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Optimum inspection interval for hidden functions during extended life

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ABSTRACT

The methodology proposed in this paper aims to provide a mathematical model for defining optimal Failure Finding Inspection (FFI) interval, during the extended period of the replacement life. It considers the maintenance strategy of “a combination of FFI, and a discard action after a series of FFI”. A cost function (CF) is developed to identify the cost per unit of time associated with different FFI intervals, for the proposed extended period of life, i.e. postponement period. The Mean Fractional Dead Time (MFDT) concept is used to estimate the unavailability of the hidden function within the FFI intervals. The proposed method concerns as-bad-as-old (ABAO) inspection and repairs (due to failures found by inspection). This means that the unit keeps the state which it was in just before the failure that occurred prior to inspection and repair. It also considers inspection and repair times, and takes into account the costs associated with inspection and repair, the opportunity cost of lost production due to maintenance downtime created by inspection and repair actions, and also the cost of accidents due to the occurrence of multiple failure.
Keywords: Cost Function; Failure Finding Inspection; Hidden Failures; Hidden Function, Maintenance, Maintenance Cost, Reliability-Centered Maintenance (RCM); Optimization, Optimal Inspection.

Notation and Acronyms

α	*scale parameter*
β	*shape parameter*
φ	*demand rate for the hidden function per hour*
C_A	*cost of an accident*
C_I	*cost of inspection*
C_{OC}	*opportunity cost of lost production*
C_r	*cost of repair*
C_{Rep}	*cost of Discard*
FFI	*Failure Finding Inspection*
$F(t)$	*unreliability function*
$F_N(t)$	*conditional probability of failure within the* N^{th} *extended FFI*
$F_k(t)$	*conditional probability of failure within the* k^{th} *scheduled FFI*
$h(t)$	*rate of occurrence of failure (ROCOF)*
$H(t)$	*cumulative ROCOF*
K	*Number of scheduled FFI interval*
MFDT	*Mean Fractional Dead Time*
N	*number of extended FFI cycles*
$R(t)$	*reliability function*
t	*local time within* N^{th} *inspection cycle*
T	*inspection interval*
T_I	*inspection time*
T_K	*scheduled discard life with* $T_K=T_S.K$
T_N	*extended life for discard (postponement period) with* $T_N=T_P.N$
T_P	*FFI interval for the extended life period*
T_R	*repair time*
T_S	*original scheduled FFI interval*

1. Introduction

One aspect of maintenance programme analysis by Reliability-Centered Maintennace (RCM) methodology, is to develop tasks to preserve and assure the availability of hidden functions (or off-line functions). These types of functions are used intermittently or infrequently, so their failure will not be evident to the operating crew during the performance of normal duties[1-3]. Examples are the failure of a pressure relief valve, fire detector, fire extinguisher or emergency shutdown system. Termination of the ability to perform a hidden function is called hidden failure.

Hidden failures are analyzed as part of a multiple failure. A multiple failure is defined as “a combination of a hidden failure and a second failure or a demand that makes the hidden failure evident”. Hence, hidden failures are not known unless a demand is made on the hidden function as a result of an additional failure or second failure, i.e. a trigger event, or until a specific operational check, test or inspection is performed[4, 5].

Depending on the criticality and consequences of multiple failures and the demand rate, a specific level of availability of the hidden function is needed. Obviously, the probability of a multiple failure can be reduced by reducing the unavailability of the hidden function by performing a maintenance/ inspection task.

It should be noted that through the inclusion of some technologies, such as Prognostics & Health Management (PHM) and Built in Test systems, the performability of inspection task can be improved. Therefore, integration of RCM and PHM approaches should be considered during design phase, to enable PHM-based inspection and test for Hidden Failures, if applicable.[6]

Hidden failures are divided up into the "safety effect" and the "non-safety effect" categories. The failure of a hidden function in the "safety effect" category involves the possible loss of the equipment and/or its occupants, i.e. a possible accident. The failure of a hidden function in the "non-safety effect" category operational interruption or delays, a higher maintenance cost, and secondary damage to the equipment).

According to (RCM) [2, 3] a scheduled Failure Finding Inspection (FFI) may be necessary to detect the functional failure of hidden functions that has already occurred, but is not evident to the operating crew. The FFI tasks are developed to determine whether an item is fulfilling its intended purpose. In fact, if a hidden failure occurs while the system is in a non-operating state; the system's availability can be influenced by the frequency at which the system is inspected[7]. If the inspection finds the system inoperable, a maintenance action is required to repair it.

When the unit is aging, it may not be possible to find a single FFI task which, on its own, is effective in reducing the risk of failure to a tolerable level for the whole life cycle. This is due to the fact that, in a long run, the unavailability of hidden functions within the inspection interval may exceed the acceptable limit (see Section 3). Therefore, as a common practice, when a single FFI task is not effective, it is necessary to employ a replacement task after a series of FFI to reduce the risk of multiple failures. See [8, 9]for further study.

In fact, due to operational restrictions, or a lack of resources, sometimes the operators cannot ground the equipment, to perform the replacement task, as scheduled. In these cases, the operators are willing to postpone the replacement task to the earliest possible opportunity so that their operation will not be affected. However, such a postponement would affect the risk and may incur unacceptable economical, operational or safety risks. Identification and quantification of the operational risk of system failure is a great challenge. This is due to a long list of uncertainties related to the large number of contributory factors, the inadequacy of in-service information, and a lack of understanding of the influence of failures.[10]

When the operational risk of a hidden failure is identified, a trade-off analysis is needed to evaluate whether the extension of the discard life is acceptable. If the life extension justified, the inspection interval, also should be adjusted for the period of life extension. Moreover, safety and risk management should also be based on cost-benefit analyses performed to support decision making on safety investments and the implementation of risk reducing measures. Cost-benefit analysis is seen as a tool for obtaining efficient allocation of resources, by identifying which potential actions are worth undertaking and how these actions should be carried out. Aven and Abrahamsen [11] argue that by adopting the cost-benefit method the total welfare will be optimized.

The methodology proposed in this paper aims to provide a mathematical model for defining optimal FFI interval, during the extended period of the replacement life. It considers the maintenance strategy of "a combination of Failure Finding Inspection (FFI), and a discard action after a series of FFI". A cost function (CF) is developed to identify the cost per unit of time associated with different FFI intervals, for the proposed extended period of life, i.e. postponement period. The Mean Fractional Dead Time (MFDT) concept is used to estimate the unavailability of the hidden function within the FFI intervals, see [12]. The proposed method concerns as-bad-as-old (ABAO) inspection and repairs (due to failures found by inspection). On the other hand, the unit keeps the state which it was in just before the failure that occurred prior to inspection and repair. It considers inspection and repair times, and takes into account the costs associated with inspection and repair, the opportunity cost of lost production due to maintenance downtime created by inspection and repair actions, and also the cost of accidents due to the occurrence of multiple failure. The rest of the paper is constructed as follows. In section 2, probabilistic models for repairable units is discussed and in section 3, proposed analytical model is presented. The paper ends in chapter 4 with the conclusion. This paper is an extended version of Ahmadi, et al. [13].

2. Probabilistic Models for Repairable Units

A system can be classified into non-repairable and repairable ones. Non-repairable systems are those which are not repaired when they fail to perform one or more of their functions satisfactorily, and are instead discarded[14]. The discard action does not necessarily mean that the unit cannot be repaired. In some cases repair actions are not economically effective since a repair would cost almost as much as acquiring a new unit[15]. Repairable units are those which, after failing to perform one or more of their functions satisfactorily, can be restored to satisfactory performance by any method other than replacement of the entire system[14].

The quality or effectiveness of the repair action is categorized in three categories[14, 16, 17]:

1) Perfect repair, i.e. restoring the system to the original state, to a "like–new" condition,
2) Minimal repair, i.e. restoring the system to any "like-old" condition,
3) Normal repair, i.e. restoring the system to any condition between the conditions achieved by perfect and minimal repair.

Based on the quality and effectiveness of the repair action, a repairable system may end up in five different possible states after repair, i.e. an as-good-as-new, an as-bad-as-old, a better-than-old but worse-than-new, a better-than-new, and a worse-than-old condition [16, 17].

While perfect repair rejuvenates the unit to the original condition, i.e. to an as-good-as-new condition, minimal repair brings the unit to its previous state just before repair, i.e. an as-bad-as–old condition, and normal repair restores the unit to any condition between the conditions achieved by perfect and minimal repair, i.e. better than old but worse than new condition. However, states four and five may also happen [14-18]. For example, if through a repair action a major modification takes place in the unit, it may end up in a condition better than new; and if a repair action causes some error or an incomplete repair is carried out, the unit may end up in a worse-than-old condition[15, 18]. Failures occurring in repairable systems are the result of discrete events occurring over time. These situations are often called stochastic point processes.

The stochastic point process is used to model the reliability of repairable systems, and the analysis includes the homogeneous Poisson process (HPP), the non-homogeneous Poisson process (NHPP), the renewal process (RP) and the generalized renewal process (GRP), the last of which was introduced by Kijima[19].

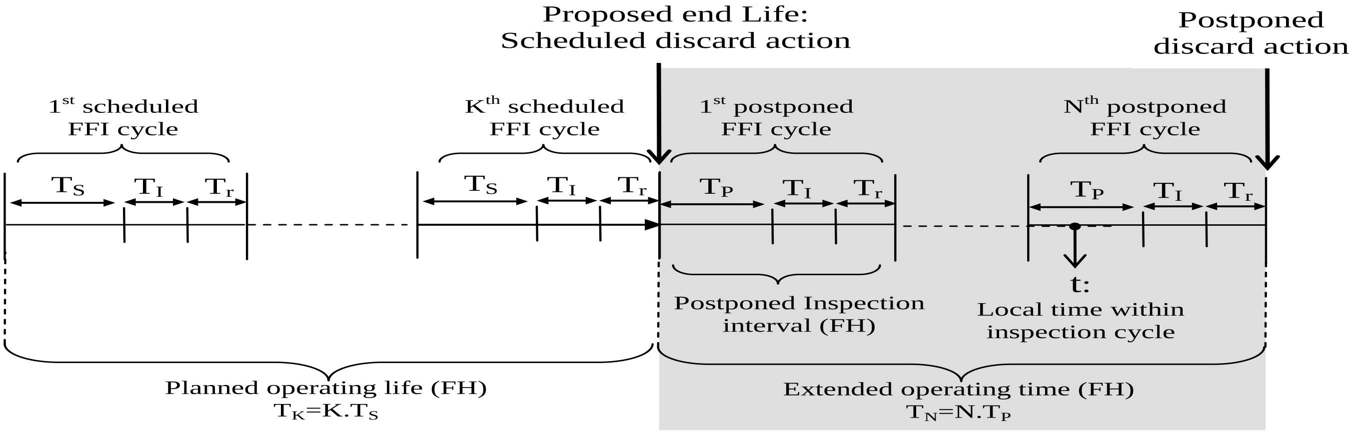

Figure 1: Schematic description of inspection and discard cycles

In actual fact, the failure of a unit may be partial, and a repair resulting from findings during FFI may be a partial repair and mostly concern adjusting, lubricating or cleaning the item. These types of repairs provide only a small additional capability for further operation, and do not renew the unit or system and may result in a trend of increasing failure rates[9, 17, 20].

In this study, it is considered that repair resulting from findings during FFI will be partially repaired, i.e. minimal repair, and hence the unit returns to an "as-bad-as-old" state after inspection and repair actions. On the other hand, the unit keeps the state which it was in just before the failure that occurred prior to inspection and repair, and the arrival of the ith failure is conditional on the cumulative operating time up to the (i-1)th failure. Hence, in the present study the power law process has been selected to model the reliability of such repairable units.

Under the imperfect repair assumption, the rate of occurrence of failure (ROCOF) and associated cumulative ROCOF of the power law are defined as [14]:

$$h(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} \quad (1)$$

$$H(t) = \left(\frac{t}{\alpha}\right)^{\beta} \quad (2)$$

where α and β denote the scale and shape parameters, respectively. Consequently, the failure probability (unreliability) and reliability functions at time "t" are defined as:

$$R(t) = e^{-H(t)} = \left[\exp-\left(\frac{t}{\alpha}\right)^{\beta}\right] \quad (3)$$

$$F(t) = 1 - R(t) = 1 - e^{-H(t)} = 1 - \left[\exp-\left(\frac{t}{\alpha}\right)^{\beta}\right] \quad (4)$$

In fact, if during the scheduled FFI, the unit is found functional (i.e. is found survived) at the (k-1)th inspection, the conditional probability of failure at any time "t", within the kth inspection cycle is given by:

$$\mathrm{F}_k(t) = 1 - \exp\left[\left(\frac{(K-1)T_S}{\alpha}\right)^{\beta} - \left(\frac{(K-1)T_S + t}{\alpha}\right)^{\beta}\right] \quad (5)$$

where "t" denotes the local time within the Nth inspection cycle, and T_S denotes the original scheduled FFI interval.

3. PROPOSED ANALYTICAL MODEL:

Figure. 1 shows a schematic description of maintenance events during the planned and extended operating times. During the initial planned operating life, the system must be grounded to perform an inspection task each time an accumulated number of operating hours "T_S" has been reached. The first inspection is performed after "T" hours, and consequently the Kth inspection will be performed after "T_K" operating hours. The inspection task takes T_I hours and, in the case of a finding which leads to a repair, the repair takes T_r hours. Hence, an inspection cycle includes T, T_I and T_r. An expected operating hours, "T_K", is considered as the unit's planned operating life, and is divided into K inspection cycles with the inspection interval TS, so that TK=K.TS .

Based on the selected interval, when the item reaches the maximum allowable operating hours, after which it will exceed the allowable risk limit, the item should be restored to its original condition. In this situation, when the operator wants to have an extension time, T_N, added to the planned operating life (i.e. age, T_K) the conditional probability of failure at time "t" after T_K and within the Nth inspection cycle of the extended operating time, is given by:

$$\mathrm{F_N}(t) = 1 - \exp\left[\left(\frac{T_K + (N-1)\mathrm{T}_P}{\alpha}\right)^{\beta} - \left(\frac{T_K + (N-1)\mathrm{T}_P + t}{\alpha}\right)^{\beta}\right] \quad (6)$$

where, under the postponement strategy, "t" denotes the local time within the Nth inspection cycle, and T_P denotes the FFI interval; both under postponement strategy.

The unavailability of hidden functions is usually measured by the mean fraction of time during which the unit is not operational as protection, i.e. the Mean Fractional Dead Time (MFDT)[5] . If dormant failures occur while the system is in a non-operating state, the system availability can be influenced by the frequency at which the system is inspected. Note that inspection, cannot improve the reliability, but can only improve the function availability[7]. According to Rausand and Vatn [12] and Vaurio [21], the function unavailability at time "t" within the Nth inspection cycle is equal to the conditional probability function, i.e. $F_N(t)$. Consequently, the mean interval unavailability within the N^{th} inspection cycle of the extended operating time, with FFI at every "T" hours, is given by:

$$\mathrm{MFDT}_{(T,N)} = \frac{1}{T}\int_0^T F_N(T)\,dt \tag{7}$$

3.1. Cost function for postponed FFI

The analytical model presented in this paper is based on the following assumptions:

1) The inspection and repair times associated with findings through FFI, i.e. T_i and T_r (constant values), do not change with operating time.
2) The failures concerned are not evident to the operating crew and hence do not interrupt the operation when they occur.
3) The failures are completely detectable by inspection/testing.
4) The inspection does not create failure by the nature of the tasks involved.
5) The maintenance crew do not create failure by their own actions.
6) Postponement of a restoration task does not increase the degradation and restoration costs.

The following cost parameters are considered for cost modelling of FFI in the postponement scenario:

- *Direct cost of inspection task, C_i.* This is considered as a deterministic value and as a constant in consecutive inspection cycles.
- *Cost of possible repair due to a finding, C_r.* As the system is undergoing aging, the probability of failure will change in consecutive inspection cycles. Hence, the expected repair cost within the N^{th} postponed inspection cycle can be estimated as: $C_r \,.\, F_N(T_P)$.
- *Cost of an accident, due to multiple failures, C_A.* The expected value of C_A in the N^{th} inspection cycle depends on the expected time during which the function is not available between two postponed inspections, "$\mathrm{MFDT}_{(T,N)}.T$", and the demand rate for the unit, φ, i.e. "$C_A \,.\, \varphi \,.\mathrm{MFDT}_{(TP,N)}\,.T$". For the "safety effect" category, C_A refers to the cost of accidents, e.g. the possible loss of the equipment and/or its occupants. For the "non-safety effect" category, C_A may entail possible economic consequences due to the undesired events caused by a multiple failure (e.g. operational interruption or delays, a higher maintenance cost, and secondary damage to the equipment.
- *Opportunity cost of the system's lost production, C_{oc}.* This cost is associated with the total system downtime due to inspection and repair, i.e. T_i and T_r. The expected value can be estimated as: $C_{OC}\,.[T_I+T_r.F_N(T)]$

Summing up, the total cost for a series of "N" inspection cycles under the FFI postponement strategy can be estimated by:

$$C_{T_P,N} = \left[\sum_{i=1}^{N}\left(\begin{array}{l} C_I + C_r.F_i(T_P) + C_{oc}.\left[T_I + T_r.F_i(T_P)\right] \\ +\left[C_A.\,\phi\;.\mathrm{MFDT}_{(T_P,i)}.T_P\right]\end{array}\right)\right] + C_{\mathrm{Rep}} \tag{8}$$

and the cost rate function for the extended period of time can be estimated by:

$$CRF_{(T_P,N)} = C_{(T_P,N)}\big/T_N \tag{9}$$

MAPLE Software is used to enable variation of the parameters of (9) to identify the cost per unit of time considering a target operating time of T_K=6000Hrs and T_N=1000, with the selected values of α=1000, β=3, $C_A.\varphi$=1, C_I=10, C_r=15, C_{OC}=100, T_i=0.2, and T_r=0.3, C_{Rep}=2000.

As Fig. 2 shows, the CRF is not sensitive around absolute T_{op} (i.e. 21,7Hrs), meaning that a range of inspection intervals, i.e. T_{op} ε [19Hrs-25Hrs], is reasonably acceptable.

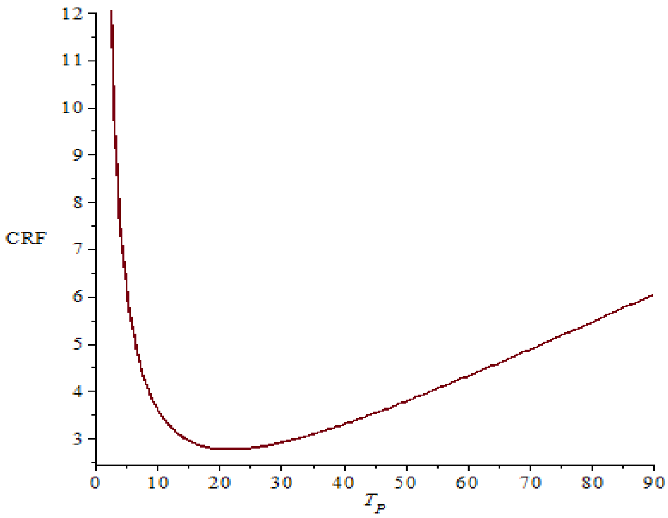

Figure 2: Cost rate vs. optimal inspection interval

In fact, the CRF is just the additional cost per unit of time for the extended operating life. If the number of inspections during extended life period tends to infinity (N →∞), then the CRF can be expected as:

$$\lim_{N\to\infty} CRF_{(T_P,N)} =$$

$$\underbrace{\lim_{N\to\infty}\frac{NC_I}{NT_P}}_{\frac{C_i}{T_P}} + \underbrace{\lim_{N\to\infty}\frac{C_r}{NT_P}.\sum_{i=1}^{N}F_N(T_P)}_{\underbrace{\frac{C_r.N}{NT_P}}_{\frac{C_r}{T_P}}} + \underbrace{\lim_{i\to\infty}\frac{C_{oc}}{NT}\sum_{i=1}^{N}T_i}_{\underbrace{\frac{C_{oc}.NT_i}{NT_P}}_{\frac{C_{oc}.T_i}{T_P}}}$$

$$+\underbrace{\lim_{N\to\infty}\frac{C_{oc}.T_r}{NT_P}.\sum_{i=1}^{N}F_N(T_P)}_{\underbrace{\frac{C_{oc}.T_r.N}{NT_P}}_{\frac{C_{oc}.T_r}{T_P}}} + \underbrace{\lim_{N\to\infty}\frac{C_A.\,\phi.T_P}{NT_P}.\sum_{i=1}^{N}MFDT_{(T_P,N)}}_{\underbrace{C_A.\,\phi}} + \underbrace{\underbrace{\frac{C_{\mathrm{Res}}}{NT_P}}_{N\to\infty}}_{0} \tag{10}$$

$$\lim_{N\to\infty} CRF_{(T_P,\mathrm{N})} = \frac{C_i + C_r + C_{oc}.(T_i + T_r)}{T_P} + C_A.\,\phi$$

3.2. Optimum interval

In fact, the operating time T_N is divided into N inspections with the interval T_P, so that $T_N=N.T_P$. The following equations are valid under certain conditions for $F_N(T)$, as proved by Vaurio [21]:

$$\sum_{i=0}^{K} F_i(T) \cong H(T_K) \quad (11)$$

$$\sum_{i=0}^{K} \overline{F_i(T)} = \sum_{i=0}^{K} MFDT_{(T,K)} \cong H(T_K)/2 \quad (12)$$

$F_i(T)$ represents the conditional probability of having just one failure in the i^th^ inspection interval, provided that the unit found functional at (i-1)^th^ inspection. $H(T_K)$ represents the mean number of failures over an interval of $(0,T_K)$. Hence, it is evident that $H(T_N)$ overestimates $\Sigma F_i(T)$. It shold ne noted that for larger α, and smaller β values, the $H(T_K)$ and $F_N(T)$ become more close, and tend to lead to more accuracy in the estimation, while smaller α and larger β values tend to lead to more deviation in the estimation. Moreover, selecting larger T values increases the inaccuracy in the estimation.

Similarly, we can express the mean number of failures within the extended operation time T_P as follow:

$$\sum_{j=0}^{N} F_i(T) \cong H(T_K + T_P) - H(T_P) \quad (13)$$

$$\sum_{i=0}^{N} \overline{F_i(T)} = \sum_{i=0}^{N} MFDT_{(T,K)} \cong \frac{H(T_K + T_P) - H(T_P)}{2} \quad (14)$$

Likewise, using the method introduced by Vaurio [21], by substituting (13) and (14) into (9), and denoting $T_P.N$ as T_N, the following CRF can be derived as a function of the inspection interval T, and the operating time T_P:

$$\begin{aligned} CRF_{(T,N)} = & \frac{C_I}{T_P} + \frac{C_r\,[H(T_K+T_N) - H(T_K)]}{T_N} \\ & + \frac{C_{oc}.T_I}{T_P} + \frac{C_{oc}.T_r.[H(T_K+T_N) - H(T_K)]}{T_N} \\ & + \frac{C_A.\phi.T_P.[H(T_K+T_N) - H(T_K)]}{2T_N} + \frac{C_{Rep}}{T_N} \end{aligned} \quad (15)$$

The optimum inspection interval, T_{OP}, that can minimize the CRF, by a fixed operating time, T_N (Hrs), can be found through this derivative $[CRF_{(T,N)}/dT_P]=0$:

$$T_{OP} = \left[\frac{2T_N.(C_I + C_{oc}.T_I)}{C_A.\varphi.[H(T_K+T_N) - H(T_K)]}\right]^{1/2} \quad (16)$$

In accordance with (16), and considering a target operating time of T_K=6000 Hrs, extended operating time of T_N=1000, and with the selected values used in Fig. 2, the optimum inspection interval is estimated to T_{op}=25.xx Hrs, which leads to CRF=$0.209 with N=40 [1000/25=40].

However, in case of the “safety effect” category of failure, the postponement process requires adequate proof of risk reduction which satisfies the risk limits. Since such a postponement may increase the cost rate and affect economy negatively, another tradef-off analysis is needed to evaluate whether the postponement idea is economical and is acceptable or not.

In the case of the “safety effect” category of failure, the limit conditions for the risk of multiple failures may be dictated through the policies authorities or the companies themselves. Considering R_{max} as the maximum permissible risk limit for the probability of multiple failures, then the postponement process needs to limit the risk of failure under the following supplementary constraint:

$$\phi.\,MFDT_{(T,N)} \le R_{max} \quad \Rightarrow \quad MFDT_{(T,N)} \le R_{max}/\varphi \quad (17)$$

4. Conclusions

This paper introduces a methodology to identify the optimum Failure finding inspection (FFI) interval during extended operating life of a nunit. In this study, a cost function (CF) has been developed to identify the cost per unit of time associated with different FFI intervals, for the proposed extended period of life. Moreover, a mathematical model has been defined to obtain the optimal FFI interval, during the extended period of the replacement life. Following this methodology, the optimum FFI interval that generates the lowest cost per unit of time, can be obtained.

The interval unavailability behavior has been discussed and MFDT has been used to identify the limits by which the risk of failure can be eliminated. This approach makes it possible to recognize the real effect of postponement on the total maintenance cost and to evaluate whether performing postponement provides benefits.

The results shows when the extended life is increasing, the FFI inspection interval within the extended life, should be reduced to arrive in at a minimum cost. Following proposed methodology the capability to take correct and effective decisions on the postponement of discard tasks will be improved.

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