Lubrication at Impact Loading

by

ROLAND LARSSON
Lubrication at Impact Loading

av

Roland Larsson

Akademisk uppsats som för avläggande av teknisk licentiatexamen kommer att presenteras i
F341, onsdagen den 13 april, kl 10.00.

Licentiate Thesis 1994:08L
ISSN 0280-8242
ISRN HLU-TH-L--94/8-L-SE
Lubrication at Impact Loading

by

ROLAND LARSSON

1994:08 L
ISSN 0280 - 8242
ISRN HLU - TH - L -1994/8 - L - SE
This thesis comprises the following papers:


C. Roland Larsson and Jan Lundberg, 1993, “A simplified solution to the combined squeeze-sliding lubrication problem.”, accepted for publication in Wear.
INTRODUCTION

Much energy is used to compensate for frictional losses and losses associated with friction, such as cooling losses. The traditional and most common way to reduce friction is to lubricate, *i.e.* a fluid or a solid lubricant is used to separate two moving surfaces from each other. Lubrication also protects the surfaces from wear.

Even though experience of lubrication is very long, friction and wear cause very high costs. From the point of view of both costs and the environment, it is desirable to reduce these losses to a minimum.

In Sweden, the total value of electric power consumption is of the order of SEK 30 thousand million per year. The total value of petrol and diesel fuel consumption, within the transport sector, is of the order of SEK 50 thousand million per year. It is very hard to estimate what percentage of these costs is caused by friction, but a conservative guess would be 10%. This means, for the cases mentioned above, that SEK 8 thousand million is spent every year just to compensate for frictional losses.

Costs due to wear are more difficult to estimate, but total costs due to both friction and wear are often said to be about 7% of GNP. This corresponds to approximately SEK 100 thousand million per year in Sweden alone.

The science of wear, friction and lubrication is called tribology. Research within the field of tribology aims to cut costs due to friction and wear. Increased knowledge of the mechanisms of lubrication is crucial in order to reduce friction and to prevent lubricating film failures. Some questions of importance are:

- What are the important properties of a lubricant?
- How do conditions at the lubricated contact influence lubricating capability?
- How are lubricant properties to be measured?
- Is it possible to use mathematical modelling of the conditions at a lubricated contact?

It is important to understand why a lubricating film fails and it is important to be able to predict such failure. Finally, it is important to be able to determine the magnitude of the losses due to friction.

It is possible to distinguish between two different types of full-film lubrication: hydrodynamic lubrication and elastohydrodynamic lubrication (EHL). The latter is the kind of lubrication found in ball bearings and gears. In elastohydrodynamic lubrication, two non-conformal surfaces are separated by
the lubricant. The contact area is very small and a very high pressure arises. The deformation caused by this high pressure is much larger than the thickness of the lubricating film, which is normally less than 1 μm.

EHL research activities are usually both experimental and theoretical. In experiments it is possible to measure, for example, lubricant film thickness and friction at each particular moment and in every situation. It is also possible to measure rheological properties such as viscosity. These experimental results are only valid over a limited range of the involved parameters. Mathematical modelling is, therefore, necessary to obtain a more general understanding of the lubrication process. On the other hand, mathematical models are always only approximate and rough descriptions of a very complex reality, but even if an analysis does not give a quantitatively correct answer it often gives a qualitatively good answer.

Load and relative velocity between the lubricated surfaces are major factors with regard to developing a lubricating film that is thick enough to prevent metal-to-metal contact.
Most of the investigations performed in the field of elastohydrodynamics are only adequate for stationary conditions, by which is meant constant load and constant velocities. In many situations, however, dynamic effects are significant and it is no longer possible to apply stationary conditions.
Typical examples of lubrication at dynamic conditions can be found in gears between meshing teeth. Other examples are the cam and follower mechanism of a car engine and ball or roller bearings that are subjected to vibration.

Experimental and theoretical investigations become much more complex when the parameter of time is involved. This is the reason fewer studies are carried out of lubrication at non-steady conditions.
In this thesis, which is mainly theoretical, lubrication at transient conditions is studied. Two different cases are studied: the pure impact case and the case of lubrication at combined squeeze and sliding motion.
Pure impact

In the pure impact case, two surfaces, separated by a lubricating film, move towards each other without any rolling or sliding motion, see Fig. 1.

![Pure impact diagram](image)

**Figure 1.** Pure impact.

At pure impact, the lubrication effect originates from the “squeeze effect”. When the surfaces approach each other the lubricant is squeezed out from the contact region. Due to shear stresses within the fluid and between the fluid and the surfaces, the lubricant cannot leave the contact infinitely fast and a pressure arises in the thin lubricant film which is trapped between the approaching surfaces.

The pure impact case has been studied by several investigators, for example Christensen [1-2] and Safa&Gohar [3]. In their investigations the case of dropping a ball onto a lubricated surface was studied both experimentally and theoretically. Even though the ball dropping case is rare in reality, it includes all the effects of pure impact which are of interest at the same time as experimental investigations may be carried out relatively easily. Christensen [2] studied the case of dropping a ball in a simplified way. He assumed the pressure at the contact centre to be constant. This is, from an engineering point of view, not a realistic assumption. To study the ball dropping case it is necessary to solve the equation of motion of the ball. The ball inertia is thus taken into account.

Peiran & Shizhu [4] made a numerical analysis of the ball dropping problem. They solved the ball’s equation of motion to determine the ball’s position during impact. Unfortunately, they only presented results regarding the beginning of the impact. The rebound was not studied at all. Comprehensive and realistic analyses of the ball dropping problem are presented in both
Paper A and Paper B.

In experiments and theoretical analyses it is of interest to study formation of pressure in the contact and the thickness of the lubricating film. The film thickness study shows whether the surfaces come into contact with each other or not. Study of the influence on pressure and film thickness of different properties of the lubricant and the bounding surfaces is also a major task. The pressure in a thin lubricating film during impact can be solved using a non-linear partial differential equation known as Reynolds equation:

\[
\frac{\partial}{\partial r}\left( \frac{r h^2 \rho \partial p}{12 \eta \partial r} \right) = r \frac{\partial}{\partial t}(\rho h)
\]  

(1)

The film thickness \((h)\) depends on the deformations of the surfaces. This deformation depends on pressure. The non-linearity of the problem is then obvious. To determine pressure the film thickness has to be known and to determine film thickness the pressure has to be known.

Viscosity \((\eta)\) and density \((\rho)\) also depend on pressure. The viscosity increases greatly when the pressure increases and at pressures of 0.5-1 GPa the lubricant may reach a solid state.

It is not possible to find an analytical solution to Reynolds equation (1), and a numerical treatment is therefore necessary. Usually, the method of finite differences are used. The substantial non-linear relation between pressure, film thickness and viscosity also makes it necessary to use a very stable and iterative solution method.

In order to obtain a solution with reasonable accuracy, Reynolds equation must be discretized into a large number of nodal points, and a large set of equations must be solved. For this, it is essential to have a fast solver. It should also be noted that the time dependency makes it necessary to solve the set of non-linear equations at several time steps and to solve the equation of motion simultaneously.

Such a fast and efficient solver can be developed by using multigrid techniques, described by, for example, Briggs [5]. Multigrid is a numerical technique which enhances the convergence rate and efficiently reduces the computation time.

Multigrid techniques within the field of EHL are described by Lubrecht [6] and Venner [7]. The multigrid method is used in both Paper A and Paper B.

Paper A presents the results of an investigation of how pressure and film thickness varies during impact and rebound of the dropped ball. Reynolds equation (1), the elastic deformation, the lubricant property equations and the equation of motion are solved simultaneously at a large number of time steps.
during impact and rebound.
It was shown that the pressure in the lubricant film exceeded the corresponding pressure at dry impact. This means the pressure will be higher if the surfaces are lubricated and there may be a greater risk of surface failure due to plastic deformation. During rebound it was shown that the pressure along the periphery of the circular contact area reached a very sharp peak. At the very end of the total impact time, just before the ball left the lubricated surface, a very high pressure was reached at the centre of the contact, Fig. 2.
Safa&Gohar [3] presented experimental evidence of this pressure "spike". Their experimental results and theoretical results from Paper A are shown in Fig. 2. The correspondence between experiment and theory is good.

![Figure 2. Pressure at the contact centre during impact and rebound.](image)

In paper B the ball dropping case is analysed further. The influence of parameters such as initial impact velocity, ball mass, lubricant layer thickness, lubricant properties and surface properties on pressure, impact force and film thickness was studied.
It was, for example, shown that the minimum film thickness increased if the initial impact velocity was higher. It was shown that if impact velocity or ball mass are smaller than a certain critical value, the ball does not rebound at all. Furthermore, a phase shift in time was observed between the time of maximum impact force and the point of time where film thickness reached its minimum value.
Lubrication at combined squeeze and sliding motion

In machine parts such as gears and bearings, impact motion is combined with sliding or rolling motion, Fig. 3.

![Figure 3. Combined squeeze and sliding motion.](image)

This combined motion is unfavourable as regards lubrication. In, for example, heavily loaded roller bearings, failures have been observed at a position within the bearing where the roller is both sliding and approaching the outer and inner races. Due to friction the rotation of the roller decreases when it comes into the unloaded part of the bearing. This causes a sliding motion between the roller and the races. When it enters the loaded region, the gap between the outer and inner race narrows and an impact or squeeze motion arises.

Lundberg et al. [8-10] have studied lubrication at combined normal approach and sliding motion. Their experimental equipment simulated the case of combined motion by letting a rotating roller impact on a lubricated surface. They found that at constant impact velocity increased sliding velocity increased the risk of lubricant film breakdown. This result contradicts results found by studying a stationary loaded contact. Both theoretical and experimental investigations of steady loaded contacts show that a lubricant film becomes thicker as the sliding velocity increases; that is, there is a decreased risk of film failure.

Lundberg et al. also found that lubricant film failure usually occurs at the end of the total impact time, just before the roller rebound from the lubricated surface. One might ask why the breakdown does not occur right at the beginning of the impact or at the point of time when the impact force reaches its maximum.
Reasons as to why increased sliding velocity enhances the risk of lubricant film failure and why failure occurs at the very end of impact are put forward and discussed in Paper C.

A lubricated contact has both elastic and damping properties. Elasticity originates from, for instance, deformation of the surfaces and damping from viscous forces in the lubricating film. The phase shift in time between maximum force and minimum film thickness, i.e. failure, is explained by studying a system with one degree of freedom which includes mass, stiffness and damping properties. If such a system is impacted by any force, it is well known that maximum force and maximum deformation are shifted in time. In Paper C this model is used to serve as a qualitative explanation of the phenomena observed in the experiments.

A more advanced model is also used in the theoretical study presented in Paper C. This model shows that minimum film thickness occurs towards the end of the impact time. The model takes account of the behaviour of the lubricant film and it also accounts for the elasticity of the surfaces. It was not, however, possible to explain why increasing sliding velocity makes the film thinner when this model was used. Several factors were not included in the model and it is likely to believe that some of these factors may explain this. Such factors are non-Newtonian behaviour, temperature effects, inertia effects and starvation effects.

ACKNOWLEDGEMENTS

This work has been carried out at the Division of Machine Elements at Luleå University of Technology and was supported financially by the Swedish Research Council for Engineering Sciences (TFR). Professor Erik Höglund is my supervisor and he provided most valuable support during my studies. I would like to express my gratitude to him. I also acknowledge my co-author Assoc. Prof. Jan Lundberg for his infectious enthusiasm and the interest he has shown in my studies. Finally, my thanks to all my colleagues at the Department of Mechanical Engineering and especially to those at the Division of Machine Elements.

REFERENCES


Paper A
Numerical Simulation of a Ball Impacting and Rebounding a Lubricated Surface

Roland Larsson and Erik Höglund
October 1993

Division of Machine Elements
Luleå University of Technology

S-971 87 Luleå, SWEDEN

ABSTRACT

The case of a ball bouncing on a flat surface covered by a thin lubricant layer is analysed theoretically. Both impact and rebound are studied. A Newtonian lubricant and perfect elastic solids are assumed. As long as the ball approaches the flat surface the pressure in the contact increases and a lubricant entrapment is formed at the centre of the contact. When the ball begins to leave the surface, cavitation occurs. At the periphery of the contact a pressure spike is formed. Just before the ball leaves the lubricated surface, very high pressure values arise at and near the contact centre. These results are compared with the case of non-lubricated impact. It is found that the pressure in the contact at lubricated impact is higher than in the case of dry impact. Due to the elastic and damping properties of the lubricant film and the impacting surfaces, a time delay is observed between the time of maximum impact force and minimum film thickness. Comparing the theoretical results with experimental results, presented by other authors, shows good correlations.
NOTATIONS.

\( a_i, b_i \) Discretization coefficients.
\( b \) Reference Hertzian radius at load \( w_0 \).
\( D_{ij} \) Influence coefficients for deformation calculation.
\( E' \) Effective elastic modulus, \( 2\left(1-\nu_1^2\right)/E_1+(1-\nu_2^2)/E_2\)^{-1}.  
\( F, \bar{F} \) Parameters used in Reynolds equation.
\( h \) Film thickness or grid size.
\( h_0 \) Rigid separation constant.
\( h_c \) Film thickness in contact centre.
\( h_{00} \) Thickness of lubricant layer.
\( H \) Non-dimensional film thickness.
\( K \) Constant in Reynolds equation.
\( m, M \) Ball mass, with and without dimension.
\( p, P \) Film pressure, with and without dimension.
\( p_H \) Reference Hertzian pressure at load \( w_0 \).
\( P_H \) Non-dimensional pressure at dry impact.
\( r \) Radial co-ordinate.
\( R \) Ball radius.
\( t, T \) Time, with and without dimension.
\( T_c \) Total impact time at dry impact.
\( \Delta T \) Time step.
\( v_c, V_c \) Normal velocity of the ball’s centre of gravity, with and without dimension.
\( v_{c0}, V_{c0} \) Initial normal velocity of the ball’s centre of gravity.
\( w, W \) Impact force, with and without dimension.
\( w_0 \) Reference load, \( w_0 = mg \).
\( w_H \) Impact force at dry impact.
\( X \) Non-dimensional radial co-ordinate.
\( Z_R \) Constant in Roelands’ pressure-viscosity relation.
\( \alpha \) Pressure-viscosity coefficient.
\( \delta \) Total surface deformation.
\( \delta_z \) Surface deformation at dry impact.
\( \delta_{z0} \) Maximum surface deformation at dry impact.
\( \Delta \) Non-dimensional surface deformation.
\( \eta, \bar{\eta} \) Viscosity, with and without dimension.
\( \eta_0 \) Viscosity at ambient pressure.
\( \rho, \bar{\rho} \) Density, with and without dimension.
\( \rho_0 \) Density at ambient pressure.
1. **INTRODUCTION.**

An elastohydrodynamically lubricated contact is often subjected to non-steady loading conditions. Examples of situations where such conditions are present are e.g. a rolling element bearing running with high vibration amplitudes and a gear mesh. One important type of non-steady or transient loading is that of the normal approach of two elastic bodies which are separated by a thin lubricant layer. Even with moderate normal velocities, very high pressure levels are reached in the contact. Excessive pressure can cause permanent plastic deformations and thus damage the surfaces. It may also cause too thin lubricating film and give rise to metal to metal contact, causing excessive wear.

Experimental investigations of a situation where two bodies are approaching normally, i.e. pure squeeze action, show that a dimple is formed in the centre of the contact (Sanborn & Winer, 1971). That means that a lubricant entrainment is formed. Minimum film thickness occurs at the edges of the dimple. The same sort of phenomena are shown in theoretical studies (Christensen, 1970).

Safa & Gohar (1986) made experimental studies of the case of dropping a ball onto a lubricated surface. Using thin film transducers, they were able to measure the pressure distribution in the contact during impact and rebound of the ball. They found that the pressure at a certain position (e.g. in the centre) in the contact, reached two maxima, Fig. 1. During impact the pressure increases. When the ball starts to rebound from the lubricated surface the pressure decreases again, but in the end of the total impact time a second pressure peak occurs. The secondary peak can be of higher amplitude than the first peak, i.e. the primary peak.

![Figure 1. Principal behaviour of the pressure in the contact centre during impact and rebound.](image)

Christensen (1970) made a numerical study of a lubricant film squeezed between two spherical surfaces. He did not, however, study a natural process such as the case of dropping a ball onto a lubricated surface. He assumed the pressure at the contact centre to be constant. This is from an engineering point of view no realistic assumption. To study the ball dropping case it is necessary to solve the ball’s equation of motion and thus taking the ball inertia into account.

Peiran & Shizhu (1989) made a numerical analysis of the ball dropping problem. They solved the ball’s equation of motion to determine the ball’s position during impact. Unfortunately they only presented results for the beginning of the impact. The rebound was not studied at all. The primary peak as described by Fig. 1 was not reached in their analysis.

In this report the case of both impact and rebound is studied theoretically. The real physical problem is analysed, i.e. pressure and film thickness are computed at different stages of the bounce. The ball’s equation of motion is solved simultaneously and thus including ball inertia in the analysis.
2. THEORETICAL MODELLING.

The phenomena studied is the situation of a spherical ball which is dropped onto a flat lubricated surface. The initial velocity of the ball, when it comes into contact with the lubricating film, is assumed to be $v_{co}$. The contact region is circular, and due to the pure squeeze situation, both pressure and film thickness are concentrically distributed around the contact centre.

2.1 Governing equations.

By assuming isothermal conditions and a Newtonian lubricant, and by neglecting inertia effects within the lubricating film, the pressure distribution can be obtained from Reynolds equation in polar coordinates

$$\frac{\partial}{\partial r} (Fr \frac{\partial p}{\partial r}) = r \frac{\partial^2}{\partial t^2} (\rho h) \tag{1}$$

where $F$ is

$$F = \frac{\rho h^3}{12\eta} \tag{2}$$

The radial co-ordinate, $r$, has its origin in the centre of the contact. To solve the pressure from (1), film thickness and lubricant properties, such as viscosity and density, have to be known. Using the conventional parabolic approximation, the film thickness can be expressed as:

$$h(r, t) = h_0(t) + \frac{r^2}{2R} + \delta(r, t) \tag{3}$$

where $h_0$ is the "rigid separation", i.e. the distance between the ball and the surface at the centre of the contact, for the situation where the ball and the flat surface are rigid. The deformation of both surfaces, $\delta$, is determined by the pressure.

The relationship between viscosity and pressure can be expressed as (Roelands, 1966):

$$\eta = \eta_0 e^{(ln\eta_0 + 9.67) \left[ 1 + (1 + 5.1 \times 10^{-9}p)^{Z_R} \right]} \tag{4}$$

where $Z_R$ can be obtained if the pressure-viscosity coefficient, Barus' $\alpha$, is known:

$$Z_R = \alpha / [5.1 \times 10^{-9} (ln\eta_0 + 9.67)] \tag{5}$$

According to (Dowson-Higginson, 1966) the relation between density and pressure can be written as:

$$\rho = \rho_0 \left(1 + \frac{0.6 \times 10^{-9} p}{1 + 1.7 \times 10^{-9} p} \right) \tag{6}$$
The boundary conditions for the pressure distribution are:
\begin{align*}
p (r, 0) &= 0 \quad (7a) \\
p (r \to \infty, t) &= 0 \quad (7b) \\
\frac{\partial p (0, t)}{\partial r} &= 0 \quad (7c) \\
p (r, t) &\geq 0 \quad (7d)
\end{align*}

The condition \( (7d) \) represents the cavitation boundary condition. Subambient pressure is not permitted.

2.2 Ball motion

\[ m \ddot{z} (t) = w (t) - mg \]  \hspace{1cm} (8)

where \( z \) is a co-ordinate describing the position of the ball’s centre of gravity, and can be defined as:
\[ z (t) = R + h_0 (t) \]  \hspace{1cm} (9)

Using equation (9), the equation of motion (8) can be written as:
\[ h_0 (t) = \frac{w (t)}{m} - g \]  \hspace{1cm} (10)

Equation (10) is used to determine new values of the rigid separation constant, \( h_0 \).

The load capacity of the lubricating film is obtained from:
\[ w (t) = 2 \pi \int_{0}^{\infty} p (r, t) r \, dr \]  \hspace{1cm} (11)
2.3 Non-dimensional form.

To transform the problem into a convenient non-dimensional form, the following dimensionless parameters have been found suitable:

\[ X = \frac{r}{b} \quad H = \frac{hR}{b^2} \quad P = \frac{p}{p_H} \quad \bar{\eta} = \frac{\eta}{\eta_0} \quad \bar{\rho} = \frac{\rho}{\rho_0} \]

\[ T = \frac{tE'}{\eta_0} \quad \bar{F} = \frac{FR^3\eta_0}{\rho_0b^6} \quad \Delta = \frac{\delta R}{b^2} \quad M = \frac{mb^2E'}{R^3\eta_0^2} \quad W = \frac{w}{E'R^2} \]

where \( p_H \) is the reference Hertzian pressure:

\[ p_H = \frac{3w_0}{2\pi b^2} \tag{12} \]

and \( b \) is the reference Hertzian radius:

\[ b = \left( \frac{3Rw_0}{2E'} \right)^{1/3} \tag{13} \]

The reference load \( w_0 \) is chosen as the gravity of the mass i.e.

\[ w_0 = mg \tag{14} \]

The dimensionless form of acceleration, \( a, \) and ball velocity, \( v_c, \) is thus:

\[ A = \frac{aRn_0^2}{b^2E'} \quad V_c = \frac{v_cRn_0}{b^2E'} \]

giving

\[ G = \frac{gRn_0^2}{b^2E'^2} \]

Reynolds equation is in non-dimensional form:

\[ \frac{\partial}{\partial X}(XF\frac{\partial P}{\partial X}) = KX \frac{\partial}{\partial T}(\bar{\rho}H) \tag{15} \]

where

\[ K = \frac{2\pi}{3W_0} \quad \bar{F} = \frac{\bar{\rho}H^3}{12\bar{\eta}} \]
The film thickness equation:

\[ H(X, T) = H_0(T) + \frac{X^2}{2} + \Delta(X, T) \]  \hspace{1cm} (16)

and the equation of motion is:

\[ \ddot{H}_0 = \frac{W}{M} - G \]  \hspace{1cm} (17)

The load capacity of the film in non-dimensional form is:

\[ W = 3W_0 \int P(X, T) X dX \]  \hspace{1cm} (18)

2.4 The deformation of the solid surfaces.

The deformation of the ball and the flat surface is treated as elastic and quasi-static. The assumption of quasi-static theory is valid if the duration of the impact is long enough to permit stress waves to be transmitted through the ball several times (Johnson, 1985). Johnson gives an expression which may be used to determine whether quasi-static theory is a reasonable approximation:

\[ T_c \frac{T_p}{T} = 0.4 \frac{R v c_0}{b^* c_0} \]  \hspace{1cm} (19)

where \( b^* \) is the maximum value of the Hertzian radius during impact and \( c_0 \) is the speed of wave propagation in the solids. If the quotient in equation (19) is much less than unity, conditions are quasi-static.

To calculate the static deformation due to a pressure distribution \( P(X) \), influence coefficients \( D_{ij} \) are introduced. The deformation can thus be computed at discrete points \( i \) as a sum of the deformation contributions from all pressure points \( j \):

\[ \Delta_i = \sum_j D_{ij} P_j \]  \hspace{1cm} (20)

where the influence coefficients, \( D_{ij} \), are computed according to (Peiran & Shizhu, 1989). The expressions for \( D_{ij} \) are given in Appendix.
3 NUMERICAL SOLUTION.

3.1 Discretization.

Equation (15) has to be solved numerically. The non-linearities introduced by equations (4,6,16) make it necessary to use an iterative solution method. The Reynolds equation (15) has to be discretized and solved at a finite number of locations along the radial coordinate axis $X$. The discretized form of equation (15) can be written as:

$$a_i P_{i+1} - (a_i + b_i) P_i + b_i P_{i-1} = \frac{K X_i}{\Delta T} \left[ (\bar{p} H)_i^k - (\bar{p} H)_{i-1}^k \right]$$

(21)

where

$$a_i = \frac{1}{2h^2} \left[ (\bar{F}X)_i + (\bar{F}X)_{i+1} \right]$$

$$b_i = \frac{1}{2h^2} \left[ (\bar{F}X)_i + (\bar{F}X)_{i-1} \right]$$

where $h$ is the mesh size, i.e. $h = X_i - X_{i-1}$. The superscript $k$, denoting discretization in time, has been omitted except in the right-hand side of equation (21). All pressure values, $P_i$, and all coefficients, $a_i$ and $b_i$, are referred to time step $k$.

At $X=0$, equation (15) has no meaning. To take account of the boundary condition (7c), the following equation is introduced at $i=1$, i.e. $X=0$:

$$- 3P_1 + 4P_2 - P_3 = 0$$

(22)

Equation (22) is derived by fitting a second degree polynomial to the first three grid points and applying boundary condition (7c) in $X=0$.

The rigid separation, $H_0$, in each time step can be determined by using the rigid separation, the ball velocity and the ball acceleration in the previous time step. Using Taylor expansion, $H_0$ can be approximated as:

$$H_0^k = H_0^{k-1} + V_c^{k-1} \Delta T + H_0^{k-1} \frac{(\Delta T)^2}{2}$$

(23)

where $\dot{H}_0$ is found from equation (17).

3.2 Multigrid techniques.

The numerical solution of a discretized system of equations (21), (22) and (23), generally requires an iterative procedure such as Gauss-Seidel relaxation. However, such an iterative process is usually only effective in reducing non-smooth error components. For smooth error components, on the other hand, it is extremely ineffective, which leads to a very slow overall convergence speed, and excessive computing times. This problem can be overcome by using multigrid techniques. Using a set of coarser grids in what is called a coarse grid correction cycle these techniques ensure that each component of the error is resolved at a grid at which it is non-smooth and consequently relaxations are effective, i.e. the smoother the component the coarser the grid at which it is resolved. Based on the path followed from finest to coarsest grid and back again, different forms of correction cycles can be distinguished, for example V or W cycles, see (Brandt, 1984).
In addition to this convergence acceleration, coarse grids can also be used as a tool to generate an accurate first approximation. This procedure can also be applied recursively and is generally referred to as a FMG algorithm. Such an algorithm, incorporating W cycles, has been developed for the solution of the problem discussed in this paper and the FMG process has been applied at each time step.

From the above it may be clear that a (stable) relaxation procedure, which effectively reduces high frequency components, is the core of a multigrid solver. Such a relaxation is described below for the present case. For more information with respect to multigrid techniques the reader is referred to the introduction written by Briggs (1987). A more detailed description including many applications can be found in (Brandt, 1984). Finally an extensive description of the application to EHL problem has been given by Lubrecht (1987) and by Venner (1991).

3.3 Relaxation.

Two relaxations methods have been used. Gauss-Seidel relaxation for cases with relatively low load and distributive Jacobi relaxation for the high load cases, (Venner, 1991). In the case of Gauss-Seidel relaxation, the pressure is updated immediately in each point as:

\[ p_i^{new} = p_i^{old} + \delta p_i \]  \hspace{1cm} (24)

where \( \delta p_i \) can be obtained from:

\[ \delta p_i = r_i \left( \frac{\partial L_i}{\partial p_i} \right)^{-1} \] \hspace{1cm} (25)

where \( r_i \) is the residual of equation (21) and \( L \) is an operator also derived from equation (21) by taking the film thickness equation into account:

\[ r_i = \frac{KX_i}{\Delta T} \left[ (\tilde{\rho}H)_i^k - (\widetilde{\rho}H)_i^{k-1} \right] - a_i p_{i+1} + (a_i + b_i) p_i - b_i p_{i-1} \] \hspace{1cm} (26)

\[ L = a_i p_{i+1} - (a_i + b_i) p_i + b_i p_{i-1} - \frac{KX_i \tilde{\rho}_i}{\Delta T} \left[ H_0 + \frac{X_i^2}{2} + \sum_j D_{ij} p_j \right] + \frac{KX_i (\tilde{\rho}H)_i^{k-1}}{\Delta T} \] \hspace{1cm} (27)

The denominator of equation (25) is obtained by differentiating equation (27) with respect to \( p_i \), where \( a_i, b_i, H_i \) and \( \tilde{\rho}_i \) are also functions of \( p_i \). In this analysis the derivative \( \partial \rho_{ij}/\partial p_i \) has been neglected since it is assumed to be small compared with the other derivatives involved. For the case of distributive Jacobi relaxation, the new pressure at \( i \), is computed as:

\[ p_i^{new} = p_i^{old} + \delta p_i - \delta p_{i+1} \] \hspace{1cm} (28)

where \( \delta p_i \) can be found from:

\[ \delta p_i = r_i \left( \frac{\partial L_i}{\partial p_i} - \frac{\partial L_i}{\partial p_{i-1}} \right)^{-1} \] \hspace{1cm} (29)

where the derivatives are obtained in the same way as for the Gauss-Seidel method. The Jacobi method implies that the pressure distribution is not updated until all grid points have been visited.
For $i=1$, i.e. $X=0$, the change $\delta P_i$ can for both methods be written as:

$$\delta P_i = -\frac{1}{3} (3P_1 - 4P_2 + P_3)$$

(30)

The criterion for the selection of Gauss-Seidel or distributive Jacobi methods is based on the minimum value of all coefficients $a_i$ and $b_i$. If this minimum value is smaller than a prescribed limit the distributive Jacobi method is used; otherwise the Gauss-Seidel relaxation method is used. As soon as a new pressure distribution is found, lubricant properties and film thickness are updated. Due to the non-linearities, underrelaxation has to be introduced in equations (24) and (28). Typical underrelaxation factors used in this analysis are 0.6 for the Gauss-Seidel relaxation and 0.25 for the distributive Jacobi relaxation. It should also be noted that it is only necessary to make a few relaxations on each grid.

The cavitation boundary condition (7d) is handled as follows. In the case of the Gauss-Seidel method, a negative pressure $P_i$ is replaced by zero. If, in the case of distributive Jacobi method $P_i<0$, then $\delta P_i$ is recalculated to receive $P_i=0$.

At the cavitation boundary there is one more boundary condition. To maintain flow continuity on the cavitation boundary, the pressure gradient $\partial P/\partial X$ has to be zero. This condition is, however, automatically fulfilled by using the numerical method described above.

3.4 Solution process.

Equation (15) has to be solved in a finite region $[0,X_{\text{max}}]$, where the boundary condition (7b) is assumed to be valid in $X=X_{\text{max}}$. It is important that a sufficiently large value of $X_{\text{max}}$ is selected in order to reduce the influence of introducing the simplification of an upper limit of $X$. That means the effect on the impact force, $W$, has to be negligible.

The initial value of $H_0$ selected, i.e. the thickness of the lubricant layer $H_{oo}$, must be large enough to make $W$ small at $T=0$ compared with the maximum value of $W$. Otherwise, if $H_{oo}$ is too small, it is necessary that the limited wetting of the ball, which occurs at the initial stage of impact, is taken into account.

At the initial stage of impact, $T=0$, the ball has just reached the lubricant layer on the flat surface. The load capacity is, therefore, $W(0)=0$, and the surfaces are not deformed. The acceleration of the ball is only due to gravity and the initial ball velocity is $V_0$.

The time stepping process is as follows:

1) Compute $H_0$ at time step $k$ by using equation (23).
2) Solve iteratively Reynolds equation (15) and the film thickness equation (16).
3) Compute load capacity, $W$, from equation (18).
4) Proceed to next time step, $k+1$, and repeat 1-4. Use the pressure distribution from the previous time step as an initial condition.

This procedure is repeated until the ball has rebounded and the pressure in the lubricant film has vanished.
4. **DRY IMPACT.**

For comparison, the dry impact case is also studied. Elastic and quasi-static Hertzian theory is assumed. The impact force will be (Johnson, 1985):

\[ w_H(t) = \frac{2}{3} \sqrt{R} E \delta_z(t)^{3/2} \quad (31) \]

where the deformation in the contact centre, \( \delta_z(t) \), is approximated as:

\[ \delta_z(t) = \bar{\delta}_z \sin \left( \frac{\pi t}{T_c} \right) \quad (32) \]

where \( T_c \) is the total impact time for a dry contact

\[ T_c = 2.87 \left( \frac{4m^2}{RE^2v_{c0}} \right)^{1/5} \quad (33) \]

and \( \bar{\delta}_z \) is the amplitude of \( \delta_z \)

\[ \bar{\delta}_z = \left( \frac{15m v_{c0}^2}{8E' \sqrt{R}} \right)^{2/5} \quad (34) \]

The quotient between the dry impact pressure at the contact centre and the reference Hertzian pressure, \( p_H \), can be written as:

\[ P_H = \left( \frac{w_H(t)}{w_0} \right)^{1/3} = \left( \frac{W_H(T)}{W_0} \right)^{1/3} \quad (35) \]

The ball is assumed to continue to accelerate from \( h=h_0 \) until it reaches the dry glass surface. The dry impact force, therefore, starts to rise some microseconds later than in the lubricated case.
5. RESULTS AND DISCUSSION.

The case studied here is a numerical simulation of the experiments performed by Safa & Gohar (1986). A 25.4 mm steel ball is dropped onto a lubricated glass surface. The elastic modulus of the ball is assumed to be 210 000 MPa and that of the glass 70 000 MPa. The Poisson ratio is 0.3 and 0.25 respectively. The values of the various input parameters are shown in Table 1.

Table 1. Computational data.

<table>
<thead>
<tr>
<th>m [kg]</th>
<th>R [m]</th>
<th>v_o [m/s]</th>
<th>h_00 [m]</th>
<th>E' [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.066</td>
<td>0.0127</td>
<td>-0.313</td>
<td>30x10^-6</td>
<td>1.1x10^11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>η_0 [Pas]</th>
<th>α [Pa^-1]</th>
<th>Δt [μs]</th>
<th>b [m]</th>
<th>P_H [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.43</td>
<td>2x10^-8</td>
<td>1.0</td>
<td>4.822x10^-5</td>
<td>132.95</td>
</tr>
</tbody>
</table>

The upper limit of the computational region was in the beginning chosen as, X_max = 15. When more than half of the region was cavitated, X_max was reduced to 7.5, 3.75 and so on.
Six different grid sizes (levels) were used. The finest grid was made up of 513 nodes and the coarsest of 17 nodes.

The results are shown in figures 3 to 15. The impact force during impact and rebound, for both the dry and the lubricated case, is shown in Fig. 3. It is seen that the total impact time is approximately 241 μs in the lubricated case and 148 μs in the dry impact case.
The dry impact force starts to rise at about t=96 μs, which is the time required to fall the distance h_00. Maximum impact force is reached at different times for the two different cases. In the lubricated case, the maximum was reached at t=145 μs and for the dry impact at t=170 μs.

![Figure 3. Impact force in the lubricated and in the dry impact case and experimental results from (Safa & Gohar, 1986).](image-url)
Comparing theory and practice gives poor correlation, as seen in Fig. 3. The force-time behaviour is qualitatively similar but the maximum impact force in the experiments was approximately three times less the theoretical impact force. This experimental value seems, however, to be underestimated, because the impulse \( \int \! wdt \) is too small and is not sufficient to give the ball an upward motion.

As a consequence of the behaviour of the impact force, the maximum deformation, shown in Figure 4, reaches its peak value earlier in the lubricated case than in the dry impact case. This peak value is approximately the same for both cases, although the impact force is greater in the non-lubricated case. The maximum deformation is found to be 16 \( \mu \)m.

![Figure 4. Deformation in the centre of the contact during the total impact time. Lubricated and dry impact.](image)

The velocity of the ball’s centre of gravity is shown in Fig. 5. The initial impact velocity is -0.313 m/s. After rebound the ball velocity is 0.257 m/s. Thus a restitution coefficient of 0.82 can be obtained. The ball reaches its turning point, i.e. \( V_c = 0 \), at \( t = 158 \, \mu \)s.

Fig. 6 shows the rigid separation \( (H_r) \), the film thickness at the contact centre \( (H_c) \) and the minimum film thickness \( (H_{min}) \). The smallest value of \( H_{min} \) was found at \( t = 176 \, \mu \)s and is 2.9 \( \mu \)m. Maximum impact force and absolute minimum film thickness did not coincide in time. The phase shift in time was 31 \( \mu \)s. This phase shift is expected because both damping and elastic properties are present in the system. Lundberg et al (1990) studied the case of impact with a superimposed sliding motion. They found that lubricant film breakdown always occurred towards the end of the total impact time and not when the impact force reached its maximum. A phase shift in time was thus detected between impact force and film thickness, which corresponds well to the theoretical results obtained by this analysis.
Figure 5. Ball velocity versus impact time.

Figure 6. Rigid separation, film thickness in contact centre and minimum film thickness during the total impact time.
Figures 7-10 show how the pressure and film thickness distributions vary with time. A pressure spike develops when the surfaces begin to separate. It should be noted that the pressure spike is not fully converged. The finest grid is too coarse to describe the pressure spike accurately. This implies that the height of the spike is somewhat erroneous.

During the impact a fluid entrapment is formed in the centre of the contact, figures 9 and 10. This entrapment grows both in depth and in radial direction as long as the ball approaches the lower surface. When the ball starts to separate, the surfaces begin to rebound and the entrapment decreases.

Figure 7. Pressure distributions at different times.

Figure 8. Pressure distribution for lubricated impact with time as parameter.
Figure 9. Film thickness distributions at different times.

Figure 10. Film thickness distribution for lubricated impact with time as parameter.

Fig. 11 shows the maximum existing pressure during the lubricated bounce. At the pressure spike very high pressure levels are reached towards the end. The spike grows higher and higher and reaches a maximum height of about 67pH. Of course, in reality the pressure can never reach these values due to several effects, which have not been taken into account within this analysis. The assumption of perfect elastic material is hardly valid when the pressure rises much above 3 GPa as it does in the spike region. Plastic flow may occur, which naturally influences the film thickness and, consequently, the pressure.
Non-newtonian effects prevent the pressure gradients to be very high, and thus very sharp pressure spikes cannot develop. Newtonian conditions probably hold in this case except in the spike region where these high pressure gradients are found. It is likely that the lubricant’s shear stress limit is exceeded in this region. The shear stresses that prevent the lubricant from flowing out from the central entrapment are thus overestimated and the lubricant trapped in the central dimple will more easily flow out. Therefore, the damping effect of the lubricant film is lower in reality. It is likely that the lubricant’s shear stress limit is exceeded in this region. The shear stresses that prevent the lubricant from flowing out from the central entrapment are thus overestimated and the lubricant trapped in the central dimple will more easily flow out. Therefore, the damping effect of the lubricant film is lower in reality.

It is seen in Fig. 11 that there are irregularities in the $P_{\text{max}}$ diagram. These irregularities show the degree of error in pressure spike height. The errors at the pressure spike do not, however, significantly affect the overall results such as impact force, ball velocity and pressure in the contact centre.

![Figure 11. Maximum pressure in the contact versus impact time.](image)

Fig. 12 shows the pressure at the contact centre. In the lubricated case a primary peak is formed at $t=163\ \mu s$ and a very sharp secondary peak towards the end. The dashed curve in Fig. 12 shows the pressure in the contact centre for the non-lubricated case. This pressure is also the maximum pressure for the case of dry impact.

The experimental results (Safa&Gohar, 1986), giving a secondary peak in the contact centre pressure, are proved theoretically in this analysis. There is good agreement between experimental and theoretical pressure at the contact centre (Fig. 12), although the elastic modulus of the glass used by Safa & Gohar is not known precisely. The experimental curve is matched into the diagram to receive the primary peak at the same point of time as the theoretical curve.

Studying figures 11 and 12, it is interesting to note that the pressure is higher for the lubricated impact than for the dry impact. The risk of plastic deformation, i.e. damage, of the surfaces is thus higher in the lubricated case. Both Christensen (1962) and Rabinowitz (referred to by Bowden&Tabor, 1986) have experimentally noted that dents caused when spheres collide are deeper, and thus more severe, if a lubricant layer is applied. These results are in good agreement with the present theoretical results.
From Fig. 13 it can be concluded that the pressure distribution is more concentrated when a lubricant layer is applied, while the dry impact pressure is lower in magnitude but is distributed over a larger area. This is also the reason why the maximum deformation (Fig. 4), which is the contact centre deformation, is approximately the same for both lubricated and dry impact. Pressure at large values of $X$ has great influence on the contact centre deformation.

The shape of the pressure distribution for the lubricated case also explains why Christensen (1962) found that dents, caused by colliding spheres, sometimes have a smaller diameter than the corresponding dry impact dent.
Fig. 14 shows how the cavitation boundary moves during the rebound. Cavitation occurs at $t=166$ µs. The cavitation boundary moves very rapidly towards the contact centre. The cavitation boundary location is defined as the first grid point where the pressure has fallen to zero, i.e. the cavitation boundary can only be located at the discrete $X$ co-ordinates defined by the grid points. This explains the irregularities in the $X_{cav}$-curve.

**Figure 14.** The location of the cavitation boundary during impact and rebound.

Fig. 15 shows the pressure distributions at $t=170$ ms both experimentally and theoretically. The difference is considerable. No pressure spike is found in the experimental pressure distribution. It should be remembered that the transducer used in the experiments has an active region with a size of about 15 µm. This means that the experimental pressure values are some kind of mean pressure and thus it is not possible to find the relatively narrow pressure spike.

**Figure 15.** Pressure distributions after 170 µs. Present analysis and experimental from (Safa&Gohar, 1986).
An amusing effect arises when comparing the pressure in the contact centre, Fig. 12, with the pressure distribution in a line contact at pure rolling, see for example (Hamrock, 1991). Striking similarities exist. But in this case the variable on the horizontal axis is time instead of a spatial coordinate. This means that the right-hand side of Reynolds equation (1), $\partial h/\partial t$ and $\partial h/\partial x$ respectively, behaves in the same way in both cases. Comparing contact centre film thickness with the film thickness distribution in a line contact at pure rolling gives the same effect, Fig. 6.

Quasi-static theory remains valid because the quotient in equation (19) is about $8 \times 10^{-4}$, which is far below unity.

According to (Bowden&Tabor, 1986) there is no risk for significant influence of thermal effects in this case. In an experiment two spherical bodies collided while the temperature in the contact was measured. The impact velocity was 1 m/s, but only a few degrees temperature rise was observed. In the case studied in this report the velocity is only 0.313 m/s and the heating effect is thus even lower.

5.1 Convergence

To determine if the pressure distribution at each time step has converged, the following difference norm is used, see (Lubrecht, 1987) and (Venner, 1991):

$$ERR(h, H) = H \sum |P^H_i - I_h^H P^h_i|$$

(36)

where $h$ and $H$ are the mesh sizes on a fine and a coarse grid respectively. $P^H$ is the solution received after a number of $W$ cycles on the coarser grid and $P^h$ is the solution received after a number of $W$ cycles on the finer grid. The parameter $I_h^H$ is an intergrid operator which transfers the solution on the finer grid to the coarser grid.

If a number of $W$ cycles are performed, the algebraic errors decrease, and the $ERR$ function is dominated by the discretization error. Since a solution where the error is dominated by the discretization error is as good an approximation to the "exact" continuous problem as an exact solution to the discretized problem, a converged solution is obtained.

All solutions presented here, has a numerical error per time step which is much smaller than the discretization error.

In addition, accuracy compared with the continuous solution, was checked by comparing a characteristic parameter, such as the minimum film thickness, as a function of time for a sequence of solutions obtained using increasingly finer mesh size and time step. From this analysis, it was concluded that the error in the results presented here is less than 1.5%. However, the error in pressure spike height is, as mentioned above, higher.
6. CONCLUSIONS.

By solving the elastohydrodynamic problem for the pure squeeze case, i.e. a ball impacting a thin lubricant layer, and assuming Newtonian lubricant and perfect elastic solids, the following conclusions can be drawn.

- The maximum impact force is less if the surfaces are lubricated than if the surfaces are without lubricant. This is due to the damping effect of the lubricant film.

- A phase shift in time between impact force maximum and film thickness minimum arises when a lubricant is applied. This is due to the influence of lubricant film damping on the dynamic behaviour of the ball.

- Maximum pressure is greater if a lubricant is applied compared with the case of no lubrication. The pressure distribution is concentrated at a smaller area in the lubricated case.

- Cavitation occurs when the ball starts to separate from the lower surface. The cavitation boundary moves rapidly in towards the contact centre. A pressure spike will be formed along the peripheral outlet of the contact.

- The pressure in the secondary peak may be higher than that in the primary peak. This indicates that it is the final part of the bounce process which is most critical with regard to plastic deformation damage.

- The risk of plastic deformation of the surfaces of two curved bodies is greater if a lubricant is applied between them than if it is not.
7. ACKNOWLEDGEMENT.

The authors would like to thank Dr. C H Venner, University of Twente the Netherlands, for interesting discussions about numerical analysis and in particular multigrid techniques. These discussions have been of great value. Thanks are also due to the Swedish Research Council for Engineering Sciences (TFR) for their financial support.
8. REFERENCES.


The influence coefficients for the deformation calculation are computed according to (Peiran–Shizhu, 1989). The deformation at node $i$ is computed as

$$
\Delta_i = \sum_j D_{ij} P_j
$$

(20)

where

$$
D_{ij} = \frac{4}{\pi} (X_{j+1/2}) \quad (i = j = 1)
$$

$$
D_{ij} = \frac{4}{\pi} (X_{j+1/2} - X_{j-1/2}) \quad (i = 1, j \neq 1)
$$

$$
D_{ij} = \frac{8}{\pi^2} X_j f_1 (X_{j+1/2}, X_i) \quad (i \neq 1, j = 1)
$$

$$
D_{ij} = \frac{8}{\pi^2} \left[ X_{j+1/2} f_2 (X_i, X_{j+1/2}) - X_{j-1/2} f_2 (X_i, X_{j-1/2}) \right] \quad (i < j, j \neq 1)
$$

$$
D_{ij} = \frac{8}{\pi^2} \left[ X_{j+1/2} f_2 (X_i, X_{j+1/2}) - X_j f_1 (X_{j-1/2}, X_i) \right] \quad (i = j, j \neq 1)
$$

$$
D_{ij} = \frac{8}{\pi^2} \left[ X_j f_1 (X_{j+1/2}, X_i) - X_j f_1 (X_{j-1/2}, X_i) \right] \quad (i > j, j \neq 1)
$$

where the functions $f_1, f_2, X_{j+1/2}$ and $X_{j-1/2}$ are defined as

$$
f_1 (X_1, X_2) = \int_0^{\pi/2} \left[ 1 - \left( \frac{X_1}{X_2} \right)^2 \sin^2 \theta \right]^{1/2} d\theta - \left( 1 - \left( \frac{X_1}{X_2} \right)^2 \right) \int_0^{\pi/2} \left[ 1 - \left( \frac{X_1}{X_2} \right)^2 \sin^2 \theta \right]^{-1/2} d\theta
$$

$$
f_2 (X_1, X_2) = \int_0^{\pi/2} \left[ 1 - \left( \frac{X_1}{X_2} \right)^2 \sin^2 \theta \right]^{1/2} d\theta
$$

$$
X_{j+1/2} = (X_j + X_{j+1}) / 2 \quad X_{j-1/2} = (X_j + X_{j-1}) / 2
$$
Paper B
**Elastohydrodynamic lubrication at impact loading.**

Roland Larsson and Erik Höglund

February 1994

Division of Machine Elements
Luleå University of Technology

S-971 87 Luleå, SWEDEN

**ABSTRACT**

Experimental and theoretical studies of elastohydrodynamically lubricated contacts normally assume static or quasi-static conditions. Non-steady conditions are, however, very common, e.g. in machine elements such as ball bearings, gears and cam-follower mechanisms.

In this paper the case of a ball impacting a flat lubricated surface is investigated theoretically. This case implies transient conditions and the lubricating effect is due to pure squeeze action in the contact.

Pressure and film thickness distributions are computed during impact and rebound. The results of the analysis show the effects of ball mass, initial impact velocity, lubricant properties and the thickness of the applied lubricant layer on, for example, minimum film thickness, maximum impact force and maximum pressure.

Increasing impact velocity increases the minimum value of film thickness achieved during the total impact time. The damping capacity of the lubricating film is very high at low impact velocity and small ball mass. In fact, the damping is so high that no rebound occurs if the velocity or the ball mass are smaller than certain critical values.

The thickness of the lubricant layer has very little influence on the results if it is thicker than a certain value. If the pressure-viscosity coefficient is increased, the film becomes thicker.
1. INTRODUCTION.

The case of colliding lubricated bodies and other situations of impact loading are problems which have not yet been fully studied. These problems are encountered in, for example, ball bearings subjected to vibrations. Meshing gear teeth and cam-follower mechanisms are also examples of machine elements where impact loading is commonly present, even though the contact geometry may differ from the spherical type and sliding motion is present in these cases.

In the field of fluid mechanics several researchers have paid attention to particle collision in the study of two-phase particle/fluid flows, e.g. (Davis et al., 1986). In cases such as filtration and coagulation it is, for example, interesting to predict whether a solid particle will stick or rebound to other particles or boundary surfaces. Lubricated spheres are good approximations to these particles.

In the field of tribology most of the research has concerned steady loading. Impact loading has, however, been studied by, for example, Christensen who carried out both experimental and theoretical investigations (Christensen 1962 and 1970). In his theoretical analysis he did not solve how pressure and film thickness varied during impact, because he studied the impact at only one moment of time by using the pressure at the contact centre as a given parameter. In his investigations he found that very high pressures arose in the lubricating film between the colliding surfaces. The pressure could reach levels higher than in the corresponding non-lubricated case. Experimentally he found that dents produced by colliding spheres could be deeper and thus more severe if the contact was lubricated than if it was not.

Safa & Gohar (1986) made an experimental investigation of the pure impact problem. They used thin film transducers to measure the pressure in the contact during impact. By studying the pressure at the contact centre they found that the pressure reaches two peaks during the total impact time. The first peak corresponds to the stage of impact where the impact force reaches its maximum. At the very end of the rebound process, immediately before the ball leaves the lubricated surface, a sharp contact centre pressure peak was found.

A first attempt to theoretically solve the pure impact problem in a realistic way was made by Peeran & Shizhu (1989). Unfortunately, their analysis only dealt with the beginning of the impact. Thus they were not able to compare maximum film pressure to the corresponding dry impact pressure. Neither could the pressure peak at the end of the total impact time be found.

Larsson & Högland (1994) solved the pressure and film thickness during impact and rebound. They assumed Newtonian lubricant and perfectly elastic solids. They concluded that the maximum pressure in the lubricant film could reach levels higher than in the corresponding dry impact situation. They also gave theoretical evidence of the existence of the secondary pressure peak reported by Safa & Gohar (1986).

Furthermore they showed that cavitation occurs during rebound and a pressure spike forms along the periphery of the contact region.

A problem related to the pure impact case is the situation of combined motion of both normal approach and sliding. This is a load case common in many lubricated contacts. Lundberg et al. have studied this problem in several papers (Lundberg et al., 1990, 1991, 1992a and 1992b). They let a rotating roller impact a lubricated ball. During impact they were able to measure impact force and any existence of metal to metal contact, i.e. lubricating film breakdown.

Two interesting results were found. First they noticed that if metal to metal contact occurred, it usually occurred at the end of the total impact time. Assuming metal contact corresponds to the point of time where the film thickness reaches its smallest value, this means that there is a phase shift in time between maximum impact force and minimum film thickness. Metal to metal contact actually occurred when the impact force was almost zero.

Another interesting result was that increasing the amount of sliding motion resulted in a higher
risk of lubricant film breakdown. This means that the sliding motion actually makes the film thickness decrease if squeeze action is simultaneously present.

The first phenomena was explained by Larsson & Lundberg (1994) by modelling the lubricating film by its damping and elastic properties. In dynamic systems containing both damping and elasticity, it is well known that maximum force and maximum displacement are shifted in time and thus it is natural that minimum film thickness occurs at another point in time than the maximum impact force. Larsson & Lundberg also used a simplified theoretical model of the problem. They used a piezoviscous lubricant model and assumed the surfaces to be rigid. This analysis showed that minimum film thickness occurred towards the end of the total impact time as seen in the experiments.

The analysis performed by Larsson & Höglund (1994) also showed the existence of a phase lag between maximum force and minimum film thickness.

In this paper the impact problem is investigated further. Pure squeeze action is assumed by letting a ball impact a thin lubricant layer. The theoretical treatment follows the investigations performed by Peiran & Shizhu (1989) and Larsson & Höglund (1994). The ways in which minimum film thickness, maximum impact force, maximum pressure and phase shift between maximum force and minimum film thickness are influenced by initial impact velocity, ball mass, lubricant properties and properties of the bounding surfaces are studied.
2. THEORETICAL MODELLING.

The case of a ball impacting a flat lubricated surface is studied, Fig. 1, and pressure distributions during impact and rebound of the ball are computed. Since the contact region between the ball and the flat surface is circular, and due to the pure squeeze situation, both pressure and film thickness are concentrically distributed around the contact centre.

Figure 1. Definitions for the impacting ball problem.

The pressure distribution within the lubricating film is obtained from the Reynolds equation written in polar coordinates:

\[
\frac{\partial}{\partial r}\left( r \rho \frac{h^3}{12 \eta} \frac{\partial p}{\partial r} \right) = r \frac{\partial}{\partial t} (\rho h)
\]  

(1)

The radial co-ordinate, \( r \), has its origin at the centre of the contact. By solving the equation of motion, the motion of the ball is determined. For the impacting ball case it is:

\[
\ddot{h}_0(t) = \frac{w(t)}{m}
\]  

(2)

where \( h_0 \) is the rigid separation between the ball and the flat surface. The rigid separation can also be seen as a parameter which describes the motion of the ball's centre of gravity.

To compute the pressure in the contact and motion of the ball, equations (1) and (2) are solved simultaneously.

The impact problem is divided into two parts, the initial impact stage and the high pressure stage.

2.1 The initial impact stage.

At the initial moment of impact the ball has reached the lubricant layer and begins to squeeze the lubricant film away. The pressure is low and an isoviscous incompressible lubricant model is appropriate. Due to the low pressure, the deformations of the ball and the flat surface are negligible. The film thickness can thus be written as:

\[
h(r, t) = h_0(t) + \frac{r^2}{2R}
\]  

(3)

where the last term is the well-known parabolic approximation.
The boundary conditions to equation (1) are:

\[ p(r, 0) = 0 \]  \hspace{1cm} (4a)
\[ p(r_{\text{max}}, t) = 0 \]  \hspace{1cm} (4b)
\[ \frac{\partial p}{\partial r}(0, t) = 0 \]  \hspace{1cm} (4c)

By studying Fig. 1 and using equation (3), \( r_{\text{max}} \) is defined as:

\[ r_{\text{max}}(t) = \sqrt{2R(h_{00} - h_0(t))} \]  \hspace{1cm} (5)

Using equations (2) and (5) and the boundary conditions (4), the pressure is solved from equation (1) as:

\[ p(r, t) = 12\pi v_c(t) R^3 \left[ \frac{1}{4R^2 h_{00}^2} - \frac{1}{(2Rh_0 + r^2)^2} \right] \]  \hspace{1cm} (6)

where \( v_c \) is the velocity of the ball’s centre of gravity. Note that the ball velocity is assumed to be negative during impact. The force is found by integrating the pressure distribution (6):

\[ w(t) = \int_0^{r_{\text{max}}} p(r, t) 2\pi r dr = -6\pi \eta v_c(t) R^2 \frac{(h_{00} - h_0)^2}{h_{00}^2 h_0} \]  \hspace{1cm} (7)

The equation of motion for the ball (2), is in the initial stage of impact solved numerically by using equation (7). The numerical scheme is straightforward. New values of the velocity, \( v_c \), and the rigid separation, \( h_0 \), are obtained from:

\[ v_c^k = v_c^{k-1} + h_0^{k-1} \Delta t \]  \hspace{1cm} (8)
\[ h_0^k = h_0^{k-1} + v_c^{k-1} \Delta t + h_0^{k-1} (\Delta t)^2 / 2 \]  \hspace{1cm} (9)

The time stepping procedure continues until the impact force, \( w \), corresponds to a maximum deformation of 1% of \( h_0 \). A good measure of maximum deformation is the dry impact maximum deformation (Johnson, 1985):

\[ \delta_{\text{max}} = \left( \frac{3w_{\text{max}}}{2E R^{1/2}} \right)^{2/3} \]  \hspace{1cm} (10)

### 2.2 The high pressure stage.

When the pressure continues to increase it is no longer realistic to neglect surface deformations. The viscosity and the density are also affected by the pressure. This stage is denoted as the high pressure stage. The bulk of the computing effort is within this stage. Details of how to solve the problem in the high pressure stage are given in (Larsson & Höglund, 1994) and will only briefly be described.
The assumptions for the high pressure stage are:

- Newtonian lubricant.
- Viscosity-pressure relation according to Roelands (1966):

\[ \eta = \eta_0 e^{(\ln \eta_0 + 9.67) \left[ -1 + (1 + 5.1 \times 10^{-9} p)^{0.5} \right]} \]  

(11)

- Density-pressure relation according to Dowson-Higginson (1966):

\[ \rho = \rho_0 \left( 1 + \frac{0.6 \times 10^{-9} p}{1 + 1.7 \times 10^{-9} p} \right) \]  

(12)

- Inertia effects within the lubricant film are neglected.
- Isothermal conditions prevail during the total impact time.
- Cavitation occurs when the pressure falls below zero. Subambient pressures are set to zero, i.e. \( p \geq 0 \).
- The solids deform perfectly elastically.
- Deformations are treated in a quasi-static way, neglecting wave propagation in the solid surfaces.
- Smooth surfaces.

In this stage of impact it is no longer possible to express the impact force, \( w \), in an explicit form as in equation (7). The Reynolds equation (1) has to be solved numerically and due to the non-linearities introduced by the pressure dependence of viscosity, density and film thickness an iterative solution method has to be used. Two different relaxation methods are used. Gauss-Seidel relaxation in low pressure regions and distributive Jacobi relaxation in the high pressure regions (Venner, 1991).

To increase the convergence rate, multigrid technique is used. Multigrid techniques in the field of elastohydrodynamics are fully described by Venner (1991) and Lubrecht (1987).

A time stepping scheme is used. The pressure distribution is solved at each time step and the impact force is found by integration. From equations (8) and (9), are \( v_c \) and \( h_0 \) obtained for the next time step and a new pressure distribution can be computed. This process is repeated until the ball has rebounded and the film is completely cavitated.
3. **DIMENSIONAL ANALYSIS.**

To reduce the number of parameters it is convenient to use a non-dimensional form. There are seven parameters involved in the impacting ball problem. These are:

- Ball radius, $R$.
- Ball mass, $m$.
- Effective elastic modulus, $E'$.
- Initial impact velocity, $v_{c0}$.
- Lubricant viscosity at ambient pressure, $\eta_0$.
- The pressure-viscosity coefficient of the lubricant, $\alpha$.
- Lubricant layer thickness, $h_{oo}$.

A dimensional analysis shows that the number of parameters can be reduced to four. The following set of non-dimensional parameters are used within this investigation:

$$
H_{oo} = \frac{h_{oo}}{R}, \quad V_0 = \frac{v_{c0} \eta_0}{ER}, \quad M = \frac{mE'}{R\eta_0^2}, \quad G = E'\alpha
$$

(13)

Using the same dimensional analysis, interesting results such as impact force, pressure and film thickness can be written in dimensionless form:

$$
W = \frac{w}{ER^2}, \quad P = \frac{p}{E}, \quad H = \frac{h}{R}
$$

(14)

In this analysis $H_{oo}$, $V_0$, $M$ and $G$ are varied to study how they influence maximum impact force, maximum pressure, minimum film thickness, phase shift and the coefficient of restitution.
4. DRY IMPACT.

For comparison, the dry impact case is also studied. Hertzian theory is assumed. Johnson (1985) gives the expressions for maximum deformation, maximum impact force and maximum contact pressure at dry impact. These expressions are written here in terms of the dimensionless parameters stated above.

The maximum deformation is:

\[
\hat{\Delta}_{\text{max}} = 1.2859 M^{0.4} V_0^{0.8}
\] (15)

Maximum impact force is expressed as:

\[
\hat{W}_{\text{max}} = \frac{2}{3} \hat{\Delta}_{\text{max}}^{3/2} = 0.9721 M^{0.6} V_0^{1.2}
\] (16)

The maximum pressure at dry impact is obtained from:

\[
\hat{p}_{\text{max}} = \frac{1}{\pi} \left( \frac{3}{2} \hat{W}_{\text{max}} \right)^{1/3} = 0.3610 M^{0.2} V_0^{0.4}
\] (17)

Both \( M \) and \( V_0 \) include the lubricant viscosity, \( \eta_0 \). This may seem odd since no lubricant is present, but the viscosity vanishes if the definitions (13) are used within all the expressions (15)-(17).
5. RESULTS AND DISCUSSION.

Sixteen different cases are studied within this investigation. Input data for each case, in terms of the four dimensionless parameters \( H_0 \), \( V_0 \), \( M \) and \( G \), are given in Table I in the Appendix. The main results are summarized in Table II in the Appendix. Table 1 shows how the dimensionless parameters are varied.

Table 1. Computational range

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0 )</td>
<td>0.00125</td>
<td>0.025</td>
</tr>
<tr>
<td>( V_0 )</td>
<td>1.3\times10^{-12}</td>
<td>4.3\times10^{-11}</td>
</tr>
<tr>
<td>( M )</td>
<td>7.5\times10^{12}</td>
<td>1.5\times10^{14}</td>
</tr>
<tr>
<td>( G )</td>
<td>1800</td>
<td>4600</td>
</tr>
</tbody>
</table>

The upper limit of the computational region, \( r_{\text{max}} \), is chosen at the beginning of the high pressure stage to be the same as at the end of the initial impact stage. When more than half of the region is cavitated, or the pressure at the contact centre is more than \( 10^4 \) times larger than the pressure at \( r=r_{\text{max}}/2 \), \( r_{\text{max}} \) is reduced to half its value and so on.

Six different grid sizes (levels) are used within the high pressure stage for each case. The finest grid is made up of 513 nodes and the coarsest of 17 nodes. The number of time steps varies from case to case but normally 250-300 steps are used within the high pressure stage.

All results presented in the Appendix are estimations of the “exact” solution of the different cases. This is done by comparing converged solutions for different grid sizes.

5.1 Typical results.

Figure 2 shows the results of the analysis of case no. 3 and also the results of the corresponding dry impact case. The beginning of the high pressure stage is marked with an asterisk.

It is seen in Fig. 2a that the impact force is lower in the lubricated case than in the dry impact case. This is expected since the lubricant film provides damping capacity.

Figure 2b shows the pressure at the contact centre versus impact time. The pressure increases very rapidly, reaches a maximum and starts to decrease again. At the very end of the total impact a sharp pressure peak occurs as depicted in (Larsson & Höglund, 1994). The occurrence of this secondary pressure peak has been proved experimentally by Safa & Gohar (1986).

Figure 2c shows the minimum film thickness, the film thickness at the contact centre and the rigid separation versus impact time. It is seen that the minimum value of \( H_{\text{min}} \) occurs at a later time when the impact force reaches its maximum. This phase lag between maximum impact force and minimum film thickness corresponds well to the experimental observations made by Lundberg et al (1990).

Figure 2d shows the velocity of the ball’s centre of gravity. Due to lubricant film damping the velocity after rebound is lower than the initial impact velocity. A coefficient of restitution can be defined as the ratio between the velocity after rebound and the initial impact velocity. In this case (no. 3) the coefficient of restitution is 0.84.
Figure 2. Detailed results for case no. 3. The asterisk * marks the beginning of the high pressure stage. a) impact force vs. time, b) pressure at contact centre vs. time, c) minimum film thickness ($H_{\text{min}}$), film thickness at contact centre ($H_c$) and rigid separation ($H_0$) vs. time, d) velocity of the ball's centre of gravity vs. time.

Figure 3 shows the pressure distributions at different points of time during impact and rebound. During impact the pressure increases in magnitude. When the ball changes its direction of motion, at $T=6.75 \times 10^{-8}$, the rebound stage begins. Along the periphery of the contact the surfaces begin to separate and the pressure decreases rapidly giving rise to cavitation. Inside the contact the pressure is still very high due to the fact that the surfaces still approach each other due to the rebound of the surface deformations. Along the periphery of the pressure zone a pressure spike is formed. This pressure spike grows higher and higher until the very end of the total impact time.

Figure 4 shows the film thickness distributions at the same points of time as in Fig. 3. A lubricant entrapment is formed at the centre of the contact. The diameter of the entrapment becomes larger and larger as long as the ball approaches the flat surface. When the ball changes its direction of motion the dimple at the contact centre decreases and a sharp corner arises at the location of the pressure spike due to the rapidly changing pressure level. The minimum film thickness occurs at the edges of the dimple.
Figure 3. Pressure distributions at different times (case no. 3).

Figure 4. Film thickness distributions at different times (case no. 3).
5.2 Minimum film thickness.

The effects of the four different dimensionless parameters on minimum film thickness, \( (H_{\text{min}})_{\text{min}} \), are shown in Fig. 5. \( (H_{\text{min}})_{\text{min}} \) represents the absolute minimum value of film thickness during the total impact time. As seen in Fig. 5a, if the initial impact velocity increases \( (H_{\text{min}})_{\text{min}} \) becomes greater.

The reason why increased impact velocity increases the film thickness can be explained as follows. Absolute minimum thickness occurs during rebound, i.e. when the central dimple decreases. This causes the trapped fluid to flow out from the dimple. Simultaneously both force equilibrium and flow continuity have to be fulfilled.

When the initial impact velocity increases, the impact momentum becomes greater and thus increase also pressure and impact force. Higher pressure means a more viscous lubricant and because of the exponential dependence of pressure, it becomes more difficult to squeeze the lubricant away from the dimple as the pressure increases. Fulfilling flow continuity means to make it possible for the entrapped fluid to leave the dimple as fast as determined by the rebound of the surfaces. To compensate for the increase in flow resistance due to higher viscosity, the film thickness at the edges of the dimple has to be larger. This also means that the minimum film thickness will increase if the pressure in the contact centre increases, i.e. \( (H_{\text{min}})_{\text{min}} \) increases if \( V_0 \) increases.

The pressure gradient also increases when the pressure becomes higher, which increases the flow rate. But this effect is not strong enough to compensate for the dramatic increase in viscosity.

\[
H_{\text{oo}} = \frac{h_{\text{oo}}}{R},
\]

\[
V_0 = v_0 \frac{R \alpha}{E'},
\]

\[
M = m \frac{E'}{R_0^2},
\]

\[
G = E' \alpha
\]

Figure 5. Absolute minimum film thickness versus a) initial impact velocity, b) lubricant layer thickness, c) ball mass, d) material parameter.

From a dynamic point of view it can be described in another way. At low values of \( V_0 \) the pressure in the lubricant film is low, which makes the film dominated by damping effects. During the impact most of the deformation takes place in the lubricant film, which makes the impact well damped as seen in Fig. 6a. When \( V_0 \) increases, the pressure also increases. This causes the viscos-

12
ity to increase rapidly and the lubricant film tends to be very hard to compress due to its very large damping and stiffness. Therefore most of the deformation takes place in the surfaces and the impact becomes more elastic, as seen in Fig. 6a.

![Graphs showing the effect of the lubricant layer thickness.](image)

Figure 6. The ratio between maximum impact force with and without lubricant (o), the ratio between $P_{cl}$ and the maximum Hertzian pressure $P_{max}$ at dry impact (*) and coefficient of restitution (x) vs. a) initial impact velocity, b) lubricant layer thickness, c) ball mass, d) material parameter.

Figure 5b shows the effect of the lubricant layer thickness. It can be seen that if the lubricant layer is thick enough, the minimum film thickness is not influenced by $H_{00}$ at all. In this case $H_{00}$ has to be at least 0.0025, but this value is of course dependent on the other three parameters $V_0$, $M$ and $G$.

In Fig. 6b similar effects are seen. If $H_{00}$ is sufficiently large the influence on maximum pressure, maximum impact load and coefficient of restitution is small.

A study of Fig. 5c reveals an interesting behaviour of ($H_{min}/min$. The film thickness reaches a maximum at approximately $M=1.5\times10^{13}$. From Fig. 6c it can also be seen that the coefficient of restitution reaches zero at $M>0$.

These phenomena can be explained by studying a simple model of the system as shown in Fig. 7. The dashpot in Fig. 7 represents the damping capability of the lubricating film and the spring represents the elastic properties of the film and the bounding surfaces. The spring stiffness is highly non-linear. If $M$ is small the momentum of the impact becomes small. Consequently the film pressure also becomes low. At low pressure the elastic effects are negligible since deformation, lubricant compression and pressure influence on viscosity are small and thus the stiffness is very small. Using $k=0$ in the model shown in Fig. 7 causes no rebound of the ball. The ball approaches the flat surface asymptotically slow and reaches the surface after an infinitely long time. Thus ($H_{min}/min$ tends to zero as $M$ is below some critical value.
When $M$ increases the stiffness in the system also increases and the elasticity makes it possible for the ball to rebound and thus $(H_{\min})_{\min}$ will be greater than zero. However, if $M$ is large, in this case larger than say $5 \times 10^{13}$, the opposite effect of increased $M$ occurs. When $M$ increases, the momentum increases which also means that the pressure has to increase. The impact velocity is not changed, this means that the film has to be more compressed to give rise to an increase in pressure. Consequently $(H_{\min})_{\min}$ is reduced as $M$ is increased further.

Now the peak in Fig. 5c can be explained. Increased $M$, when $M$ is relatively small, means increasing film thickness. Increased $M$, when $M$ is large, means decreasing film thickness. To make both of these effects possible, there has to be a maximum value of $(H_{\min})_{\min}$ somewhere in between.

Finally the effect of the material parameter $G$ can be seen in Fig. 5d. The lubricating film becomes thicker when $a$ is increased. The explanation is similar to the effect of increased $V_0$. Increased $a$ means a more viscous fluid in the central entrapment and it becomes more difficult to empty this entrapment. This is because a larger amount of lubricant stays within the contact and so the film becomes thicker.

5.3 Phase shift in time.

The phase shift in time between maximum impact force and absolute minimum film thickness is shown in Fig 8. The non-dimensional time $T_w$ denotes the space of time from the point of maximum impact force is reached to the point when the ball leaves the flat surface and the impact force becomes zero. $\delta T$ denotes the phase shift and the ratio $\delta T/T_w$ describes whether it is the damping effects or the effects of elasticity which dominate. If this ratio is zero the system is not damped at all and if it tends to go to unity it is critically damped and the elastic effects are negligible compared to the damping effects.

It can be seen from Figs. 8a and 8c that this ratio approaches unity if $V_0$ and $M$ are small. In a linear system $V_0$ should not affect the degree of elasticity, but in this highly non-linear case decreased $V_0$ results in a less stiff lubricant film and the damping becomes more effective. From these results it is concluded that it is possible to determine whether the ball will stick to the flat surface or not. If the impact velocity or the ball mass are below their critical values the ball will stick to the flat surface.
Figure 8. Phase shift in time between maximum impact force and absolute minimum film thickness as a function of a) initial impact velocity, b) lubricant layer thickness, c) ball mass, d) material parameter.

5.4 Maximum pressure.

Figure 6 shows the ratio between the pressure $P_{cl}$ and the maximum Hertzian pressure $\hat{P}_{max}$ for dry impact. The pressure $P_{cl}$ is the primary peak pressure, i.e. the pressure which corresponds to maximum impact force. At the pressure spike, pressure values far above this were found. However, the Newtonian lubricant model used in this analysis is not adequate in the vicinity of the pressure spike. It is probably the case that the shear stress limit of the lubricant is exceeded in this region. This reduces the pressure gradients and consequently sharp pressure spikes as shown in Fig. 3 cannot exist. It is therefore concluded that there is little to gain from the study of the maximum spike pressure.

Figure 6 shows that it is only $G$ which influences the pressure ratio $P_{cl}/\hat{P}_{max}$ to a greater extent. It is also seen that the pressure can be up to 40% higher if a lubricant is applied than when it is not. Cristensen (1962) noticed that the dents became deeper if a lubricant was applied when the ball impacts the surface. That result correspond well to the present analysis, since increased pressure gives rise to increased amount of plastic flow and thus more severe surface damage.
6. CONCLUSIONS.

The conclusions drawn from this investigation can be summarized as follows.

- The ball impact problem can be described by four non-dimensional parameters. Initial impact velocity \(V_0\), lubricant layer thickness \(H_{oo}\), ball mass \(M\) and material parameter \(G\).

- The absolute minimum film thickness increases if the initial impact velocity increases.

- If the lubricant layer is thick enough the parameter \(H_{oo}\) has little influence on film thickness, impact force and pressure.

- If the material parameter, \(G\), is increased the film thickness also increases.

- If \(V_0\) or \(M\) are smaller than their critical values no rebound occurs. The ball continues to approach the surface while the velocity decreases. Theoretically it will take an infinitely long time for the ball to reach the surface.

- The parameters \(V_0\) and \(M\) dominate the influence on the damping properties of the lubricating film. Low values of initial velocity or mass imply a good damping capacity. High values of these parameters mean low damping effects.

- The absolute minimum film thickness reaches a maximum at a certain ball mass. If the ball mass decreases or increases from this optimum, the film thickness becomes thinner.

- The phase shift between maximum impact force and minimum film thickness increases if initial impact velocity or ball mass decreases. This is due to the increased importance of damping.

7. ACKNOWLEDGEMENT.

The authors wish to acknowledge the financial support of the Swedish Research Council for Engineering Sciences (TFR).
8. REFERENCES.


Dowson D and Higginson G R, 1966, "Elastohydrodynamic lubrication, the fundamentals of roller and gear lubrication", Pergamon, Oxford.


Larsson R and Lundberg J, 1994, "A simplified solution to the combined squeeze-sliding problem", Accepted for publication in Wear.


NOTATIONS.

$E'$ Effective elastic modulus, $2\left(1-v_1^2\right)/E_1+(1-v_2^2)/E_2\right)^{-1}$, Pa.

$G$ Material parameter, $G=E'\alpha$.

$h$ Film thickness, m.

$h_0$ Rigid separation constant, m.

$h_c$ Film thickness in contact centre, m.

$h_{00}$ Thickness of lubricant layer, m.

$H$ Non-dimensional film thickness.

$H_{00}$ Non-dimensional thickness of the lubricant layer.

$m$ Ball mass, kg.

$M$ Non-dimensional ball mass.

$p, P$ Film pressure, with and without dimension, Pa.

$P_{cl}$ Non-dimensional maximum film pressure corresponding to maximum impact force.

$P_{\text{max}}$ Non-dimensional maximum pressure at dry impact.

$r$ Radial co-ordinate.

$r_{\text{max}}$ Location on the $r$-axis where the pressure is assumed to be zero.

$R$ Ball radius, m.

$t, T$ Time, with and without dimension, s.

$T_w$ Space of time between the points of time for $W=W_{\text{max}}$ and $W=0$.

$\Delta t$ Time step, s.

$V_0$ Non-dimensional initial impact velocity.

$v$ Velocity of the ball's centre of gravity, m/s.

$v_0$ Initial impact velocity of the ball's centre of gravity, m/s.

$w, W$ Impact force, with and without dimension, N.

$\dot{W}_{\text{max}}$ Maximum impact force at dry impact.

$\alpha$ Pressure-viscosity coefficient, Pa$^{-1}$.

$\delta$ Total surface deformation, m.

$\delta_{\text{max}}, \dot{\delta}_{\text{max}}$ Maximum surface deformation at dry impact, with and without dimension.

$\delta_{\text{z}}$ Maximum surface deformation at dry impact.

$\Delta$ Non-dimensional surface deformation.

$\eta$ Viscosity, Pas.

$\eta_0$ Viscosity at ambient pressure, Pas.

$\rho$ Density, kg/m$^3$.

$\rho_0$ Density at ambient pressure, kg/m$^3$. 
APPENDIX.

In Table I the input data for the different cases are shown. In Table II the results of all cases are summarized.

**Table I. Input data.**

<table>
<thead>
<tr>
<th>Case no.</th>
<th>$H_{oo}$</th>
<th>$10^{12}XV_0$</th>
<th>$10^{-13}XM$</th>
<th>$G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.005</td>
<td>1.30</td>
<td>7.475</td>
<td>3500</td>
</tr>
<tr>
<td>2</td>
<td>0.005</td>
<td>4.35</td>
<td>7.475</td>
<td>3500</td>
</tr>
<tr>
<td>3</td>
<td>0.005</td>
<td>8.70</td>
<td>7.475</td>
<td>3500</td>
</tr>
<tr>
<td>4</td>
<td>0.005</td>
<td>21.7</td>
<td>7.475</td>
<td>3500</td>
</tr>
<tr>
<td>5</td>
<td>0.005</td>
<td>43.5</td>
<td>7.475</td>
<td>3500</td>
</tr>
<tr>
<td>6</td>
<td>0.005</td>
<td>8.70</td>
<td>14.95</td>
<td>3500</td>
</tr>
<tr>
<td>7</td>
<td>0.005</td>
<td>8.70</td>
<td>1.49</td>
<td>3500</td>
</tr>
<tr>
<td>8</td>
<td>0.005</td>
<td>8.70</td>
<td>2.99</td>
<td>3500</td>
</tr>
<tr>
<td>9</td>
<td>0.005</td>
<td>8.70</td>
<td>0.75</td>
<td>3500</td>
</tr>
<tr>
<td>10</td>
<td>0.005</td>
<td>43.5</td>
<td>0.75</td>
<td>3500</td>
</tr>
<tr>
<td>11</td>
<td>0.005</td>
<td>8.70</td>
<td>7.475</td>
<td>4600</td>
</tr>
<tr>
<td>12</td>
<td>0.005</td>
<td>8.70</td>
<td>7.475</td>
<td>1800</td>
</tr>
<tr>
<td>13</td>
<td>0.010</td>
<td>8.70</td>
<td>7.475</td>
<td>3500</td>
</tr>
<tr>
<td>14</td>
<td>0.025</td>
<td>8.70</td>
<td>7.475</td>
<td>3500</td>
</tr>
<tr>
<td>15</td>
<td>0.0025</td>
<td>8.70</td>
<td>7.475</td>
<td>3500</td>
</tr>
<tr>
<td>16</td>
<td>0.00125</td>
<td>8.70</td>
<td>7.475</td>
<td>3500</td>
</tr>
</tbody>
</table>
Table II. Results.

<table>
<thead>
<tr>
<th>Case no.</th>
<th>$W_{max}$</th>
<th>$P_{ci}$</th>
<th>$(H_{min})_{min}$</th>
<th>$\delta T/T_w$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.31x10^{-7}</td>
<td>0.27x10^{-4}</td>
<td>0.96</td>
<td>0.148</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3.14x10^{-6}</td>
<td>0.0077</td>
<td>0.59x10^{-4}</td>
<td>0.30</td>
<td>0.708</td>
</tr>
<tr>
<td>3</td>
<td>8.64x10^{-6}</td>
<td>0.0106</td>
<td>0.72x10^{-4}</td>
<td>0.22</td>
<td>0.844</td>
</tr>
<tr>
<td>4</td>
<td>2.90x10^{-5}</td>
<td>0.0154</td>
<td>0.89x10^{-4}</td>
<td>0.16</td>
<td>0.933</td>
</tr>
<tr>
<td>5</td>
<td>6.96x10^{-5}</td>
<td>0.0202</td>
<td>1.01x10^{-4}</td>
<td>0.13</td>
<td>0.966</td>
</tr>
<tr>
<td>6</td>
<td>1.43x10^{-5}</td>
<td>0.0120</td>
<td>0.58x10^{-4}</td>
<td>0.19</td>
<td>0.910</td>
</tr>
<tr>
<td>7</td>
<td>1.87x10^{-6}</td>
<td>0.0073</td>
<td>0.98x10^{-4}</td>
<td>0.47</td>
<td>0.451</td>
</tr>
<tr>
<td>8</td>
<td>4.01x10^{-6}</td>
<td>0.0087</td>
<td>0.90x10^{-4}</td>
<td>0.32</td>
<td>0.680</td>
</tr>
<tr>
<td>9</td>
<td>5.09x10^{-7}</td>
<td>0.75x10^{-4}</td>
<td>0.96</td>
<td>0.080</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.40x10^{-5}</td>
<td>0.0135</td>
<td>2.02x10^{-4}</td>
<td>0.26</td>
<td>0.784</td>
</tr>
<tr>
<td>11</td>
<td>8.60x10^{-6}</td>
<td>0.0112</td>
<td>0.93x10^{-4}</td>
<td>0.21</td>
<td>0.849</td>
</tr>
<tr>
<td>12</td>
<td>8.69x10^{-6}</td>
<td>0.0087</td>
<td>0.46x10^{-4}</td>
<td>0.30</td>
<td>0.825</td>
</tr>
<tr>
<td>13</td>
<td>8.34x10^{-6}</td>
<td>0.0105</td>
<td>0.72x10^{-4}</td>
<td>0.23</td>
<td>0.821</td>
</tr>
<tr>
<td>14</td>
<td>8.04x10^{-6}</td>
<td>0.0103</td>
<td>0.71x10^{-4}</td>
<td>0.23</td>
<td>0.794</td>
</tr>
<tr>
<td>15</td>
<td>8.99x10^{-6}</td>
<td>0.0107</td>
<td>0.71x10^{-4}</td>
<td>0.22</td>
<td>0.875</td>
</tr>
<tr>
<td>16</td>
<td>9.49x10^{-6}</td>
<td>0.0110</td>
<td>0.56x10^{-4}</td>
<td>0.16</td>
<td>0.931</td>
</tr>
</tbody>
</table>

1) Only one pressure peak was detected. This peak occurred at the very end of the total impact time.
Paper C
A simplified solution to the combined squeeze–sliding lubrication problem

Roland Larsson and Jan Lundberg
Division of Machine Elements, Luleå University of Technology, S-971 87 Luleå (Sweden)

(Received July 7, 1993; accepted December 9, 1993)

Abstract

The unfavourable elastohydrodynamic lubrication situation in combined squeeze and sliding motion has been analysed both theoretically and experimentally. In experiments a rotating roller impacted and rebounded on a lubricated surface. It was found that oil film breakdown always occurs at the end of the impact time, when the contact force is low. It has also been found that there exists an upper limit for the sliding velocity. Below this limiting velocity no oil film breakdown occurs. This paper is an initial attempt to explain theoretically why oil film breakdown takes place towards the end of the impact, and why an increasing sliding velocity reduces the capability of the oil film to separate the lubricated surfaces.

If the oil film's elastic and damping behaviour are taken into consideration it can be shown that a considerable phase shift between maximum contact force and oil film breakdown will arise. It has been found that the squeeze action dominates the pressure formation in the contact and thus the hydrodynamic effect of sliding motion is moderate. Furthermore, several effects, such as non-newtonian behaviour, surface roughness, temperature rise, starvation and deformations, which are not included in the theoretical model, may decrease the oil film thickness if the sliding velocity increases.

1. Introduction

Lubrication of non-conformal surfaces under non-steady conditions is a subject which has not been examined in any depth in the literature. Non-steady conditions are however very common. To be able to design a lubricated contact, to choose an appropriate lubricant or to decide how to apply the lubricant, it is important to know how an oil film behaves under non-steady conditions. One example of a such a situation is a lubricated contact which operates under a combined squeeze and sliding motion, for example heavily loaded large roller bearings. When such bearings are running, only a segment of the bearing is loaded, depending on the direction of the load. This means that the rollers will stop rotating around their axes because of friction, after leaving the loaded part of the bearing. When the non-rotating roller comes into contact with both races again, a combined squeeze and sliding motion arises. The combined motion can cause a breakdown of the oil film between the roller and the races, which may result in surface damage and possible failure of the bearing. Such damage is a problem in propeller shaft roller bearings in ships. Another example is gear meshes where the teeth come into contact in a similar squeeze–sliding motion. A third example is ball and roller bearings subjected to impact loading, e.g. in crushers and sieves.

The squeeze–sliding lubrication problem has been studied experimentally by Lundberg and co-workers in several papers [1–4]. They investigated how different parameters such as sliding velocity, viscosity, surface roughness and limiting shear stress influence the risk of oil film breakdown. In the experiments a rotating roller was impacting a lubricated surface. The contact force during impact was measured. Any oil film breakdown, i.e. metal-to-metal contact, could also be detected. The contact force (curve A) during the impact is shown in Fig. 1, where a limited oil film breakdown (curve B) has been detected. Breakdown occurs at the end of the contact time, i.e. when the contact force is low. It may seem strange that the breakdown does not occur

Fig. 1. Waveforms showing oil film breakdown during impact. Curve A, contact force as a function of time; curve B, voltage transient at film breakdown.
Another interesting result is that increasing the sliding velocity actually prolongs the time of oil film breakdown (Fig. 2), i.e. the oil film thickness will decrease if the sliding velocity increases. This phenomenon contradicts the theories for elastohydrodynamic contacts with steady loading, where increasing sliding velocity will increase the film thickness. Lundberg et al. defined a limiting sliding velocity. If the sliding velocity is below this limit, no oil film breakdown occurs.

Theoretical investigations of the combined motion with both squeeze and sliding motion are very rare. Iivonen and Hamrock [5] performed an advanced elastohydrodynamic analysis of the problem, taking into account the non-newtonian behaviour of the lubricant. They solved the non-steady Reynolds equation and found that the film thickness increased if the sliding velocity increased. However, they discussed starvation as a possible explanation for the experimental results. Unfortunately, they approximated the variation of load with time with a sine function instead of the more appropriate method of simulating the impact by solving the equation of motion of the roller. Furthermore, the amplitude of the load was very low and thus the effect of deformation was small. Owing to the low load, the limiting shear stress was never reached in the oil film and the non-newtonian model was more or less unnecessary.

Yang and Wen [6–7] carried out another interesting theoretical investigation. They studied pure squeeze motion by simulating the dropping of a ball on a lubricated surface. The phase-shift in time between maximum contact force and minimum film thickness was very small and contradicts the experimental results of Lundberg et al.

Bedewi et al. [8], Osborn and Sadeghi [9], and Kuroda and Hamrock [10] have also studied non-steady elastohydrodynamic lubrication. Unfortunately none of them used loading conditions comparable to the conditions in the experiments performed by Lundberg et al.

In this paper, which is mainly theoretical, two different questions emanating from experimental results will be examined:

1. Why does oil film breakdown always occur at the end of the contact time?
2. Why does the oil film thickness decrease when the sliding velocity increases?

Simplified models will be used, and comparisons with experiments will be made.

2. Experiments

2.1. Experimental method

A sketch of the experimental rig is shown in Fig. 3. The rig uses a 25 mm ball-bearing ball (1) which is
impacted by a bearing outer ring (3), with a diameter of 69.5 mm and a width of 18 mm. The $R_e$ values for the ring and ball were 0.025 $\mu$m and 0.062 $\mu$m respectively. The ball is mounted on a triaxial force transducer which gives a force–time signal to a transient recorder. A pendulum (2) with adjustable counterweights is used to give the ring a normal velocity $V_n$. The roller is driven by an air nozzle in order to get a sliding speed $V_s$. In this way both sliding velocity and squeeze velocity can be varied independently. The sliding velocity can be varied between 0 and 100 m s$^{-1}$, and the normal velocity between 0.015 and 0.75 m s$^{-1}$, giving a contact force amplitude between 30 and 6300 N. Both the squeeze and sliding velocities are measured immediately before each impact by using IR detectors. A voltage of 0.1 V is applied between the ring and the ball, and connected to the same transient recorder as the force transducer. Thus, asperity interaction gives a voltage peak of varying duration on the recorder, as shown in Fig. 2.

The procedure for investigating the influence of squeeze velocity was to keep the squeeze velocity constant and, step-by-step, approach the sliding velocity giving the first sign of asperity contact, such as the peak on curve B shown in Fig. 1. The transition between no asperity contact and the first sign of asperity contact takes place at the limiting sliding velocity $V_s^*$. The limiting sliding velocity was measured at different initial squeeze velocities $V_{s0}$. The test oil was a poly-$\alpha$-olefin with kinematic viscosity $\eta = 0.479 \pm 0.006$ Pa s at the test temperature, which was held between 21.6 °C and 23.0 °C. A limiting shear stress pressure coefficient $\gamma$ of 0.042 $\pm$ 0.002 was used, where $\gamma$ is the constant which describes the linear relationship between limiting shear stress $\tau_l$ and pressure.

Also the relation between the contact force and the normal velocity $V_n$ of the roller centre are studied by measuring the contact force and the velocity of the roller at the same time. The roller centre velocity is measured by using an accelerometer mounted on the roller centre.

In the experimental apparatus, which is completely described by Lundberg et al. [1], the ellipticity parameter $k$ was 1.22, the curvature sum $R_{10}$ was 0.0053 m and the effective radius $R$ in the sliding direction was 0.0092 m.

2.2 Experimental results

The results from the experiments are presented in Figs. 4 and 5. Figure 4 shows the limiting sliding velocity $V_s^*$ as a function of the initial squeeze velocity $V_{s0}$ just
before impact. By studying Fig. 4 it can be seen that the limiting sliding velocity increases when the initial squeeze velocity decreases. Figure 5 shows the contact force (curve A) and the normal velocity of the roller centre (curve B) during impact. It can be seen that there is a time delay between the point when the contact force reaches its maximum (point D) and when the normal velocity is zero (point C). It can also be seen that the velocity \( V_{n0} \) just before impact is much higher than the rebounding velocity \( V_{out} \) of the roller centre. This is due to energy loss in the system during impact.

3. Theoretical modelling

It is difficult to include all possible effects in a theoretical model to describe squeeze—sliding lubrication. But to get a qualitative knowledge of what is happening during the impact of the rotating roller on a lubricated surface, simplified models can be used. In this investigation, two models have been used. The first one is a very simple one-degree-of-freedom (1-DOF) model where the oil film is modelled as linear spring and damping elements, Fig. 6. The second model is more advanced, Fig. 7, using a hydrodynamic model to describe the oil film behaviour. In the theoretical analysis, dimensions have been used, which makes it easier to compare theoretical and experimental results.

3.1. Linear model

Assume that the roller with mass \( m \) is dropped onto the oil film with an initial velocity \( V_{n0} \), see Fig. 6. When the roller comes into contact with the oil film, the equation of motion for the roller–oil film system will be

\[
m \ddot{x} + c \dot{x} + kx = 0
\]  

(1)

Gravity forces have been neglected, since the acceleration of the roller is much larger than the acceleration due to gravity. If the position \( x \) of the roller is assumed to be zero at the beginning, the initial conditions will be

\[
x(0) = 0
\]

\[
x(0) = V_{n0}
\]

From eqn. (1) and the initial conditions, the position of the roller can be obtained as

\[
x(t) = x_0(t) \sin \omega t
\]  

(2)

and the film thickness will be found from

\[
h(t) = h_{oo} - x(t)
\]  

(3)

where \( h_{oo} \) is the initial thickness of the lubricant layer. The contact force can be solved from

\[
F(t) = c \dot{x} + kx = F_0(t) \sin(\omega t + \varphi)
\]  

(4)

Both \( x_0 \) and \( F_0 \) are decaying exponential functions. The phase angle can be written as

\[
\sin \varphi = \frac{c}{2\sqrt{km - c^2}}
\]  

(5)

3.2. Non-linear hydrodynamic model

A more advanced model is shown in Fig. 7. In this model the non-linear behaviour of the oil film is taken into consideration. The roller in the test apparatus is simulated by a roller with mass \( m \), which is dropped onto a lubricated surface while it is rotating. The initial squeeze velocity is \( V_{n0} \) when it comes into contact with the oil film layer of thickness \( h_{oo} \). The following approximations and assumptions have been made in the model:

- A rigid line contact is assumed. The flexibility of the surfaces is modelled as a linear stiffness \( k_w \), see Fig. 7.
• The lubricant is assumed to be newtonian and incompressible.
• The expression used for the pressure–viscosity relationship is the Barus expression:
  \[ \eta = \eta_0 e^{\alpha p} \]
• Oil film inertia is neglected.
• At the cavitation boundary \( p = 0 \) and \( dp/dx = 0 \), i.e. boundary conditions for a moving cavitation boundary have not been used.
• Isothermal conditions prevail.
  The equation of motion for the roller’s centre of gravity will be
  \[ m\ddot{h}_o = W(h_o, \frac{dh_o}{dt}, t) - mg \tag{6} \]
where \( h_o \) is the minimum distance between the surfaces and \( dh_o/dt \) is the relative squeeze velocity between the surfaces. \( W \) is the load capacity of the oil film due to the film pressure. \( W \) can be found from the integral of the pressure distribution
  \[ W = \int p(x) \, dx \tag{7} \]
where \( x_c \) is the location of the cavitation boundary. The oil film pressure can be obtained from Reynolds’ equation
  \[ \frac{d}{dx} \left( \frac{h^3}{\eta} \frac{dp}{dx} \right) = 6V \frac{dh}{dx} + 12 \frac{dh}{dt} \tag{8} \]
where \( h_o \) is
  \[ h_o(t) = h_o(t) + \frac{W}{k_o} \tag{10} \]
To solve eqn. (8), Grubin’s transformation for “reduced pressure” is used (see, for example, ref. 11):
  \[ \frac{dq}{dx} = \frac{dp}{dx} e^{-\alpha p} \tag{11} \]
Using eqn. (11), eqn. (8) will be
  \[ \frac{d}{dx} \left( h^3 \frac{dq}{dx} \right) = 6V \eta_0 \frac{dh}{dx} + 12 \eta_0 \frac{dh}{dt} \tag{12} \]
Equation (12) can be solved using the same boundary conditions as in eqn. (8), i.e.
  \[ q(x \to -\infty) = 0 \quad q(x_c) = 0 \quad \frac{dq}{dx} (x_c) = 0 \tag{13} \]
If eqn. (12) is integrated twice, and by using the boundary conditions of eqn. (13), \( q(x) \) will be
  \[ q(x) = \frac{ax}{8R\eta_0 h} - \frac{(bx + d)}{4R h^2} \]
  \[ + \frac{a}{2h_0^3/2R h_0} \left( \frac{x}{\sqrt{2R h_0}} + \alpha \right) \tag{14} \]
where
  \[ a = 3\eta_0 \left( 2V_s R - 3\frac{V_a x^2}{h_0} - \frac{12 h_0 R x}{h_0} \right) \]
  \[ b = 3\eta_0 \left( 2V_s R + \frac{V_a x^2}{h_0} + \frac{4 h_0 R x}{h_0} \right) \]
  \[ d = 24 \eta_0 h_0 R^2 \tag{15} \]
The location of the cavitation boundary \( x_c \) has to be solved numerically from the boundary condition \( q(x_c) = 0 \). If pure squeeze motion is assumed, the reduced pressure distribution can be written deduced from eqn. (12) as
  \[ q(x) = -\frac{6 R \eta_0 h_0}{h^2} \quad h_0 \leq 0 \tag{16} \]
If the sliding velocity is zero and if the surfaces are separating from each other, the reduced pressure distribution is due to cavitation, assumed to be zero. From the transform of eqn. (11), the pressure will be obtained as
  \[ p = -\frac{1}{\alpha} \ln(1 - \alpha q) \tag{17} \]
The solution procedure will be:
1. Assume constant acceleration of the roller’s centre of gravity between time steps. The acceleration of the roller’s centre of gravity will be found from eqn. (6).
2. Solve \( h_0 \) from eqn. (10). Compute \( dh_0/dt \) by using \( h_0 \) in the previous time step.
3. Compute \( q(x), p(x) \) and \( W \) from eqns. (14), (17) and (7).
4. Repeat 2 and 3 until the load capacity \( W \) of the oil film and the minimum film thickness \( h_0 \) satisfy eqns. (6), (7) and (10).
5. Advance to next time step and repeat 1–4.
   The time-stepping procedure is repeated until the surfaces begin to separate, i.e. when the minimum value of \( h_0 \) has been passed.

4. Theoretical results
   A line contact was used in the non-linear hydrodynamic model. In the experiments an elliptical contact was used. The parameters in the theoretical model
have, however, been chosen to simulate the experimental results as accurately as possible. The roller radius was 35 mm. Assuming a roller density of 7850 kg m\(^{-3}\) gives it a mass of 30.2 kg m\(^{-1}\). The effective radius of the roller was chosen as the effective radius in the sliding direction in the experimental apparatus, giving \(R=0.0092\) in. The linear stiffness \(k_s\), which simulates the deformation in the contact, was calculated from (see, for example, ref. 12)

\[
k_s = \frac{\pi E'}{2} \left[ \frac{1}{2 + \ln\left(\frac{4r_{in}}{b}\right) + \ln\left(\frac{4r_{out}}{b}\right)} \right]
\]

By using \(E' = 226\) 000 MPa and assuming oil film forces between 1000 and 100 000 N m\(^{-1}\), a variation of \(k_s\) between \(1.9\times10^{10}\) and \(2.5\times10^{10}\) N m\(^{-2}\) was obtained. A mean value of \(k_s = 2.2\times10^{10}\) N m\(^{-2}\) was used in the analysis. The lubricant viscosity was varied between 0.3 and 0.5 Pa s. The pressure-viscosity index \(\alpha\) was assumed to be \(1.5\times10^{-4}\) Pa\(^{-1}\). The sliding velocity was varied between 0 and 20 m s\(^{-1}\) and the initial normal, or squeeze, velocity between \(-0.15\) and \(-0.25\) m s\(^{-1}\), where a negative velocity means downward motion and a positive velocity means upward motion. The thickness of the lubricant layer, \(h_{oo}\), was chosen to be thick enough to make its influence on the results negligible, i.e. the forces in the oil film are very small just at the moment when the roller comes into contact with the oil film, compared to the maximum oil film force. Therefore it is not necessary to take account of starvation at the beginning of the impact.

In Fig. 8 the oil film force \(W\) is shown as a function of time during an impact with combined squeeze and sliding motion. The initial squeeze velocity was \(-0.25\) m s\(^{-1}\) and the sliding velocity \(5\) m s\(^{-1}\). The lubricant viscosity was 0.5 Pa s, applied in a layer 0.4 mm thick. Figure 8 also shows the minimum film thickness \(h_0\) and the motion of the roller centre \(h_c\). It can be seen that the minimum film thickness arises about 0.06 ms later than the oil film force maximum. In Fig. 9 the relative squeeze velocity \(V_s\) between the surfaces during impact is shown for the same case as in Fig. 8. The normal velocity of the roller's centre \(V_n\) is also shown in Fig. 9. Owing to the deformation of the elastic support \(k_s\), the velocities \(V_s\) and \(V_n\) are not equal. It should be noted that the gap between the surfaces \(h_0\) is still decreasing, i.e. \(V_s < 0\), when the roller centre begins to move upwards, i.e. when \(V_n = 0\) at about \(t = 1.72\) ms.

In Fig. 10 the location of the cavitation boundary \(x_c\) is shown for the same case as in Figs. 8 and 9, and in Fig. 11 the pressure distributions at different times
5. Comparison and discussion

5.1. The linear model

To answer the question of why film breakdown always occurs at the end of the impact, the simple linear 1-DOF model shown in Fig. 6 can be studied. If the damping capacity of the oil film is sufficient compared to the stiffness and roller mass, eqn. (5) states that a considerable phase shift in time between maximum oil film force and minimum film thickness will arise. The reason why the breakdown occurs towards the end of the impact can therefore be explained by considering the oil film force due to damping as being of the same order of magnitude as the oil film force due to the elastic behaviour of the oil film.

The oil film damping originates from the viscous behaviour of the lubricant, i.e. the squeeze effect. The lubricant cannot be squeezed out from the contact region infinitely fast and, owing to the shear stresses in the lubricant and between the lubricant and the bounding surfaces, a damping force will arise. Theo-
Alternatively the \( dh/dt \) term in Reynolds' equation provides for the damping forces.

The elastic behaviour of the oil film is due to several effects. Theoretically it is included in the load-carrying wedge effect of the oil film, i.e. the \( dh/dx \) term in Reynolds' equation. However, if the Sommerfeld boundary conditions are used and cavitation is not taken into consideration, the stiffness and load-carrying capacity due to \( dh/dx \) will vanish. The compression of the lubricant, viscoelastic behaviour and the deformation of the lubricated surfaces also contribute to the elastic effects.

5.2. Non-linear hydrodynamic model

By using a hydrodynamic model of the oil film, it is possible to calculate the oil film forces due to both squeeze and sliding motion. The results from the non-linear model, presented in Fig. 7, show the same effects as the linear model. As can be seen in Fig. 8, the oil film force \( W \) reaches a maximum at a time before \( h_0 \) reaches its minimum. By comparing the experimental results in Figs. 1 and 5 with the theoretically derived curve in Fig. 8, it can be seen that the correspondence between theory and experiments is obvious. This proves the damping and elastic forces to be in the same order of magnitude.

The reason why the risk of oil film breakdown increases as the sliding velocity increases cannot be explained by using a hydrodynamic model. As seen in Fig. 12, the minimum film thickness will always increase if the sliding velocity is increased, which contradicts the experimental results. But, as seen in Fig. 12, the effect of the sliding motion will be small for sliding velocities in the range of 0-10 m s\(^{-1}\). In the case of \( V_{sw} = -0.25 \text{ m s}^{-1} \) and a viscosity of 0.5 Pa s (curve B in Fig. 12), the experimental value of the limiting sliding velocity is about 8 m s\(^{-1}\) \([3]\). The increase of \( (h_o)_{min} \) is in the theoretical analysis only about 60% (see Fig. 12, curve B) if the sliding velocity increases from 0 to 8 m s\(^{-1}\). The reason for this is that the formation of oil film pressure is dominated by the squeeze effect, i.e. the effect of sliding will be small.

Figure 12 also indicates, by comparison of curves A and B, that increasing initial squeeze velocity lowers the relative effect of sliding motion. Experimentally the limiting sliding velocity \( V_{sw} \) will decrease if the initial squeeze velocity is increased, see Fig. 4. This means that at high initial squeeze velocities the hydrodynamic lift due to the sliding motion will be negligible at \( V_s = V_{sw} \).

The experiments (Fig. 4) indicate that increasing the initial squeeze velocity reduces the limiting sliding velocity and may cause oil film breakdown. It is obvious that the minimum oil film thickness will decrease if the squeeze velocity, and thus also the contact force, is increased for a given value of the sliding speed. This is also shown theoretically in Fig. 12, where curve A means an initial squeeze velocity of \(-0.20 \text{ m s}^{-1}\) and curve B a velocity of \(-0.25 \text{ m s}^{-1}\).

Several effects have been neglected in the model; for example, the non-newtonian behaviour of the lubricant, surface roughness, temperature effects, starvation effects and deformations. Some of these effects can qualitatively be shown to have a negative effect on the film thickness if the sliding velocity is increased.

If the lubricant is assumed to have a limiting shear strength \( \tau_L \), the lubricant can only transfer shear stresses less than or equal to \( \tau_L \). The limiting shear strength is reached very fast at the high oil film pressure in the contact. Figure 15(a) shows the shear stress distribution on the lubricant, in a circular contact or a circular asperity, during pure squeeze motion. The shear stresses on the lubricant try to prevent the oil from being squeezed out from the region. In Fig. 15(a) the limiting shear stress \( \tau_L \) is reached. If a sliding motion is superimposed, the magnitude of the shear stresses cannot increase. They can only change direction, as seen in Fig. 15(b). Increasing sliding velocity will orientate the shear stress more and more into the sliding direction. The component of the shear stresses in a direction perpendicular to the sliding direction will thus decrease, and the lubricant can more easily be squeezed out from the contact and, if enough time is available, a breakdown will occur.

Similar effects are induced by a temperature rise in the oil film due to increasing sliding velocity, for example. Increasing temperature will lower the viscosity and the \( \alpha \) value, giving a thinner oil film.

Iivonen and Hamrock [5] discussed starvation as a possible explanation for why increasing sliding velocity increases the risk of lubricating film breakdown. Starvation can be a problem if the applied lubricant layer is too thin.

Oil film inertia is also neglected. Owing to oil film inertia, it will not be possible to reach a fully lubricated condition in an infinitely short time. Studies of a thin oil film between two parallel surfaces, while giving one
of the surfaces an impulsively started sliding motion relative to the other surface (see, for example, ref. 13), shows however that a fully lubricated condition is reached very fast, in less than 1 μs. This shows that the simplification of neglecting oil film inertia is acceptable.

To explain why increasing sliding velocity will increase the risk of oil film breakdown, it is necessary to get a better understanding of how these effects, not included in the theoretical model, influence the oil film thickness.

6. Conclusions

From the theoretical investigation and the experiments, several conclusions can be drawn.
- Experimentally it has been shown that increasing initial squeeze velocity causes the limiting sliding velocity decrease.
- The reason why there is a considerable phase shift between maximum oil film force (or contact force) and oil film breakdown can be found if damping and elastic behaviour are attributed to the oil film. The damping effect of the oil film is high and thus, according to eqn. (4), the maximum oil film force will occur when the squeeze velocity is high, i.e. not when the minimum oil film thickness is reached. This is also seen in Fig. 8.
- Owing to deformations of the lubricated surfaces, the roller centre begins to move upwards earlier than the surfaces begin to separate.
- Using the same conditions as in the experimental apparatus, the theoretical analysis has shown that the squeeze motion dominates the formation of oil film pressure.
- The reason why there is a limiting sliding velocity cannot be found by using the simple model in this analysis. It has been shown, however, that the hydrodynamic influence due to sliding motion is small compared to the effects due to the squeeze motion. Effects not included in the model, such as limiting shear strength, starvation and increase in temperature, may therefore reduce the film thickness when the sliding velocity is increased.

Acknowledgments

The authors wish to thank Professor Erik Höglund and Dr. Ove Isaksson at Lulea University of Technology for valuable discussions. Thanks are also due to the Swedish Research Council for Engineering Sciences (TFR) for their financial support.

References


Appendix: Nomenclature

- \( a \) constant
- \( b \) constant or half width of hertzian contact \( R^{1/2}B/W/\pi \)
- \( c \) damping coefficient of the oil film (N s m\(^{-1}\))
- \( d \) constant
- \( E' \) effective elastic modulus (Pa)
- \( F \) oil film force, in the case of 1-DOF model (N)
- \( F_0 \) oil film force, decaying exponential function (N)
- \( F_c \) maximum contact force (N)
- \( g \) acceleration of gravity (m s\(^{-2}\))
- \( h \) oil film thickness (m)
- \( h_0 \) minimum oil film thickness (m)
- \( h_c \) coordinate describing the motion of the roller's centre of gravity (m)
- \( h_{oo} \) thickness of the applied layer of lubricant (m)
$k_s$ oil film stiffness (N m$^{-1}$)

$k_s$ stiffness simulating the elasticity due to the deformation of the surfaces (N m$^{-2}$)

$m$ mass of roller (kg, kg m$^{-1}$)

$p$ oil film pressure (Pa)

$q$ reduced pressure (Pa)

$R$ radius of roller (m)

$r_{sa}$ radius in sliding direction of solid $a$ (m)

$r_{sb}$ radius in sliding direction of solid $b$ (m)

$t$ time (s)

$V_n$ relative squeeze velocity between the surfaces (m s$^{-1}$)

$V_{no}$ initial relative squeeze velocity between the surfaces (m s$^{-1}$)

$V_{nc}$ normal velocity of the roller's centre of gravity (m s$^{-1}$)

$V_s$ sliding velocity (m s$^{-1}$)

$V_{sl}$ limiting sliding velocity (m s$^{-1}$)

$W$ oil film force or oil film load capacity per unit width (N m$^{-1}$)

$x$ coordinate describing the motion of the roller (m), or coordinate in sliding direction (m)

$x_c$ location of cavitation boundary (m)

$x_0$ describes the motion of the roller, decaying exponential function (m)

$\alpha$ pressure-viscosity coefficient (Pa$^{-1}$)

$\varphi$ phase angle between force and motion

$\eta$ absolute viscosity (Pa s)

$\eta_0$ absolute viscosity at $p=0$ (Pa s)
From the point of view of both costs and the environment, it is desirable to reduce friction and wear to a minimum. Lubrication reduces friction and protects the surfaces from wear. But more knowledge of the mechanisms of lubrication is needed.

In this thesis lubrication at non-steady loading is studied. The case of a ball dropped onto a lubricated surface is analysed numerically. Pressure and lubricant film thickness are computed during impact and rebound.

It was shown that a very large pressure occurs along the periphery of the contact area during the rebound stage.

The case of combined sliding and squeeze motion was also studied. This is the load case encountered in many bearings and gears. The combined motion is unfavourable as regards lubrication. It has been investigated how sliding velocity and squeeze velocity influence on lubricating capability.

Nyckelord, högst 8 / Keywords, max 8
tribology, elastohydrodynamic, lubrication, squeeze, impact, sliding, multigrid