Probability of failure in Turbine Exhaust Cases subjected to crack propagation

Sannolikheten för brott i Turbine Exhaust Cases utsatta för sprickpropagering

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Abstract
In the aviation industry the most important factor is safety. GKN are producing the Turbine Exhaust Case (TEC) which is a component in an airplane engine. The welds in an airplane engine are a risk for a breakdown because of fatigue failure due to crack propagation. Since the number of cracks and the size of them are unknown the life expectancy of a TEC is unknown. Instead using deterministic models when calculating the life expectancy one another way is to do the calculations with a probabilistic model. The random variables in the probabilistic model are the number of cracks, size and position of each crack, the variation in materials and the offset between the plates that are welded. By running a Monte-Carlo simulation the probability of failure can be estimated for a specific number of flights. The simulation is validated against a known theoretical case to prove that the method is valid. The aim with the thesis is to have a fast process for as many simulations as possible but the work process is shown to be too slow due to the program that does the crack propagation calculations.
**Sammanfattning**

Nomenclature list

TEC  Turbine Exhaust Case
OEM  Original Equipment manufacturer
EWB  Engineering Work Bench
NDT  Non-destructive testing
K    Stress intensity factor
σ    Stress
a    Crack size
N    Number of cycles
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1. BACKGROUND

In the aviation industry the most important factor is safety. If a failure would occur while flying midair the chances that the accident is fatal is high. Having a record of safe trips assures safety and it would encourage people to travel by air rather than other modes of travel.

The company GKN Aerospace Sweden is co-operating with the OEM aircraft engine companies Rolls-Royce, Pratt & Whitney, Snecma and General Electric for civil aircraft engines. GKN is making components in engines and has now components in over 90% of all new bigger aircrafts all across the world [1].

One of the components that GKN are producing is the Turbine Exhaust Case (TEC). In order to optimize the TEC GKN has developed a method called Engineering Work Bench (EWB). The principle behind EWB is to use variations of a standard TEC and then evaluate the best one. The TEC is modeled in the finite element program ANSYS and from a standard model the variations in TEC geometries are generated. An example of a variation of a TEC model is shown in appendix A. The TEC is the middle part of the structure showing the struts. The variations could be different dimensions, weld locations, number of struts and their angles etc. Due to the variations the different models will have different loads, geometry etc. For producing a TEC it needs to pass the requirements from production, aerodynamics and mechanics. The main cause of the EWB is to see if the model first passes the requirements and then there is an evaluation process of which TEC is the most optimal. Because of the vast information that EWB needs to handle in order to grade a TEC there is a need for making the process as automatic as possible. Three different TEC models can be seen in appendix B, C and D.

The TEC is assembled by separate castings, forgings and other material forms. The pieces are joined by welding. Common phenomena in welds are cracks and due to cracks which limits the useful life. In EWB, one important part in the mechanical requirements is the evaluation of the fatigue life.

2. Aim of thesis

The aim with the thesis is to replace a deterministic way of calculating the fatigue life of a TEC with a probabilistic way. Instead of calculating the worst case scenario of fatigue failure, another way is to calculate the probability of fatigue failure. A probabilistic analysis offers the possibility of relaxing the deterministic assumptions and possibly predicts a longer useful life. By validating the method against a theoretical case it is possible to confirm that the method is acceptable.
3. THEORY

3.1 Introduction

Two test samples that by the naked eye look identical show a great difference in fatigue life. This may be due to differences in defects in the sample, the size and quantity of them. With non-destructive tests (NDT) one can measure the size and the quantity of the cracks. When dimensioning a TEC one has to do assumptions. In a deterministic model the assumption for crack size is the largest crack size one risk to miss in a NDT inspection. So if a crack was found that has a smaller size than the assumed crack size the smaller crack would theoretical be safe. If one measures as many cracks possible that are present in the weld one can create a model for the assumed crack size. The most optimal way would be to have detailed information in each individual i.e. crack sizes, geometry and analyze each detail with its properties. This is not possible so one has to use models for the input to the calculation models i.e. distribution functions. So in order to get a probabilistic model one has to collect data by measure as many cracks as possible and try to fit a theoretical model. It is not possible to know which detail that has which defect but it is known that the defects are connected to a theoretical model. With the probabilistic model the aim is to investigate how many combination one get with crack sizes, bad material etc. The aim is to determine how many % of these would not pass the requirements for usage. One can also identify what one can do to increase for instance the fatigue life in order to lower the probability of failure. When it is not possible to affect the crack sizes one can for instance change the geometry to lower the stress levels.

In figure 1 one can see the relationship between difference in initial crack size and difference in life time in terms of number of cycles.

![Figure 1: The uncertainties in initial crack size gives a variation in calculated life.](image-url)
In this thesis it is assumed that the crack sizes follows an exponential distribution curve and the assumption is based on that it is more logical to have a higher number of small cracks than larger cracks and that the quantity of different sizes follows an exponential curve.

So it is stated that one of the reasons of the uncertainty of the lifetime in number of cycles is the uncertainty in the initial crack size. There are other uncertainties as well e.g. geometry, material etc. One way to quantify or observe the uncertainty is by using random variables as input to a lifing model with deterministic base equations.
3.2 Crack propagation

3.2.1 Stress intensity factor
One useful parameter in fracture mechanics is the stress intensity factor. The stress intensity factor describes the stress state at a crack tip. Irwin’s paper shows that the stress intensity factor is a function of the stress and crack size, see equation 1. The critical stress intensity factor \( K_{IC} \) is a material property. For LEFM (Linear Elastic Fracture Mechanics) failure will occur when the stress intensity factor has reached the critical stress intensity factor, seen in equation 2 [2].

\[
K = f(\sigma, a) \quad \quad 1)
\]
\[
K \geq K_{IC} \quad \quad 2)
\]

The stress intensity factor can be described in three independent modes. The three modes can be seen in figure 2. The first mode is opening (tensile), the second mode is the in-plane shearing or sliding mode and the third mode is the tearing or anti-plane shear mode. Mode I will be assumed in this work.

![Figure 2: The stress intensity has three different modes, the first mode is the most common [3].](image-url)
3.2.2 Crack growth

The fatigue crack growth curve can be divided into three regimes and this can be seen in figure 3. The first regime is at low stress intensity (ΔK) levels, and it is around the threshold level ΔK\text{th}. Load cycles below the threshold level gives no crack growth. The threshold is defined as da/dN < 10^{-9} m/cycle [4]. The second regime is where the crack growth is stable. The third regime is where the crack growth rate accelerates and K approaches the fracture toughness as seen in equation 2 and ends in a fracture [3].

**Figure 3:** The three regions in crack growth, the second region is the important region if crack growth needs to be controlled [3].

3.2.2.1 Region I
The first region is near the threshold region and indicates a threshold value ΔK\text{th}, below this value no crack growth occurs. Microstructure of materials, mean stress and environment mainly control the first region [3].

3.2.2.2 Region II
In the second region the crack growth rate can be described with help of Paris law and can be seen in equation 3. Since the curve is linear so is the crack growth more predictable than the other regions. In the equation 3 the term on the left is the crack growth rate, i.e. the increasing crack length for each load cycle. Parameters C and n in equation 3 are material constants. ΔK is the stress intensity factor range. It is important to note that the most common trend for the
crack growth rate is that it increases with increasing crack length if the stress remains constant.

\[
\frac{da}{dN} = C\Delta K^n
\]  

3) The stress intensity factor range for a through crack in a large plate can be seen in equation 4 where \(\Delta \sigma\) is the stress range \((\sigma_{\text{max}} - \sigma_{\text{min}})\).

\[
\Delta K = \Delta \sigma \sqrt{\pi a}
\]  

4) By inserting equation 4 in 3 and integrating with the starting crack size \((a_0)\) and final crack size \((a_N)\) one arrives at the equation 5.

\[
a_N = \left[ \frac{2-n}{2} + \left( \frac{2-n}{2} \right) NC \left( \Delta \sigma \sqrt{\pi} \right) \right]^{\frac{2}{2-n}}
\]  

5) As described, the region II corresponds to a more stable macroscopic crack growth. The fatigue crack growth behavior in this region is less dependent on the microstructure and mean stress than the first region [2-3].

3.2.2.3 Region III
The crack growth rates are very high now when they approach instability. The rate is now so high that only a little part of the total life will be spent in this region. The region is primarily controlled by the fracture toughness [3].

3.2.3 Mean stress effect
The fatigue crack behavior is typically influenced by the mean stress. The mean stress is often expressed by the stress ratio \(R\) and can be seen in equation 6.

\[
R = \frac{K_{\text{min}}}{K_{\text{max}}}
\]  

6) With increasing positive R-ratio the threshold will be lowered and the crack growth at a smaller stress range. A bigger R-ratio will lower the threshold more than a smaller R-ratio. This can be seen in figure 4. A negative R-ratio will have the opposite effect, it will increase the threshold and delay the start of the crack growth. [3,5]
3.2.4 SN-curve and Palmgren-Miners cumulative damage law

S-N diagram or Wohler diagram is a plot of stress versus the number of cycles to reach failure and an example can be seen in figure 5. It is a tool to predict when fatigue failure will occur dependent on the cyclic stress amplitude. The curve is achieved by testing samples with different stresses and recording the number of cycles when failure occurs, different materials and geometries have different curves. At low stress range there is no failure because the stresses do not exceed the fatigue threshold, this is called fatigue limit.

When a structure is subjected to a range of stress amplitudes one way to calculate the number of cycles to failure is the Palmgren-Miners cumulative damage law and can be seen in equation 7. It is assumed that $D = 1$ results in failure. For each stress amplitude $\sigma_i$ there is certain number of cycles $N_i$ when failure occurs. If it only has a number of cycles $n_i$ which is below $N_i$ failure will not occur but there will be some damage. By adding these damages it results in a total damage, an example of this can be seen in figure 6 [2,6].

\[
D = \sum_{i=1}^{k} \frac{n_i}{N_i} = 1 \quad 7)
\]
3.2.5 Rainflow count

The rainflow counting method is a commonly used in fatigue load spectrum where the aim is to reduce the spectrum of varying stresses into a set of simple stress reversals. Together with the Palmgren-Miners rule the advantage is that it can be used as a tool to simplify the calculations of the fatigue life when the loading is complex [2].
3.3 Analysis model of cracks in welds

Now all the needed information is explained in order to do a calculation of the fatigue life in a weld. Two plates joined by a weld can be seen in figure 7. A crack with the size $a_0$ is found in the welds and the plates are subjected to a cyclic stress.

![Figure 7: Two plates joined by a weld, a crack exist in the weld.](image)

As described in section 3.2.2.1, if the stress intensity exceeds the threshold for the stress intensity then crack will grow which. In Region II the crack will grow according to Paris law which can be seen in equation 5. If the cyclic stress remains the same and the crack grows the stress intensity will approach the critical stress intensity.

As described in equation 1 the stress intensity factor is a function of the stress range and the crack size but if the cyclic stress remains the same it will only be a function of the crack size. According to equation 2 it will be fatigue failure when the stress intensity factor exceeds the critical stress intensity factor. Since the stress intensity factor is a function of the crack size it means that it will be failure when the crack size exceeds the critical crack size, this is shown in equation 8. The critical crack size can be achieved by solving the equation 4 for crack size. The critical crack size can be seen in equation 9.

\[
\text{Failure: } a_N > a_c \quad 8)
\]

\[
a_c = \frac{1}{\pi} \left( \frac{K_{IC}}{\sigma_{\text{max}}} \right)^2 \quad 9)
\]
4. THE PROBABILISTIC MODEL
The random variables that were used in this thesis were:

- Initial crack size
- Crack position
- Number of cracks
- The variation in material
- Geometry

This section will describe the respective model for each random variable in order to achieve a probabilistic model to calculate the probability of failure.

4.1 Probability of failure of existing crack
The first step to calculate the probability of failure is to calculate the probability of failure if a crack exists. As can be seen in equation 5 the final crack size, \( a_N \), is a function of the initial crack size, \( a_0 \). As described earlier the initial crack size is modeled by an exponential distribution function and can be seen in equation 10 [7].

\[
f_a(a_0) = \frac{1}{\bar{a}} \exp \left( - \frac{a_0}{\bar{a}} \right)
\]  

In the equation 8, \( \bar{a} \) is the average defect size and is the only parameter in the exponential distribution.

In order to determine the number of load cycles allowed until the crack size has reached its maximum value a failure criterion is introduced and it can be seen in equation 11.

\[
d = a_N - a_c
\]  

In equation 11, \( a_c \) is the critical crack size and \( a_c \) is the crack size after \( N \) load cycles. When \( d \) is zero or positive the crack size will have reached the critical crack size and failure will occur. Assuming Linear Elastic Fracture Mechanics (LEFM) is applicable and assuming a through crack in an infinite plate as an example, the following equations can be derived. Starting with equation 12 where the critical crack size is shown. In figure 8 a tough crack in an infinite plate is shown.

\[
\sigma
\]

\[
2c
\]

\[
\sigma
\]

Figure 8: A through crack in a center of an infinite plate, \( a = 2c \).
\[
K_{IC} = \sigma \sqrt{m_c} \rightarrow a_c = \frac{1}{\pi} \left( \frac{K_{IC}}{\sigma} \right)^2
\]

In equation 12, \(K_{IC}\) is the fracture toughness and \(\sigma\) is the maximum stress. The probability of no failure can be seen in equation 13.

\[
P(d < 0) = P(a_N - a_c < 0) = P(a_N < a_c)
\]

For a specific stress and number of cycles the final crack length can symbolically be written as a function of initial defect size. By inserting that function into equation 13 that results in equation 14.

\[
P(a_N < a_c) = P(g(a_0) < a_c) = P(a_0 < g^{-1}(a_c))
\]

By inserting equation 10 into equation 14 one gets an expression for the probability of surviving \(N\) cycles at a certain load range \(\Delta \sigma\), this result in equation 15.

\[
P \left( a_0 < \left( a_c^2 - \gamma N \right)^{2-n} \right) = \int_0^{\left( a_c^2 - \gamma N \right)^{2-n}} \frac{1}{\bar{a}} \exp \left( -\frac{a_0}{\bar{a}} \right) da_0
\]

In equation 15, \(\gamma = C \left( \frac{2-n}{2} \right) \left( \Delta \sigma \sqrt{\pi} \right)^n\). By solving the integral in equation 15 one obtains the probability of surviving at a certain number of load cycles \(N\) and this can be seen in equation 16.

\[
P(d < 0) = P(a_N - a_c < 0) = P(a_N < a_c) = 1 - \exp \left( -\frac{\left( a_c^2 - \gamma N \right)^{2-n}}{\bar{a}} \right)
\]

When the probability of surviving is known one can also calculate the probability of failure at the same number of load cycles, this can be seen in equation 17.

\[
P_f = 1 - P(d < 0)
\]
4.2 Probability of occurrence of crack
By inspecting several test samples at the same location one will see that in some test samples cracks occur and in some samples there will be no cracks. So it is not possible to assume that cracks are always present, only occasionally. The position of crack in weld is also important shown in figure 9.

![Diagram showing tensile and compressive stress with welding profile, crack size and location](image)

Figure 9: Crack size and location influence the life of the weld, the location is important due to the stresses.

In figure 9 the crack size and crack location are random and as it can be seen there are different stress profiles. In stress profile 2 the location of the crack would not matter at all because the load is same over whole profile. On the other hand, profile 3 it has a tensile stress peak and a crack near that location would have a higher risk of contributing to failure than the rest of the profile. For a deterministic analysis the crack growth life would be the same for all three stress profiles when investigating at the same stress. The crack size after a certain number of cycles in equation 5 is a function of the stress.

The probability of failure with initial the crack size is already explained but the distribution of number of cracks and their position need explanation. Therefore another distribution is needed to model the occurrence of cracks. So, in summary, the functions that will be combined are:

- Distribution function to model the crack size [mm]
- Distribution function to model the occurrence of cracks [number of cracks / meter weld]

So the failure of a weld is the combination of the distribution functions for crack sizes and occurrence, it can be seen in equation 18 and means that the probability of failure given that a crack exists multiplied by the probability that the crack actuality exists.
\[ P_f = P(d > 0 | d)P(d) \]

It is assumed that the cracks appear randomly along the weld, so a good distribution of the quantity of cracks is the Poisson distribution. The Poisson distribution is useful when events occur randomly in time or space at a known average rate and independently of each other, this can be seen in equation 19 [8].

\[ P_x(k) = \exp(-\nu_o x) \frac{(\nu_o x)^k}{k!} \]

In the equation 19, \( \nu x \) is the intensity and \( P \) is the probability of having \( k \) events. In this case the frequency of events is the frequency of cracks where \( \nu \) is the number of cracks per meter weld and \( x \) is the weld length in meter.

In the Poisson distribution the probability of no events or no cracks (\( k=0 \)) can be calculated and this can be seen in equation 20.

\[ P(k = 0) = \exp(-\nu_o x) \frac{(\nu_o x)^0}{0!} = \exp(-\nu_o x) \]

Therefore the probability of at least one event or crack can now be calculated and can be seen in equation 21.

\[ P(k > 0) = 1 - \exp(-\nu_o x) \]

Next step is to combine the probability of having a crack with the probability of having a failure. Because of the randomness in crack location and the stress profile the argument in equation 19 is replaced by equation 22.

\[ \nu(x) = \int \nu_o P(d(x) > 0) dx \]

The equation 22 is the total probability of failure for position \( x \) and by inserting equation 22 back into equation 21 the probability of failure for the weld is obtained, this can be seen in equation 23.

\[ P = 1 - \exp(-\int \nu_o P(d > 0) dx) \]

The probability of failure at each \( x \)-coordinate can be seen in equation 17 and by inserting that equation in equation 23 one arrives at the probability of at least one failure for the weld and this can be seen in equation 24.

\[ P(\text{at least one failure}) = 1 - \exp \left( -\int_0^L \nu_o P \left[ \frac{\left( \frac{2-n}{a_c^2} - \gamma N \right)^{\frac{2-n}{2}}}{\bar{a}} \right] dx \right) \]

The parameter \( \gamma = C \left( \frac{2-n}{2} \right) \left( \Delta \sigma(x) \sqrt{\pi} \right)^n \) includes the stress that varies along the weld.
4.3 Geometry

When two sheets are welded together it is likely that there will be an offset between the centerlines of the two sheets. The offset will introduce an additional bending momentum in the structure, this can be seen in force equilibrium in figure 10 and in equations 25-27. The offset varies along the weld. This is a problem when welding relatively thin sheets, if the sheet had a thickness much larger than the offset it would not be a problem but unfortunately in this case it is a problem that needs to be considered.

![Figure 10: The offset between the plates will add a bending momentum.](image)

The load $P_1$ and momentum $M_1$ are assumed to be known together with the thickness $t_1$, $t_2$ and offset $e$ then one can calculate the stresses as can be seen in equations 25-28.

$$P_2 - P_1 = 0$$  \hspace{1cm} 25) \\
$$M_2 - M_1 - P_2 e = 0$$  \hspace{1cm} 26) \\
$$\sigma_1 = \frac{P_1}{A_1} + \frac{M_1 z}{I_1}$$  \hspace{1cm} 27) \\
$$\sigma_2 = \frac{P_1}{A_2} + \frac{(M_1 + P_1 e) z}{I_2} = \frac{P_1}{A_2} + \frac{M_1 z}{I_2} + \frac{P_1 e}{I_2}$$  \hspace{1cm} 28)

Introduce the equations for moment of inertia, membrane stress and bending stress which can be seen in equations 29, 30, 31 [9].

$$I = \frac{B t^3}{12}$$  \hspace{1cm} 29) \\
$$\sigma_m = \frac{P}{A}$$  \hspace{1cm} 30) \\
$$\sigma_b = \frac{M_{z_{\text{max}}}}{I}$$  \hspace{1cm} 31)

By combining the equation 28 with equations 29, 30, 31 one gets equation 32.

$$\sigma_2 = \sigma_m + \sigma_b + 6\sigma_m \frac{e}{t}$$  \hspace{1cm} 32)

The offset cannot be of infinite size due to the geometry and tolerances. The offset is assumed to have a uniform distribution between a lower and upper limit. The lower and upper limit could be for instance -0.5 mm and +0.5 mm offset between the centerlines.
4.4 Variation in material

Another uncertainty in fatigue life calculations is the variation in material. Due to the material samples may have different defects, the heat treatment may affect the samples different or how different people measure the results etc, there will always be some sort of error.

The more samples that are examined the closer one can anticipate how the material will behave. In the fatigue life calculations the crack growth rate parameter \( \frac{da}{dN} \) needs to be examined for each material in different temperatures. So in order to do good calculations this parameter needs to be close to reality.

The crack growth rate for a material in a certain temperature is tested by investigating the crack growth rate for three different R-ratios. For each R-ratio, four samples are used therefore there are twelve samples in total. Due to limitations in time and economy more samples are not investigated. For each R-ratio that is tested the test will stop at a certain crack size to see how many cycles it took for the sample to arrive at that crack size. In figure 11 three different R-ratios is shown.

![Figure 11: Three different R-ratios are tested four times each.](image)

Test results from each R-ratio are then fit into the NASGRO equation for crack growth rate and can be seen in equation 33. The NASGRO crack growth equation is a complicated equation that accounts the R-ratio, the near-threshold and near-instability regions with many parameters. This equation will serve as a model for the crack growth rate [10].

\[
\frac{da}{dN} = C \left[ \left( \frac{1-f}{1-R} \right) \Delta K \right]^n \left( \frac{\Delta K_{th}}{\Delta K} \right)^p \left( 1 - \frac{K_{max}}{K_c} \right)^q
\]  

33)

By putting the equation 33 and the measured samples in the same figure the difference for each sample in terms of crack growth rate is visualized clearly, this can be seen in figure 12.
The NASGRO equation is fit to the test data using the least square method. The comparison between the integrated equation 33 and measured variation in life at a certain crack size can be seen in figure 13.

\[
\log(N) = \log(\bar{N}) + \varepsilon
\]

From the equation 34 the error can now be calculated and this can be seen in equation 35). The error is a sum of multiple errors and they could be for instance error in material, sample or measurement.

\[
\log(N) - \log(\bar{N}) = \varepsilon \Leftrightarrow \log\left(\frac{N}{\bar{N}}\right) = \varepsilon
\]
5. METHOD

5.1 Monte-Carlo Simulation
Due to the random variables like number of cracks, crack size, crack position, offset and variation in materials one approach is using the Monte-Carlo simulation. Monte Carlo is a method to simulate mathematical systems which can have several variables that cannot be solved with integrals. Since the simulation method is used by random variables, a more general case can be described by doing multiple simulations so the more simulations that provides results the more accurate gets the method [11]. In equation 36 one can calculate how many simulations the Monte-Carlo simulation needs to run in order to get a close to exact answer. In the equation 36, n is the number of simulations, Pf is the probability of failure and ε is the error [12].

\[
\frac{1}{n} \geq \frac{Pf - 1}{\varepsilon^2}
\]

So for an example, if the parameters were; Pf = 0.001 and ε = 0.01 then the number of simulations would be, n = 9990000. So in order to get exact results it would need 9990000 simulations with 1% error at the probability of failure 0.1%.

5.2 Flowchart of the process
A summary for the preparatory work and Monte-Carlo simulation is shown in figure 14. Over the dashed line it is the preparatory work and below the dashed line it is the Monte-Carlo simulation.

![Flowchart](image)

The preparatory work starts with read the model file of the TEC in ANSYS. For the nodes that represent the welds the program Align_stress was run to get the stresses in each node. The program RFC was run to get the load cycles. The program FASTGRO was run to do simple life calculations to see which nodes that are vulnerable to crack propagation and a MATLAB code

Figure 14: Flowchart of the process, above the dashed line is the preparatory work and below is the MC-simulation.
called GetWelds.m was then run to collect all the vulnerable nodes and see which welds that were vulnerable and then print out files with the node coordinates for the vulnerable welds.

The MC-simulation was divided into three steps. The first step was a MATLAB code called CreateCracks.m that read an input file of how many simulations and welds that will be investigated. The code then reads the coordinates and calculates the weld length. Then the code generates how many cracks that are expected on the weld length. For each generated crack it generates a size and position on the weld length. Then the code added the offset between the welded plates in the load files achieved from RFC. Lastly it created an input file for the program NASGRO® which calculates the life for each crack. In the input file it is the crack size and load file that varies for each crack.

The second step was the crack propagation calculations for each crack in the program NASGRO. The third and last step was a MATLAB code called ReadResults.m that reads all the results file from NASGRO. For each life result the variation in material was added. Then it loops for all the number of cycles that were investigated and the probability of failure for that number of cycles were calculated. The code ends with plotting the probability of failure against all the number of cycles that was requested by the user.

5.3 Programs

The programs that are used in the preparatory work for the Monte-Carlo simulation and programs used in the simulation will be described in this part of the thesis.

5.3.1 ANSYS

ANSYS Mechanical APDL is a commercial program used for finite element analysis. The only input is the model file of the TEC that will be evaluated.

5.3.2 Align_stress

Align_stress is a pre-processor created by GKN that reads a stress and temperature input file (.cns) from ANSYS. The .cns file contains information about stresses and temperatures for different time steps. For node the program will calculate stresses in a (by user defined) direction. Typically are stresses perpendicular to a weld sought for. For each selected node and creates an output file with the stress in a defined direction.

Input information

- What stress direction that the program should transfer to from the global coordinate system, the stress normal to the weld line is the relevant stress so the input when the program asks for the stress type the answer in this thesis is NORMAL.
- If it is a shell or a solid model, the model that is used in this thesis is a shell model.
- The node numbers along the weld together with the node number of the node next to the node along the weld. The program will take those two nodes together with next node number along the weld to create a plane, the normal to this plane is what is sought for.
• Input for “.cns files” are the stresses etc, for a shell model two .cns files are needed, one for the top stress and one for the bottom stress because the shell model has both stresses at the same node.
• The .cns files and the node numbers do not include the node coordinates and these are needed to get the direction of the weld. An extra file is needed for these coordinates.
• Next input file is the name for the output file.
• Last input file is the name for the output file that is formatted for a RFC-program.

Output information

The output information will be the stresses in a defined direction and one of the files will be for the RFC-program. A output file can be seen in appendix E.

5.3.3 FASTGRO
FASTGRO is a program created by GKN and is a simplified life calculation. The program receives the input data of the stresses that are the output from Align_stress and a material data file. The material data file contains SN-curves for multiple temperatures and stresses, see chapter 3.2.4. So the program takes the stresses from the stress file and interpolates the cycle to failure with help of the SN-curves in the material data file.

5.3.4 RFC
RFC is a program created by GKN that does a Rain Flow Count on the stress data. As described in the theory section a rain flow count is needed in order to translate an arbitrary stress vs. time history into load cycles. The program will start with help of flags and in this case three flags are needed; name of the input file from Align_stress, name of the output file and what type of sorting. Four types of sorting exist and in this case it is number four, all the types can be seen below.

• -s1 Sort on mean stress divided by stress range
• -s2 Sort on bending stress divided by membrane stress
• -s3 Multiple nodes and sort on mean stress divided by stress range
• -s4 Multiple nodes and bending stress divided by membrane stress

5.3.5 NASGRO (NASFLA)
NASGRO is a commercial program that does crack growth calculations. By choosing geometry, material, loads the program then calculates the number of load cycles until failure. For the program to do the calculations each crack must have its own input file with the relevant data. NASGRO use the output from RFC files as input for the calculations.

5.3.6 MATLAB
MATLAB is a program commonly used for numerical computations. In MATLAB the random variables are generated and create the input files for NASGRO for each crack that is generated. It is also used to read the output files to calculate the probability of failure.
5.4 Work process

The work process for an evaluation of a TEC can be divided into two steps. The first step was the preparatory work of the simulation and the second step was the simulation.

5.4.1 Preparatory work

The information one needed was the node numbers with their stresses, temperatures and coordinates. The coordinates were important for two reasons; firstly, the coordinates were needed so that a direction could be defined in order use the program Align_stress and secondly, so that the total weld length could be calculated which was needed later for the simulation. The stresses for each temperature in every node were also needed in order for NASGRO to do the crack growth calculations. A material data file is needed for the FASTGRO calculations. The files that are provided are:

- A .rst file which contained the model of the TEC created in the ANSYS program.
- Text file with the node coordinates for the weld
- Text file with the weld directions for each node
- Material data file
- Input data files for Align_stress and FASTGRO

The preparatory work started with opening the model in ANSYS. The next step was to run the programs Align_stress and FASTGRO. With the program Align_stress the stresses in in the normal direction were defined. With the output file from Align_stress the program FASTGRO did the calculations of life expectancy for all the nodes in the welds.

The reasons to do the FASTGRO calculations were to save time and not do unnecessary work which means that NASGRO does not do life calculations for nodes that are safe. The output file with the results from the FASTGRO calculations is long so a short draft of one can be seen in appendix F, where the first column are the node number and the third column are the estimated number of cycles when failure was reached. As can be seen many of the lifetimes are 1E30 number of cycles and do crack propagation investigation in those nodes is unnecessary work and time consuming. So all welds under a certain limit should be investigated, this is illustrated in figure 15 where the life is plotted against the weld length. \( N_c \) is the critical number of cycles, below this limit the node is vulnerable.
The next step was to run the developed MATLAB code called GetWelds. The code starts with reading an input file which contains all the names of the files it should read in and the requested random variables used in the simulation. The code reads the FASTGRO output file. The file with all the nodes and which node belongs to which weld were also read and a small part of that file can be seen in appendix G. The code then identified the vulnerable welds and for each weld it created a file for the node coordinates in the weld. A file of a weld with its nodes and their coordinates can be seen in appendix H. Other output files are the files WELDDATA and RANDOMVARIABLES which can be seen in appendix I. WELDDATA contains the information of how many simulations and welds. RANDOMVARIABLES contains information of the parameters for the random variables.

Next step was to run the RFC program, which did the rain flow count of the stresses that were achieved in Align_stress and the output was a stress file for each node. Since all the RFC files were not used as some were for the welds that were not vulnerable to failure therefore it was wise to manually delete the irrelevant ones to save some memory on the storage.

The information that was obtained from this step was which of the welds are vulnerable to failure. Also, the node numbers of welds with their coordinates and stress acting on them were also found out.

5.4.2 Simulation
The simulation comprised of three steps:

1. Generated cracks along the welds and created NASGRO input files for all cracks.
2. Run NASGRO for all cracks to calculate the number of cycles to reach failure.
3. Read the results from all the NASGRO output files and calculated the probability of failure.

5.4.2.1 First step
The MATLAB code CreateCracks.m was written to produce a crack size and a crack location in order to do the NASGRO calculations. The code reads the user provided input file called WELDDATA.dat. This file contains information about how many simulations that it is requested and how many welds it was going to analyze inside each simulation. It also reads a
file called RANDOMVARIABLES which contains the parameters for the parameters for the distribution functions.

Next step in the weld analysis was to read the node coordinates. The distance between the nodes was calculated and then added together so the whole weld length was known. With the Poissons distribution the number of cracks was generated. The code read how many cracks that were generated and started a loop so that for each crack a size was generated with exponential distribution and a position. The position was a random number between 0-1 times the weld length so its position was random along the weld. The assumption was made that the distance between the nodes was so small that the stresses between two nodes were relatively close so an interpolation of stresses is not needed. Therefore for each crack the code calculated which node was the closest from its given position.

The code then reads a template NASGRO input file, which can be seen in appendix L, and changed the crack size and name of stress file. Then it writes a new NASGRO input file with this information with the name Weld_Y_Crack_Z_X. In the name, X was which simulation, Y was which weld and Z was which crack in order to track them. The next step was to read the stress files from the RFC program. The offset varies along the weld so it was generated by a random number between a given lower and upper limit. Since the crack was given a position it knew which load file to read, it read the load file for the same node it was given as its position. The RFC files were generated with the name rfc_out_node_sidenode_Y_X with X being the simulation number and Y the weld number. After this the loops ended.

For each crack that has been generated, a row has been added in a matrix with the information about the number of simulation, weld and crack. The last step was that a file called Weld_Crack.dat was created with this information so that the last code that read the results knew which files to read.

A flowchart for the code can be seen in appendix M.

5.4.2.2 Second part
The program NASGRO calculates the number of cycles it took to reach fatigue failure. The program calculated one crack at a time and after each calculation it created an output file for the crack which included the number of cycles until fatigue failure.

5.4.2.3 Third part
The code ReadResults.m was created with the purpose to read the result files from NASGRO and then calculate the probability of failure. The code started with reading in Weld_Cracks.dat with the reason that then it knew which results files were the relevant ones. The code then created a file for the future failures. The code then looped over all simulations and then started read in all cracks and checked how many number of cycles it took for the crack to reach its critical crack size. The error in variation in material was added to the life i.e. number of cycles by a log-normal distribution, see equation 35. For each failure the crack was printed onto the file that was created in the beginning of the code with the information of the number of cycle it took to reach failure with the error, which variation in material that was added and which simulation it belongs to. After all cracks had been investigated the loop ended. The code then read the file with all failures and started a loop with the number of cycles that should
be investigated. The code than for each simulation read the failed cracks for each simulation and saw if the simulation passed the investigated number of cycles. If it did not pass the limit it added a value on the variable k, so k=k+1 if no simulation has had a failure at the investigated number of cycle then k=k+0. When all simulations had been checked the equation 37 was used and can be seen below, which for each number of cycle calculates the probability of failure. Then the loop ended and a plot can be seen of the probability of failure against the number of cycles.

\[
\text{Probability of failure} = \frac{\text{number of failures}(k)}{\text{number of simulations}}
\]

A flowchart for the code can be seen in appendix N.
6. CODE VALIDATION

6.1 Method

In order to see if the Monte-Carlo simulation method is acceptable it needs to be tested against a known case. If the simulation method shows results close to the known case then it can be concluded that the method is acceptable. The input for the theoretical case and the working method is the same in order to solve the same problem.

The known case was using the equation 24 with a simple geometry without variation in offset or material. With help of MATLAB the equation 24 can be solved by using trapezoidal numerical integration. The simple geometry was a big plate with flanges made in ANSYS and can be seen in figure 16 and the stress input was taken from the path where the weld was supposed to be. The weld was located in the middle of the plate from one side to the other. The model is constrained in the bottom. On the top of the model a tension is applied and on the corners compression is added. This is done in order to vary the stress through the stress profile along the weld. In order to prove that the simulation was an acceptable method compared to the equation 24 the loads, geometry and material needed to be identical. The parameters were $C = 1e^{-12}$, $n = 3$ and $\lambda = 0.01$ cracks/meter weld.

The simulation was run 5 times with 200 simulations in each run. This means a total of 1000 simulations.
6.2 Results
The results for each numbers of cycles can be seen in table 1 and for better visualization it can also be seen in figure 17. The axes in the figures are probability of failure for all the simulations against number of cycles. One can see that the results from the analytical method and the simulations are quite close and it could be arranged that the simulation method is acceptable.

Table 1: Comparison between the theoretical results to the results retrieved from the MC-simulations

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In figure 18 the theoretical equation is compared to the average of the simulations. One can see that they are substantially aligned from start to finish. This visualization gives a clear evidence that the method can be concluded to be acceptable and that 1000 simulations is enough to validate the method.

Figure 17: Comparison between the theoretical results to the results retrieved from the MC-simulations.
Figure 18: Comparison between the theoretical result to the average of the MC-simulations, and as can be seen the lines are aligned and therefore it can be concluded that the method is acceptable.
7. EXAMPLE SIMULATION ON A TEC

7.1 Method

One random TEC from EWB was investigated. The TEC investigated can be seen in appendix C.

The investigation of a TEC was divided into three steps. In the first step there will only an investigation of random variables i.e. number of cracks, size and position for each crack. The next step the variation in materials was added for each crack. The third step the random offset was added for each crack. That was done to see how the random variables affect the probability of failure.

Each investigation was made 10 times with 4000 simulations each which gave a total of 40000 simulations. The parameters that were used for the cracks are $\lambda = 1$ crack per meter weld and average crack size $\bar{a} = 0.3$ mm. The mean deviation in the variation in materials was 0 % and the standard deviation was 25%. The offset lower and upper limits were -0.5 mm and 0.5 mm.

Since the variation in material was added in the result reading code it was tested 10 times for the same results files to see how much it varied. The offset cannot be varied in the same way, but one simulation of a different offset was tested just to see how the probability of failure trend changed with changed offset. An increase in number of cracks per meter weld was also be investigated in order to see if the probability of failure trend changed if the numbers of cracks are increased, this was done in the first step.

When there was a crack present it always lead to fatigue failure at some point if the stresses are high enough. Since all the cracks are in the vulnerable all the cracks that are investigated reached failure. It was tested for the different steps when the last crack reaches failure. So the code was modified a little bit just for this test. Instead of using the equation 37 the equation 38 will be used and can be seen below.

$$Ratio\ of\ cracks\ failed = \frac{number\ of\ failures(k)}{number\ of\ cracks}$$  \hspace{1cm} 38)

When it has reached 1 it means that all cracks have reached failure. By adding the random parameters it should take longer time for each step. The first and second step can be compared to each other in the same figure because they share the same number of cracks since the test can be done in same simulation, just to run the “ReadResults.m” once with the variation in materials and one without. The offset cannot since it has different load files but the trend can be compared between the steps.
7.2 Results

7.2.1 Preparatory work
In the preparatory work for the chosen TEC there were 114 welds that were calculated in FASTGRO and 6 of them showed at least one node in the danger of crack fatigue within the limit of one million cycles. The welds in danger of fatigue failure were between two and four nodes long.

7.2.2 Random number of cracks, size and position
In figure 19 the results from the random distribution in how many cracks and for each crack its size and position. The axes in the figures are probability of failure for the TEC against number of cycles. As can be seen in figure 19 the variation in probability of failure between the simulations increased at the higher number of cycles.

![Graph](image-url)  
Figure 19: Probability of failure plotted against number of cycles obtained from ten separate MC-simulations together with the average curve of these ten simulations.
7.2.3 Variation in materials

In figure 20 the variation in material is added to each crack. The axes in the figures are probability of failure for all the simulations against number of cycles. Compared to the step one the spread in probability of failure is a little bigger at the low number of cycles but a little lower at the higher number of cycles.

![Graph showing probability of failure against number of cycles with variation in materials added. The graph includes axes for probability of failure and number of cycles, with each simulation represented by a different colored line, and the average curve also shown.](image-url)

*Figure 20: Probability of failure plotted against number of cycles obtained from ten separate simulations together with the average curve when the variation in materials is added.*
7.2.4 How variation in materials affects the results

Comparison between first and second step can be seen in figure 21 and figure 22. Where step one was the investigation of random number of cracks and for each crack random size and position. The second step was the investigation where the variation of materials was added to the first step. The axes in the figures are probability of failure for all the simulations against number of cycles. In figure 21 the original curve is from the first step and the other ten are from the second step where the effect of variation in material was added. So the difference between the two steps can be visualized. Comparison in figure 22 is the average of the variation in material to the original curve and it can be seen how the variation of the material affects the probability of failure. One can see that for low number of cycles the probability of failure increases when the variation in material was added. But at high number of cycles the probability of failure decreases when the variation in material was added.

![Figure 21](image-url)  
*Figure 21: Comparison between the simulation results with and without the effect of variation in materials.*
7.2.5 Add offset
The variation in offset is added and the results can be seen in figure 23. The axes in the figure are probability of failure for all the simulations against number of cycles. The interval is almost constant through the number of cycles. Compared to the step one and two there is a clear difference in probability of failure at low number of cycles.

Figure 22: Comparison between the simulation results with and without the effect of variation in materials.

Figure 23: Probability of failure plotted against number of cycles when the effect of offset is added.
7.2.6 Comparison between the steps

Comparison between the average curves from the three different steps can be seen in figure 24. The axes in the figure are probability of failure for all the simulations against number of cycles. The trend is clear, after each random variable that are added the probability of failure increase for low number of cycles and decrease for high number of cycles.

Figure 24: The average curves from the three steps are compared to each other.
**7.2.7 Change offset**

The trend of changing the offset can be seen in figure 25. The offset was changed from ±0.5 mm to ±0.1 mm. The axes in the figure are probability of failure for all the simulations against number of cycles. It is clear that the probability of failure is decreased with decreased range in offset. By decreasing the offset from 0.5 to 0.1 the probability of failure decreases approximate one percent except at very low number of cycles then it was two percent. It is also clear that the offset has a big impact on the probability of failure. In figure 26 the new curve with the new offset is compared to the first and second step.

![Figure 25: The two offsets are compared to each other, decreased range in offset gives decreased probability of failure.](image-url)
Figure 26: The new offset is compared to the average curve of the two other steps.
7.2.8 Increase the expected number of cracks

The parameter expected cracks per meter is increased to 5 cracks per meter and the results can be seen in figure 27. The axes in the figures are probability of failure for all the simulations against number of cycles. As can be seen by comparing the figure 19 where the expected number of cracks are one per meter to figure 27 where the expected number of cracks per meter are five, the trend is the same when the number of cracks are increased, the only thing that is different is that the probability of failure increases.

![Figure 27: The expected number of cracks was increased and the trend shows to be the same as before the increase.](image-url)
7.2.9 Ratio of failed cracks
In figure 28 one can see the comparison between the first and second step (without and with the effect of variation in material) of when all the cracks has reached failure. The curves are the ratio of cracks that has failed against the number of cycles. As expected the second step takes longer time for all cracks to reach failure.

![Graph showing ratio of failed cracks](image)

*Figure 28: Ratio of failed cracks in the two first steps are compared, the same cracks are investigated.*
All the cracks have not reached failure in figure 28 for the second step. In figure 29 it can be seen when all the cracks have reached failure for the second step. The effect of variation in material is that the last failed cracks fail at a high number of cycles.

![Graph showing the ratio of failed cracks vs. number of cycles](image)

**Figure 29:** The ratio of failed cracks in the second step.
In figure 30 the third step can be seen. All the cracks reach failure faster than the second step. This is because half of the offset loads are negative and will lower the tensions in the weld and the other half will increase the tensions in the weld and therefore make the weld more vulnerable to fatigue failure.

![Graph showing the ratio of failed cracks plotted against number of cycles when the effect of variation in material and offset are included.](image)

**Figure 30:** Ratio of failed cracks plotted against number of cycles when the effect of variation in material and offset are included.
8. DISCUSSION
The most important part of the thesis is the code validation because that is the evidence that this is an acceptable method. The result from the code validation shows that the simulation shows similar results to the theory which concludes that the Monte-Carlo simulation method is acceptable. A comparison between the simulations shows that the higher number of cycles that are investigated the higher the uncertainty of the method is. With only five simulations tested in the code validation the curves between the average and theory start to deviate at 500000 cycles. This is because the uncertainty in probability of failure increases at a higher number of cycles and in order to have the two curves close to each other all the way more simulations would be needed. The high numbers of cycles does not have the same importance as the low number of cycles because it is not reasonable to have an airplane go so many flights due to safety reasons without inspection.

The work process was originally manually done in the preparatory work in ANSYS to retrieve the nodes that represented the welds. This was not preferable due to the time consuming work to manually picking the nodes that represents the welds. The FASTGRO program was not run with all the unnecessary nodes that have a long life expectancy. That would mean a lot of work and therefore a lot of time would be spent collecting unnecessary information. The automatic way then made it possible for automatic work in ANSYS and also run the FASTGRO program. This means less time will be spent in retrieving the nodes and not dealing with the unnecessary nodes with a long life expectancy. But the code CreateCracks was originally made for the manually work process so when the automatic way was introduced the input files were different so the code GetWelds were made in order to read in the FASTGRO results and see which welds that is in the danger zone and construct the node files so that CreateCracks can read the files and do calculations. This means that the work process of the codes are not optimal but due to time limit there was not time enough to do it in a better way but there are probably room for development in the work process.

By examine figure 24, it is shown that after each variable that was added the probability of failure increased at the lower number of cycles. On the other hand, at the higher number of cycles the probability of failure will decrease. This means that the variance increases for when a random variable is added.

For each case of TEC some of the cracks will lead to failure even at the first cycle and this is due to high tensions. The high tensions could be for two reasons; the programs that handle the tension calculations could do something wrong or that there is high tensions in the TEC due to bad geometry. While checking the load steps in ANSYS it was observed that the highest von Mises stress in the TEC was 2.43 GPa so it is concluded that the TEC that was investigated suffered from bad geometry. Due to the high probability of failure this TEC would not be passed in EWB for production.

Comparison between this TEC that is not passed with another TEC would be interesting but due to the lack of time this is not included in the thesis. But as said the important part of the thesis was the automatization of the working process with the development of the codes and the validation against the theory. Therefore, a test with one more TEC was not prioritized.
If there are 50 TECs that need to be investigated the simulations need to be done in short time. The preparatory work takes little time. In the simulation the MATLAB codes takes little time but if there is a lot of cracks that should be investigated then NASGRO calculations take a long time and is the bottleneck of the process. The time it takes for NASGRO to do the calculation for all the cracks makes the whole work process inefficient. However, since the code validation has proven that the method is theoretically correct the results achieved from the Monte-Carlo simulation can be used as a reference to adjust further simplifications and assumptions to make the calculations faster. So if the future simplifications show the same results as the Monte-Carlo simulation it means that the simplification is correct.

The time it takes to do 4000 simulations ten times is about a working day. The time for the codes in the simulation did vary a bit. The code CreateCracks took about 3-4 minutes when the adding of the offset was included and the code ReadResults took about one minute. This was for around 380 cracks. The NASGRO calculations for all those cracks took around 45 minutes. For evaluation of maybe 50 different TECs this is way too slow.

The MC-simulations can be used to see trends in probability of failure if the random parameters are changed to a lower or higher value. For instance, to see which of the random parameters that influences the probability of failure most, as seen in results when the offset is changed from 0.5 to 0.1. The offset parameter has a big influence on the probability of failure.

The standard deviation parameter used in the logarithmic random parameter in the variation in material parameter was accidental wrong in the results. It was supposed to be 0.25 but it was accidental 1 which means that the standard variance was four times larger than expected. One can see in the figures 21 and 22 that the variation in material has a little effect on the probability to failure. So if the correct standard deviation were used the effect would have been even smaller. So for demonstrating purposes it was good that the standard deviation was accidentally wrong.

One drawback of working with MATLAB is that the each time it was restarted it would generate the same random variables so in order to do the simulations the program or the computer was not allowed to be shut down. If a computer breakdown had happened in the last simulations it would take time to run the whole simulation again to be at the same spot.

If a number of simulations is run multiple times it is possible to make a good confidence interval calculations to see that the scatter is not to deviate in any direction. Lack of time made it not possible to do good confidence interval analysis but using 4000 simulations in each round does not seem to give a huge amount of scatter between the simulation runs.

This working process may be an excellent method if a supercomputer is available. Then the NASGRO calculations would be much faster and therefore the total time would be a lot shorter.
9. FUTURE WORK
Due to lack of experience in coding and changes in the method during the thesis the codes that are used in the work process can be made smoother which may bring less work and faster calculations.

Now the simulation are done in three steps, by using a more advanced programming language like PYTHON it is possible to program so that the simulation is done in one step.

No the work process only calculates for crack that exists. Calculations for crack initiation could be added to the process.

10. CONCLUSION
The aim with the thesis was to replace a deterministic way of calculating the fatigue life of a TEC with a probabilistic way. The method was a Monte-Carlo simulation with random variables. The method was easy to use and the result easy to interpret. Some important results discovered were:

- Number of simulations is important. The higher amount of simulations, the better the simulation gets.
- The number of random variables affects the probability of failure, and the random variables also affects the probability of failure differently.

This method with Monte-Carlo simulation is an acceptable method to calculate the probability of failure and this is proven by the code validation against the theory. But the simulation is too time-consuming due to a bottleneck in the crack propagation calculations that are done in the program NASGRO. But the results achieved from this method can be used as a reference to make further simplifications to reduce the calculating time.
11. ACKNOWLEDGEMENTS

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I would also like to thank Björn Månsson at GKN for taking his time for helping me with some of the programs.
12. REFERENCES
Appendix A
Appendix B
### Appendix E

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Appendix G

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### Appendix H

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Appendix I

1. Read in input file
2. Read in FASTGRO results
3. See which nodes are below the acceptable limit
4. See which welds that are below the acceptable limit
5. Print the nodes and their coordinates for the vulnerable welds
6. Read in file with all the nodes and which weld the node belongs to
### Appendix J

| 1000 | 1 | 6 |
### Appendix K

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Appendix L

FLAGUI [NASGRO(R) v6.21 final]

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[calcmode] 0
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[geobox3] 1
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  2  1  1
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  1  0.5  0
  2  1  -1
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[userornde] 0
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Appendix M

Read in data file

First simulation

First weld
Read in nodes and coordinates
Calculate the weld length

First crack
Poisson distribution of number of cracks

Add position
And find closest node
Exponential distribution of initial crack size

Create indata file for NASGRO

Next crack
Read RFC file and add offset tension

Next weld

Next simulation

Print the name for all the cracks onto a file
Appendix N

Read file with crack names

Read first result file

Add variation of materials

Print result onto a file

Next result file

Investigate first number of cycles

First simulation

Read files with result

Next simulation

If any failure, k=k+1

No failure, k=k

Calculate probability to failure, Pf = k/sim

Print result onto a file

Next number of cycles to investigate