Fast wave heating of cyclotron resonant ions in tokamaks

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**Abstract**

Heating a tokamak plasma by externally generated waves in the ion cyclotron range of frequencies often accelerates resonant ions to well above the thermal energies. Due to the low collision frequency in reactor relevant tokamaks the energetic ions will follow almost unperturbed guiding centre orbits that may deviate significantly from the magnetic flux surfaces. In this thesis the distribution functions of resonant ions, the ion cyclotron current drive (ICCD) and the RF-induced rotation are studied, and shown to be sensitive to the toroidal mode spectrum of the wave field. The studies have been performed both numerically using the SELFO code, which solves the quasilinear evolution of the distribution functions of energetic ions and the wave field self-consistently, and experimentally in the tokamak experiment JET.

The experiments show that the choice of heating scenario can have consequences for the stability of magneto-hydrodynamic (MHD) modes, the confinement of energetic ions and parasitic power losses. Ion cyclotron resonance heating can therefore be used for several purposes; for electron and ion heating, for current- and rotation-profile control, for providing a seed current in bootstrap-current dominating discharges, and for controlling MHD modes.

**Descriptors**

Fusion plasma, tokamak, RF heating, fast wave heating, fast wave current drive, ion cyclotron resonance, ICRH, ICRF, ICCD, RF induced rotation, RF induced transport, RF induced pinch, RF pinch, finite orbit width, fast wave current drive, self-consistent calculations, Monte Carlo methods
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List of papers

This thesis is based on the work presented in the following papers:


VI. “Destabilisation of Fast Ion Induced Long Sawteeth by Localised Current Drive in the JET Tokamak” L.-G. Eriksson, A. Mueck, O. Sauter, S. Coda, M.J. Mantsinen, M.L. Mayoral, E. Westerhof, R.J. Butterly, D. McDonald, T. Johnson, J.-M. Noterdaeme, P. de Vries and JET-EFDA contributors, Accepted for publication in Physical Review Letters,

VII. “Analysis of a quasilinear model for ion-cyclotron interactions in tokamaks” T. Johnson, T. Hellsten, and L.-G. Eriksson, To be submitted to Physics of Plasmas,

E. Rachlew, E. Tengfors, A. Tuccillo, A. Walden, B. Volodymyr, K. Zastrow, and JET-EFDA contributors., Draft of manuscript to be submitted to Nuclear Fusion,

Papers with contribution from the author not included in this thesis. (Published in refereed journals)


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(Oral conference contributions)

XXV. “Experimental Evidence for RF-induced transport of Resonant 3He Ions in JET” T. Johnson, T. Hellsten, M. Mantsinen, L. C. Ingesson, V. Kiptily, T. Bergkvist, S. Conroy, J. Hedin, M-L. Mayoral, F. Nguyen, J-M. Noterdaeme,
S. Sharapov, and contributors to the EFDA-JET workprogram, 7th IAEA TCM on Energetic Particles in Magnetic Confinement Systems, October 8 - 11, 2001, Göteborg, Sweden


(Conference contributions)


XXVIII. “Teaching Numerical Methods for Partial Differential Equations over the Internet” André Jaun, Johan Hedin, Thomas Johnson, Michael Christie, Lars-Erik Jonsson, and Mikael Persson. 20th World Conference on Open Learning and Distance Education, Duesseldorf, Germany, April 2001

XXIX. “RF-induced Pinch of Resonant He4 Minority Ions in JET” T. Johnson, T. Hellsten, M. Mantsinen, V. Kiptily, S. Sharapov, M-L. Mayoral, F. Nguyen, J.-M. Noterdae, J. Hedin, and contributors to the EFDA-JET workprogram, 14th Topical Conference on Radio Frequency Power in Plasmas, 7-9th May, 2001, Oxnard, Ca, USA

XXX. “Sawtooth and NTM Seed Island Control by ICRF Current Drive on JET” L. Mayoral, O. Sauter, F. Nguyen, E. Westerhof, M. J. Mantsinen, T. Hellsten, T. Johnson, and contributors to the EFDA-JET workprogram, 14th Topical Conference on Radio Frequency Power in Plasmas, 7-9th May, 2001, Oxnard, Ca, USA


XXXV. “Self-Consistent RF Modelling of Beam and ICRF heated plasmas” M Laxåback et al, Theory of Fusion Plasmas, Varenna, Italy, 2002


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1 Introduction

Mankind is facing a growing energy demand\textsuperscript{1}. With present technology the only candidates to fill this demand are fossil fuels, which may have a strong impact on our environment [4]. Today fossil fuels provide 80\% of the global energy production [1], with the dominating contribution coming from conventional oil (40\%). However, the forseen depletion of conventional oil resources [2, 3, 5] may have severe economical consequences [5]. One alternative energy resource that could fill the growing demand is thermonuclear fusion.

Thermonuclear fusion is the energy resource that powers the sun by burning hydrogen into helium. On earth it could provide a practically inexhaustible energy resource [6, 7]. For a fusion power reactor the most interesting nuclear reaction is between the heavy hydrogen isotopes $^2\text{H}$ and $^3\text{H}$ also known as deuterium D and tritium T.

$$^2\text{H} + ^3\text{H} \rightarrow ^4\text{He} + n + (17.6 \text{ MeV}) \quad (1.1)$$

For this reaction to have a significant cross section temperatures of about $10^8$ K are needed. The deuterium and tritium will then be ionized in the form of a plasma. By strong magnetic fields and auxiliary heating a toroidal plasma can be heated to the relevant temperatures and confined on time scales relevant for thermonuclear fusion [8].

This thesis described work on wave heating of magnetically confined plasmas in the Ion Cyclotron Range of Frequencies (icrf). The main part of the work is based on numerical modelling and analysis performed with the FIDQ [9] and the SELFO [10, 11] codes. In particular, the distribution functions of energetic ions following wide quasi-periodic orbits, their impact on global plasma parameters like currents and rotation, and their importance for plasma stability have been studied.

The first chapter gives an brief introduction to the field of fusion plasma physics, with emphasis on the motion of charged particles. The second chapter describes how the fast magnetosonic wave is used to heat fusion plasmas. Here the focus is on resonant wave-particle interactions and the quasi-linear diffusion that generates energetic ions. In the third chapter numerical methods and there applications to wave heating problems are described.

\textsuperscript{1}During the past 30 years the increase have been 2.2\% per year [1], while OECD/IAE predict a 2\% increasing in their predictions for the upcoming 15 years [2, 3]. An 2\% increase per year results in a 100\% increase in 35 years.
1.1 Plasmas physics

A plasma constitutes of an ensemble of charged particles, e.g. electrons and ions, with a high conductivity shielding off long range electrostatic fields. This counteracts charge-separation and makes the plasma almost neutral, quasi-neutral, when measured on scale lengths longer then the Debye length \( \lambda_D = \sqrt{\frac{e_0 T_e}{e n_e}} \). Here \( e_0 \) is the permittivity of vacuum, \( T_e \) is the electron temperature, \( e \) is the elementary charge, and \( n_e \) is the electron density. On scale lengths \( \leq \lambda_D \) particles interact electrostatically through Coulomb collisions. These collisions are dominated by “long range” (~ \( \lambda_D \)) interactions that yield very small changes in velocity \( |\Delta v|/|v| \ll 1 \).

To illustrate how weak one Coulomb collision is we shall consider an electron in a 10 keV \(^4\)He plasma with electron density \( 3 \times 10^{19} \). The electron will collide with a large number of ions as it travels a distance \( \lambda_D \) during a time \( 2 \times 10^{-12} \)s. The time scale for changes in the velocity \( |\Delta v|/|v| \sim 1 \) is \( 2 \times 10^{-4} \)s. Consequently, the electron have to travel \( 10^8 \) Debye lengths before its velocity is significantly altered by the Coulomb collision. Still, for each Debye length that the electron travels it collides with a large number of ions.

1.1.1 The motion charged particles

The motion of a charged particle with mass \( m \) and charge \( Ze \) in a static fusion plasma is often well described by Newton’s equation of motion

\[
m \frac{dv}{dt} = Ze(\mathbf{E} + \mathbf{v} \times \mathbf{B}).
\]

where \( \mathbf{B} \) is the magnetic field, and \( \mathbf{E} \) is the electric field which often provide a relatively weak force. On shorter time scales this equation describes a uniform motion parallel to the magnetic field, and a cyclotron motion perpendicular to the magnetic field. The latter is a gyration around the magnetic field lines with the cyclotron frequency \( \Omega = ZeB/m \), and with a gyro radius, or Larmor radius \( \rho = |\mathbf{v} \times \mathbf{B}|/\Omega = v_\perp/\Omega \) for \( \mathbf{b} = \mathbf{B}/B \). On time scales much longer then the \( 1/\Omega \) the presence of electric fields, inhomogeneities \( \nabla B \), and curvature of the magnetic field \( \mathbf{k} = (\mathbf{b} \cdot \nabla)\mathbf{b} \), generate a drift perpendicular to the magnetic field. The drift motion is usually described in terms of the gyro-centre, or the guiding centre \( \mathbf{r}_g \) [12-15]

\[
\mathbf{r}_g = \mathbf{r} + \mathbf{v} \times \mathbf{b}/\Omega \quad , \quad \frac{d\mathbf{r}_g}{dt} = \mathbf{v}_g \mathbf{b} + \mathbf{v}_D \quad (1.3)
\]

\[
\mathbf{v}_D \approx \frac{1}{ZeB} \mathbf{b} \times (\mu \nabla B + mBv_\parallel \mathbf{k} + Ze\mathbf{E})
\]

where \( v_\parallel \) is the parallel velocity and \( \mu = m v_\parallel^2/(2B) \) is the magnetic moment.
1.1. Plasmas physics

With the help of Lagrangian mechanics the symmetries of the equation of motion can be explored [16-19]. In an electro-magnetic field with electrostatic potential $\Phi$ and magnetic vector potential $A$ the equation of motion can be written as the

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{r}} - \frac{\partial L}{\partial r}$$

(1.4)

with the Lagrangian

$$L(\dot{r}, r, t) = \frac{m|\dot{r}|^2}{2} + Z e A(r, t) \cdot \dot{r} - Z e \Phi(r, t).$$

(1.5)

Let us now study an axisymmetric torus with toroidal angle $\phi$, as shown in figure 1.1. Due to axisymmetry $\partial_\phi L = 0$, and according to equation (1.4) there exists a constant of motion

$$P_\phi = \frac{\partial L}{\partial \phi} - R m v_\phi + Z e \psi$$

(1.6)

called the canonical toroidal angular momentum. Here $\psi = RA_\phi$.

When the particle experience only small variations of the electro-magnetic fields during one gyro period, then the Lagrangian is, to first order in Larmor radius, independent of the gyro angle $\phi$ (see e.g. chapter 6.3 in reference [18]). The magnetic moment

$$\mu = \frac{m v_\perp^2}{2B} \approx \frac{\partial L}{\partial \Phi_\perp} + O(\rho^2)$$

(1.7)

is therefore an adiabatic invariant of motion. In this thesis the $\mu$ is often replaced by the normalized magnetic moment $\Lambda - B_0 \mu / E$.

If the Lagrangian has no explicit dependence on time, i.e. is autonomous, also the energy $E - m v^2 / 2 + Z e \Phi$ is an invariant of motion. Consequently, for the strongly magnetized (small Larmor radius) axisymmetric torus there exists three invariant of motion $(E, \mu, P_\phi)$ and the motion is integrable [21].

For autonomous integrable systems there exists a set of action-angle variables $(J, \Theta)$ such that the Hamiltonian $H = H(J)$ [16]. The components of $J$ are invariants of motion and $\dot{\Theta} - \delta_\Theta H$ are independent of the angles $\Theta$. The motion is therefore quasi-periodic, which have important consequences for the confinement of a plasma for fusion applications. If a particle is confined for one quasi-period, it will stay confined. Only Coulomb collisions and electro-magnetic perturbations changing $(E, \Lambda, P_\phi)$ can induce a transport.

1.1.2 Kinetic models of plasmas

To model a plasma a statistical approach may be used where uncorrelated particles of species $\eta$ have a probability density distribution func-
Figure 1.1 Illustration of the toroidal coordinate systems \((R, Z, \phi)\) and \((r, \phi, \theta)\), where \(R\) is called the major radius and \(r\) the minor radius. The curve \(r = 0\) is called the magnetic axis and can alternatively be written as \((R, Z) = (R_0, 0)\). In a bounded torus the maximum minor radius is \(a\). The magnetic field components are \(B_\phi\) and \(B_0\) and \(I_p\) is the current. (Taken from [20])
1.1. Plasmas physics

tion $f_\eta (\mathbf{v}, \mathbf{r}, t)$, which follow the continuity equation in $(\mathbf{v}, \mathbf{r})$

$$
\frac{\partial f_\eta}{\partial t} + \mathbf{v} \cdot \nabla f_\eta + \mathbf{F} \frac{\partial f_\eta}{\partial \mathbf{v}} = 0
$$

(1.8)

$$
\dot{\mathbf{x}} = \mathbf{v}, \quad \dot{\mathbf{v}} = -\frac{Ze}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B})
$$

called the Vlasov equation. From it's solution the currents and charge densities can be calculated and substituted into Maxwell's equations that determine $\mathbf{E}$ and $\mathbf{B}$. For the axisymmetric torus with prescribed electric and magnetic field, the evolution of $f_\eta$ described by Eq. (1.8) can be given in terms of the action angle variables and a label $\sigma$ to distinguish orbits with the same triplet of actions (or invariants)

$$
f_\eta (J, \mathbf{\theta}, t; \sigma) = f_\eta (J, [\mathbf{\theta} + (t - t_0) \partial_j H], t_0; \sigma).
$$

(1.9)

Instead of resolving the dynamics on length scales of the Debye length $\lambda_D$ the Coulomb collisions can be treated statistically. For long range binary collisions a Fokker-Planck equation can be derived [15,18,22,23]

$$
\frac{\partial f_\eta}{\partial t} + \mathbf{v} \cdot \nabla f_\eta + \mathbf{F} \frac{\partial f_\eta}{\partial \mathbf{v}} - \sum_v C_{\eta v}(f_\eta, f_v)
$$

(1.10)

where $C_{\eta v}$ is the Coulomb collision operator describing collisions between particles of species $\eta$ and $v$. In Cartesian velocity coordinates $\mathbf{v}$ the Coulomb collision operator can be written as a diffusion operator in velocity space [18]

$$
C_{\eta v}(f_\eta, f_v) = \frac{\partial}{\partial \mathbf{v}^i} \left( \Lambda_{\eta v} f_\eta + \frac{\partial}{\partial \mathbf{v}^j} \left( D_{\eta v}^{ij} f_\eta \right) \right)
$$

(1.11)

where

$$
\Lambda_{\eta v} = \left( \frac{Z_\eta Z_v e^2}{m_\eta e_0} \right)^2 \ln \Lambda_{\eta v}
$$

$$
A_{\eta v}^i = \left( 1 + \frac{m_\eta}{m_v} \right) L_{\eta v} \frac{\delta \varphi_v}{\delta \mathbf{v}^i}, \quad D_{\eta v}^{ij} = -L_{\eta v} \frac{\partial^2 \psi_v}{\delta \mathbf{v}^i \delta \mathbf{v}^j}
$$

(1.12)

$$
\varphi_v \equiv -\frac{1}{4\pi} \int \frac{f_v(\mathbf{u})}{|\mathbf{v} - \mathbf{u}|} d^3u, \quad \psi_v \equiv -\frac{1}{8\pi} \int f_v(\mathbf{u})|\mathbf{v} - \mathbf{u}| d^3\mathbf{u}
$$

Further, $\ln \Lambda_{\eta v}$ is the Coulomb logarithm, and $\psi_v$ and $\varphi_v$ the Rosenbluth potentials.

In low collisionality tokamak plasmas the particle bounce, or transit time $\tau_B$ is much shorter than the time scale for collisions $\tau_c$. Since one is in general only interested in the evolution of the distribution functions on the longer time scale, the distribution functions can be expanded as $f = \tilde{f} + (\tau_B/\tau_c) \tilde{g} + \ldots$. Orbit averaging will then reduce the six dimensional Fokker-Planck equation to a three dimensional equation for $\tilde{f}(J; \sigma)$, or $\tilde{f}(E, \Lambda, \mathbf{P}_\theta; \sigma)$ [24–26].

$$
\frac{\partial \tilde{f}_\eta}{\partial t} + \mathbf{J} \cdot \nabla \tilde{f}_\eta = -\sum_v C_{\eta v}(\tilde{f}_\eta, \tilde{f}_v)
$$

(1.13)
where the orbit average of a quantity $X$ is defined by

$$< X > (t) = \int X(\theta(t'), t) dt'$$

(1.14)

The second term on the left of Eq. 1.13 contain for example the quasi-linear effects from wave-particle interactions.

The orbit averaged Fokker-Planck equation (1.13) also need boundary conditions at the surfaces in $J$-space (or $(E, A, P_{ph})$-space) where $\sigma$ may change [27]. These surfaces appear where the mapping from $J$ onto orbits bifurcates, e.g. at the trapped-passing boundary.

1.1.3 Magnetohydrodynamics

The global behaviour of plasmas can often be described by simpler equations than the kinetic ones. In magnetohydrodynamics (MHD), the plasma is described as a fluid with a mass density $\rho$, a velocity $v$, a current density $J$, and a pressure $p$, which is situated in an electro-magnetic field [28]. This model can be used to describe e.g. a plasma equilibria as a balance between $J \times B$ forces and the pressure gradient (and possibly initial forces), or to describe the evolution of global plasma wave modes.

An important result from the ideal MHD equations, without resistive or viscous dissipation and with an Ohms law $E + v \times B = 0$, is that the plasma is frozen into the magnetic field. This means that if two fluid elements are connected by a magnetic field line at one instant in time, then they will stay connected by a magnetic field line. An example is the two dimensional magnetic field with an $x$-point at $(x, y) = (0, 0)$, as shown in figure 1.2a. The topology of this configuration will be preserved under perturbation consistent with the ideal MHD.

Including a small, but finite resistivity $\eta$ the magnetic field evolution is described by Ohms law and Faraday's law yielding

$$\frac{\partial B}{\partial t} - \nabla \times (v \times B) + \frac{\eta}{\mu_0} \nabla^2 B,$$

(1.15)

In most parts of the plasma the resistive dissipation in the third term of equation (1.15) is negligible. However, for horizontally compressing and vertically decompressing flows, as illustrated by the arrows in figure 1.2b), a narrow current sheet (gray area) can be generated in which the resistive dissipation is strong due to the sharp gradients [19].

Consider a thin infinite tube of plasma stretched out along a magnetic field line in figure 1.2b). Let it flow from the left toward the separatrix. The part of the fluid that enters the current sheet is no longer frozen into the magnetic field which may break up and reconnect. The part of the tube that is outside the current sheet, still frozen into the magnetic field, is therefore split into two parts, that flows horizontally upwards and downwards (see the arrows in figure 1.2b).

In general a small but finite resistivity give rise current diffusion on a slow resistive time scale $\tau_R \sim \mu_0 L_c^2/\eta$, where $L_c$ is the characteristic
scale length. In a toroidal geometry the resistivity allows a perturbation to break the magnetic topology of closed magnetic field lines at so called rational surfaces. The new topology typically takes the form of a chain of magnetic islands, as illustrated in figure 1.2c).

*Tearing modes* is an important class of modes that generate magnetic islands. They grow on time scales \( \tau_R^{3/5} \tau_A^{2/5} \) (where \( \tau_A \) is the Alfvén transit time [19]) much shorter then the current diffusion. The reason is that resistive effects dominate only in very thin layer around the rational surfaces, while ideal MHD describe the plasma behaviour outside this layer.

**1.2 Magnetic Confinement Devices: The Tokamak**

The tokamak is an axisymmetric magnetic confinement concept with a strong toroidal magnetic field induced mainly by currents in external coils, and a weaker poloidal magnetic field provided mainly by currents in the plasma [29]. See figure 1.3. The traditional way to drive the plasma current is by ramping a current in a toroidally winded coil. This induces a plasma current which is proportional to the time derivative of the coil current, like in the secondary winding of a transformer. The ramping of the coil current puts a limit to the time the plasma current, and also the plasma, can be sustained. Since it beneficial if a fusion reactor can
generate a continuous supply of power, alternative ways of driving the plasma current are of great interest. A large noninductive current can be generated by the plasma pressure gradient \cite{18,30}. This current is called the *bootstrap current*\footnote{The bootstrap current is named after the reported ability of Baron von Munchenhausen to lift himself by his bootstraps.} and is a consequence of the toroidal geometry and the long mean free path of the electrons in a hot fusion plasma. However, the bootstrap current cannot generate all current needed to confine the plasma. In particular at the magnetic axis the bootstrap current vanishes and the transport equilibria are difficult to maintain \cite{18}.

Luckily, noninductive current drive can also be obtained from neutral beam injection \cite{31,32}, wave heating schemes such as the ion or electron cyclotron current drive \cite{33-39} and lower hybrid current drive \cite{40,41}, and from the \(\alpha\)-particles \cite{42}.

In a tokamak the magnetic field lines follow helical paths as they go round the torus. The ratio of toroidal to poloidal turns of the magnetic field is called the *safety factor* \(q\); in general higher values of \(q\) lead to greater stability. In conventional tokamak scenarios \(q\) is about 1 in the centre and increase to about 5 at the wall. For irrational values of \(q\) the field lines cover surfaces ergodically, so called *flux surfaces*. These surfaces span the entire plasma volume, except for discreet surfaces

\textbf{Figure 1.3 Schematic illustration of a tokamak. Taken from reference [20].}
1.2. Magnetic Confinement Devices: The Tokamak

with rational values of \( q \), rational surfaces, where the field lines form closed curves. The flux surfaces can be labeled by the flux function \( \psi \), which is proportional to the poloidal flux \( 2\pi \psi(R, Z) \) through any surface bounded by a circle of constant \( R \) and \( Z \).

Although the easiest way to build an experimental reactor today may be using the tokamak concept, alternative concepts may prove to be superior as power plants. One of many alternative schemes for magnetically confined fusion is the stellarator, which is a toroidal device that is not toroidally symmetric. It has the advantage that no plasma current is required for equilibrium. However, the lack of symmetry makes nested flux surfaces difficult to obtain, and requires high precision in the design which may make a reactor expensive.

1.2.1 Equilibria

In a fluid MHD model the tokamak equilibrium is a force balance between the pressure gradient and the \( \mathbf{J} \times \mathbf{B} \) force. Normalizing the magnetic field \( B_n - B/B_0 \sim 1 \) and the pressure \( p_n - p/p_0 \sim 1 \) the balance reads

\[
\beta (\nabla \times \mathbf{B}_n) \times \mathbf{B}_n - \nabla p_n. \quad \text{Here } \beta = 2\mu_0 p_0 B_0^2 \text{ is the ratio between the plasma pressure and the magnetic pressure. Since the magnetic field strength is limited by the stress it induces in the coil system, and high plasma pressure is beneficial for fusion energy production, the } \beta \text{ value is an important quality parameter in a fusion plasma.}
\]

Representing the magnetic field by \( \mathbf{B} = F(\psi) \nabla \phi + \nabla \phi \times \nabla \psi \), where \( F = R R \phi \), the force balance can be written as the Grad-Shafranov equation for \( \psi(R, Z) \).

\[
\frac{R}{\partial R} \frac{1}{\partial R} \frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial Z^2} - \mu_0 R^2 \frac{\partial p}{\partial \psi} - F \frac{\partial F}{\partial \psi} = 0 \quad (1.16)
\]

The equilibrium is characterized by two free functions, here the pressure \( p \) and the function \( F \), and the boundary conditions. A typical equilibrium from the real tokamak is shown in figure 1.4.

A particularly simple equilibrium can be found for a low \( \beta \) plasma with high aspect ratio \( R_0/a \), where \( R_0 \) and \( a \) are the minor and major radius of the plasma respectively (see figure 1.1). The equilibrium has concentric circular flux surfaces centered around the magnetic axis \( R = R_0 \) and \( Z = 0 \) (see figure 1.1), i.e. \( \psi = \psi(r) \) where \( r = \sqrt{(R - R_0)^2 + Z^2} \). Further, the poloidal magnetic field is relatively small such that \( B_\phi = B \approx B_0 R_0/R \). This equilibrium is used for the numerical modelling of ion cyclotron resonance heating discussed in chapter 3.2.

1.2.2 Stability and transport

Compared with many of the early magnetic confinement schemes the tokamak is a relatively stable system. Still a number of instabilities may
Figure 1.4 Equilibrium from the JET tokamak. The dotted lines are flux surfaces, i.e., surfaces of constant $\psi$. The dashed line marks the positions of the X/RH antennas.
1.2. Magnetic Confinement Devices: The Tokamak

appear; some of them leads to a global collapse of the plasma, a disruption, while most of them only degrades the performance. The stability can often be effected by ICRH. The current gradient, which is the force driving e.g. kink and tearing instabilities, can be effected by localized current drive [43-46], or by changing the current penetration by localized heating [47,48]. The production of energetic particles with a toroidal precession frequency \( \omega_p \), much higher then the mode frequency, can have a stabilizing effect, since they average the potentials of the perturbations. The free energy associated with gradients in the pressure of fast ions may also destabilize Alfvén eigenmodes, fishbone oscillations and energetic particle modes [19,49,50]. In a reactor these mode could be triggered by fusion born \( \alpha \)-particles [50].

Long wave length plasma instabilities are most likely to occur at surfaces with rational values of \( q = m/n \), where \( n \) and \( m \) toroidal and poloidal mode numbers, respectively. On such a surface the phase of the perturbation is constant along a magnetic field line, thereby minimizing the stabilizing effect from bending the magnetic field lines.

The most frequently observed instability in tokamak experiments is the sawtooth instability, which may be triggered when central \( q \)-value falls below one. In the most simple theoretical picture due to Kadomtsev the ideal internal kink mode becomes unstable. The unstable mode generates an \( n - m - 1 \) magnetic island at the \( q - 1 \) surface, that grows until it has swallowed all plasma inside it. During the sawtooth instability, or sawtooth crash, cold plasma from outside the \( q - 1 \) surface moves into the centre, and a rapid drop in the measured temperature and soft x-ray is observed. This also leads to a redistribution of the plasma current that flattens the current gradient and stabilizes the internal kink.

After the crash an equilibrium is formed and, due to heating, the central temperature starts to rise and the current profile becomes more and more peaked. Ultimately the ideal internal kink mode becomes unstable again and a new rapid sawtooth crash follows.

Strong stabilization of the sawtooth have been observed in the presence of energetic ions generated by ICRH and other heating schemes [51]. Further, localized ion cyclotron current drive (ICCD) has been shown to be able to both stabilize and destabilize the sawtooth [45,46].

The sawtooth oscillations not only limits the plasma performance by reducing the central temperature, but also by triggering other modes like the neo-classical tearing modes [52,53]. This mode may become unstable in high \( \beta \)-plasmas where a large fraction of the current is due to bootstrap currents. For the instability to occur an initial magnetic island is required; a seed island that may be generated by a sawtooth crash. In the island the pressure gradient is flattened, and hence the bootstrap current reduced. The loss of bootstrap current has a destabilizing effect on the island itself, which may grow until it saturates. The neo-classical tearing mode may put a strong limit on the \( \beta \)-value in a tokamak reactor. It is therefore of great interest to find control tools that can stabilize the NTM directly, or indirectly by controlling the seed island. Experimental
results concerning the NTM control through ICCD destabilization of the sawtooth, reducing the size of the seed island, are reported in paper X, XI and XXXVII.

Another type of modes that can degrade the tokamak performance and limit the $\beta$-value are the resistive wall modes. In general these modes are unstable for a resistive wall and stable for a perfectly conducting wall. However, if the plasma is rotating at a speed high enough so that the mode does not have time to penetrate the wall before being displaced one wave length, then the mode may be stabilized for a resistive wall. It is therefore of great interest to find ways to generate rotation [54, 55], e.g. by $\text{ICRH}$.

The transport in a tokamak is dominated by turbulent processes with a Bohm-type diffusivity $\propto T/B$ [56]. However, experiments have shown that with strong auxiliary heating, the transport can be strongly reduced by so call transport barriers. These barriers may appear at the plasma edge, forming the high confinement mode (H-mode) [57], or inside the plasma forming an internal transport barrier ITB [58]. To trigger and sustain an ITB careful control of the plasma is essential. In particular the $q$-profile, the power deposition profile, and the plasma rotation profile are believed to have a strong impact on the ITB [59, 60].

The ion cyclotron resonance heating scheme enables us to heat locally, to control the partition of power between electrons and ions, to drive currents, and to induce rotation. This flexibility may make it an important tool for future design of advanced scenarios with internal transport barriers in a fusion reactor.

1.2.3 The JET experiment

The Joint European Torus (JET) in Abingdon, England, is the largest tokamak in the world. It has a major radius $R_0 = 3$ m, and a minor radius $a = 1$ m. The poloidal cross section of the flux surfaces in a typical JET equilibrium are almost shaped like a $D$ by external poloidal field coils. Compared to a circular poloidal cross section the $D$ shape improves the confinement by allowing a larger plasma current. JET has a central magnetic field $B_0 \approx 2 - 4$ T, and a plasma current $I_P \approx 2 - 4$ MA, which is sufficient to confine $\alpha$-particles generated by deuterium-tritium reactions described in (1.1). To reach reactor relevant conditions high auxiliary heating power is installed; 20 MW NBI, 10 MW $\text{ICRH}$, and 5 MW LHCD.

About $8 \times 80$ m of transmission lines are used to transmit the electromagnetic waves from the generators to the four $\text{ICRH}$ antennas installed in JET. Each antenna has four vertical parallel current straps, see figure 1.5. Depending on the relative phase between the oscillating currents in the strap, wave fields with different toroidal mode spectra can be excited. The three most common ways to phase the antenna straps are with $180^\circ$ phase difference between adjacent straps, called dipole phasing, or with $+90^\circ$ or $-90^\circ$ phase difference. The latter two phasings
1.2. Magnetic Confinement Devices: The Tokamak

![Image of Tokamak]

Figure 1.5 A photo taken inside of the JET tokamak. Two of the four ECH antennas can be identified by the four vertically parallel current straps, each covered by a Faraday screen; bars with an angle of about 14° to the horizontal plane.

generate waves with asymmetric toroidal mode spectra; for the +90° and −90° phasings about 80% of the wave energy propagates co and counter to the plasma current, respectively. The other 20% propagates in the opposite direction.

1.2.4 Ion orbits and the invariants of motion

In a JET plasma with a temperature of 10 keV in a magnetic field of strength \( B_0 \sim 3 \, \text{T} \), both the ion Larmor radius \( \rho_i \sim 6 \, \text{mm} \), and the electron Larmor radius \( \rho_e \sim 0.1 \, \text{mm} \) are small. The guiding centre drift velocity, discussed in chapter 1.1.1, allows a particle to move perpendicular to the magnetic field. This is most important for ions with low parallel velocity, for which orbits become trapped on low side side of the torus. These orbits are denoted banana orbits, or trapped orbits. For a 10 keV ion at a minor radius \( r = 0.3 \, \text{m} \), and \( q = 2 \), the drift perpendicular to the flux surfaces gives the orbit a width in the poloidal plane that can be approximated by

\[
\delta_\phi = 2q \frac{m v}{ZeB_0} \sqrt{1 - \frac{\Delta R_0}{R_0 + r}} \sim 4 \, \text{cm}
\]  

(1.17)

when the \( \delta_\phi \ll r \). Thus, the thermal ions are well confined, and their orbits approximately follow the magnetic field lines. However, C3H can
generate ions with energies of several MeV, with orbit widths that are comparable to the minor radius. Their dynamics during ICRH have important consequences for the heating, current drive, and rotation, which can be used to control the plasma stability and performance.

For the low β high aspect ratio equilibria discussed in section 1.2.1 a triplet of invariants of motion \((E, \Lambda, P_\theta)^3\) can be mapped onto zero, one or two guiding centre orbits. A map of the topology of the guiding centre orbits of 100 keV protons in a JET like plasma is given in the figure 1.6, in which the \((E, \Lambda, P_\theta)\)-space, where \(P_\theta \equiv P_\phi/m\), is divided into nine regions. Typical orbits corresponding in these regions are shown in figure 1.7.

The angle between direction of the magnetic and the particle velocity is called the pitch angle

\[
\xi \equiv \frac{v_\perp}{v} = \pm \sqrt{1 - \Lambda B/B_0} = \pm \sqrt{1 - \Lambda R_0/R}
\]  

(1.18)

Due to the conservation of the magnetic moment the magnitude of the pitch angle will decrease when the ion moves into a region with higher magnetic field, i.e. lower \(R\). The minimum magnitude \(|\xi| = 0\) is obtained

\footnote{Here the characteristic magnetic field used to define \(\Lambda\) is chosen to be the field at the magnetic axis \(B_0\), i.e. \(\Lambda \sim (B_0/B)(v_\perp/v)^2\).}
at $R = \Delta R_0$, where the toroidally velocity is reversed ($v_{\phi} \approx v_{\|} = v \tilde{\xi}$). This point is therefore called the toroidal turning point, or just the turning point, and the orbit are called toroidally trapped, or just trapped. If the width of the orbit is much smaller than the minor radius $\delta_\parallel \ll r$, then the orbit is denoted banana orbit and the turning point the banana tip. This gives us an intuitive picture of $\Lambda$ as the value of $R/R_0$ at the trapped particle turning point.

Since $v_{\phi}$ vanishes at the turning point, the toroidal angular momentum is given by $P_{\phi} = Ze\psi$. We can then interpret $P_{\phi}/(Ze)$ as the value of the flux function at the turning point of a trapped particle.

Ions with low energy have a small kinetic momentum and the conservation of the canonical toroidal angular momentum $P_{\phi} = Ze\psi(r)$ will confine their orbits to flux surfaces. Here $\psi$ is normalized to zero at the magnetic axis, and it increases approximately quadratic in $r$ for flat current profiles, i.e. $r \sim \sqrt{\psi}$.

Since the magnetic momentum $Ze\psi$ is the dominant term in $P_{\phi}$ for low energies, the vertical axis in figure 1.6 roughly represents $\psi(r)$, “perturbed” by the kinetic toroidal momentum $R m v_{\phi}$. The magnitude of this perturbation can be illustrated by the curve $C_2$ that marks all orbits that goes through the magnetic axis (where $\psi = 0$) for $v_{\phi} \in (-v, v)$.

Figure 1.6a shows a $\{\Lambda, P_{\phi}\}$-plane with passing orbit in region $I$, and trapped orbit in region $\text{VII}$. They are approximately separated by the in line $C_1$ obtained from the condition that the toroidal turning point is

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure17.png}
\caption{Typical guiding centre orbits corresponding to the different regions in figure 1.6. (Taken from reference [61])}
\end{figure}
in the midplane $Z = 0$. Due to the kinetic toroidal momentum the co-current passing ion $v_+ < 0$ have $P_\phi < Z e \psi$ and can be found in the region $I$, $II$, $VI$, and $VIII$. These ions intersect the wall at the lower dashed line in figure 1.6a. In the same manner the counter-current passing ions $v_+ > 0$ have $P_\phi > Z e \psi$, and can be found in the region $I$, $II$, $III$, $IV$. They intersect the wall at the upper dashed line in figure 1.6a.

Zooming in on the near axis region ($P_\phi \sim 0$) for particles with high $v_+/v$ ($\Lambda \sim 1$) we obtain figure 1.6b. This figure reveals the presence of non-standard trapped orbits that encircle the magnetic axis in region $V$, and non-standard passing orbits that do not encircle the magnetic axis in region $VIII$. Both these types of orbits are called potato orbits since their width are of the same order as the minor radius [62]. If a co-current ($v_+ < 0$) passing orbit in region $VIII$ approach the line $C_4$ it will shrink into a point and vanish as it “enter” region $IX$. The limiting orbits are called stagnation orbits as they do not move in the poloidal plane, i.e. their motion is entirely toroidal.

Finally, in figure 1.6c) the line $C5$ marks the smallest $P_\phi$ for counter-current orbits, equivalent to line $C4$ for co-current passing orbits. When a trapped ions reach line $C6$ from the right it will bifurcate into two orbits, one passing and one trapped. This bifurcation, denoted the trapped-passing boundary, appears when at the poloidal turning point reaches the midplane.

Topologically there are only three regions in the invariant space; the region with zero solutions (region $IX$), the regions with one solution (regions $V$-$VIII$), and the regions with two solutions (regions $I$-$IV$). These region are separated by the stagnation lines $C_4$, $C_5$ and $C_6$. Further, there is only one boundary where the orbits bifurcates. This is at line X'type stagnation line $C_6$ where the poloidal turning point is in the midplane. At the other topological boundaries appear at the O-type stagnation line $C_4$ and $C_5$ where the orbits of co- and counter-current passing ions shrink into a point.

### 1.2.5 Orbit averaged collision operator

In the following we will consider collisions between a test particle and Maxwellian background. The collision operator, Eq. (1.11), can then be separated into two one dimensional diffusion processes describing the scattering of the pitch angle, and the energy scattering and slowing down. When collision operator is averaged over a guiding centre orbit both the pitch angle scattering and the energy scattering and drag becomes two dimensional.

To illustrate this we shall study an ion located at a major radius $R$. If it changes its pitch angle $\xi \rightarrow \xi + \Delta \xi$ the invariants of motion will change

$$\Lambda \rightarrow \Lambda + R/R_0(2\xi \Delta \xi^2 - 2\xi \Delta \xi)P_\phi \rightarrow P_\phi + R v \Delta \xi \quad (1.19)$$

The new invariants are distributed along a characteristic line parameterized by $\Delta \xi$. This is the one dimensional pitch angle scattering process.
1.2. Magnetic Confinement Devices: The Tokamak

Figure 1.8 The accessible $\Lambda$ and $P_\phi$ due to pitch angle scattering on an orbit. The ion has initial invariants $\nu = 0.01$, $P_\phi = 3$, and $\Lambda = 0.8$, marked by a star. Depending on the initial position along the orbit ($R \in [R_{\text{min}}, R_{\text{max}}]$), pitch angle scattering will distribute the orbits along lines (the dashed lines) in $(\Lambda, P_\phi)$.

If an ion undergoes a collision somewhere along a guiding centre orbit it will be scattered along one of the characteristic line, i.e. it will be distributed on a surface that is spanned by all $R$ along the orbit and all changes in the pitch angle $\Delta \xi$. In figure 1.8 the characteristic line for all $R_{\text{min}} \leq R \leq R_{\text{max}}$ are shown.

The Coloumb collisions in a fusion plasma are weak. Thus, a large number of collisions are needed for significant changes of the invariants. The orbit averaged pitch angle scattering process is therefore well described by a symmetric diffusion tensor. However, when implementing simulation tools with finite time stepping the curvature of the characteristics becomes important.
2 Ion Cyclotron Resonance Heating

Ion Cyclotron Resonance Heating (iCRH) is one of the most common ways to heat present day tokamak experiments and is foreseen to be important in the experimental reactor ITER\(^1\) [63–66]. The advantages with iCRH is that the waves are easy and cheap to generate, and relatively easy couple to the plasma. Further, there is a flexibility in the choice of resonant ions species and their concentrations, as well as the possibility to control both the resonance position by changing the frequency and the toroidal mode spectrum by phasing the current straps in the antennas (see section 1.2.3). The ability to control the effective temperature of the heated energetic ions allows one to collisionally heat background ions, electron, or both. Finally, iCRH can provide localised current drive and rotation and is often used both to provide heating and to control the plasma performance [67].

The process of iCRH starts at the RF-generators, that are connected to antennas by transmission lines. The antenna currents do not directly couple to any plasma wave, since these are all evanescent (exponentially decaying) in the low density plasma directly in front of the antennas. Luckily, the waves tunnel through the low density region and couple to the fast magnetosonic wave, or \textit{fast wave}, that transports the wave energy to the plasma core. Here cyclotron resonance interactions with the ions, and electron Landau damping (ELD) and transit time magnetic pumping (TTMP) with electrons, will dissipate the wave. The fast wave may under some conditions be mode-converted into slower waves, but this is outside the scope of this thesis [68].

After introducing the equations describing plasma waves in section 2.1, the fast wave propagation, polarization and absorption is discussed in section 2.1.2. The response of the resonant ions is then studied from a ballistic point of view in 2.2. The section describes the conditions for resonance and quasilinear diffusion coefficients are derived. The orbit averaged response of the resonant ions is then discussed in section 2.2.1, and the full kinetic equations are given in 2.3. The sections 2.4 and 2.5 concludes the chapter with a discussion of the theory for driving rota-

\(^1\)ITER International Thermonuclear Experimental Reactor. "Iter" means "the way" in Latin.
tion and currents by iCRH.

2.1 The Wave Equation

Small amplitude waves induce a current \( J = \sigma(E) \), where the linear operator \( \sigma \) is the conductivity acting on the electric field \( E \). The time derivative of Ampere’s law and the rotation of Faraday’s law yields the wave equation

\[
\frac{\partial^2 E}{\partial t^2} + \frac{1}{\epsilon_0} \frac{\partial \sigma(E)}{\partial t} + c^2 \nabla \times \nabla \times E = \frac{1}{\epsilon_0} \frac{\partial J_{\text{ext}}}{\partial t}
\]  

where \( J_{\text{ext}} \) are externally induced currents, e.g. currents in an antenna, \( c = 2.9979 \times 10^8 \text{ m/s} \) is the speed of light, \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \) the permeability of free space, and \( \epsilon_0 = 1/(\mu_0 c^2) \) is the permittivity of free space.

In a homogeneous plasma the Fourier transform in space and time \((x, t) \rightarrow (k, -\omega)\) of \( \sigma \) is an algebraic tensor [69] and the wave equation can be written as [22]

\[
D \varepsilon = -\frac{i}{\omega \epsilon_0} J_{\text{ext}}
\]

\[
D = \epsilon + \left( \frac{kc}{\omega} \right)^2 \left( I - \frac{kk}{k^2} \right)
\]

Here the conductivity is included in the dielectric tensor

\[
\epsilon = I - \frac{i}{\omega \epsilon_0} \sigma,
\]

where \( I \) is the unit tensor. The Hermitian part of the dielectric tensor describes the wave propagation, or more rigorously the reversible plasma response, while the anti-Hermitian part describes the wave damping, or the irreversible plasma response.

2.1.1 Dispersion relation for waves in the ICRF

A dispersion relation for the eigenmodes of the wave equation 2.2 are obtained when the determinant of \( D \) is zero. In the ion cyclotron range of frequencies this relation is simplified by the mobility of the electrons that short circuited the parallel electric field. To give an explicit expression for the dispersion relation, a coordinate system is introduced in which the magnetic field is in the \( z \) direction and the wave vector is in the \( x \) direction. The dispersion relation then reads

\[
n^2_x = \epsilon_{yy} - n^2_n - \frac{\epsilon_{xy}}{\epsilon_{xx}} \frac{\epsilon_{yx}}{n^2_n}
\]  

(2.4)
2.1. The Wave Equation

where \( n_{\perp, \rho} = c k_{\perp, \rho}/\omega \) are the refractive indexes parallel and perpendicular to the magnetic field, respectively. Note that in general the dielectric tensor is a function of the refractive indexes.

Away from cut-offs \( (k \rightarrow 0) \) and resonances \( (k \rightarrow \infty) \) the fast wave can be studied with a dielectric tensor for a cold multi species plasma \[21,68\].

\[
\begin{align*}
\varepsilon_{xx} &= \varepsilon_{yy} = \frac{1}{2} (\varepsilon^+ + \varepsilon^-) \\
\varepsilon_{yx} &= -\varepsilon_{xy} = \frac{i}{2} (\varepsilon^+ - \varepsilon^-) \\
\varepsilon^z &= 1 - \frac{1}{\omega} \sum_n \frac{\omega_n^2}{\omega^2} \\
\end{align*}
\]

Here \( \omega_n = \sqrt{n_\eta Z_\eta^2 e^2/(m_\eta \epsilon_0)} \) are the plasma frequencies of the individual particle species.

A “warm” plasma is described by the distribution functions of the ions and the electrons \( f_\eta(v_\parallel, v_\perp) \), where \( \eta \) labels the species, and where \( v_\parallel \) and \( v_\perp \) are the velocities parallel and perpendicular to the magnetic field, respectively. Their response to electromagnetic perturbations can be described by the linearized Vlasov equation, from which a warm plasma dielectric tensor can be derived \[22,68\].

\[
\varepsilon(k, \omega) = \left(1 - \frac{\omega_p^2}{\omega^2}\right) \left(1 - \sum_n \frac{\omega_n^2}{\omega^2} \sum_{n=-\infty}^{\infty} \int d^3v \frac{\Pi_n(v_\parallel, v_\perp; n)}{n\Omega_\eta + k_\rho v_\parallel - \omega L(f_\eta)} \right)
\]

where

\[
L(f_\eta) = \left(\frac{n\Omega_\eta}{v_\perp} \frac{\partial}{\partial v_\perp} + k_\rho \frac{\partial}{\partial v_\parallel}\right) f_\eta
\]

\[
\Pi_n(v_\parallel, v_\perp; n) =
\begin{pmatrix}
\frac{n\Omega_\eta}{k_\perp} J_n' & iv_\perp \frac{n\Omega_\eta}{k_\perp} J_n J_n' & v_\parallel \frac{n\Omega_\eta}{k_\perp} J_n \\
iv_\perp \frac{n\Omega_\eta}{k_\perp} J_n J_n' & v_\parallel^2 (J_n')^2 & -iv_\parallel v_\perp J_n J_n' \\
v_\parallel \frac{n\Omega_\eta}{k_\perp} J_n & iv_\parallel v_\perp J_n J_n' & v_\parallel^2 J_n^2
\end{pmatrix}
\]

and

\[
J_n = J_n(\rho) \quad J_n' = \frac{dJ_n}{d\rho} \quad \rho = \frac{k_\perp v_\perp}{\Omega_\eta}
\]

Here the total plasma frequency is \( \omega_p = (\sum_n \omega_n^2)^{1/2} \), and \( J_n \) is the \( n \)th Bessel function.

The operator \( L \) is a derivative in the direction \( (n\Omega_\eta/v_\perp, k_\rho) \) in the velocity space \( (v_\parallel, v_\perp) \). It trace out characteristics in velocity space to which the particles are constrained during acceleration by a wave with constant \( n/k_\rho \).

When the distribution functions are Maxwellian the velocity space integrals can be evaluated explicitly, as done in reference \[68\].
2. Ion Cyclotron Resonance Heating

Figure 2.1 a) Fast wave dispersion relation evaluated in the midplane $Z = 0$. The dashed line corresponds to the cold plasma model described in equation 2.5 and the solid line to the warm plasma model described in equation 2.6 evaluated with a Maxwellian distribution function of all plasma species. The dotted lines represent the cyclotron resonance of $^3$He, while the cyclotron resonance of D is outside the plasma ($R_{CD} < R_0 - a$). Note that the singularity in the dielectric tensor components $\epsilon_\perp$ does not give rise to a singularity in the dispersion relation. The plasma is "JET"-like; $R_0 = 3.0m$, $a = 1.0m$, $B_0 = 3.5T$, $n_e(r) = 3 \times 10^{12} (1 - (r/a)^2)^m m^{-3}$, $n_{H3}/n_D = 0.1$, $T_e = T_D = T_{H3} = 15keV$, $f_{RF} = 40MHz$, $n_\phi = 15$. b) Dispersion relation evaluated close to the perpendicular ion-ion Alfvén resonance. The cut-off/resonance pair obtained with a cold plasma model vanishes in a sufficiently warm plasma. (c) The fast wave polarization. In the ICRF the fast wave mainly right hand polarized, $|\mathbf{E}_+ / \mathbf{E}_-| < 1$. In particular cold plasma theory predicts that $\mathbf{E}_+ = 0$ at the cyclotron resonances, and $\mathbf{E}_- = 0$ at the perpendicular ion-ion Alfvén resonance. In a warm plasma both $\mathbf{E}_+$ and $\mathbf{E}_-$ remain finite.
2.1. The Wave Equation

2.1.2 The fast magnetosonic wave in a tokamak plasma

Fast wave solutions of the dispersion relation in the midplane of a JET-like plasma is illustrated in figure 2.1a) and 2.1b). Here propagating waves corresponds to positive values of $n^2$, while negative values corresponds exponentially decaying electric fields; the wave is said to be evanescent. In the low density regions close to the walls $(R - R_0 = \pm a)$ the fast wave is evanescent. Consequently the fast wave have to tunnel through this region to get from the antennas into the high density plasma.

On the high field side of the $^3$He resonance the cold plasma dispersion relation has a resonance and a cut off in $n^2$. This “resonance/cut off” layer appears only when ions with different cyclotron frequency are present, in this case deuterium $D$ and helium $^3$He. The position and width of the layer depends on the relative concentration of the two species. For higher helium concentrations the layer moves towards the deuterium resonance, i.e. towards the higher magnetic field, and vice versa. However, as shown by the solid line in figure 2.1b), the warm plasma effect eliminate the resonance, and may also for small concentration and high temperature (as used in the example) eliminate the cut off. The warm plasma response is nonlinear in $n^2$, since the dielectric tensor depends on the $k_\perp$. The nonlinearity introduces new modes, like the ion-Berstein mode [68], to which the fast wave may be mode converted.

To describe the polarization of the fast wave the electric field is decomposed into two circularly polarized wave components $E_\pm = (E_x \pm iE_y)/2$. The polarization is given by [68]

$$\frac{E_+}{E_-} = -\frac{\epsilon_{xx} + i\epsilon_{xy} - n_\perp^2}{\epsilon_{xx} - i\epsilon_{xy} - n_\perp^2} \quad (= -\frac{\epsilon_+ - n_\perp^2}{\epsilon_- - n_\perp^2}) \quad (2.9)$$

This relation is illustrated by the dashed line in figure 2.4c). At the ion cyclotron resonance the $\epsilon^+ \to \infty$ and the fast wave is right-hand polarized $E_+ = 0$, i.e. rotates counter to the ion cyclotron motion! This has the consequences that heating a one species plasma at $\omega \sim \Omega$ often become very inefficient. Instead the heating is usually performed either at a harmonic cyclotron resonances $\omega \sim n\Omega$, or at the fundamental cyclotron resonance $\omega \sim \Omega_\eta$ of a minority species $\eta$. If the minority concentration is small and the temperature high, then $E_+$ will be sufficiently strong at the resonance to yield strong damping. The polarization close to the resonance is illustrated by the solid line in figure 2.4c). In general a minority heating scheme can be defined as a heating scheme for a two (or more) ions species plasma where the minority concentration is low enough, and the temperature is high enough, that the polarization and dispersion relation is unaffected by the minority species.

Heating schemes at harmonic cyclotron resonances require a finite temperature. In this case the finite Larmor radius ($k_\perp \rho > 0$) makes the electric field strength vary over the gyro orbit, resulting in a gyro averaged acceleration (see chapter 11.3 in [68]).
Figure 2.2 Electric field for $n_\theta = -15$, calculated with the LION code [70, 71], which is described in chapter 3.3.

A wave field with a single toroidal Fourier mode $n_\theta = -15$ has been calculated with the LION code, described in chapter 3.3, and is shown in figure 2.2. It is a typical wave field for hydrogen heating with a high temperature and a low minority concentration in a plasma with circular cross section.

2.2 Resonant particle-wave interactions

The energy gained by a single charged particles accelerated by a plane wave with circularly polarized perpendicular components $\mathcal{E}_\perp$ and parallel component $\mathcal{E}_\parallel$ is

$$\dot{E} = Z e \Re [\mathbf{v}_\perp \cdot \mathbf{E} + \mathbf{v}_\parallel \cdot \mathbf{E}]$$  \hspace{1cm} (2.10)

where $\Re$ denotes the real part and where

$$\mathbf{v}_\perp \cdot \mathbf{E} = v_\perp \sum_{n=\pm\infty} (\mathcal{E}_+, J_{n+1} + \mathcal{E}_-, J_{n+1}) e^{i\nu(t,n)}$$

$$\mathbf{v}_\parallel \cdot \mathbf{E} = v_\parallel \mathcal{E}_0 e^{i\nu(t,0)}$$  \hspace{1cm} (2.11)

Furthermore, $J_m = J_m(k_\perp \rho)$ is the $m$th Bessel function, and $\nu(t, n) = \int_0^t d\tau (\omega - n\Omega_\parallel - k_z v_z)$ is the relative phase between the gyro motion
2.2. Resonant particle-wave interactions

and the wave oscillation. For most \( v_i \), this acceleration will oscillate harmonically in time \( t \), yielding no net contribution. However, at a local resonance

\[
\dot{v}(t, n) = n\Omega_\eta + k_i v_i - \omega = 0 \tag{2.12}
\]

the acceleration or deceleration is constant in time. Note that this resonant factor appear also in the denominator of the dielectric tensor in equation (2.6). A detailed analysis of the local resonance condition is given in paper VII.

For fast wave heating in a tokamak the local resonances at \( n = 0 \) is most likely to be satisfied for electrons. This resonances condition states that the phase velocity of the wave equals the parallel velocity of the particle, i.e. in the reference frame of the particle the phase of the wave phase is stationary. There are two types of damping associated with this resonance; one is due to the parallel electric field and is call electron Landau damping (ELD)\(^2\) and the other is due to the Lorentz force and is called Transit Time Magnetic Pumping (TTMP). For the scenario of fast wave electron heating close to the cyclotron frequency in a Tokamak the accelerations by parallel electric field and the Lorentz force are coupled such that they partially cancel, reducing the overall damping.

For fundamental resonance, \( n = 1 \), as well as harmonic resonance, \( n < 1 \), the strength of the interaction depend on the Larmor radius compared to the perpendicular wave length \( k_i \rho \) that appear in the Bessel functions. In a fusion plasma the fast wave often have long wave lengths, \( k_i \rho \ll 1 \). Minority heating at the fundamental resonance, which is the \( n = 1 \) local resonance, is therefore efficient. Although the Larmor radius is small the harmonic resonance heating, \( n > 1 \), can still give a significant heating. There are two reasons for this; firstly the harmonic heating can be performed with a resonant majority ion species, thereby increasing the number of resonant particles. Secondly, when changing from fundamental to harmonic heating you either increase the wave frequency, i.e. shorten the wave length, or reduce the magnetic field, i.e. increase the Larmor radius.

Resonances in a torus

In an axisymmetric toroidal systems a conductivity tensor was derived by Kaufman [2–4], by expanding the wave field in harmonic functions of the angles from the action-angle coordinate system introduced in chapter 1.1. Associated with each angle is a periodic motion; the gyro, the toroidal, and poloidal motion. The resonance condition in the axisymmetric torus is

\[
\omega = n\Omega + N_\phi \Omega_\phi + N_\theta \Omega_\theta \tag{2.13}
\]

\(^2\)The term Landau damping corresponds to the linear damping mechanism, where the interactions are decorrelated before the non-linear effect become significant [68].
where \((n, N_\phi, N_\theta)\) are three integers, \((\Omega, \Omega_\phi, \Omega_\theta)\) are angular frequency associated with the cyclotron, toroidal and poloidal motion respectively.

In a quasi-homogenous approximation of cyclotron interactions \((n > 0)\) in an axisymmetric torus all wave damping appear at local resonances, described in Eq. (2.12). The energy \(\Delta E\) gained when an ion moves through a local resonance can be calculated by integrating the acceleration given by Eq. (2.11). Assuming \(v_\perp, k_\perp, \) and \(E_e\) to be constant close to the resonance yields

\[
\Delta E = \Re \left[ Z e v_\perp (E_e J_{n-1} + E_e J_{n+1}) \right] \Pi(v) \tag{2.14}
\]

where \(\Pi\) is the phase integral

\[
\Pi(v) = \int d\tau e^{i\nu(\tau,n)} \tag{2.15}
\]

To evaluate the phase integral it is often sufficient to expand \(\nu(\tau, n)\) to second order in \(\tau\) around the local resonance. This integral is solved numerically as illustrated in figure 2.3. By extending the integration interval to infinity the integration can be performed using the stationary
2.2. Resonant particle-wave interactions

phase method [72], which yields

\[
\Pi(\nu) \approx \int_{-\infty}^{\infty} d\tau \ e^{i\nu \tau^2/2} \bigg|_{\nu=0} = \sqrt{\frac{2\pi}{\nu}} e^{i\nu} \bigg|_{\nu=0}
\]  

(2.16)

This approximation gives high accuracy, except when the phase velocity is close to stationary \( \nu |_{\nu=0} = 0 \). Here higher order terms have to be included in the expansion of \( \nu \). When retaining terms up to third order the phase integral can be expressed in terms of the Airy function \( Ai \) [27, 73]

\[
\Pi(\nu) \approx \int_{-\infty}^{\infty} d\tau \ \exp \left( \nu + \frac{1}{2} \nu t^2 + \frac{1}{6} \nu t^3 \right), \ \{ \tau \equiv t - \frac{\nu}{\nu}, \ \nu_0 \equiv \frac{\nu^3}{3\nu^2} + i\nu \}
\]

\[
= \int_{-\infty}^{\infty} d\tau \ \exp \left[ i \left( \frac{1}{6} \nu \tau^3 - \frac{1}{2} \nu^2 \tau \right) \right] e^{i\nu_0} \bigg|_{\nu=0}, \ \{ s \equiv \left( \frac{\nu}{2} \right)^{1/3} \ \tau \}
\]

\[
= \int_{-\infty}^{\infty} ds \left[ \exp i \left( \frac{1}{3} s^3 + zs \right) \right] \left( \frac{2}{\nu} \right)^{1/3} e^{i\nu_0} \bigg|_{\nu=0}, \ \{ z \equiv -\frac{1}{2^{2/3} \nu^{4/3}} \}
\]

\[
= 2\pi \left( \frac{2}{\nu} \right)^{1/3} Ai(z) e^{i\nu_0} \bigg|_{\nu=0}
\]

(2.17)

Also this approximation of the phase integral may become invalid when both the phase velocity \( \nu \) and the phase acceleration \( \overset{.}{\nu} \) vanishes. This situation is discussed in paper VII.

Decorrelated resonant ion cyclotron interactions in a torus

The change in energy during a time \( T \gg \{ \Omega_0^{-1}, \Omega_0^{-1} \} \) can approximately be written as a sum over all local resonances crossed during this time.

\[
\Delta E_{t-t+T} = \sum_i \Delta E_i = \sum_i \Re \left[ \Delta E_{i0} e^{i\nu_i} \right]
\]

(2.18)

Here the energy gained at the \( i \)th resonance \( \Delta E_i \) is given by equation (2.14). The relative phase between the wave and gyro motion at the local resonance \( \nu_i \), corresponds to \( \nu |_{\nu=0} \) in the equations (2.15), (2.16), and (2.17).

Consider now the situation where the initial phase is random, i.e. a uniformly distributed random number \( \nu_1 \in (0, 2\pi) \). All subsequent phases are deterministically given by the unperturbed equation of motion \( \overset{.}{\nu}_j = \nu_i + \Delta\nu_{i,j} = \nu_i + \int (n\Omega_i + k_{i,v_i} - \omega)dt \). The expectation value
2. Ion Cyclotron Resonance Heating

for the energy gained is zero and the variance is

\[ < \Delta E_{[t-t+T]} \Delta E_{[t-t+T]} > = \frac{1}{2} \sum_{i,j} \Re \left[ \Delta E_{0i}^* \Delta E_{0j} e^{i \Delta \psi_{ij}} \right] \] (2.19)

If the phase difference between two interactions at the same resonance point is \( 2\pi N \), where \( N \) is an integer, then the toroidal resonance condition in equation (2.13) is fulfilled. If not the variance vanishes when \( T \) becomes large. A quasilinear diffusion coefficient can then be defined that approximates the one developed by Kaufmann

\[ D_Q = \frac{1}{2T} < \Delta E_{[t-t+T]}^* \Delta E_{[t-t+T]} > \] (2.20)

The quasilinear theory requires that the acceleration is linear, \( \Delta E \ll E \).

This is achieved by successive decorrelation of the phases \( \psi_i \), which can be achieved either by collisions [74], or by non-linear effects [25, 68, 75]. Decorrelation can be introduced by assuming the phase between the local resonances to be stochastic \( \psi_j = \psi_i + \Delta \psi_{ij} + \Delta \tilde{\psi}_{ij} \). Here \( \Delta \psi_{ij} \) are the deterministic parts of changes in phase given by the Eq. 2.15 and \( \Delta \tilde{\psi}_{ij} \) are coefficients of an antisymmetric decorrelation matrix of random variables with zero mean. The variance of the gained energy is then

\[ < \Delta E_{[t-t+T]} \Delta E_{[t-t+T]} > = \frac{1}{2} \sum_{i,j} \Delta E_{0i}^* \Delta E_{0j} + \frac{1}{2} \sum_{i,j} \Re \left[ \Delta E_{0i}^* \Delta E_{0j} e^{i \Delta \psi_{ij} < e^{i \Delta \tilde{\psi}_{ij}} >} \right] \] (2.21)

When the variances of the coefficients \( \tilde{\psi}_{ij} \) are very small \( < (2\pi)^2 > \) equation (2.19) is obtained. In the opposite limit, where the interactions are completely decorrelated, all cross terms vanishes and the quasilinear diffusion coefficient reads

\[ D_{RF} = \frac{\pi}{4} \frac{1}{T_B v} \left( Z e v_{\perp} |E_{\perp}, I_{N-1} + E_{\perp}, I_{N+1}| \right)^2 \] (2.22)

This is the diffusion coefficient used in the FIDO code.

2.2.1 Transport of orbits by wave particle interactions

Resonant particle-wave interactions breaks one of the symmetries in the Lagrangian associated with the invariants \( \{ E, \Lambda, P_p \} \). The change of the three invariants are therefore related. The relations can be derived from the equation of motion and Faraday’s law for a homogenous magnetic field when the parallel electric field \( E_{\parallel} = 0 \).

\[ \dot{E} = v \cdot (m \dot{v}) = Z e E_{\perp} \cdot v_{\perp} \]

\[ m \dot{v}_{\parallel} = \frac{1}{B} B \cdot (m \dot{v}) = \frac{k_{\parallel}}{\omega} Z e E_{\perp} \cdot v_{\perp} \] (2.23)

\[ \dot{v} = \frac{1}{B^2} (B v - v_{\parallel} B) \cdot (m \dot{v}) = \frac{1}{B_0} \Lambda_{res} Z e E_{\perp} \cdot v_{\perp} \]
where $\Lambda_{res} = nZ_e B_0 / (m \omega)$. In the low $\beta$ high aspect ratio equilibrium the poloidal magnetic field is weak, i.e. $v_n = v_\phi$. If the upshift of $k_n$ due to the poloidal mode structure is neglected $k_n = k_\phi = n_\phi / R$, then the changes in the invariants due to resonant interactions can then be related by

$$\Delta P_\phi \approx \frac{n_\phi}{\omega} \Delta E, \quad \Delta \Lambda = \frac{\Lambda_{res} - \Lambda}{E + \Delta E}$$  \hspace{1cm} (2.24)

The two equations define characteristic lines in $(E, \Lambda, P_\phi)$ to which an ion is constrained during the interaction with one mode, described by $(n_\phi, \omega)$. These characteristics are a toroidal version of those described by the differential operator $L$ in equation (2.6) for a homogeneous plasma. Some characteristics are illustrated in figure 2.4.

Equation (2.24) can be understood from the interpretation given in section 1.2.4 that $\Lambda$ and $P_\phi$ corresponds to $R / R_0$ and $Z e \psi$ at the trapped ion turning point, respectively. When energy is absorbed at a cyclotron resonance the magnetic moment $\mu$ increases. The normalized magnetic moment $\Lambda (\equiv b_0 \mu / E)$ incooperate both the change in $E$ and $\mu$. It changes such that $\Lambda$ approaches $\Lambda_{res}$ asymptotically when the energy increases.
2. Ion Cyclotron Resonance Heating

The limit $\Lambda = \Lambda_{\text{exc}}$ corresponds to orbits with their turning point exactly on the resonance $\omega = n\Omega$. As an example, heating a passing ion in region $I$ will increase the $\Lambda$ such that the ion becomes trapped, typically in region $VII$, with its turning point on the resonance.

The equation (2.24) for $P_\phi$ describes the toroidal momentum absorbed from the wave. For a trapped ion this momentum will displace the trapped ion turning point across the flux surface $\Delta \psi = \Delta P_\phi / (Ze) = n_\phi \Delta E / (Ze\omega)$. For $n_\phi > 0$ the turning point is displaced outward, and vice versa. This process is called the RF-induced pinch, or the inward RF-induced pinch, for $n_\phi < 0$ and the or inverted, or outward RF-induced pinch for $n_\phi > 0$.

A number of orbits along a characteristics with $n_\phi > 0$ is shown in figure 2.5a). The turning points of the initial orbit (dashed line) is displaced toward the resonance (vertical line) and radially outwards, as the ion is heated.

For an ion being heated by modes with $n_\phi < 0$, illustrated in figure 2.5a), the turning points of the orbit are pinched until either, it is detrapped into a counter-current passing ion at line $C_6$ (figure 1.6), or it is detrapped into a co-passing ion at $C_1$. Since line $C_6$ is restricted lower values of $\Lambda$ (in figure 1.6 line $C_6$ exists only for $\Lambda < 0.94$) all orbits with higher $\Lambda$ will be detrapped into counter-passing ones. This is called an asymmetric detrapping [62,76] and was shown to be important for ion cyclotron current drive [38]. If a detrapped ion is heated further, the increasing $\nabla B$ and curvature drifts in equation (1.3) will displace the orbit towards the Low magnetic Field Side of the torus (LFS), i.e. towards higher $R$. See paper II.

In figure 2.5b) an initially trapped 0.5 MeV ion (dashed line) in region $VII$ moves along the characteristic from equation (2.24). When passing from region $V$ to $V$ the orbit is detrapped into a counter-current passing one illustrated by the solid curve enclosing the magnetic axis $(R,Z) = (3,0)m$. Finally, at 4MeV the orbit is displaced towards the LFS and does no longer encircle the magnetic axis, nor does it cross the cold resonance $\omega = n\Omega$. The ion may still be resonant through the Doppler shift.

2.3 The distribution function during ICRH

The evolution of the distribution function during ICRH can be described by an orbit averaged Fokker-Planck equation (1.13) for $\dot{f}(E,\Lambda,P_\phi,\sigma)$, in which the second term is replaced with a quasilinear diffusion term.
2.3. The distribution function during ICRH

Figure 2.5 Orbits on ICRH characteristics. The solid vertical line represents the cold resonance $\omega = n\Omega$, and the dotted circles are flux surfaces. The dashed orbits correspond to an ion with energy 0.5 MeV, the dotted-dash 1 MeV, and the solid lines 2, or 4 MeV. The orbits that intersect the cold resonance are orbits of 2 MeV ions, and the one that does not is an orbit of a 5 MeV ion.
\[ < Q > \] describing the particle-wave interactions

\[
\frac{\partial \tilde{f}_n}{\partial t} = \sum_{\nu} < C_{\eta\nu}(\tilde{f}_\eta, \tilde{f}_\nu) > + < Q(\tilde{f}_\eta) >
\]

\[ < Q(\tilde{f}) > = \sum_{n \phi} \mathcal{D} \mathcal{L} \tilde{f} \tag{2.25} \]

\[
\mathcal{L} = \frac{\partial}{\partial E} + \frac{\Lambda_{res}}{E} \frac{\partial}{\partial \Lambda} + \frac{n_\phi}{\omega} \frac{\partial}{\partial \phi}
\]

Following the derivations in e.g. [26] the diffusion coefficients \( \mathcal{D} \) should be calculated for a decorrelation time much longer then the bounce time as done in equation (2.20). However, we shall (like in the FIDO code) use the diffusion coefficient for decorrelated interaction \( D = D_{res} \) from equation (2.22).

The particle-wave interactions give rise to a diffusion along the characteristics described by \( \mathcal{L} \). Thereby, strongly anisotropic non-thermal distribution functions can be created with a large number of ions with turning points close to the resonance. The collision operator acts to restore the isotropic Maxwellian. However, for velocities well above the thermal energy the ion-ion collision frequency decreases as \( \sim v^{-3} \). Ions of species \( \eta \) will collide as frequently with background ions as with the electrons when their energies reach the Stix critical energy

\[
E_c \approx 14.8T_e \left( \frac{m_\eta}{m_p} Z_{eff} \right)^{2/3}, \quad Z_{eff} = \frac{\sum_{\nu} n_\nu Z_\nu^2}{\sum_{\nu} n_\nu Z_\nu} \tag{2.26}
\]

where \( m_p \) is the proton mass and \( n_\eta \) are the ion densities. Above this energy the ion-electron collisions dominate, resulting in a steady deceleration of the ions called slowing down. However, the pitch angle scattering from ion-electron collisions is very weak and the distribution function of resonant ions may develop strong anisotropy.

The typical distribution function of a resonant species during ICRH has an almost unperturbed bulk of thermal particles. Above the thermal velocities the distribution function often develops a tail of energetic ions with a much higher effective temperature then the thermal bulk, \( \partial_x \ln(f_\eta) \gg T_\eta \). Above the critical velocity the tail is increasingly anisotropic and well above the critical velocity the distribution is confined to a layer around \( \Lambda = \Lambda_{res} \). The RF-induced pinch can move this layer towards higher or lower \( \phi \) for positive or negative \( n_\phi \), respectively.

### 2.4 Ion cyclotron driven rotation

When the toroidal mode spectrum of the fast wave is asymmetric net momentum is transferred from the antennas to the plasma. The toroidal
torque, $T_{rf}$, and power, $P_{rf}(n_\phi, \omega)$ from a mode $\{n_\phi, \omega\}$, are related by equation (2.24).

$$T_{rf} = \sum_{n_\phi, \omega} P_{rf}(n_\phi, \omega) \frac{n_\phi}{\omega}$$  \hspace{1cm} (2.27)

Heating in JET using the $\pm 90^\circ$ phasing described in section 1.2.3, gives the net torque $T_{rf} \approx \pm (6/\omega) \sum P_{rf}$. This momentum is first transferred to the resonant ions that subsequently transfer the momentum to the bulk plasma. We shall now discuss the mechanisms behind this momentum transfer for trapped and passing ions.

When an ion following a thin passing orbit absorbs momentum from the wave the toroidal velocity is changed $\Delta v_\phi = n_\phi \Delta E/(\omega R m)$. This momentum is then transmitted to the bulk plasma ions and electrons by collisions. A trapped ion reacts different to toroidal acceleration. When the orbit is thin the averaged toroidal velocity is very small both before and after the interaction with the wave. However, the position of the turning point is displaced $\Delta \psi = n_\phi \Delta E/(\omega Z e)$. This introduces a radial ion current, $j_{fast}$ yielding a charge separation, and a radial electric field. The bulk plasma ions quickly react and set up a polarization current $j_p \approx -j_{fast}$ that cancels the RF-induced radial ion current. In this process the bulk plasma gains a momentum $R \mathbf{j} \times \mathbf{B} = R j_p B_0$ as it is accelerated by the radial electric field [77].

For ions with wide orbits, trapped or passing, absorption of momentum from a wave field give rise to both a radial current and a change in the orbit averaged toroidal velocity. The momentum is then transferred to the bulk through both the $R \mathbf{j} \times \mathbf{B}$-torque and collisional friction.

Even when no net momentum is supplied to the plasma, the momentum exchange between fast ions with wide orbits and the background plasma may still drive rotation [77–81]. An example similar to that by Perkins et al [79], is the slowing down of a fast ion that is generated at a flux surface $\psi_0$ with no kinetic toroidal momentum $v_\phi = 0$, i.e. on a turning point. Assume now that the temperature of the background plasma is very high along the orbit, except for outermost part of the orbit at $\psi = \psi_{out}$ where the temperature is very low. All collision can then be considered to take place at $\psi_{out}$, and the final thermalized thin orbit will be confined to the flux surface $\psi_{out}$. The resulting frictional torques are deposited at $\psi_{out}$ and are in the co current direction. The radial ion current $j_{fast}$ due to the motion of the ion from $\psi_0$ to $\psi_{out}$, induces a polarization current in the plasma $j_p \approx -j_{fast}$, that accelerates the plasma by a $j_p \times \mathbf{B}$ force in the counter current direction. The sum of these two torques has a bipolar structure with a smooth counter current torque for $\psi \in (\psi_0, \psi_{out})$, and a narrow co current torque at $\psi = \psi_{out}$.

To model the toroidal rotation a momentum conservation equation is used [18]

$$\partial_t <Rm_\eta n_\eta v_{\phi,\eta}> = <\mathbf{j}_\eta \cdot \nabla \psi - R^2 \nabla \phi \cdot (\nabla \cdot \mathbf{n}_\eta) + RF_{\phi,\eta}>$$  \hspace{1cm} (2.28)
where $< \cdot >$ denotes the flux surface average, $\vec{v}_{\phi,i}$ is the toroidal velocity of species $\eta$, $\vec{\pi}$ is the stress tensor, and $F_{\phi,i}$ is the $\phi$ component of the friction force. Further, $\sum_\eta j_\eta \cdot \nabla \psi$ is the torque from the radial polarization current, which for circular concentric flux surfaces can be written as $-f_{r,\eta} R B_\eta$.

The stress tensor could be described by neoclassical theory [82,83]. However, experimental results indicate that a the momentum transport is dominated by anomalous processes [84]. The viscous torque is therefore approximated by a diffusion term similar to that found for classical momentum transport in a cylinder (see section 5.3 in reference [18])

$$< R^2 \nabla \phi \cdot (\nabla \cdot \vec{\pi}_\eta) > = \frac{1}{r} \frac{\partial}{\partial r} \left( r < R m_{\eta} n_{\eta} \chi > \frac{\partial < \vec{v}_{\phi,\eta} >}{\partial r} \right) \quad (2.29)$$

In a torus with minor radius $a$ and a momentum confinement time $\tau_M$ (the ratio of plasma momentum to input torque at steady state), then an approximate anomalous viscosity can be estimated as $\chi = a^2/(2\tau_M)$. More advanced approximations of the anomalous viscosity are used in e.g. reference [79] and in paper VI.

### 2.5 Ion cyclotron current drive

The toroidal rotation velocity of a minority ion species can be calculated from the solution of the orbit averaged Fokker-Planck equation (2.25).

From the same solution the radial current $j_r$ and the frictional torque $T_e$ can be obtained and the rotation of the background plasma can be calculated using equation (2.28). We shall now derive a model for the electron rotation which in combination with the ion rotation give the total current drive during ion heating. The following derivation is similar to the one in references [45,85,86].

To estimate the current driven by ICRH we consider a plasma with a minority ion species $\eta$ in a background plasma with ion species $i$. In the centre of mass frame the velocities $\vec{v}_\eta$ and $\vec{v}_i$ are related by

$$0 = m_{\eta} n_{\eta} \vec{v}_\eta + \sum_i m_i n_i \vec{v}_i \quad (2.30)$$

where the mass of the electrons is assumed to be small. If all species $i$ have the same velocity $\vec{v}_i = \vec{v}_0$, then

$$\vec{v}_0 = -c_m \vec{v}_\eta, \quad c_m = \frac{m_{\eta} n_{\eta}}{\sum_i m_i n_i} \quad (2.31)$$

Following now to the static collisional force balance for the electrons in a torus. Assuming the Larmor radius to be small the Fokker-Planck equation (1.10) can be averaged of the gyroangle yielding the *drift kinetic equation*

$$\bar{v}_e \nabla \cdot f_e = C_{ee} (f_e, f_e) + C_{e\eta} (f_e, f_\eta) + \sum_i C_{ei} (f_e, f_i) \quad (2.32)$$
2.5. Ion cyclotron current drive

Here the radial gradients and the electric fields are neglected for convenience. The solution of this equation consists of two parts; the passing electron with a velocity determined by the collisional force balance, and the trapped electron that are stationary in the centre of mass system. The two parts are separated by a boundary layer that generate a friction between the trapped and passing electrons. An effective fraction of trapped electrons \( f_T \) is therefore introduced that are stationary in the centre of mass frame. The remaining passing electrons are approximately unaffected by the toroidicity.

The magnitude of the collision operator for passing electrons is proportional to the difference between the velocity of the passing electrons \( v_e \) and the ions.

\[
0 = Z_\eta^2 n_\eta (v_e - v_\eta) + \sum_i Z_i^2 n_i (v_e - v_i)
\]  

(2.33)

By combining equations 2.31 and 2.33 the total driven current can be calculated in term of the minority current density \( j_\eta = Z_\eta e n_\eta v_\eta \).

\[
\begin{align*}
  j_{RF} &= Z_\eta e n_\eta v_\eta + \sum_i Z_i e n_i v_i - (1 - f_T) e n_e v_e \\
     &= j_\eta \left( 1 - \frac{Z_\eta}{Z_{\text{eff}}}(1 + c_m) + f_T \left[ 1 - \frac{Z_\eta}{Z_{\text{eff}}} - c_m \frac{\sum_i Z_i^2 n_i}{Z_{\text{eff}} Z_\eta n_\eta} \right] \right)
\end{align*}
\]

(2.34)

where

\[
n_e = Z_\eta n_\eta + \sum_i Z_i n_i, \quad n_e Z_{\text{eff}} = Z_\eta^2 n_\eta + \sum_i Z_i^2 n_i
\]  

(2.35)

An approximate expression for the effective fraction of trapped electrons was derived by Connor and Cordey [85]

\[
f_T \approx 1.46 A(Z_\eta) \sqrt{r/R_0}
\]

(2.36)

where the function \( A \) is described in the same reference. Note that when the fraction of electrons is small and \( Z_\eta = Z_{\text{eff}} \), then the current is completely quenched. In JET this may be the case for minority \( ^3 \text{He} \) heating close to the centre of the plasma.

The difference between the minority current \( j_\eta \) and the total current \( j_{RF} \) is often called drag current.

2.5.1 Mechanisms for ICCD

The possibility of ion current drive was first proposed by Fisch [36, 86, 87]. He recognized that Doppler shifted absorption of a wave field with an asymmetric toroidal mode spectra heats ions with \( v_i > 0 \) on one side of the cold resonance, and \( v_i < 0 \) on the other. Therefore there are more energetic ions and less thermal with \( v_i > 0 \) on one side and vice versa. The collisional isotropization is faster for the thermal than
energetic ions. The minority ions will then rotate in same direction as the fast ions while the background plasma will rotate in the opposite direction. The resulting current has a bipolar shape centred around the cold resonance.

The finite orbit width of the heated ions provide other mechanisms for ICCD [37, 38]. The toroidal velocity of a trapped ion is along the plasma current on the outer “leg” of the orbit, i.e. outside the turning point, and counter on the inner “leg”. A gradient in the density of such orbits produce a diamagnetic current. Since ICRH heats the plasma only close to the resonance strong gradient of heated trapped particles are often present and significant currents may be obtained.

Consider two identical trapped ions. Let one be detrapped into a co-, and the other into a counter-current moving orbit. Although the two orbits can have the same canonical toroidal angular momentum the counter current moving orbit is always “inside” (smaller \( r \)) the co current moving ion. Thus, part of the diamagnetic current appear as a current of passing ions [37].

The RF-induced pinch displaces the energetic ions inwards, or outwards, depending on the direction of the wave propagation. This displacement appear also in the the pressure profile of energetic ions and in the driven current profile. The RF-induced pinch may also enhance the production of energetic ions on potatoe orbits that moves preferentially in the co-current direction. Detrapping of a trapped potatoe orbit at sufficient high \( \Delta \) produces exclusively co-current passing ions; the assymmetric detrapping described in section 2.2.1. This systematic production of co-current passing ions generates an ion current enhancing the plasma current [38].

Another current drive mechanism was proposed by Chang [42], and is a result of the RF-induced transport discussed in reference [88]. This mechanism is most efficient when the ion is heated on the high or low field side of the magnetic axis, and slowed down by collisions on the opposite side.
3 Numerics and codes

In the Monte Carlo method for the advection-diffusion equation the “density” function is discretized by a set of quasi particles, such that the number density of the particles represents the function. The evolution of the quasi particle position is described by an Itô stochastic differential equation, or Langevin equation, which is related to the advection-diffusion equation, as described in section 3.1. The stochastic differential equation is then discretized in time, by for example an Euler scheme. However, the convergence of the discretization of the Euler scheme is slow, and more advanced schemes are often useful (see section 3.1.1).

In section 3.2 the FIDO code is presented, which solves the orbit averaged Fokker-Planck equation (2.25) using a Monte Carlo method. The wave equation for the fast magnetosonic wave is solved with the LION code, described in section 3.3, by a finite element method. A self consistent treatment of the wave damping and the minority heating is obtained in the SELFO code (see section 3.4), which couples the evolution of the distribution function from the FIDO code, and wave field from the LION code.

3.1 The Monte Carlo method for the advection-diffusion equation

Consider an N-dimensional stochastic Markov processes $X(t, \omega)$, where $X(t, \cdot)$ is a random variable at each time $t$ and $X = X(\cdot, \omega)$ form a sample path, trajectory, or realization for each $\omega$. The evolution of the $i$th component $X_i$ can be described by an Itô stochastic differential equation (or Langevin equation)

$$X_i(t + dt, \omega) - X_i(t, \omega) \equiv dX_i(t, \omega) = a_i dt + b_{i,j} dW_j(t, \omega)$$

(3.1)

where summation over repeated indexes is implicit. Here the drift $a_i = a_i(X(t, \omega), t)$ describes the deterministic, and the $b_{i,j} = b_{i,j}(X(t, \omega), t)$ the stochastic motion. $W(t, \omega)$ is a vector of uncorrelated Wiener processes, i.e. the differentials $dW_j(t, \omega) \equiv W_j(t + dt, \omega) - W_j(t, \omega)$ are independent Gaussian random variables with zero mean and variance $dt$. Note that since $b_{i,j}(X(t, \omega), t)$ is uncorrelated with $dW_j(t, \omega)$ equation (3.1) is indeed a Markov process.
The density distribution function \( f(x, t) \) of \( X(t) \) follows the Kolmogorov-Fokker-Planck equation (see chapter 4.7 in reference [89]) which in Cartesian coordinates \( x_i \) reads

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i} (a_i f) + \frac{\partial^2}{\partial x_i \partial x_j} \left( \frac{1}{2} b_{i,n} b_{j,n} f \right)
\] (3.2)

equivalent to an advection diffusion equation

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i} \left( a_i f + f \frac{\partial}{\partial x_j} \left( 2 b_{j,n} b_{i,n} \right) \right) + \frac{\partial}{\partial x_i} \left( \frac{1}{2} b_{i,n} b_{j,n} \frac{\partial f}{\partial x_j} \right)
\] (3.3)

This relation between the advection-diffusion equation and the Itô stochastic differential equation enables us to find approximate solutions to the Fokker-Planck equation by sampling a statistically significant number of trajectories according to equation (3.1); the Monte Carlo method. In a plasma these trajectories resembles the motion of a real ion or electron under the forces of electric and magnetic fields and collisions. However, due to the large number of ions and electrons in a plasma, the trajectory of one quasi-particle has to represent a very large number of real particles.

Discretization, or sampling, of the Itô stochastic differential equation (3.1) for a quasi-particle with trajectory \( X_i(\cdot, \omega) \) \((i = 1..N)\) can be done with an Euler scheme similar to that for an ordinary differentiation equation.

\[
\Delta X_i(t, \omega) = \Delta t \left( a_i(X_i(t, \omega), t) + b_{i,n}(X_i(t, \omega), t) \mathcal{W}_n(t, \omega) \right)
\] (3.4)

A sample path of the Wiener process can be obtained from \( \mathcal{W}_n = \sqrt{\Delta t} \xi_n \), where \( \xi_n \) is a vector of uncorrelated normally distributed, Gaussian, random numbers with zero mean and unit variance. The density distribution corresponding to the ensemble of \( M \) quasi particles \( X^{(m)} \) \((m = 1..M)\) is \( f(x) = \sum_{m=1}^{M} \delta(X^{(m)} - x) \).

The stepping algorithm in equation (3.4) can be visualized by inserting a Dirac distribution, i.e. an ensemble of identical particles, into the one dimensional advection diffusion equation (3.3). As time evolves the Dirac distribution will smooth out due to diffusion and be translated by the advection. After a short time \( \Delta t \) the solution will be a Gaussian function centered around \( X + a \Delta t \) with a width \( b \sqrt{\Delta t} \), see figure 3.1. When sampling the Wiener process \( \mathcal{W} \) in equation (3.4) for an ensemble of \( M \) quasi particles \( X^{(m)} \) Gaussian function is represented by the density of the quasi particles.

### 3.1.1 Convergence and higher order schemes

There are two different types of convergence for a discretization of a stochastic differential equation. If the sample path of the approximate solution \( \hat{X}(\cdot, \omega) \) converges to the sample path of the exact solution for each \( \omega \), then \( \hat{X} \) has a strong convergence. If the density distribution
3.1. The Monte Carlo method for the advection-diffusion equation

![Diagram showing probability density function](image)

**Figure 3.1** The initial condition is a Dirac distribution $P(0,x) = \delta(X - x)$ at time $t = 0$. It is represented by the vertical arrow at $x = X$. After a time $\Delta t$ the probability density $P(\Delta t, x)$ is a smooth Gaussian function centered around $X + a \Delta t$ with a width $b \sqrt{\Delta t}$.

The function $\hat{f}$ of $\hat{X}$ converges to the distribution $f$ of $X$, then $\hat{X}$ has a weak convergence. The rate of strong convergence $\gamma$ and weak convergence $\beta$ can be defined as

\[
< |\hat{X}(t, \omega) - X(t, \omega)| > \leq K \Delta t^\gamma 
\]

and

\[
| < g(\hat{X}(t, \omega)) > - < g(X(t, \omega)) > | \leq K \Delta t^\beta 
\]

for some constant $K$ and any smooth function $g$. Here $< \cdot >$ denotes the ensemble average. Under regularity constraints for $a$ and $b$ the Euler scheme has a strong convergence rate $\gamma = 1/2$ and a weak convergence rate $\beta = 1$, see theorem 10.2.2 and 14.1.5 in reference [89].

The slow "strong convergence" of the Euler scheme can be improved as shown by Milstein [90]. For the $i$th component $X_i$ the Milstein scheme reads

\[
\Delta X_i = \alpha_i \Delta t + b_{i,n} \Delta W_n + b_{i,n} \frac{\partial b_{i,m}}{\partial x_j} \Delta I_{n,m} 
\]

where $\Delta I_{n,m}$ are double Itô integrals [89]. Its diagonal element can be written as $\Delta I_{n,n} = 1/2 (\Delta W_n^2 - \Delta t)$, and the sample path can be chosen as $\Delta W_n = \xi_n \sqrt{\Delta t}$. The off diagonal elements cannot easily be expressed.
in terms of \( \Delta W_n \) and \( \Delta W_m \). An approximate sample path of \( \Delta t_{n,m} \) is

\[
\Delta t_{n,m}^p = -\frac{1}{2} \delta_{n,m} \Delta t + \Delta t \left( \frac{1}{2} \xi_n \xi_m + \sqrt{\rho_p} \left( \mu_{n,p} \xi_m - \mu_{m,p} \xi_n \right) \right) + \Delta t \left( \frac{1}{2\pi} \sum_{k=1}^p \frac{1}{k} \left( X_{n,k} \left( \sqrt{2} \xi_m + \eta_{m,k} \right) - X_{m,k} \left( \sqrt{2} \xi_n + \eta_{n,k} \right) \right) \right)
\]

where \( \delta_{n,m} \) is the Kronecker delta,

\[
\rho_p = \frac{1}{12} - \sum_{k=1}^p \frac{1}{2\pi k^2},
\]

\( \xi_n, \mu_{n,p}, X_{n,k}, \) and \( \eta_{m,k} \) are independent Gaussian random variables with zero mean and unit variance. Furthermore, \( \xi_n \) should be the same random variables as used for \( \Delta W_n \) in equation (3.7).

The importance of the last term in the Milstein scheme can be seen for scattering of the pitch angle \( \xi \) in \( \Lambda(\xi) = (1 - \xi^2)B_0/B \) and \( P_\phi(\xi) = R m v \xi B_\phi/B + Z \psi \). This process has Milstein and the Euler schemes given by

\[
\Delta \Lambda = \frac{1}{2} \frac{B_0}{B} \left( 3\xi^2 - 1 \right) \frac{Y}{v^2} \Delta^* + \zeta \sqrt{2} \frac{B_0^2}{B^2} (1 - \xi^2) \xi \frac{Y}{v^2} \Delta t
\]

\[
\Delta P_\phi = -R m \xi \frac{Y}{v} \Delta^* - \zeta R m \frac{\xi}{|\xi|} \sqrt{2} (1 - \xi^2) v \Delta t
\]

where \( \zeta \) is a Gaussian random variables with zero mean and unit variance. For the Milstein scheme \( \Delta^* = \xi^2 \Delta t \) and for the Euler scheme \( \Delta^* = \Delta t \).

Since only the pitch angle is scattered the diffusion should be constrained to the characteristic \( \left( \Lambda(\xi), P_\phi(\xi) \right) \), \( \xi \in \{-1, 1\} \), the solid lines in figure 3.2. In the same figure it is shown how a quasi particle moves for different \( \xi \) in \( \{-1, 1\} \). Starting from the points marked with a star “*”, one step with the Milstein and Euler schemes distributes the particles on the dashed and dotted lines, respectively. In the upper figure we see how the dashed line from the Milstein scheme is curved in a similar way as the characteristic to which the motion should be constrained. However, the dotted line from the Euler scheme is a straight line very far from describing the characteristic in the figure 3.2a.

Monte Carlo methods are particularly efficient for solving advection and diffusion in several dimension. They give a perfect conservation of mass, while conservation of momentum and energy is less trivial to conserve [91-95]. The problem is to represent self-collisions, which introduces a correlation between the test particles. In this thesis only minority heating schemes are studied in which self-collisions are negligible.
3.1. The Monte Carlo method for the advection-diffusion equation

Figure 3.2 Pitch angle scattering in the \((\Lambda, P_\phi)\)-plane for two different initial conditions. The stars "*" are the starting points. The solid line is the characteristic to which the time continuous motion is constrained. The dashed and dotted lines are the sets of possible endpoints for one step with the Milstein or Euler scheme with random numbers \(\xi \in (-\sqrt{3}, \sqrt{3})\), respectively. (a) illustrates how the two schemes represent a curved characteristic and (b) illustrates the time stepping close to a boundary (here \(\Lambda = 0\)) where the diffusion coefficient vanishes.
3.2 The FIDO code

The FIDO code [9] solves the orbit averaged Fokker-Planck equation (2.25) by a Monte Carlo method [26, 96]. It uses an Euler discretization scheme. The solution of the Fokker-Planck equation is a distribution function in the invariants of motion \((E, \Lambda, P_{\Phi}, \sigma)\). Each combination of invariants can be mapped onto an orbit in the \((v_\perp, v_\parallel, r, \theta)\)-space, from which flux surface averaged profiles can be calculated, such as the density, the current (of flows), and the collisional power and momentum transfer to electrons and ions.

A major problem with early versions of the FIDO code where the thermal particles. They where many, and required very short time steps. Through the recently implemented weighting scheme [97] these particles can be represented by fewer quasi-particles, while the rare but important energetic ones can be represented by more quasi-particles.

Another recent improvements of the code are the implementation of a recycling boundary condition, through which particles lost to the wall where replaced by thermal particles. This has enabled studies of steady state distributions functions which has been important in the study of RF-induced rotation [98, 99].

Finally, neutral beam sources [97], poly-chromatic wave spectra [100] and particle acceleration by MHD modes [101] have been implemented. Improvements that have enabled more detailed comparison with and analysis of experimental data [100, 102, 103].

3.2.1 Projection of JET plasma parameters to FIDO input

The FIDO code uses the high \(\beta\) large aspect ratio equilibrium discussed in chapter 1.2.1. This equilibrium is quite different from most JET equilibria. One of many consequence of the approximate equilibrium is that in order to have the same flux function \(\psi\) in FIDO as in JET the \(q\)-profile will have to be different! Consequently, when defining plasma parameter for FIDO modelling of a JET scenario only some parameters can be the same in the two equilibria. A set of such invariants can be specified by:

1. The intersection of the cold resonance \(\omega = n\Omega\) with the midplane \((Z = 0)\) should be invariant.

2. The density and temperature profiles in the midplane on the low field side of the magnetic axis \((Z = 0\) and \(R \geq R_0\)) should be invariant.

3. The frequency of the launched wave should be invariant, which implies similar wave lengths of the fast wave.

4. The averaged power absorbed per particle should be invariant. Since the densities are the same, while the volume of a JET plasma is larger, the total heating power should be smaller in FIDO.
5. The poloidal flux $2\pi \psi$ should be invariant. This quantity determines the neoclassical transport, the RF-induced pinch, and the finite orbit width effects. However, with this assumption the ratio between the poloidal and toroidal orbital periodicities are not invariant, which can cause problems when studying the interactions between fast ions and MHD modes [101].

3.3 The LION code

The LION code [70, 71] solves the wave equation (2.1) using a finite element method [104] in an axisymmetric torus. The electric wave field is separated into independent mode described by frequency and toroidal mode number

$$\mathcal{E}(\mathbf{r}) = \sum_{n_{\phi}, \omega} \mathcal{E}_{n_{\phi}, \omega}(R, Z) e^{i(\omega t - n_{\phi} \phi)}$$

(3.12)

The contribution to the dielectric tensor from each species, the susceptibility, can either be supplied as input data, or be calculated within the code from susceptibility of a quasi-homogeneous plasma, equation (2.6). In the latter case the distribution function is taken to be Maxwellian with a parallel temperature $T_{\parallel}$ and a perpendicular temperature $T_{\perp}$. Further, in the calculations of the susceptibility the up-shift of the parallel wave number is neglected $k_{\parallel} = n_{\phi} / R$. The LION code gives an adequate description of the fast wave propagation in absence of mode conversion.

3.4 The SELFO code

Wave-particle interactions may, due to the long relaxation times, create non-Maxwellian distribution functions that can have an important impact on the dispersive properties of the plasma. The evolution of the wave field and the distribution functions of the resonant ion species may therefore need a self-consistent treatment.

The SELFO code [10, 11] couples the FIDO code and the LION to calculate the power absorption and tail formation self-consistently. From an initial distribution function $f_p(v_{\perp}, v_{\parallel}, R, Z)$ and $k_{\perp}(R, Z)$ the susceptibilities of the resonant species $\eta$ are calculated. These are then given as input to the LION code, which calculates the electric wave fields, a new $k_{\perp}(R, Z)$, and the power absorbed by the different species. The wave is used to evolve the distribution function a short step in time using the FIDO code. This process can now be repeated by calculating new susceptibilities from the new distribution function, and so forth.
4 Summary of the included papers

4.1 Ion current drive, sawteeth, and neoclassical tearing modes

Between autumn 2000 and spring 2003 a number of experimental sessions at JET were dedicated to the study of ion cyclotron current drive (ICCD) and to develop tools for controlling the stability of neo-classical tearing modes by ICCD. In preparation of these experiments numerical simulations were performed with the SELFO code. The results are included in this thesis as paper I. The ICCD obtained for heating at the fundamental proton cyclotron frequency was similar to previous results from the FIDO code [37,38]. New results were however obtained for ICCD at the second harmonic cyclotron frequency of the protons. Furthermore, both paper I and paper IV include analysis of how the driven current is distributed in energy and minor radius, i.e. the total current is obtained by integrating $dl = J(r,E) dr dE$, showing that different current drive mechanisms dominate at different energies.

The simulations were performed with plasma parameters similar to those reported from previous ICCD experiments in JET by Start et al [43,45]. These experiments were designed to study the mechanism for ICCD discovered by Fisch [36,86,87]. Instead of measuring the actual ICCD the resonance position was carefully chosen such that changes in the magnetic shear, $s = r \partial_r \ln(q)$, at the $q = 1$ surface would stabilize, or destabilize the sawtooth instability [44].

Comparison with the experiments [43,45] shows that the ICCD obtained from the SELFO code, and from the Fisch model, are very different. A careful analysis shows that the changes in magnetic shear at the $q = 1$ surface, due to the currents from SELFO, are consistent with the experimental results. However, the exact position of the $q = 1$ surface is critical for predictions of the sawtooth stability.

The sawtooth stabilization observed with the $+90^\circ$ phasing could in principle be due to either ICCD, stabilization by energetic ions, or reduced current diffusion due to electron heating. However, sawtooth destabilization could only be explained by ICCD (unless the plasma density is very low [105]). The destabilization was observed in a set of
4. Summary of the included papers

carefully designed experiments described in paper VI. Sawtooth destabilization by ICCD could be used for controlling the neo-classical tearing modes, which are expected to limit the achievable $\beta$ in a tokamak reactor.

ICCD control of neoclassical tearing modes proved successful, and were reported in papers X, XI, XII, XXX, XXXIII, and XXXIV. The experiments were mainly performed at the second harmonic proton cyclotron resonance. This ICCD scenario was analysed in paper I and XI.

4.2 RF-induced transport of resonant $^3$He ions in JET

In the spring of 2001 experiments were performed in JET with minority $^3$He heating in $^4$He plasmas. In these experiments the $\gamma$-ray emission from $^{12}$C($^3$He,py)$^{14}$N reactions were measured along 19 lines of sight spanning a mesh over the poloidal cross section (see paper XIII). From these measurements tomographic reconstructions were made and published in paper II. The results showed a strong asymmetry between the distribution functions of $^3$He ions with MeV energies for heating with $+90^\circ$ and $-90^\circ$ phasing. For the $-90^\circ$ phasing the tomographic reconstructions were consistent with a population of trapped $^3$He ions with their turning point close to the unshifted cyclotron resonance, which agrees qualitatively with the predictions from two-dimensional theory [27]. However, for the $+90^\circ$ phasing the intensity of the $\gamma$-emission was stronger and the tomographic reconstructions were consistent with asymmetrically detrapped co-current passing ions; most of them located on the low field side of the unshifted cyclotron resonance. The distribution functions observed for $+90^\circ$ and $-90^\circ$ phasing are shown to be in qualitative agreement with predictions from paper IX.

The strong asymmetries between heating with $+90^\circ$ and $-90^\circ$ phasing were also observed in the fast ion energy content, and the stability of the sawtooth and the Alfvén eigenmodes.

4.3 Bulk plasma rotation induced by ICRH

During recent years there have been an interest in using ICRH for driving plasma rotation, since rotation can influence the stability of resistive wall modes [54, 55] and is believed to suppress turbulence [59, 60]. It have been shown that even in absence of toroidal momentum input plasma rotation can be driven [77-80, 106].

In papers III, IV, V, XXIII and XXIV the ICRH induced toroidal rotation is studied in the presence of toroidal momentum input from fast magneto-sonic waves. A steady state torque balance can then be written for the minority ions $v$ including the following terms (neglecting the toroidal electric field); the torque from the wave field $T_{\text{eff}}$, from the wall $T_{\text{wall}}$,
from collisions with ion and electrons $T_{\nu\eta}$, and the torque from radial ions currents $j_{\nu}$, and the torque from radial ions currents $j_{\nu} \cdot \nabla \psi$. For the background plasma the torque balance is given by Eq. 2.28 where the polarization currents $\sum_{\eta} j_{\eta} = -j_{\nu}$ and the collisional friction force $F_{\phi,\eta} = -T_{\nu\eta}/R$.

In preparation for these studies the SELFO code was upgraded, to have recycling boundary conditions and diagnostics to monitor the momentum exchange in the plasma, including biasing from the boundary conditions.

To measure the toroidal rotation in paper V a diagnostic beam of neutral deuterium was fired into the plasma in pulses of 200 ms. The rotation profiles could then be calculated from the Doppler shift of the charge exchange spectrum for C$^{+6}$ [107]. By considering only the first spectrum detected after the application of the beam, the rotation profiles could be considered unperturbed by the beam [108].

The rotation was measured in JET discharges with 5.5 MW ICRH for $+90^\circ$ and $-90^\circ$ phasing, and in a reference discharge with $+90^\circ$ phasing where 2 MW of ICRH power was replaced by lower hybrid heating. The three pulses gave different rotation profiles that can be interpreted as follows. In all pulses there is an underlying co-current rotation that is not controlled by the antenna phasing. A second component of the rotation is a direct consequence of the absorption of momentum from the ICRF wave field. This component is in the co-current direction for $+90^\circ$ phasing, and counter-current for $-90^\circ$ phasing. The rotation in the pulse with both ICRH and lower hybrid heating lies between the rotation in the other two discharges. Thus, the results are consistent with rotation driven by momentum input from the wave field. SELFO modelling of the discharges with $+90^\circ$ and $-90^\circ$ phasing show differences in the rotation profiles similar to the experimental results.

The modelling shows that for $+90^\circ$ phasing fast ions following co-current passing potato orbits absorb momentum from the wave and transfer it to the background plasma. Fast ions are also observed from $\gamma$-spectrometry. The spatial distributions of the $\gamma$ observed experimentally and predicted with the SELFO code are similar, but the intensity is weaker in the prediction. The reason for this discrepancy may be due to differences in the transport of thermal particles in the experiments and in the modelling.

4.4 Distribution functions of cyclotron-resonant ions

Much effort have been invested to increase our understanding and intuition of how the distributions of resonant ions behave. This includes studies of the topology of guiding centre orbits at different energies, the conditions for resonant absorption, and the properties of the quasilinear diffusion coefficient discussed in paper VII. The connection to global
measures like current drive and torques delivered to the bulk plasma are discussed in the papers I, III and IV.

Paper VII is written to quantify the importance of the factors in the quasilinear diffusion coefficient. It provides a detailed investigation of the local resonance condition, including analysis of how the location of resonance is displaced during the heating. Furthermore, it is shown that the phase velocity is not stationary at the turning point of trapped ions.

In general the quasilinear diffusion coefficient can be separated into a sum of toroidal modes, where each term describe a one dimensional diffusion process along characteristics in the space of invariants of the unperturbed motion. The properties of the guiding centre orbits, the resonance positions, and the factors yielding the diffusion coefficient have been evaluated along typical characteristics. This evaluation shows that the diffusion coefficient deviate significantly from the $D \propto v^{2n}$ scaling, due to inhomogeneous polarization of the electric field (see figure 4.1), sensitivity of phase-integral to the shape of the guiding centre orbit and the resonance location, and to finite Larmor radius effects.

The perhaps most important conclusion drawn from paper VII is the fact that at marginal resonances the diffusion coefficient becomes discontinuous. At the discontinuity the distribution function can change its slope, or “effective temperature” from $[\partial E \ln f]^{-1} \sim 1$ MeV to the electron temperature $T_e \sim 10$ keV. A new Monte Carlo scheme is therefore developed that can treat problems with discontinuous diffusion coefficients. It uses the method of imagine to reflect the appropriate number of test particles at the boundary where the diffusion coefficient is discontinuous.
4.5 Fast wave scenarios in ITB plasmas

Paper VIII concerns experiments with fast wave electron heating and current drive in plasmas with internal transport barriers (ITBs). In the experiments the single pass damping was low, i.e. only a small fraction of the coupled power was absorbed during one transit in the plasma. The fast wave electron heating were therefore plagued by parasitic absorption, which could be measured from the balance between the energy supplied by the heating systems (ICRF, LHCD, NBI, and ohmic heating), the power radiated from the plasma (measured by bolometers) and the power going into the divertor (measured by thermocouples). The bursting line radiation observed from Beryllium gives an indication that rectified RF-sheath potentials are participating in the parasitic absorption.

Fast wave direct electron heating were shown be efficient in plasmas with ITBs and reversed shear. An advantage of direct electron heating is that there are no fast ions generated, thereby avoiding monster sawteeth that can trigger neo-classical tearing modes.

4.6 The author's contributions to the included papers

- Paper I: The author performed the numerical modelling, the analysis and wrote the paper.
- Paper II: The experimental results on RF-induces transport of fast $^3$He ions was first published in the conferences proceeding XXIV and XXIX. These papers were written by the author, who did the theoretical interpretation, the numerical modelling, and parts of the analysis of experimental data. An extended report written by Mervi Mantsinen is included in the thesis as paper II.
- Paper III: The author developed the numerical tools for evaluating the torque transferred from the resonant ions to the bulk plasma, did the numerical modelling, and contributed to the theoretical content of the paper.
- Paper IV: The author performed the numerical modelling, the analysis and wrote the paper.
- Paper V: The author performed the numerical modelling and participated in the preparations and the execution of the experiments.
- Paper VI: At JET a large effort was made to find experimental evidence for ICCD. In this effort the author contributed by predictions in paper I, by SELFO modelling for a large number of JET discharges, and by analysis of experimental data. The evidence for ICCD is reported in paper VI. However, the scenario used in the reported experiments proved difficult to model with the SELFO code (several
attempts were made). The authors contribution is therefore mainly in the preparations of the experiments.

• Paper VII: The author initiated and performed this study.

• Paper VIII: The author did predictive numerical modelling, analysis of experimental data and participated in planning and execution of the experiments.

The main contribution to many of the papers included in this thesis have been in modelling and analysis of the ICRH using the FIDO and the SELFO codes. In order to model the experiments and to study e.g. ICRH-induced rotation, significant efforts have been invested in developing the codes.
5 Conclusions

Predictive numerical modelling of ion cyclotron current drive (ICCD) and rotation have been performed with the SELFO code. ICCD is shown to have a more complex structure than previously reported with several mechanisms competing. Typically different mechanisms dominate in different parts of the phase space. The works on RF-driven rotation show how momentum is transferred from the RF wave to the bulk plasma via resonant ions. Although the resonant ions have wide guiding centre orbits the profiles of the torque to the bulk plasma is often similar to that absorbed from the wave field.

The experimental work presented in this thesis have been performed in JET and cover four subjects; RF-induced transport, RF driven rotation, ion- and electron-current drive by the fast magnetosonic wave. The studies on the RF-induced transport include measurements of the spatial distribution of resonant energetic $^3$He ions. When comparing discharges where the wave field have different toroidal mode spectra strong differences in the distribution functions were observed that are consistent with numerical predictions. These differences affect the the fast ion energy content and the stability of sawteeth and Alfvén eigenmodes.

The influence of directed ICRF waves on the toroidal plasma rotation has been experimentally demonstrated. The ICRF power and momentum are transferred from the wave to the bulk plasma via fast resonant ions. Thus the result provides evidence for the influence of ion cyclotron resonance heated fast ions on the plasma rotation.

Experimental results show that ICCD can be used to destabilize the sawtooth instability. This may provide an essential tool for avoiding neo-classical tearing modes in a tokamak reactor.

Results from fast wave electron current drive (FWCD) and heating experiments in plasmas with internal transport barriers have been presented. In the experiments the single pass damping was low and parasitic power losses were evident. The beryllium line radiation gives an indication that rectified RF sheath potentials are contributing to the parasitic losses.
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