Analysis of Student Loan Asset-Backed Securities

Construction of a Valuation Model using a Trinomial Interest Rate Tree

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Abstract

Student debt in the U.S has grown rapidly over the last decades. A common practice among lenders is to pool the loans into securities that are sold off and traded between institutional investors. Since these securities have no market price this thesis aims to develop a valuation model. A time discrete approach is used, based on the Hull-White short-rate model to create a trinomial interest rate tree. This tree serves as a basis for the discounting of future cash flows generated from a specific student loan asset-backed security. In order to assess the credit risk, the student loan market and potential speculative bubbles are discussed.

The model is applied on the "Navient Student Loan Trust 2015-2" and each tranche’s intrinsic value and yield to maturity is calculated. Since the model lacks proper quantification of the credit risk, the result is a valuation model that is best used when valuing asset-backed securities that can be deemed risk-free.
Sammanfattning


Modellen appliceras på ”Navient Student Loan Trust 2015-2” och det diskonterade värdet samt avkastning för varje specifik tranche beräknas. Då modellen saknar en kvantifiering av kreditrisken, är resultatet en modell som är mest applicerbar vid värdering av tillgängssäkrade värdepapper som kan bedömas som riskfria.
Acknowledgements

We want to extend our warmest thanks to Peter Touma for giving us the idea behind this thesis. We would also like to express our gratitude to Henrik Segerberg and Nordea Markets for allowing access to a Bloomberg terminal. In addition, we would like to thank Fredrik Armerin for providing us with inspiration and guidance.
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Chapter 1

Introduction

1.1 Background

Prior to the financial crisis of 2007-08 the packaging and selling of home mortgages created a bubble that affected the world economy in an unprecedented way. There are many similarities between the mortgage and the student loan market and there is an increasing fear that the student loan market will be next to trigger a financial crisis.\[1\] Student debt in the U.S has risen sharply over the last decades, amounting to over $1.2 trillion\[2\]. Furthermore, student loans are not collateralized, meaning that there is no underlying lien to back the loan against. The value of the loans are therefore fully dependent on the future income of the students.

Student loan asset-backed securities, or SLABS for short, are securities consisting of numerous student loans pooled together. The SLABS deliver scheduled coupon payments much like an ordinary bond. The selling of SLABS allows lenders to move their credit risk to several investors. In theory this allows for a more efficient loan market and creates better means for students to finance their education.

1.2 Problem Statement

SLABS are traded "over the counter" between institutional investors. Consequently there is no market price, which makes it unknown to individuals who are not affiliated with the asset-backed security (ABS) industry. Furthermore, the student loan market is rapidly growing and its sustainability can be questioned.\[1\] As such, this thesis attempts to answer the following questions:

- Based on a short-rate model, what is the intrinsic value of a typical SLABS?
1.3 Aim

This thesis aims to develop a pricing model for a specific type of ABS, namely a student loan trust. It will also provide an insight into the student loan market in the U.S. The results could be used as a framework by an institutional investor interested in trading in these securities.
Chapter 2

Theoretical framework

2.1 Characteristics of Asset-Backed Securities

An asset-backed security is a financial instrument that is backed by a loan on a non-mortgage asset. The underlying asset can for example be credit card debt or company receivables. Whereas the holder of a mortgage-backed security (MBS) has claim to the real estate property that the loan is backed on, the ABS holder’s claim is often to a less tangible asset. In the case of SLABS the holder has claim to the future income of the indebted students.[3]

A student loan trust is a collection of several student loans that are bundled together by a bank and sold with the help of several underwriters.[4] The issuing of new trusts occurs seldomly, roughly once a year, and the amount of student loans in each security can be exceedingly high, often in the range of billions of dollars. This creates an irregular market where the prices are set "over the counter" by the underwriting banks.

2.1.1 Payments

Even though the bank that issues the trust has sold off the student loans it will still act as a servicer in terms of collecting payments from the students. The payments are of two types; interest payments and principal payments.

- Principal payments lower the notional amount loaned and often follow an established plan agreed upon between the student and the bank. However the students have the option of paying off their loan quicker than planned, this is called prepayment.

- Interest payments are made monthly or quarterly and are payments equaling the remaining loan the student has times the coupon rate agreed upon.

It is important to note that prepayment reduces the future cash flow from the security since the loan pool will be reduced prematurely leading to a loss in
interest payments. [3][4]

2.1.2 Tranches

An important feature of an ABS is that the issuer separates the security into different tranches. A tranche is a section of the security that has a specific seniority, which means that it has a different risk level relative to the other tranches. The most senior tranche of an ABS, called A-1, will receive interest and principal payments before the other tranches. First after the holders of the A-1 notes have been paid, payments will start to flow towards the subordinate tranches, A-2 and onwards, creating a "waterfall" of payments, as seen in figure 2.1 [5].

It is important to understand how the fragmentation of the security into tranches changes the risk an investor faces. An investor that purchases notes of the lowest seniority will bear the risk of default and prepayment losses for all the tranches in the security. On the other hand a buyer of the most senior notes will only face a loss after all the other tranches has lost their entire amounts. Because of this, a buyer of a more senior note pays a premium over a subordinate note buyer. [5]

![Figure 2.1: Example of payment distribution for a student loan trust](image)
2.2 Risk-Neutral Measure

A common way of pricing financial derivatives is to assume that the value is equal to the discounted expected future payoffs under a risk neutral measure. Risk-neutral pricing is a powerful and convenient computational tool as it does not incorporate investors risk premia. As such their individual risk preferences for different instruments will not be taken into account. This has the benefit of creating a more general model which works by simply discounting future payoffs.[3]

2.3 Short-Rate Modelling

In a risk neutral world, the price of a financial derivative depend on the short rate, $r$ (continuously compounded). The short rate is the interest rate at which money can be borrowed or lent for an infinitesimally short period of time. By establishing the evolution of the short rate, the future evolution of interest rates can be described. An interest rate derivative with the payoff $f_T$ at time $T$ has a value at time $t$ of

$$E_Q[e^{-\bar{r}(T-t)} f_T | F_t],$$

(2.1)

where $\bar{r}$ is the average value of $r$ between $t$ and $T$. Define $P(t, T)$ as the price at time $t$ of a zero-coupon bond that pays $1$ at time $T$:

$$P(t, T) = E_Q[e^{-\bar{r}(T-t)} | F_t].$$

(2.2)

Define $R(t, T)$ as the continuously compounded interest rate between $t$ and $T$. Then

$$P(t, T) = e^{-R(t,T)(T-t)},$$

(2.3)

so that

$$R(t, T) = -\frac{1}{T-t} \ln P(t, T)$$

(2.4)

from equation 2.2 we obtain

$$R(t, T) = -\frac{1}{T-t} \ln E_Q[e^{-\bar{r}(T-t)} | F_t].$$

(2.5)

This equation enables that the term structure of interest rates can be obtained once the short-rate process has been defined.[3]

2.4 Hull-White’s Short-Rate Model

In 1990 John Hull and Alan White introduced a model to describe the movement of the short rate.[6] The model is an expansion of the short rate process described
by Vasicek. In its most general form the model describes the short rate as satisfying the following stochastic differential equation.

\[ dr(t) = [\theta(t) - \alpha r(t)]dt + \sigma dW(t) \] (2.6)

Where \( r(t) \) is the instantaneous rate and \( W(t) \) is a Wiener process. The model has three important properties; 1) Drift 2) Mean reversion and 3) Volatility.

1. Drift is defined by the first term in Hull-White’s model, namely \([\theta(t) - \alpha r(t)]dt\). The \( \theta(t) \) function calibrates the short rate so that the future rates given by today’s term structure corresponds to the expected value of the short rate. Mathematically the function is defined as follows

\[ \theta(t) = F_t(0,t) + \alpha F(0,t) + \frac{\sigma^2}{2\alpha} (1 - e^{-2\alpha t}) \] (2.7)

Where \( F(0,t) \) is the instantaneous forward rate for maturity \( t \) seen at time zero and the subscript \( t \) in \( F_t(0,t) \) denotes a partial derivative with respect to \( t \). The last term \( \frac{\sigma^2}{2\alpha} (1 - e^{-2\alpha t}) \) is usually very small so the drift can often be estimated to \([F_t(0,t) + \alpha (F(0,t) - r(t))].\)

2. Mean reversion simulates the observed behaviour of how interest rates tend to revert from extreme values. At any given time the short rate will revert to the time dependent level \( \frac{\theta(t)}{\alpha} \) with speed \( \alpha \).

3. Volatility \( \sigma(t) \) is a measure of the magnitude of \( W(t) \) which is a Wiener process under the risk-neutral measure. This simulates the random movements of the interest rate.

### 2.5 Trinomial Interest Rate Tree

In ABS-valuation, one common methodology is the tree model. In general, this is a model that shows the different paths an underlying asset’s value may take over its lifespan. When simulating the interest rate movement, a time-discrete representation of the stochastic process for the short rate is used. This is depicted as a trinomial tree. This means that each node has three possible branches leading to another node. Each branch has a different probability of occurring and the time between each time step is defined as \( \Delta t \). The main advantage of using a trinomial tree, rather than a binomial, is the extra degree of freedom. It makes the characteristics of the interest rate process more accurate.

The rate \( R \) is assumed to have the characteristics as the short rate \( r \). In each node the rate is denoted, by \( R_{node}(t,j) \) with \( t \) being the current time step and \( j \) the state.
2.5.1 First Step - Tree for $R^*$

The tree generation process described in this section follows the steps described in John C. Hull’s book “Options, Futures and Other Derivatives” [3]. The process starts with defining $R^*$ characterised by

$$dR^* = \alpha R^* dt + \sigma dW(t)$$

and in each node,

$$R_{node}^*(t,j) = R_{node}^*(t,0) + \Delta = R_{node}^*(t,0) + j\sigma\sqrt{3\Delta t}.$$  \hspace{1cm} (2.9)

Typically the branching from a node is characterized as a) in figure 2.5.1. This standard branching is used if the state $j$ is in an acceptable range, $-j_{max} < j < j_{max}$.

![Figure 2.2: Different types of branching](image)

If a node is in a state $j$ that is high or low enough a special kind of branching is used, shown in b) and c). This occurs specifically when $j = -j_{max}$ (upwards branching) or $j = j_{max}$ (downwards branching). $j_{max}$ is defined as the smallest integer that is equal or greater than $0.184/\alpha$.[3].

The probability of moving along a branch for each type of branching must satisfy the following equations where $p_u$, $p_m$ and $p_d$ are the probabilities for moving along the upper, middle and lower branch respectively.

\[
\begin{align*}
 p_u \Delta R - p_d \Delta R &= -\alpha j \Delta R \Delta t \\
 p_u \Delta R^2 + p_d \Delta R^2 &= \sigma^2 \Delta t + \sigma^2 j^2 \Delta R^2 \Delta t^2 \\
 p_u + p_m + p_d &= 1
\end{align*}
\]  \hspace{1cm} (2.10)
For the standard type of branching a) the equations are solved using
\[ \Delta R = \sigma \sqrt{3} \Delta t \]
\[ p_u = \frac{1}{6} + \frac{1}{2} (\alpha^2 j^2 \Delta t^2 - \alpha j \Delta t) \]
\[ p_m = \frac{2}{3} - \alpha^2 j^2 \Delta t^2 \]
\[ p_d = \frac{1}{6} + \frac{1}{2} (\alpha^2 j^2 \Delta t^2 + \alpha j \Delta t) \]

For the upwards type of branching in b)
\[ p_u = \frac{1}{6} + \frac{1}{2} (\alpha^2 j^2 \Delta t^2 + \alpha j \Delta t) \]
\[ p_m = -\frac{1}{3} - \alpha^2 j^2 \Delta t^2 - 2\alpha j \Delta t \]
\[ p_d = \frac{7}{6} + \frac{1}{2} (\alpha^2 j^2 \Delta t^2 + 3\alpha j \Delta t) \]

And lastly the downwards type c)
\[ p_u = \frac{7}{6} + \frac{1}{2} (\alpha^2 j^2 \Delta t^2 - 3\alpha j \Delta t) \]
\[ p_m = -\frac{1}{3} - \alpha^2 j^2 \Delta t^2 + 2\alpha j \Delta t \]
\[ p_d = \frac{1}{6} + \frac{1}{2} (\alpha^2 j^2 \Delta t^2 - \alpha j \Delta t) \]

Now an interest rate tree for \( R^* \) can be created where the root node \( R^*(0,0) \) is set to zero and arbitrary values for \( \alpha \) and \( \sigma \) are used.

2.5.2 Second Step - Term Structure Calibration
The \( R^* \) tree needs to be calibrated to today's term structure. This means that the tree will be transformed to a tree for \( R \), with abuse of notation. The difference between the two rates is denoted \( \delta(t) \) so that
\[ \delta(t) = R(t) - R^*(t) \]  

(2.14)

The instantaneous change in \( \delta(t) \) can therefore be written

\[ d\delta(t) = dR(t) - dR^*(t) \]

\[ = [\theta(t) - \alpha R(t)] dt + \sigma dW(t) + \alpha R^* dt - \sigma dW(t) \]

\[ = [\theta(t) - \alpha R(t) + \alpha R^*(t)] dt \]

\[ = [\theta(t) - \alpha (R^*(t) - R(t))] dt \]

\[ = [\theta(t) - \alpha \delta(t)] dt. \]  

(2.15)

Using equation (2.14) \( \delta(t) \) can be rewritten as

\[ \delta(t) = F(0, t) + \frac{\sigma^2}{2\alpha^2} (1 - e^{-\alpha t}). \]  

(2.16)

Let's introduce \( Q_{t,j} \) as the value of an Arrow-Debreu security that pays $1 if node \((t, j)\) is reached and zero otherwise. The tree is now adjusted by finding a \( \delta_m \) so that a zero-coupon bond \( P_{m+1} \) maturing at time \((m + 1)\Delta t\) will be correctly priced in every node.

\[ P_{m+1} = \sum_{j=-n_m}^{n_m} Q_{m,j} e^{-(\delta_m + j\Delta R)\Delta t} \]  

(2.17)

\( n_m \) is the width of the tree at time \( m\Delta T \) which is the same as the number of nodes on each side of the node with state \( j = 0 \). Solving for \( \delta_m \) gives

\[ \delta_m = \ln \left( \sum_{j=-n_m}^{n_m} Q_{m,j} e^{-j\Delta R \Delta t} - \ln P_{m+1} \right) / \Delta t \]  

(2.18)

Moving forward in the tree begins with calculating the Arrow-Debreu price for the next time step

\[ Q_{m+1,j} = \sum_k Q_{m,k} q(k,j) e^{-(\delta_m + k\Delta R)\Delta t} \]  

(2.19)

\( q(k,j) \) is the probability of moving from node \((m,k)\) to node \((m+1,j)\). \( k \) is chosen so that all nodes are summed where \( q \) is non-zero.

### 2.6 \( \alpha \) and \( \sigma \) Calibration

In Hull-White’s short-rate model, the function \( \theta(t) \) is adjusted to make the model consistent with the initial term structure. The remaining parameters \( \sigma \) (volatility) and \( \alpha \) (mean-reversion) are determined by comparing the market price of an interest rate derivative with the theoretical price given by the interest
rate tree. This is known as the calibration process. A commonly used instrument is interest rate caps. The objective of the calibration is to choose the parameters so a “goodness-of-fit” measure is minimized. A solid measure of this is the summed squared residual $\sum_{i=1}^{n} (U_i - V_i)^2$.  

(2.20)

Where $U_i$ is the market price and $V_i$ is the theoretical tree price. An interest rate cap is characterized as a portfolio of put-options which provides a payoff of $L\lambda_{\text{max}}(R_K - R_k, o)$.  

(2.21)

Where $L$ is the notional amount exchanged and $\lambda$ is the day count fraction. $R_K$ is the actual interest rate, whilst $R_k$ is the strike rate. Furthermore, as caps are essentially options, the market price, $U$, can be determined with Black’s formula. With abuse of notation, suppose that $r$ is the risk-free interest rate and the future’s price $F$ of the underlying is lognormal with the implied volatility $\sigma$. Black’s formula states that the price of an interest rate cap with maturity $T$ is

\[
U = L\lambda e^{-rT}[FN(d_1) - R_kN(d_2)]
\]

\[
d_1 = \frac{\ln(F/R_k) + \sigma^2 T}{\sigma \sqrt{T}}
\]

\[
d_2 = \frac{\ln(F/R_k) - \sigma^2 T}{\sigma \sqrt{T}} = d_1 - \sigma \sqrt{T}.
\]

Once the market price is calculated, the calibration parameters $\sigma$ and $\alpha$ can be determined by minimizing the goodness-of-fit function. This is usually done with a root-minimizing optimization algorithm. The algorithm changes $V$, calculated with Hull White’s model until the goodness-of-fit no longer improves significantly. The results are calibrated $\sigma$- and $\alpha$-parameters that can be input in the trinomial tree to generate a more accurate interpretation of the forward rates.

### 2.7 Discounted Cash Flows

After creating the calibrated interest rate tree the structure can be used to value future cash flows given by a security. This is done by backwards induction, where the starting point is the last cash flow given by the security. Then the discounted value of the last cash flow with respect to branching probabilities is calculated for all nodes in the previous time step. Added to this is the node specific cash flow for the new time step and in this manner all cash flows are added by moving backwards in the tree.

\[
V_{t,j} = (p_uV_{t+1,j+1} + p_mV_{t+1,j} + p_dV_{t+1,j-1})e^{-R_{t,j}\Delta t} + C_{t,j}
\]

(2.23)
Equation 2.23 shows the principle for the backwards induction where $V_{t,j}$ is the discounted value of all the future cash flows at node $(t,j)$ and $C_{t,j}$ is the cash flow given specifically at node $(t,j)$.
Chapter 3

Methodology

3.1 Qualitative Methodology

3.1.1 Literature Studies

Scientific literature and articles were used in order to gain a deeper understanding of the asset-backed securities market. Obtaining information regarding the mathematical theory has not been difficult as they are well defined financial instruments.

The mathematical theory behind the valuation model is mainly based on the information given in Hull’s book "Options, Futures and other Derivatives"[3].

The model was constructed in Matlab. The official website Mathworks contributed with code and pre-made functions, both for the tree generation and the calibration.

3.1.2 Data Collection

Once we had chosen a specific loan trust to valuate, we gathered data on maturity, principal and rates of return from the base prospectus. The base prospectus is a legal document published by the issuing bank which contains the data an investor needs to make an informed investment decision. The prospectus is public information and can be found on the U.S. Securities and Exchange Commission’s website.[4]

The trinomial interest tree requires a term structure as input. American treasury bonds were used and their rates were collected from the U.S. Department of Treasury’s website[9]. The volatility of interest rate caps is required for the calibration of the trinomial tree, this was gathered from the financial information software Bloomberg.
CHAPTER 3. METHODOLOGY

3.2 Navient Student Loan Trust 2015-2

The valued student loan trust was the "Navient Student Loan Trust 2015-2". It was chosen mainly because it has a recent date of issuance. As previously mentioned, each trust consists of tranches and each tranche will be valued individually. The value of the trust is simply the sum of the value of the tranches. However, the value of the whole trust is of little relevance as investors are usually interested in a portion of a single tranche which matches their risk preferences.

<table>
<thead>
<tr>
<th>Navient Student Loan Trust 2015-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tranche</td>
</tr>
<tr>
<td>-----------------------------------</td>
</tr>
<tr>
<td>Floating Rate Class A-1 Notes</td>
</tr>
<tr>
<td>Floating Rate Class A-2 Notes</td>
</tr>
<tr>
<td>Floating Rate Class A-3 Notes</td>
</tr>
<tr>
<td>Floating Rate Class B Notes</td>
</tr>
</tbody>
</table>

The maturity depends on the rate of prepayment. Hence, the tranches were valued for different constant prepayment rates (CPR). Below are tables for each respective tranche that display the percentages of original principal remaining at certain CPR.

### Tranche A-1

<table>
<thead>
<tr>
<th>Distribution Date</th>
<th>0%</th>
<th>2%</th>
<th>4%</th>
<th>6%</th>
<th>8%</th>
<th>10%</th>
<th>12%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closing Date</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>June 2015</td>
<td>94%</td>
<td>92%</td>
<td>91%</td>
<td>89%</td>
<td>88%</td>
<td>86%</td>
<td>85%</td>
</tr>
<tr>
<td>June 2016</td>
<td>70%</td>
<td>63%</td>
<td>57%</td>
<td>50%</td>
<td>44%</td>
<td>37%</td>
<td>31%</td>
</tr>
<tr>
<td>June 2017</td>
<td>44%</td>
<td>34%</td>
<td>23%</td>
<td>13%</td>
<td>3%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>June 2018</td>
<td>16%</td>
<td>3%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>June 2019</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
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### Tranche A-2

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<tr>
<td>June 2020</td>
<td>7%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>June 2021</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>
3.3 Trinomial Tree Generation

The pricing model was set-up in Matlab, it supports many of the crucial functions for this analysis. A trinomial interest rate tree was generated with time step $\Delta t = 1$ year. It displays the forward rate for each year. The $\theta(t)$ function in the Hull-White model (equation 2.6) was calibrated by inputting today’s term structure. U.S. treasury bonds were used in this case as they could serve as an alternative investment, and can henceforth be used for discounting. Furthermore, they also have a maturity for up to 30 years. This is useful since SLABS have a long maturity as well. There are treasury bonds with maturity of 1, 2, 3, 5, 7, 10, 20 and 30 years.
3.4 Interpolation of Treasury Yield Rates

The term structure given by the U.S. treasury rates lack spot rates for certain maturities, for example there is a gap between 3 and 5 years in maturity. In the tree generation, it is required that a spot rate is supplied for every time step. As a remedy, interpolation was used to fill the gaps. There are several methods of interpolation, in this case the cubic spline method was used, it has the benefit of small interpolation errors. Other methods such as quadratic and cubic interpolation were also considered but they appear to be less representative of the treasury yield curve.
3.5 Calibration

Up until now it has been assumed that the parameters $\sigma$ and $\alpha$ are known, however they are not and their value affect the generated forward rates greatly. The parameters have to be decided by a method of calibration. As previously discussed in section 2.5 and 2.6 this is done by comparing the market price with the theoretical value of an interest rate derivative. Interest rate caps were used in this case and volatility data was collected from Bloomberg, as seen in figure 3.3. This data was used to acquire the market price of the interest rate caps, referenced as $U$ in section 2.6. The theoretical value, $V$, can be determined with the treasury yield rate. The treasury yield rate and the volatility cap data was gathered from the same day. The calibration results are presented in figure 3.4 and 3.5. As a result, the value of the $\sigma$- and $\alpha$-parameters were retrieved and added into the Hull-White model.
Figure 3.3: Volatility of interest rate caps
SSR =

1.4608

<table>
<thead>
<tr>
<th>Black76</th>
<th>Calibrated Caplets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1597</td>
<td>0.7540</td>
</tr>
<tr>
<td>3.0511</td>
<td>3.6663</td>
</tr>
<tr>
<td>6.6848</td>
<td>6.9860</td>
</tr>
<tr>
<td>9.3454</td>
<td>9.3675</td>
</tr>
<tr>
<td>11.4306</td>
<td>11.1801</td>
</tr>
<tr>
<td>12.5077</td>
<td>12.1358</td>
</tr>
<tr>
<td>12.5262</td>
<td>12.2477</td>
</tr>
<tr>
<td>12.2675</td>
<td>12.1527</td>
</tr>
<tr>
<td>12.2366</td>
<td>12.2283</td>
</tr>
</tbody>
</table>

CapPrice Black76 ................:  80.13265
CapPrice HW analytical..........:  80.91943
CapPrice HW from capbyhw ...:  81.59598

Figure 3.4: Caps-fitting

Figure 3.5: Optimization data
3.6 Time Step, Annualisation and LIBOR-Rate Spread

The tree was created with time step $\Delta t = 1\text{ year}$. This is advantageous since the tree will not become unreasonably large. However the interest and principal payments are collected monthly and must therefore be annualised. The interest coupon for each month is dependent on the current monthly LIBOR-rate. The generated interest rate tree is based on the U.S. Treasury rates, thus an assumption about the spread between these two rates is necessary. The difference between the 3-month LIBOR-rate and the 3-month U.S. Treasury rate is commonly called the TED-Spread. As seen in figure 3.6 the TED-Spread can differ drastically especially in periods of economic turmoil, such as the financial crisis of 2007-08 (marked grey). In our calculations we have assumed the monthly TED-Spread to 0.3%\cite{10}. To annualize the cash flows we have estimated the monthly LIBOR to be $\frac{1\text{ year LIBOR}}{12}$ this contradicts the liquidity preference behaviour described by Keynes\cite{11}, but seems to hold quite well historically. The cash flows can be approximated to

$$1\text{ month LIBOR} = \frac{1\text{ year Treasury Bill} + \text{Average TED Spread}}{12}.$$  (3.1)

![Figure 3.6: Historic TED-spread\cite{12}](source: Federal Reserve Bank of St. Louis research.stlouisfed.org myf.red/g/4iaR)
3.7 Credit Risk

There are several risk factors that affect the value of the loan trust. To begin with, the trust does not provide predictable payments. Accordingly, the return of the investment might not be what was expected. This is due to the fact that loan terms can be modified. Furthermore, the student loan trusts rely on payments from the students. If they were to fail to make their payments, the investor would experience a loss. This risk increases when moving down in sub-ordination of the tranches. The lower seniority tranches have greater risk and consequently a higher coupon rate.\[4\] To fully analyze the risk aspects to a satisfactory extent would be a complicated process, hence the risk associated with this was not taken into account. This means that the intrinsic value presented in the results section is only applicable if there is no investor loss due to failure of student payments.

Additionally, there is also the risk of prepayment. How the remaining loan pool is affected due to different constant prepayment rates (CPR) is presented in section 3.2.\[4\] This was taken into account in the results section and the intrinsic value was calculated with different CPR.

3.8 Discounted Cash Flows

As mentioned in section 2.7, the discounting is achieved through backwards induction, one time step at a time. Below follows a few examples of the discounted cash flows for the most senior tranches at different CPR rates.

<table>
<thead>
<tr>
<th>CPR (%)</th>
<th>0</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>109.3793</td>
<td>110.4882</td>
<td>108.9592</td>
<td>73.8559</td>
<td>45.6362</td>
<td>16.3833</td>
</tr>
<tr>
<td>1%</td>
<td>110.4818</td>
<td>108.9517</td>
<td>73.8500</td>
<td>45.6324</td>
<td>16.3820</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>110.4739</td>
<td>108.9444</td>
<td>73.8443</td>
<td>45.6287</td>
<td>16.3806</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>110.4726</td>
<td>108.9372</td>
<td>73.8386</td>
<td>45.6251</td>
<td>16.3793</td>
<td></td>
</tr>
<tr>
<td>4%</td>
<td>110.4716</td>
<td>108.9301</td>
<td>73.8331</td>
<td>45.6215</td>
<td>16.3779</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>110.4710</td>
<td>108.9231</td>
<td>73.8276</td>
<td>45.6180</td>
<td>16.3766</td>
<td></td>
</tr>
<tr>
<td>6%</td>
<td>110.4704</td>
<td>108.9163</td>
<td>73.8222</td>
<td>45.6145</td>
<td>16.3754</td>
<td></td>
</tr>
<tr>
<td>7%</td>
<td>110.4700</td>
<td>108.9106</td>
<td>73.8172</td>
<td>45.6111</td>
<td>16.3741</td>
<td></td>
</tr>
<tr>
<td>8%</td>
<td>110.4697</td>
<td>108.9050</td>
<td>73.8123</td>
<td>45.6077</td>
<td>16.3729</td>
<td></td>
</tr>
<tr>
<td>9%</td>
<td>110.4693</td>
<td>108.9005</td>
<td>73.8075</td>
<td>45.6043</td>
<td>16.3716</td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>110.4690</td>
<td>108.8961</td>
<td>73.8030</td>
<td>45.6010</td>
<td>16.3704</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.7: Discounted cash flows for a $100 nominal in the A-1 tranche with no prepayment (CPR = 0%)
CHAPTER 3. METHODOLOGY

$\begin{array}{cccccc}
106.2064 & 107.2841 & 91.8028 & 45.1552 & 3.0716 \\
0 & 107.2762 & 91.7956 & 45.1515 & 3.0713 \\
0 & 107.2684 & 91.7884 & 45.1479 & 3.0711 \\
0 & 0 & 91.7814 & 45.1444 & 3.0708 \\
0 & 0 & 91.7745 & 45.1409 & 3.0706 \\
0 & 0 & 0 & 45.1374 & 3.0703 \\
0 & 0 & 0 & 45.1340 & 3.0701 \\
0 & 0 & 0 & 0 & 3.0699 \\
0 & 0 & 0 & 0 & 3.0696 \\
\end{array}$

Figure 3.8: Discounted cash flows for a $100 nominal in the A-1 tranche with CPR = 8%

$\begin{array}{cccccc}
127.0173 & 128.2979 & 123.1629 & 117.9804 & 112.7466 & 107.4579 \\
0 & 128.2966 & 123.1586 & 117.9738 & 112.7385 & 107.4491 \\
0 & 128.2954 & 123.1545 & 117.9674 & 112.7306 & 107.4405 \\
0 & 0 & 123.1504 & 117.9611 & 112.7228 & 107.4320 \\
0 & 0 & 123.1463 & 117.9548 & 112.7151 & 107.4236 \\
0 & 0 & 0 & 117.9487 & 112.7075 & 107.4154 \\
0 & 0 & 0 & 117.9427 & 112.7000 & 107.4073 \\
0 & 0 & 0 & 0 & 112.6927 & 107.3993 \\
0 & 0 & 0 & 0 & 112.6855 & 107.3914 \\
0 & 0 & 0 & 0 & 0 & 107.3837 \\
0 & 0 & 0 & 0 & 0 & 107.3761 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}$

Figure 3.9: Discounted cash flows for a $100 nominal in the A-2 tranche with no prepayment (CPR = 0%)
Chapter 4

Results

This bachelor thesis has been focused around SLABS, but the developed model can essentially be used for any asset-backed security. The main result is the model itself, as it can be used in many different scenarios, rather than the calculated values. Several tranches from one recently issued SLABS will serve as example of its application.

The tables below present the intrinsic value and the yield to maturity, both explained in the terminology chapter, of the tranches in the Navient Student Loan Trust 2015-2. The values are calculated for a nominal amount of $100. The intrinsic value is calculated from the date of issuance (April 8, 2015). The results are furthermore given for different CPR. The tables are presented in declining seniority, a lower rank of seniority is equivalent to a higher credit risk. The credit risk is unaccounted for in the model, this means that the intrinsic value and the yield would be lower if the credit risk was to be included.

<table>
<thead>
<tr>
<th>Tranche A-1</th>
<th>Monthly interest rate = 1-month LIBOR+0.28%</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPR (%)</td>
<td>Maturity (Years)</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
</tr>
</tbody>
</table>
### Tranche A-2
Monthly interest rate = 1-month LIBOR + 0.42%

<table>
<thead>
<tr>
<th>CPR (%)</th>
<th>Maturity (Years)</th>
<th>Intrinsic Value ($)</th>
<th>Yield to Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>127.02</td>
<td>3.99%</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>125.04</td>
<td>4.47%</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>122.44</td>
<td>4.05%</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>120.54</td>
<td>4.67%</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>119.26</td>
<td>4.40%</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>117.30</td>
<td>3.99%</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>116.11</td>
<td>4.98%</td>
</tr>
</tbody>
</table>

### Tranche A-3
Monthly interest rate = 1-month LIBOR + 0.57%

<table>
<thead>
<tr>
<th>CPR (%)</th>
<th>Maturity (Years)</th>
<th>Intrinsic Value ($)</th>
<th>Yield to Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
<td>155.65</td>
<td>4.92%</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>153.33</td>
<td>4.75%</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>151.28</td>
<td>4.6%</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>149.01</td>
<td>4.43%</td>
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<tr>
<td>8</td>
<td>8</td>
<td>145.83</td>
<td>4.72%</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>143.95</td>
<td>4.55%</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td>141.93</td>
<td>4.38%</td>
</tr>
</tbody>
</table>

### Tranche B
Monthly interest rate = 1-month LIBOR + 1.50%

<table>
<thead>
<tr>
<th>CPR (%)</th>
<th>Maturity (Years)</th>
<th>Intrinsic Value ($)</th>
<th>Yield to Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
<td>265.19</td>
<td>10.84%</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>265.19</td>
<td>10.84%</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>265.19</td>
<td>10.84%</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>265.19</td>
<td>10.84%</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>248.50</td>
<td>11.38%</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>248.50</td>
<td>11.38%</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td>248.50</td>
<td>11.38%</td>
</tr>
</tbody>
</table>
Chapter 5

Discussion

5.1 Reasonability of Results

The reasonability of the results is difficult to quantify since there is no market price to use as benchmark. This causes the intrinsic value and the yield to maturity to be only numbers that are hard to review critically. However, one aspect of the SLABS that has been considered is that it is essentially a collection of loans. The present value of the loan should be larger than the principal. This applies for rational markets but might not always hold true. Nonetheless, this is a criterion of the model that has set the lower limit of the value of each respective tranche. The upper limit is troublesome to pinpoint. The intrinsic value that has been established only takes into account the risk of prepayment and how it would affect the value. Hence, it could serve as an upper limit of the fair price as additional risk will only lower the selling price. The inaccuracy of the results increases in lower tranches as they have a higher credit risk that is not taken into account.

5.2 The Shortcomings of Financial Mathematics

The developed model in this thesis has its foundation in Hull-White’s book "Options, Futures and Other Derivatives". It is within the area of financial mathematics, a field that has been heavily criticized, especially since after the financial crisis of 2007-08. A common assumption within this area is that investors make rational decisions, this however ignores the effect of psychological, social, cognitive and emotional factors on economic decisions. Complementary to this, there are also market inefficiencies such as unfair advantage, market regulations and a lack of transparency. These are factors that are usually not included in current financial modelling. If this were to lead to incorrect pricing, it could enable an unwarranted market growth with a pricing that strongly deviates from the intrinsic value, resulting in an economic bubble. As such, the practical usage of mathematical models in finance can be debated. This leads
to the question; can the price of a complex financial instrument such as an ABS really be established by the simplified models currently in use? If not, it would render much of current practice irrelevant and misleading.

5.3 Further Development

To improve the model, credit risk should be taken into account. This has not been attempted since the credit risk is difficult to quantify. There is historical data available on student loan default which could be used, but the future development of the student loan market is hard to predict. Moreover, the tranche structure makes credit risk increasingly difficult to apply on asset-backed securities.

As an addition to the results section it would be interesting to discuss the calculated values with a seller of SLABS. A discussion of that sort would make it possible to further analyse the reasonability of the results. If the work is correctly executed in this thesis, the intrinsic value plus a premium should represent the price for the tranches with low risk.
Chapter 6

Bubble Formation in Higher Education

6.1 Background

Education has since long been seen as a financial investment that pays a high return in the form of lifetime income. Students take on debt to finance this seemingly safe bet, often without adequate examination of the expenditure. With increasing tuition fees the cost of education is rising sharply. There is speculation that there is a higher education bubble forming in the U.S. The concern is that the supply of college graduates in many fields will exceed the demand for their skills. The result of a higher education bubble would be graduate unemployment and underemployment which in turn would lead to student loan default. Up until now, this thesis has had its focus on developing a mathematical model for the pricing of SLABS. As the underlying asset of the instrument is student loans, the development of education is highly relevant to the future value of the SLABS. Henceforth, there is a need to discuss the premises of American education. Consequently, the formation of economic bubbles in education will be the topic of discussion in this chapter.

6.1.1 Definition

There are differing opinions on what exactly constitutes a bubble. Therefore the term is often used inconsistently in press and academic papers. A generally accepted foundation is that bubbles represent situations where market prices wildly differ from what can be inferred from fundamentals. Harrison and Kreps suggested that a speculative bubble occurs when the right to resell a stock makes investors willing to pay a premium as opposed to if they were obliged to hold on to the asset forever. Shiller adds in the behavioural mechanisms
of bubble creation, where each individual investor’s choice affects the next in an information cascade. Siegel\textsuperscript{[16]} proposes a mathematical definition. A period of rising or falling prices can be classified as a bubble if the realised return of an asset over a future time period differs more than two standard deviations from the expected return. Important to note is that this definition allows for negative bubbles where the asset price is much lower than expected return dictates. Furthermore this definition is only applicable ex-post, after the event. Asness\textsuperscript{[17]} introduces a general definition that is well applicable in many contexts, namely that ”the term bubble should indicate a price that no reasonable future outcome can justify”.

Both Siegel and Asness definitions require an understanding of the expected or reasonable value of the asset. As discussed in section 5.2 it is hard to capture the complexity needed in a financial model to make it accurate.

### 6.2 General Bubble Theory and their Formation

The cause of bubble formation is still difficult to define with today’s economic theory. The main idea is that bubbles form as a result of weak financial policy and excessive monetary liquidity in the financial system. When interest rates are low, investors tend to lever their capital by borrowing from banks and invest in different assets\textsuperscript{[18]}. Bubbles are often only identified ex-post, when a sudden drop in prices appear. It has the characteristics of an initial rapid price increase and when the bubble eventually bursts, the prices plummet\textsuperscript{[19]}.

In his book ”Irrational Exuberance”\textsuperscript{[14]}, Robert J. Shiller gives a comprehensive account of the extreme market climate in year 2000 right before the IT-bubble burst. He points out that three kinds of factors are especially important when analysing bubble formation;

1. Precipitating Factors
   Factors outside the market that shape its development.

2. Amplification Factors
   Factors that strengthen the effect that the precipitating factors have on the market behaviour. These factors can create patterns in which price changes spurs further price changes.

3. Cultural Factors
   Sentiment of the whole community that can enforce irrational market behaviour.

Some economists propose that bubbles are related to inflation and therefore believe that the factors causing inflation also cause economic bubbles. Others state that they originate from communication of key economic players. Another school of thought present bubbles as an unavoidable effect of asset pricing based on recent returns, without regard for economic fundamentals.\textsuperscript{[19]} Despite
CHAPTER 6. BUBBLE FORMATION IN HIGHER EDUCATION

their undefined origin, there are common denominators that characterize the economic bubbles

- Unusual changes in financial measures relative to their historical levels.
- Elevated usage of debt (leverage) to purchase assets.
- Higher risk lending and borrowing behavior. Such as borrowing to those with low credit scores.
- Rationalization of borrowing, where the decisions are based on future expected payoff instead of the ability to repay.
- A high presence of media coverage related to the asset.
- Movable risk between different actors.
- International trade imbalances, resulting in an excess of savings over investments.
- A lower interest rate environment.

6.3 Psychological Mechanisms in Bubble Creation

Most financial models are based upon the "Efficient Market Hypothesis". This hypothesis assumes that investors make rational decisions. The extreme market behaviour during financial bubbles is therefore not well explained if one where to assume that markets are efficient. One way of explaining irrational investment behaviour is by looking at the psychological mechanisms that affect the decision making.

6.3.1 Anchoring

Anchoring is an important cognitive trait used when lacking sufficient time or information to make an informed decision. An interesting example is how stock prices are observed. If a certain stock price has been moving around $10 for a long time it is assumed that this is the correct price. Thereby anchoring the thinking about this stock in terms of what the price should be. This thinking often fully excludes any information about the fundamentals of the company. Anchoring can lead to an irrational investment behavior as investors are lead by a "gut feeling" rather than actual information.

6.3.2 Herd Behaviour and the Information Cascade

When faced with a decision one often assumes that the decision made by a larger group is correct. Deutch and Gerard even showed that when the larger group makes an obviously incorrect statement we are compelled to question our own
reasoning and agree with the group. Most times this herd behaviour is perfectly rational, a larger group of individuals probably have better means to assess the information at hand and make an informed decision. However, this thinking can quickly deteriorate, an example of this is when an information cascade forms. A visualizing example of this is the restaurant problem.

Imagine two empty restaurants next to each other. The first hungry customer walks by and tries to assess which one is the better restaurant, as he does not know enough to make an informed decision he simply chooses one at random. The next customer is in a similar situation, however, this customer acknowledges that there is a customer already dining at one of the restaurants. It is perfectly rational for him to choose this restaurant since the first customer might have been better informed. In this pattern one of the restaurants may become full at the end of the night while the other stays empty. Even though the empty restaurant might have been superior. Note that every customer acted rationally but as a group they made an irrational and ill-informed decision. Analogously, investment patterns can occur as an accumulated pattern of ill-informed decisions.

6.3.3 Overconfidence

Overconfidence is an effect present in many professions. It is a miscalibration of subjective probabilities. Overconfidence could emanate when naive investors suddenly make a sizeable return. They become overconfident in their skills and it drives them to invest more. Furthermore, those who have greater market experience also tend to be more confident.

Overconfident investors often trade in risky assets, without consideration of the possible consequences. Moreover, to finance this they can borrow money to further increase potential payoff. This elevates overconfidence as it gives credit to their expertise when successful. Overconfidence of investors is widely believed to be a cause of irrational pricing. The result of this can be a strong deviation from the fair value causing herding, imitation and can contribute to the formation of economic bubbles. The cause of this phenomenon has no general explanation but social dynamics and adverse selection could certainly promote it.

6.3.4 Short Term Rationality

An important feature of irrational markets is that they can stay irrational for long periods of time. Even if an investor realizes that an asset price wildly exceeds what can be justified, it can be difficult for her to monetize on this by shorting the asset, simply because the irrational behaviour can continue for a long time. As John Maynard Keynes concluded "markets can stay irrational longer than you can stay solvent."
6.4 Student Loan Market Discussion

As previously mentioned, the exact cause of economic bubbles is still under debate. As such, drawing any concrete conclusions is not an easy task. When looking at the student loan market, two opposing schools of thought can be established. One that believe that a bubble is forming in the education loan market and another that do not concur with this.

6.4.1 Arguments for Bubble Formation

There are several indicators that imply overvaluation on the higher education loan market and there are also striking similarities with the housing bubble that caused the financial crisis of 2007-08.

To begin with, one of the strongest arguments that imply overvaluation on the student loan market in the U.S. is the increase of tuition in relation to the consumer price index (CPI).

As seen in figure 6.1, cost of schooling has risen faster than inflation the last decades. The cost of higher education will at one point outperform the benefit and it could mean that an economic bubble is in the making. Whether or not the higher education market is overvalued today or will be in many years is however difficult to tell, but an increase at this rate is unsustainable. This opinion is controversial. The main arguments against a higher education bubble is that the student loan default rate is low, the unemployment rate among college graduates is low and their financial well-being is increasing. Be that as it may, these arguments ignore the characteristics of bubbles. Most signs of bubbles are only visible ex-post. Once the bubble reaches its peak, the value
of higher education will tumble quickly. The financial well-being of future students cannot be determined by historical data. Nonetheless, the students with a relevant education which can provide a lucrative income will indeed be well off financially, despite high debt. The issue is the drop-outs and the scholars of academic fields which supply more students than the economy demands. Their conditions of repayment are poor and at a certain level of debt, their income will not be enough, hence they will be unable to repay their student loans.

The securitization of student loans serve as a strong precipitating factor as it helps distributing the credit risk over several agents. The high demand for education allows schools to charge increasingly higher tuition, while lenders can reduce their risk by selling the loans in the form of SLABS to different investors. As a result, the lenders can be less careful about their practices, as their returns are not correlated to the borrowers ability to pay back. Securitization is a strategy used by private companies, which most students turn to if their federal student loans are insufficient. The private companies have a higher interest rate on their loans which makes it increasingly difficult for the students to repay. With a rising student debt, more will turn to private lenders with higher interest rate and consequently make securitization more widespread.

The government amplify the effects on the market by giving student loans to borrowers that are unlikely to repay. In the case of default, borrowers are subject to significant consequences, including damaged credit rating, ineligibility for additional student aid, payment of collection costs, wage garnishment and legal action. Providing means for education promotes economic development in general, but can also put the borrower in debt with little hope of repayment. Weak lending standards was present in the housing bubble as well, but is the cause of even greater risk in the student loan market as the loans are not backed up by a tangible asset.

Furthermore, classic education carries a strong traditional and cultural importance. Adolescents are encouraged to participate in the educational tradition of going to college, just as their parents and grand parents. This factor further strengthens the students willingness to invest heavily in expensive education. Historically this has been a wise decision which causes students to rationalize their unreasonable debt obligations. The market has changed and so has the conditions, which contributes to a questionable sustainability of the student loan market.

6.4.2 Arguments against bubble formation

There are strong arguments that support that education still is and will remain a solid investment, even at the cost of a high debt level. Even though the tuition fees are growing at an unsustainable rate, the return on college education is higher than ever, and the gap between those with and without a bachelor’s degree has never been larger. Furthermore, rapid advancements in online
education is creating a valid alternative to classic college education. In 2013, 6.7 million U.S students were enrolled in online courses. It is reasonable to assume that the importance of higher education online will continue to grow as traditional education becomes more expensive.

The student loan system works well as long as it is the government doing the lending and the majority of outstanding student debt is still public. The interest rate of public student loans is low and the government is less susceptible to insolvency in the case of major student loan default. As long as this is the case, the government can control a potential bubble and keep it from bursting. Those believing in a higher education bubble also rarely acknowledge the significant public benefit of higher education. Therefore, a federal aggressive student loan strategy could be beneficial for the economy overall, rather than destructive.

The fact that there are many similarities between the housing and the proposed higher education bubble could also serve as an argument against a bubble formation. Many investors will not repeat the same mistakes as in the early 2000s. They are also more cautious today regarding trading in asset-backed securities, leading to a smaller but albeit growing market today. As a result, the consequences of a potential bubble would be less severe.

6.4.3 Conclusions from the Student Loan Market

This section has highlighted the different point of views regarding the future of the student loan market. The advocates of a higher education bubble raise several valid points that indicate an economic bubble, correlated to the bubble indicators displayed in section 6.2. Troubling increases in financial measures, borrowing rationalization, securitization, risky lending and a low interest environment certainly provide a foundation for bubble formation. However, most student loans are still federal. The U.S. government have means to control the market and reduce the consequences. The increase in debt levels is indeed unsettling, but the benefits of education are still prominent. In addition, online education is on the rise and it provides a cheaper alternative. Our conclusion is that it is unlikely that the student loan market in its own will form a bubble, but it may create an unstable market structure that is susceptible to changes in the overall economy.
Chapter 7

Conclusions

The mathematical model presented in this thesis could serve as a basis for ABS valuation. There are however limitations to the model, as credit risk should be incorporated to make the valuation of the lower tranches more correct. The highest tranche A-1 has such a high credit rating that the model should provide a fair valuation. Assuming that high seniority tranches are risk free can lead to disastrously poor investment decisions, this was particularly evident in the financial crisis of 2007-08. In addition to this, the state of the student loan market should be considered before trading in SLABS. As discussed in chapter 6, there are several signs that indicate potential economic bubble formation. In particular, the rate in which the cost of higher education has risen over the last decades is unsustainable. In the case of economic turmoil and increased student loan default rate, the value of SLABS would be affected greatly.
Chapter 8

Terminology

1. Arrow-Debreu security
   A contract that agrees to pay one unit of a numeraire if a particular state occurs at a particular time in the future. It pays zero in all other states.

2. Day count convention
   The day count convention determines the number of days between two coupon payments and also how interest accrues over time. When a security is sold between interest payment dates, the seller is eligible to some day count fraction of the coupon amount.

3. Interest rate cap
   An interest cap is an interest rate hedging derivative. The cap is comprised of series of European call options (caplets) on an underlying interest rate. An investor that has taken a floating rate loan and is worried that the rate may rise over a certain level can buy an interest rate cap with the same nominal amount $N$ as the loan.

4. Intrinsic value
   Refers to the value of an asset determined through fundamental analysis, without reference to its market value. It is usually calculated by discounting the future income generated by the asset to obtain the present value.

5. LIBOR-rate
   The London Interbank Offered Rate, abbreviated LIBOR, is an interest rate that is used as a benchmark globally. It is calculated as the average of the rates that the leading London banks would be charged if they were to loan from other banks. The rate is given for maturities; 1-day, 1-week, 1-month, 2-months 3-months, 6-months and 1 year.

6. Liquidity Preference Theory
   This theory introduced by John Maynard Keynes states that an investor requires a premium for securities with longer maturities. This means that
a holder of a 3-year treasury note should receive a higher rate than a holder of a 1-year note.

7. Yield to maturity
The expected return on an asset until maturity, quoted yearly. For example, a bond that in total gives 10% return over 5 years has a yield to maturity of 2%.

8. Zero-coupon bond
A zero-coupon bond is the most simple form of a bond. The bond does not generate interest payment (coupons) and at maturity the holder receives the bond’s face value.
Bibliography


[31] Us abs issuance and outstanding, 2016.