concentration factor propagates along with the debond at its tip, which results in a monotonically increasing probability of fiber failure. Also, debonding makes uneven and jagged crack propagation more likely, in which case the damage becomes more spatially distributed with a more homogeneous stress field. A large variability in fiber strength has basically the same break distributing effect. With a more homogeneous stress distribution, the variability in strength decreases, which can explain the smaller scatter in fatigue life of composites that show a more homogeneous distribution of damage.

4.3 Fiber bridged cracking

Fiber bridging in composites

In CMCs, fiber bridging is a common feature (cf. Warren, 1992; Sørensen & Talreja, 1993; Liu et al., 1997). Since the matrix is brittle, and has a lower strain to failure than the fibers, fiber bridged cracks will readily appear upon static loading in the fiber direction. Since both the matrix and the fibers have a linear elastic behavior, and the fiber bridged cracks are large and generally bridged by many fibers, CMCs are well suited for modeling activities. CMC was the material in mind when the first comprehensive models for bridged crack growth was presented (e.g. Marshall et al., 1985; McCartney, 1987). The bridged crack model can be coupled with a microstructural model for debonding and frictional sliding of the bridging fibers to provide a link from the microscopic features to the macroscopic mechanical behavior (Budiansky et al., 1986; Budiansky & Amazigo, 1989). For an extensive overview of modeling of bridged cracking, see Cox & Marshall (1994). Microstructural parameters can be linked directly to macroscopic cracking with a minimum of empirical fitting parameters. Well-defined physical models have unfortunately primarily been conceived for relatively simple material systems with linear elastic and brittle constituents, and with a regular geometry. In volume, polymer matrix composites are by far more widely used, but they show a more complex fracture process. An adequate comprehensive physical fracture model encompassing irreversible damage accumulation on the microlevel and ensuing macroscopic behavior may not be feasible and useful for PMCs for design purposes. Firstly the microscopic mechanisms have to be identi-
fied. Secondly, the propagation of the fatigue damage has to be quantified, which makes great demands on the precision of non-destructive measurement techniques on a microscopic level. Modeling difficulties may arise if several interacting mechanisms are simultaneously active. For engineering design purposes, an empirical and more straightforward description lies close at hand. In spite of these adversities, sensible modeling of the operative micromechanisms can yield important information for microstructural tailoring for optimal end use properties. Even though the ultimate objective of accurate fatigue life prediction of PMCs may not be fulfilled by micromechanical modeling, the task should ardently be endeavored.

In MMCs, fiber bridged cracks appear in fatigue (Bakuckas & Johnson, 1992; Majumdar & Newaz, 1995; Zheng & Ghonem, 1995) and can be measured to plot crack growth curves. In MMCs, the bridged cracking can be interpreted as the progressive mechanism that dominates in the sloping part in a plot of fatigue life data (Talreja, 1995). Modeling of fiber bridged cracking in fatigue has mainly been developed for through-thickness cracks in MMCs (McMeeking & Evans, 1990; Bao & McMeeking, 1994).

Polymer matrix composites with randomly-oriented short fibers, such as glass mat thermoplastics (GMT) often show extensive fiber bridging in static loading. In particular, this is the case when the matrix is ductile, and the interface is weak, e.g. for glass fiber/polypropylene (Karger-Kocsis et al., 1995; Lindhagen & Berglund, 1997a). Around the crack tip there is a large damage zone with debonded fibers, and yielding matrix. The bridging fibers in the crack wake have debonded, and are being pulled out from the matrix. Physical modeling of crack propagation is not directly amenable here, since the fracture process is very complex, including inclined bridging fibers, pull-out, viscoelastic deformation of the matrix in a large heterogeneous damage zone. Empirical or semi-empirical approaches are more widely used in conceiving failure criteria (e.g. the flaw model, damage zone criterion, point stress and average stress criteria). However, concepts from cohesive zone models based on fiber bridging can be useful also for these materials (e.g. Lindhagen & Berglund, 1997b).

In PMCs with continuous aligned and closely packed fibers, observations of fiber bridged cracking is not as common. The polymer matrix has generally a higher strain to failure than the reinforcing fibers, and bridging would therefore not be occurring under static conditions. The glass and carbon fibers themselves are quite insensitive to fatigue in non-corrosive environments, whereas cracks can propagate in the polymer matrix even at low fatigue amplitudes. If these matrix cracks can propagate in mode I around fibers, the cracks will become bridged by these fibers. The effectiveness of the bridging as a toughening mechanism is governed by the amount of slipping
4.3. FIBER BRIDGED CRACKING

of the bridging fibers to the surrounding matrix.

Fiber bridged cracking is harder to capture experimentally in CFRPs than in many other composites with polymer matrices. Unidirectional composites with high volume fractions of fibers are brittle materials, that require a very small number of broken contiguous fibers to result in final failure even if only moderate loads are being applied (Jamison, 1982; Bader, 1988; Dharani et al., 1992). This makes it cumbersome to locate and monitor the growth of a fatigue crack. Bridging occurs in unidirectional polymer matrix composites, and is an important toughening mechanism. In aramid fiber/epoxy laminated with aluminum plies, large zones of fiber bridging have been observed (Ritchie et al., 1989; Wilson & Wilson, 1991). Bridged cracking has also been observed in carbon fiber/polyetherimide (Luke et al., 1996) and in glass fiber/epoxy model composites (Botsis et al., 1995). Bridging of ridged bonded glass fibers in epoxy and vinyl ester resins has been detected by Shih & Ebert (1987). Baron (1992) has observed fiber bridged cracks in the $0^\circ$ plies initiated by transverse cracks in the $90^\circ$ ply in carbon fiber/epoxy cross-ply laminates.

One important parameter in the description of fiber bridged cracking is the bridging or surface traction law. It relates the closing traction exerted by the bridging fibers on the crack surfaces, $p$, to the crack opening displacement (COD) in the z-direction, $w$. The bridged zone, $r_0 \leq x \leq r$, will result in a partial closure of the crack with a squeezed COD profile as depicted in Figure 4.9. The best way to estimate the bridging law would be to directly measure it, but in certain cases this might not be feasible, and one must rely on an indirect method. Direct methods can be the destructive pull-out

Figure 4.9. Crack opening profile with a cohesive zone described by the bridging law $p(w)$. 
test of bridging fibers, or the non-destructive Raman spectroscopy method for non-amorphous fibers (see e.g. Galiotis, 1991; Andrews & Young, 1993), where the local stress in the bridging fibers may be measured during loading. Bennett & Young (1997) have measured the local strain in bridging aramid fibers arranged in a monolayer in an epoxy matrix. Indirect methods rely on the measurement of another physical property which can be used to estimate bridging stress. For large cracks with large bridging zones, crack growth results in an increase in global tensile compliance. This can be formulated in terms of the $J$-integral for certain geometries. The bridging law is expressed as $p(w) = \frac{\partial J}{\partial w_0}$, where $w_0$ is the COD at the original crack tip, and $J$ is determined from changes in compliance (Li et al., 1994). For unidirectional PMCs, the changes in compliance during crack growth is hardly measurable since the bridged cracks are generally small and scarce compared to the specimen volume. An alternative approach to the $J$-based technique has been presented by Cox & Marshall (1991a), where the bridging law can be estimated from the COD profile and the global stress. This method requires the weight or Green's function of the crack geometry, and is sensitive to noise in the measured data. In this work, a variant of the Cox-Marshall method will be developed and employed. For a crack geometry where the weight function is not given in a closed form, the COD and surface traction for a reference case of a crack with the same dimensions can be used to determine the bridging law.

If the nature of the bridging phenomenon were known, the shape of the bridging law could be anticipated, which would reduce the number of parameters, and hence increase the efficiency of the estimation from a mathematical viewpoint. Different shapes of the bridging laws stem from different deformation mechanisms as schematically drawn in Figure 4.10. Here, $p$ and $w$ designate indices related to the bridging traction and the COD, respectively.

If a crack has grown in high-cycle fatigue, the bridging tractions probably evolve slowly and barely change from cycle to cycle with only a small hysteresis (reversible loading and unloading). Fatigue shake-down reduces the hysteresis losses (Suresh, 1991), and the traction-displacement relation will eventually take a linear Hookean form as sketched in Figure 4.10a.

If fibers are broken in the matrix close to the crack they will be pulled out as the crack opens, which results in a decreasing bridging law. The further the fiber has been pulled out, the lower is the stress in the bridging fiber. This behavior is shown in Figure 4.10b, where the pull-out can be caused by interfacial debonding followed by frictional sliding, or by shear yielding of the matrix material close to the fiber, depending on the interfacial strength.

For an unbridged crack in a ductile material, Figure 4.10c, a plastic zone forms at the crack tip. According to the Dugdale model, this scenario
4.3. Fiber Bridged Cracking

(a) Linear
\[ p = kw \]
linear springs, reversible loading

(b) Softening
\[ p = k(w_0 - w) \]
pull-out of broken fibers from matrix

(c) Dugdale
\[ p = p_0 \]
plastic zone at crack tip

(d) Sliding or yielding
\[ p = k \sqrt{w} \]
sliding or yielding with constant shear stress

(e) Average behavior
\[ p(w) \]
several of above mechanisms

Figure 4.10. Bridging laws relating the surface traction to the crack opening with concomitant mechanisms.
corresponds to a virtual crack which is larger than the original crack by the length of the plastic zone. At the tip of this virtual crack, the surface traction is constant and equals the yield stress.

Under static loading conditions with unbroken fibers in Figure 4.10d, \( p \) is proportional to \( \sqrt{w} \) if the interfacial shear stress is constant. This can be assumed to be the condition when frictional sliding or shear yielding takes place. The proportionality coefficient, \( k \), can be related directly to material properties of the constituents and the interface (e.g. Cox & Marshall, 1991b).

If the above mechanisms are all present in one crack and interact, the bridging law takes a non-monotonic curve in average. This is schematically shown in Figure 4.10e. Also, if there are many bridging fibers and they have a distribution in strength, the bridging law has the same type of appearance (Thouless & Evans, 1988). Olsson & Giannakopoulos (1993) have investigated the \( R \)-curve behavior for a number of the bridging laws outlined in Figure 4.10.

**Surface crack propagation**

In fatigue, semi-elliptical surface cracks are a common form of damage (see Figure 4.11). They initiate from a surface defect and propagate inwards into the material and along the free surface at the same time. Surface cracks are quite often responsible for final failure of composites in fatigue (Majumdar & Newaz, 1995; Soboyejo et al., 1995). There are more defects on the surface of a structure compared to the interior due to inevitable damage caused during manufacturing and handling. The maximum stress-intensity factor for a semi-circular (half penny shaped) surface cracks is \( \sim 30\% \) larger than that of a fully embedded circular (penny shaped) crack for equal crack surface area (e.g. Ewalds & Wanhill, 1984). This is another reason why surface flaws are more critical to failure than internal flaws. The stress intensity factor for a quarter-circular edge crack is larger than that of a semi-circular surface crack for equal crack surface areas, and the edge crack is therefore more likely to cause ultimate failure. However, a considerable part of failures are initiated from semi-elliptical cracks for unnotched specimens, which has been observed for short-fiber reinforced polyamide (Casado et al., 1997). For test coupons of unidirectional longitudinal composites, edge flaws usually cause splits with sprawling segments, and the observable fatigue cracks tend to initiate and grow on the surface rather than at the edge (Gamstedt & Talreja, 1997).

For semi-elliptical cracks in a PMC, it is usually difficult to determine the shape of the crack prior to failure. The length of the crack, \( 2a \), is readily measured directly on the surface, but to find its depth, \( b \), an experimental technique is required which can analyze the interior of the material. X-
Figure 4.11. Semi-elliptical surface crack in a material subjected to tension.
ray tomography may be used with a contrast-enhancing penetrant (Eriksson et al., 1997), but for small cracks (< 100 μm) in a composite with a heterogeneous microstructure, the resolution is currently not accurate enough. Another difficulty is that the crack must be opened in situ to make the crack detectable. Post mortem fractography has not been able reveal any information on the geometry of the semi-elliptical surface cracks for unidirectional CFRPs (Gamstedt & Talreja, 1997), since the fracture surface is very uneven and the precrack is very small compared to the dimensions of the cross section. The stored strain energy in CFRPs is large because of the high longitudinal stiffness. When failure occurs, the stored elastic energy is abruptly released, which results in a dynamic disruptive movement that causes the brittle composite to fracture in different planes, and multiple fragments are detached from the coupon. This makes the identification of the fatigue precrack virtually impossible. Therefore, the shape of the semi-elliptic crack has either (i) to be calculated provided the crack growth rate equation is known, or (ii) to be estimated based on sound physical reasoning and experimental data.

If the shape is to be determined from a growth rate equation, an assumption of the dimensions of the initial defect that causes the fatigue crack has to be made. Above a certain threshold value, the rate equation of stable fatigue crack propagation can be formulated, and is e.g. known as the Paris law,

$$\frac{da}{dN} = C (\Delta K)^m,$$

where \(N\) is the number of elapsed cycles, \(\Delta K\) is the range of the stress intensity factor, and \(C\) and \(m\) are parameters that characterize the fatigue crack growth rate. The crack may have a curved front such as the semi-elliptical surface crack, where \(\Delta K\) is dependent on the position along the front, and \(da\) represents an increment in crack length in the direction normal to the crack front. Only mode I cracking is considered.

By integration of Equation (4.29) with the unknown dimensions of the incipient flaw as starting limit, the shape of the fatigue crack can be estimated. Paris law is an empirical relation, and there are a number of variations of this formulation to describe the growth rate (see e.g. Carlson & Kardomeateas, 1996). It must be ascertained that the chosen rate equation provides a reasonably good description, which can only be done by comparison of the goodness-of-fit with experimental data. For an inaccurate growth law, any initial error will propagate itself, and result in a substantially larger error in the crack dimensions after integration. Furthermore, the stress singularity is of another type at the free surface compared to the interior crack front (Benthem, 1977). The stress state is plane close to the surface, whereas the
strain state is plane in the interior for thick samples. The stress state influences the mode of cracking, and therefore also the evolution of crack shape during fatigue.

Fatigue cracks usually initiate at a surface, and in composites it is likely that they initiate from one single fiber break at the surface (Soboyejo et al., 1995), forming a crack that can be regarded as semi-circular. For a semi-circular surface crack where \( b \ll t \), the stress intensity factor at the crack tip intersection with the free surface (points A and A') is \( \sim 10\% \) larger than that of the innermost point B of the crack front where the propagation direction is normal to the free surface (e.g. Ewalds & Wanhill, 1984). From this, one would anticipate that the crack grows faster along the surface, \( i.e. \frac{da}{dN} > \frac{db}{dN} \), and that the crack takes a prolate elongated shape with \( a > b \) at an initially accelerating rate. In practice semi-elliptical surface cracks with thick ligaments tend to grow with a constant aspect ratio, \( a/b \), ranging from 1.0 to 1.2 (Newman & Raju, 1981; Chermahini, 1993; Casado et al., 1997). For a composite material, the heterogeneity of the microstructure will result in a more erratic growth pattern, and the shape of the incipient crack from one or a few fiber breaks is not easily determined. The aforementioned difficulties make it precarious to use a fracture mechanics model to predict the evolution of the shape of a surface crack during fatigue, and such an approach will therefore be refrained from in this study. If the evolution of the crack shape can not adequately be calculated from a rate equation, a shape has to be assumed. Since the small cracks have previously shown to have an almost semi-circular shape, it is henceforth assumed that the surface cracks grow with a constant aspect ratio equal to unity, \( i.e. a(N) \equiv b(N) \).

Fracture mechanics model

A schematic picture of the semi-circular surface crack in a half-space is found in Figure 4.12; (a) in cross section, and (b) from the surface with the COD, \( w_s(x) \). The total displacement between opposite points on the two fracture symmetric surfaces is \( 2w_s(x) \). The far-field applied stress, \( \sigma_0 \), is applied in the \( z \)-direction normal to the crack plane. On part of the crack surface a cohesive traction is applied, and the crack opening profile takes a deformed shape. In polar coordinates \((\rho, \phi)\), the total crack surface, \( S \), and the bridged crack surface, \( S' \), are

\[
S = \left\{ (\rho, \phi); \ 0 \leq \rho \leq r, \ 0 \leq \phi \leq \frac{\pi}{2} \right\}, \quad (4.30)
\]

and

\[
S' = \left\{ (\rho, \phi); \ r_0 \leq \rho \leq r, \ 0 \leq \phi \leq \frac{\pi}{2} \right\}. \quad (4.31)
\]
For simplicity, only a quarter of a circle \((0 \leq \phi \leq \pi/2)\) is considered since the y-z plane forms a plane of symmetry. The quarter crack \(S\) is bounded by the crack front contour \(\Gamma\). Since there are no data on the interior COD of the partially bridged crack, it is assumed that the crack opening is rotational symmetric around the z-axis in the half plane, i.e.

\[
w(\rho, \phi) = w_s(\rho),
\]

where \(w\) is the COD at any point in \(S\). Since it is assumed that the crack propagates self-similarly, the symmetry of the crack opening is the first that comes into mind. However, this assumption is not fully consistent with a uniform propagation rate, because the stress intensity factor is slightly larger at the crack surface \((\phi = 0 \text{ or } \pi)\) than at the innermost point of the crack front \((\phi = \pi/2)\). Any error stemming from this inadvertency is believed to be small compared to the imprecision in measurement data.

The first step in the calculation of the stress intensity factor of the bridged crack is to obtain the bridging law, \(p(w)\). This can be achieved by use of the Betti-Rayleigh reciprocal theorem and the superposition principle. A relation between two sets of surface pressures and accompanying displacements fields, \((\sigma, w)\) and \((\sigma_r, w_r)\), can be formulated as

\[
\iint_S \sigma_r w \, dS = \iint_S \sigma w_r \, dS,
\]

where the surface differential is \(dS = \rho \, d\rho \, d\phi\) for a polar coordinate system. The known reference stress field is chosen to be the elementary case of constant stress, \(\sigma_r(\rho, \phi) = \sigma_0\), and the concomitant crack opening is \(w_r(\rho, \phi)\). If the global stress \(\sigma_0\) is applied to the body with a bridged crack, a crack opening profile \(w_s\) can be measured, from which \(w(\rho, \phi)\) is determined in Equation (4.32). As discussed earlier, a linear bridging law can be surmised in the fatigue case of a bridging fiber. With the constitutive relationship in Figure 4.10 a, the underlying crack surface pressure is

\[
\sigma(\rho, \phi) = \begin{cases} 
\sigma_0 - k w(\rho, \phi) & \text{in the bridged region}, \\
\sigma_0 & \text{in the unbridged region}.
\end{cases}
\]

It is assumed that the bridging traction is smoothly distributed along the crack surfaces. For few bridging fibers, it is more correct to assume a discrete spatial distribution of surface traction located at the positions of the fibers themselves. Östlund (1995) showed that a continuum traction zone can be an injudicious assumption for a fiber bridged composite. Discretely spatially distributed tractions zones give more accurate values of the effective stress intensity factor. Nevertheless, the continuum approach will be advocated in
Figure 4.12. Rotational symmetric semi-circular surface crack viewed (a) in cross section, and (b) from surface.
the present study since the exact positions of the bridging fibers could not be directly discerned, and the scatter in measured data does not justify detailed model refinements.

It should be noted in Equation (4.34) that $\sigma(\rho, \phi)$ is rotational symmetric since $w(\rho, \phi)$ is rotational symmetric in the considered half plane. However, the presence of the free surface implies an unsymmetric stress distribution field. This incompatibility is disregarded in the present investigation.

The constant $k$ is the sought spring constant, and can be determined from Equations (4.33) and (4.34) as

$$k = \frac{\sigma_0 \int_S (w_r - w) \, dS}{\int_S w w_r \, dS},$$

where $\| \cdot \|_S$ and $\langle \cdot, \cdot \rangle_S$ denote a norm and a scalar product over the surface $S$ for all positive displacement fields, respectively. The norm is the volume comprised by the opened crack appurtenant to the norm’s argument.

The expression of the spring constant $k$ has a physical interpretation by considering an unbridged and bridged crack with the same applied stresses $\sigma_0$. The reference stress, $\sigma_r$, and the corresponding displacement field, $w_r$, are given and fixed. If the cohesive action is strong, the measured volume of the opened crack will differ significantly from the reference displacement, i.e. the numerator $\|w_r - w\|_S$ is large, and then the spring stiffness $k$ will be large. Furthermore, if the COD of the bridged crack is similar to that of the unbridged crack in region $S'$, the denominator $\langle w, w_r \rangle_{S'}$ will be large, and $k$ will be small.

For a bridged crack to be able to open, there has to be some slippage of the bridging fibers relative to the matrix close to the crack surfaces. Two plausible slip mechanisms are (i) debonding and frictional sliding, and (ii) shear yielding of the matrix material close to the fiber. The slip will increase the COD, and the induced damage will decrease the effective longitudinal Young’s modulus in the zones close to the crack surface. The Betti-Rayleigh reciprocal theorem requires equal material properties for the two loading cases, which is not the case after slip damage. Nonetheless, it is here assumed that the bridging traction acts directly on the crack surfaces, and that there is no slip damage close to the bridging fibers.

In opening of the unbridged reference crack, the material is assumed to be perfectly elastic, and an approximate analytical expression of its displacement field has been calculated by Fett (1988) for a semi-elliptical crack. Fett’s expressions will be used for $w_r$ in the present model. The calculations are based on the stress intensity factor solutions of Newman & Raju (1981). The
4.3. FIBER BRIDGED CRACKING

displacement field for the reference load, \( \sigma_r \), can be expressed as the series

\[ w_r(\rho, \phi) = \sum_{i=0}^{2} C_i(\phi) \left( 1 - \frac{\rho^2}{r^2} \right)^{i+1/2}, \]  

(4.36)

where \( C_i(\phi) \) is a set of functions determined from a number of physical boundary conditions. These functions are deduced from \( K_r \) which vary along the crack front:

\[ K_r(r, \phi) = \sigma_0 Y_r(\phi) \sqrt{r}, \]  

(4.37)

where \( Y_r \) is a geometric factor. For a semi-circular surface crack in a semi-infinite body \( Y_r = 1.17 \). The variation along the crack front \( \Gamma \) is described by

\[ \omega(\phi) = 1 + 0.1(1 - \sin \phi)^2. \]  

(4.38)

The functions \( C_i(\phi) \) are expressed as linear combinations of \( \omega(\phi) \). The accuracy of the displacement field in Equation (4.36) has been validated in the determination of energy release rates and comparison with reference data (Fett, 1988).

The next step is to determine the effective stress intensity factor at the surface crack tip where the effect of bridging is taken into account. The energy stored in the opened bridged crack can by calculated by integration of the work done by the surface pressure distribution over the displacement fields:

\[ U(r) = \int \int_S \sigma w \, dS, \]  

(4.39)

where \( \sigma \) and \( w \) depend on the position in \( S \). With the bridging law in Equation (4.34), the stored energy can be rewritten as

\[ U(r) = \sigma_0 \int_S w \, dS - k \int_{S'} w^2 \, dS = \sigma_0 ||w||_S - k\langle w, w \rangle_{S'}. \]  

(4.40)

If all the energy expended during crack growth is stored in the body due to the opening of the bridged crack, a general relation can be expressed between the energy release rate, the stress intensity factor, and the derivative of the stored energy \( U \) with respect to an arbitrary crack surface increment \( A \). Since the growth of the semi-circular crack is self-similar, the increment \( A \) will be in the radial direction, with an energy release rate \( G_A \) for an infinitesimal increment. At all points along the crack front, the direction of propagation will be perpendicular to the tangent of the crack front contour \( \Gamma \), and the driving force can be expressed in the associated stress intensity factor, \( K_A \). A state of plane stress can be assumed at the surface, but at the point of maximum depth (\( \phi = \pi/2 \)) plane stress cannot be assumed due to the constraints that suppress Poisson contraction at the crack front. For the
considered crack configuration and propagation mode, the energy release rate can be expressed as

\[ G_A = \frac{K_A^2}{\beta E} = \frac{\partial U}{\partial A}, \tag{4.41} \]

where \( E \) is the effective orthotropic Young’s modulus, and \( \beta \) is a factor that depends on the variation from plane stress to triaxial stress along \( \Gamma \). An integral formulation of Equation (4.41) for a three-dimensional crack with an arbitrary front was first presented by Rice (1972). The expression of \( K_A \) can be obtained, and can be interpreted physically. From Equation (4.40), it turns out that \( K_A \) is large if the volume comprised by the opened crack increases rapidly as the crack grows. On the other hand, if the bridged zone volume in the opened crack increases rapidly during crack growth, so will \((w,w)_s'\), which means that the increase of \( K_A \) is moderated. In other words, the bridging effect restrains the stress intensity.

With a self-consistent formulation of Equation (4.33), a relation between the energy release rate and the stored energy can be rewritten for the semicircular crack configuration as

\[ \iint_S G \, dS = \int_{r_0}^r \left\{ \int_{\Gamma} \frac{K^2(r', \phi)}{\beta(\phi) E} \, d\Gamma \right\} \, dr' = U(r), \tag{4.42} \]

where \( K \) is the stress intensity factor, and \( r_0 \) is the initial crack radius which corresponds to the unbridged region. If no bridging fibers are broken during fatigue before ultimate failure, the transition from the unbridged to the bridged part of the crack will remain at a constant radius \( r_0 \). The effective Young’s modulus \( E \) is calculated from the orthotropic elastic constants listed in Table 1, by use of the following relation derived by Lekhnitskii (1963):

\[ E = 2E_L \left\{ 2 \left( \sqrt{\frac{E_L}{E_T}} - \nu \right) + \frac{E_L}{G_T} \right\}^{-\frac{1}{2}}. \tag{4.43} \]

The function \( \beta(\phi) \) represents the change in stress state along the crack front. Based on thorough investigations, Wang (1995) proposed the empirical expression

\[ \beta(\phi) = \frac{1 - \nu^2(1 - \sin \phi)^4}{1 - \nu^2}, \tag{4.44} \]

where \( \nu \) is the major Poisson’s ratio. This formulation meets the limit stress state conditions of plane stress at \( \phi = 0 \) and \( \phi = \pi \), and plane strain at \( \phi = \pi/2 \).
4.3. FIBER BRIDGED CRACKING

To determine the stress intensity factor at the surface, \( K(r, \phi = 0) \), the above integral equation can be simplified to

\[
\int_0^{\pi/2} \frac{K^2(r, \phi)}{\beta(\phi)} r \, d\phi = E \frac{\partial U}{\partial r},
\]

(4.45)

where \( U(r) \) is calculated from cubic spline interpolation of the discrete experimental results.

The variation of \( K(r, \phi) \) along \( \Gamma \) is unknown for the bridged crack. Since the crack is semi-circular and the surface pressure is rotational symmetric, it can be assumed that the variation of \( K(r, \phi) \) along \( \Gamma \) is proportional to the variation of \( K_T(r, \phi) \) along \( \Gamma \) for the unbridged crack in Equation (4.37). This assumption was made by Wang (1993) for a semi-elliptical crack subjected to an elementary polynomial load distribution, and rendered good correspondence with the stress intensity factor calculated with a finite element method. Since the crack is small compared to the dimensions of the body, an ansatz of a separable function can be made for the fix applied stress \( \sigma_0 \) where the variation along the crack front contour is described by \( \omega(\phi) \) in the same form as in Equation (4.37):

\[
K(r, \phi) = \sigma_0 Y \omega(\phi) \sqrt{r},
\]

(4.46)

where \( Y \) is a geometric factor which is also dependent on the bridging traction. Substitution into Equation (4.45) results in

\[
Y = \frac{1}{\sigma_0 r \Omega} \sqrt{\frac{2E}{\pi} \frac{\partial U}{\partial r}},
\]

(4.47)

where \( \Omega \) is the root-mean-square value of the function \( \omega(\phi) \) weighted by the stress state effect along the crack front contour \( \Gamma \):

\[
\Omega = \sqrt{\frac{2}{\pi} \int_0^{\pi/2} \omega^2(\phi) \frac{1}{\beta(\phi)} \, d\phi}.
\]

(4.48)

When \( Y \) is calculated, the effective stress intensity factor, \( K_{\text{eff}} \), can be determined in Equation (4.46).

The chosen point of reference for comparisons of growth rates is point A in Figure 4.11, since it is the distance between points A and A' that has been experimentally measured. The interior shape of the crack has not been measured and hence postulated to be semi-circular, which means that a selection of e.g. the maximum depth point B as a point of reference would be more doubtful. However, it should be borne in mind that the stress field
is not a square root singularity at a perpendicular intersection of the crack front and a free surface (Benthem, 1977). When the crack front meets the free surface at a certain oblique angle, the square root singularity is recovered which gives rise to the largest energy release rate, and hence the most stable crack configuration (Hills et al., 1996). Despite the idiosyncrasy of the surface stress state, it is assumed that the crack front meets the free surface perpendicularly, and the stress intensity factor is used in its conventional form in the growth rate equation.

In tension-tension fatigue, the crack grows with a certain rate, \( \frac{da}{dN} \), which depends on the range of the effective stress intensity factor, \( \Delta K_{\text{eff}} \). Since no COD measurements were made on the minimum stress level of the fatigue cycle period, it is assumed that the \( K(r, \phi) \) is proportional to the applied stress, \( \sigma_0 \). The stress ratio is \( R = \sigma_{\text{min}} / \sigma_{\text{max}} \), and the stress intensity factor range at the crack front vertex (point A) is

\[
\Delta K_{\text{eff}} = (1 - R) K(a, 0),
\]

which can be determined explicitly from the solutions of Equations (4.46) and (4.47):

\[
\Delta K_{\text{eff}} = 1.1(1 - R) \sigma_0 Y \sqrt{a}.
\]

In order to compare the influence of bridging to fatigue crack growth, a rate relation can be determined for the crack where the bridging has not been taken into consideration. In this case, stress intensity formulae from the literature (Newman & Raju, 1981), cf. Equation (4.37), can be used to calculate the nominal stress intensity factor at the crack front vertex of an unbridged crack, viz.

\[
\Delta K_{\text{nom}} = (1 - R) K_r(a, 0),
\]

which can be rewritten explicitly as

\[
\Delta K_{\text{nom}} = 1.29(1 - R) \sigma_0 \sqrt{a}.
\]

With a plot of the rate curve, with \( \frac{da}{dN} \) versus \( \Delta K_{\text{eff}} \) and \( \Delta K_{\text{nom}} \), respectively, the bridging effect can be discriminated. A comparison of the rate curve for the matrix material would assess the influence of bridging as a toughening mechanism. The fatigue crack growth data of the neat matrix material needs to be converted to apply for the situation in a composite material. This can be done by energetic considerations of the fracture process of a matrix crack (McCartney, 1987), which leads to

\[
\Delta K_m = \sqrt{\frac{V_m E}{E_m}} \Delta K'_m,
\]
where $K_m$ and $K'_m$ are the stress intensity factors in the composite and in the neat matrix material, respectively. The volume fraction of matrix is $V_m$, and the Young's modulus of the matrix material is $E_m$. In this manner, data of crack propagation of the matrix material can be re-calculated for simple matrix cracking in the composite material, and subsequently compared with experimental data to estimate the influence of bridging on retardation of fatigue crack propagation.

For semi-crystalline polymers, the mechanical properties of the matrix materials differ in the composite compared to their neat form due to morphology differences such as transcrystallinity induced by the presence of fibers. For thermosets cured at high temperatures and thermoplastics with high glass transition temperatures, residual thermal stresses will influence the behavior of the composite matrix. Chemical shrinkage due to cross-linking during cure also gives rise to residual stresses. Since the local behavior of the matrix in a composite is difficult to characterize, it is assumed that the fatigue crack growth rate in the composite matrix is the same as that of the neat matrix material.

It is noteworthy that an elastic fracture mechanics model such as the one that is presented here is not amenable to ductile materials. The risk of obtaining misleading results is in particular large for cracks were the process zone ahead of the crack tip is fairly large compared to the dimensions of the crack. However, the polymer matrix material was an epoxy which is brittle compared to other polymer matrices. Furthermore, the objective is not to conceive an accurate model to describe and predict crack propagation, but rather to evaluate the contribution of bridging as a toughening mechanism in a linear elastic framework of fatigue crack growth in a CFRP.

Interpolation of the COD profile and the elastic energy was performed by cubic splines by the Matlab™ software package (The MathWorks, Inc.). Spline interpolations are known to give smooth and accurate values of first order derivatives, and are not liable to numerical instability with extreme oscillations between the nodes. The analytic calculations were performed with the Maple™ software (Waterloo Maple, Inc.).

**Results and discussion**

The fatigue life diagram of the CFRP material is presented in Figure 3.3. A fatigue life diagram is basically a modified $S-N$ curve plotted with peak initial strain on the ordinate, which has been partitioned into three different regions each pertinent to dissimilar fatigue damage mechanisms (Talreja, 1993). The scatter in static strength is relatively large, which indicates a brittle behavior. As a consequence, the scatter in life in the sloping progressive part of the
fatigue life diagram (Region II) is also large. The slope of this region is comparatively small, which signifies that the material is relatively fatigue resistant. Therefore, the damage growth rate should be small and dependent on the applied load level. Crack retarding mechanisms contribute to a slow damage growth rate. One such crack retarding mechanism could be crack tip shielding by bridging fibers.

In the studied carbon fiber/epoxy composite, no observable cracks were detected in the pristine state. At the first application of load, there was an accumulation of distributed fiber breaks. From these fiber breaks, matrix cracks grew perpendicular to the load and fiber direction. After about 1,000 cycles, the cracks took deformed shapes with squeezed fiber tips (see Figure 3.4). This implies that cohesive tractions are acting on the crack surface striving to close the crack. If there were no cohesive tractions, the crack opening profile would have taken a near elliptical shape. Superficial fiber breaks were clearly visible with apparent displacements with the replication technique, and there were no fiber breaks in the squeezed part close to the crack tip. With a volume fraction of fibers of about ~ 63%, the fibers are bound to be closely juxtaposed. The squeezed formation of the crack opening profile can only be explained by crack bridging fibers leading to the observed cohesive effect.

Bridged cracks with squeezed fiber tips were not observed initially, but became omnipresent and more pronounced during the course of fatigue application. The cracks were scarce and dispersed, and did not interact or coalesce unless they were in the extreme vicinity of one another (cf. Figures 3.7 and 3.8). The few close and colinear cracks that were observed stemmed from surface scratches. The cracks were generally distributed and far apart.

In Figure 4.13, scanning electron images of sections of etched CFRPs subjected to fatigue are found. The bridging 0° fibers are clearly visible. The cracks have initiated in the adjacent 90° ply, and propagated into the 0° ply. Baron (1992) have made similar observations of bridging carbon fibers in epoxy matrices. Bridging in CFRPs is a frequently occurring feature, and the squeezed crack tips by bridging in Figure 3.4 is not an artifact, but an authentic physical mechanism. Some bridging fibers could however be optically discerned in special cases. They originated from surface flaws, usually scratches, and had extremely superficial fiber exposed as ridges on the laminate surface. An example is shown in Figure 3.5 after 1,000 fatigue cycles.

In Figure 4.14 the growth of four individual fiber bridged cracks is plotted with respect to the logarithm of the number of cycles. The linear regression lines are fitted by the least square method of a power function. Since the growths show a near linear relationship in a linear-logarithmic plot, the
Figure 4.13. Crack bridging $0^\circ$ fibers initiated from transverse cracks in the $90^\circ$ ply (courtesy of Dr. M. Luke).
growth rate decreases continuously during fatigue. The reason for this must be a crack retarding mechanism caused by the composite microstructure. In monolithic materials, fatigue cracks are known to obey Paris law in Equation (4.29) and accelerate during fatigue.

The growth rates vary from crack to crack, which can be attributed to the irregularity of the microstructure. The fibers are not evenly spaced in a well-defined geometric order, and the propagating crack will therefore encounter obstacles intermittently with uneven intervals in a trap-release fashion, which makes the propagation somewhat jerky in an accelerating-decelerating manner. Therefore, the fitted power function was used to determine the growth rate, da/dN, for a given crack size, a. Each set of data is pertinent only to its proper crack, and the deduced parameters are not generic. A large number of cracks have to be analyzed if the variability in growth pattern should be taken into account.

In growth rate plots, the stress intensity factor range is generally the property assigned to the x-axis. Conventional use of the stress intensity factor is warranted for a crack normal to the major material axis of a transversely isotropic material. It necessitates the effective orthotropic Young's
4.3. Fiber bridged cracking

Table 4.1. Static mechanical properties of the tested CFRP.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress to failure, $\sigma_u$</td>
<td>1.63 GPa</td>
</tr>
<tr>
<td>Strain to failure, $\varepsilon_u$</td>
<td>1.27%</td>
</tr>
<tr>
<td>Longitudinal Young’s modulus, $E_L$</td>
<td>129 GPa</td>
</tr>
<tr>
<td>Transverse Young’s modulus, $E_T$</td>
<td>9.7 GPa*</td>
</tr>
<tr>
<td>In-plane shear modulus, $G_T$</td>
<td>4.2 GPa*</td>
</tr>
<tr>
<td>Major Poisson’s ratio, $\nu$</td>
<td>0.29</td>
</tr>
</tbody>
</table>

* The unknown values have been selected from similar composite systems.

modulus, $E$, in Equation (4.43). For the elastic properties listed in Table 4.3, $E = 42$ GPa which is considerably less than the measured longitudinal Young’s modulus, $E_L = 129$ GPa.

The crack belonging to the data of the largest crack in Figure 4.14 (hollow triangles) was analyzed in more detail. Firstly, the bridging effect was disregarded, and the nominal crack intensity factor range, $\Delta K_{\text{nom}}$, was calculated and plotted with respect to the corresponding crack growth rate. The results are compared with crack growth of the matrix material in Figure 4.15. The fatigue crack growth data of the epoxy material was taken from Hertzberg & Manson (1980), and recalculated for the composite with Equation (4.53). It is apparent that the unbridged crack shows a marked crack retardation behavior. Even though the stress intensity factor range increases as the crack grows, the propagation rate decreases rapidly. The somewhat naïve usage of the nominal stress intensity of a pretended unbridged crack does not account for any toughening or propagation retarding mechanism.

The crack opening profile was measured for the same crack at five different occasions. As an example, its profile is plotted in Figure 4.16 together with the calculated crack opening profile of an unbridged crack of equal size at its largest extent. The discretized points come from the microscopically measured CODs. The interconnecting lines are cubic spline interpolations, which provide a set of continuous functions to be used in the deduction of the bridging law. At the crack tip, the boundary condition is free, but at the crack center, the derivative is set to zero. In Figure 4.16 the COD is larger for the unbridged hypothetical crack, and the bridged crack is deformed close to the tip due to the cohesive effect. The displacements have been normalized with respect to the maximum COD for the unbridged crack at the crack center ($\rho = 0$). The bridging fibers start at around 13 $\mu$m from the crack center and
act on the fracture surface out to the crack front. The point of inflection of the COD of the bridged crack remained close to the initial crack size, which was taken as the starting point of the bridging zone. The COD values were on the micrometer length scale, and are therefore liable to noise from imprecise measurements. The presented data should not be taken as accurate material parameters, but should be regarded as qualitative measures of the effects of the observed bridging mechanism. The key objective is to separate the crack retardation by bridging from other toughening mechanisms to evaluate its influence on the fatigue resistance of the material.

In Figure 4.17, the stiffness constant of the bridging law is plotted with respect to the number of elapsed cycles. The stiffness constant, $k$, is the ratio between the cohesive surface traction and the local COD, and was calculated from the measured COD profile with Equation (4.35). There is a cycle dependent degradation of the stiffness, which results in an increasingly less efficient closure of the growing bridged zone. This can be attributed to the accumulation of irreversible damage occurring in at the interface and in the matrix material in the vicinity of the fiber. The carbon fibers themselves do not in general show any fatigue sensitivity, and have excellent long term properties (Farquhar et al., 1989). Chan (1993) has offered the explanation of a reduction in the interfacial friction by repeated sliding at the interface.
The influence of the rate of decay of the bridging elements on the propagation of bridged cracks has been numerically investigated by Cox & Rose (1994). By use of a reference displacement field for a prescribed applied stress, the effective stress intensity factor can be calculated with the expression in Equation (4.47) substituted into Equation (4.46). The bridging action is thus taken into consideration. For each of the datum points, the COD profile was discretized and used as input in the fracture mechanics model. In Figure 4.18, the crack growth rate curves for the hypothetical unbridged crack and the bridged crack are plotted together. When the bridging effect is included, the rate curve takes a steeper slope since the effective stress intensity factor is reduced due to the cohesive action of the fibers. The curve is shifted somewhat to the left in the growth rate plot closer to the curve of the matrix material, even though the discrepancy still is significant. The exponent in the growth rate law in Equation (4.29), which corresponds to the slope in the da-dN curve, is $m = 4.8$ for the epoxy material. For the composite, it is $m = -44$ for $\Delta K_{\text{nom}}$ in the unbridged crack, and $m = -85$ for $\Delta K_{\text{eff}}$ for the bridged crack. Ghosn et al. (1992) have also observed a decreasing effective stress intensity factor with increasing crack length for a fiber bridged composite. For a bridged two-dimensional through-thickness crack, the stress intensity factor increases monotonically with increasing crack length, and takes a constant value for sufficiently large cracks (Cox & Lo, 1992). For
Figure 4.17. Degradation of bridging stiffness during fatigue.

A decelerating crack to show a physical positive slope in a da/dN-ΔK plot would require a monotonically decreasing ΔK_{eff} which seems unlikely since an increment of a bridged crack should give a small but positive contribution to ΔK_{eff}.

The mismatch in the curves of the composite and the neat epoxy indicate that there must be other active toughening mechanisms than bridging alone. The outlined model does not take the viscoelastic/plastic nature of the polymer matrix into account. A schematic picture of a fiber bridged crack with viscoelastic yielding at the crack tip, and debonding and shear yielding along the bridging fibers is presented in Figure 4.19. Most of the bridged cracking models in the literature have been developed with CMCs and MMCs in mind. A ceramic matrix is brittle and has a near linear elastic behavior. Cracks in a metallic matrix have relatively small plastic zones. In a PMC, the yielded zone in front of the crack zone should be large compared to MMCs. Furthermore, polymer behavior shows strong dependencies on time and temperature, and autogeneous heating is not unlikely in repeated loading. Costanzo & Allen (1994) have discussed the limitations of a description of crack growth by a Paris law in terms of ΔK for a material with viscoelastic behavior.

Another toughening mechanism is crack front bowing around obstructive
4.3. Fiber Bridged Cracking

Figure 4.18. Fatigue crack growth curves for the bridged and unbridged crack.

Figure 4.19. Viscoelastic yielding in a fiber bridged crack.
fibers (see Figure 4.20). This phenomenon has been investigated analytically by Bower & Ortiz (1991), and verified experimentally by e.g. Mower & Argon (1993), Botsis et al. (1995) and Goto et al. (1996) with model composite specimens. Hull (1993) has discussed the tilting and twisting for a growing crack that encounters an obstacle based on differential geometry. As the crack approaches fibers, the stress state changes due to presence of the fibers which have different elastic properties. The crack front will take a multiply curved shape, and the total crack front length will be larger than that of a locally straight crack. The formation of a longer crack front will act as a toughening mechanism, which makes the material more resistant to crack propagation.

It can still be concluded that the bridging phenomenon plays an important role in the fatigue resistance of the material. Another fatigue damage mechanism is growing debond and shear yield zones which give rise to new fiber breaks (cf. Figure 4.1 b). Previous studies (Gamstedt & Talreja, 1997; Gamstedt et al., 1997b) show that the latter fatigue mechanism results in a more fatigue sensitive behavior for CFRPs and glass fiber reinforced polypropylene. The main difference is that the fiber-matrix interface is weaker in the cases where longitudinal damage growth occurs. For materials with a higher interfacial strength, the cracks initiated from distributed fiber breaks grew perpendicular to the fiber and load directions, and were arrested by adja-
cent fibers or retarded by bridging fibers. An improvement of the interface properties which suppresses the longitudinal damage growth and forces crack growth to be planar would prolong the fatigue life. A drawback of increased interfacial strength and stress transfer efficiency is embrittlement of the material and lower observed static strains to failure.

An important objective of the identification, quantification and modeling of a fatigue crack mechanism is that it is a necessary input to the prediction of fatigue life. The growth rate must be completed with a failure criterion to predict fatigue life. For a unidirectional composite, failure occurs when the fibers start to break unstably. There were only a handful observed progressive fiber breaks on the $15 \times 20$ mm replica films during the course of fatigue in the present material. At the first application of load, a number of fibers were broken due to the distribution of fiber strength. As the fiber bridged cracks propagated, there were only a few scarce fiber breaks until imminent failure. For a composite that exhibits fiber bridged cracking, it has been proposed that ultimate failure occurs when the bridging fibers start to break (Begley & McMeeking, 1995). Once the outermost fiber in the crack wake breaks, the stress in the next fiber becomes too large, and it fails also, etc. In CFRPs, the number of contiguous broken fibers sufficient to cause final failure even at moderate stress level is relatively low (Jamison, 1982; Bader, 1988). This supports the failure criterion based on breakage of bridging fibers, if debonding can be ruled out. When the stress in the bridging fibers exceeds the local strength, ultimate failure ensues. The determination of the evolution of stresses in the bridging fibers must first be done experimentally, since the relaxation of the fiber stresses due to fatigue dependent slip is not tractable to modeling. The variation in strength along the fibers must also be taken into consideration. Considering the weakest link behavior of brittle materials such as carbon fibers, the segment that is bridging the opened crack is extremely short, and has therefore a high strength. Since the stress decreases gradually in the bridging fiber at some distance from the opened crack, it might break in a weak segment away from the crack. This behavior together with viscoelastic deformation and crack bowing require to be investigated experimentally before conclusive guidelines for successful modeling endeavors can be made.

**Synopsis on fiber bridging**

Tension-tension fatigue tests have been performed on a carbon fiber reinforced epoxy (AS4/8552). The fatigue damage micromechanisms were observed by a surface replication technique. At first application of load, distributed fiber breaks accumulated in the composite from which matrix cracks
grew perpendicular to the fiber and load direction. These surface cracks took a deformed shape with squeezed cracks tips caused by bridging fibers. The bridging action has a toughening effect, as it shields the crack tip and reduces the effective stress intensity factor which is the driving force for crack propagation. Fiber bridging has been observed as a toughening mechanisms for a number of different types polymer matrix composites after fatigue application.

A fracture mechanics methodology has been adopted to quantify the bridging behavior. Assuming that the shape of the surface crack is semicircular, a reference solution of the crack opening displacement field was used to compute the bridging law, and subsequently the effective stress intensity factor. A discrimination of the bridging effect was thus done by plotting a $da/dN-\Delta K$ curve, where the growth of a fiber bridged crack in terms of the unbridged nominal stress intensity factor range shows a decelerating trend due to various toughening mechanisms. The corresponding effective stress intensity factor range takes a lower value, but the crack growth rate curve still shows crack retardation, and there is a notable difference to that of the growth rate curve of the neat epoxy material. This discrepancy has to be attributed to other toughening mechanisms such as crack front bowing and viscoelastic yielding. However, it remains clear that the fiber bridging has a beneficial effect on the fatigue resistance of the composite. For composite materials with poor fatigue properties, the results suggest that changes in the microstructural materials design that promote the bridging mechanism would result in better fatigue performance.

4.4 Fatigue life prediction

The lion’s share of the fatigue life prediction schemes presented in the scientific literature concerns degradation of a macroscopic property, i.e. residual stiffness or strength, until final failure occurs. A fatigue damage model based on the microscopic mechanisms would require less costly testing, give better understanding, and provide indications for improvements of material properties. The models presented in this work are but a small step towards the ultimate goal of fatigue life prediction. It should be emphasized that a mechanism based fatigue model implies ample experimental work to identify and quantify the operative fatigue damage mechanisms, from which kinetic relationships intrinsic to the given composite material could be deduced. For life prediction, the kinetic equation must be complemented with a failure criterion. In the case of a shear lag model, failure takes place when the fibers start breaking unstably (Beyerlein & Phoenix, 1997b). This can be
predicted directly by the model itself since the stress at all locations is given by the solution. In a fiber bridged crack model, it is likely that final failure takes place when the first bridging fiber breaks (Begley & McMeeking, 1995). Acoustic emission investigations of fatigue in unidirectional composites have showed activity in the first few cycles, followed by no or little activity until imminent failure where fibers started to break again (Bhat & Murthy, 1993). The absence of successive continuous fiber breakage and debonding during the course of fatigue strengthens the hypothesis of failure when the bridging fibers start to break. Since the failure criterion would be based on the local stress of the bridging fibers, the relaxation process of the fiber stress should be described, which requires considerable further experimental research.

Micromechanical models for damage accumulation and life prediction in creep or static fatigue have been more frequently reported than micromechanical fatigue. Creep is also of great practical importance in durability of structural materials. Typically, a power law is applied to describe the viscoelastic constitutive equation of one of the constituents (Phoenix et al., 1988; Otani et al.; 1991, Du & McMeeking; 1995). Creep is, like fatigue, a time-dependent process which results in irreversible damage or deformation, and may finally lead to failure. The mechanisms responsible for creep may very well be similar to those active in fatigue, although they propagate at different rates. Macroscopically, the applied load can be fluctuating (fatigue) or constant (creep), which microscopically gives rise to similar sets of basic damage mechanisms such as matrix yielding, debond growth, successive fiber breakage etc. Creep models for life predictions have been developed for global load sharing (Du & McMeeking, 1995; Fabeny & Curtin, 1996), in which distributed fiber breaks are unloaded equally by the all surviving fiber in the same plane. Stress-life curves for creep in MMCs have been successfully predicted with these models. Since the rate of creep or debond growth in the matrix can be expressed arbitrarily, the models can be reformulated for a fatigue scenario. It is doubtful, though, if the approach of global load sharing can be used for PMCs, since the assumption of global load sharing may be too crude.

In PMCs, the ratio between the maximum shear stress in the matrix to the characteristic fiber strength is relatively large compared to MMC materials (Curtin, 1993), and local stress concentration close to fiber breaks will therefore be high. A local load sharing approach would therefore be more suitable for PMCs. In global load sharing, the fatigue limit can be estimated by the strength of the fiber bundle with no matrix. For PMCs, this is a largely over-conservative estimation. The composite will fail at lower stress levels due to localization of stress and damage. Local load sharing models for damage accumulation and life prediction become more detailed than
those for global load sharing, since the individual damage sites have to be considered, as well as cluster formation and coalescence. A promising technique based on lattice Green functions has been presented by Zhou & Curtin (1995), in which successive failures of interacting fiber segments with spatially dependent load transfer can be modeled in a three dimensional lattice. Ibnabdeljalil & Curtin (1997) employed the lattice Green method to develop a technique to scale strength distributions for small composite volumes to larger ones with the microscopic damage evolution taken into account. If the kinetic laws pertaining to the fatigue damage mechanisms could be incorporated into the creep models, fatigue life predictions would be feasible. With a kinetic law dependent on the applied load level, life predictions for variable amplitude loading would be possible without resorting to further generalizations to the linear Miner-Palmgren rule or the non-linear Marco-Starkey recursive model (Adam et al., 1994).
Chapter 5

Damage mechanisms in transverse plies – Tension-compression loading

5.1 Background

The previous chapters have been devoted to fatigue damage mechanisms in longitudinal plies only, since they constitute the critical element of a multidirectional laminate. However, laminates used in applications are usually composed of plies with a variety of fiber directions. In terms of ply properties, the longitudinal $0^\circ$ ply is one extreme case, and the other limit would be the transverse $90^\circ$ ply. Plies with intermediate fiber direction angles generally have mechanical properties that fall between those of $0^\circ$ and $90^\circ$. As pointed out earlier, the $0^\circ$ plies have the largest influence on the fatigue performance in a multidirectional laminate in tensile fatigue, since it is the main load carrying constituent and the last one to fail. However, the off-axis plies have an influence, and for a more comprehensive understanding of fatigue of composite structures, these plies need to be taken into consideration. This is in particular the case for compressive loading, and for applications where stiffness retention is of importance. Therefore, this chapter will deal with fatigue damage mechanisms in transverse plies subjected to tension-compression loading – not as a divergence from the main topic of this thesis, but as a first step towards a generalization to fatigue damage mechanisms in a multidirectional laminate.

Transverse plies are an almost ubiquitous element in multidirectional composite laminates for structural applications. These plies increase the stiffness and strength in the transverse direction and prevent the laminate
Figure 5.1. A transverse crack may cause delamination and out-of-plane buckling in compression, and fiber breakage in adjacent plies in tension.

from splitting. When biaxially loaded, they increase the strength for judicious laminate lay-ups. However, the transverse plies are the first to show cracks since they have lower strains to failure compared with plies with other fiber directions. For most applications, the transverse cracks are initially subcritical, but they will reduce stiffness and induce further damage accumulation which eventually causes final failure. Transverse cracks may bring about fiber breakage in the adjacent plies or local delamination which can result in buckling failure in compression (Reifsnider & Jamison, 1982; Jamison et al., 1984; Dillon & Buggy, 1995). Bredemo (1992) detected fiber breaks in adjacent plies along delaminations that originated from transverse cracks in fatigue of filament wound tubes. The failure modes initiated by transverse cracking are schematically depicted in Figure 5.1. For pressure vessels, transverse cracks by themselves may cause failure since the vessel can cease to function due to weepage (Jones & Hull, 1979).

5.2 Effects of tension-compression loading

Tension-compression (T-C) fatigue is a common type of load waveform for service applications (e.g. in vibrating structures such as aircraft wings). The greater part of all laboratory fatigue testing is however performed under tension-tension (T-T) cycling. The reason for this unbalance in testing may be explained by difficulties in testing, such as buckling of specimens and lack of control of the stress state. For composite materials, T-C fatigue has
shown to be more deleterious than T-T fatigue. The $S-N$ curves plotted with the peak maximum stress or strain on the ordinate show significantly steeper slopes for load wave forms containing compressive load excursions. This has been shown to be the case for multidirectional laminates containing off-axis plies (Rosenfeld & Huang, 1978; Rosenfeld & Gause, 1981; Curtis & Dorey, 1986), and in particular for those containing transverse plies (Owen, 1974a; Ryder & Walker, 1977; Schütz et al., 1981; Gathercole et al., 1994; Boyum & Mall, 1995; Nyman, 1996). Detrimental effects of compressive load excursions has also been observed for pure unidirectional transverse composites (Rotem & Nelson, 1989; El Kadi & Ellyin, 1994).

The physical reason for the adverse effect of T-C loading of composite materials has not been clarified entirely. For multidirectional laminates, delamination followed by out-of-plane ply buckling would certainly play an important role on the ply-level scale. On the scale of an individual fiber or a group of fibers, the underlying physics remains more unclear.

For materials with a more homogeneous microstructure (e.g. metals or alloys), fatigue crack growth is characterized by self-similar propagation of single large cracks until the largest one of them achieves a critical size and catastrophic failure takes place. This propagation is primarily governed by crack opening under remote tensile loading. A compressive load excursion would close the crack tip and effectively result in negligible propagation of the crack in comparison with corresponding tensile load fluctuations. However, for a heterogeneous material like a composite that consists of phases with differing elastic properties, distributed microcracks of various sizes and shapes would form at an early stage of loading. The interfaces between the constituents would be particularly prone to cracking due to the inherent stress concentration and residual stresses from processing. Considering the cracks between media of dissimilar elastic properties, it is likely that some of these microcracks would open under global compression, especially those oriented along the load direction. In a multidirectional composite laminate with continuous fiber plies, cracks with an effective opening under compression could be incipient delamination on the ply level and large fiber-matrix debonds on the microscopic level. For notched composite specimens, it has been reported that cracks grow parallel or perpendicular to the load direction depending on whether the load is on the compressive or tensile side, respectively (Stinchcomb & Reifsnider, 1979).

The formation of transverse cracks can be initiated by microcracks which grow under compressive loading. It is not far-fetched to suggest that this type of microcracks would be interfacial debonds with a fairly large circumferential size. Debonding is known to be the micromechanism incipient to transverse cracking. As the debonds at the interface between the fibers and matrix
coalesce and grow unstably, a transverse crack forms (Harrison & Bader, 1983). Even if it is assumed that transverse cracking is initiated in the matrix through yielding or cavitation, it has been shown that this will take place at the fiber-matrix interface for uniaxial transverse loading (Asp et al., 1996). This crack initiation will subsequently immediately cause a partial debond.

In this chapter, a mechanism is proposed that accounts for the inferior fatigue behavior in T-C compared with T-T of laminates containing transverse plies. The mechanism is debond crack propagation under compression for sufficiently large debond angles. As depicted in Figure 5.2, crack opening zones form at the crack tips under global compression. The interfacial crack can now propagate more easily than if there were contact zones at the crack tips for equal tensile far-field loading (Owen, 1974b; París et al, 1996; Varna et al., 1997b). The validity of this mechanism hypothesis has been investigated by a number of experiments. On the macroscopic level, fatigue testing of cross-ply laminates has been performed to investigate the effect of T-C loading on the formation of transverse cracks. On the mesoscopic level, the role of debonding in the development of transverse cracks has briefly been investigated. On the microscopic level, model specimens with single transverse fibers have been tested to examine the debond growth under low cycle T-C fatigue loading. The proposed mechanism has also been validated by finite element analysis.

5.3 Methods of analysis

Materials

Two types of specimens were manufactured for this investigation; a cross-ply laminate with a [0,90]s lay-up and a model specimen with a single fiber in the transverse direction. The matrix used for the cross-ply laminates was a flexible epoxy vinyl ester (VE) Norpol Cor Ve 8515 from Jotun Polymer AS, with a styrene content of 32 – 36% by weight. The glass fiber was a 2400 Tex fiber from Owens Corning of type R25H. The mean diameter of the fiber was 23 μm and the fiber volume fraction in the laminates 60%.

Casting procedure

The vinyl ester resin was cured with 2% by weight methylethylketone peroxide (MEKP) as catalyst and 2% by weight cobalt-octoate (1% Co) as accelerator. For injection of the resin, 0.2% by weight inhibitor NLC-10 was used
to obtain a sufficiently long time to gelation.

The cross-ply laminates were produced by the resin transfer molding (RTM) technique, using a similar mold and peripheral equipment as has been described by Lundström et al. (1994). A turning machine was used to wind the fibers onto a steel plate mandrel. The fiber preform was placed in the mold and resin was injected. The total thickness of the $0^\circ$ and $90^\circ$ layers were 0.5 mm and 1 mm, respectively. The injection pressure was 0.3 MPa. The pressure at the outlet side of the mold was lower than atmospheric. Connecting a vacuum pump to the outlet of the mold has been shown to reduce the laminate void content (Lundström & Gebart, 1994). The mold and resin temperature were ambient during injection.

The single fiber transverse specimens were manufactured in a similar way as is described in a study by Zhang et al. (1997). The specimens were prepared by mounting the fibers on a 1 mm thick steel frame, using a double-sided adhesive tape. The frame was then placed between two flat Teflon coated aluminum mold plates, separated by spacers of 2 mm thickness and provided with a silicon tube sealing. Positioning in the thickness direction was achieved by 1 mm thick steel slivers on the tape. After closing the mold, the resin was carefully poured into the vertically positioned mold and cured.

To reduce the amount of entrapped air from the mixing operations, the resin was vacuum treated for a minimum of 5 minutes before injection or pouring into the molds. Post-curing of the laminates and plates was performed for 2 hours at $80^\circ$C.
Material characterization

The glass fiber content in the cross-ply laminates was measured by weighing the material prior to and after burning at 550°C for 2 hours. Micrographs of fiber-matrix debonding were taken with an Olympus AHBT optical microscope. The specimens that were examined from the edge were polished in several steps with silicon carbide papers, followed by several steps on a Texmet surface, using diamond sprays with successively finer particles. Final polishing was performed on a Microcloth surface with a SiO₂ suspension. A Cambridge Camscan SH-80D scanning electron microscope (SEM) was used to take photograph from the edge of a single transverse fiber specimen in tension. The specimen was coated with a thin gold layer in a Blazer SCD050 sputter coater.

Mechanical testing

All tests were made at ambient room temperature. The fatigue tests of the cross-ply laminates were performed in an Instron 1272 servo-hydraulic tensile machine. The specimens were prepared according to ASTM standard D3410-87. Their dimensions were $2 \times 12 \times 110$ mm, with a gage length of 12 mm. The specimen edges were polished by 1200 grit followed by a 2400 grit silicon carbide paper. A sinusoidal load wave form was used and the maximum load level was 70% of the load for transverse crack initiation and growth in static tension. The T-T stress ratio was $R = 0.1$ and T-C stress ratio was $R = -1$. The frequency was kept low enough to prevent autogeneous heating. The specimens were photographed intermittently during fatigue testing and the total length of the transverse cracks were measured with respect to the number of elapsed load cycles based on these photographs.

The single fiber specimens were tested in a horizontal miniature mechanical tensile stage (Minimat) from Rheometric Scientific Ltd. The experimental set-up is schematically depicted in Figure 5.3. The test machine was mounted on the $x$-$y$ table of a microscope. Interfacial debonding between fiber and matrix was observed in situ by optical microscopy, where light in the transmission mode was used. The microscope was equipped with a video camera, as well as with a conventional photographic camera. The miniature tensile machine was manually controlled, but the load and the displacement were continuously recorded by and stored in a computer. The stress amplitude was 50 MPa which is lower than the yield stress of the vinyl ester. Interfacial debonding was observed during step-wise loading. After each load cycle, the fiber-matrix debonds were photographed, video recorded and stored in a computer. After the tests, the debond size and geometry were quantified by
Figure 5.3. Schematic illustration of the experimental set-up for monitoring debond growth in a single transverse fiber specimen.

Finite element analysis

The analysis of the crack opening profiles was made using the commercial finite element code (ANSYS 5.3). A unit cell with an axis of symmetry through the fiber and plane normal strain along the opposite boundary (see Figure 5.4). Contact elements of uniform length were used along the debond crack of varying length. The CONTA48 are point-to-surface contact elements with no friction. A uniform axial stress corresponding to the global stress used in the single transverse fiber tests was applied. The elastic properties of the fiber and the matrix were set to those of the single transverse fiber tests. The fiber volume fraction was set to 20% with ~ 10,000 PLANE82 quadrilateral-triangular plane strain elements. The automatic mesh generator was used, since no effort to determine crack tip properties such as energy release rates was made in this study. For a more quantitative finite element analysis, the virtual crack closure technique may be used to determine the energy release rates in modes I and II based on the nodal forces and displacements for a small crack increment (see e.g. Dattaguru et al., 1994; Beuth, 1996). Here, the existence of opening or contact zones along the debond is highlighted. The extent of the contact and opening zones were determined from the normal component of the nodal forces with Cauchy’s formula.
5.4 Results and discussion

Transverse cracking

In T-C fatigue of multidirectional laminates, transverse cracking, delamination and subsequent out-of-plane buckling would probably be the most important failure mechanism. In T-T fatigue, the failure mechanisms are different, viz. fiber breakage in the vicinity of transverse cracks as schematically shown in Figure 5.1. The common denominator for the two loading modes is transverse cracking, which is a precursor to ultimate failure. If the compressive load excursions have an influence on the formation of transverse cracks, they would also have an influence on the ultimate failure and hence on the fatigue life. For pure 90° specimens, failure occurs when the first transverse crack appears. If the plies are regarded as continua, there is no evident difference in damage mechanisms for T-T and T-C fatigue for this type of laminate, since failure is imminent when the first crack forms. If the fibrous microstructure is taken into account, there should be some difference on the microscopic level prior to transverse cracking since T-C fatigue is known to lead to a more rapid degradation compared with T-T loading (Rotem & Nelson, 1989; El Kadi & Ellyin, 1994).

As mentioned previously, T-C fatigue testing can present experimental difficulties like buckling during the compressive part of a cycle. To avoid...
this, the specimens have to be thick and have a short gage length. One disadvantageous effect of this specimen geometry is that a non-uniform and multiaxial stress state will develop despite the application of a uniaxial load. In this study these effects are not taken into consideration, since the direction of the largest principal stress will be parallel to the direction of the applied load and the main goal is to study the causes of observed effects of compressive loading.

In order to determine the difference in damage development on the ply level in T-C fatigue compared with T-T fatigue, cross-ply laminates were manufactured and tested in fatigue. The same maximum load was used for the two cases, i.e. the amplitudes were the same but the range was twice as large for the T-C case compared with the T-T case. Photographs were taken of the specimens intermittently every decade of elapsed cycles. There were no visible delaminations that originated at tips of the transverse cracks at the ply interface. The tests were terminated at 100,000 cycles and at this point the amount of transverse cracks was considerably larger for T-C as can be seen in Figure 5.5. The total transverse crack length of the specimens was measured and normalized with the gage length. These lengths were plotted with respect to the number of elapsed cycles (see Figure 5.6). Even though the peak load was the same for the two loading modes, the total length of transverse cracks grew to be more than twice as large for T-C compared with T-T. This phenomenon must be explained by an underlying micromechanism that is active under compression and results in further damage growth.

To single out a mechanism that is responsible for the formation of transverse cracks, static tensile tests of cross-ply laminates were performed. During traction, in-situ microscopy was done at the edge of the transverse ply at high magnification. It was observed that the first damage to develop was interfacial debonding. As the specimen was further strained the debonds coalesced and formed a transverse crack. The debonds appeared at a low load and were widespread and easy to find. The ensuing transverse cracks were sparse and emerged stochastically at different locations. It was therefore difficult to track individual debonds as they linked up and formed transverse cracks. However, a few micrographs of the crack coalescence process were caught and one of them is presented in Figure 5.7. A schematic picture of the coalescence of debonds into a transverse crack is found in Figure 5.8. This sequence of damage development has also been observed in carbon-fiber reinforced plastics under monotonic and cyclic loading (Harrison & Bader, 1983). Pyrz & Bochenek (1995) have presented a model to simulate the process of transverse cracking based on a discretized network of an arbitrary two-phase microstructure. Experimental results from CFRP laminates by Wood & Bradley (1996) show that transverse cracks initiate at debonds in
Figure 5.5. Transverse cracks in cross-ply laminates at 100,000 cycles in (a) T-T fatigue, and in (b) T-C fatigue.
transition regions between resin rich and fiber rich areas. There is reason to believe that the residual stresses are in tension in this region.

This qualitative analysis shows that debonding is the initiating mechanism to transverse cracking. A next step would be to investigate the influence of load cycling on debond growth on a microscopic scale in order to shed some light on the reasons why T-C fatigue show more transverse cracking than T-T fatigue (cf. Figure 5.5).

**Debond propagation**

To quantify the debond growth for single glass fibers, a method used by Berglund and co-workers (Zhang *et al.*, 1997; Varna *et al.*, 1997) was employed. The single transverse fiber test method is made by molding a single glass fiber into a transparent polymer matrix. The specimen is cut out and transversely loaded under a microscope with transmitted light. When debonds develop, dark zones appear where the cracks are open. Debonds can also be observed directly from the surface of the specimen edge with reflected light, but in that case full debonding may occur when the residual stresses from processing are released (Hu *et al.*, 1995). Therefore, the debonds have
Figure 5.7. Fiber matrix debonds (a) form, and (b) coalesce to form a longer transverse crack.
5.4. RESULTS AND DISCUSSION

Debonding Transverse crack

**Figure 5.8.** Schematic drawing of the formation of debonds and subsequent link-up and transverse cracking.

been measured from the dark crack zones with transmitted light and the fiber aligned perpendicular to the directions of sight and illumination (see Figure 5.9). A similar method has been used for metal matrix composites with ultrasonic imaging by Hu (1996).

A single fiber composite is not a real composite in the sense that there are no neighboring fibers that influence fracture processes etc. Even though the fibers and the matrix are the same as for the intended application, the absence of neighboring fibers will strongly affect the stress state around the fiber under investigation. The benefits of a model specimen test is that the material system is well controlled, and material parameters can be more easily quantified. In particular for the single transverse fiber test, the debonds can be directly observed and measured. In a composite specimen with the same high volume fraction of fibers as intended for a service application, any debond would be obscured by other fibers and defects. For a multifiber composite, debonds would therefore not be measurable with the transmitted light technique.

Some scanning electron micrographs were initially taken to show that there exists a debond opening zone for a single transverse fiber subjected to a tensile load, one of which is presented in Figure 5.10. Similar debond appearances have been observed at the surface of transverse fibers in MMCs in tension (Boyum & Mall, 1995). In compression, no clear debond opening zone appeared due to the complex stress state caused by the free surface. Optical microscopy with transmitted light was therefore opted for. The observed debonds initiated from the free surface and grew inwards. Some debonds also initiated in the interior of the specimen creating a pattern of debonds on alternating sides of the fiber (see Figure 5.11). The debond angle also
Figure 5.9. Single transverse fiber test with a debonded fiber.

Figure 5.10. A single fiber loaded horizontally in transverse tension with large debonds on either side of the fiber.
Results and Discussion

Figure 5.11. A partially debonded fiber in transverse tension with debonds on alternating sides of the fiber.

varied significantly from position to position along the fiber, which can be explained by the high local variation of interfacial strength. As pointed out by Zhang et al. (1997), many measurements from single transverse fiber tests have to be made to obtain a reliable value of e.g. the critical energy release rate of the interface. Since the objective of this study was to reveal the active mechanisms and not to characterize any interfacial parameters, only a limited number of positions for measurement of typical debond growth were selected.

The image of a debond under tension is shown in Figure 5.12a. The dark stripe along one side of the fiber is the opened debond. In compression, the debond closes at the equator and opens towards the poles. A picture of a debond with opening zones at the crack tips under compression is presented in Figure 5.12b. A once debonded interface which has been closed again shows a more diffuse nuance and can be distinguished from crack opening zones and pristine adhesive zones. Schematic pictures of the cross section of
a partially debonded fiber in tension and compression are found Figure 5.13. They reflect the crack profiles observed experimentally along the fiber in Figures 5.12a and 5.12b.

The Poisson effect and the fact that the fiber is stiffer than the matrix, result in an opening zone in compression and a contact zone in tension for a sufficiently large debond. The debond angle, $\theta$, at which a contact zone develops in tension has been estimated to be around 60° for a glass fiber in an infinite epoxy matrix (Varna et al., 1997a). The calculations were made with a boundary element method and with an analytical model. Conversely, it can be expected that the angle, $\theta$, at which an opening zone develops in compression would also be around 60°. Considering debond propagation for angles larger than this value, it would be in pure shear mode for global tension, since the crack tip is closed. In compression, there would be crack tip opening and the debond can propagate both in mode I and mode II. It is well known that cracks in polymers, ceramics and their interfaces are more susceptible to propagation in mode I than in mode II. For this type of materials, the effective critical energy release rate including friction is generally several times lower for mode I than for mode II, *i.e.* $G_{Ic} \ll G_{IIc}$.
5.4. RESULTS AND DISCUSSION

For these reasons, the debond growth rate should be larger for compressive loading than for the same load in tension for sufficiently large debond angles. The debond propagation should consequently be faster in T-C fatigue than in T-T fatigue.

The effect of T-T and T-C fatigue on debond growth was likewise investigated for single transverse fiber specimens with transmitted light. The angle was monitored for a number of debonds with respect to the number of elapsed load cycles. The debond angle was measured at the peak tensile load. A number of T-T load cycles were applied, followed by T-C cycles. The evolution of the debond length with number of cycles is plotted in Figure 5.14. Two samples were subjected to 10 cycles in T-T fatigue with a stress ratio of $R = 0$, followed by a couple of cycles in T-C with $R = -1$ and the same peak stress value. As can be seen in Figure 5.14, a debond angle of around 70° forms at the first cycle, whereupon the angle increases slightly and stabilizes at a marginally higher value. At the application of compressive load cycles there is a sudden and distinct increase in the debond angle and subsequent further propagation. This clear increase in debond length as compressive load cycles are applied is explained by the effective crack tip opening of a partially debonded fiber as discussed in the previous section. In order to ascertain that the observed leap in debond angle was not caused

Figure 5.13. Drawing of crack opening and contact zone for a transversely loaded fiber in tension and compression.
by an external perturbation, two more specimens were tested in fatigue with 20 cycles in T-T followed by a couple of cycles in T-C. A sudden jump in debond angle at the change of loading mode was likewise observed in these tests, which supports of the proposed mechanism. For all tests, the debond angle grew during the first few cycles and then stabilized at a constant level. This asymptotic behavior can be explained by the formation of a contact zone at the crack tips in tension. The pure mode II energy release rate is not sufficiently large to result in any considerable debond growth. It is not until the onset of alternating compressive loading, that debond propagation resumed since the mixed mode I and mode II energy release rate attains a sufficiently large value.

**Finite element model**

It is assumed that the debond length is much longer in the axial direction than in the circumferential direction. This was the case for the single transverse
Table 5.1. Elastic properties of the composite constituents.

<table>
<thead>
<tr>
<th>Property</th>
<th>E-glass</th>
<th>Vinyl ester</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus, $E$</td>
<td>76 GPa</td>
<td>2.2 GPa</td>
</tr>
<tr>
<td>Poisson's ratio, $\nu$</td>
<td>0.21</td>
<td>0.34</td>
</tr>
</tbody>
</table>

fiber tests, except initially at the formation of the debond. For a few degrees of debond angle, the debond grew axially and took a prolate elongated shape. The geometric configuration can thus be reduced to two dimensions along the cross section of a debonded fiber. In the interior of the specimen, where the debond progression has been measured, there is a state of plane strain. A direct interpretation of the single fiber transverse specimen would require a finite element model with one fiber surrounded by a matrix material of infinite extent. Since the objective is not to quantify any interfacial material parameters from the experimental results of the single transverse fiber test, a fiber volume fraction of 20% has been chosen for the modeling, which is more similar to applied composite materials. The boundary element method (BEM) has successfully been employed in the calculation of crack propagation of a single fiber in an infinite matrix subjected to transverse tension (París et al., 1996; Varna et al., 1997b). The BEM has proved advantageous to finite element analysis in cases when stress singularities are present and when the geometry is infinite. In this investigation, the existence and extent of a crack opening zone for a sufficiently large debond under global compression for a single fiber in a finite volume of matrix were studied with finite element analysis.

Both the fiber and the matrix are supposed to behave linear elastically. This is certainly not the case for the polymer matrix which is visco-elastic and generally yields before fracture. In this numerical investigation, it was nevertheless assumed that the matrix is linear elastic, since low stresses were used and the stress-strain curve for the various multiaxial stress states was not known for this polymer. The constraint imposed on the matrix by the surrounding fibers results in a triaxial stress state, in which the polymer has a more brittle and linear elastic behavior than were the stress state uniaxial. The elastic properties were the same as those of the single transverse fiber experiment (see Table 5.1).

The residual stresses were not taken into account in this study, even though the used material, as well as most polymeric composites in general, show residual stresses from processing. Thermosets usually exhibit chemical shrinkage after cure. Moreover, there is thermal shrinkage of the matrix
since the cure temperature is higher than service temperature. These two forms of shrinkage result in an effective residual compressive stress over the interface of an average fiber. To open a debond at such an interface, it is required that the tensile stress over the interface should overcome the residual compressive stress. The energy release rate is substantially larger in tension than in compression for the same absolute applied stress. In other words, it would be easier to overcome the residual compressive interfacial stresses in tension compared with compression. This means there would be a shift in T-C stress ratio \( R < 0 \) to lower values (a larger relative contribution from the compressive part of the load cycle) for the most deteriorating fatigue damage development for higher residual compressive stresses. However, the residual stress state will depend on the microstructural geometry. In a regular arrangement, say square packing, the interfacial residual stresses are compressive. In a disordered geometry as in real composites, tensile interfacial stresses may develop close to resin rich areas (Wood & Bradley, 1996). In this investigation, the goal was only to identify the governing mechanism and the effects of residual stresses and varying stress ratios were therefore neglected. The above assumptions and simplification have been made in order to make the testing and finite element modeling feasible. It is believed the assumptions are judicious, and do not impair the validity of the mechanistic explanation of the observed phenomena.

In Figure 5.15, an enlargement of the crack opening zone under global compression is presented. This is in accordance with the observed crack opening zones from the single transverse fiber tests. Since a finite matrix size was chosen in the finite element modeling and boundary conditions with uniform strains at the unit cell edges were prescribed, the composite would be composed of symmetric unit cells with debonded fibers. The neighboring unit cells in a square packed composite are thus marred by equal cracks on the reflected side of the fiber. Neither the packing geometry nor the symmetry in debonding is likely to occur in a conventional composite material, but the configuration is more realistic than that of a single transverse fiber in a large volume of bulk matrix. The quintessence is that crack opening in compression occurs in compression.

In Figure 5.16 the opening zone angle, \( \alpha - \theta \), is plotted with respect to the total debond angle, \( \alpha \), under compression for debond angles between 0° and 90°. Crack tip opening occurred for debond angles larger than 55°. The opening zone increased for increasing debond angles, with a transition from contact zone to opening zone invariably between \( \theta = 55° \) and \( \theta = 60° \). The existence of a crack opening zone for large transverse debonds under compression has thus also been validated by finite element analysis.
5.5. **Damage development and failure**

A recapitulation of the performed tests and a link from debonding on the microstructural level to the fatigue life behavior of macroscopic coupons has been established and is qualitatively presented in Figure 5.17. The starting point for the study is the fatigue performance of multidirectional composites containing transverse plies and especially unidirectional transverse composites, which are subjected to T-T and T-C loading. As stated previously, the compressive load excursions have a significant deteriorating effect on the fatigue performance. From a mechanistic viewpoint, this can be explained in terms of micromechanisms and their effects on different size scales.

There are two columns in Figure 5.17, one represents T-C fatigue and the other represents T-T fatigue. There is a notable difference in the crack profile of a single debond in compression and in tension. In previous case, there is an opening zone at the crack tip for large debonds which facilitates propagation. In the latter case, there is a contact zone and the debond is constrained to propagate only in mode II. At an early stage of debonding for sufficiently small debond angles, the crack tip is opened in tension which results in a fairly rapid propagation also for T-T fatigue. This enhanced propagation
rate is also present in T-C fatigue, since it is assumed to have the same peak tensile stress. A noticeable acceleration for the T-C case would commence when the debond angle reaches the point where an opening zone at the crack tip forms. The average debond size would consequently be larger in T-C compared with T-T for a given number of elapsed cycles. The debonds are known to initiate transverse cracks in conventional composite materials, either from coalescence of adjacent debonds or from initiation at the largest debond with the most severe stress state. The transverse crack density is therefore larger in T-C. For pure unidirectional transverse laminates, the specimens fail at the formation of the first transverse crack. The transverse cracks have a detrimental effect on the fatigue performance of a multidirectional laminate. In tension, the transverse cracks act as stress raisers on the adjacent load carrying plies and in compression, delamination from transverse cracks may cause out-of-plane buckling (cf. Figure 5.1). For multidirectional composites containing transverse plies, the mechanisms of crack tip opening of debonds in compression has therefore a contributing effect in the shorter fatigue lives and steeper S-N slope for T-C fatigue compared with T-T fatigue. For unidirectional transverse composites, the observed mechanism has a more direct effect in the fatigue properties since transverse cracking leads to imminent
failure.

Since tension-compression fatigue is a widespread loading mode for composite applications, a suppression of the investigated mechanism can be profitable in terms of prolonged fatigue life. One way to improve the fatigue performance would be to increase the fracture toughness of the fiber-matrix interface, which would subdue debond propagation. This can be done by selecting a suitable fiber sizing or a chemical modification of the polymer matrix to improve interfacial adhesion.

In summary, composite laminates containing transverse plies degrade more rapidly in T-C compared to T-T fatigue. The reason for this is the more rapid debond growth around the transverse fibers in T-C loading, due to effective crack tip opening under global compression for sufficiently large debonds. The opened debonds in compression have been observed experimentally in a model specimen with a single transverse fiber and validated by a linear elastic finite element model. Due to these debonds, transverse cracks initiate at an earlier stage for T-C loading and hence the premature appearance of multiple transverse cracking. Unidirectional transverse specimens fail immediately upon formation of the first transverse crack, whereas for multidirectional laminates, the transverse cracks have a detrimental effect on the fatigue performance. In this way, the proposed and observed debonding mechanism have a contributing effect on the adverse effect of compressive load excursions in fatigue.
### Figure 5.17. Schematic diagram of link from micromechanisms to macroscopic fatigue behavior.
Chapter 6

Final remarks

6.1 Conclusions

Fatigue in composites

Composite materials have gained increased use structural applications in the last few decades. Due to the inherent difference in microstructure, design with composite materials has to be treated differently compared to more conventional metallic materials. An investigation of fatigue mechanisms is prompted by the fact that most failures in service of load carrying structures can be attributed to fatigue. For design purposes, static considerations prevail, despite the importance of fatigue failures. This is partly due to lack of understanding of the underlying fatigue mechanisms, based on which design models could be conceived. A comprehensive model from the detailed micromechanisms to estimations of lifetime of macroscopic composite structures is a formidable task, a much further work is required to accomplish this quest. However, an understanding of the fatigue damage mechanisms could as a first step provide a qualitative link between the active mechanisms and the macroscopic fatigue behavior. Also, a mechanistic study would indicate the weaker constituents, and supply the materials engineer with useful information for materials selection. Newer composite materials with tougher matrices have not as good fatigue properties as expect. For instance, carbon fiber reinforced PEEK shows superior static properties with higher fracture toughness and higher resistance to delamination compared to carbon fiber reinforced epoxy, but in fatigue performance the order is reversed. One objective of this work has been to explain this phenomenon.

Previous studies have shown that the longitudinal plies of a multidirectional laminate control the fatigue life in tension-tension loading. If the fatigue behavior of longitudinal laminates has been characterized, the fatigue
behavior of multidirectional laminates can be assessed based on the influence of damage in the off-axis plies on the critical longitudinal plies. Emphasis is therefore placed on unidirectional longitudinal laminates in the present study.

If the fatigue life data is plotted with maximum initial logarithmic strain with respect to the logarithm of the number of cycles to failure, the data points can be partitioned into three different regions, each pertinent to different damage mechanisms. Such a plot is termed fatigue life diagram, and was originally presented by Talreja (1981a). With fatigue life diagrams, composite materials with different lay-ups, constituents and processing procedures can be easily be compared in the same framework. A statistical method has been devised with which fatigue life diagrams can systematically be deduced. The estimated parameters and the fatigue life diagram itself can be used to compare the fatigue performance of different composite materials in a quantitative manner.

**Damage mechanisms**

Experiments were done on two unidirectional commercial CFRP materials; CF/epoxy and CF/PEEK. They both had the same kind of AS4 carbon fibers, whereas the matrix was a thermoset or a thermoplastic. Previous work has shown a slightly weaker fiber-matrix interface for the PEEK system which has a more ductile matrix. The operative micromechanisms were monitored by intermittent application of surface replica films during the course of fatigue testing. The macroscopic behavior was characterized by means of fatigue life diagrams. Scanning electron fractography was also undertaken of the failed specimens. The tension-tension fatigue tests with a stress ratio of $R = 0.1$ were performed with a servo-hydraulic tensile machine. CF/PEEK showed shorter fatigue lives, and more rapid fatigue degradation compared to CF/epoxy. Microscopically, the fatigue damage was distributed and comprised cumulative fiber breakage and abundant debonding or matrix cracking along the fibers. The debonds emanated from flaws or fiber breaks, a led to stress redistribution of the load in the neighboring fibers. Since the strength is distributed along the fibers, the growing debonds gave rise to new fibers breaks, which in turn initiated further debonding etc. In CF/epoxy, the damage sites were sparse and localized. Because of the distribution in fiber strength, some fibers were broken at the first application of load. From these fiber breaks, matrix cracks propagated perpendicular to the fiber and load direction, and subsequently took a deformed shape with squeezed crack tips caused by cohesive forces from bridging fibers.

Moreover, unidirectional GRP samples were tested under the same load...
Maleic anhydride modification of the PP matrix has shown to improve the static properties due to its higher interfacial strength. Therefore GF/PP and GF/MA-PP composites were tested also in fatigue, and the replica technique was used to monitor the fatigue mechanisms. In the GF/PP with the weaker interface, there was extensive fiber breakage combined with propagating large scale debonds, which was entailed by further fiber breakage and debonding. In the GF/MA-PP the damage consisted of few fiber break sites with matrix cracks growing in the transverse direction which were arrested by the adjacent fibers. No successive fiber breakage was recorded in this case. On a macroscopic scale, the residual axial stiffness of the GF/PP degraded continuously during fatigue, whereas the GF/MA-PP showed a ‘sudden death’ behavior with little decrease in stiffness. The fatigue lives were approximately a decade shorter for the composite with the weaker interface.

Scanning electron microscopy of the fracture surfaces did not reveal any apparent differences between static and fatigue loading. This indicates that the fatigue cracks prior to catastrophic failure are small, i.e. the critical flaw size for the applied load levels was small, probably in the order of ~ 10 contiguous broken fibers. Since the critical flaw size is small, the fracture surface is dominated by catastrophic static fracture rather than non-critical fatigue cracks. Fractography of GF/PP showed adhesive failure with fibers deprived of adhering matrix resin, whereas GF/MA-PP exhibited cohesive failure with residual matrix material bonded to the fibers. This shows that the maleic anhydride modification gives rise to a stronger interfacial bond, which in turn results in better fatigue performance.

Based on the observations of both the CFRP and GRP systems, it can be concluded that an increase in resistance to debonding or longitudinal matrix crack growth leads to improved fatigue performance. Both materials showed a transition in mechanism. In the materials with shorter fatigue lives, extensive matrix cracking or debonding along the fiber direction accompanied by stochastic fiber breakage were the dominating mechanisms. In the materials with the better fatigue performance, the damage was limited to few localized sites with fiber breaks from which matrix cracks grew transverse to the fibers, and sometimes formed fiber bridged cracks. The reason for the difference in observed mechanisms and the ensuing macroscopic fatigue behavior can be attributed to the resistance to fatigue crack propagation along the fibers, either by interfacial debonding or matrix cracking. If fatigue is the most important design parameter, it can thus be recommended to select a composition of composite constituents that suppresses longitudinal debonding or longitudinal matrix cracking.
Modeling of damage propagation

The two principal mechanisms identified by experimental observations were thus (i) debonding originating from fibers breaks which resulted in new fiber breaks, and (ii) matrix crack propagation perpendicular to the fiber direction resulting in fiber bridged cracks. The propagation of these two basic damage mechanisms has been modeled, in order to gain understanding of the basic relations between microstructural properties and the nature of the mechanisms. From the modeling results, some qualitative conclusions could be drawn concerning materials selection and for improved fatigue properties.

A parametric study of the influence of debond propagation on the stochastic breakage of fibers was performed. The fundamental damage configuration of a single broken fiber from which debonds propagates was investigated with a shear lag model. As the debond grows the axial stress in the adjacent fibers become moderated and more spatially distributed. A Monte Carlo simulation showed that the mean axial distance between fiber breaks increases with increasing debond lengths. The same distance also increased with increasing variability in strength along the fibers. Composites that are prone to debonding when subjected to fatigue, and that have fibers with a large scatter in strength would therefore become more notch insensitive. A smaller scatter in fatigue life has been observed for composites with stress peak soothing debonds, which concertns with the modeling result that the variability in scatter in stress at fiber failure decreases with increasing debond lengths.

The other observed fundamental type of damage propagation was fiber bridged cracking. The bridging law was determined from the crack opening profiles, and showed a softening trend during fatigue. A crack growth curve in terms of $da/dN-\Delta K$ was plotted with the nominal stress intensity factor. The crack growth curve with respect to the effective stress intensity factor where the bridging action was taken into consideration fell closer to the curve of the neat matrix material, but the difference was still considerable. Both curves with the nominal and effective stress intensity factors had downward decelerating slope. These results show that there has to be other crack retarding mechanisms than crack bridging itself, possibly crack front bowing and matrix yielding.

Tension-compression fatigue

It has been observed that multidirectional composites are more sensitive to fatigue in tension-compression (T-C) loading than tension-tension (T-T) loading. A study of the T-C mechanisms is motivated by the fact that the reasons for the adverse effects of the compressive load excursions have not
6.2. Suggestions for future investigations

Experiments

The fatigue damage mechanisms in a few polymer matrix composite materials have been discussed in this thesis, but much further work remains to be accomplished to convey a more complete understanding of mechanisms and their relation to macroscopic fatigue properties. From the author’s perspective, some guidelines for future investigations will be suggested in this final section.

To undertake further research, the objective lodestar should be made clear. It should first be emphasized that the primary driving force for fatigue investigations should directly or indirectly be oriented to engineering applications. A more clear picture of the fatigue damage mechanisms is important for the material scientist working with microstructural design and manufacturing processing in order to avoid the onset of the deleterious mechanisms. As an extension of investigating the mechanisms, the ultimate goal is to predict fatigue lifetime based on modeling of the observed mechanisms.
In a distant future, such a description could be incorporated in design tools. To conceive judicious and physically correct models, further experimental work on other materials should be performed. The surface replica technique has proven itself quite useful in mapping the evolution of fatigue damage on a microscopic level. If this method could be complemented with a quantitative measurement technique like laser Raman spectroscopy, the details of damage evolution could be investigated with greater accuracy. For instance, the progression of matrix damage, i.e., matrix yielding, microcracking or debonding at the interface, can indirectly be assessed by quantification of the local stresses along the fibers. This progression is difficult to examine with other non-destructive evaluation techniques.

The irregular microstructure of commercial composite materials with high volume fractions of fibers makes modeling endeavors of fatigue damage processes difficult and therefore risky. To judge how well a model describes physical measurements and thereafter to calibrate the model would be significantly more tractable for results from model composite tests. Experimental results from fatigue tests of a monolayer of carbon fibers would give impetus to such a model evaluation.

A natural extension to fatigue in longitudinal laminates would be to move on to multidirectional laminates, which are by far more widely used in applications than unidirectional laminates. The longitudinal plies have shown to strongly influence the fatigue properties of multidirectional plies. However, the interlaminar and intralaminar cracks in the neighboring plies influence the local stress distribution in the longitudinal plies, and hence also the fatigue durability. For example, the influence of transverse cracks in off-axis plies and delaminations on the evolution of the stress state in the longitudinal plies needs to be addressed. From this stress evolution profile, the fatigue properties of the entire laminate could then be estimated based on fatigue life data for longitudinal laminates only.

Modeling

Further modeling of the fatigue damage accumulation is useful in several aspects. Parametric simulations can be undertaken to elucidate the interaction of constituent properties in fatigue degradation, which could provide valuable insight for microstructural tailoring for optimal fatigue performance. Macroscopic design for finite fatigue lives also motivates the conception of a micromechanical model, even though much more careful experimental work is required to achieve this goal. The design models currently in use are in general empiric and relies on costly characterization of fatigue life curves for every new type of material and stacking sequence. If the degree of empiricism
of these macroscopic models could be refined or replaced by micromechanical models based on microstructural properties, the amount of fatigue testing could be reduced. Due to the large scatter in fatigue life for composite materials, a statistical approach is a necessity. In design, it is desirable to prescribe a given probability of survival to given degree of confidence.

In the literature, there exists quite a few detailed micromechanical models for static and creep failure of unidirectional composites, which could be extended to include fatigue damage accumulation. This requires an addition of a kinetic expression for the cycle dependent growth of damage specific to fatigue, e.g. an equation for debond growth depending on the local matrix shear stresses, which in turn demands a method to extract the parameters for the kinetic law from experimental data.

More specifically, the presented shear lag model could be generalized to embrace the successive breakage of fibers in a composite monolayer through continuous debond growth during fatigue, leading to eventual failure. Calibration of such a model to experimental data would hopefully validate the model and characterize the microstructural properties and kinetic relations in a microcomposite framework. Parametric investigations could then indicate critical microstructural parameters, which would be valuable in materials selection. Next, one should strive for a generalization to three dimensional models, which could more accurately describe the actual damage accumulation process in composite materials used in commercial applications.

The bridging model can be refined to account for spatially discrete fiber bridging, debond or matrix yield propagation along the bridging fibers, as well as crack front bowing. This improvement should reconcile the discrepancy in crack growth curves between the bridged crack and a crack in the neat matrix material. With the supplement of a failure criterion, e.g. rupture of one of the bridging fibers, life prediction would then be feasible.

A unification of the bridging and shear lag models is also desirable. Both scenarios are based on the same set of submechanisms, which interact differently. Although, it is ultimately the fibers that control failure in both cases, since they constitute the main load carrying members. However, a description that comprises both phenomena demands a deeper understanding of the basic mechanisms and how to model their progression than what is currently the case. To relate the models to life prediction of test coupons, it is a necessity to scale the results in volume for the statistical occurrence of a larger and more critical damage site.

Finally, vigorous attempts to extend the models for service applications are endorsed. In a distant perspective, this should include multidirectional laminates, variable amplitude loading, compressive loading, environmental fatigue, which all influence the fatigue behavior of composite material when
used in real applications.
Appendix A

Nomenclature

The symbols and abbreviations that are used in this thesis are listed here. The page number after each symbol and explanation indicates where the symbol was introduced.

A.1 List of symbols

\( A \)  
Cross-sectional area of a fiber, p. 94
Arbitrary crack surface increment, p. 125

\( a \)  
Half surface crack length, p. 118

\( b \)  
Maximum depth of a surface crack, p. 118

\( C \)  
Coefficient in the fatigue crack growth rate equation, p. 31

\( C_i(\phi) \)  
Functions describing the crack opening displacement field of an unbridged semi-circular surface crack, p. 125

\( c \)  
Exponent in the fatigue degradation model, p. 31

\( D \)  
Hypothetical damage metric characterizing a complex damage state in a composite, p. 30

\( E \)  
Effective orthotropic Young's modulus, p. 126

\( E_f \)  
Young's modulus of the fiber, p. 94

\( \Delta G \)  
Energy release rate range, p. 9

\( G_m \)  
Matrix shear modulus, p. 95

\( G_A \)  
Energy release rate associated with crack increment \( A \), p. 126

\( h \)  
Ply thickness, p. 94

\( K \)  
Stress intensity factor, p. 127

\( \Delta K \)  
Stress intensity factor range, p. 9
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$K_I$</td>
<td>Mode I stress intensity factor, p. 10</td>
</tr>
<tr>
<td>$K_A$</td>
<td>Stress intensity factor associated with crack increment $A$, p. 125</td>
</tr>
<tr>
<td>$K_c$</td>
<td>Material constant characterizing the fracture toughness, p. 30</td>
</tr>
<tr>
<td>$\Delta K_c$</td>
<td>Critical stress intensity factor range, p. 9</td>
</tr>
<tr>
<td>$\Delta K_{\text{eff}}$</td>
<td>Effective stress intensity factor range, p. 128</td>
</tr>
<tr>
<td>$K_f$</td>
<td>Influence weight of the fiber break, p. 97</td>
</tr>
<tr>
<td>$K_{k,m}$</td>
<td>Influence weight of matrix shear couple number $k$, p. 97</td>
</tr>
<tr>
<td>$\Delta K_m$</td>
<td>Stress intensity factor range in the composite matrix, p. 129</td>
</tr>
<tr>
<td>$\Delta K'_m$</td>
<td>Stress intensity factor range in the neat matrix material, p. 129</td>
</tr>
<tr>
<td>$\Delta K_{\text{nom}}$</td>
<td>Nominal stress intensity factor range, p. 128</td>
</tr>
<tr>
<td>$K_r$</td>
<td>Stress intensity factor for an unbridged crack, p. 125</td>
</tr>
<tr>
<td>$\Delta K_{\text{th}}$</td>
<td>Threshold value of the stress intensity factor range, p. 9</td>
</tr>
<tr>
<td>$k$</td>
<td>Factor in the fatigue degradation model, p. 31</td>
</tr>
<tr>
<td></td>
<td>Slope coefficient in the fatigue life relation, p. 39</td>
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<tr>
<td></td>
<td>Spring constant of the Hookean bridging law, p. 122</td>
</tr>
<tr>
<td>$L$</td>
<td>Likelihood function, p. 34</td>
</tr>
<tr>
<td></td>
<td>Fiber segment length for statistical interpretations, p. 102</td>
</tr>
<tr>
<td>$l$</td>
<td>Intercept term of the fatigue life relation, p. 39</td>
</tr>
<tr>
<td>$m$</td>
<td>Exponent in the fatigue crack growth rate equation, p. 31</td>
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<tr>
<td>$N$</td>
<td>Number of elapsed load cycles, p. 31</td>
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<tr>
<td>$N_{I,p}$</td>
<td>Lower boundary of the life scatter band at $\varepsilon_{I,p}$, p. 40</td>
</tr>
<tr>
<td>$N_{{I,1-p}}$</td>
<td>Upper boundary of the life scatter band at $\varepsilon_{I,p}$, p. 40</td>
</tr>
<tr>
<td>$N_{\text{III},p}$</td>
<td>Lower boundary of the life scatter band at $\varepsilon_{\text{III},p}$, p. 40</td>
</tr>
<tr>
<td>$N_{{\text{III},1-p}}$</td>
<td>Upper boundary of the life scatter band at $\varepsilon_{\text{III},p}$, p. 40</td>
</tr>
<tr>
<td>$N_{\text{end}}$</td>
<td>Number of cycles at censored life, p. 35</td>
</tr>
<tr>
<td>$N_f$</td>
<td>Number of cycles to failure, p. 9</td>
</tr>
<tr>
<td>$N_{\text{lim}}$</td>
<td>Number of cycles at the endurance point, p. 35</td>
</tr>
<tr>
<td>$n$</td>
<td>Total number of data samples, complete and censored, p. 34</td>
</tr>
<tr>
<td>$P_f(\cdot)$</td>
<td>Probability of failure, p. 102</td>
</tr>
<tr>
<td>$P_f(\sigma)$</td>
<td>Cumulative distribution function of the strength, p. 29</td>
</tr>
<tr>
<td>$P_n(\xi)$</td>
<td>Normalized stress in fiber number $n$, p. 95</td>
</tr>
<tr>
<td>$P_s(\sigma)$</td>
<td>Probability of survival, p. 29</td>
</tr>
<tr>
<td>$p$</td>
<td>Percentile value, p. 36</td>
</tr>
<tr>
<td>$p^*$</td>
<td>Stress scaling factor, p. 95</td>
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### A.1. List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$p(w)$</td>
<td>Bridging law, p. 116</td>
</tr>
<tr>
<td>$p_r(\sigma)$</td>
<td>Probability density function, p. 29</td>
</tr>
<tr>
<td>$p_n(x)$</td>
<td>Stress in fiber number $n$ at position $x$, p. 94</td>
</tr>
<tr>
<td>$q$</td>
<td>Number of samples that failed at $N_f \leq N_{lim}$, p. 35</td>
</tr>
<tr>
<td>$R$</td>
<td>Fatigue stress ratio $\left(\sigma_{\text{min}}/\sigma_{\text{max}}\right)$, p. 4</td>
</tr>
<tr>
<td>$R_p$</td>
<td>Reliability for $p$th percentile, p. 36</td>
</tr>
<tr>
<td>$r$</td>
<td>Number of complete data samples, p. 34</td>
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<tr>
<td>$r_0$</td>
<td>Crack radius at the onset of fiber bridging, p. 121</td>
</tr>
<tr>
<td>$S$</td>
<td>Total crack surface area, p. 121</td>
</tr>
<tr>
<td>$S'$</td>
<td>Area of the fiber bridged part of a crack surface, p. 121</td>
</tr>
<tr>
<td>$s$</td>
<td>Standard deviation of stress at fiber failure, p. 110</td>
</tr>
<tr>
<td>$\text{sgn}(\cdot)$</td>
<td>The signum function, p. 96</td>
</tr>
<tr>
<td>$T_n(\xi)$</td>
<td>Normalized shear stress in matrix bay number $n$, p. 95</td>
</tr>
<tr>
<td>$U$</td>
<td>Stored crack opening energy in a bridged crack, p. 125</td>
</tr>
<tr>
<td>$U_n(\xi)$</td>
<td>Normalized axial displacement in fiber number $n$, p. 95</td>
</tr>
<tr>
<td>$u_n(x)$</td>
<td>Axial displacement in fiber number $n$, p. 94</td>
</tr>
<tr>
<td>$w$</td>
<td>Fiber spacing, p. 94</td>
</tr>
<tr>
<td>$w_r$</td>
<td>Reference crack opening displacement field, p. 122</td>
</tr>
<tr>
<td>$w_s$</td>
<td>Surface crack opening displacement profile, p. 121</td>
</tr>
<tr>
<td>$Y$</td>
<td>Geometric factor in $K$, p. 127</td>
</tr>
<tr>
<td>$Y_r$</td>
<td>Geometric factor in $K_r$, p. 125</td>
</tr>
<tr>
<td>$Z_i$</td>
<td>Random variable number $i$ from a uniform distribution in $[0,1]$, p. 106</td>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$\alpha$</td>
<td>Debond length, p. 104</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Shape parameter of the Weibull distribution of fiber strength, p. 29</td>
</tr>
<tr>
<td>$\beta(\phi)$</td>
<td>Variation in stress state along the crack front, p. 126</td>
</tr>
<tr>
<td>$\bar{\beta}$</td>
<td>Effective value of $\beta$ along the crack front, p. 126</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Crack front contour, p. 122</td>
</tr>
<tr>
<td>$\Gamma(\cdot)$</td>
<td>Gamma function, $\Gamma(n) = (n-1)!$ for integers, p. 103</td>
</tr>
<tr>
<td>$\Gamma_n(\xi)$</td>
<td>Normalized shear strain in matrix bay number $n$, p. 95</td>
</tr>
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</table>
APPENDIX A. NOMENCLATURE

\( \gamma \) Exponent for non-linear scaling of fiber strengths, p. 102
\( \delta \) Length of an infinitesimal fiber segment, p. 104
\( \delta_{ik} \) Kronecker's delta function, p. 97
\( \varepsilon_{I,p} \) Lower boundary of the scatter band in Region I, p. 37
\( \varepsilon_{I,1-p} \) Upper boundary of the scatter band in Region I, p. 37
\( \varepsilon_{III,p} \) Lower boundary of the scatter band above Region III, p. 38
\( \varepsilon_{III,1-p} \) Upper boundary of the scatter band above Region III, p. 38
\( \varepsilon_{\text{max}} \) Maximum strain level, p. 9
\( \Lambda \) Stress in fiber imposed by unit a unit compressive stress at the fiber break, p. 96
\( \lambda(\xi) \) Stress profile imposed by the fiber break, p. 98
\( \xi \) Normalized axial coordinate, p. 95
\( \rho \) Radial coordinate in a semi-circular surface crack, p. 121
\( \Sigma(\xi) \) Stress concentration profile along the adjacent fiber, p. 98
\( \Sigma_0 \) Scale parameter of the Weibull distribution of fiber strength, p. 102
\( \Sigma_e(\alpha, \xi) \) Maximum stress profile for debond length \( \alpha \), p. 104
\( \Sigma_f \) Normalized fiber strength, p. 102
\( \Sigma_u \) Normalized location parameter of the three parameter Weibull strength distribution
\( \sigma \) Surface pressure field, p. 122
\( \sigma^* \) Stress normalization factor, p. 95
\( \sigma_0 \) Weibull scale parameter, p. 29
Applied far-field stress, p. 121
\( \sigma_e \) Initial strength at \( N = 0 \), p. 31
\( \sigma_{\text{max}} \) Maximum cyclic stress, p. 4
\( \sigma_{\text{min}} \) Minimum cyclic stress, p. 4
\( \sigma_r \) Reference surface pressure field, p. 122
\( \sigma_{\text{res}} \) Residual or instantaneous strength, p. 30
\( \sigma_u \) Weibull location parameter, p. 29
\( \tau_n(x) \) Shear stress in matrix bay number \( n \), p. 94
\( \Phi(\cdot) \) Cumulative distribution function of the standard normalized distribution, p. 33
\( \Phi_k \) Stress at the fiber break position imposed by a shear load couple, p. 97
A.2. List of abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>AE</td>
<td>Acoustic emission</td>
</tr>
<tr>
<td>ASTM</td>
<td>American society for testing and materials</td>
</tr>
<tr>
<td>BEM</td>
<td>Boundary element method</td>
</tr>
<tr>
<td>CMC</td>
<td>Ceramic matrix composite</td>
</tr>
<tr>
<td>CF</td>
<td>Carbon fiber</td>
</tr>
<tr>
<td>CFRP</td>
<td>Carbon fiber reinforced plastic</td>
</tr>
<tr>
<td>COD</td>
<td>Crack opening displacement</td>
</tr>
<tr>
<td>DSC</td>
<td>Differential scanning calorimetry</td>
</tr>
<tr>
<td>DCB</td>
<td>Double cantilever beam</td>
</tr>
<tr>
<td>DEN</td>
<td>Double edge notched</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite element method</td>
</tr>
<tr>
<td>GF</td>
<td>Glass fiber</td>
</tr>
<tr>
<td>GLS</td>
<td>Global load sharing</td>
</tr>
<tr>
<td>GMT</td>
<td>Glass mat thermoplastic</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>GRP</td>
<td>Glass fiber reinforced plastic</td>
</tr>
<tr>
<td>LEFM</td>
<td>Linear elastic fracture mechanics</td>
</tr>
<tr>
<td>LLS</td>
<td>Local load sharing</td>
</tr>
<tr>
<td>LRS</td>
<td>Laser Raman spectroscopy</td>
</tr>
<tr>
<td>MA-PP</td>
<td>Maleic anhydride modified polypropylene</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum likelihood</td>
</tr>
<tr>
<td>MMC</td>
<td>Metal matrix composite</td>
</tr>
<tr>
<td>NDE</td>
<td>Non-destructive evaluation</td>
</tr>
<tr>
<td>PEEK</td>
<td>Poly(ether ether ketone)</td>
</tr>
<tr>
<td>PEI</td>
<td>Poly(ether imide)</td>
</tr>
<tr>
<td>PI</td>
<td>Polyimide</td>
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<td>PMC</td>
<td>Polymer matrix composite</td>
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<tr>
<td>PP</td>
<td>Polypropylene</td>
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<tr>
<td>RTM</td>
<td>Resin transfer molding</td>
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<td>SEM</td>
<td>Scanning electron microscopy</td>
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<td>SEN</td>
<td>Single edge notched</td>
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<td>SIF</td>
<td>Stress intensity factor</td>
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<td>Tension-compression</td>
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<td>T-T</td>
<td>Tension-tension</td>
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<td>UTS</td>
<td>Ultimate tensile strength</td>
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<td>WF</td>
<td>Weight function</td>
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References


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