

Sensitivity Analysis of an LQ Optimal Multivariable Controller for a Fine Coal Injection Vessel

Wolfgang Birk and Alexander Medvedev

Abstract—This paper deals with a sensitivity analysis of a linear quadratic optimal multivariable controller for a fine coal injection vessel used in the blast furnace process. The multivariable controller from a previous work is briefly presented and the closed-loop system is studied by means of a sensitivity analysis. Effects of disturbances and uncertainty on the closed-loop system are studied based on analysis of the singular values of the sensitivity and the complementary sensitivity functions, the relative gain array, and the minimized condition numbers. Finally, the sensitivity analysis is validated by the use of logged data from test operation at the coal injection plant at SSAB Tunnplåt AB, Luleå, Sweden.

Index Terms—Coal injection, linear quadratic control, multivariable control, sensitivity analysis.

I. INTRODUCTION

NOWADAYS, iron producers are reducing production costs by replacing the expensive energy carrier coke with other cheaper alternatives. In Luleå, Sweden, SSAB Tunnplåt AB is partly substituting coke by fine coal, which is 40% cheaper, in their iron production. The economical benefits of pulverized coal injection (PCI) are discussed in [1].

Since fine coal, in its pure form, is highly inflammable even under normal conditions, it is difficult to supply it to the process. Therefore, it is important to keep the fine coal isolated from the air, which can be done by using a pneumatic conveying device (see [2], [3]), where the transportation gas is nitrogen or at least has a higher rate of nitrogen compared to that of the air. The fine coal injection vessel is a part of a coal injection plant for a blast furnace, where fine coal is pneumatically conveyed to the blast furnace and finally injected at tuyere level. The coal injection plant at SSAB Tunnplåt AB (Fig. 1), which has been used for the experiments, is described in more detail in [4].

A drawback of substituting coke with fine coal is that it can result in blast furnace instabilities if coal flow outages appear [5]. Hence, a tight and reliable control of the fine coal flow from the injection vessel to the blast furnace becomes necessary.

A combined model-based control and leakage detection system for a fine coal injection plant has been developed in

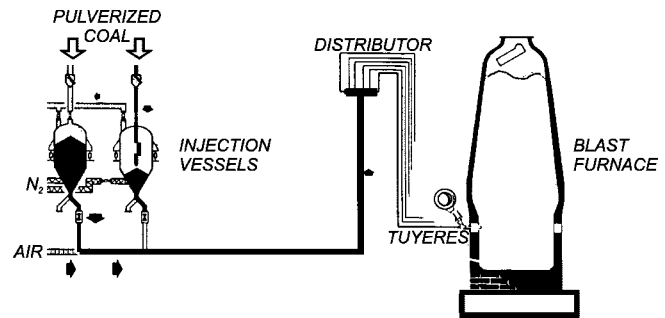


Fig. 1. Coal injection plant (injection vessels, distributor, and blast furnace).

[6]. The designed controller is a linear quadratic (LQ) optimal multivariable controller for the control of the fine coal flow out of the injection vessel. Since the controller is exposed to plant dynamics alternations because of components wear-out, repair, and replacement, as well as noise, it is necessary to analyze the robustness against these effects. Typically, sensitivity analysis is used to obtain the necessary information. In [7], some schemes for sensitivity analysis in the multivariable case are given and used for the subsequent sensitivity analysis.

Following up results of experiments and test operation is an essential part of an industrial project, since controller performance specifications have to be checked. From a theoretical point of view, such an analysis helps to improve controller designs and pinpoints possible shortcomings in the control strategy. It can also motivate further research in the area. This paper discusses such a followup in order to validate theoretical results of a sensitivity analysis.

The paper is organized as follows. In Section II, the controller design is presented. Section III discusses the sensitivity analysis and creates a framework for the followup. Finally, in Section IV, the acquired data from the tests are used to validate the antecedent analysis.

II. LQ OPTIMAL MULTIVARIABLE CONTROLLER

The multivariable controller is a part of the loop structure depicted in Fig. 2 and is a result of a previous study in [6].

Besides the state vector feedback controller, a Kalman filter, a feedforward controller, and an actuator saturation are present in the closed-loop system. Both Kalman filter and multivariable controller design are based on an identified multiple-input multiple-output (MIMO) model of the process dynamics.

The structure of an injection vessel can be described by Fig. 3, and is principally a pressurized tank process. During the injection of coal, the valves u_I and u_V are closed. Consequently, the two actuators u_N and u_C can be used to control the discharge

Paper PID 99-28, presented at the 1999 Industry Applications Society Annual Meeting, Phoenix, AZ, October 3-7, and approved for publication in the IEEE TRANSACTIONS ON INDUSTRY APPLICATIONS by the Metal Industry Committee of the IEEE Industry Applications Society. Manuscript submitted for review October 7, 1999 and released for publication January 28, 2000. This work was supported by Norrbottens Forskningsråd through the project "Reliable Process Control" at the Center for Process and System Automation (ProSA).

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Publisher Item Identifier S 0093-9994(00)04390-5.

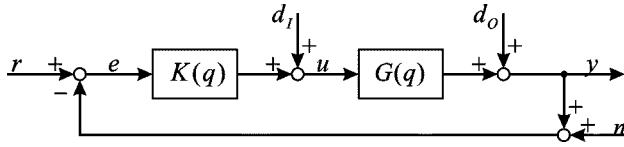


Fig. 4. Closed-loop system for sensitivity analysis.

where

$$e(k) = r(k) - y(k) = \begin{bmatrix} p_{\text{ref}}(k) - p(k) \\ m_{\text{ref}}(k) - m(k) \end{bmatrix}.$$

The measurement signal, vector $y(k)$, contains the pressure $p(k)$ and the net mass $m(k)$, whereas the reference signal vector contains the pressure set point $p_{\text{ref}}(k)$ and the net mass trajectory $m_{\text{ref}}(k)$.

According to the *separation principle*, the Kalman filter and LQ controller dynamics are not coupled, which allows separate study of their dynamical behavior. As the feedforward controller is designed for steady state and driven by an external signal [coal flow set point $\dot{m}_{\text{ref}}(k)$], the influence of this controller on the closed-loop dynamics can be neglected. Furthermore, it is assumed that the control signals are not saturated.

The resulting closed-loop structure for the controller and the plant is given in Fig. 4, where $K(q)$ denotes the controller in (4), $G(q)$ is the process model in (2), and q is the forward-shift operator. Additional to the loop structure in Fig. 2, disturbance and noise inputs are considered.

III. SENSITIVITY ANALYSIS

Three sensitivity functions are considered:

- complementary sensitivity function T , which is the transfer matrix from reference input r to the output y ;
- input sensitivity function S_I , which describes the transfer matrix from the disturbance input d_I to the control error e ;
- output sensitivity function S_O , which is similar to S_I , but for the disturbance input d_O .

For the sake of simplicity, the operator s is dropped.

Analyzing the block structure in Fig. 4 the following sensitivity and complementary sensitivity functions are obtained:

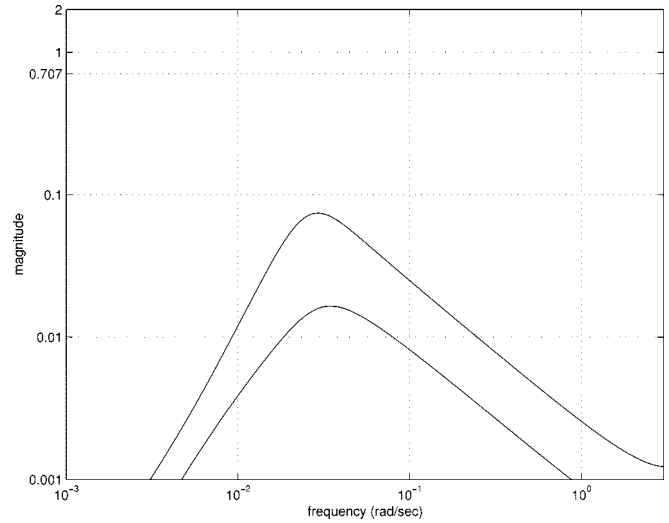
$$T = GK(I + GK)^{-1} \quad (5)$$

$$S_I = -G(I + GK)^{-1} \quad (6)$$

$$S_O = -(I + GK)^{-1}. \quad (7)$$

It can be mentioned that the sensitivity function for the noise input n to the control error e is identical to S_O , and the sensitivity function from the reference input r to the control error is equal to $-S_O$. Obviously, an analysis of these two more functions would not contribute with more information.

While the sensitivity and the complementary sensitivity functions are directly analyzed in the scalar case, the quantities of interest in the multivariable case are the singular values of these functions. However, the role of the sensitivity and the complementary sensitivity functions in both cases are similar, since

Fig. 5. Singular values of the input sensitivity function S_I .

their magnitudes are usually used to measure stability robustness with respect to modeling uncertainties.

Of great interest in the sensitivity analysis are the suprema of the singular values of the sensitivity functions. High peaks can lead to instability of the closed-loop system under perturbation and should be avoided. In the controller design, the characteristics of the singular values can be used to achieve a closed-loop system with minimized peaks in the singular values. Although the magnitudes of the sensitivity and complementary sensitivity functions are not a good measure for the gain of the MIMO system, information on the character of the cross couplings in the MIMO system can be obtained and exploited in the design process.

Finally, the sensitivity of the plant toward element-by-element uncertainty and input uncertainty is analyzed using the relative gain array (RGA) and the minimized condition numbers for the plant and controller.

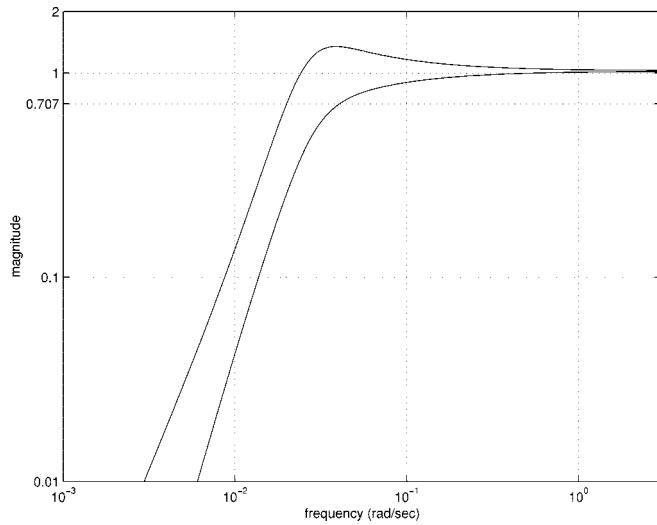
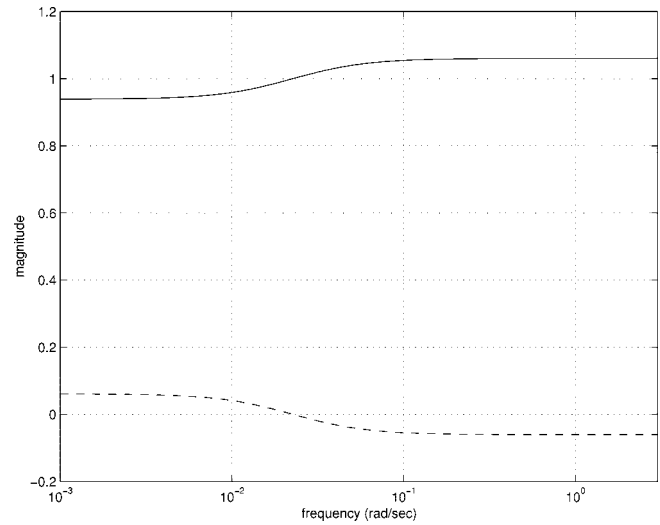
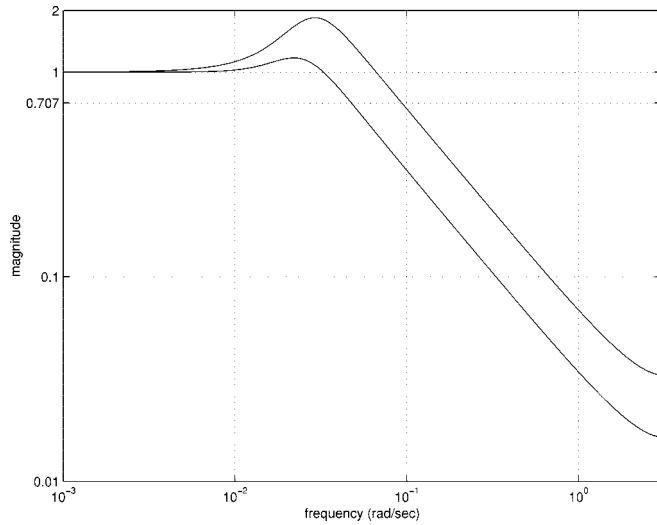
A. Input Sensitivity Function

Fig. 5 shows the singular values of the input sensitivity function S_I . Obviously, the magnitude is very small compared with the other sensitivity or complementary sensitivity functions (Figs. 6 and 7, respectively). Since disturbances of magnitude larger than one decade are unlikely to occur, the sensitivity to disturbances at the plant inputs can be neglected.

B. Output Sensitivity Function

Information on the bandwidth of the system with respect to output disturbance attenuation can be obtained from the singular value plot of the output sensitivity function (Fig. 6). Since the bandwidth for a multivariable system depends on the input/output directions, the lowest bandwidth value for output disturbance attenuation should be chosen. To determine the bandwidth of a system, the definition in [9] is used. According to Fig. 6, disturbances at the frequencies up to 0.02 rad/s can be attenuated.

Furthermore, the peak value of the sensitivity function (1.35) is a quite small value, which indicates that output disturbances can lead to a slight overshoot in the transient behavior.

Fig. 6. Singular values of the output sensitivity function S_O .Fig. 8. RGA of G .Fig. 7. Singular values of the complementary sensitivity function T .

C. Complementary Sensitivity Function

Similarly, the bandwidth for the reference tracking is defined, but here the higher value is chosen. Another important factor is the roll-off, i.e., a high negative slope above the crossover frequency.

Fig. 7 shows the singular values for T . The bandwidth for reference tracking is about 0.095 rad/s and the closed-loop system has a high negative slope of -2 above the crossover frequency. The peak value of T is also rather small and is 1.85. According to [7], recommended peak values are less than two.

D. Sensitivity to Uncertainty

In [7], the RGA and the minimized condition numbers are used to evaluate sensitivity to uncertainty. In this case, the MIMO model for design is obtained from direct identification of the plant dynamics. Because of nonlinearities, as well as disregarded dynamics, the model only approximates the plant behavior. Hence, uncertainty has to be taken into consideration.

The RGA is given by

$$\text{RGA}(G) = G \times (G^\dagger)^T$$

where \times is the Schur product and \dagger denotes the pseudoinverse. The RGA is computed at discrete frequency points. Here, the RGA is a symmetric 2×2 matrix of the form

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix}.$$

Large values in the RGA indicate that G will loose rank if the element g_{ij} in G is multiplied by a factor $(1 - (1/\lambda_{ij}))$, where λ_{ij} is the respective entry in the RGA. Thus, the RGA should contain small values.

In Fig. 8, the entries of the RGA are displayed. Because of the above-given form of the RGA, only two values are plotted. The values are small and, therefore, the closed-loop system should not be sensitive to element-by-element changes. An important property of the RGA is that sign changes of entries over the frequency axis indicate the presence of right-half-plane (RHP) zeros in G or at least in one subsystem of G . As pointed out in [10] and [11], such nonminimum-phase zeros lead to fundamental performance limitations of the closed-loop system. Since one RGA entry is changing sign (see Fig. 8), at least one subsystem of G has an RHP zero and, thus, performance limitations of the closed-loop system exist.

The minimized condition numbers $\gamma_I^*(G)$ and $\gamma_O^*(K)$ are a measure of robust performance to diagonal input uncertainty. According to [7], these condition numbers can be derived as follows:

$$\begin{aligned} \gamma_I^*(G) &= \min_{D_I} \gamma(G D_I) \\ \gamma_O^*(K) &= \min_{D_O} \gamma(D_O K) \end{aligned}$$

where D_I and D_O are scaling matrices. The minimized condition number is the result of a minimization of the condition number over all possible scales.

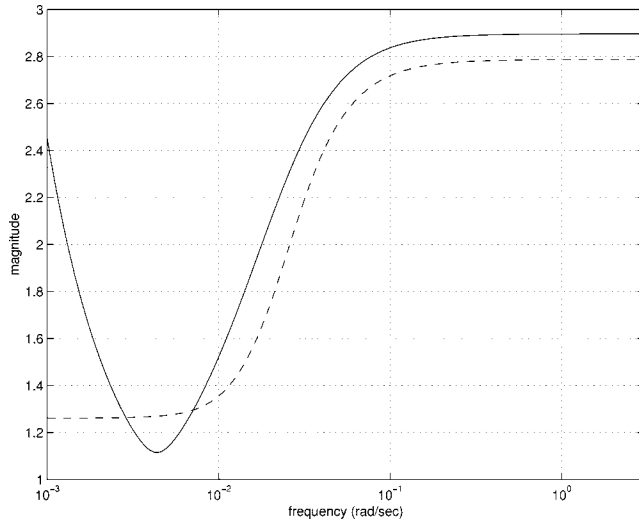


Fig. 9. Minimized condition numbers: $\gamma_I^*(G)$ solid, $\gamma_O^*(K)$ dashed.

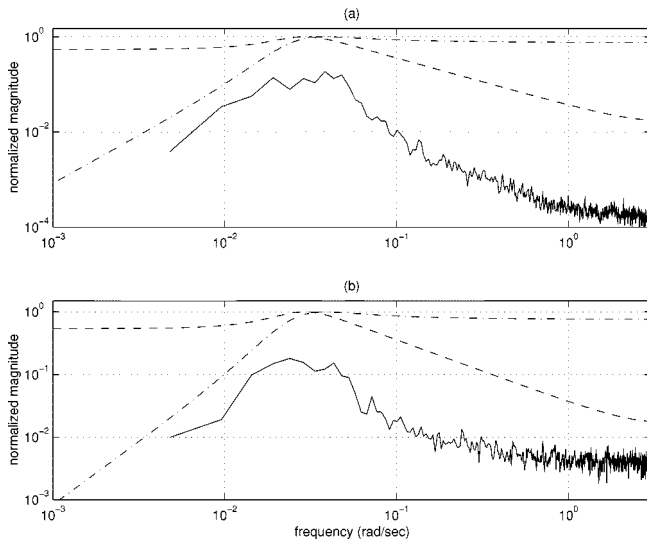


Fig. 10. Power spectral density (solid) versus maximum singular value of T (dashed) and S_O (dashed-dotted). (a) Power spectral density of the pressure error. (b) Power spectral density of the mass error.

For the closed-loop system to be insensitive to input uncertainty, the values of $\gamma_I^*(G)$ and $\gamma_O^*(K)$ should be around 2 or smaller in the crossover region. Fig. 9 shows both condition numbers. In the crossover region (between 0.024–0.065 rad/s), the minimized condition numbers are obviously around 2 (between 1.94–2.76), which yields a relative insensitivity of the closed-loop system to input uncertainty.

IV. FOLLOWUP

The LQ optimal multivariable controller has been tested during a period of two weeks at the coal injection plant. During this period, data have been logged. Since two injection vessels are injecting coal alternately, the time for one injection phase is limited and depends on the coal flow set point. Thus, the frequency range is not only bounded from above with half the sampling frequency (π rad/s), but also from below. The minimum recordable frequency is approximately 0.002 rad/s.

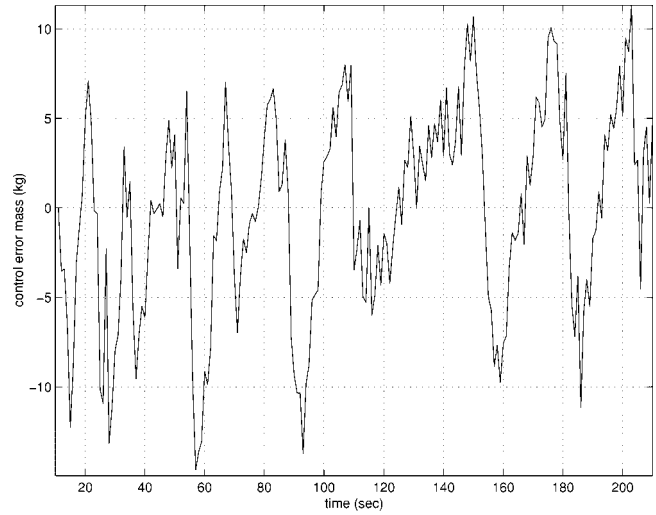


Fig. 11. Control error in the mass for the malfunctioning position control of the valve.

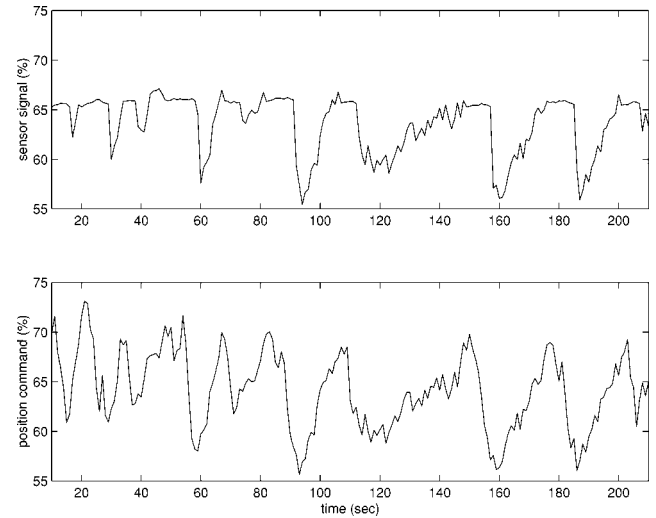


Fig. 12. Simulated malfunction of the position controller of the u_C valve. (a) Position signal from the sensor. (b) Control signal to the valve.

Several injection series are randomly selected, and the power spectral density is estimated. Then, the expectation value for every frequency point is estimated and plotted versus the maximum singular value of the output sensitivity and complementary sensitivity function (Fig. 10).

Obviously, the peaks in the maximum singular values of S_O and T are in the same frequency range as the ones in the power spectral density of the mass and the pressure deviation. Accordingly, the sensitivity analysis for S_O and T is validated by these results.

Since the input sensitivity function S_I contributes only with a comparably small magnitude, pure data analysis is not sufficient to validate the analysis. Therefore, a disturbance at the plant inputs has to be introduced in an experiment.

In practice, disturbances at the plant inputs are introduced by adding an extra signal to the input of the actuators, which can be achieved by sending a falsified position signal to the valve's position controller. Here, the position sensor signal is simply saturated before reaching its usual upper bound. Therefore, the

transmitted position signal is incorrect if the valve has opened more than this artificially introduced upper bound. As a result, the valve's position controller is reacting on a nonexistent position error and, thus, is opening the valve completely as soon as the physical position of the valve exceeds the artificial bound.

Such a situation can occur when the valve's position sensor malfunctions or a transmitting buffer amplifier saturates.

During the experiment, the closed-loop system did not lose its stability, but performance losses could be observed (see Fig. 11). In Fig. 12, the position signal from the valve and the control signal to the valve's position controller are displayed.

Since these rather extreme input disturbances have not invalidated the analysis of the input sensitivity function, it can be concluded that the analysis is reliable.

In order to test the closed-loop sensitivity for input uncertainty and element-by-element changes in practice, the design is based on a model for one injection vessel, but also run on the second injection vessel, which is equipped with a larger sized pressure control valve (u_N). During test operation, no performance losses due to this fact could be recognized.

V. CONCLUSIONS

In this paper, an LQ optimal multivariable controller for a fine coal injection vessel was analyzed. The indications from the theoretical sensitivity analysis are validated through evaluation of data acquired during test operation of the controller at the coal injection plant at SSAB Tunnplåt AB, Luleå, Sweden. It is shown that the observed behavior of the closed-loop system during operation agrees reasonably well with the results of the sensitivity analysis. Therewith, the controller design is validated and reliable enough to be considered for permanent installation. The commercially available control and leakage detection system *SafePCI* is now equipped with the above controller.

ACKNOWLEDGMENT

The authors would like to thank SSAB Tunnplåt AB, Luleå, Sweden, for making the coal injection plant and maintenance personnel available to them.

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