Nous avons donc d’après (6)

\[ W_0 = \frac{\varepsilon}{2} \Phi(0). \]  

(8)

Les résultats obtenus nous conduit à de considérations suivante. Toutes les espaces caractérisées par les formes métriques (2), où les coefficients satisfont à la condition \( g_{00} = -\frac{1}{\varphi^2} \), ne peuvent pas influencer la structure du champ. L’énergie est infini, analogiquement à l’énergie Coulombienne dans l’espace de Minkowski.

Dans le cas où la métrique est telle que le potentiel \( \Phi(r) \) est fini dans l’origine \( r = 0 \), nous obtenons l’énergie finie du champ de singularité ponctuelle. Par conséquent, nous voyons qu’il est possible d’obtenir l’énergie finie de l’électron ponctuel choisissant une métrique adéquate de l’espace Riemannien. Ceci vient du fait, qu’on peut substituer les forces agissantes dans l’espace de Minkowski par une métrique d’entrellement choisie de l’espace de Riemann, dans lequel nous considérons les phénomènes physiques.

**ELECTRIC SINGULARITIES IN THE GRAVITATIONAL FIELD**

Antoni Raabe

*Chair for Mechanics, University of Lviv*

We offer to the readers an article by Antoni Raabe, an Assistant of the Chair for Mechanics of the University of Lviv, which was to be published in the *Scientific Notes...* of the Lviv University in 1940, however, the journal never appeared in print (see below). The author’s texts based on the manuscript in Ukrainian and French are reproduced with the spelling and style preserved. The article may be of interest from both historical and methodological point of view.

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**ANTONI RAABE (1915–1942)**

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Raabe himself noticed (cf. [1]) that in 1937 he sent his paper on wave mechanics of material systems of points to Prof. Louis de Broglie (1892–1987). Moreover, in 1939 he did a paper on geometrization of the theory of mesotron, part of which was to be published in the autumn of 1939. This work was presented at the theoretical physics seminar of the Lviv University.

In the last months before the war, he lived in Vilnius and then he came to Lviv. Since January 1940 he was an assistant at the Chair for Mechanics of the Ivan Franko State University of Lviv, which was headed by Juliusz Paweł Schauder (1899–1943). In Lviv, Raabe continued his cooperation with Weyssenhoff who worked at the Lviv Polytechnic Institute at the time. In May 1941, Raabe was admitted to pass his candidate (doctoral) exams in the theory of relativity, theoretical physics, dialectical and historical materialism, and the German language, which were supposed to take place in the academic year 1941/42.

In the summer of 1941, following the German occupation of Lviv, he returned to Cracow, together with Weyssenhoff, where they continued collaborating on the relativistic theory of spin particles and spin fluid, which was a continuation of Mathisson’s and Weyssenhoff’s work [2, 3]. His three joint papers with Weyssenhoff were presented at a secret meeting of physicists in Warsaw in 1942, but could not be published during the German occupation. They appeared after the war:


The results of these works, especially from [WR47a], were later on cited by many authors [4, see especially p. 46]. In the summer of 1942, Raabe was arrested and sent to the Auschwitz concentration camp (now Oświęcim), where he was murdered on 7 September 1942.

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ЕЛЕКТРИЧНІ ОСОБЛИВОСТІ В ГРАВІТАЦІЙНОМУ ПОЛІ

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Пропонуємо увагу читачів статтю Антоні Раабе, асистент кафедри механіки Львівського університету, яка мала бути опублікована в "Наукових записках Львівського університету" 1940 року, однак сам журнал зник так і не вийшов. Авторські тексти на підставі рукописів українською та французькою мовами подані зі збереженим правопису і стилю. Стаття може бути цікавою як з історичного, так і з методичного перспектив.

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Відомо, що існує взаємний вплив електромагнітного і гравітаційного полів. З теорії Максвела–Еїнштейна випливають залежності цих піль, з якості сторони електромагнітне поле є жерелом гравітаційного, з другої сторони метрика простору змінює форму рівнянь електромагнітного поля. В досліджувати остання залежність та вплив метрики простору на власну енергію-пунктової електричної особливості.

Виходячи з рівнянь Максвела в просторі Рімана

$$\partial_\alpha \sqrt{-g} F^{\alpha\beta} = 0,$$

де $g$ — детермінант $|g_{\alpha\beta}|$ коефіцієнтів метричної форми простору. Розглядаємо статичний випадок з сферичною симетрією, це значить, що коефіцієнти $g_{00} = \frac{1}{1 - \frac{\rho}{R}}$ від часу, та єдиною не рівною нулю складовою тензора електромагнітного поля є $F^{10}$, тобто не залежне від часу.

Тому що існує сферична симетрія, приймаємо наступну метричну форму простору

$$ds^2 = g_{00} c^2 dt^2 + g_{11} dr^2 - r^2 (d\phi^2 + \sin^2 \theta d\varphi^2)$$

(2)

таким чином

$$\sqrt{-g} = \sqrt{-g_{00} g_{11}} \frac{r^2}{r^2 \sin \theta}.$$

Рівняння Максвелла приводяться тоді до одного рівняння

Les équations de Maxwell se réduisent dans le cas examinier à une seule équation

$$\partial_1 \sqrt{-g} F^{10} = 0$$

(3)

якого розв'язком є функція

$$F^{10} = \frac{\varepsilon}{\sqrt{-g}},$$

dout la solution générale est

$$F^{10} = \frac{\varepsilon}{\sqrt{-g}},$$

Но, варіація енергії взаємного впливу електромагнітного поля та гравітаційного поля буде

$$\delta T_{\text{EM}} + \delta T_{\text{GR}} = \delta \int \sqrt{-g} \left( \frac{\varepsilon^2}{\sqrt{-g}} - \frac{\varepsilon^2}{\sqrt{-g}} \right) \, d^4x.$$
де ε є сталою інтегрування, яка означає східні пунктової особливості. Інакше можемо написати

\[ F^{10} = \frac{\varepsilon}{r^2 \sqrt{-g_{00}g_{11}}} . \]  

З огляду на приняті форму (2), обчислення індексів тензора \( F^{\alpha\beta} \) виглядатиме

\[ F_{10} = g_{00}g_{11} F^{10} \]

на основі (4), маємо звідзь

\[ F_{10} = \frac{\varepsilon}{r^2 \sqrt{-g_{00}g_{11}}} . \]

З переходом в нескінченність гравітаційне поле прямує до нуля, тобто \( g_{00} \rightarrow 1, g_{11} \rightarrow -1 \), і тому електричне поле \( F_{10} \) асимптотично наближається до Кулонбовського.

Потенціал поля (5) можемо визначити з помочю інтегралу

\[ \Phi(r) = \int_r^\infty F_{10} \, dr = \varepsilon \int_r^\infty \frac{\sqrt{-g_{00}g_{11}}}{r^2} \, dr. \]

Прийняна гравітаційного поля до нуля при переході в нескінчність для нам збіжність інтегралу.

Перейдімо тепер до повної енергії електромагнітно-го поля

\[ W = \frac{1}{4\pi} \int \sqrt{-g} T^{00} \, d\tau, \]

де інтегрування розтягується по цілому просторі, \( T^{\alpha\beta} \) є тензор густоти енергії електромагнітного поля: \( T^{00} \) є тензор густоти енергії поміж простором і часом.

Le compositant covariant du champ électrique à la forme

\[ F_{10} = g_{00}g_{11} F^{10} \]

Quand la distance entre le point examiner et la singularité tend vers l'infini le champ gravitique devient zero, c. t. a. \( g_{00} \rightarrow 1, g_{11} \rightarrow -1 \), et par conséquent le champ électrique tend asymptotiquement vers le champ Coulombien.

Le potential électrostatique est donné par l’intégrale

\[ \Phi(r) = \int_r^\infty F_{10} \, dr = \varepsilon \int_r^\infty \frac{\sqrt{-g_{00}g_{11}}}{r^2} \, dr. \]

Le fait que le champ gravifique tend vers zero dans l’infini, nous donne la condition suffisante de la convergence de cette l’intégrale.

Nous considérons, maintenant, l’énergie totale du champ électrique

\[ W = \frac{1}{4\pi} \int \sqrt{-g} T^{00} \, d\tau, \]

оù \( T^{00} \) est tensor de densité de l’énergie-impuls du champ électromagnétique

\[ T^{00} = F^{\alpha\mu} F_{\alpha\mu} - \frac{1}{4} \delta^{\alpha\beta} F^{\mu\nu} F_{\mu\nu}. \]

В статичному випадку маємо

\[ T^{00} = \frac{1}{2} F_{10} F^{10}. \]

На основі (4) i (5), одержимо

\[ W_0 = \frac{1}{8\pi} \sqrt{-g} F^{10} \, d\tau = \frac{1}{8} \int_0^\infty \int_0^{2\pi} \int_0^r r^2 \sin \theta \sqrt{-g_{00}g_{11}} F^{10} F_{10} \, dr \, d\theta \, d\varphi = \frac{\varepsilon^2}{2} \int_0^\infty \frac{\sqrt{-g_{00}g_{11}}}{r^2} \, dr. \]
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