Design Rules for Multipath Simulator Antenna Array

Paul Hallbjörner
Abstract

Multipath simulators are a technique for characterizing wireless terminals, such as for instance mobile phones. A multipath simulator distributes the measurement signal over an array of antennas encircling the test object. When building a multipath simulator, the antenna array should be given an optimal design with respect to the characteristics of the test objects, the parameters to be measured, and the desired measurement accuracy. This report presents mathematical relations which can be used for optimizing the antenna array. The relations are mostly derived theoretically using simple assumptions, and in some instances found from numerical simulations. The report assumes an array with the antennas evenly distributed on a circle with the test object at the center, and a measurement signal power which is uniformly distributed over the array. Considered measured quantities include power, correlation coefficient, and the statistical distribution of amplitudes.

Key words: Multipath simulator, Design rules, Measurement accuracy

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1 Objectives

Accurate characterization of mobile terminal antennas using multipath simulators (MPS) relies on a properly designed MPS antenna array. This report studies different measurement errors related to the antenna array. The study should establish relations which ensure that within a given tolerance...

- The same average transferred power is measured regardless of test object location within the test zone and orientation in azimuth.
- The same correlation coefficient is measured regardless of test object location within the test zone and orientation in azimuth.
- A Rayleigh distribution of the transferred amplitude is achieved.

Results are in the form of simple formulas relating the design parameters of the antenna array to test object parameters and measurement accuracy.

2 Assumptions and Variables

The array is always assumed to be circular with the antennas evenly distributed over the circle. These constraints together with four variables according to Table 1 define the array. Four input parameters according to Table 2 are used when deriving the relations for the MPS antenna array. Other parameters that relates to the surroundings of the MPS are listed in Table 3.

Table 1. Variables defining the MPS antenna array.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>Array radius</td>
<td>m</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of array antennas*</td>
<td>1</td>
</tr>
<tr>
<td>$W_a$</td>
<td>Array antenna beam width</td>
<td>rad</td>
</tr>
<tr>
<td>$G_a$</td>
<td>Array antenna power gain</td>
<td>1</td>
</tr>
<tr>
<td>$M$</td>
<td>Multiplicity*</td>
<td>1</td>
</tr>
</tbody>
</table>

*N is the number of antennas per polarization and MIMO branch. The total number of array antennas is $MN$.

Table 2. Input parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Test object maximum dimension</td>
<td>m</td>
</tr>
<tr>
<td>$r$</td>
<td>Test zone radius</td>
<td>m</td>
</tr>
<tr>
<td>$G_i$</td>
<td>Test object power gain</td>
<td>1</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Relative power error ($\Delta P/P$)</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3. Parameters used to describe the room.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>Signal power</td>
<td>W</td>
</tr>
<tr>
<td>$I$</td>
<td>Interference power</td>
<td>W</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Power standard deviation</td>
<td>W</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Power mean</td>
<td>W</td>
</tr>
</tbody>
</table>
The centre of the test zone always coincides with the centre of the MPS antenna array. Figure 1 illustrates the setup. The test object is always placed entirely within the test zone. A uniform power distribution over the MPS array (2-D environment) is assumed in all cases. For simplicity, the derivation is sometimes done under assumptions of feeding a single array antenna, but those cases assume subsequent superposition of signals from all array antennas.

An MPS antenna array sometimes has two polarizations, typically with every other linear vertical and every other linear horizontal. The presented formulas should then be applied to each polarization separately with its set of array antennas, ignoring the antennas of the other polarization. Study of errors such as polarization mismatch and cross-polarization decoupling is omitted, a reasonable approach as long as the polarizations used in the antenna array are linear vertical and horizontal, but not necessarily if they are slanted linear or circular.

Test object radiation pattern variations are limited by the maximum dimension of the test object. With a maximum dimension of \( D \), the worst case in terms of rapid variations in the radiation pattern is with two equally strong radiators separated by \( D \). The half-power beam width can then be as small as in (1). Variations can be slower than this, and at the other extreme the radiation can be constant.

\[
W_t = \frac{\lambda}{2D}
\]  

(1)

Figure 1. Geometry of antenna array, in this case with \( N=8 \). The array antennas are always evenly spaced around the circle, and the test zone is always centred at the centre of the array. The test object should always fit within the test zone, i.e. \( D<2r \).

3 Method

Relations between the input parameters and the MPS antenna array variables, under the given assumptions, are derived using basic formulas for transferred power and geometry. When necessary, numerical simulations in Matlab are used to establish the relations. Resulting relations are subdivided into categories in the following, for the purpose of providing a better overview. The categories are "transferred power", "correlation coefficient", "Rayleigh distribution", and "multiplicity". For a specific MPS design, one or several of these could be relevant, or perhaps all of them. Whether the Rayleigh
distribution is relevant at all depends very much on whether the MPS should simulate a realistic environment, because such environments rarely show a Rayleigh distribution.

All relations are given as mathematical expressions with the array variables (Table 1) on the left-hand side, and the input parameters (Table 2) on the right-hand side. Some expressions do not include \( p \). The reason is that sometimes the relation is quite insensitive to \( p \) over a certain range and then \( p \) suddenly becomes unreasonably large. There is then no point in quantifying the error but instead just give the limit for the useful range.

After having derived the relations, two design examples are looked at. In the first, an MPS is designed for characterization of mobile handsets. In the second, an MPS is designed for characterization of laptops. These design examples demonstrate which of the relations are most difficult to satisfy, and thereby what to focus on in the MPS antenna array design.

4 Transferred Power

A number of different error sources are studied, each providing a constraint for the antenna array design.

4.1 Far-Field Condition

This is concomitant to avoiding an uneven illumination of the test object. The far-field condition should be met for each MPS array antenna versus the test object.

A worst case is assumed with the test object having two equally strong in-phase radiators within the maximum dimension \( D \). Amplitude and phase illuminations are treated separately. When studying amplitude illumination, the test object is located at the centre of the test zone. This is not a worst case, but moving the test object around within the test zone makes a minor difference, and is also treated separately in a subsequent section. When studying phase illumination, the error can be much larger when moving the test object to extreme positions within the test zone, compared to keeping it at the centre. Therefore, the test object is in this case placed at the edge of the test zone. Figure 2 illustrates how the two radiators are placed in the different cases.

Transferred power is calculated assuming that the signals from the two radiators are added in-phase. This power is compared to a reference case with a small test object placed at the centre of the test zone. The difference in transferred power constitutes the measurement error.
Figure 2. Cases used to derive relations for the far-field criterion. All three cases assume a test object of two equally strong in-phase radiators. The transferred power is compared to the reference case. Case a gives an uneven amplitude excitation of the test object. Cases b and c give uneven phase excitation of the test object for different assumptions regarding $D$ versus $r$. One array antennas is fed. The far-field condition should be met for each array antenna.

Amplitude error in endfire case (Figure 2a) results in the relation

$$R \geq \frac{D}{\sqrt{2p}}$$  \hspace{1cm} (2)

Phase error in broadside case with $D<r$ (Figure 2b) gives

$$R \geq \frac{\pi D}{\lambda \sqrt{p}}$$  \hspace{1cm} (3)

and with $D>r$ (Figure 2c)

$$R \geq \frac{\pi^2}{2\lambda \sqrt{p}}$$  \hspace{1cm} (4)

4.2 Array Antenna Beam Width

Worst case here is a test object in the form of a small antenna at the edge of the test zone, with reference case at the centre of the test zone, which is where calibration is assumed to be performed.
Figure 3. Case for array antenna beam width. Test object is a point radiator at maximum offset sideways. The reference case is with test object at the centre. One array antennas is fed, and the relation should apply for each array antenna.

The resulting relation is

$$RW_a \geq \frac{\pi R}{2 \sqrt{p}}$$

(5)

4.3 Multiple Reflections between Array and Test Object

A receiving antenna re-radiates a power which is equal to the received power. Power transfer between two antennas will be affected by multiple reflections between the two antennas. For one of the array antennas communicating with a test object at the centre of the test zone, the condition for keeping the error due to this effect below $p$ is

$$\frac{R}{\sqrt{G_a}} \geq \frac{\lambda}{2\pi} \sqrt{\frac{G_t}{2p}}$$

(6)

4.4 Test Object Offset

The test object is a small antenna at the edge of the test zone, and the reference case is with the small antenna at the centre of the test zone. All array antennas are fed.
Figure 4. Case for test object offset. Test object is a point radiator at maximum offset in one direction. The reference case is with test object at the array centre. All array antennas are fed.

With an omnidirectional test object, moving closer to some array antennas will be to a large extent compensated for by moving away from other array antennas. The condition is therefore benign, and is found to be

\[ R \geq \frac{r}{\sqrt{p}} \]  

(7)

Directive test objects are much more critical. In worst case, the test object is communicating with one or a few array antennas in only one direction. Moving in this direction is then not compensated for as in the omnidirectional case. The situation is equivalent to feeding only one array antenna, and the relation becomes

\[ R \geq \frac{2r}{p} \]  

(8)

Assuming a directive test object implies that the test object is no longer small, and the maximum offset can therefore not equal \( r \). However, for oblong test objects radiating in the broadside direction and with \( D \) smaller than \( 2r \), the maximum offset can be nearly \( r \).

### 4.5 Test Object Rotation

Worst case is a directive test object. If the test object half power beam width is equal to or greater than the angular spacing between neighbouring array antennas as seen from the test object, there will be no variations in transferred power during rotation. If it is smaller, there will be variations, which can be very large. Using the equality as condition, with the worst case (i.e. smallest) test object beam width, renders the condition

\[ N \geq \frac{4\pi D}{\lambda} \]  

(9)
Figure 5. Case for test object rotation. The test object consists of two equally strong in-phase radiators separated by $D$, with different rotations compared to the reference case. All array antennas are fed.

### 4.6 Disturbances from the Room

Reflections from the walls of the room in which the MPS is located will interfere with the measurement signal and cause an error. Reflections from antenna stands and feed cables as well as leakage from cables are other examples of disturbances which can be included in the same category. By "disturbances from the room" should thus be understood all disturbances emanating from sources other than the MPS antenna array itself.

In order to quantify the error, a statistical approach is used, with a two-sigma criterion for the error, i.e.

$$p = \frac{2\sigma}{\mu} \quad (10)$$

where $\sigma$ is the standard deviation of the power and $\mu$ is the mean power. For signal power $S$ and total disturbance (interference) power $I$,

$$\frac{\sigma}{\mu} = \sqrt{1 + \frac{2S}{I}} \approx \sqrt{\frac{2I}{S}} \quad (11)$$

The required signal-to-interference ratio is thus

$$\frac{S}{I} \geq \frac{8}{p^2} \quad (12)$$
4.7 Mutual Coupling within the Array

Interference resulting from coupling within the MPS array is analyzed. Since an antenna re-radiates a power equal to the received power, the signal from each array antenna will reflect in the other array antennas. The reflected signals appear at the test object as a disturbance, the magnitude of which can be derived using theoretical formulas. The disturbance level is then treated in the same way as disturbances from the room, i.e. it is related to the normalized standard deviation \( \sigma/\mu \).

Assuming that coupling between neighbouring array antennas is the dominating contribution, which typically is the case with omnidirectional array antennas and a large number of array antennas, the relation is

\[
\frac{R}{G_a N} \geq \frac{\lambda}{16 p}
\]

(13)

If the array antennas are directive with low radiation sideways, coupling to opposing array antennas is likely to be dominating. In that case,

\[
\frac{R}{G_a \sqrt{N W_a}} \geq \frac{\lambda}{16 p}
\]

(14)

Both cases should of course be considered hence both (13) and (14) should be satisfied. If the MPS array consists of antennas of different polarizations, a relation should be calculated for each polarization separately.

The coupling between array antennas can be reduced by various techniques, if necessary. Absorbing shields between the array antennas, shunt circuit elements between the array antenna ports, and array antenna element pattern suppressions in the directions of other array antennas, are all examples of such techniques. Each of these works best between neighboring antennas. Only a lower \( G_a \) can reduce the coupling between opposing antennas (for fixed values of \( R \) and \( N \)).

5 Correlation Coefficient

Measurement of correlation coefficient is analyzed numerically in Matlab. The correlation coefficient used here is the magnitude of the correlation coefficient calculated from complex amplitudes. A required accuracy of ±0.1 is anticipated. Accuracy is thereby not included as a variable in the derived formulas.

Test object offset and rotation are not very critical. By far the most critical condition is to have a sufficient number of array antennas within the test object coverage area. Different types of orthogonality between the test object antennas (space, angle, polarization) are studied. The reference case is the theoretical case of a continuous power distribution over angle and from an infinite distance. Two types of errors can occur with the MPS compared to the reference case. First, the fact that the MPS provides a discrete power distribution instead of a continuous can make a difference. Second, the limited \( R \) can make a difference, because angles and distances to the diversity antennas are affected in different ways.
5.1 Space Diversity

A test object consisting of two omnidirectional diversity antennas separated by $D$ represents the typical space diversity case. In Figure 6, the two diversity antennas are marked with a red and a yellow dot, respectively.

![Space diversity case](image)

Figure 6. Space diversity case. Two diversity antennas marked with red and yellow, each a small antenna separated by $D$. All array antennas are fed. The reference case is the theoretical case of a continuous power distribution over angle and from an infinite distance.

The relation for a correlation coefficient error of less than 0.1 is found to be

$$N \geq \frac{8D}{\lambda}$$

(15)

Only a very small dependence on $R$ is found, and the relation is practically the same even if the desired accuracy is ±0.2 instead.

5.2 Angle Diversity

A test object consisting of two directive diversity antennas at varying angles relative to each other is assumed. Each diversity antenna consists of two equally strong in-phase radiators separated by $D$. The diversity antennas are rotated relative to each other. The setup is illustrated in Figure 7, with the two diversity antennas marked with red and yellow dots, respectively.
Figure 7. Angle diversity case. Two diversity antennas marked with red and yellow, each a pair of equally strong in-phase radiators separated by $D$, centered at the test zone centre. The two test objects are at varying angles relative to each other. All array antennas are fed. The reference case is the theoretical case of a continuous power distribution over angle and from an infinite distance.

The relation is the same as for space diversity, i.e.

$$N \geq \frac{8D}{\lambda}$$

(16)

Also in this case it is seen that there is only small dependence on $R$, and that the relation is approximately the same if the desired accuracy is $\pm 0.2$.

### 5.3 Polarization Diversity

In order to study polarization diversity, both space and angle diversity need to be eliminated. Therefore, a setup is assumed with two omnidirectional antennas both located at the test zone centre. They are assumed to have different and varying polarizations.
The requirements on the MPS array depend on the rate of change in test object polarization over angle. For practical test objects, this rate is typically slow, so about 5-10 array antennas is estimated to be sufficient. Should a test object with unusually rapid changes in polarization be at hand, the value of $N$ needs to be greater.

6 Rayleigh Distribution

Rayleigh distribution is accomplished by adding several signals. Approximately five signals should be added in order to give an accurate Rayleigh distribution, if the signals are equally strong. If the signals are weighted in amplitude, the required number of signals is effectively higher. For the MPS array, signals can be translated to array antennas, but it should be kept in mind that the magnitude of a signal also depends on the test object radiation pattern. Consequently, at least five array antennas within the test object coverage area are needed. Hence, for directive test objects,

$$N \geq \frac{20\pi D}{\lambda}$$  \hspace{1cm} (17)$$

Omnidirectional test objects basically cover all array antennas, but in practice they often have some variations in the coverage. In order to compensate for the variation, the number of signals should be approximately doubled, and consequently

$$N \geq 10$$  \hspace{1cm} (18)$$
7 Multiplicity

7.1 Polarization

Above presented relations are valid for an array of the same polarization, i.e. $N$ is the number of array antennas with one polarization. An MPS antenna array that contains two orthogonal polarizations therefore has $2N$ array antennas, i.e. a multiplicity $M=2$. Design conditions should then be studied for each set of $N$ antennas separately. An exception can be made for disturbances from the room, where all $2N$ antennas can be treated as one array, but then again these relations do not contain the variable $N$. Note that it is possible for all antennas in the array to have the same polarization in all directions only if the polarization ellipse has a vertical or horizontal main axis (common examples of this are: linear vertical, linear horizontal, and circular). Slant linear polarization, for instance, applied equally to all array antennas, will appear as different polarizations depending on the direction.

The derived expressions assume no polarization mismatch of the test object versus the MPS array antennas. Polarization must therefore be considered when using the expressions. This is to say that the test object radiation characteristics can be different when calculating relations for different polarizations. Especially the test object gain $G_t$ can have vastly different values depending on the polarization. As long as this is observed, no extra multiplicity for the MPS array antennas is necessary because of the polarization properties of the test object. (The factor-two multiplicity for dual polarized MPS arrays described in the previous paragraph still applies, however.)

7.2 MIMO

Presented relations assume that all MPS array antennas are included when the signal is said to be distributed uniformly over the array. An MPS to be used for $2 \times 2$ MIMO tests, however, has two connectors towards the base station emulator, and each connector couples to only half of the array antennas. The variable $N$ is then the number of array antennas for one of the MIMO branches. An MPS that supports $M \times M$ functionality needs a factor $M$ more array antennas, i.e. its array should consist of $MN$ antennas.

The derived relations should be applied for each MIMO branch of the MPS array separately. One exception to this is disturbances in the room which can be considered for all MIMO branches together, but which does not contain the variable $N$ anyway. Another exception is coupling within the array where all MIMO branches can also be treated as a single array, as long as all array antennas have the same polarization.

8 Example

A design example is included to illustrate the use of the presented relations. It also serves the purpose of highlighting the main design problems for MPS antenna arrays. A total power accuracy of $\pm 1$ dB is assumed. With approximately 10 error sources combined according to RSS, the acceptable error for each error source is $\pm 0.3$ dB, which corresponds to $p=0.069$.

A typical handheld mobile terminal is envisioned as test object. It is assumed to include dual antennas for diversity/MIMO functionality. Input parameters are as follows:
- \( f = 700-2700 \, \text{MHz} \)
- \( D = 0.1 \, \text{m} \)
- \( G_t = 1 \)
- Test object cannot be directive
- Dual antennas for diversity/MIMO
- \( r = 0.1 \, \text{m} \)
- \( p = 0.069 \)

These values are applied to the relations, with the following results:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Condition Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power, far-field condition</td>
<td>( R &gt; 0.5 , \text{m} )</td>
</tr>
<tr>
<td>Power, array antenna beam width</td>
<td>( RW_a &gt; 0.6 , \text{m} )</td>
</tr>
<tr>
<td>Power, multiple reflections</td>
<td>( R/sqrt(G_o) &gt; 0.2 , \text{m} )</td>
</tr>
<tr>
<td>Power, test object offset</td>
<td>( R &gt; 0.4 , \text{m} )</td>
</tr>
<tr>
<td>Power, test object rotation</td>
<td>( N &gt; 11 )</td>
</tr>
<tr>
<td>Power, disturbances from room</td>
<td>( SI &gt; 32 , \text{dB} )</td>
</tr>
<tr>
<td>Power, array coupling, low-gain array antennas</td>
<td>( R(G_oN) &gt; 0.4 , \text{m} )</td>
</tr>
<tr>
<td>Power, array coupling, directive array antennas</td>
<td>( R(G_o sqrt(NW_a)) &gt; 0.4 , \text{m} )</td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>( N &gt; 7 )</td>
</tr>
<tr>
<td>Rayleigh distribution</td>
<td>( N &gt; 10 )</td>
</tr>
<tr>
<td>Multiplicity</td>
<td>( M = 4 )</td>
</tr>
</tbody>
</table>

Based on this, a design is decided according to the following:

- \( R = 1 \, \text{m} \)
- \( W_a = 2 \, \text{rad} \)
- \( G_o = 0 \, \text{dBi} \) (Low-gain antennas with 12 dB suppression sideways)
- \( N = 16 \)
- Anechoic room with absorbers of good quality

Comments to the design:

The condition for multiplicity is not met. This is intentional, and it is motivated by the need to avoid a very large number of array antennas. Moreover, the array coupling to opposing antennas cannot be suppressed enough. The actual error with the chosen design is \( p = 0.15 \), corresponding to ±0.7 dB. Although this is more than ±0.3 dB, it is still below the total error of ±1 dB, and many of the other error sources are well below ±0.3 dB to compensate.

**9 Conclusions**

Formulas are presented for determining the optimal design of a multipath simulator antenna array to be used for antenna characterization. The formulas are based on assumptions regarding the test object and test zone dimensions, as well as the desired measurement accuracy. Formulas for the necessary signal-to-interference ratio in the setup are also presented.

Examples with different test objects illustrate that it is very difficult to design an MPS that can accurately characterize test objects which are large compared to the wavelength. It is the possibility of rapid changes in the test object radiation pattern that causes problems.
Test objects that are small (approximately one wavelength or less), or test objects that for other reasons can be guaranteed to have only slow variations in the radiation pattern, can be measured with good accuracy. The main design problem with the MPS array is then the mutual coupling within the array, which causes interference. For any kind of test object, functionality that necessitates a multiplicity of MPS array antennas is problematic, since more array antennas further increases the disturbance level from coupling within the array.

10 References


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